

Distributed Fusion Estimation for Stochastic Uncertain Systems with Network-Induced Complexity and Multiple Noise

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Abstract—This paper investigates an issue of distributed fusion estimation under network-induced complexity and stochastic parameter uncertainties. First, a novel signal selection method based on event-trigger is developed to handle network-induced packet dropouts as well as packet disorders resulting from random transmission delays, where the H_2/H_∞ performance of the system is analyzed in different noise environments. In addition, a linear delay compensation strategy is further employed for solving the complexity network-induced problem, which may deteriorate the system performance. Moreover, a weighted fusion scheme is used to integrate multiple resources through an error cross-covariance matrix. Several case studies validate the proposed algorithm and demonstrate satisfactory system performance in target tracking.

Index Terms—Distributed fusion estimation, network-induced complexity, transmission delays, stochastic and deterministic uncertainty.

I. INTRODUCTION

ENERGY Internet has been a new and advanced paradigm established by effective integration of energy and information network infrastructures involving traditional centralized generation [1], distributed energy resources [2], advanced communication [3], smart metering [4], intelligent computing [5], and smart management systems [6]. The structure of Energy Internet has witnessed promising solutions with improved efficiency to holistic energy flow problems, while accommodating the transformation of traditional power networks. However, owing to the requirements of growing system scale, in order to address the challenges in system scalability

and reliability, a new distributed networked control system (DNCS) needs to be developed.

In the existing DNCS structure, the data or signals are exchanged between three system components (e.g. sensors, controllers and actuators) using a shared communication network. Furthermore, the system component is physically distributed and interconnected with the others in order to coordinate their tasks for achieving the desired objectives [7]. Thus, distributed controllers are capable of coordinating their behaviors by transmitting/receiving information to/from other controllers within a certain neighboring area. The communication networks are introduced to the distributed systems [8, 9] and this inevitably produces network-induced complexity (such as random transmission delays, packet dropouts, packet disorders, and missing/fading measurements), which may significantly deteriorates the system performance [10]. Therefore, it is a challenging issue for Energy Internet to incorporate distributed information with network-induced complexity in controller design and time-sensitive applications [11].

In engineering practice, noise can be categorized into three types: bounded and stochastic uncertainty noise, energy-bounded noise and uncertain white noise [12]. In ship control applications [13], for example, we may expect different environmental changes (e.g. winds, waves and currents) during the ship navigation, leading to bounded and stochastic disturbance uncertainties for the control units. At the same time, sensor noise may be of unknown characteristics but bounded power, while the instrument input can be mixed with uncertain Gaussian white noise. To effectively establish a working system, stochastic system uncertainty is investigated using a multiplicative noise model [11, 14]. The correlated noise (i.e. auto and cross-correlated noise) [14, 15] is considered when they are handling the estimation problem over the networks. Moreover, the bounded noise [16, 17] is introduced in a realistic networked system to evaluate the measurement error and external disturbances. On the other hand, filtering, in particular H_∞ filtering, plays an important role in the field of signal processing and communications. However, considering signal over sensor networks with transmission delays in the communication process, H_∞ filtering [8, 18] is insensitive to the uncertainty. To solve the parametric uncertainty problem, robust filtering based on finite-horizon Kalman filter [19] is proposed to reduce the conservativeness and maintain the performance of the filter.

For the transmission over networks, the one-step prediction compensation is used for most of existing systems due

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to the bandwidth constraint to handle transmission delays and/or packet dropouts. Such that the augmented state scheme is applied to obtain optimal estimation [20]. A model for multi-sensor fusion system was established by using the full-rank decomposition approach and to describe the random observation delays and packet dropouts [21]. Moreover, to break the resource constraints (i.e. bandwidth or energy), a measurement re-organization method [22] has been developed to simplify the system model with delays. Another classic method is the modelling of delayed system by sequences of Bernoulli random variable distributions [20, 23], where the measurement model for each sensor is augmented by different characteristics. Due to random delays for the involved sensors, the local estimator based on the compensation or augmentation method possesses high computation cost.

To reduce the unnecessary resource consumption, the delayed system is established by introducing the event-triggered scheme. The containment control for multi-agent system was investigated under the centralized event-triggered containment algorithm to deal with constant time delays [24]. Li *et al.* [25] presented an event-triggered sampled-data stabilization for switching linear systems to obtain the average dwell time conditions. Moreover, packet disorders inevitably appear due to transmission delays. Several works on dealing with packet disorders have been developed. To store the data packets, the signal selection method of zero-order-holder (ZOH) chooses and receives the most recent arriving data packets [26], whilst the signal sequence re-ordering method using the logic ZOH scheme [8, 27, 28] was introduced to choose the latest time-stamped data packets. It is worth pointing out that the logic ZOH based on event-trigger mechanism is widely applied in the field of networked control systems (NCSs). However, the issue of the fixed-time event-triggered consensus was studied for the multi-agent systems [29]. Therefore, handling signals with or without packet disorders, as well as estimating filter parameters are difficult problems. In addition, with the aid of the event-triggered scheme, designing the distributed fusion estimation is a complex problem.

For the sake of alleviating the negative influence during transmission, some promising estimation and filtering methods have been proposed for distributed systems. Liu *et al.* [30] introduced a two level weighted fusion scheme using the reorganized innovation sequence as well as filtering error cross-covariance, and presented a distributed weighted fusion estimation method with transmission delay and cross-correlated noise. Different distributed fusion schemes, and event-triggered Kalman consensus filter [31] was used to deal with the distributed estimation. Considering reducing energy consumptions, the finite-horizon filtering [22, 32] derives the filter parameters from the upper bound for the covariance matrix of the estimation error. Furthermore, for a class of discrete stochastic systems, a finite horizon state estimation is investigated, which using the event-based modelling strategy to actively dropped the disordered packet [33]. To guarantee optimal estimation performance, Chen *et al.* [12] studied the distributed fusion estimation to handle admissible parameter uncertainties. The information fusion estimation method based on bio-inspired computation was presented for the designing

dynamic 3D positioning system [34]. Since the distributed estimation is converted into a convex optimization problem, the estimated state has better robustness for the transmission performance.

For a class of stochastic uncertain networked systems, this paper intends to design the local filter with packet dropouts and disorders resulting from random transmission delays, as well as multiple noise resources. Due primarily to the difficulty in handling the disordered and missing data packets, dealing with stochastic and deterministic parameter uncertainties simultaneously. Motivated by the above analysis, the purpose of this paper concentrates on establishing a new system description, analyzing system performance and designing local/fusion estimator. The main contributions of this paper are summarized as follows:

(i) Suppose that the proposed system model contains energy-bounded noise and uncertain white noise. Due to the difficulty of estimating the appropriate filter parameters, the logic ZOH scheme is widely used in NCSs [8, 27, 28, 35, 36]. Based on the signal selection method for the logic ZOH, the event-triggered condition is introduced, and a system model is established to effectively deal with the network-induced complex and improve system performance. The proposed system model synthetically describes the network-induced phenomena and simplifies the characterization of the complex network environments.

(ii) For the energy-bounded noise and uncertain white noise, studying the optimization problem is difficult as the objective function is of significant nonlinear items [12, 17, 37]. The assumption on the parameter uncertainty can be satisfied by analyzing the mixed H_2/H_∞ estimation performance. Subsequently, different from using sequences of Bernoulli random variables distribution [20, 38–40], a simplified local estimation scheme in the augmented state-space with the re-organized measurements is presented, aiming to consume fewer network resources and provide the appropriate filter parameters.

(iii) To handle packet dropouts and missing data, instead of adopting the augmented state approach [20, 41] of compensating every step delay, a linear delay compensation for packet dropouts is firstly presented to suppress the growing error accumulation and alleviate the computational burden by re-ordering measurement sequences. Moreover, to further improve the estimation accuracy, the missing data packets due to the packet dropouts are collected using the one-step prediction method.

The rest of this paper is organized as follows: Establish the stochastic uncertain model using the logic ZOH scheme in Section 2. Section 3 shows a local estimator and analyses the estimation performance. Section 4 elaborates the distributed fusion estimation approach. A numerical example is presented in Section 5 and Section 6 draws the conclusions.

Notations: Throughout this paper \mathbb{R}^r and $\mathbb{R}^{r \times r}$ denote the r and $r \times r$ -dimensional Euclidean space, respectively. $E(\cdot)$ represents the mathematical expectation operator, and the superscript T is the transpose. M is a real symmetric matrix satisfying $M > 0$, while M^{-1} indicates that the inverse of M . Moreover, $\text{tr}(M)$ is the trace of M , and the symmetric terms in a symmetric matrix are denoted by $*$. I represents

the identity matrix with an appropriate dimensions. $l_2[0, \infty)$ represents the space of a square integral function on $[0, \infty)$, and $\delta_{k,l}$ denotes the Kronecker function (i.e. $\delta_{k,l} = 1$ if $k = l$, and $\delta_{k,l} = 0$ if $k \neq l$).

II. PROBLEM FORMULATION AND ANALYSIS

A. System Description

A considered object is measured by L sensors through the stochastic uncertain system (Fig.1 shows that the local estimation is solved by the spatially distributed sensors, subsequently, it is sent to the fusion center).

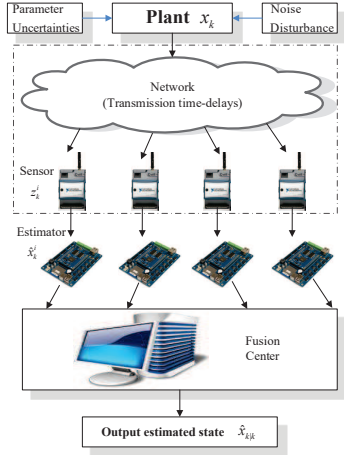


Fig. 1. Structure of distributed fusion estimation.

This process is described by the following linear discrete-time model:

$$x_{k+1} = \left(A_k + \mathcal{F}_k F_k E_k + \sum_{\vartheta=1}^h A_{\vartheta} \varpi_{\vartheta,k} \right) x_k + B_k w_k + G_k v_k, \quad k = 1, 2, \dots \quad (1)$$

$$z_k^i = \left(C_k^i + \mathcal{H}_k^i F_k E_k^i \right) x_k + B_k^i w_k + G_k^i v_k, \quad i = 1, \dots, L \quad (2)$$

where $x_k \in \mathbb{R}^r$ represents the system state, and $z_k^i \in \mathbb{R}^{m_i}$ refers to the measurement output from the i^{th} sensor at the time instant k . It is assumed that the initial state x_0 with mean μ_0 as well as covariance P_0 , which is uncorrelated with other noise. $A_k \in \mathbb{R}^{r \times r}$, $B_k \in \mathbb{R}^r$, $G_k \in \mathbb{R}^r$, $C_k^i \in \mathbb{R}^{m_i \times r}$, \mathcal{F}_k , E_k , \mathcal{H}_k^i , E_k^i , B_k^i and G_k^i are known time-varying matrices. Note that F_k represents the parameter uncertainty satisfying $F_k F_k^T \leq I$. $w_k \in l_2[0, \infty)$ denotes the energy-bounded noise. The multiplied term $\left(\sum_{\vartheta=1}^h A_{\vartheta} \varpi_{\vartheta,k} x_k \right)$ given in Eq.(1) presents the stochastic parameter uncertainty. $\varpi_{\vartheta,k} \in \mathbb{R}$ ($\vartheta = 1, \dots, h$) and v_k with covariance matrices $\theta_{\vartheta,k}$ and R_k denote the Gaussian white noise. Note that they are mutually uncorrelated and independent. In practice, it is difficult to obtain accurate covariance $\theta_{\vartheta,k}$. Therefore, the definition of the lower bound θ_{ϑ}^L and the upper bound θ_{ϑ}^U should be carefully determined, which satisfy the following constraint:

$$\theta_{\vartheta}^L \leq \theta_{\vartheta,k} \leq \theta_{\vartheta}^U, \quad (3)$$

where the lower and the upper bounds are known.

In this paper, for the given bounds θ_{ϑ}^L and θ_{ϑ}^U ($\vartheta = 1, \dots, h$), the following three conditions are simultaneously satisfied [12, 17]:

Condition 1. When $w_k \equiv 0$ and $v_k \equiv 0$, the error estimation system is asymptotically mean-square stable.

Condition 2. Satisfying the zero-initial condition and $v_k \equiv 0$, each local estimation error (e.g. e_k^i) with an arbitrary energy-bounded noise w_k conforms to the following inequality:

$$\sum_{k=0}^{\infty} E \left(\left(e_k^i \right)^T e_k^i \right) < \gamma_i^2 \sum_{k=0}^{\infty} E \left(w_k^T w_k \right), \quad (4)$$

where γ_i denotes the H_{∞} disturbance attenuation level bound (DALB) for the estimation error. Note that e_k^i ($i = 1, \dots, L$) represent the difference between the actual and estimated states, which is defined in Eqs.(17) and (18).

Condition 3. Each local estimation error e_k^i with v_k and $w_k \equiv 0$ (i.e. the Gaussian white noise and energy-bounded noise) guarantees that the upper bound of the H_2 performance

$$J^i = \lim_{k \rightarrow \infty} E \left(\left(e_k^i \right)^T e_k^i \right) \quad (5)$$

is minimal. Note that the relation between noise w_k and v_k are mutually uncorrelated and independent.

Remark 1. The aforementioned performance requirements for the local estimation of each subsystem are analyzed using the unified formulation (Conditions 1-3). Thus the filter parameters for the local estimation will be separately derived. Comparing with the distributed fusion estimation reported in [9, 11, 31], the proposed approach considers multiple noise (e.g. white noise and energy-bounded noise) simultaneously. In this sense, the noise assumption and modelling in Eqs.(1) and (2) indicates that the distributed fusion estimation approach can be implemented in practice, and will be made more generalised in the future development.

B. System Modelling for Signal Selection Scheme

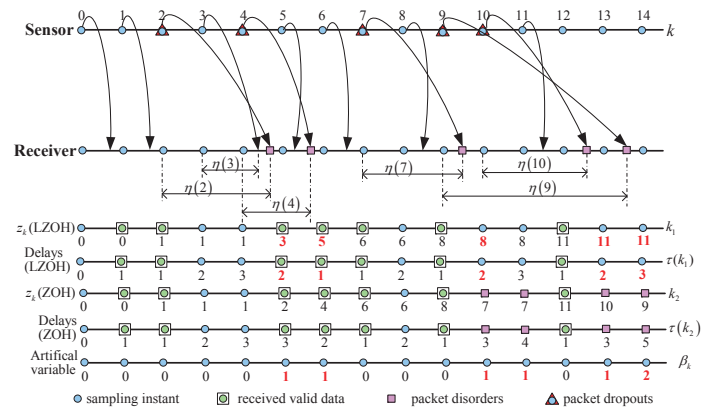


Fig. 2. Sorting procedure of the packet sequence for logic ZOH and ZOH.

Due to the resource constraints, the network congestion unavoidably interrupts the system evaluation. During the time-stamped data packets transmission, the signal selection scheme is introduced based on the logic ZOH, and the latest data packet is chosen before being transmitted. Therefore, for the processor, the stored signals on the receiver will be updated until it receives the latest data packet, while other data packets are discarded. Furthermore, the packet disorders during the transmission are dropped via the logic ZOH [27]. A typical scenario describing the network-induced complexity is shown

in Fig.2, and the detailed analysis is shown in Appendix A. Meanwhile, due to the limited service quality over the network, data packets are transmitted from the sensor to the estimator through a communication network in a single packet manner. Therefore, it is possible that an early leaving packet may arrive at the processor late, which leads to packet disorders. Since the disordered packets usually contain incorrect temporal signals, they should be discarded [35].

Fig.2 illustrates the relationship between the states and variables. From the previous discussion, to formulate the relationship of current time k and time-stamp, we define time-stamps k_1 and k_2 as those when we have the received data packets and the most recent transmission signal, which satisfies:

$$k = \tau(k_1) + k_1 = \tau(k_2) + k_2. \quad (6)$$

Remark 2. Suppose that the transmission delays are represented as $\eta(k_1)$ and $\eta(k_2)$ from the sensor to the processor with and without logic ZOH respectively, while $0 \leq \eta(k_1) \leq \eta(k_2) \leq N$ (N denotes the maximum of the transmission delay) is satisfied. Subsequently, the transmission delays for the received data packets are denoted as $\tau(k_1) \in \mathbb{N}$ and $\tau(k_2) \in \mathbb{N}$ at the sampling time instant k . For the given Eq.(6), the sampling time instant for the received data packets consists of two components: the time-stamp and the transmission delay.

Note that the received latest data packets approximate the current signal, and variable β_k is used for expressing the relationship between k_1 and k_2 , that is:

$$k_1 = k_2 + \beta_k. \quad (7)$$

In which β_k is determined according to the time-stamp before being transmitted. Since the time-stamp k_2 for receiving the most recent data packet is not proceeding the latest one k_1 , $\beta_k \geq 0$ is satisfied. In terms of the transmission delays obtained from Eqs.(6) and (7), $\tau(k_1) = \tau(k_2) - \beta_k$ is rewritten, and $\tau(k_1) \leq \tau(k_2)$ is satisfied.

According to the definition of symbol k_2 and the ZOH scheme, it should be pointed out that if no new signal input the ZOH, its output keeps as the constant. For instance, at a specific time instant k' (e.g. $k' = \{9, 12, 13\}$ in Fig.2) as well as time-stamp $t(k_2)$, if

$$t(k_2 + 1) = k' + 1 - \tau(k_2 + 1) < k' - \tau(k_2) = t(k_2), \quad (8)$$

the received signal at time instant $k' + 1$ is based on the previous signals with time-stamp $t(k_2)$, and more updated signals with time-stamp $t(k_2 + 1)$ are not used even if they are available by the ZOH signal selection scheme. Therefore, the most recent transmission signal is kept until the latest packet is used no matter whether it is a packet disorder or not [27].

Then, using the logic ZOH in order to detect the packet disorders, the random transmission delay satisfies:

$$\tau(k_1 + 1) \leq \tau(k_1) + 1 \quad (9)$$

and

$$t(k_1 + 1) = k' + 1 - \tau(k_1 + 1) \geq k' - \tau(k_1) = t(k_1). \quad (10)$$

This implies that the transmission signal at time $k' + 1$ is based on the outdated data packet at time-stamp $t(k_1)$, and a more updated data packet at the time-stamp $t(k_1 + 1)$ is used if it is available at the processor. The estimation accuracy of system is further improved, and the computational burden is

also reduced due to the data packets with less transmission delays [35].

As the logic ZOH is event-driven, a function $f(\cdot, \cdot)$ for the event generator is defined to determine the event-triggered condition as follows:

$$t(k_1 + 1) = t(k_1) + \min_{j>0} \{j | f(\sigma_k, \delta) > 0, j \in \mathbb{N}\}, \quad (11)$$

in which

$$f(\sigma_k, \delta) = \sigma_k^T \Omega \sigma_k - \delta \hat{x}_{k_1}^T \Omega \hat{x}_{k_1} \quad (12)$$

and

$$\sigma_k \triangleq \hat{x}_{k_1+j} - \hat{x}_{k_1}. \quad (13)$$

Note that \hat{x}_{k_1+j} represents the estimation at the latest event time $k_1 + j$, and \hat{x}_{k_1} is the current filter with the time-stamp k_1 . Furthermore, Ω is a weighted matrix, which is symmetric positive-definite, and $\delta \in [0, 1)$ is a scalar threshold.

Remark 3. In [42], the event-triggered condition is defined in Eq.(11), and the range of the threshold parameter δ is set to be $[0, 1)$ in this paper, which satisfies the following relationship:

$$f(\sigma_k, \delta) = \begin{bmatrix} \hat{x}_{k_1+j}^T & \hat{x}_{k_1}^T \end{bmatrix} \mathbb{Q} \begin{bmatrix} \hat{x}_{k_1+j} \\ \hat{x}_{k_1} \end{bmatrix} \quad (14)$$

where $\mathbb{Q} \triangleq \begin{bmatrix} \Omega & -\Omega \\ -\Omega & (1-\delta)\Omega \end{bmatrix}$. Note that $\delta \hat{x}_{k_1}^T \Omega \hat{x}_{k_1}$ is a positive scalar owing to $\Omega > 0$. On the one hand, if $\delta \geq 1$, \mathbb{Q} is non-positive definite. That is $f(\sigma_k, \delta) \leq 0$ for any \hat{x}_{k_1+j} and \hat{x}_{k_1} , the event-triggered condition given in Eq.(12) is never triggered. On the other hand, if $\delta < 0$, \mathbb{Q} is positive definite, namely, the event-triggered condition in Eq.(12) is always triggered. Consequently, the scalar threshold parameter satisfies $\delta \in [0, 1)$.

Next, the measurement re-organization approach is investigated to simplify the system description as well as alleviate the computational burden. For the i^{th} subsystem using the logic ZOH, when the processor receives the data packet z_t^i with time-stamp t and transmission delay $\tau^i(k_1)$, the stored signal y_k^i is re-organized as

$$y_k^i = z_t^i, \quad (15)$$

where $t = k - \tau^i(k_1)$ in Eq.(6). Note that the processor (i.e. estimator) receiving the time-stamped data packets may accompany the information with packet delays and dropouts at each sampling time [22]. It suggests that using the logic ZOH based on event-trigger, the signals with packet dropouts and packet disorders are translated into random transmission delays using the time-stamped data packets.

Remark 4. For the delay systems, the well-known innovation re-organization approach was proposed in [43, 44], which designed $l + 1$ different standard Kalman filtering with the l -step time delayed measure. To further reduce the computation cost, the proposed measurement re-organization approach given in Eq.(15) is used to handle the random transmission delays, which is translated into a unified form considering the current time instant as well as the time-stamp before being transmitted. Therefore, we design the optimal state estimation for each subsystem to develop a unified finite horizon filtering. More importantly, the network-induced events such as packet dropouts and packet disorders resulting from the random transmission delays are translated into the re-organized measurement sequence, and a signal selection method based

on event-trigger is applied to further simplifying the temporal process.

Taking into account the stored measurement sequence with network-induced complexity, the issue of distributed fusion estimation is transferred into probing the optimal state estimation $\hat{x}_{k|k}$, which is compensated and fused by local estimation $\hat{x}_{t|t}^i$ with transmission delay $t = k - \tau^i(k_1)$.

III. LOCAL ESTIMATION DESIGN

In this section, considering the system noise in Eq.(1) and the measurement noise in Eq.(2) are different, the augmentation method for the state-space model is involved [12]. Meanwhile, the local estimator $\hat{x}_{k|k}^i$ ($i = 1, \dots, L$) is derived by using the Schur complement lemma to analyze the estimation performance [17].

A. Augmentation Method for State Vector

Firstly, the received data packet y_k^i is obtained from Eqs.(2) and (15), that is

$$y_k^i = (C_t^i + \mathcal{H}_t^i F_t E_t^i) x_t + B_t^i w_t + G_t^i v_t, \quad i = 1, \dots, L. \quad (16)$$

It is assumed that the processor has a sufficient capability to find the optimal estimated state $\hat{x}_{t|t}^i$ from the stored valid signals $\{y_0^i, \dots, y_{k-1}^i, y_k^i\}$.

The objective of the distributed fusion estimation is achieving the optimal estimation $\hat{x}_{k|k}^i$ from the upper bound of the local error covariance matrix. Thus, the local state is estimated employing the re-organized data packets:

$$\hat{x}_{t|t}^i = \hat{x}_{t|t-1}^i + K_t^i (z_t^i - \hat{C}_t^i \hat{x}_{t|t-1}^i), \quad (17)$$

$$\hat{x}_{t+1|t}^i = \hat{A}_t^i \hat{x}_{t|t-1}^i + L_t^i (z_t^i - \hat{C}_t^i \hat{x}_{t|t-1}^i), \quad (18)$$

where $\hat{x}_{t|t}^i$ and $\hat{x}_{t+1|t}^i$ denote the filter and predictor, respectively. Then the local estimation errors are defined by $\tilde{e}_t^i \triangleq x_t - \hat{x}_{t|t-1}^i$ and $e_t^i \triangleq x_t - \hat{x}_{t|t}^i$. Time-varying matrices \hat{C}_t^i , K_t^i , \hat{A}_t^i and L_t^i are the solved filter parameters.

In order to derive the upper bounds for the covariance matrices of the estimation errors, the augmentation method for state vectors including error and estimated state are defined as:

$$\tilde{\Psi}_t^i \triangleq \begin{bmatrix} \tilde{e}_t^i \\ \hat{x}_{t|t-1}^i \end{bmatrix}, \quad \Psi_t^i \triangleq \begin{bmatrix} e_t^i \\ \hat{x}_{t|t}^i \end{bmatrix}. \quad (19)$$

Subsequently, the augmented system model for the state-space shown in Eqs.(1) and (17)-(19) is further discussed in Appendix B. Note that the deterministic uncertainty F_t , energy-bounded noise w_t as well as Gaussian white noise v_t and $\varpi_{\vartheta,t}$ appear in Eqs.(B.4) and (B.5). It is difficult to solve the accurate estimation using the covariance matrices $\tilde{\Theta}_t^i$ and $\tilde{\Sigma}_{t+1}^i$. Therefore, this paper investigates an alternative method to solve the upper bounds for the errors. Furthermore, the filter parameters are optimized by minimizing the error covariance.

B. Performance Analysis

To analyse the system performance, the estimation error from the local estimation $\hat{x}_{k|k}^i$ can be rewritten as:

$$\begin{aligned} \chi_{k+1}^i &= (A_{t3}^i + \Delta A_{t3}^i) \chi_k^i + \left(\sum_{\vartheta=1}^h (\varpi_{\vartheta,k} A_{\vartheta,t3}) \right) \chi_k^i \\ &\quad + B_{t3}^i w_k + G_{t3}^i v_k, \\ \tilde{e}_k^i &= D_{t3}^i \chi_k^i, \end{aligned} \quad (20)$$

where $\chi_k^i \triangleq \begin{bmatrix} x_k \\ \tilde{e}_k^i \end{bmatrix}$, and the parameters are defined as:

$$\begin{aligned} A_{t3}^i &= \begin{bmatrix} A_k & 0 \\ A_k - \hat{A}_k^i + L_k^i (\hat{C}_k^i - C_k^i) & \hat{A}_k^i - L_k^i \hat{C}_k^i \end{bmatrix}, \\ \Delta A_{t3}^i &= \begin{bmatrix} \mathcal{F}_k F_k E_k & 0 \\ (\mathcal{F}_k - L_k^i \mathcal{H}_k^i) F_k E_k & 0 \end{bmatrix}, \quad A_{\vartheta,t3} = \begin{bmatrix} A_{\vartheta} & 0 \\ A_{\vartheta} & 0 \end{bmatrix}, \\ B_{t3}^i &= \begin{bmatrix} B_k \\ B_k - L_k^i B_k^i \end{bmatrix}, \quad G_{t3}^i = \begin{bmatrix} G_k \\ G_k - L_k^i G_k^i \end{bmatrix}, \quad D_{t3}^i = \begin{bmatrix} 0 & I \end{bmatrix} \end{aligned} \quad (21)$$

First of all, the following bounded real lemmas for the discrete-time stochastic system χ_k^i must be satisfied.

Lemma 1. [12] *For the stochastic uncertain system, the discrete-time model is established as follows:*

$$\begin{aligned} x'_{k+1} &= (A' + \Delta A' v'_k) x'_k + B' w'_k, \\ z'_k &= C' x'_k, \end{aligned} \quad (22)$$

in which v'_k is the Gaussian white noise with covariance matrices R'_k , and $w'_k \in l_2[0, \infty)$ denotes an energy-bounded noise. For a given scalar γ' and the H_∞ performance, the following inequality

$$\sum_{k=0}^{\infty} E((z'_k)^T z'_k) < (\gamma')^2 \sum_{k=0}^{\infty} E((w'_k)^T w'_k) \quad (23)$$

is held. With the aid of the system performance in Eqs.(22) and (23), if and only if there exists a matrix $X' > 0$, such that the inequality

$$\begin{aligned} (A')^T X' A' + (C')^T C' + R'_k (\Delta A')^T X' (\Delta A') - X' \\ + (A')^T X' B' ((\gamma')^2 I - (B')^T X' B')^{-1} (B')^T X' A' < 0. \end{aligned} \quad (24)$$

is satisfied.

Lemma 2. [12] *Let $\Gamma_1^T = \Gamma_1$, Γ_2 and Γ_3 be real matrices with appropriate dimensions, and λ_k satisfying $\lambda_k^T \lambda_k \leq I$. Then*

$$\Gamma_1 + \Gamma_3 \lambda_k \Gamma_2 + \Gamma_2^T \lambda_k^T \Gamma_3^T < 0 \quad (25)$$

is held, if and only if there exists a positive scalar $\sigma > 0$, such that the following inequality is satisfied:

$$\begin{bmatrix} -\sigma I & \sigma \Gamma_2 & 0 \\ * & \Gamma_1 & \Gamma_3 \\ * & * & -\sigma I \end{bmatrix} < 0. \quad (26)$$

On the basis of Lemmas 1 and 2, the mixed H_2/H_∞ performance will be analysed in this section, while the local estimation $\hat{x}_{k|k}^i$ satisfies Conditions 1-3 presented above.

Theorem 1. *For a given level bound γ_i , the requirements of Conditions 1 and 2 are held. If and only if a matrix $X_i > 0$ exists, the inequality in Eq.(24) is obtained as*

$$\Delta_i = \begin{bmatrix} -X_i & X_i (A_{t3}^i + \Delta A_{t3}^i) & X_i B_{t3}^i \\ * & (D_{t3}^i)^T D_{t3}^i + \Upsilon_{t3} - X_i & 0 \\ * & * & -\gamma_i^2 I \end{bmatrix} < 0, \quad (27)$$

where $\Upsilon_{t3} \triangleq \sum_{\vartheta=1}^h \varpi_{\vartheta,k} A_{\vartheta,t3}^T X_i A_{\vartheta,t3}$, and parameters A_{t3}^i , ΔA_{t3}^i , $A_{\vartheta,t3}$, B_{t3}^i , G_{t3}^i as well as D_{t3}^i are obtained using

Eq.(21). In this case, the upper bound of H_2 performance can be calculated by

$$J^i \leq \text{Tr} \left\{ R_k \left(G_{t3}^i \right)^T X_i G_{t3}^i \right\}, \quad (28)$$

where the matrix X_i satisfies the inequality within Eq.(27), while G_{t3}^i is denoted in Eq.(21) and R_k is the covariance of v_k .

Proof. The proof is derived and shown in Appendix C. \square

Depending on the Gaussian white noise $\varpi_{\vartheta,k}$ in Eq.(1) as well as γ_i ($i = 1, \dots, L$) given in Condition 2, the optimal filter parameters satisfying Conditions 1-3 are able to be determined from the solution of the following optimization problem:

$$\min \text{Tr} \left(R_k \left(G_{t3}^i \right)^T X_i G_{t3}^i \right), \quad (29)$$

which is subject to the constraints of $F_k F_k^T \leq I$, Eqs.(3) and (27). However, solving the optimization problem from Eq.(29) is intractable, and the following three conditions should be analysed:

- (i) For the objective function shown in Eq.(29), the matrix term $\left(G_{t3}^i \right)^T X_i G_{t3}^i$ is nonlinear.
- (ii) The uncertain parameters F_k and $\theta_{\vartheta,k}$ are included in the matrix inequality of Eq.(27).
- (iii) $X_i A_{t3}^i$ and $X_i B_{t3}^i$ shown in Eq.(27) are nonlinear.

For the sake of obtaining an efficient solution of the optimization problem in Eq.(29), the convex optimization issue will be considered under a certain relaxation condition. The derivation is also shown in Appendix C.

IV. DISTRIBUTED FUSION ESTIMATION FOR LOGIC ZOH

For the stochastic uncertain system modelled in Eqs.(1) and (16), based on the robust finite horizon filtering, this paper studies a distributed fusion estimation approach (RFHDFE) by the aid of designing the local estimation $\hat{x}_{k|k}^i$. Then, a weighted robust fusion estimation is presented by investigating a convex optimization problem to improve the accuracy of the distributed estimation.

A. Upper Bound for Estimation Error

To design the local estimation, it is necessary to examine the appropriate filter parameters, which are derived from the error covariance matrix. Since the proposed system model contains uncertain parameters and energy-bounded noise, it is difficult to obtain the accurate estimation for state. Therefore, the guaranteed upper bounds based on the augmentation method are presented by minimizing the estimation error. Then, Lemmas 3 and 4 for solving inequality constraints are introduced by transforming the upper bound of the error covariance.

Lemma 3. [22] Suppose that matrices A , H , E , F and X have certain dimensions, and satisfying $FF^T \leq I$. If there exists an arbitrary positive constant $\alpha > 0$, and $\alpha^{-1}I - EXE^T > 0$ is satisfied, which is a symmetric positive-definite matrix. Thus, the following inequality is held:

$$\begin{aligned} & (A + HFE)X(A + HFE)^T \\ & \leq A(X^{-1} - \alpha E^T E)^{-1}A^T + \alpha^{-1}HH^T. \end{aligned} \quad (30)$$

Note that $(X^{-1} - \alpha E^T E)^{-1} = X + XE^T(\alpha^{-1}I - EXE^T)^{-1}EX$ is transformed from the matrix inversion lemma.

Lemma 4. [22] Assuming that $X > 0$ and $Y > 0$ are symmetric positive definite matrices. At the time instant k , the functions satisfy the conditions $s_t(X) = s_t^T(X) \in \mathbb{R}^{n \times n}$ and $h_t(X) = h_t^T(X) \in \mathbb{R}^{n \times n}$. If there exists $Y > X$, such that the functions meet $s_t(Y) \geq s_t(X)$ and $h_t(Y) \geq h_t(X)$, the solutions M_t and N_t are derived as follows:

$$M_{t+1} = s_t(M_t), \quad N_{t+1} = h_t(N_t), \quad M_0 = N_0 > 0 \quad (31)$$

satisfying $M_t \leq N_t$.

Theorem 2. The unified form of $(A + HFE)X(A + HFE)^T$ from $\tilde{\Theta}_t^i$ in Eq.(B.4) and $\tilde{\Sigma}_{t+1}^i$ in Eq.(B.5) based on Lemmas 3 and 4 satisfies the following case: if there exists a positive scalar $\alpha_t > 0$ and a symmetric positive-definite matrix Σ_t^i both of which satisfy the inequality $\alpha_t^{-1}I - E_{t2}^i \Sigma_t^i (E_{t2}^i)^T > 0$, the upper bounds of the error $\tilde{\Sigma}_t^i \leq \Sigma_t^i$ and $\tilde{\Theta}_t^i \leq \Theta_t^i$ will be available. Then, the upper bounds of Θ_t^i and Σ_{t+1}^i are computed by the following recursive equations:

$$\begin{aligned} \tilde{\Theta}_t^i & \leq A_{t1}^i \Sigma_t^i (A_{t1}^i)^T + \alpha_t^{-1} H_{t1}^i (H_{t1}^i)^T + B_{t1}^i E (w_t w_t^T) (B_{t1}^i)^T \\ & + A_{t1}^i \Sigma_t^i (E_{t1}^i)^T \left(\alpha_t^{-1} I - E_{t1}^i \Sigma_t^i (E_{t1}^i)^T \right)^{-1} E_{t1}^i \Sigma_t^i (A_{t1}^i)^T \\ & + G_{t1}^i R_t (G_{t1}^i)^T = \Theta_t^i, \end{aligned} \quad (32)$$

$$\begin{aligned} \text{and} \\ \tilde{\Sigma}_{t+1}^i & \leq A_{t2}^i \Sigma_t^i (A_{t2}^i)^T + \alpha_t^{-1} H_{t2}^i (H_{t2}^i)^T + \Sigma_t^i A_{\vartheta,t2} A_{\vartheta,t2}^T \theta_{\vartheta,t} \\ & + A_{t2}^i \Sigma_t^i (E_{t2}^i)^T \left(\alpha_t^{-1} I - E_{t2}^i \Sigma_t^i (E_{t2}^i)^T \right)^{-1} E_{t2}^i \Sigma_t^i (A_{t2}^i)^T \\ & + B_{t2}^i E (w_t w_t^T) (B_{t2}^i)^T + G_{t2}^i R_t (G_{t2}^i)^T = \Sigma_{t+1}^i. \end{aligned} \quad (33)$$

Proof. The derivation process is obtained from Lemmas 3 and 4. \square

Based on Theorem 2 and the finite horizon filtering, we define the error covariance matrix in the following form:

$$\Sigma_t^i = \begin{bmatrix} \bar{\Sigma}_t^i & 0 \\ 0 & P_t - \bar{\Sigma}_t^i \end{bmatrix}, \quad (34)$$

where $\bar{\Sigma}_t^i = E \left(\tilde{e}_t^i (\tilde{e}_t^i)^T \right)$ and $P_t = E \left(x_t x_t^T \right)$.

Then, the upper bounds of the error from the augmented state vectors given in Eqs.(B.4), (B.5), (32) and (33) are defined as follows:

$$E \left(e_t^i (e_t^i)^T \right) = [I \ 0] \tilde{\Theta}_t^i \begin{bmatrix} I \\ 0 \end{bmatrix} \leq [I \ 0] \Theta_t^i \begin{bmatrix} I \\ 0 \end{bmatrix} = \bar{\Theta}_t^i \quad (35)$$

and

$$\begin{aligned} E \left(\tilde{e}_{t+1}^i (\tilde{e}_{t+1}^i)^T \right) & = [I \ 0] \tilde{\Sigma}_{t+1}^i \begin{bmatrix} I \\ 0 \end{bmatrix} \\ & \leq [I \ 0] \Sigma_{t+1}^i \begin{bmatrix} I \\ 0 \end{bmatrix} = \bar{\Sigma}_{t+1}^i. \end{aligned} \quad (36)$$

Now, the H_∞ performance of Condition 2 under Lemma 1 and Eqs.(35)-(36) is derived as follows:

$$\sum_{k=0}^{\infty} E \left(\left(\tilde{e}_k^i \right)^T \tilde{e}_k^i \right) \leq \sum_{k=0}^{\infty} \bar{\Sigma}_k^i < \gamma_i^2 \sum_{k=0}^{\infty} E \left(w_k^T w_k \right). \quad (37)$$

Therefore, for a given H_∞ DALB γ_i , the inequality in Eq.(37) holds, if and only if a matrix $\tilde{\chi}_{k+1}^i$ for the estimation error using the augmented state vector $\tilde{\Psi}_k^i$ in Eq.(19) can be rewritten as:

$$\tilde{\chi}_{k+1}^i = \left(A_{t4}^i + \Delta A_{t4}^i \right) \tilde{\chi}_k^i + \left(\sum_{\vartheta=1}^h A_{\vartheta,t4} \varpi_{\vartheta,k} \right) \tilde{\chi}_k^i \quad (38)$$

$$+ B_{t4}^i w_k + G_{t4}^i v_k,$$

$$\tilde{e}_k^i = D_{t4}^i \tilde{\chi}_k^i,$$

where $\tilde{\chi}_k^i \triangleq \begin{bmatrix} x_k \\ \tilde{\Psi}_k^i \end{bmatrix}$, and the parameters are defined as:

$$\begin{aligned} A_{t4}^i &= \text{diag} \{ A_k, A_{t2}^i \}, \\ \Delta A_{t4}^i &= \text{diag} \{ \mathcal{F}_k F_k E_k, H_{t2}^i F_k E_{t2}^i \}, \\ A_{\vartheta,t4} &= \text{diag} \{ A_{\vartheta}, A_{\vartheta,t2} \}, \\ B_{t4}^i &= \text{col} \{ B_k, B_{t2}^i \}, G_{t4}^i = \text{col} \{ G_k, G_{t2}^i \}, \\ D_{t4}^i &= [0 \ I \ 0]. \end{aligned} \quad (39)$$

Note that $A_{t2}^i, H_{t2}^i, E_{t2}^i, A_{\vartheta,t2}, B_{t2}^i, G_{t2}^i$ are defined in Eq.(B.3). Then, based on Theorem 1 and Eq.(C.5), the inequality is satisfied

$$\begin{bmatrix} -\tilde{X}_i & \tilde{X}_i (A_{t4}^i + \Delta A_{t4}^i) & \tilde{X}_i B_{t4}^i \\ * & (D_{t4}^i)^T D_{t4}^i + \Upsilon_{t4} - \tilde{X}_i & 0 \\ * & * & -\gamma_i^2 I \end{bmatrix} < 0, \quad (40)$$

where $\Upsilon_{t4} \triangleq \sum_{\vartheta=1}^h \varpi_{\vartheta,k} A_{\vartheta,t4}^T \tilde{X}_i A_{\vartheta,t4}$.

On the other hand, the upper bound of the H_2 performance in Condition 3 is derived as follows:

$$J^i \leq \text{Tr} \left\{ R_k \left(G_{t4}^i \right)^T \tilde{X}_i G_{t4}^i \right\}. \quad (41)$$

The proof is similar to that of Theorem 1 and the inequality derivation of Eq.(C.7) from Eq.(C.5).

Next, in order to obtain $\bar{\Sigma}_t^i$ and P_t in Eq.(34), Theorem 3 is presented depending on the optimal filter and the predictor, as well as the filter parameters for the i^{th} local estimation shown in Eqs.(17) and (18).

Theorem 3. For the i^{th} subsystem, the time-stamped measurement z_k^i is transmitted using the logic ZOH. Meanwhile, the received data packet is denoted as y_k^i with transmission delay $\tau^i(k_1)$ at the time instant k . Set $t = k - \tau^i(k_1)$, and $\alpha_t > 0$ be a positive scalar. Therefore, the matrices $\bar{\Sigma}_t^i$ and P_t have the positive definite solutions, which are defined as follows:

$$\bar{\Theta}_t^i = \bar{\Sigma}_t^i \left(I + \left(E_t^i \right)^T \left(\tilde{M}_t^i \right)^{-1} E_t^i \bar{\Sigma}_t^i \right) - \Lambda_t^i \left(\Xi_t^i \right)^{-1} \left(\Lambda_t^i \right)^T, \quad (42)$$

$$\bar{\Sigma}_{t+1}^i = \tilde{\Delta}_t^i - L_t^i \tilde{\nabla}_t^i, \quad (43)$$

$$\begin{aligned} P_{t+1} &= A_t \left(P_t^{-1} - \alpha_t E_t^T E_t \right)^{-1} A_t^T + \alpha_t^{-1} \mathcal{F}_t \mathcal{F}_t^T \\ &+ \theta_{\vartheta,k} \sum_{\vartheta=1}^h A_{\vartheta} P_t A_{\vartheta}^T + B_t E \left(w_t w_t^T \right) B_t^T + G_t R_t G_t^T. \end{aligned} \quad (44)$$

In which

$$\begin{aligned} \Lambda_t^i &= \left(I + \bar{\Sigma}_t^i \left(E_t^i \right)^T \left(\tilde{M}_t^i \right)^{-1} E_t^i \right) \bar{\Sigma}_t^i \left(C_t^i \right)^T, \\ \Xi_t^i &= C_t^i \bar{\Sigma}_t^i \left(I + \left(E_t^i \right)^T \left(\tilde{M}_t^i \right)^{-1} E_t^i \bar{\Sigma}_t^i \right) \left(C_t^i \right)^T \\ &+ \alpha_t^{-1} \mathcal{H}_t^i \left(\mathcal{H}_t^i \right)^T + B_t^i E \left(w_t w_t^T \right) \left(B_t^i \right)^T + G_t^i R_t \left(G_t^i \right)^T, \end{aligned}$$

$$\begin{aligned} \Delta_t^i &= A_t \left(I + \bar{\Sigma}_t^i \left(E_t^i \right)^T \left(\tilde{M}_t^i \right)^{-1} E_t^i \right) \bar{\Sigma}_t^i \left(C_t^i \right)^T \\ &+ \alpha_t^{-1} \mathcal{F}_t \left(\mathcal{H}_t^i \right)^T + B_t E \left(w_t w_t^T \right) \left(B_t^i \right)^T + G_t R_t \left(G_t^i \right)^T, \\ \nabla_t^i &= C_t^i \left(I + \bar{\Sigma}_t^i \left(E_t^i \right)^T \left(\tilde{M}_t^i \right)^{-1} E_t^i \right) \bar{\Sigma}_t^i \left(C_t^i \right)^T \\ &+ \alpha_t^{-1} \mathcal{H}_t^i \left(\mathcal{H}_t^i \right)^T + B_t^i E \left(w_t w_t^T \right) \left(B_t^i \right)^T + G_t^i R_t \left(G_t^i \right)^T, \\ \tilde{\Delta}_t^i &= A_t \left(I + \bar{\Sigma}_t^i \left(E_t^i \right)^T \left(\tilde{M}_t^i \right)^{-1} E_t^i \right) \bar{\Sigma}_t^i A_t^T + \alpha_t^{-1} \mathcal{F}_t \mathcal{F}_t^T \\ &+ B_t E \left(w_t w_t^T \right) B_t^T + G_t R_t G_t^T + \sum_{\vartheta=1}^h A_{\vartheta} P_t \left(A_{\vartheta}^T \right) \theta_{\vartheta,t}, \end{aligned}$$

and

$$\begin{aligned} \tilde{\nabla}_t^i &= C_t^i \left(I + \bar{\Sigma}_t^i \left(E_t^i \right)^T \left(\tilde{M}_t^i \right)^{-1} E_t^i \right) \bar{\Sigma}_t^i A_t^T \\ &+ \alpha_t^{-1} \mathcal{H}_t^i \mathcal{F}_t^T + B_t^i E \left(w_t w_t^T \right) B_t^T + G_t^i R_t G_t^T. \end{aligned}$$

Meanwhile, it satisfies $P_t^{-1} - \alpha_t \left(E_t^i \right)^T E_t^i > 0$, $\tilde{M}_t^i = \alpha_t^{-1} I - E_t^i P_t \left(E_t^i \right)^T$ and $\tilde{M}_t^i = \alpha_t^{-1} I - E_t^i \bar{\Sigma}_t^i \left(E_t^i \right)^T > 0$, respectively.

Furthermore, the re-organized local estimation given in Eqs.(17)-(18) is facilitated by using the following filter parameters:

$$\hat{C}_t^i = C_t^i \left(I + \bar{\Sigma}_t^i \left(E_t^i \right)^T \left(\tilde{M}_t^i \right)^{-1} E_t^i \right), \quad (45)$$

$$K_t^i = \Lambda_t^i \left(\Xi_t^i \right)^{-1}, \quad (46)$$

$$\hat{A}_t^i = A_t \left(I + \bar{\Sigma}_t^i \left(E_t^i \right)^T \left(\tilde{M}_t^i \right)^{-1} E_t^i \right), \quad (47)$$

$$L_t^i = \Delta_t^i \left(\nabla_t^i \right)^{-1}. \quad (48)$$

Proof. This theorem comes from the upper bounds for minimizing the estimation error in the covariance matrices. Appendix D presents the procedure of proof. \square

B. Linear Compensation for Packet Dropouts

To solve the local state estimation $\hat{x}_{k|k}^i$, this section applies the optimal estimation $\hat{x}_{t|t}^i$ to deriving the state $\hat{x}_{k|t}^i$ at the current time instant k , which is obtained from the maximum of the transmission delay N as well as the current received valid signal. So the predictor $\hat{x}_{t+1|t}^i$ is introduced to solve the linear compensation, that is:

$$\hat{x}_{k|t}^i = \left(1 - \frac{\tau^i(k) - 1}{N} \right) \hat{x}_{t+1|t}^i, \quad (49)$$

where $\tau^i(k)$ denotes the transmission delay of the received valid signal. The acknowledgment (ACK) time-stamped signal z_k^i is assigned the highest transmission priority due to the timing, whilst the transmission delay is negligible to obtain the estimated state $\hat{x}_{k|t}^i$ [45]. Thus, the filter $\hat{x}_{t|t}^i$ and predictor $\hat{x}_{t+1|t}^i$ are used for compensating the estimated state $\hat{x}_{k|t}^i$ given in Eq.(49).

Remark 5. Note that it is an approximate estimation using the linear compensation method for the estimated state $\hat{x}_{k|t}^i$ with delay-free in Eq.(49). The filter $\hat{x}_{t|t}^i$ derived from the latest data packet with time-stamp is used to estimate the current state $\hat{x}_{k|k}^i$ from the compensated value $\hat{x}_{k|t}^i$. To deal with the packet dropouts, the data transmission delays for each step will be ignored until the next received data packet.

Due to the growing error accumulation and computational burden using the traditional one-step predication, we propose the linear compensation for packet dropouts, which has the advantage of simplifying the state estimation and highlighting the valid transmission signals. Therefore, the proposed method is able to reduce the computational complexity, and then alleviate the negative effect owing to the random transmission delays. More importantly, define $s = k + 1 - \tau^i(k + 1)$ with transmission delay $\tau^i(k + 1)$ at the next sampling time $k + 1$, when $s > t + 1$, the missing data packets are produced due to the signal selection scheme. Finally, the estimated state $\hat{x}_{s|s}^i$ is calculated by using the one-step prediction, which meets the artificial delay $1 < \tau^{st}(k) = s - t \leq N$. For the missing data packets, the re-organized state estimation sequence will be compensated, and the estimated values are listed as follows:

$$\left\{ \hat{x}_{t+1|t+1}^i, \dots, \hat{x}_{t+\tau^{st}(k)|t+\tau^{st}(k)}^i \right\}. \quad (50)$$

Therefore, the objective of the linear compensation with the random transmission delays, as well as the one-step prediction for the missing packets, is designed the fusion estimation of each local subsystem subsequently.

C. Distributed Fusion Estimation Based on Local Attributes

Theorem 3 presents a local state estimation, meanwhile, the current state is estimated by using the proposed linear compensation for packet dropouts. In addition, to improve the state estimation accuracy from each subsystem, this section investigates the distributed fusion estimation method with the aid of the weighted fusion criterion.

Lemma 5. For the linear discrete-time systems described with Eqs.(1) and (2) at time instant k , based on the established signal selection model and linear compensation for packet dropouts, the upper bounds for cross-covariance matrix $\bar{\Theta}_t^{i,j}$ of the filtering error and cross-covariance matrix $\bar{\Sigma}_{t+1}^{i,j}$ for the prediction error between the i^{th} and j^{th} subsystems have the following recursive expression:

$$\begin{aligned} \bar{\Theta}_t^{i,j} &= \left(I - K_t^i C_t^i \right) \bar{\Sigma}_t^{i,j} \left(I - K_t^j C_t^j \right)^T \\ &+ K_t^i \left(\hat{C}_t^i - C_t^i \right) \left(P_t - \bar{\Sigma}_t^{i,j} \right) \left(K_t^j \left(\hat{C}_t^j - C_t^j \right) \right)^T \\ &+ \alpha_t^{-1} K_t^i \mathcal{H}_t^i \left(K_t^j \mathcal{H}_t^j \right)^T + K_t^i G_t^i R_t \left(K_t^j G_t^j \right)^T \\ &+ K_t^i B_t^i E \left(w_t w_t^T \right) \left(K_t^j B_t^j \right)^T \\ &+ \left(\bar{\Sigma}_t^{i,j} + K_t^i \left(\hat{C}_t^i \left(P_t - \bar{\Sigma}_t^{i,j} \right) - C_t^i P_t \right) \right) \\ &\times \left(E_t^j \right)^T \left(\tilde{M}_t^{i,j} \right)^{-1} E_t^j \\ &\times \left(\bar{\Sigma}_t^{i,j} + K_t^j \left(\hat{C}_t^j \left(P_t - \bar{\Sigma}_t^{i,j} \right) - C_t^j P_t \right) \right)^T, \end{aligned} \quad (51)$$

and

$$\begin{aligned} \bar{\Sigma}_{t+1}^{i,j} &= \left(A_t - L_t^i C_t^i \right) \bar{\Sigma}_t^{i,j} \left(A_t - L_t^j C_t^j \right)^T \\ &+ \left(A_t - \hat{A}_t^i + L_t^i \left(\hat{C}_t^i - C_t^i \right) \right) \left(P_t - \bar{\Sigma}_t^{i,j} \right) \\ &\times \left(A_t - \hat{A}_t^j + L_t^j \left(\hat{C}_t^j - C_t^j \right) \right)^T \\ &+ \alpha_t^{-1} \left(\mathcal{F}_t - L_t^i \mathcal{H}_t^i \right) \left(\mathcal{F}_t - L_t^j \mathcal{H}_t^j \right)^T \\ &+ \sum_{\vartheta=1}^{\bar{h}} A_{\vartheta} P_t \left(A_{\vartheta}^T \right) \theta_{\vartheta,t} \end{aligned}$$

$$\begin{aligned} &+ \left(B_t - L_t^i B_t^i \right) E \left(w_t w_t^T \right) \left(B_t - L_t^j B_t^j \right)^T \\ &+ \left(G_t - L_t^i G_t^i \right) R_t \left(G_t - L_t^j G_t^j \right)^T \\ &+ \left(\left(A_t - L_t^i C_t^i \right) \bar{\Sigma}_t^{i,j} \right. \\ &\quad \left. + \left(A_t - \hat{A}_t^i + L_t^i \left(\hat{C}_t^i - C_t^i \right) \right) \left(P_t - \bar{\Sigma}_t^{i,j} \right) \right) \\ &\times \left(E_t^j \right)^T \left(\tilde{M}_t^{i,j} \right)^{-1} E_t^j \\ &\times \left(\left(A_t - L_t^i C_t^i \right) \bar{\Sigma}_t^{i,j} \right. \\ &\quad \left. + \left(A_t - \hat{A}_t^j + L_t^j \left(\hat{C}_t^j - C_t^j \right) \right) \left(P_t - \bar{\Sigma}_t^{i,j} \right) \right)^T \end{aligned} \quad (52)$$

where $\tilde{M}_t^{i,j} = \alpha_t^{-1} I - E_t^i P_t \left(E_t^j \right)^T$ and $M_t^{i,j} = \alpha_t^{-1} I - E_t^i \bar{\Sigma}_t^{i,j} \left(E_t^j \right)^T$.

To obtain the distributed fusion estimation $\hat{x}_{k|k}$, the considered system develops the linear minimum covariance, as well as the optimal weighted fusion from the local estimation. Therefore, the designed estimator from Eq.(1) is described as the following convex optimization form:

$$\hat{x}_{k|k} = \sum_{i=1}^L \Omega_k^i \hat{x}_{k|k}^i. \quad (53)$$

To have the optimal value for the distributed fusion estimation [11, 41, 46], if and only if the weighted matrix in Eq.(53) is composed by

$$\Omega_k = \left[\Omega_k^1, \dots, \Omega_k^L \right] = \left(I_0^T \Pi_k^{-1} I_0 \right)^{-1} I_0^T \Pi_k^{-1}, \quad (54)$$

where $I_0 = \underbrace{\left[I_r, \dots, I_r \right]^T}_L$ is an $rL \times r$ -dimensional matrix,

and $\sum_{i=1}^L \Omega_k^i = I_r$. Moreover, the covariance matrix based on

the linear compensation is defined as $\Pi_k^{i,j} \triangleq E \left(e_k^i \left(e_k^j \right)^T \right)$, and

$$\Pi_k = \begin{bmatrix} \Pi_k^{1,1} & \dots & \Pi_k^{1,L} \\ & \ddots & \\ \Pi_k^{L,1} & \dots & \Pi_k^{L,L} \end{bmatrix} = \left(\Pi_k^{i,j} \right)_{rL \times rL} \quad (55)$$

is a symmetrical positive-definite matrix. Set $\tilde{\Pi}_k = \left(\Pi_k^{-1/2} I_0 \right)^T \left(\Pi_k^{1/2} I_0^i \right)$ and $I_0^i = \underbrace{[0, \dots, I_r, \dots, 0]}_L$ be the

i^{th} element in I_0 . It is worth noting that on the basis of the cross-covariance of $\tilde{\Pi}_{k|k} \triangleq E \left((x_k - \hat{x}_{k|k}) (x_k - \hat{x}_{k|k})^T \right)$, the corresponding optimal fusion estimation is designed by $\tilde{\Pi}_{k|k} = \left(I_0^T \Pi_k^{-1} I_0 \right)^{-1}$, which satisfies $\tilde{\Pi}_{k|k} \leq \Pi_k^i \leq \bar{\Theta}_k^i$ under Eq.(35), if $i = j$; otherwise, we have $\tilde{\Pi}_{k|k} = \Pi_k^i \leq \bar{\Theta}_k^i$ by setting $\Omega_k^i = I_r$ and $\Omega_k^j = 0$ ($j = 1, \dots, L, j \neq i$).

Remark 6. The stochastic uncertain system presents the signal selection scheme to handle network-induced complexity simultaneously. Meanwhile, since the received data packets include the latest signals, the packet disorders are actively avoided. That is, a linear compensation method for the packet dropouts is obtained from each local estimation $\hat{x}_{k|k}^i$. Furthermore, to achieve information exchange between any two subsystems, we propose the convex optimization problem based on the weighted fusion criterion, and the optimal estimated state is available by using the linear minimum covariance matrix.

Thus, the fusion estimation possesses higher estimation accuracy than each local subsystem.

Summarizing the throughout calculation process of the distributed fusion estimation, the solution approach is illustrated in Fig.3.

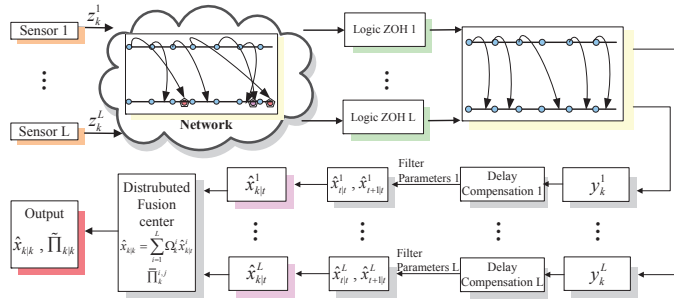


Fig. 3. Computational procedure of RFHDFE.

D. Stability Analysis

To analysis the steady-state local estimators, the definition and formalism of Lemma 6 is shown.

Lemma 6. [40, 41, 47] For the re-organized system shown in Eqs.(1) and (16), the H_2 performance is represented in Eq.(41), and $\tilde{X}_i \leq X_i$ is satisfied under Eq.(40). Given the covariance matrix $E(\tilde{e}_{k+1}^i(\tilde{e}_{k+1}^i)^T)$ by Eq.(36) with any initial value $E(\tilde{e}_0^i(\tilde{e}_0^i)^T) \geq 0$ due to the external disturbance, the error covariance will converge to the positive semi-definite solution $\tilde{\Sigma}^i$. According to the upper bound for the covariance matrix of the estimation error, the inequality $E(\tilde{e}_{k+1}^i(\tilde{e}_{k+1}^i)^T) \leq \tilde{\Sigma}_{k+1}^i$ is satisfied due to Eq.(43), as well as $Tr(E(\tilde{e}_{k+1}^i(\tilde{e}_{k+1}^i)^T)) \leq Tr(\tilde{\Sigma}_{k+1}^i)$, which is rewritten as follows:

$$\begin{aligned} \tilde{\Sigma}_{k+1}^i &= (A_k - L_k^i C_k^i) \left(I + \tilde{\Sigma}_k^i (E_k^i)^T (M_k^i)^{-1} E_k^i \right) \tilde{\Sigma}_k^i A_k^T \\ &+ \alpha_k^{-1} (\mathcal{F}_k - L_k^i \mathcal{H}_k^i) \mathcal{F}_k^T \\ &+ (B_k - L_k^i B_k^i) E(w_k w_k^T) B_k^T \\ &+ (G_k - L_k^i G_k^i) R_k G_k^T + \sum_{\vartheta=1}^h A_{\vartheta} P_k (A_{\vartheta}^T) \theta_{\vartheta,k}. \end{aligned} \quad (56)$$

Let the set filter parameters be $\tilde{L}^i = \lim_{k \rightarrow \infty} L_k^i$ given in Eq.(48), $\tilde{\Delta}^i = \lim_{k \rightarrow \infty} \tilde{\Delta}_k^i$ and $\tilde{\nabla}^i = \lim_{k \rightarrow \infty} \tilde{\nabla}_k^i$ in Eq.(D.9).

Therefore, $\tilde{\Sigma}^i = \lim_{k \rightarrow \infty} E(\tilde{e}_{k+1}^i(\tilde{e}_{k+1}^i)^T)$ is determined by $\tilde{\Sigma}_k^i$. We know that matrix A_k is stable, and $\rho(A_k)$ denotes the spectrum radius of matrix A_k . In this situation, the estimated state $\hat{x}_{k+1|k}$ in Eq.(18) is asymptotically stable.

Note that the stability of matrix A_k for the steady-state filter is necessary. In addition, the error covariance matrix $E(\tilde{e}_{k+1}^i(\tilde{e}_{k+1}^i)^T)$ is influenced by energy-bounded noise and uncertain white noise. If matrix A_k is unstable, the error covariance matrices of the estimation errors will be unbounded.

Meanwhile, the covariance matrices of the filtering errors are solved by $E(e_k^i(e_k^i)^T) \leq \bar{\Theta}_k^i$ shown in Eq.(35), and $\bar{\Theta}_k^i$ is determined by $\tilde{\Sigma}_k^i$ according to Eq.(42). Therefore, the steady-state cross-covariance matrices for the filtering errors are converging.

Finally, the proposed distributed fusion estimation based on local estimation is the steady-state filter under Eqs.(53) and (54). Note that $\hat{x}_k^i|_k = [0 \ I_r]^T \Psi_k^i$ in Eq.(19) is the local steady-state filter for the state x_k , and Ω_k is the steady-state weighted matrix. Moreover, Π_k in Eq.(55) is the steady-state covariance matrix for the filtering error. Therefore, the variance matrix $(I_0^T \Pi_k^{-1} I_0)^{-1}$ is the distributed fusion estimator.

Remark 7. The proposed distributed fusion estimation (i.e. RFHDFE) approach is not required to be calculated at each step, so that it is easy to be implemented in engineering application. In addition, the proposed RFHDFE alleviates the computational burden. Since it has the computational cost with the order of magnitude $O\left(\sum_{i=1}^L \left(\frac{\tau^i(k)}{N} (m_i + r)\right)^2\right)$, where r expresses the dimension of the state, m_i represents the measurement dimension and L denotes the number of the subsystems. Therefore, this paper investigates the distributed fusion estimation based on the local estimation possesses less computation complexity and reduces the energy consumption.

V. NUMERICAL SIMULATION

We use simulations to illustrate the effectiveness of the proposed RFHDFE approach. Considering that a moving target tracking system is measured by three sensors, which is described by the following state-space and measurement models, respectively, and the related systems are modeled in [11, 12, 22]:

$$x_{k+1} = (A_k + \mathcal{F}_k F_k E_k + A_1 \varpi_{1,k}) x_k + B_k w_k + G_k v_k, \quad k = 1, 2, \dots \quad (57)$$

$$z_k^i = (C_k^i + \mathcal{H}_k^i F_k E_k^i) x_k + B_k^i w_k + G_k^i v_k, \quad i = 1, 2, 3 \quad (58)$$

where the system parameters are set as follows:

$$A_k = \begin{bmatrix} 0.9 & T & T^2/2 \\ 0 & 0.9 & T \\ 0 & 0 & 0.9 \end{bmatrix}, \quad A_1 = \begin{bmatrix} 0.02 & 0.03 & 0.01 \\ 0.06 & 0.05 & 0.02 \\ 0.05 & 0.03 & 0.01 \end{bmatrix},$$

$$B_k = \begin{bmatrix} 0.1 \\ 0.3 \\ 0.2 \end{bmatrix}, \quad B_k^1 = \begin{bmatrix} 0.1 \\ 0.3 \\ 0.2 \end{bmatrix}, \quad B_k^2 = \begin{bmatrix} 0.2 \\ 0.4 \\ 0.5 \end{bmatrix}, \quad B_k^3 = \begin{bmatrix} 0.4 \\ 0.5 \\ 0.2 \end{bmatrix}$$

$$G_k = \text{col}\{T^2/2, T, 1\}, \quad G_k^i = B_k^i (i = 1, 2, 3),$$

$$\mathcal{F}_k = [0.1 \ 0.1 \ 0.1]^T, \quad E_k = [0.02 \ 0.02 \ 0.02],$$

$$C_k^1 = [0.6 \ 0.8 \ 1], \quad C_k^2 = [1 \ 0.8 \ 0.5],$$

$$C_k^3 = [0.3 \ 1 \ 0.7], \quad \mathcal{H}_k^1 = \mathcal{H}_k^2 = \mathcal{H}_k^3 = 0.8.$$

Symbol T denotes the sample period and is set to be 0.1s. The defined state $x_k \triangleq (s_k \ \dot{s}_k \ \ddot{s}_k)^T$ includes the position, velocity and acceleration, respectively, of the moving target at the time instant kT , and the maximum of the transmission delay is set $N = 5$. The parameter uncertainty $F_k = \sin(0.6k)$ is a time-varying matrix. v_k and $\varpi_{1,k}$ are the Gaussian white noise with covariance $R_k = 0.09$ and $\theta_1^L \leq \theta_{1,k} \leq \theta_1^U$, where the covariance satisfies $\theta_1^L = 0.01$ and $\theta_1^U = 0.02$. The DALB $\gamma_1 = \gamma_2 = \gamma_3 = 0.2$, and the energy bounded noise w_k is given by $w_k = 2 \cos(0.6k)$.

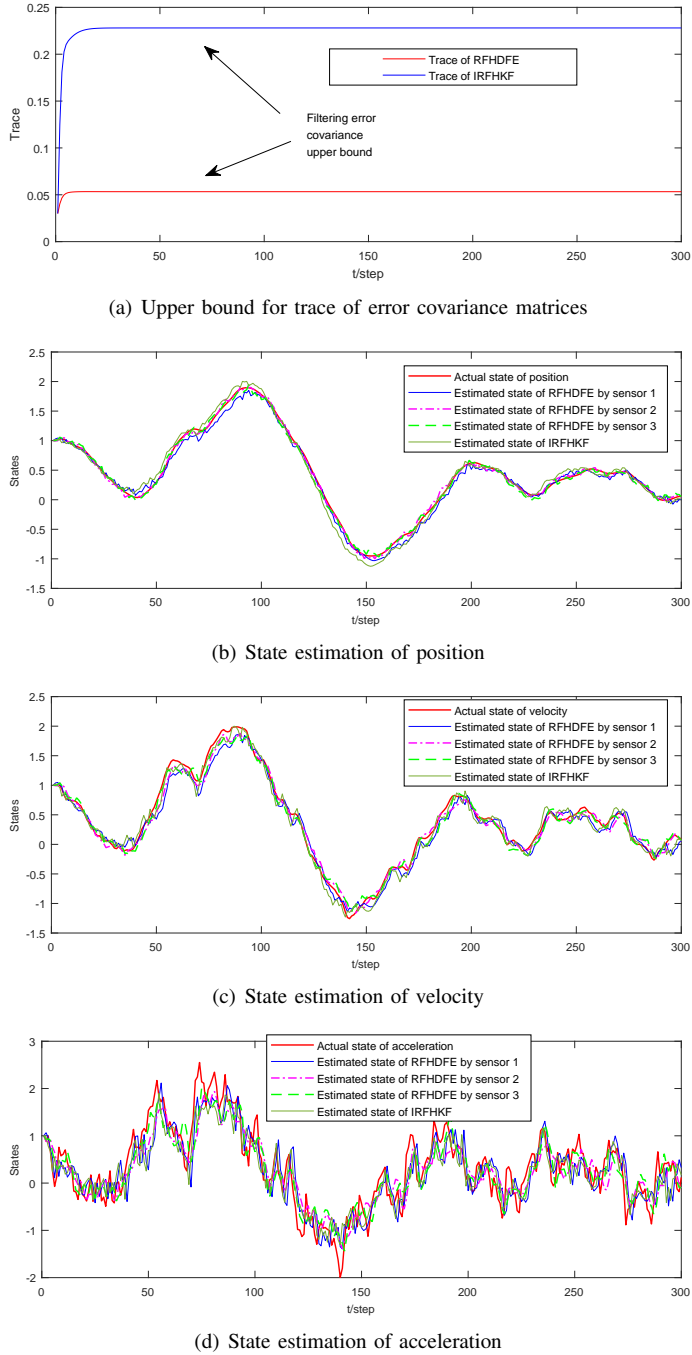


Fig.4. Comparison of covariance matrices between RFHDFE and IRFHKF.

Without losing generality, to solve Theorem 3 with appropriate filter parameters, the initial values are set as $\hat{x}_{0|0} = \mu_0 = E(x_0) = [1 \ 1 \ 1]^T$, $P_{0|0} = 0.01I_3$ as well as $\alpha_k = 3$. Fig.4 demonstrates the traces for the covariance matrices of the estimation errors. The estimated state $\hat{x}_{k|k}$ using the proposed RFHDFE with delay-free is compared against the improved robust finite-horizon Kalman filtering (IRFHKF) approach [22]. From Fig.4 (a), the proposed RFHDFE approach has a less upper bound than IRFHKF for the covariance matrices of the estimation errors. Note that due to the weighted fusion criterion, the fused upper bound for the cross-covariance matrix of the error is much smaller than that

of the other covariance matrices, and the fusion estimation criterion is optimal. Comparing the estimation accuracy of the both methods shown in Figs.4 (b)-(d), it implies that the proposed RFHDFE method results in an appropriate estimator, which is suitable for deriving the optimal filter parameters and obtaining the precise estimation. Taking the energy bound noise and unknown state-dependent noise into account, the dynamic tracking trajectory is very close to the actual one.

The comparison of the mean square error values (MSEs) [11, 41] are provided by Monte-Carlo shown in Table 1, to indicate the accuracy improvement of the proposed RFHDFE approach with network-induced complexity and multiple noise in different noise environments. Note that the comparison of MSEs includes the states of position, velocity and acceleration, as well as their estimated values and runtime for each method, respectively. It shows that the MSEs of the fusion estimation are smaller than each subsystem, and also smaller than the IRFHKF method. It means that the RFHDFE approach has the capacity for obtaining better estimation performance than each one and the IRFHKF method in the following aspects: smaller MSEs and less time consuming.

Table 1 Comparison of mean square error values

Scheme	Subsystem	MSEs			Time (s)
		Position	Velocity	Acceleration	
RFHDFE	Sensor 1	0.0041	0.0073	0.0335	0.0624
	Sensor 2	0.0025	0.0066	0.0289	0.1560
	Sensor 3	0.0018	0.0047	0.0245	0.0936
	Fusion	0.0004	0.0006	0.0005	0.3432
IRFHKF		0.0124	0.0148	0.0322	1.8564

To illustrate the effectiveness of the system, the corresponding tracking results of the estimation for the moving target are shown in Fig.5. The simulation results come from the proposed (RFHDFE) approach with packet disorders and the IRFHKF method, which are obtained from Eqs.(57) and (58). More importantly, the actual and the estimated states with or without packet disorders are shown in Figs.5 (a)-(c), and it is a further verification that using the logic ZOH is capable of identifying and discarding packet disorders induced from random transmission delays. Meanwhile, Fig.5 (d) verifies the system stability by showing the traces for the covariance matrices of the estimation errors depending on each filter and the fusion estimation using the proposed RFHDFE method and the IRFHKF method.

The range of the performance indicator is presented in Table E.1 of Appendix E, which means that for the proposed approach, the upper bounds for the covariance matrices of the estimation errors are less than those of the IRFHKF method. Note that the estimation performance is improved, and the estimation accuracy is higher if the upper bound for error covariance is close to the lower bound.

The distributed estimation results for the states are shown in Fig. E.1. In the simulations, the re-ordering error covariance criteria are used to compare the dynamic tracking performance between the RFHDFE and IRFHKF methods. It suggests that the proposed RFHDFE approach has better estimation accuracy than the IRFHKF method with the random transmission delays, packet dropouts and disorders simultaneously. Since the energy bound noise, unknown state-dependent noise, and the actual estimation errors are lower than the upper bound, for

the actual estimated values, the proposed RFHDFE method has better performance including accurate system states and rapid convergence to the steady-state.

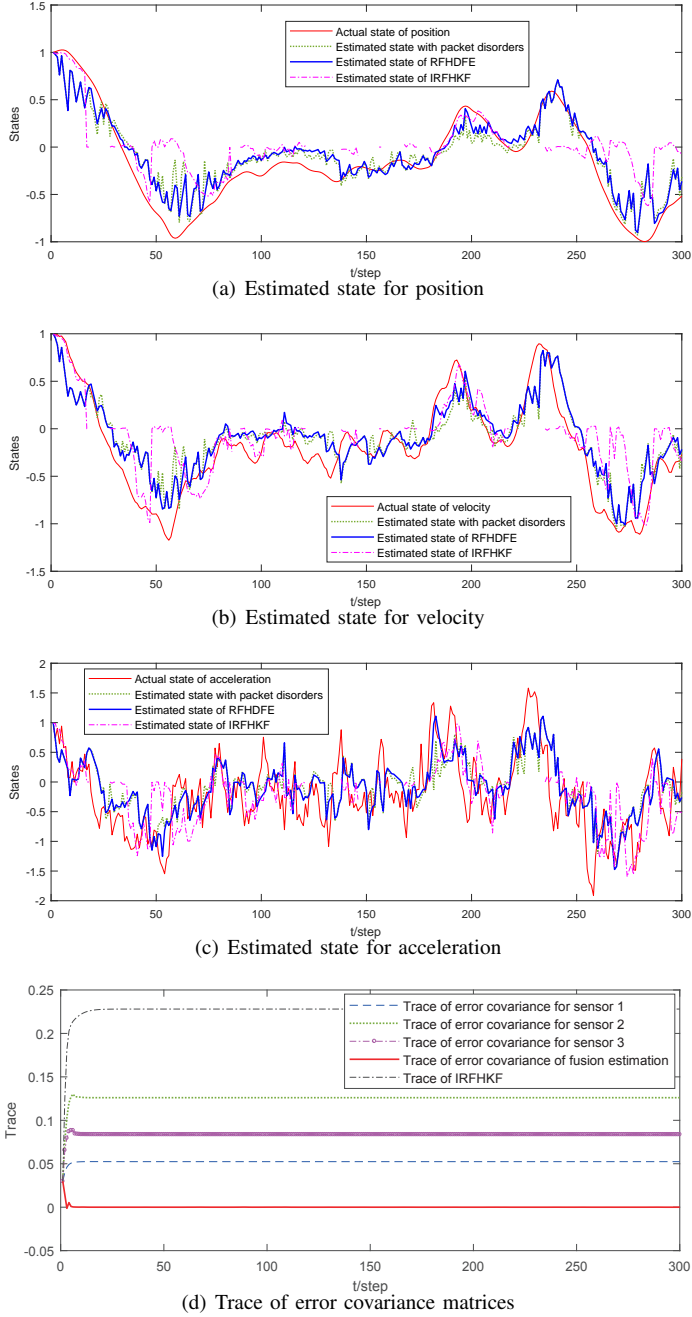


Fig. 5. Estimated results with or without packet disorders.

VI. CONCLUSION

In this paper, the distributed fusion estimation problem considering a class of stochastic uncertain systems with network-induced complexity, parameter uncertainty and mixed noise disturbance have been investigated. To handle packet dropouts and packet disorders generated from random transmission delays, the data packets selection and system modelling have been implemented using the logic ZOH. For the admissible uncertainty, the problem of the local estimation has been investigated by analyzing the mean-square errors and the mix

H_2/H_∞ performance by means of the Schur complement lemma. To estimate the state with packet dropouts, a linear compensation method has been proposed, and the optimal state estimation has also been presented to overcome the influence of the limited communication capacity and suppress the computational burden. In addition, the upper bound of the estimation error has been designed developing the local finite horizon filtering. Subsequently, the distributed state estimation approach has been integrated with the weighted fusion criterion. Moreover, the proposed distributed fusion estimation has the advantage of higher accuracy than each local subsystem by the performance analysis of MSEs. Finally, the numerical simulations for the target tracking systems have been given to illustrate the advantages and effectiveness of the proposed RFHDFE method, in particular at the system state and the higher accuracy of the estimated value.

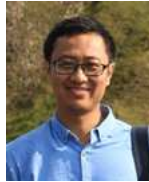
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