Probabilistic criteria for lateral dynamic stability of bridges under crowd loading

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Abstract

Probabilistic conditions for lateral stability of bridges are proposed, based on output from the inverted pendulum pedestrian model from the field of biomechanics. Statistical variations of the parameters defining the model are studied based on real statistical data of the English population. Variability of the self-excited forces is quantified for crowds of different velocities and critical conditions are identified for bridge natural frequencies below 5Hz. Allowance is made for the influence of the bridge mode shape and number of pedestrians in the crowd and their spatial distribution. This allows realistic worst case conditions among different loading scenarios for a particular structure to be found.

Keywords: bridges; human-structure interaction; biomechanics; inverted pendulum model; lateral vibrations; dynamic instability

1. Introduction

The problem of pedestrian-induced lateral vibrations is especially pertinent to bridges, which, due to the trend of building lighter and longer structures, have become increasingly vulnerable to dynamic pedestrian loading. Among well documented cases of bridges susceptible to excessive lateral vibrations are the London Millennium Footbridge (LMF) [1], the Singapore Airport's Changi Mezzanine Bridge (CMB) [2], the Clifton Suspension Bridge (CSB) [3] and the Pedro e Inês Footbridge (PIF) [4]. The measured responses of these bridges to crowd actions are characterised by divergent amplitude lateral vibrations which develop rapidly with a small increase in the number of occupants, which cannot be explained considering pedestrian forces exerted on stationary ground only, thus suggesting the existence of self-excited (or 'motion-dependent') forces arising from bi-directional human-structure interaction. (Excessive vibrations of bridges due to pedestrian loading can also occur in vertical direction (e.g. [5, 6]) and human-structure interaction is also likely to occur on vertically oscillating ground [18], but this problem is outside of the scope of this paper.)

The origin of the self-excited forces has most commonly been explained as the pedestrians synchronising to the movement of the structure, adjusting the frequency and phase of their footsteps in a manner to increase its motion ('lock-in'), a phenomenon allegedly reinforced by interpersonal synchronisation occurring unintentionally in crowds. However, many loading models based on these propositions stand in direct contrast to some recent observations. Specifically, no evidence of synchronisation was detected from measurements on the CMB [2] and CSB [3], yet rapid increases of lateral displacement amplitudes were clearly observed. Interestingly, the measured responses of these two bridges are compatible with the model based upon a linear relationship between the local velocity of the deck and the lateral pedestrian force, derived by Arup from the tests on the LMF [1], although the values of the pedestrian negative damping parameter (the coefficient of proportionality) differ in each case. Moreover, a lack of synchronisation was found from the latest experimental campaign aimed at measuring forces from pedestrians walking on a laterally oscillating instrumented treadmill [7]. However, self-excited forces were identified, with the most important component centred at the treadmill vibration frequency, which was generally different from the walking frequency. Therefore, the model derived by Arup seems to be valid (although the nature of the underlying mechanism, at least in the case of small amplitude vibrations, might have been misunderstood at the time), but it requires further generalisation.

For that purpose a fundamental biomechanically-inspired inverted pendulum pedestrian model (IPM) has been applied to study lateral pedestrian-structure interactions [8, 9]. In this model, while supported on one leg, the pedestrian acts passively under the influence of gravity and any acceleration of the supporting surface, which can be considered as an external perturbation. Lateral balance is maintained by means of a foot placement control law at the transition from one foot to the other (without assuming synchronisation of footstep timing to the bridge motion), whereby the foot is placed further or less far out to the side on each step to stabilise the pedestrian's lateral balance depending on the their lateral velocity at the time it is placed (e.g. if falling too fast to the right, the foot is placed further to the right). Experimental evidence was recently presented by Hof *et al.* [10] showing this is the primary response to lateral perturbations while walking. Outputs of the IPM have been found to be consistent with the measured lateral forces of pedestrians on stationary ground [8], the self-excited forces identified from the laboratory tests by Ingólfsson *et al.* [7], and the measurements on the LMF, CMB and CSB [9].

To put the findings from the IPM in the context of existing modelling approaches, formalised design recommendations and other proposed models are briefly reviewed and some of their shortcomings highlighted. Utilising real statistical data, the distributions of the parameters defining the IPM are then analysed. Taking these into consideration, probabilistic dynamic stability criteria are derived for a given number of pedestrians on a bridge, accounting for their spatial distribution with relation to the mode shape.

2. Existing design recommendations and modelling approaches

Elementary recommendations for the design of structures for the actions of pedestrians are included in Eurocodes 0 and 5 [11, 12] and ISO 10137 [13], dealing with the evaluation of serviceability against vibrations of walkways for human occupancy. All these standards propose some design parameters expressed in terms of acceleration for lateral frequencies typically below 2.5Hz. However, measurements on the CMB [2] and CSB [3] have revealed that quantifying the acceleration alone may not capture the potential for instability, since when certain conditions are met the acceleration amplitude can grow rapidly from very low levels. Eurocode 1 [14] acknowledges the complex nature of pedestrian action and states that appropriate loading models and comfort criteria may be defined in the National Annexes. In broad terms, a periodic force with a frequency range between 0.5 and 1.5Hz is to be assumed in the lateral direction.

A lateral pedestrian load model is presented in the reports from two major European research projects focusing on human-induced vibrations: Human Induced Vibrations of Steel Structures (HIVOSS) [15] and Advanced Load Models for Synchronous Pedestrian Excitation and Optimised Design Guidelines for Steel Footbridges (SYNPEX) [16]. However, this model ignores the influence of the feedback from the movement of the structure on pedestrian behaviour and instead, for calculation of the structural response, it suggests application of the first harmonic load contribution only, characteristic of walking on stationary ground (0.04 fraction of body weight), and application of an increased first harmonic load factor when synchronisation with the vibration occurs (0.055 or 0.075 fraction of body weight for acceleration amplitudes lower or higher than $0.5m/s^2$, respectively). Synchronisation lies at the centre of the guidelines from the French Ministry of Transport and Infrastructure (Sétra) [17]. An acceleration limit of $0.1m/s^2$ is proposed, beyond which the probability of synchronisation increases and large amplitude lateral vibrations can develop. However, the negative damping model proposed by Arup is consistent with the data collected on the LMF, CMB and CSB down to very low vibration levels, well below this proposed limit. (Synchronisation may however occur for larger vibration amplitudes, which is beyond the scope of the current paper, which deals only with the initiation of the lateral instability.) A number of other modelling approaches have been proposed in which synchronisation and parametric resonance are employed as driving mechanisms of lateral vibrations, which are reviewed in [18-22]. However, these models are often based on uncertain forcing assumptions and parameters are often chosen to fit the data.

An alternative source of the additional self-excited forces was suggested by Barker [23] who formulated a pedestrian model comprising a lumped mass, equal to the whole pedestrian body mass (at the centre of mass, CoM), moving along the bridge in a straight line, from which the lateral forces are derived by resolving its action through an inclined massless leg. He found that, without assuming synchronisation, averaged over all possible phase angles, pedestrians put energy into the vibrating bridge, even for pedestrian pacing frequencies different from the bridge frequency. The results from this model, calibrated by the Arup model [1], constitute the basis of the recommendations in the UK

National Annex to Eurocode 1 (UKNA) [24] for avoidance of unstable lateral responses due to crowd loading [25], shown in Fig. 1.

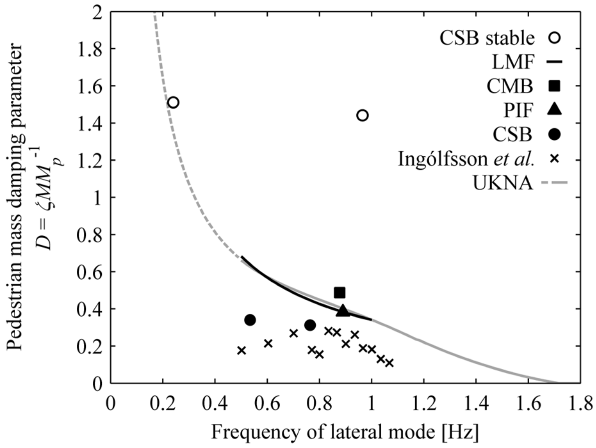

Fig. 1. Lateral stability boundary taken directly from UKNA (grey curve whose dashed part indicates uncertain values). Also presented are the results from site measurements on four bridges: the LMF [1] (for frequency range of 0.5-1Hz – black curve), CMB [2] (■), CSB [3] (● – unstable modes, ○ – stable modes) and PIF [4] (▲), and results of laboratory investigations [7] for amplitude of 4.5mm (×).

The stability boundary (grey curve) is defined in terms of the pedestrian mass damping parameter (similar to the Pedestrian Scruton Number proposed by McRobie & Morgenthal [26] and equivalent to half the Pedestrian Scruton Number adopted by Newland [27]) relating the modal mass of the bridge, M, the modal mass of pedestrians, M_p, and the structural damping ratio, ζ:

$$D = \zeta \frac{M}{M_p} \tag{1}$$

where M_p is defined as:

$$M_p = \int_0^L m\phi^2 \mathrm{d}s \tag{2}$$

where m is the mass of pedestrians per unit length, L is the length of the bridge, ϕ is the lateral mode shape and s is the distance along the bridge. To avoid dynamic instability in a given lateral vibration mode, the pedestrian mass damping parameter for that mode, with the relevant pedestrian mass, should lie above the stability boundary. For comparison, also presented are estimates of values on the stability boundary from the LMF [1], for bridge natural frequencies of 0.5-1Hz, the CMB [2] and CSB [3] (two unstable modes), derived for these three bridges through inverse dynamics (by identifying the forces from the motion of the bridge and finding the constant of proportionality with the velocity).

Also shown is a value on the stability boundary from the PIF [4], derived from crowd loading tests which validated the critical number of people necessary for the onset of instability, N_{cr}, as specified by the formula established by Arup [1] for a uniform distribution of pedestrians:

$$N_{\mathrm{cr}} = \frac{4\pi\zeta f_n M}{k}\frac{L}{\int_0^L \phi^2 \mathrm{d}s} \tag{3}$$

where f_n is the natural frequency and k is the negative damping coefficient per pedestrian, taken as 300Ns/m as derived from tests on the LMF. Also shown in Fig. 1 are estimates of the stability boundary from the latest experimental measurements from people walking at their preferred speed on a laterally oscillating treadmill [7] (the effect of different walking speed was not investigated) and two CSB stable points (open circles) which must lie above the actual stability boundary. All of the points and curves shown from bridges and the treadmill, except for the two CSB stable points, are estimates of the stability boundary itself. In all but one case (CMB) the UKNA design curve envelopes the estimated stability boundary values from measurements, so it would seem reasonable. However, much uncertainty remains.

According to the authors of the UKNA, reliable test measurements are available for frequencies 0.5-1.1Hz [28], to which stability boundaries were calibrated (rather than derived directly from the model). The extensions of the curve beyond this range are based upon Barker's [23] theoretical model of the response assuming crowded walking at the mean pacing frequency of 2Hz and with the model parameters set to fit the experimental results obtained at Imperial College London (to the best of the authors' knowledge, unpublished), while taking half of the pedestrians to be correlated with the bridge motion. Other limitations of the model include neglecting the effect of motion of the pedestrian CoM induced by feedback from the bridge motion, the assumption that pedestrians place their feet without regard to the current kinematics, at a constant lateral distance from the vertical projection of the CoM at each step, and lack of consideration of a wider range of pedestrian parameters and the effect of their distributions (including masses and leg lengths and in particular walking frequencies, which decrease with crowd density). Moreover, since it assumes constant bridge and pedestrian masses per unit length, the UKNA does not allow for inclusion of variable traffic situations (e.g. different distributions of pedestrians on the bridge and their relation to the mode shape) in the case of lateral vibrations. Clearly, the current recommendations have some merit, but further improvements are necessary. This prompted an aspiration to redefine the stability boundary based on findings from a more fundamental, biomechanically-inspired inverted pendulum model of the pedestrian behaviour, which is a more rigorous and justified extension of Barker's [23] model.

A stochastic model for pedestrian lateral loading has been proposed by Ingólfsson & Georgakis [29] based on results from measurements of pedestrian forces on a laterally oscillating instrumented treadmill [7]. In their model the magnitudes of the self-excited forces were presented as functions of the ratio of the lateral vibration frequency, f_b to the pedestrian lateral walking frequency, f_p. However, no evidence has been presented showing that the pedestrian self-excited forces are dependent on this ratio but independent of its individual components which could justify such a parametric simplification. Moreover the distribution of pedestrian walking frequencies, which was found to greatly influence the critical number of pedestrians necessary for instability, was assumed arbitrarily. Most importantly, in contrast to the stochastic load model of Ingólfsson & Georgakis [29] the probabilistic criteria proposed here do not require extensive computational effort for the assessment of dynamic stability. Instead, simple formulae are given which can be quickly applied by the designer.

3. Revised loading model – the Inverted Pendulum Model (IPM)

As in the model devised by Barker [23], the IPM consists of the CoM placed on top of a massless rigid leg (Fig. 2). However, a number of improvements are introduced to account for the main shortcomings of the former model.

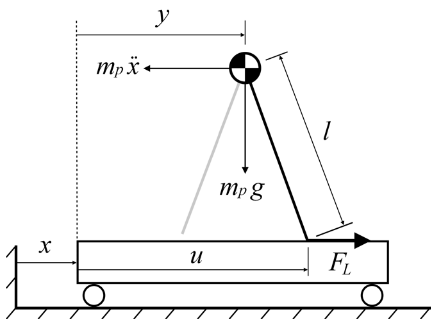

Fig. 2. Inverted pendulum pedestrian model (IPM) subjected to lateral bridge vibrations.

The IPM is built to mimic anthropomorphic properties of upright locomotion and is scaled to preserve spatio-temporal parameters of human gait. It is commonly used in the field of biomechanics [30] (but with stationary ground assumed). For simplicity, here it is restricted to the frontal plane (i.e. the vertical plane perpendicular to the direction of progression). To truly capture the behaviour of a pedestrian walking on an laterally oscillating bridge, the motion of the CoM due to the effects of gravity and the acceleration of the bridge are considered, as are the equal and opposite lateral forces on the CoM and bridge. The lateral force on the bridge, F_L, is found using the Lagrange-d'Alembert principle as (for detailed derivation of the model see Macdonald [8]):

$$F_L = -m_p(\ddot{x} + \ddot{y}) = m_p \Omega_p^2 (u - y) \tag{4}$$

where m_p is the pedestrian mass, x is the displacement of the bridge, y and u are, respectively, the displacement of the CoM and the position of foot placement relative to an arbitrary point on the bridge, $\Omega_p = \sqrt{g/l}$, where g denotes acceleration due to gravity and l is the inverted pendulum length, and dots over symbols represent the derivatives with respect to time. Sinusoidal motion of the bridge is assumed in the analysis to define the self-excited forces on the bridge from the pedestrian, as a function of bridge frequency.

A foot placement control law is adopted for the pedestrian to remain balanced, which is the most efficient strategy for correcting postural instability in the presence of lateral perturbations [31]. In agreement with experimental findings of human gait, the pedestrian adjusts the width of each step according to the state of the CoM at the time of foot placement, such that [32]:

$$u_n = y_n^0 + \frac{\dot{y}_n^0}{\Omega_p} + (-1)^n b_{\min} \tag{5}$$

where subscript n represents the step number, superscript 0 denotes the value of a parameter at the beginning of current step, and $b_{\min}$ is a constant distance known as the margin of stability. The applicability of the above formula in the presence of lateral perturbations has been recently confirmed in an experimental study in which an external lateral impulse was applied to subjects walking on an instrumented treadmill [10]. Some uncertainty remains in the way people perceive self-motion while walking on moving ground. In the analysis an assumption was made that in this case sight provides the most important sensory information, hence the relative velocity (with respect to the bridge) was adopted in the foot placement control law (Eq. (5)) (see [8] and [9] for a discussion of this). The IPM gives reasonable estimates of the lateral forces on the stationary ground with components at the lateral walking frequency, f_p, and its odd harmonics. On laterally vibrating ground it is capable of producing additional self-excited components of force, without having to synchronise to the ground motion. These components appear as lines on both sides of the odd harmonics of the pedestrian lateral walking frequency, f_p (Fig. 3).

Instead of frequency and phase tuning, the pedestrian frequency is assumed to be unaffected by the bridge motion which only causes step width adjustments. Importantly, this agrees with the experimental results from tests in which subjects walked at their preferred speed on a laterally oscillating treadmill [7]. It should be pointed out that a limitation of the experiments was that the walking speed was selected initially for no lateral motion and was then fixed for all subsequent tests with motion. It is therefore not proven how pedestrians may behave if instead able to adjust their speed freely, which hence could involve changing their walking frequency. However, such a change is more likely for larger vibration amplitudes, which are more perceptible, so for the small vibrations relevant to the initial onset of the dynamic instability addressed in this paper, the assumption is believed to be valid.

From the output of the model, the critical component of the force which is at the bridge vibration frequency (f_b; Fig. 3 – thick line), can be divided into a component in phase with bridge velocity and a component in phase with bridge acceleration. These components are found to be proportional to the bridge velocity and acceleration themselves, so they can be expressed as the equivalent pedestrian added damping, ΔC (equivalent to $-k$), and equivalent pedestrian added mass, ΔM, [8, 9] which can be easily included in the bridge equation of motion (Section 5). The self-excited components of the interaction force from treadmill experiments have been treated similarly (but with the opposite sign convention) by Ingólfsson and co-workers [7, 29].

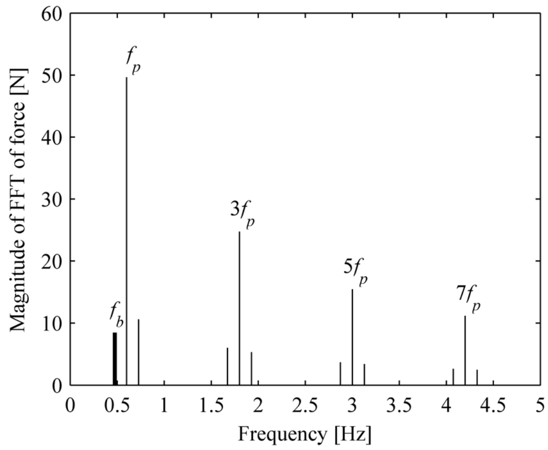

Fig. 3. Fourier decomposition of the force derived from the IPM from a typical pedestrian (m_p = 76.2kg, l = 1.153m) walking with lateral frequency of 0.6Hz in the presence of lateral ground motion with a frequency of 0.47Hz (i.e. approximately as for the first lateral mode of the central span of the LMF) and amplitude 2mm.

It was found previously by Macdonald [8] that the equivalent added damping and mass derived from the IPM are strongly dependent on the bridge vibration frequency but are independent of the amplitude of the bridge vibration and the margin of stability, $b_{\min}$. Also they are directly proportional to the pedestrian mass, m_p. An extensive parametric study of the effects of the leg length, pedestrian walking frequency and bridge vibration frequency on the equivalent added damping and mass has been conducted [9]. The variation of ΔC and ΔM with bridge vibration frequency, for a typical pedestrian walking at different frequencies, are presented in Fig. 4. Note these results are expected averages over long walking time periods. Also presented in Fig. 4(a) are the results from site measurements on the LMF [1], CMB [2] and CSB [3] which fall within the range of values predicted by the IPM for low walking frequencies.

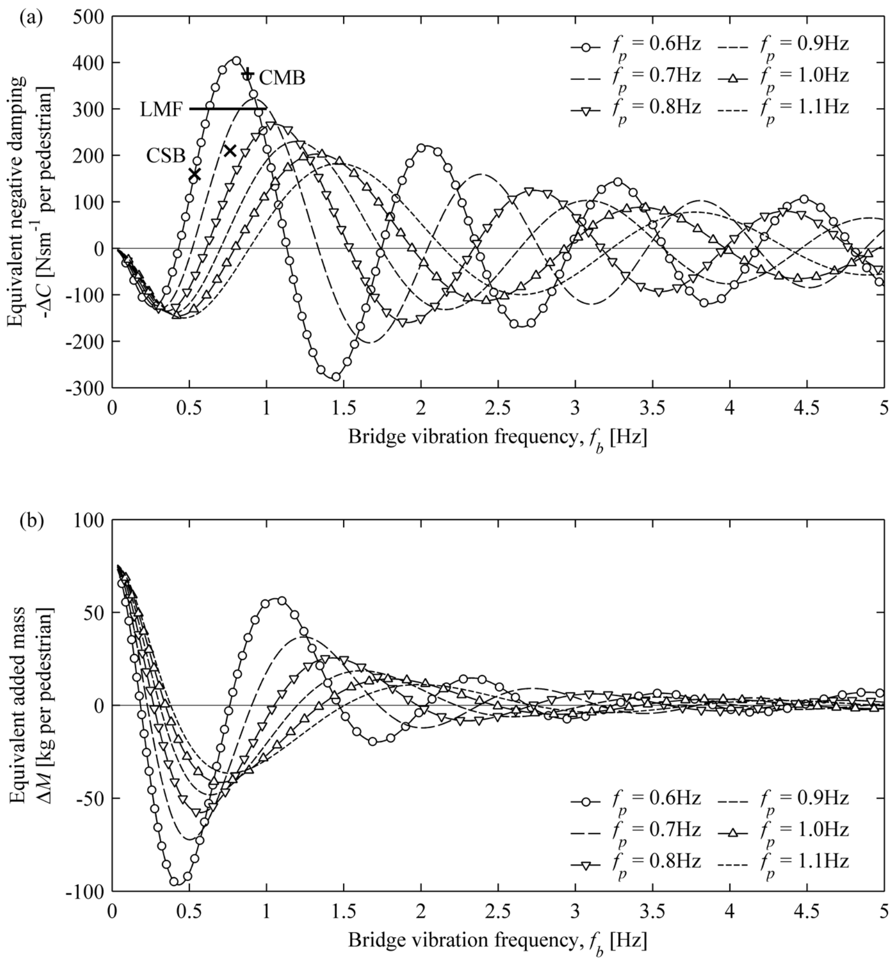

Fig. 4. (a) Equivalent negative damping and (b) equivalent added mass per pedestrian for lateral walking frequencies from 0.6Hz to 1.1Hz for a typical pedestrian (m_p = 76.2kg, l = 1.153m). Also presented in (a) are the results from site measurements on three bridges: LMF (thick black line,[1]), CMB (+,[2]) and CSB (×,[3]).

The IPM is believed to be the only current model (other than Barker's simpler version of it [23]) that, when subjected to lateral ground motion, gives self-excited forces at the bridge vibration frequency compatible with Arup's negative damping model [1] without synchronisation of the footsteps (as observed on the CMB and CSB). Furthermore, the results from the IPM are in very good agreement with the measurements on full-scale bridges [9], which are available for bridge frequencies in the range 0.5-1.0 Hz. They are also in line with the observation on the CSB that the frequencies of the two excited lateral modes increased under the action of pedestrians, and it is the only current model that can explain the simultaneous excitation of multiple lateral modes observed on the LMF and CSB [9]. Furthermore, the results are in reasonable agreement with those from the tests of people walking on a laterally oscillating instrumented treadmill [9], conducted by Ingólfsson *et al.* for treadmill frequencies in the range 0.33-1.07 Hz [7]. The advantage over Ingólfsson *et al.*'s [7] empirical self-excited forces is the IPM's ability to explore parameter variations, such as the effect of different walking frequencies, leg lengths and pedestrian masses. Therefore, the model was applied in a procedure that aimed to capture the variability of pedestrian loading in crowds and define probabilistic stability criteria. Importantly, the proposed framework is not only valid for pedestrian loading model derived from the IPM but can be easily applied with results obtained from any other models or from experimental investigations.

4. Distribution of pedestrian parameters

In order to define probabilistic stability criteria some descriptive statistical measurements relevant to the entire statistical population need to be known. Quantitative information of central tendency and dispersion of data can be specified by mean and standard deviation, respectively. Therefore, the purpose of the procedure outlined below was to estimate the mean and standard deviation of the equivalent added damping and the mean of the equivalent added mass from individual pedestrians. This was achieved by adopting the distribution of pedestrian parameter values defining the IPM, corresponding to a representative sample of the English population aged 16 and over obtained from the latest available UK National Health Service survey [33].

The individuals' gender, mass and height records were extracted from the pool of data and filtered according to the reliability parameter assigned for each category. This left approximately 12400 individual data records available for further processing (referred to hereafter as the statistical population). Rather than extracting statistics of the data and using them for further analysis assuming a certain distribution, the equivalent added damping and mass were estimated for each individual and the results were analysed statistically. Thus the actual distributions of the parameters of the statistical population, including their inter-dependence, and the non-linearity of their effect on the equivalent added damping and mass were included in the results. To capture the parameters of the IPM characteristic of typical crowds, the plausible velocities (or pedestrian densities, as these parameters become correlated with densification of traffic) of the crowd also had to be established, since the pedestrian damping was found to depend on pacing rate hence velocity as well as well as the basic pedestrian parameters. The following procedure was adopted to satisfy these requirements.

The leg lengths of pedestrians were obtained from the relationships established (although not given explicitly) for each gender by Pheasant [34]:

$$l_{\mathrm{m}} = 0.7028h - 0.3091 \quad \text{and} \quad l_{\mathrm{f}} = 0.6797h - 0.2781 \tag{6a,b}$$

where l_{m} and l_{f} are the leg length for males and females, respectively, h is the height, and the values of all parameters are expressed in metres. The equivalent inverted pendulum lengths were then calculated according to the formula adopted by Hof [35]: $l = 1.34 \times (\text{leg length})$. The range of plausible walking velocities considered in the study spanned between 0.5m/s and 1.7m/s. The lower boundary was conservatively chosen considering a maximum density of approximately 2people/m^2. Although it can be easily imagined that in dense crowds pedestrians can advance at much lower velocities, the mechanics of walking are in these cases often impaired and near-periodicity is lost due to the close proximity of other pedestrians. As a consequence, the duration of the double-support phase of gait (the period when both legs are in contact with the ground) increases and the walkers move in an unsteady manner. Velocities higher than 1.7m/s are also achievable, but they are uncomfortable for walking in practice and at this point people often break into a jog [36]. The individuals' lateral walking frequencies in the plausible range of walking velocities were obtained from the relationship established by Dean [37]:

$$f_p = 1.3502\frac{v^{0.5}}{h} \tag{7}$$

where v is the forward velocity expressed in metres per second and h is the height expressed in metres. For a fairly dense crowd in which all pedestrians are constrained to walk at the same velocity Eq. (7) gives a distribution of lateral walking frequencies depending on the pedestrian height. The applicability of this formula can be demonstrated by comparison with the results from measurements of behaviour of 800 pedestrians walking on two footbridges by Pachi & Ji [38]. In that study it was established that the average pedestrian frequency was 1.8Hz and the standard deviation was 0.11Hz for average pedestrian velocity of 1.3m/s. For the same walking velocity, using data from more than 12400 representative individuals from the statistical population in conjunction with Eq. (7), the calculated average walking frequency was found as 1.84Hz and the standard deviation as 0.11Hz. Considering all walking velocities and all individuals from the statistical population, the pedestrian lateral walking frequencies spanned between 0.47Hz and 1.28Hz.

The values of equivalent added damping and mass for each individual (using his/her height, leg length and mass) at thirteen walking velocities (0.5m/s to 1.7m/s in 0.1m/s increments), for the bridge vibration frequency range of 0.05Hz to 5Hz were obtained by interpolating from base curves similar to those in Fig. 4, but normalised by pedestrian mass, for walking frequencies of 0.4Hz to 1.3Hz in 0.1Hz increments and leg lengths of 0.66m to 1.12m in 0.115m increments, distributed such as to cover the whole range of this parameter established for the entire statistical population. (An alternative method of finding the equivalent added mass and damping for each pedestrian would be to use the recently derived analytical solution of the model [39], which yields almost identical results to those from the adopted numerical method). The mean and standard deviation of all the individual results for ΔC (critical for stability) and the mean of ΔM were then found ($\mu_{\Delta C}$, $\sigma_{\Delta C}$ and $\mu_{\Delta M}$, respectively) at each combination of crowd velocity and bridge vibration frequency.

4.1 Equivalent added damping

The mean equivalent added damping for all thirteen crowd velocities is presented in Fig. 5(a). To make it non-dimensional and for convenience for subsequent use it is normalised by $2\omega_b\bar{m}_p$ (giving $\tilde{\mu}_{\Delta C}$), where $\bar{m}_p$ = 76.2kg is the average pedestrian mass of the statistical population and $\omega_b = 2\pi f_b$. $-\tilde{\mu}_{\Delta C}$ is comparable with the pedestrian mass damping parameter (Fig. 1) (after accounting for the added mass effect (Section 5) and the number of pedestrians on the structure (Section 6)). It can be seen that $\tilde{\mu}_{\Delta C}$ is very dependent on the crowd velocity, v, due to Eq. (7) and the dependence of ΔC on f_p (Fig. 4(a)). The maximum value of $-\tilde{\mu}_{\Delta C}$ in Fig. 5(a), at a given bridge vibration frequency, corresponds to the most detrimental expected added damping for any speed of crowd. For the values of the pedestrian mass damping parameter, D (Eq. (1)), in Fig. 1 from the four bridges and the treadmill tests, the actual crowd velocity may not have been the most critical for the relevant bridge frequency, so the given measured values are lower bounds of the worst case envelope. If the crowd had a different speed, the damping contribution from the pedestrians could be more detrimental (or beneficial).

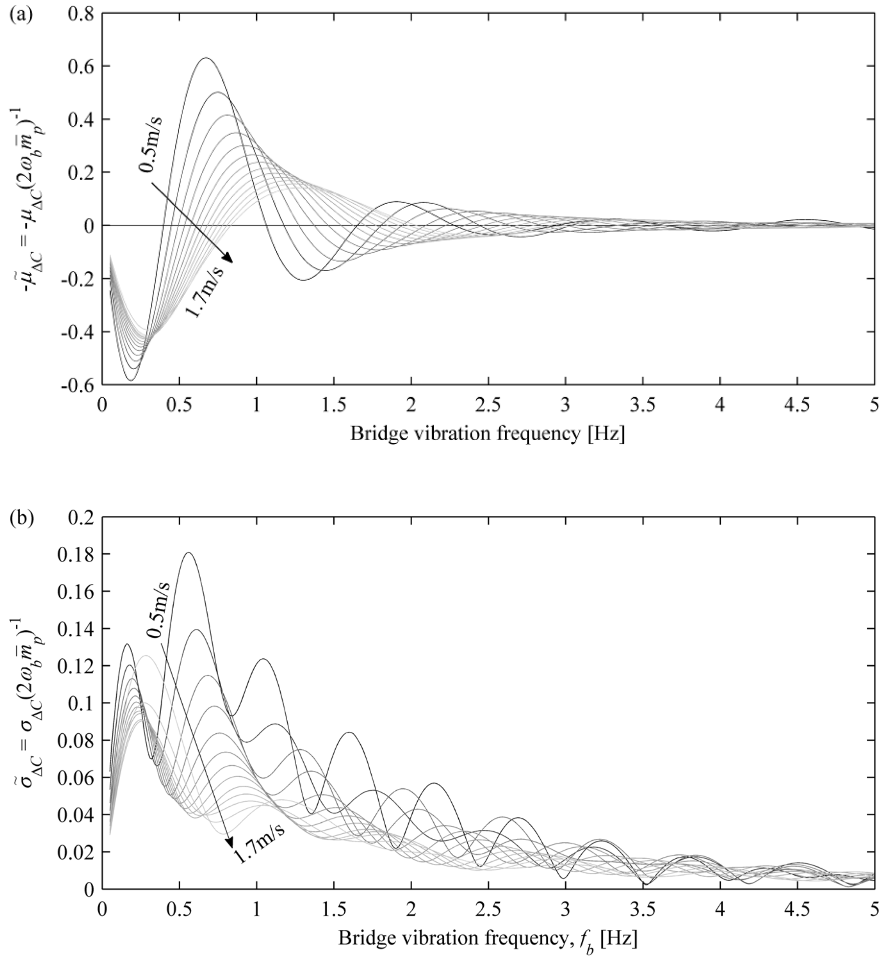

Fig. 5. (a) Normalised mean equivalent added damping from more than 12400 representative individuals of the English population aged 16 and over, for walking velocities from 0.5m/s to 1.7m/s at 0.1m/s intervals, and (b) corresponding normalised standard deviation of equivalent added damping.

It needs to be pointed out that uncertainties exist in the limiting pedestrian damping values for bridge vibration frequencies below 0.5Hz. This is due to the lack of reliable data on pedestrian balance adjustments on laterally oscillating ground for walking velocities below 0.5m/s and the complexity of gait changes in these cases, previously discussed (Section 4). However, no reliable measurements from bridges on which modes below approximately 0.5Hz have been excessively excited due to crowd action have been reported to date.

The standard deviations of the equivalent added damping values from the statistical population, $\sigma_{\Delta C}$, for each crowd speed, normalised by the same factor as for $\tilde{\mu}_{\Delta C}$, are presented in Fig. 5(b) (i.e. $\tilde{\sigma}_{\Delta C}$).

4.2 Equivalent added mass

The second component of the self-excited force, the equivalent added mass, needs to be accounted for due to its potential effect of changing the natural frequency of the system which in turn can cause a change in ΔC (see Section 5). The mean equivalent added mass of all individuals from the statistical population normalised by the average pedestrian mass ($\bar{m}_p$ = 76.2kg), for all considered crowd velocities, is presented in Fig. (6) (i.e. $\tilde{\mu}_{\Delta M}$).

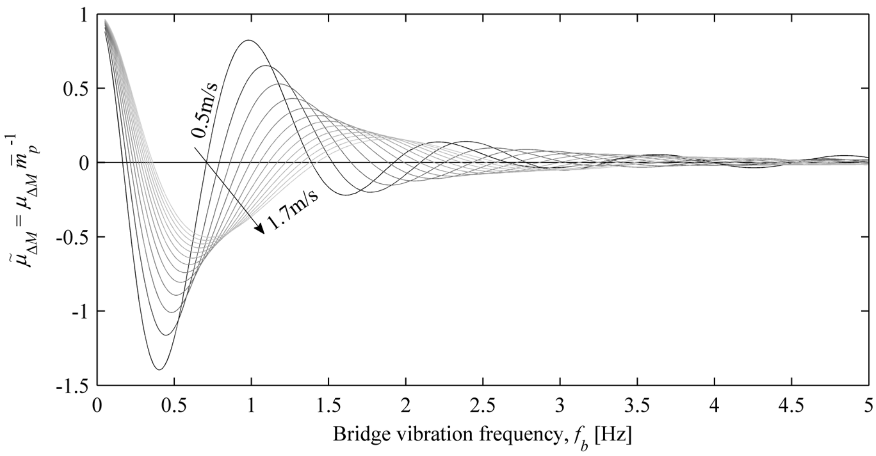

Fig. 6. Normalised mean equivalent added mass from more than 12400 representative individuals of the English population aged 16 and over, for walking velocities from 0.5m/s to 1.7m/s in 0.1m/s intervals.

5. Critical stability parameters

In the previous section statistical measures of the equivalent added damping and mass were found for all individuals from the statistical population for different combinations of crowd velocity and bridge vibration frequency. However, for each bridge vibration frequency only the critical crowd velocity needs to be considered, but the added mass can shift the bridge vibration frequency, which needs to be taken into account. To identify the critical conditions a mathematical expression describing the interacting crowd-bridge system is first defined. Considering a single lateral mode of the structure, assuming it to behave linearly, dynamically loaded by a crowd, the equation of motion can be written as:

$$\left[M+\sum_{i=1}^{N_p} \Delta M_i(\omega_b)\phi_i^2\right]\ddot{X}+\left[C+\sum_{i=1}^{N_p} \Delta C_i(\omega_b)\phi_i^2\right]\dot{X}+KX=F_{\text{ext}} \tag{8}$$

where N_p is the number of pedestrians on the bridge, C and K are, respectively, the generalised damping coefficient and stiffness corresponding to the structural mode, X is the generalised displacement of the structural mode, ΔM_i and ΔC_i are the effective added mass and damping from the i-th pedestrian, ϕ_i is the modal amplitude at the location of the i-th pedestrian and F_{ext} contains all components of the pedestrian loading apart from that at the bridge vibration frequency (see Fig. 3). To analyse the dynamic stability of the crowd-structure system only the self-excited forces at the bridge vibration frequency are considered (F_{ext} = 0) [9, 40]. If the total damping becomes negative, divergent amplitude vibrations will develop. However, the added damping from the pedestrians is frequency-dependant. The focus in this section is, therefore, on establishing the influence of the equivalent added mass, included as the sum of contributions from all pedestrians on the bridge in the first brackets of Eq. (8), on the equivalent added damping, included as the sum of contributions from all pedestrians on the bridge in the second brackets of Eq. (8). This allows the mean and standard deviation of the critical negative damping to be found which will be utilised in probabilistic stability criteria in Section 6.

5.1 Frequency shifts

The equivalent added mass can have the effect of modifying the natural frequency of the crowd-bridge system. This is especially pronounced for high pedestrian to bridge mass ratios ($q = M_p/M$), hence, most likely, for approximately uniformly distributed dense crowds. For bridges where continuous dense crowds can be expected, the UKNA [24] suggests application of a vertical live load of 5kN/m^2. Taking an average pedestrian mass of $\overline{m}_p$ = 76.2kg this corresponds to the static weight of 6.7 people/m^2. Obviously, at this density walking is largely constrained, if not impossible, hence the IPM is not applicable. However, keeping in mind that the maximum mass ratio observed among all herein quoted bridges during lateral pedestrian-induced instability periods was approximately 0.23 [1] (assuming uniform distribution of pedestrians on the bridge), a maximum value of 0.5 was chosen in the analysis. For each pedestrian to bridge mass ratio from 0.1 to 0.5, in 0.1 increments, the ratios of the expected angular response frequency (i.e. natural frequency of the combined pedestrian-bridge system) to the angular natural frequency of the bridge itself, $\omega_n = 2\pi f_n = \sqrt{K/M}$, were found ($r = \omega_b/\omega_n$) at each considered crowd velocity, over the whole range of considered bridge vibration frequencies. This was possible by considering the natural frequency and damping coefficient on the stability boundary of the combined system in Eq. (8) (as in Newland [27] and Bocian *et al*. [9]). Hence:

$$\omega_n = \omega_b\sqrt{1 + \tilde{\mu}_{\Delta M}(\omega_b)q} \tag{9}$$

$$\zeta = -\tilde{\mu}_{\Delta C}(\omega_b)qr \tag{10}$$

where $\tilde{\mu}_{\Delta M}$ is the normalised mean equivalent added mass presented in Fig. 6. Note, from Eq. (10) it follows that for small mass ratios ($r \approx 1$) the expected pedestrian mass damping parameter D (Eq. (1)) is equivalent to $-\tilde{\mu}_{\Delta C}$ from Fig. 5(a), with $f_b = f_n$.

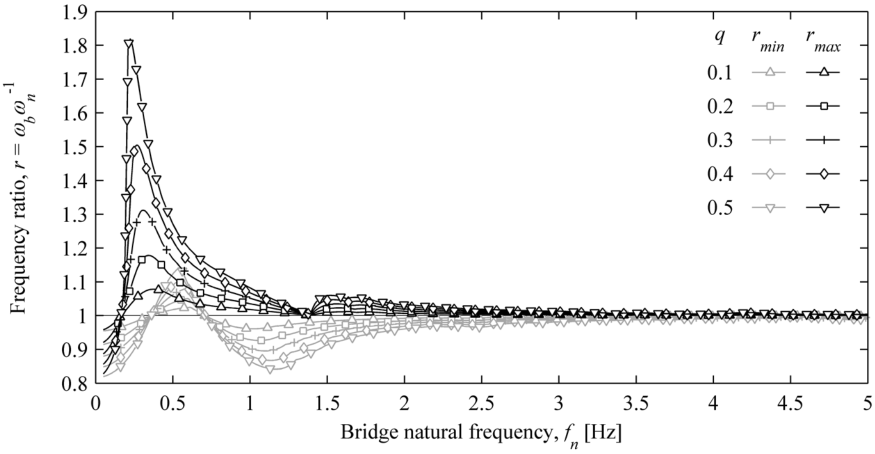

Fig. 7. The envelopes of maxima (black curves) and minima (grey curves) of response frequency to structural natural frequency ratios (r) based on mean equivalent added mass, for pedestrian to structure mass ratios (q) from 0.1 to 0.5.

The envelopes of minima, r_{min}, and maxima, r_{max}, of the frequency ratio r established for each mass ratio, for any crowd velocity, are presented in Fig. 7. These do not generally correspond with critical conditions for dynamic instability, but they show the possible ranges of natural frequency shifts of the system in the presence of the pedestrians. It can be seen that large mass ratios cause non-linear shifts in the vibration frequency of the structure which can either become lower or higher (e.g. as identified on the CSB [3]) than the structure's inherent natural frequency, depending mainly on the crowd velocity. For the maximum mass ratio of 0.23 [1] mentioned above, the expected change in vibration frequency does not exceed 21%.

5.2 Critical negative damping

Having found relationships between the response and natural frequencies for all combinations of considered walking speeds, v, and mass ratios, q, from Eq. (9), it is possible to define the normalised critical mean negative added damping and its standard deviation, as a function of the structural natural frequency, using Eq. (10). Since different walking velocities may determine the critical mean equivalent added damping at different bridge vibration frequencies, the maximum detrimental $-\tilde{\mu}_{\Delta C}$ was found for each bridge natural frequency and then the corresponding standard deviation, $\sigma_{\Delta C}$, was identified. The critical $-\tilde{\mu}_{\Delta C}$ for different mass ratios is presented in Fig. 8(a). It can be seen that increasing the mass ratio widens the frequency range for which detrimental pedestrian damping can be expected and, in general, increases the magnitude of $-\tilde{\mu}_{\Delta C}$ for f_n below 1.4Hz. For f_n above 1.4Hz the differences between the values of critical $-\tilde{\mu}_{\Delta C}$ for different mass ratios are negligible and they can be approximated by the same curve. In the derivation of the results in Fig. 8(a) using Eqs. (9) and (10), only the mean value of ΔM was considered and not its statistical variation. To allow for this and for the range of mass ratios likely in practice, from Fig. 8(a) an empirical envelope of $-\tilde{\mu}_{\Delta C}$ for a mass ratio of 0.5 is proposed for subsequent use as follows (with f_n expressed in Hz):

$$\begin{cases} -\tilde{\mu}_{\Delta C}(f_n) = \dfrac{-0.3297f_n^5 + 1.149f_n^4 - 0.8517f_n^3 - 0.8094f_n^2 + 1.1f_n - 0.1937}{f_n^3 - 2.121f_n^2 + 1.393f_n - 0.1063} & \text{for } 0.21\text{Hz} < f_n < 1.4\text{Hz} \\ -\tilde{\mu}_{\Delta C}(f_n) = 0.48\mathrm{e}^{-0.84f_n} & \text{for } f_n \geq 1.4\text{Hz.} \end{cases} \tag{11}$$

The normalised standard deviation of the equivalent added damping, $\tilde{\sigma}_{\Delta C}$, corresponding to the crowd velocity giving the critical value of $-\tilde{\mu}_{\Delta C}$, for each bridge frequency and mass ratio, is presented in Fig. 8(b). Note the discrete values of $\tilde{\sigma}_{\Delta C}$ in Fig. 7(b) do not give smooth curves, which is caused by the critical $-\tilde{\mu}_{\Delta C}$ (and corresponding $\tilde{\mu}_{\Delta M}$) falling on different walking velocity curves and the related different frequency shifts (Eqs. (9) and (10); see Section 5.1). Also shown in Fig. 8(b) is a fitted curve that envelopes the empirical points, given by (with f_n in Hz):

$$\begin{cases} \tilde{\sigma}_{\Delta C}(f_n) = -0.83f_n^2 + 0.79f_n & \text{for } 0.21\text{Hz} < f_n < 0.56\text{Hz} \\ \tilde{\sigma}_{\Delta C}(f_n) = 0.27\mathrm{e}^{-1.83f_n} + 0.12\mathrm{e}^{-0.57f_n} & \text{for } f_n \geq 0.56\text{Hz.} \end{cases} \tag{12}$$

The fitted $\tilde{\sigma}_{\Delta C}$ usually overestimates the magnitude of $\tilde{\sigma}_{\Delta C}$ derived empirically, thus exaggerating the dispersion of the data, which can be considered as a conservative measure when defining probabilistic stability criteria.

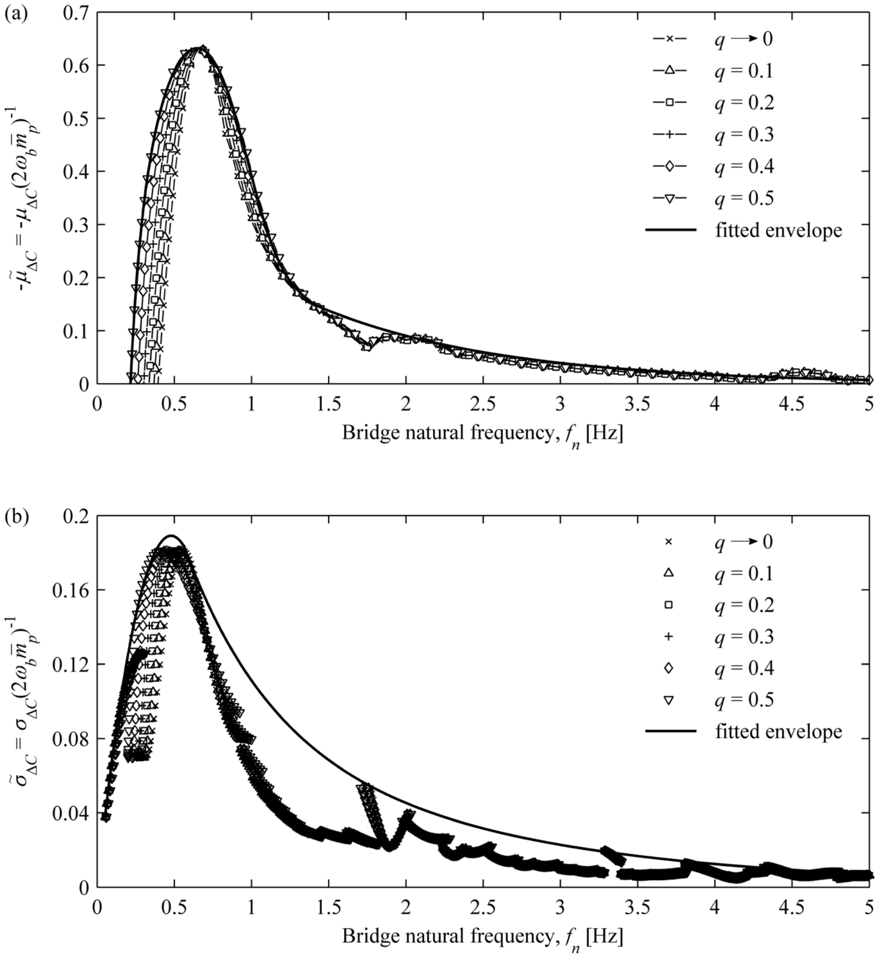

Fig. 8. (a) Critical $-\tilde{\mu}_{\Delta C}$ for different mass ratios after making allowance for the added mass effect, and (b) normalised standard deviation corresponding to the critical equivalent added damping for the same mass ratios, plotted against bridge natural frequency.

6. Probabilistic stability criteria for crowds of pedestrians distributed over a structure

In the above section the normalised mean and standard deviation (Fig. 8(a) & (b), respectively) of the equivalent added damping (critical for stability and accounting for the added mass effect) were established for individual pedestrians from the statistical population. In this section the sum of the effects from any number of pedestrians on the bridge (N_p) is considered making provisions for the variability of their parameters and their distribution with relation to the bridge mode shape. As N_p increases, obviously the expected total pedestrian damping increases in magnitude, but the relative spread of the results decreases. It is useful to bear in mind that not all combinations of N_p and the critical value of $-\tilde{\mu}_{\Delta C}$ may be realistic for a particular bridge. For example, high walking velocities which might define the critical $-\tilde{\mu}_{\Delta C}$ in some bridge frequency ranges might not be achievable if the traffic becomes too dense. The proposed probabilistic stability conditions address a uniform distribution of pedestrians, a random distribution of pedestrians and a distribution where all pedestrians are at the maximum of the mode shape. The first case is applicable to structures where high density pedestrian traffic is expected. The second case is more conservative for the same number of pedestrians, but it is applicable for less dense crowds, and the third case is the most conservative distribution of pedestrians possible, but it is not realistic for large crowds.

For dynamic stability, it must be satisfied that the total damping is positive, i.e.:

$$C + \sum_{i=1}^{N_p} \Delta C_i(\omega_b)\phi_i^2 > 0. \tag{13}$$

By recalling that the damping ratio is defined as $\zeta = C/(2M\omega_b)$, Eq. (13) can be rewritten as:

$$\zeta M > \frac{-\sum_{i=1}^{N_p} \Delta C_i(\omega_b)\phi_i^2}{2\omega_b}. \tag{14}$$

The numerator of the expression on the right side of the inequality in Eq. (14) is the weighted sum of damping contributions from all pedestrians on the bridge. Considering the random nature of the pedestrian parameters, the probabilistic stability condition is:

$$\zeta M > \frac{1}{2\omega_b}\left\{-\mathrm{E}\left[\sum_{i=1}^{N_p} \Delta C_i(\omega_b)\phi_i^2\right] + z_\alpha \sigma_{\sum_{i=1}^{N_p} \Delta C_i(\omega_b)\phi_i^2}\right\} \tag{15}$$

where $\mathrm{E}[\bullet]$ denotes the expected value of an arbitrary random variable •, $\sigma_\bullet$ is the standard deviation of •, and z_α is the parameter corresponding to the $100(1-\alpha)$ percent one-sided upper confidence interval of the normal distribution.

The assumption of the applicability of the normal distribution is made on the basis of the central limit theorem, stating that the mean (or sum) of a random sample drawn from a population with any distribution, provided this distribution has finite variance, is approximately distributed as a normal random variable [41]. Note the confidence limit is defined for a random sample of N_p pedestrians on the bridge at any one time. People will move on and off the bridge so the sample is continuously changing. Hence for long duration events, the confidence limit will be exceeded for a proportion α of the time. Therefore α needs to be made sufficiently small so that the confidence limit is expected to be exceeded for too little time for vibrations to build up appreciably. Also note that the "external forcing" component of loading (F_{ext} in Eq. (8)) can put energy into bridge (or take it out) over short periods, which can have a similar short term effect as negative (or positive) damping (but in long term it averages out). Ingólfsson *et al*. [7] had very large scatter of measured added damping values, especially for $f_p \approx f_b$, probably due to this effect, based on measurements records of 30 seconds duration.

The standard deviations in the following mathematical expressions defining the probabilistic stability criteria were obtained by considering the distribution of the variance, $\sigma_{\bullet}^2$:

$$\sigma_{\bullet}^2 \;=\; \mathrm{E}[(\bullet - \mathrm{E}[\bullet])^2]. \tag{16}$$

For known positions of pedestrians, given that ΔC_i and ϕ_i are independent, it can be shown that Eq. (15) can be expressed as:

$$\zeta M > \frac{1}{2\omega_b}\left\{-\mu_{\Delta C}\sum_{i=1}^{N_p}\phi_i^2 + z_\alpha\sigma_{\Delta C}\sqrt{\sum_{i=1}^{N_p}\phi_i^4}\right\}. \tag{17}$$

This holds for all possible distributions of pedestrians and all possible mode shapes. The summations for discrete pedestrian positions can be approximated by integrals for an equivalent continuous distribution.

Hereafter parameters Φ_j are introduced for brevity, denoting non-dimensional integrals of the j-th power of the mode shape:

$$\Phi_j = \frac{1}{L}\int_0^L \phi^j \,\mathrm{d}s\,. \tag{18}$$

These can be evaluated for any known mode shape, but for sinusoidal mode shapes defined as:

$$\phi = \sin\frac{a\pi s}{L}, \tag{19}$$

where a is the number of half sine waves, for any integer a:

$$\Phi_2 = \frac{1}{2} \qquad \text{and} \qquad \Phi_4 = \frac{3}{8}. \tag{20a,b}$$

6.1. Uniform distribution of pedestrians

For a uniform distribution of pedestrians, Eq. (17) becomes:

$$\frac{\zeta M}{M_{p_{\mathrm{nom}}}} > -\tilde{\mu}_{\Delta C} + z_\alpha\tilde{\sigma}_{\Delta C}\frac{1}{\Phi_2\sqrt{N_p}}\sqrt{\Phi_4} \tag{21}$$

where the nominal modal mass of pedestrians can be taken as:

$$M_{p_{\text{nom}}} = N_p \overline{m}_p \Phi_2 \,. \tag{22}$$

It is noteworthy that for uniformly distributed mass of the bridge, $M = m_b L \Phi_2$ where m_b is the bridge mass per unit length, so in Eq. (21) $M/M_{p_{\text{nom}}}$ is independent of the mode shape and is simply equal to the ratio of the physical bridge mass to nominal physical pedestrian mass (per unit length or total). Then the only dependence on the mode shape is through Φ_2 and Φ_4 on the right hand side of the equation.

Using the mean term ($-\tilde{\mu}_{\Delta C}$) only, Eq. (21) is equivalent to Eq. (3), but here allowance is made for the added mass effect and $-\tilde{\mu}_{\Delta C}$ (equivalent to k in Eq. (3)) is given as a function of bridge frequency over a wider range (Fig. 8(a) and Eq. (11)). In addition, the variability of pedestrian parameters is allowed for with the standard deviation term ($\tilde{\sigma}_{\Delta C}$, Fig. 8(b) and Eq. (12)), with a chosen confidence interval, given by z_α.

For sinusoidal mode shapes Eq. (21) becomes:

$$\frac{\zeta M}{M_{p_{\text{nom}}}} > -\tilde{\mu}_{\Delta C} + z_\alpha \tilde{\sigma}_{\Delta C} \sqrt{\frac{3}{2N_p}} \tag{23}$$

and

$$M_{p_{\text{nom}}} = N_p \overline{m}_p / 2 \,. \tag{24}$$

6.2. Random distribution of pedestrians

For a random distribution of pedestrians, in Eq. (15) ϕ_i (the mode shape amplitude at the position of each pedestrian) is a random variable, as well as ΔC_i. For a uniform probability distribution function for the pedestrians' positions along the bridge, Eq. (17) becomes:

$$\frac{\zeta M}{M_{p_{\text{nom}}}} > -\tilde{\mu}_{\Delta C} + \frac{z_\alpha}{\Phi_2 \sqrt{N_p}} \sqrt{\tilde{\sigma}_{\Delta C}^2 \Phi_4 + \tilde{\mu}_{\Delta C}^2 \left(\Phi_4 - \Phi_2^2\right)} \,. \tag{25}$$

Note that here $M_{p_{\text{nom}}}$ is still the deterministic nominal modal mass of pedestrians, as defined in Eq. (22) for a uniform distribution, rather than the actual modal mass which is a random variable depending on the actual distribution of pedestrians.

For sinusoidal mode shapes, Eq. (25) becomes:

$$\frac{\zeta M}{M_{p_{\mathrm{nom}}}} > -\tilde{\mu}_{\Delta C} + z_\alpha \sqrt{\frac{1}{2N_p}\left(\tilde{\mu}_{\Delta C}^2 + 3\tilde{\sigma}_{\Delta C}^2\right)}\,. \tag{26}$$

Comparing Eqs. (25) & (26) with Eqs. (21) & (23), clearly the expected damping demand for uniformly and randomly distributed pedestrians is the same, but for a random pedestrian distribution it has higher variance.

6.3. All pedestrians at the maximum of the mode shape

The worst possible distribution of pedestrians is when all of them are positioned at the maximum antinode of the mode shape (or antinodes, if there are more than one with equal magnitude) so all their damping contributions have maximum weighting, given by ϕ_{max}^2, where ϕ_{max} is the maximum amplitude of the mode shape. Often ϕ_{max} is normalised to unity, but most importantly it has to be consistent with scale of the mode shape in M and Φ_2. This scenario is only realistic for small crowds. In this case Eq. (17) becomes:

$$\frac{\zeta M}{M_{p_{\mathrm{nom}}}} > \frac{\phi_{max}^2}{\Phi_2}\left(-\tilde{\mu}_{\Delta C} + z_\alpha \tilde{\sigma}_{\Delta C}\frac{1}{\sqrt{N_p}}\right). \tag{27}$$

Again, for consistency of presentation, $M_{p_{\mathrm{nom}}}$ is still the nominal modal mass of pedestrians defined in Eq. (22) (based on a uniform distribution), rather than the actual modal mass of pedestrians which in this case is somewhat larger. From Eq. (27), for sinusoidal mode shapes, the distribution of pedestrians such that all of them are at the maximum antinode causes a mean damping demand twice as large and a standard deviation of damping demand $\sqrt{8/3}$ times as large as in the case of the same number of pedestrians uniformly distributed.

6.4. Example application

To quantify the effects of the statistical variations studied in this paper for a typical bridge, an example is considered of a bridge having a sinusoidal mode shape, occupied by a maximum of 100 pedestrians. Stability boundaries are found for a serviceability limit state confidence limit of 99% ($z_{0.01}$ = 2.326). The proposed pedestrian mass damping parameter values required for stability, for the three different pedestrian distributions considered, are presented in Fig. 9 from Eqs. (23), (26) and (27) and using the envelope curves for $-\tilde{\mu}_{\Delta C}$ (Eq. (11)) and $\tilde{\sigma}_{\Delta C}$ (Eq. (12)). For comparison the corresponding recommendations from the UKNA [24] are also included. Note the pedestrian mass damping parameter is expressed here as a function of the nominal pedestrian mass, $M_{p_{\mathrm{nom}}}$, from Eq. (22) and not the actual pedestrian mass, M_p, from Eq. (2). Hence for a sinusoidal mode shape and with all the pedestrians at the antinode(s), the values of D (Eq. (1)) from the UKNA [24] need to be doubled. The values of pedestrian mass damping parameters for 50% (mean) and 99% confidence limits are denoted D_{50} and D_{99}, respectively.

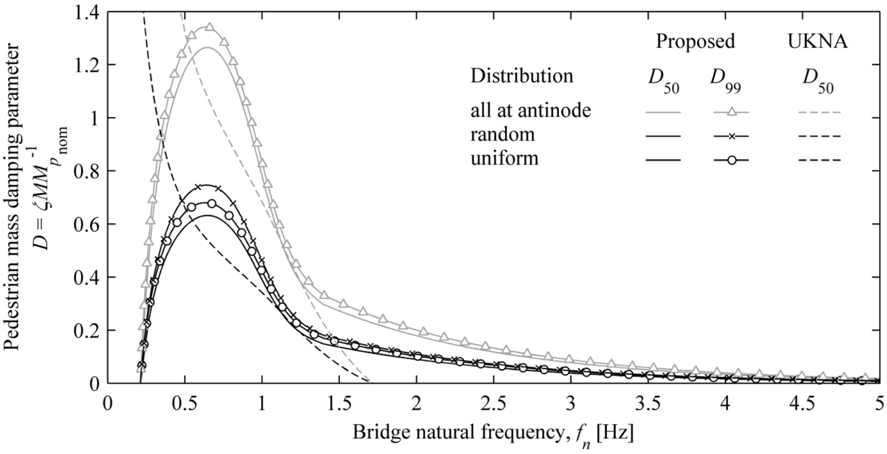

Fig. 9. Proposed values of the pedestrian mass damping parameter required for stability for 50% (mean, D_{50}) and 99% (D_{99}) confidence limits for uniform and random pedestrian distributions, and a distribution where all pedestrians are located at the maximum of the mode shape, with corresponding recommendations from the UKNA [24], for a bridge with a sinusoidal mode shape occupied by 100 pedestrians.

As expected, the most lenient criterion is found for the uniform distribution. A more onerous criterion needs to be met for a random distribution of pedestrians. In the case of all pedestrians concentrated at the maximum of the mode shape, the values of the pedestrian mass damping parameter lie considerably above these from the other two cases. However, if the considered bridge were such that the presence of 100 pedestrians represented dense traffic conditions, it is likely a lower number of pedestrians could be physically accommodated at the maximum antinode(s).

Applying the 99% confidence limit causes increases in the required pedestrian mass damping parameter which are at least 6%, 18% and 5% larger than the corresponding D_{50} for the uniform, random and maximum antinode distributions, respectively. The recommendations from the UKNA [24] are typically less onerous except for frequencies below approximately 0.55Hz, for which the UKNA pedestrian mass damping parameter curve was defined based on uncertain assumptions (see Section 2). Also the UKNA [24] suggests that the lateral loading from a crowd of walking pedestrians does not need to be considered for structures with lateral natural frequencies above approximately 1.7Hz. In contrast, the IPM predicts that this limit is closer to 5Hz and that pedestrian mass damping parameter below this value can be significant. Regrettably there are no suitable data quantifying pedestrian loading currently available for vibration frequencies above approximately 1.1Hz which could help in verification of the predictions of the IPM. It is noteworthy that this renders the stability conditions proposed by UKNA above this limit equally uncertain. Further empirical data would be beneficial to close this gap. The proposed stability criteria are more onerous than the UKNA since they allow for the most critical walking velocity for each bridge frequency and the statistical variation of pedestrian parameters, as well as being based on a more rigorous underlying pedestrian model.

For a larger number of pedestrians on the bridge the relative spread of the equivalent added damping from all the pedestrians would be smaller, therefore D_{99} would be closer to the corresponding D_{50} (from Eqs. (21), (25) and (27)). Conversely, a smaller number of pedestrians would allow more adverse effect from the statistical variation and would require relatively more restrictive stability criteria.

For real bridges the mode shapes are not necessarily sinusoidal although they may have a similar form. Eqs. (17), (21), (25) and (27) are applicable for any lateral mode of any bridge, with Φ_2 and Φ_4 given by Eq. (18). To give an indication of the effect of the mode shape on the results, values have been calculated for the first four lateral modes of the Clifton Suspension Bridge, as measured on site and reported by Macdonald [3]. The values of Φ_2 are 0.693, 0.468, 0.499 and 0.477 and of Φ_4 are 0.537, 0.354, 0.387 and 0.343 for each mode, c.f. 0.5 and 0.375 respectively for sinusoidal modes. The mode shape makes no difference to the mean values of pedestrian mass damping parameters in any case, but the statistical variation of the results is affected. Relative to the results for sinusoidal mode shapes, still for 100 pedestrians on the bridge, the 99% confidence limits are modified by -3.2% to +0.3% for a uniform pedestrian distribution, -5.3% to +1.5% for a random pedestrian distribution and -28% to +6.7% for all pedestrians at the maximum antinode. Hence, apart from the case of all pedestrians at the maximum antinode, it seems that the probabilistic stability criteria are not very sensitive to the mode shape. The effect of the number of pedestrians is always to reduce the spread of the equivalent added damping by a factor of $\sqrt{N_p}$.

7. Conclusions

In this paper probabilistic stability criteria for structures subjected to lateral crowd actions have been presented, based on analysis of the inverted pendulum pedestrian model from the field of biomechanics. The model is capable of providing self-excited forces on bridges, which can be quantified as equivalent added damping (sometimes negative) and mass to the bridge and is consistent with measurements both on full-scale bridges and in laboratory tests. The model shows high dependence on the bridge and pedestrian frequencies, so these parameters have both been varied in the analysis. In the derivation of the stability criteria real statistical data from the English population have been utilised to obtain distributions of pedestrian parameters defining the model. The variability of the self-excited forces in the plausible range of crowd velocities has been quantified through statistical measures and the worst conditions identified for structural frequencies in the range 0.05Hz to 5Hz. The analysis has accounted for the most critical crowd velocity for each bridge frequency and the effective added mass from the crowd, which can modify the vibration frequency and hence broaden the instability region. Hence envelopes of the mean and standard deviation of the equivalent added damping per pedestrian, as a function of bridge natural frequency, have been determined.

Considering the total effect of a crowd of pedestrians the probabilistic stability criterion has been defined in general, for any distribution of pedestrians and any mode shape. It has then been developed for three different traffic situations, which could determine the structural damping requirements in these different cases, namely uniform, random and concentrated (at the most onerous position) distributions of pedestrians. The influence of the number of pedestrians occupying the structure has also been addressed in that for larger crowds the effect of the statistical variability reduces, relatively speaking. The analysis allows for the effect of the bridge mode shape, although it has been found that the results are generally not very sensitive to it. The final results are presented in terms of the minimum mass damping parameter of the structure required to prevent lateral dynamic instability of the bridge with a certain confidence limit, as a function of bridge natural frequency. The resulting criteria are similar to but slightly more onerous than the UKNA [24] for bridge frequencies above approximately 0.5Hz, since they allow for the most critical walking velocity for each bridge frequency and the statistical variation of pedestrian parameters.

The proposed stability criteria come from the output of the presented fundamental biomechanical pedestrian model rather than from empirically fitting model parameters to the limited number of measured bridge responses. Although the utilised statistical data are specific to the English population, the outlined procedure could be readily applied with any other relevant distributions of pedestrian parameters. Furthermore, any advancement in the determination of the pedestrian loads on laterally oscillating structures can be easily incorporated within the proposed probabilistic framework to further refine the stability criteria. Therefore the outcome of this study is believed to represent a step forward towards improvement of the existing recommendations concerned with the design of structures against the destabilising lateral walking forces from crowds.

Acknowledgements

MB is supported by an EPSRC Doctoral Training Account studentship.

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