

THE REFORM OF ENGLISH MATHEMATICAL  
EDUCATION IN THE LATE NINETEENTH  
AND EARLY TWENTIETH CENTURIES

By

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John Perry (1850-1920)

Engineer and Educational Reformer

he will go down to fame as an original  
and constructive teacher who laid the  
foundation of a new era. He made  
mathematical teaching practical....  
[Armstrong, 1920, p.752]



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Abstract

This thesis is a case study in the growing field of curriculum history. The principal focus is on mathematics in a general secondary education, and particularly during the period 1900-1914 of major curricular upheaval. However, relevant aspects of the nineteenth-century background are discussed; a number of the longer term features of change are traced up to the 1940s; and some related developments in infant, elementary, higher elementary, technical, and teacher, including university, education are also considered.

It is argued that major change was a consequence of an accumulation of circumstances of different kinds in the years around the turn of the century. In particular, important developments in the structure and patterns of the educational system and administrative control are considered, as well as features of the 'new education' movement which have particular implications for teaching methods in mathematics. In addition, the influences of scientific and technical education are shown to be of central importance, the latter influence being associated with what became known during the period as the 'Perry movement.' The newer ideals for a more 'useful' and 'practical' mathematical education and the extent of their realization are explored in detail.

In the implementation of change, the work of various individuals and organizations, operating locally, nationally and internationally, is discussed, and the Mathematical Association in particular, as well as the Board of Education. The importance of examining bodies and the teaching force itself for the scope and character of actual change is strongly emphasized.

It is also shown that reaction to the direction of change, evaluation of progress, and refinement of the thinking in mathematical education were further distinctive products of the period of major reform in English mathematical education.

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ABBREVIATIONS

AMA	Assistant Masters' Association
AIGT	Association for the Improvement of Geometrical Teaching
APSSM	Association of Public School Science Masters
ATTI	Association of Teachers in Technical Institutions
BE	Board of Education
BAAS	British Association for the Advancement of Science
DSA	Department of Science and Art
HMC	Headmasters' Conference
ICTM	International Commission on the Teaching of Mathematics
ICM	International Congress of Mathematicians
LCC	London County Council
MA	Mathematical Association
NCTM	National Council of Teachers of Mathematics
NUET	National Union of Elementary School Teachers
NUJMB	Northern Universities Joint Matriculation Board
SSEC	Secondary School Examinations Council

To avoid seriously overworking the use of ibid a policy has been adopted that all simple page references refer back to the last source cited in the text. The use of ibid is thereby confined to the footnotes.

## Chapter 1

### Introduction

#### Historical Approaches to Curriculum Change

Recent interest in the historical study of curricula is but one aspect of the growing international interest, particularly American, in the field of curriculum studies generally. Relationships between perspectives in the history of education and in curriculum theory have been usefully explored in a paper by Marsden [1979]. His general review of the literature, though necessarily incomplete in some details, does give a good indication of the specific topics in curriculum history which have attracted attention in the present century. As well as coverage of aspects of the curriculum in elementary schools, the specific subjects of English, history, geography, classics, religious studies, science, domestic studies, music, art, craft, and physical education have also warranted various studies. Curiously, mathematics is nowhere mentioned, although as Marsden [p.80] rightly suggests it is science education that has attracted the greatest interest to date. Twentieth-century dissertations and theses on the history of education include a number of studies on all the previously mentioned subjects, as well as a few on mathematics [History of Education Society, 1979b]. An ambitious attempt to consider curriculum change generally, since



1800, has been made in a recent book by Gordon and Lawton [1978].

This has been fairly described by Marsden [1979, p.80] as 'an important ground-clearing operation,' though the insights provided into the complexity of change concerning content and methods in particular areas of the curriculum are, predictably, limited.<sup>1</sup>

Researches which adopt a more limited curricular focus in scope and time are likely to be more penetrating, and here it is the recent work in the field of science education which appears to be currently leading the field in curriculum history.

For historians of scientific and technical education a wide range of sources and approaches has been valuably surveyed in bibliographical papers by Brock [1975a], Heward [1980] and Jenkins [1980]. Scientific and technical education provides common ground for historians of science and education, and the former have paid much attention in recent years to education as a major contributory factor in scientific and technological developments. This partly explains the strength of the interest in science education, though these bibliographies suggest that this interest does not extend to the field of mathematical education despite the obvious links. Two recent and contrasting books on school science education deserve mention for their methodology.

Layton [1973] has focused on the intensive efforts of individuals and pressure groups during the mid-nineteenth century to establish science as a subject in the education of the working classes. Brock [1975a, p.69] has referred to this as 'an exemplary study which will provide a model for research into other periods of intensive curriculum change.' By contrast, Jenkins [1979] has provided a set of largely unrelated studies of different aspects of twentieth-century science

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1 Curriculum change in the subjects of English, mathematics, religious and moral education, home economics, social studies and science is considered in a single chapter.

education in England and Wales, primarily with reference to secondary schools. Jenkins' study is supported by much bibliographical detail, though science is treated as exclusive of mathematics, and no links between the two domains are considered. Jenkins' attitude appears to reflect the more general tendency of historians of science to eschew mathematics, which Grattan-Guinness [1976] has detected. However, by contrast with historians of science, it appears that historians of mathematics have largely ignored the educational field. May's [1973] huge international bibliography of the history of mathematics includes some items concerning mathematical education, as well as the place of the history of mathematics within mathematics teaching, but overall the coverage is very slight.<sup>2</sup> As Grattan-Guinness [1976, p.326] and others have similarly remarked 'virtually no work is done on the educational and social aspects of the history of mathematics, whereas such questions are well-established in the other sciences.' However, it should be added that over the last few years historians of mathematics have shown some interest in sociological perspectives and a new historiography is emerging, as the content of Historia Mathematica since 1976 illustrates [Bos and Mehrtens, 1977; Mehrtens, 1980]. These new directions may result in greater attention being paid to educational contexts though there is little evidence of this to date.<sup>3</sup> It is perhaps significant that a recent general survey of the literature in mathematics includes separate contributions by different writers on the history of mathematics and on mathematical education [Grattan-Guinness, 1977; Howson, 1977].

As part of the growing interest in curriculum studies in recent years, both science and mathematics education have emerged as specialist

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2 Only four pages out of around seven hundred pages are devoted to aspects of mathematical education.

3 A paper by Joan L. Richards (Harvard) on 'Mathematics and a Liberal Education in England, 1840-1865' is to appear in Historia Mathematica.



fields of interest within which different academic perspectives may be applied. Various features of the study of mathematical education have been distinguished in a useful literature survey by Howson [1977], which includes a section on historical studies. An important book by Griffiths and Howson [1974] considers the interrelationships between mathematics, society and curricula, and discusses important 'determinants' and 'instruments' in curriculum change. Systematic historical analysis is not attempted, but historical material is incorporated to substantiate the authors' views of curriculum change. A recent collection of papers on mathematical education edited by Wain [1978] includes contributions on sociological perspectives [Williams, 1978], the diffusion of innovations [Howson, 1978], and general curriculum theory [Layton, 1978]. The output of theses and dissertations on mathematical education has also expanded rapidly since the mid-1960s, as the valuable bibliography of Frobisher and Joy [1978] demonstrates, and a number of historical studies have appeared.<sup>4</sup> The range of more detailed sources for the history of mathematical education remains to be considered.

A useful bibliography on the history of mathematics including mathematical education has recently been prepared for the Mathematics Teacher Education Project [Wain and Woodrow, 1980]. In particular, much of the scholarly work of Peter Wallis on British mathematical education before circa 1850 is included.<sup>5</sup> The bibliography strongly reflects the historical interests of its compiler/s and is less helpful for developments since 1850.

The teaching of mathematics in Scotland up to the end of the eighteenth century has been considered in a doctoral thesis and

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4 New dissertations and theses on science (including mathematics) education are listed biennially in Studies in Science Education.

5 See also the papers by Wallis and Wallis [1980] on British female philomaths in the eighteenth century, and Robson [1949] on the period 1600-1850.

subsequent book by Wilson [1935; Frobisher and Joy, 1978]. Developments in nineteenth-century Scottish mathematical education have been considered in a doctoral thesis by Gray [1952].<sup>6</sup> Gray discusses changes in the organizational context as well as relevant social, economic, educational and mathematical factors in relation to curricula. The thesis is important for its analysis of the way in which the early nineteenth-century practical mathematical tradition in Scotland, which blended a wide range of theoretical and applied mathematics, was gradually eclipsed and replaced by two distinct forms of mathematics in education. The first of these, which came to dominate a general education, was narrow in scope, predominantly theoretical, and sustained by the universities, the examination system, and the teaching force itself. The second, a much broader and less theoretical form, gradually evolved within technical education, and this form came to be devalued outside the technical field as 'illiberal,' being regarded as only narrowly useful and having insufficient theoretical underpinning.<sup>7</sup> In addition to this Scottish work, certain American sources, some of which consider international developments, also deserve mention.

Two American historians of mathematics, Florian Cajori and David Eugene Smith, are exceptional for their early interest in mathematical education, exhibited in their writings from the late nineteenth century onwards. Two books by Cajori [1890, 1917] consider both the history and the teaching of mathematics, including reference to some international trends. As well as producing many pioneering methods books, pedagogical papers, and textbooks, from the 1890s, Smith [1909] was also a leading figure within the International Commission on the

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6 Gray's [1952] thesis is deposited at Edinburgh, and not Exeter, as stated by Frobisher and Joy [1978].

7 The clash of these two alternative paradigms in England from the late nineteenth century onwards will be a major focus for the present thesis, particularly in Chapter 5.

Teaching of Mathematics (ICTM) from 1908 [NCTM, 1970a, p.210].<sup>8</sup> The published output stimulated by the ICTM over the next ten years includes a wealth of detailed historical and comparative source material. In particular, Smith and Goldziher's [1912] international bibliography of over eighteen hundred items on mathematical education, covering the period 1900-1912, is a valuable source,<sup>9</sup> and the American contribution also includes comparative papers by Smith [1913], Brown [1915], Taylor [1915], Kandel [1915] and Archibald [1918] on teaching methods, curricula in secondary schools, curricula in commercial and technical schools, the training of elementary teachers, and the training of secondary teachers respectively. These American sources contain some helpful interpretations of developments in England, supplementing the many papers produced for the ICTM by English writers, and collected together as two volumes of the Board of Education's (BE) [1912a, 1912b] Special Reports on Educational Subjects. Supplementing the Special Reports is a German historical survey of English secondary education by Wolff [1915]. It runs to over two hundred pages and is a key source for understanding developments from the 1860s. Further historical and comparative material was produced by the NCTM after the First War.

The NCTM's [1926] First Yearbook surveyed American progress in the first quarter of the twentieth century, and the Fourth Yearbook [NCTM, 1929] covered international developments from 1910, including those in England. The Fourteenth Yearbook [NCTM, 1939] compared English and American secondary mathematical education and teacher training.

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8 The American National Council of Teachers of Mathematics (NCTM) was founded in 1920. Smith's prolific and varied published output was the subject of a detailed bibliography by Frick [1936], in the first volume of Osiris, which was also dedicated to Smith. The work of the ICTM will be discussed in Chapter 3.

9 The bibliography covers mainly articles and excludes textbooks. It was compiled over a short period and is therefore far from complete as regards English periodicals.



A very ambitious historical account of mathematical education in the United States and Canada is provided by the Thirty-second Yearbook [NCTM, 1970b], compiled by a team of writers.

The Thirty-second Yearbook reviews developments over separate sub-periods between the sixteenth century and the 1960s, and within separate educational sectors. Various recurring 'issues' and the deeper mathematical, educational and societal 'forces' giving rise to them, and contributing to their resolution, are highlighted and catalogued throughout. The use of a writing team produces some overall lack of coherence, and more light is shed on theories and prescriptions than on actual classroom practices, though an accompanying set of Readings [NCTM, 1970a] helps here and also includes a number of biographical sketches. Nothing on a comparable scale has been produced with regard to English conditions,<sup>10</sup> though certain individual contributions deserve mention.

Three worthwhile masters' theses on general developments in secondary mathematical education have been produced. Retter [1936] considers the previous half century, and his work is important for its proximity in time to an important period of reform from circa 1900 to the First World War. Redhead [1953] provides a fairly general comparative study of twentieth-century developments in Germany, France, America and Britain. Talbot [1955] explores developments from the 1850s, with a specific focus on 'fusion' of mathematical content.<sup>11</sup>

In addition to biographical material, a number of historical studies, primarily papers and unpublished dissertations or theses, have considered specific features of mathematical education, particularly geometry and arithmetic. Relevant sources in these areas will be cited

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<sup>10</sup> Despite the apparent lack of interest in English conditions, mathematical education in China has warranted a detailed American study, by Swetz [1974].

<sup>11</sup> See also the paper by Talbot [1956]. 'Fusion' as a curricular tendency will be considered in Chapter 7.

when the specific aspects are considered in the chapters which follow. Less detailed, but sometimes illuminating, are the various sketches by individuals of general developments, predominantly in secondary schools, up to a given point in time.

Two important early twentieth-century papers are those prepared by Godfrey<sup>12</sup> [1908, 1912a] for the International Congress of Mathematicians (ICM) and for the journal Science Progress. These provide valuable insights into early progress in public and other secondary schools from one of the leading figures in the reform movement. The papers by HMIs Fletcher [1912], Strachan [1918] and Carson [1929]<sup>13</sup> are valuable for their perspectives on change in grant-aided secondary schools, based on wide-ranging evidence. Also useful for developments in public schools is the paper by Fawdry [1924],<sup>14</sup> significantly titled 'Reformed Mathematical Teaching.' The most prolific output of sketches is that provided by Siddons<sup>15</sup> [1936, 1948, 1952a, 1952b, 1956]. His personal experiences of public schools and the work of the Mathematical Association (MA) are valuable, and he is a commonly cited secondary source, though his data and judgements are sometimes unreliable. A reasonably balanced public school view is provided by Bushell's [1947] paper on 'A Century of School Mathematics.' Two recent and worthwhile sketches have been prepared for general reports on secondary mathematical education by the Ministry of Education [1958] and the Assistant Masters' Association (AMA) [1973]. It remains in this introduction to raise certain methodological questions and to outline the focus and approach that will be adopted for this thesis.

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12 For Godfrey see the Appendix. He is the subject of a paper by Geoffrey Howson [1973a], which has been revised for inclusion in a forthcoming book. The book will consist of a collection of biographies and readings concerning leading figures in the history of mathematical education. I am grateful to Dr. Howson for this information.

13 For Fletcher and Carson see the Appendix.

14 For Fawdry see the Appendix.

15 For Siddons see the Appendix. The limitations of his interpretations will become clear in Chapters 3 and 4.

Curriculum history raises some major methodological problems. For a given subject at a given time, one task is to characterize the state of the subject - in other words to analyze its scope, or possible non-existence, its content, and its methods, which may vary across different types of educational institution. Here it is important to distinguish life in 'typical' as opposed to 'innovatory' institutions and classrooms. Another task is to chart the way in which the state of a subject changes over time, particularly within critical periods of reform. In this context it is important to distinguish desired or recommended changes from actual changes in teachers' practices. Primary sources which can be used as indicators of states and changes include textbooks, exercise books, educational aids, schools' schemes of work, examination syllabi and papers, inspectors' and examiners' reports, contemporary theses and dissertations, the pedagogical literature (including periodicals), and investigations, reports and recommendations of pressure groups and 'official' bodies. It must be admitted here that states and changes in teaching methods are more difficult to capture than those in scope and content. Furthermore, sources tend to give more data concerning desired states than prevailing states, and this may lead to distortion in historiography.

As well as describing states and changes, yet another aspect of curriculum history is the analysis of the causes of change, or lack of it, in any period. At least two levels of analysis are possible here. The more tangible level involves description of the mechanisms of control and change, involving various interested parties, including individuals, distinguishable groups, central and local government, the universities, examination bodies, publishers and educational suppliers. At a deeper level the search is for important underlying factors determining or inhibiting change, which may relate to shifting and possibly conflicting views of a subject, to the nature of the



educational system and the prevailing educational 'climate,' and to the wider social, economic and political milieu. In particular, sociological factors affecting the conception of school subjects have recently attracted considerable interest.

Although some historians of mathematics have become particularly interested in the social context of mathematical developments, crude sociological reductionism in historiography has generally been rejected [Mehrtens, 1980]. However, sociological perspectives are particularly relevant when considering the development of mathematics in education and such key questions as why this is taught rather than that, and why it is taught in this way rather than in that way, in various educational institutions. Layton's [1978, p.121] assertion that 'school subjects have been, and are, socially determined to some considerable extent' applies as much to mathematics as to any other subject, and questions of power and control are central issues in curriculum history, particularly over periods of curricular upheaval. However, as Layton suggests, there are very few detailed historical studies of curriculum change and resistance to change over key periods in the fields of mathematics and science.<sup>16</sup> The present thesis is offered as one such study in the field of mathematics.

The discussion will be restricted predominantly to England, though some important international activity will be considered in Chapter 3. The period chosen is necessarily somewhat flexible in its extent. This will allow some consideration of the late nineteenth-century circumstances prior to the years of greatest innovatory activity, between 1900 and 1914. Furthermore, to evaluate adequately the success or otherwise of the reform movement it will sometimes be necessary to consider developments up to as late as the 1940s, although curricula

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<sup>16</sup> Layton's [1973] own book is exceptional, and on recent curriculum development in science see also Waring [1979].

had largely stabilized by around 1925. General secondary education will form a particular focus throughout, but it will not be treated exclusively. The reform of mathematical education involved the educational system in all its parts, and cannot be fully understood without reference to these various parts. Thus certain developments in infant, elementary, higher elementary, technical, and teacher, including university, education will be considered.<sup>17</sup> The period chosen is also a vitally important one for the history of English educational administration and curricular control, aspects of which will be considered in Chapter 2. The activities of various pressure groups and certain major early reforms will be discussed in Chapter 3. An evaluation of the particular contribution of the MA to the reform movement and the dissemination of innovations will be undertaken in Chapter 4. The influence of scientific and technical education, changes in teaching methods, and the progress of the 'Perry movement' will be explored in Chapter 5. Changes in the general 'climate' of educational thought and developing theoretical perspectives in mathematical education will be considered in Chapter 6. Two important curricular tendencies, 'broadening' and 'unification' (or 'fusion') of content, will be probed in Chapter 7. Some discussion of the transformation of the three traditional branches, arithmetic, algebra and geometry, will be undertaken in Chapter 8. Finally, in Chapter 9, some concluding remarks will be made concerning general problems of curriculum change, based on the particular material and analysis in the earlier chapters.

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17 The mathematical education of girls will not be discussed separately from that of boys, though aspects of 'differentiation' will arise in Chapters 2, 4 and 6. The whole subject warrants a separate study. See, for example, Clements [1979] for an Australian viewpoint.

## Chapter 2

### The Administrative Context

#### Government Curricular Controls,

#### the Examination System and the Teachers

The very last thing I desire to do is to impose on teachers my ideas of methods. Anything of the nature of a standardized method in English schools is unthinkable. The Board of Education, as I knew it, never issued decrees in matters affecting the faith and doctrines of our educational system; it confined itself to making suggestions. [Westaway, 1931, p.xi]

This statement from a former secondary HMI would, around the time it was made, have fairly reflected the attitude of the central authority for English education to detailed curricular controls in the fields of elementary, secondary, vocational and teacher education [Selby-Bigge, 1927, pp.159-174]. These accepted limits for central educational administration in relation to curricula contrast sharply with the practices up to the 1890s of the Department of Science and Art (DSA) in relation to vocational and diverse forms of education, other than strictly elementary; the Education Department in relation to elementary and elementary teacher education; and, to a lesser extent, the Charity Commission in relation to endowed schools. Both the DSA and the Education Department adopted the powerful controlling mechanism of examination linked with inspection, for the purpose of allocating grants [BE, 1924, pp.9-45]. The important shifts of administrative policy influencing curricula from the 1890s, and particularly after the creation of a single central authority, the Board of Education (BE) in 1899, and the passing of the 1902 Education Act,<sup>1</sup> will be considered in this chapter for the fields of elementary,

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1 The nineteenth-century administration of the DSA, Education Department and Charity Commission, which culminated in the establishment of the BE, have been considered in detail by Bishop [1971]. Central administrative restructuring after the 1899 Act, and local educational administration before and after the 1902 Act have been considered by Gosden [1962, 1966].



technical, secondary and teacher education in turn. These administrative demarcations in English education were strictly drawn during the early years of this century, separately administered, and the blurring of educational categories was thereby discouraged. However, such categories were problematic in the late nineteenth century, when conflicting interests were involved, and the important general curricular implications should not be overlooked.

These issues concern primarily the relationships between elementary, secondary and technical education, and the important implications of their rigid definition and separation for the twentieth-century progress of scientific and technical education have rightly been emphasized by Cane [1959, pp.63-64] and by Heward [1980, pp.107-109]. There are also important implications for the scope and character of mathematical education in different types of educational institution. Although the Education Department took some measures to extend mathematical education in elementary schools beyond the rudiments of arithmetic in the various Standards, shortly to be discussed, it was the DSA that did much through its examinations in science and art, and its arrangements for science schools and classes, to stimulate the study of both science and mathematics beyond arithmetic, not only in day and evening technical schools and classes, but also in schools of a 'higher elementary' type, grammar schools, pupil-teacher centres, and even training colleges [Rich, 1933, pp.196-197]. The determining factors in the defining of secondary and technical education, and their separation, and the motives and activities of interested parties like the Headmasters' Conference (HMC) need not be pursued here, but relevant sources have been discussed by Heward [1980, pp.102-109] and by Jenkins [1980, pp.46-47]. Technical instruction came to be very widely interpreted after the Acts of 1888-1890 and it expanded rapidly in various forms over the last two decades of the nineteenth century

[BE, 1938, pp.50-62]. What is important for this thesis are the consequences of the resulting definitions and split of secondary and technical education for curriculum development in mathematics.<sup>2</sup> The BE continued to examine in science and art, but its operations became confined to the field of strictly vocational and predominantly part-time evening education [BE, 1911, pp.18,325]. Thus, one stimulus for the broadening of the mathematics curriculum in the upward-striving elementary system was removed.<sup>3</sup> Furthermore, any curricular initiatives in science and art taken by the BE became limited in their direct influence.<sup>4</sup> For the various types of secondary school under the BE's regulations it was still the universities, through their examinations, who largely determined the scope and content of secondary mathematical education.

Administrative policies served also to sharpen the distinction between elementary and secondary education, and quasi-secondary developments out of the elementary system, such as the higher grade schools, were inhibited [BE, 1926a, pp.17-35]. The concept of 'two nations' in English education persisted, linked only by a formal and narrow ladder [Lawson and Silver, 1973]. Simon [1965, p.246] has referred to the policy of no 'crossing the lines' after 1902, with the expansion of secondary education meaning in effect the permitting of more elementary pupils up the ladder, via selection on the basis of examination performance. Thus Abbott<sup>5</sup> [1917, p.34] was prompted to refer to:

the case of mathematics under the existing atrophied system of state education, in which the instruction of the majority of the

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2 The 'Perry movement,' to be discussed in subsequent chapters, had its roots in strictly technical education, but subsequently became immensely important for mathematical education generally.

3 'Broadening' of content as a general curricular tendency will be discussed in Chapter 7.

4 The new subject of 'practical mathematics' from 1899 is an important example, to be discussed in detail in Chapter 5.

5 For Abbott see the Appendix.

nation ceases abruptly at the age of thirteen or fourteen; it is usually impossible to attempt anything more than ordinary arithmetic.

What seems clear is that to some extent it was the nature of the administrative arrangements that were hammered out around the turn of the century, and their underlying social assumptions, that created the situation to which Abbott drew attention, as well as reinforced the distinction between a 'liberal' and a 'technical' mathematical education, which has contributed to the latter's generally inferior status in the present century.<sup>6</sup> The Royal Commission [Bryce, 1895a, p.73] had referred to the possible 'warping' of curricula through the DSA's arrangements, but added:

More general in its nature ... is that conflict which goes on in so many schools between the attempt to educate - to train the mind - and the attempt to teach something of immediate practical utility.

The separation of secondary and technical education was one administrative response to this conflict, and the various early Regulations for Secondary Schools, attempted to tackle this conflict at the level of overall curricular balance. Within mathematical education the conflict persisted, and subsequently will be revealed as an important one for the development of mathematical curricula generally.

#### Elementary Codes, Examinations, Schemes and Suggestions

From the time of the Revised Code of 1862, until the 1890s, the arrangements for the annual examination of pupils for the purpose of allocating grants, based on the various published Codes and Instructions to Inspectors, exerted a powerful and dominant influence on the curriculum in elementary schools.<sup>7</sup> As Selleck [1968, p.33] has

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6 The importance of the dichotomy will become clearer from Chapter 3.

7 For the major changes in the nineteenth-century Codes and Instructions see Birchenough [1930] and Gordon and Lawton [1978, pp.11-21], and for some discussion of the social and political context of change in elementary curricula in the period 1870-1907 see Gordon [1977].



emphasized, 'This rigid observance of the Code ... is of the utmost importance for any study of the curriculum in the late nineteenth century,' and he has provided a useful general characterization of the 'instrumentary education' under the system of payment for results in this period [pp.24-45].

The attitudes, motives, and conflicts which resulted in the final form of the Revised Code have been discussed by various writers, and the different interpretations need not be considered here.<sup>8</sup> The Code of 1862 specified six Standards for the three Rs, as a framework for the assessment by HMIs of pupils from the age of six. The syllabus in arithmetic was narrowly confined to the four rules for whole numbers, money, weights and measures, and culminated in 'practice' (evaluating costs from given rates) and 'bills of parcels' at Standard VI.<sup>9</sup> HMIs implemented the system with varying degrees of conviction, and Matthew Arnold was notably critical of its effects at an early stage [BE, 1924, pp.14-17]. Reporting in 1869, Arnold referred to 'Government arithmetic' in scathing terms as 'a modification of the science peculiar to inspected schools' [BE, 1908a, p.128].

To make the best of the system, by maximizing pupils' performance in the written tasks set by HMIs, the teachers were supported by the publishers, who operated in a highly competitive but expanding market, particularly after the 1870 Act. As a Report for the BE [1928a, pp.11-12] subsequently remarked, 'the indirect influence on publishers of successive Codes and Instructions to Inspectors ... was very great' and sales 'depended upon the skill with which publishers were able to assist teachers in meeting official requirements.' Thus both the publishers'

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8 For a recent discussion of various interpretations see Marcham [1979].

9 The Standards for 1862 are reproduced in Gosden [1969, pp.34-35], who also includes a cameo of life in a village school under the system in the 1860s [pp.40-44], as well as some of HMI Matthew Arnold's early criticisms [pp.36-39].

Year ending 31 August.	Number of Scholars presented for Examination										
	Geography.	Grammar.	History.	English Literature.	Physical Geography.	Algebra.	Euclid.	Mensuration.	Mechanics.	Latin.	French or German.
1872	59,774	13,426	16,465	11,685	1,036	2,234	17	33	—	23	273
1873	61,361	20,338	16,762	19,817	653	3,681	—	53	—	46	174
1874	62,070	23,330	15,256	126,831	1,088	4,653	77	89	37	36	422
1875	63,431	29,202	17,710	39,211	2,037	5,913	73	23	—	63	673
1876	47,200†	26,017†	10,957†	34,031	8,553	5,360†			299	452	1,457
1877	—	—	—	44,790	18,936	3,326			534	616	1,791
1878	—	—	—	58,966	23,126	3,520			834	654	1,021
1879	—	—	—	80,137	29,459	5,251			1,621	864	2,577
1880	—	—	—	113,193	34,223	5,623			2,107	831	3,336
1881	—	—	—	127,313	31,382	6,174			2,453	1,026	3,791
1882	—	—	—	117,772	31,207	6,422			3,033	956	3,770
1883	—	—	—	107,435‡	22,521‡	8,256 {	4,750‡		4,136	720	4,723
1884	—	—	—	—	—	634					
1885	—	—	—	—	—	24,727	2,010		3,320	454	5,011
1886	—	—	—	—	—	25,347	1,269		3,766	363	5,178
1887	—	—	—	—	—	25,393	1,247		4,072	342	5,040
1888	—	—	—	—	—	25,103	995		6,343	363	5,519
1889	—	—	—	—	—	26,443	1,006		7,292	371	6,102
1890	—	—	—	—	—	27,461	923		9,651	350	6,728
1891	—	—	—	—	—	30,033	977		11,662	360	7,255
1892	—	—	—	—	—	31,349	870§	1,489§	15,559	347	8,493
1893	—	—	—	—	—	23,542	927	2,202	12,000	260	8,845
1894	—	—	—	—	—	31,437	1,279	3,762	20,773	247	10,161
1895	—	—	—	—	—	33,612	1,399	4,013	21,532	276	11,341
1896	—	—	—	—	—	33,237	1,453	5,614	23,806	250	12,859
1897	—	—	—	—	—	41,846	1,534	6,859	24,955	126	12,901
1898	—	—	—	—	—	47,225	2,059	8,619	26,110	112	15,681
1899	—	—	—	—	—	51,031	2,471	10,828	27,229	224	15,591
1900**	—	—	—	—	—	111,456	5,032	24,843	52,324	359	37,049

\* By the Minute of the 20 February 1867 incorporated into the Code of 1863, extra grants, limited to 8s. a school, were offered, provided *inter alia* that the time-table of the school provided "for one or more specific subjects of secular instruction beyond" reading, writing, and arithmetic. The Code of 1871 first offered "a grant of 3s. per subject for every day scholar presented in Standards IV.-VI. who passes a satisfactory examination in not more than two of such subjects." In the four months ending 31 August 1871, 19,455 scholars were presented in specific subjects.

† By the Code of 1875 grammar, history, and elementary geography were from the 31 March 1876 to be considered class subjects. Algebra, Euclid, and mensuration became "mathematics." The grant for specific subjects was raised to 4s. per subject, and children who had passed Standard VI. were allowed to take up three subjects.

‡ By the Code of 1882 English and physical geography were, from the 30 April, 1882, added to the class subjects. Any scholar in Standards IV.-VII. was allowed to be presented in not more than two specific subjects. Mathematics were divided into "algebra" and "Euclid and mensuration."

§ By the Code of 1890 Euclid and mensuration were recognised as separate subjects.

|| By the Code of 1892 the grant for specific subjects was, after the 31 August, 1892, "2s. or 3s. for each scholar presented in any specific subject." No scholar could be presented in more than two specific subjects, and the Department was to decide "which, if either, of these sums of 2s. or 3s. shall be paid, after considering the report and recommendation of the Inspector."

¶ Elementary science, experimental mechanics, fruit culture, geology, history, horticulture, hygiene, manual instruction and applied drawing, natural philosophy, navigation, physiotherapy, practical science, political economy, and social economy.

\*\* For any school year beginning after 31 August 1897, in place of the grant payable on examination of individual scholars "all the hours during which any registered scholar has received instruction . . . shall be added together, and a grant of 1s. or 6d. shall be paid for every complete 24 hours." (Code of 1897.)

Illustration 1 Some 'Specific Subjects' under the Elementary Code, 1872-1899 [8E, 1900a, pp.114-115]

output, commonly in the form of a series of printed cards in arithmetic for each Standard, and their advertising, reflected the various detailed shifts in the official Codes and Instructions [Ellis, 1971]. The general effects of the annual examination have been well summarized as follows:

its uniform character allowed no scope for variety ... its results were minutely and unreasonably related to the Government grant, and, perhaps worst of all, ... its demands left no time over for teachers and inspectors to think of anything else. [BE, 1924, p.16]

In arithmetic in particular, the mechanical working of the prescribed algorithms was relentlessly practised, to optimize performance on the crucial day of examination, and understanding, which the system did not test, was not surprisingly neglected. Partly as a response to feedback from HMIs, as well as pressure exerted by the School Boards from 1870, and organizations such as the National Union of Elementary School Teachers (NUET), various adjustments were made to the Education Department's powerful mechanism of curricular control.<sup>10</sup> Only those changes which are important for mathematical education will be considered here.

One of the early effects of the Revised Code was the serious narrowing of the elementary school curriculum, both in its overall range and within particular subjects. To attempt to overcome this problem, grant-earning 'specific subjects' were introduced in a Minute of 1867, and 'class subjects' were added in the Code of 1875.<sup>11</sup> (See Illustration 1.) Furthermore, within the basic three Rs, higher levels of attainment were demanded in the Code of 1871, which replaced the syllabus for Standards I-V by the previous syllabus for Standards II-VI, thereby dropping the old Standard I, and introduced a new

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<sup>10</sup> For some discussion of the curricular influence of the School Boards see Gordon and Lawton [1978, pp.209-212]. The NUET dropped the qualification 'elementary' in 1889, and its particular efforts to reform the Codes, to influence the School Boards, and to gain the appointment of teachers as HMIs have been considered by Tropp [1957].

<sup>11</sup> Mathematics was affected only by the arrangements for 'specific subjects.'



Standard VI, which brought simple proportion, as well as, vulgar and decimal fractions into the syllabus for arithmetic [Committee of Council, 1871, p.cix].<sup>12</sup> Illustration 1 shows in tabular form the progress of the various 'specific subjects' under the Codes in the period 1872-1899. Notable are the modifications to the classification of the mathematical subjects for grant-earning purposes, with algebra, Euclid and mensuration separately recognized from 1890, as had previously been the case up to 1875. Algebra, which could be taught mechanically on the lines of arithmetic, was not surprisingly much more popular than the study of a couple of Books of Euclid throughout this period [BE, 1900a, pp.558-559].<sup>13</sup> During the 1890s there was a rapid gain in popularity for mensuration, which was again a much more approachable branch than pure Euclidean geometry.<sup>14</sup> The general doubling of the number of entries in 1899 was a consequence of the ending of individual examination of pupils to determine grants. Further broadening of the scope of the work within the three Rs was one objective of the Code from 1882.

The 1882 Code introduced a higher Standard VII, and prescribed the additional topics in arithmetic of averages, percentages, discount and stocks, following the conventional commercial orientation in this branch [Committee of Council, 1882, pp.132-133]. Also some attempt was made to encourage more intelligent learning through the introduction of 'merit grants,' based on an HMI's general impressions of a school.

Elementary education in all its aspects was subjected to a thorough investigation by the Cross Commission in the period 1886-1888.

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12 The previous absence of these topics illustrates the narrowness of the original Revised Code.

13 Algebra covered the conventional development of the four rules; greatest common measure (GCM), lowest common multiple (LCM), simple equations in one and two unknowns, and simple quadratics. The Cambridge textbook writer H.S. Hall catered specially with a textbook for the 'specific subject' of algebra under the Code [Hall and Wood, 1900].

14 Mensuration covered triangles and parallelograms, the circle, parallelopiped, sphere, right cone and cylinder.

Major differences of opinion resulted in the issue of a majority and a minority report. The minority favoured the ending of the system of payment for results and the adoption of a system of payment for means, with greater local administrative responsibility.

The need for some relaxation of the system was generally admitted by the Cross Commissioners, to reduce the over-emphasis on the lower Standards, to bring less rigidity into the teaching, and to encourage more breadth and variety in curricula. The need for more practical and vocationally relevant curricula was emphasized, and this applied to arithmetic in particular, which also warranted the subsequently common plea for more understanding and less rote manipulation of symbols [Birchenough, 1930, pp.150-155,384-385].<sup>15</sup> However, until the 1890s, the system still persisted in its most severe form in spite of teachers' discontentment, and the growing enlightenment in educational thinking generally.<sup>16</sup>

The newer educational arguments reflected the more general reaction to the earlier Victorian preoccupation with efficiency and competition through examination, as well as the increasingly critical attitude to the mental and moral disciplinary value of the 'instrumentary' education of the masses [Gordon and Lawton, 1978, pp.179-199; Selleck, 1968, pp.45-69].<sup>17</sup> Also important for the break-up of the system was

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15 A master's thesis on arithmetic teaching by Owen [1959, pp.79-185] shows that in the 1880s some teachers devised schemes, published in the Teachers' Aid, which were well in advance of the requirements of the Code, particularly for the lower Standards.

16 Ballard's detailed description of arithmetic under the system around 1890 is reproduced in Gordon and Lawton [1978, pp.92-93]. The annual examination comprised three very elaborate mechanical calculations and one problem, all on a very narrow syllabus for each Standard. The rules were the teachers' main concern, with problems either totally neglected or practised during the last quarter of the year. Two items right out of four secured a pass, so it was possible to avoid the problems entirely, and still leave some margin for error. For Ballard see the Appendix.

17 Mental discipline and its theoretical underpinning in faculty psychology will be explored in Chapter 6.



	Standard I.	Standard II.	Standard III.	Standard IV.	Standard V.	Standard VI.	Standard VII.
*Reading	To read a short passage from a book not confined to words of one syllable.	To read a short passage from an elementary reading book.*	To read a passage from a reading book.*	To read a passage from a reading book, or history of England.*	To read a passage from some standard author, a reading book, or a history of England.*	To read a passage from one of Shakespeare's historical plays, or from some other standard author, or from a history of England.*	To read a passage from Shakespeare or Milton, or from some other standard author, or from a history of England.*
†Writing	Copy in manuscript characters a line of print, commencing with a capital letter. Copy books (large or half text hand) to be shown.	A passage of not more than six lines, from the same book, slowly read once, and then dictated word by word. Copy books (large and half text hand) to be shown. In the Second and Third Standards the examination in Spelling may be separated from the exercise in Writing, and an examination in word-building leading to correct spelling may be substituted if the managers wish. Notation and numeration up to 1,000. Simple addition and subtraction of numbers of not more than three figures. In addition not more than five lines to be given. The multiplication table to 8 times 12. Short exercises in mental arithmetic may be given in the examination of all quantities, and should be preparatory to the work of the next higher Standard. In the intelligence of the teaching may be tested by sums worked orally by the class with	Six lines from one of the reading books of the Standard, slowly read once and then dictated. Copy books (capitals and figures, large and small hand) to be shown. The former rules, with long division. Addition and subtraction of money.	Eight lines of poetry or prose, slowly read once, and then dictated. Copy books to be shown.	Writing from memory the substance of a short story read out twice; spelling, handwriting, and correct expression to be considered. Copy books to be shown.	A short theme or letter on an easy subject; spelling, handwriting, and composition to be considered. Copy books to be shown.	To read a passage from one of Shakespeare's historical plays, or from some other standard author, or from a history of England.*
‡Arithmetic (Scheme A.)	Notation and numeration up to 1,000. Simple addition and subtraction of numbers of not more than three figures. In addition not more than five lines to be given. The multiplication table to 8 times 12. Short exercises in mental arithmetic may be given in the examination of all quantities, and should be preparatory to the work of the next higher Standard. In the intelligence of the teaching may be tested by sums worked orally by the class with	Notation and numeration up to 100,000. The four simple rules to short division. The multiplication table and the pence table to 12s.	The former rules, with long division. Addition and subtraction of money.	Compound rules (money) and reduction of common weights and measures.‡	Practice, bills of parcels, and single rule of three by the method of unity. Addition and subtraction of proper fractions, with denominators not exceeding 12.	Fractions, vulgar and decimal; simple proportion, and simple interest. [Questions involving recurring decimals will not be put to girls.]	Averages, per-centages, and stocks.
Do. (Scheme B.)	The four simple rules. Divisors and multipliers not exceeding 6. No number higher than 99 to be employed in the questions or required in the answers.	Reduction and compound rules (money). Divisors and multipliers not exceeding 12. Sums of money in the questions and answers not to exceed 10s.	Simple rules and compound rules (money). Divisors and multipliers not exceeding 99. No number higher than 99,999 to be employed in the question or required in the answer. Sums of money in the questions and answers not to exceed 99s.	Reduction and compound rules applied to the following weights and measures (length, weight, capacity, time). In length, yards, feet, and inches; in weight, tons, cwt., qrs., lbs., ozs.; in capacity, gallons, quarts, pints; in time, days, hours, minutes, seconds—are the only terms that will be required. Divisors and multipliers not to exceed 99.	Vulgar fractions (simple fractions only). Practice. Bills of Parcels. Common weights and measures. Mensuration of rectangles.	Decimal fractions (excluding recurring decimals). Simple proportion or single rule of three by the method of unity. Calculation of simple interest upon a given principal. Mensuration of rectangular solids.	Vulgar and decimal fractions. Averages and per-centages. Investments of savings. Consols.
Do. (Scheme S.)		STAGE I. To include Standards I. and II.		STAGE II. To include Standards III. and IV.	STAGE III. To include Standards V., VI., and VII.		
		The four simple rules; the multipliers and divisors not to exceed 12. The numbers employed in the processes and in the answers not to exceed 999.		The four simple rules, the compound rules (money) and tables of weight, time, and capacity). The multipliers and divisors not to exceed 99. The numbers employed in the processes and in the answers not to exceed 9,999.	Vulgar fractions (simple fractions only). Decimal fractions (not recurring). Compound rules (tables of length and area). Practice, bills of parcels. Single rule of three by the method of unity. Calculation of simple interest on a given principal.		

**N.B.—In Welsh districts—**  
 (a.) Bilingual reading books and copybooks may be used and the instruction translated into English of an easy piece of Welsh written on the blackboard.  
 (c.) In the Arithmetic set to Standards I.-IV. inclusive the problems should relate to the History of England and Wales.

\* Reading with intelligence will be required in all the Standards, and increased fluency and expression in successive years. Two sets of reading books must be provided for Standards I. and II., and three, one of which should relate to English history, for each Standard above the second. The Inspector may examine from any of the books in use in the Standard, and in Standard III. and upwards, from any book or passage suitable for the purpose which he may select. The intelligence of the reading will be tested partly by questions on the meaning of what is read.  
 † The writing and arithmetic of Standards I. and II. may be on slates or paper, at the discretion of the Managers; in Standard III. and upwards it must be on paper.  
 ‡ The Inspector may examine scholars in the work of any Standard lower than that in which they are presented, and in arithmetic should require the scholars of the Fourth Standard and upwards to add columns of pounds, shillings, and pence within a specified time in order to show readiness and accuracy.

generally may be bi-lingual.  
 board, or of a story read twice, shall, if the Managers so desire, be substituted for be in both English and Welsh.  
 Wales.

§ The tables to be learned include those weights and measures only which are in ordinary use, viz.:—  
 Weight.—The ton, hundredweight, quarter, stone, pound, ounce, and drachm.  
 Length.—The mile, furlong, rod or pole, chain, yard, foot, and inch.  
 Area.—The square mile, acre, rood, square pole or perch, the yard, foot, and inch. (Boys only.)  
 Capacity.—Quarter, bushel, peck, gallon, quart, and pint.  
 Time.—Year, month, week, day, hour, minute, and second.  
 The Inspector should satisfy himself that the principles of arithmetic are properly taught in the school. The work of girls will be judged more leniently than that of boys, and, as a rule, the sums set will be easier. The scholars in Standards V., VI., and VII. should know the principles of the metric system, and be able to explain the advantages to be gained from uniformity in the method of forming multiples and sub-multiples of the unit. As a preparation for this it will be useful to give in Standard IV. elementary lessons on the notation of decimal fractions.



the rapid development of elementary schools after 1870, as well as the growing concern of the State for education other than elementary. The administrative burden, particularly on HMIs, became such that 'the system at length broke down by its own weight' [BE, 1924, p.17].

Annual examination was withdrawn by stages between 1895 and 1900, and a system of 'block grants' replaced that of payments for results in various aspects of the curriculum. The new enlightenment of the 1890s is also reflected in the Education Department's policy with regard to arithmetic.

Various Circulars and Instructions to Inspectors from around 1890, and particularly Circular 322 of 1893, contributed to the spread of Kindergarten methods in infant education. The Education Department also encouraged the extension of activity methods upwards to Standard III [BE, 1933, pp.26-30; Garlick, 1898, pp.353-356].<sup>18</sup> The Code of 1894 introduced some major changes in the requirements for arithmetic up to Standard IV.

A new Scheme B was introduced in 1894 as an alternative to the previous syllabus, designated Scheme A, with yet a further Scheme S provided for small schools. (See Illustration 2.) The alternative Schemes were introduced to permit a more gradual development of the subject, with much smaller numbers in the early stages and a more varied treatment. The Instructions explained that many experienced teachers now felt the need for such a change and added 'My Lords desire that teachers who adopt this view of arithmetical teaching should be at full liberty to give effect to it' [Committee of Council, 1894, p.420]. 'My Lords' also emphasized the value of varied oral work, mental exercises, the understanding of principles and processes, and intelligent problem solving [Committee of Council, 1894, pp.419-420]. Other notable features of the 1894 Code are the elements of

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<sup>18</sup> Activity methods will be discussed in Chapter 5.

differentiation between boys and girls, which persisted into the twentieth century; the introduction of some mensuration in Scheme B; and the new requirement that the principles of the metric system should be taught, and preferably also decimal notation at Standard IV.<sup>19</sup> The introduction of Scheme B was a progressive step by the Education Department, but it was taken at a time when central curricular control through individual examination was about to be phased out. The process of transition from prescription and examination to suggestion and inspection was completed by the new BE in the early years of this century.

After 1900, although the 'specific subjects' might still be taught, syllabuses were no longer centrally prescribed and the financial incentive for teaching beyond the range of elementary subjects was removed.<sup>20</sup> From 1904, the Code merely specified in generalities the components of a basic elementary education, treated as a whole, and allowed the possibility of local variations in scope, content and methods. The following extract illustrates the kind of detail regarded as appropriate, and it also summarizes some of the newer emphases in arithmetic:

Arithmetic, including practical work in measuring and weighing, oral exercises, written exercises, which should be of a varied character and should not infrequently involve the application of more than one arithmetical operation, and, in the higher classes, practice in describing the processes used. Elementary instruction in mensuration should be given, and the advantages of a decimal system of weights and measures should be explained to the older scholars. The use of literal symbols in working simple problems may with advantage be taught in the higher classes. [BE, 1906a, p.2]

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19 By 1900, mensuration was included in Schemes A and B, but for boys only [BE, 1900a, p.539]. The metric system had been introduced into the much earlier Code of 1871, but had dropped out three years later [Committee of Council, 1871, p.cix; 1874, p.cl]. The pressure for metrication was much stronger in the 1890s, and these issues will be explored in more detail in Chapter 8.

20 With the ending of central curricular controls, and the encouragement given to local initiatives, it is much more difficult to capture the states of and changes in elementary curricula.



## SCHEME No. II.

It is supposed that the bulk of the children will be promoted at the end of every year, but that there will also be promotion of a fair number of bright children at the end of every term.

### FIRST YEAR.

#### *First Term.*

The four simple rules and their application to shillings and pence, so that no number greater than 30, and no sum of money greater than 2s. 6d., is employed. The addition sums to consist of not more than two lines. Construction of tables within the above limits. Shopping, and the actual measurement of length in inches and tenths.

#### *Second Term.*

Recapitulation of the above with the limits 60 and 6s.  
Practical work extended to include ruling of lines of given length.

#### *Third Term.*

Recapitulation of the above with the limits 60 and 6s.  
Practical work as before.

### SECOND YEAR.

#### *First Term.*

Recapitulation and the extension of money sums so as to include sums less than £1 without farthings.  
Construction of tables.  
Practical work extended to include halves, quarters, and eighths of an inch.

#### *Second Term.*

Recapitulation and extension to numbers less than 1000.  
Multiplication of shillings and pence by numbers not greater than 6 so that no sum of money greater than £1 is employed.  
Estimation of lengths in addition to ruling and measuring.

#### *Third Term.*

Recapitulation and extension of the above so as to include sums of money less than £10, and also multipliers and dividers not greater than 12.  
Estimation of lengths continued. Division of paper into halves, quarters, eighths.

### THIRD YEAR.

#### *First Term.*

Recapitulation and extension of previous work so as not to include numbers greater than 9999.  
Multiplication and division by numbers that are reducible to factors not greater than 12.  
Measurements of lines in inches and fractions of an inch.  
Addition and subtraction of inches and tenths of an inch.  
Practical use of weights and measures.  
Other practical work suggested by the scheme of Hand and Eye training.

#### *Second Term.*

Recapitulation and extension of previous work so as to include sums of money not greater than £100 (without farthings).  
Practical work as before with the addition of centimetres and millimetres.

#### *Third Term.*

Recapitulation and extension of the above to farthings.  
Decimals (tenths only).  
Fractions with the denominators deducible from the rulers i.e., 2, 4, 8; 3, 6, 12; 5, 10.  
Mental multiplication and division by 10 and 100.  
Practical work as before.



It should be emphasized that in arithmetic Scheme B continued to be very influential and was still used by the BE as a syllabus for Certificates of Proficiency for the labour market until 1905 [Ballard, 1912, pp.3,26]. For this purpose it was then replaced a year later by a new Syllabus of Arithmetic with a strong emphasis on mensuration [BE, 1906a, pp.v-vi, 42-44]. However, the BE's general policy was now to encourage variety and experimentation, and, to this end, the first handbook of Suggestions was published in 1905 [BE, 1905a].<sup>21</sup>

The Suggestions encouraged teachers to go beyond the previous Schemes A and B, and three new schemes were published 'to stimulate teachers to draw up their own schemes suitable to the local needs and to the abilities of the school staff' [BE, 1905a, pp.44-45]. Over the first decade of this century Scheme B continued to be the basis for most textbooks intended for elementary schools, and some referred to Scheme B in their titles [British Museum, 1906, pp.61-63; 1911, pp.62-64]. The London County Council (LCC) [1911, pp.121,128] in a survey during 1907-1908 found some schemes in advance of Scheme B, following the BE's Suggestions, with the earlier teaching of fractions and decimals, practical and graphical work, and more mensuration in boys' schools; and work on the lines of the BE's Scheme II in girls' schools. (See Illustration 3.)

The general pattern of change in London is confirmed by Ballard's [1912, pp.9-10] survey of seventy senior departments (see Illustration 4), and one headmaster referred to 'almost a revolution' concerning aims and methods over the previous decade [Spencer, 1912, p.31], though older teachers particularly were experiencing discomfort in the process of adaptation. Teachers' conservatism in spite of the new circumstances created for curriculum development is understandable, for as

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21 The Suggestions were subsequently revised and reissued as a new handbook from time to time. The section on arithmetic was first rewritten and issued as Circular 807 in 1912 [BE, 1912c].

# MATHEMATICS IN THE SENIOR DEPARTMENTS.

Before dealing with the senior departments, it may be well that I should explain the range of experience upon which my report is based. First, comes my knowledge of the schools in my own district; secondly, visits made to typical schools in other parts of the Metropolis; thirdly, conferences with my colleagues; and fourthly, an examination of the syllabuses and methods in use in 70 departments taken entirely at random from the various inspectorial districts. Of these 70 schools, 30 were boys', 30 girls', and 10 mixed departments. The list comprised both Council and Non-provided Schools; in fact, every variety of school except the Higher Grade and Central.

Some of the results of my investigations are herewith tabulated:—

Subject or Method.	Number of Cases in which it is taught or used in the 70 Schools.
Scheme B. Syllabus (modified) - - - -	60
Practice - - - - -	51
Recurring Decimals - - - - -	37
Compound Interest - - - - -	33
True Discount - - - - -	17
Stocks and Shares - - - - -	36
Square Root - - - - -	33
Cube Root - - - - -	11
"Rule of Three"—taught by method of unit exclusively - - - - -	24
"Rule of Three"—taught by proportion as well - - - - -	46
Tots - - - - -	36
Simple Graphs - - - - -	47
Practical Arithmetic taken systematically	44
Multiplication in which the pupil starts with the highest denomination in the multiplier - - - - -	3
Both methods of multiplication (starting with the unit figure, and starting with the highest denomination) - - - -	14
L.C.M. and H.C.F. taught by repeated divisions only - - - - -	22
L.C.M. and H.C.F. taught by factors only	24
" " " both methods - - - - -	24
Factors used for extraction of square and cube roots - - - - -	17

Illustration 4 Some Results of  
Ballard's [1912, p.9] Survey

Edmond Holmes, a former HMI, forcibly remarked 'for thirty-three years they had been treated as machines and they were suddenly asked to behave as intelligent beings.'<sup>22</sup>

The period 1895-1905 was also a difficult one for HMIs, whose role had to be gradually redefined, and not without some early 'uncertain effort and doubtful aims' [BE, 1924, p.18]. After 1902, HMIs also had to work in conjunction with the new LEAs and their officials, and in particular the LCC was notably active in the field of curriculum development before the First World War, providing conferences and courses on various aspects of the teaching of mathematics [LCC, 1911; Nunn, 1912a, pp.302-303].<sup>23</sup> In addition to the formal work of inspection, HMIs too became involved in the dissemination of innovations, through publications, including the Suggestions, courses and informal contacts.[BE,1924, pp.18-24]. The early years of this century were certainly critical ones for elementary curriculum development, but there are two important inhibiting factors to be considered, namely the arrangements for scholarships and free places, and the growing concern for standards in the three Rs, from around 1910, which prompted initiatives from both the central and local authorities.

Aspects of the general development of the competitive scholarship ladder linking the elementary and secondary systems from the 1860s have been considered by Gordon [1977, pp.49-52], and by Gordon and Lawton [1978, pp.195-204].<sup>24</sup> Supplementing those scholarships provided privately, by some schools' endowments, and by the DSA, local authority provision developed from generally small beginnings in the 1890s, largely through the working of the Technical Instruction

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22 Quoted in footnote [Selby-Bigge, 1927, p.134].

23 The LCC's [1911] Report, following a Conference on arithmetic teaching, 1906-1908, runs to 134 pages. It was largely the work of Branford, with some assistance from Nunn. For Branford and Nunn see the Appendix.

24 For more detail of developments, 1890-1912, see also BE [1913a, pp.3-33].



Acts,<sup>25</sup> and developed further after the 1902 Act, particularly through the efforts of the new LEAs and the stimulus provided by the BE for the preliminary education of elementary teachers in secondary schools. Further expansion followed the BE's introduction in 1907 of the scheme which required grant-aided secondary schools to provide normally twenty-five per cent of their places as 'free places' for elementary pupils [BE, 1938, p.305].

Written examinations in the basic subjects of English and arithmetic, at around the age of eleven, became the LEAs' generally accepted principal means of determining pupils' suitability for a secondary education.<sup>26</sup> Although it was argued that this policy would minimize the backlash effects on the elementary curriculum as a whole, clearly there are important implications for mathematical education [Association of Education Committees, 1930, p.27]. One effect was the narrowing of the mathematics curriculum so that arithmetic was the sole area of consideration. Even before the First World War one elementary teacher, while describing his own innovatory scheme, complained that 'work in the later years is very much curtailed by the miserable requirements of some educational authorities for scholarships to be held in the Secondary Schools' [R.W. Jones, 1912, p.371].

Between the Wars, the BE's [1931, p.139] Consultative Committee reported that 'There is a general agreement among our witnesses that too much time is given to arithmetic in primary schools,' and that this situation was 'often maintained by the importance attached to the subject in free-place and scholarship examinations' [pp.139-140]. As well as inhibiting the broadening of the mathematics curriculum, a large number of witnesses also criticized the emphasis on English and

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25 The LCC's scholarship system developed rapidly in the 1890s, and specimen arithmetic papers were included by the prolific arithmetic textbook writer Pendlebury [1899, pp.185-188] in his very popular A 'Shilling' Arithmetic.

26 Various tactics in selection are discussed in BE [1926a, pp.132-139].

arithmetic for the resulting distortion of the overall balance of the curriculum [pp.101-102]. These examinations also generated competition between teachers and schools, as well as pupils, and encouraged rigid and early streaming of pupils and even the streaming of the teachers themselves [Wingrove, 1976, p.17]. As with payment for results, the system led to coaching of pupils for the requirements in mechanical and problem arithmetic, and, furthermore, mediocre pupils could sometimes be coached to a relatively high level of performance [BE, 1931, p.102].

As well as receiving the attentions of the Consultative Committee, the whole system of selection was considered by a Departmental Committee [BE, 1920a], and was also the subject of a Memorandum from the BE [1928b]. Such examinations were developed as the major administrative device for operating the narrow ladder linking the elementary and secondary systems. Unfortunately, the intention merely to provide a uniform qualifying hurdle was not implemented in practice. A Committee of Enquiry in 1927 summarized the situation thus:

obviously, under present conditions, with the lack of secondary school places, so far from being a merely qualifying examination, it has become a competitive test of great severity, and, with the considerable variation in the number of school places available in different parts of the country, the standard needed for success varies from district to district in a most distracting way.<sup>27</sup>

This general pattern of developments in the secondary system exacerbated the various effects which have already been considered. Another potentially narrowing and distorting influence on mathematical education was the growing concern for standards in the three Rs.

There is evidence of concern for the standards of arithmetical accuracy of London elementary school pupils by 1910. Ballard, an LCC Inspector, reported in 1911, 'On the Alleged Decline in Arithmetic Accuracy in Elementary Schools' [Gordon and Lawton, 1978, p.93],

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27 Quoted in Association of Education Committees [1930, p.23].



and he returned to this issue in his paper for the Special Reports in the following year [Ballard, 1912, p.16].<sup>28</sup> The BE's [1912c, p.9] revised Suggestions also commented:

In the reaction against a dogmatic treatment of 'rules' and an undue insistence on neat and mechanical cyphering, it may be that accuracy in arithmetic is sometimes insufficiently regarded. It is not, however, in any way incompatible with intelligence....

Birchenough [1930, p.205] has referred to 'The pessimistic utterances before and after the War about the deterioration in the three R's' and to an enquiry in Lancashire, during 1913, concerning the 'alleged deterioration.' The complex problem of judging standards, and contributory administrative factors such as the size of classes, the quality of the teaching force, and the provision of buildings and equipment will not be pursued here. However, what is clear is that the concern of both LEAs and the BE continued in the 1920s.

Concern for standards in rural schools prompted Leicestershire County Council [1923] to publish their own Suggestions for arithmetic. This LEA's motivation is clear from some of the introductory remarks:

There is no doubt that the Arithmetic teaching in the Elementary Schools has in recent years developed very strongly on what is called the 'intelligence' side.... While continuing to develop intelligence we want to make it more effective by improving the mechanical efficiency of the children in dealing with numbers. [Leicestershire County Council, 1923, p.6]

Particularly significant is the investigation by HMIs, during 1924, of some fifty-five thousand children's performance on mental, mechanical and problem arithmetic tests at Standards V and VII. Over 1250 schools were involved and the results were subsequently published in detail [BE, 1925a].

This exercise was conducted twenty-five years after the ending of the annual examination by HMIs, and would have been a delicate matter as regards relations between the BE, the LEAs and the teachers.

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28 Historical parallels with the recent concern for standards have been drawn by McIntosh [1977], an LEA adviser, and by T.J. Fletcher [1977], an HMI.



Predictably, the general conclusions were rather vague, and interspersed with qualifying remarks, particularly concerning the need to improve the quality of educational provision. However, the BE's action symbolizes an educational climate less sympathetic to curriculum development than had been the case before the War.<sup>29</sup>

### Science and Art Examinations

In addition to the Government's powerful curricular control of elementary schools, exercised by the Education Department until the 1890s, the Government also came to influence the teaching of various 'science and art' subjects in a variety of educational contexts, through the operations of the DSA, and particularly its examinations in science from 1860. There have been various studies of aspects of the nineteenth-century work of the DSA, though with a focus predominantly on administration rather than curricula [Brock, 1975a, p.90; Heward, 1980, pp.97-99, 109].<sup>30</sup> Its operations are important for understanding the development of scientific and technical education, including mathematics. The DSA's subsequent influence on mathematics in the higher grade schools has been considered in a dissertation by Hitchens [1978].

Even before the Revised Code, the DSA itself had adopted a system of payment for results in annual examinations on the principles of science and art, intended for members of the industrial classes, taught in the evening by suitable persons certificated by the Department. Practical geometry was offered in the first list of science subjects, in a Minute of 1859, and pure mathematics was added in 1864. The geometry, both plane and solid, was essentially

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29 The Consultative Committee's [BE, 1931, pp.140-141] attitude to primary arithmetic was also notably reactionary, and in conflict with the Committee's generally progressive educational stance.

30 For a valuable summary of aspects of Butterworth's doctoral research on the DSA see Butterworth [1978], to appear in R. Macleod (ed.), Days of Judgement (Driffield).

geometrical drawing and was divorced from the theoretical study of Euclid. On the other hand, the mathematics included purely demonstrative Euclidean geometry and the conventional academic subject-matter of arithmetic and algebra, which reflected the DSA's policy of promoting education in general scientific principles rather than instruction in narrowly technical applications. Both practical geometry and pure mathematics in its various 'stages' subsequently became very popular subjects [Butterworth, 1978, pp.4-7, 18-19].<sup>31</sup>

The scale of the DSA's work, based at South Kensington, grew rapidly from the 1870s, in response to the general demand for various forms of technical education, and the number of science subjects grew to around two dozen. Furthermore, the Department also became gradually involved in the payment of grants for instruction in day-time classes, and arrangements for the conduct of day 'organized science schools' were introduced in 1872, to encourage more coherent courses in science and art than just the teaching of isolated subjects in 'science classes.' Thus day schools also had the option of adopting the arrangements for either 'science classes' or a 'school of science' in order to benefit from the DSA's grants. The DSA thereby came to influence the curriculum in schools providing various forms of education other than elementary, and particularly the higher grade schools, but also the less prestigious and financially needy secondary schools. Initially, a 'school of science' had to provide a three-year course on approved lines with at least fifteen hours of science and art subjects, and grants paid partly for results and partly for attendance. These conditions were such that grammar schools tended to organize 'science classes' rather than a 'school of science,' the latter arrangement being favoured by the higher grade schools [Gosden, 1966, p.48; Cane, 1959, pp.59-61].

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31 Mathematics examination papers are reproduced in Hitchens [1978, pp.73-78].

School.		English Subjects.				Languages.				Sciences.		Art Subjects.			Commercial Subjects.		Physical Exercises.	Other Subjects.	Total No. of Hours per Week.	Remarks.			
Grade.	Name.	Form.	Religious In-	English.	History.	Geography.	Arithmetic.	Greek.	Latin.	French.	German.	Mathematics.	Physical Science.	Drawing.	Manual In-struction.	Music and Singing.	Book-keeping.	Shorthand.					
			struction.																				
3rd Grade	LEEDS CENTRAL HIGHER GRADE BOARD SCHOOL (BOYS' DEPART- MENT).	4th year's course.	-	1 1/2	1	1	-	-	4 1/2	3	-	-	C. 3 1/2 P. 2	2	-	-	-	1	1	hrs.	25 1/2	The school includes an organ-ized science school. Boys in Form I., 2nd year's course, give 2 hours, and boys in Form II., 1 hour a week to German in addition to the regular 2 1/2 hours. Boys taking the 3rd year's course give 1 1/2 hours to manual in-struction on Saturday.  (1) Geometry. M. = Mechanics.	
		3rd year's course.	1	1	1	-	-	-	1 1/2	2	2	-	3 1/2 C. 2 P. 1	2	-	1 1/2	-	1	1	hrs.	25 1/2		
		2nd year's course—Form I.	1	1	1	-	-	-	1 1/2	2	2	-	3 1/2 C. 2 P. 1	2	-	1 1/2	-	1	1	hrs.	25 1/2		
		II.	1	1	1	-	-	-	1 1/2	2	2	-	3 1/2 C. 2 P. 1	2	-	1 1/2	-	1	1	hrs.	27 1/2		
		1st year's course—Form I.	1	1 1/2	1	-	-	-	-	1	1 1/2	-	-	C. 3 1/2 P. 3	1 1/2	-	1	-	1	1	hrs.		25 1/2
		II.	1	2	1	1	-	-	1	2	-	4 1/2	1 1/2	-	1	-	1	1	1	hrs.	25 1/2		
		III.	1	1 1/2	1	1	-	-	1	2 1/2	-	3	1 1/2	1 1/2	1	-	1	1	1	hrs.	25 1/2		
-	SHEFFIELD CENTRAL HIGHER GRADE SCHOOL (BOYS' DEPARTMENT).	Ex-VII. 1 (2nd and 3rd years).	1 1/2 or 2	1 or 1 1/2	1	-	-	-	1	1 1/2	-	-	C. 3 1/2 M. 2 1/2 P. 1 1/2	1 1/2 1 1/2 Mec.	1 1/2	-	-	-	11	11(1)	25 1/2	The two highest forms con-stitute an organized science school, all forms below Ex-VII. 2 are elementary.  Mec. = Mechanical drawing. (1) Commercial correspon-dence.  A drilling lesson is given once in three weeks.	
		Ex-VII. 2 (1st year better section).	1	-	1	1	-	-	-	-	-	-	1 1/2 1 1/2 Mec.	1 1/2 1 1/2 Mec.	-	-	-	-	1	1	25 1/2		
		Ex-VII. 3 (commercial section).	1	-	1	1	-	-	-	-	-	-	-	-	-	-	-	-	-	1	11(1)		25 1/2
			1	-	1	1	-	-	-	-	-	-	-	-	-	-	-	-	-	1	1		25 1/2

\*NOTE.—The curriculum of this school is inserted here in full, but some of the work done, particularly in the 4th year's course, is distinctly above that of a Third Grade School.

Illustration 5 Timetables of Specimen Higher Grade Schools  
[Royal Commission (Bryce), 1895b, pp.414-415]



By 1894 there were nearly one hundred day 'schools of science,'<sup>32</sup> and the DSA's activities in the field of secondary education were a particular concern of the Bryce Commission, which reported in 1895. The effects of the DSA's arrangements on the overall balance of secondary curricula is well illustrated by the specimen timetables collected for the Commission. Summarizing generally, for boys' schools of the first and second grades the number of hours of mathematics in a week, including arithmetic, varied between four and seven for non-specialist classes, with additional hours provided for intending army officers and university aspirants specializing in mathematics. Predictably, schools of the first grade were not influenced by the DSA, though the example of Leeds Modern School illustrates the effects of the DSA on the curricula of some second grade schools. The top three forms of this school constituted an 'organized science school' providing thirteen, thirteen and a half, and fifteen and a half hours of mathematics and science, with as much as nine hours of mathematics in the top form [Royal Commission (Bryce), 1895b, pp.404-412]. For boys' schools of the third grade there is rather more variation in the amount of mathematics, down to as little as three hours of arithmetic only, but with much more generous provision for science and mathematics, including practical geometry, in higher grade schools under the DSA [pp.412-415]. (See Illustration 5.) In the case of girls' schools the amount of mathematics generally varied between two and five hours, with a greater tendency for this to be predominantly or wholly arithmetic. However, the influence of the DSA even extended to girls' education, as the example of Leeds Central Higher Grade Board School illustrates. The girls' department included an 'organized science school,' with five hours of mathematics boosted by three and a half

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32 The total number of 'classes' was around twenty-five times this figure. The bulk of the DSA's 'schools' and 'classes' continued to be held in the evening [Butterworth, 1978, p.21].

hours of practical geometry in the first year [pp.416-423].<sup>33</sup>

The Commission noted that the DSA had become essentially the central authority for technical education, as well as a government examining body in the vaguely defined secondary field. As regards the latter, the Commission drew attention to various limitations in its work as an educational authority. Its operations were too centralized and concerned principally with payments for results, with little attention being paid to educational means via the mechanism of inspection. Furthermore, it was too specialized, being an examining body only for science and art subjects, and with syllabi and methods geared principally to part-time adult evening education and not to day-time secondary education on 'liberal' lines. In addition to these educational arguments, and as in the case of the Education Department during the 1890s, the DSA's administrative burden as an examining body had increased alarmingly, and, even before its demise, some modification of its policies became necessary [Bishop, 1971, pp.190-201].

From 1894 the regulations for a 'school of science' were modified. The minimum specified time for science and art subjects was reduced to thirteen hours; grants were now to be paid partly on the results of inspection, with payment for results restricted to the more advanced stages; and inspection was to be extended to literary as well as scientific subjects, with a minimum allotment of ten hours per week. The DSA's general policy of payment for results was also gradually phased out, with some resulting diminution in the total number of examination entries by 1900 [Gosden, 1966, p.48; Butterworth, 1978, p.15]. It should be added that during the 1890s many more day schools of various types incorporated 'science classes,' in normally

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33 The DSA allowed the girls to substitute botany for practical geometry in the second year, which exemplifies differentiation and demonstrates at least some flexibility in the Department's operations.



up to at most four subject, rather than a 'school of science.'<sup>34</sup> In the 'classes,' pure mathematics and inorganic chemistry were the most popular subjects outside the technical schools, where practical geometry was the most popular choice, with mathematics and chemistry also popular additional subjects. Thus, in general, the DSA's policies strengthened the place of mathematics in schools below the first grade, but the character of the work undertaken is another matter. As HMI Fletcher [1912, p.93] subsequently remarked:

the great stress laid on Science, though in some ways it has helped to improve the Mathematics, has probably on the whole tended to obscure the need for improvement. Care had to be taken at one time that the demand for Science should not be treated as met to some extent by what was then the more familiar subject, Mathematics.

One early problem for the BE was to accommodate and gradually transform the concept of a 'school of science,' through its regulations for secondary schools. The BE also inherited the DSA's major role as an examining body in the field of day and evening further education in science and art, and early twentieth-century developments in this respect will be considered first.

The DSA's work in the field of further education was continued by a newly created Technological and Art Branch of the BE from 1903. Regulations distinct from those for elementary, secondary and teacher education were published annually, and included the arrangements for the examinations in science and art. The BE was not alone in the field of technological examining, which was also catered for by bodies like the City and Guilds of London Institute and the Union of Lancashire and Cheshire Institutes [Montgomery, 1965, pp.91-96, 212-227]. Teachers in this field eventually became nationally organized as the Association of Teachers in Technical Institutes, in 1904, and could now exert more effective pressure on the various examining bodies to

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<sup>34</sup> For example, for the year 1898-99, there were 159 day 'schools of science' and 533 day 'science classes,' though involving only forty per cent more students [Cane, 1959, p.59].



produce a system better adjusted to local and industrial needs, and to the teaching in the technical institutions themselves.<sup>35</sup> The ATTI pressed for improvements in the content and form of examinations, and, more radically, for the ending of purely external examining, such as was conducted by the BE in particular [Webb, 1915a, p.7]. As part of the BE's general policy of abandoning detailed and centralized curricular controls, and partly as a result of the ATTI's pressure, the BE itself gradually withdrew from technological examining. Examinations in the elementary 'stages' were discontinued in 1911 and all the BE's science examinations were phased out by the end of the First World War.

Other examining bodies continued their operations; the formation of new regional bodies was encouraged, as well as greater involvement of the institutions themselves; and the BE now pursued a policy of endorsement for 'grouped course' rather than individual subject certificates. Furthermore, from 1921 a system of national certificates and diplomas was developed jointly by the BE and certain professional bodies, such as the Institution of Mechanical Engineers, with the examinations conducted by the technical institutions themselves, or provided by an approved body [BE, 1924, p.30]. Thus the BE maintained some measure of control over the general arrangements for technological examining, even though the detailed centralized control of the old 'South Kensington' examinations had been eliminated.

The BE's policy of gradual withdrawal from actual examining meant that its direct influence on the curricula of technical institutions declined in the early twentieth century. However, by coincidence, in the same year as the BE Act was passed (1899), the DSA introduced a new subject called 'Practical Mathematics,' as an

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35 'Institutes' became 'Institutions' in 1907, and Abbott, a member of the Association (ATTI), was a leading figure in its organization, as well as in mathematical education [ATTI, 1954, pp.5-9].

alternative to the conventional pure mathematics. There were only 212 papers worked in the new subject's first year, compared with around twelve and a half thousand in practical geometry and twenty-four thousand in mathematics [BE, 1900b, p.35]. Although the total number of entries for these three subjects gradually declined, as a consequence of the BE's policy with respect to the DSA's examinations and the desirable range of their influence, there was a dramatic swing towards the new subject and away from pure mathematics over the first decade of this century [Brock and Price, 1980, pp.376-377]. The engineer John Perry<sup>36</sup> was the inspiration behind the new subject, which subsequently enjoyed enormous success within technical education, and the progress of the much wider 'Perry movement' will be explored in the later chapters of this thesis. It is sufficient here to quote some of Abbott's [1912a, p.10] opening remarks in his paper for the

#### Special Reports:

It is probably correct to say that nobody has done more to influence the teaching of Mathematics in this country during the last 15 or 20 years than Professor Perry; and in no branch of the work has he brought about greater changes than in the teaching of Mathematics to technical students. Until the introduction of his 'Practical Mathematics,' the mathematical teaching of students in Technical Institutions followed, for the most part, the ordinary conventional academic routine.... But all this was changed by the introduction of Professor Perry's 'Practical Mathematics'....

Thus, in sharp contrast to its general administrative policies regarding curricula, the BE itself became instrumental in major curriculum change within technical mathematical education.<sup>37</sup>

36 For Perry see the Appendix.

37 The BE's examinations in pure mathematics appear to have left much to be desired in the same period. The following bizarre item of 1904, with pungent comment, was included in the Mathematical Gazette's 'Pillory' for examination questions:

'The external measurements of a closed box are 36 inches, 2.2 feet, and .506 yards. Find the cubic space within if the wood of which it is made has a uniform thickness of one-tenth of a foot.'... Note the useful 'it,' the mixture of units, and the recurring decimal. English grammar, ordinary common sense, and physical possibility smashed in one question! Can anyone beat this? [Aleph, 1910, p.280]



### The Board of Education and Secondary Education

The BE and the new LEAs from 1902 inherited an unco-ordinated system of grant-aided secondary education in various forms conducted in a variety of institutions such as endowed grammar schools, higher grade schools, technical day schools and pupil-teacher centres, with curricula in many cases tailored to the DSA's system of grants. In addition, there existed schools independent of central and local funding, and headed by the influential boys' public schools of the HMC, which were closely linked with the ancient universities. It was now the joint responsibility of the new central and local authorities to transform the existing patchwork into a coherent and well-defined system of aided secondary schools. The Cockerton Judgement of 1901 had already affected the future of the higher grade schools, few of which became higher elementary schools, following the Minute of 1900, as opposed to municipal secondary schools under the BE. Higher elementary schools represented a blurring of educational categories, which was discouraged both legislatively and administratively. However, another form of quasi-secondary education was provided by the central schools from 1911, particularly in the London area, and the general development of post-primary education, other than in secondary schools, was stimulated by the Education Act of 1918. The whole subject was considered in detail by the Consultative Committee, which reported in 1926 [BE, 1926a, pp.26-69].<sup>38</sup>

Day technical schools and classes had also thrived under the DSA, but were somewhat uncomfortably accommodated within the BE's administrative framework, being dealt with under the regulations for further education of the Technological Branch from 1905. However, the

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38 The development of central schools, and senior elementary classes and schools, culminated in the notion of a secondary modern school. Senior School Mathematics was the subject of an Educational Pamphlet of the BE [1935].

development of junior technical education was such that these schools became the subject of separate regulations from 1913. Some technical schools took an alternative route and became municipal secondary schools under the BE [BE, 1938, pp.82-86; Millis, 1925, pp.98-118].

The BE's policy for the development of curricula in grant-aided secondary schools, with particular reference to the place of science, has been explored in some detail by Jenkins [1979, pp.2-14]. For the two-year period, 1902-1904, the existing 'schools of science' continued to operate on the general lines laid down by the DSA from 1895 (see p.30), being merely renamed 'Secondary Day Schools (Division A),' which now numbered over two hundred, with grants determined almost wholly by capitation and attendance. However, grants were also made available for 'Secondary Day Schools (Division B),' these being predominantly endowed grammar schools, which had to provide at least nine hours of science and art subjects per week. Given the necessarily higher running costs for 'Division A' schools, a higher rate of grant was provided than for 'Division B' schools, which numbered over two hundred and fifty in 1904, exceeding the number of 'Division A' schools. Thus the BE continued to accommodate the existing 'schools of science,' though no longer basing the grants on examination results. It is the 1904 Regulations, however, which have attracted the most attention from historians and been the subject of somewhat conflicting interpretations.

As a bifurcation of the DSA, a Secondary Branch of the BE, separate from the Technological Branch, had already been established to implement the BE's policy for the development of 'secondary schools' as units, which were required to provide a 'general' education, beyond elementary, and extending at least to the age of sixteen, with payment for means, involving inspection, and not payments for results. The BE's own single subject examinations were no longer intended for secondary schools, and, in any case, the many endowed schools which became



secondary schools under the BE were aligned with the public schools and the universities, the latter acting as the major secondary examining bodies [Montgomery, 1965, pp.88-90].<sup>39</sup> The 1904 Regulations no longer distinguished two types of secondary school, 'Division A' and 'Division B,' and instead required all schools to teach a range of compulsory subjects, with certain timetable requirements such as the allocation of 'not less than 7½ hours to Science and Mathematics, of which at least 3 must be for Science' [BE, 1904a, p.18]. The general concept of a secondary school was now aligned with the schools of 'Division B,' which had rapidly developed from 1902..

For historians, the major controversies concern the importance of the Regulations per se, and the role of Robert Morant in particular, for the future direction of secondary education along so-called 'liberal' rather than 'vocational' and non-university orientated lines.<sup>40</sup> The various arguments will not be pursued in detail here, though it does seem that the instrumental importance of the Regulations is easily exaggerated. 'Division A' schools were subsequently accommodated within the mainstream development of general secondary schools under the BE, but only gradually. Under the 1904 Regulations it was still possible to earn additional grants for pupils pursuing 'special courses' in science at an advanced level, with at least thirteen hours per week of mathematics (including theoretical and practical geometry), science and drawing [pp.19-20]. Presumably, these special arrangements, which persisted up to the 1906 Regulations [BE, 1906c, pp.3-4], were designed primarily to soften the financial squeeze on earlier

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39 The BE's regulations for further education, 1906-1907, still permitted examination of 'external candidates,' although the examinations were intended only for students and teachers within further education. A high additional fee might be charged for 'external candidates' [BE, 1906b, pp.28-34]. Secondary schools sometimes preferred the examinations of the College of Preceptors, as the fees were half those of the universities [Fletcher, 1911, p.451].

40 See, for example, the general discussions in BE [1938, pp.66-73], and the papers by Banks [1954] and Eaglesham [1962].

'Division A' type schools. However, for the 1907 Regulations the whole business of detailed timetable engineering was abandoned, including the differential grants for special courses, though a list of compulsory subjects, including mathematics, was still given, and the BE's approval of a school's general scheme was still a condition of receiving a grant [BE, 1907a]. A subsequent Circular of the BE explained that the earlier specific time allocations were 'of great service in practically impressing on schools the necessity of a certain breadth and solidity in the education given,' but were only a 'temporary expedient,' and were dispensed with as soon as their purpose had been accomplished [BE, 1913b, p.4]. The Circular added that the BE 'could do no greater dis-service to education than by attempting to check the spirit of exploration, experiment, and inquiry which should exist in every school' [p.5].

Around 1900 there was clearly much public concern for curricular distortion in certain quarters [Jenkins, 1979, pp.4-6], though Cane [1959] has argued from quantitative data that the scientific and technical bias tended to be exaggerated, and, in any case, much of the bias favoured mathematical subjects, which were well-established anyway, at least in boys' schools. Generally, the Regulations appear to have followed the dominant conception of a secondary education, once this had become divorced from elementary and technical education, and along lines established by the universities, the public schools, and the older grammar schools. Developments were such that, by 1907, the BE found it no longer necessary to dictate either the hours to be devoted to specific subjects, or what was to be put into the hours, through the mechanism of its own examinations. However, the BE still generally insisted upon mathematics, beyond arithmetic, in a four-year course up to the age of sixteen, but the important question of differentiation between the sexes could not be ignored.



The Regulations from 1905 allowed some general differentiation in the timetables for girls' schools where the total time of instruction was less than for boys [BE, 1923a, p.39]. From 1909 the BE granted the specific concession that housecraft might be substituted for science or mathematics, other than arithmetic, for girls over fifteen years of age [BE, 1913b, pp.29,31]. HMI Fletcher [1912, p.98] explained that the BE had adopted a 'waiting observant attitude' concerning differentiation, and, while the general policy was not to differentiate, greater flexibility regarding the scope of mathematics in girls' schools was granted in some cases. The BE's desideratum and the general trend was to provide at least two years of geometry and algebra for all girls, and often more, up to the expected minimum for boys of 'Algebra to progressions or the binomial theorem, and the equivalent of Euclid I.-IV. and VI.' [p.99]. The specific concession concerning housecraft persisted after the War, and the BE [1923a, p.46] still allowed some girls to drop mathematics or science, in certain circumstances, though by 1923 it was found that 'such applications are not so frequently made at the present time as they were in former years.' The BE's [p.124] Consultative Committee still suggested that in mathematics 'there might well be some differentiation between boys and girls both in subject-matter and in methods of teaching,' and claimed that this view was 'widely held by competent authorities.' The main issue in differentiation now concerned the character of girls' mathematical education rather than its scope [p.139]. From 1918 the latter was much influenced by the requirements of the First Examination for both boys and girls. Bringing some coherence into the system of secondary examinations was a major component of the BE's policy concerning curricular control. Before discussing examinations, some points remain to be made concerning the relatively weaker curricular controls exerted by HMIs and the BE's publications, apart from the Regulations.

From 1904, free inspection of grant-aided secondary schools under the BE's Regulations expanded rapidly, and was conducted by a newly organized Secondary Branch of the Inspectorate, led by the first Chief Inspector of Secondary Schools, W.C. Fletcher, a Cambridge mathematician and former Headmaster of the Liverpool Institute [BE, 1915a, pp.28-33; Siddons, 1959]. From 1906, public and private schools were also offered free inspection if they wished to be officially classed as 'efficient,' and most of the leading public schools subsequently took up this option [Selby-Bigge, 1927, p.144]. Operating on a district basis, HMIs acted as the eyes and ears of the BE, working in conjunction with the LEAs, governing bodies, and the schools. Given their wide and varied experience, HMIs, through their contacts with schools, provided one means for the dissemination of ideas. Their most powerful direct influence on the schools was exercised through the mechanism of 'Full Inspections,' instituted by Fletcher, and conducted on average quinquennially, by a team of HMIs over a period of up to five days [BE, 1915a, pp.33-41]. HMIs also became involved, anonymously, in the writing of various official guidelines for the secondary curriculum, as well as the provision of short courses for secondary teachers, begun in 1916 [BE, 1926b, p.154]. An alternative measure of 'efficiency' to that provided by HMIs was pupils' performance in examinations conducted by various bodies independent of the BE, and the relation between inspection and examination was a major concern of the BE in the early twentieth century.

From the 1850s it was the examination of pupils which became the principal means of promoting fair competition, determining 'efficiency,' and raising educational standards. Although the university examining bodies did make some provision for inspection, it was examination results and not educational means that were the dominant concern in the



nineteenth century. The limited scope of examinations as instruments of educational advance only slowly became clear, and, when the BE began to undertake inspection on a large scale, the problem of examinations and their effects rose to prominence. Furthermore, with the growing involvement of the BE in inspection, the universities gradually confined their operations to examination. Thus the flexible mechanism of inspection and the narrower but powerfully influential mechanism of examination were controlled by different bodies, unco-ordinated with each other [BE, 1911, p.435; BE, 1915a, pp.41-44]. A Report of the BE [1915a, p.16] admitted that secondary examinations 'grew up as a congeries of unrelated efforts,' and that the divorce from public inspection 'remained for long, and still to some extent remains, a grave weakness in the Secondary School system.' Although some co-ordination between the examining bodies was eventually achieved by 1918, the relationship with inspection remained problematic. The scope and content of secondary mathematics continued largely to be determined by examination requirements beyond the BE's control, though the various publications of the BE which concerned the curriculum were also an important influence, albeit less powerful, and particularly with regard to aims and methods.

Before the First World War, the BE issued a number of Circulars on specific subjects in the secondary curriculum [BE, 1913b, p.37]. The first such Circular was devoted to geography, and the first Circular concerning mathematics was issued in 1909 as Circular 711, Teaching of Geometry and Graphic Algebra in Schools [BE, 1909a]. Such Circulars were published anonymously, but it was Fletcher who was responsible for Circular 711 and the subsequent Circulars on mathematics, up to his retirement in 1926.<sup>41</sup> Circular 711 took a

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41 According to Siddons [1959, p.86], Fletcher sent drafts of these Circulars to selected persons for criticism. Bushell [1944, p.84] numbered Fletcher 'among the leaders of the revolt, and the guides in the new Mathematics.'

progressive line concerning the role of intuition and experiment in geometry, and powerfully argued for a broader axiomatic basis in deductive geometry for schools. In this, the Circular was ahead of the requirements of examining bodies, but it pointed the way forward, and Siddons [1959, p.86] justifiably claimed that its effect was 'very far-reaching and led up to the reforms that have since been made in the teaching of geometry.' The Circular also sought to clarify the place and scope of graphs in the mathematics curriculum.<sup>42</sup> In the light of experience, Circular 711 was superseded by two, more detailed Circulars, 851 and 884, both issued in 1914, and covering geometry and graphs respectively. Circular 884 was further revised and reissued in 1925 [BE, 1914a, 1914b, 1925b].

These widely circulated publications were important instruments of change, which had a direct influence on secondary teachers, who were largely untrained, on textbook writers, and on examining bodies.<sup>43</sup> The BE also issued general Circulars on the secondary curriculum in 1913 and 1922, the latter following the work of four special Committees from 1915, which considered and reported on the subjects of science, modern languages, classics and English [BE, 1913b; 1938, pp.81-82]. Mathematics was not investigated in this way, but had already been the subject of intensive international investigation, following the establishment of the International Commission on the Teaching of Mathematics (ICTM) in 1908. The BE co-operated in this venture and devoted two of its Special Reports, issued by the Office of Special Inquiries and Reports, to The Teaching of Mathematics in the United Kingdom, which included over nine hundred pages of individually prepared papers [BE, 1912a, 1912b]. The Office was also responsible

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42 Geometry and graphs, and the BE's Circulars, will be discussed in subsequent chapters.

43 Scrutiny of the prefaces of geometry textbooks in the MA's Library confirms the importance of the influence on textbook writers.



for various Educational Pamphlets of the BE, and one of these comprised a translated Syllabus of Mathematics for the Austrian Gymnasien, the issue of which was another contribution to the work of international comparison at this time [BE, 1910]. However, the BE withdrew its financial support for the British contribution to the ICTM in 1914 [Greenhill, 1914, p.259]. The Pamphlets and Special Reports were intended to be informative, and to stimulate debate and experimentation, whereas the Circulars were more authoritative in character. In relation to secondary schools, the BE's Consultative Committee reported in detail on examinations and practical work before the War, and on differentiation and general developments between the Wars [BE, 1911, 1913c, 1923a, 1938].

Although the various Reports of the Consultative Committee had important administrative implications, the pronouncements on curricula tended to be rather general and somewhat visionary in character. The Reports provide much valuable data for the curriculum historian, but seem relatively unimportant as specific instruments of curriculum change, certainly in mathematics.<sup>44</sup> Secondary examinations not surprisingly occupied the Committee's attentions at an early stage, and this subject has very important implications for the development of curricula generally.

#### Examinations and Secondary Education

The complex and controversial subject of examinations has warranted a number of detailed investigations. The Consultative Committee's Report of 1911 ran to over six hundred pages, which is some measure of the importance and complexity of examinations, just with reference to secondary schools. The International Institute Examinations Enquiry in the 1930s was wider in scope, and the resulting published output included a bibliographical survey of

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<sup>44</sup> See also the section on mathematics in The Education of the Adolescent [BE, 1926a, pp.214-220].

English examinations, 1900-1932, covering more than seventy periodicals, and well demonstrated the general level of concern for the subject in this period [Champneys, 1934]. A more recent and comprehensive general survey of the development of English examinations as 'administrative devices' has been provided by Montgomery [1965]. The development of examinations affecting English secondary schools in the periods 1850-1900 and 1900-1945 has been considered by Roach [1971, 1979].<sup>45</sup> Petch [1953] has provided an inside view of the work of one examining body, the Northern Universities Joint Matriculation Board (NUJMB), and Brereton [1944], of the Cambridge Local Examinations Syndicate, a more polemical view of the educational role of examinations as a 'unifying influence' on the secondary curriculum.<sup>46</sup>

A useful historical paper by Morris [1961] considers examinations as devices serving the diverse purposes of motivation, social engineering and administration, as well as educational purposes. One examination may serve a variety of purposes, some purposes originally unintended, and even conflicting purposes. The difficulty of combining educational and administrative purposes is well illustrated by the examinations of the Education Department and the DSA, already discussed (see pp.15-22,27-33). Secondary scholarship examinations were both an administrative device and a tool of social engineering (see pp.24-25). Furthermore, what should have been a uniform qualifying hurdle became a severe competitive test, with varying standards nationally. Examinations for entry to public service and the professions similarly served principally administrative and social rather than educational purposes. The discussion which follows will be restricted to those developments which have important implications for secondary mathematical education from the late nineteenth century.

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45 See also Roach [1976] on examining and history teaching.

46 See also Brereton [1969] in a recent international survey, The World Year Book of Education 1969: Examinations.



The general English mania for examinations as a panacea for educational and social deficiencies and injustices developed rapidly from the mid-nineteenth century. However, the complex system of examinations which came to affect secondary schools developed along arbitrary and unco-ordinated lines. By 1900, secondary schools were being influenced in various ways by the different examinations of the universities, particularly Oxford, Cambridge, and London, the College of Preceptors, the DSA, and the Civil Service Commission, as well as those of a variety of professional bodies and specialist organizations [BE, 1911, pp.316-328].

The competitive scholarship examinations of the ancient universities were a particularly important early influence on the advanced work undertaken in public and endowed schools, whose prestige became measured by scholarship successes. This system also had an important effect on the whole work of the school, for as Macaulay and Greenstreet [1912, p.211] remarked in their paper on mathematical scholarships:

by an almost irresistible tendency the lines laid down for secondary training have been adapted to the needs of the future University student rather than to the requirements of the average boy or girl.

Also, some mathematics was included in the pass examinations for all degrees, termed Responsions or 'Smalls' at Oxford, and the Previous or 'Little-go' at Cambridge. From around 1890 both these examinations could be taken by secondary school pupils before entering the university [BE, 1911, pp.321-322]. The mathematical requirements for Responsions were surprisingly limited to arithmetic and either some Euclid or some algebra [Hawkins, 1912, p.439].<sup>47</sup>

Like Responsions and the Previous, London Matriculation, from

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47 In 1901, one public schoolmaster referred to Oxford as 'a far worse offender' than Cambridge, as regards the character of its elementary mathematical papers [Hurst, 1901, p.371].

1838, was instituted as a general university qualifying examination, but it subsequently came to be used by more ambitious secondary schools as a prized certificate in itself, for pupils from the age of sixteen [Cane, 1959, pp.57-58]. Arithmetic, algebra to progressions, and the subject-matter of Euclid I-IV were the compulsory requirements in mathematics up to 1922.<sup>48</sup> Over the first decade of this century London widened the scope of its examining, to cater specifically for schools' needs, and provided school certificates below, above and at matriculation standard, with inspection also undertaken [BE, 1911, pp.15-16]. However, the popularity and prestige of 'London Matric' persisted, and its effects were strongly attacked by Hawkins [1912, pp.446-448] in the Special Reports. He estimated that over thirty per cent of candidates took the examination just for the certificate; the failure rate was up to sixty per cent; special coaching of selected pupils was essential; and yet the mathematical syllabus was strictly limited in scope, with the demands greatly stretched 'by setting difficult and catchy questions of a puzzling and confusing type' [p.448]. Another teacher complained 'we are kept down and deadened by these terrible literary problems which have nothing to do with mathematics' and that 'the examiners still exact a terrible amount of technical ability.'<sup>49</sup> Hawkins [p.461] concluded:

The want of sympathy which this University shows towards the teaching profession is a survival from the days when London University considered teaching as quite outside its sphere.

Between the Wars, London continued to be inflexible in its matriculation requirements and at odds with other examining bodies [Montgomery, 1965, p.131, 135-139]. Furthermore, there is abundant evidence that London persistently showed greater resistance to pressure for change in its basic mathematical requirements than other bodies

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48 London Matriculation's syllabus changes in mathematics, from 1844 to 1936, have been summarized by Retter [1936, pp.129, 167, 207].

49 E.H. Butt in Godfrey [1910a, p.242].



[MA, 1930a; BE, 1932].<sup>50</sup> The ancient universities responded to the schools' general needs much earlier than London by providing Local Examinations from 1858.

Roach [1971] has made a detailed study of the nineteenth-century development of the Locals, and, as he points out 'During approximately forty years they were the only system of secondary-school examinations with a national constituency' [p.viii]. These examinations were intended for endowed, proprietary and private schools serving the middle classes i.e. predominantly second grade secondary schools, and were controlled by the Oxford Local Examinations Delegacy and the Cambridge Local Examinations Syndicate. They were provided to stimulate middle class education along 'liberal' rather than narrowly vocational lines, and to give parents some guarantee of general educational standards from an impartial and respected authority. The Locals were neither class nor school based, but were individual, competitive examinations taken at local centres. By 1900, the Locals were offered at four levels, Preliminary, Junior, Senior, and Higher, covering a wide age range from fourteen to nineteen, and open to girls as well as boys [BE, 1911, pp.10-14, 316-317]. From 1877, the Senior Locals could be used to gain exemption from Responsions and the Previous via an approved choice of subjects. Beyond such rudiments as English and arithmetic, candidates for the Locals were offered considerable latitude in subject choice for a certificate. Thus, in the Oxford Locals up to 1900, religious knowledge, English and French were more popular than mathematics [Cane, 1959, pp.55-56]. Both the Delegacy and the Syndicate very gradually became more sympathetic to the schools' own needs. By 1905, school-based examining and inspection was offered, with school certificates and special certificates for the army, and these arrangements were variously linked

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50 The detail will be pursued in Chapter 4.

with the well-established Locals [Montgomery, 1965, pp.48-51]. Schools of the upper classes, i.e. first grade secondary schools, also established close and quite distinctive links with the ancient universities via examinations.

The threat of State intervention in secondary education led to the establishment of the HMC in 1869, for schools having close ties with Oxford and Cambridge. This bond was strengthened by the establishment of the Oxford and Cambridge Schools Examination Board (the Joint Board) in 1873, specifically for the inspection and examination of public schools. As Roach [1971, p.236] has suggested, social stratification was a key factor, and 'a new layer had been inserted between the Universities and the Locals.' The examinations were opened to girls from 1879, though the strong link with boys' public schools persisted. By 1905, Lower, School, and Higher Certificates were offered, generally for the sixteen to eighteen age range [BE, 1911, pp.14-15,317]. Unlike the Locals, all the examinations were school-based, less competitive in character, and served jointly the needs of individuals for certification, and of whole classes for more general educational purposes [Story, 1912, pp.553-554]. The Higher Certificate was a grouped subject certificate, commonly used to gain exemption from Responsions and the Previous. This partly explains the fact that between 1880 and 1900 elementary mathematics was the most popular subject choice, and, furthermore, its popularity increased over this period [Cane, 1959, pp.54-55].<sup>51</sup> In sharp contrast to the London Board, it is also clear that the Joint Board was notably sympathetic to progressive tendencies in mathematical education, particularly after 1910 [Siddons, 1928; BE, 1932].<sup>52</sup> The general pattern of

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51 For some mathematical syllabus changes of the Joint Board, 1874-1936, see Retter [1936, pp.128, 163, 214].

52 The links between the Joint Board, the MA and the HMC, and Godfrey's role in particular, will be explored in later chapters [King, 1981].



overall strategy [Webb, 1915b, pp.12-13].<sup>55</sup> As regards social status, the College's examinations catered principally for private and small proprietary schools i.e. schools of the third grade, and, in particular, many girls' schools [Cane, 1959, pp.58-59]. Furthermore, the examinations were taken, on average, by the relatively young age range from twelve to sixteen [BE, 1911, p.317]. Up to the turn of the century the College's examinations were numerically more important than the Locals, though inferior in status. Certificates were awarded in three classes, and a 'Lower Forms' examination was provided from 1895. The competitive aspect featured prominently, with class lists in order of merit and special awards published regularly in the College's journal, the Educational Times [Montgomery, 1965, pp.63-65]. As a compulsory subject, arithmetic featured conspicuously in these examinations.<sup>56</sup> However, from around 1895, the College's examining in secondary education entered a period of decline. The Preliminary Locals provided new competition for the College in the examining of younger pupils, but, more serious was the BE's general policy concerning examinations and the control of curricula. As well as discouraging through its Regulations the examining of younger pupils, the BE persistently refused to give the College official recognition as a secondary examining and inspecting body.<sup>57</sup> The College's relatively low social status and lack of university respectability appear to have been the major contributory factors here. By 1910 it was the Locals and the various matriculation examinations which dominated the secondary field, though the College's examinations were 'sometimes preferred' [Fletcher, 1911, p.451], being cheaper and regarded as slightly easier [Hawkins,

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55 The College's general educational work will be considered further in Chapter 3.

56 Pendlebury's [1899, p.v] arithmetic textbook catered specifically for the College's examinations, with specimen papers included, as well as for the Locals.

57 Significantly, in the historical introduction to the Consultative Committee's Report, the College's examining warranted just one brief paragraph [BE, 1911, pp.9-10].

1912, pp.439-441; Story, 1912, pp.553-554]. The general pattern of secondary examining was further complicated by the existence of a large number of preliminary examinations for specific vocations, particularly those conducted by the Civil Service Commission and the various professional bodies [BE, 1911, pp.7-9,19-25]. The multiplicity of examinations and their effects on curricula was a major concern of the BE in its early years.

The pattern of developments in England was such that, by the 1890s, the idea of a centralized secondary examination system, closely linked with inspection, could not be seriously entertained. A system such as that in Scotland, where the Education Department was responsible for inspecting and examining for secondary certificates, and in Prussia, where secondary curricula were powerfully influenced by the centralized Arbiturienten Examen, might have been desirable, but was now not possible, given the strength of the universities' hold on the examinations for secondary schools [Story, 1912, pp.556-557]. All that could now reasonably be achieved was some standardized general system of certification which would remain 'basically in private hands' and be 'unique among European countries' [Roach, 1971, p.256]. The early twentieth-century developments which eventually culminated in the settlement of 1918 have been considered in some detail by Roach [1979].

The question of examinations was first referred to the BE's Consultative Committee in 1902, and in the following year a Conference was held involving the Committee and the Universities of Oxford, Cambridge, London, Durham, Victoria and Birmingham [Petch, 1953, pp.54-55]. The Committee had proposed, and subsequently widely publicized, a system of 'Leaving Certificates.'<sup>58</sup> Victoria University was generally sympathetic, and the new NUJMB submitted its own scheme for leaving examinations. However, the two ancient universities, acting in

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58 The proposals were published in the Mathematical Gazette in particular [BE, 1904b].



conjunction, raised fundamental objections, particularly to what appeared to be an attempt to adapt the Prussian model to English conditions [BE, 1903, pp.60-62]. Given the various administrative and financial difficulties, the proposals were subsequently shelved by the BE, and the general issues were not again referred to the Committee until 1909. However, from 1907 the BE's [1907a, p.3] Regulations did generally prohibit the examination of pupils in grant-aided schools below the age of fifteen.

The Consultative Committee reported on an impressive scale after two years of intensive investigation [BE, 1911]. The main areas of concern administratively were the multiplicity of examinations and the various bodies' independence; the complexity and incompleteness of the system of 'equivalents' between examinations; competitive excesses reinforced by advertisement; the BE's inadequate role; and the separation of inspection and examination. Regarding examinations in relation to curricula, concern focused on the restriction of content, methods and experimentation; the disintegration of classes; the teachers' inadequate role; the absence of uniform standards; and the generally narrowing influence. The analysis of inadequacies was detailed and powerful, but changing the system in practice was more problematic, and some of the Committee's proposals appear to have ignored the deeply entrenched nature of the existing system. On the desired links with inspection, Brereton [1944, pp.85-86] subsequently remarked that the Committee 'failed to lay bare the realities of the situation' and that unity 'is no more a reality to-day than it was in 1911.'

The Committee clearly regarded a uniform examination system as an instrument for reinforcing the concept of a liberal secondary education, previously embodied in the 1904 Regulations, which was the subsequent objective of the grouped subject system from 1918

[BE, 1911, p.xii].<sup>59</sup> However, the BE proceeded cautiously. There were still major financial difficulties [p.iii], and the Committee's proposal for an Examinations Council with executive powers was rejected as unrealistic [Petch, 1953, p.66]. Following consultations with the universities, teachers' associations, and LEAs, the first Circular [BE, 1914c] on the proposed new scheme was issued in 1914, though partly as a consequence of the War the system did not become operable until four years later.

The full details of the scheme were spelt out in a series of Circulars up to 1918. Two examinations were to be provided, one to test a general secondary education around the age of sixteen, and termed the First Examination, and the other, the Second Examination, to test two further years of more specialist studies. The First Examination corresponded closely with the existing School Certificate examinations of the various university bodies.<sup>60</sup> Candidates were to be assessed in three compulsory groups of subjects, one of which was science and mathematics, and the whole form rather than the pupil was to be the unit for the examination. Two levels of pass were specified, with a simple pass intended to be slightly below matriculation standard. A pass with credit was to correspond to matriculation standard, for the benefit of the universities and various professional bodies. Unfortunately the latter came to be used as an end in itself for its higher esteem, and the two purposes of certification for a general secondary education and qualification for the universities and various professions conflicted between the Wars, with predictable effects on secondary curricula. The First Examination followed the lines of the much older Senior Locals, and

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59 As Brereton [1944, p.87] has pointed out, between the Wars the grouped system was inconsistent with the BE's subsequent principle that examinations should follow and not control curriculum.

60 The First and Second Examinations were commonly referred to as the School and Higher School Certificates respectively.



was thus far from new in conception. Science and mathematics was one of the optional specialist groups for the Second Examination. For grant-aided secondary schools, the BE's subsequent policy was to limit examining of a general secondary education to the First Examination, conducted annually. This had a serious effect on the range of influence of the universities' Preliminary and Junior Examinations and of the examinations of the College of Preceptors particularly.. The BE's role was that of a co-ordinating authority, assisted by a Secondary School Examinations Council (SSEC), representing the interests of the universities, the LEAs and the teachers. Provision was also made in the scheme for the use of schools' own special syllabuses and teachers' estimates of attainment. The seven approved bodies in 1918 were the Cambridge Syndicate, the Oxford Delegacy, the Joint Board, the NUJMB, and the Boards of London, Durham and Bristol. The Central Welsh Board was also approved two years later [BE, 1915b, 1917]. As Roach [1979, p.52] has remarked, it was the settlement of 1918 which was 'to dominate the secondary schools during the inter-war period.'

The main function of the SSEC was to maintain parity of standard between the approved bodies' examinations. However, it was only an advisory body for the BE, and could make suggestions to the examining bodies, but had no power to implement change. The SSEC provided a forum for discussion, reported regularly, and occasionally conducted more detailed 'investigations' of the examining bodies' operations. 'Investigations' were undertaken in 1918, 1924 and 1931, the results being communicated to the examining bodies confidentially, with the first separate publication in 1932 [BE, 1932, p.5]. Brereton [1944, p.96] has suggested that 'its faith in the outworn practice of publishing official reports and hoping that reforms will follow has not been justified.' As Petch [1953, pp.76-79] has emphasized there were also methodological problems involved in comparing the standards

of different types of paper set on different syllabuses, this degree of uniformity not being a feature of the scheme. The SSEC's weakness as an instrument of change is well illustrated by the character of its reporting on mathematics in 1932.

By 1930, mathematics was taken by 'almost all' the boys and 'a large proportion' of the girls, with around ninety-five per cent of candidates taking mathematics, which was usually compulsory for matriculation. Although syllabuses now varied 'but slightly,' London was out of step with the other seven bodies [BE, 1932, p.114]. As well as insisting on a pass in mathematics or arithmetic, London omitted similarity (Euclid VI) and trigonometry, and was criticized for its narrow attitude, though not named. By contrast, two bodies, again unnamed, were commended for the distinctive character of their papers. The NUJMB divided their papers into two sections of short and straightforward, and longer and more difficult questions.<sup>61</sup> The Joint Board set mixed papers, whereas the other boards set separate papers on the branches of mathematics [pp.116-117].<sup>62</sup> Thus the detail concerning syllabuses and papers remained outside the control of the BE and its SSEC, and largely in the hands of the universities. However, provision was made for special syllabuses.

The principle of teachers' involvement in examinations has a long history. The College of Preceptors' ideal of participation by teachers in examining was abandoned at an early stage, given the need in the mid-nineteenth century for some objective measure of standards, as embodied by the Locals [Montgomery, 1965, pp.63-64]. However, this theme re-emerged later in the nineteenth century as part of the ancient universities' arrangements for school-based examining [pp.50-51]. The newer universities gradually followed this lead in the early twentieth

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61 This innovation of the NUJMB was brought in soon after the War, following pressure from several headmasters [MA, 1930a, p.151].

62 The Joint Board set mixed papers from 1921 [King, 1981].



century, catering in principle for the use of special syllabuses, on payment of an additional fee, and involving teachers as examiners for school certificates [Petch, 1953, p.56]. Following the Consultative Committee's Report, the BE officially encouraged teacher participation in the forms of consultation and representation, provision for special syllabuses, and the use of teachers' estimates of attainment [BE, 1914c]. This was with the First Examination only in mind, and the universities continued to act in a detached and wholly external manner as regards more advanced and scholarship examinations. Teachers' attitudes to participation have been usefully discussed in a paper by Gillham [1977]. Teachers generally appear to have been 'reluctant beneficiaries,' both as regards the use of schools' own assessments and special syllabuses.<sup>63</sup> As regards the little use made of special syllabuses, the general pattern was confirmed by Brereton [1944, p.104]. Various causal factors have been discussed by Gillham such as acceptance of the status quo and the external authority of the universities; teachers' inexperience in syllabus construction; and lack of knowledge of this provision. On the universities' side, Gillham draws attention to poor advertising, the administrative and financial hurdles, and their general lack of enthusiasm for this mode of examining. In spite of teachers' general conservatism, and the persistence of Victorian attitudes to the function of examinations, there is some evidence of individual initiatives in mathematics.

At a meeting of the College of Preceptors in 1902, J.J. Findlay [1902, p.185] referred to the special syllabus provision of the Joint Board and the Central Welsh Board. At least one school had used this provision to get Euclid abolished entirely, before examining bodies

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63 Various pressure groups, particularly the MA, and their relationship with examining bodies will be considered in Chapters 3 and 4.

had generally moved in this direction.<sup>64</sup> Some twenty years later this provision was used by a group of nine schools to permit the use of logarithms, these being still generally prohibited by the London Board, which subsequently changed its regulations [MA, 1930a, pp.147-148]. The NUJMB introduced some calculus into additional mathematics for the First Examination in 1928, following the action of nine schools which had utilized a common special syllabus two years earlier [MA, 1932a, p.100]. In connection with the introduction of logarithms, and the early treatment of trigonometry and calculus, HMI Carson [1929, pp.26-27] emphasized that these developments were not forced on the schools by examining bodies, but rather 'the act of the schools themselves' prompted examining bodies, with varying degrees of sympathy, to alter their requirements to accommodate persistent classroom tendencies. Godfrey [1912a, p.170] too, whilst remarking that 'English education is dominated by examinations,' significantly added that 'The most serious opposition to a general movement ... will come from teachers; examiners generally cede to a fairly universal demand' [p.173]. Certainly, at the turn of the century, HMI Brill's [1901, p.278] remark that 'the teaching of no other subject has been more effectively cramped by [examinations] than that of mathematics' reflected a rapidly growing feeling, and the early dramatic changes in geometry, which were soon to follow, will be discussed in the next chapter. However, even given the examining bodies' generally conservative attitudes, their reluctance to eliminate ageing content, and the tendency to 'the elaboration of detail,' bringing with it 'undesirable accretions' [Fletcher, 1912, pp.92-93], it is important not to exaggerate the extent to which examinations prevented curriculum change, and were out of step with the majority of teachers' preferred

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64 Findlay was very probably referring to the Cardiff Intermediate School,, where he was the headmaster and introduced a new approach to geometry throughout the school [Woodall, 1902]. For Findlay see the Appendix,



practices. The major difficulties sprang rather from the infrequency and discontinuity of change in examination requirements, and the unique independence of English examining bodies, even after 1918, which contrasts sharply with the Scottish pattern [Gibson, 1912]. One non-university examining body, the Civil Service Commission, hitherto only briefly mentioned, was distinctive in its operations, and its work had important implications for the reform of mathematical education, particularly in public schools.

The Civil Service Commission was established in 1855, and, by 1870, virtually all Civil Service appointments were made via open competitive examinations. In the same year the Commissioners also took over the examinations for entrance to the military academies of Sandhurst and Woolwich. Candidates for the various appointments covered a wide range from fifteen upwards, with considerable variation in the calibre expected. These examinations served principally administrative purposes, and the need for special preparation had a distorting effect on secondary schools' organization and curricula. To compete with the special Civil Service and army 'crammers,' and prevent the early leaving of pupils, public and proprietary schools set up special classes. Certain schools, such as Rugby, Eton, Cheltenham and Wellington, became specifically known for the special provision they made for army candidates. More generally, the Commission's examinations were one factor in the development of separate classical and modern 'sides,' thereby reducing the traditional classical dominance of public schools' curricula [Montgomery, 1965, pp.29-31]. With growing pressure from headmasters, the Civil Service Commission altered its scheme of examinations during the 1890s, to reduce the harmful effects on the schools [BE, 1911, pp.7-9]. These examinations were conducted on a large scale over the first decade of this century, though the numbers of secondary school pupils involved

is not clear [pp.318-319]. Palmer [1912, p.256], in the Special Reports, strongly criticized most examinations in arithmetic, though commended the Commission for its reforms, which 'not only allow but encourage the teaching of Arithmetic on satisfactory lines.' Bushell [1947, p.84] too later acknowledged that 'a debt is owing to the mathematical examiners of the Civil Service Commissioners.'<sup>65</sup> The Commission's army examinations directly affected only a small number of older pupils, but Godfrey [1912b, p.431] judged that 'The Army requirements during the last 10 years have no doubt had a powerful reaction on the character of the teaching throughout English public schools.' During the early stages of the Perry movement, one school-master remarked:

Whatever faults they [the army examinations] have had in the past, I know of no papers which have gone farther towards encouraging - I might almost say enforcing - some of Professor Perry's proposed reforms.<sup>66</sup>

The army requirements and those for the second division of the Civil Service encouraged a more practical and broader mathematics curriculum, including mechanics and the calculus [Eggar, 1912, pp.338-339; Jackson, 1912, p.367]. The early twentieth-century arrangements for the education of naval cadets were also distinctive and have implications for innovation in mathematical education.

In 1903, the Royal Naval Colleges at Osborne and Dartmouth began to accept naval cadets at the early age of twelve to thirteen years [BE, 1911, pp.23-24]. These two institutions enjoyed two advantages over public and other secondary schools as regards the opportunities for curriculum development. Firstly, all the boys were being prepared for a common vocation, with fairly well-defined educational

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65 D.B. Mair, a Cambridge mathematician and educationalist, joined the Commission in 1896, and was a leading figure in the reform of its examinations [Times, 23rd May 1942, p.7]. For Mair see the Appendix.

66 E.M. Langley in Perry [1901a, p.44]. The Commission's mathematical influence will be considered further in Chapters 5 and 7.



prerequisites. Secondly, the cadets were not subjected to any external examinations, but to a common internal examination at sixteen to seventeen years of age, and one devised to follow a curriculum designed from scratch on the basis of educational and vocational considerations. Thus, in mathematics, trigonometry, mechanics and calculus were all included and treated on pioneering lines [Ashford, 1912]. The Colleges were essentially operating as laboratories for curricular experimentation, without external constraints. The detail was described by Mercer [1912] of Dartmouth in the Special Reports. The naval mathematics curriculum was certainly atypical in the early twentieth century, but it pointed the way forward in a number of respects. Godfrey became Headmaster at Osborne in 1905, following Ashford [1906], who became Headmaster at Dartmouth, and was a pioneer in mechanics teaching [Howson, 1973a, p.167]. Mercer [1906, 1910] produced innovatory textbooks on elementary trigonometry and calculus, based on the work of both Colleges.<sup>67</sup> Most teachers were, of course, less gifted and working in far from ideal circumstances. The production and quality of the teaching force generally has still to be considered.

The teaching force itself is a fundamental factor determining the state of the curriculum at a given time, and also determining the character of change over time, particularly where the concern is for the realities of life in typical classrooms. It is through the ordinary teacher's interpretations of the intentions of the producers of classroom materials, particularly textbooks, and the framers of examination syllabuses and papers, as well as the providers of advice of various kinds, that ideals and their general realization are mediated. The forms of preliminary general education and specific vocational training are two important elements governing teachers' interpretations. There

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67 The detail will be explored in Chapter 7. The naval scheme is reproduced in Griffiths and Howson [1974, pp.159-178]. For Mercer see the Appendix.

have been a considerable number of general historical studies of various aspects of teacher education and professionalization. The subject has warranted a bibliographical survey by Tempest [1960], one on the institutions themselves by Berry [1973], and a large number of theses and dissertations have appeared [History of Education Society, 1979b, pp.100-109].<sup>68</sup> A grant-aided investigation of general developments in England and Wales was conducted in the 1920s, resulting in a large-scale report by Jones [1924]. Not surprisingly, the BE's publications included detailed surveys of various aspects of teacher education, and, specifically, the pupil-teacher system [1907b], elementary teacher training [1914d, 1925c], and the training of secondary women teachers [1912d].

Turning specifically to the preparation of mathematics teachers, it appears that in the early twentieth century the general level of concern for the various issues involved was much greater in America and on the Continent, particularly in Germany, than in England. Some American comparative work in this field, which was one focus chosen by the ICTM, was mentioned in Chapter 1 (see p.6). Out of nearly one hundred items on this subject listed in Smith and Goldziher's [1912, pp.81-85] Bibliography only three concern specifically English conditions.<sup>69</sup> Notable individual exceptions to the general English pattern before the War were Professor G.H. Bryan [1907a, 1908a, 1912a],<sup>70</sup> and two leading innovators in teacher education generally, Nunn [1912a, 1919], at the London Day Training College, and Findlay [1913], at Manchester University.

The discussion which follows will be restricted to a brief outline of general administrative developments, but also including material which sheds some light on the reform of mathematical education.

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68 A number of sources are listed in Jenkins' [1980, pp.48-49,82-83] survey. The useful doctoral theses on general developments by Rich [1933] and Tropp [1957] have been published.

69 Two of these items were, in fact, articles in foreign journals.

70 For Bryan see the Appendix.



The preparation of elementary and secondary teachers involved fundamentally different nineteenth-century traditions, and will therefore be considered separately.

### Elementary School Teachers

From the mid-nineteenth century, the Education Department exercised a tight control over the system of entry to and training in the colleges, through a system of inspection and examination, with payments for results, and uniform syllabuses published annually in the Elementary Code. To stimulate the supply of candidates for the colleges, and, at the same time, to contribute to the supply of assistant teachers in the schools, a pupil-teacher system had been in operation since 1846. Here too the Department conducted annual examinations for entrance to and during a pupil-teachership, linked with inspection and payments. After 1870, School Boards began to undertake central instruction of pupil-teachers in special pupil-teacher centres, which spread rapidly throughout the country after 1875 and into the 1890s [Rich, 1933; BE, 1907b, pp.8-9]. Up to 1890, the normal pattern of training was residential in a denominational training college for two years, but thereafter the universities became involved.

During the 1890s an alternative pattern of training developed in newly established day training colleges, linked to university institutions. An important new principle was established - that of internal examining of students in academic subjects - and it became possible to train and study concurrently for a degree over three years. The involvement of the universities had a beneficial effect upon the quality of training in the older colleges. There was a further broadening of interest and involvement in elementary training after 1902, when the new LEAs also entered the field and established new colleges [BE, 1914d, pp.4,28-32]. Towards the end of the century, the pupil-teacher system became the subject of mounting concern.

A Departmental Committee considered the pupil-teacher system in detail over the period 1896-1898. Whilst not seeking to abolish the system, the Committee made various recommendations for its improvement, which were subsequently implemented. By 1902, the period of pupil-teachership was limited to three years, from a minimum age of fifteen, and annual examinations linked with payment for results were abolished. However, the BE still conducted examinations for acceptance as a pupil-teacher, and for entry to the colleges, though suitable university examinations were also accepted [BE, 1907b, pp.10-13]. The BE's examinations for entry to a college reflected the normal conception of a general education, which would subsequently be extended in the colleges, and assessed in the BE's certificate examinations.

In 1901, the mathematical requirements for college entrance, through the King's Scholarship Examination, were limited in scope, and exemplify differentiation. The 'principles and practice' of arithmetic 'including the measurement of rectangular areas and solids' was compulsory [BE, 1901b, p.495]. For men only, Euclid I and II was prescribed, and algebra up to quadratics 'together with simple questions on ratio, proportion, variation and arithmetical progression' [p.495]. The subject-matter being conventional, no further detail was necessary. Most of the successful candidates were being prepared for this examination in the pupil-teacher centres, which provided a form of secondary education, though cramming for college entrance was the major objective. The centres had developed before a coherent system of secondary education existed, but it became the BE's general policy to merge the preliminary education of intending teachers within the developing system of maintained secondary schools, and to eliminate the system of special preparation for the teaching profession [BE, 1907b, pp.18-19]. The various changes were



**13.—ARITHMETIC :—**

Excluding Cube Root, Scales of Notation, Foreign Exchanges, True Discount, Troy Weight, and Apothecaries' Weight.

Any question on Stocks will be of a simple character, and will not involve a knowledge of "Brokerage."

Candidates must understand the principles of the Metric System.

**14.—ALGEBRA (for Men only) :—**

As far as, and including, quadratic equations of one unknown quantity and simple simultaneous equations of two unknown quantities, with easy problems leading up to these equations.

When questions are set on graphs, squared paper will be provided.

**15.—GEOMETRY (for Men only) :—**

Candidates may be examined either (A) in Euclid or (B) on the Syllabus set forth below ; they cannot, however, be examined both on this Syllabus and in Euclid.

**A.—EUCLID :** Books I. and II., with simple geometrical exercises,

N.B.—Euclid's definitions will be required, and no axioms or postulates except Euclid's may be assumed. The actual proofs of propositions as given in Euclid will not be required, but no proof of any proposition occurring in Euclid will be admitted in which use is made of any proposition which in Euclid's order occurs subsequently.

or

**B.—(SYLLABUS FOR CANDIDATES WHO DO NOT TAKE EUCLID).**

Every Candidate taking this Syllabus must be provided with a ruler graduated in inches and tenths of an inch, and in centimetres and millimetres, a small set square, a protractor, compasses furnished with a hard pencil point, and a hard pencil.

Figures should be drawn accurately with a hard pencil.

Questions may be set in which the use of the set square or of the protractor is forbidden.

Any proof of a Proposition will be accepted which appears to the Examiners to form part of a logical order of treatment of the subject. In the proof of theorems and deductions from them, the use of hypothetical constructions is permitted.

**PRACTICAL GEOMETRY :—**

The following constructions and easy extensions of them :

Bisection of angles and of straight lines.

Construction of perpendiculars to straight lines.

Simple cases of the construction from sufficient data of triangles and quadrilaterals.

Construction of parallels to a given straight line.

Construction of angles equal to a given angle.

Division of straight lines into a given number of equal parts.

Construction of a triangle equal in area to a given polygon.

Construction of tangents to a circle.

Construction of common tangents to two circles.

Construction of circumscribed, inscribed, and escribed circles of a triangle.

Candidates will be expected to be acquainted with the forms of the cube, the rectangular block, the sphere, the cylinder, and the cone.

**THEORETICAL GEOMETRY :—**

The substance of the theorems contained in Euclid, Book I., Propositions 4-6, 8, 13-16, 18, 19, 26-30, 32-41, 43, 47, 48, and Book III., Propositions 3, 14-16, 18-22, 31. Questions upon these theorems, easy deductions from them, and arithmetical illustrations will be included.

spelt out in the Regulations for the Instruction and Training of Pupil-Teachers, which was first published separately from the Elementary Code in 1903.

The 1903 Regulations raised the minimum age for a pupil-teacher to sixteen, and encouraged LEAs to cater for pupil-teachers in secondary schools, to convert centres to ordinary secondary schools, and to provide secondary scholarships for intending teachers [BE, 1907b, pp.16-18]. The BE's mathematical requirements for the King's Scholarship Examination in the 1904 Regulations followed the general pattern of examination reform in this subject.<sup>71</sup> The arithmetic requirements were simplified, graphical work was brought into algebra, and a new syllabus for geometry, both theoretical and practical, was provided as an alternative to Euclid, and followed closely the wording adopted for the Cambridge Previous, though not the same scope [MA, 1905a, p.14]. A little mathematics, beyond arithmetic, was now offered as an alternative for women. (See Illustration 6.) Through its examinations, the BE sought further to improve the preliminary mathematical education of teachers with a new scheme from 1906 for the renamed Preliminary Examination for the Elementary School Teachers' Certificate.

For the Preliminary Examination, in two parts, a further modified syllabus in arithmetic was compulsory in Part I, with elementary mathematics offered as an option for both men and women in Part II, which generally had been 'deliberately remodelled with a view to giving an advantage to those who have been educated and not crammed' [BE, 1907b, p.21]. There was much concern at this time for candidates' lack of understanding, particularly in arithmetic [pp.20-21], where the syllabus was also considerably simplified. (See Illustration 7.) The mathematics in Part II followed the lines of the 1904 Regulations, with Euclid III added in the traditional geometry option [BE, 1906d, pp.33-34].

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71 General examination reforms will be considered in Chapter 3.



*Defective spelling or handwriting will be taken into account in estimating the value of a Candidate's work.*

*The use of rulers will not be allowed except for Mathematical questions where actual measurements are necessary.*

## PART I.

### I.—Reading:—

To read with clear enunciation, ease, and intelligence, from a work of a standard prose author and a work of a standard poet.

### II.—Repetition:—

To repeat 100 lines of Shakespeare or some other standard English author with clearness and force, and knowledge of the meaning.

In place of 50 lines of English, candidates from Welsh districts may substitute 50 lines from a standard Welsh author.

The exercises in Reading and Repetition will be performed at the Examination Centre and not during the visits of Inspectors to Public Elementary Schools.

### III.—Penmanship:—

To set copies in large and small hand.

### IV.—Composition.

### V.—Arithmetic:—

The Theory and Practice of Arithmetic.

The following will be excluded:—

Troy and Apothecaries Measures.

The rules for finding Square and Cube roots.

N.B.—Candidates may be asked to determine the square (or cube) roots of numbers that can readily be expressed as the product of the squares (or cubes) of small factors.

Practice.

Ratio.

Proportion except by the unitary or fractional method.

Stocks and Shares.

True Discount.

Scales of Notation.

Foreign Exchanges.

Recurring Decimals and Complicated Fractions.

The metric system will only be applied to measuring length, area, and volume.

Questions may be set on the mensuration of rectangular surfaces and solids.

The use of algebraic symbols will be permitted.

As a rule, (a) the questions will not involve long operations or complicated numbers, (b) the answers to money sums will not be required beyond the nearest penny.

The papers will be sufficiently long to allow the candidates some latitude in the questions selected, but no limit will be placed on the number of questions which may be attempted.

The general pupil-teacher system was the subject of a detailed Report in 1907.

The Report strongly opposed the separate existence of centres as crammers for college entrance, to which secondary pupils were often being transferred [BE, 1907b, p.19]. There were particular problems in mathematics:

Sometimes ... scholars, who at the Secondary School have been taught Geometry in accordance with the new method, are set when they enter the Centre to study Euclid. Often, again, a scholar who has made considerable progress in some subject such as French or Algebra is put back to begin with the rudiments. [p.19]

The Report advocated the early transfer of intending teachers from elementary to secondary schools, to allow time for a broader preliminary education. It was generally found at this time that ex-elementary pupils 'have not touched Mathematics outside Arithmetic; some of the boys have begun a very little Algebra, the girls scarcely ever, and neither boys nor girls have begun Geometry' [p.21]. Also, the general value of pupil-teachership as a form of apprenticeship was now being seriously questioned, and an alternative system of bursarships, introduced in 1907, was advocated [p.26]. Given also the rapid expansion of the secondary system, the demise of pupil-teachership was now inevitable. Except in some rural areas the system was largely eliminated by the First World War [Jones, 1924, p.28]. However, for some years the BE's Preliminary Examination continued to be an important one for secondary schools.

The Report of 1911 found that many intending teachers took an alternative university examination, but the number taking the Preliminary Examination was still 'considerable' and 'the examination must be recognised therefore as one which has a considerable effect upon Secondary Schools' [BE, 1911, p.25]. The numbers were declining, though Fletcher [1912, p.90] still felt it necessary to criticize the syllabus for its omission of Euclid VI. This was rectified in the



following year, when a complete break was made with Euclid's system of theorems, and the BE instead adopted in full the Schedule of the Cambridge Previous for theoretical geometry, as part of the elementary mathematics option [BE, 1913d, pp.9-11; MA, 1905a, pp.14-16].

By 1920, over eighty per cent of intending teachers were passing through a full secondary course, and the First Examination could now be used for college entrance [BE, 1925c, p.11]. A Departmental Committee reported on elementary training in 1925, and advocated a complete break with the system of apprenticeship, some residues of which still remained. The Committee also recommended 'That the Preliminary Examination ... should not be approved as a qualifying examination for pupils in Secondary Schools' [pp.158-159]. This Examination was phased out over the next three years, bringing to an end the BE's direct influence on the preliminary education of elementary teachers [BE, 1926b, p.139]. The BE also withdrew as an examining body for the academic work in the colleges, though the process spanned some twenty-five years.

The BE inherited the Education Department's tight control over two-year training college courses, through a system of academic and professional examinations, closely linked with inspection and the payment of grants. In mathematics, there were separate requirements for men and women, along conventional lines in arithmetic, algebra and Euclid, with some mathematics beyond arithmetic as an alternative to further arithmetic or domestic economy for women [BE, 1901b, pp.452-474]. Since 1870, the colleges had also utilized the DSA's examinations, to earn additional grants, and the BE still offered various science subjects as further options. In 1900, practical geometry was second in popularity to physiography, with over seventeen hundred papers worked; whilst only twenty-three papers were worked in pure mathematics

[Jenkins, 1979, pp.216-217].<sup>72</sup> As a member of a Departmental Committee, Perry had actually proposed his new scheme of practical mathematics for the basic training college course, but it was far too radical an alternative for adoption outside the technical field. However, Perry's tactics are interesting, and he made them clear in his address to the British Association for the Advancement of Science (BAAS) in 1901:

courses of instruction adopted in training colleges are very likely to be adopted in primary schools, in continuation schools, and in many secondary schools. I have been allowed by the Science and Art Department to introduce this method of mathematical teaching.... It is obvious, therefore, that what I am doing may have far-reaching consequences.... [Perry, 1901a, p.1]

Predictably, the changes in the BE's mathematical requirements for the colleges followed the general trend set by the university examining bodies. In 1904, the BE first published Regulations for the Training of Teachers and for the Examination of Students in Training Colleges, separate from the Elementary Code. A generally more liberal curricular policy was implemented, the changes in elementary mathematics following the same lines as those in the King's Scholarship Examination (see p.63), with a slightly wider algebra syllabus, including the use of logarithmic tables, and the subject-matter in geometry extended to Euclid VI [BE, 1904d, pp.29-31]. There were lower expectations for women, who could still avoid mathematics beyond arithmetic. This differentiation had some important implications for the reform of mathematics teaching in the elementary schools. As a Report of the LCC [1911, pp.110-111] noted, only those women who had opted to study mathematics would be fit to implement a broader and more progressive course, and such women were still 'comparatively few in number.' It should also be added that, although the quality of both college entrants and the lecturers gradually

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72 Jenkins may have misinterpreted the statistics. The science and mathematics subjects he lists were additional options, the basic curriculum being governed by other requirements, which he does not discuss.



improved [BE, 1925c, p.82], it took some time for the changes in mathematics to be generally adopted. Bryan [1908a, p.226] referred to the persistence of the teaching of Euclid, which still remained in the syllabus as an alternative to the 'new geometry.' However, Euclid as an alternative was eliminated in the Regulations of 1913, which also brought in a number of major modifications [BE, 1913e, pp.xiii, 65-67].

By the First World War, given the improved preliminary education of intending teachers, it had become possible to introduce greater freedom into the Regulations concerning academic subjects [BE, 1925c, p.90]. Differentiation was ended, and mathematics was no longer a compulsory subject, but was optional at two levels. These changes followed the recommendations of the Training College Association, which had also proposed specimen syllabuses in mathematics, including a very ambitious one by Nunn [1912a, pp.295-297, 303-307]. Nunn included both trigonometry and some calculus in his syllabus for the pass level, and, like Perry ten years earlier, sought to implement change through the BE's examinations. Again his scheme was far too radical for general adoption, though a little trigonometry was introduced into the otherwise only slightly modified mathematics syllabuses.<sup>73</sup>

By around 1920, for the two-year courses, there had developed a varied pattern of involvement of the universities, the colleges and the BE in examining the academic and professional elements, though the BE in the early 1920s continued to play a dominant role [Jones, 1924, p.99; BE, 1926b, p.147]. However, a Departmental Committee [BE, 1925c, pp.108, 163] strongly recommended the transfer of general responsibility to the universities, working in conjunction with the

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73 The syllabus for professional mathematics reflected the reformed views concerning elementary school mathematics, and specified 'methods and apparatus; practical instruction and its relation to handwork; use of literal symbols and graphs by older children; mensuration and geometrical drawing' [BE, 1913e, p.72].

colleges on a regional basis. A new scheme, supervised by a widely representative Central Advisory Committee, was formulated by the end of the decade and the BE's own detailed curricular controls were abandoned [Montgomery, 1965, pp.235-236].

The university departments had always been subject to less central control, and this freedom had been extended to the professional aspects after 1911, when a consecutive pattern of three years plus one year of elementary teacher education was recognized as an alternative to the concurrent three-year pattern [BE, 1925c, p.20; Nunn, 1912a, p.301]. Subsequently, the three-year courses declined in popularity, and the universities tended to restrict themselves to consecutive four-year courses for elementary as well as secondary teacher education [Jones, 1924, p.114; BE, 1926b, p.147]. Also, it should be emphasized that alternative routes into elementary school teaching existed, apart from those already discussed. As late as 1921-1922, more elementary teachers were certificated but not college-trained, uncertificated, or classed as 'supplementary,' than were college-trained and certificated, which is an important factor to recognize when judging the general quality of the teaching force [BE, 1925c, p.175]. Another aspect of training was the provision of in-service courses for elementary teachers by the LEAs, the larger authorities such as the LCC being particularly active at an early stage (see p.23), and the general provision increased after the War [BE, 1909b, p.80; 1925c, pp.119-120; 1926b, p.153].

As in the cases of elementary, secondary and further education, so also, but more gradually, in the case of teacher education the BE transformed its role. A Report of the BE [1914d, p.7] referred to this general process as:

a momentous change in policy, which can be traced in almost every branch of educational administration. The State has abandoned the attempt to direct education by prescribing in



detail the subjects and methods of instruction for individual students.... It leaves teachers free to attack, each in his own way, the detailed problems of curriculum and method....

Thus the BE gradually restricted its concern to the general conditions and finance of education in all aspects. The process was completed soon after the War, and the total volume of the BE's regulations was reduced from around one hundred and ninety pages to a mere thirty in 1926 [Selby-Bigge, 1927, p.168]. However, in the field of secondary teacher education the central authority never held a significant measure of control, and the general trends here remain to be considered.

### Secondary School Teachers

In discussing the quality of the secondary teaching force it is profitable to make a number of distinctions. Firstly, there is a teacher's academic education, which may or may not be supplemented by some form of professional training. Secondly, there are both formal and informal modes of education and training, both pre-service and in-service. Thirdly, the status of a teacher's school is an important variable, and, finally, there are some important differences between the sexes. It was in the case of lower status secondary schools that pioneering efforts were made to cater for the various aspects of teacher education and training.

The College of Preceptors' school examinations have already been considered (see pp.48-49), and this provision by the College was one consequence of its general ambition to create an autonomous teaching profession. Other aspects of the College's general strategy included the provision of a teacher's diploma; the arrangement of lectures and meetings; the publication of its own journal, the Educational Times, which included reports of meetings, articles, letters, book reviews, mathematical problems and solutions, and numerous advertisements for books and other materials; the appointment

of the first professor of education in England; and persistent efforts to establish a scheme for the registration of teachers [Rich, 1933, pp.250-269; Gosden, 1972, pp.235-244]. The College was a pioneer in in-service teacher education, and did much to bring to prominence general educational issues and detailed questions of pedagogy.<sup>74</sup> However, its wider ambitions to control entry to the profession and to regulate educational standards through its own examinations for teachers and pupils were not fulfilled, though it continued to act as a valuable agent for the dissemination of ideas in the early twentieth century [Webb, 1915b, pp.12-12].<sup>75</sup> Its diplomas were never popular, and an experiment involving its own teacher training college failed in the 1890s. Significantly, women were first admitted to the College's Council in 1869, and it was not men but women who generally took an early interest in the provision of secondary training facilities.

It was the lack of university opportunities for women in the 1870s that stimulated interest in alternative forms of academic education and professional training in departments attached to schools, and in other suitable institutions. This development was part of the wider movement for the improvement of girls' education, and up to 1890 secondary training facilities generally were restricted to women, whose career opportunities could thereby be enhanced [Rich, 1933, pp.261-263; BE, 1912d, pp.1-3]. However, as part of the wider opening up of examinations to the female sex, from around 1880 women began to obtain academic degrees at London, Cambridge, and Oxford, and this provided an alternative high status and direct route into secondary teaching, which was the traditional

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74 Findlay had close early links with the College, and Richard Wormell, a pioneer in science and mathematics education, was another of its leading members. For Wormell see the Appendix.

75 The College's specific contribution to mathematical education will be pursued in Chapter 3.



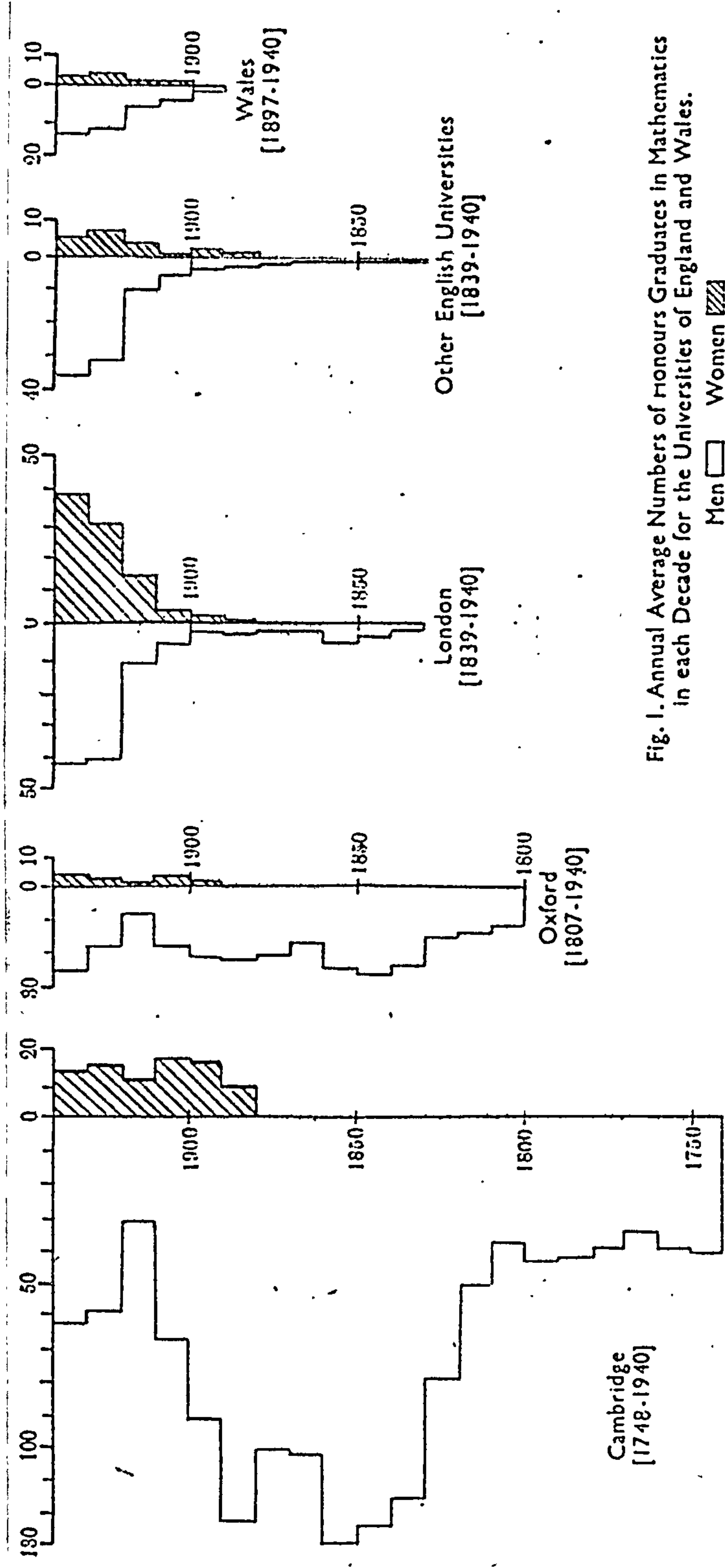


Fig. 1. Annual Average Numbers of Honours Graduates in Mathematics in each Decade for the Universities of England and Wales.

Illustration 8 Output of Men and Women Honours Graduates in Mathematics [Chapman, 1946, facing p.68]

pattern for men [BE, 1923a, pp.31-33]. The subsequent output of women honours graduates in mathematics, up to 1940, is shown in Illustration 8. As regards the academic calibre of women mathematics teachers, the changing pattern was well summarized by Story [1912, p.544]:

at first the difficulties were great owing to the fact that the supply of well-qualified teachers was wholly inadequate ... the equipment of those teaching before [1881] was in the main meagre: a smattering of arithmetic and algebra, with little knowledge of principles, and perhaps two books of Euclid learnt more or less by rote. In the twenty years following the opening of the Tripos Examination to women, 250 students took honours in Mathematics at Cambridge alone, and it was with the entrance into the teaching profession of this band of highly qualified women that the organisation of mathematical teaching may be said to have begun.

Story's [p.545] survey of one hundred and eighty schools linked with the Association of Head Mistresses found approximately one third of the senior mathematical mistresses had taken the Mathematical Tripos, and another third a London pass degree, though very few had undergone professional training. Story [p.545] considered that this disinclination to supplement degree studies with professional training had made 'the curriculum in Mathematics at girls' schools, as well as at boys,' a preparation for the future mathematical student at the Universities, rather than an education for the average pupil.' Although the total supply of London mathematics graduates increased rapidly after 1900, it was Cambridge which traditionally dominated higher mathematical education, and there are important implications for secondary schools, particularly those of higher status.

The products of the Mathematical Tripos system were much sought after by the higher status secondary schools during the nineteenth century, and also subsequently. For the leading public schools in the 1860s, the Clarendon Commission reported that 'it is easy to obtain Mathematical Masters of high ability who have had a University education' [Royal Commission (Clarendon), 1864, p.15]. However, until the 1890s, as will be shown, professional training for men was



generally regarded as superfluous. The character of mathematical education at Cambridge was thus an important factor influencing the perspectives on school mathematics of many leading teachers of the subject, as well as textbook writers.<sup>76</sup>

The history and effects of the Cambridge Mathematical Tripos have been considered by a number of writers, and the finer detail will not be repeated here. A full account of all aspects of Cambridge mathematics has been provided by Ball [1889, 1912] and developments concerning the Tripos from the late nineteenth century have been considered by Berry [1912], who has well captured the highly competitive character of this examination with its distinctive 'order of merit' for honours in three classes, the 'wranglers,' and the senior and junior 'optimes:'

The system obviously involved great confidence in the accuracy of marking attainable; and its prevalence and popularity might be ascribed in part to the general British interest in sport. University studies were reduced as far as possible to a semblance of a race; the candidates were the horses, their teachers the trainers, the examiners the judges. In Cambridge great interest was always felt in these competitions, especially in the Mathematical Tripos, large crowds always attending when the list was read out. Even the general public, usually little interested in University matters, were involved; the lists were telegraphed to the newspapers, short biographies of the Senior Wrangler and other high wranglers were published.... [pp.184-185]

Further distinctive features of the Tripos system were the divorce of teaching from examining, the former being principally in the hands of a few specialist 'coaches,' and the non-involvement of

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76 A scrutiny of school textbooks in the MA's Library indicates that over the first quarter of this century successful textbook writers generally possessed a Cambridge and public school background. Prominent writers and schools were Hall, Stevens, Fawdry and Palmer (Clifton); Borchardt, Perrott, Baker and Bourne (Cheltenham); Godfrey and Durell (Winchester); Siddons (Harrow); Barnard (Rugby); and Pendlebury (St. Paul's). The Appendix shows that a number of the leading figures in school mathematical education were Cambridge Wranglers, led by Fletcher, Carson and Mair (2nd Wranglers), and Godfrey (4th Wrangler). However, the powerful thinkers Branford, Nunn and Findlay were not Cambridge products.

professors in undergraduate work. As late as 1905, the Sadleirian Professor, A.R. Forsyth, was prompted to remark:

I live in a place where examinations seem to be the breath of life, and where they are utilised for the purpose of arranging students in an artificial order of merit, and in order to see who is the best gymnast in over-coming obstacles set by people who had nothing to do with the teaching. That is examination run mad.<sup>77</sup>

The complex question of the effects of the system on the state of English mathematics will not be considered here. Forsyth [1935] subsequently described the traditional pattern in some detail, re-emphasizing his criticisms, whilst G.H. Hardy [1926] was notably outspoken and argued powerfully that the Tripos and other difficult honours examinations, like Oxford Greats, were 'fundamentally vicious and should be abolished.'<sup>78</sup> Predictably, such a drastic remedy was not applied. However, turning to the schools, the traditional Tripos system had some important implications.

Firstly, the severely competitive character of the Tripos tended to be adopted for school mathematics examinations, with detailed results used by the schools for advertisement. Secondly, many teachers would have been relatively unsuccessful products of a system geared to produce high 'wranglers,' and one likely to distort an individual's mathematical and educational values.<sup>79</sup> Finally, and critically for both Cambridge and the schools, from the 1880s the Tripos entered a period of declining popularity, at a time when in Hardy's [1926, p.137] view it stood 'in difficulty, complexity, and notoriety, at the zenith of its reputation.' According to Berry [1912, p.190], this steady and continuous decline, which was not arrested until after the First World War, provided one argument for

77 Forsyth in Perry [1906, pp.28-29]. For Forsyth see the Appendix.

78 Hardy in Heywood [1925, p.330]. For alternative, less critical, views see Ball [1912] and Pearson [1936]. For Hardy see the Appendix.

79 The resulting élitist tendency in school mathematics will be considered further in Chapter 6. Many of the individuals in the Appendix were relatively successful products of the system.



some major reform of the system. (See Illustration 8.) As Berry [pp.190-191] pointed out, mathematics was now encountering new competition from other subjects:

Between 1882 and 1909 ... the number of successful candidates in Part I., had fallen from 116 to 74; the corresponding numbers in Part I. of the Natural Sciences Tripos, were 54 and 174, while the class list of the Mechanical Sciences Tripos contained 7 names when it began in 1894 and 31 in 1909. It seemed clear that mathematicians were drifting away to other subjects....

After much campaigning, by individuals such as Forsyth, Hardy, and E.W. Hobson, who became Saldeirian Professor in 1910, a new, more flexible, scheme in two parts was narrowly approved in 1906, and, in particular, the controversial order of merit was subsequently abolished [Piaggio, 1931, p.464].<sup>80</sup> The mathematical examinations at Oxford were also reformed before the First World War, with some shift of emphasis, as at Cambridge, away from an insistence on specialized techniques and the elaboration of detail, and towards a deeper understanding of general principles [Dixon, 1912]. Although the output of Oxford mathematics graduates remained fairly stable up to 1910, there was general concern for the total supply of mathematics graduates for teaching in the early twentieth century.

What had been a relatively favourable general state of supply and demand, in the case of assistant masters for secondary schools, changed significantly towards the turn of the century. As one speaker remarked during a Conference at Cambridge in 1902, 'The demand for competent assistant masters is steadily growing, the supply is as steadily diminishing' [Cambridge University, 1902, p.113]. Another speaker referred to 'a very decided dearth in the market,' which was 'felt mainly in Mathematics, and was partly accounted for by the falling off in the Mathematical Tripos' [p.122]. There was also the more general problem of competing, in terms of salaries and conditions,

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80 For some discussion of the character of the campaign, which spread to the columns of the Times, see Hassé [1951]. For Hobson see the Appendix.

with other professions, particularly the Civil Service, engineering and manufacturing [p.126].<sup>81</sup> As with science teaching, the supply of academically competent mathematics teachers was inadequate for the expanding secondary school system in the early twentieth century [Jenkins, 1979, pp.215-223]. What is more, it was science which had 'drawn many teachers away from the study of Mathematics' [Fletcher, 1912, p.97]. There were particularly serious consequences for mathematics, as this period up to the War was also one of general curricular upheaval. The position in maintained secondary schools was considered in detail by HMI Fletcher [1912], who emphasized that 'When all difficulties of tradition and organisation are allowed for, the chief difficulty remains - the poverty of much of the teaching' [p.94].

A comprehensive survey by HMIs found that, out of a total of around seven thousand teachers in six hundred and fifty English schools, two thousand five hundred taught some mathematics beyond arithmetic, though Fletcher [p.95] admitted 'It is ... a matter for grave concern that half of our mathematical teachers have had no instruction in Mathematics beyond what they have had at school.' He also referred to a curious 'doctrine' [p.97] particular to mathematics:

It seems to be readily assumed that because Mathematics has been part of every teacher's own school work every one is competent to teach the subject, at least in its lower stages.

As well as considering academic attainment in the sense of some mathematics to degree level, not taken externally, Fletcher also considered experience of the calculus as another criterion, which yielded the following percentages:

	Some mathematics teaching	Some degree mathematics	Some calculus
Men	44	47	44
Women	27	52	33
	% of all teachers	% of the 1st column	

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81 Salaries varied considerably, and were relatively unattractive for lower status secondary schools in particular [Archibald, 1918, pp.58-60].



He judged that barely sixteen per cent of those teaching mathematics had high qualifications, and that in around twenty per cent of the schools surveyed there was no teacher sufficiently qualified to adopt a leadership role in mathematics. Fletcher stressed that poor qualifications resulted in conservatism, over-reliance on textbooks, and a tendency to misinterpret the spirit of reforms, however communicated [p.93]. Palmer [1912, pp.225-226] made similar remarks, with reference to teachers in all but a small proportion of privileged schools, and added that 'the unsatisfactory teacher so often shows a strong liking for the unsatisfactory text-books.'

After the War, the supply of honours mathematics graduates was boosted by the contributions from London and the newer universities (see Illustration 8), and the proportion of graduates, in all subjects, teaching in grant-aided secondary schools steadily improved [Jones, 1924, p.223; MA, 1932b, p.332]:

Year	1908	1921	1931
Men	58	70	84
Women	37	54	66

Percentages

Such figures refer to all graduates, not just those with specialist honours, and include an important contribution from graduates who had also been trained for elementary teaching, in a university department, and often competed for higher status secondary appointments [BE, 1925c, p.112; Jenkins, 1979, p.219].<sup>82</sup> Academic competence itself was necessary but not sufficient for 'efficiency' in the teaching of mathematics, as Fletcher [1912, p.95] pointed out, particularly during a period of reform. Furthermore, university mathematical education generally continued to be governed by tradition, being competitive in

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82 These statistics do not reveal the extent to which general improvements were matched by improvements in the academic calibre of mathematics teachers.

character, following the Cambridge pattern, albeit modified, still dominated by examinations, and out of step with the reform movement at school level, where educational considerations now loomed large.<sup>83</sup> The important question of forms of professional training and support for secondary teachers remains to be explored.

Broadly speaking, for much of the nineteenth century, the strength of the desired need for training, to supplement academic study, varied inversely with the status of school to which a teacher aspired. By contrast with the situation in elementary, girls' and lower status private schools, for boys' schools of the upper and middle classes a sound academic education was generally regarded as sufficient, and, as Rich [1933, p.249] has argued, 'professional training, as such, had to stand or fall on its own merits.' The HMC showed some interest in the matter during the 1870s and a Teachers' Training Syndicate was set up at Cambridge in 1879, to provide facilities and certification, but the subsequent uptake by men was very poor. Oxford did not follow Cambridge until 1896, by which time the new university day training colleges were also beginning to extend their provision to the secondary field, and secondary diplomas became more widely available [BE, 1912d, pp.4-5; Rich, 1933, pp.231-274]. Before 1900, there was also more concerted pressure for secondary training from the Associations of Headmistresses (1874) and Assistant Mistresses (1884), as well as the newer Association of Headmasters (1890) and the AMA (1891), and the subsequent pattern of one-year postgraduate training in a university institution became generally accepted in principle. Around twenty secondary departments were established by 1900.

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83 See Heywood [1925], who contrasted the widespread and sustained attack on school mathematics with the case of university mathematics, which he claimed 'has not up to the present ... been systematically studied by any corporate body in this country' [p.322], adding that 'reform is urgently needed' [p.323]. Hardy [p.330] was generally in sympathy with Heywood's challenging views.



Up to the 1890s, one important factor concerning the low esteem in which secondary training was held was the association of training with the State-controlled elementary colleges, and their narrow courses in 'applied psychology' and class management, emphasizing mechanical methods and rules of thumb, and, furthermore, dominated by the system of payments for examination results. As Findlay [1898, p.347] remarked, 'The primary teacher does not study Education in the proper sense of the word at all; he is not required to do so.' Four years later, Findlay [1902, p.185] referred to mathematicians' 'contempt for the theory of education and its exposition in training colleges.' The new involvement of institutions of university standing, and broader views of the study of education and pedagogy were important prerequisites for a general change in attitude.<sup>84</sup> As one speaker at the Cambridge Conference remarked, 'the narrow technical view is inevitable if the University or other liberal institution does not dominate the situation' [Cambridge University, 1902, pp.40-41]. Sir John Gorst strongly argued at the same time for the control of secondary training curricula by the universities and not central or local government [pp.129-130]. However, the need for training was still far from generally accepted around the turn of the century.

In a paper for the Special Reports, Hendy [1898, p.387] stressed that 'The strong point of English masters at the present time is their comparatively high intellectual equipment.' However, as Findlay [1898, p.353] pertinently remarked, with reference to mathematics:

the teacher of algebra has learned his mathematics and may have become a competent mathematician, without having reflected upon the nature of algebra and of the mode by which this knowledge takes its place in the mind.... It is remarkable that this inquiry is commonly regarded as superfluous.

Subsequently, in his powerful paper for the College of Preceptors on

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84 Developing educational perspectives will be considered in Chapter 6.

'The Teaching of Elementary Mathematics: Impending Reforms,' Findlay [1902, p.185] argued that the contempt for educational theory and training in some quarters was particularly unfortunate at a time ripe for curriculum development in mathematics. More generally, Sir John Gorst judged from his wide experience of schools that there was 'a good deal of revolutionary probability hanging over the teaching profession,' though, as H.E. Armstrong, a science educationalist remarked, most teaching was 'far too technical, except for those who wish to become professional experts. Teachers are not guided by pedagogical considerations.'<sup>85</sup> Nevertheless, pedagogical issues in mathematics rose to prominence in the early twentieth century and involved all secondary teachers, though formal postgraduate training developed only slowly.

Efforts to enforce secondary training through a system of registration failed, though the BE provided some financial stimulus from 1908 [Gosden, 1972, pp.235-257]. Separate Regulations for the Training of Teachers for Secondary Schools [1908b] were published for postgraduate one-year courses, following the main lines for existing diplomas, with school experience and specialist methods components [Nunn, 1912a, p.302]. These grant-aided courses catered for very small numbers up to the War, the annual average for men being under forty and for women around four times this figure. The numbers improved considerably after the War, as well as the proportion of men. [Jones, 1924, p.121; BE, 1926b, p.149]:<sup>86</sup>

Year	1921	1925
Men	78 (30%)	299 (37%)
Women	180 (70%)	509 (63%)

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<sup>85</sup> Gorst and Armstrong in Cambridge University [1902, pp.121,85]

<sup>86</sup> For the detailed statistics up to 1925 see Jenkins [1979, p.222]. The last figure for men given by Jenkins should be 299 not 259.



By 1925, the BE [1926b, p.150] could claim that training was now established as a 'normal procedure,' though for the public and larger grammar schools the principle was still 'by no means accepted ... as incontestible.' Between the Wars, the public schools in particular continued to attach little importance to preliminary training, being still content with high academic qualifications [Rich, 1933, p.248; Brereton, 1944, p.92].<sup>87</sup> However, for grant-aided schools around half the men were trained by 1931 [MA, 1932b, p.332]. For mathematics teachers, the MA did little to promote or influence directly the character of formal training, though it did itself provide various less formal means of professional support, to be discussed in Chapter 4 [Milne, 1920, p.83; NCTM, 1939, p.166].<sup>88</sup> Some other forms of support for secondary teachers in-service also deserve mention.

The growing involvement of various interested parties in mathematical education, including the teachers themselves, acting collectively will be considered in the next chapter. The developing pedagogical literature and particularly the output of textbooks also contributed to the new directions in mathematics teaching, as did the provision of lectures, meetings for discussion, and more ambitious courses. The early work of the College of Preceptors has already been mentioned (see pp.69-70), and both Oxford and Cambridge were involved in in-service work by 1900. Exceptional at this time was the provision of a course of twenty Saturday-morning lectures by Branford [1901a] on The Teaching of Elementary Mathematics, as part of Victoria University's extension programme. The BE inherited the DSA's provision of short summer courses for teachers of science and art.

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87 Siddons' system of 'apprenticeship' for new mathematics teachers at Harrow was one form of professional training described in the NCTM's [1939, p.152] Yearbook.

88 For discussions of the character of the educational and mathematical training components between the Wars see Archibald [1918], Milne [1920], MA [1932b], and NCTM [1939]. The account in MA [1932b] was prepared by the BE for the resumed ICTM.

The BE's summer courses were held at the Royal College of Science, South Kensington, and catered principally for teachers seeking to improve their academic competence in science and mathematics. The courses were aligned with teachers in further education rather than secondary and were dropped during the First World War. Perry contributed courses on mechanics, practical geometry, graphics, and, notably, the teaching of practical mathematics [BE, 1904e, p.18; 1909b, p.81; 1913a, p.136; 1926b, p.153]. The BE [1926b, p.154] did not provide courses for secondary teachers specifically until 1916. This first course was for modern languages, though two years later summer courses were held for mathematics teachers at Manchester Grammar School and the London Day Training College [BE, 1919, p.71]. Such courses proved popular, with the demand for places greatly exceeding the supply [BE, 1926b, p.154]. Mathematics courses in the 1920s were also held at Oxford, Cambridge and Durham, and a detailed account of the course at Durham, organized by Professor Milne of Leeds University, was included in the Mathematical Gazette [Westcott, 1920; BE, 1923b, p.66; 1924, p.138; 1925d, p.123]. Particular foci at Durham included the teaching of trigonometry, logarithms, early calculus, mathematical laboratories, and correlation with other subjects such as geography.

These features of mathematical education had all risen to prominence before the War, and will be explored in subsequent chapters. However, the earliest major concern was for geometry teaching, and the organizational context of change in the teaching of this major branch of elementary mathematics will be considered first.



### Chapter 3

#### Organizations and the Reform of Mathematical Examinations

Although major and general reforms in secondary mathematical examinations were not implemented until after 1900, organizational structures, which provided the means of exerting pressure and disseminating ideas, had developed during the nineteenth century. Indeed, organized pressure for reform can be traced to the 1860s, though it was not until around 1900 that pressure again mounted, this time on a much larger scale, and within a general educational 'climate' more sympathetic to change than hitherto.<sup>1</sup> The developing structures which later played an important part in the more successful early twentieth-century movement will be explored in this chapter, though no detailed evaluation of the nineteenth-century attempts to reform mathematical education will be attempted here. The largely unsuccessful efforts to reform geometry teaching in the period 1860-1901 have been explored in a paper by Brock [1975b], with particular reference to the work of the Association for the Improvement of Geometrical Teaching (AIGT). The AIGT became the MA in 1897, and a detailed discussion of this important Association's contribution to the reform of mathematical education will be undertaken in the next chapter.<sup>2</sup>

#### Nineteenth-Century Organizations

Both the powerful controlling mechanism of examinations and the involvement of Royal Commissions in secondary education developed over the second half of the nineteenth century. The general state of mathematical education over the first half of the century is difficult

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1 The changing educational 'climate' will be analyzed in Chapters 5 and 6.

2 Mathematical and pedagogical issues in geometry will be probed in Chapter 8.

to determine and warrants a separate study.<sup>3</sup> During this earlier period, the autodidactic tradition was still an important feature of education, before the development of State intervention. Organizations such as the Society for the Diffusion of Useful Knowledge, from 1826, and the Central Society of Education, from 1837, undertook pioneering work in the dissemination of ideas, particularly through their various publications.<sup>4</sup> It was these Societies in particular that provided the mathematician Augustus De Morgan with the means to make an important early contribution to the development of mathematical education [Price, 1975].<sup>5</sup> In as early as the 1830s, De Morgan produced articles for the Quarterly Journal of Education suggesting some modifications in the treatment of Euclid [National Education Association, 1912, pp.23-25]. However, De Morgan's individual initiative appears to have been an isolated one in mathematical education at this time, and not part of a wider movement for reform. In the field of science education, the organized involvement of scientists can be traced to around 1850.

The BAAS was formed in 1831, and the background to the creation of this important academic organization for science and mathematics has been considered in a paper by Orange [1972].<sup>6</sup> The BAAS was organized in Sections with distinguishing letters, headed by Section A, devoted to Mathematical and Physical Sciences. Various modifications and additions to the classification of the sciences in Sections were subsequently made. In 1849, the idea of a permanent committee of members in Parliament was adopted.

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3 See, for example, Howson's [1974] article, significantly entitled 'Mathematics - the Fight for Recognition,' and the useful historical chapter in BE [1923a].

4 Theses have appeared on both these Societies [History of Education Society, 1979b, pp.29-30]

5 A biography of De Morgan by Joan Richards is in preparation, and he is also the subject of one of Howson's biographical chapters in a forthcoming book (see Chapter 1, note 12).

6 A general survey of the first ninety years of the BAAS was provided by Howarth [1922], and, more recently, theses have been produced by Mikhail [1964] and Collins [1979a].



This Parliamentary Committee is the subject of a paper by Layton [1976], who refers to it as 'the first formally constituted and effectively organized pressure group of scientists to express a view on educational matters' [p.25]. The Committee produced reports and made representations in the 1850s and 1860s, though Layton has judged that in science education it actually achieved comparatively little in this period, and its efforts faded [p.37]. Educational matters were also sometimes raised within particular Sections, possibly resulting in the establishment of a special committee, which subsequently undertook investigations and reported on the issues involved. A notable early example here was the creation of a Committee to consider the promotion of school science education, following a paper given to Section D (Biology) in 1866 [p.36]. Their Report in the following year was also issued as a White Paper, and was reprinted by the Taunton Commission [Layton, 1973, p.73].<sup>7</sup> It was the Clarendon and Taunton Commissions of the 1860s which, in particular, also brought to prominence some major deficiencies in secondary mathematical education.

Wilson [1921, p.241] judged the publication of the Taunton Report [Royal Commission, 1868] to be:

the important and decisive event.... this Report made plain to all a fact, in which schoolmasters had hitherto acquiesced as a decree of fate, that boys might have worked for years at Euclid, and even know Euclid perfectly, and yet know next to nothing of the spirit or method or the results of Geometry. Time was in fact wasted over Euclid.... A better method was wanted.

The possibility of a general shift away from the strictly Euclidean treatment of geometry led to the publication of some 'new geometries' in 1868, including one by Wilson. At this time it was the College of Preceptor's journal, the Educational Times, which provided

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7 The Committee included J.M. Wilson, who was soon to become closely involved also in efforts to improve geometry teaching. Wilson is the subject of another of Howson's biographical chapters.

an important early means of communication concerning the new possibilities. A letter from Wilson [1868a, p.60] claimed:

many good teachers depart very widely from Euclid, and in fact teach geometry to a great extent independently and then teach Euclid. But there are scores of schools where boys learn and say their Euclid like declensions.

Furthermore, as a reviewer of Wilson's textbook pointed out, 'Euclid's Elements are not used in public instruction by any nation other than our own' [College of Preceptors, 1868a, p.86]. The reviewer of another 'new geometry' added:

we hear that a third is nearly ready. Several more will doubtless be produced soon, for the same purpose; and we hope the time is not far distant when some one or more of these will succeed in giving general satisfaction, and be recognised as text-books by the examining 'powers that be' in our various Universities. [College of Preceptors, 1868b, p.134]

These early initiatives came principally from the schoolmasters themselves, before the formation of the AIGT, and the success of the movement thus initiated depended crucially upon the general response of mathematicians and the universities.

The London Mathematical Society was founded by De Morgan, Thomas Archer Hirst and others in 1865, primarily to further the interests of mathematicians [Collingwood, 1966].<sup>8</sup> However, in 1868, Wilson was offered the Society's platform, to publicize his views concerning geometry teaching. Significantly, he began:

It will not, I hope, be thought beyond the province of a Mathematical Society to discuss the merits and demerits of so distinguished a text-book of Geometry as Euclid's Elements. The position still occupied in education by this book has lately attracted renewed attention from our distinguished mathematicians, from the Commissions of Enquiry into Schools, and from many who are engaged in education. [Wilson, 1868b, p.126]

Through his paper, which was subsequently published in the Educational Times, Wilson sought to obtain the Society's endorsement for the need

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<sup>8</sup> Hirst was sympathetic to reform in geometry, and was an inspiration behind some of the new geometries [Brock, 1975b, pp.24-25].



to change and of his own views concerning its desirable direction, and he suggested the sending of a memorandum from the Society to examining bodies, particularly Cambridge. In any case, he argued somewhat oddly that 'we may reform geometrical teaching slowly, by writing and teaching modern geometries, and compelling the Universities to admit our pupils' [p.128]. Hirst later argued a similar case to the AIGT [Bourne, 1921, p.245]. Wilson [1921, p.241] subsequently recollected that he was 'well "heckled"' at the London meeting, and his own textbook was 'severely handled' on mathematical grounds by De Morgan in a review for the Athenaeum. General support for Wilson from the London Mathematical Society was not forthcoming in 1868. This appears to have set the subsequent pattern for the Society of non-involvement in mathematical education, and of exclusive concern for the advancement of the subject [Collingwood, 1966]. However, in 1869, the innovatory tendencies in geometry teaching attracted the attention of mathematicians within the BAAS.

Professor J.J. Sylvester [1870, p.6] in his Presidential Address to Section A in 1869, admitted 'I should rejoice to see ... Euclid honourably shelved or buried "deeper than did ever plummet sound" out of the schoolboy's reach.' By way of apology, he added:

The early study of Euclid made me a hater of geometry, which I hope may plead my excuse if I have shocked the opinions of any in this room (and I know there are some who rank Euclid as second in sacredness to the Bible alone, and as one of the advanced outposts of the British Constitution) by the tone in which I have previously alluded to it as a school-book. [p.8]

At Cambridge in particular, as Forsyth [1935, p.170] subsequently recalled, 'There was a special devotion to an ancient and limited geometry: to Euclid, not the old man, but to the Simson-Todhunter presentation.' Nevertheless, an academically powerful Committee of the BAAS was set up in 1869 'for the purpose of considering the possibility of improving the methods of instruction in elementary geometry' [BAAS, 1870, p.lxxvii]. The Committee included, among seven Professors,

Sylvester, Hirst, and Cayley, the Sadleirian Professor at Cambridge until 1895, and also Wilson, with William Kingdon Clifford as Secretary. Potentially, this was an important initiative from the BAAS, and taken before the schoolmasters themselves had become organized in their opposition to the continuing use of Euclid as a school textbook. However, the Committee did not take any early lead in the matter, and did not report until 1873, by which time other developments had taken place [BAAS, 1874, pp.459-460]. Meetings would probably have been restricted to the period of the annual gathering of the BAAS, and no doubt many of the members had academic preoccupations far removed from elementary geometry.<sup>9</sup> Furthermore, a critical factor in the Committee's non-productivity would undoubtedly have been the fundamental differences of opinion amongst its members, making the framing of a policy for geometry teaching along other than strictly Euclidean lines impossible at this time. Hirst and Sylvester were sympathetic, but Cayley was a staunch admirer of Euclid's text, and an uncompromising opponent of reform until his death in 1895.<sup>10</sup> Professor Kelland, the Senior Wrangler of 1834, was also, according to Wilson [1921, p.242] 'wholly and openly opposed to the change.' The strength of the academic opposition, particularly at Cambridge, is confirmed by the publication in the 1870s of Todhunter's [1873, pp.136-192] powerful defence of Euclid, and Dodgson's [1879] Euclid and his Modern Rivals, in which various alternative geometries were stylishly exposed as logically inferior to the ancient classical text. Clearly, a number of the mathematicians who became involved in the

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9 It is perhaps significant that Wilson [1921, p.242], a Committee member, could not recollect any of its business, admitting that 'Whether that Committee ever met or reported I do not know.'

10 Cayley is quoted in Siddons [1936, p.18] as having once remarked that 'the proper way to learn geometry is to start with the geometry of  $n$  dimensions and then come down to the particular cases of 2 and 3 dimensions.' As Bushell [1947, p.84] remarked 'Cayley, of course, was out of touch with realities.'



discussions at this time were quite out of touch with school conditions, and preoccupied with the academic rather than pedagogical issues raised. In 1870, with the mathematicians divided, the school-masters themselves took the initiative.

The situation in 1870 was subsequently well summarized by Wilson [1921, p.242]:

correspondence took place among mathematicians and mathematical masters with the aim of securing sufficient uniformity of treatment to make possible the conducting of Examinations. That was the real difficulty. Some of us, including myself, had hoped that either the British Association, or an association of University Professors would draw up for us an authoritative Syllabus to which text-books might be written. But those august bodies made no sign. We therefore resolved to form an Association for ourselves, since the need was urgent....

At this time it was the new scientific journal, Nature, which provided an important means of communication for those anxious to form some organization. The matter was first raised in Nature in March of 1870, taken further by Rawdon Levett of King Edward's School, Birmingham, and Wormell of the Cowper Street Middle Class School, Finsbury, and the result at the end of the year was the notice of a forthcoming conference [Nature, 1921, p.639]. Thus the first meeting of the AIGT took place in January of 1871.<sup>11</sup> In his Presidential Address to Section A of the BAAS in 1873, Professor H.J.S. Smith [1874, p.5], a sympathetic member of the BAAS's own Committee, remarked:

For some years past this Section has appointed a committee to aid the improvement of geometrical teaching in this country ... we have advanced at least one step in the direction of an important and long-needed reform. The action of this Section led to the formation of an Association for the improvement of geometrical teaching.

Smith's remark should perhaps be qualified by adding that it was the action of the Section, together with the resulting inaction of its Committee, which led to the formation of the AIGT.<sup>12</sup> The HMC also

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11 The title 'Association for the Reform of Geometrical Teaching' was changed to the AIGT at the first Conference [Siddons, 1948, p.160]. The notice of the Conference is reprinted in MA [1971, pp.123-124].

12 This conclusion is inconsistent with that of Brock [1975b, p.26].

considered the question of geometry teaching in as early as 1870.

It was the work of the Clarendon and Taunton Commissions in the 1860s, and the resulting threat of legislation concerning the secondary teaching force and examinations, which produced a cohesive response from the headmasters of certain leading endowed schools. The first Conference was held at Uppingham in 1869, and was principally concerned with the recent threats to these schools' independence in matters involving the teachers, examinations and curricula.<sup>13</sup>

The second Conference at Sherborne in 1870, however, spent some time considering aspects of the curriculum, specifically the teaching of English, natural sciences, and geometry [Percival, 1969, p.53]. The question of the wider use of alternative geometries was raised, and the Committee was instructed to confer with government departments, the universities, and other examining bodies. However, given the involvement of the BAAS and the AIGT at this time, the HMC subsequently decided not to pursue the matter until these organizations had themselves clarified their positions in relation to the issues raised [Roche, 1972, p.450].

During the 1870s, the HMC furthered its schools' interests in the fields of examinations and teacher training. Pressure from the HMC led to the establishment of the Oxbridge Joint Board in 1873 (see p.47) and the Cambridge Teachers' Training Syndicate in 1879 (see p.77). Curricular issues were also raised from time to time, and, in particular, the AIGT's proposed geometry syllabus was first discussed by the HMC in 1878, and the detail was included in the appendices of the annual Report. By this time, Wilson had moved from Rugby to become Headmaster at Clifton, and he took a leading part in the HMC's discussions concerning geometry. Like other examining bodies, the

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13 For the history of the HMC see the paper by Baron [1955], and the thesis by Roche [1972]. For the origins of the HMC see also Percival [1969].



Joint Board had not responded to the pressure for reform, which it was felt would lead to 'confusion and inaccuracy without any compensating advantages in the power of working riders.'<sup>14</sup> Interestingly, Wilson suggested that schools might 'slip the new methods in when they were preparing a mathematical syllabus for use both in the "school work" examination and the award of certificates,' which was a reference to the Joint Board's early and distinctive provision for school-based examining [Roche, 1972, pp.451-452]. The general problem of geometry examinations was a major concern of both the AIGT and the Committee of the BAAS in the 1870s.

The publication of various new geometries brought with it a variety of competing approaches, and a major objective of the AIGT was 'to unify the teaching on the new lines,' and, thereby, to overcome the problems of examining in geometry as opposed to Euclid [Wilson, 1911, p.19]. Under Hirst as its first President, and with a membership predominantly of schoolmasters, the first important tactical decision of the AIGT involved a choice between the preparation of either a textbook or a syllabus as the first priority.<sup>15</sup> Wilson [1921, p.243], in particular, favoured the former course, and put his own textbook at the AIGT's disposal, but, as he later remarked, 'the Committee, I now think wisely, chose the latter.' On tactical grounds, this decision has been recently criticized by Combridge [1971a, pp.3-4], and by Brock [1975b, p.31]. However, as Brock has pointed out [p.34], nearly half the Committee of the BAAS became members of the AIGT also. Given this overlap of membership, the AIGT would have been familiar with the attitudes of the Committee of the BAAS, whose support was a

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14 'Riders' were problems, additional to those actually solved as part of the logical development of the subject, termed the 'bookwork,' which could be learnt purely by rote.

15 The AIGT's first published list of members, in 1870, contained 28 names, with 24 schoolmasters, and, in the following year, there were 61 names with 52 schoolmasters [Siddons, 1936, p.13].

necessary condition for progress in the campaign for examination reforms. Significantly, when this Committee first reported in 1873, and discussed the matter of plurality of textbooks, the idea of replacing Simson's Euclid by some other standard textbook was rejected, and the Committee judged that 'it does not seem probable that a good book could be written by the joint action of selected individuals' [BAAS, 1874, p.459]. The Committee favoured a new uniform syllabus to facilitate examining, which the BAAS itself might authorize, leaving the teachers to choose suitable textbooks within a freely competitive market.<sup>16</sup> By this time, the AIGT had slowly and painstakingly prepared a new syllabus to replace Euclid I-IV, which the Committee judged to be 'decidedly good so far as it goes' [p.460]. The Committee was reappointed to consider the AIGT's full syllabus and 'to discuss the advisability of giving to it the authority of the British Association' [p.460]. On other matters, the Committee supported the idea of a propaedeutic course in practical geometry, though not the use of redundant axioms in the formal deductive course. The use of intuition and experiment in geometry, and the general question of the relation between demonstrative and practical geometry attracted relatively little general attention at this time.<sup>17</sup> Academic questions and the problems of examining geometrical proofs dominated the situation. Euclid's pure mathematical ideals still provided the paradigm for school geometry. Predictably, the AIGT laboured greatly and at length over Euclid V (ratio and proportion), and its complete syllabus for plane geometry up to Euclid VI was not published until 1875. The Committee of the BAAS [1877, p.9] reported again in 1876, and generally approved the AIGT's syllabus:

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16 With hindsight, given the independence of both examining bodies and publishers in England, the idea that one new textbook in geometry might be adopted as a standard, to replace the publishers' various editions of Euclid, seems somewhat fanciful.

17 These aspects of geometry, which loomed much larger around the turn of the century, will be pursued in Chapters 5 and 8.



it should be considered in detail by authorized representatives of the Universities and the other great examining bodies of the United Kingdom with a view to its adoption.

It was thus six years before the AIGT was in a position to pressurize the various examining bodies, with the support of a testimonial from the BAAS for its alternative, albeit conservative, syllabus. However, as a writer in the School World [1904, p.56] subsequently remarked:

This recommendation seems to have fallen rather flat. There was then no general feeling of unrest and of misgiving that all was not well with our system of education comparable to that which lay behind Prof. Perry's vigorous attack on our antiquated methods twenty-five years later....

The generally poor response from the major examining bodies has been discussed by Brock [1975b, pp.28-29]. In addition to any failings in the tactics of the AIGT itself, which Brock has considered, there are other factors which inhibited progress in this earlier period.

The continuing academic support for Euclid as a textbook has already been discussed (see pp.86-87). However, by 1900, educational arguments had risen to prominence, in which the standpoint of the learner was now a major concern, and the defence of pure Euclid as a mental discipline was being challenged. Furthermore, the development of science and technology brought with it a renewed emphasis on the utility of mathematical knowledge, and the teaching of practical geometry and measurements in particular. These new demands eroded the classically dominated secondary curriculum, within which Euclid had been comfortably accommodated.<sup>18</sup> In addition, the nature of the English examination system needs to be considered.

Firstly, there were a number of examining bodies, each of which was autonomous and had to be separately approached. No mechanism existed for imposing a common policy for examining in geometry. Secondly, these bodies were generally conservative, and sought to

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<sup>18</sup> These fundamental factors in the reform of mathematical education will be explored in Chapters 5 and 6.

cater for the prevailing practices of the majority of teachers, who accepted the status quo in geometry [School World, 1904, pp.56-57]. The AIGT remained a small and sectional group, whose syllabus, with or without the approval of the BAAS, is likely to have been regarded by the examining bodies as a threat to their autonomy. It is significant that what the various bodies did agree on, after 1900, was the desirability of freedom for teachers and textbook writers to experiment in deductive geometry, which was these bodies' response to a much wider general movement for reform. This is a very different matter from agreeing to adopt one standard and detailed syllabus, devised by a narrowly representative group like the AIGT. Indeed, a standard sequence of theorems to replace Euclid's was never subsequently adopted by the examining bodies in this country. It does seem that in this earlier period no possible tactics of the AIGT could have been adopted to impose a new uniformity for examinations in geometry. Having achieved little in the 1870s, the AIGT turned to the alternative tactic of textbook production in the 1880s.

The writing of a textbook by Levett and others, based on the AIGT's syllabus, was again a very slow and pedantic business, spanning some five years, and the treatment was still dominated by academic rather than pedagogical considerations, with a view to impressing sceptical mathematicians like Cayley. Godfrey [1923, p.328] subsequently well captured the flavour of the AIGT's work, writing at the time of Levett's death:

Perhaps the best illustration of 30 years' change of standpoint is that the first theorem in the Elements of Plane Geometry is 'All right angles are equal to one another;' the proof occupies a page and a half.<sup>19</sup>

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<sup>19</sup> Both Godfrey [1923] and Siddons [1936, pp.13-18] were pupils of Levett, and both generously acknowledged his influence. However, it appears that Levett's influence on these two outstanding pupils was principally mathematical rather than pedagogical. For example, in geometry, Godfrey [p.238] remarked 'I do not remember that we used ruler and compass in Levett's classroom.'



As has already been argued, the various examining bodies would not have been favourably disposed to the adoption of the AIGT's textbook, or any other for that matter, as a new uniform standard. The result of the AIGT's new initiative was subsequently well summarized in the School World [1904, p.57]:

The publication of the complete work afforded a favourable opportunity for addressing the Universities and Civil Service Commissioners, but the replies amounted to little more than this:- Euclid's sequence and axioms being retained, any proof will be admitted. The consequence was the issue of a number of new editions of Euclid's Elements, cleared to a great extent of his cumbrous verbiage and well supplied with exercises and addenda. These have also had their use in preparing for the present freedom, and in rendering less abrupt the transition from Potts or Todhunter to the manuals now claiming attention.

Thus the AIGT's success as a pressure group for the reform of examinations and textbooks in geometry was very limited in the nineteenth century. However, its activities and interests as a subject association did widen during the 1880s.

Apart from the work in elementary plane geometry, committees also considered solid geometry, higher plane geometry, and geometrical conics, as well as mechanics, and various syllabuses were published [Siddons, 1948]. Significant here was the tendency of the AIGT to broaden the range of its interests by considering the more advanced rather than more elementary aspects of mathematical education. A new feature from 1883 was the presentation of papers on various subjects, which were subsequently published in the annual reports. Here again, speakers often chose to consider aspects of more advanced geometry and mechanics, though notable exceptions in the 1880s were the Presidential Addresses by R.B. Hayward [1886] on 'The Correlation of the Different Branches of Elementary Mathematics' and Professor G.M. Minchin [1889] on 'The Vices of our Scientific Education' [Wolff, 1915, pp.59-65].<sup>20</sup> Another exceptional paper was given by G. Heppel [1893] on 'The Use of

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20 These two Addresses were also published in Nature.

History in Teaching Mathematics.<sup>21</sup> Under Hayward's Presidency, and with the somewhat wider appeal of its activities, the AIGT's membership increased from just over one hundred to around two hundred during the 1880s. However, the official organ of the Association, the Mathematical Gazette, was not started until 1894. Three years later, the AIGT's wider concerns were eventually reflected in the adoption of a new name, the Mathematical Association. The MA provided one means for the dissemination of ideas and the exertion of pressure for reform, but, as will become clear, the important agitation around 1900 was not created by this Association. More important in this respect were the developments involving the BAAS.

After the far from intensive, and, in any case, non-productive efforts of the Committee of the BAAS on geometry in the 1870s, this Association did not take any further collective initiative in mathematical education for the rest of the nineteenth century. However, as with the AIGT, there were isolated individual contributions to the debate concerning the teaching of elementary mathematics, and notably the Presidential Addresses to Section A by Professors George Chrystal and Olaus Henrici in the 1880s. Henrici's Address was given in 1883, when he was also President of the London Mathematical Society. He had already been responsible for the production of a rival to Euclid, Elementary Geometry, Congruent Figures [Henrici, 1879], whilst also being Professor of Pure Mathematics at University College London.<sup>22</sup> In his Address, he criticized strongly the teaching of geometry, with its divorce of the theoretical and practical aspects. He criticized the AIGT in particular for its conservatism and neglect of newer ideas

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21 Heppel's subject will be discussed in Chapter 6. He also gave interesting papers to the College of Preceptors in the 1880s [Heppel, 1887], and his progressive thinking in algebra is discussed in Price [1974, pp.57-59; 1976a].

22 Henrici's and the AIGT's textbooks were both attacked by Dodgson [1885]. Much later, W.J. Dobbs [1902, p.730], a markedly progressive thinker, referred to Henrici's little book as one which 'should be in the hands of every teacher.' For Dobbs see the Appendix.



in geometry, such as symmetry. In the case of algebra, he deplored its rigidly mechanical treatment, without a foundation of basic principles, and also its divorce from the problems of science and everyday life. Professor Chrystal [1886, p.892] launched a scathing attack upon examinations as 'the canker which turns everything that is good in our educational practice to evil.' His own particular interest was in algebra, which he claimed was taught as 'neither an art nor a science, but an ill-digested farrago of rules, whose object is the solution of examination problems' [p.892]. He referred to the recent work of De Morgan, Hamilton, Grassmann, and others, which had 'thrown a flood of light on the elements' [p.891], but such advances had not influenced either elementary or higher algebra, as embodied in textbooks and examinations. He concluded:

The result ... is that science, in the hands of specialists, soars higher and higher into the light of day, while educators and the educated are left more and more to wander in primeval darkness. [p.893]<sup>23</sup>

The speeches of both Henrici and Chrystal were strikingly powerful for this period, but nothing tangible resulted. However, as Chrystal pointed out, the younger generation was becoming increasingly disenchanted with the existing system, which was sustained by powerful conservative interests [p.894]. Nevertheless, the traditional pattern in English mathematical education did not come under sustained attack until around the turn of the century. As had been the case around 1870, various individual initiatives and agitation in the media preceded more organized activity.

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23 Chrystal's own textbooks in advanced algebra were a source of inspiration for school algebra teaching from the late nineteenth century. Heppel [1895, p.25] remarked that 'A "Chrystal for Beginners" is sadly wanted,' and Chrystal [1898] obliged with an innovatory Introduction to Algebra for secondary schools and technical colleges. The textbook writer, Barnard [1912, p.320] acknowledged that he was 'vastly indebted' to Chrystal, 'in common with most teachers of algebra' [Barnard, 1907; Barnard and Child, 1908]. Significantly also, two leading figures in school mathematical education, Mair and Branford, were students of Chrystal at Edinburgh. Mair entered Edinburgh University at sixteen, proceeding to Cambridge subsequently.

### Mounting Pressure and Major Examination Reforms

Towards the latter part of the nineteenth century, there was a general expansion in the periodical literature concerning education. The first number of the Journal of Education appeared in 1879, and, according to Tropp [1958, p.154] it became the leading nineteenth-century journal of secondary education 'in spite of various attempts to found competing periodicals.' It was to this periodical that Branford [1899a, 1899b, 1899c, 1900a, 1900b, 1901b] contributed an extraordinary series of articles concerning his own innovatory ideas for elementary geometry teaching. The first number of the Mathematical Gazette appeared in 1894, and Macmillan started a new monthly, School World, in 1899.<sup>24</sup> The first editorial of the School World [1899a, p.1] acknowledged that its birth was a reflection of the growing concern for curricular matters, and a general policy of relevance to the secondary classroom was adopted. In its first year, series of articles were published on the teaching of algebra and geometry, by Professors Mathews and Minchin respectively, the latter resulting in some lively correspondence concerning Euclid as a school textbook [School World, 1899b]. Minchin [1899] had already contributed to the scientific journal Nature on the provocative question of 'Geometry versus Euclid.' Indeed, some of the earliest agitation concerning mathematics teaching is to be found not in the educational press but in technical journals such as the Electrician, Engineer and Engineering, as well as in Nature [Perry, 1900a, p.vii; MacGillivray, 1902]. As Findlay [1902, p.184] pointed out, many of the ideas concerning change were far from new:

They have long been familiar, in their main outlines, to the many teachers, 'thoughtful teachers,' who think for themselves. Prof. Perry himself tells us that he has been prophesying in vain since 1880, and the columns of the Educational Times and

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24 The School World provided strong competition for the Journal of Education and ran until 1918. It was subsequently incorporated in the latter.



the Journal of Education contain repeated sketches of reform, worked out in great detail.

However, Findlay [p.184] judged that the important thrust came in 1900:

with articles by Prof. Perry in Nature ... and it is my impression that the editors of that journal (closely associated with an enterprising magazine called the School World, issued from the same publishing house) recognized that the time was ripe for stirring up both the scientific world and the teaching world on this subject.

Perry had already achieved a breakthrough in the field of technical education through the DSA's adoption of his scheme of practical mathematics from 1899, and Perry's lectures had also been published by the DSA [1899] (see pp.32-33). However, he had much wider ambitions to extend the influence of his ideals to elementary, secondary and teacher education (see p.66). A strong general statement from Perry [1900b] on the teaching of mathematics was published in Nature, together with his scheme for the DSA, and, before the end of 1900, a good deal of correspondence had been generated [Beard, 1900; Heaviside, 1900; Mair, 1900; Stromeyer, 1900; Woolen, 1900]. Also in 1900, a collection of statements from Perry [1900a] on various aspects of education, with particular reference to engineering, was published under the title England's Neglect of Science, and this book was subsequently given a sympathetic review by Minchin [1901] in Nature. Thus, by the end of 1900, major issues in mathematical education were again rising to prominence in the educational and scientific literature. However, organized support for reform was a necessary prerequisite for the success of the movement which had been initiated.<sup>25</sup>

The ineffective role of the BAAS in relation to mathematical education in the 1870s and 1880s has already been considered (see pp.86-96). The organization of the BAAS in Sections catered primarily for academic rather than educational matters, the latter being treated

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25 Interestingly, the MA and the columns of the Mathematical Gazette remained unruffled at this time concerning these early initiatives for reform.

incidentally within particular Sections, and without adequate representation from the school-teachers themselves. The late nineteenth-century efforts to form a new Section L, devoted wholly to education, have been considered in a paper by Collins [1979b]. Armstrong was at the centre of important developments here, particularly during the Bradford Meeting of 1900, and his campaign for a new Section was supported by Perry amongst others. It was at the Bradford Meeting that Perry 'preached his gospel' concerning mathematics teaching, though, like Henrici and Chrystal before him, his audience was principally composed of mathematicians and scientists, and his contribution was not widely publicized [Moylan, 1901, p.39]. However, Perry clearly saw the creation of a new Section L, which might co-operate with existing Sections, as a means of generating a wider interest in and influence upon both the education of engineers and mathematical education generally, through the BAAS. More generally, it was hoped that school-teachers might become more involved in the work of the BAAS, through its Section L, though the subsequent influence of this Section on the world of education remained slight [Collins, 1979, pp.237-241].

The inaugural meeting of the new Section L, for educational science, was held at Glasgow in 1901. The business of the Section in its subsequent years ranged widely, and embraced various issues concerning examinations, curricula, and the teaching force.<sup>26</sup> However, a notable early initiative from the new Section was the arrangement of

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26 The subject of examinations was a persistent concern of the Section [Champneys, 1934, pp.25-28]. Occasionally, small grants for research were made available to the Section. For example, a Committee, set up in 1911, reported on the influence of school-books on eyesight, in 1912 [Howarth, 1922, p.280; BAAS, 1912, p.cxxxix; BAAS, 1913, pp.295-318]. Their recommendations concerning the use of large type, particularly for younger pupils, were adopted by Godfrey and Price [1915, p.v] in their Arithmetic, and, by 1928, the BE's [1928a, p.21] Consultative Committee found that the majority of publishers had adopted these recommendations for elementary school-books.



## PRACTICAL MATHEMATICS.

### ELEMENTARY STAGE.

*Arithmetic.*—The use of decimals; the fallacy of retaining more figures than are justifiable in calculations involving numbers which represent observed or measured quantities. Contracted and approximate methods of multiplying and dividing numbers whereby all unnecessary figures may be omitted. Using rough checks in arithmetical work, especially with regard to the position of the decimal point.

The use of  $5204 \times 10^5$  for 520400 and of  $5204 \times 10^{-3}$  for 5204. The meaning of a common logarithm; the use of logarithms in making calculations involving multiplication, division, involution and evolution. Calculation of numerical values from all sorts of formulae.

The principle underlying the construction and method of using a common slide rule; the use of a slide rule in making calculations. Conversion of common logarithms into Napierian logarithms. The calculation of square roots by the ordinary arithmetical method. Using algebraic formulae in working questions on ratio and variation.

*Algebra.*—To understand any formula so as to be able to use it if numerical values are given for the various quantities. Rules of Indices.

Being told in words how to deal arithmetically with a quantity, to be able to state the matter algebraically. Problems leading to easy equations in one or two unknowns. Easy transformations and simplifications of formulae. The determination of the numerical values of constants in equations of known form, when particular values of the variables are given. The meaning of the expression "A varies as B."

Factors of such expressions as  $x^2 - a^2$ ,  $x^2 + 11x + 30$ ,  $x^2 - 5x - 66$ .

*Mensuration.*—The rule for the length of the circumference of a circle. The rules for the areas of a triangle, rectangle, parallelogram, circle; areas of the surfaces of a right circular cylinder, right circular cone, sphere, circular anchor ring. The determination of the area of an irregular plane figure (1) by using a planimeter; (2) by using Simpson's or other well-known rules for the case where a number of equidistant ordinates or widths are given; (3) by the use of squared paper whether the given ordinates or widths are equidistant or not, the "mid-ordinate rule" being used. Determination of volumes of a prism or cylinder, cone, sphere, circular anchor ring.

The determination of the volume of an irregular solid by each of the *three* methods for an irregular area, the process being first to obtain an irregular plane figure in which the varying *ordinates* or widths represent the varying *cross sections* of the solid.

Some practical methods of finding areas and volumes. Determination of weights from volumes when densities are given.

Stating a mensuration rule as an algebraic formula. In such a formula any one of the quantities may be the unknown one, the others being known.

*Use of Squared Paper.*—The use of squared paper by merchants and others to show at a glance the rise and fall of prices, of temperature, of the tide, &c. The use of squared paper should be illustrated by the working of many kinds of exercises, but it should be pointed out that there is a general idea underlying them all. The following may be mentioned:—Plotting of statistics of any kind whatsoever, of general or special interest. What such curves teach. Rates of increase. Interpolation, or the finding of probable intermediate values. Probable errors of observation. Forming complete price lists by shopkeepers. The calculation of a table of logarithms. Finding an average value. Areas and volumes, as explained above. The method of fixing the position of a point in a plane; the  $x$  and  $y$  and also the  $r$  and  $\theta$ , co-ordinates of a point.

Plotting of functions, such as  $y = ax^n$ ,  $y = ae^{bx}$ , where  $a$ ,  $b$ ,  $n$ , may have all sorts of values. The straight line. Determination of maximum and minimum values. The solution of equations. Very clear notions of what we mean by the roots of equations may be obtained by the use of squared paper. Rates of increase. Speed of a body. Determination of laws which exist between observed quantities, especially of linear laws. Corrections for errors of observation when the plotted quantities are the results of experiment.

In all the work on squared paper a student should be made to understand that an exercise is not completed until the scales and the names of the plotted quantities are clearly indicated on the paper. Also that those scales should be avoided which are obviously inconvenient. Finally, the scales should be chosen so that the plotted figure shall occupy the greater part of the sheet of paper; at any rate, the figure should not be crowded in one corner of the paper.

*Geometry.*—Dividing lines into parts in given proportions, and other illustrations of the 6th Book of Euclid. Measurement of angles in degrees and radians. The definitions of the sine, cosine and tangent of an angle; determination of their values by drawing and measurement; setting out of angles by means of a protractor when they are given in degrees or radians, also when the value of the sine, cosine or tangent is given. Use of tables of sines, cosines and tangents. The solution of a right angled triangle by calculation and by drawing to scale. The construction of a triangle from given data; determination of the area of a triangle. The more important propositions of Euclid may be illustrated by actual drawing; if the proposition is about angles, these may be measured by means of a protractor; or if it refers to the equality of lines, areas or ratios, lengths may be measured by a scale and the necessary calculations made arithmetically. This combination of drawing and arithmetical calculation may be freely used to *illustrate* the truth of a proposition.

The method of representing the position of a point in space by its distances from three co-ordinate planes. How the angles are measured between (1) a line and plane; (2) two planes. The angle between two lines has a meaning whether they do or do not meet. What is meant by the projection of a line or a

plane figure on a plane. Plan and elevation of a line which is inclined at given angles to the co-ordinate planes. The meaning of the terms "trace of a line," "trace of a plane."

The difference between a *scalar* quantity and a *vector* quantity. Addition and subtraction of vectors.

Slope of a line; slope of a curve at any point in it. Rate of increase of one quantity  $y$  relatively to the increase of another quantity  $x$ ; the symbol for this rate of increase, namely,  $\frac{dy}{dx}$ ; how to

determine  $\frac{dy}{dx}$  when the law connecting  $x$  and  $y$  is of the form  $y = ax^n$ . Easy exercises on this rule.

In setting out the above syllabus the items have been arranged under the various branches of the subject.

It will be obvious that it is not intended that these should be studied in the order in which they appear; the teacher will arrange a mixed course such as seems to him best for the class of students with whom he has to deal.

Illustration 9 Perry's Scheme of  
Elementary Practical Mathematics  
[Perry, 1900b, pp.319-320]



a joint discussion with Section A on the teaching of elementary mathematics. Sir John Gorst, the first President of Section L, chaired the meeting, which began with a paper presented by Perry, who, no doubt, had much to do with the creation of this new platform for the dissemination of his ideas.

Perry launched a scathing attack on the existing state of affairs in elementary mathematics before a distinguished audience of around two hundred at Glasgow [Muirhead, 1901, p.81]. Following the adoption of his scheme of practical mathematics by the DSA, Perry presented a syllabus on similar lines intended for teachers' training colleges, and for a general mathematical education. Perry [1901a, p.2] emphasized:

I recommend this method, not only for classes of engineer apprentices, not only for children in Board schools, not only for the average British boy, but for the boys of very acute intellect ... as well as for the few boys ... who are likely to become great mathematicians.

It should be added that he also regarded the scheme as an appropriate one for girls. This was a very radical conception of elementary mathematics at the time and it warrants reproducing in full. (See Illustration 9.) Notable innovatory features were the emphasis on decimals, with approximation, and the use of logarithmic tables and the slide rule in arithmetic; the wide use of formulae in algebra, as well as the study of functions and graphs using squared paper, with an early introduction to ideas in the calculus; the prominence of practical methods in mensuration and geometry, with the deductive ideals of Euclid virtually eliminated; and the inclusion of simple trigonometry, some work in three dimensions, and vectors. Perry also advocated a mixed treatment of the various elements i.e. a unified mathematics course, and preferably correlated with science. A three-hour discussion followed Perry's address, and involved, amongst others, Professors Forsyth of Cambridge and Hudson of London, Sylvanus Thompson of Finsbury Technical College, Greenhill of the Royal



Artillery College, Woolwich, Lodge and Minchin of the Royal Indian Engineering College, as well as Sir John Gorst. A wide range of interests in secondary and higher, technical and military education were represented at Glasgow, and a tension between the study of mathematics for its own sake and for its utility emerged clearly during the discussion.<sup>27</sup> However, there seemed to be general agreement on the need for more practical teaching methods, and 'general assent, even on the part of the "official" mathematicians, to the idea of "abandoning Euclid" - a notable fact' [Muirhead, 1901, p.82]. In the case of geometry, other countries had already abandoned Euclid, and the position in England was becoming increasingly untenable [National Education Association, 1912, pp.5-32]. Perry's radical alternative threw into sharp relief the general deficiencies in English school mathematics, which were well summarized by Love [1902, p.457], Professor of Natural Philosophy at Oxford, in Nature, who referred to:

the pedantic and unpractical character of the habitual teaching of mathematics in schools. This character belongs both to the methods of presenting particular subjects and to the order in which the subjects, and the parts of the subjects, are studied. The methods and the order now in vogue are not, of course, a system devised purposely; they have been arrived at gradually, and are sanctioned by tradition.... The mathematics of most of our elementary text-books is felt to be almost as much out of touch with modern mathematics as with everyday life.

The crisis of confidence for the mathematical community was further exacerbated by the growing pressure for greater utility in the mathematics curriculum, to serve not only the engineers, but also the teachers of school science, which was a 'modern' subject in the curriculum, and one which had made considerable advances over the 1890s

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27 For a brief summary of the proceedings see Nature [1901, p.592]. Perry's contribution and scheme have recently been reproduced in NCTM [1970a, pp.220-245]. Greenhill [1900, p.584], who had reviewed Perry's Six Lectures on practical mathematics sympathetically and at length in Nature, contrasted the 'Outwit' principle of the practical man, exemplified by Perry, with the 'Inwit' principle which 'interests and commends itself to the philosophic contemplative mind,' and added that 'an increasing gap is arising between the two lines of thought.'

[Findlay, 1902, p.184]. The situation was well captured by Professor Bryan [1902, p.90] of the University College of North Wales, writing in the School World:

Formerly our schools and colleges were given over mainly to the study of classical and literary subjects, and mathematics was looked upon as a portion of an arts course. The rigid deductive system was undoubtedly admirably suited to the object then held in view. With the development of experimental science new teachers have been appointed all over the country for physics, chemistry and biology, but next to nothing has been done to meet the greatly increased demand for mathematical teaching thus produced [Bryan's stress]. The same teachers who provided efficiently for the teaching of mathematics on the classical side have now thrust upon them an influx of new pupils having quite different requirements. The mathematical master is thus placed in the position of a 'tweeny-maid' or 'buffer' between opposing forces - the classical side and the modern side, and between these two stools it will be a great credit to him if he does not fall to the ground.

As Findlay [1902, p.184] put it, 'The teachers of science demand that mathematics shall be reformed ... and they look to the British Association for help.'<sup>28</sup>

The major difficulty which clearly emerged during the discussion at Glasgow was the existing requirements of examinations. As Love [1902, p.458] pointed out:

the first step will necessarily be the conversion of examiners and of the bodies that make regulations for the conduct of examinations. The future of mathematical teaching in this country is in their hands.

For Perry, the formidable tactical problem of implementing major change through the complex secondary examination system was highlighted by W.P. Workman of the Kingswood School, Bath:

you will have to get the local examiners, the Joint Board, the College of Preceptors, and the University of London all to agree to issue an alternative 'practical mathematics' syllabus before secondary schools can safely move. [Perry, 1901a, p.77]

The American Smith provided Perry with the following penetrating remarks concerning curriculum change:

The scheme is certainly better than the present one. But the obtaining of books, the training of teachers, the conversion of examiners, these are serious matters, and cannot be

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28 The relationship between mathematics and science will be considered further in Chapters 4 and 5.



accomplished in one generation. [Perry, 1901a, p.91]<sup>29</sup>

In any case, Perry's scheme was so fundamental a departure from the traditional conception of elementary mathematics that there was no chance of it being adopted in toto for the examination of either intending teachers or secondary school pupils at this time (see p.66)<sup>30</sup> However, the question of examinations in geometry had now again become a major concern.

Although the university examining bodies generally refused to depart from their insistence on Euclid's order for deductive geometry in the nineteenth century, the DSA had taken the initiative in the 1880s [Hitchens, 1978, p.71]. As Rumsey [1903, p.11] pointed out in the School World:

The pioneers in the movement were ... the heads of the Science and Art Department, who many years ago decided not to make a knowledge of Euclid's Elements a sine qua non for securing a pass in geometry.

What is more, to facilitate examining, the DSA instructed candidates as follows:

Unless you expressly state the contrary, it will be assumed that you have read GEOMETRY in Euclid, and you will be expected to follow Euclid's sequence, otherwise you must state what text-books you have used in geometry. [p.11]

This was an important concession by the DSA in its pure mathematics papers, though the geometry remained wholly theoretical [Hitchens, 1978, pp.73-76].<sup>31</sup> By contrast, the papers in plane and solid geometry were wholly practical and quite separate from the pure mathematics [Godfrey, 1908, pp.256-257]. The Civil Service Commissioners had also moved at an early stage, as regards the

29 Smith's [1900] early methods book, The Teaching of Elementary Mathematics, had been given a full and enthusiastic review by Perry [1901b] in Nature.

30 Perry's scheme has many features in common with the alternative Jeffery Syllabus of 1944 for School Certificate Mathematics, and various links will be explored in Chapters 7 and 8.

31 Questions might require the description and proof of a construction, but not its actual performance.

theoretical and practical aspects of geometry. For their military examinations during the late 1890s:

There was a frequent admixture of drawing and mensuration questions with those in formal geometry; and finally, in 1901, a regulation dispensing with Euclid's order of propositions was issued by the board. [Rumsey, 1903, p.11]

Minchin [1903, p.60] also acknowledged the early encouragement given to drawing and measuring by the Civil Service Commission, and added:

even in the examination for the Indian Civil Service the employment of graphic constructions and measurement is now firmly established.... Indeed, the requirements of the Civil Service Commissioners are almost sufficient, without the aid of the masters in the Public Schools, to ensure a substantial change in the teaching of Geometry.

As Godfrey [1906a, p.76] also remarked, the military requirements as well as those for naval entrance 'gave a most valuable lead,' and along lines in sympathy with Perry's practical ideals.<sup>32</sup> However, further initiatives and organizational developments were necessary before the secondary examining bodies generally adapted their requirements.

Following the discussion at Glasgow, and at the joint request of Sections A and L, an academically powerful Committee of the BAAS was appointed, with Forsyth as Chairman and Perry as Secretary. Its terms of reference were:

To report upon improvements that might be effected in the teaching of Mathematics, in the first instance in the teaching of Elementary Mathematics, and upon such means as they think likely to effect such improvements. [Perry, 1901a, p.ix]

As had been the case thirty years earlier, the Committee was largely composed of academics, and included Professors Hudson, Thompson, Chrystal, Henrici, Lodge, Minchin, Greenhill, and Gibson, the last mentioned of Glasgow University and the West of Scotland Technical College. Given the Committee's membership, with its strong representation of scientific, technical and military interests there

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32 See p.58. The army requirements will be considered in more detail in Chapters 5 and 7.



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# SPECIAL MATHEMATICAL NUMBER.

THE

# School World

A Monthly Magazine of Educational Work and Progress.

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[SIXPENCE

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Illustration 10 Front Cover (half size) of a  
Special Number of the School World [1902b]



would have been considerable sympathy for Perry's motives and support for major reform in school mathematics. W.D. Eggar, a science and mathematics teacher at Eton, was one representative for the public schoolmasters on the Committee.<sup>33</sup> There was considerable individual and collective activity before the next meeting of the BAAS, when the Committee reported.

Before the end of 1901, Macmillan, the publishers of Nature and the School World, also published in one volume Perry's Glasgow address and proposed scheme, details of the discussion, and a collection of written remarks from various interested parties, together with some further responses from Perry [1901a]. Perry's address was also reproduced in full in the School World for October and November of 1901 [School World, 1902a, p.61]. Perry's volume was reviewed sympathetically and at length in Nature by Love [1902]. In March of 1902, a 'Special Mathematical Number' of the School World [1902b] was produced, as one of a series which had already covered English and Science. The list of contributors is not without interest. (See Illustration 10.)<sup>34</sup> In the same month, Perry [1902a] replied to some of his critics in a letter to Nature. The following typifies the general reaction in some quarters:

the syllabus ... is admirably adapted for a technical training. In practical mathematics, where mental training is of minor importance, exigencies of time will compel the teacher to omit explanations, or only give them roughly, for his chief object is to enable his pupils to apply mathematical results, as distinct from reasoning, to problems in engineering, science, or kindred subjects.

On the other hand, the average boy's mathematical education up to the age of fifteen or sixteen is an absolutely different matter. [Beard, 1900, p.466]

However, Perry also argued a case for his scheme on pedagogical

---

33 For Eggar see the Appendix. Professor Love [1902], another strong supporter for Perry, also joined the Committee, which reported a year later [School World, 1902d].

34 As a major publisher of mathematical textbooks, it would have been in Macmillan's own interests to encourage major reforms, thereby creating a new market for textbooks.



grounds, and, as one schoolmaster in the Journal of Education was quick to point out, although critics might object to Perry's scheme 'on the ground that it is suited only as a preparation for technical training' they should also acknowledge that 'our present school course is not a preparation for anything' [Moylan, 1901, p.39].<sup>35</sup>

Various individual responses in periodicals considered the tactical possibilities raised by the establishment of a Committee of the BAAS. Findlay [1902, p.185] expressed some of his concerns to the College of Preceptors:

It is useless, it is unjust, for this or any other Committee to try and induce examining bodies to draft new styles of syllabus or examination-paper before the schools are ripe for change; but they can, and should, encourage 'freedom and variety.'<sup>36</sup>

Others were concerned that freedom and variety would produce chaos in the schools. Under the pseudonym 'Experientia Docet' [1902, p.159], one writer in the School World felt 'we are likely to be landed in what Professor Lamb called "a disastrous muddle" unless extreme caution is observed in the initial stages of reform.'<sup>37</sup> The writer also remarked:

there are many who view with apprehension the possibilities of wide-reaching change planned by a Committee whose members - however eminent in their own sphere - have for the most part had very little experience in teaching elementary mathematics to young students. [pp.158-159]

The ability of the teaching force generally to cope with major innovations was also questioned, and, as this writer also pointed out:

The frequent changes in a staff of teachers have to be taken into account, and it is impossible to ignore all the difficulties and limitations arising out of a pupil's successive stages as he passes from one class to another through the hands of different teachers. [pp.159-160]

By the end of 1901, the columns of the Mathematical Gazette were also

35 These issues will be probed more deeply in Chapter 5.

36 On the use of the special syllabus provision in this connection see pp.55-56.

37 Professor Lamb of Owens College, Manchester, had contributed to Perry [1901a, p.91].

beginning to reflect the renewed interest in reform.

In 1901, before the MA took any organized initiative, articles were published in the Gazette by Muirhead [1901], and by the public schoolmasters Langley [1901], Godfrey [1901], and Siddons [1901]. As Godfrey [1906a, p.76] later admitted, in the Cornhill Magazine, 'The Mathematical Association ... awoke as one out of sleep,' and, indeed, the MA was forced in 1902 to react to new circumstances created outside its own narrow confines. There are some interesting contrasts between the early reactions of Godfrey and Siddons. There was some degree of inconsistency in Siddons' [1901, p.108] reaction to Perry's scheme:

It may be admirably adapted to the wants of training colleges, but seems quite impracticable for public schools, though ... there are several suggestions that might well be adopted.

As with some other critics, Siddons [p.110-111] also associated Perry's technical bias with an exclusive concern for 'a certain mechanical power,' and inferred 'No thought is given to the development of brain power; in fact such a scheme would deaden all power of thinking.' On the question of abandoning Euclid's sequence, Siddons [p.109] adopted a conservative view:

The result would be chaos; there would immediately be ten thousand text-books treating the subject in ten thousand different orders....

Given the nature of the English examination system, Siddons had quite rightly ruled out the possibility that a new standard order might generally be adopted, and, not contemplating freedom, concluded 'Why not then be contented with Euclid's order...? Surely this allows enough latitude for reform' [p.110]. However, as Godfrey [1906a, p.76] observed, in retrospect:

This question of 'Euclid's order' has been the key of the position. Till this was taken, no substantial victory was possible.

Nevertheless, Godfrey showed both diplomacy and sensitivity in relation to the new situation, at an early stage. In his first contribution to the Mathematical Gazette, he began:



TERM.	GEOMETRY.	ARITHMETIC.	ALGEBRA.
I.	Experimental work with ruler, compass, protractor, and set squares; paper cutting.	Prime factors, leading to H.C.F. and L.C.M.; fractions; drill in decimal notation; addition and subtraction of decimals; multiplication and division of decimals by single figure; applications of fractions and decimals to concrete quantity; unitary method.	Transition from Arithmetic to Algebra; negative number and its application to quantity; numerical evaluation; use of brackets; drill in notation of Algebra; addition, subtraction, and multiplication, regarded as operations with brackets; positive integral indices; simplest equations and problems.
II.	Euclid I. 1, to I. 26 (omitting 2, 3, 7). Definitions as required. Throughout the Geometry course plenty of easy riders. Geometrical drawing and exercises in measurement to go on side by side with theory.	Multiplication of decimals beginning with left-hand figure of multiplier; division of decimals; decimalisation of money; how recurring decimals occur when we turn a fraction into a decimal; metric system; revision of Term I.	Simple numerical equations in one variable, to be solved in all cases by the use of the four axioms, solutions to be verified always; symbolical expression; problems; easy factors; revision of Term I.
III.	Euclid I. 27—I. 43; part of Book III.	Areas of rectangles, parallelograms, triangles, trapezia, and irregular figures, chiefly by means of squared paper; square and cube root by prime factors; graphical treatment of square root; volumes; simple interest.	Cartesian coordinates; plotting simple graphs; plotting tables of statistics; simultaneous simple equations illustrated by graphs (literal equations to be postponed); problems on simultaneous equations; easy factors and fractions.
IV.	Euclid III., with the exception of Propositions 35, 36, 37.	Rule for square root; how to deal with $\frac{1}{\sqrt{2}-1}$ , etc.; contracted multiplication of decimals, with applications (practice; simple and compound interest; involution, etc.).	Factors, leading to H.C.F. and L.C.M.; long division; detached coefficients; method for H.C.F. when factors are not obvious; harder fractions; manipulation of quadratic surds; graphs.
V.	Euclid II., and Propositions 35, 36, 37 of Book III.	Contracted division of decimals, with applications (inverse interest, present worth, discount, etc.); ratio and percentage.	Connections between factors and equations; quadratic equations with problems; reducing degree of equation when one root is obvious.
VI.	Euclid IV., Propositions 1-5, 10, 11, 15. The whole book to be treated as a collection of problems in geometrical drawing.	Revision of contracted multiplication and division; stocks and shares.	Remainder theorem; arithmetic and geometric progressions, without formulae; ratio and proportion.
VII.	Euclid VI., omitting Euclid's treatment of proportion, and substituting an algebraic treatment.	Miscellaneous problems.	Indices, so far as needed to introduce common logarithms; arithmetical problems involving use of logarithm tables; variation; revision of progressions, introducing formulae for same; recurring decimals.
VIII.	Revision of Books III. and VI.	Numerical trigonometry of the acute angle, using sine, cosine, tangent; problems of heights and distances; solution of right-angled triangles, at first by natural sines, etc., afterwards by logarithmic sines, etc.; solution of general triangles by drawing perpendiculars and thus dividing into right-angled triangles. Irrational equations; simple cases of simultaneous quadratics, especially such as can be illustrated by graphs; literal equations; easy permutations and combinations.	

Illustration 11 Godfrey's [1901, p.107]  
Early 'Compromise' at Winchester

After Professor Perry's stimulating denunciation at Glasgow, many of us must be wondering how far are we really able to mend our ways at public schools? [Godfrey, 1901, p.106]

He presented his own 'Compromise,' which had been implemented at Winchester during the previous eighteen months, even given the existing examination constraints. (See Illustration 11.) He added, 'we hope that Professor Perry, in an indulgent mood, would not condemn it utterly' [p.107]. Perry would certainly have supported the inclusion of experimental geometry, geometrical drawing and measuring; the emphasis on decimals, the metric system and approximations in arithmetic; the varied use of squared paper for areas and graphical work in algebra; and the inclusion of logarithmic tables, and other tables for numerical work in trigonometry. Godfrey also led a collective initiative involving a number of younger public schoolmasters, before the MA took any organized steps.

The so-called 'Letter of the Twenty-Three Schoolmasters' appeared in January 1902, in Nature and the Mathematical Gazette, and in the February number of the School World [1902a].<sup>38</sup> In October 1901, the controlling Council of the MA discussed what action it might take in response to the initiative of the BAAS.<sup>39</sup> Younger public schoolmasters such as Godfrey were not represented on the Council at this time, though Minchin and Lodge were both members of the Council and the Committee of the BAAS. The Committee's Chairman, Forsyth, had no associations with the MA at this stage [MA, 1920a]. The MA's Council was not disposed to take an early lead. For example, Dr. Macaulay felt that 'it would be better that suggestions should come from a higher quarter and be discussed by the Association,' and that 'schoolmasters were not the best persons to suggest reforms in teaching.' The Council decided that 'no steps should be taken for the present unless the :

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38 See Nature [1902], Vol.65, No.1681, Jan., pp.258-259, and Mathematical Gazette [1902], Vol.II, No.31, Jan., pp.143-146.

39 MA [1932c], minute 3/10/01.



Association should be approached by the British Association Committee.'<sup>40</sup> However, Forsyth had already approached Godfrey independently suggesting that he should forward proposals to the Committee [Siddons, 1952b, pp.4-5]. Forsyth had become Sadleirian Professor at Cambridge on the death of Cayley in 1895, and, no doubt, Godfrey must have gained his confidence in part as a consequence of a very successful academic performance at Cambridge in the 1890s [Siddons, 1924, p.137; Piaggio, 1931, pp.462-463]. Godfrey [1906a, p.76] later referred to Forsyth's 'broad-minded attitude' concerning the needs of ordinary pupils, as opposed to future 'wranglers,' and the sympathetic attitude of such a leading Cambridge mathematician was a further factor conducive to change at this time.

Siddons [1902a] sketched the background to the schoolmasters' letter in the School World, referring to a 'Committee of twenty-two masters' who had signed a letter reflecting a 'partial opinion' from nine different public schools. He continued:

The 'Committee' referred to never met; in fact, it never existed. One of the twenty-three (for there were twenty-three) wrote the letter and sent it to a few of his personal acquaintances (including myself), nearly all of whom signed it; the two or three who did not do so objected to details and asked for alterations to be made, but time did not permit.<sup>41</sup>

The numerical discrepancy concerning the letter is explained by the fact that one of the original signatories, S.T.H. Saunders of Merchant Taylors' School, was omitted in published versions [Siddons, 1955]. Levett, and Langley, the latter a member of the AIGT from 1882, signed the letter, but the majority were public schoolmasters under thirty.<sup>42</sup>

The substance of this letter deserves reproducing in full. (See

40 MA [1932c], minute 3/10/01.

41 Siddons [1924, p.137] later confirmed that Godfrey wrote the letter, but after Godfrey's death he claimed to have shared in its compilation [Siddons, 1936, p.19, 1952b, pp.4-5].

42 Siddons [1952a, p.153]. Only eight signatories were also members of the MA, though F.W. Dobbs (not W.J. Dobbs), Price and Tuckey did join in 1902 [MA, 1920a]. Hurst [1901] and J.W. Marshall [1902] had independently contributed to Nature concerning reform.

proportionals in  
the triangle into right-angled triangles. Only two trigonometrical identities should be introduced—

$$\sin^2\theta + \cos^2\theta = 1, \text{ and } \frac{\sin\theta}{\cos\theta} = \tan\theta.$$

In short, the work should be arithmetic, and not algebra.

Formal algebra cannot be postponed indefinitely; perhaps now will be the time to return to that neglected science. We might introduce here a revision course of algebra, bringing in literal equations, irrational equations, and simultaneous quadratics illustrated by graphs, partial fractions, and binomial theorem for positive integral index. Side by side with this it ought to be possible to do some easy work in mechanics. Graphical statics may be made very simple; if it is taken up at this stage, it might be well to begin with an experimental verification of the parallelogram of forces, though some teachers prefer to follow the historical order and start from machines and parallel forces. Dynamics is rather more abstract; a first course ought probably to be confined to the dynamics of rectilinear motion.

It is not necessary to discuss any later developments. The plan we have advocated will have the advantage of bringing the pupil at a comparatively early stage within view of the elements of new subjects. Even if this is effected at the sacrifice of some deftness in handling  $a$ ,  $b$  and  $c$ , one may hope that the gain in interest will be a motive power of sufficient strength to carry the student over the drudgery at a later stage. Some drudgery is inevitable, if he is ultimately to make any use of mathematics. But it must be borne in mind that this will not be required of the great majority of boys at a public school.

We beg to remain, gentlemen,

Yours faithfully,

G. M. BELL, Winchester.	R. LEVETT, King Edward's
H. H. CHAMPION, Upping-	School, Birmingham.
ham.	J. W. MARSHALL, Charter-
H. CRAHTRIE, Charterhouse.	house.
F. W. DOBBS, Eton.	L. MARSHALL, Charterhouse.
C. GODFREY, Winchester.	C. W. PAYNE, Merchant Tay-
H. T. HOLMES, Merchant	lors' School.
Tailors' School.	E. A. PRICE, Winchester.
G. H. J. HURST, Eton.	D. S. SHORTO, Rugby.
C. H. JONES, Uppingham.	A. W. SIDDON, Harrow.
H. H. KEMBLE, Charter-	R. C. SLATER, Charterhouse.
house.	H. C. STEEL, Winchester.
T. KENSINGTON, Winchester.	C. O. TUCKER, Charterhouse.
E. M. LANGLEY, Bedford	F. J. WHIPPLE, Merchant
Modern School.	Tailors' School.



## THE TEACHING OF MATHEMATICS IN PUBLIC SCHOOLS.

THE following letter has been sent to the Committee appointed by the British Association to report upon the teaching of elementary mathematics.

GENTLEMEN,—At the invitation of one of your own body, we venture to address to you some remarks on the problems with which you are dealing, from the point of view of teachers in public schools.

As regards geometry, we are of opinion that the most practical direction for reform is towards a wide extension of accurate drawing and measuring in the geometry lesson. This work is found to be easy and to interest boys; while many teachers believe that it leads to a logical habit of mind more gently and naturally than does the sudden introduction of a rigid deductive system.

It is clear that room must be found for this work by some unloading elsewhere. It may be felt convenient to retain Euclid; but perhaps the amount to be memorised might be curtailed by omitting all propositions except such as may serve for landmarks. We can well dispense with many propositions in the first book. The second book, or whatever part of it we may think essential, should be postponed till it is needed for III, 35. The third book is easy and interesting; but Euclid proves several propositions whose truth is obvious to all but the most stupid and the most intellectual. These propositions should be passed over. The fourth book is a collection of pleasant problems for geometrical drawing; and, in many cases, the proofs are tedious and uninteresting. No one teaches Book V. A serious question to be settled is—how are we to introduce proportion? Euclid's treatment is perhaps perfect. But it is clear that a simple arithmetical or algebraical explanation covers everything but the case of incommensurables. Now this case of incommensurables, though in truth the general case, is tacitly passed over in every other field of elementary work. Much of the theory of similar figures is clear to intuition. The subject provides a multitude of easy exercises in arithmetic and geometrical drawing; we run the risk of making it difficult of access by guarding the approaches with this formidable theory of proportion. We wish to suggest that Euclid's theory of proportion is properly part of higher mathematics, and that it shall not in future form part of a course of elementary geometry. To sum up our position with regard to the teaching of geometry, we are of opinion—

- (1) That the subject should be made arithmetical and practical by the constant use of instruments for drawing and measuring.
- (2) That a substantial course of such experimental work should precede any attack upon Euclid's text.
- (3) That a considerable number of Euclid's propositions should be omitted; and in particular
- (4) That the second book ought to be treated slightly, and postponed till III, 35, is reached.
- (5) That Euclid's treatment of proportion is unsuitable for elementary work.

Arithmetic might well be simplified by the abolition of a good many rules which are given in text-books. Elaborate exercises in vulgar fractions are dull and of doubtful utility; the same amount of time given to the use of decimals would be better spent. The contracted methods of multiplying and dividing with decimals are probably taught in most schools; when these rules are understood, there is little left to do but to apply them. Four-figure logarithms should be explained and used as soon as possible; a surprising amount of practice is needed before the pupil uses tables with confidence.

It is generally admitted that we have a duty to perform towards the metric system; this is best discharged by providing all boys with a centimetre scale and giving them exercise in verifying geometrical propositions by measurement. Perhaps we may look forward to a time when an elementary mathematical course will include at least a term's work of such easy experiments in weighing and measuring as are now carried on in many schools under the name of physics.

Probably it is right to teach square root as an arithmetical rule. It is unsatisfactory to deal with surds unless they can be evaluated, and the process of working out a square root to five places provides a telling introduction to a discourse on incommensurables; furthermore, it is very convenient to be able to assume a knowledge of square root in teaching graphs. The

same rule is needed in dealing with mean proportionals in geometry.

Cube root is harder and should be postponed until it can be treated as a particular case of Horner's method of solving equations approximately.

Passing to algebra, we find that a teacher's chief difficulty is the tendency of his pupils to use their symbols in a mechanical and unintelligent way. A boy may be able to solve equations with great readiness without having even a remote idea of the connection between the number he obtains and the equation he started from. And throughout his work he is inclined to regard algebra as a very arbitrary affair, involving the application of a number of fanciful rules to the letters of the alphabet.

If this diagnosis is accepted, we shall be led naturally to certain conclusions. It will follow that elementary work in algebra should be made to a great extent arithmetical. The pupil should be brought back continually to numerical illustrations of his work. The evaluations of complicated expressions in  $a$ ,  $b$  and  $c$  may of course become wearisome; a better way of giving this very necessary practice is by the tracing of easy graphs. Such an exercise as plotting the graph  $y = 2x - \frac{x^2}{4}$

provides a series of useful arithmetical examples, which have the advantage of being connected together in an interesting way. Subsequently, curve-tracing gives a valuable interpretation of the solutions of equations. Experience shows that this work is found to be easy and attractive.

With the desire of concentrating the attention of the pupil on the meaning rather than the form of his algebraical work, we shall be led to postpone certain branches of the subject to a somewhat later stage than is usual at present. Long division, the rule for H.C.F., literal equations and the like will be studied at a period when the meaning of algebra has been sufficiently indicated by arithmetical work. Then, and not till then, will be the time to attend to questions of algebraic form.

But at no early stage can we afford to forget the danger of relapse into mechanical work. For this reason it is much to be wished that examining bodies would agree to lay less stress upon facility of manipulation in algebra. Such facility can generally be attained by practice, but probably at the price of diminished interest and injurious economy of thought. The educational value of the subject is sacrificed to the perfecting of an instrument which in most cases is not destined for use.

To come to particulars, we think that undue weight is often given to such subjects as algebraic fractions and factors. The only types of factors which crop up continually are those of  $x^2 - a^2$ ,  $x^2 \pm 2ax + a^2$ , and, generally, the quadratic function of  $x$  with numerical coefficients.

In most elementary algebra books there is a chapter on theory of quadratic equations in which a good deal of attention is paid to symmetric functions of roots of quadratics. No further use is to be made of this until the analytical theory of conics is being studied. Might not the theory of quadratics be deferred until it can be dealt with in connection with that of equations of higher degree?

Indices may be treated very slightly. The interpretation of negative and fractional indices must of course precede any attempt to introduce logarithms; but when the extension of meaning is grasped, it is not necessary to spend much more time on the subject of indices; we may push on at once to the use of tables.

It will be seen that our recommendations under the head of algebra are corollaries of two or three simple guiding thoughts, the object in view being—to discourage mechanical work; the means suggested—to postpone the more abstract and formal topics and, broadly speaking, to arithmetise the whole subject.

The omission of part of what is commonly taught will enable the pupil to study, concurrently with Euclid VI., a certain type of diluted trigonometry which is found to be within the power of every sensible boy. He will be told what is the meaning of sine, cosine, and tangent of an acute angle, and will be set to calculate these functions for a few angles by drawing and measurement. He will then be shown where to find the functions tabulated, and his subsequent work for that term will consist largely in the use of instruments, tables and common sense. A considerable choice of problems is available at once. He may solve right-angled triangles, work sums on "heights and distances," plot the graphs of functions of angles, and make some progress in the general solution of triangles by dividing

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Illustration 12.) In the main, it was an expanded version of Godfrey's 'Compromise,' five of the signatories being Winchester masters. The letter did not press strongly for the abandonment of Euclid's order, and one teacher felt that it contained 'many suggestions which, surely, all mathematical masters will gladly endorse; but are they not too timid in their attack upon Euclid?' [Kingdon, 1902]. Otherwise, the letter exemplified the progressive standpoint in public schools, and this teacher hoped that some scheme to replace Euclid would be forthcoming from the Committee of the BAAS. A writer in the School World [1902a, p.62] judged that, in spite of the letter's somewhat cautious stance concerning Euclid, it would 'strengthen the hands of the committee very considerably,' and drew attention to the fact that organized opposition to change had not been forthcoming. However, at this time there was also some criticism of the circumstances which had led to the letter's production. Hawkins [1902] of Haileybury College complained that teachers had not been officially consulted by the Committee up to March of 1902, which the editors of the School World confirmed, adding that:

It was, of course, quite permissible [sic] for one of the members to invite a statement of views from practical school-masters; but the invitation and the response must be regarded as of the nature of individual actions, and no more than this is implied in the letter. [Hawkins, 1902, p.118]

However, plenty of other views were being expressed at this time [School World, 1902b]. From an advanced standpoint, Euclid had been exposed as logically imperfect. Berry [1902, p.83] of Cambridge remarked 'As every mathematician knows, modern criticism has detected flaw after flaw in Euclid's logic.' Thus, as Bertrand Russell [1902, p.165] subsequently pointed out in the Mathematical Gazette, the claim that Euclid was 'an invaluable training to the youthful powers of reasoning ... vanishes on a close inspection' because:

His definitions do not always define, his axioms are not always indemonstrable, his demonstrations require many axioms of which he is quite unconscious.



As in the 1870s, a key question was the need or otherwise for some standard order of geometrical theorems. In a paper given to the Association of Principals and Lecturers in Training Colleges, Minchin [1902, p.102] concluded:

I agree that there must be some such standard, although even this is disputed by some reformers. I do not see how we can do without it, but I deny the necessity for having Euclid.

Fletcher [1902a, p.103], however, in an address to the Association of Headmasters argued that Euclid's order was a hindrance, and added 'I am not suggesting the imposition of any other system; until we are free to experiment we cannot get at the best.' He managed to convince this Association to adopt a strong resolution:

That the Association desires to press upon the Universities and other examining bodies the desirability of greater elasticity in their regulations, and is of opinion that to insist upon adherence to the order of propositions in Euclid is mischievous. [Fletcher, 1902a, p.102]

The MA was also goaded into action early in 1902.

Langley [1901, p.105] argued in the Mathematical Gazette that the MA:

should take advantage of the upheaval at Glasgow to press some of their immediate aims.... I do not ... think that we need to look upon Prof. Perry as an opponent but rather as a valuable ally.... Our cause may be advanced considerably if we make a proper use of the present opportunity.

He concluded:

I venture to hope that some official action will be taken ... and that some of the younger members of the Association with whom the future lies will throw themselves heartily into the work. [p.106]

The MA discussed the general question of reform at its Annual Meeting in January of 1902. Lodge argued:

I think this meeting should not conclude without appointing a strong committee to co-operate with the British Association Committee and assist it in every way possible. That Committee has already had a valuable communication signed by 23 schoolmasters, and has asked me to express to this Association its request for the fullest co-operation and advice. [MA, 1902a, p.132]

Thus the Council of the MA resolved that the first Teaching Committee should be established, to include Godfrey and other schoolmasters on the Council, as well as Minchin and Lodge, who were also on the Committee of the BAAS. To facilitate meeting, the full Committee of around thirty members of the MA was largely composed of public schoolmasters from the London area, with a significantly younger element, as well as a number of older members of the AIGT.<sup>43</sup> Thus, as Siddons [1924, p.137] subsequently acknowledged:

The Letter from 23 Schoolmasters ... was the main cause of the appointment of the Mathematical Association Teaching Committee.

With the meetings normally chaired by Lodge or Minchin, and with Siddons as Secretary, there was a much greater sense of urgency in the Committee's work in 1902 than had been the case with the AIGT thirty years earlier [Siddons, 1936, p.19]. The Committee's detailed recommendations in geometry were published in the May number of the Mathematical Gazette [MA, 1902c], and their progress was reported upon by Siddons [1902a, 1902b] in the School World, and by Godfrey [1902a] in Nature. A draft had been sent to all schools listed in the Public Schools Year Book. Significantly, the Committee decided to avoid the problem of a new order of theorems, and still followed Euclid's. Siddons [1902a, p.197] explained:

We consider it would be unwise to propose further change at present than the introduction of a course of geometrical drawing and measurement, and the shortening of the course of Euclid 'by judicious omission and readjustment.'

Godfrey [1902a, p.202] referred to the report as 'an attempt, on conservative lines, to simplify the study of geometry and to make it interesting.' He added, 'If the attempt is judged to be successful, now is the time to make examiners unstop their ears.' As it turned out, examiners generally chose to go much further than the MA's Committee, and, as Godfrey [1920, p.20] subsequently admitted:

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43 MA [1932c], minute 18/1/02, and MA [1902b], inside front cover.



*I.—The Aim of Mathematics.* We consider to be, in the first place, to furnish the mind with those concepts of number and form which give quantitative definiteness to all branches of knowledge.

We believe that incidentally mathematics afford a certain formal training of the mind, although the subject can claim no monopoly in this respect.

*II.—Arithmetic.* (1) The method of teaching in the early stages should be inductive and concrete. Actual measuring and weighing should be introduced as early as possible.

(2) Decimals should be treated as an extension of the ordinary notation, their nature being illustrated by actual metric weights and measures. Multiplication and division of a decimal by a decimal would, we think, have to follow vulgar fractions.

(3) The decimalisation of English money and English weights and measures should be practised frequently.

(4) Approximate methods should be gradually introduced after the treatment of finite decimals. They should be taught with due regard to rigidity of proof. Appreciation of the degree of approximation should be continually insisted upon.

(5) If "commercial arithmetic" is to be taught at all, the subject-matter should receive more adequate and correct treatment, and the examples should be drawn from transactions as they actually occur.

*III.—Algebra.* (1) The foundation of algebra should be "literal arithmetic," *i.e.*, algebra should at first be arithmetic generalised.

(2) The minus sign should receive its extended meaning from copious illustrations; and illustrations, not rigid proof, should also be resorted to for the purpose of the "rule of signs."

(3) Algebra should often be applied to geometry.

(4) Logarithms should form an important section of the subject. We believe that the graphic method could be very usefully employed in this connection.

(5) We desire to deprecate the waste of time so commonly practised in mere manipulation of symbols.

*IV.—Geometry.* (1) We are strongly of opinion that the ordinary deductive geometry should be preceded and continually supplemented by concrete and inductive work.

(2) Whilst "mensuration" might possibly be taught in connection with physics and arithmetic, we believe that the value of geometry would be enhanced by practical applications of the propositions as they occur.

(3) We feel very strongly that Euclid's text is very unsuitable for teaching geometry. But we are impressed with the difficulty of abolishing its use in the face of external examinations. Under the circumstances, we can only hope that examining bodies, even if they insist on Euclid's sequence, will allow greater latitude in methods of proof, and give greater prominence to easy "riders" and applications of geometry.

Illustration 13 The Preliminary Report of  
the AMA on the Teaching of Elementary  
Mathematics [School World, 1902c]

On the eve of our liberation the M.A. published a report on Geometry teaching, a very conservative report, as it was considered impracticable to secure the abolition of the sequence. This report became obsolete in 1902....

The Committee also reported provisionally on arithmetic and algebra in the July number of the Mathematical Gazette, suggesting various omissions and simplifications, more understanding, links between the branches, and the extensive use of graphs [MA, 1902d].<sup>44</sup> The AMA also produced a statement for the Committee of the BAAS, which was published in its Circular to Members and in the School World. (See Illustration 13.) On the question of Euclid, the AMA adopted an attitude of cautious optimism, and avoided the somewhat pedantic detail provided by the MA.<sup>45</sup> The Committee of the BAAS reported to Section L at Belfast in September 1902.

The Report of the BAAS's Committee was published in the Mathematical Gazette and the School World [1902d] in 1902, and in the Annual Report of the BAAS [1903, pp.473-480], in the following year.<sup>46</sup> It was drawn up by the Chairman, Forsyth, who began by referring back to the work of the BAAS in the 1870s. At that time, attempts to get examining bodies to adopt a new uniform scheme for geometry had failed. However, in the 1870s the need for uniformity was taken as axiomatic, whereas the Committee now felt:

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44 At this time, Godfrey [1902b, 1902c, 1902d] produced detailed and illuminating suggestions for the teaching of the three branches and simple trigonometry, with the needs of preparatory schools particularly in mind. These articles were published in the School World, and not the Mathematical Gazette, which was still far from being a lively journal for ordinary school mathematics. A content analysis of the Gazette will be discussed in Chapter 4.

45 W.J. Dobbs was a leading mathematical master within the AMA, and presented a paper on geometry in September, which was published in the Journal of Education [Dobbs, 1902]. Here he worked out in some detail his visionary belief that 'considerable use should be made throughout of the idea of motion' [Dobbs' stress, p.730], building also on some of Henrici's ideas.

46 See Mathematical Gazette [1902], Vol.II, No.35, Oct., pp.197-201. The Gazette omitted the Appendix of two suggested schemes of experimental geometry, provided by Eggar and Perry.



if the teaching is to be improved, it seems to be a preliminary requisite that examinations should be modified; and, where it is possible, these modifications in the examinations should leave greater freedom to the teacher, and give him more assistance than at present. [School World, 1902d, p.389]

Thus, the crucial change of standpoint was the rejection of the need for uniformity, though the Committee offered some general guidelines for the earlier stages of geometry, including two suggested schemes..

As regards a propaedeutic practical course, the Report stressed:

This practice should be adopted, whether Euclid be retained, or be replaced by some authorised text-book or syllabus, or if no authority for demonstrative geometry be retained. [p.389]

Thus, the possibility of retaining Euclid's order was not ruled out, though the Committee felt:

it is not necessary that one (and only one) text-book should be placed in the position of authority in demonstrative geometry; nor is it necessary that there should be only a single syllabus in control of all examinations. Each large examining body might propound its own syllabus.... [p.390]

Furthermore, the Report went on:

In every case, the details of any syllabus should not be made too precise. It is preferable to leave as much freedom as possible, consistently with the range to be covered; for in that way the individuality of the teacher can have its most useful scope. It is the competent teacher, not the examining body, who can best find out what sequence is most suited educationally to the particular class that has to be taught. [p.390]

Thus, the Committee boldly argued for freedom and variety in teaching geometry, and gave educational considerations a high priority, in sharp contrast to the academic deliberations of the 1870s. The Report avoided fine detail, leaving this to teachers and examiners, but made a number of general recommendations:

The Report stressed the need for examinations to test more than just recall of textual material in geometry, and suggested the inclusion of practical tasks, riders, and related arithmetical and and algebraic exercises. Some recommendations were also made concerning these other two branches. Manipulative excesses and

unnecessary complications were deprecated, correlation between the two branches was favoured, as well as an emphasis on the use of tables, graphical methods and formulae. The Report concluded by suggesting that, through pruning and simplification, the curriculum could be broadened to include introductory elements of trigonometry, co-ordinate geometry and even calculus [pp.390-391]. Thus, there appears to have been a fair amount of sympathy within the Committee for Perry's conception of elementary mathematics, though certainly not for the abandonment of systematic demonstrative geometry. At Belfast, a paper from Siddons followed the Report's presentation, and a discussion ensued involving, in particular, Forsyth, Perry, Eggar and Godfrey.<sup>47</sup> According to Siddons [1952b, pp.6-8], it was at Belfast that he and Godfrey were advised by Forsyth to write a geometry textbook ignoring Euclid's order, as Forsyth anticipated major concessions in future Cambridge examinations. Siddons [p.8] also claimed that 'Cambridge gave the lead to the other examining bodies and, naturally, this caused a flood of new text-books.' These conclusions will be shown to be rather curious, though they are shared by other writers.<sup>48</sup>

Early reforms in technical, naval and military examinations have already been described (see pp.103-104). In addition, in as early as May of 1902, important changes in the Oxford Locals were announced in Nature, to come into force in 1903. The new syllabus for geometry stated:

Questions will be set so as to bring out as far as possible a knowledge of the principles of geometry, a smaller proportion than heretofore consisting of propositions as enunciated in Euclid. Any solution which shows an accurate method of geometrical reasoning will be accepted.... Geometrical proofs of the theorems in Book ii. will not be insisted upon.<sup>49</sup>

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47 See note 46. [p.201].

48 See, for example, Piaggio [1931, p.463], and, more recently, Brock [1975b, p.31] and Howson [1973a, p.165].

49 Quoted in Perry [1902b, p.81]. Godfrey [1906a, p.76], looking back on general reforms, referred to the 'dreadful Book II' as a 'slough which has engulfed so many pilgrims.' This Book subsequently became 'a pleasant geometrical illustration of certain well-known formulae in algebra.'



H.T. Gerrans of the Oxford Delegacy was on the Committee of the MA which chose not to depart from Euclid's order. However, he had presumably heard enough from school-teachers to be convinced that major change would not be precipitate. He subsequently remarked that 'the universities had in the past deferred such alterations because of their doubt as to whether the schools were ready for change.'<sup>50</sup>

Siddons [1902b, p.253], however, reflected the dominant public school view within the MA, and claimed:

Most teachers feel that a standard order is essential in England, because boys are constantly passing from master to master, besides passing from preparatory to public school, and also because of our system of examinations. (The Oxford Local Authorities and the Civil Service Commission do not seem to consider the latter reason to be of any weight.)

Changes in the requirements for London Matriculation were also announced in 1902, and discussed in detail by C.H. French [1902] in the October number of the School World. He noted:

there is a marked tendency on the part of some of the most important examining bodies to modify their syllabuses so as to recognise modern methods... This is so in the case of the Oxford Locals as well as the London Matriculation.... Cambridge still remains, for the most part, unmoved; but it is hardly conceivable that this attitude will be maintained for long. [p.363]

The new London syllabus in arithmetic emphasized the use of the metric system, contracted methods of approximate computation, and practical applications. The introduction of graphs and the construction and use of formulae were the main changes in algebra. Interestingly, French [p.365] remarked:

Here graphs [his stress] is the word which has caught the popular eye, witness an advertisement in a morning paper: 'Wanted immediate preparation in mathematics for the London Matric., graphs necessary.'

The changes in geometry followed the lines of the Oxford

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50 Quoted by A.T.S. [1903, p.260], reporting on the London Conference of Science Teachers in January of 1903, which chose to consider mathematical education in detail, and attracted 'a larger attendance than in any previous year' [p.259], amounting to over four hundred.

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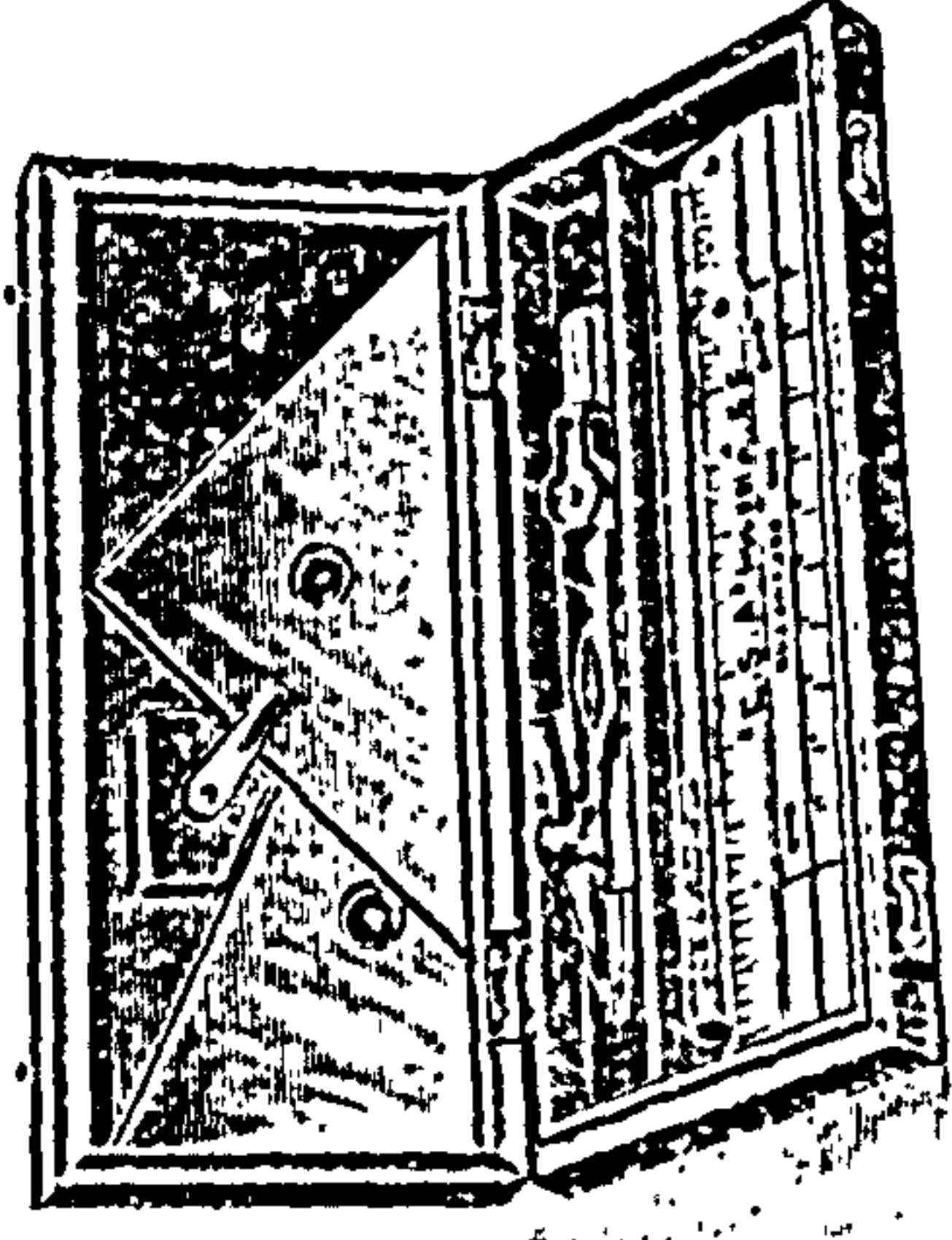
1 Nickel Set Square (unbreakable),  $60^\circ$ .

Price 1s. 9d. per Box. { Price for orders of 6 dozen, 1s. 7d. per Box.  
" " " 12 dozen, 1s. 6d. per Box.

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Illustration 14 Instruments for Practical Geometry in  
the Cambridge Locals [College of Preceptors, 1903a]



Locals.<sup>51</sup> Before the end of 1902, the Cambridge Local Examinations Syndicate also announced changes in their requirements which were publicized in Nature [Perry, 1902b] and the School World [1902e].

The Cambridge Syndicate had appointed a special committee, including Forsyth and Hobson, to consider mathematics in the Locals [Siddons, 1902b, p.253].<sup>52</sup> Initially, late in 1902, detailed changes were announced for the Preliminary and Junior Locals, but for the Senior Locals also it was stated that:

Any proof of a proposition will be accepted which appears to the examiners to form part of a logical order of treatment of the subject. In the proof of theorems and deductions from them, the use of hypothetical constructions is permitted.<sup>53</sup>

The syllabus covered theoretical and practical geometry, with a set of instruments prescribed. (See Illustration 14.) The syllabus for theoretical geometry referred to the 'substance' of portions of Euclid, using his numbering, and there were many omissions including the whole of Euclid II. The requirements also specified 'Questions upon these theorems, easy deductions from them, and arithmetical illustrations! [School World, 1902e, p.465]. Specimen papers were published in December of 1902, and subsequently reproduced in the School World [1903a], which also published a full discussion of the new teaching possibilities raised by the reforms in the Cambridge

- 51 French [1902, p.366] also referred to some of the 'numerous' new introductory textbooks in geometry, including Minchin's Geometry for Beginners, and Hamilton and Kettle's [1900] A First Geometry Book: A Simple Course of Exercises Based on Experiment and Discovery. For an innovative deductive course he recommended Fletcher's [1902b] Elementary Geometry, covering Euclid I-IV and VI.
- 52 Forsyth began his association with the MA as its President for 1903-1905, and he claimed the MA's work had helped in the reforms at Cambridge [MA, 1903a, p.259]. Hobson became President of the MA for 1911-1913 [Piaggio, 1931]. These two Sadleirian Professors also campaigned strongly for the reform of the Mathematical Tripos (see pp.73-74).
- 53 Quoted in School World [1902e, p.464]. Euclid did not assume a construction in a proof unless the construction had already been established formally in the deductive development. Allowing 'hypothetical constructions' meant that constructions could be freely brought in as hypotheses when desirable. This was a major pedagogical concession.

Locals.<sup>54</sup> The Cambridge Previous was not affected by these changes, though another Syndicate, again involving Forsyth, worked on reforms here, from the end of 1902. However, Oxford again moved before Cambridge in the reform of its requirements for Responsions.

In January of 1903, the School World [1903b] announced that a notice signed by Gerrans, Chairman of the Board of Studies, had been circulated. The new regulations stated that 'Any method of proof will be accepted which shows clearness and accuracy in geometrical reasoning,' to cover Euclid I-III, with various omissions, and allowed algebraic arguments in Euclid II. Given the peculiar alternative of Euclid or algebra in Responsions, the changes in geometry subsequently had a surprising effect in practice, which was described by Professor Turner [1912, p.398] in the Special Reports:

Seeing that the new methods of geometrical teaching were designed to help the weaker brethren, it was expected that when they were introduced geometry would be preferred to algebra. But exactly the contrary happened; geometry was deserted and nearly all the candidates chose algebra. On making inquiry I found the reason to be that for the purposes of passing an examination it is undoubtedly easier to cram a dull boy with a few facts in the old way (which is still unreformed in algebra) than to teach him how to reproduce his knowledge of geometry.... Many of the boys in question formerly learnt their Euclid by heart, and this resource has been taken from them. They now learn a few rules in algebra by heart. Whether this does them much good is another question.<sup>55</sup>

A similar kind of attitude to mathematics in the Previous was described by Rumsey [1903, p.11], before changes here were also implemented:

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54 See Deakin [1903], who also provided a general review of a number of the new geometry textbooks. Significantly, the Cambridge Schedule suggested the use of books on geometrical drawing for the practical geometry, though at this time most of these books served the technical, naval and military fields. However, practical geometries with a wider appeal had also recently appeared, such as Eggar's [1903a] Practical Exercises in Geometry.

55 H.H. Turner, Savilian Professor of Astronomy at Oxford, was presumably the 'H.H.T.' who became a friend of Perry, and compiled his obituary for the Royal Society, which, curiously, was published in 1926, six years after Perry's death [H.H.T. (Turner), 1926].



In equal circles (or, in the same circle) (i) if two arcs subtend equal angles at the centres, they are equal; (ii) conversely, if two arcs are equal, they subtend equal angles at the centres.

In equal circles (or, in the same circle) (i) if two chords are equal, they cut off equal arcs; (ii) conversely, if two arcs are equal, the chords of the arcs are equal.

Equal chords of a circle are equidistant from the centre; and the converse.

The tangent at any point of a circle and the radius through the point are perpendicular to one another.

If two circles touch, the point of contact lies on the straight line through the centres.

The angle which an arc of a circle subtends at the centre is double that which it subtends at any point on the remaining part of the circumference.

Angles in the same segment of a circle are equal; and, if the line joining two points subtends equal angles at two other points on the same side of it, the four points lie on a circle.

The angle in a semicircle is a right angle; the angle in a segment greater than a semicircle is less than a right angle; and the angle in a segment less than a semicircle is greater than a right angle.

The opposite angles of any quadrilateral inscribed in a circle are supplementary; and the converse.

If a straight line touch a circle, and from the point of contact a chord be drawn, the angles which this chord makes with the tangent are equal to the angles in the alternate segments.

If two chords of a circle intersect either inside or outside the circle the rectangle contained by the parts of the one is equal to the rectangle contained by the parts of the other.

*Proportion: Similar Triangles.*

If a straight line is drawn parallel to one side of a triangle, the other two sides are divided proportionally; and the converse.

If two triangles are equiangular their corresponding sides are proportional; and the converse.

If two triangles have one angle of the one equal to one angle of the other and the sides about these equal angles proportional, the triangles are similar.

The internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle, and likewise the external bisector externally.

The ratio of the areas of similar triangles is equal to the ratio of the squares on corresponding sides.

*Triangles and Rectilinear Figures.*

The sum of the angles of a triangle is equal to two right angles.

If the sides of a convex polygon are produced in order, the sum of the angles so formed is equal to four right angles.

If two triangles have two sides of the one equal to two sides of the other, each to each, and also the angles contained by those sides equal, the triangles are congruent.

If two triangles have two angles of the one equal to two angles of the other, each to each, and also one side of the one equal to the corresponding side of the other, the triangles are congruent.

If two sides of a triangle are equal, the angles opposite to these sides are equal; and the converse.

If two triangles have the three sides of the one equal to the three sides of the other, each to each, the triangles are congruent.

If two right-angled triangles have their hypotenuses equal, and one side of the one equal to one side of the other, the triangles are congruent.

If two sides of a triangle are unequal, the greater side has the greater angle opposite to it; and the converse.

Of all the straight lines that can be drawn to a given straight line from a given point outside it, the perpendicular is the shortest.

The opposite sides and angles of a parallelogram are equal, each diagonal bisects the parallelogram, and the diagonals bisect one another.

If there are three or more parallel straight lines, and the intercepts made by them on any straight line that cuts them are equal, then the corresponding intercepts on any other straight line that cuts them are also equal.

*Areas.*

Parallelograms on the same or equal bases and of the same altitude are equal in area.

Triangles on the same or equal bases and of the same altitude are equal in area.

Equal triangles on the same or equal bases are of the same altitude.

Illustrations and explanations of the geometrical theorems corresponding to the following algebraical identities:

$$k(a+b+c+\dots) = ka + kb + kc + \dots,$$

$$(a+b)^2 = a^2 + 2ab + b^2,$$

$$(a-b)^2 = a^2 - 2ab + b^2,$$

$$a^2 - b^2 = (a+b)(a-b).$$

The square on a side of a triangle is greater than, equal to, or less than the sum of the squares on the other two sides, according as the angle contained by those sides is obtuse, right, or acute. The difference in the cases of inequality is twice the rectangle contained by one of the two sides and the projection on it of the other.

*Loci.*

The locus of a point which is equidistant from two fixed points is the perpendicular bisector of the straight line joining the two fixed points.

The locus of a point which is equidistant from two intersecting straight lines consists of the pair of straight lines which bisect the angles between the two given lines.

*The Circle.*

A straight line, drawn from the centre of a circle to bisect a chord which is not a diameter, is at right angles to the chord; conversely, the perpendicular to a chord from the centre bisects the chord.

There is one circle, and one only, which passes through three given points not in a straight line.



the universities are perhaps not greatly to blame for the lack of reforming spirit which has hitherto existed in their dealings with the examinations for poll degrees and Little Go. The dons know very well that these examinations, however well appointed, will never be taken seriously by the candidates.

Rumsey hoped for changes soon, for otherwise 'a very awkward situation will be created in some of our largest public schools' [p.11]. As Langley [1903, p.359] subsequently remarked, up to this time:

the success of reformers, though considerable, was still partial, in its range ... because those trained in the great Public Schools who did not look forward to enter the services, might still be taught to regard the reproduction of Euclid's text as the ultimate aim of geometrical teaching.

The Cambridge Syndicate finally reported in May of 1903, following six months of deliberations, and the reforms were publicized in Nature [1903a] and the School World [1903c] in June. Detailed Schedules A and B for practical and theoretical geometry in the Previous were provided, with no order imposed in the case of the latter, and the propositions were listed in groups. The spirit of the new requirements in geometry followed the lines of the Cambridge Locals. (See Illustration 15.) In arithmetic, algebraic methods were permitted, and graphs and four-figure logarithms were brought into algebra. Langley [1903, p.359] judged that the changes in the Previous virtually brought to completion the success of the movement for examination reforms, and, furthermore, the changes affecting pass degrees had important implications for the schools also:

For the pass men of Oxford and Cambridge supply to a great extent the teaching staff of the secondary schools.... We may now feel assured that this cause of geometrical stagnation has been removed. [p.361]<sup>56</sup>

By 1905, the MA's [1905a, p.1] Teaching Committee judged that the geometry requirements of the Previous 'form now the standard for the school teaching of Geometry in this country.' However, this standard

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56 The Oxbridge Joint Board subsequently adopted the Cambridge Schedules [Retter, 1936, p.163], and the BE also followed the lines of Cambridge in its examinations for teacher training (see pp.63,66).

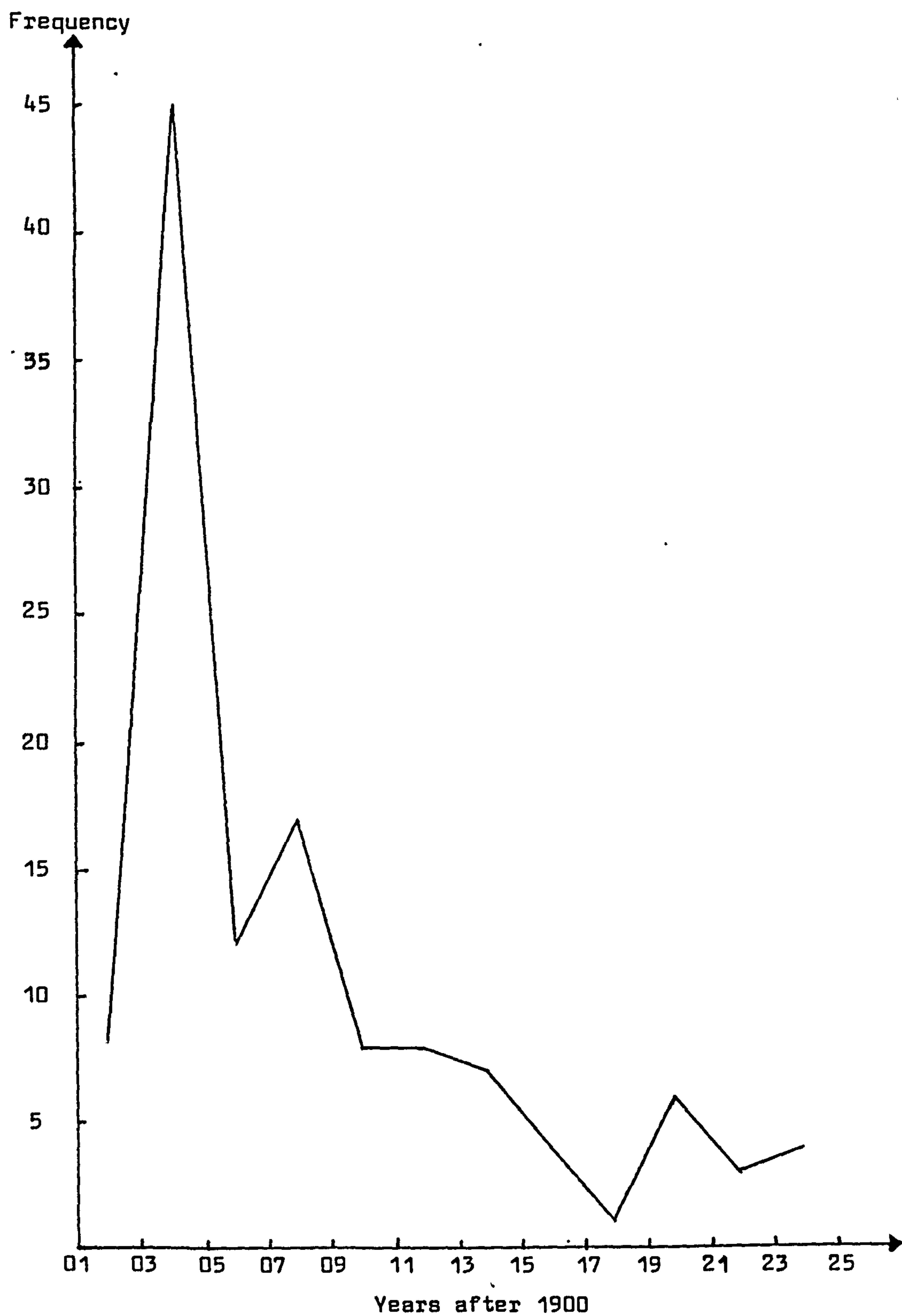


Illustration 16 Output of New Geometries  
in the Early Twentieth Century



was painstakingly worked out as a culmination of activity in the reform of mathematical examinations, and was not a breakthrough for freedom from Euclid in geometry teaching, where Cambridge was far from being in the vanguard of progress. 'The Reform in Mathematical Education' was also enthusiastically publicized in the journal Engineering during 1903 [Correspondent, 1903a, 1903b], and before the end of the year the School World [1903d, 1903e] also reported major developments in Scotland and America.<sup>57</sup> As well as creating a new demand for geometrical drawing instruments and aids of various kinds, including squared paper, examination reforms also stimulated a remarkable surge in the output of new textbooks from authors and publishers competing in a highly competitive but profitable new market.<sup>58</sup>

The output of new geometries, whether preliminary, practical, theoretical, or some combination of these features, was quite extraordinary in the early years of this century, particularly in the two-year period 1903-1904, during which there appeared around thirty-five per cent of the total output of first editions over the first quarter of this century. (See Illustration 16.)<sup>59</sup> Many of the new textbooks were reviewed in the pedagogical literature, particularly in 1903, and the general situation in the schools around this time must have

57 Within the chaotic English examination system some examining bodies remained unmoved until as late as 1905. The Law Society and the Surveyor's Institution still required a knowledge of Euclid in their preliminary examinations, until the MA took specific action through correspondence in each case [Siddons, 1906].

58 The new practical tendencies in teaching will be explored in Chapter 5.

59 Being a copyright deposit library, the British Museum [1906, 1911, 1918, 1922, 1927, 1967] Collection was used to measure the output of first editions [Esdaile, 1946]. Separately published parts, such as those provided in geometry by the hugely successful Hall and Stevens, and advertised in their textbooks [1904a, 1904b], have not been counted, nor have answers or keys. The date for a book also published in parts over a period has been taken as the first date by which it had appeared complete in some form. The geometries counted include some new Euclids, in 1901, and some technically orientated geometries, including some treating aspects of solid geometry.

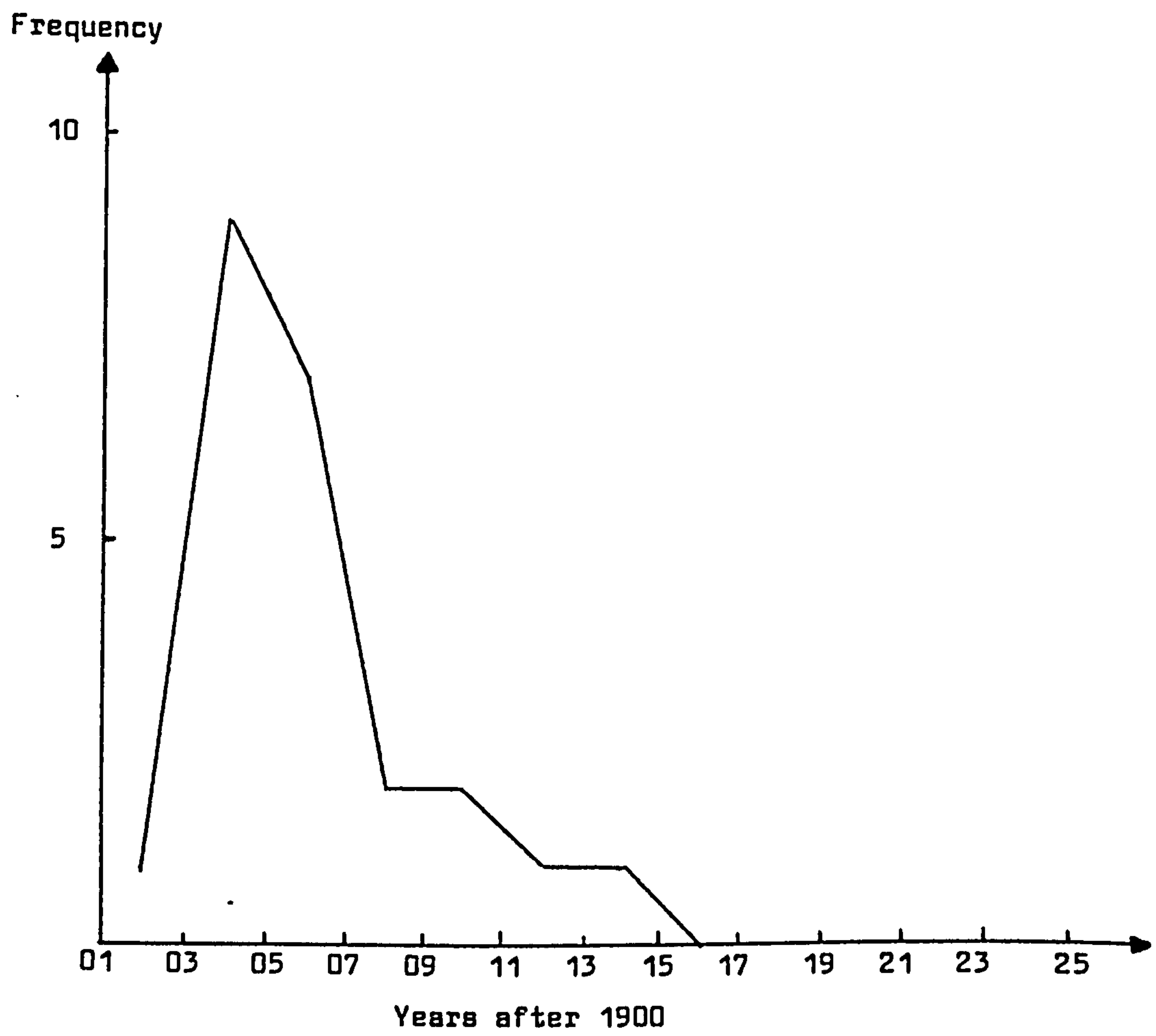


Illustration 17 Output of Early Twentieth-Century  
Publications on Graphs



been a stimulating but confusing one for many teachers seeking new directions in an unprecedented climate of freedom.<sup>60</sup> It is also clear that many textbooks had already been published, or were in an advanced stage of production, before Cambridge published its Schedules for the Previous in June of 1903. As has been shown, considerable energy for reform had been generated in the years 1901 and 1902. One other extraordinary publishing phenomenon concerned the new emphasis on graphical methods in algebra.

A number of publications on graphs, the majority containing relatively few pages, suddenly appeared on the market from the end of 1902, as supplements to larger textbooks on algebra. (See Illustration 17.)<sup>61</sup> Hall's [1903] A Short Introduction to Graphical Algebra was compiled on a seaside holiday at short notice to meet a 'sudden demand.' Other writers soon followed with similar contributions, but the output tailed off sharply before the First World War. The demand was, of course, stimulated by the new examination requirements, and, indeed, French and Osborne's [1906] contribution was entitled Matriculation Graphs, with London's requirements particularly in mind.<sup>62</sup> The treatment of graphs subsequently became integrated within algebra textbooks, though there was a tendency in the early years of this century to treat 'graphical algebra' as some new branch of mathematics. As Palmer [1912, p.235] subsequently remarked:

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60 For example, Langley reviewed many new geometries in the Mathematical Gazette during 1903 [MA, 1933, pp.106-160], and the monthly School World's coverage was very rich. See also Harrison [1903a] in Nature and Nature [1903b].

61 The British Museum Collection was again used to measure the output.

62 French and Osborne [1903] had already produced a similar booklet, three years earlier. Professor Gibson's [1904] An Elementary Treatise on Graphs was an exceptionally rich treatment devised to convince readers that there was much more to graphical work than the narrow exercises which were being overdone in the schools, in preparation for examinations. Graphical methods will be discussed in Chapters 5 and 7.

Graphical work was introduced into the Algebra course as an instrument of teaching; it provided an effective means of illustration. But to many text-book writers Algebra is only a loose congeries of miscellaneous subjects thrown together for convenience' sake, and 'graphs' is but another added to the collection. It may be confined to a single chapter, and this chapter the last, as is the case in at least one well-known text-book....

Palmer was referring to Hall and Knight's highly successful Elementary Algebra for Schools [1885], to which Hall later grafted on his contribution concerning graphs, for the 1907 edition, thereby missing many of the graphical possibilities in the rest of the book.<sup>63</sup>

Individual and collective discussion of the progress of reform in the early stages of upheaval accompanied the changes in mathematical examinations and the production of new textbooks.

At Belfast, in 1902, Perry delivered another powerful address, to open a joint discussion between Sections G (Engineering) and L (Education) of the BAAS on the education of engineers. Again he turned some of his attention to the general teaching of mathematics, and, given the encouraging progress already made, he looked forward optimistically to further achievements:

It seems probable that at the end of another five years no average boy of fifteen years of age will have been compelled to attempt any abstract reasoning about things of which he knows nothing; he will be versed in experimental mathematics, which he may or may not call mensuration; he will use logarithms, and mere multiplication and division will be a joy to him; he will have a working power with algebra and sines and cosines; he will be able to tackle at once any curious new problems which can be solved by squared paper; and he will have no fear of the symbols of the infinitesimal calculus.... Five years hence it will be called 'elementary mathematics.' Four years ago it was an unorthodox subject called 'practical mathematics....' [Perry, 1903a, p.719]

Certainly the general trend in secondary schools was towards Perry's broader and more practical conception of 'elementary mathematics,' but the developments were slow and complex, except in the cases of practical geometry and graphs.<sup>64</sup> Following Findlay's [1902] stimulating paper

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63 Graphs in relation to algebra will be considered in Chapter 8.

64 The detail will be pursued in subsequent chapters.



given to the College of Preceptors on the question of 'impending reforms,' this group considered during 1903 some of the new possibilities, which could now be implemented. Lodge [1903a, 1903b] delivered lectures on both elementary geometry and algebra, which were published in the Educational Times, and a general discussion on the former was also reported [College of Preceptors, 1903b]. Here, one perceptive member reflected upon some of the deeper factors which had contributed to reforms in mathematics teaching:

the change was inevitable.... When teachers began to teach science - as well as other subjects - scientifically and with more due regard to the pupils' share in education, when the demands of the laboratory and the workshop, of technical schools and colleges, made themselves felt, reform was bound to come in the teaching of mathematics, as of logic and languages and other subjects. It is part of a general movement. [p.465]<sup>65</sup>

However, in the following year, discussion had shifted to the provocative question 'Is the New Geometry Worth Preserving?' [College of Preceptors, 1904]. Here various comments were made concerning the apparent chaos caused by the profusion of new textbooks and the absence of an agreed sequence to replace Euclid's. In January of 1905, the MA [1905b] also heard a paper from W.H. Wagstaff on 'The New Geometry,' and a discussion followed. The weight of opinion seems to have been against the imposition of a new standard sequence. Siddons, in particular, now argued:

If they were to have the best possible order, they must not have a standard order. If they had such a standard order, no one would attempt to improve on it. [p.149]

Forsyth expressed his opinion forcibly:

He trusted that neither this Association nor any body of teachers would ever attempt to re-institute and re-establish one definite order of teaching in Geometry to be mechanically imposed on all. Teachers now have their freedom; let them value their freedom, even if it brought a little chaos. [p.152]

Godfrey [1906a, p.76] was optimistic in the following year:

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65 This particular assessment will be developed in Chapter 5.

chaos tarries; examiners find no difficulty, and the result of freedom has been an effective agreement among writers of text-books as to order and content. So long as freedom is permitted, so long will progress be possible.

The sequence issue, nevertheless, persisted as a prominent concern in some quarters, and the AMA's involvement after the First World War will be considered in the next chapter.<sup>66</sup> Before the War, there were a number of organizational developments which helped forward the movement for reform in mathematical education, following the major breakthrough already discussed. The important work of the MA will be evaluated in the next chapter. Some other less sustained but significant contributions to reform will be surveyed first.

#### Organizations in the Early Twentieth Century

Having helped to advance the movement for geometrical reform, the BAAS turned to the teaching of elementary mechanics and its improvement, and a Committee was established at Southport in 1903 [BAAS, 1904, p.cii]. C.S. Jackson of Woolwich RMA<sup>67</sup> and Siddons were members, along with a number of mathematicians and scientists, including Perry [1903a, p.719], who had remarked at Belfast, 'I am sorry to say that the teaching of mechanics and mechanical engineering through experiment is comparatively unknown.'

With a view to reforming mechanics teaching, Perry copied the main lines of his strategy at, and following, the Glasgow Meeting, for the 1905 Meeting in South Africa. Perry opened a discussion involving Section A, at Johannesburg, with Forsyth in the Chair. Again, Macmillan published in one volume Perry's address and reply, details of the discussion and a number of written remarks. The Committee of the BAAS had not yet formally reported, though a Committee of the MA [1904] had already made suggestions in the Mathematical Gazette. Perry had also taken the initiative, as Secretary of the

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66 The sequence issue will also be reviewed in Chapter 8.

67 For Jackson see the Appendix.



BAAS's Committee, and 612 secondary schools listed in Whitaker's Almanac had been approached concerning the relations between mathematics, physics and mechanics teaching. Only 187 schools replied of which 178 said they taught some mechanics. Two-thirds taught mechanics as part of mathematics and experimental physics, one-fifth taught it only under physics, and the remainder only under mathematics. Thirty per cent said that different masters taught the mathematical and experimental aspects.<sup>68</sup> At the 1905 Meeting, the MA's suggestions, with which Perry sympathized, were circulated, and, amongst others, Professors Forsyth and Bryan contributed to the discussion. Written remarks were subsequently forthcoming from Professors Worthington, Armstrong, Larmor, Love and Minchin, as well as Eggar, Jackson, Godfrey and Siddons.<sup>69</sup> In the discussions concerning mechanics there are some interesting parallels with geometry, though implementing change in the former case proved to be much more problematic for the reformers.<sup>70</sup>

Forsyth recalled his own experiences at school:

I was the only boy in that particular class who was supposed to be prepared for mechanics.... I was merely given a book, made to read it and I learnt nothing. [Perry, 1906, p.28]

Jackson well captured the parallels with geometry:

The habitual omission in geometry of the experimental stage renders in many cases a student's conception of technical terms vague in the extreme. This is now universally conceded and has been remedied.

The omission of the preliminary experimental stage in mechanics has been far more complete than was ever the case in geometry. Had geometry ever been taught as mechanics has been often taught, we should have had editions of Euclid in which the cost of diagrams and the space occupied by them would have been saved. [pp.33-34]

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68 Perry [1906, pp.8-9], which also includes the MA's final proposals [pp.10-14], as well as a paper by Ashford [1906] of Dartmouth on 'The Teaching of Mechanics by Experiment' [pp.65-74], delivered to the BAAS at York in 1906, and subsequently published in the School World.

69 Perry [1906, pp.vii-viii]. Godfrey by this time had moved from Winchester to Osborne. Perry circularized his address widely, but received only eleven replies.

70 Mechanics teaching will be discussed further in Chapters 5 and 7.

Minchin drew attention to a major barrier to progress, namely the classical dominance of public schools, with their general hostility to the experimental methods of science [p.39], and Armstrong judged that teachers required detailed prescriptions, implemented through the examination system, as in the case of Perry's practical mathematics under the BE. He concluded 'Nothing short of compulsion will bring about the reforms we need in this country' [p.46]. However, as Ashford pointed out in the following year, there were major factors to consider in relation to the teaching force itself.

Ashford drew attention to the fact that mechanics at public schools was traditionally the preserve of the mathematical masters, before science teaching became established, and mathematicians 'naturally treated it as a deductive, not an inductive or experimental, science.... It would have implied something like a slur on the power and accuracy of mathematics to appeal to experiment' [Perry, 1906, pp.65-66]. The development of physics teaching had brought with it the introduction of some experimental mechanics, though Ashford added 'it has been subservient to the mathematical treatment.... No attempt is usually made to attack the subject inductively' [p.66]. By contrast, he referred to the technical institutions where 'we see exactly opposite methods. There mechanics is treated frankly as an experimental science,' though he added pupils 'often miss the special training given by the public-school course of mechanics' [p.66]. Pertinently he asked:

Is it a Utopian ideal that these two lines should converge, and in their junction should admit of the development of a method of teaching mechanics which will provide for the three desiderata, a knowledge of the facts and principles of the science, a strengthening of the reasoning power, and a training in the proper way to study natural phenomena in order to fathom the underlying causes? [p.66]

Generally, the merging of these two distinct traditions continued to be a utopian ideal, though in the very special circumstances at



Osborne and Dartmouth innovatory methods were implemented.<sup>71</sup> The BAAS's Committee did not report on mechanics until 1907, and then only briefly, with some of the MA's earlier suggestions reproduced. The Committee firstly recommended that 'Practical and theoretical mechanics ought, if possible, to be taught by the same person; mechanics and mathematics ought not to be treated as distinct from each other' [BAAS, 1908, p.97]. However, as regards tactics to implement change, the Committee was silent and confined itself to idealizations, unlike the earlier Committee under Forsyth which had boldly tackled the reality in geometry teaching. The curriculum in elementary schools also attracted the attention of the BAAS, from 1903.

In 1903, a Committee with Sir Philip Magnus as Chairman, and both Perry and Armstrong as members, was appointed 'To report upon the Course of Experimental, Observational, and Practical Studies most suitable for Elementary Schools' [BAAS, 1904, p.cvi]. The Committee was specifically interested in the areas of science, nature study, domestic science, art and manual instruction, and 'practical and experimental arithmetic and geometry,' as well as the correlation of subjects. Fact-finding was a first priority, and a letter requesting information was circularized and published in various periodicals.<sup>72</sup> The Committee reported on nature study in 1904, and continued its investigations with the aid of a grant [BAAS, 1905, pp.352-360]. A fairly full Report was presented to Section L at York in 1906.

The Committee argued a general case for the particular emphasis on practical methods in its terms of reference, and looked for progress through the reduction of class sizes, improvements in teacher training, and the appointment of sympathetic inspectors. The Committee claimed that 'It is in the teaching of arithmetic that the greatest reform is

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71 Details of this will be pursued in Chapter 7.

72 See Educational Times [1903], Vol.LVI, No.512, Dec., p.503 and MA [1903b].

needed' [BAAS, 1907, p.440], and a Sub-Committee, including Perry and Armstrong, reported separately on arithmetic and mensuration, their conclusions also being reported in the School World [1906]. The Sub-Committee, no doubt influenced by Perry, chose to survey generally the progress in secondary mathematical education, and included extracts relevant to elementary schools from the earlier Reports of the BAAS, the MA, and a Committee of Institution of Civil Engineers. which reported in 1906, as well as made reference to the BE's [1905a] Suggestions. There was plenty of agreement in principle at this time, but, as the Report pointed out with reference to the Suggestions in particular, 'What is now required is that the principles ... should be raised from the plane of pious opinion to that of actual practice' [BE, 1907, p.451]. The Sub-Committee claimed 'it is well known that in most schools no change will be made until it is insisted upon as essential to efficiency,' and that 'The futility of issuing a book of maxims without insisting that they shall be acted upon is obvious to anyone familiar with teachers and schools' [p.451]. These were strong words, directed at the BE in particular, but they were not supported by realistic proposals for measuring and ensuring 'efficiency,' without returning to some of the evils of payment for results. Furthermore, the criticisms concerning published recommendations as instruments of curriculum change applied equally well to much of the educational work of the BAAS itself. This particular Sub-Committee recommended the use of imitation coins, rulers in metric and imperial units, balances and capacity measures, squared paper, geometrical instruments, and logarithmic tables for the higher standards; the correlation of arithmetic and science; and the extension of arithmetic into simple algebra and graphical work, as well as practical geometry [p.459].<sup>73</sup>

Two years after this Report, Perry himself expressed his general

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73 These features will be more fully explored in Chapters 5 and 7.



frustration with the progress of reform.

In an address of 1908, Perry complained:

It astonishes me to see how little comprehension there seems to be of the proposals made by the British Association committees. We recommended experimental geometry with common-sense reasoning, and everybody seems to think we asked for a babyish use of rulers and compasses following a series of propositions. We asked for interesting work in weighing and measuring, and care is taken that all such work is made as uninteresting as possible. We recommended some work with graphs on squared paper, and some teachers do nothing but graphs, and there are dozens of school books to help on the craze.... Some teachers think that squared paper was invented merely to illustrate the solution of certain simultaneous or quadratic equations. [MA, 1909a, p.13]

Perry was prone to exaggeration, but distortion of curricular objectives was a major problem in early twentieth-century reforms. However, as in the case of Armstrong, Perry appears to have taken a rather simplistic view of the way forward by suggesting 'it almost seems that at present we must impose some system of teaching so complete in every detail that any teacher can follow it exactly' [p.13]. It was the MA and not the BAAS which played a major part from around this time in the development of English mathematical education, the BAAS having made its most important strategic contribution in the critical years 1901-1902. The organization of the BAAS was ill-adjusted to tackle the details of school curricula, as Perry in particular came to realize. Before turning to other organizational developments, Hobson's Presidential Address to Section A in 1910 deserves mention, as it captured succinctly some of the general reactions within the mathematical community to the practical tendencies in mathematics teaching.

Hobson [1911, p.521] granted that:

Of late years a new spirit has come over the mathematical teaching in many of our institutions, due in no small measure to the reforming zeal of our General Treasurer, Professor John Perry.

However, he referred also to 'some serious dangers' [p.521] in the newer practical tendencies, which in some cases had been exaggerated and distorted. In the case of both geometry and mechanics, Hobson desired a balanced treatment, recognizing the practical and experimental

aspects, and also the deductive aspect of the mathematician. He concluded that:

a mixed treatment of geometry, as of mechanics, must prevail in the future, as it has done in the past, but that the proportion of the observational or intuitional factor to the logical one must vary in accordance with the needs and intellectual attainments of the students, and that a large measure of freedom of judgement in this regard should be left to the teacher. [p.522]

He also referred to 'some signs of reaction against the recent movement of reform in the teaching of geometry' [p.521], though he again strongly advocated freedom for the teacher from any uniform sequence, which might be desired for examination purposes.

Before the end of the first decade of this century, international co-operation in mathematical education had also developed, on an unprecedented scale. The ICTM was formed in 1908, at the Fourth ICM in Rome, following a proposal from the American Smith [1909, p.465], who gave a paper on mathematics teaching to one Section of the Congress, devoted to 'Philosophy, History and Didactics.' Smith raised various general questions which he felt the ICM 'might profitably consider through the medium of committees representing the leading education countries.' Godfrey [1908] also gave a paper at Rome on mathematics teaching in English public schools, and general discussions at the Congress on secondary mathematical education in Germany, France, England, America, Austria, Hungary, Italy, Switzerland, and Greece were reported by Gibson [1908] in the School World.<sup>74</sup> The ICTM was an enterprising new departure and the various countries were required to appoint Sub-commissions, and to prepare reports on the state of mathematical education for the Fifth ICM at Cambridge in 1912. Within ten years, the ICTM had stimulated an enormous published output, which is some measure of the world-wide concern for mathematical education in this period. Eighteen countries published 178 reports, amounting

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74 On Smith see pp.5-6. For Godfrey's paper, together with an appendix covering his scheme of mathematics teaching at Winchester see ICM [1909, pp.449-464].



to over twelve thousand pages. Germany's output was by far the most prolific, amounting to 5393 pages, with the American contribution about one-fifth of Germany's, and Britain's about one-sixth [Archibald, 1918, p.3].<sup>75</sup>

For the British effort, the BE was approached and agreed to appoint an Advisory Committee, to act also as the British Sub-commission. The British reports would then be published as Special Reports from the Office of Special Inquiries and Reports. The Advisory Committee included Godfrey and Ashford of Osborne and Dartmouth, Jackson of Woolwich as Secretary, as well as Greenhill, Larmor, Love, Hobson and Hardy, all Fellows of the Royal Society, and Gibson [BE, 1912a, p.iii]. Thus the schools were not directly represented on the Committee, though its educational sympathies were strong. In the period 1908-1912 various meetings were held, and the various international exchanges soon revealed that pedagogy was in a more advanced state on the Continent in comparison with England.<sup>76</sup> The balance between intuition and rigour in mathematics teaching was chosen as the first major area for international comparisons. Godfrey [1912c, p.245] became enthusiastically involved in the work, and admitted 'To my knowledge there has never been anything quite on the same scale before.' It was planned that Professor Felix Klein, President of the ICTM, should present the various countries' reports to open the meeting at Cambridge. Godfrey [1912d, p.292] judged Klein to be 'perhaps the most prominent among the leading mathematicians of our day who have devoted themselves to the improvement of mathematical education,' and Klein's published output concerning pedagogy and teacher education was certainly

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75 The reports are held by the Library of the MA at Leicester.

76 On the BE's output related to the work of the ICTM see pp.41-42. Up to the War, the School World, Nature, and the Mathematical Gazette all included articles on international trends, particularly in Austria and Prussia. The important influence on English thinking in algebra and calculus teaching will be explored in Chapters 7 and 8.

impressive, even before the establishment of the ICTM [Smith and Goldziher, 1912, pp.15,30,40,76,83]. The MA became involved in planning an exhibition at Cambridge to illustrate all aspects of mathematical education in this country, which it appears to have managed to achieve with some difficulty [MA, 1912a; Abbott, 1912b]. In the build-up to the meeting at Cambridge, the Educational Times devoted a leader to a review of 'Reform in Mathematical Teaching' [College of Preceptors, 1912]. Reports from the various countries, amounting to around nine thousand pages, were presented at Cambridge, and Jackson [1912a, p.385] referred to the value of the ICTM's work in the following terms:

The problem of combining a coherent and logical scheme of mathematical education with due attention to the average boy and to the value of topics arising from familiar daily life in developing interest and an appreciation of accuracy, is a difficult one, but is in process of solution. The labours of the Commission - in particular of the German and American contributors - have forwarded the process of solution in many details.

The recent emphasis on 'intuition and experiment' formed part of the solution, and Smith [1913] summarized the interpretations and trends here in Austria, France, Germany, Switzerland, America and England, drawing upon the quantitative data in various presented papers, including one by Godfrey [1912b] concerning English secondary schools. Pedagogical papers for the ICM at Cambridge were also given by the mathematician and philosopher Alfred North Whitehead [1913a] on general aims, by Nunn [1913a] on calculus in schools, and by Carson [1913a] on mechanics teaching.<sup>77</sup> Both the two volumes of Special Reports, which comprised the British submission for the ICTM, and the work of the Commission generally, were well publicized at this time, in meetings and periodicals.<sup>78</sup> Specific American contributions, up

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77 A pamphlet describing Carson's [1912a] innovatory Scheme of Mathematical Teaching in Tonbridge School was also exhibited at Cambridge. For Whitehead see the Appendix.

78 For reviews of the Special Reports in the School World see Dale [1911, 1912] and the School World [1913a], and in Nature see Mair [1912a, 1912b, 1913]. For the work of the ICTM see Jackson [1912a, 1912b], Godfrey [1912c], Abbott [1912c], and Nature [1912].



to 1918, were mentioned in Chapter 1 (see p.6). The arrangement chosen for Smith and Goldziher's [1912] international bibliography is not without interest. In addition to sections on the traditional branches of mathematics, there are sections devoted to descriptive geometry and geometrical drawing, graphs, logarithms and the slide rule, the function concept, the correlation and applications of mathematics, the history of mathematics in teaching, and the preparation of teachers of mathematics. Many of these international interests in mathematical education were shared in England, and will be explored in subsequent chapters. The extent of the German contribution to the entries in this bibliography is also a notable feature, and one specific contribution to the huge German output for the ICTM deserves mention.

The British Special Reports, being two collections of papers from over forty different authors, were felt by some other countries to provide only illumination for local parts of the system, whereas more global and objective overviews would have proved more valuable for purposes of international comparison. Accordingly, a young German teacher, Georg Wolff, was given the task of filling this gap, with reference to secondary education. He came to England in 1913, and his findings were published two years later as a lengthy report, in German, including much historical detail and penetrating analysis of English conditions. [Wolff, 1915].<sup>79</sup> As the American Archibald [1918, p.60] remarked:

To anyone outside of England this work has met a great need because of the inadequacy of the reports published by members of the International Commission from the United Kingdom.

Wolff's Report is certainly a very valuable source for the curriculum historian.

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79 For a tribute to Wolff, who died in 1977, see Fletcher [1978].

The work of the ICTM continued into the War years, the British effort being sustained by the MA, who formed a small committee, including Godfrey, to continue the work, after the BE withdrew its financial support in 1914 [MA, 1914a]. For the meeting at Paris in 1914, the areas of calculus in schools and mathematics in higher technical education were chosen for investigation. A framework of questions concerning calculus teaching was prepared by the ICTM [MA, 1914b], and Godfrey [1914] again undertook the necessary investigations in England, with the co-operation of nearly one hundred public schools. At the meeting in Paris it was decided to take the work of comparison into the field of teacher training. It was felt that this move would 'constitute in a certain sense the crown of the labours of the commission' [Archibald, 1918, p.4]. A questionnaire was again prepared, but only two reports based on it were published during the War, by Germany and Belgium. However, using largely the evidence gathered up to 1914, the Americans continued this work and reported on elementary and secondary training in various countries, including England [Kandel, 1915; Archibald, 1918]. Because of the War, the Sixth ICM was cancelled, and the first Congress after the War, in 1920, showed little concern for pedagogical questions. The ICTM did not resume its activities until 1928, at Bologna [Howson, 1973a, p.180]. The work concerning teacher training was resumed, with a revised questionnaire issued to those countries who had not previously reported. The teacher-training field was surveyed at Zürich in 1932, the reports being published in the following year in L'Enseignement Mathématique.<sup>80</sup> The early momentum of the ICTM had clearly been lost, partly as a consequence of the War, though in any case much of the planned work had already been completed. The ICTM was a pioneering venture in international co-operation in mathematical education, the scale of the published output it stimulated

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80 NCTM [1939, p.4]. The brief English contribution was also published in the Mathematical Gazette [MA, 1932b].



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# Journal

OF THE

## Association of Teachers of Mathematics

For the South-Eastern Part of England.

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No. 1.

DECEMBER, 1911.

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Three numbers of the Journal are issued in each year. The Papers and Discussions are concerned solely with the teaching of Elementary Mathematics in Schools of every type. Copies may be obtained from the Hon. Secretary, The School, Tonbridge.

*Price One Shilling and Sixpence.*

Illustration 18 Cover of the First Number of the Journal  
of the Association of Teachers of Mathematics  
for the South-Eastern Part of England



is very impressive indeed, and this material is of considerable value for the historian of curriculum change. Its work was largely an exercise in evaluating the progress of reform, undertaken whilst many countries, including England, were still in a transitional state in mathematical education. Granted the choice of Cambridge for the first meeting, and the publicity given to the ICTM in England, most teachers would still not have been affected, and were certainly not involved in its deliberations. In particular, the scale, character and timing of the published output would have limited its influence on classroom practices. However, the ICTM did facilitate the dissemination of some ideas from other countries to England, through the involvement of innovators such as Godfrey in particular.<sup>81</sup> A very different venture which operated at a local level in this country remains to be considered.

On the initiative of Carson, an Association of Teachers of Mathematics for the South-Eastern Part of England was formed in 1911. It was quite independent of the MA, and intended to embrace all types of school in the area around Tonbridge School, a public school in Kent, where Carson was the Head Mathematical Master.<sup>82</sup> As he emphasized in the first editorial of the Journal of the Association:

In so far as any type of school or teacher is unrepresented among its members, in so far will it fail in its object of asserting the unity of purpose which should animate all those who teach the subject.... England is almost alone in preserving an unnatural separation between the professed mathematician and the teacher of Elementary Mathematics. [Carson, 1911, p.1]

In the first year there were forty-six members, with Carson as Chairman, and altogether eight from Tonbridge School itself. (See Illustration 18.) The Inaugural Meeting was held at the School, where Whitehead delivered the first Presidential Address on 'The Place of

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<sup>81</sup> See note 76.

<sup>82</sup> Tonbridge was also an innovative school in the field of elementary science teaching [Jenkins, 1979, p.32]. The Association's creation could be regarded as a local response to the relatively narrow range of work and influence of the MA over the first decade of this century. The gradually broadening scope of the MA will be considered in the next chapter.



Elementary Mathematics in a Liberal Education.'<sup>83</sup> Given Carson's sympathies, and the fact that mistresses formed half the membership, his opening remarks must have struck a rather discordant note:

The subject of my address to-day is the consideration of the part which the elements of mathematics should play in a liberal education for the generality of boys up to the age of nineteen. The boys I mean are, of course, those who are capable of a liberal education.... I exclude the residuum of boys, and am thinking of those only with fair brains and decent interests. Happily for England these constitute the great majority of the ordinary students who pass on to our Universities. [Whitehead, 1911a, p.2]

Three numbers of the slim journal were planned for issue each year, and in the second number Carson [1912b] publicized the Congress at Cambridge, and pressed for the need for an increased membership: 'The Association cannot fulfil the programme which has been suggested if the number of members is less than one hundred.' In the same year, Jackson [1912b] gave a paper to the Association on the work of the ICTM, and the Special Reports were publicized in the Journal of the Association [1912, pp.31,42-43], which also included a stimulating paper by Carson [1912c] on the teaching of elementary arithmetic.<sup>84</sup> However, Carson [1912d] expressed some concern over the future value of the Journal which:

can only become of real use if it is used by members for correspondence and the discussion of problems.... The low standing of the teaching profession in this country is in part due to the fact that the body of teachers has been inarticulate, and it rests with teachers themselves to remedy the defect.

Carson left Tonbridge in 1913 to take up a Readership at the University of Liverpool, but he continued for a short time to have links with the Association. Bell, with a monopoly of cover advertising, published the Journal at a reduced price from the fifth number and Carson continued to contribute with papers on 'Mathematics and the Ordinary Man' [1913b] and 'The Use of Technical Terms' [1914a].

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83 Whitehead produced a number of papers on similar lines up to 1917.

84 For parts of Carson's paper see also Price [1976b]. The papers in the Special Reports could be purchased separately.

However, in 1914 he ceased his active connection with the Association and commented:

The existence of the Association is now fully justified, and it has every prospect of an active career which will be of real profit to its members. [Carson, 1914b, p.2]

However, Carson appears to have underestimated the importance of his own towering contribution to the health of the Association. As Proctor [1914, p.1], the new editor of the Journal remarked:

Mr. Carson having, unfortunately for us all, severed his active connection with the Journal, the Committee had to find a successor.... The difficulty of following such a man as Mr. Carson will be obvious to all members.... Mr. Carson was such a host in himself that he could fill the Journal without assistance....

The eighth number of the Journal was the last, and it seems that the Association's activities did not survive the War.<sup>85</sup> Thus it was a short-lived but innovatory contribution to the local organization of teachers of mathematics in this country. The national and local contribution of the MA to the reform of mathematical education is the subject of the next chapter.

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85 Tropp [1958, p.158] lists the Journal, with dates 1911-1914, and only eight numbers are bound together in one volume held by the Library of the MA.



## Chapter 4

### The Mathematical Association

The MA, which started as the AIGT in 1871, was the first of the secondary school subject associations to be formed. Over twenty years were to elapse before the next group of subject teachers, the Modern Language Association, was formed, in 1892, principally to advance the status of modern languages teaching, at a time when classical studies still dominated the curriculum in many schools [Schoolmasters Yearbook, 1905, p.218]. The Geographical Association of public schoolmasters was formed in the following year, and its membership was broadened in the early years of this century [pp.221-222]. Science masters in public schools were the next to organize themselves, as the Association of Public School Science Masters (APSSM), following initiatives in 1900, though the membership was not broadened to embrace other secondary science masters until 1919 [Schoolmasters Yearbook, 1926, p.31].<sup>1</sup> The APSSM shared the Modern Language Association's concern for the subject's status, as well as for pedagogy. In 1903, the Classical Association was constituted, and this development was principally a response to the growing threat of modern languages, science, and practical subjects, to the status of classics in the curriculum [Schoolmasters Yearbook, 1905, pp.225-226]. The Historical Association was formed in 1906, and, finally, the English Association was formed in 1907 [Gordon and Lawton, 1978, p.172].

By 1915, Webb [1915a, p.14], in her general survey of teachers' organizations, judged that:

the advancement of the technique of classics, history, English or mathematics is not exclusively, or even primarily the duty of the Head Masters' Conference or the N.U.T., but of the Classical

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1 Women science teachers formed a separate organization, which did not merge with the renamed Science Masters' Association until as late as the 1960s [Jenkins, 1979, p.71]. The School Science Review of the Association, started in 1919, published reviews of some mathematical textbooks. For sources concerning the history of associations for science education see Jenkins [1980, p.58]. A detailed historical study by Layton is currently in preparation. Links between the APSSM and the MA will be discussed later in this chapter.

Association, the History Association, the English Association, and the Mathematical Association respectively....

It should be added that status as well as technique might be involved, and that in a period of major change, such as was discussed in the last chapter, it is important to look well beyond the confines of an established subject teachers' association for the important organizational developments.

To date, there has been no detailed study of the contribution of the MA to mathematical education in its various aspects. Siddons [1936, 1948, 1952a, 1956] has produced a number of sketches of the work of the AIGT and the MA, which give a rather narrow inside view, from one public schoolmaster. More recently, and prompted by the recent Centenary of the MA, Combridge [1971b, 1971c, 1972] has also produced brief accounts, though, given his close affiliations to the MA, he has deliberately avoided any appraisal of its work. However, he has valuably summarized much factual material drawn from the Reports of the AIGT and the Mathematical Gazette.<sup>2</sup> A valuable Index to the Gazette for the period 1894-1931 has been produced, including some helpful classification by content as well as author [MA, 1933]. Also, various special numbers of the Gazette include historical material, reprints of earlier contributions, and portraits of eminent members [MA, 1913a, 1929a, 1948, 1971]. Finally, in addition to the Archives, the splendid Library of the MA should be mentioned for its rich holding of primary sources, particularly mathematical textbooks.<sup>3</sup>

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2 Combridge [undated], held in the Archives of the MA at Leicester, which contain much valuable source material, though it still needs to be sorted and classified. I am grateful to Mr. Combridge and Mr. Gray, Executive Secretary of the MA, for this information and their co-operation.

3 For the history of the MA's Library see Goodstein [1974]. The Library thrives upon the donations of members, particularly on their death, and, notably, Godfrey's complete collection is included as part of the Godfrey Memorial [MA, 1924a]. A published list of the Library's contents is now somewhat dated [MA, 1962]. The whole collection has recently been completely re-classified and catalogued with the rest of the books in the Library of the University of Leicester.



The rather narrowly focused nineteenth-century work of the MA's predecessor, the AIGT, and its limited achievements, were discussed in the last chapter (see pp.88-95). Following its disappointments as a pressure group in the 1870s and 1880s, the AIGT in the 1890s was uncontroversial in its activities, and, as the MA around 1900, this group became caught up in a new state of agitation for reform, to which it eventually responded (see pp.107-113). There followed various developments in the organization of the MA as it again adjusted itself to the role of a leading body in mathematical education. Its early twentieth-century work in relation to preparatory, public, other secondary, and girls' schools, as well as elementary, technical, teacher, and science education will be considered in this chapter. One feature of the organization of the MA was the publication of its own organ, the Mathematical Gazette, from 1894. The content of the Gazette provides a useful indication of the concerns of the Association, and will be considered first.

#### General Organizational Developments

The first quarto number of the Gazette appeared in April of 1894, with three issues in each year to the end of the century.<sup>4</sup> The first editorial declared that the journal was intended to act as one means for the dissemination of ideas in teaching, and added:

we intend to keep strictly to 'Elementary Mathematics:' while not absolutely excluding Differential and Integral Calculus, our columns will, as a rule, be devoted to such school subjects as Arithmetic, Algebra, Geometry, Trigonometry, and Mechanics.<sup>5</sup>

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- 4 From 1896 the Gazette was issued in octavo, with six numbers annually from 1900 to 1931. From 1932, under a new editor, the print size was increased, with five numbers annually, though it was intended to maintain the average yearly content [MA, 1931a].
- 5 Langley [1894]. The front cover of the first number is reproduced in MA [1948, facing p.97]. Langley edited the first six quarto numbers. Between 1896 and 1898, F.S. Macaulay, F.W. Hill, and W.J. Greenstreet shared the editing, which was then taken over for thirty years by Greenstreet [MA, 1933, p.x]. Macmillan published the first six numbers only, and Bell took over subsequently.

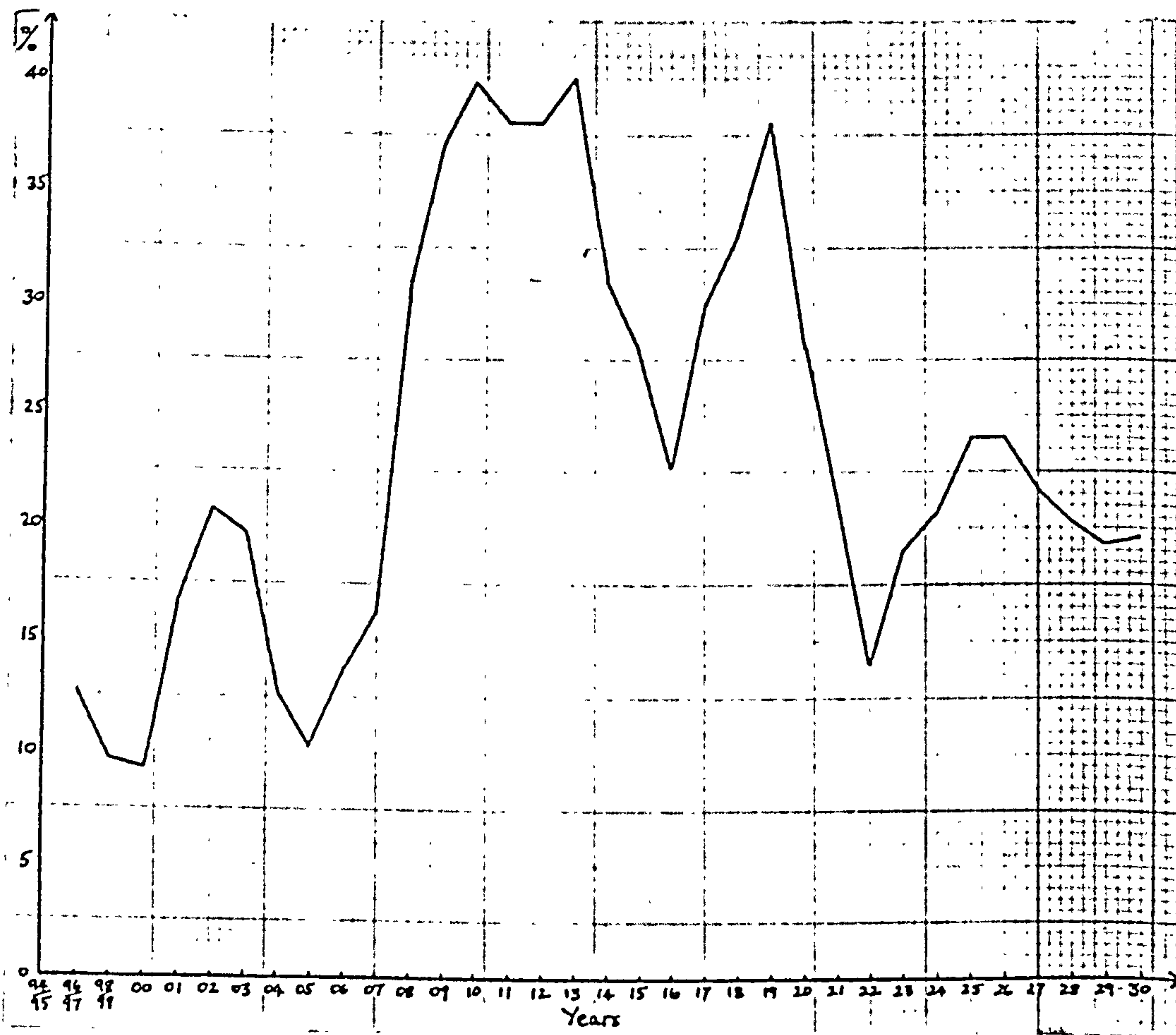


Illustration 19 Content Concerning Mathematical Education in the Mathematical Gazette (three-year moving averages)



Illustration 19 shows the extent to which the Gazette concerned itself with matters related to mathematical education, up to 1930, and includes correlation with other subjects, examinations, teacher education and educational research. The other main features in this period were book reviews, mathematical notes and articles, problems and queries, and historical articles.<sup>6</sup> The graph clearly shows that the extent to which the original editorial intentions became realized in the content of the Gazette varied considerably. T.A.A. Broadbent [1946, p.188], who edited the journal from 1931, referred to the period 1900-1914 as 'perhaps the golden age of the Gazette,' adding that the period 1918-1930 'shows a less bright and vigorous Gazette' [p.188]. Certainly, the greatest curricular emphasis comes in the earlier period, with a small peak between 1901 and 1904, stimulated by the developments already discussed, and two much larger peaks between 1907 and 1920, separated by the War years.

Illustration 20 shows in more detail the way in which the various main features of the Gazette were distributed. Notably, up to 1900 the major feature was the publication of examination and other problems, and their solutions, following the practice of the older Educational Times [Archibald, 1929, pp.396-398], and various early English mathematical serials. Thus, in 1901, an Eton master referred in Nature to the possible future role of the MA in reform in the following terms:

it is a large problem which it has to face, and it is to be hoped that its publication will not fall too much into the way of merely publishing solutions of interesting and sometimes recondite conundrums. [Hurst, 1901, p.371]

Similarly, Bryan [1902, p.89], in the School World, referred to the Gazette as a 'hardened sinner' in its partiality for 'pigs in clover' riders,<sup>7</sup> and he quoted the following as an example:

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<sup>6</sup> The Gazette also included occasional obituaries, plus notices and details of the MA's business. Correspondence was never a strong feature in this period.

<sup>7</sup> 'Pigs in clover' was a game of marbles rolling on a tilted board into recesses or pockets.

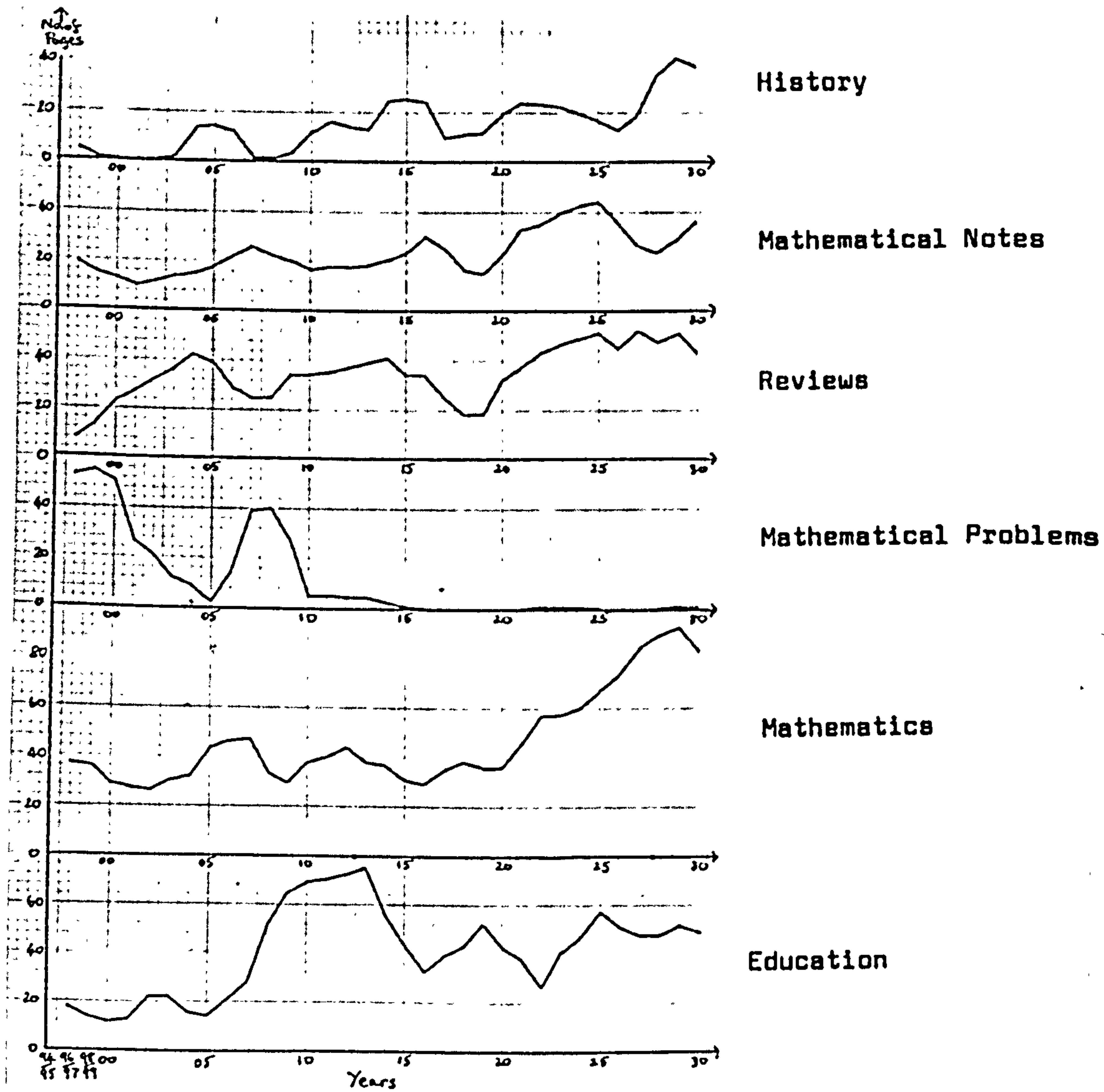


Illustration 20 The Mathematical Gazette's  
Main Features (three-year moving averages)



'ABC is a triangle; three geese whose masses are proportional to  $b^2+c^2-a^2$ ,  $c^2+a^2-b^2$ ,  $a^2+b^2-c^2$ , set out to walk along the sides with velocities proportional to  $\sin^3 B - \sin^3 C$ ,  $\sin^3 C - \sin^3 A$  and  $\sin^3 A - \sin^3 B$ ; prove that the locus of their centre of gravity is given by the trilinear equation.....' (some complicated formula no doubt)....  
[Bryan's parenthesis]

This feature of the Gazette faded rapidly in popularity with the growth of interest in curriculum development after 1900, and became virtually extinct after the publication of a special set of solutions to problems in the Gazette during 1907 and 1908.<sup>8</sup>

Unlike Nature and the School World, the Gazette was not used as an instrument in the early agitation for reform at the turn of the century, discussed in the last chapter (see pp.97-106). Illustration 19 shows that there was some, but not a great response in the columns of the Gazette, in the period 1901-1904. Significantly, in 1906, the editor was prompted to make an appeal for a shift in the journal's character towards a greater concern for pedagogy, and he requested suitable articles and discussions for publication [Greenstreet, 1906]. Furthermore, when Bryan became President of the MA for the years 1907-1909, he vigorously pursued the general issue of the balance between mathematical and pedagogical concerns.

In his first Presidential Address on 'The Neglected British Teacher. A Plea for Organisation in Mathematics' Bryan [1907a, p.28] admitted that within the MA's membership:

the greater proportion ... are mathematicians who are qualified to benefit our Association by the contribution of papers to our Gazette, and by the expression of their opinions on questions connected with the teaching of mathematics; and the large body of mathematical and science masters who would derive benefit from joining our ranks do not appreciate the advantages to be derived from membership.

As Bryan [p.30] pointed out, one problem was the MA's journal, for schoolmasters were being 'frightened off by the somewhat forbidding

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<sup>8</sup> See Mathematical Gazette [1907], Vol.IV, No.63, April; [1908], Vol.IV, No.68, Jan.; [1908], Vol.IV, No.75, Nov.

appearance of certain papers published in the Gazette.' At this time, Bryan [1907b] also gave a bold paper on 'The Future of the Mathematical Association' to which there were a number of reactions. One member complained about the Gazette:

with the exception of the reports and discussions of the reports of the committee for the reform of Elementary Mathematics, I have rarely found anything of general interest. The whole Gazette, it appears to me, is taken up with special solutions of problems of too advanced a type for the ordinary mathematical work of a school.<sup>9</sup>

Bryan [1907b, p.76] well captured the situation to which he felt the MA ought now to respond more positively, by promoting discussion and the dissemination of innovations on a much greater scale than hitherto:

I have never been sanguine, that the so-called 'reform of mathematical teaching' would leave us much better off than we were before, and the present is an opportune time for discussing such questions. Personally I consider that reform of teaching means something more than mere tinkering with syllabuses, the publishing of school geometries, and the writing off of publishers' losses on the innumerable Euclids that came out just before the change. The teaching of algebra is at present in a hopeless chaos....

The change in the character of the Gazette from around 1907, which Illustration 19 shows, reflects the significant response of the MA to the needs of the situation. In the period 1907-1914 a number of stimulating papers and discussions on various aspects of elementary mathematics teaching were published in the Gazette, including major contributions from Godfrey and Siddons on algebra, Perry and Forsyth on correlation with science, Carson on aims and general pedagogy, and the young Durell on arithmetic and early calculus teaching.<sup>10</sup> In this period up to the War, the content of the Gazette indicates that there was particular concern for the teaching of algebra and simple trigonometry, and much interest in the possibilities of teaching calculus to

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<sup>9</sup> C.H. Blomfield in Bryan [1907b, p.75].

<sup>10</sup> MA [1933, pp.8-9]. After 1920, Durell became an enormously successful school textbook writer for Bell, covering all the branches, and combining with various other writers, including Palmer, Wright, Fawdry, Siddons, Tuckey and Robson [Bell, 1934]. For Durell see the Appendix.



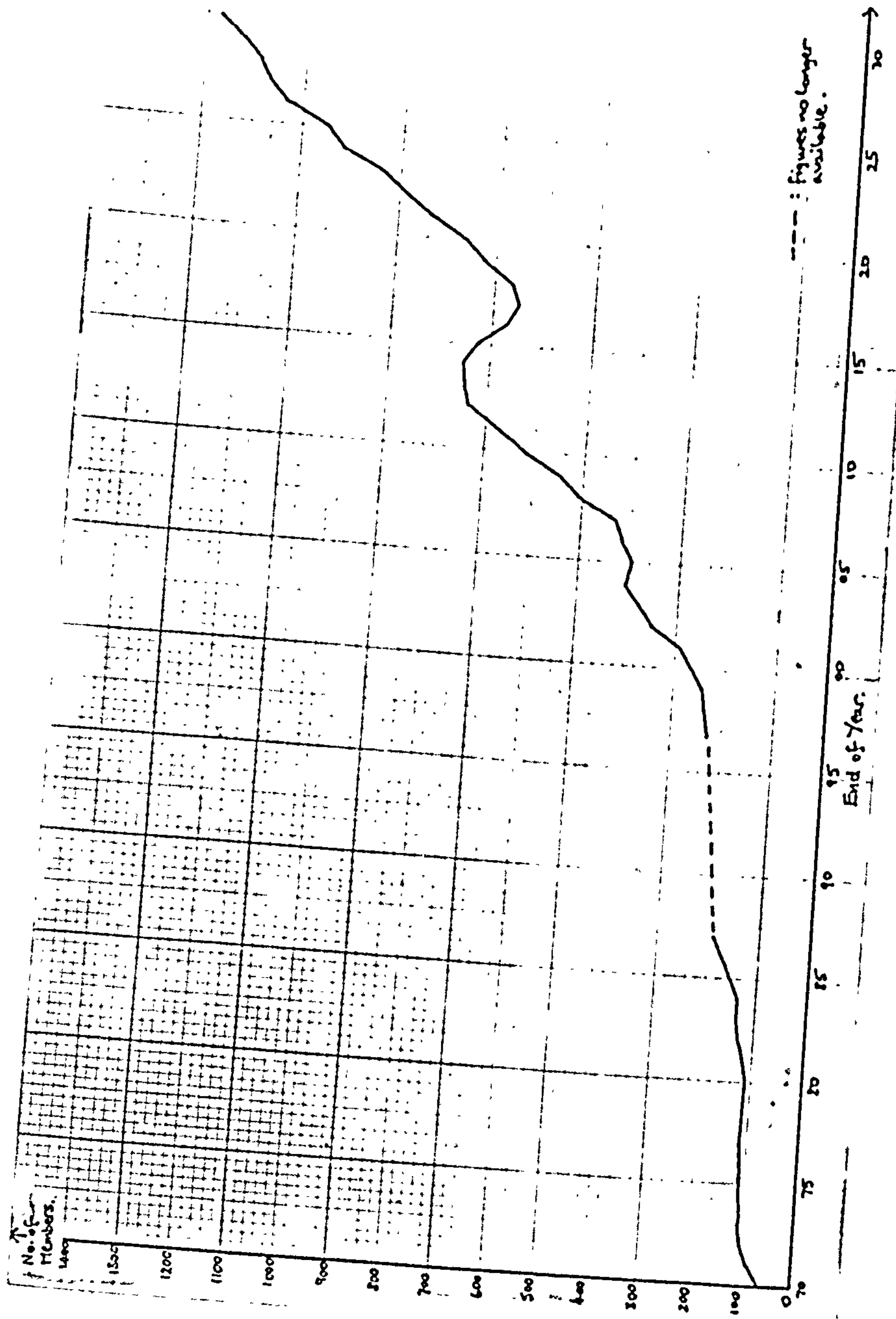


Illustration 21 Membership of the AIGT and the MA

non-specialists, partly stimulated by the work of the ICTM. Illustrations 19 and 20 also show that during the 1920s there was an overall decline in curricular interests, and an increase in the mathematical content of the Gazette. Indeed, a discussion on 'The Proper Function of the Mathematical Gazette' was held in 1926, and, whilst there was some concern that the mathematics was taking over, there was not the urgency to shift greatly the emphasis, as had been the case twenty years earlier [MA, 1926a].<sup>11</sup> The changing character of the Gazette also affected the membership of the MA, and developments here will now be considered.

Illustration 21 shows the growth in the membership of the Association up to 1930.<sup>12</sup> The graph shows the slow rate of growth for the membership of the AIGT from 1873, after the initial gains, and a first surge in the membership of the MA after 1900 when 'Teachers feeling the necessity of being in touch with the new movement, joined the Association in considerable numbers' [MA, 1913a, p.27]. However, the growth rate was not sustained in the years 1905 and 1906, which were quiet ones for the MA. The Council of the MA admitted in 1906 that it 'has very little to report concerning the year 1905. It has been very quiet, and barren of events.' [MA, 1906, p.281]. Greenstreet [1906, p.283] pertinently remarked:

We are not sure that this may not be meant for a piece of biting criticism, suggesting that though more remains to be done we have not done it.

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- 11 Ten years later, Siddons [1936, p.25], who was the first school-master to be elected President of the MA, referred in his Presidential Address to 'grumbles that the Mathematical Gazette is too "high-brow" and does not perform the function for which it was started,' and to the 'fear of highbrow criticism' [p.26] as a factor inhibiting the production of articles on elementary mathematics teaching.
- 12 The data was taken from MA [1920a], Siddons [1948, p.160], and Cambridge [undated, pp.16-17]. From 1904, the figures were given in the Annual Reports of the Council, published in the Gazette. Detailed data could not be found for the years 1888-1896, though, according to Siddons [1948, p.160] the numbers were 'generally between 180 and 190,' and see also Wolff [1915, p.60].



As well as drawing attention to the shortcomings of the Gazette as a journal for school-teachers, it was Bryan who also did much to sustain the membership's growth rate in the period 1907-1914, by drawing attention to other important contributory factors. In addition to the burden of subscription, Bryan [1907a, p.30] referred to the MA's general image as one factor deterring a master from joining, because 'He has not regarded membership of the Association as suitable for "the likes of him."' Moreover, 'He has never had the Association brought before his notice in such a way as to make him think of joining it.' Thus, in 1907, the Council agreed to circularize colleges and schools of various grades, to draw attention to the benefits of membership.<sup>13</sup> Bryan [p.31] also pressed the MA to broaden the character of its membership, as well as increase its size, since:

it is important that the body responsible for framing or adopting recommendations should be representative, and should include all those who are affected by the proposed changes.

To further this aim, Bryan pressed for the establishment of local Branches of the MA, to facilitate discussion involving teachers from all types of school. He showed the way forward and became President of the first such Branch, formed at Bangor in 1907. The Gazette announced:

there is reason for congratulation in the fact that the Branch has been formed for 'the discussion of matters relating to the teaching of mathematics in schools, etc., of all grades [original stress].' Here is an important link in the long chain that has yet to be forged before the teaching profession in this country becomes one organic whole.<sup>14</sup>

Bryan [1909, pp.44-45] further publicized the need for Branches in his last Presidential Address. In 1909, regulations were produced, and the possibility of affiliation for the newly formed Southampton and District Mathematical Society was pursued [MA, 1909b, 1909c].

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<sup>13</sup> MA [1932c], minutes 10/4/07, 31/3/08.

<sup>14</sup> MA [1908a]. There are parallels between Bryan's thinking and initiative, and Carson's in the South-East (see pp.135-137).

In 1910, the first meeting of the large London Branch of the MA was held at the Polytechnic, Regent Street, with Jackson as President, and Nunn and Abbott as two of the officers [MA, 1910a]. Over two hundred were present for a discussion of the BE's [1909a] important Circular 711 on geometry teaching, which was opened by Siddons, and, in particular, included contributions from Nunn and Carson [MA, 1910b]. In the same year, the first Australian Branch of the MA was established at Sydney [MA, 1911a, p.17]. A number of other Branches were formed after the War, and by 1930 there were twelve in existence, including three in Australia [MA, 1930b, p.36]. The development of local Branches was an important element in the broadening of the MA's influence. The development of the MA's central organization also showed increasing flexibility over the early twentieth-century period.

Up to 1900, the business of the MA was conducted by a governing Council, which reported at the Annual General Meeting, held at the beginning of each year. Committees generally worked very slowly, most of the groundwork being undertaken by one or two individuals, and there were no local Branches [School World, 1904].<sup>15</sup> The background to the establishment of the first so-called Teaching Committee, in 1902, was discussed in the last chapter (see pp.111-113). This Committee represented only the interests of public schools for boys, though some younger and forward-looking teachers were involved, and, in particular, Godfrey, Siddons, Barnard and Tuckey also became involved with the work of the Council from around this time.<sup>16</sup> The new Committee worked energetically in 1902, though along conservative lines in the case of geometry, which was the major concern at this time. However, the

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15 As a pioneering subject teachers' association in the nineteenth century, the MA lacked suitable existing organizational models upon which to build. Possibly the BAAS provided some inspiration here, in spite of its different objectives.

16 MA [1932c], minute 4/11/02. Around a quarter of the twenty-nine members were under thirty [Siddons, 1952a, p.154].



LIST OF REPORTS PUBLISHED BY THE MATHEMATICAL ASSOCIATION:

- (1) REPORTS ON THE TEACHING OF ELEMENTARY MATHEMATICS, 1902-1908 (Geometry, Arithmetic and Algebra, Elementary Mechanics, Advanced School Mathematics, the Course required for Entrance Scholarships at the Universities, Mathematics in Preparatory Schools), 6d. net. [Out of print; see (10).]
- (2) REVISED REPORT ON THE TEACHING OF ELEMENTARY ALGEBRA AND NUMERICAL TRIGONOMETRY (1911), 3d. net.
- (3) REPORT ON THE TEACHING OF MATHEMATICS IN PREPARATORY SCHOOLS (1907), 3d. net. [Out of print; see (13).]
- (4) REPORT ON THE CORRELATION OF MATHEMATICAL AND SCIENCE TEACHING, by a Joint Committee of the Mathematical Association and the Science Masters' Association (1909, reprinted 1917), 6d. net.
- (5) A GENERAL MATHEMATICAL SYLLABUS FOR NON-SPECIALISTS IN PUBLIC SCHOOLS (1913), 2d. net. [Out of print.]
- (6) CATALOGUE OF CURRENT MATHEMATICAL JOURNALS, etc., with the names of the Libraries in which they may be found (1913), 40 pp., 2s. 6d. net.
- (7) REPORT ON THE TEACHING OF ELEMENTARY MATHEMATICS IN GIRLS' SCHOOLS (1916), 1s. net. [Out of print.]
- (8) REPORT ON THE TEACHING OF MECHANICS (*The Mathematical Gazette*, No. 137, Dec. 1918), 1s. 6d. net. [A new report is being prepared, 1928-9.]
- (9) REPORT ON THE TEACHING OF MATHEMATICS IN PUBLIC AND SECONDARY SCHOOLS (*The Mathematical Gazette*, No. 143, Dec. 1919), 2s. net. [Revised and Reprinted 1928; 1s. 6d. net.]
- (10) REPORTS ON THE TEACHING OF ARITHMETIC AND ALGEBRA, ELEMENTARY MECHANICS AND ADVANCED SCHOOL MATHEMATICS (1922), 1s. net. Reprinted from (1).
- (11) REPORT ON THE TEACHING OF MECHANICS IN GIRLS' SCHOOLS (1923), 6d. net.
- (12) REPORT ON THE TEACHING OF GEOMETRY IN SCHOOLS, 1923 (3rd ed. 1920), 2s. net.
- (13) REPORT ON THE TEACHING OF MATHEMATICS IN PREPARATORY SCHOOLS (1924). Reprinted 1928; 1s. net.
- (14) REPORT ON THE TEACHING OF MATHEMATICS TO EVENING TECHNICAL STUDENTS (1926), 1s. net.
- (15) A LIST OF BOOKS SUITABLE FOR SCHOOL LIBRARIES (1920), 1s. net.
- (16) A FIRST LIST OF BOOKS AND PAMPHLETS IN THE LIBRARY OF THE MATHEMATICAL ASSOCIATION, 1926, 3s. 6d. net.
- (17) REPORT ON THE MATHEMATICS IN ENTRANCE SCHOLARSHIP EXAMINATIONS AT PUBLIC SCHOOLS, 1926, 1s. net.
- (18) SIR ISAAC NEWTON, 1642-1727, commemorative of the Bicentenary of the death of Sir Isaac Newton; 1927, 10s. 6d. net.
- (19) A Second List of Books and Pamphlets in the Library of the Mathematical Association, 1929, 3s. 6d. net.

Illustration 22 Reports of Committees  
of the MA [1929b, pp.52-53]

interests of the MA and its Teaching Committee widened sufficiently over the next ten years to warrant a complete overhaul of the structure.

In 1912, the new circumstances were summarized thus:

In view of the growth and development of the Association in recent years, and the wider interests which it now represents, it is desirable that the Teaching Committee should be re-constituted on a broader basis. [MA, 1912b, p.199]

The new regulations provided for a General Teaching Committee and three Special Committees, for Public Schools, i.e. schools in the Public Schools Year Book, Other Secondary Schools for Boys, i.e. boys' secondary schools not in the Public Schools Year Book, and Girls' Schools. The final constitution of the General Committee provided for representatives from these three types of school, as well as from preparatory schools, technical schools, teacher training and the universities [MA, 1914c, p.11]. The new structure certainly represented a considerable advance in ten years, though the activity of the new Committees was to some extent hampered by the War. A further modification was introduced from 1923, when the needs of public and other secondary schools were merged in one Committee for boys' schools, a separate Committee still catering for girls' schools [MA, 1921, p.226].

The various committees of the MA produced an impressive number of reports on different aspects of mathematical education over the first quarter of the century. (See Illustration 22.) It should be added that during the War reports on features of arithmetic teaching [MA, 1915a, 1916a], and on the teaching of mechanics [MA, 1918a], were published in the Gazette, but not separately. Much of the activity over the first decade of the century focused upon the circumstances in preparatory and public schools for boys, and the MA's involvement here will be discussed first.

#### Preparatory and Public Schools

At the turn of the century, there were around one hundred schools linked with the HMC, and listed in the Public Schools Year Book,



which was first published in 1889 [Percival, 1969, pp.73-75]. The number of so-called Conference or public schools increased to around one hundred and fifty over the first quarter of the century [Schoolmasters Yearbook, 1926, pp.10-12]. These privileged schools greatly influenced, and were served by, the preparatory schools, which became organized through the Association of Preparatory Schools, for Head Masters from 1892.

The HMC was particularly concerned with general administrative rather than curricular matters in the critical years leading up to the 1902 Education Act, and, at the turn of the century, the mathematics curriculum in both preparatory and public schools was unreformed, with geometry teaching still dominated by Euclid.<sup>17</sup> The BE [1900c] devoted an early volume of its Special Reports to the specific question of boys' preparatory schools, and, in the discussion on mathematics there was no hint of the developments that were soon to follow [Allum, 1900]. However, only two years later, Godfrey [1902b, 1902c, 1902d] provided a rich collection of progressive suggestions for preparatory school mathematics, and boldly stated: 'To put the matter bluntly, the average boy ought not to do any Euclid at his preparatory school' [1902b, p.201]. The preparatory schools could, of course, not move until the public schools altered their own entrance requirements, which largely dictated the curricula of these feeder schools. With the administrative arrangements settled, and the public schools' independence preserved, the HMC, the Association of Preparatory Schools, and the MA all became involved in the details of reform in mathematical education in the early years of this century.

In 1903, the Association of Preparatory Schools discussed the arrangements for a new Common Entrance Examination, the details of which were being worked out by the HMC. The new examination was to

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17 For administrative developments involving the HMC see Baron [1955].

include papers in arithmetic, algebra, and geometry, the teaching of this branch being also discussed by the Association of Preparatory Schools at this time [Schoolmasters Yearbook, 1905, pp.197-200]. In 1903, this Association approached the MA for advice concerning the best policy for geometry teaching, at a time of rapid change in the public schools, which were now coming under the influence of reformed examination requirements. Accordingly, the MA's Teaching Committee decided to send a letter to the headmasters of all public schools asking:

how far they would recognise and insert questions in Practical and Theoretical Geometry in their Entrance Examinations and Scholarship Entrance Examinations, in accordance with the syllabus recommended by the Cambridge Syndicate.... The object of seeking this information is to afford guidance to Preparatory Schools in the teaching of Geometry. [MA, 1903c, p.349]

The letter was signed by Forsyth, and the replies clearly demonstrated:

how very real a reform is being made; every school seems to encourage practical work, and, except in one case, to grant the freedom which the Universities have considered it wise to give. [pp.350-351]

The preparatory schools also responded rapidly to the new opportunities in geometry. Godfrey [1906a, p.77] remarked:

So far has the movement gone that out of seventy boys entering a public school [Winchester] in September, 1905, not one admitted that he had learnt 'Euclid.'

The Association of Preparatory Schools also pressed the MA for more detailed guidance in arithmetic, algebra and geometry. Up to 1905, the MA [1905a] had reported on the three branches, mechanics, and advanced school mathematics, but provided only brief statements of what ought to be taught, amounting in all to only twelve pages. The MA's Committee, with the assistance of two representatives of the preparatory schools and the benefit of responses to a circular sent to over four hundred preparatory schools, reported in 1907 [MA, 1907a]. The Report of ten pages provided quite detailed guidance concerning teaching methods for the preparatory schoolmaster, building upon the lines of earlier recommendations. The Report was also intended to influence the



examinations for entrance to public schools. By this time 'a great proportion' [p.1] of schools used the Common Entrance Examination, instituted in 1904, and the Committee judged that 'there is reason to hope that the recommendations will be accepted as a guide in future papers' [p.1].<sup>18</sup> From around this time the HMC also became involved in preparatory and public school mathematics.

The Curriculum Committee of the HMC, with the co-operation of the preparatory schools, presented a Report to the Conference in 1909 which included a 'Syllabus of Mathematical Teaching for Boys aged 9 to 16, for Non-Specialists,' the distinction between specialists and non-specialists being a notable feature at this time [MA, 1911b, p.1]. This scheme was circulated to all public schools and members of the Association of Preparatory Schools, though as Kitchener [1912, p.107] remarked in the Special Reports 'a large number have either not troubled to look at it, or have not studied it with care,' but otherwise the scheme was generally favoured. The age range deliberately cut across the age of school transfer, and the syllabus was adopted by the Common Entrance Examination Board as a basis for their mathematics papers [pp.106-107]. As regards content, notable features were the broad basis of assumption allowed in the earlier stage of deductive geometry, following the BE's [1909a] recommendations, with the Cambridge Schedules used for reference, and the inclusion of logarithms and 'numerical trigonometry' for older pupils [pp.113-118].<sup>19</sup> Nevertheless, co-ordinating the work of preparatory schools was still a major problem, in spite of

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18 Osborne RNC, where Godfrey was now the Headmaster, used entrance examinations conducted by the Oxbridge Joint Board.

19 The layout of the scheme and its content suggest that Godfrey may have had much to do with its construction, building on his 'Compromise' at Winchester (see pp.107-108). According to Siddons [1924, p.138], Godfrey served on committees of the HMC, and, from 1905, 'had the ear of the Headmasters Conference which of course had great weight with examining bodies' [1952a, p.157].

the efforts of the HMC and the MA. Lack of specialist leadership was one particular difficulty, and, as Kitchener [1912, p.108] pertinently remarked:

headmasters in their busy lives lay aside reports which are sent to them and allow them to grow dusty on their shelves, if indeed they are not at once consigned to the waste-paper basket.

However, support for the HMC's scheme from many leading public schools was confirmed by a Report from the new Public Schools Special Committee of the MA [1913b, p.1], which recommended it as a basis for non-specialists.<sup>20</sup> The Special Committee continued, through and after the War, to take steps to influence the preparatory and public schools.

During the War, strenuous efforts were made to achieve greater uniformity in the schools regarding methods of decimal computation, and the detail was reported in the Gazette [MA, 1915a]. After the War, the problem of public schools' differing scholarship requirements loomed large, and the Special Committee reported on the matter [MA, 1922a]. The main problem here was that, for their competitive scholarship examinations, some public schools were going significantly beyond the scope of the 9-16 syllabus for non-specialists, reissued by the HMC and Association of Preparatory Schools in 1916, and which the majority on the Special Committee sought to impose as a reasonable maximum [p.178]. However, some schools remained unmoved, the headmasters again pressed the MA to take further action, and the Boys' Schools Committee reported on the situation [MA, 1926b].

The earlier general recommendation was reaffirmed, with the 9-16 syllabus reproduced [pp.8-9], which it was felt would, as a basis, 'provide a more certain test of mathematical ability than a wide

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20 Durell (Winchester) was Secretary of the Committee, which included Godfrey (Osborne), Barnard (Rugby), Borchardt (Cheltenham), F.W. Dobbs (Eton), Fawdry (Clifton), Lodge (Charterhouse, previously Cooper's Hill), Mercer (Cheltenham, previously Dartmouth), Palmer (Christ's Hospital), Siddons (Harrow), and Tuckey (Charterhouse). Most members of this Committee were, or became, major textbook writers.



schedule of bookwork and so defeat the crammer' [p.6]. Clearly, some public schools were making quite unreasonable and misguided demands upon able pupils under fourteen, but the MA could not enforce change, and, no doubt, this particular examination problem persisted. Two years earlier, the Boys' Schools Committee had also reported on preparatory school mathematics generally [MA, 1924b], though, on its own admittance, the Report was 'evolutionary rather than revolutionary' [p.4], and it largely reiterated the recommendations of 1907, though some of the enlightenment in the BE's [1909a] 'important Circular 711' [p.4] on geometry was incorporated.

Although the specific problems of preparatory and public schools were a major early concern of the MA, the question of the mathematical education of girls rose to prominence in the early twentieth century and gradually forced its attentions upon this Association.

### Girls' Schools

In a recent survey of the literature concerning the scientific education of girls Jenkins [1980, p.38] concludes:

The differentiation of the science curriculum between the sexes was well-established by the early years of the century and has proved to be a remarkably enduring feature of secondary education in England and Wales.

Jenkins does not discuss mathematics, though his conclusion may be fittingly applied here also, and various features of differentiation in mathematics were indicated in Chapter 2.

Differentiation involved lower expectations for girls in elementary schools, and for women entering and undergoing training in the colleges, as well as the lesser provision for mathematics in girls' secondary schools, and the inferior mathematical qualifications of women teachers of the subject.<sup>21</sup> However, the general nineteenth-century movement for

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21 Jenkins [1979, pp.170-214] has discussed in some detail the twentieth-century scientific education of girls, including a wealth of quantitative data, with some references to mathematics. For some recent sources concerning the general nineteenth-century background see also Burstyn [1977].

Professor E. W. HOBSON, M.A., Sc.D., F.R.S. (Christ's College, Cambridge).  
 Professor W. H. HUDSON, M.A., LL.M. (King's College, London).  
 Professor H. H. TURNER, M.A., D.Sc., F.R.S. (New College, Oxford).  
 P. ABBOTT, B.A. (The Polytechnic, Regent Street, London).  
 O. W. ADAMS, M.A. (Halleybury College).  
 Miss ASHCROFT, M.A. (University College, Reading).  
 W. M. BAKER, M.A. (Cheltenham College).  
 S. BARNARD, M.A. (Rugby School).  
 Miss BRACINTON (Brighton and Hove High School).  
 G. W. BREWSTER, M.A. (Oundle School).  
 J. F. CAMERON, M.A. (Gonville and Caius College, Cambridge).  
 G. ST. L. CARSON, B.A., B.Sc. (Tonbridge School).  
 J. V. COATES, B.Sc. (Alleyn's School, Dulwich).  
 F. W. DOBBS, M.A. (Eton College).  
 W. J. DOBBS, M.A. (L.O.C. Secondary School, Holloway).  
 O. V. DURELL, M.A. (Winchester College).  
 J. ELLIOT, M.A. (County School, Market Drayton).  
 H. D. ELLIS, M.A.  
 T. J. GARSTANG, M.A. (Bedales School, Petersfield).  
 H. T. GERRANS, M.A. (Secretary of the Oxford Local Examinations Delegacy).  
 Miss GLAUERT, B.A. (Clapham High School).  
 O. GODFREY, M.A. (Royal Naval College, Osborne).  
 W. J. GREENSTREET, M.A.  
 Miss E. R. GWATKIN, M.A. (The Queen Mary High School, Liverpool).  
 F. W. HILL, M.A. (City of London School).  
 O. S. JACKSON, M.A. (R.M.A., Woolwich).  
 J. P. KIRKMAN, M.A. (Bedford Grammar School).  
 E. KITCHENER, M.A. (Greenbank Preparatory School, Liverpool).  
 A. LODGE, M.A. (Charterhouse School).  
 O. C. LYNAM, M.A. (Oxford Preparatory School).  
 F. S. MACAULAY, M.A., D.Sc. (St. Paul's School).  
 D. MAIR, M.A.  
 G. B. MATHEWS, M.A., F.R.S. (St. John's College, Cambridge).  
 L. S. MILWARD, M.A. (Malvern College).  
 J. S. NORMAN, M.A. (Sevenoaks Preparatory School).  
 T. P. NUNN, M.A., D.Sc. (London Day Training College).  
 G. W. PALMER, M.A. (Clifton College).  
 C. W. PARKES, B.A., B.Sc. (Merchant Taylor's School).  
 O. PENDLEBURY, M.A.  
 H. C. PLAYNE, M.A. (Bancroft's School, Woodford).  
 Miss PUNNETT, B.A. (London Day Training College).  
 H. SAVERY, M.A. (Marlborough College).  
 Miss SHELTON, M.A. (Sydenham High School).  
 A. W. SIDONS, M.A. (Harrow School).  
 Miss M. STORR, M.A. (Goldsmiths' College).  
 Miss L. STORY, B.A. (Royal School, Bath).  
 C. O. TUCKER, M.A. (Charterhouse School).  
 F. J. W. WHIPPLE, M.A. (Merchant Taylor's School).  
 Hon. Secretary of the Committee.  
 A. W. SIDONS, Harrow-on-the-Hill.

Illustration 23 Teaching Committee of the MA  
 in 1911 [MA, 1911b, inside front cover]



members, within the new Branches. By 1910, the thriving London Branch contained 93 'Associates,' over three quarters of these being women [MA, 1920a]. Women were also beginning to contribute to the now more pedagogically orientated Gazette, and to some of the MA's stimulating discussions up to the War.<sup>23</sup> As Miss Punnett remarked, during a discussion on arithmetic, 'It seems a pity to leave unanswered the Chairman's invitation - or challenge - to someone to speak on behalf of the girls' [Durell, 1911, p.40]. More women were also being included on the Teaching Committee, which was rapidly becoming unwieldy, though much more representative than the Committee of 1902. (See Illustration 23.) The Reports of the MA were also, by the end of the decade, beginning to acknowledge the existence of girls, in the following terms:

The Committee have in view the requirements of girls as well as those of boys, and the word 'boy' being used for convenience of expression, what is here predicated of boys is to be extended to girls, except where the context renders such an extension of meaning obviously unsuitable. [MA, 1910c, p.1]

This note appeared in a Report on algebra and trigonometry, thereby boldly placing the mathematical expectations for girls on a par with those for boys. Miss Punnett, in particular, applauded this initiative, and added:

Many opinions have been expressed for and against the theory that boys and girls are equal in this matter - it is evident, in fact, that the time is not ripe for a definite and unanimous conclusion on this point. It is very difficult to say to what extent the present comparative inferiority of the mathematics in girls' schools is merely due to an ancient tradition to that effect. The best way of solving the problem would seem to be to put the mathematics for boys and girls, for the present, on the same footing and to be guided by experience in making the necessary modifications in the work of the girls. [MA, 1911c, pp.54-55]

Plenty of contrasting views were certainly being expressed around this time concerning secondary mathematical education for girls.<sup>24</sup> In the Special Reports, papers by Story [1912], Gwatkin [1912], Miss Sara

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23 See, for example, Epps [1907] on practical arithmetic and Barwell [1913] on history of mathematics in the school curriculum.

24 Some of these issues will be considered further in Chapter 6.

Burstall [1912a] (Manchester High School), and Mrs Henry Sidgwick [1912] (Newnham College, Cambridge) considered various aspects of the matter.<sup>25</sup> Miss Ford [1912] (Roedean) gave a paper on the subject to the South-Eastern Association, as did Miss Burstall [1912b] to the London Branch of the MA, and a lively discussion followed. In 1912 also, the new Girls' Schools Special Committee of the MA was created, as part of the general overhaul of the MA's organization for committee work. Not all women shared Miss Punnett's optimism at this time. Miss Burstall [1912a, p.576] reflected upon the way in which mathematics had forced its attentions upon girls' schools, through the demands of university examinations, with some support from the mental discipline argument, since 'It was said ... that women could not reason; they were to be taught how by studying Euclid' [1912b, p.207]. Furthermore, the curriculum in girls' schools had become increasingly crowded, the situation being exacerbated by the shorter school day, and Miss Burstall concluded that less mathematics, as well as science, than for boys was justifiable [Jenkins, 1979, pp.171-176]. By contrast, in the discussion at the London Branch, Miss Punnett argued that:

those who wished to give mathematics a quite subordinate position ... should at least wait until the experiment had been fairly tried of teaching it on lines really adjusted to the interests and activities of girls. [Burstall, 1912b, p.211]

Miss Burstall [1912a, p.576] also drew attention to another difficulty, related to the character of the reform movement as 'a revolt, led by the engineers,' which had exposed mathematical education as:

too academic and unpractical for the needs of real life, that is, the needs of boys.... Practical Mathematics of various kinds is introduced, and school Mathematics is more definitely directed towards the demands of careers which do not concern girls.... It is closely related to the characteristic industries and activities of men.... It has no such relation to the characteristic activities of women: it is not required for any vocation which the ordinary woman will follow. Its only value is as a mental discipline....

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25 For some discussion of these papers see Clements [1979].



Certainly the main thrust of the new arguments based on utility were male-orientated, but nonetheless, other women responded more positively to the new possibilities for girls, as well as boys. For example, Miss Story [1912, p.551] noted that:

The Board of Education Circular ... and the publications of the Mathematical Association, have done much to stimulate interest in graphic methods which are now largely used in girls' schools.

She also referred to the newer practical tendencies in arithmetic and geometry for girls [pp.548-553]. The whole question of elementary mathematics for girls was considered by the MA's new Special Committee, which reported in 1916.<sup>26</sup>

The Special Committee's Report of twenty-six pages was the fullest one yet produced by a Committee of the MA, and was the work of seventeen members, with Miss Gwatkin as Chairman and Miss Punnett as Secretary. Not surprisingly, the Committee was opposed in principle to differentiation in any major respects, and also the 'need of unity' between the branches of the subject was stressed [MA, 1916b, pp.2-3]. Two aspects in geometry were distinguished under the heads 'investigational' and 'formal,' with the former envisaged as coming first, and the two proceeding in parallel from around the age of thirteen. The Committee followed the recommendations of the MA's [1911b] Report on algebra and numerical trigonometry, favouring the commencement of the latter at the age of fourteen. The Report was principally an elaboration of a scheme for the eight to sixteen age range, with plenty of illuminating detail on the lines of the earlier Report produced for preparatory schools [MA, 1907a]. The Committee also proposed a syllabus for 'Arithmetic of Citizenship' suitable for older girls, based upon an innovatory scheme drawn up by Nunn in 1911 [pp.4,21-22], and developed in the demonstration schools

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26 This Report was publicized in the School World [1917a], which also published another woman's view during the War [Parker, 1915a].

of the London Day Training College [Durell, 1911, p.40].<sup>27</sup> Overall, the Report of the Girls' Schools Committee well demonstrates the sensitivity of some of the leading women teachers of mathematics at this time.

Soon after the War, a general Report from the MA [1919a, p.3] on secondary mathematics included no elements of differentiation, and merely pointed out that 'Throughout ... the word BOY is to be taken as referring to pupils of either sex.' In 1923, the Girls' Schools Committee reported on mechanics teaching, five years after a Committee had generally considered the matter [MA, 1918a, 1923a]. The Report was a response to widespread criticism of the character of mechanics teaching in girls' schools; the Committee circulated over four hundred schools, and 172 replied. Over a quarter taught no mechanics at all, though nearly a half taught some experimental mechanics to younger girls as part of physics. Sixty per cent taught some more advanced mechanics, commonly including some experimental work, but often to only one or two girls. The consensus of opinion in the schools was against teaching mechanics to all girls, given the reality of the inexperience of many mathematical mistresses in physics and experimental work, and the inadequacy of the facilities and the education of mistresses for science teaching, to which the BE's [1923a, pp.104, 124-125] Report on differentiation had also drawn attention [MA, 1923a, pp.3-4]. Nevertheless, the Committee argued a case for an introductory course of practical mechanics for all girls as part of mathematics or science, and provided a suggested scheme [pp.5-6]. The possibilities for correlation were raised, and a syllabus, with experiments, was also provided for a post-matriculation course.

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27 For some discussion of this new alternative to the traditional and outdated mish-mash of commercial arithmetic, suitable for boys and girls, see LCC [1911, pp.21-22] and Strachan [1918, pp.221-224]. The syllabus was revised by Nunn [1925, pp.30-31], and reissued after the War, together with a scheme of household arithmetic for girls, pioneered by Miss Punnett before the War [pp.29-30].



In 1923 also, the BE's [1923a, pp.101-104] Report had drawn attention to the widespread view that girls' attainments in mathematics and physical sciences were inferior to boys, which was supported by the evidence of teachers, inspectors and examiners, and that, in these subjects, there might even be differences in educable capacity. Inferior teaching and provision for mathematics in elementary and secondary schools, parental attitudes, girls' lower motivation, and the fewer opportunities for exploiting utility and correlation were all mentioned as contributory causal factors. This Report concluded that some differentiation in mathematics was justified, and it prompted an interesting response from the Girls' Schools Committee of the MA (see p.38).

The MA's [1926c] Committee undertook its own fact-finding and analysis, which was published as a Memorandum in the Gazette. Lower attainments could not be denied, though the use of the term 'inferiority' was deprecated, and, more positively, the Committee sought means for improving the situation. In addition to working to overcome some of the contributory factors already mentioned (see p.152), the Committee's survey revealed some other possible areas for improvement:

while in boys' schools the Mathematics of a class was regarded as a whole, very often in girls' schools, especially in middle school divisions, definite periods were assigned to Arithmetic, Algebra and Geometry, and these were not necessarily taken by the same mistress. The former plan is much more likely to produce satisfactory work. [p.13]

The Committee [p.14] also found that algebra and geometry were often begun too late. In addition the survey found that only one third of the mistresses teaching mathematics held an honours degree in mathematics, and only one half were trained, these being chiefly the less academically qualified (see pp.70-80). Generally, the Committee found inequality of educational opportunity for girls in mathematics, producing a vicious circle in relation to the quality of the teaching force. Optimistically, the Committee concluded:

While it is possible that girls will in general not achieve as much in Mathematics as boys, it should be possible ... to disprove the well-worn but unsound statement that 'Girls can't do Mathematics.' [p.15]

The excessive strain on girls caused by the School Certificate arrangements, and particularly the credit system, was also mentioned by the Committee. In the 1920s, the grouping system as a whole was a particular concern of women teachers, who sought to improve the status of the group of practical subjects within the School Certificate, these being traditionally an important feature of girls' education [Jenkins, 1979, pp.178-179]. The possibility of allowing candidates to replace the mathematics and science group by a group of other subjects was actually discussed by the MA in 1929, following a paper by Miss Gwatkin [1929], though, predictably, no official action was taken. In the same year the Girls' Schools Committee again reported on elementary mathematics.

Like the Report on preparatory schools in the 1920s, the Report on girls' schools [MA, 1929b] leant heavily on its predecessor, and provides evidence that a stage of consolidation in reform had been reached, though elements from the Report on geometry of 1923 were incorporated [MA, 1923b], as well as a number of appendices suggesting a rich range of books, articles and reports.<sup>28</sup> The Reports on the various branches of mathematics were also an important feature of the MA's work, and their background and influence have still to be considered.

#### The General Teaching Committee

There are a number of distinguishable aspects concerning the influence of the MA's Teaching Committee on the progress of mathematical education. Firstly, the Committee sought to influence examination syllabuses and papers, by publishing and circulating Reports, and also

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28 The syllabus for practical mechanics was reproduced [MA, 1929b, pp.44-45], as well as a revision of Nunn's syllabus for 'Arithmetic of Citizenship' [pp.41-43], compiled by a master from Bedales, who gave a paper on the subject to the MA in the following year [Gimson, 1930].



through formal and informal contacts with the various bodies involved. Secondly, the Reports acted indirectly upon teachers through their influence on textbook writers, some of whom were actually involved in the Committee's work. Finally, the Reports were a contribution to the growing literature concerning pedagogy, particularly for teachers within the increasing, and gradually more representative, membership of the MA, as well as for use in courses of professional training. A number of the Reports also provided a valuable focus for discussion at meetings of the Association.<sup>29</sup>

Siddons [1952a, p.156] has suggested that the character of the MA's Reports changed around 1918, from a principal concern for bare syllabus detail, particularly with reference to examinations, to a wider concern for questions of aim and method, with the interests of practising teachers predominant. Certainly, the Reports after 1918 tended to be longer and more concerned with the finer details of teaching method. However, some broadening of the character of the Reports goes back to around 1907, when the Report on preparatory schools was first published, and the Reports in the period 1908-1918, which was generally a stimulating one for the MA, certainly did not neglect wider questions of principle (see pp.146-147). The Committee's work on the three branches in 1902 resulted in the publication of a final Report before the end of the year [MA, 1902b], which was one contribution to the much wider movement for reform at this time, discussed in the last chapter. Although the geometry recommendations were far from being revolutionary in character, in relation to the freedom granted by examining bodies, a number of new geometries did follow closely the MA's scheme.<sup>30</sup>

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29 A number of such discussions are usefully listed in MA [1933, pp.8-10].

30 See, for example, the subsequently very popular Baker and Bourne [1902, 1903], and Hall and Stevens [1904a, 1904b]. More innovatory, but also successful, were the textbooks of Godfrey and Siddons [1903], and Barnard and Child [1903].

## REPORT OF THE M.A. COMMITTEE ON ARITHMETIC AND ALGEBRA.

1. The Committee consider that there is considerable danger of the true educational value of Arithmetic and Algebra being seriously impaired by reason of a tendency to sacrifice clear understanding to mere mechanical skill.

2. In view of this the Committee recommend :

- (a) that *easy and easy* examples should be frequently used in both Arithmetic and Algebra ;
- (b) that great stress should be laid on fundamental principles ;
- (c) that, as far as possible, the rules which a pupil uses should be generalisations from his own experience ;
- (d) that, whenever practicable, Geometry should be employed to illustrate Arithmetic and Algebra, and, in particular, that graphs should be used extensively ;
- (e) that many of the harder rules and heavier types of examples, which examinations alone compel us to retain in a school curriculum, should be postponed.

With these as guiding principles the Committee are led to make the following suggestions :—

### ALGEBRA.

17. The Committee suggest that beginners should be taught the use of letters to denote numbers, by substituting letters for numbers in suitable illustrations (involving integers) of the fundamental laws underlying Arithmetic and Algebra ; they should also be shown how results and processes with which they are familiar in Arithmetic and Geometry can be expressed generally by the use of letters (for example, before beginning Algebra pupils will have been taught how to solve questions on interest and areas without formulae ; from these and similar examples they should be led at an early stage of Algebra to construct formulae for themselves).

18. That the fundamental laws of Algebra should be inferred from examples in Arithmetic and Geometry by a rough induction ; formal proofs should be postponed.

19. The Committee consider that the use of the minus sign to denote negative quantity should be introduced at the very beginning of Algebra.

20. That, instead of the usual difficult substitutions, practice should be given at an early stage in substituting numbers for letters in identities, and in testing the roots of equations, not necessarily of the first degree, and in plotting graphs of functions.

21. That long multiplication and division should be postponed till after problems on  $x, y$  equations :—the method of detached coefficients should be taught, and the analogy with the corresponding processes in Arithmetic should be pointed out.

22. That the methods used in solving equations should be based on and frequently referred back to first principles (the four axioms) ; and that roots should be tested by substitution.

23. That *graphs* should be introduced as early as possible and be used extensively (certainly in connection with  $x, y$  equations, and quadratic equations).

24. That complicated fractions, hard H.C.F. and L.C.M., and literal equations should be postponed till after easy quadratics.

25. That irrational roots of quadratic equations should be often worked out to a few significant figures.

26. That the algebraic method of proving such theorems as those of Euclid, Book II., should be explained, and that problems similar to Euclid II. 11 should be given among quadratic problems, and that other applications to Geometry which assume a knowledge of Euclid, Books I. and III., should also be given.

27. That in *surd*s the chief stress should be laid on numerical evaluation, including rationalisation of denominators.

28. That complicated questions on fractional indices should be avoided.

29. That fractional indices should be illustrated by logarithms taken from four figure tables or from graphs made by the pupils themselves.

30. That work with imaginary quantities should, as far as possible, be avoided in elementary Algebra.

31. That the comparison of the algebraic definition of proportion with that given by Euclid, together with all such phrases as "duplicate ratio," "compounding ratios," etc., and the examples illustrating them, should be dropped out of elementary Algebra.

32. That algebraic work should be checked by the consideration of simple special cases and the substitution of numbers.



Apart from the new graphical emphasis, the recommendations in arithmetic and algebra were not startling, and largely concerned cuts and simplifications. (See Illustration 24.) As Minchin [1903, p.62] remarked at this time, 'The teaching of Algebra has not provoked such differences of opinion, or occupied so much attention, as the teaching of Geometry.' However, as with geometry, a number of textbook writers referred to the MA's early recommendations in their prefaces.<sup>31</sup> Fifteen hundred copies of the complete Report were soon exhausted, and another two thousand were printed early in 1903.<sup>32</sup> Following this early flurry, the output of the Committee over much of the rest of the decade was modest in its scope and influence, during a period of consolidation for the major reforms in geometry teaching [MA, 1908b]. The question of a possible new sequence for deductive geometry was discussed, but the Council subsequently ruled that the MA should not take any initiative here.<sup>33</sup>

A Sub-Committee, set up in 1903 to consider mechanics teaching, included Minchin, Jackson, Siddons and Tuckey, and first reported in the following year [MA, 1903c, p.349; 1904]. The BAAS also became involved with this aspect of mathematical education around the same time, but there was not the same general urgency for reform as in the case of geometry, despite the obvious shortcomings and parallels mentioned in the last chapter (see pp.124-127). The Teaching Committee also reported briefly on advanced school mathematics in 1904, and on university entrance scholarships in 1907, in both cases with a view to directly influencing the ancient universities' requirements.<sup>34</sup> In 1908, all the Reports of the Committee, including the fullest one yet on preparatory schools, and the Cambridge Schedules in geometry, were collected together

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31 See, for example, Tuckey [1902], Baker and Bourne [1904], and Borchardt [1905]. Baker, Barnard, Godfrey, Siddons, and Tuckey all served on the MA's Committee in 1902.

32 MA [1932c], minute 24/1/03.

33 MA [1905b]; MA [1932c], minute 21/6/07.

34 MA [1932c], minute 22/11/04; MA [1908b].

and published in one volume [MA, 1908b]. One feature of examination reforms in arithmetic had also become a concern of the MA by this time.

From around 1900, engineers, such as Perry and the Institution of Civil Engineers in particular, had pressed for a wider emphasis on approximations, including contracted methods of computation [BAAS, 1907, pp.445-448]. Such methods were uncritically accepted at first by examiners and textbook writers, and, by 1907, Robinson [1907, p.162] could claim that 'Examiners have placed in the forefront of their demands a knowledge of these methods, and regularly complain in their reports that such is not forthcoming.' However, these 'much-belauded improvements' [p.162] became the subject of the first teaching controversy to warrant a sustained debate in the columns of the Gazette, the issue first being raised by Godfrey [1906b, p.320], who confirmed that such methods 'used to be one of the planks of the mathematical reformer's platform.' Although some still defended these methods [Lodge, 1907], Godfrey and others revealed that such methods were outdated in practice, logarithms and the slide rule being preferred [Saunders, 1907]. The MA devoted a 'good discussion' to the matter [MA, 1907b], which was referred to the Teaching Committee, the result being a letter to examining bodies requesting that they should not insist on such methods, given their dubious value [MA, 1907c]. Slowly, these methods fell into disuse, though London ignored the MA's letter [Godfrey, 1912, p.435; Story, 1912, p.549].<sup>35</sup> As well as circulating Reports and letters, by the end of the decade the MA had also developed another tactic to pressurize examining bodies, namely a 'Pillory' in the Gazette for notorious examination questions [Aleph, 1910]. However, more important by 1910 was the growing concern for algebra teaching, which was still very much burdened by aimless manipulative excesses, despite the newer arithmetical and graphical emphases.

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35 London's new Matriculation requirements in 1921 no longer mentioned contracted methods [Retter, 1936, p.207].



Again it appears to have been Godfrey [1910a] who stirred the MA into action by delivering a stimulating paper, which was followed by a discussion on 'What is Educational and What is Technical?' in the teaching of this branch. The important distinction here was between the needs of the future user or mathematical specialist, and what could be justified in a general education. This paper was referred to the Teaching Committee, who published a Report on the subject in the same year [MA, 1910c]. This was discussed at the next Annual Meeting, and, with minor amendments, including a change of title, a second Report was published in 1911 [MA, 1911b, 1911c]. The Report embodied much of Godfrey's thinking, and its style represented a new departure for the MA [Price, 1974, pp.80-83]. As one teacher remarked at the time:

I very warmly welcome this report for the innovation it makes in putting first the principles which should influence our teaching of mathematics. It places in front of every other consideration the question, 'Why should we teach mathematics? What is our aim in mathematical education in the schools?'<sup>36</sup>

In addition, the Committee hoped:..

to influence the demands of examining bodies in such a way that the teacher will have freedom to put to better use much of the time at present spent on the elaboration of algebra in elementary classes. [MA, 1911b, p.1]

As in various recommendations a decade earlier, a case was argued for systematically cutting and simplifying, to enable broadening of the curriculum, possibly to include elements of mensuration and solid geometry, mechanics, and calculus, but, particularly, to incorporate some numerical trigonometry in a general education. The Report [p.1] remarked that 'Custom, represented by public examinations, has at present the effect of withholding that opportunity,' and concluded 'we hope that we shall have the hearty co-operation of examiners, upon whose policy so much depends' [p.8]. From around this time, there is evidence of increasingly close links between the MA and certain examining bodies.

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36 G.F. Daniell in MA [1911c, p.52], who had served on a Committee of the BAAS which reported on school science in 1908 [Jenkins, 1979, p.58].

The Joint Board was traditionally closely linked with the public schools and the HMC, and had already, in 1910, made a significant addition to its syllabus for algebra in additional mathematics, by including rates of change and gradients of a graph [Retter, 1936, p.128]. In the following year, the Joint Board held a meeting with representatives of the HMC, including Godfrey, and the Headmasters' Association, to consider the MA's proposals concerning algebra and numerical trigonometry [King, 1981]. The MA also around this time communicated with the Joint Board concerning its experimental questions in trigonometry.<sup>37</sup> Given the favourable relations with the Joint Board, the Committee decided to contact other bodies, including London, in this particular case on the question of allowing the use of logarithmic tables.<sup>38</sup> More generally, a letter from Siddons was circularized to various examining bodies, inviting co-operation, and referring to the MA's Reports which:

have received the attention of Examining Bodies, who, in more than one important instance, have referred proposals for changes in their regulations and syllabuses to the Committee for consideration and advice. The Committee, encouraged by these instances to believe that their co-operation is recognised as helpful, have directed me to bring the existence of the Committee to the notice of bodies who conduct the examination of schools in Mathematics.<sup>39</sup>

Before the end of 1912, the Committee was working closely with the Oxford Delegacy and the Cambridge Syndicate for the Locals, and was now using its Reports, with the exception of the 1902 Report on geometry, as a basis for all recommendations.<sup>40</sup> Godfrey was also working from the inside, as an examiner for the Joint Board at this time, and up to his death in 1924.<sup>41</sup> The BE's [1914a, p.3] second Circular on geometry

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37 MA [1938], minute 4/5/12.

38 Ibid., minute 20/7/12.

39 Ibid., letter 5/10/12.

40 Ibid., minute 30/11/12.

41 King [1981]. After Godfrey's death, another examiner for the Joint Board commented 'he had much to do with the organisation of the examination as it now stands; he was one of the Board of Examiners for many years' - Mr. Thompson in Siddons [1928, p.71]. Soon after the War, the Joint Board adopted new syllabuses, as well as distinctive mixed papers from 1921 [Retter, 1936, pp.128,163,214].



referred to the MA's support for the broadening of the basis in deductive geometry, which had been sympathetically received in the case of the Oxford and Cambridge Locals. In 1913, the MA's Committee forwarded a number of proposals to the London Board, to bring this examining body into line with those at Oxford and Cambridge, where negotiations were proving fruitful [MA, 1914d, p.228]. Specifically, the Committee sought no insistence on contracted methods; the broadening of arithmetic to include simple mensuration; the use of logarithms; simpler algebra, with some alternative numerical trigonometry; and some calculus in the optional more advanced syllabus.<sup>42</sup> However, in the case of London the Committee reported that 'the Senate is not prepared, at present, to make any alterations in these syllabuses' [MA, 1914d, p.228]. Siddons made some interesting remarks at this time concerning the Committee's tactics:

The direct method of attack is not always the most suitable, as I think the Committee has found to some extent. It is always necessary to enquire privately of somebody who knows what is the best way to do it.... I am afraid at London University we were not successful. We took a lot of trouble to find out what was the proper method of attack, and nobody seemed to know, and the method we were ultimately advised to take has not proved successful. [MA, 1914e, p.264]

Co-operation was, however, forthcoming from the Civil Service Commission, and Sub-Committees during the War pursued the problem of London's requirements, and worked on the syllabuses of the NUJMB.<sup>43</sup> In 1918, a further Sub-Committee was appointed 'to report periodically on the current examination papers of public examining bodies, so far as mathematical subjects are concerned' [MA, 1918b, p.187]. By this time, and following the Report on algebra, Reports on other aspects of mathematical education had also been produced.

The Report on algebra and trigonometry was clearly a valuable weapon of the MA, after 1910, in its negotiations with examining bodies. Some new algebra textbooks also acknowledged the stimulus of this Report, and relegated much of the heavier manipulative work to a separate chapter or appendix.<sup>44</sup> Fairly short general Reports were produced by

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42 MA [1938], minute 22/2/13.

43 MA [1915b, p.4]; MA [1938], minute 4/11/16.

44 See, for example, Borchardt [1911], and Godfrey and Siddons [1912, 1913], who included a treatment of logarithms, functional dependence, and the beginnings of differentiation and integration. See also the subsequently successful Durell, Palmer and Wright [1920, 1921].

the Special Committees for public and other secondary schools [MA, 1913b, 1914f], the former being discussed at the MA's Annual Meeting [MA, 1914e].

However, in 1915, the General Committee reported that:

conditions have not seemed suitable for entering upon any work of an important or controversial character. The Committee is now considering what work it can most fittingly undertake under existing circumstances. [MA, 1915b, pp.3-4]

A Sub-Committee had been formed to consider geometry in 1913, and a start was made in the following year, but a Report was not published until ten years later.<sup>45</sup> However, detailed work was undertaken in arithmetic, on the lines of the earlier work in algebra.

In the case of arithmetic, it was a paper by Durell [1911], his first to be published in the Gazette, which encouraged the MA to act. As with Godfrey in algebra, Durell made a strong case for omissions and simplifications to facilitate broadening of the curriculum. There was no prompt action, in spite of a lively discussion involving Godfrey, Siddons and Nunn in particular, though a Sub-Committee subsequently took up many of Durell's proposals, and reported during the War [MA, 1916a]. This was a convincing statement concerning desired and many overdue reforms, particularly aimed at examining bodies, though methodological questions were also considered. During the War, another Sub-Committee considered the teaching of mechanics, and reported in 1918.

In 1916, this Sub-Committee circularized schools, including some girls' schools, in England, Wales and Scotland, 'in which it might be presumed that mechanics was taught' [MA, 1918a, p.291]. Out of just under two hundred schools, eighty-five per cent taught some mechanics. One quarter taught the subject under mathematics only, one third under science only, and the remainder under both.<sup>46</sup> Idealistically, the Report recommended a course in mechanics from the age of thirteen, as

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<sup>45</sup> MA [1938], minute 22/2/13; MA [1915b, p.4].

<sup>46</sup> Compared with Perry's earlier findings (see pp.124-125) there appears to have been some decline in the proportion of schools teaching mechanics under mathematics and science.



# REPORT OF THE MATHEMATICAL ASSOCIATION COMMITTEE ON THE TEACHING OF MATHE- MATICS IN PUBLIC AND SECONDARY SCHOOLS.\*

## SUMMARY OF RECOMMENDATIONS.

1. That a boy's educational course at school should fit him for citizenship in the broadest sense of the word; that, to this end, the moral, literary, scientific (including mathematical), physical and aesthetic sides of his nature must be developed. That in so far as mathematics are concerned, his education should enable him not only to apply his mathematics to practical affairs, but also to have some appreciation of those greater problems of the world, the solution of which depends on mathematics and science.
2. That the utilitarian aspect and application of Mathematics should receive a due share of attention in the mathematical course.
3. That the mathematical course in the earlier stages should not be concerned exclusively with Arithmetic, Algebra, and Geometry; that such subjects as Trigonometry, Mechanics, and the Calculus, should be begun sooner than is now customary and developed through the greater part of the boy's school career, so as to give him time to assimilate them thoroughly, and enable him to cover more rapidly, at a later stage, the higher parts of Arithmetic, Algebra and Geometry.
4. That no boy should leave school entirely ignorant of Applied Mathematics (e.g. Mathematics relating to the Stability of Structures, Motion of Bodies, Electrical Plant, Astronomy, etc.).
5. That, while the average boy should receive careful and adequate instruction, the boy of high talent should receive special attention, as the value to the race of carefully trained superior talents is incalculable. A well-equipped secondary school should be so staffed as to be able to educate *pro viribus* both the average boy and the boy of genius.
6. That a boy who takes Mathematics as his main subject in the later part of his school life should also, in general, study Science, as well as carry on some form of literary study; and that the general

Illustration 25 Summary of Recommendations  
of the Committee of the MA in the  
Mathematical Gazette [1919], Vol. IX,  
No. 143, Dec., pp. 393-394

educational purpose underlying the choice of these various subjects should be made manifest to the boy.

7. That the time devoted to Mathematics in the Secondary Schools before the period of specialisation should be at least six periods per week (excluding time for preparation at home).

8. That the teaching and organisation of Mathematics and Physics should be in the closest possible co-ordination.

9. That every teacher of Mathematics should go through (1) a course of *mathematical training* at the University to be followed by (2) a course of *professional training* in the Theory and Practice of Teaching with special reference to Mathematics, at a Training College, the two courses not to run concurrently.

10. That the mathematical teacher should also receive training of a less intensive kind in some subject in which his Mathematics can be definitely applied, e.g. Geography, Physics, Chemistry, Engineering, Manual Training, Astronomy, etc.

11. That teachers of insufficient ability or knowledge should not be promoted to be Heads of Departments simply on the ground of long and faithful service. Long and faithful service deserves recognition, but in some other way. The Head of a Mathematical Department has to teach the advanced work—which requires knowledge—and to draw up the general school syllabus—which requires outlook.

12. That Heads of Mathematical Departments and Specialist Teachers of the higher branches of Mathematics in the Advanced Departments of the Secondary Schools should have tolerably short routines in order that they may be able to read more widely in their subject and to study its modern development (cf. 15). Only thus can the knowledge imparted to the boys be kept up to date.

13. That an External Examination Syllabus should be frequently revised by a joint body consisting of representatives of the teachers themselves and of the external examining body. Otherwise the syllabus, being stereotyped, tends to become obsolete, and teachers have to teach what they have ceased to value.

14. That it is more important to TEACH boys than to EXAMINE them; that the number of examinations at present conducted in the majority of schools should be reduced, inasmuch as the setting of examination papers consumes the master's time and energy, thereby lowering his teaching capacity, while the continuous pressure of working at examination papers for days on end is an unproductive strain on the boys.

15. That every Secondary School should be provided with a Mathematical Library, containing books of a more general character than the ordinary text-books, in order that the pupils and masters may be enabled to widen their horizon and catch a glimpse of the regions beyond.

16. That portraits of the great mathematicians should be hung in the mathematical class-rooms, and that reference to their lives and investigations should frequently be made by the teacher in his lessons, some explanation being given of the effect of mathematical discoveries on the progress of civilisation.

a 'link subject' taught by the mathematical staff. Much of the Report was devoted to appendices, with major contributions from Nunn, whose ideas had already been published in a Report on secondary science for the BAAS, in 1917. There was much visionary thinking in the MA's Report which was, no doubt, much in advance of prevailing practices.<sup>47</sup>

Early in 1918, the Teaching Committee decided to produce a general report on mathematics in public and other secondary schools, with particular reference to educational aims, the place of the subject in the curriculum, correlation across the curriculum, and examinations. Thus, the resulting Report [MA, 1919a] proved to be different in character to all its predecessors, and it well summarized the general educational thinking of leading members of the MA at this time. However, the Report's grand summary of recommendations, reproduced in the Gazette, was unlikely to have a significant impact upon practices in schools generally. (See Illustration 25.) Geometry still remained as one branch of mathematics upon which the MA had not reported in detail, since 1902.

Looking back on the conservative Report of 1902, Godfrey [1920, p.20] remarked that 'no report on Geometry has emanated from your Teaching Committee since that date.' Godfrey justified this strange omission by emphasizing the need for experimentation and variety in the new period of freedom, and added:

an Association cannot easily do more than register the average opinion of sound teachers.... I do not consider that the time has yet come for the Association to support with its authority any particular method of teaching Geometry.

However, the AMA had set up a Sub-Committee to consider geometry in 1902, which worked on much more radical lines than the MA, with inspiration

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47 Schemes by Siddons, W.J. Dobbs, and G. Goodwill were also included in the appendices.



from Dobbs [1902, p.730], and reported in 1904.<sup>48</sup> Furthermore, before the War, the BE [1909a, 1914a] had very decisively taken the initiative, through its important Circulars (see pp.40-41).<sup>49</sup> Godfrey also seems to have overlooked the fact that the MA had taken steps, shortly before the War, to consider geometry teaching. Soon after the War, both the MA and the AMA again became involved with this branch of secondary mathematics.

In 1921, the AMA received representations from many schoolmasters suggesting that much time was being wasted in the early stages of geometry teaching, and that there was a need for uniformity in examination requirements. Accordingly, the AMA circularized all boys' schools in England and Wales, as well as some girls' schools and Scottish schools, through the co-operation of the Assistant Mistresses' Association and the Educational Institute of Scotland. Ninety per cent of masters favoured a new uniform sequence of theorems, to guide ordinary teachers after twenty years of free enterprise in teaching and textbook writing. However, only just under half the mistresses desired a return to uniformity. The Committee was assisted by HMI Fletcher, and reported in 1923, their memorandum being the first of a new series on secondary subjects, published up to 1934 [AMA, 1923, 1966]. In spite of feelings in boys' schools, the Committee only provided a suggested schedule of numbered theorems, since it 'does not feel it is either desirable or possible at present to stereotype a sequence,' there being 'still room for considerable diversity' [AMA, 1923, p.5]. On the question of lightening the deductive burden, and increasing intuition, the Committee classified a

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48 Dobbs [1913] referred to this early work of the AMA in the preface to his extraordinary book, A School Course of Geometry, in which he developed fully his ideas for school geometry, breaking freely from the Euclidean tradition by employing motion rather than congruence throughout (see p.113, note 45). The present writer has failed to locate a copy of the AMA's Report.

49 Before the War, given their progressive and sophisticated views, it seems unlikely that either Dobbs or Fletcher registered the 'average opinion of sound teachers' to which Godfrey referred.

number of formal proofs as not for examination purposes, and others as not even for classroom purposes, following the lines of the BE's Circulars. By this time, some examining bodies were also responding in this respect by not requiring proofs of certain propositions [Godfrey, 1920, pp.23-24; Retter, 1936, p.163].<sup>50</sup> This tendency, particularly with regard to Euclid I, also greatly reduced the importance of the sequence issue from around this time. On the question of a uniform sequence, which had been pressed upon the AMA by its members, the MA was consistent in its opposition to uniformity, and, in 1922, a resolution to this effect was sent to the AMA, with whom the Committee were not prepared to discuss this matter.<sup>51</sup> In the same year, the MA appointed a Sub-Committee to consider geometry teaching, which worked independently of the AMA, given the difference of motivation, though in the event both the Reports in the following year rejected uniformity of sequence as neither possible nor desirable, and of exaggerated importance [MA, 1923b, p.4]. However, the general character of the MA's Report was very different to that of the AMA.

The Sub-Committee included Godfrey [1920] and Nunn [1922], both of whom had become particularly interested in developing geometry teaching, as well as Tuckey, with the mathematician E.H. Neville as Chairman, and Miss Punnett as Secretary. Godfrey's thinking at this time, and his proposals for greater flexibility in the deductive course in a general education, developing the Stages proposed in the BE's Circulars, have been discussed in a paper by Howson [1973b]. Nunn was also thinking on similar lines [Godfrey, 1920, p.24], as well as arguing a case for adopting a principle of similarity in a formal deductive treatment. The

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50 Brereton [1944, p.115] subsequently cited this development as one of the few examples of common action by the examining bodies for the First Examination, concerning the construction of syllabuses. However, a schedule with 'starred' i.e. non-examinable propositions was not implemented by London until 1937 [Retter, 1936, p.167].

51 MA [1922b]; MA [1938], minute 8/4/22.



resulting Report of seventy-four pages was unprecedented in its detail and scope from a Committee of the MA. Howson [1973b] has drawn attention to the curious fact that this Report reinforced the view that a systematic logical treatment of geometry could reasonably be undertaken before the First Examination, in spite of the misgivings of both Godfrey and Nunn on this point. Thus Howson has judged that the Report was a 'millstone' which inhibited pedagogical developments. Godfrey died in the year following the Report's appearance, before he could develop and impress his own views concerning the future of geometry teaching. Durell [1925a, p.274] who was on the AMA's Sub-Committee, and, as a successful textbook writer was close to classroom realities, expressed some strong reservations:

There is little doubt that many teachers are seriously bewildered by this Report. It is not an easy document to read and frequently it is difficult to ascertain the precise nature and application of its recommendations.

He also made some interesting remarks concerning the Report's background:

The dimensions of the Report makes it possible to doubt (a) whether the Report represents the considered judgement of all members of the sub-committee and (b) whether the approval given by the Teaching Committee was much more than a mere matter of form.....

Tuckey [1945, p.2], a Sub-Committee member, subsequently revealed that Neville and Nunn were principally responsible for the Report, the former also being the editor. In particular, Nunn's ideas for a rigorous deductive treatment using similarity were incorporated, and Tuckey [1951, p.237] also revealed that the contribution of Neville and Nunn explains the fact that about half the Report is mathematically self-indulgent. Certainly, Neville's predispositions would have been mathematical rather than pedagogical, but the lack of pragmatism in Nunn's case is perhaps surprising.<sup>52</sup> Durell [1925, p.274] contrasted

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52 Nunn [1924] and a collaborator (Neville?) wrote a textbook for deductive geometry, adopting the principle of similarity, but it was never published, and, in any case, the tendency in the schools was subsequently to turn away from the logical foundations [Tuckey, 1945, p.2; 1951, pp.236-237].

this Report's character with the MA's earlier Reports: 'they have not hitherto been used to provoke discussion, but to urge particular methods which have been tested by experience,' and he concluded: 'There is a real danger that geometrical teaching may be injured by a misunderstanding and mis-application of the views expressed.' Although the Sub-Committee's Report did help to disseminate further the idea of pedagogical stages in geometry, in terms of classroom reality there are some similarities between its work and the nineteenth-century work of the AIGT, and, as Howson has argued, it acted as a freezing influence on progress in the teaching of deductive geometry.<sup>53</sup> Also, as regards textbooks, it appears that the work of the AMA was more relevant in the 1920s, judging by the acknowledgements in prefaces.<sup>54</sup> In the same decade, the MA again turned its attention to the teaching of mechanics.

In 1927, a small Sub-Committee was appointed to consider mechanics, and a Report, comparable in size to that in geometry, was published three years later [MA, 1930c]. In this case, the Sub-Committee, which included Fletcher, clearly found it difficult to produce something of value to follow the Report of 1918, which had been inspired by Nunn. Fletcher [1904], himself an innovator in mechanics teaching in the early years of this century, openly admitted some lack of motivation for the task:

it was one thing when full of a subject and anxious to write about it to produce something; it was quite another when, not particularly interested, one was commanded by superior authority to produce something. [MA, 1931b, p.343]

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53 For a discussion of the Report's relevance in relation to the MA's Second Geometry Report of 1938, which was much more classroom orientated, see Tuckey [1951]. Why Godfrey had so little influence on the earlier Report, given his stature within mathematical education, remains a matter for speculation.

54 See, for example, Hall and Stevens [1924], Siddons and Hughes [1926], and, not surprisingly, Durell [1925b]. Gray and Smith's [1923] A New Sequence Geometry also referred to the new requirements for the Scottish Leaving Certificate, which were in line with the AMA's proposals. Parkinson and Pressland [1923], who were members of the AMA's Sub-Committee, also catered for English and Scottish requirements.



Thus this Committee saw its task as 'a much humbler one:'

The best they could do was to try and say what few commonplace things they could which they thought would be of use to ordinary teachers. [p.343]

This Report's contents and the discussion upon it are evidence that a stage of consolidation had been reached in the MA's work by this time. That the pattern was a general one is confirmed by the conservative character of two new Reports on arithmetic and algebra [MA, 1932d, 1934a]. These Reports of the Boys' Schools Committee were substantial and detailed, but were principally concerned with the elaboration of well-tried practices over what was, by this time, a well-established range of content.<sup>55</sup> The Reports on preparatory and girls' schools in the 1920s were similarly designed to consolidate practices, drawing on earlier Reports (see pp.152,159). Thus, the Report on geometry appears to have been an exceptional one in comparison with its successors, as well as its predecessors. The role of the MA as a pressure group in relation to examining bodies also declined markedly in the 1920s.

Early in the 1920s, the MA continued to negotiate with the examining bodies approved for the new First Examination, who generally continued to respond favourably, though London lagged behind [MA, 1922c, p.35; 1923c, p.246]. By this time, the Boys' Schools Committee judged that reissue of the 1913 Syllabus for non-specialists [MA, 1913b] was unnecessary as 'most of these recommendations are now embodied in any good modern syllabus.'<sup>56</sup> London also began to respond to pressure in the early 1920s. At the MA's Annual Meeting in 1920, a mistress opened a discussion on the teaching of logarithms. She referred to the support for their use from the examining bodies of Oxford, Cambridge and the Northern Universities, but drew attention to London's continuing resistance [Cook, 1920]. Nunn referred to London's policy as 'a serious and

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55 For some discussion of the Report on algebra see Price [1974, pp.125-129].

56 See Minutes of the Boys' Schools Committee of the MA, 1923-38, MA Archives, Leicester, minute 3/11/23.

mischievous thing,' apparently justified on the ground that logarithms should not be introduced because the full theoretical basis of indices was not appropriate in the syllabus [p.28]. In addition to pressure from the MA, particular schools were also circumventing this restriction by using the special syllabus provision (see p.56). Thus, in 1922 the MA's Council reported that London had issued new syllabuses incorporating 'many of the suggestions of this Association' and, in particular, the use of logarithms and properties of similar figures, and some calculus in the alternative more advanced syllabus [MA, 1922c, p.35]. At this time London also dropped its insistence on contracted methods and brought some mensuration into the arithmetic syllabus [Retter, 1936, p.207].

During the 1920s, however, London continued to be out of step with other bodies in the cases of geometry and trigonometry. Geometry was limited to Euclid I-IV, with only geometrical proofs permitted; there was no detail of required theorems provided, and no provision made for numerical trigonometry [MA, 1930a, p.147]. As one schoolmaster remarked during a discussion on the First Examination at the London Branch of the MA:

Which was best from the educational point of view: to force a pupil to learn up 60 theorems which he would soon forget, or to teach him such subjects as trigonometry? [p.154]

A London examiner had earlier remarked that he regarded trigonometry as a soft option, but the London Branch agreed to approach the University concerning its exclusion and the requirements in geometry [pp.147,154-155], and a memorandum was sent out early in 1931. The General Teaching Committee had also again approached London, and, in 1931, the London Branch was notified that from 1933 some alternative questions on trigonometry would be included in the First Examination [MA, 1931c, p.314; 1932e, p.2]. With the exception of London, the MA's work as a pressure group in relation to examination requirements was largely completed in the early 1920s. Cambridge [1979], whose links with the MA go back to



1927, recollects:

I cannot remember anything like pressure group activity during the following twenty years. As I recall it, we were chiefly concerned with improved ways of teaching recognised portions of the curriculum and with keeping an eye on Boards of Examiners in case any of their questions proved objectionable from a pedagogic point of view.<sup>57</sup>

Siddons was one of three members who acted in the case of examination papers, and he admitted 'it was difficult to know exactly how to get at the various examining bodies' [MA, 1930a, p.149]. Although the major battles had by this time been won, the arrangements for examinations continued to be a popular subject for the MA's discussions, which also involved representatives of the examining bodies. Specific topics here included the Joint Board's distinctive arrangements, with its mixed papers [Siddons, 1928]; the grouping system for the First Examination [Gwatkin, 1929]; syllabuses for the First Examination [MA, 1930a]; and the variations in the requirements for additional mathematics [MA, 1932a].

Thus far, the discussion has focused on the MA's involvement with mathematics in preparatory, public, other secondary and girls' schools, though the Association also showed some peripheral interest in the elementary, technical and teacher training sectors of education.

#### Elementary, Technical and Teacher Education

Being a specialist subject association having traditionally close links with the public schools and ancient universities, the MA was both intellectually and socially distanced from the elementary schools, their teachers, and the training colleges. The only evidence of the MA's official involvement here before the War is in the case of the BE's first revision of its Suggestions for arithmetic teaching. A special Sub-Committee was formed to prepare proposals at the BE's invitation, and the MA was fortunate in having at its disposal the expertise in elementary

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57 Curiously, as late as 1924, the Teaching Committee decided to approach Glasgow University, which still insisted on Euclid's order, 'to ask if there was any special local reason why this proviso was still included' - MA [1938], minute 4/4/24.

teacher training of Nunn and Miss Punnett.<sup>58</sup>

In 1923, the Council was asked in a letter to consider the needs of elementary schools, though no action was taken, beyond suggesting that the London Branch might take some initiative.<sup>59</sup> However, two years later the MA was requested by the BE's Consultative Committee to submit recommendations concerning the mathematical education of the adolescent in elementary and junior technical schools, although the MA's relevant experience was still very limited [MA, 1926d, p.42]. Five years later the Council reported that several members had requested that the question of mathematics in the senior elementary schools be discussed at the Annual Meeting. The Council admitted:

While feeling that the Association is as yet hardly in a position to discuss this matter effectively, the Council hopes that the Branches will, as and when opportunity offers, encourage any of their members whose experience entitles them to views on this subject to make them known as a basis for discussion. [MA, 1932e, p.3]..

The subject of mathematics in the central schools was eventually discussed at the Annual Meeting, in 1934, where papers were read by two teachers from central schools. Bushell reported that:

A great many members of the Mathematical Association had been for some little time very curious indeed to know what mathematics was being taught in the Central Schools.... [MA, 1934b, p.94]

However, the MA between the Wars showed little more than curiosity with regard to the developing forms of post-primary education. Predictably, the MA also showed no concern for technical education until the 1920s.

Before the War, H.T. Holmes [1910, p.200] drew attention to the Gazette's predilection for pure mathematics and secondary mathematics teaching, and added:

the Gazette contains little of special interest to those concerned in the work of the evening classes, which are so important and special a feature of our British system of education.

It was not until after the death of Perry, in 1920, that the MA turned

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58 MA [1932c], minute 4/11/11.

59 Ibid., minute 24/2/23.



its attention to technical education. L.B. Benny [1924, p.59] contributed a short paper to the Gazette, where he noted the sharp contrast with the MA's work in secondary education:

No effective steps have been taken to consider the mathematical needs of technical students, and the most satisfactory way of dealing with them.

This writer expressed the hope that the MA 'may feel able, in the future, to extend its activities to the realm of Technical Education' [p.59].

In the same year, Professor Piaggio [1924] of the University College, Nottingham, took the initiative by requesting views from those interested in the progress of practical mathematics, which was still the label used to denote mathematics for technical students, and he referred to the possibility of a report from the MA on the subject. The Teaching Committee admitted:

A considerable proportion of the members of the Mathematical Association are not directly concerned in the teaching of Practical Mathematics but the subject is of general interest to all teachers of Mathematics ... and it is felt that this particular problem should not be divorced from other problems of Mathematical teaching.<sup>60</sup>

Accordingly, Piaggio's paper was circulated to selected persons, and, in the same year, the ATTI considered the matter and devoted space to it in its Technical Journal. Given the level of interest demonstrated at this time, the MA formed a Sub-Committee to prepare a report [MA, 1926e, p.9].

Piaggio was Chairman of the Sub-Committee, which included two representatives of the BE's Technological Branch, Holmes being one, and co-opted representatives of the ATTI and the Association of Principals in Technical Institutions, with Tuckey the only member not specifically involved in technical education. The Report acknowledged the progress made in the early twentieth century:

chiefly owing to the efforts of the late Professor Perry. His methods have been adopted very widely. When they were first introduced, they aroused great enthusiasm.... [MA, 1926e, p.9]

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60 MA [1932c], minute 5/4/24.

However, the Report suggested that there was in the light of experience still cause for concern as 'Some very good work has been done ... and yet, in the opinion of many, the results obtained are somewhat disappointing' [p.9]. By this time, the principal concern was for the overemphasis on rules and the inattention to fundamental principles. The Committee hoped for a more unified treatment of the subject, linked with its applications, and recommended that 'the teacher should possess a sufficiently liberal knowledge to enable him to present the subject in this manner' [p.12].<sup>61</sup> The Report even suggested a little deductive geometry for junior technical students to restore the educational balance [p.102], though Piaggio judged this to be a 'somewhat controversial' recommendation [MA, 1926f, p.82]. However, some general concern for the instructional rather than educational bias in mathematics for technical students is evident in the Report and the resulting discussion upon it. The Report also hoped for and welcomed greater involvement from technical teachers in the MA's activities [MA, 1926e, p.14]. This work of the MA in the technical field was a noteworthy extension of its operations, though the MA's interest here does not appear to have been sustained between the Wars, and the Report was an isolated initiative.

Although the MA's lack of concern for elementary teacher education is not surprising, the low level of its involvement in secondary teacher education warrants some explanation. Certainly, the MA became a major agent supporting teachers of mathematics in-service, through its increasing and widening membership; the activities of its developing Branches; the work of its various Committees and their published Reports; the Presidential Addresses, papers, and discussions at the Annual Meetings; and the various articles and other features in the Mathematical Gazette.

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61 For some early reactions to Perry's practical mathematics as illiberal mathematics see pp.105-107. This curious dichotomy between academic or liberal mathematics and practical or technical mathematics will be explored further in the next chapter.



The publications of the MA were also an important contribution to the literature for methods courses in initial teacher training.<sup>62</sup> The reluctance of secondary teachers to undergo preliminary training, particularly men and those aspiring to higher status schools, was discussed in Chapter 2 (see pp.77-80). Given the character of the MA's membership in the early twentieth century, it is not surprising that this body did not become concerned with this aspect of professionalization. Lip-service was paid to the need for specific training in the MA's general Report of 1919 (see p.167), which also included a short appendix on the subject from Nunn [1919].<sup>63</sup> However, in the following year, Professor W.P. Milne [1920, p.83] of the University of Leeds referred to the movement for secondary training, and remarked:

So far as I know nothing has as yet been done by the Mathematical Association as a body in helping forward this movement, but surely the Mathematical Association will fail in its fundamental aims if it does not take a very active part in helping to decide what are the best courses of instruction to give to the student-teachers while in training.

Milne went on to outline his own proposals in what was an exceptional article on this subject in the Gazette. It appears that between the Wars the MA did little more than publish a statement from the BE, prepared for the ICTM [MA, 1932b]. A comparative study for the NCTM [1939, p.166] concluded that:

the Association has not pressed for reform in this direction with the zeal with which it has attacked the problems of improving mathematical instruction in the schools.

This study contrasted sharply the English pattern where, for public schools particularly, preliminary training was still regarded as something of a luxury, with the practices in America, where formal training was obligatory for secondary teachers, and the methods courses were much

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62 A Report of the NCTM [1939, p.155] referred to 'the discussions of the Association's Reports which form so considerable a part of the methods courses.'

63 The recommendation concerning training was omitted in the 1928 revision of the Report of 1919, presumably because it was by this time more generally accepted as desirable.

more substantial than in this country.<sup>64</sup>

One other facet of the MA's early twentieth-century work was its joint activity with the teachers of science. Such developments remain to be considered.

### Science Education

The important question of the influence of science education on mathematical education, and the opportunities for correlation between these domains will be fully explored in the next chapter. The various issues and possibilities emerged in the innovatory early twentieth-century period up to the War, and it is the organizational involvement of the MA that will be the principal consideration here.

Eggar of Eton, a founder member of the APSSM in 1900, drew attention at an early stage to the need for correlation between mathematics and physics, particularly in public schools, in an article for the School World [Eggar, 1901]. Eggar himself became involved with the Committee of the BAAS on mathematics teaching, and he provided teachers with much valuable guidance for practical and experimental work in geometry.<sup>65</sup> Three years later, it was the APSSM that heard a paper on 'The Possibility of Fusing the Mathematical and Science Teaching in the Public Schools' at their Annual Conference. In the discussion which followed, there were contributions from Jackson of Woolwich, Fletcher, recently appointed at the BE, and Siddons, all leading individuals in mathematical education [Schoolmasters Yearbook, 1905, p.223]. In the following week, Jackson [1904] also gave a paper to the MA on mathematics and physics teaching, which was published in the Gazette. The APSSM had been the first of the two Associations to raise the question of

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64 The London Institute of Education, which grew out of the Day Training College, and whose development owed much to Nunn, provided a sixty-hour mathematical methods course, four times the average for twenty-two secondary departments in the NCTM's [1939, pp.145-152] survey.

65 See his schedule for the BAAS [School World, 1902d, p.391], textbook [Eggar, 1903a], and articles for the School World [Eggar, 1903b, 1903c].



correlation, though nothing tangible emerged. However, four years later the APSSM formally approached the MA, to arrange a conjoint meeting, which it was eventually agreed should take place in 1910, and, in the meantime a special Joint Committee was set up to investigate and report upon developments and possibilities in practical aspects of geometry, measurements, physics and chemistry; links between preliminary practical mathematics and science; the use of common definitions and mutually supportive content; the introduction of logarithms and the slide rule; and the teaching of mechanics [MA, 1909d, p.42]. The Joint Committee included Jackson, Siddons, Godfrey and Mair, appointed by the MA, and Borchardt, a leading writer of mathematics textbooks, but appointed by the APSSM, who were also represented by A. Vassall, a colleague of Siddons at Harrow, and a founder member of the APSSM [MA, 1909e]. The Committee, with Godfrey as Chairman, also included two representatives of the Association of Preparatory Schools.

In addition, the MA also became involved in a Joint Conference with the Federated Associations of London Non-Primary Teachers, held at the Polytechnic, Regent St. towards the end of 1908. The scale of this Conference, which was attended by over one hundred and fifty people, was unprecedented for the MA [1909d, pp.42-43]. Apparently this venture led to some friction between the MA and the APSSM, as the Secretary of the latter objected to it as 'forestalling the action of the Joint Committee.'<sup>66</sup> One number of the Gazette was devoted wholly to a Report of this Conference on 'The Correlation of the Teaching of Mathematics and Science,' which was opened by Perry, who delivered a typically pungent address [MA, 1909a]. He launched a scathing attack on mathematical examinations, and drew attention to the great gains in correlation through the adoption of his practical mathematics for technical students. He acknowledged the recent contribution of the BAAS and the MA to the reforms in

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<sup>66</sup> MA [1932c], minute 28/11/08.

secondary mathematics, but could not resist commenting that 'all so excellent as the syllabuses of the Mathematical Association are, they seem to me to possess too much of the orthodox spirit' [p.7]. Godfrey, Jackson, Bryan, Nunn and Armstrong all contributed to the resulting discussion, where some of the problems of implementing change were clearly exposed. Godfrey felt that the MA's various recommendations would have little effect 'Unless some steps ... can be taken to give a new point of view to the mathematical master' [p.22]. Armstrong referred to the misinterpretation of his own and Perry's ideals, and to the need for training mathematics teachers in the subject's applications. Summing up, Perry judged that the recent organizational developments gave some cause for optimism, but that there were other factors to consider:

The things we hope for and ought to work for are a doubling of salaries, a diminution of the size of classes, the ousting of mere specialists, and completely getting rid of the outside examiner. [p.38]

The Joint Committee of the MA and APSSM reported in the following year.

Around thirteen hundred boys' schools had been circularized, but, disappointingly, only 279 questionnaires were completed, which somewhat limits the value of the Committee's ambitious statistical exercise. The percentage returns of fifty-five for public, twenty-two for grammar, and only twelve for preparatory schools give a good indication of the different levels of interest in correlation at this time. The Report provided much factual data concerning practices in relation to the Committee's terms of reference (see pp.179-180), including details of ages for the treatment of topics and the types of teacher involved [MA, 1909e, pp.1,7-13]. The Report was considered at a Joint Meeting of the MA and APSSM, early in 1910, which was the culmination of this Committee's work.

Forsyth chaired the Meeting, and Godfrey emphasized that the Report was not an official pronouncement from either Association, but was



presented 'on its own merits' [MA, 1910d, p.253]. Armstrong and Nunn contributed to the discussion, and the problem of examination requirements again arose, particularly with reference to the use of logarithms [p.265]. Nunn regarded the recommendations as only moderate in character, but, in any case, the question of how to implement change remained largely unsolved. Certainly, innovation as regards correlation in particular schools was a feature of this period up to the War, and the formal links between the MA and APSSM reflected the interest of particular individuals at this time.<sup>67</sup>

The importance of these links for general curriculum change, however, is another matter. Organizational activity concerning correlation was not sustained after 1910, and the various features of the relation between science and mathematics, both in theory and in practice, will be a major consideration in the next chapter.

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67 For example, two members of the Committee, Siddons and Vassall [1910], produced a textbook on Practical Measurements, which followed the syllabus published in the Report.

## Chapter 5

### The Influences of Practical, Scientific and Technical Education

In earlier chapters of this thesis, various features of control and change have been considered, with a major emphasis upon strategy and tactics in relation to curriculum development. Passing reference has frequently been made to the ideas and activities of particular individuals and organizations concerned with English mathematical education. However, no systematic attempt has yet been made to chart important changes in the general educational climate, or to consider the underlying forces which shaped the various pronouncements on curricula to which frequent attention has already been drawn. Furthermore, the complex network of relationships between ideals, pedagogy, curricular recommendations and actual classroom practices needs to be explored further, in order to do justice to this remarkable period in the history of curriculum change. The task which remains is admittedly a formidable one, and particularly so as it will become clear that many of the important changes concerned methodology rather than syllabus content in mathematics, and the former is extremely difficult to assess.

With a specific focus on elementary education in the period 1870-1914, a penetrating study of the relationship between theories and practices has been undertaken by Selleck [1968], who has valuably disentangled the various features of the 'new education' which emerged in the late nineteenth century. The influence of the new thinking on teaching methods extended well beyond the confines of the elementary schools, and affected various aspects of school and college curricula, including mathematics. Selleck has postulated that the crumbling of moral and religious certainties, and the growing foreign threats to England's industrial and commercial well-being, were important underlying factors in curriculum change, which, in particular, resulted in



a growing emphasis on science, technology and education for advancement within these domains [pp.78-101]. As Jenkins [1979, pp.29,34-37] has also emphasized, the burgeoning of science and technology brought about a tension between the older classical tradition and the more modern tendencies in secondary curricula, the development of separate classical and modern sides in public schools being one administrative consequence.<sup>1</sup>

The development of science teaching brought with it a new emphasis on heuristic methods, and there are implications for mathematics teaching, which will be considered later in this chapter [pp.41-57]. Within mathematics also, concern shifted towards its applications and its relationships with science and other subjects.<sup>2</sup> One consequence was the tendency to broaden the mathematical content of curricula.<sup>3</sup> Selleck [1968, pp.227-272] has also explored the development of Herbartianism and its pedagogical implications, which included a strong emphasis on fundamental ideas and thought processes, resulting in a more unified view of subject-matter.<sup>4</sup> Furthermore, the period from around 1890 and up to the First World War also saw growing refinement in educational thought, increasing pedagogical awareness from teachers, and the beginnings of educational research. In particular, the dominant nineteenth-century doctrine of mental and moral discipline was eroded, and the discussion of purposes became broader and more refined, with greater concern for individual differences.<sup>5</sup>

Regarding teaching methods, Selleck [1968, pp.102-151] has drawn

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- 1 On the influence of the examinations of the Civil Service Commission here see pp.57-58, and, in relation to mathematics, see the various remarks quoted in Chapter 3, pp.101-102,123.
  - 2 Correlation of studies generally will be discussed in the next chapter, though the major links with science and engineering will be explored in this chapter.
  - 3 Such broadening will be explored in Chapter 7.
  - 4 Herbart's influence in mathematics will be discussed in the next chapter, and unification in Chapter 7.
  - 5 These features of the 'new education' will be considered in the next chapter.

attention to the 'practical educationists,' who campaigned for science and other practical subjects, as well as for more enlightened teaching methods. They shared common ground with the pioneers in infant education, who argued for and developed activity methods, which exploited children's interests, and necessitated a fundamental shift in the roles of the teacher and the learner. The implications of these practical tendencies for mathematics teaching will be considered first.

### Practical Methods in Education

Some of the earliest enlightenment in general teaching methods in this country concerned the education of younger children. Children under the age of six were not directly affected by the rigours of payment for results, though the effects of the severe requirements for Standard I were also felt at the infant stage up to the 1890s.<sup>6</sup> No attempt will be made here to consider in any detail the writings and general influence of pioneers in infant education such as Pestalozzi (1746-1826), Froebel (1782-1852) and, later, Montessori (1870-1952).<sup>7</sup> The early influence of Pestalozzi's thought on arithmetic teaching in America during the first half of the nineteenth century, when efforts were made to break away from the traditional mechanical cyphering methods, has been acknowledged by the NCTM [1970b, pp.21-25, 103-106; 1970a, pp.14-23].<sup>8</sup> In this earlier period, Pestalozzi's influence also spread to this country, and his inspiration on the teaching of arithmetic to infants, up to 1890, has been considered in a thesis by

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6 BE [1933, pp.18-19, 28]. Even in the early twentieth century, progressive tendencies in infant education, particularly number work, were constrained by the effects of Scheme B at Standard I, which was commonly used for examinations at the stage of transfer from the infant to the senior department [Ballard, 1912, pp.3-4]. On Scheme B see pp.20-21.

7 For a useful sketch of the development of infant education in England and Wales from the beginning of the nineteenth century see BE [1933, pp.1-46], and for a discussion of the growth of the 'naturalist' movement in the period 1870-1914, with an analysis of its early roots, see Selleck [1968, pp.179-226].

8 Pestalozzi's methods in arithmetic still warranted a discussion in Smith's [1900, pp.78-87] general methods book.



Owen [1959, pp.66-78].

Pestalozzi adopted a 'child-centred' view of education which emphasized the importance of individual differences and the systematic development of clear ideas through the training of the senses, including some actual use of various simple objects by the child. Pestalozzi opposed the 'tyranny of words' [Hudson, 1914, p.4], and stressed the value of mental exercises. The child's motivation was regarded as an important consideration for the teacher, whose aim should also be to nurture and develop natural interests and curiosity [Selleck, 1968, p.188; NCTM, 1970b, pp.25, 104-105]. Not surprisingly, the message often became garbled in practice, notably in the case of 'object lessons,' which were a common form of rudimentary science teaching during the nineteenth century, though normally they merely involved a verbal performance in the presence of objects, or pictures of objects [Gordon and Lawton, 1978, pp.134-136]. Many of Froebel's ideas were derived from Pestalozzi [BE, 1933, p.24], in particular the emphasis on the natural growth of the child from within, as opposed to the forming of the child from without, though the character of their writings and detailed prescriptions differed.

Froebel's general philosophical framework need not be considered here, though his principle of 'self-activity' and the tactics of the Kindergarten have important links with the subsequent development of practical work not only in infant education, but also in technical education, and eventually in elementary and secondary education too. As Selleck [1968, p.203] has pointed out:

It was not in the Kindergarten only that the influence of Pestalozzi and Froebel was felt. Those alert opportunists, the practical educationists, were swift to invoke their support and to adapt their teachings. Manual trainers ... testified to their influence.

Their common slogan was 'learning by doing,' which could be applied to all kinds of curricula, and to mathematics in particular. It was

not Froebel's methods involving play, songs and games, and the ritual of the 'gifts,' but rather the 'varied occupations' such as paper folding and cutting, manipulating objects, drawing, measuring and modelling which had obvious potential for more general adoption in the teaching of arithmetic, mensuration and geometry [Garlick, 1898, p.86; Selleck, 1968, pp.194-196].<sup>9</sup>

The influence of Froebel in England developed considerably after 1870, in particular through the efforts of the Froebel Society from 1874 and the enterprise of particular School Boards. The establishment of Kindergartens, training facilities and a system of examinations for women teachers of infants were all contributory features of the movement [BE, 1933, p.25]. The Education Department also became involved in the developments from around 1880, and the rigid mechanical implementation of the methods of Froebel, notably the 'gifts' and the 'occupations,' was an official cause for concern, expressed through the Codes, Instructions and Circulars to HMIs [pp.26-28].<sup>10</sup> As in the case of 'object lessons,' teachers' actual practices were often far removed from the ideals upon which they were supposedly based. In the early twentieth century the 'naturalist' movement gained further strength, and began loosely to embrace other educational thinkers such as Mary Boole, wife of the mathematician George Boole, the American John Dewey, Maria Montessori and others, all with varying shades of opinion, but some common ideals concerning the child

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9 In particular, making patterns on squared paper was one activity for infants towards the end of the nineteenth century, which Elizabeth Williams has recently recollected. She is the subject of one of Howson's biographical studies (see Chapter 1, note 12). I am grateful to Dr. Howson for providing this example. The general use of squared paper will be discussed later in this chapter.

10 Circular 322 was particularly influential, and was the basis for the section on infant education in the Suggestions of 1905 (see p.20).



## ILLUSTRATIVE SYLLABUS IN ARITHMETIC.

### INFANT DEPARTMENT.

Time given per day, 30 minutes.

Time given per week, 2½ hours.

#### GRADE I. (3-4½ years).

No number teaching except incidentally in connection with games, toys, &c.

#### GRADE II. (4½-5½ years).

Elementary idea of number up to 6. Taught by games, concretes, drawing, &c. Number pictures to 6 arranged in bricks, cubes, counters, shells, beans, beads, &c. Number pictures drawn in dominoes, squares, circles.

#### GRADE III (5½-6½ years).

Comparison of lines, surfaces, solids. Drawing of lines equal to, longer or shorter than line drawn by teacher.

Measurement using inch and foot. Exercises to teach inch and half-inch marks on ruler. To estimate these lengths. 6 in. rulers marked by children (paper) (1 in. square tablets used). Draw, mark, and cut from 1 in. squared paper, squares and oblongs of 1, 2, 3, 4, 5, 6 in. side.

Foot, half-foot.—Exercises to teach, 1 foot = 12 inches; half-foot = 6 inches; pound and half-pound; sticks of 1, 2, 3, 4, and 6 inches used to construct foot lengths. Tested by 12 in. ruler.

Pint and half-pint.—Exercises with actual measures and water.

Half and quarter.—Taught by paper folding and cutting.

Half.—Square folded into oblongs and also into triangles.

Quarter.—Square folded into squares and also into triangles; paper pies, apples, &c., cut into halves and quarters.

Number.—To count to and from 10 by ones, twos, &c. Analysis of numbers 1 to 10 into parts and into factors. Keeping shop. No formal addition or subtraction. Very simple exercises. Concrete examples only. To make number dominoes and pictures.

Apparatus.—Strips of cardboard cut into lengths of 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 inches, one set kept by each child. Bone counters, square tablets, beads, sticks, Gift III., Gift IV., beans, shells, cardboard coins, measures, squared paper (1 in. squares). Four-inch paper squares.

Illustration 26 Practical Methods in Infant  
Mathematics [Spencer, 1912, pp.37-38]

[pp.39-41; Selleck, 1968, pp.201-220].<sup>11</sup> An illustrative scheme for infant mathematics, published in the Special Reports before the War, well demonstrates the extent to which practical methods might have inspired the teaching at this early stage, though this syllabus should not be regarded as typical.<sup>12</sup> (See Illustration 26.) The BE's [1912c, p.12] revised Suggestions recommended varied practical activities so that 'every child shall not only see, but handle, estimate, measure, and construct for himself.' Clearly, there were many exciting new possibilities emerging at the infant stage, particularly following the end of payment for results. However, as regards the development of mathematical education generally, it is the thinking and influence of the 'practical educationists' which warrants closer scrutiny.

The common interest in activity methods of learning shared by the 'practical educationists' and the infant education movement has already been mentioned (see p.185). However, as Wormell [1897, pp.82-83] pointed out:

although the educational methods which are associated with the name of Froebel have been brought very near to perfection in the Kindergarten, they are to a great extent suspended when the pupil passes from the Infant School. They reappear, however, in the schools and colleges devoted to Technical and Experimental Science.... It is very desirable that we should bridge over the gap....

Wormell referred in the latter case to the use of experimental and graphical methods, which he justified in terms of 'hand and eye' training, heurism and motivation:

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11 For a selection of Mary Boole's writings see Association of Teachers of Mathematics [1972]. Eggar [1903b, p.144], in an address to the London Conference of Science Teachers, recommended her little book, Cultivation of the Mathematical Imagination, for inspiration concerning early work in experimental geometry. Dewey's thinking, with its emphasis on active enquiry, inspired Findlay in particular, and for the American's general influence on official publications in England see Burston [1961].

12 For a discussion of principles guiding infant number work see also Raymont [1910], Vice-Principal of Goldsmith's College, in an address on this subject to a Conference of the LCC. In practice, progressive tendencies were constrained by class sizes of fifty and more, lack of resources, and the influence of the senior departments' examinations (see p.185, note 6).



The charm of these methods will always be, as in the Kindergarten, inherent in their nature. They give something to be done by the hand and followed by the eye, keeping pace with the course of thought and reasoning ... the pupil ... is constantly on a voyage of discovery, and has all the pleasure and stimulus of an original investigation. [p.83]

Perry [1896, p.51] also referred to the ideals of the Kindergarten and admitted 'I feel so strongly about the necessity for experimental or kindergarten methods of education being adopted.' Five years later he also expressed a wish that 'a good teacher will educate the hands and eyes of his pupils, so that they may become expert in guessing the weights of objects, and small and large distances' [Perry, 1901a, p.27].

Both Wormell and Perry were 'practical educationists,' in the sense of Selleck [1968, pp.102-151], who has considered the ideals of this group, and their work in relation to elementary school curricula. They shared a common concern for the maintenance of Britain's industrial supremacy and actively sought to prevent its decline.<sup>13</sup> Their strategy included the promotion of the newer subjects of drawing, manual training, science and nature study, as well as the development of more practical teaching methods across the curriculum, including mathematics. Thus, this group sought to narrow the gap between the activities of the Kindergarten and the experimental methods which were developing within technical education.

With the stimulus of industrial need, as well as the support of general educational arguments, the campaign for the spread of both manual training and drawing made considerable headway in elementary schools over the last two decades of the nineteenth century [Millis, 1925, pp.89-92; BE, 1913c, pp.136-137]. However, these developments only affected the higher Standards, and the champions of manual training, such as Magnus and Ballard, sought to extend the benefits of practical work to the lower Standards by providing systematically for such exercises as paper folding, tearing and laying, bead laying, string measurement and division, clay modelling, colour sorting and drawing

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13 This commonly used justification for the promotion of subjects like science is referred to as 'declinism' by Brock [1980, p.172].

on squared paper [Selleck, 1968, p.111]. The Sloyd movement for educational forms of handwork, from the 1880s, also contributed to the dissemination of practical activities to serve educational rather than industrial purposes [pp.113-115].<sup>14</sup> In the case of mathematical education, what is important is that activities such as paper folding, cutting, measuring, drawing, and using squared paper had the potential for adoption as part of a general teaching strategy, and not just as another subject on the timetable. Furthermore, the arguments used to justify drawing with ruler and compasses, and manual training in paper, card, wood and metal could also be used to justify new tactics involving practical activities in the mathematical classroom.

The 'practical educationists' used the generally well-established faculty psychology to support practical work as a training for such powers as observation and accuracy, but they also used a particular argument which claimed that co-ordinated activities involving the eye and the hand i.e. hand and eye training, somehow also developed the mind [Selleck, 1968, pp.116-118].<sup>15</sup> Significantly, Godfrey [1908, p.253] later referred to this argument in relation to laboratory work in mathematics, which 'satisfies the need for co-ordination between brain, eye and hand, which many teachers believe to be inherent in the nature of the British boy.' The 'practical educationists' also sharply contrasted their practical emphases with the excessively bookish tendencies in education:

So long as school life above the infant school is a somewhat dreary routine of mere book learning, with not much relation to after life, and little training of the hand and the eye, so long many of our children will leave school with a great repugnance to school teaching....<sup>16</sup>

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14 The Sloyd movement originated in Sweden. For a useful historical and comparative sketch of the development of constructional handwork in various forms see BE [1913c, pp.132-140].

15 Faculty psychology will be considered more fully in Chapter 6.

16 Quoted from a leaflet of the National Association for the Promotion of Technical Education in Millis [1925, p.92].



In relation to mathematics specifically, it is significant that, by 1902, Findlay [1902, p.185] was able to single out the following as three major pedagogical principles at a time of 'impending reforms:'

[1] Mathematical knowledge, like all other knowledge, has its foundation in the senses; abstract thought must be based on concrete experience, and in the immature mind must constantly revert to concrete experience as an aid to abstraction....

[2] Self-activity. - By 'using' squared paper, by measuring, by plotting curves, etc....

[3] Intuition. - Many mathematical truths can be apprehended and rationally employed long before they can be reduced to formal expression in a system of philosophic thought.

Findlay traced the first principle back to Pestalozzi's emphasis on Anschauung (sense-observation), and referred to the second as part of 'the stock-in-trade at this moment of every lecturer on education' [p.185]. These two principles also came to be loosely linked with the third. As he pointed out, what was required at this time was 'to determine ways and means by which these saving principles [as well as correlation] can be applied to school mathematics' [p.185]. It is not surprising that when a Committee of the BAAS was appointed in 1903, to consider elementary education, and with a membership including Magnus, Perry and Armstrong, that the terms of reference focused on 'Experimental, Observational, and Practical Studies' [BAAS, 1907, p.438]. The Committee's Report three years later worked through in considerable prescriptive detail the various principles referred to by Findlay, with reference to arithmetic, mensuration and practical geometry.<sup>17</sup>

Early twentieth-century practical trends were captured by Branford [1908, p.viii], now an LCC Inspector, who observed:

Attention is now beginning to be paid in rapidly increasing measure to mathematics on its experimental and graphical side, and is exemplified by the use of drawing-boards, improved mathematics instruments, squared paper and spherical blackboards.

Branford went on to mention one further and more dramatic sign of a changed outlook, namely the provision of mathematical laboratories 'well stocked with clay, cardboard, wire, wooden, metal, and other

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17 For further detail concerning this Report see pp.127-128.

models and material, and apparatus for the investigation of form, mensuration and movement' [p.viii].

By 1910, the international interest in practical methods is demonstrated by the ICTM's choice of 'Anschauung und Experiment' in secondary mathematics as a first major focus for international comparisons. This was translated as 'L'intuition et l'expérience' by the French, and as 'intuition and experiment' by the English and Americans [Smith, 1913, p.611]. Such methods were taken to include the graphical study of statistics, functions, vectors and statics; the use of slide rules, mathematical tables and squared paper; contracted methods of computation; geometrical drawing; practical mensuration and surveying; and practical astronomy. Godfrey [1912b, p.437] reported on a survey of English secondary schools for the ICTM and referred to 'intuition and experiment' as 'the keynote of modern mathematical education.'

In summary, he found:

The use of graphical methods in elementary algebra teaching is universal and entirely a 20th-century development. Other aspects of the same movement are the adoption of descriptive geometry by the mathematicians, the use of handy 4-figure tables, and of graphical methods in statics, and, though, in these cases, the victory is less complete than that of the 'graph' it is remarkable and equally modern. [p.437]

Smith [1913] reported on corresponding developments in Austria, France, Germany, Switzerland and America.<sup>18</sup>

One aspect of the movement towards more practical methods was the advocacy of experimental methods which, as Wormell [1897, p.83] pointed out, might, though not necessarily, involve the learner in actually finding out for himself through his own experimentation. Arguably, this process would be more motivating than watching a demonstration or being told some result, and, possibly, being instructed to test it. Such methods will now be considered.

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<sup>18</sup> These developments will be considered in more detail later in this chapter.



*Remarks by Mr. Square.*

'I have been looking through a number of very neat exercise-books in which an account is given of my humble self. I had no idea that so much could be made of me. The accounts are almost as exciting to read as "Alice in Wonderland" and make me wish to ask a great many questions.

'1. First, I should like to know what the term "Practical Geometry" on the book-covers means. I do not like words that I do not understand and shall be content to wait some time before I attempt to understand this term. It seems to me that you are studying *shapes* and *sizes* and that you are finding out all you can about me and my relations, so that you may write a story about us. I shall be content if you call your story "Mr. Square and his Relations, or Shapes and Sizes."

'2. The story-books I read are divided into chapters. Will you not write your story in story-book style? Look at a story-book and see how it is arranged. Has it not a title-page? And then, on the first page, I see a heading, "Chapter I," below this there are some words to show what the chapter is about.

'3. I suppose the heading to your first chapter will be simply "Mr. Square." Now, I should like you to begin by telling me how you first came to know me and why you are interested in finding out all about me. Remember what the Mock Turtle said to Alice: "No wise fish would go anywhere without a porpoise." I hope you know the passage and understand its meaning. If not, read it at once and read it again and again and never forget it. To attempt to do anything *without a purpose* is very very wrong.

'I suppose your teacher introduced me to you, perhaps by holding up a piece of paper, telling you it was cut in the shape of a *square*, asking you to look at it and measure it and then to find out in what different ways you could fold it. And evidently you were asked to write an account as you went on with the work of all you saw and did and thought, so that you might be able to describe me. If you had told me this at the beginning of your story it would not now be necessary for me to ask for information.

'1. I am very glad to see that you each wish to have me with you, so that you may be able to study my character closely. You will be surprised, I can assure you, as you learn to know me more and more, how much character there is in me and how very useful I am. But I cannot understand from your descriptions how you have cut me out, although I must admit that most of you have cut me out very cleverly. Did you look at me long enough to know me? I fear not. Although I look so simple, I am not easy to make.

'You say I have four sides, each 4 inches long. Following your description, I have cut out a piece of paper with four sides, each 4 inches long, but it has not my shape at all. On looking at it you will say, I think, that my corners are all of the same size and that the corners of the piece I have cut out are not square corners. I see this is so. But how, then, are square corners to be made? What is the difference between square corners and those of the shape I have made? Look at them well and try to tell me.

'5. I notice that some of you say not only that my sides are equal but also that my angles are of the same size. What are angles? Are they corners? Everybody knows this latter word, but angle is not a common word. It would have been kind if you had told me, when using the word for the first time, what it meant and where it came from. I have looked in the dictionary, and find it is from the Latin word for a corner—*angulus*. Why do English people use Latin words? Can you tell me? In future, if you can, when you use new words tell me what they mean and what language they come from.

'6. I notice that some of you not only call my corners angles but that you speak of them as right angles. What does this mean? Are other angles than mine wrong angles? I must object to be called by names I do not understand and which need so much explanation. It will be much better at present for you to call corners such as mine square corners. When you have found out how to draw my corners properly we will talk about other kinds of corners.'

shift of emphasis towards experimental methods, including heuristic methods, in the teaching of mathematics.

Heurism as a method of teaching science is normally associated with Armstrong, 'the champion of the heuristic approach' [Brock, 1975a, p.75], who, from the 1880s, campaigned for the adoption of such methods in this country [Brock, 1973]. Some of the antecedents of heurism in science have been explored by Brock [1971]. Armstrong's [1898] views were publicized by the Education Department in an early volume of the Special Reports. By 1902, Findlay [1902, p.185] could refer to the 'Armstrong method' as one of a number of pedagogical panaceas which, as he amusingly added, some ignorant teachers 'confound with a novel device in corporal punishment!' The growth of heurism in science, and the increasingly critical reactions to the method and its rationale, up to the end of the First World War, have been considered by Brock [1973] and by Jenkins [1979, pp.41-56]. What is important for mathematics teaching is that elements of heurism could be transferred from the science laboratory to the mathematical classroom. A splendid example produced by Armstrong himself for a Committee of the BAAS, which considered practical methods in elementary schools, well illustrates some of the possibilities in relation to early geometry teaching. (See Illustration 27).<sup>20</sup>

An early mathematical methods book by the American Young [1907] included separate chapters on the heuristic method and the 'laboratory method,' the latter being particularly associated with Perry. In the case of heurism, Young [p.69] referred to the writings of the Spencers, Armstrong and Wormell in particular. Regarding English conditions, Wolff [1915, pp.119-121] contrasted as opposites the traditional 'euclidische Methode' and the newer 'praktisch-heuristische Methode,'

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20 In practice, the crux was finding teachers with the ability and confidence to transform their style and teach in this way.



which he associated with Perry and Armstrong. By the First World War, the main concern was to maximize the benefits of both these pedagogical extremes, particularly in geometry teaching.<sup>21</sup> Around the turn of the century, it is significant that some of the early advocates of heurism in mathematics were familiar with the science laboratory as well as the mathematical classroom.

Wormell [1900, p.241] discussed various methods of science teaching, including the destructive Socratic method and the more constructive heuristic method which 'is applicable to other subjects - to art, to geometry, to algebra, etc.' He described in detail one interesting algebraic example of the method. By employing a set of wooden cubes to build up progressively larger squares, and through carefully graded questioning:

the mode of forming ... successive additions was deduced, and the fact that  $1+3+5+\dots+(2n-1)=n^2$  was established, and, it may be said, was discovered by the pupil. [p.241]

In the case of geometry, Wormell [1902, p.211] referred to 'what the Americans call "inventional geometry,"' as a stage between that of the early exploration of geometrical forms, drawing and measuring, and the stage of demonstrative geometry. He provided an illuminating example, based on the construction of an equilateral triangle (Euclid I, Proposition 1):

Having constructed the triangle ABC, construct equilateral triangles on each of the sides, and again on those sides, and so on. We thus learn the following:- (1) to construct a regular hexagon; (2) the hexagon is composed of six equilateral triangles; (3) the whole sheet of paper, that is to say, any plane area, may be completely covered with hexagons and therefore with equilateral triangles; (4) commencing with the side AB, if we construct ABC, then on AC, ACD, then on CD, CDE, and join EA, we see from the symmetry of the figure that we have erected a perpendicular to AB at its extremity; (5) to construct a square; (6) to cover the surface with squares, etc. [p.211]

Findlay [1902, p.186] described his own work with a first year secondary class on a correlated syllabus of practical measurements,

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21 Geometry teaching will be discussed in Chapter 8.

It is required to find  $\frac{2}{3} \times \frac{1}{7}$ .

By CRAM.—Rule: Multiply the two numerators together for the new numerator, and the two denominators for the new denominator,  $\frac{2}{3} \times \frac{1}{7} = \frac{2 \times 1}{3 \times 7} = \frac{2}{21}$ . Answer.

By DEMONSTRATION.— $\frac{2}{3} \times \frac{1}{7} = \frac{2}{3} \times (5 \times \frac{1}{7}) = (\frac{2}{3} \times 5) \times \frac{1}{7} = \frac{10}{3} \times \frac{1}{7}$   
 $= \frac{10}{3} \div 7 = \frac{10}{3 \times 7} = \frac{10}{21} = \frac{2}{21}$ . Q.E.D.

By INVESTIGATION, or by the heuristic method.—Teacher: It is required to multiply  $\frac{2}{3}$  by  $\frac{1}{7}$ ; what does that mean?

Pupils: Take  $\frac{2}{3}$ ,  $\frac{1}{7}$  of a time.

T.  $\frac{1}{7}$  of a time? Has part of a time any meaning? Can you say you have seen the King half a time?

P. (laughing). No; part of a time has no meaning.

T. Never?

P. No, never.

T. Is it sense to talk of a wheel having turned round 3 times?

P. Yes, it has made 3 turns.

T. What if I say, the wheel has turned round half a time, has that an intelligible meaning?

P. Yes, the wheel has made half a turn.

T. May it mean something else as well?

P. (after some hesitation). No; (and some add) certainly not.

T. Then shall we say that the wheel has turned half a time has one meaning, and only one meaning, viz., that it has made half a turn, or half a revolution? Would you say then that part of a time is always nonsense?

P. Not always, but sometimes.

T. Let us now return to our problem:— $\frac{2}{3} \times \frac{1}{7}$ . Suppose a wheel has a circumference of  $\frac{2}{3}$  of 1 yard. How far has it travelled after it has made one revolution?

P.  $\frac{2}{3}$  of 1 yard.

T. If it has made  $\frac{1}{7}$  of one revolution?

P.  $\frac{1}{7}$  of  $\frac{2}{3}$  of 1 yard.

T. Well, what part of 1 yard is that?

P. (No answer.)

Look at this line:



T. Suppose A B to represent a yard, what would A C be?  
 $\frac{1}{2}$  of a yard, a foot.

Let us say  $\frac{1}{4}$  of 1 yard. What would A D represent?  
 $\frac{3}{4}$  of 1 yard



geometry, arithmetic and drawing, where the pupils 'do not know whether they are "doing" Science or Mathematics.' He referred to his approach as the 'developing method,' which he characterized thus:

the pupils have been allowed an abundance of concrete examples on the blackboard, and with paper and cardboard. From these they have made comparisons and arrived at concepts. The teacher then supplies the technical terms, and the results are expressed in appropriate language, and written carefully in a note-book.  
[p.186]

At this time, Nunn [1903] was also implementing a correlated syllabus of mathematics and science at William Ellis Endowed School, before he, in common with Findlay, entered the field of teacher training. Eggar was another innovator in mathematical education, who also taught physics, and was a leading figure in the early geometrical reforms.<sup>22</sup> In the preface to his Practical Exercises in Geometry [1903a, p.v], one of the new experimental geometries, he acknowledged that:

This book is an attempt to adapt the experimental method to the teaching of Geometry in schools. The main object of this method, sometimes called 'heuristic,' is to make the student think for himself, to give him something to do with his hands for which the brain must be called in as a fellow-worker. The plan has been tried with success in the laboratory, and it seems to be equally well-suited to the Mathematical class-room.

As regards the teaching of arithmetic in elementary schools, the heuristic method also featured prominently in a paper by Adolph Sonnenschein [1902] for the BE's Special Reports. Sonnenschein distinguished three methods of teaching - 'cram,' 'demonstration,' and 'investigation' - and provided illustrative examples of the application of these methods. The discussion of the problem  $\frac{2}{3} \times \frac{5}{7}$  was particularly rich, with the heuristic dialogue extending to around four pages, which well illustrates the subtlety and complexity of this method in practice. (See Illustration 28.) No doubt, the vast majority of teachers would have been content to employ 'cram,' or, possibly, 'demonstration.' However, over the early years of this century, elements of heurism certainly filtered through to the teaching of mathematics, particularly

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22 See p.179 on Eggar's activities.

geometry, and mainly through the efforts of textbook writers such as Eggar, some of whom even included terms such as 'heuristic,' 'observational' or 'experimental' in their titles [British Museum, 1906, pp.508-509].<sup>23</sup> Barnard and Child [1903, 1908], for example, were important co-authors of new textbooks for Macmillan, and Child [1904, p.468] justified the new approach, in a lecture to a Welsh Educational Society, as follows:

it would teach the pupil to find out things for himself, i.e., how to learn and how to apply knowledge already acquired to gain further knowledge, or to practical purposes; whilst the desire for knowledge for its own sake is more likely to be fostered by a method such as this than by the old system in which 'interest' played a very subordinate part.

Heurism also became a feature of the general theories of mathematical education of Branford, Godfrey and Nunn, as well as the American Young, already mentioned (see p .194).

Branford [1908, p.156] enunciated as one of his 'subsidiary principles' that 'the pupil should be guided to the DISCOVERY of definitions, theorems or truths, rules of operation, etc., for himself,' and he used the history of mathematics to provide theoretical support for the principle. Godfrey, in a paper written around 1911, suggested that 'A schoolboy should be a kind of discoverer' [Godfrey and Siddons, 1931, p.19]. He also considered the roles of induction, deduction and intuition at different educational stages, and argued for a varying blend of the scientific emphasis on guided discovery through observation and induction with the pure mathematical emphasis on deductive proof [pp.18-25].

Nunn was, like Branford, also inspired by historical considerations and proposed a recurring three-phase cycle as a model for the learning of mathematics. The cycle involves beginning with a problem to stimulate interest - the 'heuristic phase' - followed by a 'formal

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23 One exceptional arithmetic textbook by Granville and Rice [undated] was entitled A Heuristic Arithmetic.



phase' in which the ideas and methods required to solve the problem are elaborated and possibly extended. Finally, in the 'application phase,' the newly developed ideas and methods are applied to fresh problems, thereby initiating new cycles [Price, 1976c, p.31]. Nunn [1914] adopted this model for the writing of his seminal textbook on algebra. Godfrey and Siddons [1912] also adopted the sequence - problem, reinforcement, application - for the writing of the earlier sections of their textbook on algebra.

What should be clear from the discussion thus far is that the dominant early twentieth-century view of heurism in mathematics was one of closely guided discovery by the pupils of some anticipated result by the teacher, through suitable problem posing and a sequence of leading questions. More open situations, where the process of problem solving may be more important than any more tangible product, do not appear to have been considered. However, an exceptional Irish Memorandum of 1923 did open up some wider possibilities, and the following extract, though rather lengthy, does give a good indication of the nature of this more challenging form of heurism:

Mathematical faculty can not be acquired merely by listening nor by reading what others have thought out: it is not even necessary to have assimilated a certain amount of known matter before proceeding to independent exercise of the mind on the subject. There can be no real appreciation of formal solutions of mathematical problems till the pupils themselves have been beaten by the problems: that presupposes individual or collective effort in the solution and an unprejudiced attitude on the part of the teacher. A non-committal attitude on the part of the teacher and free play to every suggestion coming from the pupils are of the utmost importance in the development of mathematical outlook. Suggested methods should not be declared wrong until they have proved themselves to be wrong, and pupils should feel that there is merit in honest effort, apart from error involved.

The development of the subject, the whole nature of the course, should depend more on the success of the efforts of the pupils than on any predetermined programme. A programme may give a broad outline of the scope of activities, but it can give no hard and fast guide to the subject-matter or order of treatment which will be most beneficial to a particular class of pupils.

Mathematical text-books, in general, deal exclusively with the final and often abstract solution of problems, independently of the conditions under which the problem arises, of earlier

tentative efforts which have culminated in the given solution, and of the psychological impulses which have stimulated the spirit of enquiry. To ignore entirely this psychological element is a mistake; the pupil's mind, if it shows activity at all, must repeat more or less closely the attitudes of mind of the original mathematicians in their original quest. [Aireacht an Oideachais, 1923, p.5]

Historical patterns in mathematics were again an important factor behind these recommendations, and the writings of Young, Branford and Nunn, amongst others, were acknowledged. The problem 'How many diagonals have figures of four, five, ten, one hundred sides?' [p.5] was discussed in some detail to illuminate the possibilities.

It is impossible to measure the extent to which heuristic methods of teaching became adopted in mathematical classrooms in the early twentieth century. The more open possibilities, which make the greatest demands on a teacher's mathematical confidence, flexibility, and ability to step down from a position of uncompromising authority, were very far in advance of their time. The more rigid, safer and potentially more ritualistic forms of heurism are likely to have had a better chance of adoption, though, in this case, as with the methods of Froebel and Pestalozzi, it is likely that the ideals of Armstrong and others were often far removed from actual practices, where the interpretations of textbook writers and teachers were the key factors. Nevertheless, the general idea of 'try it and see' had certainly come to stay in mathematical education.

Thus far, the discussion of the links between science and mathematics has been restricted to the general question of teaching and learning styles. However, the issue of correlation between the two subjects also rose to prominence, initially to serve the interests of science, but subsequently for the mutual benefit of both subjects in the curriculum.

### Science and Mathematics Teaching

Before the general development of science teaching, school



mathematics could ignore this subject's distinctive demands, and correlation as an issue was unimportant. However, as a leader in the Educational Times subsequently put it, 'With the rise of experimental science, new conditions appeared' [College of Preceptors, 1912, p.230]. This same writer also drew attention to the following judgement:

Prof. Klein of Göttingen gives 1890 as the date at which physics began to make insistent demands on mathematics, and declares that this date marks the climax of the anti-mathematical feeling. [p.230]

In this country, the important late nineteenth-century cleavage between the classical and modern conceptions of secondary education, the latter giving prominence to science and mathematics as its queen and servant, was mentioned at the beginning of this chapter. The creation of new secondary schools, modern as opposed to classical 'sides, and army classes, in response to shifting parental and vocational demands, as well as the pressurizing of the scientific community, notably the BAAS, and support of the DSA's system of grants, all contributed to the growth of secondary science teaching, together with the provision of laboratories, particularly during the 1890s [Findlay, 1902, p.184; Fletcher, 1912, pp.93-94]. In 1918, the Prime Minister's Committee on Natural Science reported:

For the last twenty years the attention paid to laboratory work has been an outstanding feature of the science teaching in English schools, and results of great value have arisen from it. [Report (Thomson), 1918, p.21]

The elementary schools also became caught up in the new enthusiasm for science during the 1890s [Selleck, 1968, p.123].

The hold of classics within the public schools was, however, still strong around 1900, as Hurst [1901] pointed out in a letter to Nature, as did Minchin [1901, p.227] whilst reviewing Perry's contribution to 'declinism,' England's Neglect of Science. Minchin referred to the continuing dominance of a scientifically ignorant

classical élite over the leadership of the public schools, and remarked 'the "modern side" does not rank high in the estimation of the public school, and science is dignified with the name of "stinks"' [p.227]. It was Bryan [1902, p.90] who around this time highlighted the general implications of this conflict for teachers of mathematics (see p.102).

The cause of public school science was helped forward by the energies of the APSSM from 1900, and, at an early stage, Eggar [1901, p.361] spelt out the pressing need for at least some correlation in 'ordinary' public schools, where:

mathematics and physics are two departments completely separate from each other; and the elementary Practical Physics, which is demanded as the foundation of a boy's scientific training, has to be taught in the physical laboratory and by the science master. This is doubtless right and reasonable: but my contention is that the science master finds himself obliged to teach practically a great many things that could be taught more effectively in the mathematical classroom, and to invert the accepted order of mathematical teaching.

Findlay [1902, p.184] also drew attention to this problem for science teachers who 'find they cannot proceed further in the improvement of science (i.e., of physics) [Findlay's parenthesis] unless mathematics is treated on sounder lines.' He also singled out correlation as one guiding educational principle for mathematical reform, and he referred to this principle as:

the outcome of the desire to rank mathematics in its place as the handmaid of science. Not, be it observed, as a tool for engineers, but in close correlation with the needs of the science syllabus right through the school. [p.185]

Clearly what was needed at this time was a new outlook from teachers of mathematics, to develop some correlation with science and also to rejuvenate the methods of teaching mathematics. However, finding teachers capable of adapting to the new demands was problematic, given the strength of Cambridge's hold on the academic education of



mathematics teachers, and the character of that education.<sup>24</sup> Thus Eggar [1901, pp.361-362] hoped for greater flexibility in the mathematics curriculum to serve the teacher of physics, but added:

many men who have been through the strict course of mathematical training which culminates in the Cambridge Tripos are perhaps inclined to over-estimate its educational value.... Rightly suspicious of anything that seems like trying to shirk difficulties and to go too fast, they object to any interference with the accepted order of subjects....

What seems significant is that individuals such as Wormell, Findlay, Eggar and Nunn, who were notable leaders in mathematical education, were able to blend a mathematical and a scientific outlook, and, at an early stage, were translating the ideal of correlation into some kind of reality. Siddons [1952b, p.3] also later admitted that 'my work for the Natural Science Tripos had changed my attitude to mathematics.' Furthermore, during the critical period for the reform of mathematical examinations discussed in Chapter 3, it was the London Conference of Science Teachers who chose to devote half of its Fifth Annual Conference, in January of 1903, to reformed methods of teaching mathematics, particularly geometry, with addresses given by Eggar [1903b] and by Siddons in particular [A.T.S., 1903].<sup>25</sup>

Some organizational developments concerning correlation between mathematics and science were surveyed in the last chapter (see pp.179-182), where it was shown that the MA did not become actively involved until 1908, by which time the major reforms in the period 1902-1904 had permeated the system. However, individuals had already taken the initiative concerning correlation, and the implications with regard to the teaching of practical measurements and the development of mathematical laboratories will be considered first.

As early as 1869 Sylvester [1870, p.6] expressed to the BAAS his enthusiasm for the possibility of school science teaching, and,

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24 The Cambridge Tripos was discussed in Chapter 2, pp.71-74.

25 On this Conference, see also p.116, note 50.

prophetically, added:

I think that that study and mathematical culture should go hand in hand together, and that they would greatly influence each other for their mutual good. I should rejoice to see mathematics taught with that life and animation which the presence and example of her young and bouyant sister could not fail to impart....

At the time this was wishful thinking, but, in the following year, Perry accepted a post as second mathematical master and lecturer in physics at Clifton College, one of the new proprietary schools which took science teaching seriously. In 1871, at the suggestion of the Headmaster, Dr. Percival, Perry established a physical laboratory and workshop at the College.<sup>26</sup> He left Clifton in 1874, and the extent of his impact on mathematics teaching and its relation with physics at the College is likely to have been limited at this time.<sup>27</sup> However, at the same school, from 1875, A.M. Worthington pioneered a course of practical work in elementary mensuration and hydrostatics which subsequently became generally adopted as a first course in practical physics, called 'elementary physical measurements,' which was widely disseminated through the DSA [Fawdry, 1912, p.400; Jenkins, 1979, pp.30-33]. Armstrong also campaigned for a simple laboratory approach to mensuration, on the lines of Worthington, and it became a commonplace in elementary schools [Eggar, 1901, p.361; Jenkins, 1979, pp.41-42]. Eggar [1901, p.361] commented:

In primary schools the new teaching is altogether beneficial. A little cheap apparatus is bought or made, and the teacher uses it to supplement and explain lessons in arithmetic.

Ideally, Eggar regarded physical measurement as 'an extension and application of mathematics,' which should not be taught by a different teacher as a separate subject, as had occurred in the case

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26 Turner [1926, p.i]. Perry [1900a, pp.73,94] claimed that the workshop and laboratory at Clifton were the first established at any school, though he later acknowledged an earlier initiative at Rossall.

27 For some discussion of the difficult question of what Perry actually achieved at Clifton see Brock and Price [1980, pp.373-374].



of many public schools [p.361]. For example, at Harrow, Ashford, who later moved to Dartmouth, introduced a course of physical measurements in 1896, but it was un-correlated with the mathematics teaching, and restricted to the science laboratory. However, in ten years the situation had been transformed, and the developments were subsequently summarized by Siddons [1912, p.404]:

Some six or seven years later [1902/1903], at the time when great changes were being made in the teaching of Mathematics, some drawing and measuring was begun in the ordinary mathematical lessons, but this was quite independent of the work ... which was still being done in the science schools. About 1906, the course of measurements and weighing previously done in the science schools by the science masters was begun in the mathematical laboratories under the supervision of the mathematical masters.

Thus, in addition to the practical tendencies in geometry, it was also the case with arithmetic and mensuration that teachers of mathematics gradually came to realize the possible benefits of a more practical and experimental approach, which their scientific colleagues were already implementing in the laboratory. With simple aids like drawing instruments, scissors, tracing paper, squared paper, string, sets of cubes, and solid models, it was possible to undertake in the mathematical classroom a good deal of the work in simple practical measurements, though weight and liquid capacity required more elaborate apparatus. Moreover, such practical work could be correlated with the mathematical teaching of fractions, decimals, length, area and volume, as well as the use of formulae and graphs [Eggar, 1901, pp.362-363]. The birth of mathematical laboratories is a further and more dramatic sign of the changed outlook at some schools, where certain enthusiastic mathematics teachers were prepared to take on more ambitious forms of experimental work.

Particularly in the case of public schools, it was not only the interest of individual mathematics teachers in exploring further the links with science which serves to explain the rather surprising

creation of school mathematical laboratories. An important stimulus was provided by the new requirements for army entrance which were announced in 1904. For the Leaving Certificate in Elementary Mathematics, practical measurement of length, area, volume, weight and angle was prescribed, as well as the Principle of Archimedes, with further practical work in hydrostatics and mechanics required for the Competitive Examinations to enter Woolwich and Sandhurst [Lodge, 1904, pp.452-453]. As Siddons [1912, p.407] later admitted:

About 1904, owing to changes in the Army entrance examination, it became necessary to teach practical mechanics to boys in the Army Class. It was then that the mathematical laboratories were equipped and some mechanics was first taught from the experimental point of view.

At Harrow, two such laboratories were established, one for measurements and the other for mechanics [p.403].

At Winchester, Godfrey and his department also developed an experimental course, to be pursued in a mathematical laboratory, following the lines of Ashford's Note-Book of Practical Physics, compiled for use at Harrow [Boyt, 1906, p.297]. Godfrey and Bell [1905a] subsequently produced A Note-Book of Experimental Mathematics, and it included 124 experiments, with the requirements for army entrance particularly in mind. In the following year, Godfrey [1906c] also wrote an interesting article for the School World on the teaching of 'experimental arithmetic,' which was his name for what was essentially a course following the lines of Worthington. As Godfrey [p.202] pointed out:

A good deal of the work can be done in an ordinary class-room. But water and a balance cannot be treated respectfully outside a room fitted more or less specially for the purpose.

Godfrey [1906c, p.202] also made some very revealing remarks concerning the new questions of organization and atmosphere for learning which were raised for mathematics teachers by laboratory work:



A bottle-washer (5s. a week) will be needed, unless the boys are allowed to help themselves to apparatus from the cupboards. But, to begin with, it is hardly safe to allow this. Difficulties of discipline do not occur; the interest is too great.

The master will have to decide whether the class is to be kept together, or whether each boy is to go at his own pace. The latter plan will get more work out of the class.... On the 'Go as you please' system, each boy must be provided with very definite printed instructions; otherwise the master's task will be an impossibility. However, there is no lack of books which cater for this demand.

Boys can work in pairs, and talking may be allowed safely (it must be confined to the matter in hand). At the outset there will doubtless be cases of petty looting (especially of mercury). A suitable 'example' will generally put a stop to this; temporary banishment from the measurement room is a severe punishment.  
[p.202]

Bell [1912] subsequently described in the *Special Reports* Winchester's mathematical laboratory and the department's syllabus, and these Reports also included detail of the laboratory work in so-called practical mathematics at other public schools.<sup>28</sup> Valuable detail concerning the general progress of work in practical measurements in the early twentieth century is provided by the findings of Committees of the BAAS, and the MA and APSSM, acting jointly.

In 1908, Committees of the BAAS [1909] reported on science in elementary schools [pp.501-525] and in secondary schools [pp.526-535].<sup>29</sup> The Committee on secondary science considered correlation with mathematics, and undertook a relatively small survey of practices, receiving replies from twenty-five grant-aided boys' secondary schools and twenty-two public schools.<sup>30</sup> By this time, the course of 'elementary physical measurements' was 'practically universal,' and was 'now contained in the earlier parts of most school-laboratory manuals of physics' [p.529]. However, it was still often divorced from the mathematics teaching, though science teachers frequently desired correlation here, and the Committee optimistically recommended that:

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28 See Fawdry [1912] of Clifton, Siddons [1912] of Harrow, and Sanderson [1912] of Oundle.

29 Eggar was a member of the Committee on secondary science.

30 The tabular summaries of the Committee's findings are reproduced in Jenkins [1979, pp.29-30].

## SYLLABUS FOR A SHORT COURSE IN PRACTICAL MEASUREMENTS.

1. Measurement of straight lines by means of scale, including the estimation of tenths of the smallest scale division.
2. Measurement of the diameter of cylinder and sphere by placing between two rectangular blocks of wood and measuring the distance of these apart; also by means of calipers (without vernier).
3. Measurement of curved lines, and determination of the value of  $\pi$  (see Appendix F).
- 4.\* Examination of the micrometer screw-gauge, and application of its use in the measurement of diameter of wire, etc.
- 5.\* Examination of the vernier, and application of the instrument to slide calipers; use of the latter, especially for finding the diameter of small spheres, etc.
- 6.\* Examination and use of the spherometer.
7. Measurement of the areas of rectangles and triangles by means of squared paper, and determination of the formulae to express these areas.
8. Measurement of areas of plane surfaces of irregular outline by means of squared paper.
9. Area of circles by means of squared paper, and confirmation of formula.
10. Determination of the formula for volumes of rectangular solids, special stress being laid upon the fact that this is the product of the area of cross section and the length.
11. Practice in reading convex and concave menisci.
12. Measurement of the volumes of dense insoluble solids by displacement of water in graduated cylinders or burettes.
13. Determination of the formulae for the volumes of non-rectangular solids by methods 10 and 12.
- 14.\* Measurement of inaccessible heights and distances, and finding the areas by triangulation, etc., the results being determined by drawing to scale.
15. Examination and use of balance and weights.
16. Determination of areas by cutting out in cardboard and weighing.
17. Determination of the weight of unit volume of various substances.†
18. Determination of the weight of 1 c.c. of water, and elementary ideas of the meaning of specific gravity.
19. The specific gravity bottle, and its use in finding the specific gravity of liquids.
20. The specific gravity of solids by means of the specific gravity bottle.
21. Proof of Archimedes' principle by "bucket and cylinder experiment."
22. " " " " method of displacement.
23. Experiments on flotation, e.g. floating a weighted rod upright in water, weighing the rod, measuring the volume immersed, and hence finding the weight of water displaced.
24. The principle of the common hydrometer, approached by repeating experiment 23 with alcohol instead of water.
25. Specific gravity of dense solids by Archimedes' principle.
26. " " liquids
27. " " solids less dense than water by Archimedes' principle.
28. Specific gravity of solids soluble in water by Archimedes' principle.
29. Experiments with liquids in U-tubes, and in communicating vessels.
30. Determination of the relative density of two liquids which do not mix, using U-tubes.
31. Density of liquids which do mix, using U-tubes and mercury, or Y-tubes.
32. Density of liquids by Hare's apparatus.
33. Determination of the diameter of tubes by partly filling with mercury and weighing.
34. Boyle's law, etc.

Illustration 29 Scheme of Practical Measurements  
from the Joint Committee of the APSSM  
and the MA [1909e, pp.15-16]



much of the work which has been done in the physical laboratory can advantageously be transferred to the mathematical classes. Mensuration, including the greater part of the work frequently described as elementary physical measurements, should be part of the mathematical teaching. [p.535]

In the following year, the Joint Committee of the MA and APSSM also reported on this matter.

The Joint Committee conducted a much larger survey of boys' schools than the Committee of the BAAS.<sup>31</sup> An introductory course of measurements, from around the age of eleven in grammar schools and two years later in public schools, was now the 'universal custom' as a preliminary to more formal physics teaching [MA, 1909e, p.10]. Nearly ninety per cent of grammar schools, but only fifty-five per cent of public schools, taught such a course to every boy, though in nearly all schools over half the boys were involved. Significantly, the work was still normally done in a laboratory under a science master, though measurements not requiring the use of balance or water were taught by the mathematical masters in eleven per cent of the public and fifteen per cent of the grammar schools. Predictably, the Joint Committee made the same general recommendation as the Committee of the BAAS, in this case concerning measurements up to, but not necessarily including, Archimedes' Principle, and they provided a detailed syllabus. (See Illustration 29.)<sup>32</sup> However, there is no evidence to suggest that many mathematical masters subsequently took on such ambitious experimental work involving water or balances, though some practical work in length and area had found a place in the mathematical classroom. At the Strand School, the science masters who took the measurements also took the mathematics, and the School's

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31 See p.181 on this survey's scope. The findings in practical physics and chemistry were in 'marked disagreement' with those of the BAAS, and are, presumably, more reliable, given the different sample sizes [MA, 1909e, p.11].

32 Siddons and Vassall [1910], who closely followed this syllabus, acknowledged the influence of the army requirements, as well as a debt to Ashford for many of the ideas.

Senior Science Master claimed that in some other schools correlation had to some degree been achieved in this way, particularly with younger pupils [Hewitt, 1908, p.298]. By contrast, and perhaps surprisingly, Story [1912, p.553] referred to the practice in girls' school of 'in many cases ... putting Elementary Physics teaching of the first year into the hands of the mathematical mistress of the form.'<sup>33</sup>

By 1912, there is evidence that the course of 'elementary physical measurements' was becoming stereotyped. Sanderson [1912, p.415] of Oundle remarked:

This method has now been in working order in schools for 25 years or more, and experience suggests that without some elasticity and freedom from syllabuses it will become as great a slave to 'logic-chopping' as anything ever was or can be in a mathematical class-room.<sup>34</sup>

Also, it seems that the existence of some mathematical laboratories was largely sustained by the army requirements, and that the level of interest did not continue after the First World War.

Fawdry [1915, p.36] of Clifton admitted:

it is in great measure due to the demands of these [army] examinations that the attention of teachers has been drawn to the advisability of including practical work in the Mathematical Course.

Fawdry [1915, 1923, 1924] persistently argued a case for laboratory work in mathematics as he claimed it simulated genuinely applied mathematics, and he also produced his own collection of Elementary Experiments in Practical Mathematics [1922]. An early M.Ed.thesis by Pickles [1926] was devoted wholly to the subject of mathematical laboratories, but Pickles admitted that his proposals were idealistic, particularly given the pressure of examinations on the time available [p.14].

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33 Story [1912, p.545] had received data from two-thirds of 'the chief secondary schools for girls in England,' and there was evidently much sensitivity to the issue of correlation from the mistresses [pp.552-553].

34 W.J. Dobbs [1910], during just one year at Oundle, produced his own textbook for practical measurements, and included the army papers for 1905-1909 [pp.123-133].



Such initiatives appear to have been isolated ones, and a fairly gloomy picture concerning correlation was painted by the Prime Minister's Committee in 1918:

it is easier to get a science master who is able to teach elementary mathematics than a mathematical master with a corresponding knowledge of Science; but few schools can spare a science master for any part of the mathematical work ... effective correlation between the two subjects is rare and is not increasing.... [Report (Thomson), 1918, p.22]

It is also significant that the mathematical laboratories at Harrow subsequently fell into disuse, and Mayo [1928], who was Siddons' Head of Department, welcomed the ending of 'elementary and pottering experimentation' [p.134], which was 'pandering to the spirit of mere utility' [p.137]. Mayo felt that the laboratories had been forced upon the school as part of an extreme swing towards utility in mathematical education, and he welcomed the return to a more strictly disciplinary view.<sup>35</sup> However, there were certainly some more general gains for the teaching of mathematics related to the teaching of science.

Godfrey [1906c] had referred to 'experimental arithmetic,' envisaging a fairly ambitious course requiring a special room. However, the idea of a practical course in arithmetic correlated with other subjects had already been explored by other teachers. Findlay [1902, p.186] had developed a first-year course within the arrangements of the Directory of the BE, inherited from the DSA. Findlay explained:

it is described as Elementary Physical Measurements in the Science Section, and First Notions of Geometry in the Mathematics Section. These two are interlaced into one course of study, associated also with Arithmetic and Drawing.  
[Findlay's stress]

Subsequently, Consterdine and Andrew [1905] produced a textbook on Practical Arithmetic for the nine to twelve age range, which they

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35 Mayo's was part of a wider reaction to the practical tendencies, to be discussed later in this chapter.

claimed was a correlated treatment of measurement, drawing, arithmetic, algebra and geometry on 'heuristic lines' [p.6]. The name 'practical arithmetic' appears to have stuck, to refer to such a course for younger pupils. In many girls' schools, Story [1912, p.550] referred to the existence of:

a practical arithmetic course, including mensuration, with an introduction to the metric system, paper-folding, cardboard modelling, and work with plasticine.

She added that:

Several text-books have now been written on courses of this kind, and various educational publishers issue suitable sets of apparatus. [p.550]

Findlay [1913, pp.204-205] interestingly described this development in terms of a shift from commercial to scientific utility:

so-called practical arithmetic has to some extent, and quite properly, dislodged from pride of place the older syllabus which found in money affairs the only applications for the notions of arithmetic.

Regarding elementary schools, the BE's [1912c, p.7] revised Suggestions referred to 'concrete methods' as having been recently 'generally recognised,' and Ballard [1912, p.12] boldly claimed:

Far and away the most significant change that has taken place during the past few years has been the introduction of what is known in the schools as practical arithmetic.

He went on to discuss some shortcomings in practice, the 'gravest defect' being its separation from theoretical and mechanical arithmetic. Other defects were the tendency to substitute demonstration for individual experimentation, and to elevate the importance of measurements per se [pp.13-14]. As Spencer [1912, p.47] remarked, 'It has been well said that measurements may become "liturgical" only and be in no sense scientific.' Findlay [1913, pp.203-204] also referred to the relationship between 'practical arithmetic' and the main business of arithmetic teaching. His analysis warrants quoting in full:



OUTLINE OF SYLLABUS.	KIND OF PRACTICAL WORK INVOLVED.	CORRELATIONS.
<p>CLASS V.</p> <p><i>Vulgar Fractions</i> (introduced through scale drawing).—(a) Equivalent Fractions. (b) Addition and Subtraction. (L.C.M. introduced.) (c) Multiplication and Division. Interchange of Vulgar and Decimal Fractions. (The term percentage will be introduced as a name for fractions which have 100 for denominator; <math>\frac{1}{100}</math> of anything is called 3 per cent.) Simple percentage problems (e.g., to find "percentage of," etc.). Problems involving fractions.</p> <p>Decimalisation of money and reverse process. Multiplication and Division of Decimals. (Cf. areas in practical work.)</p> <p><i>Applications of the Four Rules.</i>—More difficult examples in Bills and Practice methods; some examples better worked by decimalising (involving simple approximation). Reduction, i.e., changing.</p>	<p>The practical work leading to the formulation of the rules for the operation of fractions, commencing with the known divisions of the foot, metre, pound, etc. This will include division of straight lines and areas (rectangle, circle). To find what fraction the radius of a circle is of the circumference and generalisation in a formula.</p> <p><i>Angles</i> (probably arising from the study of triangular figures).—Arms of angle; copying angles; standard angle. Simple protractor made; use of ordinary protractor. Measurement of height by use of angle; construction of various triangles; triangles made like other triangles; similar triangles.</p> <p>Area of square, rectangle, parallelogram (construction of parallel lines). Formulae for areas. (Note use of <math>a^2</math>, cf. use of <math>2^3, 3^4</math>, etc., in factors.) Table of Square Measure, including 40 poles=1 rood, 4 roods=1 acre.) Graphical work. First example (of pictorial kind) from science. Graphical representation extended to problems on man walking, stone falling, etc. Term "varies" used and "variation with." Graph of variation of area of square with increase in the side.</p>	<p><i>Craft-work.</i> Scale drawing and work involving fractions during the construction of the globe.</p> <p><i>Physiography.</i> Cardinal points. Mariner's Compass. <i>Craft.</i> — Construction of simple theodolite for this purpose.</p> <p><i>Science.</i>—River water: Graph of variation of impurity with increase of distance from source.</p>

Illustration 30 The Fielden School's Mathematical Scheme for Ages 11-12 [Findlay, 1913, p.212]

It is now the custom in the Junior Classes of many schools to give three or four periods a week to Arithmetic and one or two to so-called Practical Arithmetic, the two in some cases being taken by different teachers. The Practical Arithmetic then becomes a course of elementary mensuration, entirely separate from the arithmetic teaching. The title 'Practical' Arithmetic should disappear; it belongs to the initial period of reform in Mathematics teaching, when it was important to lay stress on the necessity for the scholars themselves to engage in the measurement of quantity.... Such employment, called 'practical,' involving the use of instruments of various kinds, has been cut off from the more or less theoretical arithmetic arising out of the practical experience of the scholar. It has been regarded as a new kind of arithmetic, as something to be taken in addition to the older type of exercise, which consisted merely in learning how to operate with numerical symbols. By uniting theory and practice in one course we gain much.

Findlay was also fully implementing his ideals concerning the links between theory and practice, as well as correlation, at the Fielden Demonstration School of Manchester's Department of Education. (See Illustration 30.)

Although Findlay was working in special circumstances, and correlation had made little headway in most schools, Fletcher [1912, p.102] could still acknowledge some major gains for mathematics from the physical laboratory. In particular, he referred to the improved understanding of fractions, decimals, the concepts of area and volume, and systems of units, making mensuration 'more real and intelligible.' However, from his wide experience, he judged that the mutual influence of the subjects of mathematics and physics had not extended much beyond this range in most schools. Durell [1911, p.28] also acknowledged as a 'signal benefit' for arithmetic teaching that 'the educational value of practical applications to mensuration, physics, etc., has received due recognition.' Palmer [1912, p.236] strongly underlined this point, by contrasting the older commercially biased arithmetic with the newer more widely applicable arithmetic, which served other branches of mathematics, and science, particularly in the physical laboratory. Importantly, he judged:



It was the failure of the old Arithmetic to meet the demands of the Physics master, that perhaps more than anything led to the changes in recent years.<sup>36</sup> [p.236]

Nevertheless, in spite of some improvements in arithmetic teaching, science teachers continued to complain that pupils were weak in the use of decimals, the metric system and measurements, as well as computation [Report (Thomson), 1918, p.23]. Like measurements, mechanics was another area of overlap between the domains of mathematics and physics, where the issue of correlation again arose.

The organizational involvement of the MA and the BAAS in the question of mechanics teaching was considered in earlier chapters.<sup>37</sup> In the nineteenth century, the teaching of mechanics to mathematical specialists started before the subject was taught, more widely, on experimental lines, as part of physics. Thus, in the schools, four logical possibilities emerged. Mechanics might be taught under either mathematics or science, under both, or under neither.<sup>38</sup> In principle, a varying blend of theory, experiment and applications was possible in the teaching of this subject. However, in practice mechanics was taught wholly theoretically, when treated as part of mathematics. The reason was quite simply the examination system, particularly at Cambridge, and the character of its products, who naturally reproduced their own kind. In a surprisingly early confession to the AIGT on this subject, in 1883, W.H. Besant referred to the system which:

tends to make us forget the importance of the practical applications to daily life of a knowledge of mechanics, and the temptation is to luxuriate in the flowery and ornamental problems which sometimes form the staple of examination questions.<sup>39</sup>

Todhunter of Cambridge, in particular, was said to have claimed that 'no one who required a model or experiment to enable him to grasp a

36 This conclusion will be explored more closely in Chapter 8.

37 See pp.124-127, 157, 161, 166-167, 171-172.

38 For some early twentieth-century trends here, see the data given on pages in note 37.

39 Quoted in footnote [School World, 1904, p.57].

geometrical or mechanical theorem could ever be a mathematician.'<sup>40</sup>

To understand the enormous difficulties concerning correlation in this branch of mathematics, it is important to recognize that the influential Cambridge tradition continued, largely unchanged, in the early twentieth century. Berry [1912, p.188] pointed out that about half the mathematical student's course at Cambridge was 'applied mathematics,' though this was a rather inappropriate description, as he strongly underlined:

Normally, he goes to no experimental lectures, he does no work in a laboratory, and the experimental facts which he learns in his mathematical text-books are usually of the simplest character ... suitable for the direct application of mathematical analysis.... Physics learned in this way is naturally most unreal, and the mathematician who wishes afterwards to devote himself to physics, is at first at a great disadvantage, not only by want of familiarity with physical apparatus and physical data, but by a lack of the 'physical instinct'.... All his previous training leads him to be prepared to ignore in any problem whatever his teacher or his examiner tells him to ignore in order to obtain a particular mathematical result....<sup>41</sup>

Godfrey [1912a, p.176] acknowledged some progress from the worst extremes of artificiality but judged that 'the influence of the Cambridge school of applied mathematics in its decadence is still dominant.' Predictably, mathematical mechanics in the secondary schools followed the Cambridge pattern.

Siddons [1956, p.168] recollected his own school experiences which involved no experimentation whatever; the spurning of graphics in statics and calculus in dynamics; and a predominance of problems

<sup>40</sup> Quoted by Jackson [1904, pp.76-77].

<sup>41</sup> Berry [1912, p.188] could not resist adding a footnote:  
The typical Mathematical Tripos question on mechanics is supposed to begin: 'An elephant whose mass may be neglected....' A more serious illustration is that a mathematician frequently has difficulty in remembering whether the enormous factor  $v=3 \times 10^{10}$  ... should be put into the numerator or denominator of a formula.

The 'elephant' was also used by other writers to ridicule their own experiences in mechanics at Cambridge [Godfrey, 1912a, p.176; Bushell, 1974, p.70]. Bushell [p.70] referred to the problem of the elephant as 'the most famous one,' in a scathing attack on his own 'unreal and distorted' experiences in hydrostatics, optics, electricity and mechanics.



which were largely springboards for flights of algebraic manipulation. Godfrey [1906c, p.201] drew attention to the curious pattern which had developed in the secondary schools:

The British custom is that the physics master treats mechanics experimentally, and the mathematical master works problems of the Cambridge type. This tendency of mathematical mechanics to become lifeless is the inevitable result of the split....<sup>42</sup>

On the assumption that some of the problems in mathematical mechanics would be too difficult for the physics master, Godfrey argued that, for progress, all aspects of mechanics should be taught by the mathematical master, and others supported this point of view [Hewitt, 1908]. However, the motivation and capabilities of teachers of mathematics were major inhibiting factors, and the Joint Committee of the APSSM and the MA [1909e, p.12] found that little progress had been made in mechanics teaching.

The Joint Committee concluded that 'In no subject is the want of co-operation between the Mathematical and the Science Masters so apparent as in mechanics.' Mechanics taught as part of mathematics was a common practice, but only as a specialist examination subject, and not as part of a general mathematical education. Where mechanics was taught practically, it was normally a short and 'necessarily inadequate' course for younger pupils, under the science master, unconnected with the mathematics. The Committee argued that, in consequence, both specialists and non-specialists were being deprived of valuable educational elements [MA, 1909e, p.12]. It should be added that the requirements for competitive entry to Sandhurst and Woolwich had stimulated the fuller treatment of practical mechanics in the public schools, though only for certain pupils, and under half of fifty-five such schools reported that they regularly incorporated practical work [p.12]. Harrow, in particular, responded by setting

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42 By contrast, Godfrey [1906c, p.201] referred to the American custom where mechanics was wholly the physics teacher's concern.

up a mathematical laboratory specifically for mechanics (see p.205). Unrealistically, the Committee [p.5] recommended transferring the early practical work in statics to the mathematics department, though they did draw attention to one isolated initiative designed to improve the mathematicians' experimental capabilities. In 1909, a three-week summer course was arranged at the Cavendish Laboratory, Cambridge, specifically to give mathematical masters the opportunity to perform various physical experiments [pp.13-14].

Fletcher [1904], who had himself been an innovator in the teaching of mechanics and other aspects of applied mathematics, whilst at the Liverpool Institute, confirmed the general pattern already discussed, in the particular case of grant-aided secondary schools [1912, pp.91-92]. In such schools, there were rarely sufficient pupils to permit the class teaching of mechanics as part of mathematics, most pupils being limited to a short and early course of empirical statics, as part of science. Fletcher considered at some length the problematic relationship between mathematics and physics in schools [pp.99-103]. He drew unfavourable comparisons with Germany [p.103] where 'the Prussian "Prüfungsordnung" of 1898 still in force lays it down that "Pure Mathematics and Physics must always be taken together."' Young [1907, p.103] also drew attention to France's insistence on close correlation, made possible, as in the case of Germany, by the system of centralized curricular controls. Neither America nor England could implement change in this way, and, in the case of correlation, Fletcher's own frustration as Chief Inspector is evident. Wolff [1915, pp.170-172] not surprisingly drew attention to this country's serious inadequacies in this respect, and pertinently asked 'Was ist die Physik ohne Mathematik, was der mathematische Unterricht ohne den physikalischen Einschlag?' [p.172].



In England, important constraints on progress, already considered, were the character of university courses, the examination system and the teachers. Furthermore, within the universities, Filon [1912, pp.282-289] in the Special Reports drew attention to the decline of the nineteenth-century correlated tradition of mathematical physics; the increasing specialization within the two domains; and a shift of research interest from applied to pure mathematics. It does seem that there was little chance of genuinely applied mathematics courses or effective correlation developing at university and school level in this country during the early twentieth century, in spite of the endeavours of particular individuals and committees.<sup>43</sup> The Prime Minister's Committee on science education also considered the relationship with mathematics, and confirmed the generally gloomy picture [Report (Thomson), 1918, p.22]. Recommending correlation had become by this time a rather overworked and non-productive tactic.<sup>44</sup> Regarding teachers as 'the key to the educational situation' [p.31] the Committee turned to teacher education as a key factor, and suggested that the best way forward was to produce teachers capable of combining a mathematical and scientific outlook, and for the schools to make suitable appointments of joint-specialists [p.33]. However, this would have involved breaking the existing patterns of specialization and career advancement, which were by this time firmly established.

Ten years earlier, Perry [1908, p.461] had jokingly remarked:

the marriage of mathematics and science seems to me like that of December and May - the marriage of a man of seventy with old bachelor habits to a bright young virgin of seventeen.

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43 Innovation in mechanics teaching will be discussed further in Chapter 7.

44 See also Chapter 4, Illustration 25, for the MA's recommendations in 1919.

To borrow Perry's metaphor, it does appear that this particular marriage never took place. However, more seriously, Perry had much more to say about mathematical education and much more to do with its progress, particularly in relation to the education of engineers, and the important developments, centred on Perry, will be the focus for the remainder of this chapter.

### Mathematics for Engineers

In the older engineering colleges in the nineteenth century only civil engineering was taught, for which the mathematical requirements were limited. However, the requirements increased somewhat with the development of shipbuilding and mechanical engineering, and increased greatly with the advent of high-speed machinery and the development of electrical engineering [Perry, 1912]. Thus there arose the need for a much wider and more applicable mathematics curriculum than the conventional academic routine, and one which widely utilized experimental, numerical and graphical methods. For the engineer, what was required was swift progress, to provide a working knowledge of the more useful parts of mathematics, rather than a laborious academic treatment of the various branches along rigorous, pure mathematical lines, with utility neglected.<sup>45</sup> Along with the growing movement for technical education in the 1870s, the need to develop an alternative mathematics curriculum for technical institutions became more pressing, and this challenge was taken up at an early stage by Perry in particular.<sup>46</sup>

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45 It is beyond the scope of this thesis to consider to what extent this may be viewed as a resurgence of an earlier nineteenth-century practical mathematical tradition, which has been explored in the Scottish context by Gray (see p.5). The mathematical needs of civil engineering, architecture, surveying, draughtsmanship, navigation, mechanics and the army appear to have been prominent in this earlier period. See, for example, Gregory [1825].

46 The DSA followed the dominant academic tradition in mathematics (see pp.27-28).



During his brief stay at Clifton, from 1871, Perry began to develop some experimental mechanics, fully involving the pupils themselves in practical work (see p.203). In 1875, he went to Japan and became closely associated with William Ayrton (1847-1908), particularly in the exciting new field of electrical engineering, and the development of technical education at Tokyo [Armstrong, 1920]. In Japan, Perry and Ayrton developed the extensive use of graphical methods by technical students, including the plotting of curves on squared paper [Perry, 1900a, pp.102-103; 1913, p.79]. They returned to England in 1879, and, together with Armstrong, they put their Japanese experience to good use in the pioneering development of technical curricula in England at what became Finsbury Technical College.<sup>47</sup> Armstrong [1920, p.752] subsequently referred to the trio as the 'Finsbury Mohicans,' and it was at Finsbury that Perry worked out and tested his scheme of practical mathematics, designed to serve the needs of mechanical and electrical engineers, as well as to link closely with laboratory work. Perry also produced textbooks for engineers, which were very different in character to the standard mathematical treatments.

Perry's Practical Mechanics was published as early as 1881, and it embodied his experimental and graphical approach [Perry, 1900a, p.110]. A colleague of Perry, R.G. Blaine, produced an accompanying set of Numerical Exercises in Mechanics (1888), including four-figure logarithms and trigonometric tables [Retter, 1936, pp.42-43]. Perry also developed a treatment of calculus at Finsbury, which was very different in character to the highly specialized mathematical approach, this being beyond the reach of most students, and Perry's [1897] course was published as The Calculus for Engineers. The American mathematician E.H. Moore [1903, pp.39-40] later referred to the 'greivous' separation

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47 On Finsbury see Brock [1979], and for the important role of the City and Guilds of London Institute here, and in technical education generally, see Millis [1925, pp.53-72].

between pure and applicable mathematics, and to the consequent phenomenon of textbooks like Perry's. In 1896, Perry left Finsbury to become Professor of Mathematics and Mechanics at the Royal College of Science, South Kensington, where it was intended that he should implement his innovations in mechanics linked with practical mathematics [Armstrong, 1920, p.752; Perry, 1912, p.35]. As Perry [1912, p.35] subsequently remarked, somewhat acidly, his conception of practical mathematics was 'exceedingly different from what used to be the study of the mere mathematician on the same subjects.'

In the dissemination of Perry's ideas for technical education, the adoption of his scheme of practical mathematics by the DSA in 1899 was an important breakthrough.<sup>48</sup> Perry was appointed as an examiner, and his introductory lectures on the new subject were published by the DSA [1899]. A crop of new textbooks soon followed, by writers such as Frank Castle [1900, 1901, 1903], who came under Perry's direct influence at South Kensington, and whose new textbooks were very successful for Macmillan.<sup>49</sup> A mixed treatment of various useful mathematical topics within a single textbook contrasted sharply with the conventional production of separate textbooks in arithmetic, algebra, etc. As Sumpner [1912, p.7] remarked:

The engineering student wants to study Mathematics as one subject, not as many. But mathematical studies have been divided by the text-books into sections which have been kept too rigidly apart and have been developed as if they were quite separate subjects.

The elementary stage of Perry's scheme and its distinctive features were considered in Chapter 3 (see p.100). The general breadth of the scheme was a striking feature, as well as the complete break from the shackles of Euclid. Demonstrative geometry was largely eliminated, with the useful geometrical elements treated numerically, algebraically or graphically. As Perry confessed, 'I am afraid all this is frightfully

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48 On the DSA's initiative here see pp.32-33.

49 See also Graham [1899], Ormsby [1900], Cracknell [1900] and Millis [1903].



unorthodox. The idea of replacing geometrical philosophy by arithmetical juggling is scorned by the modern mathematician' [DSA, 1899, p.118]. On methods of teaching this scheme, Perry's ideals followed those of the 'practical educationists,' already discussed (see pp.188-191), and he was also in sympathy with the heuristic methods advocated by his colleague Armstrong (see pp.193-196). The success of the new subject under the BE's regulations for further education was quite extraordinary over the first decade of this century, and at the expense of the older established stages of pure mathematics.

From very small beginnings in 1899 (see p.33), a Correspondent [1903a, p.803] in Engineering could soon refer to 'enormous' progress, and added:

How else describe the effect at one college in these islands, at which preparation was made for ten or a dozen students in the new mathematics, and to which several hundreds came?

Interest was spreading rapidly, and in 1909 the new subject claimed six and a half thousand candidates, exceeding the six thousand for mathematics, and three and a half thousand for practical geometry. Furthermore, as Holmes [1910, p.200] pointed out, the numbers actually attending the evening courses would have been at least two or three times greater. Abbott [1912, p.10] noted that:

In 1910, in England alone, 6,964 students presented themselves in Practical Mathematics while only 2,841 presented themselves in the first three stages of Pure Mathematics.

By 1913, Perry [1913, pp.vii-viii] could boast that there were more students of science engaged in his subject than any other, and also that it was more popular with students than pure mathematics, taking the attendance levels through the year as an indicator. As well as general dissemination through textbooks, Perry contributed personally to the development of teaching methods in the new subject by giving short summer courses at South Kensington (see p.81).

Perry [1900a, p.vii; 1903a] also broadcast his general thinking concerning the education of engineers in Presidential Addresses to the Institution of Electrical Engineers in 1900, and to Sections G and L of the BAAS in 1902. On both occasions he took the opportunity to tilt at 'mere mathematicians' for their ignorance of engineering, and their failure to provide appropriate mathematical courses for engineers. The general cause was taken up in 1903 by a Committee of the Institution of Civil Engineers, under Sir William White as Chairman, which surveyed practices and reported in 1906, on the training of all classes of engineer [Thompson, 1911, p.287]. The Committee spelt out a number of implications for the schools, as well as the engineering colleges, and supported in particular the school teaching of approximations, logarithms, and simple trigonometry, which were notable features of Perry's scheme [BAAS, 1907, pp.447-448]. Perry's campaign was well publicized in the journal Engineering [Correspondent, 1903a, 1903b], where the writer also drew attention to Perry's early influence on engineering education in America and Germany [1903a, p.804]. Significantly, this Correspondent [1903a, p.803] also drew attention to Perry's view that his scheme of mathematics teaching was 'the best method of teaching all children, for whatever life intended,' and he continued:

it will be easily realised that the new system would from its very nature prove less startling to those who are trained to work hand-in-hand with Nature, and would therefore be easier to introduce to the engineering world than to any other. Practical mathematics, in fact, is a system of teaching mathematics to all persons, of all kinds and all ages.

Thus, Perry strove to extend the adoption of his ideals for practical mathematics from within the education of engineers to the wider education of students in training colleges, and pupils in elementary and secondary schools.

#### Practical Mathematics for All

As early as 1880 Perry made some suggestions for an alternative



conception of elementary mathematics, in a paper given to the Society of Arts. In particular, he argued:

It is quite possible to begin more in the middle of the subject ... teaching students to test by actual experiment the truth of many of Euclid's propositions.... Our Tom Tullivers sit brooding by the hour over a proposition in Euclid, hardening their hearts, and dulling their understandings, and we call it mental training! [Perry, 1900a, p.91]

At this time, he advocated experimental plane and solid geometry; the postponement of geometry as in Euclid; the early introduction of the uses of algebra and trigonometry; and the adoption of the metric system. Thus, twenty years later, when Perry's thinking and activities became a central part of a much wider campaign for reform, he acknowledged that his views were far from new, and had even been expressed in Japanese publications in the 1870s. Furthermore, he acknowledged that his pedagogical thought was congruent with that of Herbert Spencer, as well as his friends Ayrton and Armstrong [p.v]. Perry was not only an engineer but also a 'practical educationist,' who sympathized with the methods of the Kindergarten (see pp.188-190). His name became associated with what was known as the 'Perry movement' in Great Britain, 'Perryismus' on the Continent, and the 'Laboratory Method' in America. Such became the range of Perry's influence [Young, 1907, pp.87-121; Wolff, 1915, pp.67-84; BAAS, 1907, pp.444-450].

As was shown in Chapter 2 (see p.66), Perry first sought to extend the influence of his scheme to the training colleges, as well as to create a general state of disequilibrium by popularizing his practical mathematics as an alternative paradigm to the 'academic mathematics' of the schools. The critical organizational developments which resulted, in the period 1900-1903, were discussed in Chapter 3 (see pp.97-120). There was general sympathy at this time for the need to disencumber the curriculum of much purposeless material, and to introduce more practical methods, particularly in geometry and

mensuration, with more intermingling between the traditionally separate branches of mathematics. Greater emphasis on decimals, approximations, the metric system and the use of tables was generally supported, as well as graphical methods in algebra. There was also, in principle, much support for the broadening of the curriculum, particularly in the case of trigonometry, and the possibility of introducing some mechanics and calculus in a general education was also suggested. However, although the need to abandon Euclid as a textbook became clear, there was a general lack of support from the mathematical community for one major aspect of Perry's scheme. Perry, [1902a, p.486] confessed:

The view to which I hold most firmly of all my views about the teaching of mathematics is that demonstrative geometry ought never to be taught in schools.

This was undoubtedly a major weakness in Perry's argument concerning priorities in a general mathematical education. The need to experiment in deductive geometry was generally admitted, and examining bodies granted the necessary freedom. However, this aspect of the curriculum was still highly valued on educational rather than utilitarian grounds, as a mental training par excellence in deductive thought. An important implication here was that the possibilities for broadening the curriculum along Perry's lines were seriously limited. Perry was fairly optimistic in the early stages of reform (see p.122), and Love [1902, p.458] went so far as to claim that Perry's belief that practical mathematics was not only suitable for engineers but was also appropriate as a 'means of culture,' as well as for the future specialist, was 'widely held' in the movement's early stages. However, Findlay's [1902, p.184] judgement was more sober:

I do not think Prof. Perry and his friends quite realize how fragmentary and disjointed their own suggestions are, and what a great gulf separates their work with adults and artisans from that conducted by teachers of boys and girls in school classes.



The early examination reforms affecting secondary schools and training colleges were discussed in Chapter 2 (see pp.63,66) and Chapter 3 (see pp.115-120). As well as abandoning the insistence on Euclid, the various bodies introduced some geometrical drawing and measurement, and some graphical work involving the use of squared paper. In arithmetic, there was some simplification and increase in relevance, though neither the use of logarithms nor slide rules was generally enforced. Nor was there any general encouragement given to the teaching of simple trigonometry or calculus. Exceptionally, the requirements for the army and navy moved further and faster towards Perry's ideals [Lodge, 1904; Boyt, 1906; Ashford, 1912; Mercer, 1912]. For Perry, the major early achievements in examination reform were the encouragement given to practical and experimental geometry, and the use of graphs. By way of consolation, Perry [1902b, p.82] added that, in experimental science 'weighing and measuring, the uses of squared paper and logarithms, and the ideas of the calculus have entered in all sorts of common-sense ways.'

Perry also became closely involved with other aspects of reform over the first decade of this century, and, specifically, the teaching of mechanics (see pp.124-127); practical and experimental methods in elementary schools (see pp.127-128);<sup>50</sup> and correlation between mathematics and science (see pp.180-181). It is important to understand that Perry and the movement which adopted his name stood for much more than just a syllabus of practical mathematics. Rather, the Perry movement represented a general reaction against the nineteenth-century traditions in mathematical education, and embraced a range of ideals, which,

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50 Millis [1925, pp.92-95], of the Borough Technical Institute and previously a colleague of Perry at Finsbury, also campaigned for more practical and technically relevant teaching in arithmetic, correlated with geometry and manual work. His scheme was published as Arithmetic and Geometry: A Plea for Educational Reform [Millis, 1905].

although not new, were largely new to the mathematical community, and shared much in common with the pioneers in infant education, the 'practical educationists,' and the teachers of science for whom the character of mathematical education also became a major concern. For convenience, the various positive features of the Perry movement's rationale, which have emerged in this chapter, may be summarized as follows [Young, 1907, pp.87-121]:

- (i) take account of the pupil's motivation and interests;
- (ii) base abstract ideas on concrete experience to promote understanding;
- (iii) employ activities involving the hand and eye, and not just the ear, in conjunction with the brain, and 'graphic(al) methods' in particular;
- (iv) adopt experimental and heuristic methods -

Experiment, estimation, approximation, observation, induction, intuition, common-sense are to have honoured places in every mathematical class-room in which the laboratory method has sway. [Young, 1907, p.105];

- (v) postpone logical rigour and any early concern for the foundations, and generally restrict the formal deductive elements, admitting various forms of 'proof';
- (vi) simplify, broaden and unify the subject-matter of mathematics, ignoring traditional artificial divisions;
- (vii) correlate mathematics with science and laboratory work, and generally relate mathematics to life and its applications.

Many of these principles concerned teaching methods, which could not be enforced through the examination system, but rather any movement towards such ideals depended principally upon the attitudes and interpretations of textbook writers and teachers, who, as always, held the key to progress in classroom practices.

Some general remarks concerning the international interest in 'methods of intuition and experiment,' variously interpreted, were



made earlier in this chapter (see p.192). In this country, one important and related phenomenon concerns the language used for the titles of new textbooks in the early twentieth century. Many of these textbooks were aimed principally at the general secondary market and their titles include such terms as 'practical' in geometry, and occasionally in arithmetic, mensuration and trigonometry; 'experimental' in arithmetic, mensuration, geometry and trigonometry; 'graphic(al)' in algebra; 'numerical' in trigonometry; and, exceptionally, 'heuristic' in arithmetic and geometry, and 'observational' and 'based on experiment and discovery' in geometry [British Museum, 1906, 1911, 1918]. Textbooks which referred to 'practical mathematics' were largely aimed at the technical and military fields, but it is clear that authors and publishers judged the market to be generally receptive to the new 'practical' emphasis, particularly in geometry, where the examining bodies had made the greatest concessions. The phenomenon of 'practical arithmetic' and mathematical laboratories was discussed earlier in this chapter (see pp.203-212), and there is other evidence of the interest in practical methods in the early years of this century.

Various articles in the School World publicized the new possibilities for teachers. Eggar [1903c] surveyed drawing instruments for school use; Salmon [1903] discussed the various forms of squared paper available by this time for science and mathematics; and Professor Gibson [1905] considered the general question of mathematical tables suitable for schools. Godfrey's [1902b, 1902c, 1902d] suggestions for preparatory schools fully exploited the uses of squared paper for work on areas, fractions and decimals, as well as graphs, and he claimed 'a blackboard ruled in squares is almost indispensable' [1902b, p.203]. Two authors produced interesting articles on the subject of 'practical mathematics' for secondary schools [Bayliss, 1905; Wyatt, 1905].

Bayliss [1905, p.214] referred to 'very remarkable agreement in favour of practical methods of teaching,' but at the same time 'an equally remarkable disagreement with regard to the meaning and application of such methods.' He claimed that the details were being 'hammered out in hundreds of schools' [p.214], and actually referred to 'practical algebra,' which was an unusual way of denoting the use of mathematical tables and graphical methods in this branch. Wyatt [1905] surveyed the current possibilities, available commercially, for 'a course of modern practical mathematics carried on in an ordinary secondary school' [p.216]. In particular, he mentioned the use of a subdivided rod for decimals; wooden discs for the evaluation of  $\pi$ ; calipers and sets of solid models for measurement; a cheap form of sextant, called an 'anglemeter' for out-door work; a planimeter for area; a pantograph for similar figures; and French curves and other aids for curve drawing. The BE's [1905a, p.42] Suggestions for practical aids in elementary schools also deserve quoting in full:

every school should be provided with -

- (a) Foot rulers graduated in inches and tenths of an inch and also in centimetres and millimetres. (These should have square edges.)
- (b) Cords with feet, yards and metres marked upon them,
- (c) Imitation coins.
- (d) A pair of common scales with the smaller weights such as ounces, pounds, kilogrammes, decagrammes, and grammes.
- (e) Measures of capacity such as a pint pot.
- (f) Squared paper or tracing cloth.

Plain paper also, owing to its cheapness and easy divisibility, will be found to be of very great value for illustrations.

Thus, by around 1905, an impressive number of practical possibilities had emerged, and been enthusiastically exploited in some cases. One of the simplest aids of all was squared paper, and it had become by the First World War one of the most potent practical weapons in the mathematical education of both the tyro and the specialist. This remarkable victory deserves to be considered rather more closely.



### Squared Paper and Graphical Methods

In 1904, Laisant referred to squared paper as 'a marvellous instruction which ought to be in the hands of every one who works in mathematics from the kindergarten to the university.'<sup>51</sup> By the end of the decade, the use of squared paper had become a commonplace in elementary and secondary schools, as well as technical institutions. The BE's [1912c, p.8] revised Suggestions noted the existence of squared paper in 'any well-equipped elementary school,' and, by this time, felt it necessary to issue a warning concerning its use:

Any risk of injury to eyesight by the excessive use of squared paper should be avoided. Its use for practical geometry, etc., will only be occasional and little danger to health is likely if no paper with rulings less than one-tenth of an inch apart is used. The use of paper ruled in squares for the working of arithmetical examples has no real educational advantages and might well be discontinued.

A BE's [1909a, p.9] Circular referred to 'more or less graphic work in algebra' as 'usual' in secondary schools, and yet five years later another Circular [BE, 1914b, pp.3-4] remarked:

it is no uncommon thing now to hear it said that graphs have been overdone, and to find that by a natural reaction they have been largely or even entirely abandoned.

By 1914 in this country a clear tendency to overuse or misuse squared paper had emerged, but this aid had achieved international recognition as a valuable ingredient in every individual's mathematical education, and Smith [1913, p.614] could claim that 'Of the value of squared millimetre paper there is no question anywhere.' Significantly, Smith [p.622] also claimed:

Graphic methods of one form or another are now found in the courses in [secondary] mathematics ... in all countries, having gradually made their way from engineering, through thermodynamics and general physics, to pure mathematics.

The use of squared paper was one feature of 'graphic methods,' and the phenomenon of the gradual adoption of such paper from its exclusive use as a research tool in the early nineteenth century to its

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51 Quoted in Young [1907, p.111].

universal use for a variety of purposes in mathematical education has been explored in a paper by Brock and Price [1980].<sup>52</sup> 'Graphical methods' developed in the nineteenth century, and came to include much more than the drawing of graphs and the use of squared paper. Broadly interpreted, 'graphic(al)' means drawing, with the use of hand and eye, as opposed to 'analytic(al)', which implies symbolic manipulations. The subject of 'graphics' within technical education included 'graphic arithmetic,' which was approximate calculation by drawing, as well as 'graphic statics,' involving vectors [Harrison and Baxandall, 1899]. Practical plane and solid geometry was essentially 'graphic geometry,' though this name was not used. 'Graphical calculus' was another branch of 'graphics' designed for engineers [Cajori, 1917, p.300]. For the various uses of squared paper, Wormell [1888] provided an early textbook entitled Plotting or Graphic Mathematics, and, from 1899, the normal pattern was to include a specific chapter on this subject in the new textbooks on practical mathematics.<sup>53</sup> When graphs found their way more generally into algebra the name 'graphic(al) algebra' was commonly used (see pp.121-122), and the BE [1909a] adopted it in the title of their first Circular on this subject, but, significantly, abandoned this name for the revised Circular five years later [BE, 1914b]. The ICTM included land measurement and area measurement, using squared paper or a planimeter, within their interpretation of 'graphical methods' [Smith, 1913, p.622].

Brock and Price [1980] have shown that Perry and Ayrton pioneered the use of squared paper by students in Japan in the 1870s,

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52 It also came to be used for other purposes, such as the accurate drafting of pillow lace patterns, though the history of the dissemination of this use for squared paper warrants further investigation. My thanks are due to Mrs. M. Clark for some interesting correspondence on this point.

53 See, for example, DSA [1899], Graham [1899], Cracknell [1900], and Castle [1901]. The DSA [1899, p.27] provided booklets of squared paper for the new examinations in practical mathematics.



and subsequently implemented and refined these methods at Finsbury. Prior to the 1880s, the high cost of accurately engraved squared paper limited its use in this country to the researcher, until the development of teaching methods in technical education necessitated the cheap and plentiful availability of squared paper for a variety of purposes.<sup>54</sup> Even the DSA's examinations in pure mathematics ignored graphs and the use of squared paper [Hitchens, 1978, pp.73-76,79], and, conventionally, the subject was associated with analytic geometry, which was an advanced branch of pure mathematics. Thus, Perry [1900a, p.46] referred to the nineteenth-century pattern where:

simple exercises on squared paper ... must not be approached until one has wasted years on higher algebra and trigonometry and geometrical conics, because they belong to the subject of co-ordinate geometry.

Perry may have underestimated the progress made in some schools here, as Moylan [1901, p.39] claimed that:

Most teachers, at quite an early stage, now introduce the tracing of graphs on ruled paper, and do not postpone it till the pupil has some knowledge of analytical geometry or calculus.

However, it appears that the prominence given to the uses of squared paper in the scheme of practical mathematics under the BE helped to disseminate this tactic more widely. For example, Godfrey [1902d, p.289] drew teachers' attention to the possibilities to be found in the practical mathematics textbooks of Cracknell [1900] and Castle [1901] in particular, and he claimed with evident enthusiasm:

GRAPHS have found their way into elementary work, and are now recognised as quite the most valuable instrument in our possession for awakening interest.<sup>55</sup>

54 The idea of a graph on a squared grid goes back at least to the tenth century [Funkhouser, 1936], and for a discussion of the history of the graphical representation of quantitative data see Funkhouser [1938]. I am grateful to Arthur Morley for this particular reference. As a specialized branch of mathematics, analytic geometry seems to have developed in the seventeenth century without the use of squared paper. For a recent discussion of the roots of analytic geometry see Forbes [1977].

55 Parallel developments in America have been sketched by Cajori [1917, pp.299-301]. The Americans referred to 'cross-section paper,' and see, in particular, Moore's [1906] discussion of its potential. 'Canonize the Cross-Section Paper' was his closing remark [p.338] - his own stress.

The development of science teaching (see pp.199-201) was a further contributory factor in the growth of the teaching of graphs, from around 1890. As Salmon [1903, p.169] pointed out:

SINCE the introduction of the heuristic method into our system of education the use of squared paper has come to play a very important part in laboratory instruction, more especially in physical work.

The BE's [1914b, p.11] Circular acknowledged the early use by pupils of squared paper to determine functional relationships in science, other than by pure computation:

Generations of boys had indeed been familiarised with graphs in connection with physical observations before ever graphic work was recognised as an essential element in the teaching of algebra.

With the stimulus of altered examination requirements, however, the drawing of graphs became rife by 1905, with 'graphic(al) algebra' meaning different things to different teachers, and textbook writers generally 'cashing in' on the craze (see pp.121-122). Hall [1905, p.159] was one such successful writer, who referred to a state of 'graphomania,' which he claimed was particularly affecting younger teachers. Reaction was beginning to set in, and the BE [1909a, 1914b] in particular sought to clear the air with Circulars on this subject.<sup>56</sup> However, squared paper as a general teaching aid was not to be dislodged from the mathematical classroom. Calculating aids were another feature of practical mathematics, needed by the engineer and the scientist, and, here again, their use gradually spread more widely in mathematical education.

#### Mathematical Tables and Slide Rules

Gibson [1905, p.201] has drawn attention to the fact that 'more or less elaborate' sets of tables were a common feature of mathematical textbooks used during the first half of the nineteenth century, and also that such tables were commonly published in the form of separate booklets. These aids to calculation served the earlier practical

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<sup>56</sup> Graphs in relation to algebra, and the links with the notion of 'functionality' will be discussed in Chapters 7 and 8.



mathematical tradition. However, one particular consequence of the rise of the English examination system from the mid-nineteenth century was that 'the practice of using mathematical tables fell into desuetude' [p.201], with complete sets of tables being generally banned from mathematical examinations. Even in the case of the DSA's examinations in pure mathematics, the use of sets of tables was avoided by including in the papers themselves the logarithmic and trigonometric values necessary for particular questions, and, furthermore, these values were given to seven places of decimals [Hitchens, 1978, pp.76-78].

In the case of boys' public schools, Godfrey [1908, p.253] claimed:

Ten years ago the only tables found in the mathematical class-room were those of seven figures, which were used in the solution of triangles. These were not handy enough, and the boy never had enough practice to use logarithms with confidence.

Siddons [1956, p.166] also drew attention to the late nineteenth-century practice of only introducing logarithms to mathematical specialists in public schools, and only using seven-figure tables, such as those of Chambers. Siddons [p.167] added that four-figure tables 'seemed to be unknown to mathematical masters when I started teaching [1899].' Needless to say, he had not encountered simple tables in his own schooldays, but, significantly, he had used them in his fourth year at Cambridge, when working for the Science Tripos [p.167]. They were also used by pupils learning science at Harrow, when, as a new master, Siddons had to fight for their acceptance in mathematics. Apparently there were strong objections to the principle of allowing books of tables in the school's examinations, particularly on account of the risks concerning cheating [Siddons, 1936, p.23]. However, although the use of tables in mathematics was confined to specialist school work, and particularly in trigonometry, the use of four-figure tables by younger pupils in science appears to have been developing by 1900.

Angle.	Radians.	Sine.	Tangent.	Co-tangent.	Cosine		
0°	0	0	0	∞	1	1.5708	90°
1	.0175	.0175	.0175	57.2958	.9998	1.5533	89
2	.0349	.0349	.0349	28.6363	.9994	1.5350	88
3	.0524	.0524	.0524	19.0811	.9986	1.5184	87
4	.0698	.0698	.0698	14.3006	.9978	1.5010	86
5	.0873	.0873	.0873	11.4301	.9962	1.4835	85
6	.1047	.1045	.1051	9.5144	.9946	1.4661	84
7	.1222	.1219	.1223	8.1443	.9925	1.4486	83
8	.1396	.1392	.1405	7.1154	.9903	1.4312	82
9	.1571	.1564	.1584	6.3158	.9877	1.4137	81
10	.1745	.1738	.1763	5.6713	.9848	1.3963	80
11	.1920	.1903	.1944	5.1416	.9816	1.3788	79
12	.2094	.2079	.2126	4.7046	.9781	1.3614	78
13	.2269	.2250	.2309	4.3315	.9744	1.3439	77
14	.2443	.2419	.2493	4.0103	.9703	1.3265	76
15	.2618	.2583	.2679	3.7321	.9659	1.3090	75
16	.2793	.2756	.2867	3.4874	.9613	1.2915	74
17	.2967	.2921	.3057	3.2700	.9563	1.2741	73
18	.3142	.3090	.3219	3.0777	.9511	1.2566	72
19	.3316	.3256	.3413	2.9042	.9455	1.2392	71
20	.3491	.3429	.3610	2.7475	.9397	1.2217	70
21	.3665	.3594	.3839	2.6051	.9336	1.2043	69
22	.3840	.3763	.4010	2.4751	.9272	1.1868	68
23	.4014	.3927	.4245	2.3539	.9205	1.1694	67
24	.4189	.4097	.4452	2.2409	.9135	1.1519	66
25	.4363	.4256	.4663	2.1445	.9063	1.1345	65
26	.4538	.4434	.4877	2.0593	.8988	1.1170	64
27	.4712	.4590	.5095	1.9826	.8910	1.0996	63
28	.4887	.4696	.5317	1.9137	.8830	1.0821	62
29	.5061	.4843	.5543	1.8519	.8746	1.0647	61
30	.5236	.5000	.5774	1.7921	.8660	1.0472	60
31	.5411	.5159	.6009	1.7321	.8572	1.0297	59
32	.5585	.5320	.6249	1.6793	.8480	1.0123	58
33	.5760	.5486	.6491	1.6329	.8387	.9948	57
34	.5934	.5652	.6745	1.5826	.8290	.9774	56
35	.6109	.5824	.7002	1.5281	.8192	.9599	55
36	.6283	.5979	.7265	1.4764	.8090	.9425	54
37	.6455	.6118	.7536	1.4270	.7986	.9250	53
38	.6628	.6257	.7813	1.3799	.7880	.9076	52
39	.6807	.6393	.8098	1.3349	.7771	.8901	51
40	.6981	.6528	.8391	1.2919	.7660	.8727	50
41	.7156	.6661	.8693	1.2504	.7547	.8552	49
42	.7330	.6791	.9004	1.2103	.7431	.8378	48
43	.7505	.6920	.9325	1.0724	.7314	.8203	47
44	.7679	.7047	.9657	1.0355	.7193	.8029	46
45	.7854	.7071	1.0000	1.0000	.7071	.7854	45
		Cosine	Co tangent	Tangent	Sine	Radians	Angle

Illustration 31 The DSA's [1899, p.20]  
Four-Figure Trigonometric Tables



In a letter to Nature, Dufton [1900, p.415] claimed that 'the general introduction of practical physics into secondary schools has resulted in the teaching of logarithms to younger boys.' Dufton himself used squared paper for teaching logarithms, and he referred to the fact that several 'schools of science' were working on such lines with boys from the age of thirteen [p.415]. Godfrey [1908, p.253] also drew attention to the earlier use of simple tables by teachers of science, who 'complained that they had to do the work of their mathematical colleagues in teaching the use of logarithms.' However, he added that this was 'ceasing to be true' and that four-figure logarithmic tables were 'coming into fashion' [p.253]. As Gibson [1905, p.201] pointed out, this development in mathematics teaching was part of a wider shift of emphasis, namely that 'more attention is being paid to the arithmetical side of mathematics.' The development of practical mathematics was a further important factor in the dissemination of the use of simple tables.

From 1899, the DSA [1899] provided a valuable set of four-figure tables for the new subject of practical mathematics in its examinations, and these tables were incorporated in the textbooks which were tailored to the new requirements.<sup>57</sup> The trigonometric values were conveniently compressed onto a single page. (See Illustration 31.) Perry [1901a, p.25] could soon claim that:

since the Science and Art Department has distributed its tables of four-figure logarithms and functions of angles over the country as cheaply as grocers' advertisements, there has been a wonderful development in knowledge and use of such tables.

In articles for the School World, both Godfrey [1902d, p.290] and Wyatt [1905] referred to the BE's four-figure tables, cheaply available at 5s. per hundred leaflets [Godfrey, 1902d, p.290].<sup>58</sup> Perry [1899a,

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57 See, for example, Ormsby [1900], Cracknell [1900] and Castle [1901].

58 Secondary school textbooks also borrowed these tables. See, for example, the 1907 edition of Hall and Knight's [1885] very successful algebra, which included a new section on four-figure logarithms.

1899b], in particular, became involved in some early correspondence in Nature concerning methods of calculating logarithmic values, using squared paper and the repeated extraction of square roots [Dufton, 1900].

There was much support for the earlier treatment of logarithms, and some interest also in the use of simple tables in trigonometry, shown in the various pronouncements on mathematical education in the period 1900-1903 (see pp.100-115). However, examining bodies were not generally prepared to force the use of tables in a general education. Thus, during the Joint Meeting of the APSSM and MA [1910d, p.265] at the end of the decade, one speaker complained that 'pupils have to sit at examinations where Logarithms are not used and sometimes are even prohibited.' The general picture at this time was surveyed by the Joint Committee of the APSSM and the MA [1909e, p.12].

Up to the end of the decade 'in the great majority of cases' the topic of logarithms was still regarded as a relatively specialized one, following indices in algebra, and useful for the minority of pupils reaching trigonometry. In only 'a few schools' was this topic taken as part of arithmetic, and the Committee judged that 'It seems probable, that the use of logs, as a means of simplifying ordinary arithmetical calculations, is only incidentally touched upon' [p.12]. Up to this time, apart from the algebra examinations of the Cambridge Previous and Senior Locals, questions on logarithms were still generally confined to the additional or optional papers in mathematics [Hawkins, 1912, pp.470-530]. Various papers in the Special Reports throw some further light on attitudes and practices before the War.

Palmer [1912, p.252] claimed that the topic of logarithms was properly a part of algebra, but added that 'tables of logarithms are used so much now for arithmetical calculations that the subject is found in most text-books of arithmetic.' However, he still felt it



necessary to caution that, whilst such tables were valuable in the physical laboratory:

the Arithmetic of the large majority of people is commercial rather than scientific, and for this four-figure logarithm tables are far from adequate. [p.238]

Godfrey [1912b, p.436] summarized the pattern of use for tables in boys' secondary schools:

Different tables are used in the following descending order of frequency - logarithms, trigonometrical functions, squares and square roots, cube roots.... The extent to which the everyday use of tables has established itself is shown by the prevalent use of 4-figure tables; 7-figure tables are too cumbersome for most school purposes. 4-figure tables used to be used mainly by science masters, but have now been largely adopted by mathematicians as well.

All the public schools in his survey said they used logarithms, but only with boys from the age of sixteen in half the schools. With a generally shorter school life in the other secondary schools, the tendency was to introduce logarithms somewhat earlier, as well as trigonometric tables, though twelve per cent of these schools did not use the latter tables, and two-thirds of the public schools only used them from the age of sixteen [p.436]. In girls' schools, Story [1912, pp.549-550] found that the earlier treatment of logarithms was still at an experimental stage, though a 'good many' schools were introducing this topic, linked with indices in algebra, below the sixth form [pp.557-559].

Tables came to be included in practical mathematics, arithmetic and algebra textbooks, as well as being published separately in booklet form. In addition to the tables of the BE, popular sets of four-figure tables, before the War, were produced by Castle (1902), Knott (1905) and Godfrey and Siddons (1913) [Wolff, 1915, p.157]. These last tables subsequently enjoyed enormous success over the next forty years, and their publication just before the War was timely.<sup>59</sup> From

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59 Siddons [1952b, pp.11-12] attributed the success of these tables partly to the improved lay-out and binding. Castle's tables are still in print, and are currently being advertised nationally by the booksellers W.H. Smith. See Daily Express, Aug. 26th, 1981, p.11.

around 1910 the MA were pressing strongly for the wider use of four-figure logarithmic and trigonometric tables.

The MA pressed a renewed case for tables in three Reports before the War [1909e, p.5; 1911b, p.6; 1913b, p.3], and was now working closely with examining bodies in the framing of new syllabuses (see pp.164-165, 172-174). By 1920, elementary questions involving logarithmic tables were being included in the examinations of Oxford, Cambridge and the NUJMB, though they were still forbidden for London Matriculation, and could be avoided in Oxford Responsions [Cook, 1920, pp.28-29]. London's defence was purely academic, and, with pressure from particular schools (see p.56) as well as the MA, from 1922 London allowed, but did not enforce, some use of logarithms in arithmetic, and included this topic in the algebra syllabus also [Retter, 1936, pp.129, 207]. The possibility of using logarithms in arithmetic, and their inclusion in algebra, was subsequently the general practice for non-specialist examinations [MA, 1930a, p.149]. In grant-aided schools, HMI Carson [1929, p.23] judged that:

The only marked change in the arithmetic syllabus [since 1910] is the introduction of logarithms, usually in the second or third year of the course, for practically all pupils instead of the chosen few.

Thus the victory for logarithmic tables was complete, though the battle for this advance in a general secondary education had been fought for some twenty years.<sup>60</sup>

Perry's practical mathematics also required the use of a slide rule, and a relatively cheap 'Kensington' rule was produced by Thornton

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60 A surprising number of different ways of teaching logarithms were advocated before the First World War. See, in particular, Bryan [1908b], Mair [1909], Dobbs [1909, 1915], McLeod [1909] and Nunn [1914]. A numerical and graphical approach via functions  $a^x$ , typically for  $a=2$  and then  $a=10$ , appears to have become the popular one [Godfrey and Bell, 1905b, pp.134-144; Godfrey and Siddons, 1913, pp.253-259]. Rather surprisingly, Perry claimed this was a 'common exercise' in schools by 1900 [Dufton, 1900, p.415]. He may have been referring to the work in science.



in particular [Wyatt, 1905]. However, the use of slide rules made little progress in a general mathematical education.

With the exception of the Committee of the Institution of Civil Engineers [BAAS, 1907, pp.447-448], there was no interest in the school use of slide rules expressed in early twentieth-century pronouncements on mathematical education, though their use was required in the reformed examinations for competitive entry to the army [Lodge, 1904, p.453], which forced some public schools to teach the technique to some pupils. An early article in the Mathematical Gazette [Barrell, 1901] was devoted to this subject, and Jackson [1903] of Woolwich, in particular, argued a case for the slide rule, linked with the teaching of logarithms.<sup>61</sup> There may have been more use of slide rules in the physical laboratory. One teacher in the School World explained how to make a simple rule using semi-logarithmic paper, to use merely as a tool in science. He added 'I ... do not bother about principles - that comes under the work of the teacher of mathematics' [Cotton, 1909, p.79]. However, teachers of mathematics showed little enthusiasm for this aid.

In boys' secondary schools, the Joint Committee of the APSSM and the MA [1909e, p.12] found that 'The slide-rule seems to be by no means in general use.' Two-thirds of the public schools explained its use to senior pupils only, but less than one-third of the other secondary schools followed this practice. The Report suggested that cost was a prohibitive factor in the latter case, and the army requirements provided an important stimulus in the case of public schools. Godfrey's [1912b, p.435] findings for the ICTM largely confirmed the general pattern, and the difference between public and other secondary schools

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61 He subsequently earned the nickname 'Slide-Rule Jackson' [MA, 1917], and produced an impressive treatise on Mannheim's rule with a colleague from Woolwich, for Longmans' Modern Mathematical Series [Dunlop and Jackson, 1913].

in particular. He concluded that 'On the whole, the slide rule is used by mathematical teachers only under compulsion.' The practices of mathematics teachers were largely governed by the needs of intending army entrants and engineers, and there was little interest in the possible educational benefits of slide rules. Exceptionally, Bryan [1908b] and Nunn [1914, pp.197-351] argued for the slide rule on educational grounds, linked to the teaching of logarithms. Nunn produced an ambitious scheme linking graphs, growth problems, the use of a home-made Gunter (1581-1626) scale, which he described as 'the ancestor of the slide rule used so much by engineers to-day' [p.302], and logarithms.<sup>62</sup> However, his ideas were far too ambitious for most classrooms.

The much wider adoption of tables, though not slide rules, was part of the tendency to make school mathematics more arithmetical, which, in turn, was part of the general movement to make the subject more 'practical.' Some reaction to these newer emphases in the curriculum is detectable from around 1905.

### Reactions and Appraisals

Both geometrical drawing and graphical work in algebra had at first been enthusiastically and uncritically accepted, and such practical activities spread through the system with remarkable speed. The army requirements in particular swung dramatically towards the ideals of practical mathematics, with a strong emphasis on geometrical drawing, graphical methods in algebra and mechanics, and the use of both logarithms and the slide rule [Broomfield, 1905a, 1905b]. Significantly, the Mathematical Gazette published a scathing reaction to the Civil Service Commission's wholesale acceptance of these methods. Robinson [1906, p.336] of St. Paul's School generally remarked:

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62 For a history of the slide rule see Cajori [1909].



The policy of 'overturn, overturn' has been pushed too far: if numerical calculations, graphical representations, and geometrical drawing are not merely to be an aid, but a substitute for a systematic knowledge of the theory of Elementary Mathematics, the value of the subject as an educational instrument will be but small.

Given the nature of the new army requirements, he felt that there was 'a real danger of such a revolution' [p.336]. Analyzing the new papers, Robinson concluded that 'The questions set involve little else but numerical calculations and graphic work' [p.337], and he went on:

the examiners do not require a sound theoretical knowledge of any subject, but only the ability to apply special formulae and methods to the numerical solution of a certain class of questions which they regard as of practical importance. [p.338]

Thus, he felt candidates only required 'a few knoblets of knowledge which will enable them to solve questions of a strictly practical and "useful" kind' [p.338]. In the case of the laboratory work in weighing and measuring, he referred to it as 'dignified by the title of "Practical Mathematics,"' and he felt that such experiments 'as a part of mathematics ... seem somewhat out of place' [p.338]. These reactions, though narrowly focused on the army requirements,<sup>63</sup> reflect more general misgivings concerning the directions of change.

The School World, which had done much to publicize practical methods, also published a detailed and critical appraisal of the functions of the new activities, from the Senior Mathematical Master of Bradford Grammar School. Much of Jones' [1905] discussion concerned the proper relationship between theoretical and practical geometry. He argued that 'exercises which merely test a pupil's skill in draughtsman's work are surely for the art room' [p.288], and that, of the new geometries 'the majority seem to have been hurriedly turned out for the sake of inserting something which could be called practical' [p.288]. He dismissed activities involving drawing, measuring, folding

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63 See also Hawkins [1912, pp.461-467] for a lengthy review of the army requirements, with strong criticism of the tendency to superficiality and graphical excesses.

and cutting as not a valid substitute for the serious business of deductive proof, and claimed that 'geometry is not an experimental science' [p.288].<sup>64</sup> He also shared Hall's concern for the state of 'graphomania,' and advocated a more serious consideration of purposes here (see p.231). Before the end of the decade, some of the leaders in the reform movement were also expressing concern for the pattern of change.

Branford [1908, pp.vii-viii] warmly welcomed the newer tendencies in many schools where:

the practical and theoretical aspects of mathematics are co-ordinated and developed: where simple descriptive geometry aids and is aided by clay-modelling and drawing: where theoretical geometry and practical geometrical drawing and mensuration illustrate and assist each other: where theoretical and experimental mechanics are associated with each other and with pure mathematics: where, in fine, all the branches of elementary mathematics, pure and applied, theoretical and experimental, are commingled at appropriate times, so that the mind sees and uses its mathematical conceptions and processes as a beautiful, well-ordered, and powerful whole, instead of a thing of shreds and patches.

However, he felt it necessary to add:

It must, however, be admitted that the particular type of intellectual discipline obtainable from mathematical study on its formal, systematic, and logical side, is in considerable danger of becoming temporarily sacrificed during a too extreme swing of the pendulum of reform. [p.viii]

Godfrey [1908, p.254] also referred to the recent examination reforms in geometry, which had 'released a volume of pent-up energy,' and resulted in a tendency to overdo the practical elements. He added:

it was soon realized that this would make the subject invertebrate, that there must be a certain element of severity in every school study, and that for purposes of general education geometry must still stand or fall by the logical training it gives. [p.256]

He judged that 'the transitional period is still on us,' and that 'perhaps

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64 Jones [1905, p.287] did, however, support a preliminary course of practical measurements, which 'differs in reality very little from the first course ... we formerly gave to beginners in physics' (see pp.203-208).



it is still too soon to give a final opinion' [p.256]. He also considered the newer numerical and graphical tendencies in algebra, where again 'Following the usual law, the reform went too far,' but he judged that 'The pendulum is now swinging in the opposite direction' [p.257]. Perry also expressed his concern, and some evident impatience, regarding the gap between ideals and the form of their realization at this time (see pp.129,181), and the first mathematical Circular of the BE [1909a] was a major contribution to the refinement of pedagogy concerning practical geometry and graphical algebra. Up to the War, a number of teachers were themselves evolving a satisfactory relationship between mathematics and practical work [Jones, 1912; Lister, 1913; Bingham, 1913; Child, 1914]. Evidently, practical mathematics meant different things to different people, and in the Special Reports of 1912 the name was taken to imply laboratory work in mathematics. In the introduction to the papers on this subject, Turner [1912, p.394] remarked:

Those who prefer to keep as close as possible to the old methods have made a minimum concession by introducing a few models and a little squared paper into their classrooms; the thorough-going reformers have boldly left the class-room altogether and installed themselves in the workshop.

Fawdry [1915, p.36] adopted the laboratory interpretation of practical mathematics in a paper given to the MA, and, significantly, remarked:

Numerical evaluation of algebraic expressions, accurate constructions of geometrical problems, plotting of curves, graphical solutions, use of logarithms in computation, in fact the bulk of the methods which have been adopted in the class teaching of Mathematics largely as a result of the efforts of the Mathematical Association [sic] - these to me do not mean Practical Mathematics. Such operations can be conducted in a class-room without the use of further apparatus than a box of instruments, some squared paper, and a table of logarithms.

Fawdry's judgement was seriously at fault here, the gains to which he referred all being tangible results of the Perry movement. Furthermore,

from around 1910, there is evidence of efforts by the mathematical community to discredit practical mathematics on other grounds.

The gulf separating the Cambridge school of mathematicians, led by Forsyth, and the engineering colleges, working on Perry's lines, was considerable, in spite of the apparent consensus within the Committee of the BAAS in 1902. These differences of outlook emerged strongly in an early letter by Perry [1903b, p.390] to Nature, where he referred to Professor Greenhill's judgement that his The Calculus for Engineers was 'a series of events connected by a slight thread of continuous theory,' which suggested 'a mathematical Pickwick.' He appealed for a lead from Cambridge in the case of the higher education of engineers:

it is not merely elementary education that is going into the melting-pot. Is Cambridge going to hold aloof from the little army of men who think that the melting and solidifying processes need to be guided? Has Cambridge no interest whatsoever in the nature of the possible crystallisation? [p.391]

Regarding the important supply of mathematics teachers from Cambridge, Perry [1902c, p.203] scathingly commented:

we must do without them, and indeed severely reject them, as candidates for posts, if they will not give boys such teaching as befits the twentieth century.

Significantly, in his detailed Report on English reforms, Wolff [1915, pp.67-84] strongly contrasted two schools of thought in an interesting section titled Die Reformbewegung in England (Perry and Forsyth). A major bone of contention was that rapid progress through an ambitious range of useful content in practical mathematics necessitated a lack of attention to general principles, with the systematic theoretical development of ideas neglected.

Hobson [1911, 1912] in Presidential Addresses to the BAAS and the MA strongly underlined the need to work out a satisfactory relationship between theoretical and practical work in schools. He also made some interesting remarks concerning practical mathematics:



A perusal of some of the current treatises on 'practical Mathematics' has led me to think that in some quarters the purely practical side of Mathematics is unduly emphasized.... I do not wish in the least to depreciate the importance of Mathematics as providing the tools for a vast variety of applications.... But the most important educational aspect of the subject is as an instrument for training boys and girls to think accurately and independently.... [1912, pp.238-239]

He also added the following biting criticism:

I gather that in some of the current teaching of practical Mathematics, a kind of perverse ingenuity is exhibited in evading all discussion of fundamental ideas, and in the elimination of reference to general principles. [p.239]

Interestingly, on being asked to clarify whether he meant by practical mathematics Perry's scheme for the BE, or laboratory work, he replied 'I think I had both in my mind. I used the word "practical" to distinguish from the mathematics which deals with principles' [p.243]. By this time, a third, looser interpretation of practical mathematics as practical activities in mathematics appears to have faded.<sup>65</sup>

Perry [1912, p.34] now felt that it was 'perhaps a pity that I gave such a misleading name as practical mathematics to the reformed methods, but I wanted to differentiate them from the orthodox methods.' A teacher on the technical side at St. Dunstan's College even went so far as to refer to practical mathematics as an 'obnoxious term' [Usherwood, 1912, p.63]. The suitability of Perry's scheme in the restricted case of the education of engineers was also coming into question. By this time, engineering science was established as an alternative academic study to mathematics or natural science, and was emerging as a suitable new subject in a liberal education [Hopkinson, 1912, p.329]. During the Fifth ICM at Cambridge, Sir William White [1912, p.95] argued that the 'preponderance of opinion' favoured the handing over of the mathematical education of engineers to mathematical specialists, who could provide a broader and deeper theoretical grounding, which could subsequently be applied and adapted. This was a clear

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65 See also the reactions quoted in Chapter 3, pp.105, 107, 129-130.

rejection of the older tradition of practical mathematics for engineers, closely linked with specific applications throughout, which it was felt had inhibited this country's progress in research in particular.

Perry [1912, p.34] reacted strongly to White's 'contemptuous' reference to practical mathematics. White was a member of the governing body at Imperial College, where important changes were in the process of implementation. Perry had been asked to retire, and Henrici had already retired. Perry was reluctant to leave and claimed 'The syllabus and methods of teaching are exactly as they have been for seventeen years' [p.35]. In retrospect, this appears to have been a major failing in Perry's later years, that he persistently regarded his inflexible scheme, with its roots in the 1870s, as a panacea in the education of engineers. Perry lost this battle and was forcibly retired in 1913, to be replaced by, of all people, Forsyth, a Cambridge mathematician who shared White's views [Wolff, 1915, p.83]. However, practical mathematics was by this time very firmly established in the sphere of evening technical education (see p.220).

Perry's [1913] Elementary Practical Mathematics was published as a revision of his six lectures for the DSA [1899]. In the preface, Perry [1913, p.viii] expressed frustration with many technical teachers' interpretations of his ideals, mediated through textbooks, which dominated the teaching. He also chose to reiterate his by now exaggerated and simplistic attack on 'academic methods' generally. This book was fairly savagely reviewed in the School World [1913b], under the title 'The Apotheosis of Practical Mathematics,' and by Bryan [1913] in Nature. In the School World, Perry's major contribution to curriculum development in technical education was acknowledged, in spite of teachers' misinterpretations, though his disinclination to pursue theory was criticized. The reviewer confessed 'we have not much faith in the new royal road to mathematics,' and suggested that



'The "wooden-headed, cock-sure, academic persons," perhaps know their business a little better than Prof. Perry will allow his students to believe.'

Bryan's [1913, p.551] lengthy review began by acknowledging that the 'revolution' in mathematical education over the previous ten or fifteen years 'owes its success largely to the indefatigable exertions of Prof. John Perry.' However, as regards Perry's mathematical sensitivity, or rather his lack of it, he was far from generous:

when he comes to the bookwork, we fail to find much difference between his 'practical' mathematics and the old-fashioned 'academic' mathematics, except that his methods are less logical, less interesting, and less convincing than those now adopted by our best teachers. [p.551]

The following example well exemplifies the general attack on Perry's mathematical shortcomings:

Prof. Perry ... says that 'in many important calculations we need to use Napierian logarithms, whose base is 2.71828.'  
'Why 2.71828?' asks the intelligent student. No answer is given; and this is what Prof. Perry calls 'practical mathematics.'  
We should call it cram. But the author continues to drag in this apparently useless and meaningless symbol e throughout the book.... [p.551]

Throughout his life, Perry remained too much a practical engineer in outlook and too little a mathematician for the likes of the mathematical community.

Although the use of the name practical mathematics very slowly faded, mathematics for technical students continued after the War to be associated with an inferior treatment of the subject. Benny [1924, p.59] confessed:

I have never been able to discover exactly what [practical mathematics] is supposed to include; it appears to exclude, carefully, all elements of mathematical instruction, which render it valuable as mental training.

Piaggio [1924, p.161] made the following assessment:

Practical Mathematics ... is regarded very unfavourably by many mathematicians, and ... may be summed up as Calculus ... with as much as possible of the academic mathematics required to lead up to this and with constant reference at every stage to engineering applications.

He also claimed that 'an evil tradition has grown up that lack of logic makes an argument particularly convincing to practical men' [p.162], and he pointed to the following historical circumstances:

The pioneers in Practical Mathematics were involved in much controversy, and they denounced the ordinary mathematical course with much vehemence.... But the impetus of their attack on academic methods led them to reject the good as well as the bad, and their example has been followed by too many of their followers. [p.162]<sup>66</sup>

Thus, when the MA turned its attention to the needs of technical students, in the 1920s, the mathematical quality of the courses being provided was a major concern (see pp.175-177). As has already been shown, there was also a reaction to the wider tendency towards more practical methods in teaching, for which the Perry movement had strongly campaigned (see pp.238-242).

In a paper to the MA on 'Practical Mathematics in Schools,' Steggall [1914, p.294] exemplified the harder line in relation to practical work, and expressed the hope that:

the authorities of the Mathematical Association will keep in the future, as they have done in the past, a firm faith that the primary value of mathematical study is not to be found in such results as skill with the penny ruler, the scissors and the scales, but in the culture that all genuine mathematical study has been held to give from the days of Plato to those of Russell and Whitehead.

After the War, Mayo [1928, pp.131-139] reflected a similar point of view, and confessed that he had had little sympathy for the extreme swing from the abstract to the concrete in the teaching at Harrow. He welcomed the shift back from 'materialism' to a more disciplinary approach, aligned with a classical education [p.137] (see p.209).

Perry died in 1920 and does not appear to have received at this time general recognition for his role in the reform of mathematical

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<sup>66</sup> J.T. Combridge recalls that when he was appointed to the City and Guilds Engineering College, South Kensington (part of Imperial College), in the 1920s, Perry's legacy was still evident, particularly in practical mechanics. However, he adds 'the Directly Useful series of text books was anathema to some of my colleagues.' I am grateful to Mr. Combridge for these insights.



education.<sup>67</sup> His Obituary in Nature was written by his old friend Armstrong [1920], at a time when the pioneering Finsbury College was in a serious state of decline [Brock, 1979]. Armstrong paid the following tribute to Perry:

his real interest lay in the work of education and he will go down to fame as an original and constructive teacher who laid the foundation of a new era. He made mathematical teaching more practical.... [Armstrong, 1920, p.752]

Armstrong's emphasis on teaching methods is undoubtedly the right one, for it was here that the Perry movement achieved its major successes, as has been shown. However, after the reaction had set in, 'Perryismus' became characterized as an 'extreme' school of thought, with, at the other extreme 'the "high and dry" school, now perhaps suffering temporary eclipse, but always standing for high ideals of scholarship' [MA, 1919a, p.20]. Heywood [1925, p.326] referred to the 'Perry Controversy,' and claimed:

The division between the academic mathematicians and the practical mathematicians is not so acute as it formerly was.... But the question is far from being settled ... its importance and its potency for progress lies in the fact that it is a movement from outside.

As a 'movement from outside,' Perry's campaign had been enormously important for school mathematical reform twenty-five years earlier, and many tributes were paid to Perry before the First World War.<sup>68</sup>

Bryan [1912b, p.68] referred to the movement for the earlier treatment of calculus,<sup>69</sup> and claimed:

this and many other equally important changes owe their inception largely to what has often been described as the 'Perry movement.'

Godfrey [1908, p.252] referred to the turn of the century as a time 'ripe for change,' though he judged 'no change would come till some wave of public opinion should carry examiners and teachers together to

67 Oddly, Perry's Obituary in the Proceedings of the Royal Society did not appear until 1926 [Turner, 1926].

68 See the comments of Abbott (p.33), Hobson (p.129), and Bryan (p.245).

69 This movement will be discussed in Chapter 7.

a new position,' and he granted that 'The needful impulse came from the engineering profession.' Nunn [1911a, p.171] judged Perry's doctrine of utility to be 'the psychological principle which justified "the dethronement of Euclid" and other reforms that owe so much to the eloquence, wit, and authority of Prof. Perry.' Wolff [1915, pp.78-79] has suggested that Perry's development and advocacy of a radical alternative paradigm for mathematical education was necessary for the overthrow of the classical Euclidean tradition in England. However, although there was much early sympathy for Perry's pedagogical views, attitudes subsequently hardened to some extent, with a tendency to polarization, and his mathematical limitations became increasingly exposed. As Wolff [1915, pp.76-77] has suggested, Perry became outnumbered by the mathematical opposition, and, in his later years, his persistently argued, inflexible, and often extreme views became discredited. This conclusion is consistent with Cajori's [1917, p.290] judgement that the Perry movement 'made at once a deep and lasting impress, especially in England and America, and then spent itself.'<sup>70</sup>

In relation to the various ideals of the Perry movement summarized earlier (see p.225), it is clear that the greatest gains concerned teaching methods (see (i)-(iv)), and the achievements regarding correlation with science (see (vii)) were much more limited. The tendencies to broaden and unify the subject-matter of mathematics (see (vi)) will be considered in Chapter 7, and general shifts in the character of the three traditional branches, particularly geometry (see (v)), will be discussed in Chapter 8. However, the Perry movement was also accompanied by growing refinement in the thinking concerning mathematical education, following the polemics in the early stages of Perry's campaign, and mathematical education as a field of enquiry in its own right began to emerge. These developments form the subject of the next chapter.

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<sup>70</sup> On Perry's American influence see Moore [1903], Young [1907, pp.87-121], and NCTM [1970a, pp.246-255; 1970b, pp.39-41, 173-179].



## Chapter 6

### Educational Perspectives in Mathematics

The relationship between pedagogical thought and classroom practices is a very complex one, and, in any discussion of this subject, it is difficult and often dangerous to suggest direct causal links between theoretical prescriptions and actual changes in the curriculum. However, various arguments have been used at different times to defend established practices, and there is abundant evidence that, around the turn of the century, the general educational 'climate' was one receptive to the promulgation of a variety of innovatory ideas concerning the curriculum. There are some major distinguishable features concerning the general advance of pedagogy in England from the late nineteenth century.

Early developments concerning the study of education and its status, from the 1870s, are associated with the work of the College of Preceptors, the establishment of the Syndicate at Cambridge, and the creation of lectureships and the first professorships in this subject [Rich, 1933, pp.255-261].<sup>1</sup> Educational thought in this country developed slowly up to the end of the century, coming gradually under foreign, particularly German, influences [Lawson and Silver, 1973, p.353].<sup>2</sup> The need emerged for broader perspectives on curricula than those provided in the methods literature for elementary school teachers [Findlay, 1898; Hendy, 1898]. As Professor Adams [Cambridge University, 1902, p.34] of London University underlined, at a Conference on secondary teacher training:

It is absolutely necessary that education as a science, should justify its position in the University.... Our literature up till now has not been of the best but the need for it is all the greater on that account; our literature is greatly in need of strengthening. On the theory of education we have a great

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<sup>1</sup> On the College's early contribution see pp.69-70.

<sup>2</sup> On the foreign influences in infant education see pp.185-187. The important early twentieth-century influence of Herbart will be considered later in this chapter.

deal of good work in Germany particularly and, to some extent, in America.

With the growing involvement of the universities in the study of education and teacher training, from the 1890s, the status and quality of these aspects of professionalization gradually improved, though the struggle was a protracted one in the case of secondary schoolmasters (see pp.77-80). From 1892 the lecturers in training colleges became organized as the Training College Association, and the subsequent development of interest in the scientific pursuit of educational questions is reflected in the title of this Association's organ from 1911 - the Journal of Experimental Pedagogy and Training College Record [Webb, 1915a, p.8; Tropp, 1958, p.163]. Various other sectional interests in education became organized in the late nineteenth century.

Webb's [1915a, 1915b] valuable survey of English educational organizations clearly demonstrates the richness and complexity of the involvement of teachers and others concerned with various aspects of elementary and secondary education, which had developed by the First World War. Organizations became increasingly specialized, and became concerned for the advancement of specific subjects in the curriculum.<sup>3</sup> In mathematical education, this activity was conducted at local, national and international levels.<sup>4</sup> In addition to the College of Preceptors, other organizations like the Teachers' Guild, from 1884, sought to involve teachers generally in the advancement of their art [Webb, 1915b, pp.13-14], and the creation of the new Section L of the BAAS in 1901, devoted to 'educational science,' is a further sign of the growing level of educational concern, though, in this case, the involvement of school-teachers was limited [Collins, 1979b].<sup>5</sup> Remarkably, by the end of the first quarter of this century, the

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3 On the secondary subject associations see p.138.

4 See pp.130-137 for the work of the ICTM from 1908, and the South-Eastern Association from 1911.

5 For some of the background concerning Section L see p.99.



Schoolmasters Yearbook [1926, pp.10-50] included descriptions of as many as sixty-six educational societies and organizations, as well as a list of seventy-eight other associations. Organizational developments were naturally accompanied by developments in the educational press.

The importance of the developing educational and scientific literature for the early stages in the reform of mathematical education was emphasized in Chapter 3 (see pp.97-98). Tropp's [1958] survey clearly demonstrates the notable expansion and increasing specialization of the educational press during the late nineteenth and early twentieth centuries. Issues, ideas and developments in mathematical education were not only publicized in the columns of general educational periodicals such as the Educational Times, Journal of Education and School World, but also in more specialist journals such as Nature, the Mathematical Gazette and the South-Eastern Association's Journal, as well as in the organs of sectional associations like the AMA and the Association of Preparatory Schools, and in the newer Technical Journal (1908) of the ATTI, the School Science Review (1919) of the APSSM, and the Times Educational Supplement (1910).

In the light of the various developments mentioned thus far in this chapter, it is not surprising to find Selleck [1968, p.104] referring to the period 1890-1914 as one during which there were 'frequent references to an increase in self-criticism, to a growth in professional awareness among teachers.' These were critical years for the 'new education' which was neither a unified nor coherent movement, though contemporary references to it were commonly made [pp.102-103]. The comprehensive character of pedagogical advances from around 1890 has also been well summarized by Lawson and Silver [1973, p.356-357]:

A range of new departures was becoming evident in English education in the 1890s. There were new schools and new interests in child study and child development.... The constraints imposed by payment by results were being removed from the elementary schools, and new approaches in the infant school were beginning to have an influence

on the higher classes. Technical subjects and science were not only finding a place in schools, but were also the focus of new thinking about methods and objectives in teaching. For most of the nineteenth century the major changes in education had been in terms of supply and structure. Under new pressures, changes in the final decades also began to focus on content and method, and on children. The search for a new understanding of children and of educational processes was closely related to the wider changes of emphasis in discussions of the individual, society and social policy.<sup>6</sup>

By around 1920, it had become appropriate to refer in the general educational literature not just to the 'new education,' but to the 'new teaching' [Adams, 1918] of the 'modern teacher' [Watson-Bain, 1921], and Adams [1924, p.9] could refer to more than half a dozen recent secondary methods books which embodied the new thinking. Adams [1918, p.8] could also claim that 'the leaven of progress is working throughout the whole teaching body' and that 'we have abundant proof that teachers as a class think to-day of the technique of their work in a way they have never done before.' Adams also drew attention to one notable feature of the new outlook: 'It is recognised that in the ultimate resort a subject must be approached from the standpoint of the pupil, rather than of the teacher' [p.11].

Towards the end of the nineteenth century, although the 'new education' embraced diverse educational standpoints, it was at least united in its rejection of the older disciplinary view of education [Selleck, 1968, pp.336-337]. The character of the educational outlook which dominated throughout much of the nineteenth century, and which was applied to mathematics in particular, warrants a closer inspection.

#### Mental Discipline and Faculty Psychology

The mathematical examinations of the ancient universities,

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6 The relevance of these generalizations concerning the influence of infant, scientific and technical education, in the case of mathematics, was demonstrated in the last chapter. The wide-ranging character of educational turmoil prompted one writer to judge that major change in mathematical education was 'inevitable' (see p.123). See also p.225, for a summary of the pedagogical maxims of the early twentieth-century Perry movement.



particularly Cambridge, came to exert a formidable influence upon public and grammar schools. One consequence was the high premium placed upon examination successes, at the expense of intrinsic educational objectives.<sup>7</sup> As Macaulay and Greenstreet [1912, p.211] remarked:

The schoolmaster has for some fifty years been accustomed to the control of the Universities over curricula and methods. The kudos attached to the winning of a scholarship is considerable ... in a large number of instances it may be said that almost the sole measure of the school's success is the number of these scholarships that it wins.

General educational purposes in mathematics were overshadowed by the needs of specialists in boys' and, later, girls' schools also (see pp.44,71).<sup>8</sup> The character of mathematical education was dominated by examination requirements. A writer in the Educational Times subsequently referred to 'the monopoly that Cambridge had established' [College of Preceptors, 1912, p.229], with this university remaining largely above criticism, although it 'revelled in a liturgy of academic problems, dextrous manipulations, and examination pitfalls' [p.229]. This writer drew attention to the general distortion of mathematical education, in such forms as:

the collection of an encyclopaedia of curiosities, the elaboration of rules and processes for each particular problem, the classification of appropriate dodges, and an amount of over-teaching and over-learning which, praised as 'thorough' by the author, has been the worst form of cram. The tendency to over-appreciation of details extends to the lowest ranks. [p.229]

The attitude to the majority of pupils caught up in this system was summarized by Godfrey [1912a, p.161]:

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7 See pp.70-74 for details of the Mathematical Tripos and its influence.

8 Godfrey [1906a, p.77] openly attacked this system in the Cornhill Magazine. He feared generally that 'university dons have no sense of their responsibility for English education,' their 'chief preoccupation' being 'to select and purchase the best brains in the market,' mainly for purposes of advertisement. He added, 'In the same way, the public schools buy up the brains of preparatory schools, to turn them into university scholars.'

The aristocratic theory of education has determined the choice of subject-matter in teaching mathematics in schools..... In the past - the not distant past - the assumption was made tacitly that mathematics could appeal only to the few; that the average boy was essentially stupid and more or less a hopeless problem. This entirely false assumption arose from the aristocratic theory.

Hobson [1912, p.236] also referred to this 'anti-democratic' principle, and to the rationale which neglected individual differences and fundamental questions of content and method:

the notion of Mathematical teaching was that it should be in the main medicinal and corrective. Its advantages consisted largely in calling forth the use of faculties which are the rarest in the average boy or girl, and were therefore thought to be in special need of development. It was thought to be by no means wholly a disadvantage that [the branches of mathematics] were found hard and repulsive by the majority. It was thought that the hard discipline involved in the attempt to assimilate them developed a kind of mental grit, and involved a certain species of moral training, even when the intellectual results were small. [p.236]

Growing concern for a wider ability range in curricular planning and practices is one feature of the reform in mathematical education. Godfrey [1912a, pp.161-162] referred to it as the rejection of the older 'aristocratic theory' and Hobson [1912] as 'the democratization of mathematical education,' which meant in secondary education:

the concentration of the attention of the Educator, in a much greater degree than formerly, on the work of developing the minds of the average many, and not solely of those of the exceptionally gifted few.<sup>9</sup> [p.235]

Reports of Committees of the HMC and the MA before the First World War focused specifically on the mathematics curriculum for 'non-specialists' (see pp.150-151,163).<sup>10</sup> Their needs became more pressing with the declining hold of the disciplinary view of education, based on a notion of mental faculties.

The greek roots of faculty psychology have been traced by Burt [1938]. The embryonic ideas were refined by the German Christian Wolff

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9 Hobson's 'average many' was only the small minority of the adolescent population passing through public and secondary schools.  
 10 Greater attention to individual differences also developed in America after 1900 [NCTM, 1970b, p.193].



(1679-1754), who hierarchically organized different postulated faculties of mind such as memory, imagination, reasoning and will. Burt suggests that the popularization of faculty psychology in the nineteenth century was probably due to the phrenologists, whose theory localized many distinct faculties in different parts of the brain [pp.429-432]. As Selleck [1968, pp.46-47] has pointed out, there were three major axioms involved in the educational arguments concerning mental training. These are that separate faculties exist; that they have the potential for training through exercise; and that transfer is possible from specific to more general contexts. The analogy with physical training is close, with faculties regarded as mental muscles which can be exercised, and thereby developed for general use, through specific mental tasks involving suitable subject-matter, such as learning to reproduce perfectly a theorem in Euclid or to implement a complicated algorithm in arithmetic or algebra. Extreme forms of 'drilling' could be justified educationally as a means of strengthening the various faculties of mind, including the character.

The assumptions concerning mental training are implicit in the Reports of the two Royal Commissions [Clarendon, 1864; Taunton, 1868] on secondary education in the 1860s, as well as the later Royal Commission on this subject [Bryce, 1895a; BE, 1938, pp.129-130]. The Public Schools Commission [Clarendon, 1864, p.15] claimed that 'mathematics at least have established a title to respect as an instrument of mental discipline; they are recognised and honoured at the Universities.' The Schools Inquiry Commission [Taunton, 1868, pp.29-30] admitted that 'No one can doubt the value of geometry as an exercise in severe reasoning,' though found it less easy to justify the teaching of other branches, and referred rather feebly to algebra as affording 'admirable examples of ingenuity.' The Bryce Commission [1895a, p.135] confidently adopted the faculty training argument:

All education is development and discipline of faculty by the communication of knowledge, and whether the faculty be the eye and hand, or the reason and imagination, and whether the knowledge be of nature or art, of science or literature, if the knowledge be so communicated as to evoke and exercise and discipline faculty, the process is rightly termed education.

In elementary education, Selleck [1968, pp.45-58] has shown how faculty psychology was widely used to rationalize prevailing practices during the period of the 'instrumentary education' of the masses, from the 1860s to the 1890s, and was commonly assumed by writers of manuals of method . Although the theory was not important for the framing of the elementary curriculum, it came to play a very important part in its defence. As well as the supposed benefits for the intellectual faculties, the 'instrumentary education' was also defended as a means of character formation for the presumed unruly Victorian working classes. Thus, as Selleck [p.58] has concluded:

It was important to undermine faculty psychology because it was the most obvious intellectual prop supporting the traditional education....<sup>11</sup>

The attack on faculty psychology came in essentially two forms. The first, principally associated with J.F. Herbart (1776-1841) and his interpreters, the Herbartians, totally rejected the model of faculties and postulated an alternative model of the mind based on ideas, interests and a new view of the learning process. The attack in England was helped forward by the publication of Adams' [1897] The Herbartian Psychology Applied to Education, which has been singled out by Burt [1938, pp.432-433] and Selleck [1968, p.258] as a seminal influence on English educational thought. Nunn [1911a, p.171] referred to this book as an 'educational classic,' and boldly claimed:

the central purpose in teaching mathematics is not to 'train the power' of reasoning, of generalising, of 'mental accuracy' etc. The fallacies embodied in the persistent heresy of

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11 The disciplinary theory was also dominant in America up to around 1890, when counter-arguments emerged [NCTM, 1970b, pp.27-36, 99-102, 155-162]. The newer practical and heuristic methods in education were also partly advocated for their benefits in terms of faculty training (see pp.190, 194).



'formal training' have been repeatedly exposed, and need not detain us here. The time should soon come when an educational writer may ignore them. [p.167]

Godfrey [1912d, p.294] also referred to a 'line of division ... between the mental-and-moral-discipline teachers, and the school who, consciously or unconsciously, follow Herbart,' and judged that the former school's 'star' was 'no longer in the ascendant.'<sup>12</sup> The other form of the attack on faculty psychology focused on the evidence concerning transfer of training.

A key question was the extent to which mental faculties exercised in specific fields could be transferred to other fields. Was there any evidence to suggest that the hypothetical construct of a general faculty was a reasonable one? William James raised the general question of transfer in America during the 1890s, and Thorndike and his co-workers pursued the matter experimentally from 1901 [NCTM, 1970b, pp.186-187]. Hamley [1938, p.440] subsequently summarized the implications thus:

Others [e.g. Herbart] had disposed of 'faculty psychology' by denying that faculties exist; Thorndike achieved the same result by taking from faculties their real meaning. Henceforth all abilities became individual and specific.

These conclusions thoroughly undermined the assumption that, in arithmetic for example, a strenuous period of drill could develop at least some general faculties such as accuracy, quickness, discrimination, memory, observation, attention, concentration, judgement, reasoning and so on [pp.439-440]. The similar relentless forcing of pupils through other branches of mathematics, without considering individual differences or motivation, could no longer be simply justified in disciplinary terms.<sup>13</sup>

In the early twentieth century, the arguments concerning mental discipline were certainly not eliminated, but, rather, utilized more

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<sup>12</sup> Herbart's thought, and refinements in the discussion of educational aims in mathematics will be explored later in this chapter.

<sup>13</sup> In America, by 1920, the older disciplinary argument had been soundly discredited, and a major problem was to find alternative rationales [NCTM, 1970b, pp.193,196].

cautiously and refined [BE, 1938, pp.77-78]. For example, Godfrey [1912a, p.166] admitted:

when it is said that mathematics develop the memory, the logic and reasoning faculty, the power of generalisation - develops all these powers as applied not only to mathematics but also to general activities - well, I hope that it may all be true but I have not met with the proof.

As Godfrey pointed out, transfer from mathematics to other domains was a matter of degree, and, even in the case of obviously related subjects like science, the question was far from straightforward.

He concluded:

if it be so difficult to 'carry over' mental habits from one department to another within a school, how much more difficult is it to 'carry over' from school into after-life. Perhaps this object would be better attained (if it be attainable) by providing for it more deliberately. [p.166]

One possible means of facilitating transfer was to emphasize correlations between mathematics and its applications in other subjects. The idea of generally correlating studies across the curriculum became a prominent one in pedagogical thought, with important implications for mathematics in particular.

### Correlation

By the turn of the century, the principle of correlation or the co-ordination of studies had become an established part of educational rhetoric. Barnett [1899, pp.117-118] claimed 'The necessity for concentration or connectedness in studies should be pretty obvious,' and he pointed to the presumed benefits of better understanding and retention. He added, 'The more points of rational connexion there are between our ideas, the more knowledge are we said to have, the better we understand the world,' and 'We must ... concentrate and connect so as to save effort in details.' In theory, by applying this principle time would be saved, and so the curriculum could be broadened. Findlay [1902, p.185] referred to correlation and self-activity as principles which were 'the stock-in-trade ... of every lecturer on education.'



In relation to mathematics, he pointed out that there were two aspects to correlation. Firstly, there was the outward-looking aspect of relating mathematics to its applications, particularly in science. Secondly, there was the inward-looking aspect, which 'emphasizes the necessity of bringing the various parts of elementary mathematics in correlation with each other' [p.185].<sup>14</sup> Correlation was an attractive notion in principle, though, thinking more pragmatically, Godfrey [1912a, p.166] referred to it as 'that system so dear to educationists and such a bugbear to practical teachers.'

Selleck [1968, pp.239-241] has traced back to Ziller, a disciple of Herbart, the idea of concentration of knowledge around some core, though there are only loose links with the later and wider emphasis on correlation in various forms [pp.251-252]. By the First World War, some of the possible links between mathematics and geography, history, drawing, manual training, and, particularly, physical science had been explored [Wolff, 1915, pp.170-179]. The terms of reference for the MA's [1919a] general Report included the principle of co-ordination, and various aspects of this principle featured in the general recommendations (see p.167). In the case of elementary schools, links between arithmetic, practical geometry and mensuration, and other subjects such as manual training and science came to be explored in some detail, at least in principle [BAAS, 1907, pp.438-444; LCC, 1911, pp.57-77; Millis, 1925, pp.93-96]. The interest in correlation was world-wide, particularly concerning the links between mathematics and physics, as Smith and Goldziher's [1912, pp.73-80] Bibliography demonstrates. Over ten per cent of the items listed for the period 1900-1912 were placed in the category 'correlation and applications of mathematics.' The obvious links between mathematics and science teaching were explored in detail

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14 Within mathematics, the principle was one of unification or fusion, which might be achieved through an emphasis on fundamental ideas and general thought patterns. This aspect will be discussed in the next chapter.

in the last chapter. What is perhaps surprising is that links with other subjects like geography were also explored in the early twentieth century.

Nunn [1908] presented a paper to the College of Preceptors on 'Science in Correlation with Geography and Mathematics.' On historical grounds, Nunn argued for greater attention to the instrumental value of mathematics, and for the need to exploit problems 'that have a content of positive value and interest to the boy,' which would provide 'motives for the invention of the ordinary apparatus of mathematics' [p.177]. The status of geography in the curriculum had recently improved, and Nunn referred to it as a 'substantive' subject concerned with the study of 'the surface of the earth as the home of man' [p.177].<sup>15</sup> Even with this somewhat narrow emphasis, there were obvious possibilities for correlation with mathematics, particularly geometry and mensuration applied to the earth.

Nunn was also implementing some of his ideals concerning correlation in the demonstration schools of the London Day Training College. The work here was subsequently described by Branford and Nunn, in the impressive Report of the LCC [1911, pp.57-77], which devoted a long section to co-ordination between mathematics and other elementary school subjects. The following extract illustrates the application of Nunn's ideas for correlation:

the method of fixing the position of a point by rectangular co-ordinates may be 'invented' as a means of representing the relative positions of localities in the neighbourhood of London - a means which develops into the conventional method of latitude and longitude only when, on extending the scale of our operations, the difficulties due to the curvature of the earth make their appearance. So again, the notion of the tangent of an angle may be reached as a means of standardising the results which different members of a class have obtained in their study of the variation of the length of noonday shadows. In both these instances the problems upon which mathematical machinery designed

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15 Various aspects of the history of geography teaching have been considered in dissertations and theses [History of Education Society, 1979b, pp.118-120].



ad hoc has been brought to bear lie within the continuous province of Geography. [pp.58-59]

These problems could also be extended further, to bring in graphical work in the first case, and simple surveying in the second. This form of correlation required one teacher to exploit fully the links, though as the Report of the LCC pointed out, other forms of correlation were the parallel treatment of two different aspects of a topic by different teachers, and the sequencing of subject-matter in one subject so that it could be subsequently applied in another subject [pp.55-58]. There is some evidence that particular secondary teachers were sensitive to the issues involved, and that the curriculum in mathematics was being adjusted accordingly.

Wallis [1910] described the arrangements in one grant-aided secondary school, where discussions had taken place between the teachers of mathematics, science and geography. As a result, the mathematical masters took over much of the first-year work in physical measurements, and brought in some early work in decimals, approximations and trigonometry. In addition, to serve the geographers, 'much practice in graphical work and in the measurement of areas by means of squared paper' was provided [p.129], as well as:

much practice in solid geometry and section-making by means of the building up of some of the regular solids from stiff drawing paper and the cutting of these models for the purposes of showing sections. [p.129]

The science teachers also required the early teaching of logarithms, and the use of a simple slide rule, as well as some simple work on limiting values and gradients.

Wallis [1912] also gave a paper to the London Branch of the MA on correlation between mathematics and geography. Here he underlined the need for an earlier mathematical treatment of similarity, which was traditionally tackled relatively late, if at all, as Euclid VI, and the introduction of some simple trigonometry. These modifications

would permit a fuller treatment of the globe, maps and surveying in geography. He pointed out that geography teachers also required the early use of four-figure logarithmic tables, or a simple slide rule, and made much use of approximations. Prophetically, he referred to the value of statistics teaching, beyond work on averages, as:

The adult finds a continuous appeal made to his statistical knowledge.... The gullibility of the modern man in the face of numerical statements is a reproach to both the geographer and the mathematician. [p.450]

Helen Bartram's [1912] paper for the Special Reports provides much interesting detail concerning the links between geography and practical geometry. As well as developing many of the suggestions of Wallis, she referred specifically to the measurement of curves using a dressmaker's tracing wheel; the use of angles for directions, bearings and the points of the compass; measurement of gradients and the principle of contour lines; and the principles of latitude and longitude. There is little evidence concerning the extent of actual correlation with geography, though in the case of girls' schools Story [1912, p.553] found that in 'some schools' the two subjects were 'more or less closely' correlated, with reference in particular to the topics of similarity, graphical algebra and mensuration. There is evidence of some continuing interest in the possibilities, after the War.

A course for secondary teachers at Durham, in 1920, devoted some time to the question of correlation, including references to geography [Westcott, 1920] (see p.81), and correlation with this subject was also the focus for an article in the Mathematical Gazette in the same year [Fairgrieve, 1920]. However, as Bolton [1928, p.64] subsequently emphasized, the value of correlation here:

has been pointed out again and again, but, so far, I cannot see much more than an indication of the interaction of the two subjects. Here again we are dependent on the supply of teachers with the right training....



The premium placed on narrow specialization for secondary teaching was a major barrier to general progress, though Bolton did point out that, at Cambridge, geographers were encouraged to read Part I of the Mathematical Tripos. It seems likely that close correlation between these two subjects was a far from general phenomenon, but the existence of some interest in the possibilities is noteable nonetheless. Bolton also referred to the potential links with the workshop and manual training courses, which was another potential area for correlation.

The late nineteenth-century interest in various forms of manual training, as one component of a 'practical education,' and the early implications for teaching methods in mathematics were considered in the last chapter (see pp.188-192). There is some evidence of interest in the possibilities of correlation here, in boys' secondary schools, though less than in the cases of science and geography.<sup>16</sup>

Oundle School was well equipped with a workshop, power room and engineering laboratory, and Sanderson [1912] described, in the Special Reports, the School's 'workshop method of practical mathematics' for the ten to fifteen age range where:

the experiments, tests, measurements, designs, which form the regular part of the work in the shops involve the use of a wide range of Mathematics, and in carrying them out it will be found that a boy will learn his Mathematics as he learns the use of tools by constantly applying them. [p.412]

Thus, at Oundle, the work in the mathematical classroom was largely motivated by problems from the workshop and laboratory.<sup>17</sup> Usherwood [1912] exhibited a similar philosophy, in the case of older pupils on the engineering side at St. Dunstan's College. However, under the influence of Dewey and Branford in particular, he also argued a case for the close correlation of mathematics and the simpler forms of manual training, linked also with science [pp.64-67]. He briefly

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16 Possible links between mathematics, drawing and carpentry are considered in Wolff [1915, pp.175-179].

17 See also Sanderson's evidence for the BE's [1913c, pp.274-278] Consultative Committee.

reiterated his argument in the MA's [1919a, pp.26-27] general Report, where he also drew attention to the reality:

In but few [secondary] schools - so far as can be discovered - is there any deliberate attempt at organising a scheme in which full advantage of workshop facilities is taken.... In most Secondary schools no direct use is made of the Manual Training School in the mathematical class-rooms: the reason probably is that a definite and reasoned mathematical scheme was bearing fruit long before a workshop was contemplated, and the economical aspects of correlation have not been considered or realised. [pp.26-27]

Another inhibiting factor would have been the very different status and perceptions of the teachers of mathematics and manual training, with the former identifying themselves with the ideals of a liberal education, and the latter being closer to the needs of industry and technical education. On the question of the status of manual work, Selby-Bigge remarked in a Prefatory Note to the Consultative Committee's large Report on Practical Work in Secondary Schools:

progress must depend on the creation of an opinion in favour of making Secondary Education less bookish than it has been, and on a determination to abandon the attitude towards these branches of work. [BE, 1913c, p.iii]

However, such attitudes were firmly established and resistant to change.

The Consultative Committee's Report reflected the view, by the First World War, that simpler exercises such as paper folding, cutting and modelling should not be treated as a separate subject, labelled 'constructional handwork,' but rather be adopted as part of the teaching strategy in mathematics [p.20].<sup>18</sup> The Report referred in particular to practical work in the measurement of length, area and volume, drawing to scale, graphical statistics, and the study of plane and solid shapes [p.33]. A detailed scheme was included in the appendices [pp.92-94], as well as a scheme of modelling for elementary schools, correlated with 'practical mathematics,' 'practical science,'

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<sup>18</sup> The tendency to divorce 'practical arithmetic' from ordinary arithmetic was discussed in the last chapter (see pp.210-211).



and even history [pp.94-100]. One witness, an HMI, enthusiastically described his experiences in one girls' elementary school, where 'handwork as an educational method' followed the general pattern adopted in Buckinghamshire:

Class 2, for example, were making practical acquaintance with areas by means of diagrams drawn by every girl on her desk - understood well too; then they rose from their places and 'attacked' attendance board, partition panel, blackboard, window pane, bench, classroom door, partition door, map, etc., with measuring implements, and accurately recorded on paper their findings - businesslike way - valuable. Class 1 did similar work, but with this addition, that they first eye-judged dimensions of various objects, then recorded in writing their estimates, then verified by actual measurement what they had done: thorough work. In Class 3 - area, practical teaching by diagrams - excellent written work based on actual measurements. [p.168]

In this particular case, some of the simpler forms of 'constructional handwork' had become part of the mathematics curriculum. Various features of correlation were advocated for and exploited in some elementary schools in the early twentieth century.

The BE's [1905a, p.43] first Suggestions referred to correlation between arithmetic, drawing, elementary physics and 'hand and eye' training, as well as suggested the application of arithmetic to personal finance, and domestic economy for girls. The revised Suggestions [BE, 1912c, pp.7-9] also discussed correlation and suggested the making of some measuring instruments and geometrical models in the manual workshop. Not surprisingly, the Committee of the BAAS [1907, pp.451-459] strongly advocated co-ordination of studies, particularly between mathematics and elementary science, and even suggested the use of a general thematic approach to such topics as children's ages [pp.455-456].<sup>19</sup> There is some evidence of correlation in practice, particularly in London elementary schools, which were no doubt influenced by Inspectors such as Branford and Ballard, as well as by Nunn, and the

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19 Twenty-five years later the BE's [1931, pp.80-85] Consultative Committee discussed the 'project method,' though still in rather speculative terms.

champions of manual training, like Magnus and Millis.

The 'co-ordinating method' of the educationist was adopted by Branford [1908, pp.263-269] as one of two major governing principles in his theory of mathematical education, the acknowledgement of the need to consider stages in development being the other. He argued for correlation across the curriculum and with out-of-school life:

though it is necessary to isolate school studies from each other and the world in order to conquer in detail, it is equally necessary to be continually reuniting the parts, lest [the teacher] finally fashion a being whose intellect is as a house with many chambers lacking doors and windows alike. [LCC, 1911, p.12]

Nunn shared similar ideals and was implementing them in the London Day Training College's demonstration schools, a detailed syllabus having been developed [pp.62-77] (see pp.259-261).<sup>20</sup> The scheme ambitiously exploited problems from geography, handwork, nature study, physical measurements, mechanics, surveying, navigation and finance. Findlay was also experimenting on similar lines in the Fielden Demonstration School, at Manchester (see p.211).

Some general advances in London elementary schools were reported by Ballard [1912, pp.14-15], who referred to a 'movement' for correlating mathematics and manual training, which was 'rapidly spreading,' and sustained by conferences on this subject. The need for a correlated treatment of mathematics, manual training and science, particularly as a preparation for a technical education, was strongly pressed at conferences and in publications before the War, and, by 1925, there had been some gains, particularly in the central schools [Millis, 1925, pp.92-96]. Ballard [1912, p.15] also reported that 'The tendency towards practical measurement has promoted the development of the mathematical aspect of art, science, and geography,' with a 'few

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20 After the War, Nunn [1925] continued to advocate correlation in elementary schools, and a Consultative Committee's Report [BE, 1926a, pp.214-220] appears to have closely followed Nunn's prescriptions in mathematics, though there is also some evident frustration expressed concerning actual practices by this time.



schools' correlating arithmetic and map work, and a 'large number of schools' recording temperatures and pressures daily, the corresponding graphs being 'hung upon the walls of nearly every class-room.' He concluded 'Everywhere, in fact, we find traces of the invasion of practical arithmetic' [p.15].

The principle of correlation was one aspect of pedagogical thought, with various implications for mathematics, as has been shown. However, this was only one particular feature of the refinement of educational thinking in mathematics, which developed in the early twentieth century, in parallel with the various reforms in the teaching of this subject.

#### Studies in Mathematical Education

General advances in the study of education, and the growing pedagogical awareness of teachers, were considered earlier in this chapter (see pp.249-252). The development and increasing specialization of educational organizations and periodicals provides evidence of marked progress, and the specialist contribution to mathematical education in the early twentieth century from the BE, the BAAS, the MA and its Branches, the ICTM, and the South-Eastern Association, as well as the involvement of other educational organizations, was a major focus for Chapters 2-4. Such organizations made an important contribution to the development of the literature in mathematical education particularly during the period of reform.

The English pattern of including chapters on the teaching of the branches of elementary mathematics in general methods books, for teachers in elementary and secondary schools, was well established by the end of the nineteenth century.<sup>21</sup> However, German and American

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21 See, for example, Garlick [1898], Spencer [1897] and Barnett [1897, 1899], and, subsequently, Welton [1906] and Adamson [1907]. For elementary schools, teacher's guides, to accompany the pupils' arithmetic books, were common in the early twentieth century [Spencer, 1912, p.46].

writers produced separate books on mathematical education before this development occurred in England. Young [1907, p.4] referred to Germany as 'the home of pedagogy,' and to Klein's leadership in the field of mathematical education in particular. The NCTM [1970b, p.307] have acknowledged that 'prior to 1900 most influential ideas on mathematics teaching had been imported to America from abroad,' and this influence was reflected in the general methods literature. Germany and America were also in advance of this country in mathematical teacher education (see p.60).

Mathematical methods books written in English filtered over to this country from America. Although the NCTM [1970b, p.42] have traced the publication of such books in America to the mid-nineteenth century, they judged Smith's [1900] early contribution to be 'the first book to resemble a modern methods text.'<sup>22</sup> Young's [1907] book followed, and, significantly, he held the post of Associate Professor of the Pedagogy of Mathematics, at Chicago University. The American books by Mclellan and Dewey, Smith, and Young, were listed in English methods books.<sup>23</sup>

Branford's [1908] A Study of Mathematical Education was the first English publication of its kind, and it included much material from articles in the Journal of Education, published around 1900. As a University Press, the publishers OUP appear not to have been influenced by the limited commercial prospects of such a venture at this time.<sup>24</sup> Carson's [1913c] Essays on Mathematical Education followed, this being largely a collection of previously published articles. The publishers, Longmans, launched an ambitious and unprecedented series of books on

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22 On Smith see pp.5-6, 102-103. Mclellan and Dewey's [1895] The Psychology of Number was another notable early American contribution to mathematical pedagogy.

23 See Welton [1906, pp.482-483] and Adamson [1907, p.349].

24 OUP also published the exceptional textbooks on general mathematics by Mair [1907, 1911], who acknowledged a debt to Branford.



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Illustration 32 A Publicity Leaflet for  
Longmans' Modern Mathematical Series

mathematical education before the War.

In the early twentieth century, Longmans were struggling to regain their earlier commercial strength in the publication of school mathematics textbooks. Thus, J.W. Allen of Longmans approached his friend Nunn, with whom a series of books was planned. Of course, Nunn's interests were principally pedagogical rather than commercial, and the publishers were accordingly:

landed with books of great value in some cases to the teaching profession, but from Longman's point of view it was a wasted opportunity and a considerable loss. [Parker, 1978, p.5]<sup>25</sup>

Longmans were seeking a series of textbooks to compete with the highly successful output of Macmillan, Bell and CUP. However, the Modern Mathematical Series edited by Abbott, Jackson and Macaulay was largely a pioneering contribution to mathematical pedagogy, from a number of leaders in this field. (See Illustration 32.) The series included Dobbs [1913] innovatory textbook on motion geometry, which 'failed abominably,' a seminal methods book by Nunn [1914], with an accompanying set of exercises, though these sold 'only in trifling numbers,' and Punnett's [1914] methods book which 'fared similarly' [p.2]. The handbooks on arithmetic and geometry teaching, by Abbott and Carson respectively, do not appear to have been published. A later addition to this Series, Abbott's [1918] Numerical Trigonometry enjoyed 'moderate success' [p.2], and Boon's [1924] A Companion to Elementary School Mathematics, though initially 'a disappointing failure' [p.9], was reprinted in 1960 with immediate success. The American Miller [1916] devoted a section to the pedagogy of mathematics in his general survey of the mathematical literature, though the books by Carson and Nunn were the only English entries. By this time, a number of books had also been published which, although written by mathematicians, were

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25 I am grateful to Mary Ollis of Longman for providing me with valuable help and material concerning their school mathematical publishing history.



aimed at teachers of mathematics.

Klein's [1908] book, originally only in German, provides an early example of the twentieth-century trend, where mathematicians contributed their own perspectives on elementary mathematics. A similar such book was produced by Whitehead [1911b] in this country, and a set of Monographs on Topics of Modern Mathematics Relevant to the Elementary Field was edited by the American Young [1911]. Similar contributions were provided by Laisant [1913], in the Thresholds of Science Series, and Jourdain [1912] in the series of People's Books.

Thus, the literature in mathematical education had developed considerably by the First World War, and Milne [1918] drew attention to the need for school mathematical libraries, particularly for the teachers' use, as he claimed that in mathematics:

There is nothing more extraordinary in the educational world at the present day than the change in attitude among teachers towards the subject.... [p.209]

The MA [1926g] subsequently produced its first List of Books Suitable for School Libraries, which included American and Continental works, and many entries under the various branches of the subject, as well as history and pedagogy.

By coincidence, three additions to the English literature in mathematical education appeared in 1931, the 1920s having been a quiet period in this respect. Durell [1931] focused on algebra teaching, and the influence of Nunn [1914] is clear. Godfrey and Siddons [1931] and Westaway [1931] considered the whole of elementary mathematics. The more penetrating parts of Godfrey and Siddons' book were written by Godfrey, about twenty years earlier, and some ten years separated the idea of such a book from its eventual publication [Siddons, 1952b, pp.12-13]. Westaway [1931, p.xiv], a former HMI, acknowledged the importance of Nunn's thinking in the teaching of algebra, trigonometry and calculus, and he claimed that 'The methods he advocates are methods

which have been amply tested and found to be sound and practical.'

In a later List of Books, produced by the MA [1936], Westaway's book was described as 'The only modern attempt to survey the whole field of teaching of elementary mathematics' [p.21].

Apart from the three books just mentioned, the best known English books on mathematics..teaching, between the Wars, were first published before the War [MA, 1932b, p.335]. A Bibliography by Black [1938, p.3], devoted wholly to mathematical education, referred to the 'enormous and increasing literature of books on the teaching of mathematics.' However, many of the entries were foreign ones, and it does appear that much of the best work in this field was produced during the period of great innovatory activity up to 1914. The character of the thinking in mathematical education, which developed in the early twentieth century, has still to be considered.

The Perry movement's pedagogical principles, the historical background to these newer ideals, and the dominant nineteenth-century view of mathematical education have already been considered in this and the previous chapter. The importance of Herbart's thought for the rejection of the older disciplinary arguments was pointed out earlier (see pp.256-257). However, more positively, Herbart also provided an alternative theoretical model for educationists, with particular relevance to mathematics.

Detailed discussions of Herbart's thought and its development have been provided by Knox [1975] and by Selleck [1968, pp.227-272], who has also discussed Herbartianism as one feature of the 'new education' in England. Herbart's influence can be traced to the 1880s in America, before it was felt in this country [pp.242-244]. The NCTM [1970b, pp.110,115,305-306] have drawn attention to the visits of young Americans to Germany, who brought back various new ideas, which influenced teacher training courses.in particular, and disseminated such



features as the 'doctrine of interest,' the organization of knowledge around 'fundamental meanings,' and, more basically, the use of 'steps' in lesson planning. The German's influence in this country came to be felt during the last decade of the nineteenth century.

Findlay studied education in Germany, in the early 1890s, and became one of the earliest English Herbartians [Selleck, 1968, p.208], who sought, in particular, to enrich training courses in education with the wider and deeper German perspectives [Findlay, 1898, pp.344-346]. An early article on 'Herbart's View of the Place of Mathematics in Education' [Greenstreet, 1894] was atypical for the Mathematical Gazette, but, more generally, Adams' [1897] book did much to popularize Herbart's views (see pp.256-257), and it became 'the vade-mecum of the Herbartians,' according to Selleck [1968, p.243].

Herbart rejected the hypothetical construct of innate mental faculties, and, instead, saw the mind as initially a void, into which ideas enter and compete for dominance, following an elaborate set of laws. 'Apperception' was the key process of bringing ideas to consciousness, with compatible ideas combining to form 'apperception masses.' Herbart regarded it as a natural mental tendency for ideas to seek compatible ideas, and thereby rise to prominence, and for opposed ideas to retreat below the threshold of consciousness. New ideas could only be assimilated to already established ideas, and, overall, 'The better organized the apperception masses were, the more closely they were related to each other, then the more easily and effectively a person would think' [p.231]. Thus, a key educational objective was to cultivate in the individual's mind a large and closely connected 'circle of thought,' or totality of apperception masses.<sup>26</sup>

Although character formation and moral development were the

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26 As Knox [1975, p.274] has pointed out, there are some obvious links between Herbartian and Piagetian thought concerning the nature of intellectual functioning.

principal purposes of education for Herbart, he sought to achieve such ends through the development of the circle of thought, closely linked with the development of interest on many sides. Herbart's view of motivation enters his theory here. He did not regard interest as an extrinsic motivator to 'sugar the pill,' but rather as an intrinsic part of learning itself. Interest he saw as an end, and not a means in instruction, with its balanced and systematic development on many sides facilitating the growth of the circle of thought. This was the 'doctrine of interest,' which Adams championed in England, though it was often misinterpreted by advocates and critics alike. As Nunn [1911a, pp.171-172] remarked:

To say that in teaching we must consider the natural springs of intellectual activity is to enunciate the modern 'doctrine of interest.' This doctrine is often worst served by those who proclaim themselves its chief friends. It is consequently either feared or despised by many sound teachers.... They think that the presence of interest implies pro tanto the absence of hard work and 'discipline.' In reality the presence of interest, properly understood, is the essential condition that hard work shall be profitable and 'discipline' actual.... The critics of the doctrine constantly confound interest with pleasure.

Nevertheless, in spite of misinterpretations, the consideration of interests became an important slogan of the 'new education' movement, which opposed the older drilling and cramming methods [Selleck, 1968, pp.252-254]. The many-sided development of interest and ideas as an aim implied the need for a broad and rich curriculum overall, but it was the much smaller scale planning of lessons which came to be influenced by another aspect of Herbartianism.

Through his interpreters, five 'steps' in an instructional sequence became distinguished, these being referred to as 'preparation,' 'presentation,' 'association,' 'condensation' and 'application' in a translation of Rein's interpretation [Smith, 1900, pp.111-112]. There was a tendency to regard the 'steps' very rigidly, to misapply them, and they were an obvious new ingredient for training college courses



in teaching method [Gordon and Lawton, 1978, pp.65, 146-148].<sup>27</sup> As with other theories of education, particular features with direct classroom relevance rose to prominence, and, in practice, teaching methods were consequently forced into a straitjacket. As Knox [1975, p.273] has pointed out, Herbart's ideas became 'magnified and distorted out of all proportion through his interpreters.' However, Herbartianism, variously interpreted and applied, rapidly rose to prominence in England, and, as Selleck [1968, p.264] has suggested, it 'spoke directly to teachers - more directly, perhaps, than any of the other movements.'

In addition to its influence on the educationists Findlay and Nunn, aspects of Herbartianism were also absorbed by Godfrey [1912d, pp.294-295], who, in a discussion of purposes in mathematical education, contrasted the older view of formal training with the newer Herbartian perspective, and he referred to the former view thus:

It appeared to be indifferent to the intrinsic worth of the subject-matter of instruction, and therefore failed to enlist the interest and active co-operation of the not specially gifted pupil. [p.294]

He went on:

The theory is essentially psychological, but seems now-a-days to be forsaken by professional psychologists; at any rate, it can no longer be regarded as the fundamental motive in education. [p.295]

He turned to the Herbartian alternative to the aim of formal training, which he felt had 'to be supplemented, and in part replaced, by the motive of enriching the mind with an assimilated mass of fruitful ideas, based upon and related to the circumstances of life' [p.295] (see p.257). Godfrey also adopted the notions of apperception, association and correlation, as well as supported the doctrine of interest [Godfrey and Siddons, 1931, pp.13-15]. More generally, Herbartianism in mathematics helped to shift the emphasis in curricular thinking to a search for fundamental ideas and modes of thought, to

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27 For examples of the application of the 'Herbartian Method' to mathematics see Smith [1900, pp.111-112] and Fennell [1902, pp.194-233].

help clarify purposes and facilitate learning. This important feature of the thinking of Godfrey, Nunn and others will be explored later in this and in the next chapter.

One aspect of Herbartianism, developed by Ziller (see p.259), but barely traceable in Herbart's own writings, focused on the historical development of knowledge and possible parallels with individual development [Selleck, 1968, p.239]. Given its controversial character and distance from the classroom, unlike the 'steps,' this aspect was not enthusiastically pursued by English Herbartians [pp.250-251]. However, particular links between the history of mathematics and mathematical education did come to be explored, and deserve to be considered more closely.

History of mathematics in relation to education has been considered in a master's thesis by Rogers [1976], and by Green [1974, 1976, 1977]. A detailed discussion will not be undertaken here, though two aspects to this relationship may profitably be distinguished. In a restricted way, history can be used to enrich the teaching of mathematics by providing supplementary classroom material. More widely and deeply, history can throw light on the nature of mathematics and its development, providing possible insights concerning both educational priorities and the psychology of learning mathematics.

Some limited efforts to utilize history in teaching mathematics, during the nineteenth century, have been detected by Green [1976]. Heppel [1893] addressed the AIGT on this subject, and Wormell [1897, pp.83-84] was clearly aware of the narrower and wider possibilities. This subject was also chosen by Barwell [1913] and Katz [1913] for papers presented to the MA before the First World War, but there is little evidence to suggest that there was any general enthusiasm for the use of history in the classroom.

With a few isolated exceptions, school mathematical textbooks



ignored the historical dimension [Green, 1976, p.15].<sup>28</sup> The MA's publications also reveal very peripheral interest in this subject [p.16]. Their general Report [MA, 1919a, pp.17,28] did pay lip-service to the value of history, predominantly for classroom enrichment, though admitted 'The Historical aspect of Mathematics has never yet found its fitting place in the teaching of the schools' [p.17].<sup>29</sup> Two theses in the 1920s were devoted wholly to the history of mathematics in teaching [Ross, 1923; Ruscoe, 1925], with the 'doctrine of recapitulation,' shortly to be discussed, the chosen focus for Ross. The more theoretical aspects of the relationship appear to have fared better than the direct classroom applications in the early twentieth century.

Katz [1913, p.131] reflected on the low status of history in mathematics teaching:

Perhaps it is because mathematical history has been regarded as a pleasant storehouse of anecdote or antiquarian lore ... perhaps it is this rather pretty view of mathematical history that has prevented the subject from coming into its own and exerting the influence that is its due.

A major problem was the paucity of relevant English literature, as Katz pointed out, as did Branford [1908, pp.ix-x], very forcefully:

It is a matter for regret that no adequate work on the history of mathematics is available in the English tongue: the contributions in this sphere of Peacock, Whewell, De Morgan, Gow, Allman, Chrystal, Ball, Cajori, and others, valuable as they are, have, I believe, exerted comparatively little influence on the mass of teachers of mathematics. The great treatises are in foreign tongues, inaccessible to the majority of teachers.

However, Americans such as Cajori [1890, 1917], Smith [1900] and Young [1907], the German Klein [1908], and, particularly, Branford [1908] in this country did draw on the wider historical literature for

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28 Notable exceptions were Carson and Smith's [1914a, 1914b] algebra and geometry textbooks. They were an exceptionally talented pair of writers, though not very successful in the textbook market. Smith [1919] also produced an illustrated children's book on the history of number.

29 For the extent of the Gazette's coverage of historical material see p.141. Smith and Goldziher's [1912, pp.80-81] Bibliography, 1900-1912, included only seven entries, none being English, on 'The value of the history of mathematics in teaching.'

their own contributions to the pedagogy of mathematics. The possible psychological insights from history were pursued to a greater or lesser degree by various writers in the early twentieth century.

In 1896, Cajori [1917, p.v] began the preface to his shorter history of mathematics by quoting the doctrine that 'The education of the child must accord both in mode and arrangement with the education of mankind as considered historically.' Herbert Spencer adopted this principle, and attributed it to A. Comte, though, as Branford [1908, pp.244,326] pointed out, traces of it could be found in the writings of Plato, Pestalozzi, Goethe, Froebel, Herbart and Condillac, and, more recently, Mary Boole and the physicist Ernst Mach. It was referred to as the 'recapitulation' or 'culture epochs' theory, and it has been linked with the 'fundamental biogenetic law' of the zoologist Ernst Haeckel, which asserts that 'ontogeny recapitulates phylogeny' [Green, 1977, p.93].

Cajori [1917] does not appear to have taken the doctrine very seriously, though he certainly argued for the attention to history in pedagogy. However, Wormell [1897, pp.83-84] accepted it in fairly general terms:

The development of the general intellect through the historic centuries resembles that of the individual mind. The readiest and simplest observation and the conclusions from observation are the first arrived at, and these are used to find others more subtle and more remote. The order of subjects as they are chronologically arranged in the history of research would therefore be an approximate guide as to the order in which these science subjects should be presented to the student.

There are only slight traces of the doctrine in Smith [1900], though he incorporated much historical material in his early methods book. One example from Comte, concerning Galileo's experimental determination of the ratio of the area of a cycloid to its generating circle (3 to 1), by weighing, is quoted by Smith to justify a scientific approach in the teaching of measurement, which 'merely follows the line of historic



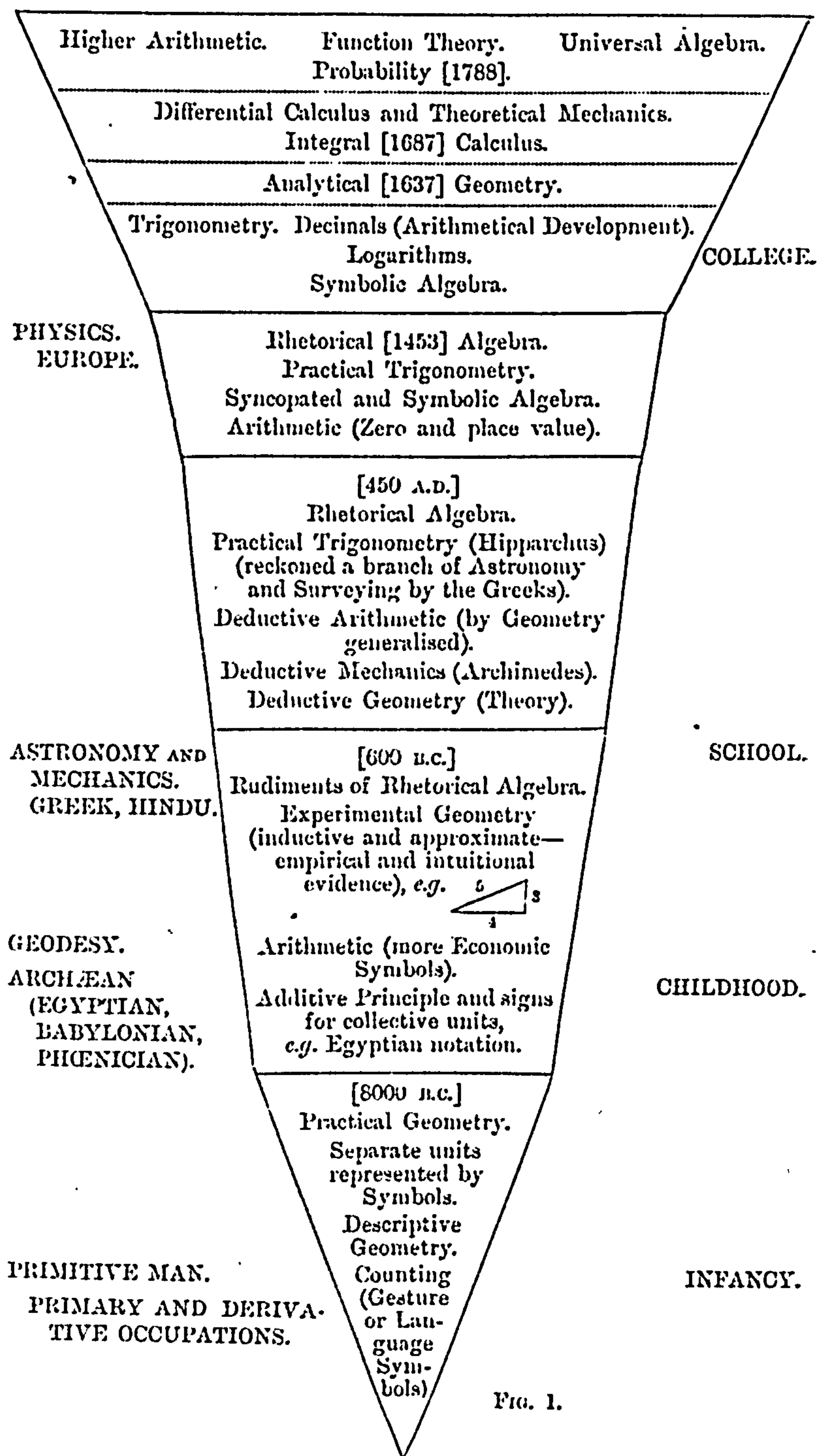


Illustration 33 Branford's Macroscopic Use of the  
Doctrine of Recapitulation [Barwell, 1913, p.73]

development, the line in which truth is first acquired by induction' [p.244].

Branford [1908] took the doctrine very seriously, though he admitted that its validity had been 'vigorously attacked' [p.244]. He argued for its value on the basis of his knowledge of history, and the results of experiments in teaching along the lines of the principle. He applied the doctrine both macroscopically, to the development of the various branches of mathematics, and microscopically, to the development of specific topics, such as place value [pp.364-365]. (See Illustration 33.) Branford also used history to justify heuristic methods (see p.197), and the principle that:

No symbol or contraction should be introduced till the pupil himself so deeply feels the need for such that he is either ready himself to suggest some contraction, or at least appreciates reasonably fully the advantage of it when it is supplied by the teacher. [pp.257-258]

The relevance of this principle to the development of algebraic symbolism was strongly argued [pp.258-261]. Branford also sketched a theory of motivation, based on a three-fold classification of historical impulses, paralleling postulated needs in the individual.

Branford's model distinguished the 'practical,' the 'scientific' and the 'aesthetic' impulses in history and the individual. The first stimulus arises from the need to resolve a practical problem; the second from the human need to organize and systematize knowledge, in order to improve the power of control over the environment; and the third from the human desire to create, and, to this end, pursue some activity for its intrinsic worth. Branford judged the first and second needs to be generally more dominant in learning than the third. His theory of motivation followed closely the earlier published theory of Nunn.

Nunn [1911a, p.168] recollected that he had first applied a classification of motives to science teaching in a paper of 1906



[Waring, 1979, pp.39-40]. In the following year he spoke on 'epistemological levels' to the Aristotelian Society, and again referred to his theory of motives, which, he claimed was based on observation, though partly inspired by the Hegelian triad of 'Thing, Law, System.' He subsequently applied these ideas to mathematics [Nunn, 1911a, 1912b], and used history to justify his view of mathematics as a creative process:

The point of immediate importance here is that mathematics is conceived not as a static body of 'truths' but in the dynamic form of an activity. In history mathematics has obviously been an activity.... To teach mathematics should be to reproduce in the school ... the essential features of this historic activity. [Nunn, 1911a, p.168]

To develop his dynamic view, Nunn distinguished three types of historical motive, termed the 'utility,' 'systematising' and 'simple wonder' motives.<sup>30</sup> These three motives correspond in the same order to the three stimuli of Branford, though the 'systematising' motive is less outward-looking than the 'scientific' impulse. Euclid's geometry and the Principia of Whitehead and Russell exemplify the 'systematising' motive in relatively pure forms.<sup>31</sup> Nunn made some assumptions concerning the relative strength of the three motives in school pupils. Whilst he recognized the appeal of 'pure wonder' in such topics as pattern in number, Nunn [1911a, p.170] judged 'utility' to be the dominant motive, up to the age of sixteen:

during the greater part of his school life [a boy] is prone to value a scientific or mathematical truth less for its 'beauty,' and less for the part it plays in a deductive system, than for its usefulness in application.

The question of usefulness for whom is not pursued by Nunn, nor is the relevance of the motive in the case of girls considered. However, Nunn [p.170] judged the 'systematising' motive to play an 'inconspicuous part before the age of 16, and in many boys has never really an

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30 Nunn's model has been discussed by Price [1976c].

31 Mair [1914] was inspired by Branford and Nunn, and outlined the latter's motivational model in the School World.

independent existence at all.' He felt that an overemphasis on this motive had caused many of the problems in school mathematical education. Like Branford, Nunn also used history to justify heuristic methods of the closely guided variety (see pp.197-198).

There are links between the thinking of Branford and Nunn, and Whitehead's [1932, pp.24-65] later 'rhythm of education,' with its three stages of 'romance,' 'precision' and 'generalisation.' A Report of the BE's [1938, pp.162-163] Consultative Committee subsequently combined Nunn's macro model of motives and cyclic model of learning (see pp.197-198) with Whitehead's 'rhythm,' in the following rich passage:

the interest of children in a subject tends to exhibit a rhythm corresponding to the rhythm of its history. There is a phase in which the subject makes its first appeal to the sense of wonder or romance. This is followed by a more sober phase in which interest fastens upon the practical utility of the new knowledge and is disciplined to precision in its application. Lastly comes the phase in which constructive logic takes the central place - the phase of generalisation or system. Now on the large scale, now on the small scale, this rhythm seems to govern the natural movements of young minds.

The question of utility has emerged as a major one in the discussion of motivation thus far. It was, of course, mathematics for utility, particularly in engineering and science, as opposed to mathematics for culture or mental discipline, which emerged as a major new slogan within the Perry movement, from around 1900. The way in which various writers developed and sought to reconcile different views concerning purposes and priorities in mathematical education warrants some further discussion.

Wormell's [1897, p.78] discussion of aims referred to 'the strengthening and training of the reasoning powers' as the ground 'most universally acknowledged in all ages' (see pp.254-256). To this he added that mathematics provided a training for various mental habits and was the 'handmaid of all sciences.' Predictably, the views



of the pure mathematician and the user of mathematics could be very different. Writing in 1902, the pure mathematician and philosopher Russell [1963, p.58] passed lightly over the instrumental value of mathematics, and claimed 'Utility ... can be only a consolation in moments of discouragement, not a guide in directing our studies.' Rather, he justified mathematics principally for its uniqueness within culture, into which pupils should be initiated:

Mathematics, rightly viewed, possesses not only truth, but supreme beauty - a beauty cold and austere, like that of sculpture, without appeal to any part of our weaker nature, without the gorgeous trappings of painting or music, yet sublimely pure, and capable of a stern perfection such as only the greatest art can show. [p.49]

Perry's [1901a, p.4] view that 'the study [of mathematics] began because it was useful, it continues because it is useful, and it is valuable to the world because of the usefulness of its results' was a sharply contrasting one at this time, with very different educational implications.

The philosopher and mathematician Whitehead [1911a, 1913a, 1913b, 1916, 1917] delivered a number of addresses in which he considered fundamental principles in mathematical education. He argued [1913a, p.453] that 'the object of a mathematical education is to acquire powers of analysis, of generalisation, and of reasoning,' which was a disciplinary view, though for Whitehead this also required getting to grips with certain fundamental mathematical ideas. Whitehead advocated a dynamic and unified view of mathematics, in the Herbartian spirit, and strongly opposed the elaboration of narrow technical details. His thinking was worked out in his splendid little Introduction to Mathematics, where the ideas of variable, form and generality, 'a sort of mathematical trinity which preside over the whole subject' [1911b, p.82], were explored, as well as other important ideas such as series, functionality and periodicity. Whitehead also stressed the

importance of the applications of mathematical ideas to nature and society, which implied the need for a broad mathematical curriculum, including aspects of similarity, trigonometry, calculus and statistics in a liberal education [1911a].

Carson also produced a number of stimulating papers on various general questions in mathematical education, in the period 1912-1914.<sup>32</sup> Carson [1913b, p.3] sought to justify a place for mathematics on distinctive grounds, and not based on utility, nor on the older disciplinary view. Thus he was placed in the following position:

In ... resigning the traditional claims of mathematics to develop, as no other subject can develop, the faculties of reasoning and expression, we have cut the ground from under our feet and are left, unless we can find another and firmer foundation, at the mercy of the teachers of natural science. [p.3]

He resolved this problem by sharing Whitehead's dynamic view of the subject as a set of characteristic fundamental ideas and modes of thought, revealing 'the glory of mathematics, as of man' as 'a glory of the mind, and not of the senses' [p.10], in which pupils should have a right to share. However, there still remained the problem of transfer in his argument that:

the purpose of mathematical education is to put the pupil in a 'mathematical way;' to permeate his whole being with the elementary principles of the science so that he will apply them spontaneously in considering any matter to which they may be relevant. [ 1913c, p.41]

Clearly, Carson was reacting to the recent emphasis on utility, which had shifted attention away from the value of mathematics per se. He cautioned:

Mathematics is coming more and more to be regarded as a mere adjunct to practical work and natural science, and this is largely the consequence of the changes and reforms of the last ten years. Is not the modern phrase, 'Science and mathematics,' which has replaced the older 'Mathematics and natural science,' a writing on the wall which all of us may read? [1913b, p.3]

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32 A number of Carson's papers were produced for the South-Eastern Association (see pp. 135-137), and some were included in his collection of Essays [1913c].



Carson also provided some much needed conceptual analysis of certain recent arguments concerning the useful and the practical aspects of mathematics.

Carson's thinking in a paper on 'The Useful and the Real' has been summarized by Price [1976b, p.20]. Carson interpreted 'real' to mean significant to the individual, and argued that this should not be confused with 'useful' in certain applications, which might be far removed from the pupils' personal realities. He also argued that what was 'real' need not be related to 'concrete' experience, but could arise from imagined situations. Certainly, Carson's arguments here would have undermined some of Perry's earlier thinking.

In another paper, Carson [1913d] explored the concept of 'intuition' at length. He advocated the exploitation of intuition in place of proof of the intuitively obvious in geometry, and also opposed the misapplication of experimental methods where intuition would better serve. For example, in the case of the equality of vertically opposite angles, or angles made by parallels and transversals, he argued:

these facts should not depend, even for their elucidation, on numerical experiments made in class-rooms or laboratories ... it is this universal acceptance of postulates without conscious experiments [intuitions] which differentiates Geometry from Physics or Chemistry.' [1912e, p.259]

A colleague at Tonbridge also referred derogatively to 'practical ... proofs' like that for the angle sum of a triangle 'from the fact that the mean of half a dozen measures of actual triangles with the protractor amounts to, say,  $179^{\circ}$ ' [Newbold, 1912, p.383]. Carson regarded such practices as 'an attack upon intuition. It replaces this natural and inevitable process by hasty generalisation from experiments of the crudest type' [Carson, 1913c, p.27]. Carson was not opposed to practical work, but his thinking was critical and constructive.

Around 1911, Godfrey also considered the question of aims in mathematical education [Godfrey and Siddons, 1931, p.4]. Godfrey was

particularly sensitive to the weakness of any kind of disciplinary argument, particularly in relation to the question of transfer (see p.258). He thus worked out a different kind of argument, based on the value of mathematics as a tool in scientific and technological advances, which he referred to as the 'outlook' value of the subject. The strength of his argument was not based on the fact that some pupils would subsequently need mathematics, beyond everyday uses, but that all pupils should appreciate something of the relevance of mathematics in its applications. He worked through the curricular implications in some detail, producing a broad and dynamic vision along the lines of Whitehead [pp.34-53; Godfrey, 1912a]. With this perspective, the place of geometry was problematic for Godfrey [1912a, p.169], it being 'a venerable monument of antiquity - on which I will lay no sacrilegious hands.' He confessed:

Formal and demonstrative geometry is not going to help us very much on the side of outlook; it must be taught as mental training, for we can hardly break with the training theory entirely, though our hold on it may be weakening. [p.169]

It was Nunn who perhaps best managed to accommodate both the inward-looking view of mathematics, exemplified by Carson, and the outward-looking view, exemplified by Godfrey, to produce a workable new philosophy for mathematical education.<sup>33</sup> Nunn passed lightly over the narrow argument of utility for everyday life and the wider utility argument, which only applied to some pupils, when not interpreted in Godfrey's sense of 'outlook.' He felt 'The universal opinion that every one ought to learn mathematics must have some better justification than this' [Nunn, 1911a, p.168]. History provided the following insights for Nunn [1914, pp.16-17]:

Mathematical truths always have two sides or aspects. With the one they face and have contact with the world of outer realities lying in time and space. With the other they face and have relations with one another.... The history of mathematics is a tale of ever-widening development on both these sides ... these

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33 For a discussion of Nunn's perspective see Price [1976c].



different currents of progress must not be thought of as independent streams. One never has existed and probably never will exist apart from the other. The view that they represent wholly distinct forms of intellectual activity is partial, unhistorical, and unphilosophical.

Initiation of pupils into this distinctive form of human achievement, as an intrinsically desirable possession, was the primary aim:

to enable the pupil to realize, at least in an elementary way, this two-fold significance of mathematical progress. A person, to be really 'educated,' should have been taught the importance of mathematics as an instrument of material conquests and of social organization, and should be able to appreciate the value and significance of an ordered system of mathematical ideas.  
[p.17]

Further weight to the argument was provided thus:

The student who is thoroughly schooled in the subject will make this spirit his own; the ideas, the mental habits and the kind of intellectual integrity proper to it will become ingrained in his nature, and he will tend to bring them into play wherever they can be applied. [1920, p.262]

However, although the argument was both stylish and persuasive, the question of transfer was still not completely avoided.

Nunn's ideals were splendidly worked out in his book [1914], which ranged widely to include material on algebra, trigonometry, calculus and statistics, and shared the ambitious curricular visions of Whitehead and Godfrey. The discussion thus far largely confirms the following conclusion of Retter [1936, pp.88-89]:

Within the seven years 1907-1914, Branford, Nunn, Godfrey, Carson and Whitehead had disentangled the main principles henceforth to guide mathematical education both in regard to procedure and to content.

However, it was certainly not a case of the schools simply following such grand theoretical prescriptions. As always, the developments were slow and complex, with ideals unrealized or only partially achieved.<sup>34</sup>

In addition to the general theory building of the early twentieth century, this period also saw the beginnings of scientific inquiry into certain narrower aspects of mathematical education, and some remarks remain to be made concerning the character of such developments.

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34 The reality will be pursued in the remaining chapters.

One feature of the late nineteenth-century development of the study of education was the emergence of the possibility of tackling some educational questions scientifically, by borrowing some of the methods of the burgeoning sciences. The creation of Section L of the BAAS in 1901 was one symbol of this new development, and the emergence of a distinguishable group of 'scientific educationists' within the 'new education' has again been ably discussed by Selleck [1968, pp.273-298]. The methods of science came to be utilized in the study of general areas such as child development, memory, intelligence, examinations and transfer of training, as well as questions arising within specific subjects in the curriculum, including mathematics.<sup>35</sup> The beginnings of work on these lines within mathematical education is another notable new feature of early twentieth-century developments.

Some of the earliest reported experimental work focused on aspects of arithmetic. W.H. Winch, an inspector, boldly claimed that 'until definite experimental methods are adopted, there is little hope for a science of pedagogy.'<sup>36</sup> He took up the challenge in arithmetic, and the second volume of the Journal of Experimental Pedagogy published the first of a series of articles by Winch [1913, 1914a, 1914b, 1915] on the question 'Should Young Children Be Taught Arithmetical Proportion?' The growing interest in arithmetical attainment, by the First World War (see pp.25-27) is also reflected in the same journal's publication of articles by Ballard [1914, 1915] on 'Norms of Performance in the Fundamental Processes of Arithmetic,' involving over eighteen thousand pupils, and, in particular, the method of subtraction by 'equal additions' was shown to be more efficient than the method of 'decomposition.' Winch followed up Ballard's work with a smaller scale in depth study, and his

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35 Selleck [1968] has discussed child development, intelligence and transfer in particular, as three of the areas where the 'scientific educationist' made 'his most notable achievements' [p.273].

36 Quoted in Selleck [1968, p.276].



research here, and in the case of proportion, was reported by Punnett [1922] in the Mathematical Gazette after the War. This article was the first of its kind in the Gazette, and the Editor suggested that this was likely to be the first of many, though this journal's coverage of such material continued to be very slight. However, in the following year, an article was published on 'Mathematical Ability - A Plea for Research' [Hamilton, 1923]. Here the variable of sex difference was one which attracted some attention before the War.

A Report of the LCC [1911, pp.26-27] considered the question of the 'relative capacity' of boys and girls in mathematics, after gathering plenty of data, including the results of a controlled experiment in arithmetic, involving five schools. This Report concluded that there were wider extremes of performance among boys; greater problem solving ability among boys; and 'greater neatness, accuracy, and conscientiousness' [p.27] in mechanical work among girls. The Report felt 'the subject merits further investigation and experiment' [p.27]. Some contributory causal factors and the general question of differentiation were considered in Chapter 4 (see pp.152-159). In the 1920s, sex differences in mathematical ability was the subject of an article in the Gazette [Sandon, 1926], and an early master's thesis [Cameron, 1923]. The growing interest in experimentation is reflected in the chosen foci for theses and dissertations in mathematical education after the War.

The following table shows the extent to which aspects of mathematical education came to be explored scientifically, in theses and dissertations between the Wars [Frobisher and Joy, 1978, pp.1-5]:<sup>37</sup>

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37 Frobisher and Joy [1978] have missed Fulford [1923], who treated geometry historically and experimentally, and so has been counted as a half scientifically. A scientific approach has otherwise been inferred from the titles.

Years	1919-1923	1924-1928	1929-1933	1934-1938
Total	5	9	12	27
Scientific Approach	3½	4	8	22
Percentage Scientific	70	44	67	81

#### Theses and Dissertations on Mathematical Education

The gradually increasing total output up to the Second World War is predictable, but, more interesting is the shifting proportion of scientific studies. In the 1920s, a number of these reflected the interest in various aspects of mathematical achievement including sex differences; psychological tests of mathematical ability; the relation between performance in writing and arithmetic; and backwardness in arithmetic.<sup>38</sup>

Thus, the new research orientation in mathematical education was only adopted on a very small scale over the first quarter of this century, but a significant start had been made. The extent to which the previously discussed wider priorities and visions in mathematical education became adopted in practice is the principal focus for the next chapter.

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38 Collar's very early thesis of 1919 on arithmetical ability, statistical aspects of which were published in the British Journal of Psychology, has been discussed by Wingrove [1976, pp.30-33].



## Chapter 7

### Unification and Broadening

Two related curricular tendencies, the broadening of the mathematical content of curricula, and the unification or fusion of related components were isolated as underlying objectives within the Perry movement (see pp.219,225). The latter tendency facilitated the former, and is also related to the general pedagogical principle of co-ordination, which was discussed in the last chapter (see pp.258-267). The developing interest in unification, and the associated emphasis on general principles and ideas in mathematical education will be considered first.

#### The Unifying Tendency

The concern for fundamental ideas, connections, and concentration in the curriculum was one feature of Herbartianism, elements of which are traceable particularly in the writings of Whitehead, Godfrey and Nunn who put forward broad and ambitious blueprints for mathematical education, before the First World War (see pp.274-275,281-285). The newer thinking contrasted sharply with the nineteenth-century convention that the various branches of mathematics should be strictly segregated.

Prophetically, Sylvester [1870, p.7] had remarked to the BAAS:

Time was when all the parts of mathematics were dissevered, when algebra, geometry and arithmetic either lived apart or kept up cold relations of acquaintance confined to occasional calls upon one another; but that is now at an end ... and we may confidently look forward to a time when they shall form but one body with one soul.

However, the pattern in the schools during the 1880s was far removed from such ideals, as Palmer [1912, p.227] pointed out:

Arithmetic was treated as a separate compartment, distinct from Algebra and Geometry. Algebraical methods were not allowed in the solution of problems, and the possibility of using geometrical or graphical methods was hardly realised.

Furthermore, Godfrey [1908, pp.252-253] drew attention to the following general pattern:

Never by any chance were numerical data or illustrations introduced into Geometry. It was very unusual to meet with any demand for numerical work in the papers on advanced mathematics set at the Universities. The result of this is that the knowledge of the mathematician trained under this system is almost entirely qualitative....

Joshua Fitch's Lectures on Teaching at Cambridge in 1880 also exemplify the segregated view of subject-matter, though the beginnings of enlightenment are detectable. For example, Fitch [1881, p.334] suggested the use of letters to generalize arithmetical properties, but cautioned 'Do not suppose that this is algebra. No one of the notions or processes proper to algebra is here involved.' He also drew attention to the potential links between Euclid II and certain algebraic identities, but again added:

since Geometry is founded entirely on the recognition of the properties of space, and Algebra and Arithmetic on those of number, it is necessary to preserve a clear distinction in the reasoning applicable to the two subjects. Except as shewing interesting analogies, the two departments of science should be kept wholly separate.... [p.341]

The independent historical roots of the various branches resulted in their initially rigid and disconnected treatment in education, which was governed principally by examination requirements and textbooks, which eschewed any intermingling of the branches [MA, 1919a, p.9]. Furthermore, the development of any one branch was systematic and thorough, with the needs of the small minority of future pure mathematical specialists dominant (see pp.252-254). Simplification and cross-fertilization of the branches were not regarded as legitimate tactics. The dominant attitude to the branches also had a generally narrowing effect on mathematical curricula. In elementary schools, the possibility of opening up arithmetic to include new elements and to exploit links with algebra and geometry was not generally entertained. In secondary schools, such branches as trigonometry and calculus were beyond the reach of the majority, as the introduction of arithmetical, geometrical and graphical elements was spurned. However, towards the



ALLIED TO ALGEBRA		ALLIED TO GEOMETRY	
1	Counting and numbering	1	Cutting paper figures, forming, moulding, or building up different shapes
2	Arithmetic	2	Kindergarten Geometry
3	Arithmetical Algebra	3	The relations and measurements of simple figures
4	Algebra	4	Euclidean Geometry
		5	Trigonometry
6	Algebraical Geometry	6	Geometrical Conics
7	Kinematics	7	Graphic Statics
8	Dynamics including Statics	8	Kinetics and General Mechanics
		9	Mathematical Physics
10	The theory of Equations	10	Solid Geometry
11	The Infinitesimal Calculus	11	Projective Geometry

Illustration 34 Wormell's [1897, p.82]  
View of the Branches of Mathematics

end of the century, the traditional inflexible view of these branches was beginning to be eroded.

Hayward [1886] addressed the AIGT on the subject of correlation between the branches, and during the 1890s correlation emerged as a prominent general pedagogical principle. Wormell [1897, pp.81-82] argued strongly for unification in mathematics, in his contribution to Barnett's [1897] Teaching and Organisation, and Barnett himself also reflected these newer emphases (see p.258). Wormell [1897, p.82] viewed the development of mathematics in two parallel streams, one algebraic and the other geometrical, and such that 'As they extend, each will grow into the other until there will exist really no essential difference between the two.' (See Illustration 34.) Significantly, Wormell placed trigonometry between the two streams, to indicate that this branch had close ties with both, and its pursuit could potentially make an important contribution to unification. More generally, the growing interest in measurements and 'practical arithmetic' was an important feature of the tendency to break down the traditional barriers (see pp.203-212). The mathematical demands of science and engineering cut across the established lines of demarcation within mathematical knowledge, and Perry's scheme of practical mathematics in particular represented an alternative conception, which ignored the older conventions, with much of the syllabus coming under the headings of mensuration and use of squared paper (see p.100). Numerical work permeated Perry's scheme, and the exploitation of links between arithmetic and mathematics was an important early advance towards unification.

Various early twentieth-century pronouncements focused on aspects of unification. The MA [1902b, p.6] adopted as one guiding principle that 'whenever practicable, Geometry should be employed to illustrate Arithmetic and Algebra,' and recommended that 'in pass



examinations in Arithmetic, the use of algebraic symbols should not be prohibited!'[p.7]. The BE's [1905a, p.44] Suggestions claimed that 'The division of arithmetic and algebra into distinct subjects is much to be deprecated,' and advocated the introduction of letters, negative numbers and simple equations as valuable extensions of arithmetic in elementary schools. The reaction to the teaching of mathematics in 'watertight compartments' extended well beyond the three elementary branches as Hall [1905, p.158] underlined:

For years teachers had been in silent rebellion against systems of examination which made it illegitimate to use algebraical symbols in an arithmetic paper; which debarred the use of the trigonometrical ratios in Euclid or Higher Pure Geometry; which sometimes insisted on geometrical methods in conics, where analysis might have been used with equal or better advantage; and which encouraged 'calculus-dodging' and other pernicious habits, all tending to an enormous waste of time without any compensating advantages. Now, by a fairly general consent, all this is to be changed....

A Committee of the BAAS [1907, p.445] was soon able to report that:

In good schools, geometry and trigonometry, have, indeed, become in their earlier stages largely arithmetical, while algebra is introduced as generalised arithmetic; and there is good ground for the belief that this rational and co-ordinated treatment of school mathematics will be generally adopted in the course of a few years.

A Report of the LCC [1911, p.83] subsequently noted that 'teachers are now freed from conventional restrictions which have prevented a free use of generalised Arithmetic,' though there was still a tendency to divorce algebra from arithmetic, and not exploit the link through 'generalised arithmetic.' However, this particular tactic was gaining in popularity [Ballard, 1909a, p.88], and was strongly advocated by the MA [1910c, 1911b] in particular.<sup>1</sup> The general opening out of arithmetic was a notable feature of reform, before the First World War.

Godfrey [1908, p.253] referred to 'The importance of numerical work in all subjects [his stress],' particularly geometry and trigonometry, as a major principle within the reform movement, and Palmer [1912, p.236]

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<sup>1</sup> For an early discussion of 'Algebra as Generalised Arithmetic' see Child [1905], who claimed that Continental practices in the beginnings of algebra were generally in advance of those in England. Some further remarks concerning algebra teaching will be made in the next chapter.

pointed out that the new concern was for arithmetic as 'a tool to be constantly used, not only in the other branches of Mathematics, but also in Science teaching.' In London elementary schools, Ballard [1912, p.10] noted as one major development that 'Arithmetic is passing into Mathematics. The claims of Geometry, Mensuration and Algebra obtain at least some recognition.' Developments in girls' secondary schools were summarized by Story [1912, p.548]:

Arithmetic, algebra, and geometry are ... taught simultaneously throughout the school, not consecutively as in America, and recently much has been done by the foremost teachers of the subject to break down the barriers between the three divisions, and to show them all as part of the same science - algebra as generalised arithmetic and 'written geometry,' geometry as 'pictorial algebra.' It is therefore increasingly the rule for the same teacher to take the three branches of the subject with a Form.<sup>2</sup>

Thus, some of the traditional barriers were certainly taken down during the early years of reform, but the developments were still a long way from the ideal of unification in mathematics. Outside technical education, the subject continued to be discussed, taught, learnt and examined in its separate branches of arithmetic, algebra, geometry, trigonometry, calculus and mechanics. Exceptionally, practical mathematics textbooks covered a broad range of mathematical content within a single volume. However, such textbooks were designed for technical students, and school textbooks generally followed the well established pattern, which teachers' conservatism also helped to sustain. Furthermore, the idea of general courses in mathematics became principally associated with technical education. Thus, in secondary education, such courses would have been regarded as overambitious, diluted, and mathematically inferior to the systematic treatment of the branches, taken separately (see pp.242-246).

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2 For some discussion of the curious American treatment of the branches 'in tandem,' and of moves towards a 'fused' treatment, with the development of 'general mathematics' courses, see Young [1907, pp.97-98, 183-187, 246-248] and Smith [1926, p.29]. These developments in America have also been discussed by the NCTM [1970b, pp.46-53, 173-179].



There are some parallels here with developments in America and in science education. The NCTM [1970b, p.185] have referred to the 'unified nature' of most 'vocational mathematics,' as opposed to the higher status 'college-preparatory' mathematics. Such associations 'contributed to the downgrading of unified mathematics.' The 'general science movement' in this country has been explored by Jenkins [1979, pp.70-106], and a major barrier to progress was the inferior status accorded to such courses, as an alternative to the established and more specialized courses in physics, chemistry and biology.<sup>3</sup> However, certain innovators in mathematical education did manage to persuade publishers to break with convention.

Mair's [1907] A School Course of Mathematics, published by OUP, covered geometry, algebra and trigonometry, and was written in the style of a discussion between teacher and pupils (see p.268, note 24). Two extraordinary books in the Longmans' Modern Mathematical Series (see p.269) achieved a considerable measure of unification. Dobb's [1913] A School Course in Geometry included trigonometry, mensuration, some co-ordinate geometry, and even squeezed in a little calculus, in addition to treating geometry dynamically using transformations. Thus Dobbs provided an ambitious course, with a central geometrical focus, and acknowledged a debt to Branford in particular. Nunn [1914, pp.v, 19-20] argued strongly for unification, and, with an algebraic focus, he implemented his ideals in a treatment which embraced trigonometry, calculus, complex numbers and statistics. Not surprisingly, these ventures of Nunn and Dobbs were commercially unsuccessful, the breadth and degree of unification in their courses being far in advance of practices in the schools. However, the idea of unification continued to attract some attention, though developments were predictably very

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3 The NCTM [1970b, p.422] have quoted one teacher as remarking 'We might just as well bind together Physics and Biology and call it "popular science"... or put all the subjects in one book and label it "Hash."'

slow and limited.

The mistresses within the MA [1916b, p.3] referred to the 'need of unity' within the three elementary branches and stressed:

in teaching it is most important to co-ordinate the work as far as possible, so that the subject is developed as a whole of which these divisions are only the different aspects.

However, the MA's [1919a, 1923b, 1932d, 1934a] Reports between the Wars paid very little attention to the issue, with detailed recommendations being confined to particular branches. Thus Retter [1936, p.92] referred to the need for secondary teachers themselves to emphasize connections, and added 'the "binding" ... is still to be achieved.' Particular publications of the BE [1926a, pp.214-220; 1935, pp.15-22; 1938, pp.235-242; 1943, pp.104-108] paid lip-service to the need for unity in elementary and secondary school mathematics, though mathematics in the schools continued to be far from unified. Even where textbooks for elementary schools were devoted to mathematics, rather than a single branch, this often amounted to no more than a treatment of the branches in separate sections [BE, 1928a, p.44], and opinion was divided on the need for general textbooks. However, very slowly, the secondary examining bodies did give some encouragement to a more flexible treatment of the subject as a whole.

The Oxbridge Joint Board's close links with the public schools, particularly through the HMC and the MA, and the important influence of Godfrey on the Board's mathematical examinations were mentioned in earlier chapters (see pp.47, 164). In 1919, it was this Board that explored the possibility of mixed papers in mathematics for the First Examination. Godfrey and Mair became involved in drafting new schedules for the mixed papers, which replaced those in the separate branches from 1921 onwards [King, 1981].<sup>4</sup> The Joint Board's distinctive arrangements

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4 Interestingly, this Board was also the first to add general science to its scheme for the First Examination, in 1921 [Jenkins, 1979, p.71].



were subsequently discussed by the MA, following a paper by Siddons [1928], in which he pointed out the benefits of this more flexible policy for examinations. The SSEC also supported the Joint Board's isolated initiative, following its investigations of the eight approved bodies during 1931 (see p.54). However, it was not until the 1940s that these bodies acted collectively concerning an alternative to examining in the separate branches.

From 1928, Durham provided an alternative mathematics scheme for the First Examination, in which much of the traditional geometry was axed to permit a broader range of content, including some trigonometry, solid geometry and numerical integration, and with functionality chosen as a unifying theme in the algebra syllabus [MA, 1930a, p.148]. However, Durham's influence was small; this was only an isolated initiative; the schools made little use of the alternative, and it was allowed to lapse in 1934 [Talbot, 1955, p.96]. Ten years later, a much more concerted effort was made to follow Durham's lead.

In 1943 and 1944, the possibility of an alternative syllabus being provided for all the First Examinations was explored by a Conference with wide representation, the matter being also discussed by the MA [1944a], and this activity culminated in the publication of the Jeffery Report [MA, 1944b]. As Redhead [1953, p.367] has suggested, the proposed alternative can be seen in a number of ways as a coming to fruition of the ideals both of the Perry movement and of the leading thinkers in mathematical education before the First World War (see pp.225,281-285). What is remarkable is the lapse of time between the enthusiastic promulgation of alternative blueprints for secondary mathematics, and the general embodiment of an alternative in examination syllabuses. The Jeffery Report's syllabus was arranged under the headings of number; mensuration; formulae and equations; graphs, variation and functionality (including some calculus); two dimensional figures

(including some co-ordinate geometry and trigonometry); three dimensional figures; and applications to problems involving drawing, trigonometry and geometry. The syllabus achieved a considerable degree of broadening by cutting and simplifying the conventional arithmetic and algebra syllabus, and by eliminating much of the demonstrative geometry. The syllabus was also arranged thematically and unification was the underlying principle here:

an overwhelming case can be made out for the fusion of mathematical subjects and in particular for a closer association of geometry and trigonometry. To encourage this, each paper should be 'mixed'.... In these papers complete freedom of method should be allowed....

Thus, the Joint Board's policy of mixed papers was now advocated for general adoption as an alternative.

The various bodies soon made alternative syllabuses available, with some variations in detail. However, the links in the chain from the original ideals underlying the new alternative, their embodiment in syllabuses and papers, the interpretations of textbook writers, through to teachers' practices, as always, raised fundamental problems. For example, successful textbook writers within the traditional branches, and notably Durell, were not surprisingly reluctant to start from scratch and provide fundamentally new general or unified mathematics textbooks, to cater for the new market. It was much easier to produce new textbooks in general mathematics which were little more than a permutation of chapters from earlier textbooks on arithmetic, algebra, etc., and were thus far from being infused with the ideals of unification.<sup>5</sup> Furthermore, teachers were generally slow to adopt the new alternative which involved breaking away from the narrower, well-trodden paths, familiar textbooks, and examination papers [Price, 1974,

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5 Durell's publishers, Bell, felt that general mathematics textbooks would have the advantages of novelty and even distribution of cost i.e. one textbook per year. Apparently, Durell was at first not sympathetic to the idea, but he was gradually won over by his publishers. I am grateful to R.J.B Glanville of Bell and Hyman for this note.



pp.135-140; Ministry of Education, 1958, pp.20-21]. Talbot [1955, p.113] found that only about one third of sixty-three grammar schools in Northumberland and Durham reported that they used the new alternative, and Montgomery [1965, p.180] found that it was not until 1962 that the alternative became more popular amongst the large number of schools taking the NUJMB's examinations.

Both Durham's scheme and the Jeffery syllabus adopted the theme of functionality, which contributed an important element of unification. This idea, which has a long history and attracted much international interest, deserves to be considered more closely.

### The Concept of Functionality

Love [1902, p.458] well summarized the general nineteenth-century lack of coherence in curricular thinking in mathematics:

The traditional order of study has tended to obscure the fundamental notions and the general drift of arguments under a cloud of secondary developments.

Love reflected the newer Herbartian emphasis on fundamental ideas, which became an important feature of early twentieth-century thinking in mathematical education (see pp.281-285). Functionality was one of these ideas, and its central importance was strongly argued by Godfrey [1910a, 1912a, 1912d] and by Whitehead [1911a, 1911b] in particular.

Interest in this idea had developed much earlier in Germany, during the 1880s, and the German reform movement which subsequently elevated functionality to a central position became principally associated with Klein (see pp.131-132), whose influence spread to America and England in the early twentieth century [Price, 1912; NCTM, 1970b, pp.174,206]. There are obvious links between variables and functional dependence and the use of squared paper, as the American mathematician Moore [1906, p.318] underlined:

I know of no medium serving to bring together so closely and so easily the three phases or dialects of pure mathematics - number, form, formula - and to lead so directly to the concept of functionality....

Moore prophetically judged that this concept 'will play a fundamental rôle in the reorganization of elementary mathematical education' [p.318].

Klein [1908, p.4] shared this view:

We, who are called the reformers, would put the function concept at the very centre of instruction, because, of all the concepts of the mathematics of the past two centuries, this one plays the leading role wherever mathematical thought is used.

Thus, the concept was a doubly attractive one for the reformers, given its potential for utility as well as for unification. Smith [1913, p.616] subsequently associated functionality with the following range of possibilities:

The notion of limit, invariability, rate, function, and graph must be so gradually introduced and must become so clearly understood that when the calculus is reached they will be met as we meet familiar friends. How to do this economically is one of the problems relating to intuitional mathematics.

Thus functionality is also related to the issue of bringing aspects of the calculus into a general education, which will be considered later in this chapter.

Clearly there are some obvious links between aspects of the German's funktionale denken and Perry's scheme of practical mathematics (see p.100). However, the power of this idea was not exploited in England until around 1910, when international influences became stronger, particularly through the ICTM (see pp.130-135). Smith and Goldziher's [1912, pp.71-72] Bibliography included a section on the 'function concept,' which shows that it was predominantly the German journal literature which first came to explore the concept in detail, particularly from around 1905. By 1910, Continental developments were beginning to be reported in this country also.

Wallace [1910] provided an account of developments in German secondary schools, for the School World, with particular reference to the introduction of calculus. Price [1912] addressed the London Branch of the MA on German reforms, and his resulting paper in the Mathematical



Gazette threw much light upon developments in Prussia from the late nineteenth century.<sup>6</sup> His discussion clearly shows that functionality in mathematical education gradually emerged from the 1880s as part of a shift of emphasis from an exclusively specialist pure mathematical treatment of forms  $f(x,y)=0$  in analytical geometry and geometrical loci, to a treatment of dynamic functions, both empirical and analytical, involving forms  $y=f(x)$ , and correlated with science. Wallace [1910, p.250] drew attention to a marked contrast concerning the adoption of aspects of functionality in English and German schools:

the general adoption in school of the graphical method of representing algebraical functions has taken place earlier in this country than in Germany; but the Germans are going a step further in showing that by the aid of this method the calculus in its earlier stages may be made a very valuable part of the school curriculum.

Price [1911] also translated for the Gazette the mathematical syllabus for the Gymnasia, which was part of the Meraner Lehrplan of 1905, and included the strong Prussian emphasis on functionality. Developments in German mathematical education up to the War were also reported in the journal Nature [1913; Pressland, 1910]. In addition, Price translated a syllabus of mathematics for the new Austrian Gymnasien, issued in 1909, which was subsequently published as a Pamphlet of the BE [1910] (see pp.41-42). Such a publication in England at this time was felt to be particularly relevant as 'the reforms suggested ... are similar to those that are now being discussed in this country' [p.i].<sup>7</sup> The Austrian syllabus adopted functionality as a central theme, from the beginnings of funktionale Anschauung through to the calculus, and also advocated the general guiding principles of simplification, unification, correlation and the consideration of stages in intellectual development.

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6 See also Young [1900] for an early American perspective on Prussian school mathematics.

7 This Pamphlet was publicized in Nature [1910] and the School World [1910].

Godfrey [1912d, pp.289-292], who became much involved with the ICTM, provided his own interpretation of functionality in a paper on algebra teaching for the Special Reports. He also presented a paper to the MA, where he strongly argued that algebra teaching should be organized around important ideas such as generalization, functionality and problem solving, and he admitted 'We insist on the technique of algebra so much that I fear we lose the spirit' [Godfrey, 1910a, p.232]. Much of his thinking became embodied in the MA's [1911b] subsequent Report on this branch (see p.163). The mistresses within the MA [1916b, pp.6-7] were also quick to stress the central ideas of variation and functionality in their Report. The place of graphical representation was by this time assured, and the main issues now concerned the scope of the treatment of variation and the possibility of introducing some ideas from calculus.

Between the Wars, the value of the function concept continued to be emphasized in America [NCTM, 1970b, pp.40-41,47,202-206], and a Report of 1923 viewed it as 'the best single concept for unifying the curriculum' [p.205]. The NCTM [1934] devoted a whole Yearbook to Relational and Functional Thinking in School Mathematics, and its author, H.R. Hamley, also discussed this subject in a memorandum published in a Report of the Consultative Committee [BE, 1938, pp.450-452].

Finally, the secondary examining bodies in this country generally recognized the importance of this theme, following the recommendations of the Jeffery Report. The Jeffery syllabus included a section on 'Graphs, Variation, Functionality' [MA, 1944b, pp.v-vi], which extended to work on gradients and areas involving curves, with various applications. The rationale for this theme followed closely the arguments promulgated internationally before the First World War. Another very striking feature of the Jeffery syllabus was its general



breadth. Developments concerning the broadening of mathematical curricula again have a long history, and various features remain to be discussed.

### The Broadening Tendency

As has been shown in earlier chapters, within the general reform movement of the early twentieth century there was much concern for pruning and simplifying mathematical curricula, to permit a more purposeful treatment disencumbered of technical details, and organized around important ideas, with a broader range of content embracing the more useful aspects of mathematics. Simplification and breadth were central features of Perry's scheme of practical mathematics, which made such remarkable progress in the restricted field of technical education (see p.220), and these principles were important ones within the Perry movement (see p.225). Various early twentieth-century pronouncements, discussed in Chapter 3, reflected the general interest in greater breadth and purpose, as did the thinking of a number of leading figures in mathematical education, discussed in Chapter 6. In relation to secondary education, the MA in particular pursued in detail some of the implications of these newer emphases, working closely with examining bodies, particularly from around 1910, as was shown in Chapter 4.

The general principle of broadening could be applied to the mathematical curriculum in various types of educational institution, with different implications. In the case of elementary schools, the main issue concerned the extent to which the curriculum should extend beyond arithmetic, to warrant the label 'mathematics,' particularly in the case of older pupils. The narrowing influence of the system of payment for results, until the 1890s, was discussed in Chapter 2 (see pp.15-21), as was the arrangement for 'specific subjects,' which encouraged some broadening, particularly in the cases of algebra and

mensuration. Following the abandonment of the system, and with the development of a gradually more flexible view of the relationship between the branches of mathematics (see pp.291-293), there is some evidence of interest in broadening and some gains here in the higher Standards, during the early twentieth century (see pp.21-23). The BE's [1905a, pp.43-44; 1912c, pp.3-4,21-26] Suggestions discussed various aspects of broadening, which might be achieved through a shift from the older commercial bias in arithmetic towards such aspects as plane and solid mensuration, practical geometry, graphical work and the beginnings of algebra. The BE accepted that the extent of broadening should be a function of local circumstances, which might imply no more than a little mensuration, and, in any case, demonstrative geometry was 'not suitable for the ordinary Elementary School' [1912c, p.24]. The most general gains appear to have been in the development of work in measurements and 'practical arithmetic' (see pp.203,210).

In London elementary schools, Ballard [1912, p.11] found that algebra as a separate subject had lost the place it had held five or six years earlier, though some 'not very successful' attempts were still being made to graft some algebra onto arithmetic. A Report of the LCC [1911, p.20] found that geometry was largely restricted to boys' schools, and was generally limited to drawing, following the pattern of the old South Kensington Art syllabus [Ballard, 1912, p.12]. Between the Wars, there is some evidence of a tendency to concentrate on arithmetic in elementary schools, at the expense of other potentially broadening elements.

Two major constraints on progress, the selection system for secondary schools, and the concern for standards of arithmetical performance, were discussed in Chapter 2 (see pp.23-27). Early leaving, large classes, inadequate facilities and inferior teachers were all persistent general problems, which also inhibited enlightenment in elementary school



mathematics [BE, 1926a, p.216]. Reports of the BE's [1926a, pp.214-220; 1931, pp.139-145] Consultative Committee criticized the traditional and continuing dominance of arithmetic at the primary and post-primary stages, and ambitious proposals were made in the latter case for a four-year course including mensuration, algebra, geometry and trigonometry. Nunn [1925] shared such optimism for the seven Standards in elementary schools, and his own detailed syllabus was published.

Significantly, he remarked:

The syllabus claims to be a syllabus in mathematics, not in arithmetic. This departure from custom is meant to imply a rejection of the illiberal tradition which has limited the mathematical studies of the elementary scholar very largely to the manipulation of number....

Such remarks provide further evidence that in practice the extent of broadening in elementary school mathematics was generally rather limited. However, the question of mathematics at the post-primary stage became an increasingly important one between the Wars (see p.34).

An early M.Ed. dissertation of 1928 was significantly entitled 'Practical Mathematics: the Approach of the Post-Primary Pupil to the Study of Mathematics' [Frobisher and Joy, 1978, p.2], and HMI Carson [1929, pp.29-30] stressed the importance of the new concept of a post-primary 'modern school,' for which an alternative, more useful, and less academic mathematics curriculum than in the secondary school was required. He added:

The subject (or method) known as practical mathematics, which thirty years ago did so much for teaching in this country, is in mind in this connection. [p.30]

The resurrection of the name practical mathematics is of particular interest here, and the BE's [1935] Pamphlet on Senior School Mathematics encouraged LEAs 'to provide courses for teachers in Practical Mathematics' [p.4], which could ignore secondary examination constraints, as well as 'the need for academic or conventional treatment' [p.3], and encourage a unified approach. In the 1930s, the MA also began to take

some peripheral interest in such developments, particularly in the case of the central schools (see p.175).

Central schools, which provided a four-year course for more able pupils within the elementary system, had been established before the First World War, mainly in the London area, and the kind of mathematics which was developing in these schools was described at an early stage by Ballard [1912, pp.19-24] and by Spencer [1912, pp.48-59] in the Special Reports. In mathematics, these schools were striving to reconcile the demands of some kind of utility, whether industrial, commercial, or domestic in the case of girls, with the need to provide a course with general 'cultural' value [Ballard, 1912, p.20]. Again, there are many links between the courses being evolved and Perry's practical mathematics, particularly in the case of boys with a future in industry. In particular, as Ballard remarked, 'Deductive geometry occupies an anomalous position' [p.21], and Spencer [1912, pp.55-56] emphasized that, with various simplifications, 'The ground covered is extensive ... but the work is intended to be simple, practical and in no way pretentious' [p.55]. Furthermore, 'All mathematical subjects are to be broadly treated and the whole made naturally related parts of one course of study' [p.56].

It is beyond the scope of this thesis to explore in any detail the historical links between the mathematics of the higher grade schools under the DSA in the nineteenth century (see pp.27-31), that of the central, junior technical and senior schools, and, eventually, of the secondary modern schools. The importance of the idea of practical mathematics, variously interpreted, as an alternative to the mathematics of the secondary schools, which was dominated by the universities, seems clear. This whole subject warrants further investigation.

Turning to the secondary schools, in the case of girls' mathematical education the principle of broadening was initially related



in the main to the extension of the curriculum beyond arithmetic, to include some algebra and geometry up to the normal maximum for non-specialists in boys' schools (see pp.29-30,38,152-159). However, the Perry movement brought with it wider possibilities for a general education, at least in the case of boys. As Fawdry [1924, p.133] remarked:

In brief, the general object of the reform was to make it possible for all boys of seventeen to leave school with some knowledge of trigonometry, mechanics, and the calculus.

Such advances required, in the first instance, the development of new approaches to the introduction of these branches, along lines very different to those traditionally followed in the case of the minority of mathematical specialists. There were special and major difficulties in the case of mechanics, which will be considered first.

#### The Teaching of Mechanics

The specific involvement of the BAAS and the MA, and the problematic relationship between mathematics, science and mechanics teaching have already been discussed (see pp.212-217). The possibility of reform in this branch can be seen as arising principally from the struggle to merge two independent nineteenth-century traditions. As Eggar [1912, p.338] remarked in the Special Reports:

Engineering and Physics, art and science, practice and theory, have advanced on independent lines, and have only recently amalgamated, and the growing importance of Mechanics as a school subject is due to this amalgamation.

In mechanical engineering, various problems of construction necessitated the blending of mathematical theory with genuine applications [p.338], and Perry in particular became involved at an early stage in the development of practical mechanics in technical education, and also, briefly, at Clifton College (see pp.218-219).

In the case of schools generally, as Eggar [1912, p.338] pointed out, 'the practical teaching of Mechanics in schools is almost entirely

a modern growth,' and it formed part of the late nineteenth-century development of science teaching and physical laboratories (see pp.200-201). In sharp contrast, mathematical mechanics was already established in the schools, but as a wholly theoretical examination subject, confined to mathematical specialists, and dominated by the Cambridge mathematical tradition. Interest in blending the mathematical and experimental approaches to school mechanics, to produce courses more appropriate for a general education, developed as part of the wider movement for correlation before the First World War.

The MA's [1904] early Report on this branch stressed the need to incorporate experimental, numerical and graphical elements, in conjunction with the analytical treatment, and over the next few years various individuals reported developments and discussed various ideas in the columns of Nature and the School World. During 1904, a letter published in Nature on 'Graphic Methods in an Educational Course on Mechanics' [Milne, 1904] produced a number of responses.<sup>8</sup> In the School World there were contributions from Minchin [1904] and Nunn [1905] on dynamics; Fletcher [1904], who outlined the practical possibilities of a trolley which he had designed; Eggar [1905] on experimental mechanics; Ashford [1906] of Dartmouth, who described his pioneering course for naval cadets from the age of thirteen, at Osborne and Dartmouth, where Fletcher's trolley was actually being used;<sup>9</sup> Schofield [1906, 1907], who discussed various pieces of apparatus for experimental work; and Tuckey [1907] on the questions of mass, weight and systems of units. Godfrey [1912a, pp.174-177] also optimistically discussed the possibility of teachers of mathematics providing correlated courses of mechanics for average pupils. However, outside

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8 Letters from four different correspondents were published, including one from W.J. Dobbs. See Nature [1904], Vol.70, No.1804, pp.81-82; No.1805, p.103; No.1806, p.125.

9 The naval colleges were in a unique position to innovate in mechanics teaching [Eggar, 1912, pp.340-341].



the naval colleges, and apart from the case of intending engineers and army entrants (see p.205) the blending of the theoretical and experimental approaches to mechanics made very little headway [Eggar, 1912, p.338].

Branford [1908, p.254] referred to the divorce of theory and practice in mechanics as a 'radical blunder ... still generally perpetrated outside the technical schools,' and he referred to Ernst Mach's The Science of Mechanics as exemplifying the way forward. In a general secondary education, mechanics was normally confined to a little practical statics as part of physics, and divorced from mathematics. Eggar [1912, p.341] was thus prompted to remark that 'In the case of the ordinary public or secondary school ... Mechanics does not form part of the general education of all boys.' After the War, Fawdry [1924, p.137] admitted that 'In mechanics our progress towards the ideal of Professor Perry is slow,' and that 'many teachers still neglect experimental work, either from lack of time or owing to the difficulty of finding a place to do it in' [p.138].<sup>10</sup> Various deeper factors which inhibited progress were discussed in Chapter 5 (see pp. 212-216).

For the First Examination, mechanics as a separate subject was taken by only four per cent of candidates in 1926, and this proportion had nearly halved by 1937 [BE, 1938, p.99]. Some mechanics was included as part of additional mathematics by three of the eight approved examining bodies, investigated in 1931 [BE, 1932, p.118]. However, in two of these cases, additional mathematics was only taken by around five per cent of the candidates. The Oxbridge Joint Board was exceptional, with one quarter of the candidates taking additional mathematics and the public schools generally chose to teach some

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10 Fawdry [1920] outlined his own ideas for mechanics in a general education in the Mathematical Gazette, and also produced a textbook for non-specialists on Readable School Mechanics [1925].

mechanics at this stage [MA, 1932a, p.103]. The SSEC reported:

Mechanics is treated by some [examining bodies] purely as a Mathematical subject, others demand some familiarity with experiment, and one regards it as a branch of Science and conducts a practical examination in the schools. [BE, 1932, p.118]

Significantly, this Report concluded:

The practice of holding a practical examination in Mechanics does not commend itself to the Investigators ... it is open to the same strong objections as practical examinations in Science at this stage, and there is in addition a danger of giving the wrong impression of the part which experimental work should play in the case of Mechanics. [p.119]

Thus, mechanics made very little progress in a general secondary education, and the treatment of this branch in the case of mathematical specialists continued to follow the nineteenth-century pattern. The ideals for correlation of innovators such as Perry, Eggar, Nunn, Fletcher, Ashford and Godfrey were never generally realized in English secondary schools. Much more successful, however, was the movement to broaden the curriculum to include aspects of trigonometry for all secondary pupils.

#### Trigonometry in a General Education

Beyond the three traditional branches, trigonometry emerged as an obvious candidate for broadening the curriculum. It was a potentially unifying branch, having links with arithmetic, mensuration, geometry, particularly similarity, and algebra, including functionality (see p.291). It also had considerable utility value for scientists, engineers and geographers, whose interests in mathematical education were becoming increasingly important. However, the traditionally highly specialized and isolated treatment, following a thorough grounding in manipulative algebra and geometrical similarity, which was not reached until Euclid VI, made this branch wholly inappropriate for the majority of pupils, who never reached this 'promised land,' and its potential utility was little exploited [Mercer, 1913, p.194; Strachan,



1918, pp.218-219].

The conventional academic approach began with the six definitions and the development was dominated by compound angles and heavy algebraic manipulation. Much of the numerical work was with special angles, involving surds, or angles measured down to seconds. Where tables were used, seven-figure tables such as those of Chambers were the norm [Siddons, 1956, p.167]. Graphical methods, involving measuring and squared paper in particular, were not exploited, nor was the functionality aspect. The popular textbooks of Hall and Knight, Lock and Loney typify this late nineteenth-century pattern [Wolff, 1915, pp.89-91,99-100].<sup>11</sup>

Schools which came under the DSA's influence were required to teach some trigonometry for the Second Stage mathematical examinations, but the approach was conventional [Hitchens, 1978, pp.76-78]. The solution of triangles was the principal topic, and wider possibilities were largely ignored [Brill, 1901, p.280]. The ideals of correlation and unification were neglected, and, around the turn of the century, pupils reaching this branch were typically still being put through a rigorous and overambitious course, dominated by the rote learning of formulae for examination purposes [Mathews, 1902].<sup>12</sup> However, the Perry movement brought with it alternative possibilities.

Perry had urged the more general introduction of some useful trigonometry in as early as 1880 (see p.222), and twenty years later his widely publicized scheme of practical mathematics (see p.100) included some simple trigonometry, treated numerically and graphically, with the aid of four-figure tables (see p.233). Godfrey's 'Compromise' at Winchester included some 'numerical trigonometry,' under this name

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11 Loney [1893, p.v] claimed 'Trigonometry consists largely of formulae and the applications thereof.' His textbook went through thirty-two new editions or reprints, up to as late as 1959!

12 Mathews [1902] drew attention to potential links with mensuration, surveying and dynamics, and to the benefits of unification.

(see p.108), and in the public schoolmasters' letter he referred to it as 'a certain type of diluted trigonometry which is found to be within the power of every sensible boy' (see pp.109-110). He also provided more detailed suggestions in the School World [1902d, pp.290-291], where he referred to the stimulus for introducing some simple trigonometry provided by the recent naval entrance papers, and to the value of the DSA's handy four-figure tables. Marshall's [1902] suggestions in Nature were also quoted as supporting 'arithmetical trigonometry,' to broaden a general education and provide a basis for the more specialist 'algebraical' treatment [p.291]. General support for simplification and broadening to include some trigonometry was also forthcoming from the MA [1902b, p.5] and the BAAS (see pp.112-115). There is an obvious parallel here between the development of preliminary practical geometry in a general education and the idea of introductory 'practical trigonometry,' though a general victory for the latter took a long time to achieve.

The new thinking produced a crop of trigonometry textbooks, the titles of which often included such words as 'beginnings,' 'beginners,' 'elements' or 'elementary,' and occasionally 'practical' or 'experimental' [British Museum, 1906, 1911].<sup>13</sup> A scrutiny of prefaces confirms the influence of the Perry movement, and, in particular, Borchardt and Perrott [1904] granted that the 'new trigonometry' was part of a general change 'so largely due to the genius of Prof. Perry.' The early influence of the naval and military requirements is also clear, and Mercer's [1906] textbook was based on his work with junior classes at Oundle and subsequently with young naval cadets.

For boys' public schools, Godfrey [1908, p.258] could soon claim that simple trigonometry for non-specialists was 'now introduced at an

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13 The word 'numerical' was not used in titles until after the publication of the MA's [1910c, 1911b] Reports [British Museum, 1918].



early stage in many schools, say at 13-15 years of age,' and the HMC's 9-16 Syllabus of 1909 embodied the new approach, with applications to surveying (see pp.150-151). However, examining bodies were still generally not prepared to force the pace in this matter [Hawkins, 1912], and, towards the end of the decade, there was a renewed movement to implement change throughout secondary schools.

The Joint Committee of the APSSM and the MA [1909e, pp.4-5] argued the case on the grounds of correlation, and, the MA [1910c, pp.6-8] strongly urged major simplifications in algebra to permit the introduction of trigonometry and mathematical tables. Significantly, the title of the MA's [1911b] revised Report incorporated the qualifying terms 'elementary' for algebra and 'numerical' for trigonometry, and referred to the latter as 'so important that it should be included in all cases' [p.7], even granted the principle of teachers' curricular freedom. There was no swift response from examining bodies generally, though the Oxbridge Joint Board did take some tentative experimental steps concerning trigonometry questions (see pp.163-164). At this time, practices varied from school to school in a typically English way.

Fletcher [1912, p.91] reported that in grant-aided secondary schools:

Of higher subjects the first and most frequently taken is Trigonometry ... in a fair number of schools it may be regarded as part of the normal course of work at least for boys.

The common introductory approach followed the more practical tendencies, though work out of doors was rare, and some important examining bodies, notably London, still regarded trigonometry as an exclusive branch of higher mathematics. Pessimistically, Nunn [1913b, p.706] was prompted to remark to the BAAS that 'few systematic attempts have been made to reform school trigonometry in the spirit of Professor Perry's teaching,' and his own ambitious ideas were embodied in his seminal

book [1914] (see p.269).<sup>14</sup> Nunn deplored the inhibiting effect of some examinations, though the BE at this time did introduce trigonometry and tables into the Ordinary Syllabus for the Teachers' Certificate Examination (see p.67). There is, however, some evidence of differentiation between the sexes regarding trigonometry in secondary schools.

Miss Punnett admitted there was 'a tendency to assume that girls are not interested in such abstract matters as sines and cosines [MA, 1911c, p.55], and, given the new approaches now being implemented in some boys' schools, she regretted this common omission in the case of girls. The pattern was confirmed by Story [1912, p.552]:

In a few schools trigonometry is begun in the Upper Fifth Form, but in the majority it is deferred until the Sixth Form or not taken at all.

Not surprisingly, the Girls' Schools Committee of the MA [1916b, pp.25-26] advocated trigonometry, with various applications, from the age of fourteen. By this time, various new approaches had become embodied in recently published textbooks.

There was a significant peak in the output of new trigonometry textbooks in the period 1911-1914, following the MA's [1910c, 1911b] Reports. The qualifying term 'numerical' was now included in a number of titles, such as those of Mercer (Dartmouth) and Price (Osborne), in particular [British Museum, 1918], and, subsequently Abbott [1918] (see p.269). Mercer [1913, 1914] outlined his approach to trigonometry for beginners in a paper delivered to the London Branch of the MA, and Piggott [1919], also of Dartmouth, acknowledged Mercer's pioneering work, in the preface to his own textbook. A scrutiny of prefaces again reveals the importance of the Perry movement's ideals (see p.225)

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14 Dobbs' [1913] extraordinary book presented a unified treatment of pure geometry, co-ordinate geometry, trigonometry and mensuration. He outlined his rationale to the London Branch of the MA - see Mathematical Gazette [1913], Vol.VII, No.106, pp.139-146; No.107, pp.167-170. He argued similarly some twenty years later [Dobbs, 1932].



for pedagogical developments in trigonometry. However, the victory for this branch in a general education was still not complete.

HMI Strachan [1918, pp.218-219] looked back at the nineteenth-century élitist pattern and claimed that 'All that is changed or changing.' The earlier treatment of geometric similarity, previously deferred to Euclid VI, had helped the movement forward, as had also the desire for some outlet in arithmetic, which was a narrow and over-worked branch. Ten years later, HMI Carson [1929, p.23] reported that the 'normal minimum curriculum' included 'in almost all cases trigonometry of a simple numerical kind,' though in the First Examination this branch was 'sometimes optional and sometimes absent.' He referred to the spread of trigonometry and logarithms as 'outstanding features' of reform, with the initiatives coming from the schools, and the examining bodies gradually responding [p.26] (see p.56). However, London continued to omit trigonometry, in spite of the MA's pressure (see pp.172-173).

By 1930, seven of the approved bodies for the First Examination catered in some way for trigonometry, as an addition to arithmetic or geometry [MA, 1930a, p.147]. The SSEC reported that it was 'now the common practice in schools to teach ... the use of numerical Trigonometry in pre-certificate Forms,' though the examination questions were alternative and could be avoided [BE, 1932, p.114]. The SSEC added that the eighth, unnamed, Body's resistance was 'much to be deprecated' [p.114], and felt that it was even appropriate to make numerical trigonometry no longer alternative. Eventually, in 1932, London changed its regulations to permit some trigonometry as an alternative in arithmetic [Retter, 1936, p.207]. More boldly, the Jeffery syllabus subsequently gave a firm place to trigonometry and its applications, fused with plane geometry [MA, 1944b, pp.vi-vii]. By this time, some trigonometry for non-specialists was well

established, but what is striking is that the process of change spanned some twenty-five to thirty years. Paralleling this development was the movement to teach calculus more widely.

#### Calculus in a General Education

As with mechanics and trigonometry, there was no possibility of teaching calculus to a wider ability range until alternative approaches to the conventional academic treatment for mathematical specialists had emerged. Again, with the rise of engineering, technical education and science teaching, there developed alternative treatments of calculus, tailored to its applications, and exploiting numerical and graphical methods in particular. Perry's [1897] textbook for engineers exemplified a radical alternative to the severely analytic approach, and he also urged that a more intuitive treatment, limited to simple functions, and stressing applications, had much wider educational potential [1900a, p.18]. Some of his basic ideas were built into his scheme of practical mathematics (see p.100). Thus, the earlier teaching of calculus became one objective within the Perry movement (see p.247), and Bryan [1903] raised this possibility at an early stage in the Mathematical Gazette.

Siddons [1936, p.24] recollected his own experiences around the turn of the century at Harrow, where calculus was limited to a small number of scholarship candidates, who were principally being drilled in the mechanics of differentiation [1956, p.168]. Siddons [1936, p.24] admitted that he first taught this branch to pupils with very different requirements in science. However, in the period 1901-1904 geometry reform dominated the situation and there is no evidence of early concerted efforts to introduce calculus more widely in the schools. Interest became greater with the development of international influences towards the end of the decade.

Godfrey [1908, p.259] reported that 'there is a strong movement



at the present day in favour of an early use of the calculus.' Gibson [1908, p.215] confirmed that 'A very keenly debated question on the Continent at present is the introduction of the elements of the calculus into secondary schools,' and that 'The trend of opinion is probably in favour of its introduction (in France this has been effected); but agreement is by no means universal.' Recent developments in Germany were reported by Wallace [1910], and the new thinking was closely linked with the growing interest in functionality, which in this country helped to sustain the Perry movement's case for extending the curriculum to include calculus (see pp.298-302).

New approaches in this country were again being pioneered with naval cadets, through the efforts of Mercer in particular, at Dartmouth. The character of the resulting textbook by Mercer [1910] was summarized by Godfrey [1912a, p.178]:

the most striking feature ... is that (in development of a suggestion originally due to Prof. Perry) all the main applications of differentiation and integration are exemplified without using any function more abstruse than  $x^n$ .

The treatment embraced kinematics, extrema, errors, and numerical and analytic integration, with various applications. The Civil Service Commission's examinations for the second division and for army entrance further stimulated innovation in calculus teaching, particularly in public schools. Jackson [1912c, p.367] referred to their 'marked effect on the introduction of the subject and on the lines which its development has taken' (see pp.58-59).

In his paper on calculus for the Special Reports, Jackson [p.369] underlined the recent shift in attitude:

Some years ago a boy would learn to find the  $n^{\text{th}}$  differential coefficient of  $e^{-ax} \cos bx$  before having any idea that the velocity of a body which fell  $16t^2$  feet in  $t$  seconds could be found by differentiation. As has been remarked by Mr. Eggar, such a course gave the impression of a cinematograph film whose time-sequence had been accidentally reversed. Professor Perry in his well-known treatise on the calculus initiated a reaction....

By this time, in England, the rigorous treatment of calculus by pure mathematicians such as Hardy, who had been strongly influenced by the Continental analysts and had impressed his views at undergraduate level [Newman, 1948], and users of this branch such as Perry and Silvanus P. Thompson, whose Calculus Made Easy was published in 1910, represented sharply contrasting extremes [Schwarzenberger, 1980]. Significantly, Jackson [1912c, p.374] remarked:

To set up separate 'practical' and 'academic' courses in the calculus will deprive each party of valuable material and will repeat a mistake from the effects of which the teaching of mechanics has not yet recovered.

Obviously, rigorous pure mathematical analysis was unsuitable as an introduction to this branch, but more practical and approachable forms of calculus had now emerged for possible adoption at school level.

In the period 1907-1914, the Mathematical Gazette devoted more space to the teaching of calculus than to any other branch (see pp.143-144), and this interest was paralleled by growing concern for new directions in algebra teaching. In particular, Godfrey [1912a, p.178] was strongly influenced by Continental developments:

The ideas ... of the calculus and a feeling of the extraordinary power of this new instrument are accessible to a student with a modest degree of manipulative skill in algebra ... it is not necessary to tread for years the weary paths of highest common factor, fractions and the like before becoming worthy to enter this rich country.

He drew attention to the introduction of some calculus on the classical side in French lycées [pp.179-180], and he argued for the strong 'outlook' value of this branch, as the culmination of a disencumbered algebra syllabus, which should be infused with the idea of functionality, to provide a hitherto lacking and unifying sense of purpose [Godfrey, 1912d, pp.289-292, 309-311]. Godfrey and Siddons [1913, pp.vi-vii] boldly included an introduction to calculus in their new algebra textbook, and, again, the Oxbridge Joint Board had lent some early support, from 1910, in its algebra syllabus for additional



mathematics [Retter, 1936, p.128].<sup>15</sup>

Durell [1912] opened a discussion on the earlier teaching of calculus at the MA's Annual Meeting, and he acknowledged the influence of Mercer, though had to admit that 'in England the movement is far from general; only a few schools have given it a serious trial.' In particular, Godfrey, Siddons and Nunn contributed to a lively discussion, though the MA took no organized action at this time. Nevertheless, particular individuals continued to outline their own experiences and ideas. The Gazette included Jackson's [1913, 1914a, 1914b] scheme for school calculus, and one from Knowles [1913, 1914], based on his experiences since 1907 with boys of sixteen in a technical day school. Nunn's [1910, 1911b, 1913a, 1914] contribution at this time was typically rich and distinctive, with historical insights resulting in an unusual treatment of integration before differentiation, and he also advocated major innovations concerning symbolism. However, in relation to the schools, many of Nunn's ideas were no doubt fanciful, and, furthermore, the BE were not persuaded to broaden their mathematical requirements for the Teachers' Certificate to this extent (see p.67).

Outside the public schools, the movement appears to have made little general headway up to the War. Fletcher [1912] actually used experience of calculus as one measure of a teacher's academic competence (see pp.75-76), and he judged that:

Few will need to teach Calculus at school, and there are plenty of well qualified teachers to deal with the few boys and girls who need to go as far as this at school. [p.95]

In girls' schools, Story [1912, p.552] merely reported that calculus was 'reserved for the scholarship candidate.' However, the ICTM chose school calculus as a major focus for international comparisons (see p.134).

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15 See p.164. Godfrey and the HMC may have stimulated the Joint Board's initiative here.

Godfrey's [1914] resulting investigations provided much valuable data concerning the progress of the movement, but only in the case of the leading public schools. Calculus for mathematical specialists was well established; for future engineers and army entrants it had developed over the previous fifteen or so years; and from the age of sixteen for non-specialists it was 'a more recent movement' [p.235].

He acknowledged:

Perhaps the most powerful stimulus is that of the engineers, as represented by Prof. Perry. The physicists have long pressed for a modicum of calculus, and prefer to take it without too much mathematical rigour.

However, Godfrey's own enthusiasm may have coloured his judgement concerning general English conditions. He claimed that '"Calculus for the average boy" is the keynote of the modern movement in mathematical teaching' [p.235], and that the movement had received 'general support' [p.239]. He added:

In a few schools a preliminary course in calculus now forms an organic part of the curriculum for middle classes; in many schools there is a tentative movement in the same direction. [p.235]

External examinations, apart from those of the Civil Service Commission, were typically not taking the lead in this matter [pp.239-240], and, significantly, Godfrey remarked that 'the main obstacle is that most powerful force in educational matter - vis inertiae.' Newer textbooks, particularly Professor Gibson's [1904] Introduction to the Calculus, and Mercer's [1910] treatment, had made some impact in the public schools, though older, more conventional treatments were still popular [p.237]. In secondary schools generally, the crowded curriculum, narrow examination requirements, the quality of the teaching force, and early leaving all inhibited the progress of the movement. However, attempts were still made to sustain the momentum through and after the War.

Bayliss [1916] argued the case for his own First School Calculus



in the School World, and the subject was discussed at the BE's course for secondary teachers at Durham in 1920 (see p.81). New textbooks, such as Durell and Fawdry's [1923] continued to appear, and the latter author claimed that the number reaching calculus taught along the newer lines at public schools was 'steadily increasing' [Fawdry, 1924, p.137]. More generally, HMI Carson [1929, pp.26-27] subsequently claimed:

This subject now stands where simple trigonometry stood fifteen years ago. It is 'in the air' for quite ordinary boys, and a small but increasing number of schools are actually attempting it with success, quite independently of any examination requirements....

Most First Examinations included some calculus, but only for additional mathematics, and thus some progress had again been achieved 'by the act of the schools themselves' [p.27] (see p.56). The SSEC found wide variations in the provision made for calculus within additional mathematics, and reiterated the pre-War arguments for simplification and broadening to produce a 'wider outlook' [BE, 1932, pp.118-119]. However, calculus was still excluded from elementary mathematics for the First Examination.

Again, the Jeffery syllabus represents the culmination of a movement, stretching over some forty-five years, to bring some calculus into a general secondary education. The Report [1944b, p.iii] claimed:

The most striking feature of the proposed syllabus is the inclusion of the beginning of calculus in the optional parts of the papers....

The underlying rationale closely recapitulated the arguments before the First World War, based principally on functionality (see p.301):

the intrinsic importance of the subject is very great. The transition from the static mathematics of the formula, which enables one quantity to be calculated when another is known, to the dynamic mathematics of the function, which considers how one thing changes with another, is one of the chief ways in which mathematics has adapted itself to the consideration of practical problems. The ideas ... grow naturally and easily

out of the consideration of graphs which rightly now occupy so important a place in the elementary teaching of the subject. [p.iii]

However, convincing schools generally of the value of this wider and far from new innovation was a very different matter, within a much larger secondary school system, and one not now caught up in any general movement for reform (see pp.296-298).

In relation to the general broadening of secondary mathematics for non-specialists, this chapter has shown that the reform movement eventually achieved a fairly complete victory for trigonometry, only a partial victory in the case of calculus, and largely a defeat in the case of mechanics. The complexity of general change and its lengthy time scale are particularly striking features of the English pattern. Throughout the period, the three traditional branches remained the staple of general secondary mathematics, and some major features of change within these branches will be considered in the next chapter.



## Chapter 8

### The Three Traditional Branches

It is not a purpose of this thesis to consider the finer detail of developments in the teaching of the various specific topics which were traditionally grouped under the headings of arithmetic, algebra and geometry, and which largely persisted in the curriculum, though possibly in modified forms. Various features of the reform of mathematical education, relating to these branches, have already arisen in the wider context of change, discussed in earlier chapters. In particular, one important development, considered in Chapter 7, was the growing flexibility in the relationship between the branches. However, there are a number of pedagogical trends particular to arithmetic, algebra and geometry, considered separately, which deserve to be brought into sharper focus. In this chapter, many of these trends will be related to the wider forces and tendencies already considered, particularly from Chapter 5. Relatively stable states existed concerning content and methods within the three branches towards the end of the nineteenth century. It was against this stability that the reform movement reacted along organized lines discussed in Chapter 3. The full history of the development of these branches into their conventional nineteenth-century forms will not be considered here, though some relevant sources will be indicated.

#### Arithmetic

Arithmetic became the most widely taught branch, aspects of which are involved in the history of infant, elementary, secondary, technical and teacher education. However, unlike the case of geometry, with the eventual abandonment of Euclid as a textbook and the adoption of practical geometry, there were no rapid and global developments in arithmetic teaching from the late nineteenth century, but rather there were a number of gradual and subtle transformations in detail.

A discussion of various treatments of arithmetic over the extensive period 1535-1935 has been undertaken by Yeldham [1936], who, in particular, draws heavily on the survey of textbooks by De Morgan [1847]. Broadly speaking, the early nineteenth-century treatment was dominated by the specific requirements of commercial utility, the approach being anti-mathematical in its predisposition for an abundance of special rules, with a neglect of general principles [Williamson, 1928]. As such, the subject was spurned by mathematicians and was educationally held in low esteem as a narrow technical accomplishment. However, over the period 1830-1870, arithmetic became generally established as an examination subject, and newer, more mathematically orientated textbooks, such as those of Colenso and Hamblin Smith, blended techniques, principles and commercial applications [MA, 1919a, p.9; Bushell, 1947, pp.71-72].<sup>1</sup> There is an interesting contrast between the continuing commercial bias in nineteenth-century arithmetic and the general neglect of utility in algebra and geometry [LCC, 1911, pp.16-17]. By the late nineteenth century the popular textbooks of Pendlebury [1886, 1899], in particular, followed closely the well established pattern.

Various developments in arithmetic under the Elementary Codes were discussed in Chapter 2 (see pp.15-23), and teaching methods during the period 1839-1890 have been explored in a thesis by Owen [1959]. A useful outline of developments from the mid-nineteenth century is provided in a paper by Flemming [1959], and Wingrove's [1976] dissertation focuses on twentieth-century arithmetic in London, with particular reference to the issue of standards. A more detailed analysis of developments in arithmetic over the period of major reform in mathematical education has been undertaken in an unpublished paper by Price [1980], which provides the basis for a number of the general

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1 Gray [1952, pp.61-62] has described a parallel general nineteenth-century shift in arithmetic in Scotland.



conclusions which now follow.

Some of the earliest enlightenment in arithmetic teaching concerned the infant stage, and gradually spread more widely to affect the teaching of older pupils (see pp.185-189). Specifically, the newer concerns here were for motivation and the learner's involvement; more oral work and mental arithmetic, as opposed to mechanical cyphering; the use of concrete aids, to provide hand and eye training in particular; more work with smaller numbers to promote understanding; and greater attention to work in measurement as well as pure number. This last trend relates to the wider 'practical education' movement in particular, and to the growing demands of scientific and technical education discussed in Chapter 5. The work on measures in arithmetic was traditionally just a feature of computation, and the remarkable rise of 'practical measurements' and 'practical arithmetic,' which brought with it a concern for processes of estimation, approximation, actual measurement and experimentation, has already been discussed (see pp. 203-212). The newer emphasis on scientific rather than commercial utility gradually brought with it such advances as a more critical approach to units, and a growing interest in metrication; a shift of interest from fractions to decimals, with improved teaching methods; a more critical attitude to unwarranted accuracy in computation, and to problems with specially chosen numbers; and a wider use of calculating aids, particularly logarithms (see pp.231-238).

The historical development of the extraordinary English system of measures has been sketched by Cajori [1917, pp.167-179]. One aspect of the reform of English and American mathematical education was the movement to make the teaching of measures more relevant and practical. Two specific goals here were the abandonment of outdated measures and the introduction of metrication, which had become the general norm in scientific work and science education, as well as in the commercial

and general educational fields in most other than English-speaking countries. With a decimal numeration system there were obvious benefits to be gained from abandoning the older system with its multiplicity of units and exchange values based on no coherent general principles. Godfrey [1906a, p.72] referred to the 'declinist' arguments of engineers like Perry (see p.189), and added 'the remedies they urge are the adoption of the metric system and the improvement of mathematical teaching.'

Support for the scientists and engineers was forthcoming from the Education Department, and subsequently the BE (see pp.21-22), and from various individuals and pressure groups discussed in Chapter 3, as well as the Decimal Association from 1890. The interest extended to the system of coinage, and the great potential saving of time in education by eliminating the learning of complicated tables and algorithms was strongly urged. In 1897, a Bill was passed which legalized the metric system in trade, and the teaching of the metric system, linked with decimals, became an established part of arithmetic. Educational periodicals and methods books included various suggestions concerning teaching methods and aids for this topic.

The British system of measures, however, still persisted, and efforts up to the end of the First World War to enforce metrication and a decimal system of coinage failed. The movement faded, and this country continued with its own unique system whose complexities continued seriously to encumber the teaching of arithmetic at all stages. Admittedly the British system had been trimmed, but the metric system was now an additional burden for mathematics teachers. On the credit side, there were some general gains from the shift of interest from fractions to decimals in arithmetic.

The teaching of fractions had become an important and extraordinarily elaborate game during the nineteenth century. The main



advances here were reduced complexity, involving less emphasis on 'skyscraper' types and multiple bracketing, with a more concrete introduction using pictures and diagrams, and a more rational treatment based on general principles, particularly equivalence. The importance of decimals, as opposed to fractions, gradually increased, with the former topic being introduced earlier. Other gains for the teaching of decimals include the exploitation of links with the numeration system for whole numbers, and with measurement and the metric system. Computational difficulty was slowly reduced and exact computation with recurring decimals gradually eliminated, with a shift to the more useful business of approximation. However, manipulative excesses in fractions and decimals tended to persist, and were sustained by examination requirements in particular.

The need for more attention to approximations in arithmetic was particularly urged by scientists and engineers.<sup>2</sup> One early general development here was the uncritical acceptance of contracted methods of calculating to a specified degree of accuracy (see p.162). Subsequently, approximations were considered on a broader front, and more critically, including estimations, checks, errors and accuracy. Interest in contracted methods faded, and the use of logarithms became firmly established (see pp.231-236). The growing interest in approximation eroded the nineteenth-century preoccupation with unwarranted accuracy and artificial exactness, and was part of a wider shift from commercial to scientific utility in arithmetic.

The commercial element in arithmetic persisted, though gradually a more rational treatment was adopted, and a number of outdated and unreal commercial topics were slowly eliminated.<sup>3</sup> Furthermore, the

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2 In his arithmetic textbook, Workman [1902, p.211] quoted Perry thus: 'When I was at school the mean distance from the earth to the sun was stated as 95,142,357 miles. I wonder why furlongs and inches were not mentioned. The best knowledge we now have of this distance is that it is not greater than 93 nor less than  $92\frac{1}{2}$  millions of miles.'

3 See, for example, pp.21-22 and p.63 for some early changes in schemes for arithmetic.

wider new alternatives of 'household arithmetic' for girls, and 'arithmetic of citizenship' began to emerge (see pp.156-157). As part of the shift from rules to principles, there was also growing resistance to the general classification of problems by type, with special rules taught for each type.<sup>4</sup> The newer interests were in more realistic problems and the cultivation of general problem solving abilities, which reflects the Herbartian emphasis on ideas and thought processes discussed in Chapter 6. There was much lip-service paid from the 1890s to the need for more common-sense, intelligence, and thinking from pupils in arithmetic, instead of the blind following of rules. Specifically, greater attention was paid to general principles, such as place value in work on whole numbers and decimals, equivalence in fractions, and ratio and proportion in commercial and other applications of arithmetic. There was also greater concern for explanations of the principles underlying various algorithms, and some processes such as extraction of cube roots, but not square roots, LCM and GCM, other than by prime factors, were eliminated. However, renewed concern for the mechanical aspects of arithmetic is detectable from around 1910, both officially, and in the chosen foci for research (see pp.25-27,286-288).

Finally, arithmetic should be viewed not only in isolation but also as a tool within the context of the mathematical curriculum as a whole. This was one feature of the unification tendency discussed in Chapter 7 (see pp.289-293). In particular, the links between arithmetic and algebra came to be exploited, as one of a number of general developments in the teaching of algebra, which will now be considered.

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4 Pendlebury [1886] included sections on work, pipes, etc.; races and games of skill; hands of a clock; mixtures, previously termed alligation; areas of rooms, carpets, etc.; areas of walls, papering, etc. He even added a section on pasture with growing grass in 1897! This pattern in textbooks persisted, with support from examination requirements.



## Algebra

In sharp contrast to geometry, the history of English algebra teaching has attracted very little attention, though an attempt to consider the historical development of traditional school algebra from its early roots has been made in a dissertation by Price [1974]. The conclusions which follow are based largely on the material in Price [1974], though they will be related to a number of especially relevant features of reform already discussed in earlier chapters.

In the early nineteenth century, some algebra became required of all students at Cambridge, and, with the general development of the examination system, algebra became firmly established in secondary schools for boys, and subsequently for girls, as part of the movement to eliminate major differentiation. A little conventional algebra also became a very popular 'specific subject' in elementary schools, particularly during the 1890s (see p.18). The treatment of major nineteenth-century writers of school algebras from Colenso, in the 1840s, through Todhunter and Hamblin Smith to the hugely successful Hall and Knight, from the 1880s, varied little, and paralleled the mathematical treatment of arithmetic, beginning with chapters on the four rules, and subsequently dealing with all the manipulative complexities of factors, GCM, LCM, fractions, involution (powers), evolution (roots), indices and surds. The solution of equations featured prominently, and, indeed, the solution of often artificial problems via equations was the only major use made of this complex machine, but normally this came only relatively late in the course [Wolff, 1915, pp.54-55,88-91; Smith, 1926, pp.20-26].

Traditionally, this branch was not related to arithmetic or geometry, its potential wider utility was spurned, and the highly abstract and specialized mathematical treatment could only be justified in a general education on the increasingly shaky ground of mental

discipline. In the early twentieth century there was a general reaction to the established pattern, and, slowly, there developed a search for new purposes to infuse the teaching of algebra to non-specialists. Some specific early advances relate to the wider unification tendency.

The development of links with arithmetic brought with it a more natural transitional approach to the early stages of algebra, through 'generalised arithmetic,' as well as the more general tendency to 'arithmeticise' this whole branch (see pp.292-293).<sup>5</sup> The concern for a closer relationship between formal manipulative algebra and the behaviour of numbers also raised important pedagogical problems concerning the generalizations of number, from the naturals to the reals, and particularly the case of negative numbers, as well as indices and surds. The possibility of a more rigorous treatment of school algebra, following the deductive ideals in geometry, and breaking away from the typically slipshod nineteenth-century textbooks, was explored and implemented in some cases [Barnard, 1907, 1912] (see p.96). However, there were pedagogical objections concerning an uncompromisingly pure mathematical approach in a general education. Nevertheless, there generally developed more mathematical and pedagogical sensitivity concerning the placement and treatment of 'directed' i.e. positive and negative numbers in particular, and their corresponding algebra.

Another tactic which exploited the links with arithmetic, and provided important motivation, was the much earlier treatment of arithmetical problems, using algebraic methods, than was the conventional practice. Other connections with geometry and mensuration also developed. Specifically, the alternatives emerged of treating certain geometrical theorems algebraically, particularly those in Euclid II (see p.115, note 49), and of exploiting the use of formulae and equations in work on mensuration. A further essentially geometrical

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5 See, in particular, the public schoolmasters' letter, p.110.



development was the rapid and widespread adoption of graphical methods in algebra, involving the use of squared paper.

The phenomenal spread of graphical methods in mathematical education was discussed in Chapter 5 (see pp.228-231). Graphs were rapidly, enthusiastically and largely uncritically accepted in the early stages of reform. Predictably, however, there was over-indulgence and much confusion concerning purposes, with little early consideration of the proper place of graphical work in algebra. It was not the experimental use of graphs, on the lines of scientists and engineers, which made a major impact in mathematical education generally, but rather the use of graphical methods to bring a practical and geometrical dimension to the study of relationships and to the solution of equations. There arose two major shortcomings here. One was the predilection for the approximate graphical solution of equations which could be fairly easily and precisely solved analytically. The other was the tendency to attack prematurely forms  $f(x,y)=0$ , which were general relations belonging to analytical geometry, and previously only considered by mathematical specialists. The subtle but important distinction between the more general relational forms  $f(x,y)=0$  and the particular functional forms  $y=f(x)$  was not generally appreciated. The latter became associated with the important concept of functionality, which subsequently provided a valuable general sense of purpose for 'graphical algebra' (see pp.298-301).

Significantly, three Circulars of the BE [1909a, 1914b, 1925b] boldly tackled the question of purposes in graphical work. As well as the aspects already discussed, the BE pressed for the acceptance of graphs as a powerful pedagogical tactic to be woven into the fabric of algebra teaching, and not merely grafted on as an additional burden, albeit a novel one, which was common in the early years of reform. The BE suggested the use of graphs to provide intuitive support for

the generalizations of number and the laws of algebra, and for the treatment of powers, roots, indices generally, and also logarithms. The work on indices leading to logarithms, supported by the use of graphs and four-figure tables, brought further enlightenment into algebra teaching (see pp.233-236).

Another development in algebra teaching, which relates to the Perry movement and the need for correlation, was the increasing importance attached to the creation, manipulation and use of algebraic formulae, with examples drawn from mensuration and science in particular. Indeed, between the Wars, an important issue in algebra teaching was whether problems leading to simple equations or the development of formulae from various situations should be used to provide a purposeful introduction to algebra, in place of the older plunge into various definitions, followed by the four mechanical rules. This was one aspect of a wider discussion of the purposes of algebra in a general education, which developed in the early twentieth century.

Although the links with arithmetical and other problems provided a sense of purpose for the beginnings of algebra, the problem of justifying the full machinery of algebra for the majority of pupils, who would never need it vocationally, still remained. In principle, Godfrey, Nunn, Whitehead and other leading thinkers in mathematical education provided a new rationale, which emphasized the important ideas and principles of this branch, in place of the older preoccupation with elaborate techniques, which could only be crudely justified as a mental discipline (see pp.281-285). Functionality and the 'outlook' value of algebra were central themes, with the wider ambition being to extend the traditionally narrow confines of algebra to reach aspects of trigonometry and the calculus. The slow twentieth-century progress of these broader visions in practice was discussed in Chapter 7, and,



even within the traditional confines of algebra, general advances were limited, and lagged a long way behind the newly emerging spirit.

HMI Strachan [1918, pp.210-217] judged that the traditional pattern of heavy manipulations in 'algebra up to progressions' [p.211] was still the norm, though he regarded the introduction of graphs as 'the thin end of the wedge for the break-up of the old wicked system' [p.212]. However, even in the case of graphs, there had been reactions before the War, as well as accusations that manipulative skills in algebra as well as arithmetic were declining, as a result of reform. Getting the balance right between techniques, ideas and applications was a long drawn out and continuing problem, there being the usual inhibiting factors.

Examinations continued to test mainly what was easiest to assess, that is manipulative skills as opposed to understanding and applications, and a tendency persisted to demand a higher level of skill than was reasonable for average pupils. Thus the needs of the growing majority of secondary pupils continued to some extent to be sacrificed to those of the minority of potential future users of algebra. However, from around 1910 the MA in particular did much to persuade examining bodies to change their requirements in algebra, so as to reflect the newer emphases on formulae, applications and somewhat easier manipulations (see pp.163-165, 172-174).<sup>6</sup> Much of the newer thinking was embodied in textbooks such as those of Godfrey and Siddons [1912, 1913], and, subsequently, the very successful Durell [Bell, 1934], though much older textbooks, particularly Hall and Knight's [1885] continued to provide strong opposition, in spite of being infused with the nineteenth-century spirit.<sup>7</sup> Teachers' conservatism was also a

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6 In the early 1920s, the algebra syllabuses of both the Oxbridge Joint and London Boards were redrafted [Retter, 1936, pp.128-129].

7 According to Mumby and Norrie [1930, p.321], Hall and Knight's algebra textbooks for Macmillan 'continued to earn such vast sums that it became a family joke to name the statues at the entrance to their country seat, Birch Grove, after the two mathematicians.'

persistently powerful force, and HMI Carson [1929, p.26] referred to 'the tacit refusal of teachers to limit the syllabus in algebra' as an outstanding feature of twentieth-century developments. The subject-matter of algebra had changed little, and still stopped at quadratics for the majority, with possibly also progressions. The main advances were in methods of teaching, and Carson judged that examination requirements were consistent with the expectations of the majority of teachers [pp.23-26]. He concluded:

The truth is probably that [algebra] needs an outlet such as logarithms provided for the work in arithmetic. Compared with the size of the tool the pupils have made, as yet, but little use of it. [p.24]

Significantly, the Jeffery Report, and the resulting alternative syllabuses after 1944, implemented a wider view of School Certificate Mathematics, based on ideas and applications, and in both arithmetic and algebra there were still plenty of opportunities for pruning and simplifying on the manipulative side. The Jeffery syllabus adopted the general themes of 'formulae and equations' and 'graphs, variation, functionality,' which had risen to prominence in the thinking concerning algebra teaching before the First World War [MA, 1944b]. Furthermore, Durell [1931] (see p.270), whose various algebra textbooks enjoyed prolonged success after 1920, was far from being a pioneer as regards general ideas, but he very successfully reconciled the demands of various examination requirements and the needs of teachers and non-specialist pupils in the expanding secondary school system, capitalizing fully upon earlier contributions to the pedagogy of algebra [Bell, 1934]. Apart from the case of graphs, the dissemination of new ideas in the teaching of this branch was, as with arithmetic, a slow and complex process within the English educational system. The most dramatic developments, which concerned the teaching of geometry, remain to be surveyed.



Geometry

## EUCLID'S ELEMENTS.

By AN ANCIENT GEOMETER.

Farewell! old Euclid: loved of yore, and may be loved again  
 When our beatific vision sees thy plane surface, plain;  
 Unfettered now, we range without thy limited confines;  
 The concept had no breadth at all. We must have broader lines.

We shun thy close restrictions, and thy ordered sequence, too:  
 The ancient Greeks might learn to think; we've other things to do;  
 Nor can we stimulate again thy sober mental joy -  
 Euclidian reasoning 'stupefies the normal British boy.'

No more we seek the famous pons when standing on the brink -  
 'Tis but a shallow stream to cross, nor need the tiro shrink;  
 Bring compasses and callipers and geometric tools,  
 And waive eternal principle for briefly stated rules.

So we close the battered volume; lay it high upon the shelf,  
 And adopt more modern methods in our eager chase for pelf:  
 Thus one more link is severed, and we hail our glad release  
 From the intellectual thralldom of the glory that was Greece.  
 [S.C., 1907]

This remarkable lament in the Journal of Education exemplifies well a new tension which developed between the classical and persisting Euclidean ideals and those of the Perry movement, which inspired the major early reforms up to 1905, discussed in Chapter 3. Given the character of developments in English geometrical teaching, it is not surprising that this branch has attracted more attention in the literature than any other.

Cajori [1917, p.281] referred to England as 'the home of conservatism in geometric teaching,' and an American Report on geometry teaching highlights this conclusion in a useful historical sketch of contrasting nineteenth-century developments in France, Germany, Italy, England and America [National Education Association, 1912, pp.5-32]. The limited nineteenth-century achievements in England have been considered by Brock [1975b], and the period of resistance to change from 1868 was discussed in Chapter 3 (see pp.84-94). Wolff [1915] has

also provided a valuable detached view of English developments in geometry teaching up to the First World War. After the War, two early masters' theses focused specifically on geometry. Fulford [1923] included much historical material, and he also undertook some early work in the experimental pedagogy of geometry (see p.287, note 37). Jackson's [1924] thesis provides a valuable historical survey of the development of geometry, from its early roots, and its teaching in England. This thesis also throws much light on both teaching practices and pedagogical thought in geometry, around the time of the publication of the Reports of the MA [1923b] and the AMA [1923] on this branch.<sup>8</sup> Aspects of the thinking which developed over the first quarter of this century have also been considered by Howson [1973b] (see pp.169-171). To appreciate fully the magnitude of change in the teaching of this branch in the early years of this century it is necessary first to consider certain wider historical aspects of geometry.

The thirteen Books, which constituted Euclid's geometry, were a monumental achievement in the logical systematization of then existing geometrical knowledge. As Eves [1972] has emphasized, the Greeks, and Euclid in particular, only indulged in 'material axiomatics', in geometry i.e. the logical organization of what were presumed to be geometrical facts concerning a physical reality. The general interest in non-Euclidean geometries did not develop until the nineteenth century, and it was only towards the end of the century that the view of geometry widened further to become one focus for 'formal axiomatics.' The concern now was to build internally coherent geometrical systems, with Euclidean geometry as one of many such systems. The new canons of rigour in the foundations of geometry revealed imperfections in Euclid, which were overcome by David Hilbert in his famous Grundlagen der

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8 The recent dissertation by Eustice [1977] on geometry teaching, 1850-1940, is less illuminating than the earlier theses of Fulford [1923] and Jackson [1924].



Geometrie of 1899, and alternative postulational treatments of Euclidean geometry soon followed.

Although these higher mathematical developments in geometry may appear to have no relevance for school geometry and its reform, Brock [1975b, p.21] has suggested that 'the appearance of other logical geometries ... helped to undermine the authority of Euclid as an educationally unique mind-trainer.' Certainly, during the important debates of 1902, very occasional reference was made to the recently exposed logical imperfections of Euclid (see p.110). However, the evidence does not suggest that either non-Euclidean geometries or foundational developments were important for the overthrow of Euclid as a school textbook. Indeed, mathematicians' reverence for Euclid's achievement persisted [Burn, 1975], and T.L. Heath [1925, p.ix], in the preface to his new edition of the Elements, remarked:

Euclid is far from being defunct or even dormant, and ... mathematicians will find it necessary and worthwhile to come back again and again, for one purpose or another, to the twenty-two-centuries-old book which, notwithstanding its imperfections, remains the greatest elementary textbook in mathematics that the world is privileged to possess.

The earlier quoted lament (see p.334) is also implicitly eulogistic concerning Euclid. Furthermore, in sharp contrast to the foundational interest in eliminating geometrical intuitions, and even eschewing diagrams, twentieth-century pedagogical trends were in quite the opposite direction, and there gradually developed a freer use of intuitions, a more flexible view of 'proof,' and a broader basis of assumption, with less interest in proving the relatively obvious. Here, Fletcher, in the BE's [1909a, p.5; 1914a, pp.8-11, 17-21] important Circulars, cleverly drew attention to the nature of foundational developments as a justification for turning away from the foundations in school geometry, and following the above-mentioned

trends instead.<sup>9</sup> The newer thinking and tendencies in teaching followed the abandonment of Euclid as a school textbook, and the general adoption of practical geometry. The nineteenth-century divorce of these two aspects of geometry, the theoretical and the practical, warrants some further discussion.

Parallels have already been drawn between developments in the teaching of geometry and mechanics (see pp.125-127,212-216).<sup>10</sup> As has been shown, there were formidable difficulties involved in making the latter more practical in mathematical education, and progress was very limited. However, whereas practical mechanics involved relatively elaborate apparatus, geometry could be made more practical through the use of very simple aids and instruments in ordinary mathematical classrooms. Nevertheless, as long as various editions of Euclid were being used in the schools, such as those of Potts, Colenso, and Todhunter, building on Simson's edition, and the various parts of Euclid provided by Hamblin Smith and Hall and Stevens in the late nineteenth century, the links with practical geometry were not exploited [Wolff, 1915, pp.50-54,88-90].<sup>11</sup> In this connection, Euclid's attitude to geometrical constructions needs to be explained.

Euclid postulated that just three constructions could be performed, with the aim of building up all other constructions from these, with justifications. These postulates were that a straight

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9 Fletcher [1923] subsequently also wrote an article for the Gazette on 'A Method of Studying Non-Euclidean Geometry,' but this was a highly speculative contribution in relation to school geometry.

10 It was also argued in Chapter 7 that a necessary condition for broadening a general mathematical education, to reach such branches as mechanics, trigonometry and calculus, was the development of more practical forms of these traditionally specialist subjects, in their early stages.

11 Euclid I-VI covered plane geometry, though Euclid's very complicated geometric theory of proportion, developed in Book V, was at an early stage regarded as unsuitable in a general education. Thus Book VI on similarity was often not reached, and the work was limited to some or all of Books I-IV (see, for example, pp.18,45,62). Books VII-IX covered various topics now classified as the theory of numbers, e.g. Euclid's algorithm for the GCM. Book X dealt with irrationals, and solid geometry was not reached until Book XI.



line can be drawn joining any two points; a finite straight line can be continuously extended in a straight line; and a circle can be drawn with a given centre through a given point. Thus Euclid allowed himself just two geometrical tools, an unmarked straight-edge, and compasses that cannot directly transfer distances in the way that modern compasses and dividers can. His straight-edge was not a measuring instrument, and his compasses would collapse if lifted from the paper. Such practically unnecessary limitations were imposed as part of the pure mathematical predilection for deriving as much as possible, in principle, from as little as possible.<sup>12</sup> Thus, in Euclid I.2. it is ingeniously shown that distances can be transferred using collapsing compasses, and this proposition is subsequently freely used. Actually performing a construction, which might be very complicated with Euclidean tools, was not Euclid's concern.

In establishing any proposition, the deductive development of the constructions was rigidly observed. Thus, in Euclid I.5., the so-called pons asinorum, where it is shown that the base angles of an isosceles triangle are equal, Euclid employed a complicated congruence argument, which was necessitated by the fact that he had not yet established that an angle could be bisected. This was established later, as I.9., to be followed by I.10. on bisecting a line segment and I.11. on erecting a perpendicular. With the abolition of Euclid as a school textbook, such constructions became freely admitted in the deductive development as 'hypothetical constructions' (see p.117). The previous formidable difficulty of the pons in pupils' early work in geometry is understandable, and Bushell [1947, p.77] recalled that this proposition was the subject of a famous Harrow School song, called

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12 Following Euclid, the potential of other sets of geometric tools, such as modern compasses only, fixed distance compasses, named 'rusty compasses,' and the like, has been explored by pure geometers. This subject was discussed in Hilda Hudson's [1916] Ruler and Compasses, in the Longmans' Modern Mathematical Series (see p.269).

'Euclid,' which was still occasionally sung in the 1940s. These early propositions in Euclid not surprisingly caused enormous difficulties for pupils beginning geometry and, as a Report of the LCC [1911, p.17] remarked, 'A general belief grew up that geometry was beyond the grasp of the average mortal.' Remarkably, the Euclidean disinterest in practical constructions persisted in English schools throughout the nineteenth century.

Siddons [1952a, p.154] recalled his own introduction to geometry, in 1889, as the verbal learning of a string of definitions, postulates and axioms, followed by Euclid's propositions from I.1., with no experimentation and no drawing. Yet the head of his school's department was Levett, a leading member of the AIGT (see p.93). As late as 1900, Mair [1900, p.389] reported in Nature:

A few weeks ago I asked some hundred boys in a well-taught school (as present teaching goes) to give a certain construction of Euclid's, and also to carry out the construction with ruler and compasses on a given line. Hardly one failed to write out the construction and proof, but only one of the hundred carried out the practical construction. Clearly our present Euclidian teaching has little to do with geometry.

The classically dominated public and preparatory schools showed the greatest conservatism. Bushell [1947, p.69] recollected his own experiences as a pupil at Charterhouse:

I left school in 1903, just when Euclid was being dethroned, and well do I recollect how in my last term Mr. Tuckey produced a ruler with strange markings on it. It was, of course, a protractor. In those days, brought up upon the purest Euclid, when protractors were banned and indeed unknown, I was dumbfounded at its appearance....

However, outside the public schools the use of protractors and other geometrical instruments became well established in the case of practical geometry, which was a subject unrelated to the teaching of Euclid.

Henrici [1879, p.ix] significantly remarked that 'Geometrical drawing belongs ... to a branch of Geometry of which Euclid knew nothing, and where Euclid's propositions are of little use,' and he



deplored the divorce of the science and art of geometry, which was a consequence of the continuing use of Euclid as a textbook (see pp.95-96). The art of geometry developed on independent lines in the nineteenth century, as the subject of 'practical plane and solid geometry,' including the descriptive geometry of Gaspard Monge, and as drawing in various forms, all supported by the science and art examinations of the DSA, discussed in Chapter 2 (see pp.27-31). The DSA did much to disseminate geometrical drawing in schools below the first grade as well as in training colleges (see p.65), and, as part of the 'practical education' movement, drawing in elementary schools made remarkable progress in the late nineteenth century, with support from the DSA and the Education Department (see pp.188-190). However, as Godfrey [1908, p.255] pointed out, geometrical drawing was:

frequently taught as a branch of fine arts rather than as mathematics ... the worst feature was that 'geometrical drawing' came to be identified with a vast collection of special and unrelated rules; the educational value of the subject had sunk to zero.

In particular, although practical solid geometry was being taught much more widely than Euclid XI, the treatment was generally unmathematical. Although the public schools were not influenced by the DSA, it should be added that the army requirements also promoted geometrical drawing for some pupils in such schools, before Euclid was abandoned [Fawdry, 1901] (see pp.103-104).

These nineteenth-century developments contributed to the dissemination of geometrical instruments, including modern compasses and rulers, protractors of various kinds and dividers, as well as more technical drawing equipment, before the reform of the universities' examinations in geometry.<sup>13</sup> HMI Brill [1901, pp.279-280] referred to the use of instruments by younger pupils in some 'schools of science,'

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<sup>13</sup> For the full range of instruments and their uses in practical geometry see, for example, Harrison and Baxandall [1899] and Harrison [1903b].

and to the existence of some relevant textbooks for preliminary practical geometry. Significantly, a reviewer of some of the new geometries of 1903 remarked in Nature [1903b, p.147]:

A PERSON may be a Cambridge Wrangler, and yet unable to make a simple graphical construction with accuracy. The ordinary schoolboy's knowledge of practical geometry is generally worthless or nil.... But this state of affairs is being rapidly changed.<sup>14</sup>

Harrison's [1903b] textbook on practical geometry contained a recommendation from Perry, who interestingly remarked that this subject was:

no longer a mere collection of rules; it is now really an educational subject ... bringing [the student] into contact with many subjects through the common-sense application of a very few general principles. [p.ix]

However, the older influence of South Kensington tended to persist in elementary schools (see p.303), and Branford [1908, p.255] could still refer to:

secondary and technical schools and classes where we find a so-called 'Practical Geometry' taught, by pure rule-of-thumb methods, out of all relation to reasonably intelligent grasp of the reason 'why'....

Of course, practical geometry rose to prominence because of its utility, but the new more generally practical tendencies in geometry teaching, particularly in the early stages, are also bound up with important developments in pedagogy [LCC, 1911, pp.17-19].

The consequences of the growing interest in practical teaching methods were more important for geometry teaching than for any other branch of mathematics in education. The importance of the 'practical education' movement and of the spread of heurism from science to mathematical education, particularly in the case of geometry, was argued in Chapter 5 (see pp.185-199). A number of the ideals of the Perry movement were particularly relevant to this branch, and the titles of some of the new geometries after 1900 reflected these wider

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<sup>14</sup> The reviewer sounds like Joseph Harrison [1903a] of the Royal College of Science, who had already reviewed some other new geometries for Nature.



influences (see pp.225-226). However, the idea of basing the teaching of geometry on a practical foundation was far from new.

In the same year as W.G. Spencer's [1860] extraordinary little book Inventional Geometry was published (see p.193) another book, by W.D. Cooley, also appeared, in which an empirical approach using paper and scissors was implemented [National Education Association, 1912, p.25].<sup>15</sup> Furthermore, before the formation of the AIGT, F.E. Kitchener's [1868] Geometrical Note-book, which included many practical exercises, was published, and this writer collaborated with Wilson in a book entitled Experimental Geometry and the Use of Simple Instruments, as an Introduction to Theoretical Geometry [Fulford, 1923, pp.120-121; Jackson, 1924, p.37].<sup>16</sup> However, an American Report has concluded that 'England was the last country actually to introduce propaedeutic courses in elementary instruction' [National Education Association, 1912, p.23], and it was shown in Chapter 3 that the nineteenth-century efforts to reform English geometry teaching were dominated by academic questions concerning axiom systems, proofs, deductive sequences and examining difficulties in demonstrative geometry (see pp.84-94). By contrast, the new agitation for reform from around 1900, discussed in Chapter 3 (see pp.97-115) capitalized upon educational arguments; the unsuitability of Euclid's level of rigour in schools was exposed, particularly in the case of younger pupils; the crude argument for Euclid as a mental discipline was undermined; and remarks such as Oliver Heaviside's [1900, p.548], in a reply to Perry, were now taken much more seriously:

[geometry] is essentially an experimental science, like any other, and should be taught observationally, descriptively and experimentally in the first place.

The new educational enlightenment, broadcast by Perry in particular,

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15 Much later, Harrison [1903a, p.577], in Nature, referred back to Spencer's book.

16 Eggar [1903a, p.vii] acknowledged Kitchener's inspiration for his own practical geometry textbook.

was incompatible with the continuing use of Euclid as a school textbook, and the necessity of combining the practical and theoretical aspects was now generally accepted. This required the granting of freedom for teachers and textbook writers to experiment with various approaches, and this requirement was now judged to be more important than the possible complications in examining deductive geometry. Educational arguments largely won the day, and, with the benefit of hindsight, it does seem that the AIGT became unavoidably involved in the wrong kind of struggle at the wrong time.<sup>17</sup>

The rapid changes in geometry teaching up to 1905, involving examination reforms, textbook writing and pedagogical discussion on an unprecedented scale, and the further dissemination of geometrical instruments and aids, have already been surveyed (see pp.115-121, 148-149, 160).<sup>18</sup> Some other general features of the new geometry also deserve mention. The shift of interest from the rote learning of propositions, including Euclid's reference numbers, to understanding brought an emphasis on problem solving, that is the working of 'riders.' Also, more attention was paid to practical, numerical and algebraic exercises, which relates to the unification tendency, and, furthermore, the links between geometry and arithmetic, algebra, mensuration, and, later, trigonometry came increasingly to be exploited (see pp.291-298). The restriction to commensurable magnitudes and the use of algebra considerably eased the passage through Euclid's propositions, particularly Book VI on similarity, and facilitated the broadening of plane geometry [Godfrey, 1908, p.255]. However, efforts to broaden further the curriculum, to solid geometry, made little headway, and Euclid's postponement of this subject until Book XI partly

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17 These conclusions are consistent with Godfrey's [1906a; 1908, pp.253-257] assessment of the background to 'The Passing of Euclid.'

18 See also pp.63-67 for the BE's changes in its examinations concerning teacher education.



accounts for this failure [Carson, 1912e]. The use of 'hypothetical constructions' brought further flexibility into the deductive treatment (see p.338). It should be stressed that the subject-matter of school geometry continued to be a selection of propositions and constructions from Euclid I-IV and VI, as in the Cambridge Schedules for example (see p.119). The major changes concerned teaching methods, and some reaction to the early enthusiasm for the practical aspects is evident from around 1905.

As with graphs, practical work in geometry was initially accepted uncritically, with little fundamental consideration of purposes, and there was much variety in practices, particularly concerning the relationship with the deductive work. Reactions to the extent of the shift of interest from the theoretical to the practical in geometry, as part of a general reaction to the new practical tendencies, were discussed in Chapter 5 (see pp.238-241,246). The so-called new geometry had become a subject for critical discussion, and, as well as concern for the practical bias, the problem of different deductive sequences also loomed large (see pp.123-124,129-130).

The early examination reforms had only slightly relaxed the burden of learning geometrical proofs, and fifty or more might still be required for an examination. Furthermore, although the use of pupils' intuitions in geometry teaching was increasing, particularly in the case of the basic results concerning angles, parallels and congruence, formal proofs along the lines of Euclid I were still being demanded, and intuition was not initially regarded as an appropriate substitute, as this meant broadening the basis of assumption. Deductive geometry could still only be justified for its disciplinary value (see pp.240,284), and the view that this would be reduced by moving further away from Euclid's pure mathematical ideals tended to persist. On the new alternatives in deductive geometry, Godfrey

[1908, p.256] remarked:

As regards sequences of theorems, no system has found favour that can be called revolutionary; there has been no bold cutting adrift from the Euclidean tradition.

Although Godfrey [1907] and others questioned the need for a new uniform sequence (see pp.123-124,130), the case for uniformity continued to be pressed up to the 1920s (see pp.168-169), and Bryan [1912c] in particular led a campaign for the re-establishment of Euclid's sequence. However, the sequence issue largely concerned the fundamental propositions, and as Godfrey [1912e, p.357] pointed out:

This endless controversy as to the best sequence in Book I arises from the impossibility of compromising satisfactorily between the claims of pedagogy and mathematical rigour.

Significantly, Godfrey added that 'The Board of Education Circular simply cuts the knot, by treating these fundamental theorems as postulates.'

The central importance of the BE's Circulars on geometry for pedagogical progress in this branch was emphasized in Chapter 2 (see pp.40-41). The MA had not taken any lead, and the BE's [1909a] first Circular adopted an attitude in advance of general examination requirements and textbooks, and also refined the thinking concerning the relationship between theoretical and practical geometry. Three stages in geometry teaching were proposed for the first time. Briefly, the first is introductory, bringing in the fundamental notions and the use of instruments; the second intuitive and experimental, leading up to the fundamental propositions; and the last deductive, but using a wide base of assumption developed in the first two stages. There were many varied reactions to this Circular, which boldly provided a general sense of direction.<sup>19</sup> A writer in Nature [1914, p.686] judged that the Circular of 1909 had exerted:

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<sup>19</sup> See School World [1909a, 1909b], MA [1910b], Godfrey [1910b] and Carson [1912], who strongly argued the case for intuition in geometry (see p.283). On the second Circular [BE, 1914a] see Parker [1915b].



a marked and unquestionably beneficial influence on elementary [meaning secondary] education. We do not know of any geometrical text-book, published since that date, which has not taken account of it.

As Retter [1936, p.137] has rightly suggested, 'It is difficult ... to overestimate the importance of the Board's Circular.'

Following the BE's [1909a, 1914a] Circulars, examining bodies gradually relaxed their requirements concerning proofs of the fundamental propositions, and clarified their general expectations in deductive geometry [MA, 1930a] (see pp.164-165, 168-169). As HMI Strachan [1918, p.208] suggested, the best way forward was to turn away from the foundations, and he added 'Probably we shall never be happy until we get right away from the Euclidean tradition.' Strachan judged the sequence issue to be relatively unimportant [pp.203-204], though the AMA and the MA became involved here in the 1920s, and the detail was explored in Chapter 4, as was the character of the MA's late and controversial contribution to pedagogy in this branch (see pp.168-171). HMI Carson [1929, p.26] noted some tendency to go beyond Euclid I-IV and VI, to study curves other than the circle and some descriptive geometry, and he guessed that 'teachers may be deciding unconsciously that further study of the axioms, is, for the present, the flogging of a dead horse.'

Between the Wars, the BE's Circulars, HMIs, the AMA, but not the MA initially, examining bodies and textbook writers all contributed to the general, but only gradual, tendency to ignore foundational difficulties and to omit the full systematizing stage of deductive geometry in the Euclidean spirit. After the Second World War, Tuckey [1951, pp.236-237] judged that the sequence issue was now 'out of date' [p.237], and that the earlier twentieth-century ambitions in deductive geometry had been generally moderated. He referred to a continuous movement:

away from the formal setting-out of a series of propositions in logical sequence as in Euclid's Elements and towards the somewhat haphazard discussion of various geometrical properties arrived at by methods which permit the use of algebra, trigonometry and perhaps accurate drawing ... at the start of the century a pupil was expected to learn the proofs of 100 theorems; about 1925 teachers were content to exact proofs of about 50 theorems, while in 1951 they consider themselves fortunate if their pupils master a dozen. [p.236]

In particular, drastic pruning of the number of required proofs, as opposed to riders, was a feature of the alternative Jeffery syllabus and Durham's early experimental scheme for the First Examination (see pp.296-297). These alternatives also brought in a little solid geometry, and, generally, had largely departed from the Euclidean tradition.

The pattern of developments shows that half a century after Perry had launched his campaign to get rid of Euclid as a school textbook, and, more radically, to eliminate formal deductive geometry, the strength of the hold of this aspect of mathematics in secondary education was markedly weakening. However, during the early twentieth-century period of major reform, deductive geometry continued to feature prominently and seriously inhibited efforts to broaden the mathematics curriculum. Nevertheless, the successful campaign for practical and experimental geometry brought with it a gradual appreciation that proof can involve different kinds and degrees of conviction, and paved the way for the creation of pedagogical models of stages in the teaching of this branch. At least there was now a better chance of many more pupils learning something of the properties of space, and of the spirit of both inductive and deductive thinking in mathematics through geometry.

It has been the major purpose of this chapter to show that the general twentieth-century trends in the teaching of arithmetic, algebra and geometry were not haphazard and unrelated in character, but largely followed the various pedagogical principles which had risen to prominence before the First World War. The general implementation of change was often slow and complicated, and, in some respects, the



process continued long after Perry's death in 1920. The strength of the Perry movement as a force gradually faded, but what is remarkable is the extent of the similarity between the trends in the thinking and practices within the three branches, and Perry's scheme of elementary practical mathematics, taken in conjunction with the ideals of the movement which he inspired (see pp.100,225).

## Chapter 9

### Conclusion

This thesis is offered as one case study in the growing field of curriculum history. It has been limited to one school subject, though the links between mathematics and other subjects, particularly science, have emerged as important ones. The study has been limited to English mathematical education, though, here again, links with other countries from around 1908 have been shown to be significant for the refinement of pedagogy, and particularly in the cases of intuition and experiment, functionality in algebra, and the earlier teaching of calculus. Much of the discussion has concerned the period 1900-1914, which was one of unprecedented innovatory activity. Fortunately, this period also generated an abundance of source material, which has been used to assess various aspects of the progress of reform. The period of largely consolidatory activity after the War has been harder to assess, with rather limited evidence concerning the state of curricula. However, the often formidable difficulty of implementing general curriculum change within the distinctive English educational system has been demonstrated in various cases, such as the broadening of elementary school mathematics; the adoption of mathematical tables; the wider teaching of trigonometry and calculus; the unification of mathematics; effective correlation with other subjects; and the teaching of mechanics. The late nineteenth-century failure to achieve a major breakthrough in geometry teaching has also not been neglected, and various contrasts here with the period of relatively rapid and general change, between 1900 and 1905, have been emphasized.

Because of the richness, variety and complexity of the developments affecting mathematical education during the early twentieth century, an analytic approach was adopted for the previous chapters. Clearly, a simple linear account of 'progress' would have been



inadequate, though the purpose of this concluding chapter is to highlight certain major features of curriculum change and to raise some issues which appear to be of more general significance.

The historian G.R. Elton [1967, p.170] has argued that 'biography is not a good way of writing history,' and his conclusion fittingly applies to the history of periods of major curriculum change. Various individuals with contrasting backgrounds have risen to prominence in previous chapters, and their lives and contribution to mathematical education have admittedly received scant treatment in the Appendix. However, it has been a deliberate tactic in the writing of this thesis to bring to prominence the endeavours of one man in particular.

This writer freely confesses his sympathies for the engineer and educational reformer John Perry, though the possibilities of resulting bias in the evidence presented and the questions chosen for pursuit have at least been recognized. Certain individuals may become principally associated with some more general endeavour in curriculum history, such as Armstrong and heurism, and Froebel in infant education. What seems extraordinary is that Perry, an engineer, a pioneer in technical education, and certainly not a member of the establishment of 'mere mathematicians,' became associated with a general 'movement' to reform mathematical education in this country and abroad. Frequent references were made to Perry and the 'Perry movement' by various writers before the First World War, who were caught up in the period of reform. This movement is certainly not a hypothetical construct of the present writer. It has also become clear that during the years of early major reform up to 1905 most of the important developments arose outside the academic mathematical community, with Cambridge, the AIGT and later MA, and the public schools exhibiting markedly conservative tendencies, as was demonstrated in Chapters 3 and 4. It was certainly not a case of a closed mathematical system engaging in curricular

renewal from within. A simplistic model such as this does not fit the reality of curriculum change in the critical years around 1900.

Before reflecting upon the character of actual change, some distinctions need to be made concerning the concept of 'curriculum,' which, even today, as Layton [1978, p.112] has remarked 'remains disturbingly vague.' Some recent comparative studies of change in mathematical education since 1960 have paid particular attention to the distinctions between the 'intended,' 'implemented,' and 'attained' curriculum [Institut für Didaktik der Mathematik (IDM), 1980, pp.267-268]. The distinctions here concern what is recommended, what teachers do, and what pupils actually learn. For the curriculum historian it is much easier to capture desired states than life in classrooms, though this writer has struggled to present both aspects throughout. There are formidable difficulties here, as much of the evidence concerns recommended or 'innovatory,' rather than 'typical' practices, and, furthermore, methodological changes are more elusive than those in scope and content. Writers in mathematical education are usually the innovators and enthusiasts, and, used as evidence, their contributions tend to produce a distorted picture of reality. Reports of HMIs and surveys of general practices provide more objective though often starker evidence.

Another very important variable in discussing the curriculum concerns the question of mathematics for whom. For example, there is not one fixed form of calculus, but a variety of approaches depending upon the teacher and learners involved. In this case, global distinctions can be made between calculus for technical students, calculus for mathematical specialists, and calculus in a general secondary education. More generally, in discussing the scope, content and methods of mathematics differences have emerged between elementary and higher



elementary schools, secondary schools, technical institutions, boys' and girls' schools, army classes and the naval colleges. The form of mathematics in education may vary considerably across different periods, countries and educational institutions. In this connection, the early nineteenth-century period, the mathematical education of girls, and the forms of mathematics which developed for adolescents outside the public and secondary schools, are three areas which appear to warrant more detailed studies. Granted that different forms of mathematics in education may exist in a given period, questions then arise concerning the circumstances which produce the dominant forms in a given period, and the relationships between the forms. For example, it appears that the form of secondary mathematics for girls gradually became congruent with the form for boys. However, a central theme in this thesis has been the exploration of the relationships between practical mathematics for technical students, mathematics for naval cadets and army candidates, and mathematics in a general secondary education. This theme provides one clue concerning the complex question of causation in the reform of English mathematical education, particularly in its early stages.

The circumstances which led up to the dramatic developments in the period 1900-1905 can be considered from a number of different standpoints, yielding different kinds of explanation. To judge that major change was 'inevitable' is, of course, an historical non-explanation. However, judgements of this kind were made in the period 1900-1905, and writers referred to a 'general feeling of unrest and of misgiving' (see p.92), and also to reform in mathematical education as 'part of a general movement' (see p.123). Such remarks suggest that the accumulation of circumstances around the turn of the century was such that major curricular upheaval was very likely, and not only in mathematics. Two major features of education conducive to curriculum change generally at this time deserve to be highlighted.

The years around the turn of the century were traumatic ones for the structure and patterns of administrative control in the English educational system, the various features of which were discussed in detail in Chapter 2. Elementary schools were freed of the constraints of the system of payment for results, and curriculum innovation was actively encouraged by HMIs and the central authority, and by the new LEAs and their officers. The new system of secondary schools was being shaped and developed from the existing patchwork, and again innovation in the curriculum for the expanding influx of pupils into the new grant-aided secondary schools was also stimulated administratively. In technical education too, the new BE became involved in the dissemination of Perry's practical mathematics through the powerful controlling mechanism of its examinations in science and art. It seems reasonable to conjecture that an educational system in a state of flux is likely to be particularly receptive to curriculum innovation. The administrative developments in secondary education were also accompanied by growing concern for the educational needs of the majority of pupils, particularly in the newer secondary schools [Carson, 1912e, p.272]. The emerging concern for 'democratization' around 1900 (see p.254) was one specific aspect of the generally developing 'enlightenment' in educational thinking.

It is no accident that major reform in mathematical education came during the period of the 'new education.' There existed a general educational 'climate' likely to be receptive to various kinds of curriculum innovation towards the end of the nineteenth century. Various features of the 'new education' relevant to mathematics were distinguished and explored in Chapters 5 and 6. The refinements here came not only in educational theory and pedagogy, but also gradually in the thinking of the professional body of teachers. Developments in the structure of and thinking in education were conducive to curriculum change generally.



Science education also developed markedly during the period of the 'new education,' with specific and important implications for mathematics.

One concern in Chapter 5 was to demonstrate that the influences of science education were real and important, and not merely some hypothetical factor in mathematical reform. Science teachers became involved in mathematics teaching and in the campaign for reform. Practical and heuristic methods of teaching filtered through to mathematics from science, with tangible results in the teaching of geometry, mensuration and arithmetic, which were pursued in Chapter 8. During the second half of the nineteenth century the only kind of utility which was important in a general mathematical education was commercial and limited to arithmetic. Scientific utility was a new consideration which rose to prominence in the late nineteenth century and helped to disturb the balance in mathematical education.

Thus far, three important new features of education which contributed to a state of imbalance in mathematical education have been isolated. The upsurge of individual and collective actions in the critical years 1900-1903 should be viewed within this wider context of receptivity to curriculum change. The nature of the campaign for reform in these years was explored in Chapter 3. Perry certainly emerges as the prime mover in this period, who exploited fully the arguments concerning democracy in education, utility for science as well as engineering, and the need for more practical teaching methods. He also exploited the tactic of advocating his scheme of practical mathematics for all, and it could not be easily dismissed as a mere curiosity suitable only for engineering students. Rather, his scheme threw into sharp relief the serious deficiencies in the traditional 'academic' pattern; it presented a real threat to the established scheme for secondary schools in particular and contributed further to the state of instability. Significantly, Godfrey, who emerged as the leading

agitator from the public schools, produced his own 'Compromise.' The attack on the old order became more organized and it intensified. In parallel with these developments, examinations were reformed, and new textbooks, teaching materials and aids were disseminated. It was certainly not a simple case of a causal chain of important events as the evidence in Chapter 3 demonstrates, though the extent to which the general developments in this period centred on the activities and thinking of just one man is very remarkable. There still remained various problems concerning the general implementation of change; there was reaction to the character of the early major reforms; and there followed a period of evaluation, and refinement in thinking and practices.

The difficulty of implementation in many cases, regarding both the time-scale and the extent of change in English schools has already been emphasized (see p.349). However, in the cases of Euclid as a textbook, and the adoption of practical, numerical and graphical methods, change was remarkably rapid and widespread by around 1905. It appears that the reforms here were long overdue in England and resistance was not a significant force to be overcome. The character of change was principally determined by teachers' interpretations of the new spirit, mediated through textbooks and the crude but very powerful influence of changed examination requirements. Here the quality of change depended fundamentally on the quality of the teaching force, and major problems concerning the supply and education of teachers were exposed in Chapter 2. The new prescriptions largely concerned methods of teaching, and implementing heurism in geometry, for example, raised unsurmountable problems. Instead, there was a general tendency to graft on some practical geometry as an additional subject, divorced from theoretical geometry, as well as to add 'practical arithmetic' and 'practical algebra' i.e. graphs, again unrelated to the existing subject-matter in the corresponding branches. To interrelate profitably the



theoretical and newer practical elements required considerable mathematical and pedagogical sensitivity, which was only thinly distributed across the teaching force. The character of actual change was such that reaction soon set in, along lines discussed in Chapter 5. Two features of reaction in the case of arithmetic deserve to be highlighted.

Like graphs, contracted methods of computation spread rapidly and widely in the early, heady years of reform. However, during the subsequent period of reaction, whereas the role of graphs was merely reviewed and refined, contracted methods were actually rejected (see p.162). Another aspect of reaction before the First World War was the growing concern for standards of mechanical manipulation in arithmetic, as well as algebra. The newer concerns for understanding and practical work had, it was claimed, reduced the attention paid to the pure manipulation of symbols. However, teachers' conservatism would probably have limited the extent of this shift in practice, with well-established textbooks and the important requirements of examinations contributing to the preservation of the status quo.

Examinations have emerged as a very powerful instrument regarding the implementation of change. The English examination system is unique, with the settlement of 1918 only a compromise, which preserved much of the independence of the university bodies, particularly concerning syllabuses and methods of examining in individual subjects. Before the First World War, the system was extremely complicated, but the important bodies were not slow to respond to the situation in mathematics, in the period 1900-1903. Examining bodies were generally conservative, in the sense of not seeking to force the pace of change in advance of persistent tendencies in the schools, but the extent of their resistance to generally desired change is easily exaggerated, and provision was made for some variety in curricula. However, after 1903, it is evident that different bodies demonstrated different degrees of sympathy with the

schools concerning the accommodation of their requirements to various developments in mathematics teaching up to the 1940s, the cases of London and the Oxbridge Joint Boards representing extremes in this respect. This would seem to be an undesirable feature of the English system. The role of publishing houses, and their output, also warrant some general remarks.

The role of publishing houses in relation to curriculum change is an interesting subject which could profitably be investigated further. The number of different houses which competed in the freely competitive and highly profitable field of school mathematics textbook publishing is remarkable. The size and variety of their output, particularly during the period 1900-1905, is also very striking. Of course, educational and commercial purposes may conflict. For example, whilst a period of curricular upheaval is clearly in the commercial interests of those houses which capture a major share in the new market, it is also profitable to sustain long runs for particularly successful books, thereby contributing to conservatism in the schools. The textbooks of Hall, with various partners for Macmillan, and Pendlebury for Bell in arithmetic, are cases in point here. Such writers merely adjusted their nineteenth-century treatments to accommodate changes in examination requirements, and they continued to provide strong competition for new textbooks by younger writers, such as Godfrey with various partners, who started from scratch and sought to implement the newer enlightenment. Teachers' choices determined the successful textbooks, and these reflected the pedagogical preferences of the teaching force which often lagged a long way behind the new ideals. The fact is that no mechanism existed for selecting the best textbooks and getting these into the schools. Furthermore, the way good or bad textbooks are actually used by teachers in classrooms determines the quality of implemented change. Again, the teaching force itself is the key factor.



One other general feature of curriculum change remains to be considered.

Along with the implementation of change through examinations, textbooks and teachers' interpretations there also developed a growing concern for evaluation and refinement in pedagogy. The work of the ICTM from 1908 may be regarded as an exercise in international evaluation of the progress of reform. From around the same time, the BE and the MA made major contributions concerning the refinement of pedagogy, and influenced examinations, textbooks, and teachers directly. The character of the MA's work here was discussed in Chapter 4. The creation of the South-Eastern Association in 1911 also provides further evidence of refinement. The way in which various pedagogical developments, within the three branches and beyond, gradually affected the character of mathematical education was explored in Chapters 7 and 8. The process of refinement in pedagogical thought was a feature of the general emergence of mathematical education as a field of enquiry in its own right, which was discussed in Chapter 6. This phenomenon may be viewed as partly a response to the major curriculum developments in the early years of this century.

It has been the principal purpose of this concluding chapter to bring into sharper focus the processes of causation and initiation, implementation, reaction, evaluation and refinement in the reform of English mathematical education. To what extent this one case study may throw some light on more general patterns of major curriculum change is not for this writer to judge. Clearly, further detailed studies of other subjects, countries and periods, such as the last twenty-five years, need to be made. Waring's [1979] stimulating study of Nuffield Science and the various studies published by the Institut für Didaktik der Mathematik [1980] are recent examples. The value of ambitious general model building in the field of curriculum history would seem to be limited at the present time. Rather, it is the uniqueness of so many aspects of the reform

discussed in this thesis which is particularly striking. Possible generalization is a major and hazardous task for the future.



APPENDIXBiographical NotesAbbott, Percival William Henry (1869-1954).

Educated privately and London University BA, 1899. One year Whitechapel Foundation School; Mathematics Master and Head of the Mathematical Department, Polytechnic School, Regent St., 1895-1919; Headmaster of the Polytechnic School, 1919-1934. Leading member of the ATTI, and the MA, and editor of the Technical Journal; member of the SSEC and Consultative Committee of the BE. Textbooks on arithmetic, mensuration, trigonometry and mathematics for technical students; an editor of Longmans' Modern Mathematical Series.

Ballard, Philip Bowwood (1865-1950).

Educated local schools Maesteg and Borough Rd. Training College; London University MA, 1905, Gold Medal, and DLit, Carpenter Medal. Experience in London Schools, 1886-1898; Headmaster, Pupil Teachers' School, Tondu, Glamorgan, 1898-1903; Inspector of Glamorgan Schools, 1903-1905; Inspector of London Schools, 1906-1930. Many publications for pupils and teachers, particularly concerning arithmetic in elementary schools; books on educational studies, including the new scientific perspectives.

Branford, Bencheru Bertrand Patrick (1868?-1944).

Educated Edinburgh University, MA. Lecturer, Victoria University, Leeds; Director of Higher Education and Principal of the Technical College Sunderland, 1901-1905; Inspector of London Schools, 1905-1929. Innovator in elementary, secondary and technical education. Publications on politics as well as mathematical education.

Bryan, George Hartley (1864-1928).

Educated Peterhouse, Cambridge University, 5th Wrangler, 1886, and Smith's prizeman, 1889; FRS, 1895, and ScD, 1896. Fellow of Peterhouse, 1889-1895; Professor of Pure and Applied Mathematics, University College Bangor until 1926. Innovative President of the MA, 1907-1909. Publications in thermodynamics and aeronautics; various mathematical textbooks in the University Tutorial Series, produced to further the work of the University Correspondence College, which catered for London University's Examinations.

Carson, George Edward St. Lawrence (1873-1934).

Educated University College, Liverpool, and Victoria University BSc; Trinity, Cambridge University, 2nd Wrangler, 1896, and 1st Class Pt. II, 1897. Lecturer, University College Sheffield, 1898-1901; Scientific Adviser, J.H. Dollmeyer Ltd., 1901-1905; Head of the Mathematical Department, Battersea Polytechnic, 1905-1908; Head Mathematical Master, Tonbridge School, 1908-1913; Reader in Mathematics and Lecturer in Education, Liverpool University, 1914-1917; HMI and Staff Inspector Mathematics. Founder of the South-Eastern Association of Teachers of Mathematics. Many published papers on mathematical education and textbooks with the American D.E. Smith.

Dobbs, William John (1868-7).

Educated Wolverhampton GS and St. John's, Cambridge University, 9th Wrangler, 1890. One year Totnes GS and Queen Elizabeth's School, Mansfield; Senior Mathematical Master, Messrs. Carlisle and Cregson's, Kensington, 1892-1909; Dundle School, 1910; Second Master and Chief Mathematical Master, Holloway County Secondary School, 1910-1919; Part-time Lecturer, Wandsworth Technical Institute from 1902, and Head of the Mathematics Department, Wandsworth Technical Institute Secondary School from 1919. Leading member of the AMA and the MA, and examiner for the Civil Service Commission from 1911. Various papers and textbooks, and particularly innovative in geometry teaching.

Durrell, Clement Vynuar (1882-1968).

Educated Felsted School and Clare, Cambridge University, 7th Wrangler, 1903, and 1st Class Pt. II, 1904. One year Gresham's School, Holt; Winchester College from 1905, and Senior Mathematical Master from 1910. Leading member of the MA and also member of the London Mathematical Society. First textbook, a collection of problems, 1906, and subsequently prolific output for Bell up to the 1960s.

Engar, William Douglas (1865-1945).

Educated Brighton College and Trinity, Cambridge University, 13th Wrangler, 1887, and 2nd Class Pt. II Natural Sciences Tripos, 1888. Two years Brighton College and four years Wellington College; Eton College from 1895, and Senior Science Master, 1911-1920. Founder member of the APSSM, and also involved with the AMA, BAAS and Physical Society, London. Textbooks in physics, mechanics, geometry and algebra.

Fawdry, Reginald Charles (1873-1965).

Educated Worcester GS and Corpus Christi, Cambridge University, 27th Wrangler, 1897; London University BSc and Oxford University Teacher's Diploma, 1899. Felsted School Army Side, 1900-1903; Clifton College, 1903-1933, and became Head of the Military and Engineering Side. Various mathematical textbooks, including some related to practical mathematics.

Findlay, Joseph John (1860-1940).

Educated Kingwood School Bath and Wadham, Oxford University, 1st Class Mathematica Moderations, 1881, and 1st Class Honours History, 1883; Jena and Leipzig, 1891-1893, study of education and PhD, 1893. Bath College, 1884-1885; Headmaster, Queen's College, Taunton and Wesley College, Sheffield, Wesleyan proprietary schools, 1885-1891; Rugby School Master, 1893-1894; Lecturer, College of Preceptors, 1895-1898; Headmaster, Cardiff Intermediate School, 1898-1903; Professor of Education, Manchester University, 1903-1925. Visited America and reported to Bryce Commission, 1894-1895; member of various educational societies and numerous papers and books on educational studies - a pioneer in elevating the status of this subject.



Fletcher, William Charles (1865-1959).

Educated Kingwood School Bath and St. John's, Cambridge University, 2nd Wrangler, 1886, and 1st Class Pt. II, 1887. Nine years Bedford GS; Headmaster, Liverpool Institute, 1896-1904; Chief HMI of Secondary Schools, 1904-1926, and principal creator of the new role for secondary HMIs. Early involvement with the NUJMB, Association of Headmasters, and Moseley Commission, 1903; leading member of the MA between the Wars and President, 1939-1943. Various papers, particularly on geometry, and author of the BE's important Circulars on secondary school mathematics; unusual contribution on 'English and Mathematics' [1924] also distributed by the BE.

Forayth, Andrew Russell (1858-1942).

Educated Liverpool Collegiate Institution and Trinity, Cambridge University, Senior Wrangler, Smith's Prizeman, and Fellow of Trinity, 1881; FRS, 1886, and numerous other academic honours. Professor of Mathematics, University College Liverpool, 1882-1883; Lecturer, Cambridge University, 1894, and Sadleirian Professor, 1895-1910; after a period in India, Professor of Mathematics, Imperial College, 1913-1923. Active member and President of the MA, 1903-1905 and 1936-1937, and London Mathematical Society, 1904-1906; leader in the reform of Cambridge's school and university examinations, and in the work of the BAAS concerning school mathematics, 1901-1905. Research publications in the theory of differential equations, complex function theory and higher geometry.

Godfrey, Charles (1873-1924).

Educated King Edward's School Birmingham and Trinity, Cambridge University, 4th Wrangler, 1895, and 1st Class Pt. II, 1896. Three years University College Cardiff, and part-time, Cambridge and King Edward's; Senior Mathematical Master, Winchester College, 1899-1905; Headmaster, Osborne RNC, 1905-1920; Professor of Mathematics, Greenwich RNC, from 1920. Dominant member of the MA, HMC and British representative on the ICTH; important influence on examining bodies, particularly the Oxbridge Joint Board. Large output of papers on many aspects of mathematics teaching, and numerous textbooks with various partners, particularly Siddons.

Hardy, Godfrey Harold (1877-1947).

Educated Winchester College and Trinity, Cambridge University, 4th Wrangler, 1898, 1st Class Pt. II, 1900, Smith's Prizeman, 1901; FRS, 1910, and numerous other academic honours. Fellow of Trinity, 1900, and Lecturer from 1906; Cayley Lecturer, Cambridge, 1914-1919; Savilian Professor of Geometry, Oxford University, 1919-1931; Sadleirian Professor, Cambridge University, from 1931. President of the MA, 1924-1926, and London Mathematical Society, 1926-1928; campaigned for the reform of the Mathematical Tripos, and radical views concerning the effects of such examinations on English mathematics; leader in the reform of university mathematics teaching, particularly in analysis; British representative on the ICTM. Prolific research output, principally in the theory of numbers and analysis, and very creative partnership with J.E. Littlewood.

Hobson, Ernest William (1856-1933).

Educated Derby School and Christ's, Cambridge University, Senior Wrangler, 1878; FRS, 1893, and various other academic honours. Fellow of Christ's, 1878, and Stokes Lecturer from 1903; Sadleirian Professor, 1910-1931. President of the London Mathematical Society, 1900-1902, and the MA, 1911-1913; campaigned for the reform of the Mathematical Tripos; British representative on the ICTM. Research principally in the theory of functions, and books on higher trigonometry, theory of series and functions, and harmonics.

Jackson, Charles Samuel (1867-1916).

Educated Uppingham School and Bedford GS, and Trinity, Cambridge University, 8th Wrangler, 1889, and 1st Class Pt. II Law Tripos, 1890. Brief period at the Bar; Instructor in Mathematics, Woolwich RMA, 1891-1916. Leading member of the MA, particularly the London Branch; British representative on the ICTM; interest in correlation and applications of mathematics, and the wider teaching of calculus; influence on examining bodies, the Civil Service Commission in particular. An editor of Longmans' Modern Mathematical Series, and his own contribution on the slide-rule; nick-named 'Slide-Rule Jackson.'

Hair, David Beveridge (1868-1942).

Educated Dollar Academy and Edinburgh University, 1884-1888, 1st Class Honours mathematics and natural philosophy; Christ's, Cambridge University, 2nd Wrangler, 1891, 1st Class Pt. II, 1892, Smith's prizeman, 1893, and Fellow, 1894. Two years teaching, Cambridge and University College London; Civil Service Commission, 1896-1933, and director of examinations from 1918. Important influence on the mathematical and other examinations of the Commission. Innovative books on elementary and higher mathematics.

Mercer, John William (1870-1951).

Educated Bradford GS and Caius, Cambridge University, 11th Wrangler, 1892, and 1st Class Pt. II, 1893. Oundle School, 1895-1904; Head of Mathematics, Osborne RNC, 1904-1905, and Dartmouth RNC, 1905-1913; Head of the Military and Engineering Side, Cheltenham College, 1913-1934. Innovator in trigonometry and calculus teaching, particularly with young naval cadets.

Nunn, Thomas Percy (1870-1944).

Educated privately and University College Bristol; London University BSc, 3rd Class Honours Physics, 1890, 1st Class Honours Logic, 1904; MA and Teacher's Diploma, 1902, DSc Logic and Methodology, 1907. Two years Halifax New School; four years Wilson's GS, Camberwell; six years William Ellis School, and became Second Master; one year Lecturer, Woolwich Polytechnic; Lecturer and Science Master, Shoreditch Technical Institute, 1903-1905; Vice-Principal, London Day Training College, 1905-1922, and Principal, 1922-1936; Professor of Education, London University, 1913-1936. Leading member of the MA, and President 1917-1919; involvement with the BAAS, Historical Association, School Nature Study Union, Aristotelian Society, British Psychological Society, and Training College Association. Publications in various fields including philosophy, science, theory of education, science education and mathematical education; rich and highly innovative contribution to the pedagogy of mathematics.



Perry, John (1850-1920).

Educated the Model School, Belfast, and foundry apprentice, Belfast, 1864-1868; Queen's College, Belfast, 1868-1870, First Class Honours Bachelor of Engineering, Gold Medal, Whitworth Scholarship, 1870, and subsequently FRS. Mathematical and Science Master, Clifton College, 1871-1874; Assistant to William Thomson (Lord Kelvin), 1874-1875; Joint Professor of Engineering, Imperial College of Engineering, Tokyo, 1875-1878; Examiner for the City and Guilds from 1879 and Professor of Mechanical Engineering, Finsbury Technical College, 1882-1896; Professor of Mathematics and Mechanics, Royal College of Science, 1896-1913. Prominent member of the BAAS and Treasurer from 1904. Many technological papers, particularly with Ayrton, in the field of electrical engineering; innovatory textbooks in mechanics, calculus and practical mathematics; innovator in teaching methods in mathematics, and leading agitator in English mathematical education, whose name became associated with the movement for reform.

Siddons, Arthur Warry (1876-1959).

Educated King Edward's School Birmingham and Jesus, Cambridge University, 5th Wrangler, 1898, 1st Class Pt. I Natural Sciences Tripos, 1899. Assistant Master in physics and mathematics, Harrow School, 1899-1936 and 1943-1946. Leading member of the MA and President, 1935-1936. Prominent textbook writer, particularly with Godfrey.

Whitehead, Alfred North (1861-1947).

Educated Sherborne School and Trinity, Cambridge University, 4th Wrangler, 1883, 1st Class Pt. II and Fellow, 1884, ScD, 1905; FRS, 1903, and various other academic honours. Lecturer, Cambridge University, 1884-1910; Lecturer in Applied Mathematics and Mechanics and, subsequently, Reader in Geometry, University College London, 1911-1914; Professor of Applied Mathematics, Imperial College, 1914-1924; Professor of Philosophy, Harvard University, 1924-1937. President of the MA, 1915-1916, and South-Eastern Association of Teachers of Mathematics, 1911-1912. Publications in the foundations of mathematics, notably with Russell, and philosophy, particularly in relation to epistemology and science; stimulating theoretical perspectives on school mathematics.

Wormell, Richard (1838-1914).

Educated as a pupil-teacher, and Borough Rd. Training College; London University MA, Gold Medal, and DSc. Mathematical Master, the Middle Class School, Cowper St., 1866-1873; Instructor in Mathematics, Greenwich RNC, 1873-1874; Head Master, the Cowper St. School, 1874-1900. Leading member of the AIGT and College of Preceptors; member of the Bryce Commission; President of the Association of Headmasters, 1895-1896; early innovator in mathematics teaching and contributor to the pedagogy of mathematics and also science.

Sources: Schoolmasters Yearbook, Mathematical Gazette, Times, Who Was Who, DNB, College of Preceptors [1914], Armstrong [1920], Turner [1926], Piaggio [1931], and Venn [1954].

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## Further Abbreviations for Some Journals

<u>BJES</u>	<u>British Journal of Educational Studies</u>
<u>ET</u>	<u>Educational Times</u>
<u>JE</u>	<u>Journal of Education</u>
<u>JEP</u>	<u>Journal of Experimental Pedagogy and Training College Record</u>
<u>JSE</u>	<u>Journal of the Association of Teachers of Mathematics for the South-Eastern Part of England</u>
<u>MG</u>	<u>Mathematical Gazette</u>
<u>SW</u>	<u>School World</u>

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