

# Open economy New Keynesian macroeconomic models and the cost channel

Thesis submitted for the degree of  
Doctor of Philosophy  
at the University of Leicester

By  
Sérgio Miguel Chilra Lagoa  
BSc (Technical University of Lisbon) and MSc (University Nova of  
Lisbon)

Department of Economics  
University of Leicester

April 2010

## Abstract

# Open economy New Keynesian macroeconomic models and the cost channel

Sérgio Miguel Chilra Lagoa

Evidence in the literature points to a puzzling initial increase in inflation after an increase in nominal interest rates. This can be explained by the fact that firms have to borrow money to pay wages in advance, i.e., by the cost channel. In this paper, the study of the cost channel is extended to an open economy with sticky prices. It is shown that a broadened concept of the cost channel has significant implications for the economy's dynamics and monetary policy, and also contributes to explain some interesting empirical evidence.

Supply side effects of interest rates and import prices on inflation have important implications for monetary policy. Usually such effects are estimated using the New Keynesian Phillips Curve (NKPC). However, the estimation of the cost channel may be distorted when import prices are omitted from that curve. To address this issue, we estimate empirically the NKPC for domestic and CPI inflations. In relation with this, we also study if imports of consumption goods are paid in advance, whether there is an immediate pass-through of exchange rates, and if imports should be treated as final consumption goods and/or as inputs in production.

Another concern of monetary policy in a monetary union is inflation differentials, since they can undermine the success of the union. Against this background, our goal is to explore the determinants of inflation differentials in twelve euro area countries, focusing on the role of the business cycle. On one hand, convergence of inflation rates and business cycles is analysed with both an unobserved component model estimated with the Kalman filter and a common factor approach. On the other hand, an econometric analysis of the determinants of inflation differentials is performed.

## Acknowledgements page

I am thankful to my supervisor Prof. Stephen G. Hall for all the help, support and useful comments. Without him this endeavour would not be possible. I am also thankful to my second supervisor, Prof. Gianni De Fraja.

I thank also to the participants of the University of Leicester's PhD workshop, March 2009, and to the participants of the Money Macro and Finance Research Group 41st Annual Conference, Bradford, 2009. Their comments have enriched Chapter 2.

My PhD thesis has benefited from the financial support of 'Fundação para a Ciência e a Tecnologia, Ministério da Ciência e Tecnologia', Portugal.

I am also thankful to the Department of Economics of University of Leicester and the Department of Economics of the ISCTE- Lisbon University Institute for all their support.

Finally, I cannot forget all the support from my family, friends and colleagues.

# Contents

<b>1</b>	<b>Introduction</b>	<b>12</b>
1.1	Industry-level and firm-level evidence on the cost channel . . . . .	13
1.2	The Phillips curve and the cost channel . . . . .	14
1.3	Estimating the cost channel with general equilibrium models . . . . .	17
1.4	Exchange rates and inflation . . . . .	19
1.5	Thesis structure and content . . . . .	21
<b>2</b>	<b>The cost channel in a small open economy New Keynesian model</b>	<b>27</b>
2.1	Introduction . . . . .	27
2.2	A small open economy model with the cost channel . . . . .	30
2.2.1	Households . . . . .	30
2.2.2	Import prices and some important identities . . . . .	34
2.2.3	Domestic firms . . . . .	38
2.2.4	Foreign country . . . . .	42
2.2.5	International Risk Sharing . . . . .	43
2.2.6	Uncovered interest parity and aggregate demand . . . . .	44
2.2.7	Marginal cost and the flexible-price output . . . . .	46
2.2.8	Deviations from the flexible-price equilibrium . . . . .	52
2.3	Policy trade-offs and optimal policy . . . . .	58
2.3.1	Policy trade-offs . . . . .	58
2.3.2	Optimal monetary policies with discretion . . . . .	58
2.4	Domestic technological shock under different policy regimes . . . . .	60
2.4.1	Calibration . . . . .	61

2.4.2	Without the cost channel . . . . .	62
2.4.3	With a cost channel . . . . .	69
2.5	Stochastic simulations and macroeconomic volatility . . . . .	77
2.5.1	Cost channel's impact on macroeconomic volatility . . . . .	79
2.5.2	Policy functions . . . . .	83
2.5.3	Interest rate smoothing . . . . .	85
2.5.4	Correlation between CPI inflation and the change in the nominal exchange rate . . . . .	86
2.5.5	The impact of imported inputs . . . . .	87
2.6	Conclusion . . . . .	90
2.7	Annex: dynamic response of the economy to shocks . . . . .	94

<b>3</b>	<b>Inflation dynamics in open economies: empirical evidence for G7 countries on the role of import prices and the cost channel</b>	<b>127</b>
3.1	Introduction . . . . .	127
3.2	Literature review on the NKPC: inflation, interest rate and import prices . . .	129
3.2.1	The New Keynesian Phillips Curve . . . . .	129
3.2.2	The New Hybrid Keynesian Phillips Curve . . . . .	132
3.2.3	The NKPC with a cost channel . . . . .	134
3.2.4	The NKPC in an open economy context . . . . .	135
3.2.5	Critics to NKPC estimations . . . . .	140
3.3	NKPC for domestic inflation . . . . .	142
3.3.1	Identifying the cost channel in an open economy . . . . .	142
3.3.2	Data . . . . .	145
3.3.3	NKPC and the relative price of imports . . . . .	146
3.3.4	Estimating the baseline NKPC . . . . .	149
3.3.5	Estimating the NKPC with open economy variables . . . . .	157
3.4	NKPC for CPI inflation . . . . .	160
3.4.1	NKPC assuming imports as final consumption goods . . . . .	161
3.4.2	NKPC assuming imports as both final consumption goods and inputs .	166
3.4.3	NKPC assuming consumer goods imports paid in advance . . . . .	170

3.5	Conclusion . . . . .	173
3.6	Annex: additional tables . . . . .	175
<b>4</b>	<b>Empirical analysis of inflation and business cycle convergence in the euro area</b>	<b>185</b>
4.1	Introduction . . . . .	185
4.2	Measurement of convergence . . . . .	189
4.3	Testing convergence over the period 1980-2008 . . . . .	195
4.3.1	Convergence of inflation rates . . . . .	196
4.3.2	Convergence of real ULCs . . . . .	198
4.3.3	Convergence of output gaps . . . . .	200
4.3.4	Comparing the convergence processes of inflation and output gap . . . . .	201
4.3.5	Relationship between convergence of inflation and output gap . . . . .	203
4.4	Common factor approach to convergence . . . . .	205
4.4.1	Inflation . . . . .	205
4.4.2	Business cycles convergence . . . . .	210
4.4.3	Correlation between the convergences of inflation rates and the business cycles . . . . .	214
4.5	Explaining inflation differentials . . . . .	215
4.5.1	Determinants of inflation differentials in the euro area . . . . .	215
4.5.2	Explaining inflation differentials using measures of divergence . . . . .	218
4.5.3	Explaining inflation differentials using the NKPC . . . . .	224
4.5.4	Inflation differentials, imperfect competition model and the NKPC . . . . .	232
4.6	Conclusion . . . . .	236
4.7	Annex: additional graphs and tables . . . . .	240
<b>5</b>	<b>Conclusion</b>	<b>257</b>
<b>6</b>	<b>Appendices</b>	<b>263</b>
6.1	Chapter 2 . . . . .	263
6.1.1	Households' FOC . . . . .	263
6.1.2	The real exchange rate and the terms of trade . . . . .	264

6.1.3	Production function, prices and consumption in deviations from the steady-state . . . . .	265
6.1.4	Elasticity of output with respect to labour 267	
6.1.5	Real uncovered interest parity . . . . .	267
6.1.6	Imports demand . . . . .	267
6.1.7	The signs of Phillips curve's coefficients . . . . .	268
6.1.8	Discretionary policy . . . . .	269
6.1.9	The coefficient of output gap on the Phillips curve: the impact of imported inputs . . . . .	269
6.1.10	The coefficient of interest rate on the Phillips curve: the impact of imported inputs . . . . .	270
6.2	Chapter 3 . . . . .	272
6.2.1	Annex: data description . . . . .	272
6.2.2	Annex: some proofs . . . . .	273
6.3	Chapter 4 . . . . .	275
6.3.1	Data description . . . . .	275
6.3.2	Kalman filter . . . . .	278
6.3.3	Principal components analysis . . . . .	285

# List of Tables

2.1	Impact of the cost channel in the response of the economy to a technological shock . . . . .	76
2.2	Impact of the cost channel on the response of the economy to an interest rate shock . . . . .	77
2.3	Variance of the main variables with and without the cost channel . . . . .	79
2.4	Variance of the main variables when the Central Bank follows a DITR with a positive weight for the output gap . . . . .	83
2.5	Policy functions and the cost channel . . . . .	84
2.6	Correlation between CPI inflation and the change in the nominal exchange rate.	86
2.7	Impact of imported inputs share on macroeconomic volatility (with the cost channel) . . . . .	89
3.1	Sensitivity analysis of expression A's value . . . . .	144
3.2	Estimates of the baseline NKPC for domestic inflation . . . . .	153
3.3	Estimates of the NKPC for domestic inflation augmented with the interest rate	154
3.4	Estimates of the open economy NKPC for domestic inflation with the relative real exchange rate . . . . .	158
3.5	Estimates of open economy NKPC for domestic inflation with the relative price of imports . . . . .	159
3.6	Estimates of the baseline NKPC for CPI inflation . . . . .	162
3.7	Estimates of the NKPC for CPI inflation augmented with the interest rate . . .	163
3.8	Estimates of the NKPC for CPI inflation with imports as consumption goods, using the change in the RER . . . . .	165

3.9	Estimates of the NKPC for CPI inflation with imports as consumption goods, assuming slow exchange rate pass-through . . . . .	167
3.10	Estimates of the NKPC for CPI inflation with imports both as consumption goods and inputs, assuming slow exchange rate pass-through . . . . .	169
3.11	Estimates of the NKPC for CPI inflation with imports of consumption goods paid in advance . . . . .	171
3.12	Estimates of the NKPC for domestic inflation augmented with the interest rate - Chowdhury et al (2006)'s set of instruments . . . . .	175
3.13	Estimates of the NKPC for domestic inflation augmented with the interest rate - Bandwidth fixed at 1 . . . . .	176
3.14	Estimates of the NKPC for domestic inflation augmented with the interest rate - Bandwidth computed according with Andrews (1991) . . . . .	177
3.15	Estimates of the NKPC for domestic inflation augmented with the interest rate - small set of instruments . . . . .	178
3.16	Estimates of the NKPC for domestic inflation augmented with the interest rate - variables in log deviation from the steady-state . . . . .	179
3.17	Estimates of the NKPC for domestic inflation augmented with the interest rate - deviations from the steady-state computed using Christiano-Fitzgerald (2003) Frequency Filter . . . . .	180
3.18	Estimates of the NKPC for domestic inflation augmented with the interest rate, using the output gap . . . . .	181
3.19	Estimates of the NKPC for domestic inflation augmented with the interest rate using the labour income share of the business sector . . . . .	182
3.20	Estimates of the NKPC for CPI inflation with imports of consumption goods paid in advance and using the change in the relative price of imports . . . . .	183
3.21	Estimates of the NKPC for CPI inflation with imports both as consumption goods and inputs, assuming slow exchange rate pass-through and with the change in the interest rate . . . . .	184
4.1	Quarters of statistically significant divergence in inflation during the euro period	197

4.2	Testing if the variance of the convergence variable of the RULC is zero in 2008Q4 . . . . .	200
4.3	R2 of the first component for three subperiods between 1980 and 2008 . . . . .	206
4.4	Correlation between the R2 of inflation and the R2 of output gap or RULC . .	214
4.5	Explaining inflation differentials using the residuals of the common factor model. GMM estimation. . . . .	222
4.6	GMM estimation of the NKPC for a panel of 12 euro area countries. . . . .	227
4.7	Determinants of inflation differentials for a panel of 12 euro area countries. GMM estimation . . . . .	229
4.8	Determinants of inflation differentials ignoring expectations. GMM estimations for a panel of 12 euro area countries. . . . .	231
4.9	GMM estimation of the ICM for inflation differentials of 12 euro area countries, 1999Q1-2008Q4. . . . .	235
4.10	Measuring inflation convergence towards Germany with time-varying parameters. Estimation with the Kalman Filter, 1980Q1-2008Q4 . . . . .	244
4.11	Measuring real ULC convergence towards Germany with time-varying parameters. Estimation with the Kalman Filter, 1980Q1-2008Q4 . . . . .	245
4.12	Measuring output gap convergence towards Germany with time-varying parameters. Estimation with the Kalman Filter, 1980Q1-2008Q4 . . . . .	246
4.13	Testing the equality of the convergence processes of inflation and output gap (part I). Estimation with the Kalman Filter, 1980Q1-2008Q4 . . . . .	247
4.14	Testing the equality of the convergence processes of inflation and output gap (part II). Estimation with the Kalman Filter, 1980Q1-2008Q4 . . . . .	248
4.15	Testing the equality of the convergence processes of inflation and output gap (part III). Estimation with the Kalman Filter, 1980Q1-2008Q4 . . . . .	249
4.16	Causality between convergences of inflation and output gap (part I). Estimation with the Kalman Filter, 1980Q1-2008Q4 . . . . .	250
4.17	Causality between convergences of inflation and output gap (part II). Estimation with the Kalman Filter, 1980Q1-2008Q4 . . . . .	251
4.18	Causality between convergences of inflation and output gap (part III). Estimation with the Kalman Filter, 1980Q1-2008Q4 . . . . .	252

4.19 Determinants of inflation and inflation differentials with fixed effects . . . . .	256
---	-----

# List of Figures

4-1	Cross section standard deviation of inflation rates after 1998 . . . . .	186
4-2	Cross sectional standard deviation of inflation rates in the euro area (countries have equal weight) . . . . .	207
4-3	Inflation co-movement during the euro period. . . . .	208
4-4	Country weights of first and second principal components of inflation . . . . .	209
4-5	Output gaps co-movement during the euro period (1999-2008) . . . . .	210
4-6	Country weights of the first principal component of output gap (1999-2008) . .	211
4-7	RULC co-movement during the euro period . . . . .	213
4-8	Country weights of first principal components of RULC (1999-2008) . . . . .	213
4-9	Inflation's differentials towards Germany, filtered state variable, 1980Q1-2008Q4 (part I). . . . .	240
4-10	Inflation's differentials towards Germany, filtered state variable, 1980Q1-2008Q4 (part II). . . . .	241
4-11	Log difference between the real ULC of each country and Germany. . . . .	242
4-12	Difference between the output gap of each country and Germany. . . . .	243
4-13	Country weights of first principal component for inflation after the introduction of the euro . . . . .	253
4-14	Country weights of first principal component for output gap after the introduc- tion of the euro . . . . .	254
4-15	Country weights of first principal component for RULC after the introduction of the euro . . . . .	255

# Chapter 1

## Introduction

One of the main purposes of this thesis is to study the role that the cost channel plays in inflation dynamics in open economies. The cost or working capital channel of monetary policy exists when firms' marginal cost depends directly on the nominal interest rate. In this case and in opposition to the traditional view, the nominal interest rate exerts *pressure* for an increase in inflation.

Traditional models postulate that monetary policy affects output and inflation through the demand side. With sticky wages and prices, a reduction in money supply increases real interest rate, producing a reduction in investment and consumption. In this context, a monetary contraction causes a reduction in inflation. However, this relation has often been contradicted by empirical evidence. Sims (1992) finds that inflation increases significantly after an increase in interest rate. In the literature, this situation is called the 'price puzzle'. Other papers found only a sluggish response from inflation: after a monetary contraction, the price level does not respond for almost two years, and then begins to decline (Christiano et al., 1994; Bernanke and Gertler, 1995; Bernanke and Mihov, 1998). Empirical studies also show that small changes in policy rates have a large effect on output (Bernanke and Gertler, 1995). Besides the presence of a balance sheet channel, this result may indicate that, in addition to a downward change in aggregate demand, a tightening of monetary policy may also induce an upward change in aggregate supply.

The empirical weakness of the traditional approach inspired some authors to propose a cost channel of monetary policy transmission. The main idea is that if firms have to pay their

factors of production before receiving proceeds from the sales, they have to borrow to finance working capital (Blinder, 1987; Christiano et al., 1997). Therefore, an increase in nominal interest rate causes an increase in firm's marginal cost and thus inflation.

Sections 1.1 to 1.4 revise the literature on the cost channel. Section 1.5 explains the main contributions of the thesis to the literature.

## 1.1 Industry-level and firm-level evidence on the cost channel

One of the first studies showing the empirical relevance of the cost channel was Barth and Ramey (2001). Using VAR models to perform an industry-level analysis of prices for the US, with monthly data between 1959 and 1996, they conclude that there were supply side effects of monetary policy. The main challenge faced was to distinguish the supply effects from the demand effects of the interest rate. The solution was found by observing that a supply shock causes a price (relative to wages) increase. Therefore, if after a monetary shock an increase in prices is observed, then there is evidence that the supply side effects of monetary policy are larger than the demand side effects.

As mentioned by Barth and Ramey (2001), besides the cost channel, one possible explanation for the empirical evidence that prices increase after an increase in the interest rate is a misspecification in the VAR model. Indeed, Sims (1992) had already noticed that the price puzzle reflects the reaction function of the policy maker. Since the central banks have information about future inflation, which is not captured by the VAR model, an important variable is omitted. In order to avoid this omission, Christiano et al. (1994) include the commodity price as a leading indicator of inflation. In this way, they eliminate the price puzzle.

Taking into account the price puzzle debate, Barth and Ramey (2001) include a commodity price index and control for the reaction function of the central bank. They estimate a VAR model for each industry and for the aggregate of manufacturing industries with the following variables: industrial production, personal consumption expenditure deflator, producer price index commodities, change in total reserves of the Federal Reserve System, Federal funds rate, industrial production in industry  $i$ , and the ratio of price to wage in industry  $i$ . The results obtained indicate the relevance of supply side effects of monetary policy for 13 of the

21 industries studied and for the aggregate manufacturing industries. However, such supply side effects are essentially a short-run phenomenon. From our point of view, this approach only enables us to identify the cost channel when there is a positive response of prices after an interest rate shock. However, when the demand effects are larger than the supply effects of interest rate, this methodology is not able to identify the cost channel.

A natural extension of Barth and Ramey (2001) consists of testing the cost channel with micro data. Gaiottu and Secchi (2006) perform such a study with data for 2000 Italian firms covering a period of 14 years. They assume a production technology with three inputs: capital, labour, and material inputs. A fraction of material inputs and labour used in production needs to be paid in advance, which means that firms have to borrow money to finance working capital. Assuming cost minimisation behaviour, it was obtained a firm level equation for the change in prices that depends on the unit labour costs, unit material input costs and nominal interest rate. Using that equation, the relevance of the cost channel is measured by examining the relationship between individual changes in interest rates and individual changes in output prices, after excluding all the aggregate effects and changes in material and labour costs. They assume that the demand channel is an aggregate or industry-level phenomenon, while the cost channel is an individual effect that depends both on the amount of working capital used by the firm and on the interest rate of each individual firm. The authors argue that this kind of firm level analysis avoids the identification problems that arise when macroeconomic data are used.

Gaiottu and Secchi (2006) conclude for the relevance of the cost channel. This channel, as expected, is related with the amount of working capital used by firms.

## **1.2 The Phillips curve and the cost channel**

Given that, as we described, an approach based on VAR models cannot distinguish clearly the supply effects from the demand effects of monetary policy, the preferred approach in the literature has been to use a more structural approach. In this strand of the literature, an influential work is that of Ravenna and Walsh (2006) that introduces a cost channel in a New Keynesian model, assuming that firms have to borrow from financial intermediaries to pay

salaries in advance. They obtain the following forward-looking Phillips curve:

$$\pi_t = \beta E_t \pi_{t+1} + \gamma \widehat{mc}_t$$

where  $\pi_t$  is the inflation rate and  $\widehat{mc}_t$  is the percentage deviation of real marginal cost from its steady-state value. The real marginal cost is the sum of real unit labour cost and nominal interest rate (both in log deviations from the steady-state). The effect of nominal interest rate on marginal cost is the only difference in relation to a standard Phillips curve. Their estimates of the Phillips curve for the US using a GMM and quarterly data from 1960 to 2001 have shown the statistical relevance of the cost channel.

Regarding monetary policy implications, Ravenna and Walsh (2006) find that with the cost channel the flexible-price output depends on the nominal interest rate. An increase in this rate increases firms' financial costs associated with paying salaries in advance. Therefore, the demand for labour decreases and thus there is a reduction in the equilibrium level of employment. In addition, the model with a cost channel produces a realistic trade-off between inflation and output gap. In contrast, without the cost channel, monetary authorities are always able to maintain output gap and inflation in their steady-state levels when the economy is affected by shocks.

Chowdnury et al. (2006) obtain a NKPC very similar to Ravenna and Walsh (2006), with the main difference being the presence of lagged inflation. Their formulation highlights that the size of the cost channel depends positively on the fraction of changes in the policy rate that are pass-through to changes in the lending rate. They estimate the interest rate augmented Phillips curve for the G7 countries and conclude for the statistical relevance of the cost channel in all countries with the exception of Germany and Japan. It is argued that the supply side effect of the nominal interest rate is not present in these two countries because their banking markets are more regulated and less competitive, implying that changes in the policy rate are pass-through to the lending rate at a slower velocity.

Beside studying the empirical relevance of the cost channel, Chowdnury et al. (2006) also evaluate its importance in the context of a New Keynesian general equilibrium model, which includes the estimated Phillips curve, an IS curve and a Taylor-type interest rate rule. They found that, from a general equilibrium perspective, supply effects of interest rate on

inflation are substantial. Under severe financial frictions (when the increase in the loan rate is larger than the increase in the policy rate), the cost channel will be stronger than the demand channel, and an increase in the interest rate may even increase inflation. Then the cost channel can explain the price puzzle. These financial frictions may come from the financial accelerator effect: a contractionary monetary policy deteriorates firms' balance sheets, which increases even further the cost of external financing, either from bank or non-bank sources (Bernanke and Gertler, 1989; Bernanke et al., 1999).

Financial frictions that vary over the business cycle and the evolution of financial markets explain that the importance of the cost channel has changed over time in the US (Tillmann, 2009a). The cost channel was more important in the pre-Volcker period and less important in the Volcker-Greenspan era. In recent years however, the importance of the cost channel increased again.

Owing to the fact that the cost channel varies over time, the central bank faces uncertainty about the true dimension of that channel. This uncertainty, makes the optimal policy implemented by a Taylor rule respond less to inflation (Tillmann, 2009b). In addition, taking into account the uncertainty about the cost channel, the optimal Taylor rule matches the estimated rule for the Federal Reserve after 1982.

A different way of assessing the cost channel takes into account that the NKPC can be written as a discounted sum of the stream of expected marginal costs. This present-value approach can be used to obtain a series for fundamental inflation. Tillmann (2008) contrasts the fundamental inflation obtained in this manner with the actual inflation. For the US, the UK, and the euro area, he shows that the cost channel improves the ability of the NKPC to explain inflation dynamics; namely it allows for the explanation of certain inflation episodes that were not accounted for by the standard curve. Also, the estimated coefficients of the nominal interest rate in the NKPC in present-value format are statistically significant.

As Chowdnury et al. (2006) highlight, the importance of the cost channel may depend on financial and banking markets' characteristics. Castelnovo (2006) adds that the relative importance of supply side effect of monetary policy also depends on habit formation in consumption and interest rate smoothing. He argues that habit formation increases the relative importance of the cost channel over the demand channel and that interest rate smoothing decreases it. Regarding habit formation, notice first that an increase in the interest rate leads to

a reduction in consumption, due to the substitution effect, and consequently output decreases. The larger the degree of habit formation, the smaller both the reduction in consumption and the impact of the demand channel in reducing inflation.

On the contrary, interest rate smoothing reduces the relative importance of the cost channel. Indeed, in the presence of interest rate smoothing, an increase in interest rate lasts for a longer period of time. Therefore, for a given level of interest rate, the larger the degree of interest rate smoothing, the lower expected inflation. This makes it more likely that an increase in interest rate will decrease inflation.

Finally, Castelnuovo (2006) also argues that for reasonable parameters' values the demand channel is more important than the cost channel, implying that inflation decreases after an increase in interest rate.

### **1.3 Estimating the cost channel with general equilibrium models**

We are now going to emphasize another strand of the literature that estimates general equilibrium models. Next, we will discuss, by this order, the papers of Henzel et al. (2009), Rabanal (2007) and Christiano et al. (2005). These papers, especially the first two, do not show a major macroeconomic role for the cost channel.

Henzel et al. (2009) develop and estimate a New Keynesian model that takes into account imperfections in the banking market. These imperfections affect the pass-through of a change in the policy rate to the loan rate, and therefore the importance of the cost channel. This idea had already been proposed by Chowdhury et al. (2006), but they assumed, without providing microfoundations, a static relationship between the policy rate and the loan rate.

In Henzel et al. (2009), banks play an active role in the cost channel, which contrasts with Christiano et al. (2005) and Ravenna and Walsh (2006), where banks act costlessly in a competitive market. Without imperfections in the banking market, banks do not play any role; they only make a neutral transmission of monetary policy.

With the purpose of obtaining a more realistic representation of banks' role in the transmission of monetary policy, Henzel et al. (2009) model the interest rate pass-through assuming monopolistic competition in the banking market and allowing banks to set the loan rate fol-

lowing a Calvo-type approach. This means that, in each period, only a random fraction of banks adjust their loan rate to a change in the central bank interest rate. Consequently, the aggregate loan rate responds slowly to changes in the policy rate, which reduces the relevance of the cost channel. In opposition, in a market-based financial system, the interest rate pass-through would be quicker, implying a larger effect of the cost channel.

Henzel et al.'s (2009) hybrid NKPC, where the loan rate affects the marginal cost, is similar to that of Chowdnury et al. (2006). The full model is estimated for the euro area (1990Q1-2002Q4), a bank-based economy, adopting a minimum distance approach, which involves minimising the distance between the impulse functions generated by the model and the ones estimated with a VAR model. With the parameters estimated in this way, the cost channel is not able to explain the price puzzle, but it helps to generate in the first quarters a constancy in the inflation rate. After a positive interest rate shock, an initial increase in inflation only occurs when small deviations from the estimated parameters are assumed: a higher nominal wage rigidity and/or a lower degree of price stickiness. These deviations are not rejected by the data.

Rabanal (2007) also constructs a New Keynesian model with sticky prices and sticky wages and a cost channel for monetary policy. Using a Bayesian method to estimate the model, which allegedly is better than the GMM, he concludes that the cost channel is small in the US, and is not able to produce an increase in inflation after an increase in interest rate.

In contrast with the other papers presented above, Christiano et al. (2005) are not concerned primarily with the cost channel. Instead, they pose a more general question: whether or not models with moderate nominal rigidities generate, in response to a monetary shock, the inertial inflation and persistent output movements that are observed in reality. To answer this question they create and estimate a dynamic general equilibrium model with wage and price rigidities. Namely, regarding the supply effects of monetary policy, they assume that firms have to borrow working capital to pay wages. The model is estimated with US data, for the period 1965Q3-1995Q3, adopting a minimum distance approach, as in Henzel et al. (2009). The estimated VAR model generates a slight short-run decrease in inflation in response to an expansionary monetary shock.

Even though overall the cost channel does not play a major role in the model's dynamic, it has two significant contributions. On one hand, it allows the model to produce an initial

decrease in inflation after a monetary expansion. On the other hand, without the cost channel, the model would need a degree of price stickiness much higher than the one observed in reality. In the face of these results, they conclude that “the working capital channel plays an important role in the benchmark model’s performance”. We can interpret this evidence as confirming the relevance of the cost channel of monetary policy.

## 1.4 Exchange rates and inflation

A common feature to all the papers in the literature is that they study the cost channel without explicitly taking into account the role of exchange rate in inflation dynamics. Simultaneously, a significant number of papers have studied the relation between the exchange rate and inflation. We can identify two distinct types of models in the literature. On one hand, there are the standard models, such as Gali and Monacelli (2005), which treat imports as consumption goods. On the other hand, there are models, as the one of McCallum and Nelson (2000), which consider imports as inputs in production. Even though both types of models use a dynamic setting, rational expectations, optimising agents, and slowly-adjusting prices of domestic goods, they have different implications regarding the relation between the exchange rate and inflation.

Starting with the standard models, in an open economy the Consumer Price Index (CPI) is composed of domestically produced goods and imported goods. Since there is a cost of adjusting the prices of domestic goods, these prices respond sluggishly to shocks. In contrast, the prices of imported goods in domestic currency adjust rapidly to shocks. For example, if a shock leads to the depreciation of domestic currency, there is an increase in the prices of imported goods in domestic currency, and a consequent increase in CPI. This depreciation also results in an increase in the current account and output gap, which constitutes an additional pressure leading to the increase in CPI. In conclusion, these models predict that the CPI is very flexible and is closely related to the nominal exchange rate. However, both of these implications are not empirically validated.

The models with imports as consumption goods and with full pass-through of exchange rate to the price of imported goods arrive at a Phillips curve where the change in the real exchange rate affects directly inflation. Kara and Nelson (2003) estimate this equation for

the UK, with data from 1964Q2 to 2001Q4, and get a wrong signal for the coefficient of the change in real exchange rate. The explanation for this result cannot be the slow pass-through of exchange rate, because that will imply a smaller coefficient, but not one with the wrong sign.

Alternatively McCallum and Nelson (2001) consider imports as inputs in production. To treat imports only as inputs in production may look strange. However, conservative estimates for the US suggest that, as a share of imports, inputs are more important than consumer goods and services (McCallum, 2001). Naturally, in a real economy, imports will be both consumer goods and inputs in production.

McCallum and Nelson (2001) obtain a more realistic description of inflation. In their model depreciation affects inflation only through the output gap, with two distinct effects. First, the usual increase in the current account is present, with the consequent increase in output. Second, exchange rate depreciation causes an increase in import prices and thus a reduction in potential output. These two effects together cause an increase in output above its potential value, implying an increase in inflation.

McCallum and Nelson's (2001) model is able to explain two empirical regularities: on one hand, the small correlation between changes in nominal exchange rate and inflation, and on the other hand, inflation persistence. The latter empirical regularity is explained by admitting that all prices in the CPI adjust gradually.

The two types of models described, with imports as consumption goods and with imports as inputs, have different implications for monetary policy. In the standard models, the monetary authority should take into account the direct impact of changes in the real exchange rate on CPI inflation. Instead, in the model of McCallum and Nelson (2001) all the relevant inflationary pressures are captured by the output gap. For instance, a large exchange rate depreciation should only lead the central bank to increase the interest rate if output increases considerably above its potential value.

Inspired by the model of McCallum and Nelson (2001), Kara and Nelson (2003) consider for the UK a Phillips curve where imports appear as final consumption goods and inputs. Admitting imports as final goods implies that the change of the real exchange rate has to be present in the Phillips curve. When imports are also inputs, the *level* of the real exchange rate also has to be added. Estimations of that particular Phillips curve show that for all the

sample, the coefficient of the level of the real exchange rate is positive but not significant.<sup>1</sup> For the sample after 1992 results become more significant. At the same time, the change in real exchange rate continues with the wrong signal. This implies that the data are consistent with the idea that imports are intermediate goods.

## 1.5 Thesis structure and content

This section highlights the main contributions to the literature made by this thesis, as well as its main results.

Since in the study of the cost channel little attention has been paid to the role of exchange rates on inflation, in Chapter 2 the cost channel is studied in a small open economy New Keynesian model. In this model the interaction between the interest rate, the exchange rate and inflation makes the analysis of the supply side effect of interest rate richer. In our framework, there are two more additional novelties. On one hand, imports are considered both inputs and consumption goods. On the other hand, the concept of the cost channel is broadened, assuming that besides wages, imports are also paid in advance. In particular, imported consumption goods are also paid in advance because they are inputs for retailers.

Our model studies two types of topics. Firstly, we study whether the effects of the cost channel in a closed economy model are also present in an open economy model. In particular, given the various effects of the nominal interest rate on the marginal cost, it is worth investigating whether, as a result of the cost channel, there is still a mitigated effect of the nominal interest rate on domestic inflation. Furthermore, we study how policy trade-offs arise and are managed, both when the monetary authority follows optimal policies and Taylor rules.

Secondly, an open economy model permits us to study the impact of the cost channel on areas that do not exist in a closed economy. We analyse how variables like CPI inflation, the terms of trade and the nominal exchange rate are affected by the cost channel. Simultaneously, we study the relative performance of different policy regimes, that in an open economy also include the Consumer Inflation-based Taylor Rule (CITR) and the exchange rate peg.

Our study makes some more contributions to the literature. One of them is the analysis, under several policy regimes, of the impact of the cost channel on the economy's response

---

<sup>1</sup>An increase in the real exchange rate corresponds to a depreciation of the domestic currency.

to a technological shock, on one hand, and on macroeconomic volatility, on the other hand. Secondly, we also study the contribution of the working capital channel to explain interest rate smoothing and the small empirical correlation between CPI inflation and the change in the nominal exchange rate. Finally, the impact of imported inputs on the economy's dynamics is analysed.

Chapter 2 concludes that the cost channel has important implications for the dynamics and monetary policy of an open economy. The assumption that imports are paid in advance introduces new effects of the nominal interest rate on the IS curve and on the equation describing CPI inflation. The new configuration of the IS curve breaks the isomorphism between the closed and open economy representations in the domestic inflation and output gap's space, which characterises standard open economy New Keynesian models.

We confirm that some closed economy results regarding the working capital channel hold in an open economy context. To begin with, the nominal interest rate affects the Phillips curve and its effect is richer than in a closed economy. Next, the cost channel leads to a trade-off between output and domestic inflation, causing the monetary authority to allow more inflation variability under optimal discretion. Lastly, after a decrease in the interest rate, domestic and CPI inflations are lower when the cost channel is present.

The simulation of the model lead us to conclude that for all the policy regimes, with the exception of the PEG, the cost channel increases the volatility of domestic inflation, output gap, and the nominal interest rate. One of the explanations for this increase in volatility is that the central bank is substantially more active replying to shocks. The best policy for the monetary authority in terms of welfare is to follow optimal commitment. Alternatively, it can commit to a domestic inflation Taylor rule, which yields better results than optimal discretion. The gains arising from commitment also justify interest rate smoothing in the context of a Taylor rule.

In addition, the cost channel explains partially why the observed contemporaneous correlation between CPI inflation and the change in the nominal exchange rate is relatively small.

Finally, the increase of imported inputs share in output with a correspondent decrease in imported consumption goods share, in general, reduces macroeconomic volatility, especially under the CITR and the exchange rate peg.

After studying the theoretical implications of the cost channel, in Chapter 3 its empirical

relevance is tested on a NKPC with open economy variables. Our goal is to fill a gap in the literature, which has neglected to consider such variables explicitly when estimating the cost channel. With open economy considerations, the marginal cost and supply side inflation depend both on interest rate and the terms of trade; and these two latter variables are typically related to each other. Therefore, only in some specific circumstances will the cost channel be correctly identified when the terms of trade are omitted from the Phillips curve. For instance, such identification will be incorrect when the nominal interest rate affects directly the terms of trade. In this way, the relevance of assessing the supply side effect of the interest rate while taking into consideration the impact of the terms of trade on inflation is clear.

Given the need to consider open economy variables in the Phillips curve to correctly estimate the cost channel, it is necessary to identify the proper way of dealing with these variables, particularly in light of the fact that a consensus does not exist about it. Some models consider imports as final consumption goods (e.g. Gali and Monacelli, 2005), while others treat imports only as intermediated goods (e.g. McCallum and Nelson, 2001). Another issue that deserves attention is the aggregate relevance of exchange rate pass-through.

The relevance of the cost channel and the way in which open economy variables enter the Phillips curve are determinants for the design of monetary policy. Clarida et al. (2001) show that with immediate exchange rate pass-through the central bank should target domestic inflation. Conversely, if the exchange rate slowly affects import prices, it maybe optimal to target CPI inflation. If imports are treated only as inputs, the concern of the monetary authority should also be CPI inflation. Finally, the advantages that the central bank has in committing and the desirable degree of exchange rate flexibility are also affected by how exchange rates affect imported goods' inflation.

Finally, we test whether or not is empirically relevant to extend the cost channel assuming that imports of final consumption goods are also paid in advance. For that we use some results from Chapter 2.

To our knowledge Chapter 3 is the first paper dedicated to studying the empirical relevance of the cost channel considering explicitly open economy variables. The research into whether or not the data confirm that imports of consumption goods are paid in advance is also new in the context of works on the working capital channel. This paper also contributes to the literature testing for the G7 countries and in the context of the NKPC the empirical relevance

of slow exchange rate pass-through. Lastly, the test for the G7 countries whether imports should be considered as inputs and/or consumption goods also adds to the literature, because until now such a test has only been attempted for the UK.

Our results indicate that open economy variables play an important role in explaining inflation dynamics, both for domestic and CPI inflations. The empirical success of the NKPC with slow exchange rate pass-through was larger than with immediate pass-through. This was valid both for imported inputs and imported consumption goods. The model of McCallum and Nelson (2000) where imports are solely considered as inputs in production is rejected by the data. Instead, a model with imports as both consumption goods and inputs has a better empirical adherence. Regarding the cost channel, there is weak evidence that the level of the nominal interest rate affects inflation. Nevertheless, there is strong evidence that the change in the nominal interest rate affects CPI inflation. This last finding can be explained by the fact that firms pay for imported consumption goods in advance.

In Chapter 3 we study inflation dynamics in the G7 countries. In this group there are three euro area countries, France, Germany and Italy. For the euro area there is evidence in the literature showing inflation divergence after the introduction of the euro (Busetti et al., 2006; Lane, 2006), that was however reversed around 2001/02 (Becker and Hall, 2009b). As highlighted by the optimum currency area literature, inflation differentials can undermine the success of a monetary union. In this context, in Chapter 4 we try to understand what factors determine inflation differentials and the correcting mechanisms at work. According to the NKPC, one of the main drivers of inflation is the business cycle, normally measured by the output gap or real unit labour cost. As a result, convergence in inflation rates should be accompanied by convergence in business cycles. Taking this into account, our main goal in this chapter is to study the relationship between the two convergence processes. Specifically, we want to analyse if divergence/convergence in inflation rates after the introduction of the euro can be explained by divergence/convergence in business cycles. We start by analysing the convergence of inflation, output gap and real unit labour cost over the period 1980-2008, using the Kalman filter to test whether or not the variance of the unobserved convergence component decreases over time. The convergence of the same variables is also analysed using a common factor approach, which is put into practice using principal component analysis. Next, we construct an econometric model to assess the determinants of inflation differentials,

among which we consider the business cycle and the cost channel. As an intermediate step in the estimation process, we also estimate Phillips curves for a panel of 12 euro area countries, which allows us to test with different data and with panel techniques the relevance of the cost channel.

Let us highlight the most innovative features of this paper and our contributions to the literature. The analysis of convergence of RULCs and output gaps using, on one hand, the Kalman filter, as proposed by Hall et al. (1997), and on the other hand, the common factor approach of Becker and Hall (2009a) is new in the literature. Also, the study of RULC has been ignored in the convergence literature, even though this indicator is important in the New Keynesian approach to inflation.

The joint analysis of the convergence processes of inflation rates, output gaps and RULCs with Hall et al.'s (1997) model has two novelties. First, we compare the rates at which the (unobserved) convergence of inflation rates and output gaps evolve over time. Second, we analyse the two-way causality between output gap and inflation convergence.

Our analysis shows that during the euro period there were periods of convergence and divergence in inflation, output gap and RULC. However, in general there was an increase in co-movement on each of that variables during the euro period. The process of convergence of the RULC seems to be the most idiosyncratic of the three convergence processes considered. On a larger horizon, between 1980 and 2008, inflation rates have converged faster than output gaps. In the same period, output gap convergence had a positive effect on inflation convergence but the opposite did not occur.

When explaining inflation differentials, an innovative feature is the use of residuals of a common factor model to measure variables' divergence. There are two more distinctive features of our study. To start with, we test how inflation and exchange rate expectations affect inflation divergence. Expectations have been mainly ignored despite their importance in explaining national inflation rates. Next, the New Keynesian framework is tested to see if provides a complete description of inflation differentials, looking at the usefulness of the Imperfect Competition Model. We are particularly concerned with the importance of both nominal ULC growth and equilibrium conditions for prices on inflation dynamics. As a by-product of the analysis of convergence, we estimate the NKPC for the euro area using panel data. This is interesting, because there is little evidence on the NKPC using panel data.

Our empirical evidence shows that expectations of both inflation and exchange rates are statistically significant for inflation differences and their introduction changes the significance of other variables. Moreover, the only business cycle indicator relevant for explaining inflation divergence is the labour costs. Also, the equilibrium conditions for prices are important for explaining differences in inflation rates. Besides, the ICM model is not encompassed by the NKPC when explaining inflation differences. Lastly, our panel data evidence supports the NKPC for national inflation rates and the existence of the cost channel.

## Chapter 2

# The cost channel in a small open economy New Keynesian model

### 2.1 Introduction

Evidence in the literature points out for a puzzling initial increase in inflation after an increase in interest rates (Sims, 1992). This can be explained by assuming that firms have to borrow money to pay wages in advance, *i.e.*, by the cost channel.

In the context of a closed economy, the study of the cost channel with sticky prices is performed by Ravenna and Walsh (2006) and Chowdhury et al. (2006). These papers show that the introduction of the cost channel generates an endogenous cost shock that creates a trade-off between inflation and output gap. In such a context, an optimal discretion monetary policy leads to larger inflation variability when the cost channel is present, due to a larger nominal interest rate fluctuation. Furthermore, Chowdhury et al. (2006) show that, when the central bank follows a Taylor rule, the cost channel makes the decrease of domestic inflation in response to a given positive interest rate shock smaller, but it amplifies the reduction in the output gap. They also conclude that under severe financial frictions, an increase in the nominal interest rate may even increase inflation in the short-run.

However, little is known about the impact of the cost channel in an open economy context, where the interaction between interest rate, exchange rate and inflation makes the analysis more complex. Our contribution to the literature is precisely to deepen this knowledge.

Therefore, we introduce the cost channel in a small open economy New Keynesian model similar to Gali and Monaceli (2005). Our model has two more novelties. Firstly, we assume imports of both inputs and consumption goods. Even though Adolfson (2001) uses it, this approach is not very common in the literature. Secondly, in an open economy we propose a broadening of the concept of cost channel, assuming that all inputs in production have to be paid in advance. As a result, wages and imports of both consumption goods and inputs imply financial costs to firms. Particularly, imported consumption goods also have to be paid in advance because they are considered inputs for retailers.

The model developed in this paper brings to light some interesting topics. It allows us to study whether or not the effects of the cost channel already described in the literature for a closed economy are also valid in an open economy. Namely, our model admits that the nominal interest rate has several effects on the marginal cost: it increases the wage bill, directly increases the price of imported consumption goods and inputs, and indirectly decreases import prices (due to its effect on the terms of trade). When considering all these effects, an interesting question is whether or not we still observe a mitigated effect of the nominal interest rate on domestic inflation as a result of the cost channel.

In an open economy model we can also study the impact of introducing the cost channel in other subjects that do not exist in a closed economy model. On one hand, we analyse the behaviours of CPI inflation, the terms of trade and the nominal exchange rate. On the other hand, it can be analysed the relative performance of different policy regimes, that in an open economy also include the Consumer Inflation-based Taylor Rule (CITR) and the exchange rate peg.

Our analysis makes some more contributions to the literature. One of them is the analysis, under several policy regimes, of the impact of the cost channel on the economy's response to a technological shock, on one hand, and on macroeconomic volatility, on the other hand. Secondly, we also study the contribution of the working capital channel to explain interest rate smoothing and the small empirical correlation between CPI inflation and the change in the nominal exchange rate. Finally, we study the impact of imported inputs on the economy's dynamics.

This paper concludes that the cost channel has important implications for the dynamics and monetary policy of an open economy. The assumption that imports are paid in advance

introduces new effects of the nominal interest rate on the IS curve and on the equation describing CPI inflation. The new configuration of the IS curve breaks the isomorphism between the closed and open economy representations in the domestic inflation and output gap's space, which characterises standard open economy New Keynesian models.

We confirm that some closed economy results regarding the working capital channel hold in an open economy context. To begin with, the nominal interest rate is present in the Phillips curve and its effect is richer than in a closed economy. Next, the cost channel leads to a trade-off between output and domestic inflation, causing the monetary authority to allow more inflation variability under optimal discretion. Lastly, after a decrease in the interest rate, domestic and CPI inflations are lower when the cost channel is present.

The simulation of the model lead us to conclude that for all the policy regimes, with the exception of the PEG, the cost channel in general increases the volatility of domestic inflation, output gap, and the nominal interest rate. One of the explanations for this increase in volatility is that the central bank is substantially more active replying to shocks. The best policy for the monetary authority in terms of welfare is to follow optimal commitment. Alternatively, it can commit to a domestic inflation Taylor rule, which yields better results than optimal discretion. The gains arising from commitment also justify interest rate smoothing in the context of a Taylor rule.

In addition, the cost channel explains partially why the observed contemporaneous correlation between CPI inflation and the change in the nominal exchange rate is relatively small.

Finally, the increase of imported inputs share in output with a correspondent decrease in imported consumption goods share, in general, reduces macroeconomic volatility, especially under the CITR and the exchange rate peg.

The remainder of the paper is organised as follows. Section 2.2 describes the model and the equilibrium equations in the space output gap and domestic inflation. Section 2.3 analyses the implications of the model in terms of policy trade-offs, detaching the optimal discretion policy. Section 2.4 studies the impact of a domestic technological shock under different policy regimes, with and without the cost channel. Section 2.5 studies the impact of the cost channel on macroeconomic volatility assuming simultaneously a technological, a preferences and a foreign output shocks. Section 2.6 concludes this study.

## 2.2 A small open economy model with the cost channel

In this section, we describe the small open economy model with the cost channel. Section 2.2.1 describes the behaviour of households. Section 2.2.2 explains how the price of imported goods is established by retailers and presents some important identities. Section 2.2.3 discusses how the determination of domestic prices by firms leads to the Phillips curve. Section 2.2.4 describes the aggregate demand in the foreign country. Section 2.2.6 explains how the uncovered interest parity and the aggregate demand are obtained. Section 2.2.7 presents the marginal cost and the flexible-price output. Section 2.2.5 discusses how the international risk sharing links the domestic and foreign outputs. Finally, in Section 2.2.8 the model is written in deviations from the flexible-price equilibrium.

### 2.2.1 Households

In our model there are two countries: the home country is denoted by the superscript  $h$  and the foreign country by the superscript  $f$ . Both countries have the same preferences and technologies. The home country uses foreign goods for both final consumption and production of domestic goods. For the sake of simplicity, it is assumed that the foreign country only uses the domestic goods for consumption.

In both countries, households consume goods, supply labour, and hold money and a portfolio of assets. Using a CES function, consumption goods are composed of home goods,  $C_t^h$ , and foreign goods,  $C_t^f$ . The home country consumption,  $C_t$ , is:

$$C_t = \left[ (1 - \gamma_c)^{\frac{1}{a}} \left( C_t^h \right)^{\frac{a-1}{a}} + \gamma_c^{\frac{1}{a}} \left( C_t^f \right)^{\frac{a-1}{a}} \right]^{\frac{a}{a-1}} \quad (2.1)$$

where  $a > 1$  measures the elasticity of substitution between domestic and foreign aggregate goods and  $\gamma_c$  denotes the share of home consumption allocated to imported goods.

In both domestic and foreign markets there exists a set of differentiated goods produced by monopolistically competitive firms of measure 1. Thus,  $C_t^h$  and  $C_t^f$  are composed of several domestic and foreign goods, respectively. In other words, as in Galí and Monacelli (2005),  $C_t^h$

and  $C_t^f$  are aggregated goods:

$$\begin{aligned} C_t^h &= \left( \int_0^1 C_t^h(i)^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}} \\ \text{and } C_t^f &= \left( \int_0^1 C_t^f(i)^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}} \end{aligned} \quad (2.2)$$

where  $\theta > 1$  determines the price elasticity of each individual good  $i$  within the categories of home and foreign goods.

A household optimally allocates spending within each category of goods. For instance, it tries to reach a given level of consumption  $C_t^h$  minimising the overall expenditure. The solution of such a problem for domestic and foreign consumption leads to the following demand functions:

$$\begin{aligned} C_t^h(i) &= \left( \frac{P_t^h(i)}{P_t^h} \right)^{-\theta} C_t^h \\ \text{and } C_t^f(i) &= \left( \frac{P_t^f(i)}{P_t^f} \right)^{-\theta} C_t^f, \end{aligned} \quad (2.3)$$

for all  $i \in [0, 1]$  and with price indices for domestic and foreign goods, respectively:

$$P_t^h = \left( \int_0^1 P_t^h(i)^{1-\theta} di \right)^{\frac{1}{1-\theta}} \quad \text{and} \quad P_t^f = \left( \int_0^1 P_t^f(i)^{1-\theta} di \right)^{\frac{1}{1-\theta}}.$$

The optimal allocation between the aggregated domestic and imported goods is characterised by the following demand curves:

$$C_t^h = (1 - \gamma_c) \left( \frac{P_t^h}{P_t^c} \right)^{-a} C_t \quad \text{and} \quad (2.4)$$

$$C_t^f = \gamma_c \left( \frac{P_t^f}{P_t^c} \right)^{-a} C_t \quad (2.5)$$

, with the consumer price index (CPI) given by:

$$P_t^c = \left[ (1 - \gamma_c) \left( P_t^h \right)^{1-a} + \gamma_c \left( P_t^f \right)^{1-a} \right]^{\frac{1}{1-a}}. \quad (2.6)$$

Since in the steady-state domestic and foreign price indices are equal,  $P_t^h = P_t^f$ , then  $\gamma_c$ , as mentioned above, is the steady-state share of imported consumption goods in total home consumption.

Combining the two last demand curves, yields:

$$\frac{C_t^h}{C_t^f} = \left( \frac{1 - \gamma_c}{\gamma_c} \right) \left( \frac{P_t^h}{P_t^f} \right)^{-a}. \quad (2.7)$$

It can be observed that the relative demand of domestic goods depends inversely on the relative price of those goods.

The total consumer expenditure is:

$$P_t^c C_t = P_t^h C_t^h + P_t^f C_t^f$$

, with total consumer expenditure on domestic goods given by:

$$P_t^h C_t^h = \int_0^1 P_t^h(i) C_t^h(i) di$$

and total consumer expenditure on foreign goods equal to:

$$P_t^f C_t^f = \int_0^1 P_t^f(i) C_t^f(i) di.$$

The utility of the representative household depends on the consumption of the composite good,  $C_t$ , and on the hours of work,  $N_t$ . Leisure is simply  $1 - N_t$ . The utility function, that is also affected by a random taste shock,  $\xi_t$ , is given by:

$$U(C_t, N_t) = \frac{\xi_t C_t^{1-\sigma}}{1-\sigma} - \chi \frac{N_t^{1+\eta}}{1+\eta}$$

where  $\chi$  determines the disutility of labour,  $\sigma$  denotes the inverse of the intertemporal elasticity of consumption, and  $\eta$  is the inverse of the elasticity of labour supply. An increase in  $\sigma$  means that the household is less willing to substitute present consumption for expected future consumption in response to an increase in the real interest rate. In other words, the household is more risk averse. Households maximise the expected present discounted value of

utility:

$$E_t \sum_{i=0}^{\infty} \beta^i \left[ \frac{\xi_{t+i} C_{t+i}^{1-\sigma}}{1-\sigma} - \chi \frac{N_{t+i}^{1+\eta}}{1+\eta} \right]$$

where  $\beta$  denotes the discount factor.

### Cash-in-advance constraint

Households face a cash-in-advance constraint. At the beginning of period  $t$  households have cash holdings of  $H_t$  and a portfolio (which includes namely, but not only, shares of both firms and financial intermediaries) held at the end of period  $t - 1$ . The nominal payoff of this portfolio,  $D_t$ , is received in cash at the beginning of period  $t$ . At that moment households also receive labour income in cash:  $W_t N_t$ , where  $W_t$  is the hourly *nominal* wage, and  $N_t$  the number of hours worked. Because at the beginning of period  $t$  firms do not have cash available, they borrow money to pay wages in advance to households. In conclusion, the cash that households have at the beginning of period  $t$  is  $H_t + W_t N_t + D_t$ . Households can use this cash to make deposits at financial intermediaries,  $DP_t$ , buy consumption goods and invest in a new portfolio at price  $E_t (\Omega_{t,t+1} D_{t+1})$ , where  $\Omega_{t,t+1}$  is the stochastic discount factor for one-period ahead nominal payoffs relevant to the domestic household, and  $D_{t+1}$  is the nominal random payoff in  $t + 1$  of the portfolio purchased in  $t$ . Therefore, the household faces the following cash-in-advance constraint:  $H_t + W_t N_t + D_t \geq E_t (\Omega_{t,t+1} D_{t+1}) + P_t^c C_t + DP_t$ . At the end of period  $t$ , the cash carried over to period  $t + 1$  is:

$$H_{t+1} = H_t + W_t N_t + D_t + I_t DP_t - E_t (\Omega_{t,t+1} D_{t+1}) - P_t^c C_t - DP_t$$

, with deposits paying interest at the end of period  $t$ , at the gross interest rate  $I_t$ . It is assumed that households have access to a complete set of contingent claims traded internationally.

### Financial intermediaries

Following Ravenna and Walsh (2006), intermediaries receive deposits from households,  $DP_t$ , and a free cash injection from the central bank,  $X_t$ . Intermediaries then lend funds to firms at an interest rate  $I_t$ . Since there are no operational costs and the market is competitive, intermediaries' profit is:  $\Pi_t = I_t(DP_t + X_t) - I_t DP_t = I_t X_t$ .

The cash injection from the central bank corresponds to the increase in money stock from  $t$  to  $t + 1$ ,  $g_{t+1}$ . Consequently, the cash injection can be expressed as  $X_t = H_{t+1} - H_t = g_{t+1} H_t$ . The existence of equilibrium in the market for loans requires that supply equals demand, with the latter determined by salaries and imports. Notice that we assume that salaries and imports have to be paid in advance. Then the equilibrium can be expressed as:  $W_t N_t + P_t^f (C_t^f + M_t) = DP_t + X_t$ , where  $M_t$  is the total demand of imported inputs by firms.

### First order conditions

Households maximise *expected present discounted utility* subject to the CIA constraint, leading to the usual FOCs (in addition to the demand function for individual goods and CIA constraint) - See Annex (6.1.1) for proof:

$$\beta \frac{P_t^c}{P_{t+1}^c} \frac{\xi_{t+1} C_{t+1}^{-\sigma}}{\xi_t C_t^{-\sigma}} = \Omega_{t,t+1} \quad (2.8)$$

$$\frac{\chi N_t^\eta}{\xi_t C_t^{-\sigma}} = \frac{W_t}{P_t^c}. \quad (2.9)$$

Notice that the FOCs depend on the CPI, because a household's utility depends on the consumption of domestic and foreign goods.

Taking conditional expectations on both sides of (2.8), it is possible to get the usual stochastic Euler equation for the optimal intertemporal allocation of consumption:

$$\beta E_t \left( \frac{I_t P_t^c}{P_{t+1}^c} \xi_{t+1} C_{t+1}^{-\sigma} \right) = \xi_t C_t^{-\sigma} \quad (2.10)$$

where  $1/I_t = E_t(\Omega_{t,t+1})$  is the price of a one period discount bond that pays off one unit of domestic currency in  $t + 1$ , and  $I_t$  is its gross return.

Moreover, the second FOC describes the equilibrium in the labour market, requiring that the marginal rate of substitution between leisure and consumption equals the real wage.

### 2.2.2 Import prices and some important identities

In this section, we are going to explain the pricing of imports and define some important identities that are going to be applied later. The lowercase letters with a hat denote variables

in percentage deviation around the steady-state.

In the home country there exists a large number of retailers that import differentiated goods from the foreign country. In this monopolistically competitive market, a retailer facing a demand for good  $i$  defined by equation (2.3), can set the price of that good in domestic currency,  $P_t^f(i)$ .

For the sake of simplicity, the local retailer only supports two costs: the price paid for the foreign good in the international market and the financial cost of paying for that good in advance. As a result, the marginal cost, in local currency, of the retailer which imports good  $i$  is

$$MC_t(i) = I_t \ddot{E}_t P_t^{f*}(i) \quad (2.11)$$

, with  $P_t^{f*}(i)$  as the price of foreign good  $i$  in foreign currency and  $\ddot{E}_t$  as the nominal exchange rate (price of foreign currency in terms of domestic currency). An increase in  $\ddot{E}_t$  means a depreciation of the home currency. Notice that the nominal interest rate affects the marginal cost because importers have to pay imports in advance.

In order to simplify the analysis, we assume a complete and immediate exchange rate pass-through of nominal exchange rate to import prices.<sup>1</sup> Therefore, it is admitted that retailers can adjust their prices every period. The retailers' decision simplifies then to maximise profit given by:

$$\Pi_t(i) = \left[ P_t^f(i) - MC_t(i) \right] C_t^f(i),$$

subject to the demand for product  $i$ , equation (2.3). The FOC for this problem is:

$$P_t^f(i) = \frac{\theta}{\theta - 1} MC_t(i)$$

, corresponding to the typical solution in monopolistic competition: firms set the price equal to a mark-up,  $\theta/(\theta - 1)$ , over marginal cost.

Aggregating the last expression for all imported goods and using (2.11), we obtain:

$$P_t^f = \frac{\theta}{\theta - 1} I_t \ddot{E}_t P_t^{f*} \quad (2.12)$$

---

<sup>1</sup>This can be seen as an extreme case of a situation where retailers have a smaller price rigidity than domestic firms.

, with  $P_t^{f*} = \left( \int_0^1 P_t^{f*}(i)^{1-\theta} di \right)^{\frac{1}{1-\theta}}$ . In deviation from the steady-state, we obtain

$$\widehat{p}_t^f = \widehat{i}_t + \widehat{e}_t + \widehat{p}_t^{f*}. \quad (2.13)$$

Importers follow precisely the same steps to establish the price of imported *inputs*. At this point, we assume two simplifying assumptions without any substantive effect on results. On one hand, imported goods are used both for consumption and production. On the other hand, the individual price elasticity for imported consumption goods and imported inputs is the same,  $\theta$ . Under these conditions, the aggregate price of imported inputs in domestic currency is also given by (2.12).

Let us define the *gross* terms of trade or, in other words, the price of foreign goods in terms of home goods as  $\delta_t = P_t^f / P_t^h$ , or in deviations from the steady-state:

$$\widehat{\delta}_t = \widehat{p}_t^f - \widehat{p}_t^h. \quad (2.14)$$

We can isolate the *direct* effect of the nominal interest rate on the terms of trade, by writing:

$$\widehat{\delta}_t = \widehat{i}_t + \widehat{\delta}_t' \quad (2.15)$$

where:

$$\widehat{\delta}_t' = \widehat{e}_t + \widehat{p}_t^{f*} - \widehat{p}_t^h. \quad (2.16)$$

We call  $\widehat{\delta}_t'$  simply the terms of trade. Next, we use the terms of trade to define some identities that will be useful later.

Assuming a steady-state with  $P_t^h = P_t^f$ , equation (2.6) becomes (see Annex (6.1.3) for details):

$$\widehat{p}_t^c = (1 - \gamma_c) \widehat{p}_t^h + \gamma_c \widehat{p}_t^f. \quad (2.17)$$

Since from the terms of trade  $\widehat{p}_t^f = \widehat{\delta}_t + \widehat{p}_t^h$ , we can write:

$$\widehat{p}_t^c = \widehat{p}_t^h + \gamma_c \left( \widehat{\delta}_t' + \widehat{i}_t \right) \quad (2.18)$$

Furthermore, it is possible to write equation (2.1) as (see Annex (6.1.3) for details):

$$\widehat{c}_t = (1 - \gamma_c) \widehat{c}_t^h + \gamma_c \widehat{c}_t^f. \quad (2.19)$$

In addition, using the terms of trade, equation (2.7) can be written as:

$$\frac{C_t^h}{C_t^f} = \left( \frac{1 - \gamma_c}{\gamma_c} \right) \delta_t^a,$$

, or in deviations from the steady-state:

$$\widehat{c}_t^f = \widehat{c}_t^h - a \left( \widehat{\delta}_t^f + \widehat{i}_t \right). \quad (2.20)$$

The Euler equation for consumption, equation (2.10), can be approximated around a zero-inflation steady-state, as:

$$\widehat{c}_t = E_t \widehat{c}_{t+1} - \frac{1}{\sigma} \left( \widehat{i}_t - E_t \pi_{t+1}^c \right) - \frac{1}{\sigma} \left( E_t \widehat{\xi}_{t+1} - \widehat{\xi}_t \right). \quad (2.21)$$

The optimal leisure-consumption choice (equation (2.9)) in deviations from the steady-state is:

$$\eta \widehat{n}_t + \sigma \widehat{c}_t - \widehat{\xi}_t = \widehat{w}_t - \widehat{p}_t^c.$$

Replacing  $\widehat{p}_t^c$  for expression (2.18), we get:

$$\widehat{w}_t - \widehat{p}_t^h = \eta \widehat{n}_t + \sigma \widehat{c}_t - \widehat{\xi}_t + \gamma_c \left( \widehat{\delta}_t^f + \widehat{i}_t \right) \quad (2.22)$$

Since domestic inflation and CPI inflation are, respectively,  $\pi_t^h = \widehat{p}_t^h - \widehat{p}_{t-1}^h$  and  $\pi_t^c = \widehat{p}_t^c - \widehat{p}_{t-1}^c$ , and taking into account equation (2.18), we can express:

$$\pi_t^c = \pi_t^h + \gamma_c \left( \widehat{\delta}_t^f - \widehat{\delta}_{t-1}^f \right) + \gamma_c \left( \widehat{i}_t - \widehat{i}_{t-1} \right). \quad (2.23)$$

This means that CPI inflation depends on domestic inflation, the change in the terms of trade and the change in the nominal interest rate. Therefore, an increase in the nominal interest rate increases *directly* CPI inflation, because it increases the gross terms of trade.

The real exchange rate is the ratio of two countries' CPIs expressed in a common currency:

$$Q_t = \frac{\ddot{E}_t P_t^{c*}}{P_t^c} \quad (2.24)$$

where  $P_t^{c*}$  is the foreign country's CPI in foreign currency. Assuming that the foreign country is large in relation to the home country, the real exchange rate is (See Annex (6.1.2)):

$$\hat{q}_t = (1 - \gamma_c) \hat{\delta}_t' - \gamma_c \hat{i}_t. \quad (2.25)$$

This expression is associated with the fact that an increase in the nominal interest rate increases the price of imported goods, causing an increase in the domestic CPI, which leads to an appreciation of the real exchange rate.

### 2.2.3 Domestic firms

As usual in many models with sticky prices, a Calvo (1983) setup will be used to describe how domestic firms set prices. In each period, a fraction  $1 - \omega$  of randomly selected firms is able to adjust prices optimally. The remaining firms update prices using the steady-state inflation rate, which in the present case is zero. This means that a fraction  $\omega$  does not adjust its prices and it can be shown that  $1/(1 - \omega)$  is the average time during which prices remain fixed on average. Then, a larger  $\omega$  implies that fewer firms adjust prices and the *average* time between price changes is bigger. Hence,  $\omega$  is a measure of price rigidity.

Firms that adjust their prices in  $t$  maximise the expected discounted value of current and future profits. Firms' future profits are affected by the price in  $t$  because there is a positive probability that prices cannot be adjusted in the future. In fact, the probability that a firm will *not* adjust its price between  $t$  and  $t + s$  is  $\omega^s$ .

A key element determining prices' adjustment is the marginal cost, which is related to the production function. Firms produce to satisfy domestic ( $C_t^h$ ) and foreign demand ( $C_t^{h*}$ ) for their products. All firms use identical technology,<sup>2</sup> with the aggregate CES production

---

<sup>2</sup>Since in empirical applications there is no firm-level data available, it is often used a representative firm hypothesis (Gali and Gertler, 1999). Thus, we assume that all firms use the same production function and have identical marginal cost. Nevertheless, with a CES production function, if it is assume that prices and thus production levels can change from firm to firm, the marginal cost will not be constant across firms. But this would not have a fundamental impact on our analysis. The only difference is that we would have in the Phillips

function given by:

$$Y_t = \left[ \alpha_N (Z_t N_t)^{1-\frac{1}{\varepsilon}} + \alpha_M (M_t)^{1-\frac{1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} \quad (2.26)$$

where  $\alpha_N \in (0, 1]$ ,  $\alpha_M = 1 - \alpha_N$ ,  $\varepsilon$  is the elasticity of substitution between inputs with  $(1 - \frac{1}{\varepsilon}) \in (-\infty, 1)$ ,<sup>3</sup>  $Y_t$  is the domestic output in  $t$ ,  $Z_t$  is the *aggregate* technological shock in  $t$ ,  $N_t$  is the number of hours of work in  $t$ , and  $M_t$  is the amount of imported inputs in  $t$ . There are two points worth mentioning. Firstly,  $M_t$  is an index of foreign goods:  $M_t = \left( \int_0^1 m_t(i)^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}}$ . Moreover, an important hypothesis is that foreign goods are used both for consumption and production, implying that the imported input  $i$  is the same good as the imported consumption good  $i$ . This hypothesis, without eliminating any significant issue, simplifies the analysis, allowing the existence of only one foreign price  $P_t^f(i)$ , relevant for consumers and firms.

Secondly, with imported inputs, output differs from value added. The latter in real terms (deflated by the domestic price index,  $P_t^h$ ) is:

$$Y_t^{va} = Y_t - \delta_t M_t. \quad (2.27)$$

In order to express the last equation in deviations from the steady-state, it is assumed that in the steady-state  $M/Y^{va} = \gamma_i$ , which using (2.27) implies  $M/Y = \gamma_i / (1 + \gamma_i)$ .<sup>4</sup> Therefore, from (2.27), it is possible to obtain:

$$\widehat{y}_t^{va} = (1 + \gamma_i) \widehat{y}_t - \gamma_i \left( \widehat{\delta}_t + \widehat{i}_t \right) - \gamma_i \widehat{m}_t. \quad (2.28)$$

In order to guarantee zero net exports in the steady-state<sup>5</sup> (which implies  $C = Y^{va}$ )  $\gamma_c$  has to depend on  $\gamma_i$ . In terms of domestic prices, the trade balance is  $NX_t = C_t^{h*} - \delta_t C_t^f - \delta_t M_t$ . Since in the steady-state  $\delta = 1$ , we have:

$$\frac{NX}{C} = \frac{C^{h*}}{C} - \frac{C^f}{C} - \frac{M}{C} = \gamma(1 + \gamma_i) - \gamma_c - \gamma_i.$$

---

Curve an aggregation constant factor, like in Gagnan and Khan (2001), but the driver of inflation would still be the aggregate marginal cost.

<sup>3</sup>With  $1 - \frac{1}{\varepsilon} = 0 \Leftrightarrow \varepsilon = 1$ , we have a Cobb-Douglas production function. If  $1 - \frac{1}{\varepsilon} = -\infty$ , i.e.,  $\varepsilon = 0$ , we have perfect complements. When  $1 - \frac{1}{\varepsilon} = 1$ , i.e.,  $\varepsilon = +\infty$ , we have perfect substitutes.

<sup>4</sup>A variable without a time subscript indicates a variable in the steady-state.

<sup>5</sup>Zero net exports in the steady-state are also assumed in Galí and Monacelli (2002), for a given initial distribution of wealth.

From the last equation, to guarantee a zero trade balance in the steady-state, the imports' share in consumption has to be:

$$\gamma_c = \gamma - \gamma_i (1 - \gamma) \quad (2.29)$$

where  $\gamma$  is the share of exports in domestic product. This equation is related with the fact that, for a given weight of exports on domestic product, an *increase* in the weight of imported inputs on value added,  $\gamma_i$ , has to be balanced by a *decrease* on the weight of imports on consumption,  $\gamma_c$ , in order to maintain a zero trade balance.

All firms have to borrow from intermediaries, at the *gross* nominal interest rate  $I_t$ , to pay wages and imports in advance. As a result the unitary *nominal* cost of labour is  $I_t W_t$  and the marginal cost of imports is given by (2.11).

Next, it is shown how the real marginal cost is obtained. Firms' problem is to minimise the real cost of production,  $I_t \frac{W_t}{P_t^h} N_t + \delta_t M_t$ , subject to the production function given by (2.26). Notice that the gross terms of trade, which already include the effect of the nominal interest rate, are the price of imports in real terms. Then, the Lagrangian of the problem is:

$$L = I_t \frac{W_t}{P_t^h} N_t + \delta_t M_t + \varphi_t (Y_t - F(N_t, M_t)),$$

where  $F(N_t, M_t)$  is the production function.<sup>6</sup>

One of the first order conditions yields:

$$\varphi_t = \frac{I_t \frac{W_t}{P_t^h}}{\frac{\partial F}{\partial N_t}}.$$

Observe that the marginal cost is given by  $\varphi_t$ <sup>7</sup> and can also be written as:

$$MC_t = \frac{I_t \frac{W_t}{P_t^h} \frac{N_t}{Y_t}}{\frac{\partial F}{\partial N_t} \frac{N_t}{Y_t}} = \frac{I_t S_t}{\gamma_t} \quad (2.30)$$

---

<sup>6</sup>Notice that is the *nominal* interest rate that is relevant because firms have to pay in advance nominal wages,  $W_t N_t$ . As usual, for cost minimisation firms take into account the real cost of total wage expenditure  $(I_t W_t N_t) / P_t^h$ .

<sup>7</sup>It represents the impact on the objective function (cost function) of an unitary increase in the restriction (output).

where  $\gamma_t$  is the elasticity of output with respect to labour:

$$\gamma_t = \frac{\frac{\partial F}{\partial N_t}}{\frac{Y_t}{N_t}}$$

and  $S_t$  is the labour income share:

$$S_t = \frac{N_t W_t}{P_t^h Y_t}. \quad (2.31)$$

With flexible prices, domestic prices are defined as a constant mark-up,  $\Phi$ , over the nominal marginal cost ( $NMC_t$ ):  $P_t^h = \Phi NMC_t$ , thus implying that the *real* marginal cost is:

$$MC_t = \frac{NMC_t}{P_t^h} = \frac{1}{\Phi}. \quad (2.32)$$

A standard result is that:

$$\Phi = \frac{\theta}{\theta - 1} > 1. \quad (2.33)$$

Therefore, an *increase* in  $\theta$ , *decreases* the steady-state mark-up.<sup>8</sup> This result comes from the fact that, as the demand elasticity of individual goods increases, *i.e.*,  $\theta$  increases, the degree of substitutability between goods also increases. Consequently, firms have *less* market power and are forced to define a lower mark-up.

However, with sticky prices the real marginal cost deviates from its steady-state value,  $(\theta - 1)/\theta$ . In this case, we have the standard New Keynesian Phillips curve for *domestic* inflation:

$$\pi_t^h = \beta E_t \pi_{t+1}^h + k \widehat{mc}_t \quad (2.34)$$

where  $\widehat{mc}_t$  is the marginal cost in *percentage* deviation around its steady-state value;<sup>9</sup> and

$$k = (1 - \omega)(1 - \omega\beta)/\omega.$$

---

<sup>8</sup>Notice that the last expression can be written as  $\Phi = \frac{1}{1 - \frac{1}{\theta}}$ .

<sup>9</sup>The percentage deviation of inflation from its steady state is simply the inflation rate, because the steady-state value of inflation is zero.

## 2.2.4 Foreign country

Recall that the foreign country is large relative to the home country. This has several implications for the foreign country. On one hand, domestic inflation and CPI inflation are equal:  $\pi^{c*} = \pi^{h*}$ . On the other hand, exports' share in demand is irrelevant, implying that consumption is equal to output:  $\widehat{c}_t^{f*} = \widehat{y}^f$ .

We assume that the foreign country only uses imported goods for final consumption.<sup>10</sup> Since it is assumed there are identical preferences for both countries, the demand that foreign consumers make for domestic goods is similar to the demand that home consumers make for foreign goods. Consequently, the expression for  $\widehat{c}_t^{h*}$  is similar to equation (2.20):

$$\widehat{c}_t^{h*} = \widehat{c}_t^{f*} + a\widehat{\delta}_t^f$$

Notice two points. Firstly, to simplify, we assume that foreign firms do not have to pay imports in advance.<sup>11</sup> Secondly, an increase in the terms of trade increases the consumption of home goods by foreign consumers. Now, since  $\widehat{c}_t^{f*} = \widehat{y}^f$ , we get finally:

$$\widehat{c}_t^{h*} = \widehat{y}_t^f + a\widehat{\delta}_t^f. \quad (2.35)$$

Another implication of assuming identical preferences in both countries, is that the Euler condition for foreign country's households is simply

$$\widehat{y}_t^f = E_t \widehat{y}_{t+1}^f - \frac{1}{\sigma} \left( \widehat{i}_t^f - E_t \pi_{t+1}^{h*} \right).$$

Defining the foreign real interest rate as  $\widehat{\rho}_t^f = \widehat{i}_t^f - E_t \pi_{t+1}^{h*}$ , we get:

$$\widehat{\rho}_t^f = \sigma \left( E_t \widehat{y}_{t+1}^f - \widehat{y}_t^f \right) \quad (2.36)$$

, with the foreign output being determined exogenously.

---

<sup>10</sup>It simplifies the analysis because domestic exports only depend on consumers.

<sup>11</sup>Such assumption should be removed if we were interested in the strategic interaction between policy makers of different countries or in the world interest rate.

## 2.2.5 International Risk Sharing

Above, it was shown the first order condition for the domestic household's consumption (equation (2.8)). Under the assumption of complete securities markets <sup>12</sup>, a similar expression will hold for the foreign representative household: <sup>13</sup>

$$\Omega_{t,t+1} = \beta \left( \frac{C_{t+1}^{f*}}{C_t^{f*}} \right)^{-\sigma} \frac{P_t^{c*}}{P_{t+1}^{c*}} \frac{\ddot{E}_t}{\ddot{E}_{t+1}}. \quad (2.37)$$

Combining equations (2.8) and (2.37), together with the definition of real exchange rate,  $Q_t$ , it follows that:

$$\frac{C_{t+1}}{C_{t+1}^{f*} Q_{t+1}^{\frac{1}{\sigma}}} = \frac{C_t}{C_t^{f*} Q_t^{\frac{1}{\sigma}}} \left( \frac{\xi_{t+1}}{\xi_t} \right)^{\frac{1}{\sigma}}. \quad (2.38)$$

Let us assume the following initial condition:

$$C_1 = v C_1^{f*} Q_1^{\frac{1}{\sigma}} \xi_1^{\frac{1}{\sigma}}$$

where  $v$  is a constant that depends on the initial relative net asset position. Using this initial condition to iterate equation (2.38), it follows:

$$C_t = v C_t^{f*} Q_t^{\frac{1}{\sigma}} \xi_t^{\frac{1}{\sigma}}.$$

In deviations from the steady-state, the latter equation is:

$$\widehat{c}_t = \widehat{c}_t^{f*} + \frac{1}{\sigma} \widehat{q}_t + \frac{1}{\sigma} \widehat{\xi}_t.$$

Using the relation between the real exchange rate and the terms of trade (equation (2.25)) and the equality  $\widehat{c}_t^{f*} = \widehat{y}_t^f$ , we get:

$$\widehat{c}_t = \widehat{y}_t^f + \frac{1 - \gamma_c}{\sigma} \widehat{\delta}_t - \frac{\gamma_c}{\sigma} \widehat{i}_t + \frac{1}{\sigma} \widehat{\xi}_t. \quad (2.39)$$

With efficient risk sharing, domestic consumption should be higher when the price of domestic

---

<sup>12</sup>With complete financial markets, there is an equilibrium price for every asset in every possible state of the world.

<sup>13</sup>See Gali and Monaceli (2002) and Monacelli (2003) for similar proof.

consumption is low relative to foreign consumption, *i.e.*,  $\hat{\delta}'_t$  is high.

## 2.2.6 Uncovered interest parity and aggregate demand

Following Gali and Monacelli (2005), with complete international financial markets, a riskless bond denominated in foreign currency will have an equilibrium price in domestic currency equal to  $\ddot{E}_t \left( I_t^f \right)^{-1} = E_t \left( \Omega_{t,t+1} \ddot{E}_{t+1} \right)$ . Combining this equation with the pricing equation for a bond denominated in domestic currency,  $I_t^{-1} = E_t \left( \Omega_{t,t+1} \right)$ , we obtain:

$$E_t \left[ \Omega_{t,t+1} \left( I_t - I_t^f \frac{\ddot{E}_{t+1}}{\ddot{E}_t} \right) \right] = 0.$$

Linearising around a perfect-foresight steady-state leads to the well known uncovered interest parity:<sup>14</sup>

$$\hat{i}_t = \hat{i}_t^f + (E_t \hat{e}_{t+1} - \hat{e}_t).$$

The real parity becomes<sup>15</sup>

$$\hat{\rho}_t^h = \hat{i}_t - E_t \pi_{t+1}^h = \hat{\rho}_t^f + \left( E_t \hat{\delta}'_{t+1} - \hat{\delta}'_t \right). \quad (2.40)$$

Let us turn now to the demand for domestic output, which in equilibrium has to be equal to the output:

$$\hat{y}_t = (1 - \gamma) \hat{c}_t^h + \gamma \hat{c}_t^{h*}. \quad (2.41)$$

We observe that consumers' demand is divided into the demand of domestic  $(1 - \gamma)$  and foreign consumers  $(\gamma)$ . Now, equation (2.41) is going to be defined using  $\hat{c}_t$ ,  $\hat{y}_t^f$  and  $\hat{\delta}_t$ . Firstly, plug in (2.20) into equation (2.19) to get:

$$\hat{c}_t^h = \hat{c}_t + \gamma_c a \left( \hat{\delta}'_t + \hat{i}_t \right). \quad (2.42)$$

Secondly, substitute the last equation and (2.35) in (2.41) to obtain:

$$\hat{y}_t = (1 - \gamma) \hat{c}_t + [(1 - \gamma) \gamma_c a + \gamma a] \hat{\delta}'_t + (1 - \gamma) \gamma_c a \hat{u}_t + \gamma \hat{y}_t^f. \quad (2.43)$$

<sup>14</sup> $\hat{e}_t$  is  $\ddot{E}_t$  in log-deviations from the steady-state.

<sup>15</sup>Notice that the real parity is defined using domestic inflation (see Annex (6.1.5) for details).

It is possible to observe that the nominal interest rate affects domestic demand. With total consumption and the terms of trade constant, if the interest rate increases, consumption of imported goods decreases in favour of the consumption of domestic goods, meaning that the demand for the domestic product increases.

Solving (2.43) for  $\widehat{c}_t$ , we obtain:

$$\widehat{c}_t = \frac{1}{(1-\gamma)}\widehat{y}_t - \frac{(1-\gamma)\gamma_c a + \gamma a \widehat{\delta}'_t}{1-\gamma} - \gamma_c a \widehat{i}_t - \frac{\gamma}{1-\gamma}\widehat{y}_t^f. \quad (2.44)$$

The next step is to replace the real UIP in the Euler equation for consumption. For that, it is necessary to express the real UIP using CPI inflation. To start with, using equation (2.23) we can write:

$$\widehat{i}_t - E_t \pi_{t+1}^c = \widehat{\rho}_t^h - \gamma_c (E_t \widehat{\delta}'_{t+1} - \widehat{\delta}'_t) - \gamma_c (E_t \widehat{i}_{t+1} - \widehat{i}_t)$$

Making use of the real UIP, the last expression becomes:

$$\widehat{i}_t - E_t \pi_{t+1}^c = \widehat{\rho}_t^f + (1-\gamma_c) (E_t \widehat{\delta}'_{t+1} - \widehat{\delta}'_t) - \gamma_c (E_t \widehat{i}_{t+1} - \widehat{i}_t)$$

Next, to eliminate the terms of trade, we use the real UIP getting:

$$\widehat{i}_t - E_t \pi_{t+1}^c = (1-\gamma_c)\widehat{\rho}_t^h + \gamma_c \widehat{\rho}_t^f - \gamma_c (E_t \widehat{i}_{t+1} - \widehat{i}_t)$$

Finally, we can replace the last equation in the Euler equation, (2.21), getting:

$$\widehat{c}_t = E_t \widehat{c}_{t+1} - \frac{1}{\sigma} \left[ (1-\gamma_c)\widehat{\rho}_t^h + \gamma_c \widehat{\rho}_t^f - \gamma_c (E_t \widehat{i}_{t+1} - \widehat{i}_t) \right] - \frac{1}{\sigma} (E_t \widehat{\xi}_{t+1} - \widehat{\xi}_t). \quad (2.45)$$

The final step is to obtain the IS curve. From equation (2.43) can easily be obtained:

$$\begin{aligned} \widehat{y}_t - E_t \widehat{y}_{t+1} &= (1-\gamma) (\widehat{c}_t - E_t \widehat{c}_{t+1}) + [(1-\gamma)\gamma_c a + \gamma a] (\widehat{\delta}'_t - E_t \widehat{\delta}'_{t+1}) \\ &\quad + (1-\gamma)\gamma_c a (\widehat{i}_t - E_t \widehat{i}_{t+1}) + \gamma (\widehat{y}_t^f - E_t \widehat{y}_{t+1}^f). \end{aligned}$$

Using equations (2.36), (2.40) and (2.45) in the last equation, we obtain the IS curve:

$$\begin{aligned} \widehat{y}_t - E_t \widehat{y}_{t+1} &= -\frac{1+w}{\sigma} \widehat{\rho}_t^h + \frac{w}{\sigma} \widehat{\rho}_t^f - \frac{1-\gamma}{\sigma} \left( E_t \widehat{\xi}_{t+1} - \widehat{\xi}_t \right) \\ &\quad + \frac{(1-\gamma) \gamma_c (\sigma a - 1)}{\sigma} \left( \widehat{i}_t - E_t \widehat{i}_{t+1} \right) \end{aligned} \quad (2.46)$$

, with  $w = (\sigma a - 1) [(1 - \gamma) \gamma_c + \gamma]$ . This is a standard demand condition for a small open economy, except for the (minus) expected change in the nominal interest rate. The reason why the nominal interest rate has an effect on output that is independent of the real interest rate (calculated with the domestic price inflation) is found in its impact on the price of imported *consumption* goods. In fact, the increase in the nominal interest rate in period  $t$  (assuming its expected value for  $t + 1$  constant) has two contradictory effects on the demand for domestic product. On one hand, the price of imported consumption goods increases, leading to a substitution of imported consumption goods by domestic goods. On the other hand, since the difference between the actual and expected interest rate becomes smaller, the expected *change* in imports price between  $t$  and  $t + 1$  is smaller, causing a smaller expected CPI inflation in  $t + 1$ . Consequently, the real interest rate for  $t$  (calculated with the expected *CPI inflation* in  $t + 1$ ) will increase, producing a decrease in consumption in  $t$  relatively to  $t + 1$ . To conclude, it can easily be seen that the effect of the nominal interest rate on the product will be positive if  $\sigma a > 1$ .

### 2.2.7 Marginal cost and the flexible-price output

In this section, we characterise the small open economy's marginal cost and flexible-price output. Expressing equation (2.31) in log deviations around the steady-state, we get:

$$\widehat{mc}_t = \widehat{i}_t + \widehat{s}_t - \widehat{\gamma}_t. \quad (2.47)$$

The labour income share, in equation (2.31), can be expressed as:

$$\widehat{s}_t = (\widehat{w}_t - \widehat{p}_t^h) - \widehat{y}_t + \widehat{n}_t. \quad (2.48)$$

Moreover, the elasticity of output with respect to labour is equal to (see Annex (6.1.4) for

details):

$$\gamma_t = \alpha_N \left( \frac{Z_t N_t}{Y_t} \right)^{1-\frac{1}{\varepsilon}}$$

, which in deviations around the steady-state becomes:

$$\widehat{\gamma}_t = \left( 1 - \frac{1}{\varepsilon} \right) (\widehat{z}_t - \widehat{y}_t + \widehat{n}_t). \quad (2.49)$$

Substituting equations (2.48) and (2.49) into equation (2.47), we obtain

$$\widehat{m}c_t = \widehat{i}_t + (\widehat{w}_t - \widehat{p}_t^h) + \frac{1}{\varepsilon} (\widehat{n}_t - \widehat{y}_t) - \left( 1 - \frac{1}{\varepsilon} \right) \widehat{z}_t.$$

Now, we can replace the real wage by expression (2.22) to get:

$$\widehat{m}c_t = \widehat{i}_t + \left( \eta + \frac{1}{\varepsilon} \right) \widehat{n}_t + \sigma \widehat{c}_t + \gamma_c (\widehat{\delta}_t + \widehat{i}_t) - \frac{1}{\varepsilon} \widehat{y}_t - \left( 1 - \frac{1}{\varepsilon} \right) \widehat{z}_t - \widehat{\xi}_t. \quad (2.50)$$

Meanwhile, it is possible to obtain an expression for the amount of labour used in production,  $\widehat{n}_t$ , as a function of output. Firstly, the production function can be expressed in deviations from the steady-state as (See Annex (6.1.3) for details):

$$\widehat{y}_t = (1 - \gamma'_i) (\widehat{z}_t + \widehat{n}_t) + \gamma'_i \widehat{m}c_t, \quad (2.51)$$

where  $\gamma'_i = \Phi \gamma_i / (1 + \gamma_i)$ . After solving the last equation for  $\widehat{n}_t$ , we still have to derive the demand for imported inputs. From the production function, we can get the conditional demand function for imported inputs:

$$M_t = \frac{1}{\alpha_M^{\frac{\varepsilon}{\varepsilon-1}}} \left( \frac{\delta_t}{\alpha_M^{\frac{\varepsilon}{\varepsilon-1}}} \right)^{-\varepsilon} \left[ \left( \frac{(W_t I_t) / P_t^h}{\alpha_N^{\frac{\varepsilon}{\varepsilon-1}} Z_t} \right)^{1-\varepsilon} + \left( \frac{\delta_t}{\alpha_M^{\frac{\varepsilon}{\varepsilon-1}}} \right)^{1-\varepsilon} \right]^{\frac{\varepsilon}{1-\varepsilon}} Y_t. \quad (2.52)$$

Notice that it has been assumed that imported inputs also have to be paid in advance, implying that its real price is  $\delta_t$ . The last expression can be simplified to:

$$M_t = \frac{1}{\alpha_M^{\frac{\varepsilon}{\varepsilon-1}}} \left( \frac{\delta_t}{\alpha_M^{\frac{\varepsilon}{\varepsilon-1}}} \right)^{-\varepsilon} MC_t^\varepsilon Y_t. \quad (2.53)$$

, because (see Annex (6.1.6) for details):

$$MC_t^\varepsilon = \left[ \left( \frac{(W_t I_t) / P_t^h}{\alpha_N^{\frac{\varepsilon}{\varepsilon-1}} Z_t} \right)^{1-\varepsilon} + \left( \frac{\delta_t}{\alpha_M^{\frac{\varepsilon}{\varepsilon-1}}} \right)^{1-\varepsilon} \right]^{\frac{\varepsilon}{1-\varepsilon}}.$$

This means that in deviations from the steady-state, we have:

$$\widehat{m}_t = -\varepsilon \left( \widehat{\delta}_t + \widehat{i}_t \right) + \varepsilon \widehat{mc}_t + \widehat{y}_t. \quad (2.54)$$

The last equation shows that outside the flexible-price equilibrium, imported inputs depend on four factors: the terms of trade, the nominal interest rate, output and marginal cost. The interpretation of these factors is straightforward. Firstly, an increase in the real price of foreign inputs, either due to an increase in the terms of trade or in the nominal interest rate, will lead firms to substitute foreign inputs for labour. Secondly, an increase in output will naturally require more imported inputs. Finally, if the marginal cost increases (without any change in the terms of trade) it means that the cost of labour has increased relative to the cost of imported inputs. Therefore, firms will use more imported inputs.

Then, plugging (2.54) into (2.51), the demand for labour can be expressed as:

$$\widehat{n}_t = \widehat{y}_t + \frac{\gamma'_i \varepsilon}{1 - \gamma'_i} \left( \widehat{\delta}_t + \widehat{i}_t - \widehat{mc}_t \right) - \widehat{z}_t.$$

Next, plugging the last expression and (2.44) into (2.50) we get:

$$\widehat{mc}_t = \frac{1 - \gamma'_i}{1 + \gamma'_i \eta \varepsilon} \left[ v \widehat{i}_t + \left( \eta + \frac{\sigma}{1 - \gamma} \right) \widehat{y}_t + h \widehat{\delta}_t - \frac{\sigma \gamma}{1 - \gamma} \widehat{y}_t^f - (\eta + 1) \widehat{z}_t - \widehat{\xi}_t \right], \quad (2.55)$$

with:

$$\begin{aligned} v &= 1 - \gamma_c (\sigma a - 1) + \frac{\gamma'_i (1 + \eta \varepsilon)}{1 - \gamma'_i}, \\ h &= \frac{\gamma'_i (1 + \eta \varepsilon)}{1 - \gamma'_i} - \frac{\gamma + w}{1 - \gamma}. \end{aligned}$$

The effects of the variables on the marginal cost will be described below. For now, it is worth mentioning that an increase in domestic output increases the marginal cost. In fact,

an increase in output demands the use of more labour and thus leads to an increase in the real wage.

To obtain the flexible-price equilibrium output we only have to take into account that the marginal cost is constant in the flexible-price equilibrium ( $\widehat{mc}_t^o = 0$ ) implying:

$$\widehat{y}_t^o = \frac{1}{\eta + \frac{\sigma}{1-\gamma}} \left[ -v\widehat{i}_t^o - h\widehat{\delta}_t^o + \frac{\sigma\gamma}{1-\gamma}\widehat{y}_t^f + (\eta + 1)\widehat{z}_t + \widehat{\xi}_t \right] \quad (2.56)$$

where  $\widehat{x}_t^o$  is the value of any variable  $x_t$  in the flexible-price equilibrium (still in deviation from the steady-state).<sup>16</sup> Notice that because prices are sticky, variables can differ from their flexible-price equilibrium values.

From the last formula, we can observe that the flexible-price output depends, on one hand, on the flexible-price values of the nominal interest rate and the terms of trade, and on the other hand, on the world output shock, the technological shock and the taste shock. Next, the impact of these variables on the marginal cost and flexible-price equilibrium output is described. In this model, an increase in the nominal interest rate has the conventional effect of increasing the cost of labour, since wages have to be paid in advance. As a consequence, firms reduce labour demand, which reduces output. However, since imports also have to be paid in advance, there are additional effects of the nominal interest rate arising from its effect on the terms of trade. These supplementary effects are described together with the effect of the terms of trade. Prior to this description, it can be noticed that, from a general point of view, the final effect of the nominal interest rate on the flexible-price equilibrium is ambiguous. However, with our calibration (that we will describe below)  $v$  is positive, implying that an increase in the nominal interest rate increases the marginal cost but decreases the flexible-price output.

Turning now to the impact of the terms of trade, it is better to start by noting that what we will say next is also valid for an increase in  $\widehat{i}_t^o$ . After this observation, it is possible to identify three effects of the terms of trade on the flexible-price output. On one hand, an increase in the terms of trade ( $\widehat{\delta}_t^o$  increases) leads firms to use less imported inputs and labour, reducing output. To begin with, when the cost of imported inputs increases, firms

---

<sup>16</sup>Notice that, with flexible-prices, shocks to the economy also imply that variables deviate from the steady-state.

substitute those inputs for labour (*first effect*). This attempt to reduce production costs is not completely successful, because the increase in labour demand leads to an increase in real wages and thus an increase in the marginal cost and a reduction in the flexible-price equilibrium output. In addition, there is another effect (*second effect*) of the terms of trade on the marginal cost, associated with the increase in CPI inflation caused by the increase on  $\hat{\delta}_t^o$ . This requires firms to increase the real wage (expressed in terms of domestic price index,  $\hat{w}_t - \hat{p}_t^h$ ) to compensate consumers for the increase in CPI inflation. Consequently, there is a decrease in labour demand, reducing the flexible-price output.

On the other hand, we have a *third effect*, which, when considered in isolation, increases the amount of labour used by firms. An increase in the terms of trade increases exports and reduces imports <sup>17</sup>, causing, with a constant output, a reduction in home total consumption. With less consumption, households are willing to work for lower real wages. Finally, this increases the amount of work used in equilibrium.

The three effects described make clear that, from a general point of view, the impact of  $\hat{\delta}_t^o$  on the flexible-price output is ambiguous. Nevertheless, with the calibration used below,  $h$  is negative, meaning that an increase in the terms of trade (depreciation) increases the flexible-price output. Regarding the effects described above, it can then be concluded that the first and second effects are dominated by the third.

Turning now to the technological shock, we observe that it has a positive effect on the flexible-price output, due to its upward impact on labour productivity. Additionally, because labour becomes more productive, the real cost of labour decreases, leading firms to use more labour and produce more.

Similarly, the world output also has a positive effect on the flexible-price output. An increase in foreign output, increases home exports and, for the same level of home product, reduces home consumption. This has a wealth effect on the labour supply, which leads to the reduction of real wage. As a result, the amount of labour used increases, causing an increase in the flexible-price output.

Finally, a taste shock (*i.e.* an increase in the value that consumers assign to consumption

---

<sup>17</sup>Notice that if we are talking only of an increase in the nominal interest rate, with the terms of trade constant, there is only a decrease in imports and no change in exports, because it was assumed that exports are not paid in advance.

relative to leisure) reduces the marginal rate of substitution between leisure and consumption, which increases labour supply and reduces the real wage. In equilibrium, this leads firms to use more labour and to produce more.

We turn now to the novelties brought by this model with respect to the determinants of the flexible-price output. In our model, as in the closed economy model of Ravenna and Walsh (2006), the nominal interest rate affects the flexible-price output. However, in the present model, besides the direct effect through the wage bill also present in the closed economy model, other effects are at work, due to the effect of the nominal interest rate on the price of imported inputs, CPI inflation and imports. Moreover, it is typical of a small open economy's model that the terms of trade affect the flexible-price output (Walsh, 2003). The new feature of this paper's model is that the terms of trade have an additional effect on the flexible-price output due to its effect on the price of imported inputs.

Above, we obtained the risk sharing condition. Now, to obtain the relation between output and the terms of trade, first incorporate equation (2.39) into equation (2.42), getting

$$\widehat{c}_t^h = \widehat{y}_t^f + \frac{1 + \gamma_c(\sigma a - 1)}{\sigma} \widehat{\delta}_t^j + \frac{\gamma_c(\sigma a - 1)}{\sigma} \widehat{i}_t + \frac{1}{\sigma} \widehat{\xi}_t.$$

Second, replacing the last equation and (2.35) into (2.41), it follows after some manipulations that:

$$\widehat{y}_t = \widehat{y}_t^f + \frac{w + 1}{\sigma} \widehat{\delta}_t^j + \frac{(1 - \gamma)\gamma_c(\sigma a - 1)}{\sigma} \widehat{i}_t + \frac{1 - \gamma}{\sigma} \widehat{\xi}_t. \quad (2.57)$$

Ignoring the taste shock, this equation makes clear that a movement in countries' relative output is associated with a change in the terms of trade or nominal interest rate. For instance, if the home country output increases in relation to the foreign output, a real depreciation will exist in equilibrium, so that the demand for home products increase.

Additionally, notice that no distortionary international risk sharing shock is considered. However, the existence of such a shock may justify that the central bank has an exchange rate objective simultaneously with a concern for domestic inflation and output gap (Kirsanova et al., 2006).

In order to express the last equation in the flexible-price equilibrium, notice three points. First, that equation (2.57) is also valid in the flexible-price equilibrium, since it does not

assume sticky prices. Second,  $\widehat{y}_t^f$  is exogenous and so does not depend on the hypothesis regarding the flexibility of prices in the domestic economy. Third,  $\widehat{\xi}_t$  is an exogenous shock that also affects the flexible-price equilibrium. Consequently, we can write:

$$\widehat{y}_t^o = \widehat{y}_t^f + \frac{w+1}{\sigma} \widehat{\delta}_t^o + \frac{(1-\gamma)\gamma_c(\sigma a - 1)}{\sigma} \widehat{i}_t^o + \frac{1-\gamma}{\sigma} \widehat{\xi}_t. \quad (2.58)$$

Now, the last expression can be used to simplify the equation for the flexible-price output, (2.56). Firstly, notice that from (2.58) it is possible to get:

$$\widehat{\delta}_t^o = \frac{\sigma}{w+1} \left( \widehat{y}_t^o - \widehat{y}_t^f \right) - \frac{(1-\gamma)\gamma_c(\sigma a - 1)}{w+1} \widehat{i}_t^o - \frac{1-\gamma}{w+1} \widehat{\xi}_t. \quad (2.59)$$

Plugging the last equation into (2.56), we obtain after some manipulations:

$$\widehat{y}_t^o = \frac{1}{\eta + \frac{\sigma}{1-\gamma} + \frac{h\sigma}{w+1}} \left[ \begin{array}{l} \sigma \left( \frac{h}{w+1} + \frac{\gamma}{1-\gamma} \right) \widehat{y}_t^f + \left[ \frac{h(1-\gamma)\gamma_c(\sigma a - 1)}{w+1} - v \right] \widehat{i}_t^o \\ + \left( \frac{h(1-\gamma)}{w+1} + 1 \right) \widehat{\xi}_t + (\eta + 1) \widehat{z}_t \end{array} \right]. \quad (2.60)$$

### 2.2.8 Deviations from the flexible-price equilibrium

In this section, the traditional formulation of the New Keynesian models will be obtained, with two main equations describing the behaviour of firms and households, the Phillips curve and the IS curve, respectively. To do this, we are going to write the model in deviations from the flexible-price equilibrium.

In the discussion below, we write the deviation of marginal cost from its steady-state value as a function of output gap. To start with, sum and subtract to equation (2.55) the two following expressions:

$$\frac{(1-\gamma'_i)v}{1+\gamma'_i\eta\varepsilon} \widehat{i}_t^o \quad \text{and} \quad \frac{(1-\gamma'_i)h}{1+\gamma'_i\eta\varepsilon} \widehat{\delta}_t^o,$$

getting:

$$\widehat{m}c_t = \frac{(1-\gamma'_i)v}{1+\gamma'_i\eta\varepsilon} \left( \widehat{i}_t - \widehat{i}_t^o \right) + \frac{(1-\gamma'_i)h}{1+\gamma'_i\eta\varepsilon} \left( \widehat{\delta}_t - \widehat{\delta}_t^o \right) + \frac{1-\gamma'_i}{1+\gamma'_i\eta\varepsilon} \left( \eta + \frac{\sigma}{1-\gamma} \right) \left( \widehat{y}_t - \widehat{y}_t^o \right). \quad (2.61)$$

where  $\widehat{y}_t - \widehat{y}_t^o$  is the output gap,  $x_t$ , and  $\widehat{y}_t^o$  comes from (2.60). From (2.57) it is possible to

obtain the expression for the terms of trade:

$$\widehat{\delta}_t^f = \frac{\sigma}{w+1} (\widehat{y}_t - \widehat{y}_t^f) - \frac{(1-\gamma)\gamma_c(\sigma a - 1)\widehat{i}_t}{w+1} - \frac{1-\gamma}{w+1}\widehat{\xi}_t.$$

Subtracting the last equation from (2.59), we get:

$$\widehat{\delta}_t^f - \widehat{\delta}_t^{fo} = \frac{\sigma}{w+1} (\widehat{y}_t - \widehat{y}_t^o) - \frac{(1-\gamma)\gamma_c(\sigma a - 1)}{w+1} (\widehat{i}_t - \widehat{i}_t^o). \quad (2.62)$$

With the last equation, we can eliminate the terms of trade from the expression of the marginal cost, getting:

$$\widehat{mc}_t = \frac{(1-\gamma'_i)}{1+\gamma'_i\eta\varepsilon} \left( \frac{\sigma h}{1+w} + \eta + \frac{\sigma}{1-\gamma} \right) (\widehat{y}_t - \widehat{y}_t^o) + \frac{1-\gamma'_i}{1+\gamma'_i\eta\varepsilon} \left( v - \frac{h(1-\gamma)\gamma_c(\sigma a - 1)}{w+1} \right) (\widehat{i}_t - \widehat{i}_t^o). \quad (2.63)$$

Using the last expression to replace  $\widehat{mc}_t$  in (2.34), and assuming like Ravenna and Walsh (2006) an interest rate peg in the flexible-price equilibrium, *i.e.*,  $\widehat{i}_t^o = 0$ , we get an expression for the Phillips curve:<sup>18</sup>

$$\begin{aligned} \pi_t^h &= \beta E_t \pi_{t+1}^h + \frac{(1-\gamma'_i)}{1+\gamma'_i\eta\varepsilon} \left( \frac{\sigma h}{1+w} + \eta + \frac{\sigma}{1-\gamma} \right) k (\widehat{y}_t - \widehat{y}_t^o) \\ &\quad + \frac{(1-\gamma'_i)}{1+\gamma'_i\eta\varepsilon} \left( v - \frac{h(1-\gamma)\gamma_c(\sigma a - 1)}{w+1} \right) k \widehat{i}_t. \end{aligned} \quad (2.64)$$

This means that our result is similar to the typical Phillips curve for a closed economy, with the only difference in the coefficients of the nominal interest rate and the output gap. In particular, we can observe that, as in Ravenna and Walsh's (2006) closed economy model, the nominal interest rate has a direct effect on domestic inflation due to the presence of the cost channel. Additionally, notice that, when the cost channel is present in an open economy model, the nominal interest rate is present both in the IS curve and in the Phillips curve.

The obtained Phillips curve nests other standard cases. To start with, if the economy is closed ( $\gamma_i = \gamma = 0$ ), then we get the standard Phillips curve with a cost channel, described

---

<sup>18</sup>In the flexible-price equilibrium, prices and the real interest rate change in response to shocks, meaning that the *nominal* interest rate does not need to change.

by Ravenna and Walsh (2006):

$$\pi_t^h = \beta E_t \pi_{t+1}^h + k \hat{v}_t + k(\eta + \sigma)(\hat{y}_t - \hat{y}_t^o).$$

Moreover, if we ignore both imported inputs ( $\gamma_i = 0$ ) and the cost channel, we get:

$$\pi_t^h = \beta E_t \pi_{t+1}^h + k \left( \eta + \sigma - \frac{\sigma w}{1+w} \right) (\hat{y}_t - \hat{y}_t^o)$$

, the standard Phillips curve for a small open economy described by Walsh (2003).

In what follows we make a detailed explanation of the impact of the nominal interest rate on domestic inflation. The first element of the coefficient of the interest rate,  $(1 - \gamma'_i) / (1 + \gamma'_i \eta \varepsilon)$ , is smaller than one,<sup>19</sup> meaning that the existence of imported inputs reduces the direct impact of the nominal interest rate on domestic inflation through its effect on the *cost of labour*.<sup>20</sup> This happens for two reasons. On one hand, since with imported inputs, labour costs represent only a fraction of the marginal cost, any change in the cost of labour will have a smaller effect on the marginal cost. On the other hand, in response to an increase in the nominal interest rate, there is a substitution of labour by imported inputs, which mitigates the increase in the marginal cost.<sup>21</sup>

In turn, the parameter  $v$  describes all the *direct* effects of the nominal interest rate on inflation that arise from the increase in the wage bill, increase in the price of imported inputs, decrease in imports of consumption goods and consequently consumption, and increase in CPI inflation (see Section 2.2.7 for a detailed description of all these effects).

Finally, an increase in the nominal interest rate leads to an appreciation in the terms of trade (see (2.62)). This produces the three effects on the marginal cost described above (see equation (2.55) and Section 2.2.7) and that are summarised by  $h$ .

In conclusion, it is possible to show that, with  $\sigma a > 1$ , the supply side impact of the nominal interest rate on domestic inflation is positive (for proof see Annex (6.1.7)). With the

---

<sup>19</sup>Notice that  $\frac{(1-\gamma'_i)}{1+\gamma'_i \eta \varepsilon} = \frac{1}{1 + \frac{\gamma'_i(1+\eta \varepsilon)}{1-\gamma'_i}}$ .

<sup>20</sup>Notice that we cannot say that imported inputs reduce the impact of interest rate on domestic inflation, because  $\gamma_i$  also affects  $v$  and  $h$ .

<sup>21</sup>Notice that the impact of the nominal interest rate on the cost of labour is not limited to its direct impact, since there are also effects through the terms of trade, as we have seen in Section 2.2.7.

standard parametrisation used in this paper, and described below, the coefficient is 0.0791.

Notice that in the closed economy case described by Ravenna and Walsh (2006) the only impact of the nominal interest rate on inflation is the increase in the wage bill, which corresponds to 1 in the parameter  $v$ .

In the same way, let us describe the impact of output gap on domestic inflation. Again, the fraction  $(1 - \gamma'_i)/(1 + \gamma'_i\eta\varepsilon)$  is present because admitting imported inputs reduces the impact that a change in real wage has on the marginal cost, either because wages are only a fraction of the marginal cost or because firms can substitute labour by imported inputs.

In addition, the effects that occur in a closed economy are also present. Firstly, an increase in the output gap requires more labour; which workers are willing to supply only if salaries increase, implying an increase in the marginal cost, translated by parameter  $\eta$ . Secondly, an increase in the output gap increases home consumption by  $1/(1 - \gamma)$ , which leads households to demand higher salaries, increasing the marginal cost. This explains the presence of the fraction  $\sigma/(1 - \gamma)$ .

Finally, in an open economy, an increase in the output gap increases the terms of trade by  $\sigma/(1 + w)$  (see equation (2.62)). As a result, all the already described effects of the terms of trade on the marginal cost take place, and they are summarised by  $h$ .

It can be shown that an increase in the output gap leads to an increase in domestic inflation - for proof see Annex (6.1.7) - and with this paper's standard parameterisation the coefficient of output gap assumes the value 0.1736.

Next, we are going to express the demand condition (2.46) in deviations from the flexible-price equilibrium. Firstly, add and subtract  $E_t\hat{y}_{t+1}^o$  and:

$$\frac{(1 - \gamma)\gamma_c(\sigma a - 1)}{\sigma} \left( \hat{i}_t - E_t\hat{i}_{t+1}^o \right)$$

to the right hand side of equation (2.46) and subtract  $\hat{y}_t^o$  to each side, obtaining

$$\begin{aligned} x_t = & E_t x_{t+1} - \frac{1 + w}{\sigma} \left( \hat{\rho}_t^h - \hat{\rho}_t^{ho} \right) + \\ & \frac{(1 - \gamma)\gamma_c(\sigma a - 1)}{\sigma} \left[ \left( \hat{i}_t - \hat{i}_t^o \right) - E_t \left( \hat{i}_{t+1} - \hat{i}_{t+1}^o \right) \right] \end{aligned} \quad (2.65)$$

where  $x_t = \hat{y}_t - \hat{y}_t^o$  and  $\hat{\rho}_t^{ho}$  is the home real interest rate in the flexible-price equilibrium

obtained from equation (2.46):

$$\begin{aligned} \widehat{\rho}_t^{ho} = & \frac{w}{1+w} \widehat{\rho}_t^f + \frac{\sigma}{1+w} (E_t \widehat{y}_{t+1}^o - \widehat{y}_t^o) + \\ & \frac{(1-\gamma)\gamma_c(\sigma a - 1)}{1+w} (\widehat{i}_t^o - E_t \widehat{i}_{t+1}^o) - \frac{1-\gamma}{1+w} (E_t \widehat{\xi}_{t+1} - \widehat{\xi}_t). \end{aligned} \quad (2.66)$$

In conclusion, we obtain a conventional IS curve for a small open economy (see for instance Walsh, 2003), with the only novelty being the impact of the nominal interest rate. Let us start by highlighting the conventional features of the IS curve. To begin with, output gap is inversely related with current and anticipated (if we solve forward) domestic real interest rate. And with  $w > 0$  or equivalently  $\sigma a > 1$ , the real interest rate has a larger impact on output gap in an open economy than in a closed economy. This is so because the increase in the domestic real interest rate produces an appreciation of the terms of trade, causing a reduction of home exports, which adds to the decrease in domestic consumption. Furthermore, as in Gali and Monacelli (2005), if  $a\sigma > 1$ , the impact of the real interest rate on output gap increases with the degree of openness, *i.e.*, with the exports fraction on output,  $\gamma$ .<sup>22</sup> This occurs due to the fact that the larger is exports share on output, the larger the reduction on demand caused by a terms of trade's appreciation, induced by an increase in the real interest rate.

Openness also causes the flexible-price domestic real interest rate to depend on the foreign real interest rate; this is the only way through which the latter rate affects home demand.

After describing the more usual characteristics of the IS curve, now we will refer to the novelties brought by the assumption that imports are paid in advance. On one hand, the expected change in the flexible nominal interest rate,  $\widehat{i}_t^o - E_t \widehat{i}_{t+1}^o$ , affects the domestic flexible real interest rate. On the other hand, the expected change in the deviation of the nominal interest rate from its flexible-price equilibrium level,  $(\widehat{i}_t - \widehat{i}_t^o) - E_t (\widehat{i}_{t+1} - \widehat{i}_{t+1}^o)$ , affects output gap (see the intuition for this result above).

Finally, to obtain the value added, substitute (2.54) into (2.28), obtaining

$$\widehat{y}_t^{va} = x_t + \widehat{y}_t^o - \gamma_i (1 - \varepsilon) (\widehat{\delta}_t^j + \widehat{i}_t) - \gamma_i \varepsilon \widehat{m}c_t \quad (2.67)$$

---

<sup>22</sup>  $\partial \left( \frac{1+w}{\sigma} \right) / \partial \gamma = (\sigma a - 1) [(1 - \gamma_c) + (1 - \gamma)(1 + \gamma_i)] / \sigma$

where  $\widehat{mc}_t$  is given by equation (3.17). The output gap in terms of value added is:

$$x_t^{va} = \widehat{y}_t^{va} - \widehat{y}_t^{vao} \quad (2.68)$$

, with:

$$\widehat{y}_t^{vao} = \widehat{y}_t^o - \gamma_i(1 - \varepsilon) \left( \widehat{\delta}_t^o + \widehat{i}_t^o \right). \quad (2.69)$$

In synthesis, the model is characterised by equations (2.23), (2.25), (2.36) (2.59), (2.60), (2.63), (2.64), (2.65), (2.67), (2.68), (2.69) and a Taylor rule or a path for the interest rate defined by the central bank.<sup>23</sup> To close the model, it is necessary to add a final equation for the terms of trade, obtained from (2.57) noticing that  $\widehat{y}_t = x_t + \widehat{y}_t^o$ :

$$\widehat{\delta}_t^f = \frac{\sigma}{w+1} \left( x_t + \widehat{y}_t^o - \widehat{y}_t^f \right) - \frac{(1-\gamma)\gamma_c(\sigma a - 1)\widehat{i}_t}{w+1} - \frac{1-\gamma}{w+1}\widehat{\xi}_t. \quad (2.70)$$

Now that the model is complete, it is the right moment to synthesise the implications of introducing the cost channel in an open economy model. Firstly, the Phillips curve depends on the nominal interest rate, as in the closed economy model of Ravenna and Walsh (2006). Secondly, assuming that imports are paid in advance introduces three more novelties: (1) the IS curve depends on the expected change of the nominal interest rate; (2) CPI inflation depends directly on the change of the nominal interest rate; and (3) the relation between the output gap and the terms of trade is also affected by the nominal interest rate.

A typical result obtained for new Keynesian models is that both open and closed economy models can be written in a canonical representation, *i.e.*, the IS curve and the Phillips curve can be represented in the space output gap and *domestic* inflation. The only differences between the open economy and closed economy curves are on the slopes and on the role of foreign shocks (see Clarida et al., 2001; Gali and Monacelli, 2005). However, the same does not happen in our model, due to the cost channel. Even though the equation for *domestic* inflation only differs from its closed economy version on the variables' coefficients, the IS curve also differs on the variables that affect it. While in the open economy version of the model the expected change in the nominal interest rate affects the IS curve, that variable is not present in the closed economy version.

---

<sup>23</sup>In Section 2.3 we will characterize the optimal behaviour of the CB under commitment and discretion.

## 2.3 Policy trade-offs and optimal policy

In this section we explain how a trade-off between domestic inflation and the output gap arises in the presence of the cost channel, and how this is managed by a central bank following an optimal discretion policy.

### 2.3.1 Policy trade-offs

In this paper's model and in contrast to a standard new Keynesian model, even in the absence of a cost shock, the central bank faces a trade-off between *domestic* inflation and the output gap. As in the closed economy model of Ravenna and Walsh (2006), such a trade-off arises due to the cost channel. When faced with a demand shock, the central bank is not able to maintain the output gap constant without changing domestic inflation. For instance, assume that the flexible-price domestic real interest rate increases due to a demand shock. To avoid a change in output gap, the central bank increases the nominal interest rate. Due to the cost channel, that has a direct effect on domestic inflation, producing a trade-off between inflation and output gap.

Furthermore, there is the more common trade-off between *CPI* inflation and the output gap. Once more, assume a positive demand shock. In order to eliminate the shock, the central bank is forced to increase the nominal interest rate. In spite of that, an appreciation of the domestic currency ( $\delta'_t$  decreases) occurs, and consequently a decrease in CPI inflation. In this regard, the only novelty of this paper's model is that the nominal interest rate also has a direct effect on CPI inflation.

In contrast, in the foreign country there is no trade-off between domestic inflation and the output gap, because the cost channel is absent. Since it is assumed that the foreign central bank's loss function depends on the variances of both domestic inflation and the output gap, its optimal policy is to put domestic inflation and the output gap equal to zero in all periods. This means that the foreign output will always be at its flexible-price level,  $\hat{y}_t^f = \hat{y}_t^{fo}$ .

### 2.3.2 Optimal monetary policies with discretion

When the central bank follows an optimal discretion policy, it is possible to obtain an expression that quantifies the trade-off between domestic inflation and the output gap. In this

context, it is assumed that the central bank minimises a quadratic loss function that depends on the variability of domestic inflation and output gap around some target values. The target values are assumed to be zero for both variables.<sup>24</sup> Furthermore, domestic inflation and not CPI inflation was included in the loss function, because domestic prices are sticky while import prices are flexible. Indeed, Clarida et al. (2001) show that with sticky domestic prices and complete exchange rate pass-through (and flexible wages), the central bank should target domestic inflation. Therefore, the quadratic loss function is:<sup>25</sup>

$$L_t = \frac{1}{2} E_t \sum_{i=0}^{\infty} \beta^i \left( \pi_{t+i}^2 + \lambda x_{t+i}^2 \right) \quad (2.71)$$

where  $\lambda > 0$  is the relative weight attached to output gap variability in the loss function.

The presence of households and firms with forward-looking expectations allows two distinct policies, one with optimal discretion and other with precommitment. While with a precommitment policy the central bank is able to affect the private sector's expectations, with optimal discretion the central bank is not able to have such an effect, because any decision it takes in  $t = 0$  can be changed in the future (is not binding).

If commitment is feasible, the central bank precommits at time  $t = 0$  to a state-contingent plan  $\left\{ \pi_t^c, \pi_t^h, x_t, x_t^{va}, \hat{i}_t, \hat{\delta}_t, \hat{\delta}_t^o, \hat{y}_t, \hat{y}_t^o, \hat{y}_t^{va}, \hat{y}_t^{vao}, \hat{m}c_t, \hat{q}_t \right\}_{t=0}^{\infty}$  to minimise the loss function (2.71) subject to (2.23), (2.25), (2.36) (2.59), (2.60), (2.63), (2.64), (2.65), (2.67), (2.68), (2.69) and (2.70) holding in all periods  $t + j, j \geq 0$ . The solution to this problem, that will be undertaken below, requires a numerical simulation of the model.

In contrast, with optimal discretion the central bank's goal is to minimise the loss function in  $t$ :

$$\frac{1}{2} \left[ \pi_t^2 + \lambda x_t^2 \right]$$

, subject to the same constraints as for the optimal commitment problem, but with expectations treated as constants. From the FOCs, the following policy equation can be obtained

---

<sup>24</sup>The fact that the target for output gap is zero means that we are assuming that, in the steady state, the difference between the efficient level of output (in the absence of monopolistic distortions) and the actual level of output is zero (Walsh, 2003). Otherwise, the central bank creates inflation bias on average. This is the reason why it is usual to assume a zero target for output gap, ensured by tax subsidies.

<sup>25</sup>Assuming that, in the steady state, the difference between the efficient level of output (in the absence of monopolistic distortions) and the actual level of output is zero (Walsh, 2003).

(all proofs are in Section (6.1.8)):

$$\pi_t^h = -\frac{\lambda}{r - \frac{\sigma u}{1+\gamma(\sigma a-1)}} x_t, \quad (2.72)$$

with

$$\begin{aligned} r &= \frac{1 - \gamma'_i}{1 + \gamma'_i \eta \varepsilon} \left( \frac{\sigma h}{1 + w} + \eta + \frac{\sigma}{1 - \gamma} \right) k, \\ u &= \frac{1 - \gamma'_i}{1 + \gamma'_i \eta \varepsilon} \left( v - \frac{h(1 - \gamma)\gamma_c(\sigma a - 1)}{w + 1} \right) k \end{aligned}$$

, and  $u > 0$  with  $\sigma a > 1$ . We can observe that the central bank faces a trade-off between the output gap and domestic inflation: when inflation increases above the target, the central bank acts to decrease the output gap. Besides, it is possible to conclude that without the cost channel ( $u = 0$ ), domestic inflation decreases *less* in response to an unitary increase in the output gap. In other words, the central bank allows more inflation variability when the cost channel is present, because it is more costly to control inflation. Indeed, in the presence of the cost channel, an increase in the nominal interest rate increases inflation directly through the Phillips curve. As a result, it is necessary to increase the nominal interest rate more, and therefore further reduce the output gap, to obtain the same reduction in domestic inflation. This result is qualitatively similar to the one obtained for a closed economy by Ravenna and Walsh (2006).

## 2.4 Domestic technological shock under different policy regimes

In this section the dynamic response of the economy to a *domestic* technological shock, assuming that the central bank follows optimal policies or simple policy rules, will be studied.<sup>26</sup> Firstly, an analysis of the dynamics without a cost channel will be carried out. Then, the cost channel will be introduced to study its implications in variables' behaviour.

---

<sup>26</sup>We focus on a technological shock because is the shock most studied in the literature.

### 2.4.1 Calibration

To study the economy's response to shocks, the model is calibrated and solved numerically.

<sup>27</sup> We use values for the parameters that are standard in the literature for a small open economy. Following Walsh (2006), we assume  $\sigma = 1.5$  and  $\eta = 1$ . The parameter  $\omega$  is set at 0.75, which is consistent with an average period of one year between price adjustments. Also, the parameter  $\beta$  is set equal to 0.99. The elasticity of substitution between differentiated goods,  $\theta$ , is assumed equal to 11, which generates, from equation (2.33), a steady-state mark-up of  $\Phi = 1.1$ . We set the elasticity of substitution between labour and imported inputs,  $\varepsilon$ , equal to 1/3, following McCallum and Nelson (1999). In turn, the elasticity of substitution between domestic and foreign consumption goods,  $a$ , is equal to 1, as in Gali and Monacelli (2005). We assume an exports share in the product,  $\gamma$ , equal to 0.4, as suggested by Gali and Monacelli (2005); and an imported inputs share in the product,  $\gamma_i$ , equal to 0.1, as suggested by Adolfson (2007). Finally, the relative weight attached to output variability in the loss function,  $\lambda$ , is set equal to 0.18. <sup>28</sup> By assumption, the introduction of the cost channel does not change the structural parameters.

Regarding the shocks hitting the economy, for technological and world output shocks we used the findings of Gali (2008). Using HP-filtered data, <sup>29</sup> he obtains the following statistical processes using labour productivity for Canada and the US output (used as a proxy of world output):

$$\begin{aligned} z_t &= 0.66z_{t-1} + \varepsilon_t^z, \sigma_{\varepsilon^z} = 0.71\%, \\ y_t^f &= 0.86y_{t-1}^f + \varepsilon_t^{y^f}, \sigma_{\varepsilon^{y^f}} = 0.78\%, \\ \text{Corr}(\varepsilon_t^z, \varepsilon_t^{y^f}) &= 0.3. \end{aligned}$$

McCallum and Nelson (1999) suggest for the IS shock a white noise process with a standard deviation of 1.11%.

In the next section we are going to focus only on the technological shock, with the other

---

<sup>27</sup>The model was solved numerically by using Matlab's routine Dynare.

<sup>28</sup>This value is slightly below the standard value of 0.25 used by other works in the literature (for example, Walsh, 2003; and Ravenna and Walsh, 2006). We have assumed the value of 0.18 to guarantee a unique equilibrium in the optimal discretion policy.

<sup>29</sup>The HP filter is used to obtain a proxy of variables in the flexible-price equilibrium.

shocks being introduced in Section 2.5.

## 2.4.2 Without the cost channel

In this section we analyse the impact of an unitary domestic productivity shock ignoring the cost channel.<sup>30</sup> The absence of the cost channel means that the nominal interest rate is not present in equations (2.23), (2.25), (2.63), (2.64), (2.67), and (2.70); and the expected change in the nominal interest rate is not present in (2.65). None of the other parameters suffers any change.<sup>31</sup> The economy's response to the shock will be analysed under five different policy regimes: optimal discretion, optimal commitment, domestic inflation-based Taylor rule, CPI inflation-based Taylor rule and an exchange rate peg (for impulse response functions see Exhibits 1, 3, 5, 7, 9 respectively, in Annex 2.7).

### Optimal discretion

To start with, the productivity shock has a contractionary impact, because it increases the flexible-price output and thus decreases the flexible-price real interest rate.<sup>32</sup> This means that there is pressure for an increase in the distance between the actual and the flexible-price real interest rate, making monetary conditions tighter. However, as mentioned earlier, without the cost channel, the central bank is able to fully stabilise domestic inflation and the output gap. This is done by decreasing the nominal interest rate in order to offset the decrease in the equilibrium real interest rate.<sup>33</sup>

The output gap based on the value added is also maintained at zero. With the full stabilization of the output gap (not expressed using the value added), the terms of trade, marginal cost and, consequently, the amount of imported inputs stay at their flexible-price levels, implying that the value added equals its equilibrium level.

---

<sup>30</sup>We study the impact of a one standard-deviation domestic technological shock with the variables expressed in log-deviations from the steady-state. For example, in the first period,  $t = 1$ , under optimal discretion a 0.71% technological shock produces approximately a 0.2% increase in CPI inflation (Exhibit 1).

<sup>31</sup>Take as an example the Phillips Curve (eq. (2.64)). Since without the cost channel the interest rate is not present in the marginal cost (eq. (2.61)) and in the international risk sharing condition (eq. (2.62)), the interest rate is also not present in the Phillips Curve. But the output gap's coefficient remains unchanged.

<sup>32</sup>Given the increase in the equilibrium output, the equality between production and demand also requires an increase in the equilibrium consumption, which is achieved through a reduction in the equilibrium real interest rate.

<sup>33</sup>For simplicity, sometimes we use the designation 'equilibrium level' to mean 'flexible-price equilibrium level'.

Given the constancy of the foreign nominal interest rate, the decrease in the domestic nominal interest rate implies, by the nominal UIP, an expectation of appreciation of the nominal exchange rate, which is attained by an initial depreciation of that rate, followed by a gradual appreciation. A similar explanation exists for the initial depreciation of the terms of trade in response to a decrease in the domestic real interest rate. However, in the long-run the shock has a null impact on the terms of trade. This behaviour of the terms of trade explains why CPI inflation initially increases and thereafter becomes negative, converging to zero in the long-run. This guarantees the stationarity of the CPI.

Finally, with no change in foreign prices and with both the real exchange rate (that depends directly on the terms of trade) and the CPI stationary, the nominal exchange rate is also stationary.

### **Optimal commitment**

The economy's response with an optimal commitment policy is equal to the one with an optimal discretion policy. This occurs as a result of the fact that expectations of inflation and the output gap are zero in both cases; since with both policies the central bank is able to fully stabilise that two variables. Consequently, the advantage of optimal commitment arising from its capacity of affecting agents' expectations vanishes.

### **Domestic inflation-based Taylor Rule**

It is known that simple instrument rules can describe well the behaviour of central banks (Taylor, 1983). Here, we assume a simple rule of the form:

$$\hat{i}_t = \hat{\rho}_t^{ho} + \alpha_\pi \hat{\pi}_t^h.$$

The nominal interest rate responds to changes in the equilibrium real interest rate that varies over time in response to real shocks (see equation (2.66)). The reason for that can be found in Woodford (2001), who argues that the optimal equilibrium is only consistent with the Taylor rule if and only if the nominal interest rate reflects one-for-one changes in the equilibrium real interest rate. Otherwise, there exist undesirable fluctuations in inflation and output gap.

To start,  $\alpha_\pi$  is set equal to the original Taylor estimate of 1.5. As the technological

shock affects output only through the real equilibrium interest rate (see equations (2.65) and (2.66)), the central bank can fully stabilise domestic inflation and the output gap, changing the nominal interest rate to reflect changes in the real equilibrium interest rate. This means that the Taylor rule can replicate the optimal commitment policy.<sup>34</sup>

### CPI inflation-based Taylor rule

Here, it is assumed that the central bank follows a CPI inflation-based Taylor rule (CITR):

$$\hat{i}_t = \hat{\rho}_t^{ho} + \alpha_\pi \hat{\pi}_t^c,$$

with  $\alpha_\pi = 1.5$ . Firstly, the technological shock decreases the real equilibrium interest rate, which after the initial impact returns slowly to zero. According to the Taylor rule, this creates a *pressure* to decrease the nominal interest rate. If such a decrease occurs, it will have the undesirable effect of depreciating the terms of trade, thus increasing CPI inflation. The technological shock also increases the flexible-price output, producing a pressure for a depreciation of the terms of trade (see equation (2.70)). Thus, to avoid a large depreciation of the terms of trade, it is necessary to increase the nominal interest rate and reduce the output gap. As a result there is an *initial* decrease in the output gap. Despite this reduction in the output gap, the increase in the flexible-price output produces an initial depreciation on the terms of trade, which causes an increase in CPI inflation. Due to the long-run neutrality of monetary policy, the terms of trade will return to their initial value, through a steady appreciation. Meanwhile, to avoid a strong decrease in CPI inflation in period 2 there is a slight depreciation of the terms of trade in that period.

The steady appreciation of the terms of trade will decrease CPI inflation, which together with a negative equilibrium real interest rate implies, by the Taylor rule, that after period 1 the nominal interest rate becomes negative. After that period, the nominal and real interest rates increase until 0, which is consistent with the steady appreciation of the terms of trade.

The reduction in the nominal interest rate in  $t = 1$  reduces the positive distance between the actual and the equilibrium real interest rate, which stimulates the output gap; that after period 1 becomes positive. And that is exactly what the central bank aims for, because as the

---

<sup>34</sup>Because domestic inflation does not change, the value of  $\alpha_\pi$  is irrelevant to obtain the optimal outcome.

technological shock's effect on the flexible-price output dissipates, it is necessary to increase the output gap to avoid a strong appreciation of the terms of trade.

Furthermore, it is both the period 1 increase in the nominal interest rate and the increase in the output gap in  $t = 2$  that explain the positive response of domestic inflation since  $t = 1$ , which in turn compensates, in some way, for the appreciation of the terms of trade after period 2, guarantying a more stable behaviour of CPI inflation.

Finally, the nominal exchange rate depreciates both on the short- and long-run. On one hand, the short-run response can be explained by the increase in both the real exchange rate (lead by the increase in the terms of trade) and the CPI. On the other hand, the long-run depreciation is explained by the long-run increase in the CPI.

One main conclusion from this analysis is that the CITR does not fully stabilise CPI inflation and the output gap. The reason for this can be found in the fact that any attempt to stabilise the output gap by changing the nominal interest rate has an effect on the terms of trade and thus on CPI inflation.

### **Comparing CPI inflation-based and domestic inflation-based Taylor rules**

Let us compare the behaviour of some key variables under CITR and DITR. Firstly, compared with a DITR, a CITR trades-off more volatility of domestic inflation and output gap for less volatility of CPI inflation.

Regarding CPI inflation, the main difference between the two regimes is that under a DITR, after an initial positive response, inflation falls to negative values, while with a CITR after an initial increase, it decays slowly to zero. This means that under a CITR there is a better stabilization of CPI inflation, which is obtained by a muted response of the terms of trade and an increase in domestic inflation. That behaviour of the terms of trade initially requires a more contractionary policy under a CITR than under a DITR, with the actual real interest rate being larger than the equilibrium one in the former case.<sup>35</sup> In fact, under a CITR the small depreciation that occurs in the terms of trade in the second quarter is obtained with a slightly positive real interest rate in the first quarter. But as the technological shock affects the economy, the central bank following a CITR adopts a more expansionary policy decreasing

---

<sup>35</sup>Record that with a DITR the actual and equilibrium real interest rates are always equal.

the real interest rate below its equilibrium level. This is the reason why under a CITR we observe, on one hand, an overshooting of the nominal interest rate, increasing initially and then decreasing over time; and on the other hand, an increase in domestic inflation.

The described response of CPI inflation, ensures the stationarity of the CPI under a DITR, while leads to a long-run increase in the CPI with a CITR.

Due to the more muted short-run response of the terms of trade, the initial depreciation of the nominal exchange rate is smaller under a CITR than under a DITR, and also exhibits a hump-shaped response. In the long-run, the terms of trade and the real exchange rate return to 0 under both regimes, implying that the long-run value of the nominal exchange rate depends on the CPI. With a DITR, since the CPI is stationary, the same happens to the nominal exchange rate. However, with a CITR the CPI increases in the long-run, driving a long-run depreciation of the nominal exchange rate, with an overshooting behaviour typical of sticky prices models.

### Exchange rate peg

With a permanent and credible exchange rate peg, the nominal exchange rate is constant:  $\widehat{e}_t = 0$ . From equation (2.16), with the nominal exchange rate and foreign prices constant, we get: <sup>36</sup>

$$\widehat{\delta}'_t = -\widehat{p}_t^h. \quad (2.73)$$

Using the last equation on (2.23), we obtain:

$$\pi_t^c = (1 - \gamma_c)\pi_t^h + \gamma_c(\widehat{i}_t - \widehat{i}_{t-1}). \quad (2.74)$$

Combining equations (2.70) and (2.73) we get an expression for the output gap:

$$x_t = -\frac{w+1}{\sigma}\widehat{p}_t^h + \frac{(1-\gamma)\gamma_c(\sigma a - 1)}{\sigma}\widehat{i}_t + \frac{1-\gamma}{\sigma}\widehat{\xi}_t - (\widehat{y}_t^o - \widehat{y}_t^f). \quad (2.75)$$

---

<sup>36</sup> Foreign prices do not change because the foreign central bank maintains inflation rate always equal to zero (see Section 2.3)

Using the last equation to replace for  $x_t$  on the Phillips curve, equation (2.64), we obtain an expression for the domestic price index:

$$\widehat{p}_t^h = \frac{1}{1 + \beta + \frac{r(w+1)}{\sigma}} \left( \beta E_t \widehat{p}_{t+1}^h + \widehat{p}_{t-1}^h + \left[ \frac{r(1-\gamma)\gamma_c(\sigma a-1)}{\sigma} + u \right] \widehat{i}_t - u \widehat{i}_t^o + \frac{r(1-\gamma)}{\sigma} \widehat{\xi}_t - r (\widehat{y}_t^o - \widehat{y}_t^f) \right). \quad (2.76)$$

Finally, assuming perfect capital mobility, the real UIP simplifies to:

$$\widehat{i}_t = \widehat{\rho}_t^f. \quad (2.77)$$

This equation resembles a policy rule where the central bank only changes the nominal interest rate to insulate the nominal exchange rate from changes in the foreign real interest rate.

In the other policy regimes an interest rate peg in the flexible-price equilibrium was admitted:  $\widehat{i}_t^o = 0$ . However, in the exchange rate peg, the equilibrium nominal interest rate has to accommodate foreign shocks, in order to guarantee a constant nominal exchange rate. By the UIP and taking into account that the foreign inflation rate is always zero, we have that:

$$\widehat{i}_t^o = \widehat{\rho}_t^f. \quad (2.78)$$

Notice that the nominal interest rate with sticky prices has exactly the same behaviour,  $\widehat{i}_t = \widehat{\rho}_t^f$ , implying that the domestic price index will not be affected *directly* by the flexible-price equilibrium nominal interest rate.

Finally, the model with an exchange rate peg can be solved using equations (2.25), (2.36), (2.59), (2.60), (2.63), (2.67), (2.68), (2.69), (2.73), (2.74), (2.75), (2.76), (2.77), and (2.78). If the cost channel is ignored, the model is composed of the same equations, but where the nominal interest rate disappears from equations (2.25), (2.63), (2.67), (2.74), (2.75), and (2.76). Furthermore,  $\widehat{i}_t^o$  is eliminated from (2.59), (2.60), (2.69) and (2.76).

At this point, we are under the correct conditions to study the impact of an unitary domestic technological shock under a PEG when the cost channel is not present. Firstly, the technological shock increases the flexible-price output, decreasing the equilibrium real interest rate. To replicate the flexible-price allocation it would be necessary to have an expansion of consumption and output, that would require a decrease in the nominal interest rate and a

depreciation of the nominal exchange rate (Gali, 2008). However, that will not occur because the central bank wants to maintain the PEG. The shock will then produce a decrease in the output gap and thus in domestic inflation. Given the constancy of the nominal interest rate, CPI inflation mirrors domestic inflation (see equation (2.74)). Moreover, the initial depreciation of the terms of trade is much more limited than in a DITR and a CITR, because it is driven only by the decline in the domestic price *index* (see equation 2.73). This smaller response of the terms of trade explains why the CPI inflation is less volatile under the PEG than under any other policy regime.

In the first quarter, given the much smaller depreciation of the terms of trade and the constancy of the nominal interest rate,<sup>37</sup> the decrease in the output gap and thus in domestic inflation is more pronounced than with a DITR or a CITR.<sup>38</sup> After that, the output gap slowly increases reaching slightly positive values, because the effect of the technological shock on the equilibrium real interest rate vanishes over time. This creates a period of inflation that in turn causes a decrease in the real interest rate, reinforcing the increase in the output gap.

Given the stationarity of the terms of trade and real exchange rate, the complete stabilisation of the nominal exchange rate requires the stationarity of the CPI and therefore of the domestic price index.<sup>39</sup> This explains why there is a period of deflation followed by a period of inflation (using both measures of inflation). Notice that the same happens in the two optimal policies, as noted by Gali (2008).

### **General similarities between policy regimes**

Even though some of the analysed policy regimes imply very different dynamics, it is possible to highlight some general conclusions on the productivity shock's impact. The two optimal policies and the DITR have identical outcomes, characterised by a full stabilisation of domestic inflation and the output gap. At the other extreme it is the exchange rate peg that produces the biggest domestic inflation and output gap's volatilities, but at the same time it ensures the lowest volatility of CPI inflation. In addition, the PEG shares some characteristics with several regimes. On one hand, the response of output gap under the PEG and the CITR is

---

<sup>37</sup>Notice also that the decrease in the domestic inflation implies an increase in the real interest rate.

<sup>38</sup>But the response of output gap under the Peg is *qualitatively* similar to the one under a CITR.

<sup>39</sup>Recall equation (6.10).

qualitatively similar, with the monetary policy being initially contractionist.<sup>40</sup> On the other hand, the PEG and optimal policies lead to the stationarity of the nominal exchange rate, the domestic price level and the CPI.

Finally, in all regimes considered, with the natural exception of the PEG, it can be observed that the terms of trade and the nominal exchange rate depreciate on impact and appreciate after.

### 2.4.3 With a cost channel

In this section, the impact that the introduction of the cost channel has on the economy's response to a technological shock under different policy regimes will be described (for the impulse response functions see Exhibits 2, 4, 6, 8, 9,10 and 11 in Annex 2.7).

#### Optimal discretion

We are going to start by analysing the optimal discretion policy. In the presence of the cost channel, the central bank is no longer able to maintain domestic inflation and the output gap constants, because when it decreases the nominal interest rate, to contradict the technological shock's effect, there is a direct negative impact on domestic inflation. This contributes to an increase in the real interest rate (the opposite of what the central bank wants), forcing the monetary authority to decrease more the nominal interest rate in comparison with the situation where there is no working capital channel, *i.e.*, no cost channel. This larger decrease in the nominal interest rate produces, overall, a larger decrease in the real interest rate, which will cause an increase in the output gap (that does not suffer any change when the cost channel is not present). However, this does not translate into an increase of domestic inflation due to the direct negative impact of the nominal interest rate on that variable, and this is consistent with the optimal policy given by equation (2.72), where domestic inflation moves in the opposite direction to the output gap.

Regarding CPI inflation, its initial increase is smaller under the cost channel because of the decrease in both domestic inflation and imported goods' inflation caused by the lower nominal interest rate. Also, do not forget that the decrease in the nominal interest rate as

---

<sup>40</sup>Evaluated by the distance between the actual and the equilibrium real interest rate.

a direct negative effect on CPI inflation. As a result of all this, the CPI decreases more in the long-run. With a smaller CPI in the long-run, there is an appreciation of the nominal exchange rate in the long-run as well. However, on impact, the nominal exchange rate is barely affected by the cost channel: there is only a slightly smaller initial depreciation. To understand the reason for that, first we write the nominal exchange rate as

$$\widehat{e}_t = \widehat{q}_t + \widehat{p}_t^c - \widehat{p}_t^{f*}. \quad (2.79)$$

Firstly, the impact of the cost channel on the terms of trade is very small, causing only a slightly larger initial depreciation in response to the slightly larger decrease in the real interest rate. Despite that, the initial depreciation of the real exchange rate is much larger with the cost channel, since that rate depends directly on the nominal interest rate (See equation (2.25)). In other words, such a big depreciation occurs because with a lower nominal interest rate, the price of imported inputs decreases leading to a decrease in the domestic CPI. Simultaneously, this also leads to a smaller initial increase in the CPI, which cancels the larger depreciation of the real exchange rate.<sup>41</sup>

In conclusion, the main effects of the cost channel when the central bank follows an optimal discretion policy are: a larger decrease in the nominal interest rate and domestic inflation, a larger increase on the output gap, a smaller initial increase in CPI inflation, a very small effect on impact and a long-run appreciation of the nominal exchange rate; and finally, despite the negligible effect on the terms of trade, the real exchange rate initially suffers a larger depreciation.

### Optimal commitment

After studying the optimal discretion policy, the optimal commitment policy will be analysed assuming that the central bank can affect agents' expectations. As under discretion, the cost channel does not allow the central bank to fully stabilise domestic inflation and the output gap. Alternatively, the central bank commits to creating domestic inflation in the future in order to

---

<sup>41</sup>Another way to understand why the cost channel has a small impact on the nominal exchange rate on impact is the following. The nominal exchange rate in  $t = 1$  is  $\widehat{e}_1 = \delta_1^c + \pi_1^h$ . At the end, the slightly larger initial increase in the terms of trade and the larger decrease in domestic inflation that occurs with the cost channel only produces a slightly smaller initial depreciation in the nominal exchange rate.

reduce inflation's volatility. Therefore, we observe that domestic inflation decreases initially; to increase afterwards, but this is not enough to produce the stationarity of the domestic price index. The initial decrease in domestic inflation rate is achieved with a reduction in the nominal interest rate which, with the cost channel, directly reduces that inflation rate. This expansionary monetary policy produces an increase in the output gap, which after three quarters leads to an increase in domestic inflation.

The cost channel has a negligible impact on the nominal interest rate, terms of trade and nominal exchange rate.

As under discretion, the real exchange rate depreciates more on impact and CPI inflation increases less on impact, implying that with commitment the CPI is almost stationary.

### **Optimal discretion versus optimal commitment**

At this point, it is possible to contrast optimal discretion policy with optimal commitment policy. Firstly, without the cost channel, the behaviour of the economy is identical for both policies, with inflation and the output gap always constant.

With the cost channel, the initial decrease in domestic inflation is smaller with commitment than with discretion. In other words, the monetary authority can better stabilise domestic inflation if it is able to precommit. The better trade-off between inflation and output gap attained by the central bank under commitment comes from its ability to commit to create future inflation (Monacelli, 2005). Since that commitment is credible, there is an increase in inflation's expectations, meaning that, according to the Phillips curve, the decrease in inflation for a given decrease in output gap is smaller. In fact, the commitment policy produces an initial period of deflation in *domestic* prices followed by a period of inflation.

Due to the better trade-off between domestic inflation and the output gap achieved under commitment, the nominal interest rate does not need to decrease as much as in the case of discretion. Consequently, CPI inflation increases more on impact with a commitment policy.

42

With the cost channel, the output gap increases more with optimal commitment than with optimal discretion, in order to ensure that domestic inflation increases after the third quarter.

---

<sup>42</sup>Recall that the nominal interest rate affects directly CPI inflation under the cost channel.

In conclusion, with commitment, the monetary authority trades-off more output variability for less domestic inflation variability.

Either with commitment or with discretion, the domestic price *level* is not stationary. But while with discretion the domestic price level decreases in the long-run, with commitment it increases, after an initial decrease. Regarding the CPI, with the cost channel a discretion policy produces a permanent decrease in that index, and a commitment policy leads almost to its stationarity. It can then be concluded that with optimal commitment both domestic and CP indices tend to exhibit a behaviour *closer* to stationarity than with discretion. This is an usual characteristic of forward-looking models with optimal commitment (see Woodford, 2003; Monacelli, 2005). The reason for that can be found in the fact that the monetary authority commits to a policy where current inflation depends on past values of output gap. Since monetary policy depends on past shocks, it is able to *partially* correct the deviations of price levels from a stationary path. In contrast, with optimal discretion any temporary shock will have a permanent effect on price indices.

Finally, the nominal exchange rate is almost stationary under commitment due to the near stationarity of the CPI. In turn, with discretion there is a nominal appreciation in the long-run explained by the long-run decrease in the CPI.

### **Domestic inflation-based Taylor rule**

For the reasons already explained above, with the cost channel a central bank following a DITR is not able to completely stabilise domestic inflation and the output gap.<sup>43</sup> In response to the reduction in the equilibrium real interest rate caused by the technological shock, the central bank reduces the nominal interest rate. With the cost channel, that directly reduces domestic inflation. According to the Taylor rule, that leads to a further decline in the nominal interest rate. The cost channel then produces a larger decline in the nominal interest rate and thus a more expansionary monetary policy, with the real interest rate below its equilibrium level. This explains the increase in the output gap that occurs after the technological shock. Even though there is such an increase in the output gap, domestic inflation decreases, due to the direct negative impact of the decrease in the nominal interest rate.

---

<sup>43</sup>To isolate the effect of the cost channel, we maintain  $\alpha_\pi = 1.5$ .

As regards consumer inflation, we observe that with the cost channel there is a smaller initial increase. This occurs because, in spite of the larger initial depreciation of the terms of trade (that we describe below), the decrease in the nominal interest rate directly reduces CPI inflation. However, in the second quarter, the appreciation of the terms of trade more than compensates for the increase in the nominal interest rate, implying a period of deflation. This deflationary period produces a long-run decline in the CPI. Consequently, the nominal exchange rate will appreciate on the long-run, which compares with a zero long-run change without the cost channel. On impact, the working capital channel has the opposite effect, implying a larger depreciation of the nominal exchange rate and the terms of trade, due to the larger decrease in the nominal interest rate.

In conclusion, the impact of introducing the cost channel when the central bank follows a DITR is to produce a larger decrease in the nominal interest rate and domestic inflation, a smaller initial increase of CPI inflation, a larger increase in the output gap, a larger initial depreciation of both the terms of trade and nominal exchange rate, and a long-run decline in both the CPI and the nominal exchange rate.

### **Consumer inflation-based Taylor rule**

The introduction of the cost channel when the central bank follows a CITR also leads to a larger volatility of the nominal interest rate. It was already explained above that in the first quarter, to avoid a large depreciation of the terms of trade, the output gap has to decrease and the nominal interest rate has to increase. In this context, there are two reasons for a larger initial movement of the nominal interest rate. On one hand, with the cost channel, such an increase in the nominal interest rate produces a direct increase in CPI inflation, implying by the Taylor rule a further increase in the interest rate. On the other hand, the fact that in the first quarter there is an expected decrease in the nominal interest rate contributes to an increase in the output gap (see eq. (2.65)). As a consequence, the central bank needs to increase further the nominal interest rate to attain a given decrease in the output gap.

In the second quarter, after controlling the increase in CPI inflation arising from imported goods, the central bank reduces the nominal interest rate to accommodate the decrease in the equilibrium real interest rate. Nevertheless, because with the cost channel the decrease in the

nominal interest rate directly decreases CPI inflation, the interest rate decreases (according with the Taylor rule) more than without the cost channel. Moreover, in a context where the nominal interest rate is more volatile and affects directly CPI inflation, it appears natural that the cost channel induces a larger volatility in the CPI inflation: on impact it increases more and after the first quarter it becomes negative.

Despite the larger increase in the nominal interest rate (that also implies a more restrictive monetary policy in terms of real interest rate), on impact, the output gap does not falls so steeply as without the cost channel (but the difference is small), because of both the positive effect of the expected decrease in the nominal interest rate and the larger expected increase in the output gap. Indeed, in the second quarter, the actual real interest rate decreases below its equilibrium level, implying a more expansionary policy than in the absence of the cost channel, which produces a larger increase in the output gap. In conclusion, the output gap with the cost channel is always larger than without the cost channel.

Regarding domestic inflation, in the first quarter it increases more with the cost channel due to the direct impact of the increase in the nominal interest rate. In the second quarter, despite the stronger increase in the output gap when the cost channel is present, we do not observe a larger increase in domestic inflation, because of the larger decrease in the nominal interest rate and its direct impact in domestic inflation. Taking the first two quarters together, we can notice that domestic inflation suffers a smaller increase in the presence of the cost channel.

Furthermore, in the first quarter the terms of trade depreciate more with the cost channel because the output gap decreases less. In the second quarter, contrary to what happens without the cost channel, the terms of trade continue to depreciate significantly, in order to minimise the decline in CPI inflation caused by the direct effect of the decrease in the nominal interest rate. That depreciation can also be explained by the positive direct effect that a decrease in the nominal interest rate has on the terms of trade when the cost channel is present.

Finally, in the second quarter the nominal exchange rate depreciates more with the cost channel, due to the larger increase in the terms of trade and real exchange rate. However, in the long-run, the nominal exchange rate depreciates less, due to the fact that CPI increases less in light of the observed deflationary period.

In conclusion, the cost channel with a CITR implies: a larger volatility of the nominal interest rate and CPI inflation; a smaller response of the domestic inflation in the first two quarters; a more positive response of the output gap; a larger initial increase of the terms of trade, nominal exchange rate and the CPI; and in the long-run, a smaller CPI and a smaller depreciation of the nominal exchange rate.

### Exchange rate peg

Due to the fact that a technological shock under an exchange rate peg does not produce any change in the nominal interest rate, in this policy regime the introduction of the cost channel does not have any impact on the dynamics of the economy. Therefore, in order to study the impact of the cost channel under a PEG, we are going to research instead the impact of a foreign output shock. Let us start by studying that shock assuming the absence of the cost channel. The increase in the foreign output decreases the foreign real interest rate. To maintain the PEG, the domestic central bank is forced to decrease the nominal interest rate. Simultaneously, on one hand the increase in the foreign output causes an increase in the flexible-price output due to the increase in home exports and labour use in the flexible-price equilibrium. On the other hand, the increase in the foreign output appreciates the equilibrium terms of trade, causing a reduction in the flexible-price output. The net effect is the reduction in the flexible-price output, implying finally an increase in the equilibrium real interest rate. This last effect, together with the decrease in the nominal interest rate, causes an increase of the output gap and thus domestic inflation. With nominal exchange rate and foreign prices constant, the increase in domestic inflation implies an appreciation of the terms of trade. Finally, CPI inflation simply mirrors domestic inflation.

The effect of the cost channel under a PEG is negligible. Firstly, there is no effect on the response of the nominal interest rate, which moves in exactly the same way as before, in order to offset the decrease in the foreign real interest rate. Secondly, the cost channel does not affect substantially domestic inflation *directly*, because the actual and the flexible-price nominal interest rate move in the same amount.<sup>44</sup> The only significant effect of the cost channel is to produce a smaller initial increase in the CPI inflation. This occurs because the

---

<sup>44</sup>The nominal interest rate still affects the domestic price level, with this effect given by  $r(1-\gamma)\gamma_c(\sigma a - 1)/\sigma$ .

Table 2.1: Impact of the cost channel in the response of the economy to a technological shock

	Policy			
	Op. Discretion	Op. Commitment	DITR	CITR
$\pi_t^h$	↓ more	↓ more	↓ more	↑ less (in first 2 quarters)
$\hat{i}_t$	↓ more	↓ less (very small impact)	↓ more	↑ more
$\hat{x}_t$	↑ more	↑ more	↑ more	Always larger
$\pi_t^c$	↑ less	↑ less	↑ less	↑ more
$\hat{\delta}_t$	↑ more (very small impact)	↑ more (very small impact)	↑ more	↑ more
$\hat{s}_t$ :Short-run	↑ more (very small impact)	↑ more (very small impact)	↑ more	↑ more
$\hat{s}_t$ :Long-run	↓ more	↑ more (very small impact)	↑ less	↑ less

decrease in the nominal interest rate reduces directly the price of imported inputs.

### Synthesis on the impact of the cost channel in the response of the economy to a technological shock

Taking into account the response of the economy to a technological shock with different policy regimes, the major impacts of the cost channel can be summarised as: a larger change in the nominal interest rate (except with the optimal commitment policy); smaller domestic inflation rates (they decrease more with optimal policies and DITR, and increase less with a CITR); a smaller initial increase in CPI inflation (except for a CITR); and a more volatile and positive response of the output gap (See Table 2.1).

Chowdwury et al. (2006) show, in a closed economy, that the cost channel dampens inflation response to an interest rate shock: after an increase in the interest rate, inflation falls less with the cost channel than without it. It is worth mentioning that, in an open economy, with the more complex effects of the nominal interest rate and in the context of a technological shock, a decrease in the nominal interest rate tends to produce lower domestic and CPI inflations with the cost channel than without it.<sup>45</sup> In order to analyse the robustness of this result, we also study the impact of an interest rate shock,  $\varepsilon_t$ :

$$\hat{i}_t = \hat{\rho}_t^{ho} + \alpha_\pi \hat{\pi}_t^h + 0.8\hat{i}_{t-1} + \varepsilon_t.$$

<sup>45</sup>With the CITR, in  $t = 1$ , when the interest rate increases, there is a larger increase in CPI inflation. In  $t = 2$ , when the interest rate decreases, there is a larger decrease in CPI inflation.

Table 2.2: Impact of the cost channel on the response of the economy to an interest rate shock

	<b>p.p.</b>	<b>%</b>
$\pi_t^h$	+0.0312	+10%
$\pi_t^c$	+0.1737	+24%
$\hat{l}_t$	+0.0468	+8.7%
$\hat{x}_t$	-0.1036	-10%
$\hat{s}_t$	-0.1338	-9.01%

The lagged interest rate was introduced to give some persistence to the shock. In Exhibits 13 and 14 in Annex 2.7, we can observe that, after a positive interest rate shock, the cost channel implies a smaller fall in CPI inflation and domestic inflation, but it amplifies the decrease in the output gap (Table 2.2).

Concerning the terms of trade, there is a larger initial depreciation, with the difference being very small under optimal policies. In the long-run, the terms of trade tend to zero with or without the cost channel. Finally, there is a larger depreciation of the nominal exchange rate in the short-run under CITR and DITR, and a negligible effect under the optimal commitment and discretion policies. In the long-run, the nominal exchange rate converges to lower values (smaller depreciation or larger appreciation), except in the optimal commitment policy where the effect is negligible. The reason why the nominal exchange rate has such a response in the long-run is that in all policy regimes (except in the optimal commitment) the CPI converges to a smaller value when the cost channel is present.

From these results, it is possible to infer that the cost channel seems to increase macroeconomic volatility. A more complete study of this issue will be undertaken in the next section.

## 2.5 Stochastic simulations and macroeconomic volatility

Until this point the focus has been on the impact of an unanticipated domestic technological shock. Now, we assume simultaneously three shocks, to the technology, preferences and foreign output, and perform stochastic simulations with several policy regimes. To assess the effect of the cost channel, in Section 2.5.1 we will compare the performance of the model with and without the cost channel. After, in Section 2.5.2 and using policy functions we will assess how the central bank reacts directly to shocks. In Section 2.5.3 the contribution of the cost

channel to explain interest rate smoothing by the central bank will be presented. Section 2.5.4 analyses how the cost channel partially explains the small empirical correlation between the change in the nominal exchange rate and CPI inflation. Finally, in Section 2.5.5 the impact of imported inputs will be analysed.

Recall that in the case of the DITR and the CITR,  $\alpha_\pi = 1.5$  was chosen in a non optimal way. Now that we are going to compare different policies in terms of welfare, the optimal value for the coefficient  $\alpha_\pi$  will be used. Assuming that the central bank can commit to the Taylor rule, an optimal value for the parameter  $\alpha_\pi$  can be obtained by minimising the weighted sum of variances of output and *domestic* inflation, with weights of respectively 0.18 and 1, subject to the equations in the model and the instrument rule:

$$\hat{i}_t = \hat{\rho}_t^{ho} + \alpha_\pi \hat{\pi}_t,$$

, with  $\hat{\pi}_t = \hat{\pi}_t^h$ , under the DITR, and  $\hat{\pi}_t = \hat{\pi}_t^c$ , under the CITR. Without the cost channel and with a CITR, it was obtained a  $\alpha_\pi = 1.9292$ . With the cost channel, we got  $\alpha_\pi = 1.6845$  with the CITR and  $\alpha_\pi = 1.9814$  with the DITR.<sup>46</sup> Notice that the impulse response functions using optimal coefficients for the Taylor rules are qualitatively similar to the ones that were obtained with  $\alpha_\pi = 1.5$ .

In the case of the CITR we observe that with the introduction of the cost channel there is a decline in the optimal reaction coefficient,  $\alpha_\pi$ . This means that the central bank reduces the adjustment in the nominal interest rate in response to an unitary change in CPI inflation. This can be explained by the fact that, with inputs paid in advance, changes in the nominal interest rate have a direct impact on CPI and domestic inflations and output gap. In this case, when in response to an increase in CPI inflation, the central bank increases the nominal interest rate, there is a direct feedback increasing consumer inflation. In turn, the central bank is led to increase the nominal interest rate even more. This process can imply a large volatility in the nominal interest rate and thus in the domestic inflation and output gap. To avoid such an outcome, the central bank has to restrain the changes in the nominal interest rate when the cost channel is present.

---

<sup>46</sup>With the DITR in the absence of the cost channel, any coefficient is optimal, since any one can ensure the total stabilisation of output gap and domestic inflation.

Table 2.3: Variance of the main variables with and without the cost channel

	interest rate			Terms of trade			Change nominal exchange rate			CPI inflation		
	no cost	cost	perc. diff.	no cost	cost	perc. diff.	no cost	cost	perc. diff.	no cost	cost	perc. diff.
<b>Optimal commitment</b>	0.1645	0.1622	-1.4%	3.6275	3.5846	-1.2%	1.2336	1.2431	0.8%	0.1426	0.1143	-19.8%
<b>Optimal discretion</b>	0.1645	0.2428	47.6%	3.6275	3.6184	-0.3%	1.2336	1.2800	3.8%	0.1426	0.1200	-15.8%
<b>Optimal DITR</b>	0.1645	0.2423	47.3%	3.6275	3.6573	0.8%	1.2336	1.3634	10.5%	0.1426	0.1165	-18.3%
<b>Optimal CITR</b>	0.2127	0.8409	295.3%	3.0205	3.2864	8.8%	0.4308	0.9054	110.2%	0.0540	0.2456	354.8%
<b>Exchange rate peg</b>	0.1100	0.1100	0.0%	1.5176	1.4796	-2.5%	0	0	-	0.0367	0.0263	-
	Domestic inflation			Output gap (output)			Output gap (value added)			Loss function		
	no cost	cost	perc. diff.	no cost	cost	perc. diff.	no cost	cost	perc. diff.	no cost	cost	perc. diff.
<b>Optimal commitment</b>	0	0.0010	-	0	0.0050	-	0	0.0083	-	0	0.0019	-
<b>Optimal discretion</b>	0	0.0083	-	0	0.0014	-	0	0.0071	-	0	0.0086	-
<b>Optimal DITR</b>	0	0.0019	-	0	0.0136	-	0	0.0202	-	0	0.0043	-
<b>Optimal CITR</b>	0.0205	0.0479	133.7%	0.1586	0.3115	96.4%	0.1042	0.2886	177.0%	0.04905	0.1040	112.0%
<b>Exchange rate peg</b>	0.0844	0.0826	-2.1%	0.7320	0.7193	-1.7%	0.5361	0.5361	0.0%	0.2162	0.2121	-1.9%

### 2.5.1 Cost channel's impact on macroeconomic volatility

We start by analysing the volatility of the main variables without a cost channel - Table 2.3. In this case, as explained in relation to the domestic productivity shock, there are three optimal policies: optimal commitment, optimal discretion and DITR. These policies ensure the full stabilisation of the output gap and domestic inflation. Likewise, we can also observe that the output gap in value added terms does not fluctuates.

In turn, the CITR and the PEG are suboptimal, because of the excess smoothness of the nominal exchange rate and the terms of trade (Gali and Monacelli, 2005). It is precisely because it does exacerbate the smoothness of these variables that the PEG is the most sub-optimal policy. It is worth mentioning that the CITR is better than the DITR at stabilising CPI inflation (this will change with the cost channel).

In the presence of the cost channel the best regime is, as expected, the optimal commitment, since it achieves the lowest value for the loss function.<sup>47</sup> The optimality of precommitment can be explained by its ability of responding to shocks maintaining a low volatility of the nominal interest rate. With the cost channel that ability becomes even more relevant since the volatility of the nominal interest rate directly affects domestic inflation and output gap's volatilities. In comparison with the optimal discretion policy, it is possible to confirm what was already seen with the technological shock: the optimal commitment policy trades-off

<sup>47</sup>Like in the determination of optimal Taylor rules, the loss function is defined as the weighted sum of domestic inflation and output gap volatilities, with a weight of 1 for domestic inflation and 0.18 for the output gap.

more output's volatility for less domestic inflation's volatility.

In comparison with the optimal discretion policy, the optimal DITR achieves a lower domestic inflation's volatility, but it implies however a higher output gap's volatility. Combining these two facts, the DITR achieves a social loss lower than the discretionary policy. This means that, without the ability to commit to a complete path of future interest rates, a weaker commitment to a Taylor rule may be better than following a discretion policy.

The cost channel does not change the fact that CITR and the PEG are the worst performing policies, with the PEG remaining at the bottom of the list. The suboptimality of those regimes is related to the fact that with the cost channel, there continues to exist an excess smoothness of the terms of trade and nominal exchange rate. For the CITR, another reason for the suboptimality can be added: the excess volatility of the nominal interest rate.<sup>48</sup>

In the literature two reasons are highlighted to target domestic inflation instead of CPI inflation (Kirsanova et al., 2006). Firstly, it is the domestic inflation that appears in the Central Bank's objective function when this function is obtained as a second order approximation of the representative consumer's utility. Secondly, targeting CPI inflation may induce instability due to the effect of interest rate on CPI inflation, through exchange rate. In our model, we observe that in the presence of the cost channel the CITR becomes a particularly bad policy. And this becomes clear when we compare the increase in the volatility of the nominal interest rate and CPI inflation caused by the cost channel under CITR and DITR. This means that the cost channel can be also seen as a reason to avoid targetting CPI inflation.

Besides that, the PEG is the only policy regime for which the loss function improves with the introduction of the cost channel. The reason for that can be found in the fact that the nominal interest rate only responds to foreign shocks. As a result, with the introduction of the cost channel, the central bank does not increase the volatility of the interest rate to achieve the stabilisation of domestic aggregates. Furthermore, even though the change in the nominal interest rate is the same, its impact on other variables is smaller when inputs are paid in advance. To understand that, take as an example a decrease in the nominal interest

---

<sup>48</sup>It may seem strange to follow a CITR when domestic inflation is in the loss function. Then, if we admit instead CPI inflation in the loss function (together with the output gap), the optimal CITR produces a volatility of CPI inflation of 0.15 and a volatility of the output gap of 0.32, yielding a loss function of 0.2076. However, the optimal DITR produces a lower value of the loss function:  $\pi_t^c + 0.18\hat{x}_t = 0.1165 + 0.18 \times 0.0136 = 0.1189$  (with these last data coming from Table 2.3).

rate: its overall effect on CPI inflation will be smaller, because paying inputs in advance will become cheaper, reducing imported inflation (see the impact of a foreign shock above for more details).

After analysing the optimality of each regime, we can turn to the *major* impacts of the cost channel in variables' volatility. As was already clear from the analysis of the technological shock, an increase in the volatility of domestic inflation, output gap and nominal interest rate can be observed. Specifically, the increase in the volatility of the nominal interest rate is relatively large with optimal discretion and DITR, around 47%, and very large with a CITR, 295.3%. The exception is the PEG, where the volatility of all variables apart from the nominal interest rate decreases, as was already expected by the analysis of the foreign output shock, carried out above. For a closed economy, Ravenna and Walsh (2006) also obtain a larger nominal interest rate volatility in response to a fiscal shock.

In contrast, the volatility of CPI inflation decreases in all regimes except in the CITR, where it *increases* significantly. And in general, with a CITR, the introduction of the cost channel implies a much larger increase in the volatility of all variables than with other policy regimes. The reason why the cost channel leads to a smaller volatility of CPI inflation in all regimes except the CITR is related with the fact that the effects of both the nominal interest rate and the terms of trade compensate each other. For example in the case of a technological shock, the decrease in the nominal interest rate contradicts the effect of the terms of trade's depreciation on CPI inflation.<sup>49</sup> On the contrary, with the CITR, the interaction between the nominal interest rate and the terms of trade actually amplify the volatility of CPI inflation. Take again as an example a technological shock. In the first quarter, in order to stabilise CPI inflation, the central bank avoids a larger depreciation on the terms of trade increasing the nominal interest rate (when in other policies there is a reduction in the nominal interest rate). However, this action is not very effective in stabilising CPI inflation, because the increase in the interest rate directly increases CPI inflation. The opposite happens in the following quarters, when the central bank decreases the nominal interest rate and there is simultaneously an appreciation of the terms of trade.

The impact of the cost channel on the volatility of the terms of trade and the nominal

---

<sup>49</sup>Notice that both the nominal interest rate and the terms of trade have a direct impact on CPI inflation.

exchange rate varies more across policy regimes. With the introduction of that channel, the volatility of the terms of trade increases slightly with the DITR and CITR, but it decreases with the other three policies. In addition, the volatility of the change in the nominal exchange rate increases significantly with the CITR and slightly with the other regimes.

In conclusion, with the exception of the PEG, it can be said that across the other four policies regimes considered, the cost channel increases the volatility of domestic inflation, output gap, nominal interest rate and the change in the nominal exchange rate. In contrast, the volatility of CPI inflation decreases in all policy regimes, except in the CITR, where it increases. The effects on the volatility of the terms of trade are more diverse.

Next, we analyse if the results are sensitive to change in some key parameters. We simulated the model changing one parameter at a time:  $\omega$  from 0.75 to 0.9,  $\gamma$  from 0.4 to 0.2,  $\sigma$  from 1.5 to 2,  $\eta$  from 1 to 2, and  $\varepsilon$  from 1/3 to 1.5 (see Exhibits 15 to 29 in the Annex 2.7).

<sup>50</sup> Changing these parameters does not change the main results. In response to a positive interest rate shock with the cost channel, domestic inflation and CPI inflation decrease less and the output gap decrease more. When we introduce the three shocks, the previous impact of the cost channel on variables' volatility is obtained. The volatility of interest rate increases, except with commitment and the exchange rate peg. The volatility of domestic inflation and output gap increases, except with the peg. Finally, the volatility of CPI inflation decreases, except with the optimal CITR and, in some cases, with the optimal discretion. Notice that for the many exercises done we never observe an increase in domestic inflation after a positive interest rate shock.

The ranking of policies continues to be the same, with the only exception being with  $w = 0.9$ , where with the cost channel optimal discretion is better than optimal DITR.

The correlation between CPI inflation and the change in the nominal exchange rate suffers always a reduction when the cost channel is introduced.

Another sensitivity analysis is to assume also the output gap in the Taylor rule. Results

---

<sup>50</sup> An increase in  $\omega$  corresponds to an increase in price rigidity. A decrease in  $\gamma$  is a decrease in the weight of exports on domestic product. An increase in  $\sigma$  is an increase in risk aversion, meaning that it decreases the substitution between current and future consumption when the interest rate increases. An increase in  $\eta$  is an increase in the inverse of the intertemporal elasticity of substitution of labour. This means that a decrease in employment decreases more real wages, i.e., the demand channel is stronger. Finally, an increase in  $\varepsilon$  is an increase in the elasticity of substitution between labour and imports in production.

Table 2.4: Variance of the main variables when the Central Bank follows a DITR with a positive weight for the output gap

	<b>no cost channel</b>	<b>cost channel</b>
<b>Interest rate</b>	0.2161	0.2391
<b>Terms of trade</b>	4.0631	3.6181
<b>Change nominal exchange rate</b>	1.4505	1.2984
<b>CPI inflation</b>	0.0208	0.1219
<b>Domestic Inflation</b>	0	0.0058
<b>Output gap</b>	0	0.0034

Note: the DITR is  $\hat{i}_t = \hat{\rho}_t^{ho} + 1.5\hat{\pi}_t^h + 0.5\hat{x}_t + 0.8\hat{i}_{t-1} + \varepsilon_t$ .

continue to be basically the same. We study a DITR with the form:  $\hat{i}_t = \hat{\rho}_t^{ho} + 1.5\hat{\pi}_t^h + 0.5\hat{x}_t + 0.8\hat{i}_{t-1} + \varepsilon_t$ . The cost channel continues to imply a smaller fall of domestic inflation and CPI inflation and a larger fall of the output gap (Exhibit 30 and 31 in the Annex 2.7). With the three shocks, there is, as before, an increase in the volatility of output gap, domestic inflation and interest rate and the volatility of the terms of trade decreases (Table 2.4). The only differences in relation to the previous results is that the variance of the change in the nominal exchange rate decreases and the variance of CPI inflation increases.

## 2.5.2 Policy functions

One explanation for the increase in volatility of some variables with the introduction of the cost channel can certainly be found in a change of central bank's behaviour. In this section, we analyse this issue in greater detail.

A DGSE model can be summarised as

$$E_t(y_{t+1}, y_t, y_{t-1}, u_t) = 0$$

where  $y$  stands for a vector of endogenous variables, and  $u$  for a vector of exogenous stochastic shocks. To solve the model, it is necessary to find a function  $y_t = f(y_{t-1}, u_t)$  that satisfies the model's equations. Such a function links the present values of endogenous variables to their last period values and to the current shocks, and has a first order approximation given by

$$\hat{y}_t = \alpha_y \hat{y}_{t-1} + \alpha_u u_t.$$

Table 2.5: Policy functions and the cost channel

Variables	Optimal discretion			Optimal Commitment		
	Without the cost channel	With the cost channel	Difference	Without the cost channel	With the cost channel	Difference
	Technological shock in t-1	-0.2226	-0.2721	-0.0495	-0.2226	-0.2158
Foreign output shock in t-1	-0.0313	-0.0476	-0.0163	-0.0313	-0.0297	0.0016
Technological shock in t	-0.3372	-0.4123	-0.0751	-0.3372	-0.327	0.0102
Foreign output shock in t	-0.0364	-0.0554	-0.019	-0.0364	-0.0345	0.0019
Preferences shock in t	0.1983	0.2238	0.0255	0.1983	0.204	0.0057
Variables	Optimal DITR			Optimal CITR		
	Without the cost channel	With the cost channel	Difference	Without the cost channel	With the cost channel	Difference
	Technological shock in t-1	-0.2226	-0.2759	-0.0533	0.0756	0.2894
Foreign output shock in t-1	-0.0313	-0.0361	-0.0048	-0.3776	-0.898	-0.5204
Technological shock in t	-0.3372	-0.418	-0.0808	0.1146	0.4385	0.3239
Foreign output shock in t	-0.0364	-0.0419	-0.0055	-0.4391	-1.0442	-0.6051
Preferences shock in t	0.1983	0.2303	0.032	0.078	0.226	0.148
delta t-1	0	0	0	-0.2	-0.5203	-0.3203
it-1	0	0	0	0	-0.5203	-0.5203

This policy function is a time-recursive approximation of the model, that can generate time-series for the endogenous variables that respect the model’s system of equations.

Regarding the central bank’s actions, the advantage of a policy function is that it characterises the direct reaction of the nominal interest to exogenous shocks. This contrasts with the Taylor rule, where the nominal interest rate only depends on a given set of variables, which indirectly reflect shocks that hit the economy.

Assuming the three shocks described above, the monetary authority’s policy functions are presented in Table 2.5. In the two optimal policies and DITR, both with and without the cost channel, the central bank responds only to exogenous shocks and reduces the interest rate in response to technological and foreign output shocks, and increases it after a shock to preferences.<sup>51</sup> This can be explained by the fact that the former two shocks decrease the equilibrium real interest rate and the later shock increases it. Notice that, without considering the cost channel, the reaction function is identical whether the central bank follows a commitment policy, a discretion policy or a DITR. This confirms what we already observed above: the three policies are identical in the absence of the cost channel.

The introduction of the cost channel in the precommitment case has a negligible effect on the reaction function. Nevertheless, the interest rate decreases less in response to a technological shock or a foreign output shock, but it increases more in response to a preference shock.

<sup>51</sup> Regarding the technological shock, this confirms the analysis carried out in Section 2.4.

In contrast, in the optimal discretion case, introducing the cost channel makes the central bank substantially more active replying to shocks, with the nominal interest rate responses to all shocks increasing (in absolute terms) substantially. The same pattern is observed with the DITR and CITR. That pattern can be explained by the fact that, without a commitment device, the policy maker needs to change the interest rate more when the trade-off between the output gap and domestic inflation worsens with the introduction of the cost channel.

### 2.5.3 Interest rate smoothing

In a model with a cost channel, the optimality of the commitment policy arises from the fact that it allows the central bank to affect agents' expectations. In order to do that, the current monetary policy has to be history-dependent. In the context of a Taylor rule, this policy inertia can be achieved by introducing lags of the nominal interest rate, a procedure that has been validated by many estimated reaction functions of the Fed (Woodford, 2001). With this idea in mind, in the context of our model, it is possible to assess if it is optimal to include the lagged nominal interest rate in the Taylor rule. Therefore, with the three stochastic shocks in the economy, and maintaining the assumption that the loss function only depends on domestic inflation and the output gap, the optimal coefficients of the Taylor rule were determined assuming that the central bank responds to the lagged interest rate:

$$\hat{i}_t = \hat{\rho}_t^{ho} + \alpha_\pi \hat{\pi}_t^h + \alpha_i \hat{i}_{t-1}.$$

When the cost channel is present in the model, the following coefficients were obtained:  $\alpha_\pi = 0.9164$  and  $\alpha_i = 0.0836$ .<sup>52</sup> <sup>53</sup> From this result it is possible to conclude that, even when the central bank does not has the nominal interest rate on its loss function, an interest rate smoothing behaviour arises in the presence of the cost channel.<sup>54</sup> That happens because with the cost channel the output gap and inflation cannot be fully stabilised, meaning that gains arise from precommitment.

---

<sup>52</sup>For US data, McCallum and Nelson's (1998) estimations obtain a value of 0.8 for the coefficient of the lagged interest rate. This means that the cost channel is only one of the factors determining the interest rate smoothing behaviour of central banks observed in the real world.

<sup>53</sup>The stability conditions of the model are fulfilled with these coefficients.

<sup>54</sup>Notice that without the cost channel, any coefficient is optimal, because all ensure the full stabilisation of the output gap and domestic inflation.

Table 2.6: Correlation between CPI inflation and the change in the nominal exchange rate.

	<b>Without the cost channel</b>	<b>With the cost channel</b>
Optimal commitment	1	0.9119
Optimal discretion	1	0.866
DITR	1	0.8842
CITR	0.9224	0.6705

Note: The value of  $\alpha_\pi = 1.5$  was used for the DITR and CITR

#### 2.5.4 Correlation between CPI inflation and the change in the nominal exchange rate

Kara and Nelson (2003) show that the empirical short-run correlation between CPI inflation and the change in the nominal exchange rate is much smaller than the one predicted by standard New Keynesian models that treat imports as consumption goods. The present model shows that the cost channel can make a contribution to explaining the small correlation between those two variables. Indeed, if we take, for example, the optimal precommitment policy, the contemporaneous correlation between CPI inflation and the change in the nominal exchange rate is 1 without the cost channel and decreases to 0.9119 with the cost channel. Further, the same pattern can be observed in all other policies (with the natural exception of the PEG) - Table 2.6.

In a model with no cost channel and with an optimal policy, the covariance between CPI inflation and the change in nominal exchange rate is exactly one because domestic inflation does not change. Consequently, CPI inflation only changes due to the terms of trade (see equation (2.23)). Furthermore, since the real exchange rate also depends exclusively on the terms of trade, the nominal exchange rate, which depends on CPI inflation and the real exchange rate, is solely determined by the terms of trade. To sum up, both CPI inflation and the nominal exchange rate depend on the terms of trade on a one-to-one basis.

When the existence of the cost channel is considered with an optimal policy, CPI inflation starts to be affected by domestic inflation and the nominal interest rate, implying a reduction in its correlation with the nominal exchange rate. Take as an example an increase in the nominal interest rate. Without the cost channel, that event will produce an appreciation

of both the terms of trade and the nominal exchange rate, and a decrease in CPI inflation. In the presence of the cost channel, an additional effect reduces the co-movement between CPI inflation and the nominal exchange rate when the interest rate increases: for a given appreciation in the nominal exchange rate, CPI inflation decreases less, since an increase in the nominal interest rate *directly* increases CPI and domestic inflations.<sup>55</sup>

However, the correlation between CPI inflation and the change in the nominal exchange rate in our model is still much higher than the one observed in the data: as an example, with annual data Kara and Nelson (2003) obtain a contemporaneous correlation of 0.491 for Germany (79-98) and 0.113 for the UK (73-98). One reason for the high correlation obtained with our model is that its dynamic is very simple. For example, it ignores the possibility of slow pass-through of movements in the nominal exchange rate to import prices.

### 2.5.5 The impact of imported inputs

Admitting imported inputs in models has important implications for the dynamics of CPI inflation. In models where imports are treated only as inputs, CPI inflation is more persistent and less correlated with changes in the nominal exchange rate (McCallum and Nelson, 2001).

In the data, the average share of imported inputs on total imports, for the period 1967 to 1998, varies considerably among G7 countries, from 0.609 for the US to 0.740 for Japan (Leith and Malley, 2007).

Based on this, it is interesting to analyse the impact of imported inputs share on the dynamics of the economy, especially on CPI inflation. To maintain a balanced current account in the steady-state, an increase in imported inputs' share on output has to be compensated by a reduction in the imported consumption goods' share on total consumption (see equation (2.29)).<sup>56</sup>

Let us start with comparative statics. In the IS curve, the direct negative effect of the

---

<sup>55</sup>With the cost channel, an increase in the nominal interest rate also implies a direct appreciation in the real exchange rate (see equation (2.25)). That, together with the direct effect on the CPI, produces an irrelevant impact of the cost channel on the nominal exchange rate's response (see (2.79)). In other words, the response of the nominal exchange rate is almost equal with or without the cost channel. For an example, see the impact of a technological shock in the nominal exchange rate.

<sup>56</sup>Maintaining  $\gamma = 0.4$ , we change  $\gamma_i$  from a minimum of 0 to a maximum of 0.66666. The last value corresponds to  $\gamma_c = 0$ .

real interest rate on output gap decreases as imported inputs share increases.<sup>57</sup> In an open economy, one of the effects of an increase in the real interest rate is to appreciate the terms of trade, leading to an increase in imports of consumption goods, which decreases the demand for domestic goods and thus decreases output gap. This effect is smaller as imported consumption goods share decreases and imported inputs share increases.

In the Phillips curve, the effect of output gap on domestic inflation decreases with imported inputs share, for  $\gamma > \bar{\gamma}$  (see Annex 6.1.9). An increase in the output gap increases wages and thus domestic inflation. As the share of wages on the marginal cost decreases, the impact of output gap on domestic inflation becomes smaller.

Still in the Phillips curve, the impact of the nominal interest rate on domestic inflation increases with  $\gamma_i$  (see Annex 6.1.10). This is the sum of several effects. One effect that reduces the impact of the nominal interest rate is related to the fact that the price of imported inputs depends on the terms of trade, and an increase in the interest rate appreciates the terms of trade, making imported inputs cheaper. Since this effect is not present in wages, as imported inputs share increases, the direct impact of the interest rate on domestic inflation tends to become smaller. On the other hand, the *dominant* effect is explained by the fact that an increase in the nominal interest rate leads to a direct increase in the price of imported inputs. As a result, imported inputs are replaced by labour,<sup>58</sup> causing an increase in wages and domestic inflation. As imported inputs become more important in production, the increase in the use of labour caused by that substitution effect becomes larger, implying a more significant increase in domestic inflation when the interest rate increases.

Turning now to the impact of imported inputs on the dynamic response of the economy to shocks, we start by analysing a technological shock under the DITR. Comparing Exhibits 6 ( $\gamma_i = 0.1$ ) and 12 ( $\gamma_i = 0.3$ ) in Annex 2.7, it is possible to observe that as  $\gamma_i$  increases, the main variables' volatility decreases. Namely we observe that, in reaction to that shock, domestic inflation decreases less and CPI inflation increases less. To understand why that happens, we start by noticing that the impact of the technological shock on the flexible-price output decreases with  $\gamma_i$ . That occurs because the *initial* increase in  $\hat{y}_t^o$  produces a depreciation in the

---

<sup>57</sup>An increase in  $\gamma_i$  is associated with a decrease in  $\gamma_c$ ; and a decrease in  $\gamma_c$  implies a decrease in  $w$  that finally reduces  $(1 + w) / \sigma$ .

<sup>58</sup>For now, we consider labour cost constant.

Table 2.7: Impact of imported inputs share on macroeconomic volatility (with the cost channel)

	interest rate			Terms of trade			Change nominal exchange rate		
	gamai=0	gamai=0.3	gamai=2/3	gamai=0	gamai=0.3	gamai=2/3	gamai=0	gamai=0.3	gamai=2/3
<b>Optimal commitment</b>	0.1828	0.1323	0.1009	4.1635	2.7056	1.6983	1.4392	0.9472	0.6099
<b>Optimal Discretion domestic inflation</b>	0.2661	0.2349	-	4.2008	3.6632	-	1.4952	1.3570	-
<b>DITR</b>	0.2612	0.2042	0.1692	4.2260	2.7221	1.6697	1.5345	1.0097	0.6392
<b>CITR</b>	1.1586	0.3913	0.1692	3.9232	2.5816	1.6697	1.1952	0.7092	0.6392
<b>Peg</b>	0.1100	0.1100	0.1100	1.7371	1.1056	0.6989	0.0000	0.0000	0.0000
	CPI inflation			Domestic inflation			Output gap (output)		
	gamai=0	gamai=0.3	gamai=2/3	gamai=0	gamai=0.3	gamai=2/3	gamai=0	gamai=0.3	gamai=2/3
<b>Optimal commitment</b>	0.1780	0.0395	0.0009	0.001	0.0009	0.0009	0.005	0.0052	0.0061
<b>Optimal Discretion domestic inflation</b>	0.1878	0.136	-	0.0074	0.0075	-	0.0016	0.0013	-
<b>DITR</b>	0.1820	0.0408	0.0025	0.0033	0.0028	0.0025	0.0076	0.0106	0.0154
<b>CITR</b>	0.4617	0.0809	0.0025	0.1047	0.0216	0.0025	0.4111	0.091	0.0154
<b>Peg</b>	0.0254	0.0287	0.0344	0.1005	0.0583	0.0344	0.8150	0.5595	0.3564
	Output gap (value added)			Loss function			cov( $\Delta s_t, \pi_t^c$ )		
	gamai=0	gamai=0.3	gamai=2/3	gamai=0	gamai=0.3	gamai=2/3	gamai=0	gamai=0.3	gamai=2/3
<b>Optimal commitment</b>	0.0050	0.0269	0.065	0.0019	0.0018	0.0020	0.9175	0.8902	-0.0419
<b>Optimal Discretion domestic inflation</b>	0.0016	0.0076	-	0.00769	0.00773	-	0.8804	0.8681	-
<b>DITR</b>	0.0076	0.0340	0.0790	0.0047	0.0047	0.0053	0.8940	0.8471	-0.0510
<b>CITR</b>	0.4111	0.1155	0.0790	0.1787	0.0380	0.0053	0.5796	0.7634	-0.0510
<b>Peg</b>	0.8150	0.2042	0.0105	0.2472	0.1590	0.0986	0	0	0

Note: For optimal discretion, instead of  $\gamma_i = 0.3$ , we use  $\gamma_i = 0.1$ , in order to guarantee the existence of a stable solution.

flexible-price terms of trade, that leads to a further increase in  $\hat{y}_t^o$ . However, the latter effect decreases with the imported inputs share. In fact, an increase in the terms of trade increases the price of imported consumption goods, leading to a decrease in consumption. As a result, the supply of labour increases, which produces an increase in the flexible-price equilibrium output. It is then clear that this effect becomes smaller as the share of imported consumption goods decreases, which is equivalent to say as the imported inputs share increases.

With a smaller increase of  $\hat{y}_t^o$  after the technological shock, the flexible-price equilibrium real interest rate decreases by less. Then, according to the Taylor rule, the nominal interest rate also suffers a smaller decrease, implying a smaller direct negative impact on domestic inflation.

Finally, CPI inflation increases less on impact because the share of consumption goods is smaller, implying that the inflationary impact of the terms of trade's depreciation is also smaller (see eq. (2.23)).

When we introduced the three shocks simultaneously and performed stochastic simulations with the cost channel, the main impact of changing the imported inputs share occurs under the CITR and the PEG: a significant decrease in the variance of both domestic inflation and output gap (Table 2.7). Under a CITR that can be explained because an increase in the

weight of imported inputs means that CPI inflation is less affected directly by the term of trade and the nominal interest rate. With a CITR, that causes a smaller volatility of CPI inflation and of the nominal interest rate. Notice that in the limiting case with zero imports of consumption goods,  $\gamma_i = 2/3$ , CITR and DITR are identical, since the CPI equals the domestic price index.

When imported inputs share increases, the distance in terms of welfare of the exchange rate peg to the optimal policies and DITR reduces. That can be explained by the fact that with an exchange rate peg, the *flexible-price* terms of trade react less to shocks as imported inputs share increases. This means that the muted response of the terms of trade that characterises the PEG imposes a smaller restriction on the economy's reaction to shocks.

The impact of increasing  $\gamma_i$  is much smaller on the optimal policies and the DITR. In general, there is a small increase in the loss function with the imported inputs share (however, the effect is not monotonic with an optimal commitment policy).

Finally, an increase in the imported inputs share reduces the covariance between the change in the nominal exchange rate and the CPI inflation, which is a similar result to that of McCallum and Nelson (2001).

## 2.6 Conclusion

In this paper the cost channel is introduced in a small open economy New Keynesian model. The concept of cost channel is broadened, assuming that, besides wages, imports of both consumption goods and inputs have to be paid in advance. This modelisation introduces new effects of the nominal interest rate and the terms of trade in the flexible-price equilibrium output. On one hand, now the nominal interest rate affects the equilibrium output through the price of imported inputs, CPI inflation and exports. On the other hand, the terms of trade have an additional effect on the flexible-price output due to its impact on the price of imported inputs.

Likewise, the assumption that imports are paid in advance introduces new effects of the nominal interest rate on the IS curve and on the equations describing CPI inflation and the terms of trade: (1) the IS curve depends on the expected change in the nominal interest rate; (2) CPI inflation depends on the change of the nominal interest rate; and (3) the relation

between the output gap and the terms of trade is also affected by the nominal interest rate. The first effect breaks the isomorphism between the closed and open economy representations in the domestic inflation and output gap's space, which characterises standard open economy New Keynesian models.

In turn, in the Phillips curve the nominal interest rate directly affects domestic inflation, as in a closed economy model with a cost channel. Notice, however, that such effect is richer in an open economy model, since the nominal interest rate directly affects not only the cost of labour, but also the price of imported inputs and consumption goods, CPI inflation and the terms of trade.

As with the closed economy, it is shown that in an open economy the cost channel leads to a trade-off between output gap and domestic inflation. That occurs due to the direct impact that the nominal interest rate has on domestic inflation. Under an optimal discretion policy, it was shown that the central bank's policy function allows more inflation variability when the cost channel is present.

The calibration of the model allows further research on the dynamics of the economy. Regarding the response to a technological shock with different policy regimes, the cost channel major impacts are: a larger change on the nominal interest rate (except with the optimal commitment policy); smaller domestic inflation rates after the shock; a smaller initial increase in CPI inflation (except for the CITR); and a more volatile and positive response of the output gap. It is worth mentioning that after a decrease in the interest rate, domestic and CPI inflations are lower with than without the cost channel. This confirms previous closed economy results regarding the mitigated response of inflation in the presence of the cost channel (Chowdury et al., 2006).

Concerning the effect of the cost channel on the terms of trade, there is a larger initial depreciation, but with a very small effect under optimal policies. In the long-run, the terms of trade tend to zero either with or without the cost channel. Finally, there is a bigger depreciation of the nominal exchange rate in the short-run under the CITR and the DITR, and a negligible effect under optimal commitment and discretion policies. In the long-run, the nominal exchange rate converges to lower values (smaller depreciation or larger appreciation), except for the optimal commitment policy where the effect is negligible. This behaviour of the nominal exchange rate occurs because in all policy regimes (except in the optimal

commitment) the CPI converges over time to a smaller value.

When shocks to the domestic technology, preferences and foreign output are simultaneously considered, it is possible to say that across all the regimes considered, with the exception of the PEG, the cost channel increases the volatility of domestic inflation, output gap, and the nominal interest rate. In contrast, the volatility of CPI inflation decreases in all policy regimes, except in the CITR, where it increases. Therefore, our results show that the introduction of the cost channel favours domestic inflation targeting over CPI inflation targeting. The effects on the volatility of the terms of trade and nominal exchange rate are more diverse.

The increase in the nominal interest rate's volatility is also clear in the policy functions without commitment. From these, we observed that the central bank becomes substantially more active when replying to shocks. This can be explained by the fact that, without a commitment device, the policy maker needs to change the interest rate more when the trade-off between the output gap and domestic inflation worsens with the introduction of the cost channel.

Also with the three shocks, without the cost channel there are three equivalent optimal policies, producing a complete stabilisation of output gap and domestic inflation: optimal commitment, optimal discretion and DITR. Instead, in the presence of the cost channel the only regime that minimises the loss function is the optimal commitment. Another result that confirms the optimality of commitment in the presence of the cost channel is the fact that the commitment to a Taylor rule is better than an optimal discretion policy. The gains arising from commitment also justify interest rate smoothing in the context of the Taylor rule when the cost channel is relevant. The policies with worse performance are the CITR and the PEG.

In addition, the cost channel partially explains why the observed contemporaneous correlation between CPI inflation and the nominal exchange rate is relatively small.

Finally, the increase in imported inputs share in output, with the associated decrease in imported consumption goods share, in general, reduces macroeconomic volatility, especially under the CITR and the exchange rate peg. This occurs in these latter regimes because the terms of trade become less important in affecting resources allocation and inflation.

We have shown that many of the cost channel's implications in a closed economy are also valid in an open economy. Moreover, that channel has significant implications for the economy dynamics and monetary policy, and also contributes to explaining some interesting empirical

evidence. For all of this, the cost channel deserves more research and attention from monetary authorities.

## 2.7 Annex: dynamic response of the economy to shocks

**Notation:**  $\delta = \hat{\delta}_t^j$ .  $\delta_0 = \hat{\delta}_t^{j_0}$ .  $di = \hat{i}_t - \hat{i}_{t-1}$ .  $dil = \hat{i}_{t+1} - \hat{i}_t$ .  $dq = \hat{q}_t - \hat{q}_{t-1}$ .  $ds = \hat{e}_t - \hat{e}_{t-1}$ .  
 $i = \hat{i}_t$ .  $mc = \hat{m}c_t$ .  $pc = \hat{p}_t^c$ .  $ph = \hat{p}_t^h$ .  $pic = \pi_t^c$ .  $pih = \pi_t^h$ .  $q = \hat{q}_t$ .  $r = \hat{i}_t - E_t \pi_{t+1}^h$ .  $rd = \hat{i}_t - E_t \pi_{t+1}^h - \hat{\rho}_t^{ho}$ .  
 $rh_0 = \hat{\rho}_t^{ho}$ .  $s = \hat{e}_t$ .  $x = x_t$ .  $xva = x_t^{va}$ .  $y_0 = \hat{y}_t^o$ .  $yf = \hat{y}_t^f$ .  $yva = \hat{y}_t^{va}$ .  $yva_0 = \hat{y}_t^{va_0}$ .  $z = z_t$ .

Exhibit 1 – Domestic technological shock under an optimal discretion policy without the cost channel

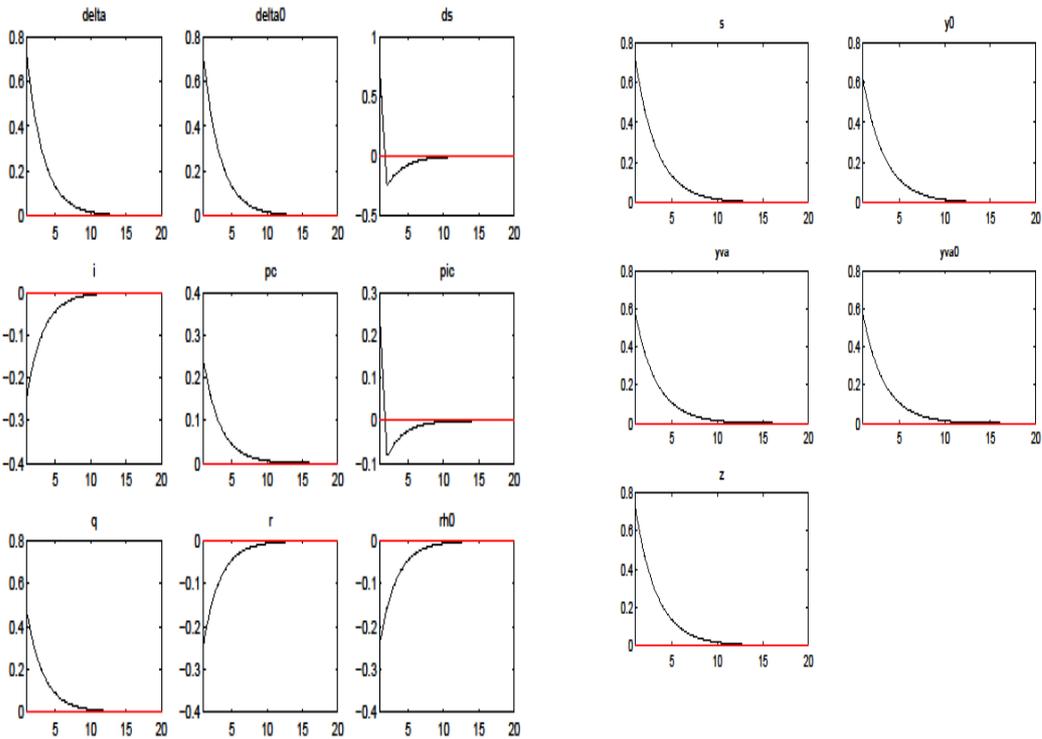


Exhibit 2 – Domestic technological shock under an optimal discretion policy with the cost channel

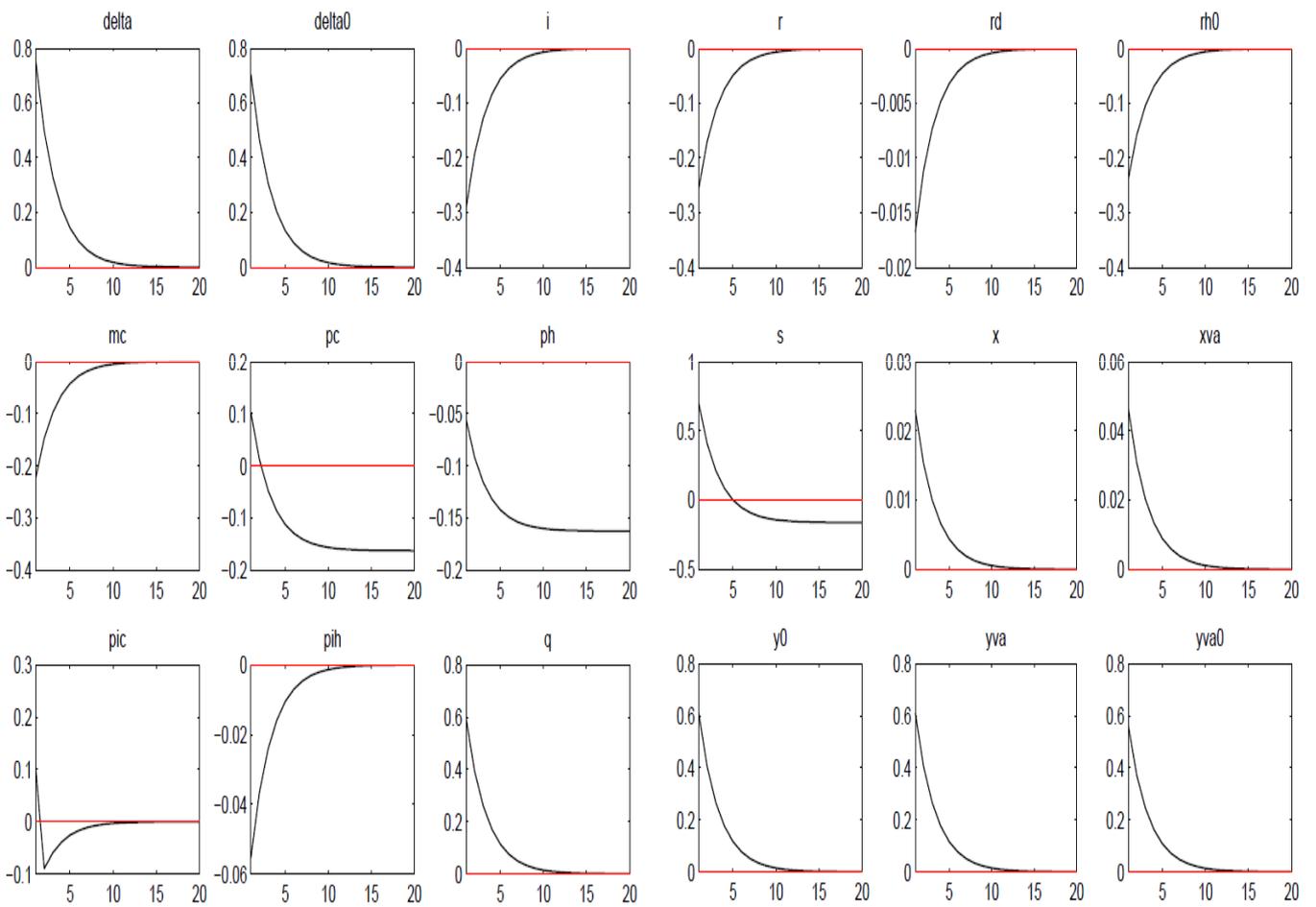


Exhibit 3 – Domestic technological shock under an optimal commitment policy without the cost channel

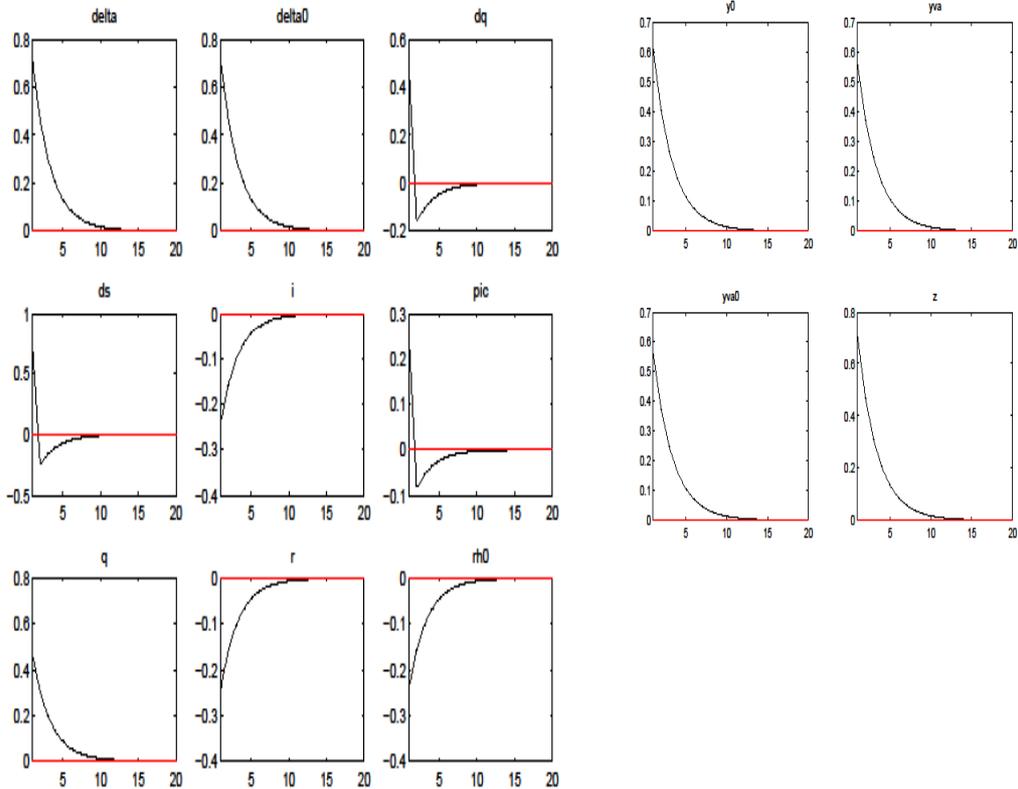


Exhibit 4 – Domestic technological shock under an optimal commitment policy with the cost channel

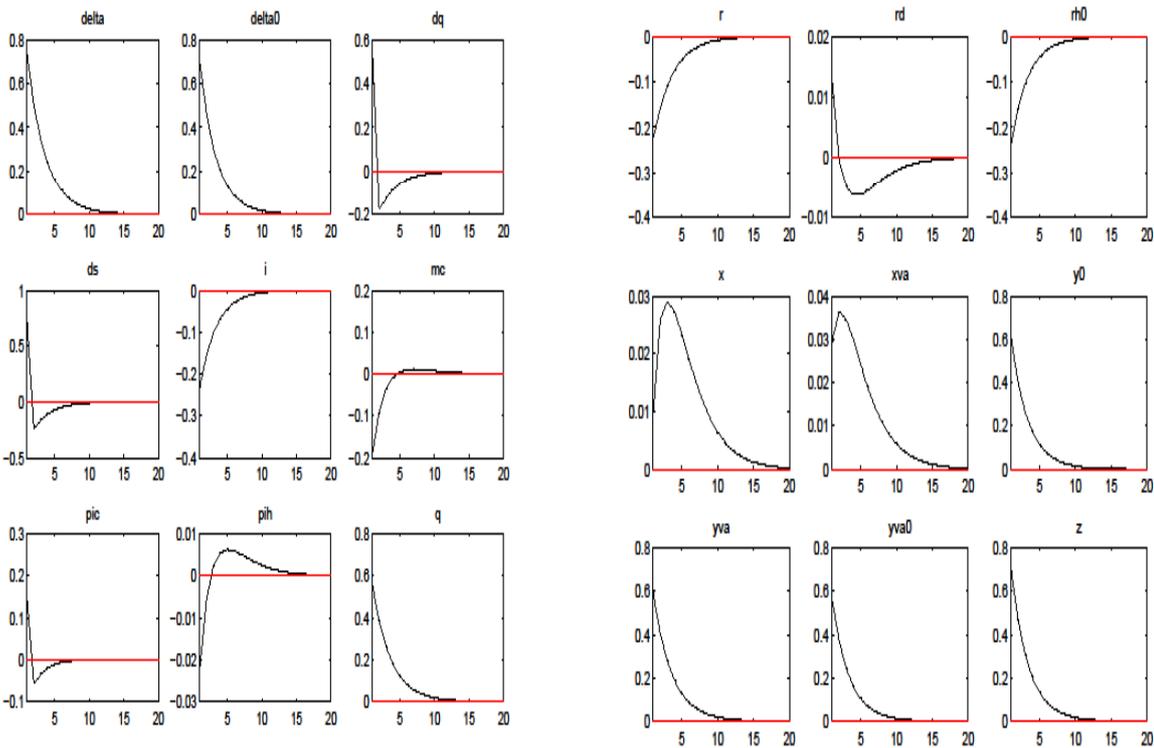
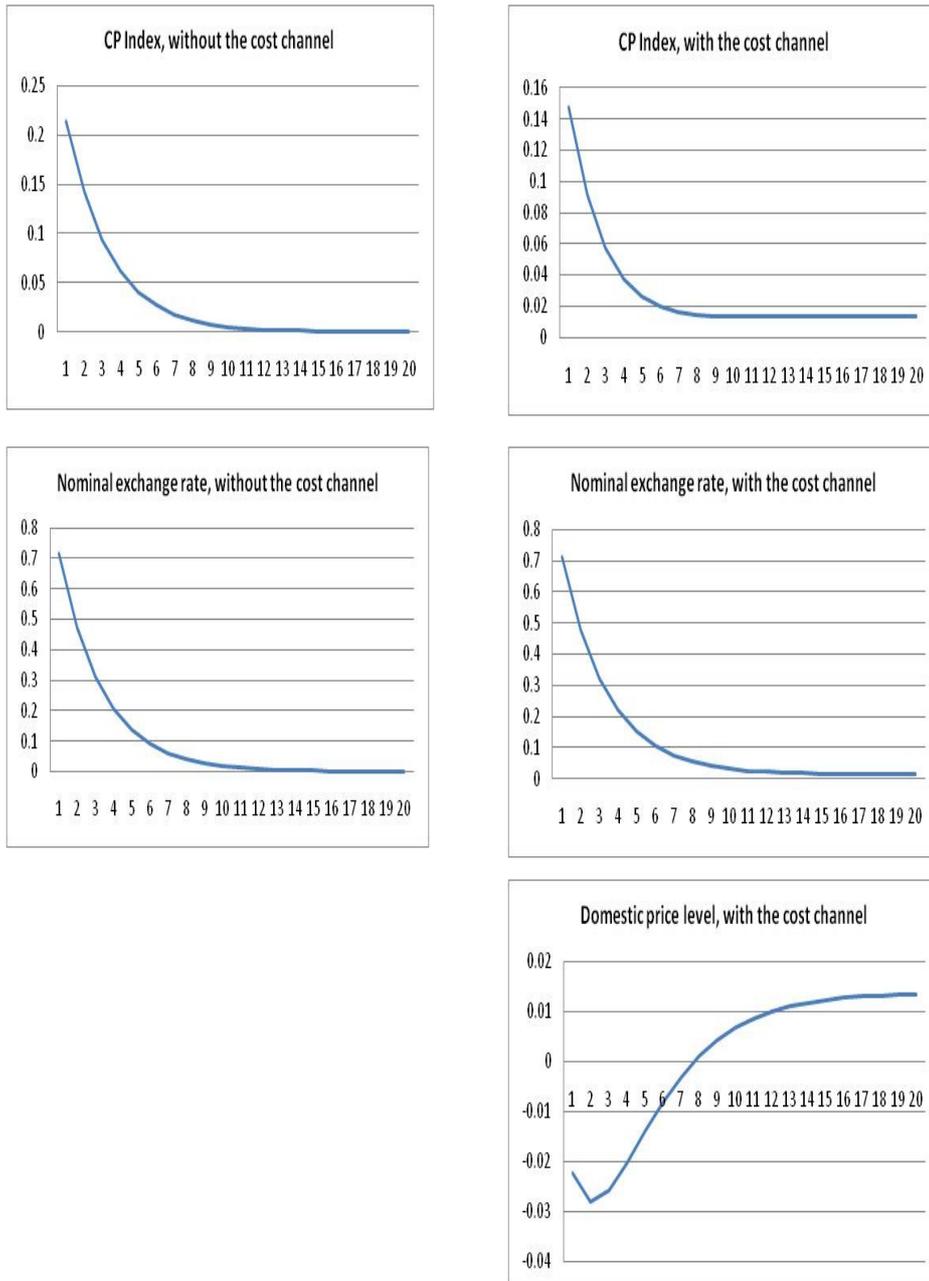


Exhibit 4a – Domestic technological shock under an optimal commitment policy with and without the cost channel: CP Index, Nominal exchange rate and Domestic price index <sup>1</sup>



Note: 1- with the cost channel, the nominal exchange rate converges to 0.013866, while without the cost channel converges to 0.

Exhibit 5— Domestic technological shock under a DITR policy without the cost channel

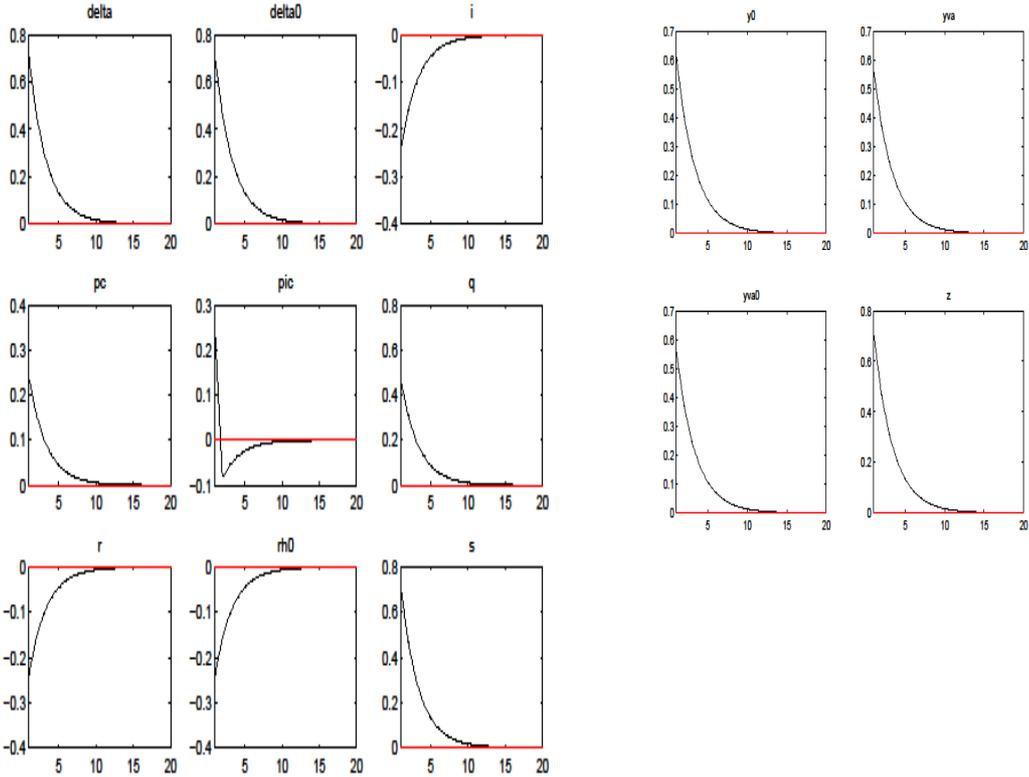


Exhibit 6– Domestic technological shock under a DITR policy with the cost channel

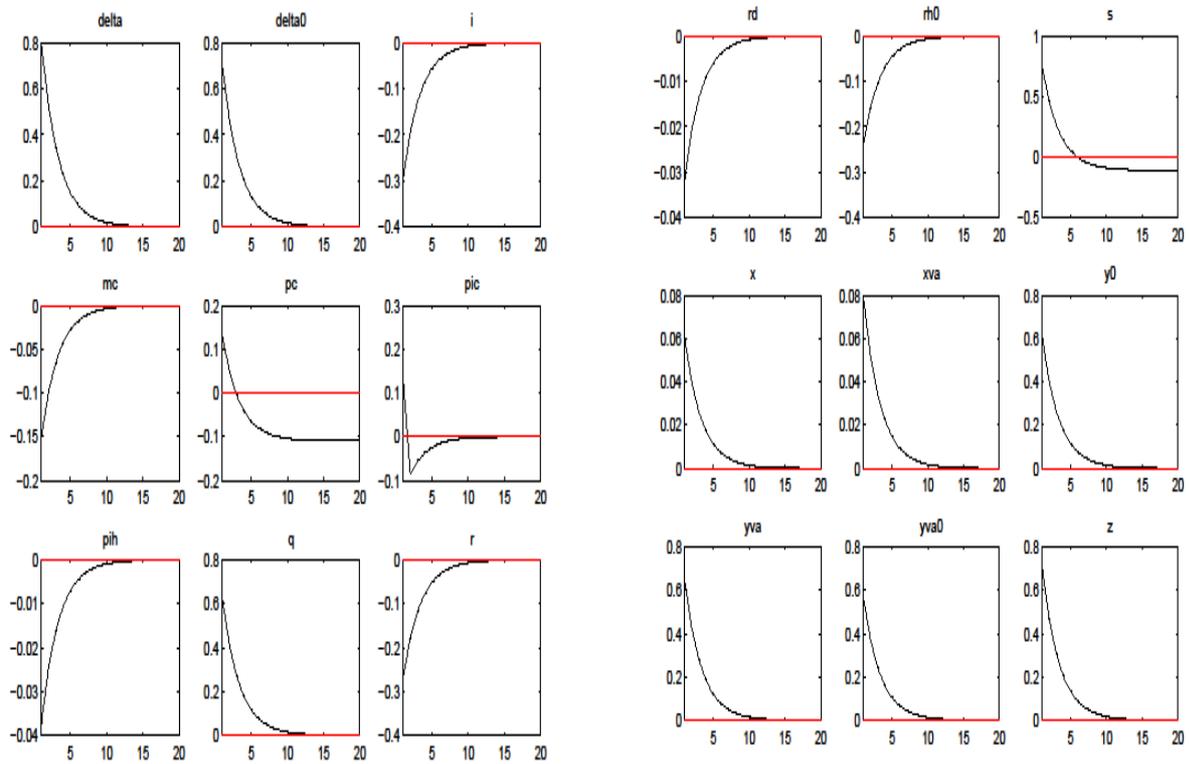


Exhibit 7– Domestic technological shock under a CITR policy without the cost channel

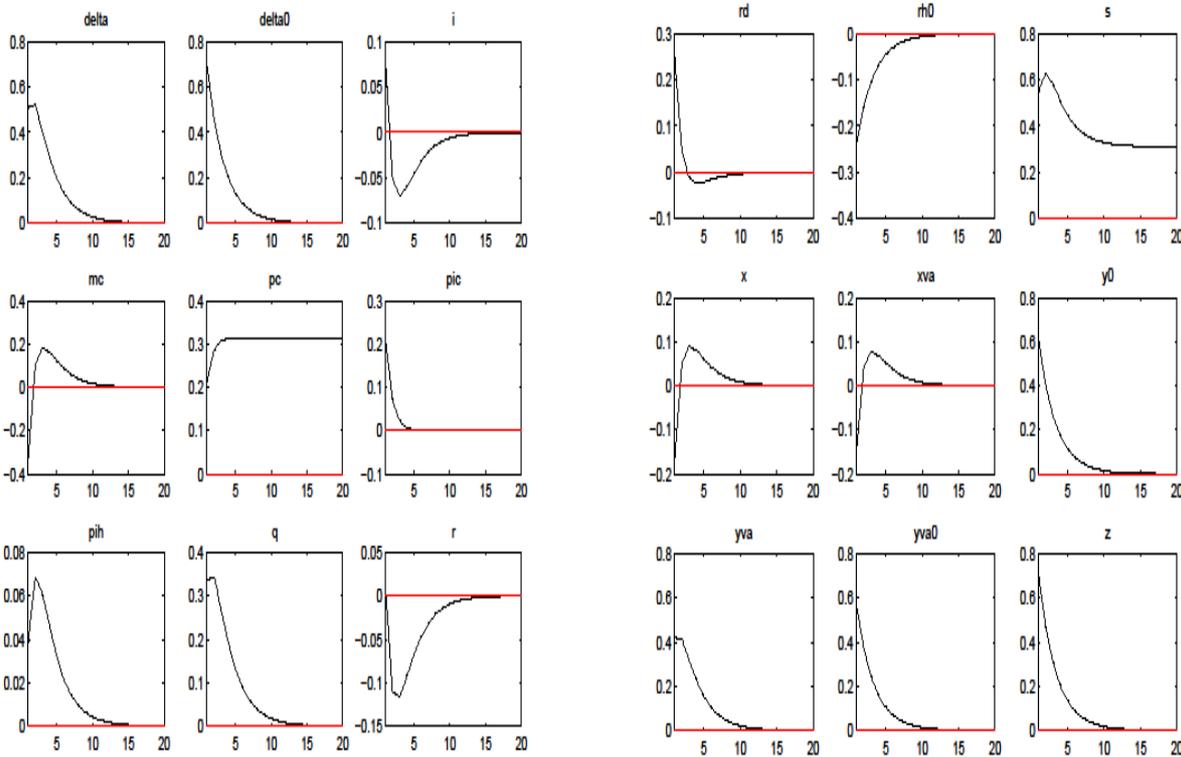


Exhibit 8– Domestic technological shock under a CITR policy with the cost channel

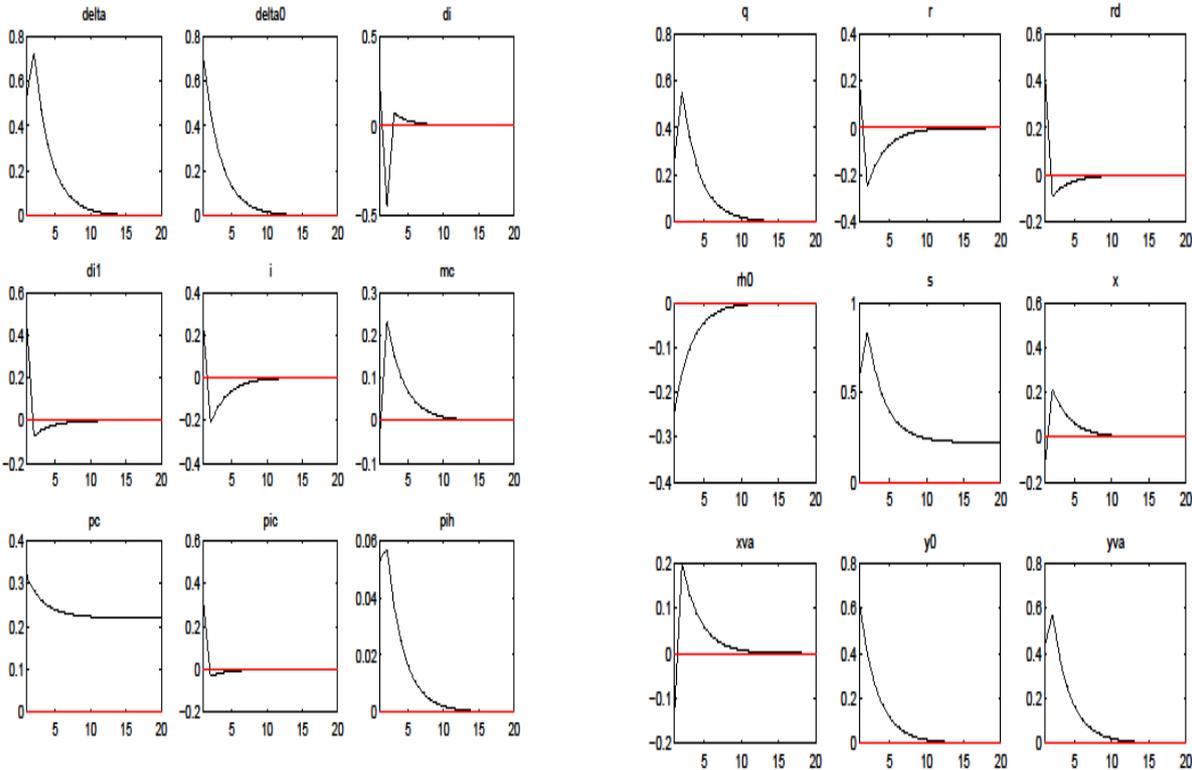


Exhibit 9– Domestic technological shock under an exchange rate peg with or without the cost channel

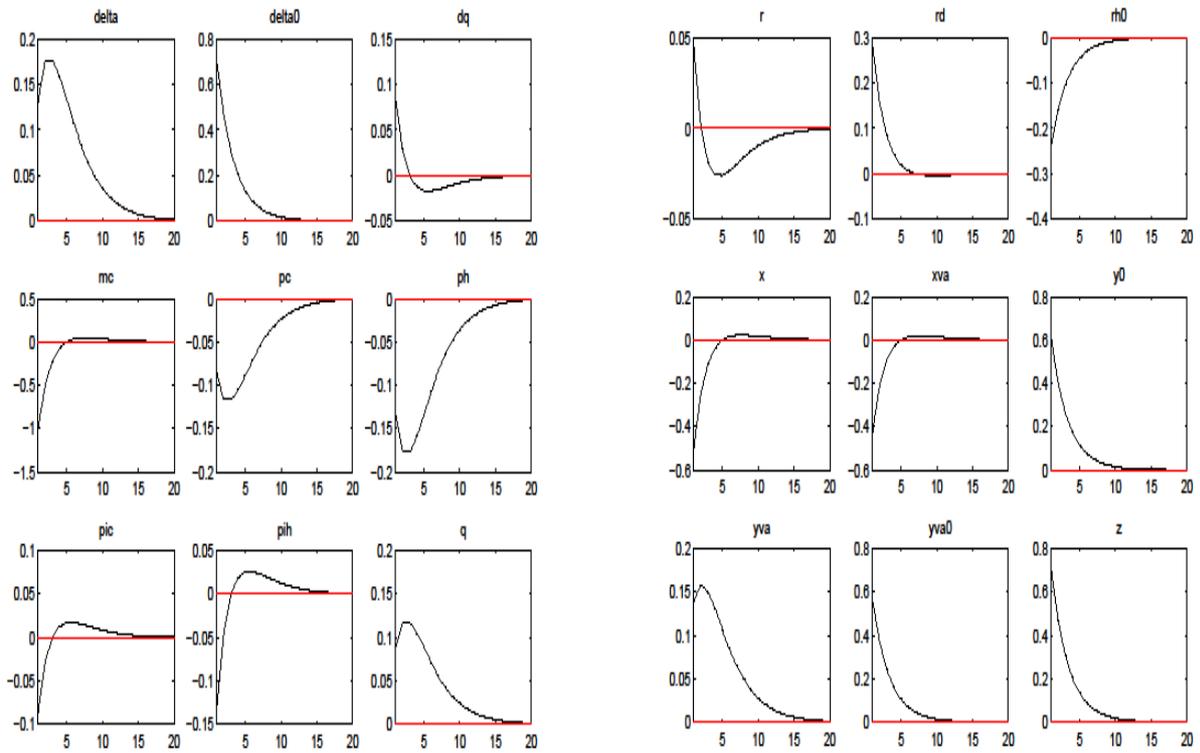


Exhibit 10– Foreign output shock under an exchange rate peg peg without the cost channel

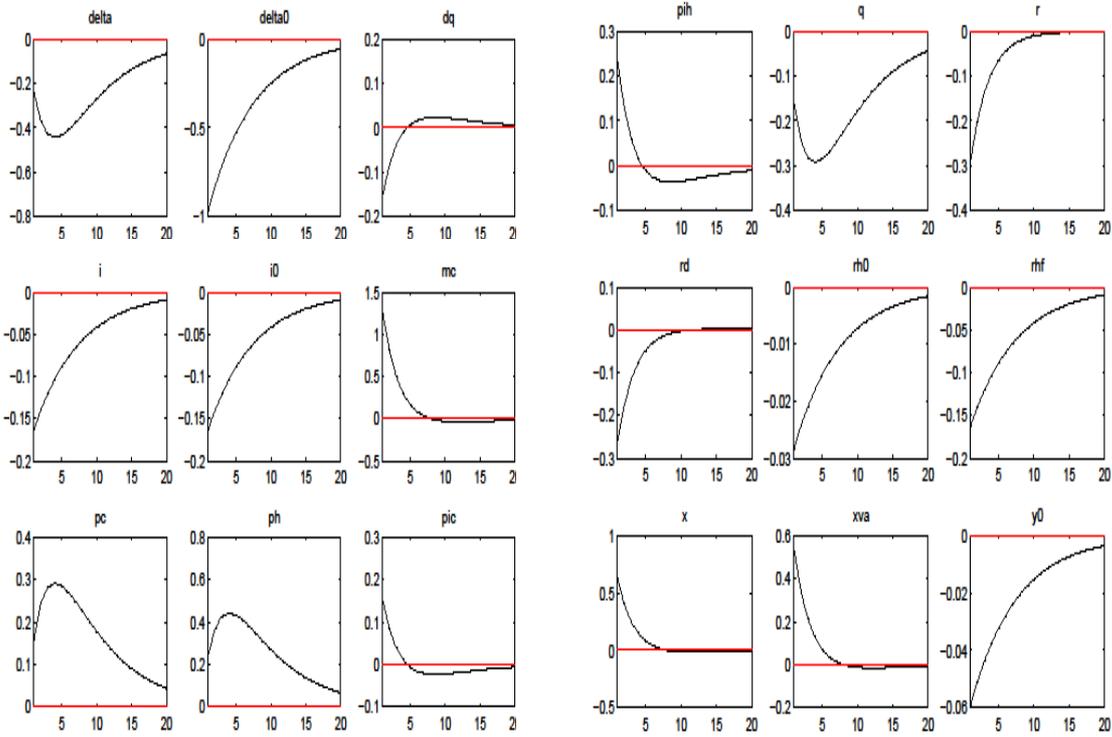


Exhibit 11– Foreign output shock under an exchange rate peg with the cost channel

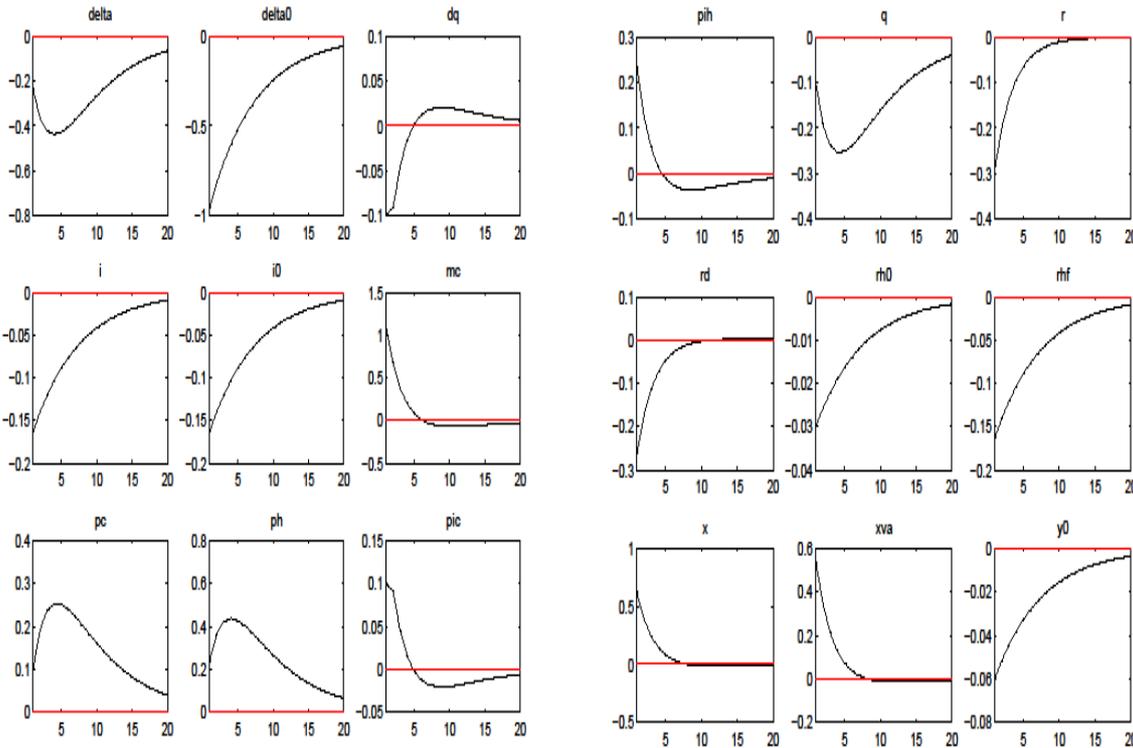


Exhibit 12– Technological shock under a DITR with the cost channel and

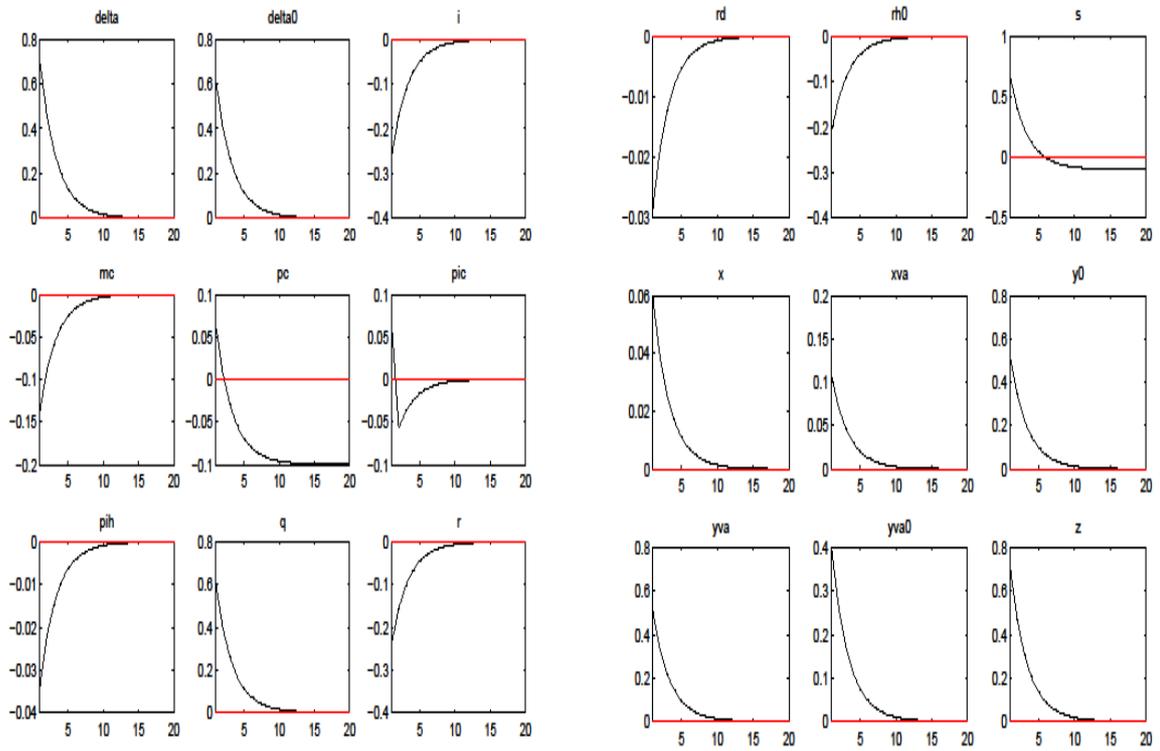


Exhibit 13– Interest rate shock under a DITR without the cost channel

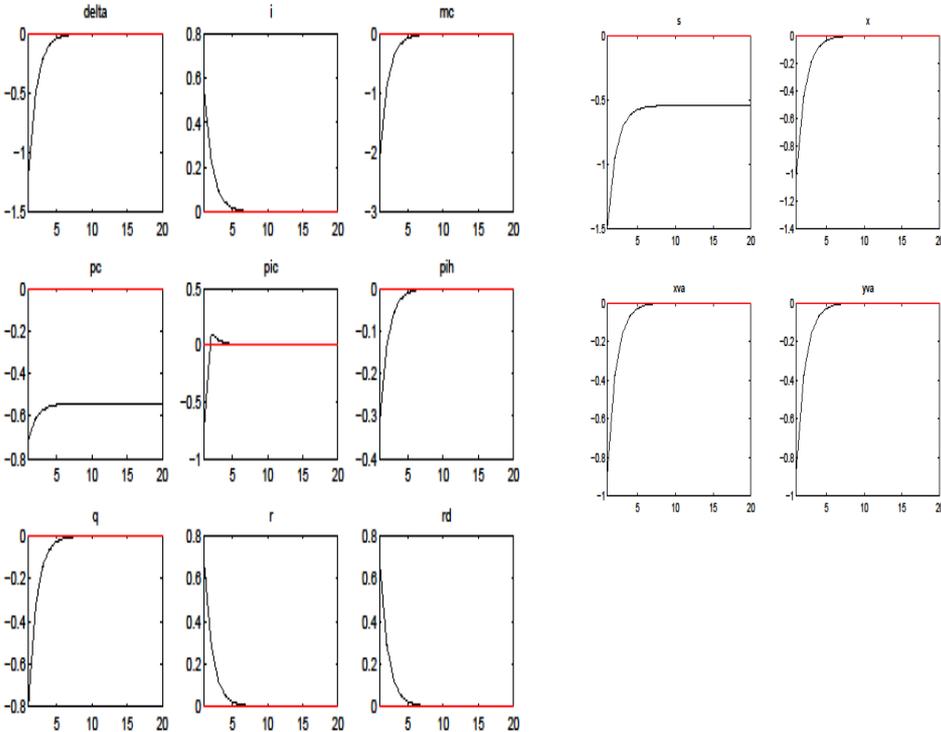


Exhibit 14– Interest rate shock under a DITR with the cost channel

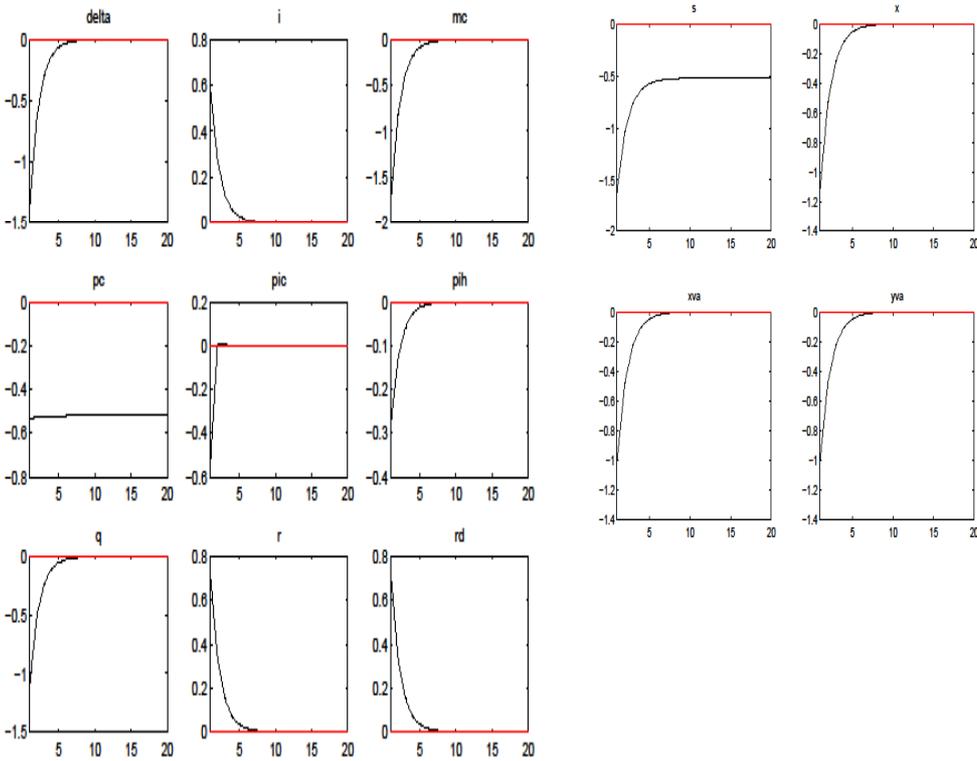


Exhibit 15– Interest rate shock under a DITR without the cost channel ( $w=0.9$ )

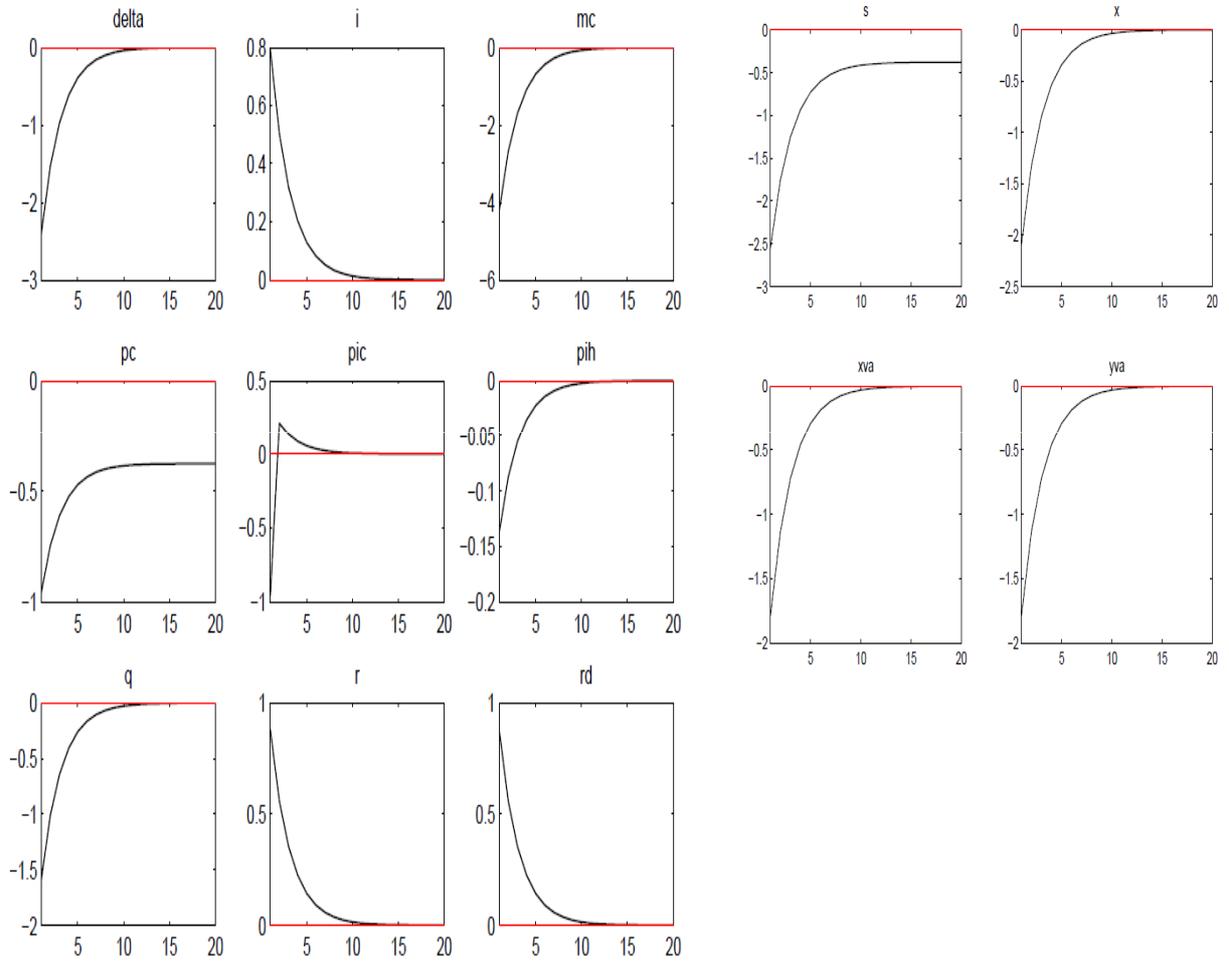


Exhibit 16– Interest rate shock under a DITR with the cost channel ( $w=0.9$ )

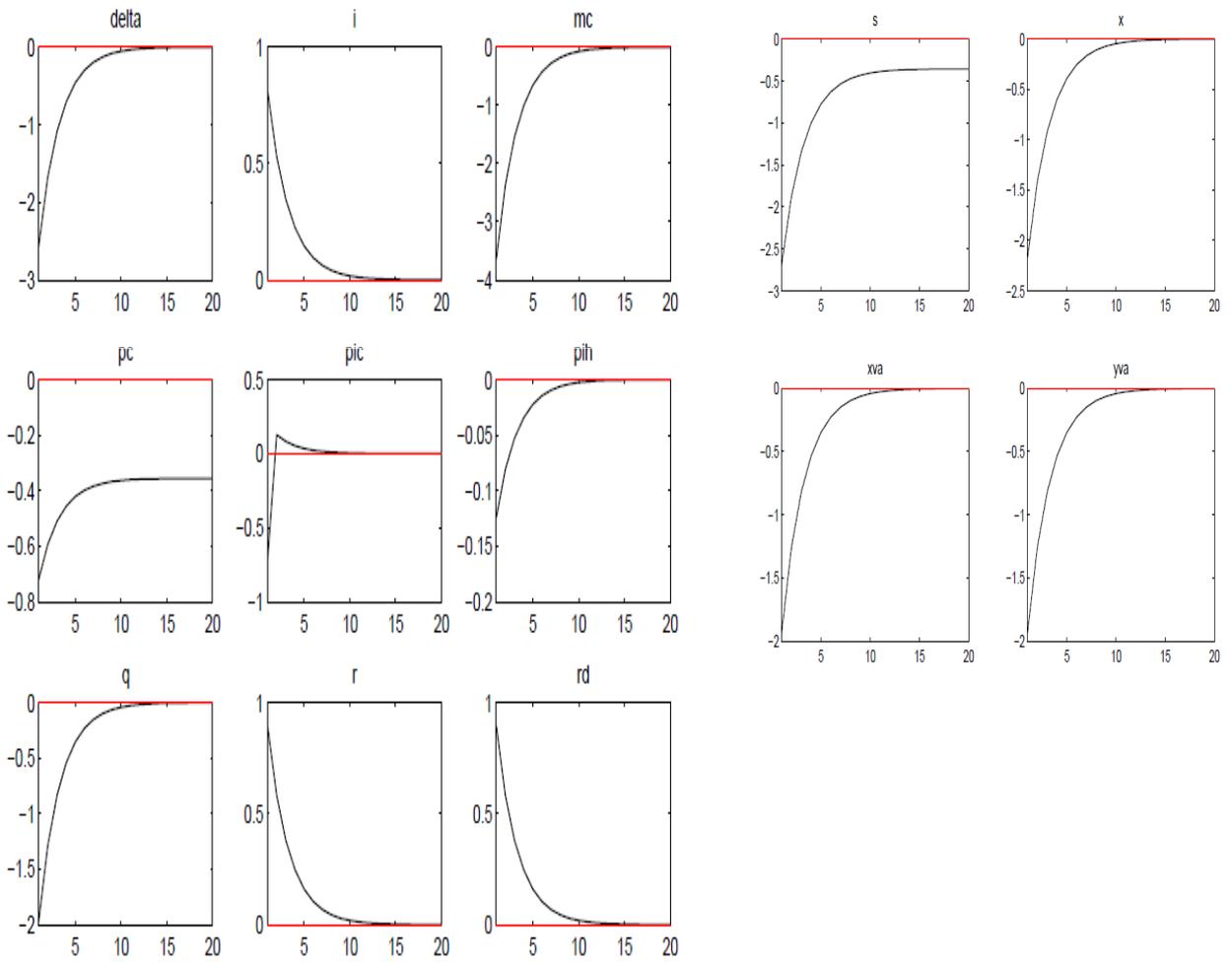


Exhibit 17– Interest rate shock under a DITR without the cost channel (lambda=0.2)

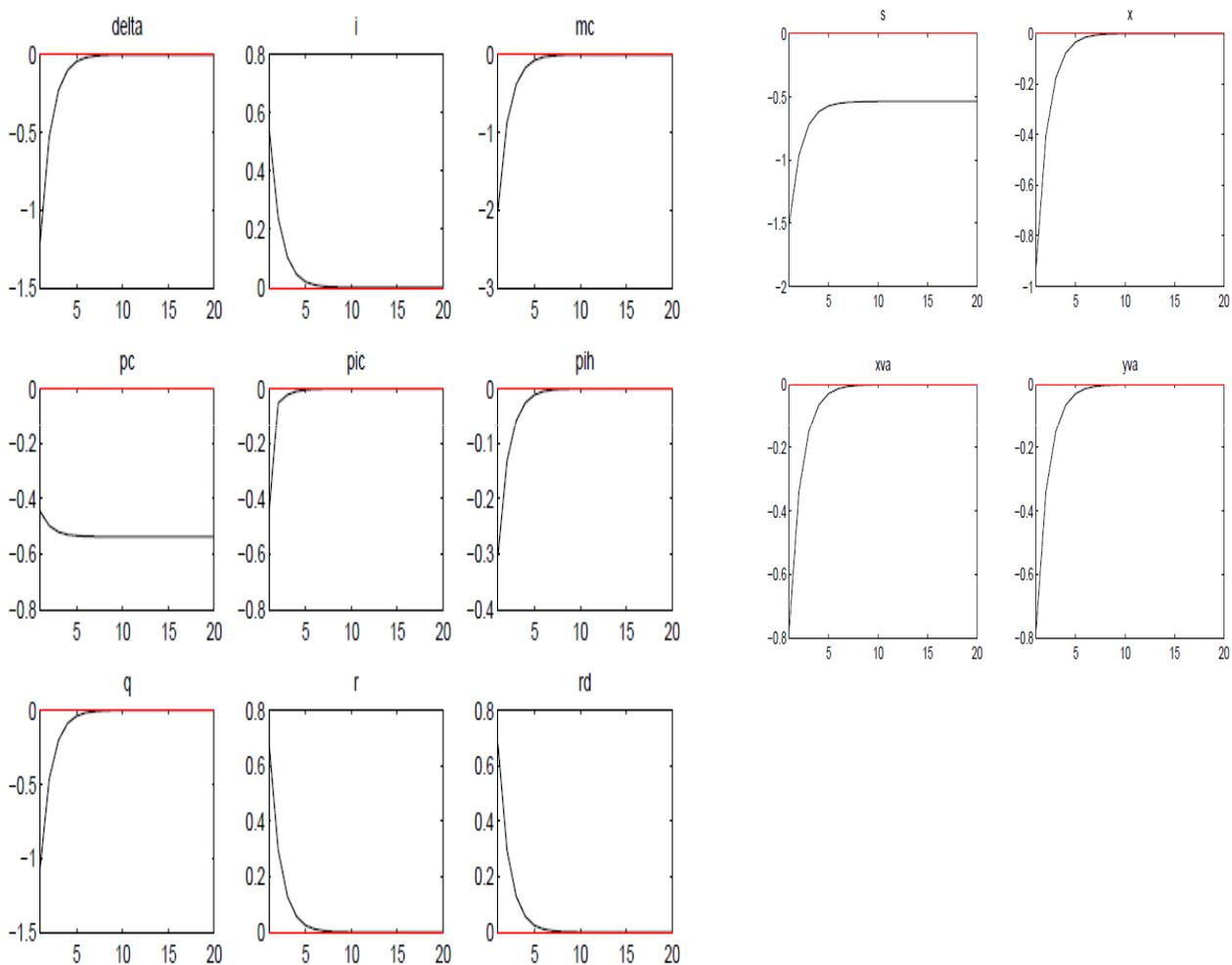


Exhibit 18– Interest rate shock under a DITR with the cost channel (lambda=0.2)

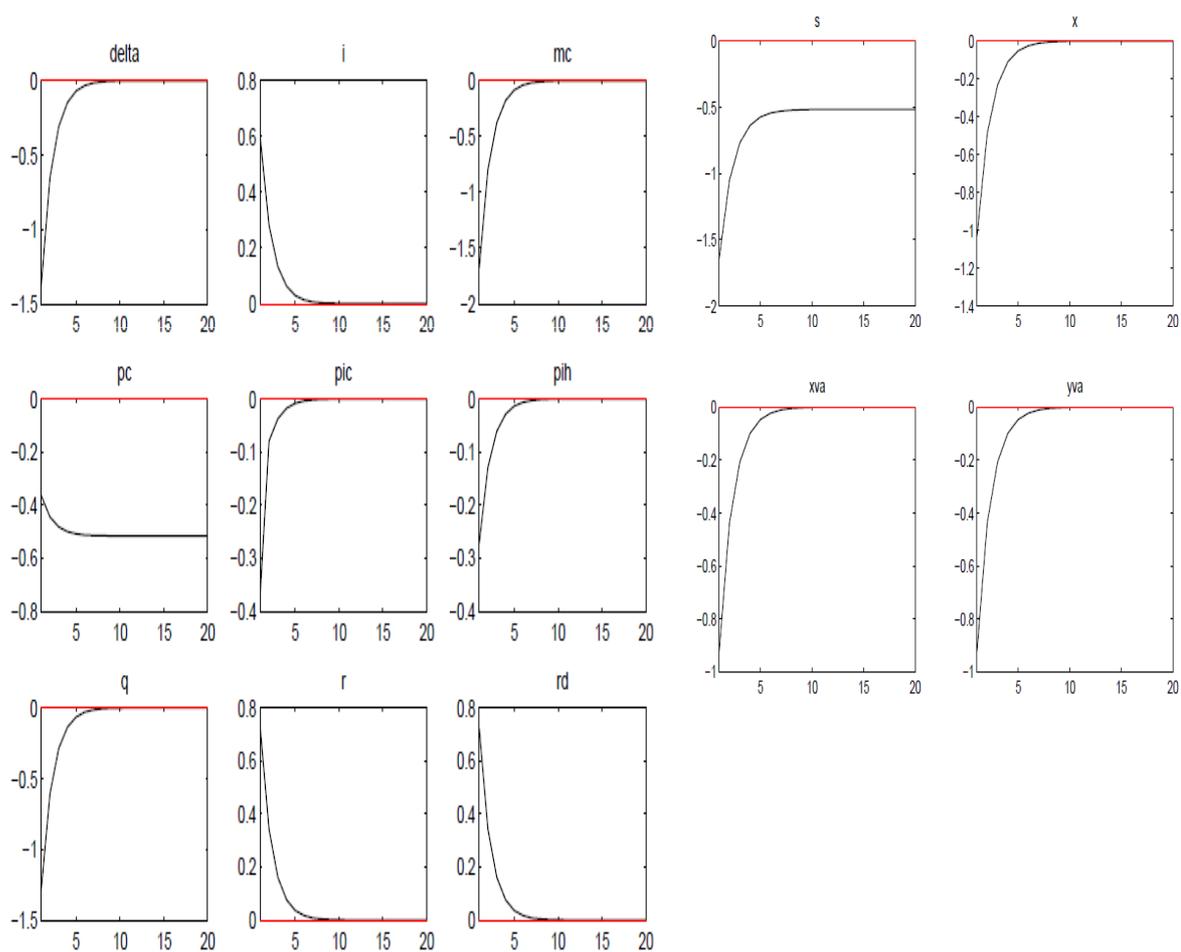


Exhibit 19– Interest rate shock under a DITR without the cost channel (sigma=2)

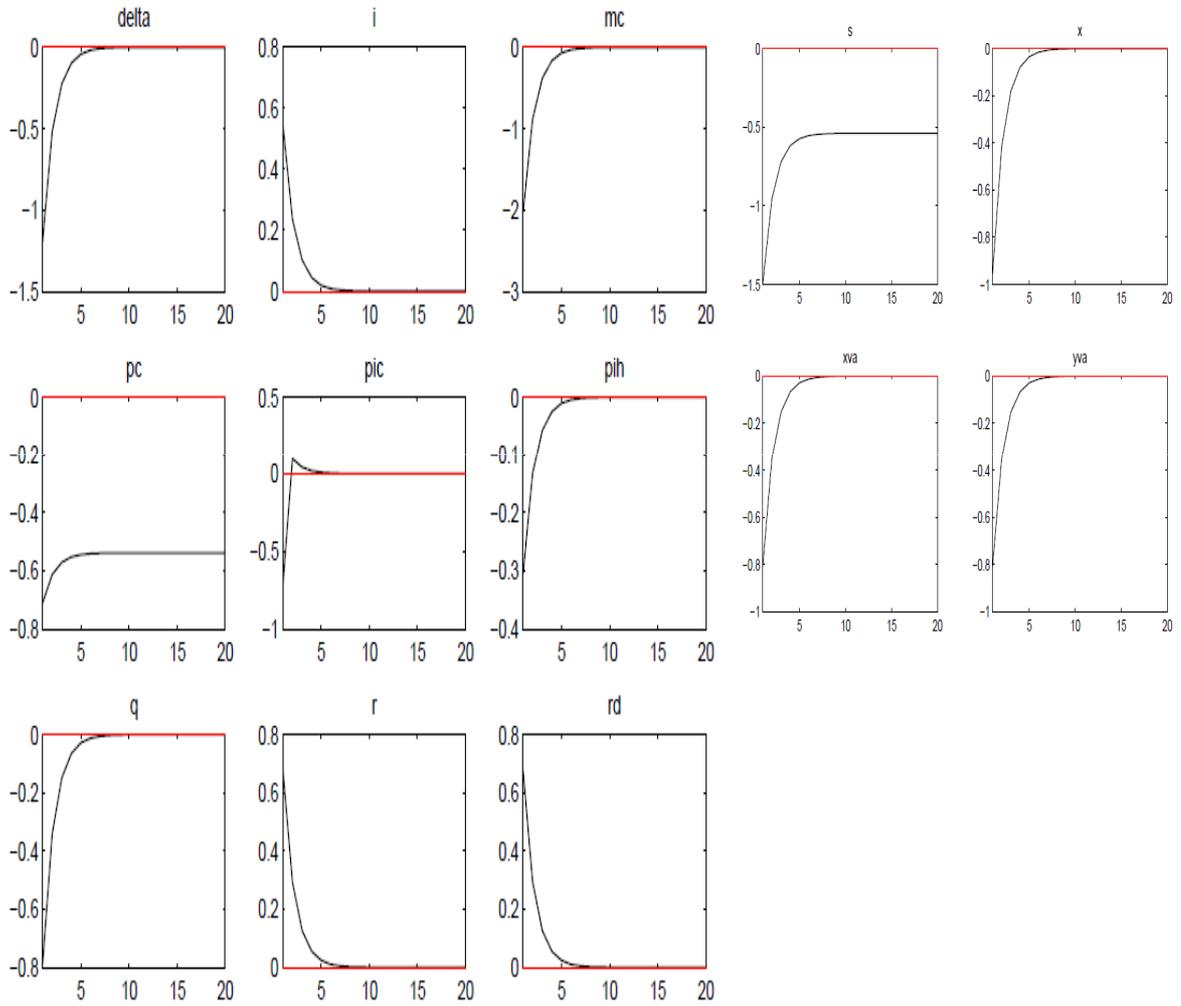


Exhibit 20– Interest rate shock under a DITR with the cost channel ( $\sigma=2$ )

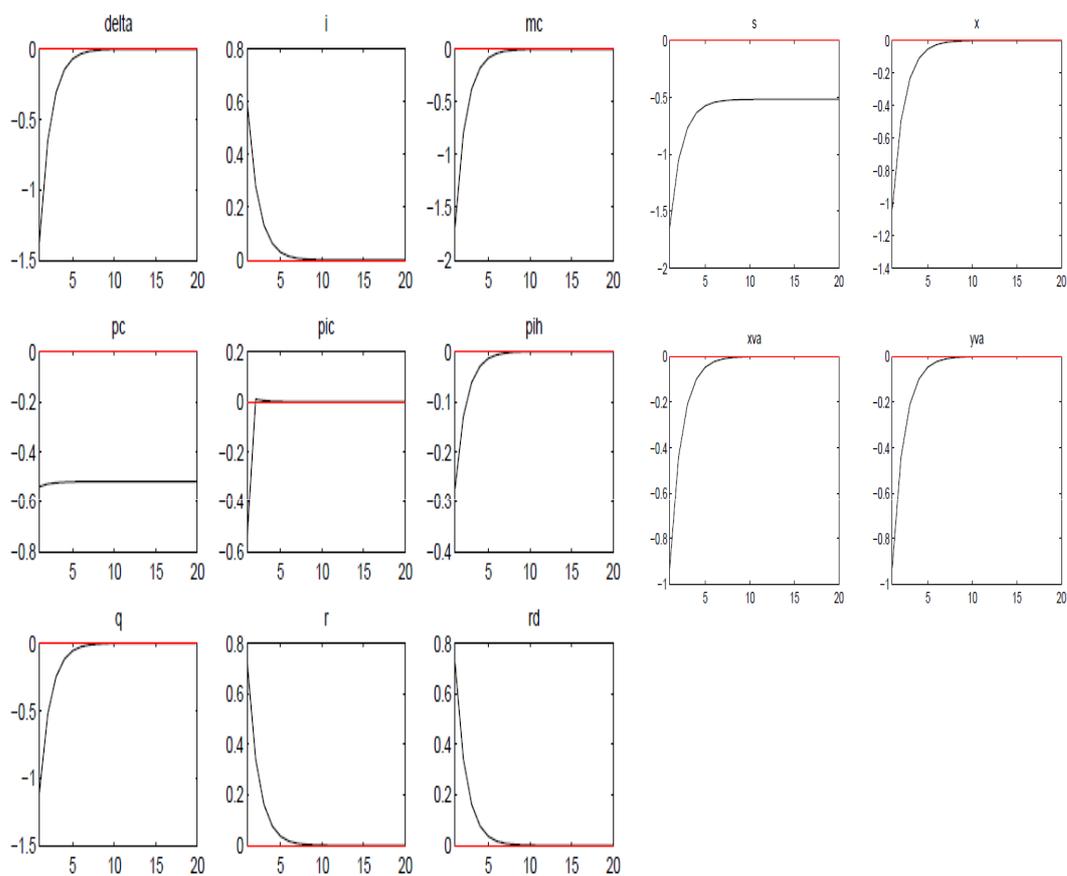


Exhibit 21– Interest rate shock under a DITR without the cost channel (eta=2)

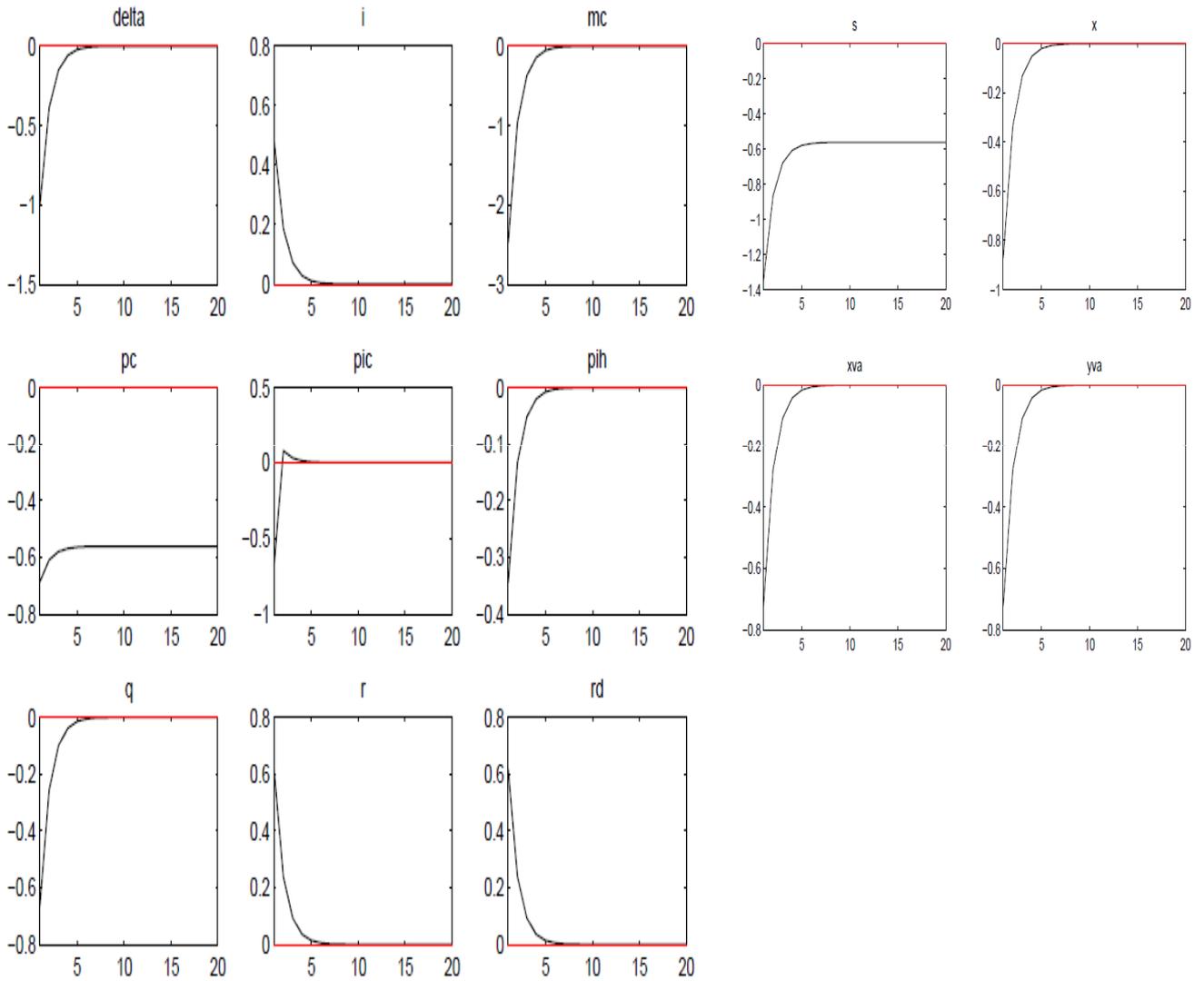


Exhibit 22– Interest rate shock under a DITR with the cost channel ( $\eta=2$ )

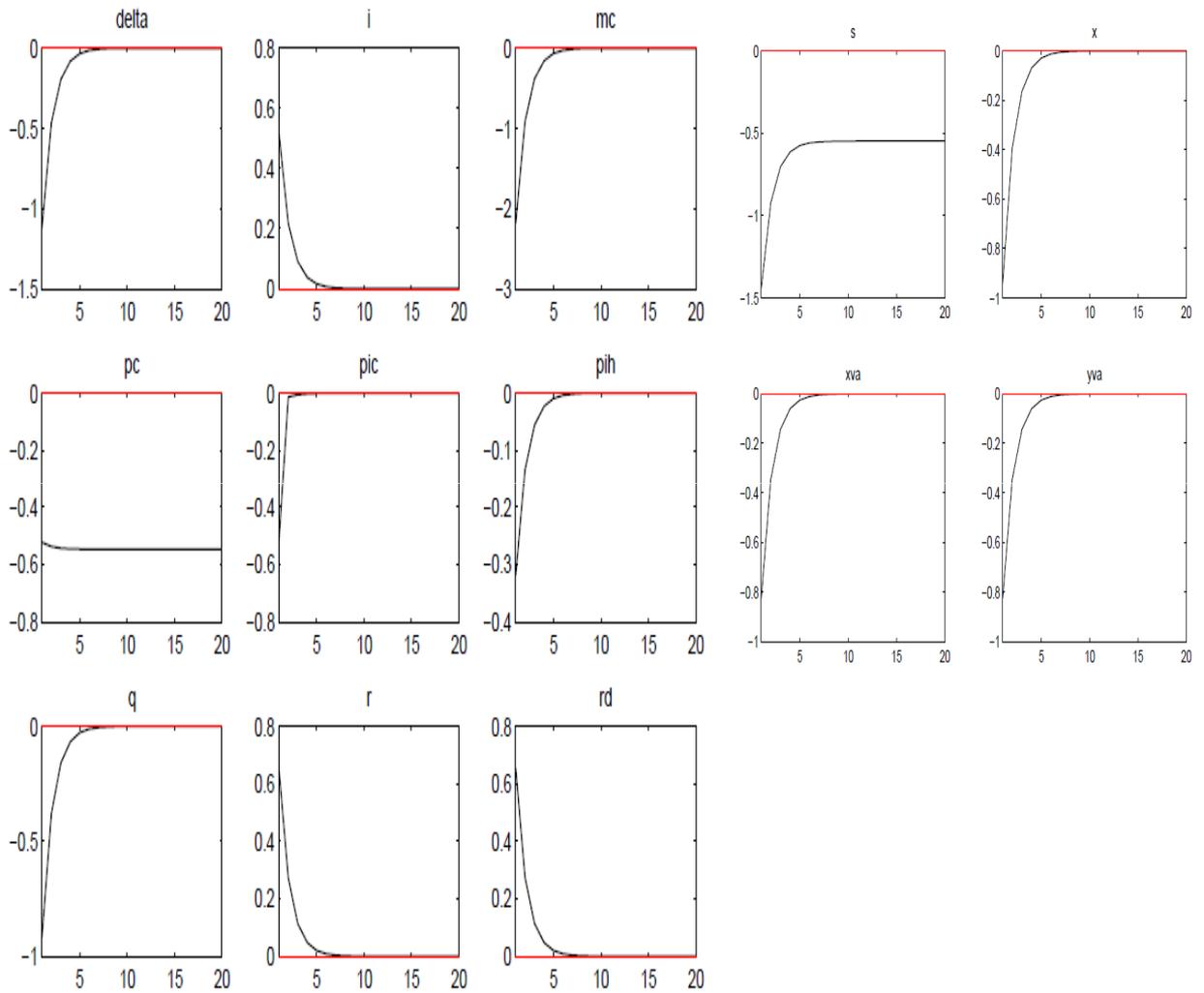


Exhibit 23– Interest rate shock under a DITR without the cost channel (epsilon=1.5)

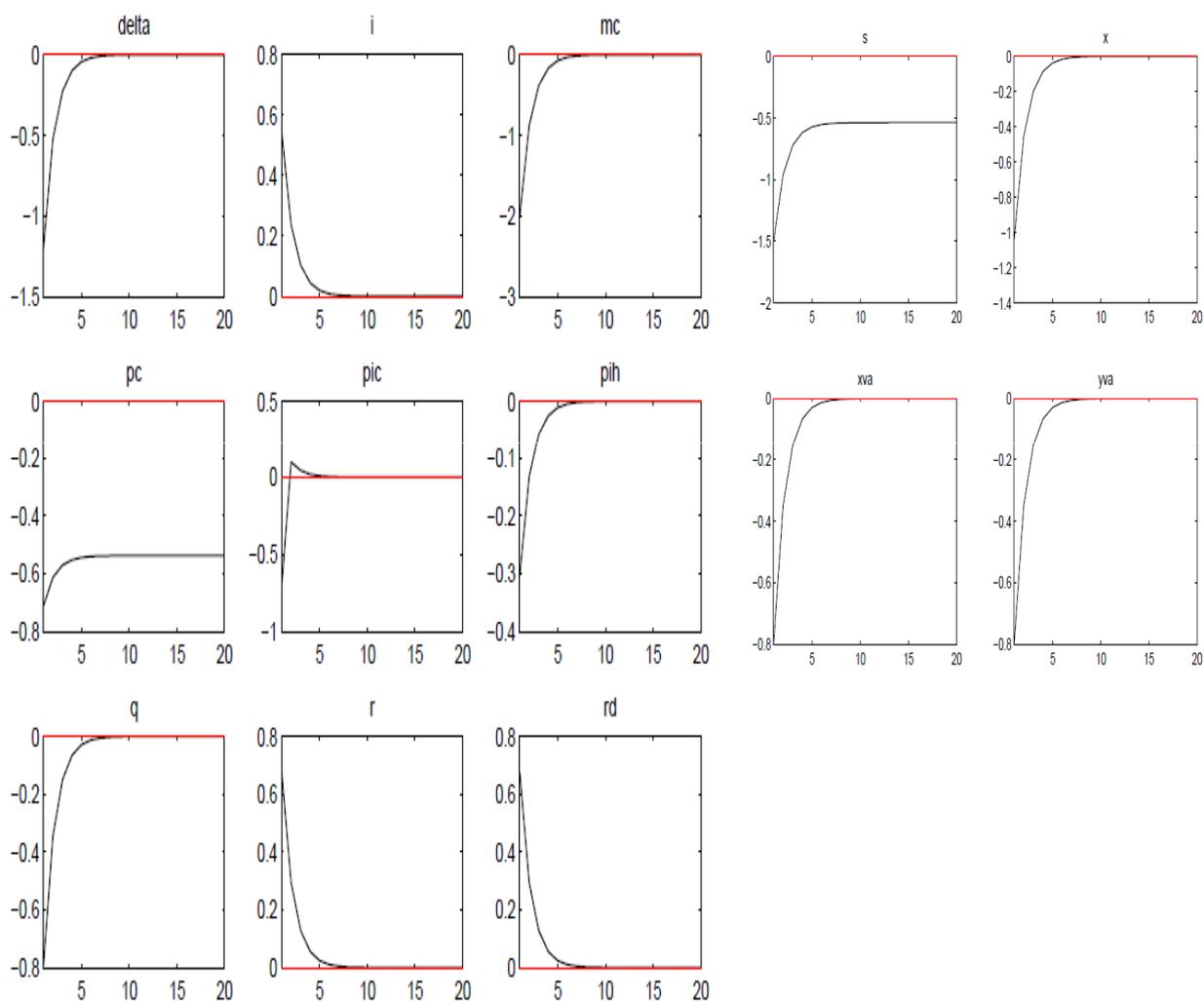


Exhibit 24– Interest rate shock under a DITR with the cost channel (epsilon=1.5)

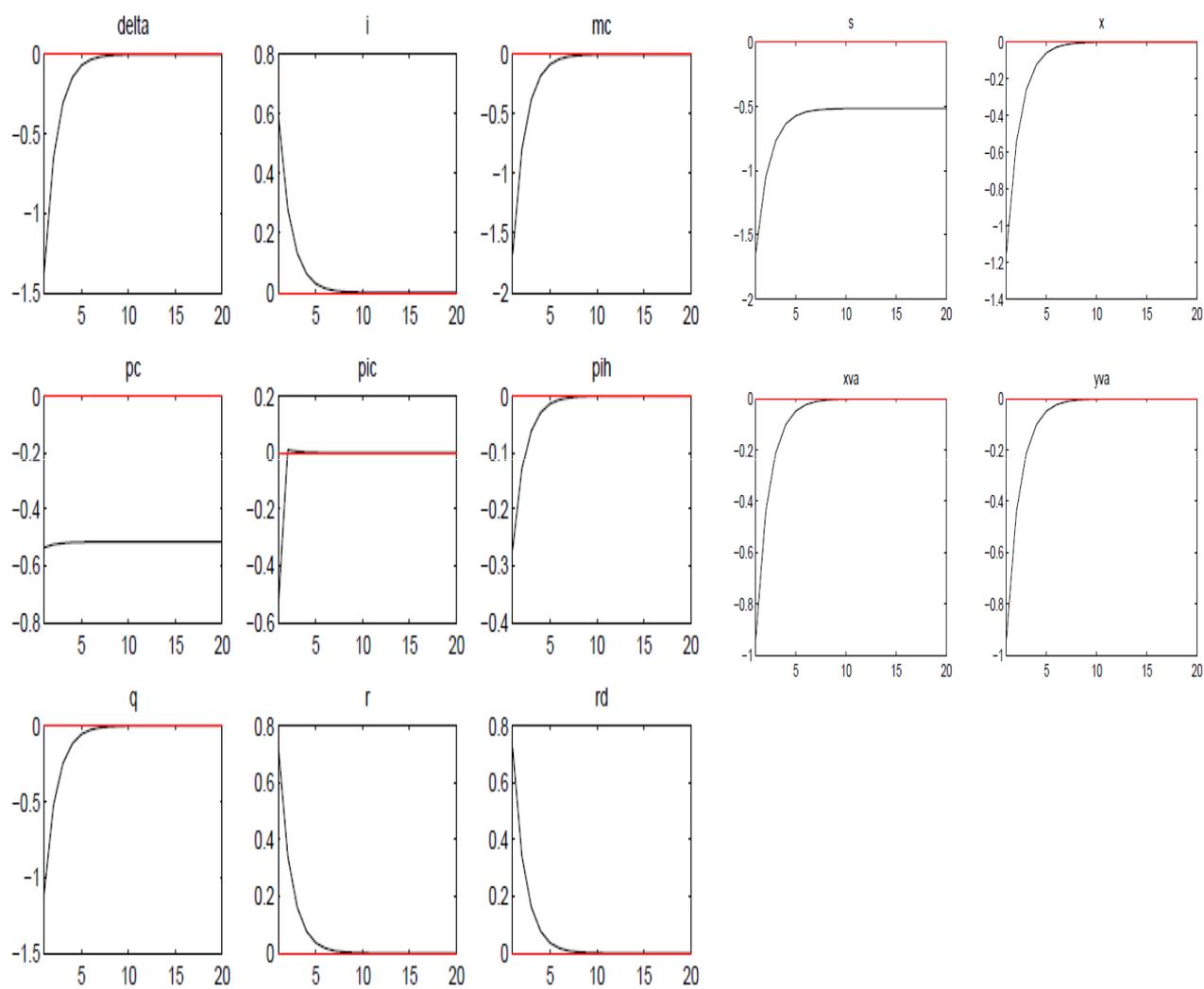


Exhibit 25– Variance of the main variables with the three shocks (w=0.9)

Variance of the main variables

	Interest rate		Terms of trade		Change nominal exchange rate		CPI inflation	
	no cost	cost	no cost	cost	no cost	cost	no cost	cost
	Optimal commitment	0.1645	0.1818	3.6275	3.6286	1.2336	1.2663	0.1426
Optimal discretion	0.1645	0.1856	3.6275	3.6376	1.2336	1.2675	0.1426	0.1152
Optimal DITR	0.1645	0.1794	3.6275	3.5991	1.2336	1.2465	0.1426	0.114
Optimal CITR	0.2923	0.5304	6.4045	7.0396	1.2026	1.4203	0.1679	0.3173
Exchange rate peg	0.1100	0.1100	0.4988	0.4802	0	0	0.0044	0.0041
	Domestic inflation		Output gap (output)		Output gap (value added)		Loss function	
	no cost	cost	no cost	cost	no cost	cost	no cost	cost
	Optimal commitment	0	0.0001	0	0.0001	0	0.0045	0
Optimal discretion	0	0.00017	0	0.000001	0	0.0045	0	0.00017
Optimal DITR	0	0.00015	0	0.00021	0	0.00159	0	0.00019
Optimal CITR	0.0166	0.0222	0.4244	0.5632	0.3491	0.4606	0.092992	0.1236
Exchange rate peg	0.0101	0.0097	1.6373	1.5891	1.199	1.1799	0.3048	0.2957

Correlation between CPI inflation and the change in the nominal exchange rate

	Without the cost channel	With the cost channel
Optimal commitment	1	0.9186
Optimal discretion	1	0.9176
DITR	1	0.9219
CITR	0.9807	0.8335

Exhibit 26– Variance of the main variables with the three shocks (gamma=0.2)

Variance of the main variables

	Interest rate		Terms of trade		Change nominal exchange rate		CPI inflation	
	no cost	cost	no cost	cost	no cost	cost	no cost	cost
	Optimal commitment	0.2161	0.2028	4.0631	4.0310	1.4505	1.4417	0.0208
Optimal discretion	0.2161	0.2989	4.0631	4.0733	1.4505	1.5043	0.0208	0.0253
Optimal DITR	0.2161	0.3139	4.0631	4.1456	1.4505	1.6244	0.0208	0.0177
Optimal CITR	0.1885	0.3633	3.9279	3.8915	1.0559	1.2110	0.0172	0.0257
Exchange rate peg	0.1100	0.1100	1.6491	1.6472	0	0	0.0694	0.0635
	Domestic inflation		Output gap (output)		Output gap (value added)		Loss function	
	no cost	cost	no cost	cost	no cost	cost	no cost	cost
	Optimal commitment	0	0.0011	0	0.0053	0	0.0107	0
Optimal discretion	0	0.0088	0	0.0015	0	0.0095	0	0.00907
Optimal DITR	0	0.0023	0	0.0145	0	0.0229	0	0.00491
Optimal CITR	0.0038	0.0063	0.0188	0.0465	0.013	0.0535	0.007184	0.0147
Exchange rate peg	0.0897	0.0896	0.6679	0.6674	0.4716	0.4809	0.2099	0.2097

Correlation between CPI inflation and the change in the nominal exchange rate

	Without the cost channel	With the cost channel
Optimal commitment	1	0.8095
Optimal discretion	1	0.658
DITR	1	0.7588
CITR	0.9106	0.7113

Exhibit 27– Variance of the main variables with the three shocks (sigma=2)

Variance of the main variables

	Interest rate		Terms of trade		Change nominal exchange rate		CPI inflation	
	no cost	cost	no cost	cost	no cost	cost	no cost	cost
	Optimal commitment	0.1706	0.172	5.4143	5.2430	1.6324	1.6522	0.1887
Optimal discretion	0.1706	0.2723	5.4143	5.2858	1.6324	1.6994	0.1887	0.1960
Optimal DITR	0.1706	0.2602	5.4143	5.2654	1.6324	1.7434	0.1887	0.1734
Optimal CITR	0.3393	1.3348	4.4828	4.6056	0.5344	1.1488	0.0645	0.344
Exchange rate peg	0.1956	0.1956	2.3211	2.1776	0	0	0.0528	0.0344
	Domestic inflation		Output gap (output)		Output gap (value added)		Loss function	
	no cost	cost	no cost	cost	no cost	cost	no cost	cost
	Optimal commitment	0	0.0009	0	0.0059	0	0.0076	0
Optimal discretion	0	0.0105	0	0.0018	0	0.0063	0	0.01082
Optimal DITR	0	0.0017	0	0.0155	0	0.0218	0	0.00449
Optimal CITR	0.02706	0.0665	0.1968	0.3926	0.1235	0.3573	0.062484	0.1372
Exchange rate peg	0.1214	0.1146	0.8868	0.8431	0.6351	0.6164	0.2810	0.2664

Correlation between CPI inflation and the change in the nominal exchange rate

	Without the cost channel	With the cost channel
Optimal commitment	1	0.9435
Optimal discretion	1	0.9068
DITR	1	0.9244
CITR	0.9069	0.5307

Exhibit 28– Variance of the main variables with the three shocks ( $\eta=2$ )

Variance of the main variables

	Interest rate		Terms of trade		Change nominal		CPI inflation	
	no cost	cost	no cost	cost	no cost	cost	no cost	cost
Optimal commitment	0.2165	0.2216	3.4758	3.4694	1.2987	1.3321	0.1501	0.1096
Optimal discretion	0.2165	0.3082	3.4758	3.5018	1.2987	1.4005	0.1501	0.1165
Optimal DITR	0.2165	0.3042	3.4758	3.5449	1.2987	1.4657	0.1501	0.112
Optimal CITR	0.1990	0.7934	2.8518	3.0131	0.4474	0.8951	0.0496	0.1965
Exchange rate peg	0.1100	0.1100	1.6148	1.5732	0	0	0.0479	0.0354
	Domestic inflation		Output gap (output)		Output gap (value added)		Loss function	
	no cost	cost	no cost	cost	no cost	cost	no cost	cost
Optimal commitment	0	0.0008	0	0.0047	0	0.0098	0	0.0016
Optimal discretion	0	0.0051	0	0.0033	0	0.0111	0	0.00569
Optimal DITR	0	0.0011	0	0.0112	0	0.0182	0	0.00312
Optimal CITR	0.0222	0.0407	0.1345	0.2573	0.0815	0.228	0.04641	0.0870
Exchange rate peg	0.1101	0.1078	0.6102	0.6006	0.4189	0.4196	0.2199	0.2159

Correlation between CPI inflation and the change in the nominal exchange rate

	Without the cost channel	With the cost channel
Optimal commitment	1	0.8746
Optimal discretion	1	0.8271
DITR	1	0.8484
CITR	0.9047	0.5546

Exhibit 29– Variance of the main variables with the three shocks (epsilon=1.5)

Variance of the main variables

	Interest rate		Terms of trade		Change nominal exchange rate		CPI inflation	
	no cost	cost	no cost	cost	no cost	cost	no cost	cost
Optimal commitment	0.1516	0.1489	3.2024	3.1381	1.0891	1.0964	0.1259	0.1033
Optimal discretion	0.1516	na	3.2024	na	1.0891	na	0.1259	na
Optimal DITR	0.1516	0.2242	3.2024	3.1884	1.0891	1.1954	0.1259	0.1042
Optimal CITR	0.2061	0.7412	2.6755	2.5507	0.3833	0.7891	0.0488	0.1812
Exchange rate peg	0.1100	0.1100	1.3153	1.2820	0	0	0.0311	0.0219
	Domestic inflation		Output gap (output)		Output gap (value added)		Loss function	
	no cost	cost	no cost	cost	no cost	cost	no cost	cost
Optimal commitment	0	0.0009	0	0.0050	0	0.0458	0	0.0018
Optimal discretion	0	na	0	na	0	na	0	na
Optimal DITR	0	0.0019	0	0.013	0	0.017	0	0.0042
Optimal CITR	0.018	0.0368	0.1427	0.2392	0.0837	0.1427	0.0437	0.0799
Exchange rate peg	0.0715	0.0699	0.6621	0.6505	0.3899	0.4211	0.1907	0.1870

Correlation between CPI inflation and the change in the nominal exchange rate

	Without the cost channel	With the cost channel
Optimal commitment	1	0.9098
Optimal discretion	1	na
DITR	1	0.8851
CITR	0.9165	0.5442

Exhibit 30– Interest rate shock under a DITR with output gap and without the cost channel

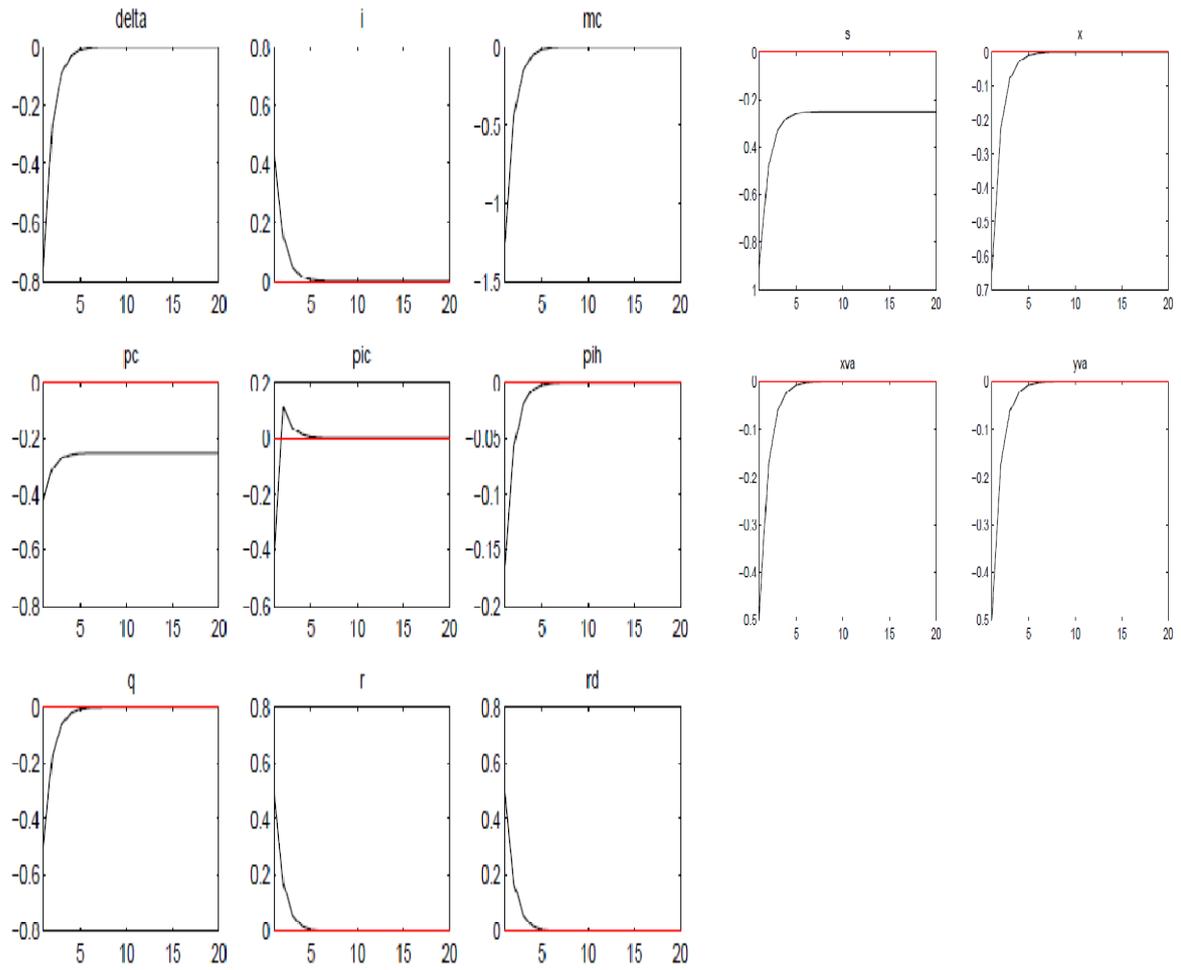
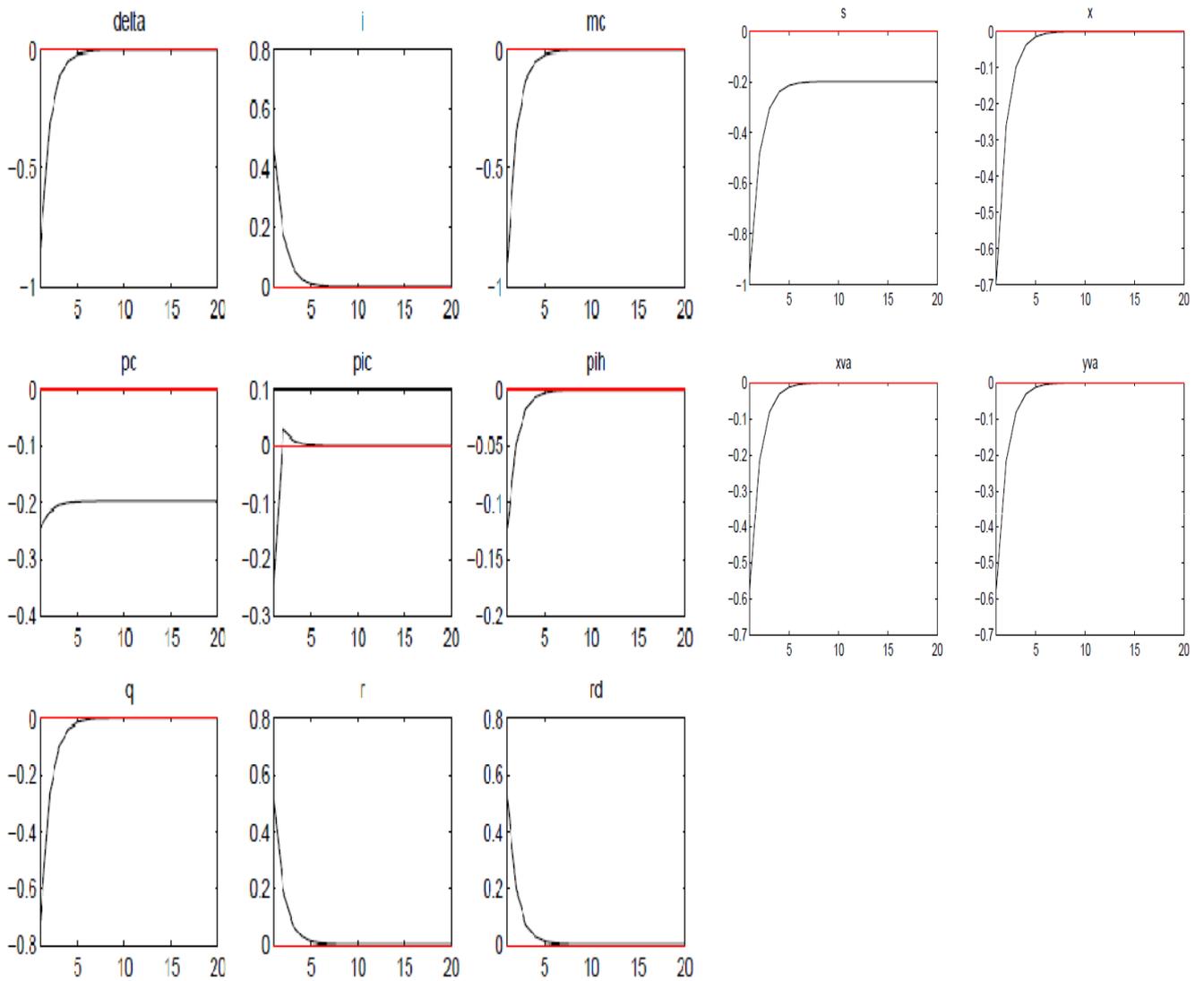


Exhibit 31– Interest rate shock under a DITR with output gap and with the cost channel



## Chapter 3

# Inflation dynamics in open economies: empirical evidence for G7 countries on the role of import prices and the cost channel

### 3.1 Introduction

The main goal of this paper is to assess the empirical relevance of the cost channel in an open economy context using the New Keynesian Phillips curve (NKPC). Ravenna and Walsh (2006) and Chowdbury et al. (2003), using a Cobb-Douglas production function, did not emphasize open economy variables in the NKPC when estimating the cost channel. In this paper, assuming a more general CES production function, such variables are considered simultaneously with the cost channel. Then, the marginal cost and supply side inflation depend simultaneously on the nominal interest rate and the terms of trade; and these variables are typically related with each other. Thus, it is only in some circumstances that the cost channel will be correctly identified when the terms of trade are ignored. For example, the cost channel will not be identified properly when the nominal interest rate affects *directly* the terms of trade. In this way, the relevance of assessing the supply side effect of the interest rate considering also the impact of the terms of trade on inflation is clear.

As we need to introduce open economy variables in the Phillips curve to properly estimate the cost channel, it is necessary to identify the correct way of doing that. However, regarding this issue a theoretical consensus does not exist. On one hand, some works on the open economy NKPC (e.g. Gali and Monacelli, 2002) assume imports as final consumption goods and ignore imported intermediate goods. On the other hand, McCallum and Nelson (2000) treat imports only as intermediate goods. In the model of Chapter 2, imports are treated simultaneously as inputs and final consumption goods. For the UK, Kara and Nelson (2002) concluded that a model where imports are modelled only as intermediate goods provides a reasonable match with the data. The goal of this chapter is to research whether or not that conclusion can be applied to other countries as well.

Another issue that will be tested when estimating the effect of import prices on the Phillips curve is the aggregate relevance of slow exchange rate pass-through. This issue is relevant for the design of monetary policy. Clarida et al. (2001) demonstrate that with immediate exchange rate pass-through the central bank should target domestic inflation. In contrast, if the exchange rate has a slow effect on import prices, it may be optimal for the monetary authority to target CPI inflation (Monacelli (2005)). The same occurs when imports are treated only as inputs. In addition, the advantages of commitment in monetary policy and the degree of exchange rate flexibility that the central bank should allow are also affected by how exchange rate affects import goods inflation.

Finally, it is tested whether or not is empirically relevant to extend the cost channel assuming that imports of final consumption goods are also paid in advance, as studied in Chapter 2.

To our knowledge this is the first paper dedicated to studying the empirical relevance of the cost channel considering explicitly open economy variables. The research into whether or not the data confirm that imports of consumption goods are paid in advance is also new in the context of works on the working capital channel. This paper also contributes to the literature testing for the G7 countries and in the context of the NKPC the empirical relevance of slow exchange rate pass-through. Lastly, the test for the G7 countries of whether imports should be considered as inputs and/or consumption goods also adds to the literature, because until now such a test has only been attempted for the UK.

Our results indicate that open economy variables play an important role in explaining

inflation dynamics, both for domestic and CPI inflations. The empirical success of the NKPC with slow exchange rate pass-through was larger than with immediate pass-through. This was valid both for imported inputs and imported consumption goods. The model of McCallum and Nelson (2000) where imports are solely considered as inputs in production is rejected by the data. Instead, a model with imports as both consumption goods and inputs has a better empirical adherence. Regarding the cost channel, there is weak evidence that the level of the nominal interest rate affects inflation. However, there is strong evidence that the change in the nominal interest rate affects CPI inflation. This last finding can be explained by the fact that firms pay for imported consumption goods in advance.

The remainder of the paper is organised as follows. Section 3.2 revises the literature on the NKPC. Section 3.3 estimates the NKPC for domestic inflation. Section 3.4 analyses the best empirical way of describing CPI inflation using the NKPC. Section 3.5 concludes the study.

## 3.2 Literature review on the NKPC: inflation, interest rate and import prices

The traditional Phillips curve relates present inflation with past inflation and the output gap:

$$\pi_t = \sum_{k=1}^p \alpha_k \pi_{t-k} + \gamma \hat{y}_t + \eta_t,$$

where  $\pi_t$  is the inflation rate,  $\hat{y}_t$  is the log deviation of output from its steady-state value (output gap), and  $\eta_t$  is an error term. One of the main problems with this equation is that its coefficients cannot be expressed as depending on technology and preference parameters. This implies that the equation is subject to the Lucas Critique: its parameters may change when the policy regime changes. To overcome this limitation, the New Keynesian approach to inflation obtained microfoundations for the Phillips curve.

### 3.2.1 The New Keynesian Phillips Curve

The New Keynesian approach to the Phillips curve uses nominal price rigidity and optimising behaviour to obtain a forward-looking dynamic for aggregate prices. This approach often uses

Calvo's (1983) assumption that identical monopolistically competitive firms optimally adjust their prices at each date with a given probability  $(1 - \omega)$  and with probability  $\omega$  firms do not change prices.<sup>1</sup> Therefore, parameter  $\omega$  is a measure of the degree of nominal rigidity. Using the Calvo (1983) setting, the aggregate price will be given by

$$\widehat{p}_t = \omega \widehat{p}_{t-1} + (1 - \omega) \widehat{p}_t^* \quad (3.1)$$

where  $\widehat{p}_t^*$  is the price fixed by firms that adjust prices in  $t$ , and each variable is expressed as a percent deviation from a zero inflation steady-state.<sup>2</sup>

Regarding firms' pricing decision, they determine the optimal price in order to maximise the expected discounted profit, knowing that in each future period that price will remain constant with some probability. It can be shown that the optimal price  $\widehat{p}_t^*$  is given by:

$$\widehat{p}_t^* = (1 - \beta\omega) \sum_{k=0}^{\infty} (\beta\omega)^k E_t (\widehat{mc}_{t+k}^n) \quad (3.2)$$

where  $\beta$  is the subjective discount factor and  $\widehat{mc}_t^n$  is the nominal marginal cost.

Combining equations (3.1) and (3.2) and assuming a linear production function, it is possible to derive the New Keynesian Phillips Curve:

$$\pi_t = \beta E_t \pi_{t+1} + \gamma \widehat{mc}_t \quad (3.3)$$

where  $\pi_t = \widehat{p}_t - \widehat{p}_{t-1}$  is the inflation rate in  $t$ ,  $\widehat{mc}_t$  is firms' real marginal cost in percentage deviation from its steady-state value in  $t$ , and<sup>3</sup>

$$\gamma = (1 - \omega)(1 - \beta\omega)/\omega. \quad (3.4)$$

Notice that  $\gamma$  is decreasing in  $\omega$ , meaning that an increase in price rigidity makes inflation less sensitive to movements in current real marginal cost. The idea behind equation (3.3) is simple. Firstly, since firms set prices as a mark-up over the marginal cost, they have to take

---

<sup>1</sup>The derivation of the Phillips curve using Calvo's setting can be found, for instance, in Goodfriend and King (1997).

<sup>2</sup>The lowercase letters with a hat refer to variables in deviation from the steady state.

<sup>3</sup>Notice that  $\gamma$  in this chapter corresponds to  $k$  in Chapter 2.

into account the present marginal cost. Secondly, since firms know that with some probability prices will be fixed for some time, they have to consider inflation in the next period when establishing current prices.

Equation (3.3) implies that the correct driving variable of inflation is the real marginal cost. However, this variable is not directly observable, and has to be derived by assuming a specific production function. It can be shown that (see Section 3.3.3), in deviations from the steady-state, the average marginal cost is equal to the labour income share:  $\widehat{mc}_t = \widehat{s}_t$ . This is done assuming that firms' marginal cost is equal to the average marginal cost. However, that will not be the case when prices are sticky and the production function is non linear. Then, in order to use the labour income share in the Phillips curve, it is necessary to correct it with an aggregation factor,  $h$ . For the Cobb-Douglas case with labour and capital, Gagnn and Khan (2001) show that the inflation equation becomes:

$$\pi_t = \beta E_t \pi_{t+1} + \gamma \left( \frac{1}{1+h} \right) \widehat{s}_t,$$

where  $h = \theta \alpha_k / \alpha_n$ , with  $\alpha_n$  and  $\alpha_k$  as the output elasticities of labour and capital respectively, and  $\theta$  as the parameter that governs price elasticity of demand for individual goods. In the discussion below, for the sake of simplicity, equation (3.3) will continue to be used.<sup>4</sup>

Gali and Gertler (1999) estimate equation (3.3) for the US and obtain good results. The coefficient of real marginal cost,  $\gamma$ , is positive and significant, and also the subjective discount factor  $\beta$  is statistically significant and within two standard errors deviations of 0.99, the typical value used in models' calibration. These authors also obtain estimates of the structural parameters  $\beta$  and  $\omega$ , estimating:

$$\pi_t = \beta E_t \pi_{t+1} + \frac{(1-\omega)(1-\beta\omega)}{\omega} \widehat{s}_t \quad (3.5)$$

The structural parameters' estimates also support the New Keynesian approach, since the values obtained for  $\omega$  and  $\beta$  are sensible and statistically significant. Namely, one of the values obtained for  $\omega$  is 0.83, which implies that prices are fixed on average between five and six quarters. This is close to the values obtained in survey evidence.

---

<sup>4</sup>In other words, we assume marginal costs equal across firms.

In the same direction, Sbordone (2002) also concludes that the NKPC is empirically successful in explaining US inflation. This author also shows that if a Cobb-Douglas technology with overhead labour is used to obtain the real marginal cost, the inflation equation's fit improves.<sup>5</sup> For Canada, the USA and Eurozone, Gagno and Khan (2005) also show that the use of a CES or a CES with overhead labour technology produces better estimates of the NKPC than a Cobb-Douglas technology. In addition, Gali and Lopez-Salido (2001) use Cobb-Douglas and CES production functions with labour adjustment cost, and obtain two distinct measures of marginal cost. They conclude that for Spain the estimated structural parameters are not very sensitive to those two alternative measures of marginal cost.

### 3.2.2 The New Hybrid Keynesian Phillips Curve

In order to explain inflation inertia, Gali and Gertler (1999) include persistence in the Phillips curve, assuming that a proportion  $\mu$  of firms set prices using an ad-hoc backward-looking rule. These firms use past inflation as a predictor of current inflation, in such a way that prices for period  $t$  are set as the average last period price ( $\bar{p}_{t-1}^*$ ) plus last period inflation:<sup>6</sup>

$$p_t^b = \bar{p}_{t-1}^* + \pi_{t-1}.$$

With this framework, Gali and Gertler (1999) obtain the following new hybrid Phillips curve:

$$\pi_t = \gamma_f E_t \pi_{t+1} + \gamma_b \pi_{t-1} + \gamma \widehat{mc}_t, \quad (3.6)$$

where:

$$\begin{aligned} \gamma &= (1 - \mu)(1 - \omega)(1 - \beta\omega)\phi^{-1}, \\ \gamma_f &= \beta\omega\phi^{-1}, \\ \gamma_b &= \mu\phi^{-1} \end{aligned}$$

---

<sup>5</sup> Overhead labour is the amount of labour that a firm has to hire independently of the level of production.

<sup>6</sup> Where  $\bar{p}_{t-1}^* = (1 - \mu)p_{t-1}^f + \mu p_{t-1}^b$ , with  $p_{t-1}^f$  denoting the price set by forward-looking firm at  $t - 1$  and  $p_{t-1}^b$  the price set by a backward-looking firm at  $t - 1$ . Backward- looking firms use last period inflation as predictor of current inflation.

and where  $\phi = \omega + \mu[1 - \omega(1 - \beta)]$ . With  $\beta = 1$ , then  $\gamma_f + \gamma_b = 1$ . Notice that the hybrid model nests within the baseline model ( $\gamma_f = \beta$ ,  $\gamma_b = 0$ ) if  $\mu = 0$ .

It is worth mentioning that the baseline model, with output gap as the driving variable, can also be justified with the Taylor's (1980) two-period contract model. In this model there is rigidity in nominal wages, which are fixed for two periods and half of all contracts are negotiated each period. This translates in price rigidity because prices are a constant mark-up over wages.

Fuhrer and Moore (1995) propose a hybrid model with  $\gamma_f = \gamma_b = 1/2$  and the output gap as the driving variable. With two period wage contracts, workers negotiate with reference to the average real wage over the life of a contract. The real wage contracted also reflects the present state of the business cycle.

Changing the structure of price rigidity in Calvo's model, Christiano et al. (1994) assume that all firms adjust prices every period but some are unable to re-optimize their prices and index prices to last period's inflation rate. They obtain  $\gamma_f = 1 - \gamma_b = \beta/(1 + \beta)$  and use real ULC as the forcing variable. In an extension of this work, Smets and Wouters (2003) allow the extent to which non-optimising firms index to past inflation rate to be a free parameter.

Rudd and Whelan (2005b) argue that none of the above papers satisfactorily describes motivations for the presence of backward-looking behaviour in inflation. Firstly, it is not obvious that Fuhrer and Moore's (1995) contracting mechanism is a reasonable approximation of the actual way in which wages are set. Also, in Gali and Gertler's (1999) approach, it is difficult to justify why there is a fraction of firms that do not optimize. Finally, Christiano et al. (1994) assume against significant empirical evidence that firms are able to adjust prices every period.

On the empirical side, Gali and Gertler (1999) for the USA estimating equation (3.6), obtain an empirical role for backward inflation, and more generally some support for the hybrid NKPC. Their most reliable estimation's method, indicates that 26% of firms use the backward rule of thumb. Furthermore, they conclude that the forward-looking behaviour is more important than the backward-looking behaviour. In other words, the estimated  $\gamma_f$  is larger than  $\gamma_b$ . With European data, Gali, Gertler and Lopez-Salido (2001) find also empirical support for the hybrid model.

However, Cogley and Sbordone (2008) show that when the drift in trend inflation is taken

into account, leading to a NKPC with changing coefficients, a purely forward-looking model is a good description of US data.

### 3.2.3 The NKPC with a cost channel

Regarding the determinants of the marginal cost, some extensions have been proposed. One of them is Ravenna and Walsh (2006) who obtain a NKPC with a cost channel, *i.e.*, with the nominal interest rate affecting directly the marginal cost. This effect occurs because firms have to borrow from banks to pay wages in advance. Assuming a firm-level Cobb-Douglas production function, real marginal cost is:

$$\widehat{mc}_t = \widehat{i}_t + \widehat{s}_t, \quad (3.7)$$

where  $\widehat{i}_t$  is the gross nominal interest rate in deviations from the steady-state.

The inflation-adjustment equation continues to be given by equation (3.3).<sup>7</sup> Substituting equation (3.7) in (3.3), the Phillips curve with a cost channel is obtained:

$$\pi_t = \beta E_t \pi_{t+1} + k \left( \widehat{i}_t + \widehat{s}_t \right) \quad (3.8)$$

where  $k = \tau\gamma$  (with  $\gamma$  from equation (3.4)),  $\tau = \alpha_n [1 + \alpha_k (\theta - 1)]$ , and  $\theta$  governs the price elasticity of demand for each good.

Assuming a labour share  $\alpha_n$  equal to 2/3 and an average mark-up of 1.1 (which implies that  $\theta$  is 11), equation (3.8) was estimated for the US GDP deflator using GMM. The empirical results have shown the existence of the cost channel.

Chowdhury et al. (2006) estimate a reduced form of an interest rate augmented hybrid NKPC:

$$\pi_t = \gamma_f E_t \pi_{t+1} + \gamma_b \pi_{t-1} + \gamma_s \widehat{s}_t + \gamma_r \widehat{i}_t \quad (3.9)$$

These authors confirmed the relevance of the cost channel in the majority of the G7 countries: only for Germany and Japan is the cost channel not statistically relevant.

---

<sup>7</sup>Now, with aggregation factor  $\tau$  to accommodate the non-linearity of the firm-level production function.

### 3.2.4 The NKPC in an open economy context

This section describes how the NKPC has been extended in the literature to account explicitly for external pressures on the inflation rate. Until this point the main reference was equation (3.3), and to obtain it no hypothesis regarding the economy's openness has been made. In other words, that equation is still valid with a Cobb-Douglas production function with imported inputs and labour. However, when a CES production function is admitted, the price of imports affects the marginal cost. With such a production technology, Gali and Lopez-Salido (2001) propose the following NKPC for the GDP deflator only with intermediated inputs:

$$\pi_t = \beta E_t \pi_{t+1} + \gamma [\widehat{s}_t + \phi (\widehat{p}_{m,t} - \widehat{w}_t)],$$

, with  $\phi = \left(\frac{1-\Phi s}{\Phi s}\right)(\varepsilon - 1)$ . The term in square brackets is the marginal cost,  $p_{m,t}$  is the nominal price of imported inputs in domestic currency,  $w_t$  is the nominal wage,  $\Phi$  is the steady-state mark-up,  $s$  is the steady-state labour income share, and  $\varepsilon$  is the elasticity of substitution between the two inputs. This equation was able to explain the recent Spanish inflation experience.

Gagnon and Khan (2005) also estimate a NKPC with imported materials as follows:

$$\pi_t = \beta E_t \pi_{t+1} + \gamma \left(\frac{1}{1+h}\right) [\widehat{mc}_t + g (\widehat{p}_{m,t} - \widehat{p}_t)]$$

where  $\widehat{mc}_t$  is the average marginal cost, different for each production technology considered but always a function of the labour income share, and  $\widehat{p}_t$  is the GDP deflator. Calibrating  $g = 0.3$ , they have found for Canada that  $\gamma$  is positive but statistically insignificant.

Similarly, Batini et al. (2005) also study the impact of imported materials on inflation using a NKPC for domestic inflation but add two novelties to the baseline inflation equation: a variable equilibrium mark-up and employment adjustment costs. Using a Cobb-Douglas technology with no capital, and admitting quadratic adjustment costs of changing both prices and employment, the authors obtain:

$$\pi_t = \beta E_{t-1} \pi_{t+1} + \alpha_1 E_{t-1} (\ln \Phi_t^*) + \alpha_1 mc_t - \beta \alpha_2 E_{t-1} \Delta n_{t+1} + \alpha_2 E_{t-1} \Delta n_t + v_t$$

where  $\alpha_1$  and  $\alpha_2$  are positive and  $\Phi_t^*$  is the equilibrium mark-up, which depends on the output gap and external competitive pressures,  $n_t$  is the level of employment and  $v_t$  is an error term. The marginal cost is  $mc_t = -\ln \alpha_n + s_t + \phi(p_{m,t} - p_t)$ , where  $\alpha_n$  is the labour share in the Cobb-Douglas production function, and  $\phi$  is a constant.<sup>8</sup> This curve nests Gali and Gertler's (1999) curve when a constant mark-up is assumed and there are no labour adjustment costs and imported inputs. Finally, Batini et al.'s (2005) equation is estimated successfully using UK data, with employment adjustment costs, the price of imports and external competitive pressures playing a relevant role in explaining inflation.

Leith and Malley (2007) construct an open economy NKPC where firms can substitute between imported intermediate goods and labour. Consequently, the price of labour relative to imported inputs affects inflation. Their curve is empirically successful since the parameters estimated for the G7 countries are plausible; namely the degree of price stickiness. The estimated parameters do not change significantly with open economy considerations.

Rumler (2007) extends the work of Leith and Malley (2007) admitting that firms use three inputs: labour, imported intermediate goods, and domestic intermediate goods. As a result inflation depends not only on the price of labour relative to imported inputs, but also on the price of labour relative to domestic inputs. When empirical estimates of the closed economy model are compared with the ones of the open economy model with imported intermediate goods,<sup>9</sup> it is found that price rigidity is lower in the open economy model. This result contradicts Leith and Malley (2007), where open economy variables have a small impact on estimated parameters. The difference in results may be explained by the fact that Rumler (2007) produces estimates for nine euro area countries, considerably smaller and more open to trade than the ones studied by Leith and Malley (2007). It is however argued that lower price rigidity in open economies occurs because firms, when faced with volatile prices of imported inputs tend to adjust their prices faster. Moreover, the open economy model with imported intermediate goods implies a lower price rigidity than the model with both imported and domestic intermediate goods. The proposed explanation for this result is that the possibility of substituting between domestic and imported inputs protects firms from fluctuations in

---

<sup>8</sup>The expression for the marginal cost is obtained assuming that the amount of imported materials depends only on the gross output and is not affected by the relative price of imports.

<sup>9</sup>This corresponds to Leith and Malley's (2007) model.

international input prices, making price adjustments less frequent. Finally, the open economy model with three inputs was found to be the one that best fits the euro area's data.

Until this point we have focused on the impact of imported input prices on domestic inflation. Instead, Kara and Nelson's (2003) main concern is the impact of imported goods prices on CPI inflation. They test several theories of inflation dynamics in an open economy using UK data. One of these theories is the monetary model, which assumes that the law of one price holds for all tradable goods. In other words, there is a complete and immediate pass-through of exchange rate and world price inflation to the domestic price of imported goods. If all domestically produced goods are tradable, then CPI inflation is:

$$\pi_t^c = \pi_t^w + \Delta e_t$$

where  $\pi_t^w$  is the consumer goods' inflation for the rest of the world and  $\Delta e_t$  is the change in the nominal exchange rate. This implies a close relation between CPI inflation and nominal exchange rate.<sup>10</sup> However, Kara and Nelson (2003) conclude that such a relation does not exist for the UK.

In opposition, there are the pricing-to-market models, which admit that firms set prices of imported consumer goods equal to the prices of domestic goods. These models imply that there is no relation between the exchange rate and both the overall inflation and the inflation of imported goods. While the former implication is confirmed by UK data, the latter is strongly rejected.

An alternative approach is to use open economy New Keynesian models. The standard model in this approach assumes imports of only consumer goods and that the price of domestic goods is sticky but the price of imports is flexible, implying that there is full exchange rate pass-through. With these assumptions, the typical Phillips curve is:

$$\pi_t^c = \beta E_t \pi_{t+1}^c + \gamma m c_t + \gamma_{\Delta q} (\Delta q_t - \beta E_t \Delta q_{t+1}) + u_t, \quad \gamma_{\Delta q} < 0, \quad (3.10)$$

where  $\Delta q_t$  is the change in the real effective exchange rate (an increase corresponds to an appreciation), and  $-\gamma_{\Delta q}$  is the share of imported goods in the CPI. The estimation of this

---

<sup>10</sup>The same strong relation prevails if the existence of non-tradable goods is admitted.

equation with UK data for 1964Q2-2001Q4 and with the marginal cost measured by the labour share delivers a  $\gamma_{\Delta q}$  insignificant and with the wrong sign.

In order to improve the empirical validity of the NKPC for an open economy, Kara and Nelson (2003) then use three different specifications: a backward-looking Phillips curve, a NKPC with import prices and a NKPC with the level of the real exchange rate.

Firstly, they investigate whether the weak empirical adherence of the Phillips curve may be explained by the failure of the forward-looking setup. Therefore, they estimate the following backward-looking curve:

$$\pi_t^c = b_0 + \sum_{i=1}^3 \beta_i \pi_{t-i}^c + \sum_{i=1}^3 \gamma_{\Delta q_i} \Delta q_{t-i} + \sum_{k=i}^3 \gamma_i mc_{t-i} + u_t.$$

This equation can be seen as a reduced form characterisation of the equilibrium arising from equation (3.10) when there are no changes in the monetary policy regime. Additionally, the last equation assumes that, on one hand, domestic prices adjust slowly to changes in the marginal cost and, on the other hand, import prices adjust slowly to changes in the exchange rate. This equation also does not adequately describe the UK data. However, in this case the problem is not in the exchange rate's coefficients, but in the marginal cost's coefficients.

As suggested by the last result, slow pass-through of exchange rates to import prices is an idea worth pursuing. Other empirical evidence follows that direction, as for example Campa and Goldberg (2002), who found an incomplete pass-through of exchange rates to import prices in the short-run that becomes gradually complete in the long-run. In such an environment, the real price of imports measures better the external pressures on inflation than the real exchange rate. Taking this into account, the following equation was estimated:

$$\pi_t^c = \beta E_t \pi_{t+1}^c + \gamma mc_t + \phi \Delta rpm_t,$$

where  $\Delta rpm_t$  is the change in the real price of imports. Using UK data, this Phillips curve's specification does not describe inflation well because  $\phi$  has the wrong sign and is statistically insignificant.

Finally, the level of the real exchange rate was introduced in the NKPC, as follows:

$$\pi_t^c = \beta E_t \pi_{t+1}^c + \gamma mc_t + \gamma_{\Delta q} (\Delta q_t - \beta E_t \Delta q_{t+1}) + \gamma_q q + u_t, \quad \gamma_{\Delta q} < 0, \quad \gamma_q < 0. \quad (3.11)$$

This specification can be justified by the existence of intermediate imported goods.<sup>11</sup> Another justification is that the real exchange rate may affect domestic price setters' mark-up over marginal cost. Nevertheless, for the full sample 1964Q2-2001Q4, a negative but statistically insignificant  $\gamma_q$  was estimated. This means that the model with imports as intermediate inputs produces a reasonably good description of the effect of external prices on UK inflation.

Other papers explain the process of exchange rate pass-through modelling firms' pricing decisions. Monacelli (2005) assumes that, like domestic producers, importers adjust prices sluggishly. His model uses local currency pricing: prices are set and sticky in the currency of the importing economy. Indeed, foreign goods are imported by firms that set prices in the local market. The law of one price is verified at the customs frontier. But since retailers solve an optimal mark-up problem with a Calvo (1983) framework, the law of one price is not valid for the final price of imported goods. The equation for the change in the price of imports for the final consumer is:

$$\pi_t^m = \beta E_t \pi_{t+1}^m + \gamma_\Psi \Psi_{m,t} \quad (3.12)$$

where  $\Psi_{m,t} = e_t + p_t^* - p_{m,t}$ ,  $e_t$  is the nominal exchange rate,  $p_t^*$  is the price of foreign goods in the world market,  $p_{m,t}$  is the price at which retailers sell foreign goods to the final consumer, and  $\gamma_\Psi > 0$  is a constant. Notice that  $\Psi_{m,t}$  can be seen as the deviation from the law of one price, *i.e.*, the deviation of world price from the domestic currency price of imports for the final consumer.<sup>12</sup> The intuition behind the effect of this term on import prices growth is straightforward. If the world price is higher than the domestic currency price of imports, retailers' profitability is negatively affected. This leads them to increase import prices to the final consumer.

Imported goods' inflation and domestic inflation determine CPI inflation:  $\pi_t^c = (1-\gamma)\pi_t^h + \gamma\pi_t^m$ , where  $\gamma$  is the weight of foreign goods in the CPI basket. Using the Phillips curve for domestic inflation:

$$\pi_t^h = \beta E_t \pi_{t+1}^h + \gamma_h \widehat{mc}_{h,t}$$

---

<sup>11</sup>They admit a CES production function including labour and imports.

<sup>12</sup>The world price is  $e_t + p^*$ .

and (3.12) in the expression for CPI inflation, the following is obtained:

$$\pi_t^c = \beta E_t \pi_{t+1}^c + (1 - \gamma) \gamma_h \widehat{m} c_{h,t} + \gamma \gamma_\Psi \Psi_{m,t}.$$

We observe that the deviation of the law of one price affects CPI inflation.

Adolfson (2001) assumes that foreign exporters fix prices in the domestic market and obtain a Phillips curve for CPI inflation very similar to that of Monacelli (2005).

Bache and Naug (2007) estimate New Keynesian import price models for Norway and the UK and obtain support for local currency pricing models, like the one of Monacelli (2005). However, these models should also include producer currency pricing.<sup>13</sup> Furthermore, expected future import price growth is significant only for Norway, and no role has been found for lagged import price growth in either countries.

### 3.2.5 Critics to NKPC estimations

Gali and Gertler's (1999) results have been criticised on the grounds that they suffer from specification bias associated with the GMM technique. Two main criticisms were made: (1) the closed form estimates are significantly different from the estimates obtained with the structural form (Rudd and Whelan, 2005a),<sup>14</sup> and (2) the Full Information Maximum Likelihood Method (FIML) is likely to be more robust than single equation techniques (Lindé, 2005). Gali and Gertler (2005) rebut these two critiques.

Regarding the first criticism, Rudd and Whelan (2005a) highlight one potential shortcoming with the GMM estimation of the structural equation (3.6). If some instruments affect inflation and are omitted from the Phillips curve, the coefficient of lead inflation can be overestimated. In response, Gali and Gertler (2005) argue that the additional lags of inflation in the Phillips curve were insignificant. However, in our opinion they have failed to test whether the other instruments were also insignificant in the inflation equation.

Bardsen et al. (2004) obtain different results with euro area data. When the lagged output gap and the fourth lag of inflation were moved from the instruments list to the regressors list, they emerged statistically significant in the NKPC and the forward component of inflation

---

<sup>13</sup>Producer currency pricing means that prices are set and sticky in the currency of the producer.

<sup>14</sup>The structural form corresponds to equation (3.5) and the closed form corresponds to equation (3.13).

became insignificant. By estimating the NKPC with UK data, Bardsen et al. (2004) also obtain the significance of two equilibrium-correction terms, one long-run wage curve and an open economy price equation; and once more the forward component of inflation became statistically insignificant.

Given the above mentioned problem with the GMM estimation of the structural equation, Rudd and Whelan (2005a) estimated the closed form of the hybrid NKPC:

$$\pi_t = \delta_1 \pi_{t-1} + \varphi \sum_{k=0}^{\infty} \delta_2^{-k} E_t mc_{t+k}, \quad (3.13)$$

where  $\delta_1$  and  $\delta_2$  are, respectively, the stable and unstable roots of the second order difference equation given by (3.6). If the hybrid model is valid, the current and expected marginal costs have to affect current inflation. However, the estimated  $\varphi$  is statistically insignificant. They also argue that  $\gamma_f$  and  $\gamma_b$  are almost completely unrelated to the forward-looking component in the hybrid inflation equation (see (3.13)). In fact, if  $\delta_1$  is less than one, as it occurs usually in practice,  $\gamma_f$  will be greater than 0.5 even if only the lagged inflation has explanatory power in equation (3.13).

According to Galí and Gertler (2005), Rudd and Whelan's (2005a) results occur because they ignored the fact that parameters in equation (3.13) are a function of the parameters in the structural hybrid model. Namely, in the hybrid Phillips curve  $\varphi = \gamma/\delta_2\gamma_f$  and  $\delta_1$  and  $\delta_2$  depend on  $\gamma_f$  and  $\gamma_b$ . So with this in mind, parameter  $\delta_1$  cannot be interpreted as a measure of the relative importance of backward-looking behaviour. Furthermore, equation (3.13) can be used to obtain orthogonality conditions to apply the GMM, and estimate directly  $\gamma_f$ ,  $\gamma_b$  and  $\gamma$ . With this approach Galí and Gertler (2005) obtain estimates for those parameters identical to the ones obtained with equation (3.6), which constitutes support for their initial conclusion that the forward-looking behaviour of inflation is dominant. In our view, they should also have tested if  $\gamma/\delta_2\gamma_f$  is zero.

Lindé (2005) argues that FIML produces more robust estimates than GMM. From a general point of view, it is not clear which technique is superior. On one hand, the GMM estimates are sensitive to the choice of instruments and can be biased in small samples and suffer from

weak instruments (Stock et al., 2002).<sup>15</sup> On the other hand, the ML technique depends crucially on the assumption of normality of the error term or on the assumed overall structure of the economy (if the FIML is used). Even though Lindé's (2005) Monte Carlo experiments show the superiority of the FIML over the GMM, they assume that the econometrician has a good *a priori* knowledge of the economy, which is unlikely to be the case (Gali and Gertler, 2005).

The ML approach solves the model forward using a forecast model for the marginal cost. Predictions for the marginal cost are obtained from a single equation, a VAR model, or a structural model. For example, Jondeau and Le Bihan (2005) estimate a VAR for the marginal cost and the short term nominal interest rate.<sup>16</sup> Using the procedure developed by Anderson and Moore (1985), the forward-looking model is written in an observable structure, *i.e.*, in a backward-looking form, which does not contain unobservable expectations. Then, the observable structure and the realisation of the data are used for evaluating the likelihood function of the model. The maximisation of the likelihood function produces estimates of the Phillips curve, after VAR's parameters have been obtained in a first step (or both estimations are performed simultaneously, when a FIML approach is used).

With US data, Lindé (2005) has found that, even though the forward-looking component was significant, the backward-looking behaviour was more important. The dominance of the lagged inflation can be explained by the use of output gap instead of the real marginal cost. In contrast with Lindé (2005), other studies using ML methods to estimate models with a NKPC indicate the dominance of forward-looking behaviour in inflation (Gali and Gertler, 2005).

### **3.3 NKPC for domestic inflation**

#### **3.3.1 Identifying the cost channel in an open economy**

After revising the literature on the NKPC, this section now highlights the identification of the cost channel in an open economy.

---

<sup>15</sup>Mavroeidis (2004) shows that the J-test's distribution is also affected by the possibility of 'weak' instruments.

<sup>16</sup>The VAR model also includes lagged values of the inflation rate.

In our model in Chapter 2 the Phillips curve for domestic prices was:

$$\pi_t^h = \beta E_t \pi_{t+1}^h + \gamma \widehat{mc}_t$$

, with

$$\widehat{mc}_t = \frac{(1 - \gamma'_i) v}{1 + \gamma'_i \eta \varepsilon} (\widehat{i}_t - \widehat{i}_t^o) + \frac{(1 - \gamma'_i) h}{1 + \gamma'_i \eta \varepsilon} (\widehat{\delta}'_t - \widehat{\delta}'_t{}^o) + \frac{1 - \gamma'_i}{1 + \gamma'_i \eta \varepsilon} \left( \eta + \frac{\sigma}{1 - \gamma} \right) (\widehat{y}_t - \widehat{y}_t^o). \quad (3.14)$$

The marginal cost is affected by the interest rate, the terms of trade and output gap, all of which are expressed in deviations from the flexible-price equilibrium.

Without imports of consumption goods being paid in advance (but still with wages and imported inputs paid in advance), the terms of trade are only related with the output gap:

$$\widehat{\delta}'_t - \widehat{\delta}'_t{}^o = \frac{\sigma}{w + 1} (\widehat{y}_t - \widehat{y}_t^o). \quad (3.15)$$

In this case the marginal cost's reduced form is:

$$\widehat{mc}_t = \frac{(1 - \gamma'_i) v}{1 + \gamma'_i \eta \varepsilon} (\widehat{i}_t - \widehat{i}_t^o) + \left[ \frac{1 - \gamma'_i}{1 + \gamma'_i \eta \varepsilon} \left( \eta + \frac{\sigma}{1 - \gamma} \right) + \frac{(1 - \gamma'_i) h}{1 + \gamma'_i \eta \varepsilon} \frac{\sigma}{w + 1} \right] (\widehat{y}_t - \widehat{y}_t^o).$$

Comparing the last equation with (3.14), it is clear that ignoring the terms of trade when estimating the PC does not affect the coefficient of the interest rate. However, this result depends on the empirical validity of equation (3.15).

Additionally, when it is assumed that imports of consumption goods are paid in advance, the terms of trade are directly affected by the interest rate:

$$\widehat{\delta}'_t - \widehat{\delta}'_t{}^o = \frac{\sigma}{w + 1} (\widehat{y}_t - \widehat{y}_t^o) - \frac{(1 - \gamma) \gamma_c (\sigma a - 1)}{w + 1} (\widehat{i}_t - \widehat{i}_t^o). \quad (3.16)$$

In reduced form, the marginal cost becomes:

$$\widehat{mc}_t = \frac{1 - \gamma'_i}{1 + \gamma'_i \eta \varepsilon} \left( v - \frac{h(1 - \gamma) \gamma_c (\sigma a - 1)}{w + 1} \right) (\widehat{i}_t - \widehat{i}_t^o) + \frac{(1 - \gamma'_i)}{1 + \gamma'_i \eta \varepsilon} \left( \frac{\sigma h}{1 + w} + \eta + \frac{\sigma}{1 - \gamma} \right) (\widehat{y}_t - \widehat{y}_t^o). \quad (3.17)$$

In this case the coefficient of the interest rate does not translate the cost channel only, but

Table 3.1: Sensitivity analysis of expression A's value

	epsilon				
	0	1/3	1	1.5	2
A	-0.0829	-0.0800	-0.7420	-0.0699	-0.0655
	gamma i				
	0	0.1	0.4	2/3	
A	-0.1090	-0.0800	-0.01803	0.0000	
	gamma				
	0	0.2	0.4	0.6	
A	-0.0104	-0.0120	-0.0800	-0.1889	
	sigma				
	1	1.5	2		
A	0.0000	-0.0800	-0.1939		
	eta				
	0	1	2		
A	-0.0829	-0.0800	-0.0771		
	a				
	0.1	1	2		
A	-0.1201	-0.0800	-0.4678		

it also captures the effect of the interest rate on the terms of trade. The sign of  $h$  is not well defined, but with our calibration it is negative and thus  $A = \frac{h(1-\gamma)\gamma_c(\sigma a-1)}{w+1} < 0$ . For different values of the parameters, expression  $A$  is always negative or zero (Figure 3.1). As a result, when estimating the Phillips curve while ignoring the terms of trade, the coefficient of the interest rate is larger than when the terms of trade are specifically taken into account. This means that the identification of the cost channel is distorted by ignoring the terms of trade in the Phillips curve's estimation. This is the reason why, below, we identify the cost channel taking into account import prices.

Following this analysis, our modelisation strategy consist in first estimate the NKPC without import prices and after introduce these variables. In this way, we are able to analyse the impact of introducing import prices on the cost channel. Other reason for using this strategy is that it allows to compare the baseline estimations with the ones obtained in the literature.

### 3.3.2 Data

The current research uses quarterly data to estimate the Phillips curve for the G7 countries.<sup>17</sup> For countries not belonging to the euro area, we analysed the period 1980-2006, and for euro area countries we restricted the study to the period 1980-98, in order to avoid any structural break arising from the introduction of the euro.

Regarding the measurement of variables in the current research, some comments are necessary. Firstly, two alternative measures of inflation will be used, the GDP deflator and the CPI. The GDP deflator, which captures inflation on domestically produced goods, is used by most papers on the NKPC. In this paper, we also use the CPI, which by measuring the price of the typical consumer's basket allows us to study the impact of imported final goods on CPI inflation.<sup>18</sup> Additionally, following some important works in the literature (see for instance, Gali and Gertler, 1999; Batini et al., 2005; Jondeau and Bihan, 2005), the current research will also use the quarterly inflation rate.<sup>19</sup>

Another central issue in our analysis is the measurement of the marginal cost. It was seen above that the marginal cost can be measured using the labour income share, which is also called real unit labour cost: ratio of total labour costs to nominal GDP (Gali and Gertler, 1999). To obtain this ratio, we used data from OECD National Accounts, as is common in the literature (see for example Leith and Malley, 2002; Chowdhury et al., 2003; Jondeau and Bihan, 2005).

Note also that to measure the short-run nominal interest rate the Treasury Bill Rate was used.

Finally, the IMF's commodity price index was only available for a period much shorter than the analysed in this paper. Therefore, we constructed an index very similar to the one of IMF, using data on prices of both fuel and non-fuel commodities.

Annex 6.2.1 describes in greater detail the data.

---

<sup>17</sup>We have chosen the G7 economies because they are a set of open economies for which it is easy to obtain data for a long period of time. This set of countries also allow an easy comparison with other papers on the NKPC and the cost channel. Notice also that the inclusion of the US, a less open economy, allows to test whether the results are crucially dependent on the degree of openness of an economy. Precisely, the study of a set of countries, instead of only one country, aims to test if models can be seen as general or country-specific.

<sup>18</sup>For example, Kara and Nelson (2003) also use the CPI.

<sup>19</sup>The CPI used to obtain the quarterly inflation rate was seasonally adjusted.

### 3.3.3 NKPC and the relative price of imports

In this section, the NKPC is used to describe domestic inflation, as measured by the change in the GDP deflator.

The NKPC assumes that the correct driver of inflation is the marginal cost. Since the marginal cost is not directly observable, it can be derived assuming a Cobb-Douglas production function with labour and imported inputs:

$$Y_t = Z_t N_t^{\alpha_n} M_t^{\alpha_m}, \quad (3.18)$$

where  $N_t$  is the number of hours of work,  $M_t$  is imported inputs, and  $Z_t$  is the aggregate technological shock. The real marginal cost can be obtained as the ratio between the real wage and the marginal product of labour,<sup>20</sup>

$$MC_t = \frac{\frac{W_t}{P_t^h}}{\frac{\partial Y_t}{\partial N_t}},$$

where  $W_t$  is the nominal wage and  $P_t^h$  is the price of domestic goods. This can be re-written as:

$$MC_t = \frac{S_t}{\alpha_n},$$

where  $S_t$  is the labour income share,  $W_t N_t / P_t^h Y_t$ . The last variable is also called real unit labour cost. Assuming  $\alpha_n$  as constant, the last equation can be expressed in percentage deviations from the steady-state as

$$\widehat{mc}_t = \widehat{s}_t.$$

It can be seen that with a Cobb-Douglas production function, the price of imported inputs does not affect the marginal cost.

However, if we depart from the unitary elasticity of substitution between inputs, and assume a CES production function, inflation is affected by the price of imports (Gali and Lopez-Salido, 2001). Recall that for Canada, USA and Eurozone, Gagno and Khan (2005)

---

<sup>20</sup>Under the assumption that firms are wage-takers (Rotemberg and Woodford, 1999).

show that the use of a CES rather than a Cobb-Douglas technology leads to an improvement in the fit of the NKPC.

We augment the curve proposed by Gali and Lopez-Salido (2001) with the cost channel. To begin with, it is assumed there is a CES production function with labour,  $N_t$ , and imported materials,  $M_t$ :

$$Y_t = \left[ \alpha_N (Z_t N_t)^{1-1/\varepsilon} + \alpha_M (M_t)^{1-1/\varepsilon} \right]^{\frac{\varepsilon}{\varepsilon-1}},$$

where  $\varepsilon$  is the elasticity of substitution between the two inputs.

The marginal cost is given by

$$MC_t = \frac{I_t \frac{W_t}{P_t}}{\frac{\partial Y_t}{\partial N_t}} = \frac{I_t S_t}{\gamma_t} \quad (3.19)$$

where  $\gamma_t$  is the elasticity of output with respect to labour, and  $I_t$  is the gross nominal interest rate.

In the discussion below, it is assumed that firms have to borrow to pay in advance for both imported inputs and wages. This means that the cost of imports is the nominal price of imports,  $P_{m,t}$ , times the gross interest rate. From cost minimisation, we have (See Annex 6.2.2 for details):

$$\frac{N_t}{M_t} = \left( \frac{\alpha_N P_{m,t} I_t}{\alpha_M W_t I_t} \right)^\varepsilon = \left( \frac{\alpha_N P_{m,t}}{\alpha_M W_t} \right)^\varepsilon, \quad (3.20)$$

Notice that both  $P_{m,t}$  and  $W_t$  are expressed in nominal terms. It is also possible to show that (See Annex 6.2.2 for details):

$$\gamma_t = 1 - \alpha_M \left( \frac{Y_t}{M_t} \right)^{\frac{1}{\varepsilon}-1}. \quad (3.21)$$

Substituting equation (3.20) into (3.21) and log-linearising, it is possible to get (Gali and Lopez-Salido, 2001):

$$\widehat{\gamma}_t = -\phi (\widehat{p}_{m,t} - \widehat{w}_t),$$

where  $\phi = \left( \frac{1-\Phi s}{\Phi s} \right) (\varepsilon - 1)$ ,  $\Phi$  is the steady-state mark-up, and  $s$  is the steady-state labour income share. Therefore, from equation (3.19) we obtain:

$$\widehat{m}c_t = \widehat{s}_t + \widehat{i}_t + \phi (\widehat{p}_{m,t} - \widehat{w}_t). \quad (3.22)$$

Replacing the latter expression in a hybrid version of the NKPC, we get:

$$\pi_t^h = \gamma_f E_t \pi_{t+1}^h + \gamma_b \pi_{t-1}^h + \gamma_s \widehat{s}_t + \gamma_i \widehat{i}_t + \gamma_{pw} (\widehat{p}_{m,t} - \widehat{w}_t) \quad (3.23)$$

where  $\pi_t^h$  is the inflation rate of goods produced and sold domestically. This curve nests the standard NKPC (with  $\gamma_i = 0$  and  $\gamma_{pw} = 0$ ) and the interest rate augmented NKPC (with  $\gamma_{pw} = 0$ ). Notice that this is a different way of writing the Phillips curve of Chapter 2.<sup>21</sup>

The impact of import prices on the marginal cost depends on  $\varepsilon$ .<sup>22</sup> If  $\varepsilon > 1$ , a decrease in the relative price of imports decreases the marginal cost, because firms substitute labour by imported inputs.

As shown in the equation (3.23), variables that affect the marginal cost are expressed in deviations from the steady-state. Typically, in the NKPC literature deviations from the steady-state are obtained as deviations from a constant mean. Important exceptions to this practice are done by Rumler (2004) and Leith and Malley (2007), who use deviations from a quadratic trend for the variables in levels.

In this paper we removed the trend from the variables in levels using the HP filter (with  $\lambda = 1600$ ).<sup>23</sup> Canova (2007) suggests precisely that in a GMM application when the data is non-stationary, it is possible to detrend the data using the HP filter. The use of this filter allows the variables' steady-state to change over time. This is useful because during the sample period inflation has decreased considerably in some countries.

A different approach to accommodate movements in trend inflation is the one proposed by Cogley and Sbordone (2008), who derive and estimate a NKPC with time-varying coefficients.

---

<sup>21</sup>From equation (3.19) we obtain an expression where the marginal cost depends on  $\widehat{i}_t$ ,  $\widehat{s}_t$  and  $\widehat{\delta}_t$ :  $\widehat{mc}_t = \frac{1}{1 - (1 - \frac{1}{\varepsilon}) \frac{\gamma_i \varepsilon}{1 - \gamma_i}} \left[ \left( 1 - (1 - \frac{1}{\varepsilon}) \frac{\gamma_i \varepsilon}{1 - \gamma_i} \right) \widehat{i}_t + \widehat{s}_t - (1 - \frac{1}{\varepsilon}) \frac{\gamma_i \varepsilon}{1 - \gamma_i} \widehat{\delta}_t \right]$ .

<sup>22</sup>As seen above, if the production function is a Cobb-Douglas, *i.e.*,  $\varepsilon \rightarrow 1$ , then the relative price of imports does not affect the marginal cost.

<sup>23</sup>The HP filter is used in the labour share, interest rate, relative prices of imports and commodities. When possible, the filter is applied for the sample starting in 1975Q1. The inflation rate is not in deviations from the steady state because the Phillips curve has one lag and one lead of this variable.

### 3.3.4 Estimating the baseline NKPC

To begin with, we estimate the standard NKPC, where the effect of the interest rate and import prices is ignored (equation (3.23) with  $\gamma_r = \gamma_{pw} = 0$ ):

$$\pi_t^h = \gamma_f E_t \pi_{t+1}^h + \gamma_b \pi_{t-1}^h + \gamma_s \widehat{s}_t. \quad (3.24)$$

This equation and others below are estimated using the GMM. With rational expectations, observed inflation is  $\pi_{t+1} = E_t \pi_{t+1} + \chi_t$ , where  $\chi_t$  is an expectation error. Then equation (3.24) in terms of realised variables is:

$$\pi_t^h = \gamma_f \pi_{t+1}^h + \gamma_b \pi_{t-1}^h + \gamma_s \widehat{s}_t + \varepsilon_t,$$

where  $\varepsilon_t$  is a linear combination of a random variable representing a measurement error,  $u_t$ , and a forecast error,  $-\gamma_f \chi_t = -\gamma_f [\pi_{t+1} - E_t \pi_{t+1}]$ . Because with rational expectations the forecast error  $\chi_t$  is not correlated with information available in  $t$  or earlier, it is correct to write the following orthogonality conditions to estimate the model by GMM:

$$E_t [\mathbf{g}_t] = E_t \left[ \left( \pi_t^h - \gamma_f \pi_{t+1}^h - \gamma_b \pi_{t-1}^h - \gamma_s \widehat{s}_t \right) \mathbf{z}_{t-1} \right] = \mathbf{0}$$

where  $\mathbf{z}_{t-1}$  is a vector ( $r \times 1$ ) of variables from period  $t - 1$  or earlier that are orthogonal to unexpected inflation in  $t + 1$ . Instruments dated from  $t - 1$  or earlier are used for two reasons (Gali et al., 2001). On one hand, some instruments may not be public knowledge in  $t$  when expectations are formulated. On the other hand, there is some error in our measure of marginal cost. Assuming that such error is not related with past information, it is correct to use instruments starting from  $t - 1$ .

Notice that because the Phillips curve is a linear model, the GMM is equivalent to a two-stage least square (2SLS). In the first step  $\pi_{t+1}^h$  is regressed on the instruments. In the second step, the fitted values from the first step are used as proxy for  $E_t \pi_{t+1}^h$ . The advantage of using the GMM instead of the 2SLS is the potential necessity to correct for autocorrelation and heteroskedasticity in the residuals.

The GMM estimation involves some important choices. Firstly, it is necessary to define the

set of instruments. We follow the common practice in the literature of using as instruments at least four lags of the regressors.<sup>24</sup> In addition, four lags of the output gap were used also. Usually, this variable is used as an instrument due to its relevance in explaining inflation.<sup>25</sup> In the context of the NKPC literature, the chosen set of instruments can be considered small. This choice has been made because in finite samples, increasing the number of instruments increases the bias of estimated parameters and reduces its variance (Ravenna and Walsh, 2006).<sup>26</sup> Also, Mavroeidis (2005) using Monte Carlo simulations shows that when the number of instruments is large, the finite-sample power of the J-test to detect misspecification in forward-looking inflation models is low.

We use the GMM Continuously Updated Estimator (CUE) of Hansen et al. (1996).<sup>27</sup> In a general formulation, this estimator minimises<sup>28</sup>

$$J(\hat{\boldsymbol{\beta}}) = n\mathbf{g}_n(\hat{\boldsymbol{\beta}})' \left( \hat{\mathbf{S}}(\hat{\boldsymbol{\beta}}) \right)^{-1} \mathbf{g}_n(\hat{\boldsymbol{\beta}}) \quad (3.25)$$

where  $\mathbf{g}_n(\hat{\boldsymbol{\beta}})$  is the sample mean of  $\mathbf{g}_t$  evaluated at  $\hat{\boldsymbol{\beta}}$  ( $k \times 1$ ):

$$\mathbf{g}_n = \frac{1}{n} \sum_{t=1}^n \hat{\varepsilon}_t \mathbf{z}_{t-1},$$

, with  $\hat{\varepsilon}_t = y_t - \mathbf{x}_t' \hat{\boldsymbol{\beta}}$ , with  $\mathbf{x}_t$  as a vector ( $k \times 1$ ).  $\hat{\mathbf{S}}(\hat{\boldsymbol{\beta}})$  is a consistent estimator of the  $(r \times r)$  matrix  $\mathbf{S} = E(\mathbf{g}_t \mathbf{g}_t')$ , the asymptotic variance of  $\mathbf{g}_n$ . The fact that the weighting matrix is an estimate of the inverse of the covariance matrix of the sample moments,  $\hat{\mathbf{S}}$ , is a necessary condition for the efficient estimate of  $\boldsymbol{\beta}$  (Hansen, 1982). Intuitively, less weight is given to the more imprecise conditions. The White's heteroskedasticity consistent estimate of  $\hat{\mathbf{S}}$  is:

$$\hat{\mathbf{S}} = \frac{1}{n} \sum_{t=1}^n \hat{\varepsilon}_t^2 \mathbf{z}_t \mathbf{z}_t'.$$

---

<sup>24</sup>As usual, we look for instruments that are correlated with the endogenous variable and not correlated with the error. Notice that we do not use the same instruments for all countries because inflation dynamics are country-specific.

<sup>25</sup>Our measure of output gap is the difference between the log of output and the H-P filtered log of output with smoothing parameter of 1600.

<sup>26</sup>And increases also the possibility of weak instruments.

<sup>27</sup>We use Eviews 6 to implement this estimator.

<sup>28</sup>For a more detailed description of the GMM estimator see for example Hayashi (2000) and Favero (2001).

If  $\widehat{\varepsilon}_t$  is obtained with a consistent estimator of  $\boldsymbol{\beta}$ , then  $\widehat{\mathbf{S}}$  is also consistent for  $\mathbf{S}$ .

In the GMM CUE of Hansen et al. (1996) the weighting matrix and  $\boldsymbol{\beta}$  are estimated simultaneously, meaning that the residuals used in  $\widehat{\mathbf{S}}$  and  $\mathbf{g}_n$  are the same. This estimator has better finite-sample properties than the two-step GMM. In particular, the CUE estimator has better performance in the presence of weak instruments (Hahn et al., 2004).

Estimations were made robust to heteroskedasticity and autocorrelation of unknown form.<sup>29</sup> Controlling for autocorrelation allows us to take into account the presence of measurement errors in the real marginal cost (Benigno and Lopez-Salido, 2002; Gali et al., 2001). The literature has extensively discussed the estimation of the covariance matrix  $\mathbf{S}$  when the error term is likely to be heteroskedastic and serially correlated. In this case, it can be shown that:

$$\mathbf{S} = \boldsymbol{\Gamma}(0) + \sum_{j=1}^{n-1} [\boldsymbol{\Gamma}(j) + \boldsymbol{\Gamma}'(j)],$$

where  $\boldsymbol{\Gamma}(j) = \frac{1}{n} \sum_{t=j+1}^n E(\varepsilon_{t-j} \varepsilon_t \mathbf{z}_{t-j} \mathbf{z}'_t)$ . The sample autocovariance of order  $j$ ,  $\boldsymbol{\Gamma}(j)$ , evaluated at a consistent estimate of  $\boldsymbol{\beta}$  without the expected value, is not always consistent for the true autocovariance. When  $j$  is large,  $\boldsymbol{\Gamma}(j)$  is composed of few terms and thus the law of large numbers cannot be applied. The solution proposed by Newey and West (1987) was to truncate the sum to obtain  $\mathbf{S}$ , eliminating the terms for which  $j$  is greater than some threshold  $p$ , obtaining:

$$\widehat{\mathbf{S}} = \widehat{\boldsymbol{\Gamma}}(0) + \sum_{j=1}^p [\widehat{\boldsymbol{\Gamma}}(j) + \widehat{\boldsymbol{\Gamma}}'(j)]$$

where  $\widehat{\boldsymbol{\Gamma}}(j) = \frac{1}{n} \sum_{t=j+1}^n (\widehat{\varepsilon}_{t-j} \widehat{\varepsilon}_t \mathbf{z}_{t-j} \mathbf{z}'_t)$ .

In our study the bandwidth  $p$  is determined using the Newey-West fixed bandwidth, which depends on the number of observations in the sample:

$$p = \text{int} [4(n/100)]^{2/9}$$

where  $\text{int}()$  denotes the integer part of the argument. This guarantees that the bandwidth

---

<sup>29</sup>Since there is evidence of negative autocorrelation in the residuals (see Table 3.2), this adjustment ensures more efficient estimates.

increases with  $n$  at a sufficiently large rate.

This procedure does not guarantee that  $\widehat{\mathbf{S}}$  is positive definite in finite samples. To solve this problem, empirical covariances were weighted with weights that decrease with  $j$ . We used the Barlett Kernel proposed by Newey and West (1987), which yields:

$$\widehat{\mathbf{S}} = \widehat{\mathbf{\Gamma}}(0) + \sum_{j=1}^p \left(1 - \frac{j}{p+1}\right) \left[\widehat{\mathbf{\Gamma}}(j) + \widehat{\mathbf{\Gamma}}'(j)\right].$$

We are now in the correct conditions to analyse the results of the estimation in Table 3.2.<sup>30</sup> In accordance with the J-statistic, we do not reject the null hypothesis of validity of instruments. Moreover, the estimated curves are sensible for several reasons. Labour income share has a positive effect on inflation for all countries, but it is statistically significant only for Italy and the UK.<sup>31</sup> The backward component of inflation is significant, with the exception of Germany and Japan.<sup>32</sup> However, the forward component dominates the backward component, and their sum is near 1. Only for France does the backward behaviour of inflation prevail. For Japan and Germany, the forward inflation coefficient is larger than 1, but is not significantly different from 1. Overall, this confirms the results Bardsen et al. (2004) and Jondeau and Bihan (2005).

The next step is to consider the interest rate augmented NKPC (equation (3.23) with  $\gamma_{pw} = 0$ ):

$$\pi_t^h = \gamma_f E_t \pi_{t+1}^h + \gamma_b \pi_{t-1}^h + \gamma_s \widehat{s}_t + \gamma_i \widehat{i}_t. \quad (3.26)$$

Solving this difference equation for inflation, our result show that present inflation depends on lagged inflation and on the discounted value of expected future marginal costs, as in Gali and Gertler (1999). Namely, with  $\delta_1$  and  $\delta_2$  as the roots of the stationary solution of the difference equation for inflation, and  $\delta_1 \leq 1$  being one stable root, the model's solution is:

$$\pi_t^h = \delta_1 \pi_{t-1}^h + \left(\frac{\gamma_s}{\delta_2 \gamma_f}\right) \sum_{k=0}^{\infty} \left(\frac{1}{\delta_2}\right)^k E_t \{\widehat{s}_{t+k}\} + \left(\frac{\gamma_i}{\delta_2 \gamma_f}\right) \sum_{k=0}^{\infty} \left(\frac{1}{\delta_2}\right)^k E_t \{\widehat{i}_{t+k}\}.$$

---

<sup>30</sup>We do not estimate the structural parameters because that often requires the calibration of some parameters, and we want to avoid imposing an homogenous calibration across countries.

<sup>31</sup>At a significance level of 15% the labour share is also significant for Germany, Japan and the USA.

<sup>32</sup>For Germany and Japan the backward component of inflation is negative and insignificant at a significance level of 5%. Thus, we restrict its coefficient to zero in both countries.

Table 3.2: Estimates of the baseline NKPC for domestic inflation

	$\pi_{t+1}$	$\pi_{t-1}$	$\hat{s}_t$	J-stat	Q-Stat (2)
Canada	0.6646*** (0.0763)	0.3414*** (0.0699)	0.0059 (0.0230)	0.0816 [0.1886]	34.555 [0.000]
France	0.4682*** (0.0812)	0.5413*** (0.0762)	0.0034 (0.0191)	0.1252 [0.4838]	28.608 [0.000]
Germany	1.0034*** (0.0240)	-	0.0573 (0.0347)	0.0854 [0.7718]	17.656 [0.000]
Italy	0.6922*** (0.0511)	0.3266*** (0.0435)	0.0627** (0.0300)	0.1405 [0.3830]	30.816 [0.000]
Japan	1.0599*** (0.0765)	-	0.0957 (0.0592)	0.0746 [0.7342]	37.440 [0.000]
UK	0.8468*** (0.0946)	0.1690* (0.0940)	0.0651** (0.0262)	0.0592 [0.7860]	33.985 [0.000]
USA	0.6676*** (0.0876)	0.3111*** (0.0844)	0.0245 (0.0155)	0.1009 [0.3655]	25.867 [0.000]

Notes: Instruments: four lags of inflation, labour income share and output gap. For Canada the lags  $t - 2$  to  $t - 4$  of output gap were not used. In general, we choose to eliminate the lags of the variables that are not sufficiently correlated with the lead inflation, based on the linear correlation and the first-stage regression. (...) contain standard errors. [...] contain p-values. \*\*\* means significance at 1%, \*\* at 5%, and \* at 10%.

The estimate of the covariance matrix of the moment conditions is robust to heteroskedasticity and serial correlation. The Newey-West fixed bandwidth and the Barlett Kernel were used.

Eviews calculates the J-statistic as  $\mathbf{g}_n(\hat{\beta})' \left( \mathbf{S}(\hat{\beta}) \right)^{-1} \mathbf{g}_n(\hat{\beta})$ . To obtain  $J$  in equation (3.25), we have to multiply the Eviews's statistic by  $n$ . The over-identified restrictions can be tested using the J-test statistic of

$$\text{Hansen (1982): } J \xrightarrow{d} \chi^2(r - k) = (\# \text{instruments} - \# \text{regressors (including constant)}).$$

Table 3.3: Estimates of the NKPC for domestic inflation augmented with the interest rate

	$\pi_{t+1}$	$\pi_{t-1}$	$\hat{S}_t$	$\hat{i}_t$	J-stat.	F-stat. 1 <sup>st</sup> stage
Canada	0.6181*** (0.0632)	0.3720*** (0.0598)	0.0070 (0.0195)	0.0016* (0.0008)	0.0909 [0.4643]	6.5423
France	0.4009*** (0.0671)	0.6092*** (0.0602)	0.0200 (0.0264)	0.0006 (0.0014)	0.1814 [0.3890]	16.8037
Germany	0.8225*** (0.0558)	0.1944*** (0.0588)	0.0426* (0.0240)	-0.0007 (0.0014)	0.1145 [0.7950]	1.9767
Italy	0.6554*** (0.0425)	0.3766*** (0.0356)	0.0077 (0.0246)	0.0040 (0.0024)	0.1732 [0.4347]	18.0363
Japan	1.0210*** (0.0752)	-	0.0607 (0.0547)	0.0003** (0.0001)	0.0830 [0.8496]	3.2839
UK	0.8226*** (0.0881)	0.1919** (0.0878)	0.0532** (0.0249)	0.0001 (0.0017)	0.0639 [0.9098]	7.7081
USA	0.6822*** (0.0559)	0.3035*** (0.0542)	0.0077 (0.0121)	0.0001 (0.0002)	0.1196 [0.4540]	21.9117

Notes: Instruments: four lags of inflation, labour income share, interest rate and output gap. For Canada the lags  $t - 2$  to  $t - 4$  of output gap were not used. See notes to Table 3.2.

We can then conclude that current inflation depends on last period inflation and on the discounted stream of expected future labour income share and interest rate.

Estimates of equation (3.26) show that the nominal interest rate had a positive effect on inflation for all countries, except Germany (Table 3.3) and the effect of that variable was statistically significant for Canada and Japan.<sup>33</sup> This indicates that the cost channel is not particularly strong in the majority of the analysed countries. These results are more negative for the cost channel as a general phenomenon than those of Chowdhury et al. (2006).<sup>34</sup>

In order to check the robustness of results, some sensitivity analysis was carried out. Firstly, we use the instruments that Chowdhury et al. (2006) use to estimate the NKPC with a cost channel. With four lags of inflation, real ULC, interest rate and real commodity prices as instruments estimates change considerably (Table 3.12, Annex 3.6). However, the

<sup>33</sup>It is also significant at 15% for Italy.

<sup>34</sup>The main aspects that may explain the differences between our results and the ones of Chowdhury et al. (2006) are: for non-euro countries we used an extended sample up to 2006, we used a CUE version of GMM, and the deviation of variables from the steady-state are obtained using an H-P filter and not the deviation from a constant.

evidence in favour of the cost channel remains almost the same: it continues to be statistically significant only for two countries. However, there are some significant changes in the interest rate coefficients at the country level. The most significant cases are Canada and France. For Canada the coefficient changes from positive and significant to negative and insignificant. For France the coefficient is always positive, but becomes significant, and the initial estimate does not belong to the two standard deviations' interval of the estimation made using Chowdhury et al.'s (2006) instruments. For Italy and Japan, there is also a change in the coefficients' significance. For Italy it becomes significant and in Japan it becomes insignificant. But in both cases the insignificant coefficient lies in the two standard deviations' interval of the significant coefficient.

Secondly, the estimator of the covariance matrix can be very sensitive to the choice of the bandwidth. Therefore, den Haan and Levin (1996) suggest using more than one way of choosing the bandwidth. In this context, we re-estimate equation (3.26) with bandwidth equal to 1, because the use of rational expectations introduces a first order MA representation in the error term. We also use the Andrews (1991) method to choose the bandwidth. This method depends on the empirical autocovariances of the moment conditions.<sup>35</sup> Once more, overall, estimates change considerably (Tables 3.13 and 3.14, Annex 3.6). Noticeable, the forward-looking component of inflation becomes more important, particularly when a bandwidth fixed at 1 is used. Concerning the cost channel, there is a slight reduction on its significance: it is significant only for Canada.

Next, in Table 3.3, the F-statistics of the first stage regression of the lead inflation on the instruments shows that for some countries the set of instruments is not very strong (the F-statistics is lower than the rule-of-thumb of 10).<sup>36</sup> In order to improve instruments' significance, they were reduced to two lags of inflation, and one lag of real unit labour costs, interest rate and output gap. This makes instruments less weak for the majority of countries, even though for two of them the F-statistic continues to be lower than 10 (Table 3.15, Annex

---

<sup>35</sup>The bandwidth selected is  $int [1.1447\hat{\alpha}(1)T]^{1/3}$ , where  $T$  is the number of observations. This method assumes that the sample moments follow an AR(1) process. Firstly, an AR(1) is fitted to each sample moment, and it is obtained the autocorrelation coefficients  $\hat{\rho}_i$  and the residuals variances  $\hat{\sigma}_i^2$ , for  $i = 1, \dots, z \cdot n$ , where  $z$  is the number of instrumental variables and  $n$  the number of equations in the system. Secondly,  $\hat{\alpha}(1)$  is function of all set of  $\hat{\rho}_i$  and  $\hat{\sigma}_i^2$ . See Eviews 6's user guide II, pg. 339, for more details.

<sup>36</sup>Notice that we use a CUE estimator that has better finite sample properties under weak instruments.

3.6). Also, the statistical significance of both real unit labour cost and interest rate does not improve and the backward inflation coefficient is only significant for the USA, contradicting some works showing the significance of this component in European countries (e.g. Gali et al., 2001; Bardsen et al., 2004; Jondeau and Bihan, 2005). Due to this and to ensure the comparability with other papers on the NKPC, in the remainder of the paper we continue, in general, to use as instruments four lags of each regressor.

Moreover, the more usual way of obtaining deviations from the steady-state was used: log deviations from the average,  $\hat{y}_t = \log(y_t) - \log(\bar{y}_t)$ , where  $y_t$  is any variable. Table 3.16, Annex 3.6, shows that evidence in favour of the cost channel becomes even weaker. This indicates that using the HP filter to obtain variables' deviations from the steady-state was a good choice. And if instead of using the HP filter, we use a Frequency (Band-Pass) Filter, estimates do not improve (see Table 3.17, Annex 3.6). The last filter isolates the cycle by specifying a range for its duration. In the empirical implementation, we define that cycles have a duration between 6 quarters (1.5 years) and 32 quarters (8 years). Then, a two-sided weighted moving average of the data is used to extract the cycles in a band. Specifically, we used the Christiano-Fitzgerald's (2003) filter, one of the most general filters that allows for asymmetry and time-varying weights.

After that, the output gap is used instead of the labour income share. Under certain conditions,<sup>37</sup> there is a log-linear relationship between the output gap ( $x_t$ ) and marginal cost (Gali and Gertler, 1999):

$$\widehat{mc}_t = \phi x_t. \tag{3.27}$$

Here, the output gap is defined as the percent difference between output and the output that would arise if prices were perfectly flexible. In Chapter 2, with flexible wages, the marginal cost in deviations from the steady-state is also a function of the output gap. This means that the NKPC can be written using the output gap, as in the traditional Phillips curve. Our empirical results using the output gap show that this variable has a negative coefficient for five countries (Table 3.18, Annex 3.6). Despite this, the cost channel is relatively more significant. The bad performance of the output gap confirms previous works showing that the

---

<sup>37</sup>Rotemberg and Woodford (1997) sticky price model assumes complete flexibility of nominal wages and absence of variable capital.

labour income share is a better driver of inflation than output gap. Gali and Gertler (1999) estimate the NKPC with output gap for the US and found a negative output gap's coefficient. Gali et al. (2001) obtain the same result for the US and euro area. These results may be explained by problems associated with the use of output gap. First, output gap is likely to be measured with error, namely due to the mismeasurement of the natural level of output. Second, the conditions under which the marginal cost is proportional to output gap may not be verified.

Maybe with a different measure of output gap, this variable would become more significant. For example, Garrat et al. (2009) obtain an output gap measure based on the Beveridge-Nelson decomposition of output using a vector-autoregressive model that includes data on actual output and on expected output obtained from surveys. They show for the US that such a measure of output gap performs well in explaining inflation in the NKPC.

Finally, we used the real unit labour cost (ULC) of the business sector,  $\hat{s}_t^b$ . Indeed, Balakrishnan and Lopez-Salido (2002) argue that the concept of labour income share may only make sense for the market sector of the economy. However, contrary to the results of Balakrishnan and Lopez-Salido (2002) for the UK, our results do not improve when considering only the real ULC of the business sector (Table 3.19, Annex 3.6). This variable becomes significant for Italy and maintains its significance for the UK. However, its coefficient becomes negative for Germany and the USA. Likewise, the cost channel does not become more significant.<sup>38</sup>

### 3.3.5 Estimating the NKPC with open economy variables

With regard to the impact of open economy variables on the effect of the nominal interest rate on inflation, we start by assuming complete and immediate exchange rate pass-through that leads us to use the real exchange rate. Indeed, notice that is possible to write:

$$\frac{P_{m,t}}{W_t} = \frac{(E_t P_{mt}^* / P_t^c)}{(W_t / P_t^c)} = \frac{RER_t}{\omega_t},$$

---

<sup>38</sup>We also used the lending rate from IMF instead of the policy rate. However, results do not change substantially, because the two rates have a high degree of correlation, due to the fact that the lending rates are prime rates.

Table 3.4: Estimates of the open economy NKPC for domestic inflation with the relative real exchange rate

	$\pi_{t+1}$	$\pi_{t-1}$	$\hat{s}_t$	$\hat{i}_t$	$\widehat{rer}_t - \widehat{\omega}_t$	J-stat.
Canada	0.5346*** (0.0629)	0.4313*** (0.0589)	0.0384* (0.0226)	0.0020** (0.0009)	0.0128** (0.0055)	0.1105 [0.5416]
France	0.3829*** (0.0513)	0.6143*** (0.0460)	-0.0203 (0.0201)	0.0017 (0.0012)	-0.0130 (0.0081)	0.1652 [0.7507]
Germany	0.7784*** (0.0471)	0.2526*** (0.0464)	0.0309*** (0.0112)	0.0006 (0.0014)	-0.0082 (0.0060)	0.1382 [0.8390]
Italy	0.6553*** (0.0725)	0.4624*** (0.0639)	0.4186*** (0.0759)	0.0033 (0.0052)	0.0594*** (0.0173)	0.1912 [0.6156]
Japan	0.9832*** (0.1108)	-	-0.0162 (0.0633)	0.0004* (0.0002)	0.0038 (0.0048)	0.0859 [0.9422]
UK	0.8910*** (0.0772)	0.1285 (0.0777)	0.0397 (0.0305)	-0.0023 (0.0021)	-0.0111 (0.0128)	0.0718 [0.9577]
USA	0.7005*** (0.0626)	0.2816*** (0.0565)	0.0241 (0.0159)	-0.00005 (0.00029)	0.0043 (0.0033)	0.1170 [0.7323]

Notes: Instruments: four lags of inflation, labour income share, interest rate, relative RER and output gap. For Canada the lags  $t - 2$  to  $t - 4$  of output gap were not used.

where  $E_t$  is the nominal exchange rate,  $P_{mt}^*$  is the price of imports in the world market,  $P_t^c$  is the domestic's CPI,  $RER_t$  is the real exchange rate (RER), and  $\omega_t$  is the real wage. Combining the latter expression and equation (3.23), we obtain: <sup>39</sup>

$$\pi_t^h = \gamma_f E_t \pi_{t+1}^h + \gamma_b \pi_{t-1}^h + \gamma_s \hat{s}_t + \gamma_i \hat{i}_t + \gamma_{qw} (\widehat{rer}_t - \widehat{\omega}_t), \gamma_{qw} > 0. \quad (3.28)$$

Estimation's results show that for Canada, Italy, Japan and the USA the coefficient of the relative RER is positive, being significant for Canada and Italy (Table 3.4). In turn, for France, Germany and the UK that coefficient is negative. In sum, the empirical fit of the open economy Phillips curve with immediate pass-through is not particularly good. Next, we analyse if it is empirically more successful to assume slow exchange rate pass-through.

Assuming slow exchange rate pass-through leads to the use of import prices instead of the RER (equation (3.23)). As can be seen from Table 3.5, the relative price of imports

<sup>39</sup>For this equation we use the inverse of the RER from International Financial Statistics (IFS) from IMF. This means that an increase in the RER corresponds to a *depreciation* of the *domestic* currency.

Table 3.5: Estimates of open economy NKPC for domestic inflation with the relative price of imports

	$\pi_{t+1}$	$\pi_{t-1}$	$\hat{s}_t$	$\hat{l}_t$	$\hat{\beta}_{m,t} - \hat{\omega}_t$	J-stat.
Canada	0.5136*** (0.0713)	0.4422*** (0.0640)	0.0380* (0.0220)	0.0009 (0.0012)	0.0251** (0.0103)	0.1098 [0.5477]
France	0.5222*** (0.0643)	0.4914*** (0.0580)	0.0162 (0.0298)	-0.0005 (0.0022)	0.0052 (0.0054)	0.1589 [0.6730]
Germany	0.5594*** (0.0278)	0.4281*** (0.0259)	0.0344** (0.0158)	-0.0015 (0.0013)	0.0212** (0.0098)	0.1239 [0.8952]
Italy	0.6172*** (0.0683)	0.4812*** (0.0604)	0.2108*** (0.0329)	-0.0048 (0.0045)	0.0309*** (0.0093)	0.1741 [0.6553]
Japan	0.9830*** (0.0986)	-	0.0072 (0.0737)	0.0003* (0.0002)	0.0141** (0.0057)	0.0801 [0.9353]
UK	0.8622*** (0.0784)	0.1363* (0.0789)	0.0305 (0.0291)	-0.0032* (0.0017)	0.0011 (0.0028)	0.1015 [0.8475]
USA	0.6671*** (0.0430)	0.3134*** (0.0411)	0.0055 (0.0126)	-0.0002 (0.0003)	0.0058 (0.0036)	0.1287 [0.6058]

Notes: For the UK the commodity price index was used instead of imports deflator. Instruments: four lags of inflation, labour income share, interest rate, relative price of imports and output gap. The exceptions are: for Canada the lags  $t - 2$  to  $t - 4$  of output gap were not used, for France the lag  $t - 4$  of the relative price of imports was not used, and for Japan the lag  $t - 4$  of the labour income share was not used.

has a positive coefficient for all countries; and for Canada, Germany, Italy and Japan, it is statistically significant.<sup>40</sup> It is worth mentioning that for the UK we use the relative commodity price (commodity price/nominal wage) instead of the imports price deflator, because the latter variable has a negative coefficient. In the estimations below, when necessary, the same procedure is followed.

In Section 3.3.1, it was mentioned that incorporating external variables in the Phillips curve may affect the estimated significance of the cost channel. From the above results, we can observe that introducing the price of imports in the Phillips curve reduced the evidence in favour of the cost channel: interest rate is no longer statistically significant for Canada and its coefficient becomes negative for France, Italy, the UK and the USA. But among the countries where the interest rate's coefficient is negative, it is only significant for the UK at 10% significance level. With the calibrated model of Section 3.3.1, it was predicted exactly

<sup>40</sup>For the USA it is also significant at 15%.

that the coefficient of the interest rate would decrease once the relative price of imports was explicitly considered in the Phillips curve.

Another important issue for monetary policy is whether or not there is a slow pass-through of exchange rates to import prices. If pass-through is immediate, the use in the NKPC of the real exchange rate or import prices will produce fundamentally the same results. In other words, using equation (3.23) or (3.28) will be equivalent. However, estimations' results show that the Phillips curve with import prices has a better empirical performance than the one with the real exchange rate. That can be seen in the fact that the coefficient of import prices has the correct sign and is statistically significant for more countries. Therefore, data support the hypothesis of slow exchange rate pass-through.

### 3.4 NKPC for CPI inflation

This section aims to explain CPI inflation using the NKPC. Once more, the empirical importance of the cost channel when import prices are considered is tested. Two more questions are studied. Firstly, we test if there is immediate pass-through of both exchange rates and world price changes to the price of imported goods. Secondly, it is analysed if imports should be treated as final consumption goods and/or inputs in production. Both of these questions are important for optimal monetary policy design. In a sticky prices environment, changes in prices lead to deviations from the flexible-price equilibrium, which is the efficient equilibrium. Therefore, the central bank has to avoid such inflationist pressures arising. In an economy with sticky domestic prices, flexible prices for imported final goods (*i.e.*, complete exchange-rate pass-through) and flexible wages, Clarida et al. (2001) show that a central bank should target domestic inflation. That occurs because only domestic prices are sticky. Furthermore, they show that the economy's representation in the space output gap and inflation is similar in closed and open economies, involving the same variables and only with differences in their coefficients. All this means that the policy design in an open and closed economy is similar.

In contrast with Clarida et al.'s (2001) model, McCallum and Nelson (2000) and Kara and Nelson (2003) suggest that when imports are treated only as inputs, all prices are sticky and therefore the policy maker should target CPI inflation. Kara and Nelson's (2003) empirical results for the UK show that imports should in fact be treated as inputs.

In order to answer the above questions, we begin by estimating the standard hybrid NKPC (Table 3.6):

$$\pi_t^c = \gamma_f E_t \pi_{t+1}^c + \gamma_b \pi_{t-1}^c + \gamma_s \widehat{s}_t.$$

The results obtained are sensible, with the coefficient of the labour income share being positive for all countries and statistically significant for Canada and Japan.<sup>41</sup> Since for Italy the labour income share has a negative coefficient, we used the output gap instead.<sup>42</sup> For Japan, the first lag of CPI inflation is negative but not significant. Then, after eliminating that lag, the first lead of inflation becomes significantly larger than 1. After this, we introduced two lags of inflation. The first lag continues to be negative and insignificant, but the second lag is positive and significant. Therefore, we choose the model with one lead and the second lag of inflation.

Comparing the Phillips curve for CPI inflation with the Phillips curve for domestic inflation, we observe that in the former case the average backward coefficient is more important, with the exception of France and Italy.<sup>43</sup>

Following this, the interest rate augmented NKPC is estimated as (Table 3.7):

$$\pi_t^c = \gamma_f E_t \pi_{t+1}^c + \gamma_b \pi_{t-1}^c + \gamma_s \widehat{s}_t + \gamma_i \widehat{i}_t.$$

We can observe that the interest rate always has a positive coefficient, except for Italy, but is statistically insignificant for all countries except for Japan.

### 3.4.1 NKPC assuming imports as final consumption goods

After testing the cost channel without open economy variables, import prices are introduced in the Phillips curve. We start by assuming imports as final consumption goods and full exchange rate pass-through. The Phillips curve proposed by Kara and Nelson (2002) is extended to include a cost channel and a backward component of inflation. Firstly, there is the domestic

---

<sup>41</sup>For the USA the labour income share is also statistically significant at 15%.

<sup>42</sup>For the same reasons, the output gap will also be used for Italy in the interest rate augmented Phillips curve and in the Phillips curve with import prices.

<sup>43</sup>For Italy the backward coefficient is insignificant in both formulations of the Phillips curve.

Table 3.6: Estimates of the baseline NKPC for CPI inflation

	$\pi_{t+1}^c$	$\pi_{t-1}^c$	$\hat{S}_t$	J-stat
Canada	0.5893*** (0.0583)	0.4081*** (0.0541)	0.0274** (0.0128)	0.1210 [0.2197]
France	0.5167*** (0.0531)	0.4774*** (0.0509)	0.0212 (0.0183)	0.1380 [0.3984]
Germany	0.7572*** (0.0535)	0.0253*** (0.0394)	0.0228 (0.0185)	0.0870 [0.7609]
Italy	0.9428*** (0.0863)	0.1058 (0.0829)	0.0212 (0.0199)	0.1209 [0.5138]
Japan	0.6998*** (0.1209)	0.2277*** (0.0813)	0.0594* (0.0349)	0.0995 [0.4097]
UK	0.5749*** (0.0622)	0.4328*** (0.0596)	0.0106 (0.0145)	0.0524 [0.8297]
USA	0.5784*** (0.0504)	0.4503*** (0.0424)	0.0395 (0.0253)	0.1007 [0.3751]

Notes: For Italy the output gap is used instead of the labour income share, and for Japan, the second lag of inflation is used instead of the first lag. Instruments: four lags of inflation, labour income share and output gap.

inflation's Phillips curve:

$$\pi_t^h = \gamma_f E_t \pi_{t+1}^h + \gamma_b \pi_{t-1}^h + \gamma_m \widehat{mc}_t. \quad (3.29)$$

Naturally, CPI inflation is

$$\pi_t^c = (1 - \gamma_m) \pi_t^h + \gamma_m \pi_t^m \quad (3.30)$$

where  $\gamma_m$  is the share of imported goods in the CPI basket, and  $\pi_t^m$  is the inflation rate of imported *consumer* goods. This equation can be written as:

$$\pi_t^c = \pi_t^h + \gamma_m (\pi_t^m - \pi_t^h). \quad (3.31)$$

Admitting full pass-through, we have  $\pi_t^m = \pi_t^w + \Delta e_t$ , where  $\pi_t^w$  is the world price of imported consumer goods and  $\Delta e_t$  is the change in the nominal exchange rate. This allows us to get  $\pi_t^m - \pi_t^h = \pi_t^w + \Delta e_t - \pi_t^h = \Delta q_t$ , where  $\Delta q_t$  is the change in the real exchange rate.

Table 3.7: Estimates of the NKPC for CPI inflation augmented with the interest rate

	$\pi_{t+1}$	$\pi_{t-1}$	$\hat{S}_t$	$\hat{i}_t$	J-stat.
Canada	0.6513*** (0.0400)	0.3395*** (0.0362)	0.0346** (0.0149)	0.0011 (0.0008)	0.1369 [0.3209]
France	0.5477*** (0.0482)	0.4542*** (0.0441)	0.0012 (0.0218)	0.0013 (0.0011)	0.1417 [0.4623]
Germany	0.7346*** (0.0469)	0.2687*** (0.0356)	0.0068 (0.0157)	0.00009 (0.00082)	0.1324 [0.6887]
Italy	0.5820*** (0.0697)	0.4244*** (0.0657)	0.0178 (0.0125)	-0.0011 (0.0013)	0.1554 [0.5427]
Japan	0.8338*** (0.0758)	0.1719*** (0.0612)	0.0055 (0.0270)	0.0003*** (0.0001)	0.1288 [0.3320]
UK	0.7931*** (0.0576)	0.2227** (0.0540)	0.0267** (0.0122)	0.0005 (0.0007)	0.0826 [0.7843]
USA	0.5900*** (0.0476)	0.4465*** (0.0425)	0.0510 (0.0289)	0.0008 (0.0008)	0.1012 [0.6244]

Notes: For Italy the output gap is used instead of the labour income share. Instruments: four lags of inflation, labour income share, interest rate and output gap. For France the lags  $t - 3$  and  $t - 4$  of the interest rate are not used, and for Japan the lag  $t - 4$  of the labour income share is not used.

<sup>44</sup> Using this result together with equation (3.31), we obtain:

$$\pi_t^h = \pi_t^c - \gamma_m \Delta q_t.$$

The last identity can be used in equation (3.29), to obtain:

$$\pi_t^c = \gamma_f E_t \pi_{t+1}^c + \gamma_b \pi_{t-1}^c + \gamma_m (\Delta q_t - \gamma_f E_t \Delta q_{t+1} - \gamma_b \Delta q_{t-1}) + \gamma \widehat{mc}_t.$$

If the cost channel is assumed, the marginal cost is:

$$\widehat{mc}_t = \hat{i}_t + \hat{s}_t,$$

---

<sup>44</sup> An increase in  $\Delta q_t$  corresponds to a depreciation of the domestic currency.

which allows us to obtain

$$\pi_t^c = \gamma_f E_t \pi_{t+1}^c + \gamma_b \pi_{t-1}^c + \gamma_s \widehat{s}_t + \gamma_i \widehat{v}_t + \gamma_{\Delta q} (\Delta q_t - \gamma_f E_t \Delta q_{t+1} - \gamma_b \Delta q_{t-1}). \quad (3.32)$$

In the data used an increase in  $q_t$  corresponds to a real *appreciation* of the domestic currency. In this case it is expected that an increase in  $\Delta q_t$  decreases inflation, since a real appreciation makes imports less expensive in domestic currency.<sup>45</sup> Therefore, in the last equation, instead of  $\gamma_m$  we have  $\gamma_{\Delta q} = -\gamma_m < 0$ .

Notice that the impact of  $E_t \Delta q_{t+1}$  and  $\Delta q_{t-1}$  on  $\pi_t^c$  is positive and contrary to the effect of  $\Delta q_t$ . In order to understand this take as example and increase in  $\Delta q_{t+1}$ . This decreases  $\pi_{t+1}^c$ , as we can observe in equation (3.32) for period  $t+1$ . But since we are interested on the impact of  $\Delta q_{t+1}$  keeping  $\pi_{t+1}^c$  constant,  $\pi_{t+1}^h$  has to increase (see equation (3.31) for  $t+1$ ). This implies that  $\pi_t^h$  also increases (equation (3.3)), leading to the increase of  $\pi_t^c$  (see equation (3.31)).

From Table 3.8, we can see that for France, Germany and the UK this model does not make sense, because the coefficient of the change in the real exchange rate is positive, and significant for the first two countries.<sup>46</sup> For the other countries the coefficient is negative as expected, but statistically insignificant.

The weak empirical performance of the model with imports as final consumption goods may be related to the assumption of full pass-through, *i.e.*, with the use of the real exchange rate. Indeed, in order to use the real exchange rate in equation (3.32), it is necessary to make two strong assumptions. First, it is assumed full and immediate pass-through of both exchange rate and world price to the price of imported goods in domestic currency. Second, the change in the real exchange rate is used as a proxy for:

$$\Delta e_t + \pi_t^w - \pi_t^h \quad (3.33)$$

It is easy to see that the change in the real exchange rate is only an approximation for equation (3.33): it is assumed that  $\pi_t^w$  is approximated by CPI inflation in the rest of the world and

---

<sup>45</sup>  $\Delta q_t = \log(q_t) - \log(q_{t-1})$

<sup>46</sup> For the UK it is significant at a 15% level of significance.

Table 3.8: Estimates of the NKPC for CPI inflation with imports as consumption goods, using the change in the RER

	$\pi_{t+1}$	$\pi_{t-1}$	$\hat{s}_t$	$\hat{i}_t$	$\Delta q_t$	J-stat.
Canada	0.6352*** (0.0376)	0.3495*** (0.0345)	0.0336** (0.0135)	0.0017** (0.0007)	-0.0049 (0.0244)	0.1464 [0.4662]
France	0.4661*** (0.0803)	0.5294*** (0.0712)	0.0335 (0.0214)	-0.00008 (0.00104)	0.1450*** (0.0302)	0.1384 [0.8754]
Germany	0.7615*** (0.0558)	0.2363*** (0.0435)	0.0192 (0.0140)	0.0003 (0.0008)	0.0405** (0.0163)	0.1430 [0.8174]
Italy	0.5278*** (0.0620)	0.4486*** (0.0605)	0.0184 (0.0132)	-0.0030*** (0.0009)	-0.0113 (0.0095)	0.1992 [0.5879]
Japan	0.9307*** (0.0901)	0.0989 (0.0659)	0.0622* (0.0339)	0.0003*** (0.0001)	-0.0116 (0.0082)	0.1392 [0.5729]
UK	0.6321*** (0.0481)	0.3776*** (0.0458)	0.0162* (0.0092)	-0.0005 (0.0007)	0.0239 (0.0152)	0.0999 [0.8282]
USA	0.6545*** (0.0440)	0.3544*** (0.0393)	0.0148 (0.0227)	0.0007 (0.0008)	-0.0142 (0.0247)	0.1169 [0.7488]

Notes: The coefficient of  $\Delta q$  is  $\gamma_{\Delta q}$ . Instruments: four lags of inflation, labour income share, interest rate, change in the RER and output gap.

$\pi_t^h$  is approximated by CPI inflation in the domestic country. <sup>47</sup>

Due to the assumptions made in order to use the real exchange rate, it may be better to use the price of imports, allowing for the possibility of slow exchange rate pass-through (Kara and Nelson, 2002). <sup>48</sup> To obtain the NKPC with imported goods' inflation, notice first that CPI inflation is given by equation (3.30), which can be re-written as:

$$\pi_t^h = \frac{1}{1 - \gamma_m} (\pi_t^c - \gamma_m \pi_t^m).$$

Plugging in the last equation on (3.29) and after some manipulations we obtain: <sup>49</sup>

$$\pi_t^c = \gamma_f E_t \pi_{t+1}^c + \gamma_b \pi_{t-1}^c + \gamma_s \hat{s}_t + \gamma_i \hat{i}_t + \gamma_m (\pi_t^m - \gamma_f E_t \pi_{t+1}^m - \gamma_b \pi_{t-1}^m), \gamma_m > 0 \quad (3.34)$$

<sup>47</sup>In our data the real exchange rate is based on the CPI.

<sup>48</sup>Kara and Nelson (2002) have used the following formulation of the PC:  $\pi_t = \beta E_t \pi_{t+1} + \alpha m c_t + \phi (\Delta p m_t - \Delta p_t)$ , where  $\Delta p m_t$  is the change in the price of imports and  $\Delta p_t$  is inflation rate.

<sup>49</sup>Regarding the coefficients of the labour income share and nominal interest rate, we obtain  $\gamma (1 - \gamma_m) (\hat{s}_t + \hat{i}_t)$ , which we simplify to a more unrestricted version:  $\gamma_s \hat{s}_t + \gamma_i \hat{i}_t$ , allowing  $\gamma_s \neq \gamma_i$ .

where  $\pi_t^m = \log(p_{m,t}) - \log(p_{m,t-1})$  is the quarterly change in import prices.

Estimating equation (3.34) shows that for all countries imported goods inflation has a positive and significant effect on CPI inflation (Table 3.9). This means that the NKPC with slow exchange rate pass-through has a better empirical adherence than the Phillips curve with immediate pass-through. In addition, our results go in the direction of Mihailov et al. (2009), which show that the expected relative change in the terms of trade is a more relevant determinant of inflation than the output gap.

Notice that for Italy only the lag of imported goods inflation is significant, with a positive sign, as expected. In opposition to equation (3.34), in this case lagged imported inflation should have a positive coefficient, because that variable does not have a direct effect on  $\pi_{t-1}^c$  and  $\pi_{t+1}^c$ .<sup>50</sup> Also, this empirical formulation does not assume that the impact of imported inflation on CPI inflation is given by equation (3.31).

With the introduction of imported goods' inflation in the Phillips curve, the coefficient of the interest rate becomes significant and changes to negative for France, Germany and the UK. In conclusion, it seems that the introduction of imports inflation reduces the empirical relevance of the cost channel.

Furthermore, when imported consumption goods are considered, the *average* backward component of inflation becomes more important for Germany, Italy, Japan and the UK. However, it remained basically the same for Canada and France, and for the US it decreased. In general, this shows that imports have an effect on the estimated degree of firms that are backward-looking. Probably, in some cases the omission of import prices, which are related to a volatile and forward-looking variable like the exchange rate, forces the forward component of inflation to be larger.

### 3.4.2 NKPC assuming imports as both final consumption goods and inputs

Besides being final consumption goods, imports are also used as inputs in production. Therefore, we tested a NKPC where imports are simultaneously inputs in production and final consumption goods. When imports are inputs, the marginal cost depends on the price of im-

---

<sup>50</sup>That can be easily seen by writing Italian's CPI inflation for  $t+1$ :  $\pi_{t+1}^c = \gamma_f E_t \pi_{t+2}^c + \gamma_b \pi_t^c + \gamma_s \hat{s}_{t+1} + \gamma_i \hat{i}_{t+1} + \gamma_m \pi_t^m$ . The same can be done for CPI inflation in  $t-1$ :  $\pi_{t-1}^c = \gamma_f E_t \pi_t^c + \gamma_b \pi_{t-2}^c + \gamma_s \hat{s}_{t-1} + \gamma_i \hat{i}_{t-1} + \gamma_m \pi_{t-2}^m$ .

Table 3.9: Estimates of the NKPC for CPI inflation with imports as consumption goods, assuming slow exchange rate pass-through

	$\pi_{t+1}$	$\pi_{t-1}$	$\hat{S}_t$	$\hat{i}_t$	$\pi_m$	J-stat.
Canada	0.6726*** (0.0414)	0.3240*** (0.0350)	0.0389*** (0.0130)	0.0004 (0.0007)	0.1482*** (0.0319)	0.1146 [0.7250]
France	0.6052*** (0.0364)	0.4399*** (0.0303)	0.0212 (0.0173)	-0.0026** (0.0010)	0.0518** (0.0259)	0.1995 [0.5123]
Germany	0.6513*** (0.0523)	0.3686*** (0.0429)	0.0428** (0.0180)	-0.0054*** (0.0015)	0.1940*** (0.0183)	0.1407 [0.8279]
Italy	0.4057*** (0.0686)	0.5878*** (0.0640)	0.0441*** (0.0129)	-0.0022 (0.0018)	0.0132*** (0.0046)	0.1616 [0.7240]
Japan	0.5788*** (0.0532)	0.3153*** (0.0519)	0.0061 (0.0303)	0.0005*** (0.0001)	0.1097*** (0.0229)	0.1539 [0.7253]
UK	0.5190*** (0.0257)	0.4873*** (0.0231)	0.0235*** (0.0077)	-0.0011** (0.0005)	0.0531*** (0.0115)	0.1183 [0.9136]
USA	0.6277*** (0.0440)	0.3918*** (0.0417)	0.0321 (0.0231)	0.0003 (0.0006)	0.1101*** (0.0340)	0.0993 [0.8316]

Notes: The coefficient of  $\pi^m$  is  $\gamma_m$ . For Italy the output gap is used instead of the labour income share, and the coefficient of imported goods' inflation corresponds only to  $\pi_{t-1}^m$ . Instruments: four lags of inflation, labour income share, interest rate, imports inflation and output gap. For Japan and UK four lags of the relative price of commodities (commodity price / nominal wage) were also used, which have proved to be good instruments.

ports, as shown in equations (3.14) and (3.22). In that case, the NKPC with a slow exchange rate pass-through is: <sup>51</sup>

$$\pi_t^c = \gamma_f E_t \pi_{t+1}^c + \gamma_b \pi_{t-1}^c + \gamma_s \widehat{s}_t + \gamma_i \widehat{i}_t + \gamma_{pw} (\widehat{p}_{m,t} - \widehat{w}_t) + \gamma_m (\pi_t^m - \gamma_f E_t \pi_{t+1}^m - \gamma_b \pi_{t-1}^m), \gamma_{pw} > 0, \gamma_m > 0 \quad (3.35)$$

Equation (3.35) allows the testing of different versions of the NKPC. On one hand, if as suggested by McCallum and Nelson (2000), imports should be treated only as inputs in production, then  $\gamma_m$  is non-significant and the coefficient  $\gamma_{pw}$  is positive and statistically significant. On the other hand, if the correct treatment of imports is both as final goods and inputs, then  $\gamma_m$  and  $\gamma_{pw}$  are both positive and statistically significant.

Table 3.10 shows that the relative price of imports has a positive effect on CPI inflation for all countries and is significant for France, Italy, and Japan. However, it is necessary to make two comments on the results. For Japan, in equation (3.35) imposing  $\gamma_m (\pi_t^m - \gamma_f E_t \pi_{t+1}^m - \gamma_b \pi_{t-1}^m)$  has proved to be too restrictive. Therefore, the model was estimated allowing free parameters:  $\gamma_{m,c} \pi_t^m + \gamma_{m,f} E_t \pi_{t+1}^m + \gamma_{m,b} \pi_{t-1}^m$ . For Germany, the best way of translating the impact of the relative price of imported inputs on inflation is using the lag of this variable. In sum, these results indicate that imports should be treated simultaneously as inputs and final consumption goods.

Let us highlight the two main empirical results obtained to this point. Firstly, imports should be treated as final consumption goods and there is a slow exchange rate pass-through. Secondly, there is also evidence that imports directly affect the marginal cost. Both of these findings support the presence of price rigidities in imports, and indicate that it may not be optimal for the central bank to target exclusively domestic inflation. Monacelli (2003) studies optimal monetary policy in a model with slow pass-through of exchange rates to import prices. In such an environment the representation of an open economy is not similar to that of a closed economy. Namely, deviations from the law of one price, and therefore the nominal exchange rate, are present in the NKPC; and it is optimal for the central bank to target CPI inflation when inflation's weight in its loss function is relatively high.

In an environment of slow exchange rate pass-through there are additional gains for the

---

<sup>51</sup>Recall that for estimation purposes, the price of imports is measured by either the imports price deflator or the commodity price index.

Table 3.10: Estimates of the NKPC for CPI inflation with imports both as consumption goods and inputs, assuming slow exchange rate pass-through

	$\gamma_f$	$\gamma_b$	$\gamma_s$	$\gamma_i$	$\gamma_m$	$\gamma_{m,f}$	$\gamma_{m,c}$	$\gamma_{m,b}$	$\gamma_{pw}$	J-stat.
Canada	0.6590*** (0.0406)	0.3263*** (0.0326)	0.0497*** (0.0125)	0.0019*** (0.0006)	0.0883*** (0.0207)	-	-	-	0.0043 (0.0066)	0.1421 [0.7094]
France	0.8426*** (0.0495)	0.2266*** (0.0408)	0.0582*** (0.0166)	-0.0043*** (0.0009)	0.0199** (0.0122)	-	-	-	0.0163*** (0.0034)	0.2059 [0.6805]
Germany	0.5665*** (0.0552)	0.4511*** (0.0423)	0.0547*** (0.0197)	-0.0071*** (0.0020)	0.1967*** (0.0200)	-	-	-	0.0058 (0.0106)	0.1496 [0.9107]
Italy	0.6211*** (0.0365)	0.3721*** (0.0328)	0.0078 (0.0099)	-0.0019** (0.0008)	-	-	-	0.0006 (0.0039)	0.0017* (0.0009)	0.1653 [0.8963]
Japan	0.5185*** (0.0615)	0.3247*** (0.0799)	0.0193 (0.0348)	0.00006 (0.0001)	0.0518*** (0.0175)	-0.0250** (0.0119)	0.0075*** (0.0024)	-0.0389*** (0.0083)	0.0075*** (0.0024)	0.1595 [0.4232]
UK	0.5181*** (0.0305)	0.4915*** (0.0284)	0.0260*** (0.0080)	-0.0013** (0.0006)	0.0552*** (0.0116)	-	-	-	0.0005 (0.0008)	0.1135 [0.9027]
USA	0.5869*** (0.0405)	0.4279*** (0.0368)	0.0203 (0.0185)	0.00001 (0.0007)	0.1510*** (0.0280)	-	-	-	0.0019 (0.0026)	0.1380 [0.7786]

Notes: For Italy, Japan and the UK  $pw_t$  is the relative price of commodities. For Germany we used  $pw_{t-1}$ .

Instruments: four lags of inflation, labour income share, interest rate, imports inflation, relative price of imports (or commodities) and output gap.

central bank to follow an optimal commitment policy (Monacelli, 2003). Monetary policy under commitment is able to affect, through the exchange rate, which is a forward-looking variable, the expected future deviations of the law of one price and consequently impact the equilibrium inflation and output gap.<sup>52</sup>

Also, if the exchange rate pass-through is not immediate, monetary policy should act to reduce the volatility of the nominal exchange rate (Monacelli, 2003). In the same direction, if the exchange rate has a slow effect on import prices, the role of exchange rate in affecting the relative prices among countries and therefore the current account is small. This means that the argument for exchange rate flexibility on the grounds that it is a substitute for domestic prices flexibility is weakened (Engel, 2002).

<sup>52</sup>The deviation from the law of one price is present in equation (3.12).

### 3.4.3 NKPC assuming consumer goods imports paid in advance

This section tests whether or not consumer goods imports are also paid in advance. If that is the case, CPI inflation takes the form (see equation (2.23))

$$\pi_t^c = \pi_t^h + \gamma_c \Delta \widehat{\delta}_t' + \gamma_c \Delta \widehat{i}_t.$$

Naturally domestic inflation is:

$$\pi_t^h = \pi_t^c - \gamma_c \Delta \widehat{\delta}_t' - \gamma_c \Delta \widehat{i}_t.$$

Using the last expression to replace  $\pi_t^h$  on the Phillips curve for domestic inflation, equation (3.26), we get after some manipulations:

$$\begin{aligned} \pi_t^c &= \gamma_f E_t \pi_{t+1}^c + \gamma_b \pi_{t-1}^c & (3.36) \\ &+ \gamma_c \left( \Delta \widehat{\delta}_t' - \gamma_b \Delta \widehat{\delta}_{t-1}' - \gamma_f E_t \Delta \widehat{\delta}_{t+1}' \right) \\ &+ \gamma_c \left( \Delta \widehat{i}_t - \gamma_b \Delta \widehat{i}_{t-1} - \gamma_f E_t \Delta \widehat{i}_{t+1} \right) \\ &+ \gamma_s \widehat{s}_t + \gamma_r \widehat{i}_t \end{aligned}$$

We next estimate the previous equation. Since the change in import prices has shown to be empirically successful, we use it instead of the change in the terms of trade.<sup>53</sup> According to Table 3.11, the change in the nominal interest rate has the expected coefficient for all countries and is always significant. Notice that for Japan and the USA the most sensible model excludes the lead and the lag of the change in the interest rate, respectively. Also, for the UK the most sensible model includes only the second lag of the change in the interest rate.

It can be argued that the last results are driven by the fact that we are replacing  $\Delta \widehat{\delta}_t'$  by  $\pi_t^m$  and so ignoring  $\pi_t^h$ . However, we confirm the previous results re-doing the estimations with  $\Delta (p_t^m/p_t^h)$  (see Table 3.20, Annex 3.6).

---

<sup>53</sup>Equation (3.36) becomes:  $\pi_t^c = \gamma_f E_t \pi_{t+1}^c + \gamma_b \pi_{t-1}^c + \gamma_m (\pi_t^m - \gamma_b \pi_{t-1}^m - \gamma_f E_t \pi_{t+1}^m) + \gamma_{\Delta i} (\Delta \widehat{i}_t - \gamma_b \Delta \widehat{i}_{t-1} - \gamma_f E_t \Delta \widehat{i}_{t+1}) + \gamma_s \widehat{s}_t + \gamma_r \widehat{i}_t.$

Table 3.11: Estimates of the NKPC for CPI inflation with imports of consumption goods paid in advance

	Canada	France	Germany	Italy	Japan	UK (1)	USA
$\gamma_f$	0.6911*** (0.0373)	0.6730*** (0.0451)	0.8601*** (0.0325)	0.3922*** (0.0390)	0.5185*** (0.0615)	0.6168*** (0.0368)	0.4839*** (0.0566)
$\gamma_b$	0.3080*** (0.0326)	0.3357*** (0.0407)	0.1619*** (0.0318)	0.5899*** (0.0364)	0.3247*** (0.0799)	0.4052*** (0.0314)	0.5216*** (0.0514)
$\gamma_s$	0.0452*** (0.0126)	0.0020 (0.0187)	0.0010 (0.0167)	0.0121 (0.0083)	0.0193 (0.0348)	0.0496*** (0.0114)	0.0481** (0.0233)
$\gamma_i$	-0.0003 (0.0008)	0.0007 (0.0009)	-0.0062*** (0.0012)	0.0003 (0.0011)	0.00006 (0.00013)	-0.0027*** (0.0009)	0.0004 (0.0006)
$\gamma_m$	0.0999*** (0.0355)	0.0629** (0.0250)	0.1236*** (0.0212)	-		0.0535*** (0.0121)	0.0943*** (0.0275)
$\gamma_{m,f}$	-	-	-	-	-0.0092 (0.0093)	-	-
$\gamma_{m,c}$	-	-	-	-	0.0501*** (0.0171)	-	-
$\gamma_{m,b}$	-	-	-	0.0050 (0.0036)	-0.0547*** (0.0123)	-	-
$\gamma_{\Delta i}$	0.0011*** (0.0003)	0.0011*** (0.0003)	0.0025*** (0.0005)	0.0007** (0.0003)	-	-	-
$\gamma_{\Delta i,f}$	-	-	-	-	-	-	-0.0020*** (0.0003)
$\gamma_{\Delta i,c}$	-	-	-	-	0.0064*** (0.0017)	-	0.0016*** (0.0004)
$\gamma_{\Delta i,b}$	-	-	-	-	-0.0005 (0.0008)	0.0008*** (0.0002)	-
J-stat.	0.0878 [0.9662]	0.1722 [0.6659]	0.1677 [0.8511]	0.1657 [0.8587]	0.1692 [0.6245]	0.1262 [0.9546]	0.1153 [0.8289]

Notes: (1) For the UK  $\gamma_{\Delta i,b}$  refers to the coefficient of  $\Delta \hat{i}_{t-2}$ . For Japan and the USA, imposing  $\gamma_{\Delta i} \left( \Delta \hat{i}_t - \gamma_b \Delta \hat{i}_{t-1} - \gamma_f E_t \Delta \hat{i}_{t+1} \right)$  is too restrictive. Therefore, for those countries we estimate the model with free parameters:  $\gamma_{\Delta i,c} \Delta \hat{i}_t + \gamma_{\Delta i,b} \Delta \hat{i}_{t-1} + \gamma_{\Delta i,f} E_t \Delta \hat{i}_{t+1}$ . Additionally, for Japan and the USA the most sensible model excludes the lead and the lag of the change in the interest rate, respectively. Instruments: four lags of inflation, labour income share, interest rate, imports inflation, change in the interest rate and output gap. Exceptions: for France the lags  $t - 2$  to  $t - 4$  of the interest rate's change is not used, and for Japan and UK four lags of the relative price of commodities are additionally used.

Previous results have shown that for France, Italy and Japan the *level* of relative import prices is statistically significant. When we add this variable to equation (3.36), the change in interest rate continues with positive and sensible coefficients (Table 3.21, Annex 3.6). However, for Italy only the contemporaneous effect of the change in the interest rate is significant.

After introducing the price of imports in the Phillips curve for domestic inflation or CPI inflation the interest rate has a negative coefficient for some countries. However, that coefficient is statistically significant only for a small number of countries: for the UK when the GDP deflator is used, and for Germany and the UK when CPI inflation is used (with the change in the interest rate). Because the negative coefficient of the interest rate in the Phillips curve is significant only for a small number of countries, not too much importance should be given to this effect. Nevertheless, we explore some possible explanations for a negative *supply* effect of the interest rate on inflation. The reasons that we point out go in the opposite direction of the traditional view of the cost channel, and help to understand why that channel is not statistically significant for some countries.

Firstly, interest rate may have a negative supply-side effect on inflation due to its effect on the equilibrium mark-up. To start with, in equation (3.3) the deviation of the real marginal cost from its steady-state is:

$$\widehat{mc}_t = mc_t - p_t - (-\Phi),$$

where  $\Phi$  is the steady-state mark-up over the marginal cost (in the steady-state:  $p = \Phi + mc$ ), which can be variable (Batini et al., 2005). Thus, when the steady-state mark-up increases, the equilibrium real marginal cost,  $-\Phi$ , decreases. In turn, this increases the gap between the actual real marginal cost and its equilibrium value, inducing firms to increase prices.

Now, we argue that the interest rate may have a negative effect on  $\widehat{mc}_t$ , through its negative effect on the equilibrium mark-up. Following Tirole (1988), it is necessary to remember that when there is collusion in an industry, the mark-up is high, and tacit collusion is only possible when firms put sufficient weight on future profits. Therefore, since an increase in the interest rate decreases the discounted value of future profits, it makes collusion more difficult. This means that markups and prices will decrease with an increase in interest rate. Besides that, an interest rate increase reduces aggregate demand, creating pressure for firms to reduce mark-ups, in order to maintain sales volume.

Secondly, credit growth can increase inflation, even when controlling for its effect on aggregate demand. In general, a large proportion of credit goes to buy property, a non-tradable good. Then, an increase in credit means a redirection of demand towards non-tradable goods, creating inflation even if it does not affect output gap.<sup>54</sup> As a result, because an increase in the interest rate reduces credit growth, it may have a negative effect on inflation, even after taking into consideration its effect on the business cycle .

Finally, an increase in the interest rate is usually associated with a decrease in expected inflation. If for any reason expected inflation is not properly accounted for in the Phillips curve, the interest rate's coefficient captures part of the effect of inflation's expectations on current inflation. This explains why that coefficient is negative. Without going into detail on the discussion of why expectations are not properly taken into account in the Phillips curve, we can make two comments. Firstly, Jondeau and Bihan (2005) show that a Phillips curve with three lags and leads emerges as a satisfactorily empirical model for inflation in the US and some European countries. Secondly, there is the fact that we assume rational expectations.

### 3.5 Conclusion

Our analysis of inflation in the seven largest industrialised nations confirms the standard features of the NKPC: both the forward- and backward-looking components of inflation are present, with the former being more important. Also, the real ULC has been shown to be the correct inflation driver.

Open economy variables play an important role in explaining inflation dynamics, both for domestic and CPI inflations. For some countries those variables are significant, while labour income share is not. Introducing the price of imports leads to some interesting conclusions. For France, Germany, Japan and the UK, the backward component of CPI inflation is larger in the open economy Phillips curve than in the closed economy Phillips curve. The empirical success of the NKPC with slow exchange rate pass-through is larger than with immediate pass-through. This is valid both for imported inputs and imported consumption goods. The model

---

<sup>54</sup>The supply of non-tradable goods responds more slowly to demand because in this sector productivity grows at a slower rate and there is less competition.

of McCallum and Nelson (2000), where imports are solely considered as inputs in production, is rejected by the data. Instead, a model with imports as both consumption goods and inputs has a better empirical adherence.

The fact that slow exchange rate pass-through is empirically relevant and that imports should also be considered as inputs has important implications for monetary policy: apart from domestic inflation, the central bank should be concerned also with CPI inflation, especially if inflation has sufficient weight on its goal; commitment in central bank actions becomes more relevant; and the volatility of nominal exchange rate becomes more harmful to social welfare.

In an open economy Phillips curve, we test two complementary versions of the cost channel. The first assumes that inputs (wages and imported intermediate goods) are paid in advance. The second version considers that trade companies pay imports of consumption goods in advance. In our sample, without considering import prices, there is some evidence in favour of the first concept of cost channel in both domestic and CPI inflation. That evidence becomes weaker when import prices are added to the Phillips curve. However, there is strong evidence that the cost channel is present in imported consumption goods.

### 3.6 Annex: additional tables

Table 3.12: Estimates of the NKPC for domestic inflation augmented with the interest rate - Chowdhury et al (2006)'s set of instruments

	$\pi_{t+1}$	$\pi_{t-1}$	$\hat{S}_t$	$\hat{i}_t$	J-stat.
Canada	0.5343*** (0.0508)	0.4578*** (0.0461)	0.0142 (0.0164)	-0.0009 (0.0011)	0.1031 [0.5614]
France	0.4617*** (0.0554)	0.5393*** (0.0507)	-0.0069 (0.0156)	0.0024*** (0.0008)	0.1401 [0.6977]
Germany	0.7398*** (0.0502)	0.3270*** (0.0418)	-0.0198 (0.0154)	0.0006 (0.0009)	0.1496 [0.6424]
Italy	0.8963*** (0.0680)	0.1314** (0.0549)	0.0887*** (0.0227)	0.0083** (0.0036)	0.1774 [0.4795]
Japan	0.8403*** (0.0613)	-	0.0776 (0.0554)	0.0002 (0.0001)	0.1010 [0.7321]
UK	0.8319*** (0.0870)	0.1611* (0.0870)	0.0268 (0.0261)	-0.0012 (0.0017)	0.0945 [0.7231]
USA	0.7868*** (0.0540)	0.1887*** (0.0485)	0.0395*** (0.0121)	0.0003 (0.0002)	0.1016 [0.6555]

Notes: Instruments: four lags of inflation, labour income share, interest rate and real commodity prices. For Canada the lag  $t - 4$  of real commodity prices was not used. Because the real commodity prices data starts in 1980Q2, implying that equations were estimated with a sample starting in 1981Q2, except for Canada where sample starts in 1981Q1.

Table 3.13: Estimates of the NKPC for domestic inflation augmented with the interest rate - Bandwidth fixed at 1

	$\pi_{t+1}$	$\pi_{t-1}$	$\hat{S}_t$	$\hat{i}_t$	J-stat.
Canada	0.7500*** (0.1020)	0.2530*** (0.0981)	0.0108 (0.0306)	0.0029* (0.0015)	0.1227 [0.4379]
France	0.5343*** (0.0784)	0.4765*** (0.0704)	0.0149 (0.0291)	0.0009 (0.0016)	0.2017 [0.2872]
Germany	0.9655*** (0.0957)	0.0671 (0.0919)	0.0332 (0.0414)	0.0001 (0.0014)	0.1263 [0.7263]
Italy	0.7618*** (0.0660)	0.2437*** (0.0526)	-0.0036 (0.0329)	0.0006 (0.0036)	0.2153 [0.2299]
Japan	0.9761*** (0.0948)	-	0.0560 (0.0652)	0.0003 (0.0002)	0.0941 [0.2916]
UK	0.8620*** (0.1126)	0.1514 (0.1094)	0.0733 (0.0493)	-0.0018 (0.0029)	0.0626 [0.9166]
USA	0.7209*** (0.0763)	0.2763*** (0.0750)	0.0125 (0.0183)	0.00009 (0.0004)	0.1570 [0.2011]

Notes: Instruments: four lags of inflation, labour income share, interest rate and output gap. For Canada the lags  $t - 2$  to  $t - 4$  of output gap were not used.

Table 3.14: Estimates of the NKPC for domestic inflation augmented with the interest rate - Bandwidth computed according with Andrews (1991)

	$\pi_{t+1}$	$\pi_{t-1}$	$\hat{S}_t$	$\hat{i}_t$	J-stat.
Canada	0.6587*** (0.0769)	0.3363*** (0.0739)	0.0136 (0.0207)	0.0019* (0.0010)	0.1018 [0.3658]
France	0.4655*** (0.0694)	0.5461*** (0.0614)	0.0144 (0.0267)	0.0010 (0.0016)	0.1944 [0.3214]
Germany	0.9092*** (0.0828)	0.1166 (0.0794)	0.0382 (0.0329)	0.0002 (0.0018)	0.1128 [0.8043]
Italy	0.7684*** (0.0633)	0.2397*** (0.0510)	-0.0028 (0.0326)	0.0008 (0.0035)	0.1969 [0.3094]
Japan	0.9889*** (0.0892)	-	0.0521 (0.0635)	0.0003 (0.0002)	0.0975 [0.7512]
UK	0.8291*** (0.0972)	0.1870* (0.0951)	0.0601* (0.0337)	-0.0004 (0.0021)	0.0636 [0.9118]
USA	0.7076*** (0.0685)	0.2834*** (0.0673)	0.0056 (0.0153)	-0.00004 (0.00037)	0.1427 [0.2818]

Notes: Instruments: four lags of inflation, labour income share, interest rate and output gap. For Canada the lags  $t - 2$  to  $t - 4$  of output gap were not used.

Table 3.15: Estimates of the NKPC for domestic inflation augmented with the interest rate - small set of instruments

	$\pi_{t+1}$	$\pi_{t-1}$	$\hat{S}_t$	$\hat{i}_t$	J-stat.	F-stat. 1 <sup>st</sup> stage
Canada	0.9498*** (0.1222)	0.0706 (0.1138)	0.0226 (0.0271)	0.0005 (0.0014)	0.023 [0.2905]	18.9669
France	0.7769*** (0.1770)	0.2510 (0.1623)	0.0168 (0.0503)	-0.0004 (0.0025)	0.0525 [0.2619]	41.9147
Germany	1.0156*** (0.0528)	-	0.0969 (0.0870)	0.0016 (0.0024)	0.0265 [0.5681]	5.2374
Italy	0.8790*** (0.1392)	0.1466 (0.1284)	-0.0005 (0.0549)	0.0008 (0.0040)	0.0031 [0.8883]	23.9695
Japan	1.0994*** (0.1347)	-	0.1115 (0.1331)	0.0004 (0.0003)	0.0229 [0.4839]	8.7184
UK	0.9979*** (0.1753)	0.0304 (0.1685)	0.0963** (0.0461)	0.00005 (0.0037)	0.0101 [0.5807]	20.2508
USA	0.6923*** (0.1188)	0.3111*** (0.1174)	-0.0028 (0.0222)	0.0003 (0.0003)	0.0010 [0.9425]	59.3504

Notes: Instruments: 2 lags of inflation, 1 lag of labour income share, output gap and interest rate. For France we used in addition lag 2 of the interest rate.

Table 3.16: Estimates of the NKPC for domestic inflation augmented with the interest rate - variables in log deviation from the steady-state

	$\pi_{t+1}$	$\pi_{t-1}$	$\hat{S}_t$	$\hat{i}_t$	J-stat.
Canada	0.6298*** (0.0684)	0.3728*** (0.0629)	-0.0030 (0.0095)	0.0001 (0.0003)	0.0897 [0.4758]
France	0.5176*** (0.0443)	0.4922*** (0.0423)	-0.0031 (0.0037)	0.0004 (0.0003)	0.1568 [0.5341]
Germany	0.8130*** (0.0533)	0.1870*** (0.0531)	0.0280* (0.0144)	-0.0014* (0.0007)	0.1060 [0.8394]
Italy	0.8089*** (0.0438)	0.2231*** (0.0365)	0.0252** (0.0102)	-0.0032 (0.0024)	0.1605 [0.5109]
Japan	0.9497*** (0.0645)	-	0.0022 (0.0087)	0.00009 (0.00006)	0.0987 [0.7420]
UK	0.9077*** (0.0980)	0.1275 (0.0943)	0.0205** (0.0098)	-0.0004 (0.0004)	0.0686 [0.8832]
USA	0.7076*** (0.0693)	0.2762*** (0.0660)	0.0017 (0.0054)	0.000003 (0.00009)	0.1105 [0.5323]

Note: Instruments: four lags of inflation, labour income share, interest rate and output gap. For Canada the lags  $t - 2$  to  $t - 4$  of output gap were not used.

Table 3.17: Estimates of the NKPC for domestic inflation augmented with the interest rate - deviations from the steady-state computed using Christiano-Fitzgerald (2003) Frequency Filter

	$\pi_{t+1}$	$\pi_{t-1}$	$\hat{s}_t$	$\hat{i}_t$	J-stat.
Canada	0.5784*** (0.0664)	0.4209*** (0.0628)	-0.0153 (0.0197)	0.0002 (0.0008)	0.0818 [0.4600]
France	0.7627*** (0.0580)	0.2791*** (0.0494)	-0.0162 (0.0244)	-0.0010 (0.0011)	0.1595 [0.5176]
Germany	0.9927*** (0.0750)	0.0301 (0.0685)	0.0209 (0.0172)	-0.0008 (0.0019)	0.0725 [0.9621]
Italy	0.9369*** (0.0594)	0.1132** (0.0512)	0.0018 (0.0300)	-0.00007 (0.0023)	0.1503 [0.5750]
Japan	0.9422*** (0.0887)	0.0422 (0.0552)	0.0124 (0.0307)	0.0002** (0.00009)	0.1053 [0.6149]
UK	0.6566*** (0.0754)	0.3627*** (0.0725)	-0.0024 (0.0203)	0.0006 (0.0012)	0.0784 [0.8173]
USA	0.7459*** (0.0667)	0.2412*** (0.0655)	-0.0108 (0.0118)	0.0051** (0.0002)	0.0889 [0.7257]

Note: Instruments: four lags of inflation, labour income share, interest rate and output gap. For Canada the lags  $t - 2$  to  $t - 4$  of output gap were not used.

Table 3.18: Estimates of the NKPC for domestic inflation augmented with the interest rate, using the output gap

	$\pi_{t+1}$	$\pi_{t-1}$	$\hat{x}_t$	$\hat{i}_t$	J-stat.
Canada	0.5789*** (0.0697)	0.3855*** (0.0655)	0.0490** (0.0229)	-0.0003 (0.0016)	0.0953 [0.4229]
France	0.2552*** (0.1014)	0.7119*** (0.0931)	-0.1210*** (0.0248)	0.0076*** (0.0020)	0.1609 [0.5085]
Germany	0.6732*** (0.0501)	0.3552*** (0.0430)	0.0324** (0.0139)	-0.0006 (0.0012)	0.1222 [0.7509]
Italy	0.5895*** (0.0608)	0.4967*** (0.0527)	-0.0927* (0.0481)	0.0262*** (0.0048)	0.1651 [0.4833]
Japan	0.9967*** (0.0866)	-	-0.0022 (0.0227)	0.0004*** (0.0001)	0.0891 [0.8134]
UK	0.7633*** (0.0879)	0.2550*** (0.0878)	-0.0279 (0.0221)	0.0015 (0.0014)	0.0898 [0.7254]
USA	0.7786*** (0.0696)	0.2015*** (0.0686)	-0.0140** (0.0069)	0.0003 (0.0003)	0.1105 [0.5249]

Note: Instruments: four lags of inflation, labour income share, interest rate and output gap. For Canada the lags  $t - 2$  to  $t - 4$  of output gap were not used.

Table 3.19: Estimates of the NKPC for domestic inflation augmented with the interest rate using the labour income share of the business sector

	$\pi_{t+1}$	$\pi_{t-1}$	$\hat{S}_t^b$	$\hat{i}_t$	J-stat.
Canada	0.6613*** (0.0847)	0.3361*** (0.0793)	0.0213* (0.0118)	0.0012 (0.0011)	0.0890 [0.4828]
France	0.3800*** (0.0809)	0.6203*** (0.0705)	0.0204 (0.0233)	0.0013 (0.0012)	0.1460 [0.6026]
Germany	0.4841*** (0.0613)	0.5456*** (0.0522)	-0.0415** (0.0190)	0.0029*** (0.0010)	0.1487 [0.5855]
Italy	0.8459*** (0.0728)	0.1645*** (0.0606)	0.0488** (0.0197)	0.0002 (0.0041)	0.1620 [0.6089]
Japan	0.9446*** (0.0890)	0.1303 (0.0804)	0.0376 (0.0330)	0.0002 (0.0002)	0.0621 [0.9195]
UK	0.8437*** (0.0891)	0.1820** (0.0863)	0.0704*** (0.0192)	-0.0009 (0.0016)	0.0644 [0.9076]
USA	0.7756*** (0.0589)	0.2095*** (0.0579)	-0.0011 (0.0099)	0.0002 (0.0002)	0.0890 [0.7248]

Note: Instruments: four lags of inflation, labour income share, interest rate and output gap. For Canada the lags  $t - 2$  to  $t - 4$  of output gap were not used.

Table 3.20: Estimates of the NKPC for CPI inflation with imports of consumption goods paid in advance and using the change in the relative price of imports

	Canada	France	Germany	Italy	Japan	UK (1)	USA
$\gamma_f$	0.6972*** (0.0393)	0.7035*** (0.0512)	0.9213*** (0.0384)	0.3002*** (0.0480)	0.5318*** (0.0762)	0.6603*** (0.0360)	0.5669*** (0.0581)
$\gamma_b$	0.3013*** (0.0345)	0.3110*** (0.0465)	0.1302*** (0.0360)	0.6887*** (0.0472)	0.1633* (0.0799)	0.3648*** (0.0314)	0.4439*** (0.0534)
$\gamma_s$	0.0432*** (0.0134)	0.0020 (0.0191)	0.0113 (0.0137)	0.0064 (0.0102)	-0.0092 (0.0297)	0.0463*** (0.0110)	0.0470** (0.0230)
$\gamma_i$	-0.0004 (0.0008)	0.0004 (0.0009)	-0.0079*** (0.0015)	0.0009 (0.0012)	0.0002 (0.00013)	-0.0028*** (0.0009)	0.0003 (0.0006)
$\gamma_{pm}$	0.1056*** (0.0289)	0.0733*** (0.0260)	0.1104*** (0.0127)	-		0.0491*** (0.0102)	0.0885*** (0.0299)
$\gamma_{pm,f}$	-	-	-	-	-0.0300** (0.0115)	-	-
$\gamma_{pm,c}$	-	-	-	-	0.0321** (0.0151)	-	-
$\gamma_{pm,b}$	-	-	-	0.0042 (0.0042)	-0.0293*** (0.0081)	-	-
$\gamma_{\Delta i}$	0.0013*** (0.0004)	0.0010*** (0.0003)	0.0023*** (0.0006)	0.0007*** (0.0002)	-	-	-
$\gamma_{\Delta i,f}$	-	-	-	-	-	-	0.0019*** (0.0003)
$\gamma_{\Delta i,c}$	-	-	-	-	0.0102*** (0.0022)	-	0.0016*** (0.0004)
$\gamma_{\Delta i,b}$	-	-	-	-	-0.0020** (0.0010)	0.0007*** (0.0002)	-
J-stat.	0.0946 [0.9277]	0.1557 [0.6913]	0.1719 [0.8349]	0.1555 [0.8995]	0.1610 [0.6798]	0.1032 [0.9875]	0.1120 [0.8480]

Notes: (1) For the UK  $\gamma_{\Delta i,b}$  refers to the coefficient of  $\Delta \hat{i}_{t-2}$ .  $\gamma_{pm}$  refers to the coefficient of the change in the relative price of imports. For Japan and the USA, imposing  $\gamma_{\Delta i} \left( \Delta \hat{i}_t - \gamma_b \Delta \hat{i}_{t-1} - \gamma_f E_t \Delta \hat{i}_{t+1} \right)$  is too restrictive. Therefore, for those countries we estimate the model with free parameters:  $\gamma_{\Delta i,c} \Delta \hat{i}_t + \gamma_{\Delta i,b} \Delta \hat{i}_{t-1} + \gamma_{\Delta i,f} E_t \Delta \hat{i}_{t+1}$ . Additionally, for Japan and the USA the most sensible model excludes the lead and the lag of the change in the interest rate, respectively. Instruments: four lags of inflation, labour income share, interest rate, change in the relative price of imports, change in the interest rate and output gap. Exceptions: for France the lags  $t - 2$  to  $t - 4$  of the interest rate's change are not used, and for Japan and UK four lags of the relative price of commodities are additionally used.

Table 3.21: Estimates of the NKPC for CPI inflation with imports both as consumption goods and inputs, assuming slow exchange rate pass-through and with the change in the interest rate

	France	Italy (1)	Japan
$\gamma_f$	0.8954*** (0.0666)	0.5609*** (0.0315)	0.5385*** (0.0674)
$\gamma_b$	0.1219** (0.0569)	0.4425*** (0.0286)	0.3577*** (0.0736)
$\gamma_s$	0.0315 (0.0249)	0.0456*** (0.0169)	0.0035 (0.0330)
$\gamma_i$	-0.0030*** (0.0009)	-0.0022** (0.0011)	0.0001 (0.0001)
$\gamma_m$	0.0585*** (0.017)	-	-
$\gamma_{m,f}$	-	-	0.0109 (0.0091)
$\gamma_{m,c}$	-	-	0.0043 (0.0135)
$\gamma_{m,b}$	-	0.0014 (0.0042)	-0.0267*** (0.0099)
$\gamma_{\Delta i}$	0.0005*** (0.0001)	-	-
$\gamma_{\Delta i,f}$	-	-	-
$\gamma_{\Delta i,c}$	-	0.0004** (0.0001)	0.0064*** (0.0013)
$\gamma_{\Delta i,b}$	-	-0.00007 (0.0001)	-0.0020** (0.0009)
$\gamma_{pw}$	0.0101*** (0.0010)	0.0018** (0.0007)	0.0066*** (0.0022)
J-stat	0.2059 [0.8778]	0.1554 [0.9620]	0.1770 [0.5065]

Notes: (1) For Italy  $pw_t$  refers to the second lag of the relative price of commodities. For the three countries  $pw_t$  is the relative price of commodities. For Italy and Japan imposing

$\gamma_{\Delta i} (\Delta \hat{i}_t - \gamma_b \Delta \hat{i}_{t-1} - \gamma_f E_t \Delta \hat{i}_{t+1})$  has shown to be too restrictive. Therefore, for these countries we estimated the model with free parameters:  $\gamma_{\Delta i,c} \Delta \hat{i}_t + \gamma_{\Delta i,b} \Delta \hat{i}_{t-1} + \gamma_{\Delta i,f} E_t \Delta \hat{i}_{t+1}$ . Likewise, for Japan we also estimated the model with free parameters for import prices:  $\gamma_m \pi_t^m + \gamma_m \Delta \pi_{t-1}^m + \gamma_m E_t \pi_{t+1}^m$ . Instruments: four lags of inflation, labour income share, interest rate, imports inflation, change in the interest rate, the relative price of commodities and output gap.

## Chapter 4

# Empirical analysis of inflation and business cycle convergence in the euro area

### 4.1 Introduction

Since the creation of the European Exchange Rate Mechanism (ERM) in 1979, there is evidence that monetary policy convergence in the euro area has been accompanied by inflation convergence. However, research indicates some inflation divergence after the introduction of the euro (Lane, 2006; Busetti et al., 2006). Such a phenomenon can also be observed in Figure 4-1. Associated with the nominal convergence demanded by the Maastricht Criteria, the cross section standard deviation of inflation rates in the euro area decreased to 0.6% in September 1999.<sup>1 2</sup> Subsequently, the standard deviation increased until it reached 1.2% in mid 2002. After these high values were reached, the downward tendency in inflation dispersion restarted, and in March 2007 the lowest level ever observed of 0.47% was achieved. In the first years of the euro (1999-2002), the countries with highest inflation rates were Greece, Ireland, the

---

<sup>1</sup>In the empirical results of this paper, “euro area” refers only to 12 countries, the original 11 plus Greece: Austria, Belgium, Finland, France, Greece, Germany, Ireland, Italy, Luxembourg, the Netherlands, Portugal and Spain.

<sup>2</sup>Data: annual inflation rates based on CPIs:  $(p_t/p_{t-12} - 1) \cdot 100$ . For each quarter, the standard deviation for the group of 12 countries was obtained.

Netherlands, Portugal and Spain.

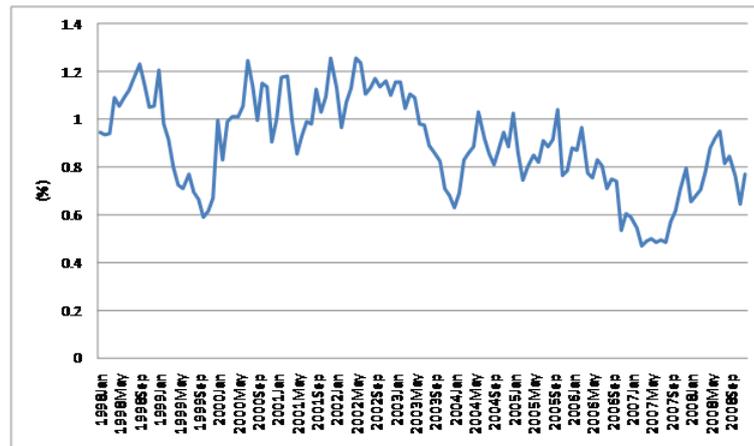


Figure 4-1: Cross section standard deviation of inflation rates after 1998

From the above analyse, we can observe that after the creation of the euro, inflation differentials have increased. In the beginning of the discussion below, we analyse the possible explanations for such a phenomenon. Firstly, inflation divergence may be due to equilibrating mechanisms. It is generally accepted that long-run relative price levels across countries depend on relative productivity or income levels. Therefore, since economic and monetary integration may lead to convergence of relative productivity and incomes, poor countries in an union will have temporarily higher inflation rates; this is known as the Balassa-Samuelson hypothesis (we will return to this in Section 4.5.1). Inflation differentials can also replace nominal exchange rate adjustments. Countries with low levels of economic growth, inflation and wage growth gain external competitiveness (Lane, 2006).

Besides equilibrating mechanisms, other explanation for inflation differentials rely on the fact that the baskets of goods used to measure CPI inflation differ from country to country. However, these differences have played a small role since the creation of the euro (ECB, 2003; Honohan and Lane, 2003).

On the other hand, the euro may produce inflation differentials with destabilizing macro-economic consequences. The convergence to the euro meant a bigger decline in real interest rates in peripheral countries. This implied a faster growth of credit, house prices, aggregate demand, and therefore inflation. This one-off expansionary shock dissipated over time; namely

because higher inflation led to the real appreciation of the currency.

A more recurrent situation in a monetary union is the existence of temporary asymmetric shocks. For example, with short-run supply rigidities, demand shocks create transitory inflation. Without a national monetary policy, the ability to deal with these shocks is limited. Inflation differentials cannot be corrected by a depreciation of the currency of high-inflation countries. In the case of deflationary shocks, countries may use expansionary fiscal policy to try to solve the problem. This can lead to a violation of the Stability and Growth Pact with negative effects on the euro area financial markets (Honohan and Lane, 2003).

The ability to deal with asymmetric shocks will be even more limited if persistent mechanisms are in place. If the labour market is not perfectly flexible, with current rather than future inflation determining wages growth, higher inflation today may lead to higher wage growth, starting an upward spiral of wage growth and inflation.

Indeed, Vines et al. (2006) show that when inflation is significantly persistent, countries in a monetary union maybe subject to large cycles after asymmetric shocks. In their model, fiscal policy can have an important role in reducing inflation differences between countries.

In addition, in a monetary union, higher than average inflation rates produce lower than average real interest rates, which may lead to both excessive debt accumulation and property prices growth, with the subsequent painful adjustment process. This can then exacerbate the differences in business cycles among European countries, widening inflation differentials even more, in a cycle of divergence (Honohan and Lane, 2003; Dullien and Fritsche, 2008).

There are however two stabilising mechanisms empirically relevant in the euro area (Hofmann and Remsperger, 2005). Firstly, GDP growth in one country has positive output spill-over effects on other countries, reducing inflation differentials. Naturally, small countries will have a limited impact on other countries. Secondly, the real exchange rate acts as a correcting mechanism: countries with higher than average inflation rates, will face a real appreciation that reduces demand and inflationary pressures. Even though this correction occurs at a gradual pace (Honohan and Lane, 2003), the effect accumulates over time, since external competitiveness depends on relative price levels (Hofmann and Remsperger, 2005).

From the above discussion, and as stressed by the optimum currency area literature, large inflation differentials can undermine the success of a monetary union. In this context, we want to understand what factors determine inflation differentials and the respective correcting

mechanisms at work. One of the main drivers of inflation is the business cycle, usually measured by the output gap or real unit labour cost (RULC).<sup>3</sup> As a result, convergence in inflation rates should be accompanied by convergence in the business cycles. Our main goal in this paper is to study the relationship between the two convergence processes. Specifically, we want to analyse if divergence (convergence) in inflation rates after the introduction of the euro can be explained by divergence (convergence) in business cycles. Indeed, the “ECB Inflation Persistence Network” concluded that the most important source of inflation differentials in the euro area was a “sustainable differential in wage growth and narrower differences in productivity growth”.

Let us highlight the most innovative features of this paper and our contributions to the literature. The analysis of convergence of the RULC and the output gap using, on one hand, the Kalman filter, as proposed by Hall et al. (1997), and on the other hand, the common factor approach of Becker and Hall (2009a) is new in the literature. Also, the study of the RULC, as an indicator of the business cycle, has been largely ignored in the convergence literature, even though this indicator is important in the New Keynesian approach to inflation.

The joint analysis of the convergence processes of inflation, the output gap and the RULC with Hall et al.’s (1997) model has two novelties. First, we compare the rates at which the (unobserved) convergence of inflation and output gap evolve over time. Second, we analyse the two-way causality between output gap and inflation convergence.

In summary, our analysis shows that during the euro period there were periods of convergence and divergence in inflation, the output gap and the RULC. However, in general there was an increase in co-movement in each of that variables during that period. The process of convergence of the RULC seems to be the most idiosyncratic of the three convergence processes considered. On a larger horizon, between 1980 and 2008, inflation rates have converged faster than output gaps. In the same period, output gap convergence had a positive effect on inflation convergence but the opposite did not occur.

When explaining inflation differentials, an innovative feature is the use of residuals of a common factor model to measure variables’ divergence. There are two more distinctive features of our study. To start with, we test how inflation and exchange rate expectations

---

<sup>3</sup>In the NKPC the inflation’s driver is the marginal cost, which can be measured using the labour income share,  $(W_t N_t / P_t Y_t)$ , also called real unit labour costs.

affect inflation divergence. Expectations have been mainly ignored despite their importance in explaining national inflation rates. Next, the New Keynesian framework is tested to see if it provides a complete description of inflation differentials, looking at the usefulness of the imperfect competition model. We are particularly concerned with the importance of both nominal ULC growth and equilibrium conditions for prices on inflation dynamics. As a by-product of the analysis of convergence, we estimate the NKPC for the euro area using panel data. This is interesting, because there is little evidence on the NKPC using panel data.

Our empirical evidence shows that expectations of both inflation and exchange rates are statistically significant for inflation differences and their introduction changes the significance of other variables. Moreover, the only business cycle indicator relevant for explaining inflation divergence is the labour costs. Also, the equilibrium conditions for prices are important for explaining differences in inflation rates. Besides, the ICM model is not encompassed by the NKPC when explaining inflation differences. Lastly, our panel data evidence supports the NKPC for national inflation rates and the existence of the cost channel.

The remainder of the paper is organised as follows. In Section 4.2 the main concepts of convergence are revised. Next, in Section 4.3 we analyse the convergence of inflation, output gap and RULC over the period 1980-2008, using the Kalman filter to test whether the variance of the unobserved convergence component decreases over time. The convergence of those three variables is analysed in Section 4.4 but using a common factor approach put into practice using principal component analysis. In Section 4.5 an econometric model is constructed to assess the relevance of the business cycle and other factors to explain inflation differentials. Finally, Section 4.6 concludes the study.

## **4.2 Measurement of convergence**

We have established above that the convergence of inflation rates is a necessary condition for the sustainability of a monetary union. However, there are many ways of measuring convergence of economic variables and it is difficult to agree on a satisfactory measure of economic convergence (Hall et al., 1997). Next, the main definitions of convergence and their implementability are revised.

Hall et al. (1997) refer to three definitions of convergence: pointwise, in expectations and

in probability. Pointwise convergence occurs when the scaled difference between two series converges to a constant:

$$\lim_{t \rightarrow \infty} (X_t - \theta Y_t) = \alpha,$$

where  $\alpha$  is a non-stochastic constant, some times set to zero, and  $\theta$  allows for scaling. This definition is too strong because it demands that in the limit the two series move exactly in the same way.

A less strict definition is convergence in expectations:

$$\lim_{t \rightarrow \infty} E(X_t - \theta Y_t) = \alpha.$$

This definition allows the difference between the two series to be random in the limit. However, based on this concept, it exists convergence between two white noise errors, which by construction are completely unrelated. This shows one weakness of this definition.

A stronger definition of convergence that rules out such cases is the convergence in probability:

$$p \lim_{t \rightarrow \infty} E(X_t - \theta Y_t) = \alpha.$$

Two sufficient but not necessary conditions for convergence in probability are:

$$\begin{aligned} \lim_{t \rightarrow \infty} E(X_t - \theta Y_t - \alpha) &= 0 \\ \lim_{t \rightarrow \infty} Var(X_t - \theta Y_t - \alpha) &= 0. \end{aligned}$$

These conditions imply that the difference between the series has to decrease gradually until it becomes a constant asymptotically. But this definition of convergence is not adequate for economic time series, because they are usually measured with error, and thus the variance of its difference will not go to zero asymptotically. This makes convergence in expectations a preferred definition.

Despite the fact that until now we only referred to convergence between two series, these definitions of convergence can be extended to a group of  $n$  series when they are applied to each pair of series:  $X_{it} - X_{jt}$ , for  $i, j = 1, 2, \dots, n, i \neq j$ .

It is easy to see that if two series are stationary, then they have converged in expectation,

but not necessarily in probability, because the majority of series are measured with some error. However, typically the discussion of convergence occurs in the context of non-stationary series. Here, it is possible to have at least three situations. Firstly, if the difference  $z_t = X_t - \theta Y_t$  is non-stationary as  $t$  goes to infinite, then there is no convergence by any of the previous definitions, since the variance of  $z_t$  will not go to zero asymptotically and there is no long-run mean to which series converge. Secondly, if  $X_t$  and  $Y_t$  are non stationary but cointegrated (and the cointegration residuals are  $I(0)$ ), then they have converged in expectation but not necessarily in probability. Many studies have used the concept of cointegration between series and the stationarity of the difference of two series to assess convergence (For example Siklos and Wohar, 1997; Holmes, 2002; Busetti et al., 2006; Gregoriou et al., 2007). Thirdly, it is possible that two series are non-stationary and non-cointegrated when the entire sample is considered, but they converge at the end of the sample. This occurs when, after an initial period of non-stationary behaviour, the difference between the variables becomes stationary due to changes in the economic environment. This means that cointegration is not a necessary condition for convergence. As Hall et al. (1997) highlight, convergence is defined as a limiting case, while cointegration is a concept that applies to entire sample.

Therefore, Hall et al. (1997) propose a more appealing way to measure convergence; which makes use of time-varying parameters and allows for convergence to take place gradually, as the series generating process evolves towards stationarity. The proposed model is then:

$$X_t - \theta Y_t = \alpha_t + \varepsilon_t \quad (4.1)$$

$$\alpha_t = \alpha_{t-1} + v_t \quad (4.2)$$

$$\varepsilon_t \sim N(0, \sigma^2)$$

$$v_t \sim N(0, \Omega_t)$$

$$\Omega_t = \phi \Omega_{t-1}, \Omega_o \text{ given.}$$

where  $\varepsilon_t$  is a random error that accounts for measurement errors. The model's central element is the unobserved component  $\alpha_t$ , which measures the convergence between series. The initial variance of  $v_t$  is given by  $\Omega_o$ . If the variance of  $v_t$  converges to zero ( $\phi < 0$ ), then  $\alpha_t$  will evolve to a non-stochastic constant, and convergence in expectation is guaranteed. A formal test

involves the null hypothesis of no convergence  $H_o : \phi = 1$ . Convergence in probability occurs also if the variance of  $\varepsilon_t$  is zero. This framework encompasses the evaluation of convergence based on cointegration tests. Indeed, an estimate of  $\Omega_o = 0$  for I(1) series means that they are cointegrated. Finally, this model is estimated using the Kalman filter, where  $\alpha_t$  is the state variable.

When studying the convergence of variables in a group of countries, it is interesting to test for each pair of countries if variables converge at the same rate. Let us take as an example the convergence of output gap and inflation in the euro area, which we will study below. It is relevant to know if the convergence of output gap and inflation occur at the same rate. To answer such a question, and taking for the sake of simplicity two variables  $Z$  and  $V$ , we estimate the following model for each pair of countries,  $i$  and  $j$ :

$$Z_t^i - \theta Z_t^j = \alpha_t^z + \varepsilon_t^z \quad (4.3)$$

$$V_t^i - \theta V_t^j = \alpha_t^v + \varepsilon_t^v \quad (4.4)$$

$$\alpha_t^z = \alpha_{t-1}^z + v_t^z$$

$$\alpha_t^v = \alpha_{t-1}^v + v_t^v$$

$$\varepsilon_t^z \sim N(0, \sigma_z^2); \varepsilon_t^v \sim N(0, \sigma_v^2)$$

$$v_t^z \sim N(0, \Omega_t^z); v_t^v \sim N(0, \Omega_t^v)$$

$$\Omega_t^z = \phi \Omega_{t-1}^z, \Omega_o \text{ given.}$$

$$\Omega_t^v = \left( \phi \cdot \phi^h \right) \Omega_{t-1}^v, \Omega_o^v = \Omega_o \cdot \Omega_o^h \text{ given.}$$

Where  $Z_t^k$  ( $V_t^k$ ) is the variable  $Z$  ( $V$ ) for country  $k$  ( $k = i$  or  $j$ ) on period  $t$ .<sup>4</sup> For each pair of countries, this model estimates simultaneously the convergence's equations for variables  $Z$  and  $V$  defined by Hall et al.' (1997) model (equation (4.3) and (4.4), respectively). The rate of convergence of the unobserved component' variance and the initial variance are allowed to be different for  $Z$  and  $V$ . This permits to compare, for the pair  $i$  and  $j$ , the rates of convergence of these two variables. If we do not reject  $H_o : \phi^h = 1$ , the two convergence

---

<sup>4</sup>All the variables refer to the pair  $i$  and  $j$ . But to simplify notation we do not put the superscript  $i$  and  $j$  on each variable.

processes occur at the same rate,  $\Omega_t/\Omega_{t-1} = \phi$ . These processes will be even more similar if the initial variance of the state variables also coincide, *i.e.*, if we do not reject  $H_o : \Omega_o^h = 1$ .

Other way of assessing convergence, involves looking at the cross-section variance of a number of series over time. When that variance declines over time and eventually goes to a constant,  $\sigma$ -convergence occurred (we have used this concept in Section 4.1). Notice that, when the time-varying parameters approach identifies convergence for a set of series, the difference between each pair in the group has converged to a constant plus an error term. As a result, the cross-section variance is also constant in the limit. This means that the two approaches to convergence produce similar results. However, Hall et al.'s (1997) measure presents some advantages, since it allows an easy implementation of a formal test of convergence and identifies the particular series that eventually have not converged.

In the economic growth literature it is also common to use the concept of  $\beta$ -convergence. According to Barro and Sala-i-Martin (1991), this concept of convergence implies that a set of time series are mean reverting. Consequently, countries with lower per-capita-income tend to grow faster. If per-capita-incomes converge to the same level, then convergence is absolute; while if a stationary difference persists between income levels, the convergence is relative. Typically, to evaluate  $\beta$ -convergence for a set of series, growth rates are regressed on the initial levels of the series. If a negative coefficient is found, that indicates convergence. Alternatively, we can regress variables' growth rates on their lagged levels (Hall et al. (1997)):

$$\log(y_{it}/y_{i,t-1}) = \alpha + \gamma \log(y_{i,t-1}) + u_{it},$$

with  $y_{it}$  as the per-capita-income of country  $i$  in period  $t$  and  $u_{it}$  as an error term. If  $\gamma$  is statistically smaller than 0, then we have  $\beta$ -convergence. Notice that this is simply a panel unit root test. Then, if series achieved  $\beta$ -convergence, they also have converged according with the definition of Hall et al. (1997), since for these authors convergence exists for a set of stationary variables.

Despite its merits, the definition of Hall et al. (1997) does not yield a measure of aggregate convergence over time. For that we should look at the definition of Becker and Hall (2009), which uses a common factor representation to assess convergence. To start with, it is assumed that the same indicator (for example inflation or RULC) observed for several countries over

time,  $x_{it}$ , with  $i = 1, \dots, p$ , is determined by a set of factors  $f_{jt}$ , with  $j = 1, \dots, p$ :

$$x_{it} = \lambda_{1i}f_{1t} + \dots + \lambda_{pi}f_{pt}, \quad (4.5)$$

where  $\lambda_{ji}$  is the weight of factor  $j$  for country  $i$ . The factors are orthogonal to each other. This general factor representation collapses to a common factor model, when the variables  $x_{it}$  have converged:

$$x_{it} = \lambda_i f_t + \epsilon_{it} \quad (4.6)$$

with  $\lambda_i \neq 0$  for all  $i$  and where  $\epsilon_{it}$  is a country specific error. Notice that  $f_t$  and  $\epsilon_{it}$  can have a time-invariant correlation structure. Variables will move perfectly together when  $\epsilon_{it}$  is zero for all  $i$  and  $t$ . This model is implemented using principal component analysis (PCA); or in other words an eigenvalue decomposition of the observed variance matrix. But if the series are in the process of converging, it is necessary to return to the multifactorial model (4.5). In this model, factors can be ordered by their explanatory power. Pointwise convergence will occur asymptotically when the first factor (the one that explains more variance) accounts for all the variance in  $x$ , meaning that  $\lambda_{2i} = \dots = \lambda_{pi} = 0$  in (4.5). In turn, convergence in expectation exists if in the limit only the first factor plus an error explain the full variance in the data, in which case we are in (4.6).

In practical terms, an ongoing convergence process can be measured by looking at the  $R^2$  of the first factor, which indicates the proportion of the variance explained by that factor. If  $R^2 = 1$ , pointwise convergence has been reached. But in general that would not happen due to measurement errors. Then, we have to analyse the  $R^2$ 's evolution over time, and as the  $R^2$  gets closer to one, the higher is the degree of convergence. Based on this idea, Becker and Hall (2009) propose to measure convergence over time looking at the evolution of the first component's  $R^2$ . If for a set of series for the period 1 to  $T$ , the  $R^2$  of the first component for the period  $T/2$  to  $T$  is larger than for the period 1 to  $T/2$ , then convergence has taken place. If for example in the first part of the sample the  $n$  variables were completely uncorrelated, the  $R^2$  will be very low, near  $1/n$ . But if in the second part of the sample, series start moving together, the  $R^2$  will increase accordingly. This example shows the ability of this approach in detecting processes of ongoing convergence.

The latter measure of convergence can also be applied when the data are integrated of order 1. Let us assume that there exists pairwise cointegration between  $n$  variables. Therefore, there is only one stochastic trend that is the first factor. The variance of this trend will grow asymptotically to infinite and dominate completely the variance in the data. As a result the  $R^2$  of the first component will go to 1. In finite samples, the  $R^2$  will measure the extent to which the stochastic trend is dominating the variance in the data.

It is worth mentioning that the approach of Becker and Hall (2009a) has the advantage of being able to detect divergence if variables are white noise errors, completely unrelated by definition. However, applying the concept of convergence in expectation to assess the convergence between those errors leads to the surprising conclusion that there exists convergence, since the series only differ by a stochastic error. But looking closer, there is no common factor, with each factor associated with an individual variable and being able to explain only a proportion  $1/n$  of the total variance. In this case, the  $R^2$  of the first factor will not grow to one asymptotically, ruling out the convergence hypothesis.

Finally, the common factor approach, besides producing a global measure of convergence, can also identify which countries are converging. That is done by looking at the weight of the first factor for each country. Complete convergence implies that for all countries the weight of the first principal component is 1 and the weight of all other components is 0.<sup>5</sup> This implies that the closer the first component countries' weights is to 1, the larger is the co-movement between countries (Hall and Becker (2009a)).

### 4.3 Testing convergence over the period 1980-2008

In this section, we analyse whether inflation, output gap or RULC of each country have converged with Germany in the period 1980-2008. Hall et al.'s (1997) approach is used to test if the variance of the unobserved convergence component decreased over time. The relationship between inflation and business cycle convergences will also be studied. Namely, the rate of convergence of the two phenomena will be compared, and the causality between them will be tested.

---

<sup>5</sup>If variables' scaling is necessary, as for example when exchange rates are involved, the weight of the first component does not go exactly to 1 for all countries when full convergence is achieved.

### 4.3.1 Convergence of inflation rates

Let us start with the study of inflation convergence from 1980 to 2008. Starting in 1980 will locate the evolution of inflation rates during the euro period in an historical context. Since Germany has been the leading economy in the euro area, we study the convergence of each country's inflation towards German inflation, analysing the difference between inflation rates of each country and Germany:  $\pi_{i,t} - \pi_{ger,t}$ , where  $\pi_{i,t}$  is the annualised quarterly inflation rate of country  $i$  in period  $t$ , and  $\pi_{ger,t}$  is inflation rate for Germany.<sup>6</sup> We analyse whether that difference evolves gradually towards stationarity, as outlined in model (4.1).

Under the null hypothesis  $\phi = 1$ , model (4.1) is non-stationary and  $\phi$  is in the boundary of the likelihood space.<sup>7</sup> So, under the null the test statistic follows a non-standard distribution. Using Monte Carlo simulations, Hall et al. (1997) suggest that  $\phi$  is asymptotically normally distributed and that the standard errors are underestimated by a factor which varies between 1.65 and 2.0.<sup>8</sup>

Looking at Table 4.10 in Annex 4.7, the null of non convergence is not rejected for Belgium, Luxembourg and the Netherlands.<sup>9</sup> In the former two cases the  $z$ -statistics is higher than 2, but in the latter case it is smaller than 1, indicating that in this last case the non rejection of the null is very clear. The reason why the null is not rejected for Belgium, Luxembourg and the Netherlands may be that, unlike for the other countries, for these three there is not a clear reduction in inflation's volatility (Figures 4-9 and 4-10, Annex 4.7). Inflation rates of these countries were already relatively more stable in the beginning of the sample and their average inflation differentials were among the lowest ones. In addition, the null hypothesis that the variance of the state variable is zero in the first period or in the last period for each of the three countries is not rejected (fifth and sixth columns of Table 4.10 in Annex 4.7, respectively). In other words, these countries already had a very high degree of convergence in 1980Q1, and for that reason the test does not identifies clearly further convergence afterwards. In addition, notice that in 2008Q4 the variance of the state variable converged to zero for the other countries as well (sixth column of Table 4.10 in Annex 4.7). In summary, in the period

---

<sup>6</sup>For data details see the Appendix 6.3.1.

<sup>7</sup>Note that with  $\phi > 1$  the model is explosive.

<sup>8</sup>The  $z$ -statistics's critical value at 5% significance for rejecting the null hypothesis (using a one-sided test:  $H_0 : \phi = 1$  vs  $H_1 : \phi < 1$ ) should be (in modulus) between 2.71 ( $=1.65*1.645$ ) and 3.29 ( $=2*1.645$ ).

<sup>9</sup>All the estimations using the Kalman filter were done with Eviews 6.

Table 4.1: Quarters of statistically significant divergence in inflation during the euro period

Country	Average of the state variable in the diverging period	No. of quarters of divergence	Quarters of divergence
Finland	1.60%	2	2000Q2-Q3
France	0.86%	3	2003Q3-2004Q1
Greece	2.12%	21	2000Q2, 2000Q4, 2001Q3-2003Q4, 2005Q1-Q2, 2006Q1-Q4, 2008Q1-Q3
Ireland	2.79%	20	1999Q2-2004Q1
Italy	1.38%	10	1999Q1, 2000Q2-Q3, 2002Q3-2004Q1
Luxembourg	0.36%	30	2000Q2-Q4, 2001Q2, 2002Q3-2008Q4
Netherlands	2.24%	12	1999Q1, 2001Q1-2003Q3
Portugal	2.28%	14	1999Q1, 2000Q2-Q3, 2001Q2-2003Q4
Spain	1.57%	40	1999Q1-2008Q4

Note: Inflation differentials are statistically different from zero when they are larger than  $2 \times RMSE$

1980-2008, there is evidence of inflation convergence in the euro area.

However, what we have just stated, does not mean that sub-periods of divergence did not exist. Such periods can be identified when the unobserved convergence variable,  $\alpha_t$ , is significantly different from zero. An estimate of that variable can be obtained using the filtered value of  $\alpha_t$ .<sup>10</sup> The root mean squared error (RMSE) can be used to assess if that estimate is statistically different from zero.<sup>11</sup> We can observe from Figure 4-9 and 4-10 (Annex 4.7) and Table 4.1 an increase in divergence (in the sense that the state variable stays significantly above zero for a certain number of periods) in some quarters after 1998 *especially* for Greece, Ireland, Italy, Luxembourg, the Netherlands, Portugal and Spain. In line with this finding, Buseti et al. (2006) identify Portugal, Greece, Ireland and Spain as a group where inflation differentials were stable after 1998, but with inflation rates relatively higher than the average. Notice that for these countries the divergence may be associated with the significant reduction in the *real* interest rate that accompanied the nominal convergence to the euro. For France and Finland there is also some inflation divergence, which however is statistically significant only for a very short period. But for all countries, except Luxembourg and Spain,

<sup>10</sup>The filtered state of  $\alpha_t$ ,  $\alpha_{t|t}$ , corresponds to the update of the one-step ahead forecast,  $\alpha_{t|t-1}$ , making use of the information in  $t$  (see equation (6.19) in Annex 6.3.2).

<sup>11</sup>See equation (6.20) in Annex 6.3.2.

the divergence is reversed latter in the sample (seventh column of Table 4.10, Annex 4.7). For Luxembourg and Spain the indicator of convergence (the final filtered value of the state variable  $\alpha_t$ ) is statistically different from zero in the last period of the sample. While for Luxembourg the indicator of divergence is very small, for Spain that indicator is considerably larger. This means that only for Spain may exist concerns regarding its long-run external competitiveness. In conclusion, inflation divergence was in general temporary in nature, not putting in danger the long-run stability of the euro area.

### 4.3.2 Convergence of real ULCs

It is well known that the business cycle affects inflation. As a result, our hypothesis is that inflation convergence in the euro area has been accompanied by convergence in the business cycles. There are however different indicators to measure the business cycle. Traditionally, output gap has been used to measure economic fluctuations. Alternatively, the New Keynesian approach argues that the correct driver of inflation is the RULC. To validate our hypothesis, in this section we study business cycles' convergence, starting by using the RULC.

There is some previous work by Dullien and Fritsche (2008) on the convergence of the growth of nominal ULC in the EMU using annual data, between 1960 and 2007. They do not reject the hypothesis of convergence for all EMU countries on two grounds. First, nominal ULC growth differentials towards the average are stationary. Second, there is cointegration between ULC growth rates of individual countries and the rest of the EMU. There is also no evidence of structural break in the convergence of nominal ULC growth rates caused by the introduction of the euro.

Using Panel Analysis of Nonstationary in the Idiosyncratic and Common components (PANIC), Fritsche and Kuzin (2007) are more pessimistic regarding ULC growth convergence in the euro area. They found that it is difficult to identify a common factor, with idiosyncratic factors explaining the majority of the variance. Besides that, countries respond to the common factor in very different ways, and it is possible to identify two groups of countries. One is the "hard currency" club, composed of Austria, Belgium, Germany, Luxembourg and the Netherlands. The other group includes Finland, Greece, Ireland, Portugal and Spain, which share common movement due to their catching-up processes.

Regarding our research, once again Germany is assumed as the leading country, and so the convergence tests were applied to the log difference between the RULC of each country and Germany. Since the data are expressed in indices, it is not correct to study the convergence towards the same level of RULC. But if two countries converge, we expect to observe their RULCs moving together, implying that RULC differentials fluctuate around a constant (not necessarily zero). However, it is possible to admit that in the beginning of the convergence process the co-movement of RULC between a high inflation country and Germany will be small. A high inflation country aiming to reduce inflation rate to the German level has to undergo an initial period of strong reduction in RULC. This will naturally imply initial divergence between the two countries. But once inflation has converged (as has occurred in euro area countries, as we have seen in Section 4.3.1), we expect that the RULC will basically grow at the same rate in both countries.<sup>12</sup>

The graphs of RULC differentials do not show a clear pattern of convergence, and after 2000 the differential actually increases for several countries (Figure 4-11, Annex 4.7). The formal test shows convergence for Austria, Finland, France, Greece, the Netherlands, Portugal, and Spain (Table 4.11, Annex 4.7). For Belgium, Ireland, Italy and Luxembourg there is no rejection of the non-convergence hypothesis.<sup>13</sup>

From the RULC's graphs of the seven countries for which the test identified convergence, we observe that the convergence process is not yet finished. To formally confirm this conclusion, a Wald test can be performed to analyse if the state variable residual's variance,  $var(\alpha_t)$ , is zero in the last quarter of the sample:  $H_0 : \Omega_{2008Q4} = 0$ .<sup>14</sup> For all countries where convergence was detected, with the exception of Portugal, this test rejected the null, confirming the incompleteness of the convergence process (Table 4.2). In fact, the variance of the state variable residual has been decreasing, but it was not yet zero in 2008Q4. This means that the RULC differentials still have a non-stationary behaviour, with convergence in

---

<sup>12</sup>This justifies the use of techniques that are able to detect ongoing convergence.

<sup>13</sup>For Luxembourg and Ireland, part of the data are interpolated.

<sup>14</sup>It is worth making two notes regarding this test. Firstly, as we are testing if the variance is zero, the test statistic has a non-standard distribution under the null. Therefore, we should use smaller significance levels. And in fact, we can reject the null for five countries with a level of significance of 1%. Secondly, as a Wald test is asymptotically equivalent to a likelihood ratio test, the null is testing more than if the variance is zero in the last period. Indeed, it is testing whether a full path of convergence leading to a zero variance in the last period of the sample does exist.

Table 4.2: Testing if the variance of the convergence variable of the RULC is zero in 2008Q4

Country	Test statistic	p-value
Austria	7.2513	0.0071
Finland	6.8000	0.0091
France	6.8956	0.0086
Greece	10.4141	0.0013
Netherlands	7.4400	0.0064
Portugal	2.8078	0.0938
Spain	4.7080	0.0300

Note: Wald test with the null hypothesis  $H_0 : \Omega_{2008Q4} = 0$  is performed for the countries for which it was obtained convergence in Table 4.11, Annex 4.7. The test statistics has a Chi-square distribution under the null.

expectation not yet achieved. But in the limit the variance will go to zero.<sup>15</sup>

In conclusion, full convergence in inflation was achieved despite the fact that RULCs are still converging. This casts some doubts over the ability of the RULC to explain inflation convergence. Therefore, in the next section we analyse output gaps' convergence.

### 4.3.3 Convergence of output gaps

In this section we study the convergence of output gaps in the euro area by analysing the difference between the output gap of each country and the output gap of Germany (Figure 4-12, Annex 4.7).<sup>16</sup> This indicator measures the synchronisation of business cycles, but it is not expected that its variance will go exactly to zero, because output gap is measured with some error. Instead, it is sensible to assume that as business cycles become more synchronised, the variance of output gaps' difference decreases.<sup>17</sup>

For all countries except Ireland, the variance of output gaps' difference has decreased in a statistically significant way between 1980 and 2008 (Table 4.12, Annex 4.7).<sup>18</sup> As noted, Ireland is the only exception, with the variance of output gaps' difference having actually

<sup>15</sup>Probably, results would not be different if convergence to the average was studied, because the RULC of Germany was highly aligned with the euro area average, as shown by the common factor approach below.

<sup>16</sup>The output gap is obtained with a HP filter ( $\lambda = 1600$ ) applied to the log of real GDP.

<sup>17</sup>Notice that for the output gaps we are not really interested in studying if there is convergence in expectation, because that is already ensured, as output gaps are stationary variables. Instead, our main goal is to understand how the variance of output gaps' difference evolves over time.

<sup>18</sup>In this test we use the standard critical values to test  $H_0 : \phi = 1$ , because the difference of output gaps is stationary even if  $H_0$  is not rejected.

increased ( $\phi > 1$ ). This result is strongly affected by the steep decrease in output gap that occurred in 2008.

We also notice that the convergence rates are quite similar among countries, ranging from -1.44% per quarter for Austria to -3.69% per quarter for the Netherlands (Table 4.12).<sup>19</sup> In addition, it is possible to identify some interesting patterns. On one hand, there is a group of countries with smaller rates of convergence: Austria, Belgium, France and Luxembourg. Probably, the output gap of these countries was already highly synchronised with Germany in 1980. On the other hand, we have the Southern countries: Greece, Italy, Portugal and Spain. These countries that in 1980 were less linked to the German business cycle have converged to it at higher rates. Also Finland that had strong trade links with the Soviet Union, has had a quick convergence towards the Germany's business cycle.

In general, since 1980, the business cycles of euro area countries have become more aligned. This was expected due to the increasing economic and monetary integration that has occurred in the euro area.

#### **4.3.4 Comparing the convergence processes of inflation and output gap**

From what we concluded above, there is strong evidence of convergence in inflation rates. On the business cycle side, there is also robust evidence of convergence between output gaps. In this context, one interesting question is whether both processes occur at the same rate. To answer that, we apply the model composed by equations (4.3) and (4.4) to inflation and

---

<sup>19</sup>The convergence rate is  $\Omega_t/\Omega_{t-1} - 1 = \phi - 1$ .

output gap:

$$\begin{aligned}
difx_t^i &= \alpha_t^x + \varepsilon_t^x \\
dif\pi_t^i &= \alpha_t^\pi + \varepsilon_t^\pi \\
\alpha_t^x &= \alpha_{t-1}^x + v_t^x \\
\alpha_t^\pi &= \alpha_{t-1}^\pi + v_t^\pi \\
\varepsilon_t^x &\sim N(0, \sigma_x^2); \varepsilon_t^\pi \sim N(0, \sigma_\pi^2) \\
v_t^x &\sim N(0, \Omega_t^x); v_t^\pi \sim N(0, \Omega_t^\pi) \\
\Omega_t^\pi &= \phi \Omega_{t-1}^\pi, \Omega_o^\pi \text{ given.} \\
\Omega_t^x &= (\phi \cdot \phi^z) \Omega_{t-1}^x, \Omega_t^x = \Omega_o^\pi \cdot \Omega_o^z \text{ given.}
\end{aligned} \tag{4.7}$$

where  $difx_t^i = x_t^i - x_t^{ger}$ , with  $x_t^i$  being the output gap of country  $i$  and  $x_t^{ger}$  the output gap of Germany. Also  $dif\pi_t^i = \pi_t^i - \pi_t^{ger}$ , with  $\pi_t^i$  as the inflation rate of country  $i$  and  $\pi_t^{ger}$  as the inflation rate of Germany.

For Belgium and Spain the convergence processes of inflation and output gap occurred at the same rate (we did not reject  $H_o : \phi^z = 1$ ).<sup>20</sup> In contrast, for Austria, Finland, France, Greece, Italy, Luxembourg and Portugal the processes were different (Tables 4.13, 4.14 and 4.15, Annex 4.7).<sup>21</sup> For these countries the convergence of inflation occurred at a faster rate than the convergence of output gap: on average 3.87% per quarter faster. The same occurs for the other two countries where the difference in the convergence dynamics of the two variables was not statistically significant. This may be explained by the Maastrich criteria that stressed the importance of nominal convergence.

For the Netherlands and Ireland both processes are clearly different, because in the Netherlands there was no clear convergence of inflation and in Ireland there was no convergence of output gap.

It is worth mentioning that the comparison between the rates of convergence of inflation rates and output gaps does not clarify if there was causality between the two processes. For instance, the two processes may have occurred at the same rate because other factors are

---

<sup>20</sup>We use a two-sided test because both  $\phi^z < 1$  and  $\phi^z > 1$  are plausible alternative hypothesis.

<sup>21</sup>For Finland and Greece the processes are different at a level of significance of 10%.

implying a common rate of convergence. Therefore, in the next section we study if there is in fact causality between both processes of convergence.

#### 4.3.5 Relationship between convergence of inflation and output gap

The convergence of inflation and the convergence of output gap may influence each other. On one hand, when a country's output gap is higher than the average output gap that creates pressure for its inflation to be also higher than average. On the other hand, inflation's convergence may affect output gap's convergence, even though the direction of the impact is unclear. It is true that if a country's inflation is growing faster than the average that will lead to a loss of external competitiveness, which may reduce output gap and lead to *convergence* of that variable. But on the contrary, high inflation leads to *lower* real interest rates, which will increase aggregate demand and lead to output gap divergence. Which of these described effects is the dominant one has to be determined empirically.

In our model one indicator of convergence is the state variable,  $\alpha_t$ . In order to study the relationship between the convergence of output gap and inflation, we made two changes to model (4.7). First, we assumed that the last period state variable of output gap may affect the current state variable of inflation (equation (4.9)). And since as we have seen the causality can be bidirectional, it was also assumed that the last period state variable of inflation may influence the current state variable of output gap (equation (4.8)). That leads to the following

model, where all equations are estimated simultaneously for each country  $i$ :

$$\begin{aligned} difx_t^i &= \alpha_t^x + \varepsilon_t^x \\ dif\pi_t^i &= \alpha_t^\pi + \varepsilon_t^\pi \\ \alpha_t^x &= \gamma_{gg}\alpha_{t-1}^x + \gamma_{ig}\alpha_{t-1}^\pi + v_t^x \end{aligned} \tag{4.8}$$

$$\alpha_t^\pi = \gamma_{ii}\alpha_{t-1}^\pi + \gamma_{gi}\alpha_{t-1}^x + v_t^\pi \tag{4.9}$$

$$\varepsilon_t^x \sim N(0, \sigma_x^2); \varepsilon_t^\pi \sim N(0, \sigma_\pi^2)$$

$$v_t^x \sim N(0, \Omega_t^x)$$

$$\Omega_t^x = \phi^x \Omega_{t-1}^x, \Omega_o^x \text{ given.}$$

$$v_t^\pi \sim N(0, \Omega_t^\pi)$$

$$\Omega_t^\pi = \phi^\pi \Omega_{t-1}^\pi, \Omega_o^\pi \text{ given.}$$

Some comments are necessary on parameters  $\gamma$ . Firstly, we allowed  $\gamma_{gg}$  and  $\gamma_{ii}$  to be different from 1 to ensure the model's stability. Furthermore, when one of the series converges and the other does not, only some values for  $\gamma$  make sense. If output gap converges and inflation does not converge, then  $\gamma_{ig} = 0$ . Otherwise, in the limit there was a non stationary component in output gap. Likewise,  $\gamma_{gi} = 0$  if output gap does not converge and inflation converges. Finally, if both series converge,  $\gamma_{ig}$  and  $\gamma_{gi}$  may or may not be different from zero.

As expected from the discussion above, our results (see Tables 4.16, 4.17 and 4.18, Annex 4.7) show that the effect of output gap convergence on inflation convergence is positive for all countries except Luxembourg. But this effect is statistically significant only for Finland, the Netherlands and Portugal.<sup>22</sup>

The sign of the effect of inflation convergence on output gap convergence varies from country to country and is never statistically significant. One explanation for this result may be that the two effects of inflation convergence on output gap convergence described above tend to compensate each other; or alternatively neither of those effects is significant. These results show that inflation differentials do not cause further divergence in output gap, which makes the destabilizing effects of inflation differentials smaller. For future work, it would be

---

<sup>22</sup>For Finland and the Netherlands the coefficient is significant at 10% only.

interesting to analyse if we obtain more significant results considering the *accumulated* effect of inflation divergence on output divergence.

In summary, in this section we concluded that from 1980 to 2008, inflation differentials in the euro area have converged in expectation, despite the emergence of some temporary divergence after the introduction of the euro. The business cycles of euro area countries, as measured by the output gap, have also become more aligned. Also, output gap convergence had a positive effect on inflation convergence, even though statistically weak for the majority of countries.

The methodology applied here does not allow for the clear identification of sub-periods of convergence and divergence in the business cycles. However, in the literature on business cycles, periods of convergence and divergence have been identified (Massmann and Mitchell, 2004). In order to be able to identify these sub-periods, we are going to apply the common factor approach in the next section.

## 4.4 Common factor approach to convergence

In this section we use the common factor approach, developed by Becker and Hall (2009a), to measure *aggregate* convergence in inflation and business cycles.<sup>23</sup> Besides producing an aggregate measure of convergence, this approach allows us to study the aggregate and country-specific convergences in short periods of time. This permits a more detailed analysis of the co-movement, especially of the output gaps and RULCs.<sup>24</sup>

### 4.4.1 Inflation

We start by analyzing the convergence of inflation rates since 1980, with emphasis on the euro period and using annualised quarterly inflation rates.

---

<sup>23</sup>In this section, we say that convergence has taken place when there is an increase in the co-movement between series. As we will see below, that does not mean necessarily that the absolute difference between series has decreased.

<sup>24</sup>This means that, as an example, instead of simply saying that between 1999 and 2008 the output gap of France has converged, we can say that in the period 1999 to 2004 it has diverged and from 2005 to 2008 it has converged.

## Analysis of long periods

Let us start by analysing long term convergence. For that we define three windows: 1980-89, 1990-98 and 1999-2008. The latter window begins when the euro was launched.

Looking at the  $R^2$  of the first component, the period 1980-89 is the one with higher co-movement between inflation rates (0.7040) - Table 4.3. In 1990-98 a decrease occurs in inflation rates' association (0.5758). Finally, during the euro area period (1999-2008) the link between inflation rates remains basically the same as in the previous window.

Table 4.3:  $R^2$  of the first component for three subperiods between 1980 and 2008

	<b>1980-89</b>	<b>1990-98</b>	<b>1999-2008</b>
Inflation	0.7040	0.5758	0.5753
Output gap	0.3560	0.6464	0.7225
RULC	0.8077	0.6850	0.6187

The decrease in the correlation of inflation rates in the period 1990-98 can be understood in the context of both the European currency crisis and the subsequent convergence process in the run-up to the euro. Indeed, during the 1990's there was a decrease in inflation differentials, as can be seen in the reduction in the standard deviation of inflation rates (Figure 4-2). To achieve this, national inflation rates had different evolutions: while some countries were converging, others had already converged.

After the creation of the single currency, the fact that inflation rates on average have not moved more closely may seem unexpected. However, as we will see below, there were periods of convergence and divergence and the last periods had a co-movement between inflation rates larger than in the 1980's. In any case, an explanation for why the convergence on average was not higher can probably be found in the dynamic interplay between inflation and output gap in a monetary area, associated with the consequences in some countries of the nominal convergence process that occurred before the euro.

We established above that the  $R^2$  of the common factor was bigger in the 1980's than both in the 1990's and during the euro period. We get the same picture if we analyse the *average* correlation between the inflation rate of the 12 euro area countries and the inflation rate of each country: the correlation has decreased from 0.8259 in 1980-89 to 0.6907 in 1999-2008.

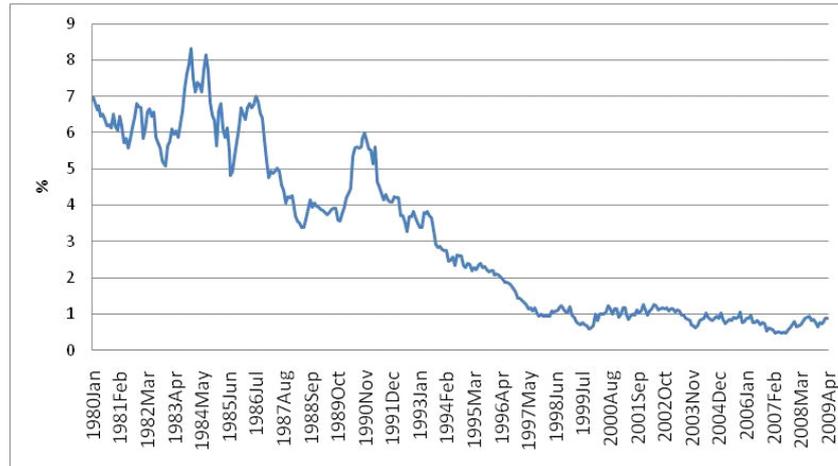


Figure 4-2: Cross sectional standard deviation of inflation rates in the euro area (countries have equal weight)

This occurs despite the fact that during the 1980's the absolute inflation differentials were larger than in the period 1990-2008.

Even though the  $R^2$  in the 1980's was higher than in the euro period, the *absolute* unexplained variance by the common factor is much higher in the former period. That can be seen by performing PCA using the covariance instead of the correlation. So, for the period 1980-89 the  $R^2$  was 0.64 and the unexplained variance was 0.010289 for a total variance of 0.028609.<sup>25</sup> For the period 1999-2008, due to much smaller inflation differentials, the total variance of inflation decreased to 0.00284 and the unexplained variance also decreased to 0.00119. However, the  $R^2$  decreased to 0.5806. In sum, in relative terms there was less co-movement, but in absolute terms the unexplained variance was approximately 10 times smaller.

### Euro area period

Now, we are going to focus on the euro period, defining several five-years windows, as can be seen in Figure 4-3. During the euro period, inflation's convergence has evolved in a U-shape. During the first three windows, 1997-2001, 1998-2002 and 1999-2003 inflation co-movement was higher than in the next three windows. The loss of co-movement that occurred in the

<sup>25</sup>Notice that it is normal that the results with the covariance and correlation are different.

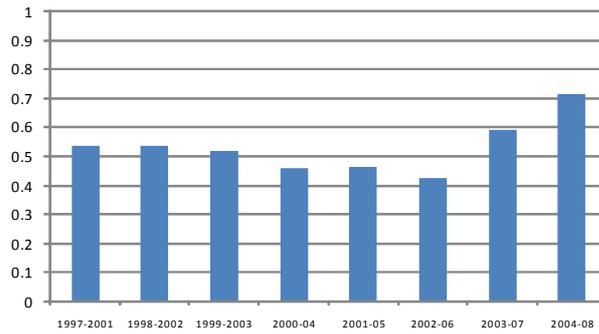


Figure 4-3: Inflation co-movement during the euro period.

windows 2000-04 and 2002-06 was recovered in 2003-07 and 2004-08, with the  $R^2$  in the latter window being 0.71, higher than in the first window 1997-2001, which was 0.53.

Notice that the periods of divergence include the years of the European slowdown and recession of 2001-04. In the opposite direction, during the last window that includes the deep recession of 2008 the convergence has increased strongly. From here we observe that the relationship between inflation convergence and the business cycle is not tight; or that other factors are affecting inflation convergence.<sup>26</sup>

Regarding the periods of divergence, the evidence just obtained is consistent with the results from Section 4.3.1. There, it was identified that, loosely speaking, between 2000 and 2004 some countries had a temporary divergence in inflation.

**Country analysis** Over the period 1999-2008, the countries less correlated by far with the average were the Netherlands and Portugal (Figure 4-4). The next group of countries less correlated is composed by Finland and Ireland. We call the group composed of Finland, Ireland, the Netherlands and Portugal the “divergent group”. Notice that in this group the only country from what we can call “core” Europe is the Netherlands. When we refer to the group "core" Europe we are, loosely speaking, talking of Austria, Belgium, France, Germany, Luxembourg and the Netherlands.

The evolution of inflation in the divergent group, especially in the Netherlands and Por-

---

<sup>26</sup>This comment should be read in the light of the fact that our sample includes few observations on the financial crisis which started in 2008.

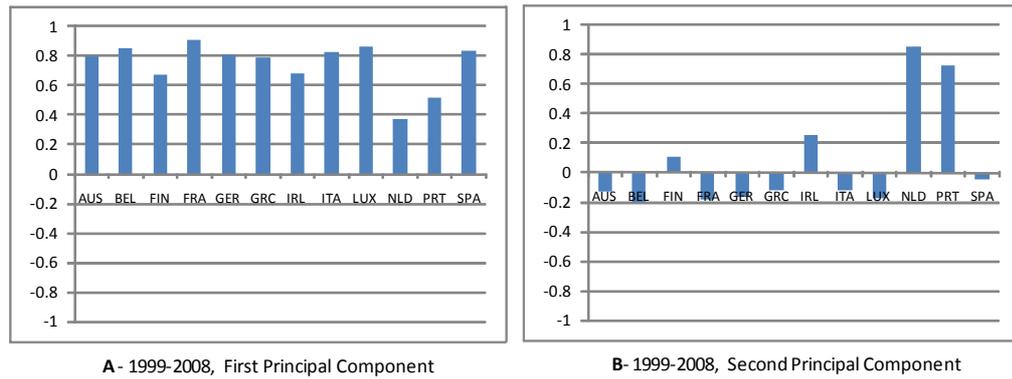


Figure 4-4: Country weights of first and second principal components of inflation

tugal, is positively and strongly correlated also with the second principal component (PC), which explains 12.42% of the variance for the period 1999-2008. So, we can conclude that this PC is associated with the observed divergence.<sup>27</sup>

Turning now to the analysis of countries' behaviour in sub-periods, we can start by the two major episodes of divergence, which occur in the windows of 2000-04 and 2002-06. In 2000-04 the major countries responsible for the divergence were Finland, Ireland, the Netherlands and Portugal (Figure 4-13, Annex 4.7).<sup>28</sup> The inflation rate of Portugal actually moved in the opposite direction to the others countries' inflation. The correlation between the inflation rates of Austria, Greece and Germany and the average rate also decreased in relation to the previous rolling window, but less than for the divergent group.

In comparison with the previous window, in 2002-06 Finland and Greece's inflation moved less with the rest of the monetary area. But the biggest reduction in correlation occurred for Italy and Belgium. For Italy a smaller decrease in correlation had already occurred in the 1999-2003 window. In addition, the inflation behaviour of the two biggest economies of the euro area, France and Germany, also became less synchronised with other countries. For France this phenomenon was already in place in the three windows since 1999 (1999-2003, 2000-04, 2001-05), even though its inflation has always kept high levels of correlation with the average. Among the large countries, Spain was the one that suffered less divergence since the

<sup>27</sup>The other PC with less explanatory power also capture divergence, but it has an eigenvalue lower than 1.

<sup>28</sup>In the sense that these countries were the ones for which weights have decreased the most.

euro was introduced.

In the windows of 2003-07 and 2004-08 convergence increased for all countries. In 2003-07, inflation rates of the countries outside the divergent group are the ones that became more aligned with the average, especially Austria, Belgium, Germany, and Italy.<sup>29</sup> In 2004-08 the increase in convergence was explained especially by the good behavior of both the divergent group (Ireland, the Netherlands and Portugal) and Greece and Italy.

In summary, apart from the countries of the divergent group, some large countries and other core European countries were also responsible for the divergence in inflation observed during the euro period.

#### 4.4.2 Business cycles convergence

In this section we analyse the convergence of business cycles in the euro area, in order to study if it is connected with inflation convergence. Firstly, the output gap will be studied and next the RULC.

##### Output gaps

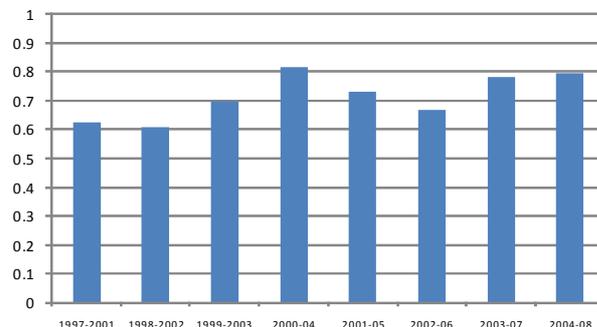


Figure 4-5: Output gaps co-movement during the euro period (1999-2008)

Between 1980 and 2008, the co-movement of output gaps in the euro area increased continuously. The  $R^2$  of the first principal component was 0.3680 in the window 1980-89 and

---

<sup>29</sup>In the sense that these countries were the ones that had the largest increase in the weights, compared with the previous period.

jumped to 0.6512 in 1990-98. Although more modest, there was also an increase in the correlation of business cycles during the euro period, with the  $R^2$  reaching 0.7225 in 1999-2008. This continuous increase in convergence is probably explained by the deepening of trade and monetary integrations. Particularly, the adoption in 1979 of a system of fixed exchange rates and the subsequent creation of a single currency implied convergence of policies that may have led to greater conformity in the business cycles. Artis and Zhang (1997, 1999) defend that this has occurred with the ERM.

In more detail, the rolling windows during the euro period show an overall trend of increasing business cycles' integration, with the correlation increasing from 0.6096 in 1998-2002 to 0.7994 in 2004-08 (Figure 4-5). But there were some windows of divergence, especially 2001-05 and 2002-06, with the proportion of variance explained by the European business cycle decreasing from 0.8079 in 2000-04 to 0.6761 in 2002-2006. For a longer period on analysis, De Haan et al. (2008) also conclude that business cycles in the euro area have gone through periods of both convergence and divergence.

Comparing the divergence of output gaps and inflation rates, the only window in which there was simultaneous divergence in both variables was 2002-06.

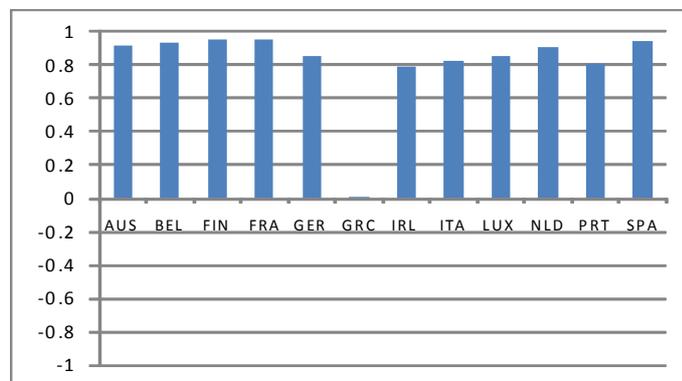


Figure 4-6: Country weights of the first principal component of output gap (1999-2008)

When we look at the behaviour of individual countries during the euro period, Greece stands out clearly as an outlier with almost zero correlation with the European business cycle (Figure 4-6). From the inflation's diverging group, Ireland and Portugal also show a smaller than average co-movement with the European business cycle, 0.79 and 0.80 respectively.

Germany and Italy's output gap also had a relatively small correlation with the European output gap: 0.85 and 0.82 respectively. We can observe that, at the country level, there was a relationship, even though not perfect, between output gap divergence and inflation divergence. Ireland and Portugal are two good examples, with a low correlation with the average in both variables.<sup>30</sup>

During the two episodes of divergence in 2001-05 and 2002-06, Finland, Ireland and Portugal were the main contributors to the reduction in the connection between European business cycles (Figure 4-14, Annex 4.7). Notice that during these two periods the output gaps of France, Italy and especially Germany also reduced their correlation with the average. Relating such evolution with inflation divergence, we observe that these large countries also had considerable divergence in inflation in 2002-06. Finland also became less synchronised with the common inflation in the windows of 2001-05 and 2002-06; but the same did not occur for Ireland and Portugal.

## RULC

In this section, we analyse the convergence of RULCs, with the aim of confirming or complementing the analysis done with the output gap. In opposition to the output gap, the synchronisation of the RULCs decreased continuously in the windows 1980-89, 1990-98, and 1999-2008 (Table 4.3). The reduction observed in the euro period is due mainly to the strong reduction in the last years, as we will see next in the detailed analysis of that period.

During the euro period, the increase in the  $R^2$  of the RULCs common factor was relatively small: from 0.5073 in the window 1999-2003 to 0.5686 in 2004-08 (Figure 4-7). However, the evolution has been characterised by cycles of convergence and divergence. In 1999-2003 and 2000-04 the  $R^2$  of the aggregate component suffered a reduction; from 0.5529 in 1998-20003 to 0.4687 in 2000-04. This reduction coincided with the decrease in inflation co-movement. Afterwards, in 2001-05 and 2002-06 there was a strong increase in convergence, with the  $R^2$  reaching 0.7802 in the latter window. Finally, in 2003-07 and especially in 2004-08 the co-movement reduces considerably, with the  $R^2$  tumbling to 0.5686 in the latter window. During these two windows we observed an opposite evolution in inflation rates and output gaps'

---

<sup>30</sup>Also Germany, Greece and Italy tend to show a less aligned behaviour with the average in both inflation and output gap.

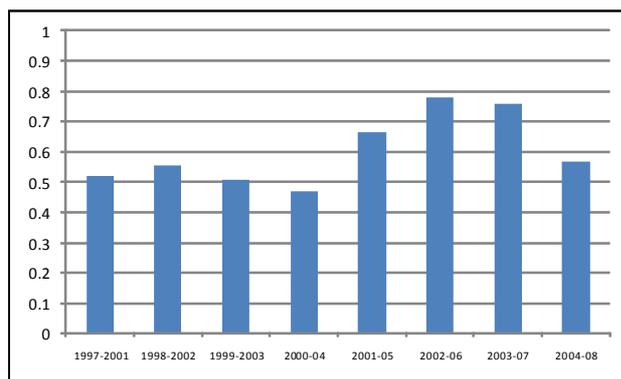


Figure 4-7: RULC co-movement during the euro period

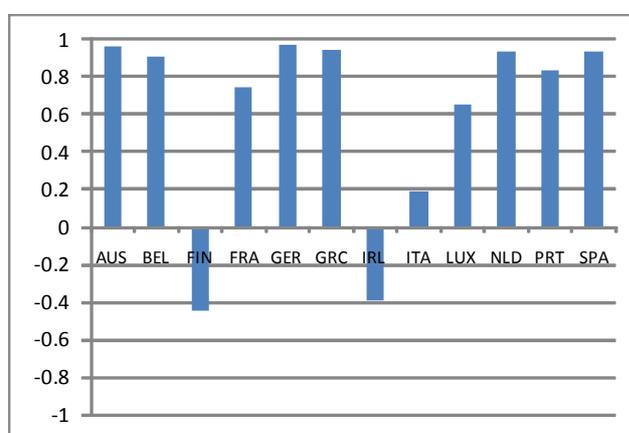


Figure 4-8: Country weights of first principal components of RULC (1999-2008)

convergence.

At the country level, for the overall euro period, the RULCs of Finland and Ireland moved in the opposite direction to the other countries' RULCs (Figure 4-8). In turn, Italy's RULC evolved in the same direction as the common factor, but with a very small correlation with it, of 0.1861. France's RULC also had a relatively small linkage with the wide evolution of the euro area's RULC, with a correlation of 0.7443: that compares with a correlation of 0.9707 for Germany.

Looking at the divergence in 1999-2003 and 2000-04, this can be explained by the diverging behavior of Finland, Italy, the Netherlands and Ireland (Figure 4-15, Annex 4.7). In turn,

the divergence in 2004-08 continues to be explicated by Italy and the Netherlands, but now Belgium and Portugal also play a role.

Once more at the aggregate and country levels we observe some connection between the convergences of RULC and inflation; however, this is not very close.

#### 4.4.3 Correlation between the convergences of inflation rates and the business cycles

Table 4.4: Correlation between the  $R^2$  of inflation and the  $R^2$  of output gap or RULC

Variables	Correlation	t-value
Inflation, Output gap	0.5754	1.4071
Inflation, Output gap (t-1)	0.1754	0.3563
Inflation, RULC	-0.1344	-0.2713
Inflation, RULC (t-1)	0.6822	1.8663

Now, we formally test the association between the convergence of inflation and the convergence of output gap or RULC. For that, we use the correlation between the  $R^2$  of those variables in six consecutive five-year windows for the period 1999-2008. We expect that as output gaps or RULCs of euro area countries tend to move closer, as shown by a larger  $R^2$ , the same will happen to inflation rates.

The correlations between  $R^2$  of inflation and output gap and  $R^2$  of inflation and one period lagged RULC are relatively high, but their statistical significance, as measured by the t-values, is not particularly strong - Table 4.4. This confirms our qualitative analysis of the association between those variables.

In summary, compared with the period 1990-98, the co-movement in European inflation rates on average did not increase during the euro period. But when looking at sub-periods after the introduction of the euro, we conclude that the link between European inflation rates has increased in the last years studied. Regarding business cycles' convergence, we obtain slightly different results depending on the indicator used. With respect to the window of 1990-98, the co-movement between output gaps has increased significantly during the euro period (1999-2008). In turn, for the RULC we need to look at *sub-periods* during the euro period, to observe a slight increase in co-movement. The comparison between the evolution of

the  $R^2$  of inflation and the  $R^2$  of output gap or RULC shows some positive correlation; that however is not particularly statistically significant. This correlation analysis is only indicative because correlation does not necessarily imply causality and there are other factors driving inflation, like the exchange rate and inflation's expectations.

## 4.5 Explaining inflation differentials

In the previous sections we observed that some relationship exists between the convergence of inflation rates and the convergence of output gaps or RULCs in the euro area. In this section, we extend the analysis of the relevance of the business cycle for inflation differentials, using a regression analysis for the period after the introduction of the euro (1999Q1-2008Q4) and comprising 12 euro area countries. In this period, inflation convergence was already very advanced for the majority of countries and therefore it makes sense to analyse the set of countries as an homogenous group. In other words, the poolability assumption in a panel is more acceptable.

### 4.5.1 Determinants of inflation differentials in the euro area

There are many possible determinants of inflation differentials in a monetary union like the euro area.

Firstly, inflation differentials can be explained by differences in countries' business cycles. Such differences may emerge due to supply shocks (*e.g.* oil price) or domestic demand shocks. The latter shocks can arise due for instance to differences in fiscal policy, country-specific non-policy demand shocks (*e.g.* taste shocks), or asymmetric effects of common demand shocks. Such asymmetric shocks can be induced by monetary policy or exchange rate movements. In fact, the common policy interest rate may have different impacts on each country, due to differences in financial and economic structures (Hofmann and Remsperger, 2005). Also, the exchange rate's evolution can cause inflation differentials, even though the euro is the common currency. That is so because differences in trade patterns may imply that national effective exchange rates respond differently to the evolution of the euro. In fact, the weight of imported consumption goods and inputs from outside the euro zone differs from country to country, as well as the trading partners.

Asymmetric demand shocks may arise due to differences in consumption patterns. These differences also imply that the weight of each sub-index of goods in the HCPI differs across countries. As a result, symmetric changes in prices of goods across the monetary union, imply different inflation rates measured by the HICP. However, this effect did not play a relevant role in explaining inflation differentials in the euro area (Hofmann and Remsperger, 2005; ECB, 2003).

On the structural side, inflation differentials in a monetary union may arise due to price level convergence, which may result from the convergence of both tradable and non-tradable prices (ECB, 2003; Hofmann and Remsperger, 2005). Tradable goods' prices convergence is originated by goods markets' integration, probably boosted by a single currency.

In turn, real income convergence, probably increased by the introduction of the euro, may also lead to convergence of non-tradable goods' prices, as explained by the Balassa-Samuelson effect. In the catching up process, productivity gains tend to occur mainly in the tradable sector, which suffers higher external competition and is more capital-intensive. As a result, in this sector, wages increase without creating inflationary pressures. Due to labour mobility among tradable and non-tradable goods sectors and competition in the goods market, wages also increase in the non-tradable goods sector. However, in this sector, where productivity growth is slower, the increase in wages has to be compensated by an increase in prices. In conclusion, catching-up countries, where the productivity differential between the tradable and non-tradable sectors is larger, will suffer larger non-tradable goods' inflation.

Honohan and Lane (2003) found that output gap, the change in the nominal effective exchange rate (NEER) and price level convergence were significant in explaining inflation differentials in the euro area for the period 1998-2001.

Rogers' (2002) results are similar to those of Honohan and Lane (2003). For the EMU-11 in 1997-2000 CPI inflation differentials were fundamentally explained by the lagged price level, output gap and trade openness. The price level had the expected negative coefficient and the two latter variables had positive coefficients. The lagged per capita GDP had also a negative effect on inflation differentials at a 10% level of significance. Regarding the significance of the price level, it was not robust enough to withstand more substantive analyses and most of inflation differences were accounted for factors other than the convergence of prices.

Angeloni and Ehrmann (2006) included one more year of data than Honohan and Lane

(2003) did, and taking into account data revisions, confirm that the exchange rate is a determinant of inflation differentials, but that its statistical significance is weak. In contrast, the significance of output gap and of the lagged price level increases in their estimates.

Honohan and Lane (2004) update their 2003 study with two more years of annual data, obtaining a sample covering the period 1999-2003. But they are not able to obtain their previous result of the significance of the *change* in the NEER to explain CPI inflation differentials. However, output gap remains significant. It seems then that the NEER is mainly affecting inflation through output gap. To make things even more complex, when using quarterly data for 1999Q1-2004Q1, they conclude that the *level* of the NEER explains CPI inflation differentials, but that the output gap does not explain. In this case, it is argued that in a monetary union national inflation rates act to correct misalignments in exchange rates; when the euro is under-valued, the increase in inflation acts as a correction mechanism, reducing external competitiveness, especially for countries more exposed to extra-euro trade.

From the above discussion, it is clear that the literature does not agree on the significance of the output gap and the NEER in explaining differences in inflation in the euro area. In order to contribute to the clarification of the relation between inflation and the business cycle, we will use the RULC as an alternative to the output gap. This variable is suggested by the New Keynesian literature as the correct driver of inflation. This literature also stresses that inflation is forward-looking, and there is also a secondary role for lagged inflation. Besides, if inflation is sufficiently persistent, temporary demand and supply asymmetric shocks may cause persistent inflation differentials (Hofmann and Remsperger, 2005).

We also use expectations and lagged values of the exchange rate to clarify the role of this variable on inflation differentials. The nominal interest rate may also play a direct role in inflation divergence if the cost channel is relevant. In this context, even changes in the common policy rate may imply differences in inflation when the importance of the cost channel differs from country to country.

Also, on the supply side, fluctuations in the price of energy cannot be ignored as a possible determinant of differences in national inflation dynamics, since there are different degrees of dependency from oil across European countries (ECB, 2003).

Finally, fiscal deficits and the real interest rate may also have contributed to inflation differentials, but probably their effect occurred essentially through the output gap. Along

this line, Honohan and Lane (2003) found that, after controlling for the output gap, fiscal positions did not have a statistically significant effect on inflation divergence in the euro area between 1999 and 2001.

#### 4.5.2 Explaining inflation differentials using measures of divergence

In Section 4.4, the convergence of key variables was analysed using a common factor approach. For each variable, equation (4.6) was estimated. The error,  $\epsilon_{it}$ , is the component not explained by the first common factor. In practice, this error is estimated by the residual obtained as the original values of the variable minus the fitted values, with the latter values computed with the first principal component. It is easy to see that  $\epsilon_{it}$  is a measure of divergence. Using such a measure, our aim is to explain divergence in inflation with divergence in other variables. The equation to estimate is:

$$\begin{aligned} r\pi_{i,t} = & \gamma_p pl_{i,t-1} + \gamma_f Er\pi_{i,t+1} + \gamma_b r\pi_{i,t-1} + \gamma_g rx_{i,t} + \gamma_{in} i_t^{euro} \\ & + \gamma_{po} po_t + \gamma_{ef} Ere_{i,t+1} + \gamma_e re_{i,t} + \gamma_{eb} re_{i,t-1} + u_{i,t} \end{aligned} \quad (4.10)$$

where  $pl_{i,t-1}$  is the price level of country  $i$  in  $t - 1$  expressed in relative terms, with the euro area 12 as a reference,  $r\pi_{i,t}$  is the residual of CPI inflation,  $rx_{i,t}$  is the residual of output gap,  $i_t^{euro}$  is the euro area interbank interest rate,  $po_t$  is the price of oil in the international market converted to euros, and  $re_{i,t}$  is the residual of the NEER's level; and  $Er\pi_{i,t+1}$  and  $Ere_{i,t+1}$  are expectations of inflation's residual and exchange rate's residual, respectively.<sup>31</sup>

Expectations of inflation are used because agents are forward-looking when establishing prices. In addition, previous studies have shown (for example Gali and Gertler, 1999; Gali et al., 2001) that a proportion of agents have backward-looking expectations, justifying the introduction of the one-period lagged inflation.

In turn, the residual of the nominal exchange rate was introduced to translate the impact of import prices on inflation. We expect that an appreciation of the euro, *i.e.*, an increase in  $re_t$ , has a negative impact on inflation ( $\gamma_e < 0$ ). The expected and lagged values of the

---

<sup>31</sup>The residuals were obtained by estimating equation (4.6) for the period 1998Q1-2008Q4.

exchange rate's residuals are introduced due to the assumption that expected and lagged domestic inflation's differentials affect present domestic inflation's differentials (this is similar to equation 3.32 for national inflation rates). The coefficients of the lag and lead exchange rates are expected to be positive ( $\gamma_{ef}$  and  $\gamma_{eb} > 0$ ).

Moreover, the lagged price level,  $pl_{i,t-1}$ , is considered as a proxy for the price convergence effect (Honohan and Lane, 2003). It is expected that countries with higher price levels experience lower inflation ( $\gamma_p < 0$ ).

Also, the euro area interest rate was introduced to capture the effect of the cost channel on inflation differentials.

Finally, the roles of output gap residuals and the price of oil are easy to understand. While the former captures the effect of business cycle on inflation differentials, the latter is introduced to assess if oil price fluctuations create differences in inflation dynamics.

The model was estimated using a panel of 12 euro area countries; the original 11 founders and Greece. The panel is unbalanced only when the loans interest rate or the price of imports are used, because before 2003Q1 there were no data available on the former variable for Luxembourg and before 2000Q1 there were no data available on the latter variable for Ireland. The estimation was done using Panel GMM because some variables are simultaneously determined (for example output gap and inflation); and also due to the presence of expectations.

In order to estimate equation (4.10), expectations are replaced by observed values under the assumption of rational expectations. This assumption implies that agents' forecast errors are not correlated with information available to them at the time expectations are formed. As a result, we can obtain orthogonality conditions to apply the GMM (see Section 3.3.4).

It is worth mentioning that we do not introduce country fixed effects for two reasons. On one hand, inflation's expectations can accommodate differences in inflation rates that remain constant for the entire sample, without it being necessary to include a constant for such a purpose. On the other hand, introducing fixed effects with a lagged dependent variable produces bias in results.<sup>32</sup>

---

<sup>32</sup>Anyway, below we show that our main results hold even if country fixed effects are introduced.

The model can be expressed in a condensed way as

$$y_{it} = \mathbf{x}'_{it}\boldsymbol{\beta} + u_{it},$$

with  $\mathbf{x}'_{it}(1 \times k) = [pl_{i,t-1} \ r\pi_{i,t+1} \ r\pi_{i,t-1} \ rx_{i,t} \ i_t^{euro} \ po_t \ re_{i,t+1} \ re_{i,t} \ re_{i,t-1}]$  and  $\boldsymbol{\beta}(k \times 1)$ . As usual, it is assumed that observations are independent over  $i$ .

Stacking the  $T$  observations for the  $i$ th individual, we simplify the notation to

$$\mathbf{y}_i = \mathbf{X}_i\boldsymbol{\beta} + \mathbf{u}_i$$

where  $\mathbf{y}_i$  and  $\mathbf{u}_i$  are  $T \times 1$  vectors and  $\mathbf{X}_i$  is a  $T \times k$  matrix.

Now, we assume that there is a matrix of instruments,  $\mathbf{Z}_i$  ( $T \times r$ ), with  $r > k$ , which satisfies  $r$  moment conditions for each individual  $i$ :

$$E(\mathbf{Z}'_i\mathbf{u}_i) = \mathbf{0}.$$

The GMM estimator looks to satisfy as close as possible these  $r$  moment conditions, by minimising the quadratic form

$$J_N(\boldsymbol{\beta}) = \mathbf{g}(\boldsymbol{\beta})' \mathbf{W}_N \mathbf{g}(\boldsymbol{\beta}) \tag{4.11}$$

with  $\mathbf{g}(\boldsymbol{\beta}) = \sum_{i=1}^N \mathbf{Z}'_i \mathbf{u}_i$  as the sample moment condition ( $r \times 1$ ) and  $\mathbf{W}_N$  as the  $r \times r$  weighting matrix.

We use the two step GMM with  $\mathbf{W}_N = \widehat{\mathbf{S}}^{-1}$ , where the matrix  $r \times r$

$$\widehat{\mathbf{S}} = \frac{1}{N} \sum_{i=1}^N \mathbf{Z}'_i \widehat{\mathbf{u}}_i \widehat{\mathbf{u}}'_i \mathbf{Z}_i \tag{4.12}$$

is a consistent estimator for  $\mathbf{S}$ , the variance of moment conditions. With the sum being performed over individuals, the weighting matrix and standard deviations are robust to arbitrary serial correlation and time-varying variances of the errors (White period method).<sup>33</sup> The errors  $\widehat{\mathbf{u}}_i$  are obtained in a first step with a consistent estimator for  $\boldsymbol{\beta}$ .

---

<sup>33</sup>See note to Table 4.5 for the formula of the variance.

The choice of good instruments for the GMM estimation is an important task. The convention in the literature is to use at least past information on the endogenous and forcing variables (Binder and Pesaran, 1995). In our case, we have to take into account that endogeneity is a potential problem for several variables. For example  $r\pi_{i,t+1}$ ,  $rx_{i,t}$ ,  $re_{i,t+1}$  and  $re_{i,t}$  can be seen as endogenous explanatory variables.<sup>34</sup> Consequently, only the lags of those variables can be used as instruments. A further reason for such a procedure is that information for period  $t$  may not yet be available when agents form their expectations. Therefore, we used as instruments the price level lagged one period, 2 lags of inflation's residuals, and one lag of each of the other explanatory variables. We added some additional instruments that proved to have a strong explanatory power in the first stage regression: one lag of the loans interest rate ( $i_i$ ) and the lags  $t - 2$  and  $t - 3$  of the difference between the CPI of country  $i$  and the euro area 12 CPI ( $diff \pi_t$ ).<sup>35</sup>

One point worth testing when choosing instruments is their weakness. We performed this test using the first stage regression of  $t + 1$  inflation on the instruments. Then we retained the F-statistic of the joint significance of the instruments. The same was done for the exchange rate in  $t + 1$ . The rule of thumb is that if the F-statistic is larger than 10, the existence of weak instruments can be ruled out (Stock et al., 2002). In the regressions of Table 4.5, this rule of thumb is fulfilled for the first stage regression of the exchange rate, but not for the inflation rate's regression. Weak instruments are common in forward-looking models with rational expectations (Mavroeidis, 2004) because agents use all available information to make expectations, implying that valid instruments will be weak.

Since the models estimated here are overidentified (number of instruments  $>$  number of regressors), the correlation between the error and the instruments can be tested with the J-test. Applying this test in all of the following regressions indicates that instruments are not correlated with the error.<sup>36</sup>

Our results show that when the output gap is used as an indicator of the business cycle, inflation's divergence is positively affected by the expected inflation's divergence for the next period, and this effect is statistically significant (Table 4.5, eq. (1)). The lagged inflation's

---

<sup>34</sup>For large countries the euro area variables can also be endogenous.

<sup>35</sup>There are some small changes in the instruments depending on the exact specification of the estimated equation. See notes to Table 4.5.

<sup>36</sup>See note to Table 4.5 for details on the J-test.

Table 4.5: Explaining inflation differentials using the residuals of the common factor model. GMM estimation.

	Eq. (1)	Eq. (2)	Eq. (3)	Eq. (4)
$r\pi_{i,t+1}$	0.74*** (0.12)	0.72*** (0.0889)	0.68*** (0.12)	0.73*** (0.10)
$r\pi_{i,t-1}$	0.094 (0.076)	0.099* (0.056)	0.11 (0.10)	0.062 (0.06)
$rx_{i,t}$	0.015 (0.218)	0.14 (0.1583)	-	-
$x_t^{euro}$	-	-	0.27 (0.28)	-
$rs_{i,t}$	-	-	-	-0.00086** (0.00040)
$re_{i,t+1}$	0.032* (0.019)	0.013** (0.0063)	0.086 (0.072)	0.038 (0.026)
$re_{i,t}$	-0.049 (0.032)	-0.010** (0.0050)	-0.15 (0.14)	-0.057 (0.047)
$re_{i,t-1}$	0.020 (0.015)	-	0.072 (0.069)	0.022 (0.022)
$i_t^{euro}$	-0.00013 (0.00018)	-	-0.0025 (0.0025)	-0.00026 (0.00029)
$po_t$	0.000087 (0.000164)	-	0.0020 (0.0020)	0.00016 (0.00028)
$pl_{i,t-1}$	-0.0038 (0.0025)	-	-0.0057 (0.0046)	-0.0064* (0.0033)
F-stat 1 <sup>st</sup> stage reg.:				
$r\pi_{i,t+1}$	8.24	8.24	7.99	7.97
$re_{i,t+1}$	136.67	136.67	136.10	134.70
J-statistic	3.98 [0.26]	6.93 [0.32]	1.99 [0.57]	3.20 [0.36]
Q (2) stat.	122.90 [0.00]	127.41 [0.00]	109.59 [0.00]	110.88 [0.00]

Notes: In all equations there were included two dummies to correct two large outliers for Greece (1999Q1=1 and 2000Q4=1). Instruments: Eq. (1): constant,  $r\pi_{i,t-1}$ ,  $r\pi_{i,t-2}$ ,  $rx_{i,t-1}$ ,  $re_{i,t-1}$ ,  $i_{t-1}^{euro}$ ,  $po_{t-1}$ ,  $pl_{i,t-1}$ ,  $pi_{i,t-2}$ ,  $i_{i,t-1}$ ,  $dif \pi_{i,t-2}$ ,  $dif \pi_{i,t-3}$  and two dummies for Greece. Eq. (2): the same as eq. (1). Eq. (3): the same as eq. (1) plus  $x_t^{euro}$  less  $pi_{i,t-2}$ . Eq. (4): the same as eq. (1) plus  $rs_{i,t-1}$  less  $pi_{i,t-2}$ . (...) contain standard errors robust to arbitrary serial correlation and time-varying variances of the errors. [...] contain p-values. "\*\*\*\*" means significance at 1%, "\*\*\*" at 5%, and "\*" at 10%.

The estimators of equation's coefficients and variances are:  $\hat{\beta}_{2SGMM} = [\mathbf{X}'\mathbf{Z}\hat{\mathbf{S}}^{-1}\mathbf{Z}'\mathbf{X}]^{-1}\mathbf{X}'\mathbf{Z}\hat{\mathbf{S}}^{-1}\mathbf{Z}'\mathbf{y}$

$$\text{and } \hat{V}[\hat{\beta}_{2SGMM}] = [\mathbf{X}'\mathbf{Z}(\mathbf{N}\hat{\mathbf{S}})^{-1}\mathbf{Z}'\mathbf{X}]^{-1}.$$

The J-statistic is obtained by evaluating the GMM objective function with the efficient GMM estimator:  $J = \left[ \sum_{i=1}^N \hat{\mathbf{u}}_i \mathbf{Z}_i \right] (\mathbf{N}\hat{\mathbf{S}})^{-1} \left[ \sum_{i=1}^N \mathbf{Z}_i' \hat{\mathbf{u}}_i \right]$ , with  $\hat{\mathbf{u}}_i = y_i - \mathbf{Z}_i \hat{\beta}_{2SGMM}$ , and is distributed as a  $\chi^2(r-k)$  under the null that the overidentifying restrictions are valid, where  $r$  is the number of instruments and  $k$  is the number of regressors. For example, for the first regression,  $r$  is 14 and  $k$  is 11, meaning that the J-statistic is distributed as  $\chi^2(3)$ . Since the aim is to minimise  $J(\beta)$ , the null is rejected for large values of the J-statistic.

The White estimator for  $\hat{\mathbf{S}}$  is based on the Panel Corrected Standard Error methodology (Beck and Katz, 1995; Eviews, 2007), where residuals are replaced by moment estimators of the unconditional variance.

divergence also has a positive effect on current inflation's divergence, but it is not statistically significant. The exchange rate's coefficients also have the right sign, with an effective depreciation of the euro increasing inflation differentials. But in this case only the expected exchange rate has a statistically significant effect at 10% of significance.

The lagged relative Purchasing Power Parity price level also has the right sign but it is significant only at a 13% significance level. In any case, the larger the country's relative price level, the smaller its inflation divergence, confirming the price level convergence hypothesis explained above. Likewise, the coefficient of the euro area nominal interest rate is not statistically significant, which is evidence against the importance of the cost channel in creating inflation differentials.

Also, the oil price does not seem to contribute to explain differences in inflation dynamics in the euro area, even though its coefficient is positive as expected.

Notice that if we remove the insignificant variables from equation (4.10), with the exception of the output gap's residual, then the remaining exchange rate variables become significant at 5% and the lagged residual of inflation becomes significant at 10% (Table 4.5, eq. (2)).

Even more surprising is the fact that output gap's residual does not have a statistically significant effect in inflation divergence, even though it has the expected positive sign. Alternatively, Borio and Filardo (2006) show that the global output gap has an important explanatory power in inflation rate equations for 16 advanced economies and the euro area. Bearing this in mind, we used the euro area 12 output gap to explain inflation differentials. However, this variable did not perform better than the national output gaps' residual (Table 4.5, eq. (3)).

It can be noticed that the models' residuals are autocorrelated. This was expected since replacing variables' expectations by observed values induces a first order moving-average structure in the error term of the estimated model (Pesaran, 1987). To tackle this problem we used standard errors robust to heteroskedasticity and autocorrelation.

Once the output gap's residual proved to be insignificant in explaining divergent inflation dynamics, we tried the RULC's residual ( $rs_{i,t}$ ) as an alternative; but still the expected link between business cycles and inflation rate divergence was not found. In this case, the RULC has a negative and significant effect on inflation differentials (Table 4.5, eq. (4)). But the lagged price level become significant.

In section 4.4.3, it was clear that at an aggregate level, inflation convergence had a closer link with the one period lag RULC than with the contemporaneous RULC. However, when period  $t$  RULC is replaced by its one period lag in equation (4.10), we still did not get a positive sign for the coefficient of the RULC.<sup>37</sup>

In summary, inflation differentials are highly affected by expected inflation differentials. The exchange rate also plays a significant role in explaining differences in inflation dynamics. Price level convergence seems to be present, but it is not significant in some regressions. However, we were unable to establish a sensible link between business cycles and differences in inflation rates evolution. This last result was unexpected because some estimates of the NKPC for the euro area countries have shown that the output gap or labour income share are important determinants of inflation (for instance, Gali et al. (2001) and Chowdhury et al. (2006)). In order to explore whether our results are specific to the methodology chosen, we next estimate the NKPC using actual variables, instead of the residuals of the common factor model. But notice that using the residuals from a common factor model emphasizes the *relative* co-movement between inflation rates, while using the difference between national inflation rates detaches the absolute difference between that rates.

### 4.5.3 Explaining inflation differentials using the NKPC

In order to highlight the difference between the factors affecting inflation and the ones affecting inflation differentials, we start by explaining national inflation rates and then analyse inflation differentials. We start by estimating the following open economy NKPC

$$\begin{aligned}\pi_{i,t} &= \gamma_f E_t \pi_{i,t+1} + \gamma_b \pi_{i,t-1} + \gamma_{mc} \widehat{mc}_{i,t} \\ &\quad + \gamma_s \Delta e_{i,t} + \gamma_{sf} E_t \Delta e_{i,t+1} + \gamma_{sb} \Delta e_{i,t-1}\end{aligned}$$

where  $\pi_{i,t} = p_{i,t} - p_{i,t-1}$  is CPI inflation in  $t$ ,  $p_{i,t}$  the log of CPI,  $\widehat{mc}_{i,t}$  is the marginal cost in percentage deviation from the steady-state,  $mc_{i,t} - mc^{ss}$  (with both marginal costs defined in logs), and  $\Delta e_{i,t}$  is the change in the log of the *nominal* effective exchange rate. The marginal cost is  $mc_{i,t} = i_{i,t} + s_{i,t} - \log(\alpha_n)$ , where  $i_{i,t}$  is the log of the nominal interest rate of country

---

<sup>37</sup>To save space, we do not report this result.

$i$ ,  $s_{i,t}$  the log of labour income share (or real marginal cost), and  $\alpha_n$  is the labour share in the Cobb-Douglas production function. The variable  $i_{i,t}$  affects the marginal cost due to the cost channel. This Phillips curve includes open-economy variables in the spirit of Batini, Jackson and Nickell (2005), with the change in the nominal exchange rate translating the impact of import prices on CPI inflation.

Further on in the discussion, when explaining inflation differentials, time fixed effects will be used to capture euro area variables. Therefore, it will not be possible to estimate the effect of variables that are equal for all countries, like the price of oil or the euro area interest rate. As a result, we have chosen to also exclude them from the Phillips curve to make results comparable. In the case of the euro area interest rate, the national lending interest rates are used as an alternative. In addition to the interbank rate, these variables include the spread charged by banks, giving a clearer picture of firms' financial costs.<sup>38</sup> Also, omitting oil price will probably not have much effect on the model, because that variable proved to be insignificant in the estimations done using the measure of divergence from the common factor model.

If the marginal cost is not expressed in deviations from the steady-state, the last Phillips curve can be written as:

$$\begin{aligned} \pi_{i,t} = & \alpha + \gamma_f E_t \pi_{i,t+1} + \gamma_b \pi_{i,t-1} + \gamma_s s_{i,t} + \gamma_{in} i_{i,t} \\ & + \gamma_s \Delta e_{i,t} + \gamma_{sf} E_t \Delta e_{i,t+1} + \gamma_{sb} \Delta e_{i,t-1} + u_{i,t} \end{aligned} \quad (4.13)$$

with  $\alpha = -\gamma_{mc} [mc^{ss} + \log(\alpha_n)]$ . Then the constant includes the *common* steady-state marginal cost.<sup>39</sup> It is also possible to define the Phillips curve using the output gap,  $x_{i,t}$ , to measure the impact of business cycle on inflation.<sup>40</sup>

When estimating equation (4.13), the poolability of the data was assumed, *i.e.*, that equation's coefficients are the same for all countries. As highlighted by Bjornstad and Nymoen

---

<sup>38</sup>The measurement of this variable suffers a change in 2003Q1. To accommodate that, we use a dummy variable, that assumes the value one for the period between 1998Q1 and 2002Q4.

<sup>39</sup>As an alternative to using a constant to capture the steady-state, we tried also to use the RULC and the nominal interest rate in deviations from their steady-state. The steady-states were obtained by applying the HP filter to the respective variables. In general, results did not change significantly. They only changed slightly in the case of the Phillips curve using the output gap, where the coefficient of this last variable became negative and statistically insignificant at 5%. These results are not reported to save space.

<sup>40</sup>Under certain conditions  $\widehat{mc}_{i,t} = x_{i,t}$ .

(2008), this assumption has advantages and disadvantages. On one hand, the pooled estimator is inconsistent and biased if the poolability assumption is not valid. On the other hand, the pooling brings efficiency gains. In the euro area, the assumption of poolability makes sense as countries in that area are relatively homogeneous, because they have been converging in nominal and real terms and share similar monetary and fiscal policy frameworks. The use of panel data with the poolability assumption is also sensible because inflation convergence is an aggregate phenomenon, involving simultaneously the dynamic evolution of a group of countries. In addition, with a panel there is no need to measure the common factors explicitly (task that always involves some aggregation problems), since they can be captured by the time dummies.

Turning now to the empirical explanation of inflation differentials, if equation (4.13) is valid for each country, it is also valid for the euro area as a whole. As a result, inflation differentials can be expressed as:

$$\begin{aligned} \pi_{i,t} = & \phi_t + \gamma_p pl_{i,t-1} + \gamma_f E_t \pi_{i,t+1} + \gamma_b \pi_{i,t-1} + \gamma_s s_{i,t} + \gamma_{in} i_{i,t} \\ & \gamma_s \Delta e_{i,t} + \gamma_{sf} E_t \Delta e_{i,t+1} + \gamma_{sb} \Delta e_{i,t-1} + u_{i,t} \end{aligned} \quad (4.14)$$

where the time dummies,  $\phi_t$ , are a linear combination of euro area variables. Here,  $pl_{i,t-1}$  is introduced to capture the price convergence effect.

Before applying the GMM as described above, it is necessary to transform equation (4.14) to eliminate the time dummies. That consists simply of subtracting (A) the average of the model over individuals from (B) the original model (Baltagi (2008)).

After explaining how regressions were done, we analyse estimations results. Table 4.6, eq. (1), shows that we can replicate the traditional features of the Phillips curve for national inflation rates. The coefficients of both the lead and lag inflation are statistically different from zero, and their sum is less than one, but it is not statistically different from one. Also the forward component of inflation is larger than the backward component. Output gap has a positive but statistically insignificant effect on inflation. The cost channel is present, with the nominal interest rate having a positive and significant effect on inflation.<sup>41</sup> Even when

---

<sup>41</sup>The p-value of the null hypothesis of “no interest rate effect on inflation between 1998Q1-2002Q4” is 0.0346. So, at a level of significance of 1% we do not reject the null hypothesis. This occurs, probably, because

Table 4.6: GMM estimation of the NKPC for a panel of 12 euro area countries.

	Eq. (1)	Eq. (2)	Eq. (3)
$c$	-0.00088 (0.00057)	-0.010 (0.023)	-0.00033 (0.00063)
$\pi_{i,t+1}$	0.83*** (0.014)	0.89*** (0.098)	0.68*** (0.111)
$\pi_{i,t-1}$	0.13* (0.073)	0.092 (0.070)	0.14** (0.068)
$x_{i,t}$	0.013 (0.014)	-	0.011 (0.015)
$s_{i,t}$	-	0.0020 0.0050	-
$\Delta e_{i,t+1}$	0.057 (0.058)	0.044 (0.057)	0.059 (0.060)
$\Delta e_{i,t}$	-0.045 (0.007)	-0.017 (0.066)	-0.057 (0.075)
$\Delta e_{i,t-1}$	0.051*** (0.018)	0.049*** (0.018)	0.049** (0.017)
$i_{i,t}$	0.00026** (0.00010)	0.00022** (0.00010)	0.00030** (0.00012)
$i_{i,t} \cdot D_t$	-0.00014** (0.000065)	-0.00011* (0.000064)	-0.00020** (0.000088)
$pi_{i,t} - pd_{i,t}$	-	-	0.0084** (0.0034)
F-stat 1 <sup>st</sup> stage			
reg.:			
$\pi_{i,t+1}$	11.72	11.53	12.37
$\Delta e_{i,t+1}$	18.33	16.63	18.46
J-statistic	2.48 [0.28]	3.45 [0.32]	3.78 [0.15]
Q (2) stat.	80.729 [0.00]	78.112 [0.00]	82.857 [0.00]

Note: See notes to Table 4.5. Panel GMM with period SUR weights and robust standard deviations. Instruments: Eq. (1): constant,  $\pi_{i,t-1}$ ,  $\pi_{i,t-2}$ ,  $x_{i,t-1}$ ,  $\Delta e_{i,t-1}$ ,  $\Delta pi_{i,t-1}$ ,  $q_{i,t-1}$ ,  $i_{i,t-1}$ ,  $pl_{i,t-1}$ ,  $p_{i,t-2}$  and one dummy,  $D_t$ , that takes the value one for the period 1999Q1-2002Q4. Eq. (2): the same as eq. (1) plus  $s_{i,t-1}$ . Eq. (3): the same as eq. (1) plus  $pi_{i,t-1} - pd_{i,t-1}$ .

we introduce the relative price of imports ( $pi_{i,t-1} - pd_{i,t-1}$ ), as suggested by the analysis in Chapter 3, the cost channel continues to be significant after 2002Q4, but not before this date (eq. (3), Table 4.6).<sup>42</sup>

Finally, the coefficients of the change in the NEER have the right signs and the coefficient associated with the lagged rate is statistically significant at 10%. If we replace the change in the NEER by the change in the REER or in the import prices deflator, we do not obtain more significant results regarding these variables.<sup>43</sup>

Recall that the cost channel was not statistically significant for the three euro area countries studied in Chapter 3. There are some possible reasons why the cost channel is more significant in this chapter. Firstly, in the current chapter it was used lending rates, while in Chapter 3 it was used policy rates. Lending rates may capture better the effective cost supported by companies when borrowing working capital, when the pass-through of interest rates is slow or incomplete. In Chapter 3 the lending rates were not used because there was no data available since 1980Q1. Other reason explaining the difference in results may lay in the fact that panel estimations bring efficiency gains. The difference may also be explained by even more simple reasons: the sample is different in terms of countries and time period studied.

When the RULC is used, results are basically the same as with the output gap. The RULC also does not have a statistically significant effect on inflation. It should be mentioned that the statistical insignificance of the output gap or the RULC in the NKPC is not unusual in the literature. In Bjornstad and Nymoen (2008), which uses panel data, the RULC has a negative sign and is not statistically significant. In a time series context, Bardsen et al. (2004) show that the significance of the wage share in Gali et al.'s (2001) study for the euro area is not robust enough to withstand small changes in the estimation methodology.

Notice that for equations (1) and (2) in Table 4.6 the instruments are not weak, according to the F-statistics of the first stage regressions.

The fact that we are able to reproduce the basic characteristics of the Phillips curve found in estimates for individual countries constitutes evidence in favour of the poolability of the

---

before 2003Q1 interest rate data are not fully harmonised across countries.

<sup>42</sup>The p-value of the null hypothesis of “no interest rate effect on inflation between 1998Q1-2002Q4” is 0.0776.

<sup>43</sup>These results are not reported to save space.

Table 4.7: Determinants of inflation differentials for a panel of 12 euro area countries. GMM estimation

	Eq. (1)	Eq. (2)	Eq. (3)	Eq. (4)
$c$	0.00010 (0.00058)	0.0068 (0.023)	0.000090 (0.00050)	0.000084 (0.00050)
$\pi_{i,t+1}$	0.96*** (0.13)	0.96*** (0.10)	0.86*** (0.13)	0.86*** (0.13)
$\pi_{i,t-1}$	-0.00062 (0.072)	-0.00030 (0.065)	0.029 (0.073)	0.030 (0.075)
$x_{i,t}$	0.0027 (0.037)	-	-	-
$s_{i,t}$	-	-0.0014 (0.28)	-	-
$\Delta ulc_{i,t}$	-	-	0.082*** (0.031)	0.080*** (0.030)
$\Delta e_{i,t+1}$	0.29 (0.20)	0.26 (0.16)	0.36** (0.16)	0.36** (0.16)
$\Delta e_{i,t}$	-0.33 (0.43)	-0.27 (0.27)	-0.43 (0.27)	-0.46* (0.25)
$\Delta e_{i,t-1}$	-0.015 (0.057)	-0.020 (0.053)	-0.016 (0.056)	-
$i_{i,t}$	0.000080 (0.000103)	0.000066 (0.000061)	0.00010 (0.000066)	0.000099 (0.000067)
$pl_{i,t-1}$	0.00014 (0.0018)	-0.000084 (0.0014)	-0.00012 (0.0014)	0.000025 (0.0014)
Time dummies	Yes	Yes	Yes	Yes
F-stat 1 <sup>st</sup> stage				
reg.:				
$\pi_{i,t+1}$	18.47	18.39	18.07	18.07
$\Delta e_{i,t+1}$	114.20	111.96	111.60	111.60
J-statistic	2.87 [0.23]	2.86 [0.41]	2.91 [0.40]	2.9 [0.57]
Q (2) stat.	107.87 [0.00]	111.13 [0.00]	103.95 [0.00]	104.33 [0.00]

Notes: See notes to Table 4.5. Instruments: eq. (1): constant,  $\pi_{i,t-2}$ ,  $x_{i,t-1}$ ,  $\Delta e_{i,t-1}$ ,  $\Delta p_{i,t-1}$ ,  $q_{i,t-1}$ ,  $i_{i,t-1}$ ,  $pl_{i,t-1}$ ,  $p_{i,t-2}$ ,  $p_{i,t-3}$ ,  $difp_{i,t-1}$  and time dummies . Eq. (2): the same as eq. (1) plus  $s_{i,t-1}$ . Eq. (3) and (4): the same as eq. (1) plus  $\Delta ulc_{i,t-1}$ .

data.

Regarding inflation differentials, results obtained from estimating equation (4.14) show that expected inflation is highly statistically significant and its coefficient is larger than in the equation for national inflation rates (Table 4.7, eq. (1)). In contrast, lagged inflation is not significant. Even though, the exchange rate does not seem to be statistically significant in explaining inflation differentials, the coefficients of the exchange rate in  $t$  and  $t + 1$  have the right signs. But the coefficient of the lagged change in NEER is wrongly signed, confirming that past dynamics does not seem to explain differences in inflation. In turn, output gap has

a positive effect but it is not statistically significant.<sup>44</sup>

Finally, the nominal national interest rate and the lagged price level are not statistically significant, with the latter variable having the wrong sign. Also, in Hofmann and Remsperger (2005), proxies of price level convergence are not significant in explaining national inflation rates. Likewise, in Rogers (2002), the lagged price level becomes insignificant in explaining inflation differences when the Arellano-Bond GMM estimator was used. The fact that such a variable is also not significant in our estimates probably means that in the euro area the level of price convergence was already high enough during the sample period. Indeed, Rogers (2007) shows that much of the price level convergence in Europe took place close to the completion of the Single Market in January 1993. But the possible future enlargement of the monetary union to more heterogeneous countries may imply that the Balassa-Samuelson effect will become significant.

Notice that, according to the F-statistics of the first stage regressions, the instruments for inflation and exchange rate are not weak.

From the above results, we can then conclude that lagged inflation rate and the nominal interest rate have a role in explaining national inflation rates, but not in explaining inflation's differences across countries.

Notice that expectations play a central role in our results. If they are ignored, we obtain results similar to Honohan (2003), with output gap, the *level* of the real exchange rate and the lagged price level having a statistically significant impact on inflation differentials (see Table 4.8).<sup>45</sup> The presence of the real exchange rate's level can be interpreted as national inflation rates acting to correct disequilibrium in that variable. Other possible interpretation is that with imported inputs in production, the *level* of the real exchange rate directly affects the marginal cost (Kara and Nelson (2003)) and equation (2.55)).

Returning to the regressions with expectations included, one intriguing result is the statistical insignificance of the output gap. It can then be asked if by using an alternative measure of the business cycle, more significant results can be obtained. Therefore, in place of the output gap, we used the RULC, but this variable was also statistically insignificant (Table

---

<sup>44</sup>We also made an estimation (not shown) with the relative price of imports,  $pi_{i,t} - pd_{i,t}$ , which had a positive but insignificant coefficient.

<sup>45</sup>This model has positive serial correlation probably because it does not capture all the dynamics present in the data.

Table 4.8: Determinants of inflation differentials ignoring expectations. GMM estimations for a panel of 12 euro area countries.

	$c$	$x_{i,t}$	$e_{i,t}$	$pl_{i,t-1}$
Coeff.	0.54***	0.11***	-0.117***	-0.031***
s.e.	(0.18)	(0.04)	(0.039)	(0.0082)
Time dummies:	Yes			
J-statistic:	2.59 [0.62]	Q (2) stat.:	291.12 [0.00]	

Notes: See notes to Table 4.5. Instruments: constant,  $\pi_{i,t-1}$ ,  $x_{i,t-1}$ ,  $e_{i,t-1}$ ,  $\dot{v}_{i,t-1}$ ,  $p_{i,t-2}$ ,  $p_{i,t-3}$ ,  $dfp_{i,t-1}$  and time dummies.

4.7, eq. (2)).

In the context of business cycle effect on inflation, there is some preliminary empirical evidence showing that wage growth is associated with different inflation dynamics in the euro area (ECB, 2003). Also, Lown and Rich (1997) are able to effectively track inflation in the 1990's using a traditional Phillips curve augmented with the growth in nominal ULC. Therefore, it was used the growth of nominal ULC instead of the output gap or RULC, and a positive and statistically significant coefficient was obtained for that variable (Table 4.6, eq. (3)). The coefficients of other variables have remained roughly the same as when the output gap was used to explain inflation differences. We can then conclude that the cyclical position affects inflation differentials if it affects the growth of nominal ULC.

In the last estimation, once more the lagged change in the NEER has the wrong sign. If we remove it, the one period expected and current NEER become significant (Table 4.6, eq. (4)). As in Honohan and Lane (2003), a depreciation of the euro in  $t$  tends to increase inflation differentials. This can be interpreted in the light of the fact that countries that have more trade outside the euro area suffer higher imported inflation when exchange rate depreciates. Different velocities of exchange rate pass-through can also explain why movements in the euro have a temporary impact on inflation differentials (Honohan and Lane, 2003). However, the exchange rate effect on inflation differentials will tend to decrease with time (Honohan and Lane, 2003). On one hand, the intra-eurozone trade will increase with the deepening of economic integration between member countries and also with future enlargements of the

euro area, namely to the UK and Eastern European countries. On the other hand, as the importance of the euro increases in the world exchange rate market, more euro area imports will be priced in euros, thus weakening the direct impact of exchange rate fluctuations on consumer prices.

#### 4.5.4 Inflation differentials, imperfect competition model and the NKPC

Given the empirical relevance of the nominal ULC, let us look at the imperfect competition model (ICM) of inflation, which defines a role for the nominal ULC. Here, we add the cost channel to the ICM model presented by Bjornstad and Nymoen (2008). This model assumes that the price of domestically produced goods,  $pd_t$ , is set as a mark-up over the unit labour cost and the nominal interest rate, and the mark-up depends on the relative price of domestic goods in terms of foreign goods,  $pi_t$ , (all variables are in logs):

$$pd_t = m_0 + m_1 (pi_t - pd_t) + i_t + ulc_t \quad (4.15)$$

where  $i_t$  is the *gross* nominal interest rate and  $m_0$  is the steady-state mark-up. In equilibrium, there is a relationship between domestic prices on one hand, and the ULC, nominal interest rate and import prices on the other hand. The nominal interest rate affects domestic prices because firms have to pay salaries in advance.

With a constant share of imports in consumption,  $1 - \gamma$ , the CPI is by definition:

$$p_t \equiv \gamma pd_t + (1 - \gamma) pi_t. \quad (4.16)$$

If we solve (4.16) for  $pd_t$  and replace the obtained expression in (4.15), we obtain after some manipulations:

$$p_t = \mu_0 + \mu_1 (i_t + ulc_t) + (1 - \mu_1) pi_t,$$

with  $\mu_0 = m_0 \mu_1$  and  $\mu_1 = \gamma / (1 + m_1)$ . Since prices often are not in equilibrium, the model should be expressed in an equilibrium correction form, where:

$$\begin{aligned} \pi_t = & \mu_0 \beta_1 + \alpha^f \pi_{t+1}^e + \alpha^b \pi_{t-1} + \beta_1 (ulc_{t-1} + i_{t-1} - p_{t-1}) \\ & + \beta_2 (ulc_{t-1} + i_{t-1} - pi_{t-1}) + \beta_3 \Delta ulc_t + \beta_4 \Delta pi_t + \beta_5 \Delta i_t \end{aligned} \quad (4.17)$$

with all coefficients  $\alpha$  and  $\beta$  positive, except  $\beta_2$  that is negative.<sup>46</sup> When the *last* period ULC plus nominal interest rate is higher than consumer price level,  $ulc_{t-1} + i_{t-1} > p_{t-1}$ , the disequilibrium is corrected with an increase in inflation in the *current* period; or in other words, with the increase in consumer prices. In turn, if in  $t-1$  the ULC plus nominal interest rate is larger than imports price,  $ulc_{t-1} + i_{t-1} > pi_{t-1}$ , then in  $t$  inflation decreases.<sup>47</sup> In the last equation, it was assumed that the dynamic part of the NKPC is valid:  $\alpha^f$  and  $\alpha^b$  are different from zero.

The open economy NKPC can be expressed in an error correction model of the price level, similar to (4.17). The initial equation is:

$$\pi_t = a^f \pi_{t+1}^e + a^b \pi_{t-1} + b \widehat{mc}_t + cz_t, \quad (4.18)$$

where  $\widehat{mc}_t = (s_t + i_t - \log(\alpha_n) - mc^{ss})$ ,  $z_t$  is a vector containing open economy variables, as for example the change in the real price of imports,  $\Delta(pi_t - p_t)$ ; and  $s_t$  is the wage share, defined as

$$s_t = ulc_t - pd_t. \quad (4.19)$$

Using (4.16), (4.18) and (4.19), and after some manipulations, we obtain:

$$\begin{aligned} \pi_t = & \alpha + \frac{a^f}{1 + \frac{b}{\gamma}} \pi_{t+1}^e + \frac{a^b}{1 + \frac{b}{\gamma}} \pi_{t-1} - \beta (p_{t-1} - \gamma ulc_{t-1} - (1 - \gamma) pi_{t-1}) \\ & + \beta \gamma \Delta ulc_t + \beta (1 - \gamma) \Delta pi_t + \beta \gamma \Delta i_t + \psi z_t \end{aligned}$$

with  $\alpha = -b(\log(\alpha_n) + mc^{ss})$ ,  $\beta = b/(\gamma + b)$  and  $\psi = (c\gamma)/(\gamma + b)$ . The last equation can be expressed as

$$\begin{aligned} \pi_t = & \alpha + \omega^f \pi_{t+1}^e + \omega^b \pi_{t-1} + \beta_1 (ulc_{t-1} + i_{t-1} - p_{t-1}) \\ & + \beta_2 (ulc_{t-1} + i_{t-1} - pi_{t-1}) \\ & + \beta_3 \Delta ulc_t + \beta_4 (1 - \gamma) \Delta pi_t + \beta_5 \Delta i_t + \psi z_t. \end{aligned}$$

---

<sup>46</sup>We do not test for cointegration regarding the two error correction terms due to the small time dimension of the sample.

<sup>47</sup>Intuitively, in this situation the price of imports is low and the markup decreases, leading to a decrease in inflation.

with  $\omega^f = \frac{\alpha^f}{1+\frac{b}{\gamma}}$  and  $\omega^b = \frac{\alpha^b}{1+\frac{b}{\gamma}}$ ,  $\beta_1 = \beta$ ,  $\beta_2 = -\beta(1-\gamma)$ ,  $\beta_3 = \beta\gamma$ ,  $\beta_4 = \beta(1-\gamma)$ , and  $\beta_5 = \beta\gamma$ . This equation imposes restrictions on the ICM:  $H_0^a : \beta_1 + \beta_2 = \beta_3$ ,  $H_0^b : \beta_4 = -\beta_2$ , and  $H_0^c : \beta_5 = \beta_3$ . If  $z_t$  includes the change in import prices, then  $H_0^b$  is no longer an imposition arising from the NKPC. The significance of the forward component of inflation is also fundamental for the validity of the NKPC.

Bjornstad and Nymoen (2008) show with an annual panel of 20 OECD countries, from 1960 to 2004, that: (1) the NKPC is encompassed by the ICM model ( $H_0^a$  is rejected), and (2) the expected rate of inflation serves as a replacement for the ICM specific equilibrium correction terms. In other words, when the equilibrium terms are included, the expected inflation's coefficient is not significant. This means that the omission of the equilibrium correction terms creates an upwards bias in the estimate of  $\alpha^f$ , explaining why the lead coefficient of inflation is significant in many estimates of the Phillips curve. Also, for the UK, Bardsen, Jansen and Nymoen (2004) show that the introduction of two equilibrium correction terms, deviations from a long-run wage curve and an open economy price mark-up, makes the forward inflation insignificant.<sup>48</sup>

Based on this discussion, the ICM is an alternative to the NKPC to explain inflation differentials. Therefore, we augmented the ICM in equation (4.17) with the lagged country's price level and estimated it using the same panel of countries as before. Table 4.9 shows that the instruments for inflation are not weak. The null hypothesis  $H_0^a$  is rejected,<sup>49</sup> meaning that the ICM model is not encompassed by the NKPC. In other words, it is better to use the ICM model than the NKPC, because the former is an unrestricted version of the latter. We can also see that the error correction variables are significant. When the *previous* period level of ULC, nominal interest rate or import prices are excessively high compared with domestic prices, firms have to increase prices to maintain mark-up over marginal costs.

In addition, previous results obtained in this paper are confirmed: the relevance of expected inflation, the change in the nominal ULC, and import prices. We observe that, even though the coefficient of the expected inflation decreases with the introduction of the error correction variables, it continues to be statistically significant. This confirms the importance

---

<sup>48</sup>But notice that both Bjornstad and Nymoen (2008) and Bardsen, Jansen and Nymoen (2004) were explaining national inflation rates, and not inflation differentials.

<sup>49</sup>P-value of 0.0063.

Table 4.9: GMM estimation of the ICM for inflation differentials of 12 euro area countries, 1999Q1-2008Q4.

	$c$	$ulc_{i,t-1} + i_{i,t-1} - p_{i,t-1}$	$ulc_{i,t-1} + i_{i,t-1} - pi_{i,t-1}$	$\pi_{i,t+1}$	$\pi_{i,t-1}$
Coeff.	-0.00027	0.0084**	-0.0079**	0.71***	0.063
s.e.	(0.0013)	(0.0040)	(0.0040)	(0.13)	(0.070)
	$\Delta ulc_{i,t}$	$\Delta pi_{i,t}$	$\Delta i_{i,t}$	$pl_{i,t-1}$	Time dummies
Coeff.	0.063**	0.071*	0.00095	-0.00060	Yes
s.e.	(0.025)	(0.041)	(0.00107)	(0.00064)	
F-stat 1 <sup>st</sup> stage reg.:	17.24	J-stat. :	6.63 [0.24]	Q (2) stat.:	108.11 [0.00]

Notes: See notes to Table 4.5. Instruments: constant,  $\pi_{i,t-2}$ ,  $x_{i,t-1}$ ,  $q_{i,t-1}$ ,  $i_{i,t-1}$ ,  $\Delta i_{i,t-1}$ ,  $p_{i,t-2}$ ,  $p_{i,t-3}$ ,  $difp_{i,t-1}$ ,  $\Delta pi_{i,t-1}$ ,  $ulc_{i,t-1} + i_{i,t-1} - p_{i,t-1}$ ,  $ulc_{i,t-1} + i_{i,t-1} - pi_{i,t-1}$ ,  $\Delta ulc_{i,t-1}$ ,  $pl_{i,t-1}$  and time dummies.

of forward inflation in explaining differences in inflation dynamics. This result to a certain extent contradicts Bjornstad and Nymoen (2008). Other point worth highlighting is that, even though the change in the nominal interest rate does not have a significant effect on inflation differentials, its level is present on the equilibrium marginal cost, which has a significant impact on inflation.

Even if we assume country fixed effects, our main results hold, both for the national inflation rates as well as for inflation differentials: respectively, equation 1 and equations 2 and 3 in Table 4.19, Annex 4.7.

The identified relevance of the nominal ULC for inflation differentials may create destabilising macroeconomic effects. Indeed, inflation differentials may lead to differences in the growth of wages that will have a further effect on inflation differentials. But there is evidence that inflation differentials have a limited effect on business cycles. We obtained in Section 4.3.5, for the period 1980-2008, that inflation differentials did not have a significant effect on output gap differentials. Also, Hofmann and Remsperger (2005), as referred to in Section 4.1, argue that the mechanisms that correct inflation differentials are relevant in the euro area.

## 4.6 Conclusion

In this paper we had two major concerns: assess the convergence of inflation rates and business cycles in the euro area and study the relationship between these convergence processes.

We started by studying the convergence of inflation, output gap and RULC towards Germany using the Kalman filter to estimate an unobserved convergence component. From 1980 to 2008, inflation differentials in the euro area have converged in expectation, despite the emergence of some temporary divergence after the introduction of the euro. This transitory diverging dynamic was more significant for Greece, Ireland, the Netherlands, Portugal and Spain.

The business cycles of euro area countries have also become more aligned between 1980 and 2008, and that was clearer when using the output gap than when using the RULC.

For the countries where convergence of output gap and inflation was identified, convergence of inflation occurred at a faster rate than the convergence of output gap. Looking at the causality between the two phenomena, while output gap convergence had a positive effect on inflation convergence, even though statistically weak for the majority of countries, it seems that the causality in the opposite direction does not exist. As a result, the destabilising impact of inflation divergence is more limited.

The methodology based on the Kalman filter does not allow us to identify clearly sub-periods of convergence and divergence in the business cycles. However, in the literature, there have been identified periods of convergence and divergence in economic fluctuations. In order to be able to identify such periods and also produce an aggregate measure of convergence, we used the common factor approach developed by Becker and Hall (2009a). This methodology allows to measure convergence in a group of years, also called windows.

Comparing the windows of 1980-89 and 1990-98, we observe that in the latter, inflation co-movement has decreased considerably. In turn, compared with the period 1990-98, the co-movement in European inflation rates did not increase on average during the euro period. But when looking at sub-periods after the introduction of the euro, we observed that the link between European inflation rates has increased strongly in the last years studied. The divergence observed in the first years of the euro was mainly explained by a group of diverging countries, which includes Finland, Ireland, the Netherlands, and Portugal. For these

countries, there was also identified a temporary divergence in inflation with the Kalman filter methodology, but some of the aggregate divergence observed was also due to large countries and other core European countries.

Regarding business cycle convergence, results depend on the indicator used. With respect to the window of 1990-98, the co-movement between output gaps has increased for *the overall* euro period, while convergence of RULCs has decreased. However, when we looked at *sub-periods* during the euro period, the co-movement between RULCs has increased slightly from the beginning to the end of the euro period.

A common characteristic of the convergence processes of the three variables studied is that during the euro period, there were windows of convergence and divergence. In general for the three variables, comparing the initial with the final 5-years windows, there was an increase in co-movement, which was smaller for the RULC. With the Kalman filter analysis, there was also observed an initial increase in inflation differentials, followed by a reduction in such differentials in the last years studied.

There was some parallelism between the evolution of the  $R^2$  of the common factor of inflation and the  $R^2$  of output gap or RULC; that however was far from being perfect. In addition, some countries that diverged on inflation also diverged on the output gap or RULC. But once more that is not true for all countries and periods.

To explore deeper the determinants of inflation's convergence, the component unexplained by the common factor was used as a measure of divergence in a model for inflation rates' differences. From such an econometric model, some interesting results were obtained. Firstly, inflation differentials are highly affected by expected differentials, while past differentials are less important. Secondly, price level convergence seems to be present, but is not significant in the majority of regressions. Also, exchange rates are important in explaining differences in inflation dynamics, with their expected value being more significant. However, the price of oil, the euro nominal interest rate and the euro area output gap do not seem to significantly explain inflation differentials. Finally, we were also unable to establish a sensible and statistically significant link between business cycles and differences in inflation rates. In order to explore whether these results were specific to the methodology chosen, we estimated the NKPC using the actual variables, instead of the residuals of the common factor model. The results obtained using the residuals were basically confirmed.

For a panel of 12 euro area countries, the estimation of the NKPC with panel data produces results similar to other studies with time-series and panel data. Inflation has both forward- and backward-looking components, with the former being more important. Exchange rates also play a role in price changes, with the lagged exchange rate having a statistically significant impact. While the cost channel is present, the output gap or the RULC have a positive effect on inflation that however is not significant.

Regarding inflation differentials, we observe that the expected inflation rate and exchange rate movements are important determinants of differences in inflation rates. Nevertheless, the past dynamics of inflation and exchange rate do not play a very relevant role, despite their significance in explaining national inflation rates. Finally, the usual measures of the business cycle, output gap and the RULC are not significant in causing differences in inflation dynamics. This confirms the analysis done with the Kalman filter and common factor models, where it was concluded that the relationship between economic fluctuations and differences in inflation rates was not very strong.

Observe that the expected inflation rate plays a fundamental role in the results. When it was introduced, the lagged price level and the output gap lost their statistical significance. It seems then that these variables were significant because they forecast inflation.

Furthermore, the growth of nominal ULC plays a significant role in explaining inflation differentials. This means that the business cycle affects inflation differentials when it causes differences in wages evolution across countries. Inflation rates differences are also affected by the lagged disequilibrium in the long-run relationship proposed by the ICM, which involves domestic prices on the one hand, and the ULC, nominal interest rate and imported goods prices, on the other hand. Also, the ICM model is not encompassed by the NKPC when explaining inflation differences. We also state that the nominal interest rate is present in the equilibrium marginal cost, which has a statistical significant effect on inflation differentials. Furthermore, the introduction of the error correction terms proposed by the ICM reduced the coefficient of expected inflation but did not eliminate its statistical significance.

In terms of policy, our results show that the management of expectations and the control of labour costs are fundamental to ensure that the convergence process of inflation is successful. The ECB should also take into account the impact of the euro on inflation differentials.

Given the relevance of labour costs, further work should be performed to assess the em-

empirical relevance of a diverging inflationary cycle arising from the interaction between labour costs and inflation rates.

## 4.7 Annex: additional graphs and tables

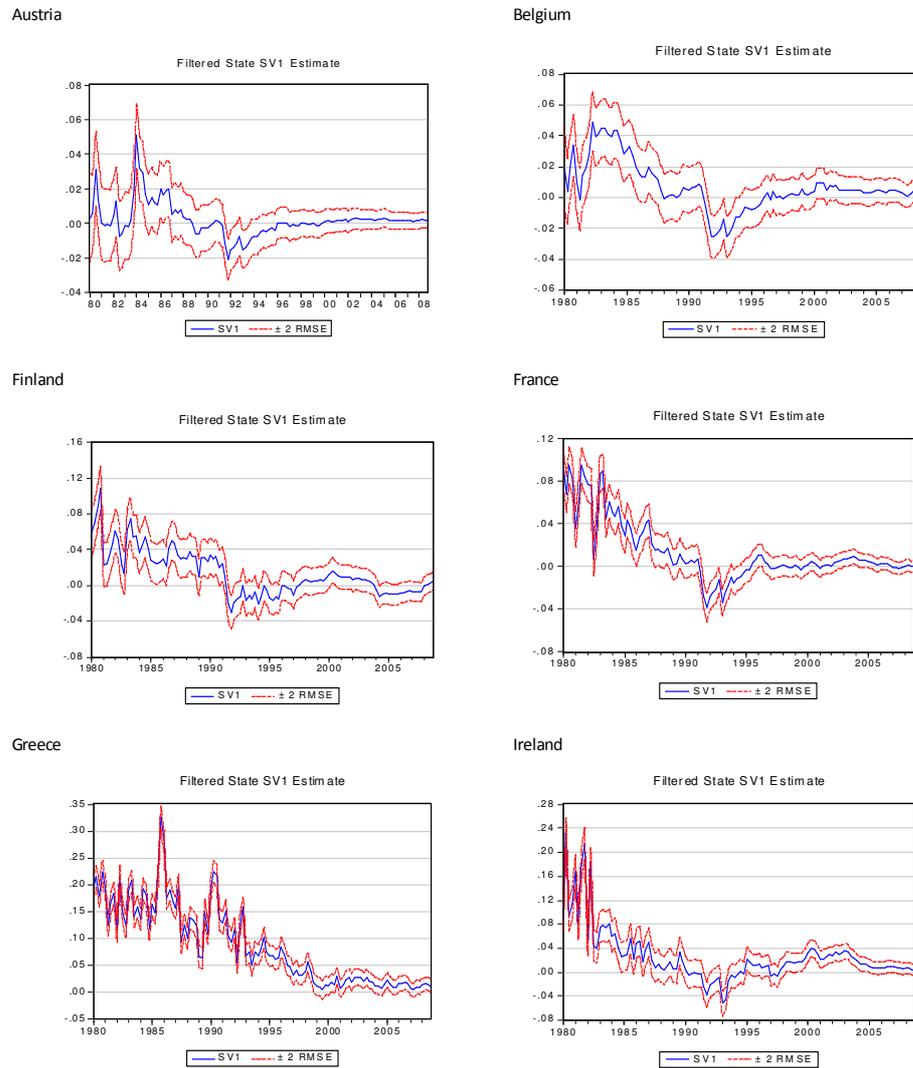
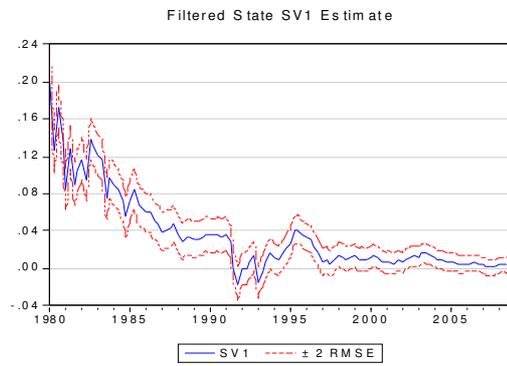
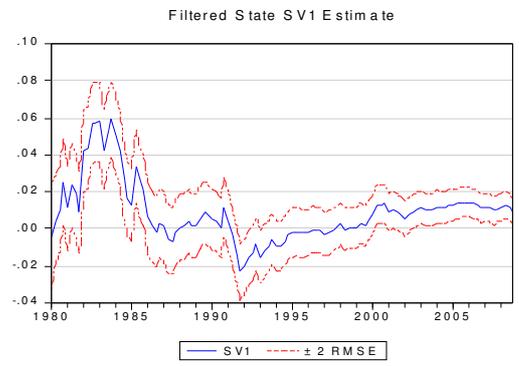


Figure 4-9: Inflation's differentials towards Germany, filtered state variable, 1980Q1-2008Q4 (part I).

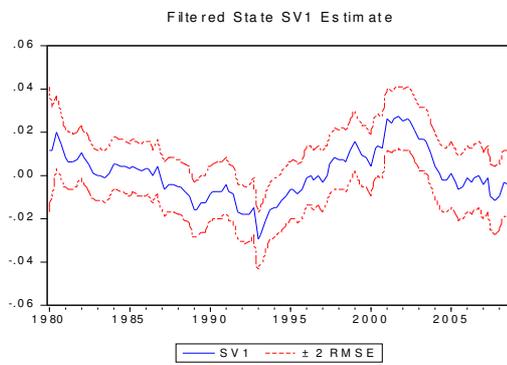
Italy



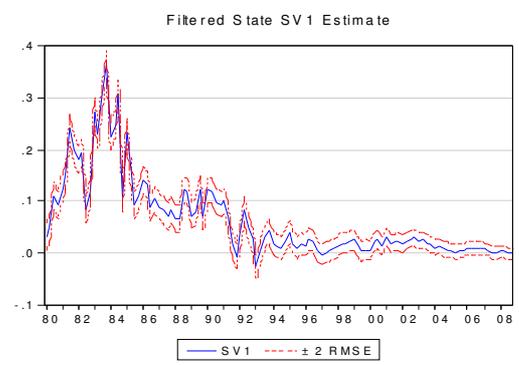
Luxembourg



Netherlands



Portugal



Spain

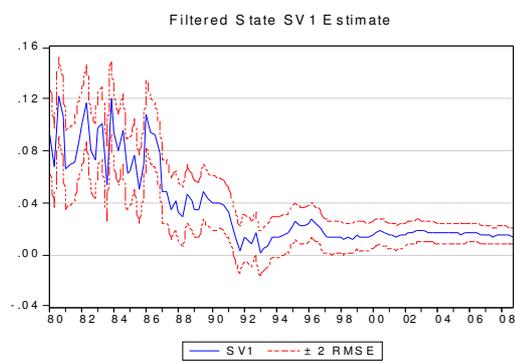


Figure 4-10: Inflation's differentials towards Germany, filtered state variable, 1980Q1-2008Q4 (part II).

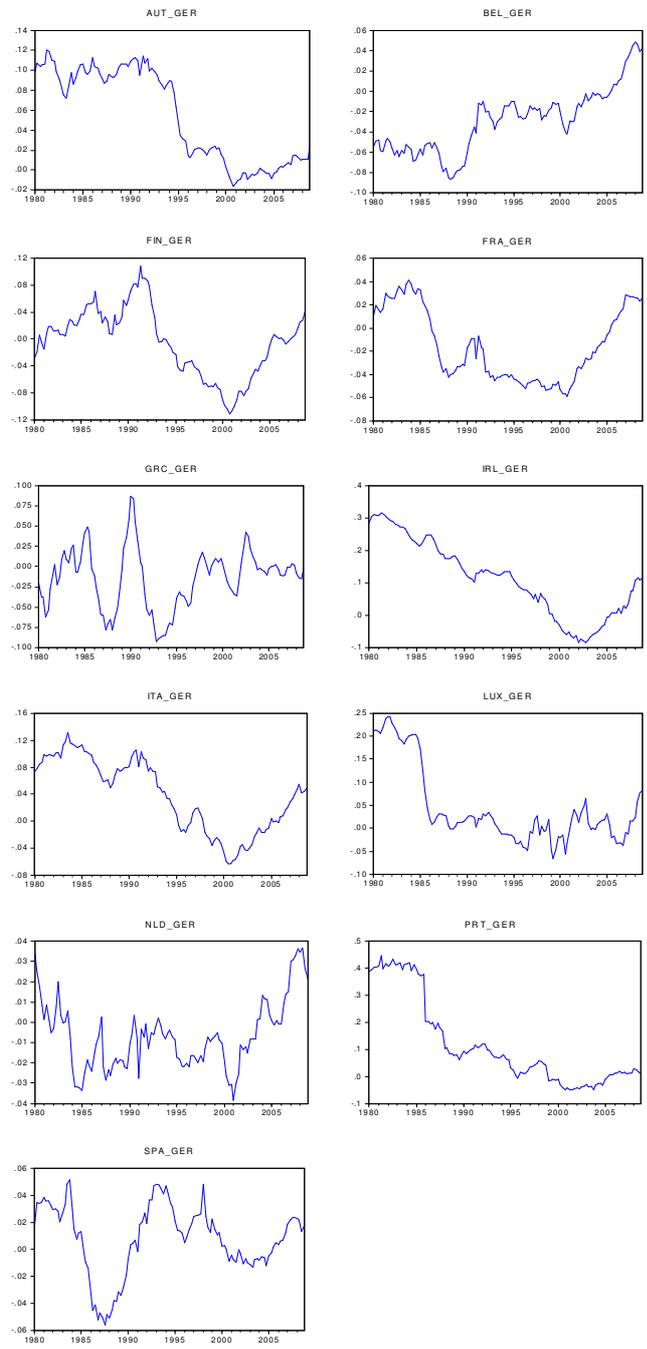


Figure 4-11: Log difference between the real ULC of each country and Germany.

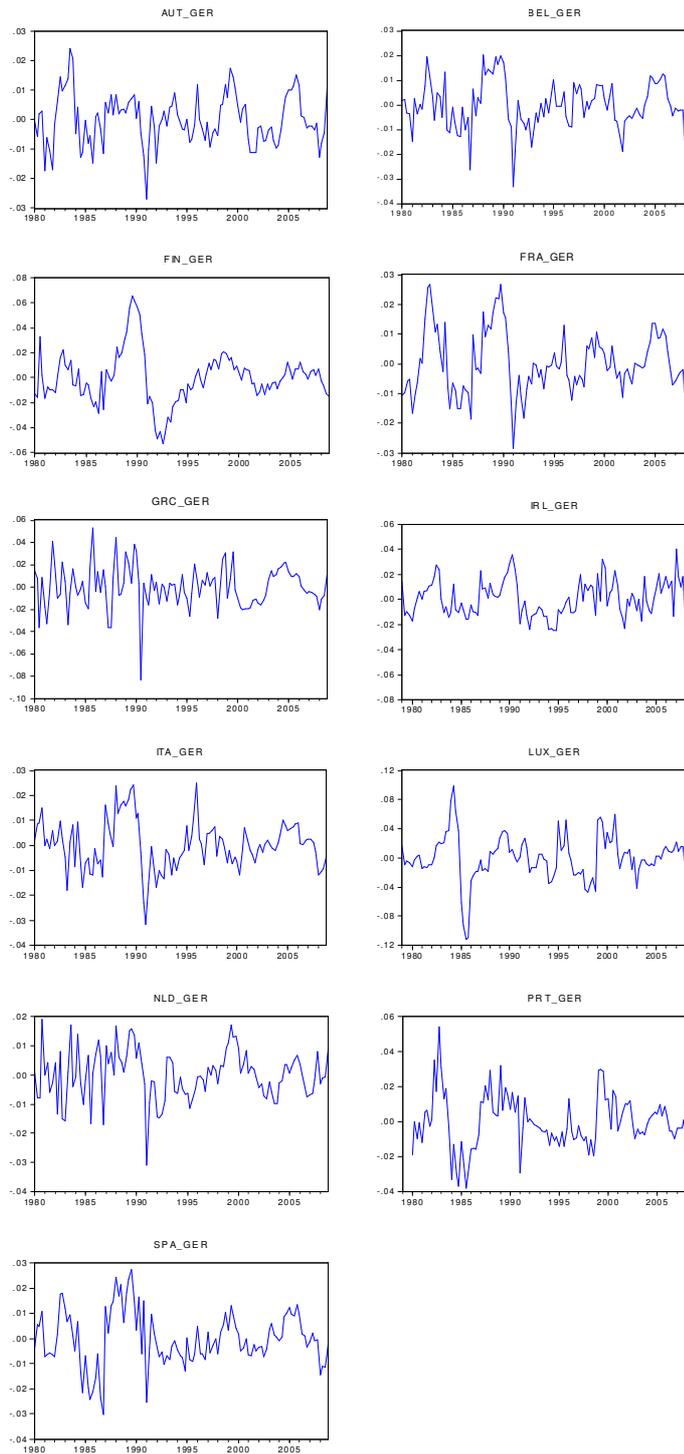


Figure 4-12: Difference between the output gap of each country and Germany.

Table 4.10: Measuring inflation convergence towards Germany with time-varying parameters. Estimation with the Kalman Filter, 1980Q1-2008Q4

	$\text{Var}(\varepsilon_t)$	$\phi$	$\phi - 1$	$\Omega_{2001}$	$\Omega_{2004}$	$\alpha_{0501 0501}$
<b>Austria:</b>						
coeff.	0.00016	0.9251	-0.0748	0.00029	0.00000038	0.0019
s.e. /RMSE	0.00021	0.0187		0.00015	0.00000073	0.0022
z stat	7.8672	-4.0004		1.9155	0.5205	0.8535
Log likelihood	311.8041					
<b>Belgium</b>						
coeff.	0.00017	0.9536	-0.0464	0.00019	0.00000082	0.0067
s.e. /RMSE	0.00025	0.0195		0.00018	0.0000012	0.0036
z stat	6.8253	-2.3779		1.0659	0.6833	1.8616
Log likelihood	307.7595					
<b>France</b>						
coeff.	0.00074	0.9337	-0.0662	0.0012	0.00000048	0.00058
s.e. /RMSE	0.00012	0.0102		0.00047	0.00000047	0.00272
z stat	5.7984	-6.4856		2.6842	1.0212	0.2142
Log likelihood	322.7840					
<b>Finland</b>						
coeff.	0.00018	0.9548	-0.0452	0.00087	0.00000042	0.0051
s.e. /RMSE	0.00030	0.0075		0.00028	0.0000032	0.0054
z stat	6.0389	-5.9777		3.0774	1.3125	0.9532
Log likelihood	284.3473					
<b>Greece</b>						
coeff.	0.00010	0.9416	-0.0583	0.0199	0.00002	0.0105
s.e. /RMSE	0.00047	0.0107		0.0094	0.000017	0.0071
z stat	2.2384	-5.4179		2.1070	1.1764	1.4732
Log likelihood	231.5578					
<b>Ireland</b>						
coeff.	0.00017	0.9316	-0.0683	0.0088	0.0000025	0.0044
s.e. /RMSE	0.00041	0.0079		0.0027	0.0000020	0.0046
z stat	4.1304	-8.6318		3.2639	1.25	0.9481
Log likelihood	257.6929					
<b>Italy</b>						
coeff.	0.00014	0.9430	-0.0569	0.0015	0.0000017	0.0053
s.e. /RMSE	0.00024	0.0099		0.00072	0.0000015	0.0041
z stat	6.0743	-5.6923		2.0814	1.1333	1.2843
Log likelihood	292.2545					
<b>Luxembourg</b>						
coeff.	0.000208	0.9468	-0.05315	0.00026	0.00000048	0.0097
s.e. /RMSE	0.00031	0.0190		0.00025	0.00000075	0.0034
z stat	6.5822	-2.7850		1.0155	0.6400	2.8391
Log likelihood	298.3712					
<b>Netherlands</b>						
coeff.	0.00020	1.0117	0.0117	0.0000067	0.000025	0.00041
s.e. /RMSE	0.00028	0.0240		0.000011	0.000037	0.0092
z stat	7.2027	0.4904		0.6090	0.6756	0.0451
Log likelihood	301.77					
<b>Portugal</b>						
coeff.	0.00017	0.9334	-0.0665	0.0155	0.0000056	-0.0007
s.e. /RMSE	0.00047	0.0097		0.0051	0.0000052	0.0061
z stat	3.7473	-6.8478		2.9842	1.0769	-0.1267
Log likelihood	230.4241					
<b>Spain</b>						
coeff.	0.00027	0.9198	-0.0802	0.0017	0.00000015	0.0137
s.e. /RMSE	0.00044	0.0223		0.0012	0.00000033	0.0031
z stat	6.0538	-3.5899		1.4171	0.4545	4.3328
Log likelihood	272.4286					

Notes: The z-statistics are for the null of each respective coefficient equal to zero, except for  $\phi$  where the null is  $\phi = 1$ .

For the final one-step ahead values of the state vector, we present the corresponding RMSE (square root of the diagonal elements of  $P_{t+1|t}$ )

Table 4.11: Measuring real ULC convergence towards Germany with time-varying parameters. Estimation with the Kalman Filter, 1980Q1-2008Q4

	$\phi$	$\Omega_{80Q1}$
<b>Austria:</b>		
coeff.	0.9874	0.000077
s.e./RMSE	0.0036	0.000020
z stat.	-3.5000	3.7584
Log likeli h.	414.3340	
<b>Belgium</b>		
coeff.	0.9939	0.000059
s.e./RMSE	0.0049	0.000015
z stat.	-1.2448	3.7884
Log likeli h.	408.8114	
<b>France</b>		
coeff.	0.9874	0.000058
s.e./RMSE	0.0033	0.000012
z stat.	-3.8181	4.7419
Log likeli h.	430.6015	
<b>Finland</b>		
coeff.	0.9862	0.00022
s.e./RMSE	0.0044	0.000059
z stat.	-3.0918	3.8358
Log likeli h.	356.5224	
<b>Greece</b>		
coeff.	0.9788	0.00056
s.e./RMSE	0.0047	0.00018
z stat.	-4.5106	3.1333
Log likeli h.	329.3325	
<b>Ireland</b>		
coeff.	1.0114	0.000065
s.e./RMSE	0.0034	0.000016
z stat.	3.3529	4.0559
Log likeli h.	345.8146	
<b>Italy</b>		
coeff.	0.9972	0.000084
s.e./RMSE	0.0052	0.000025
z stat.	-0.5202	3.3600
Log likeli h.	377.5831	
<b>Luxembourg</b>		
coeff.	1.0098	0.00017
s.e./RMSE	0.0031	0.000032
z stat.	3.1075	5.3680
Log likeli h.	294.3482	
<b>Netherlands</b>		
coeff.	0.9893	0.000089
s.e./RMSE	0.0033	0.000019
z stat.	-3.1722	4.5151
Log likeli h.	400.0428	
<b>Portugal</b>		
coeff.	0.9733	0.0012
s.e./RMSE	0.0025	0.00020
z stat.	-10.3927	6.1450
Log likeli h.	302.8765	
<b>Spain</b>		
coeff.	0.9855	0.00012
s.e./RMSE	0.0045	0.000037
z stat.	-3.1703	3.2887
Log likeli h.	394.2997	

Notes: The z-statistics are for the null hypothesis  $\phi = 1$  or  $\Omega_{80Q1} = 0$ .

Initially, we assumed  $Var(\varepsilon_t) \neq 0$ , but this variance was not significantly different from zero. Therefore, results presented here assume  $Var(\varepsilon_t) = 0$ .

For Portugal it was used a dummy ~~245~~ to accommodate an outlier in 1986Q1.

Table 4.12: Measuring output gap convergence towards Germany with time-varying parameters. Estimation with the Kalman Filter, 1980Q1-2008Q4

	$Var(\varepsilon_t)$	$\phi$	$\phi - 1$	$\Omega_{80Q1}$
<b>Austria:</b>				
coeff.	0.0000067	0.9855	-0.0144	0.000091
s.e./RMSE	0.0000053	0.004475		0.000025
z stat.	1.2486	-3.2201		3.5725
Log likelih.	394.6983			
<b>Belgium</b>				
coeff.	0.000016	0.9787	-0.0212	0.00010
s.e./RMSE	0.0000051	0.0084		0.000045
z stat.	3.1589	-2.5192		2.3059
Log likelih.	386.3252			
<b>France</b>				
coeff.	0.0000070	0.9825	-0.0174	0.00010
s.e./RMSE	0.0000041	0.0052		0.00003
z stat.	1.6891	-3.3363		3.4000
Log likelih.	396.0944			
<b>Finland</b>				
coeff.	0.0000073	0.9687	-0.0313	0.00046
s.e./RMSE	0.0000059	0.0062		0.00013
z stat.	1.2332	-5.0402		3.4552
Log likelih.	360.7888			
<b>Greece</b>				
coeff.	1.31E-17	0.9686	-0.0313	0.0022
s.e./RMSE	0.000019	0.0063		0.0006
z stat.	6.85E-13	-4.9708		3.7030
Log likelih.	283.4459			
<b>Ireland</b>				
coeff.	0.000048	1.0195	0.0195	0.000027
s.e./RMSE	0.000016	0.0058		0.000013
z stat.	3.0125	3.3526		1.9496
Log likelih.	327.8104			
<b>Italy</b>				
coeff.	4.12E-12	0.9757	-0.0242	0.00022
s.e./RMSE	0.00000417	0.0055		0.000062
z stat.	9.88E-07	-4.3576		3.6305
Log likelih.	391.7223			
<b>Luxembourg</b>				
coeff.	0.000059	0.9890	-0.0109	0.00075
s.e./RMSE	0.000044	0.0045		0.00014
z stat.	1.3537	-2.4234		5.1643
Log likelih.	263.808			
<b>Netherlands</b>				
coeff.	0.0000078	0.9630	-0.0369	0.00031
s.e./RMSE	0.0000031	0.0066		0.00010
z stat.	2.5193	-5.5602		
Log likelih.	389.9319			
<b>Portugal</b>				
coeff.	0.000047	0.9740	-0.02599	0.00024
s.e./RMSE	0.000018	0.0069		0.000096
z stat.	4.0000	-3.7203		2.5129
Log likelih.	338.901			
<b>Spain</b>				
coeff.	0.000023	0.9716	-0.0283	0.00013
s.e./RMSE	0.0000068	0.0093		0.000058
z stat.	3.4604	-3.0463		2.2108
Log likelih.	379.7455			

Note: The z-statistics are for the null of each respective coefficient equal to zero, except for  $\phi$  where the null is  $\phi = 1$ .

Table 4.13: Testing the equality of the convergence processes of inflation and output gap (part I). Estimation with the Kalman Filter, 1980Q1-2008Q4

	coeff.	s.e.	z stat.
<b>Austria:</b>			
$Var(\varepsilon_t^\pi)$	0.00016	0.000021	7.650
$Var(\varepsilon_t^x)$	0.0000067	0.0000053	1.257
$\phi^{\pi}$	0.9248	0.2004	-3.676
$\Omega_{80Q1}^\pi$	0.00029	0.00019	1.549
$\phi^z$	1.065	0.025	2.555
$\Omega_{80Q1}^z$	0.3049	0.2759	-2.519
$\phi^x = \phi^\pi \cdot \phi^z$	0.9855	$1 - \phi^x$	-0.0144
$1 - \phi^\pi$	-0.07512	$(1 - \phi^\pi) - (1 - \phi^x)$	-0.06068
Log likelih.	706.4990		
<b>Belgium</b>			
$Var(\varepsilon_t^\pi)$	0.00017	0.000027	6.387
$Var(\varepsilon_t^x)$	0.000016	0.0000054	3.090
$\phi^\pi$	0.9541	0.0197	-2.325
$\Omega_{80Q1}^\pi$	0.00015	0.00013	1.149
$\phi^z$	1.0258	0.0222	1.162
$\Omega_{80Q1}^z$	0.6577	0.6312	-0.542
$\phi^x = \phi^\pi \cdot \phi^z$	0.9787	$1 - \phi^x$	-0.0212
$1 - \phi^\pi$	-0.0458	$(1 - \phi^\pi) - (1 - \phi^x)$	-0.0246
Log likelih.	686.9323		
<b>Finland</b>			
$Var(\varepsilon_t^\pi)$	0.00019	0.000030	6.343
$Var(\varepsilon_t^x)$	0.0000092	0.0000073	1.260
$\phi^\pi$	0.9480	0.0085	-6.065
$\Omega_{80Q1}^\pi$	0.00079	0.00025	3.081
$\phi^z$	1.0199	0.0114	1.742
$\Omega_{80Q1}^z$	0.6024	0.2699	-1.473
$\phi^x = \phi^\pi \cdot \phi^z$	0.9670	$1 - \phi^x$	-0.0329
$1 - \phi^\pi$	-0.0519	$(1 - \phi^\pi) - (1 - \phi^x)$	-0.0189
Log likelih.	638.894		
<b>France</b>			
$Var(\varepsilon_t^\pi)$	0.000075	0.000013	5.831
$Var(\varepsilon_t^x)$	0.0000060	0.0000040	1.510
$\phi^{\pi}$	0.9326	0.0102	-6.556
$\Omega_{80Q1}^\pi$	0.0012	0.00047	2.698
$\phi^z$	1.0535	0.0121	4.414
$\Omega_{80Q1}^z$	0.0843	0.0382	-23.934
$\phi^x = \phi^\pi \cdot \phi^z$	0.9826	$1 - \phi^x$	-0.0174
$1 - \phi^\pi$	-0.0673	$(1 - \phi^\pi) - (1 - \phi^x)$	-0.0499
Log likelih.	718.9689		

Note: For  $var(\varepsilon_t^\pi)$ ,  $var(\varepsilon_t^x)$ , and  $\Omega_{80Q1}^\pi$  the z-statistics are for the null of each individual coefficient equal to zero. For  $\phi^\pi$ ,  $\phi^z$  and  $\Omega_{80Q1}^z$ , the z-statistics are for the null of each individual coefficient equal to one.

Table 4.14: Testing the equality of the convergence processes of inflation and output gap (part II). Estimation with the Kalman Filter, 1980Q1-2008Q4

	coeff.	s.e.	z stat.
<b>Greece</b>			
$Var(\varepsilon_t^\pi)$	0.00012	0.000053	2.262
$Var(\varepsilon_t^x)$	2.54E-10	0.0000018	0.000
$\phi^\pi$	0.9447	0.0112	-4.893
$\Omega_{80Q1}^\pi$	0.0117	0.0055	2.129
$\phi^z$	1.0251	0.0139	1.810
$\Omega_{80Q1}^z$	0.1946	0.1108	-7.268
$\phi^x = \phi^\pi \cdot \phi^z$	0.9685	$1 - \phi^x$	-0.0314
$1 - \phi^\pi$	-0.0552	$(1 - \phi^\pi) - (1 - \phi^x)$	-0.0237
Log likelih.	515.3204		
<b>Italy</b>			
$Var(\varepsilon_t^\pi)$	0.000149	0.0000248	6.008
$Var(\varepsilon_t^x)$	5.79E-14	0.00000407	0.000
$\phi^\pi$	0.9420	0.0101	-5.736
$\Omega_{80Q1}^\pi$	0.0011	0.0005	2.247
$\phi^z$	1.0357	0.0115	3.095
$\Omega_{80Q1}^z$	0.1934	0.0974	-8.280
$\phi^x = \phi^\pi \cdot \phi^z$	0.9557	$1 - \phi^x$	-0.0242
$1 - \phi^\pi$	-0.0337	$(1 - \phi^\pi) - (1 - \phi^x)$	-0.0395
Log likelih.	683.971		
<b>Luxembourg</b>			
$Var(\varepsilon_t^\pi)$	0.00020	0.000031	6.582
$Var(\varepsilon_t^x)$	0.000059	0.000039	1.536
$\phi^\pi$	0.9446	0.0190	-2.902
$\Omega_{80Q1}^\pi$	0.00021	0.00020	1.090
$\phi^z$	1.0475	0.0214	2.220
$\Omega_{80Q1}^z$	3.3394	3.0587	0.765
$\phi^x = \phi^\pi \cdot \phi^z$	0.9895	$1 - \phi^x$	-0.0104
$1 - \phi^\pi$	-0.0553	$(1 - \phi^\pi) - (1 - \phi^x)$	-0.0448
Log likelih.	562.1665		
<b>Portugal</b>			
$Var(\varepsilon_t^\pi)$	0.00018	0.000050	3.669
$Var(\varepsilon_t^x)$	0.000046	0.000011	4.071
$\phi^\pi$	0.9335	0.0098	-6.729
$\Omega_{80Q1}^\pi$	0.0152	0.0051	2.973
$\phi^z$	1.0423	0.0136	3.090
$\Omega_{80Q1}^z$	0.0169	0.0092	-105.716
$\phi^x = \phi^\pi \cdot \phi^z$	0.9730	$1 - \phi^x$	-0.0269
$1 - \phi^\pi$	-0.0664	$(1 - \phi^\pi) - (1 - \phi^x)$	-0.0395
Log likelih.	563.2096		

Note: For  $var(\varepsilon_t^\pi)$ ,  $var(\varepsilon_t^x)$ , and  $\Omega_{80Q1}^\pi$  the z-statistics are for the null of each individual coefficient equal to zero. For  $\phi^\pi$ ,  $\phi^z$  and  $\Omega_{80Q1}^z$ , the z-statistics are for the null of each individual coefficient equal to one.

Table 4.15: Testing the equality of the convergence processes of inflation and output gap (part III). Estimation with the Kalman Filter, 1980Q1-2008Q4

	coeff.	s.e.	z stat.
<b>Spain</b>			
$Var(\varepsilon_t^\pi)$	0.00017	0.000026	6.455
$Var(\varepsilon_t^x)$	0.000016	0.0000054	3.013
$\phi^\pi$	0.9535	0.0196	-2.364
$\Omega_{80Q1}^\pi$	0.00016	0.00013	1.151
$\phi^z$	1.0263	0.0221	1.192
$\Omega_{80Q1}^z$	0.6572	0.6282	-0.546
$\phi^x = \phi^\pi \cdot \phi^z$	0.9786	$1 - \phi^x$	-0.0213
$1 - \phi^\pi$	-0.0464	$(1 - \phi^\pi) - (1 - \phi^x)$	-0.0251
Log likelih.	694.0846		

Note: For  $var(\varepsilon_t^\pi)$ ,  $var(\varepsilon_t^x)$ , and  $\Omega_{80Q1}^\pi$  the z-statistics are for the null of each individual coefficient equal to zero. For  $\phi^\pi$ ,  $\phi^z$  and  $\Omega_{80Q1}^z$ , the z-statistics are for the null of each individual coefficient equal to one.

Table 4.16: Causality between convergences of inflation and output gap (part I). Estimation with the Kalman Filter, 1980Q1-2008Q4

	coeff.	s.e.	z stat.		coeff.	s.e.	z stat.
<b>Austria:</b>							
$Var(\varepsilon_t^\pi)$	0.000073	0.000048	1.5020	$\gamma_{ii}$	0.3368	0.1736	1.9406
$Var(\varepsilon_t^x)$	2.05E-14	4.52E-06	4.53E-09	$\gamma_{gi}$	0.1718	0.1607	1.0687
$\phi^\pi$	0.9722	0.0118	-2.3559	$\gamma_{gg}$	0.6320	0.0993	6.3640
$\Omega_{y0Q1}^\pi$	0.0005	0.0002	2.5000	$\gamma_{ig}$	-0.0278	0.0575	-0.4838
$\phi^x$	0.9884	0.0036	-3.2222	Log-lik	724.6549		
$\Omega_{y0Q1}^x$	0.000085	0.000026	3.2709				
<b>Belgium</b>							
$Var(\varepsilon_t^\pi)$	0.00016	0.000029	5.4982	$\gamma_{ii}$	0.8955	0.0588	15.2293
$Var(\varepsilon_t^x)$	4.85E-12	0.000012	4.00E-07	$\gamma_{gi}$	0.1404	0.1045	1.3434
$\phi^\pi$	0.9555	0.0188	-2.3670	$\gamma_{gg}$	0.6169	0.1257	4.9064
$\Omega_{y0Q1}^\pi$	0.00017	0.00013	1.3484	$\gamma_{ig}$	-0.0106	0.0603	-0.1757
$\phi^x$	0.9826	0.0072	-2.3777	Log-lik	700.4668		
$\Omega_{y0Q1}^x$	0.00014	0.000033	4.2598				
<b>Finland</b>							
$Var(\varepsilon_t^\pi)$	0.00018	0.000032	5.875	$\gamma_{ii}$	0.8320	0.0811	10.2593
$Var(\varepsilon_t^x)$	0.0000069	0.0000073	0.9471	$\gamma_{gi}$	0.0923	0.0558	1.6529
$\phi^\pi$	0.9412	0.0146	-4.0081	$\gamma_{gg}$	0.8657	0.0598	14.4652
$\Omega_{y0Q1}^\pi$	0.0010	0.00053	1.9212	$\gamma_{ig}$	0.0086	0.0571	0.1511
$\phi^x$	0.9689	0.0071	-4.3514	Log-lik	649.4147		
$\Omega_{y0Q1}^x$	0.00043	0.00014	3.0275				
<b>France</b>							
$Var(\varepsilon_t^\pi)$	0.000065	0.000014	4.5277	$\gamma_{ii}$	0.8335	0.0681	12.2338
$Var(\varepsilon_t^x)$	1.1E-16	0.0000068	1.61E-11	$\gamma_{gi}$	0.0067	0.0740	0.0908
$\phi^\pi$	0.9422	0.0117	-4.9117	$\gamma_{gg}$	0.7054	0.0906	7.7836
$\Omega_{y0Q1}^\pi$	0.0011	0.00047	2.3550	$\gamma_{ig}$	0.0098	0.0313	0.3144
$\phi^x$	0.9845	0.0054	-2.8472	Log-lik	733.2329		
$\Omega_{y0Q1}^x$	0.00010	0.000027	3.9636				
<b>Greece</b>							
$Var(\varepsilon_t^\pi)$	0.00010	0.000050	2.0315	$\gamma_{ii}$	0.9155	0.0296	30.8472
$Var(\varepsilon_t^x)$	8.92E-14	0.000066	1.33E-09	$\gamma_{gi}$	0.0576	0.1005	0.5740
$\phi^\pi$	0.9473	0.0110	-4.7778	$\gamma_{gg}$	0.4217	0.1361	3.0978
$\Omega_{y0Q1}^\pi$	0.0107	0.0051	2.0668	$\gamma_{ig}$	-0.0090	0.0172	-0.5230
$\phi^x$	0.9764	0.0102	-2.2887	Log-lik	537.5260		
$\Omega_{y0Q1}^x$	0.0010	0.00035	2.9159				

Note: The z-statistics are for the null of each individual coefficient equal to zero, except for  $\phi^\pi$  and  $\phi^x$ , where the z-statistics are for the null of each individual coefficient equal to one.

Table 4.17: Causality between convergences of inflation and output gap (part II). Estimation with the Kalman Filter, 1980Q1-2008Q4

	coeff.	s.e.	z stat.		coeff.	s.e.	z stat.
<b>Ireland</b>							
$Var(\varepsilon_t^\pi)$	0.00013	0.000052	2.5287	$\gamma_{ii}$	0.8737	0.0485	18.0010
$Var(\varepsilon_t^x)$	0.0000074	0.00002	0.374	$\gamma_{gi}$	0.0721	0.0810	0.8910
$\phi^\pi$	0.9413	0.0086	-6.7826	$\gamma_{gg}$	0.6337	0.1041	6.0874
$\Omega_{80Q1}^\pi$	0.0057	0.0018	3.0712	$\gamma_{ig}$	0.0273	0.0212	1.2837
$\phi^x$	1.0175	0.0047	3.6771	Log-lik	599.7242		
$\Omega_{80Q1}^x$	0.000048	0.000023	2.0546				
<b>Italy</b>							
$Var(\varepsilon_t^\pi)$	0.00013	0.000026	4.9808	$\gamma_{ii}$	0.9048	0.0369	24.5118
$Var(\varepsilon_t^x)$	3.87E-15	1.19E-06	3.25E-09	$\gamma_{gi}$	0.0510	0.1186	0.4306
$\phi^\pi$	0.9522	0.0105	-4.5370	$\gamma_{gg}$	0.6657	0.0688	9.6746
$\Omega_{80Q1}^\pi$	0.0010	0.00048	2.0912	$\gamma_{ig}$	0.0052	0.0222	0.2366
$\phi^x$	0.9770	0.0040	-5.6067	Log-lik	698.9543		
$\Omega_{80Q1}^x$	0.00017	0.000047	3.6610				
<b>Luxembourg</b>							
$Var(\varepsilon_t^\pi)$	0.00019	0.000035	5.5714	$\gamma_{ii}$	0.9411	0.0462	20.3288
$Var(\varepsilon_t^x)$	2.14E-14	6.84E-06	3.12E-09	$\gamma_{gi}$	-0.0063	0.0285	-0.2228
$\phi^\pi$	0.9641	0.0161	-2.2229	$\gamma_{gg}$	0.6935	0.0729	9.5083
$\Omega_{80Q1}^\pi$	0.00017	0.00014	1.1677	$\gamma_{ig}$	0.1458	0.1497	0.9741
$\phi^x$	0.9921	0.00403	-1.9501	Log-lik	572.4116		
$\Omega_{80Q1}^x$	0.00069	0.00013	5.2803				
<b>Netherlands</b>							
$Var(\varepsilon_t^\pi)$	0.00018	0.000032	5.6656	$\gamma_{ii}$	0.8887	0.0779	11.4064
$Var(\varepsilon_t^x)$	5.94E-10	5.92E-06	0.00010	$\gamma_{gi}$	0.2738	0.1426	1.9202
$\phi^\pi$	0.9854	0.0213	-0.6833	$\gamma_{gg}$	0.5517	0.1349	4.0890
$\Omega_{80Q1}^\pi$	0.000052	0.000060	0.8687	$\gamma_{ig}$	-0.0135	0.0704	-0.1915
$\phi^x$	0.9769	0.0071	-3.2167	Log-lik	708.6039		
$\Omega_{80Q1}^x$	0.00018	0.000049	3.6734				
<b>Portugal</b>							
$Var(\varepsilon_t^\pi)$	0.00016	0.000049	3.2857	$\gamma_{ii}$	0.8715	0.0446	19.5401
$Var(\varepsilon_t^x)$	9.33E-14	0.000016	5.83E-09	$\gamma_{gi}$	0.2485	0.1258	1.9755
$\phi^\pi$	0.9329	0.0108	-6.1897	$\gamma_{gg}$	0.5860	0.1033	5.6683
$\Omega_{80Q1}^\pi$	0.0154	0.0063	2.4256	$\gamma_{ig}$	-0.0105	0.0142	-0.7400
$\phi^x$	0.9805	0.0052	-3.7058	Log-lik	579.8846		
$\Omega_{80Q1}^x$	0.00035	0.000084	4.2738				

Note: The z-statistics are for the null of each individual coefficient equal to zero, except for  $\phi^\pi$  and  $\phi^x$ , where the z-statistics are for the null of each individual coefficient equal to one.

Table 4.18: Causality between convergences of inflation and output gap (part III). Estimation with the Kalman Filter, 1980Q1-2008Q4

	coeff.	s.e.	z stat.		coeff.	s.e.	z stat.
<b>Spain</b>							
$Var(\varepsilon_t^\pi)$	0.00025	0.000045	5.6000	$\gamma_{ii}$	0.9631	0.0263	36.5528
$Var(\varepsilon_t^x)$	3.71E-07	0.0000105	0.03533	$\gamma_{gi}$	0.0651	0.0934	0.6973
$\phi^\pi$	0.9333	0.0196	-3.3885	$\gamma_{gg}$	0.6284	0.1042	6.0286
$\Omega_{uu}^\pi$	0.0014	0.00080	1.7413	$\gamma_{ig}$	-0.0033	0.0237	-0.1411
$\phi^x$	0.9775	0.0089	-2.5049	Log-lik	662.1002		
$\Omega_{uu}^x$	0.00021	0.000061	3.5457				

Note: The z-statistics are for the null of each individual coefficient equal to zero, except for  $\phi^\pi$  and  $\phi^x$ , where the z-statistics are for the null of each individual coefficient equal to one.

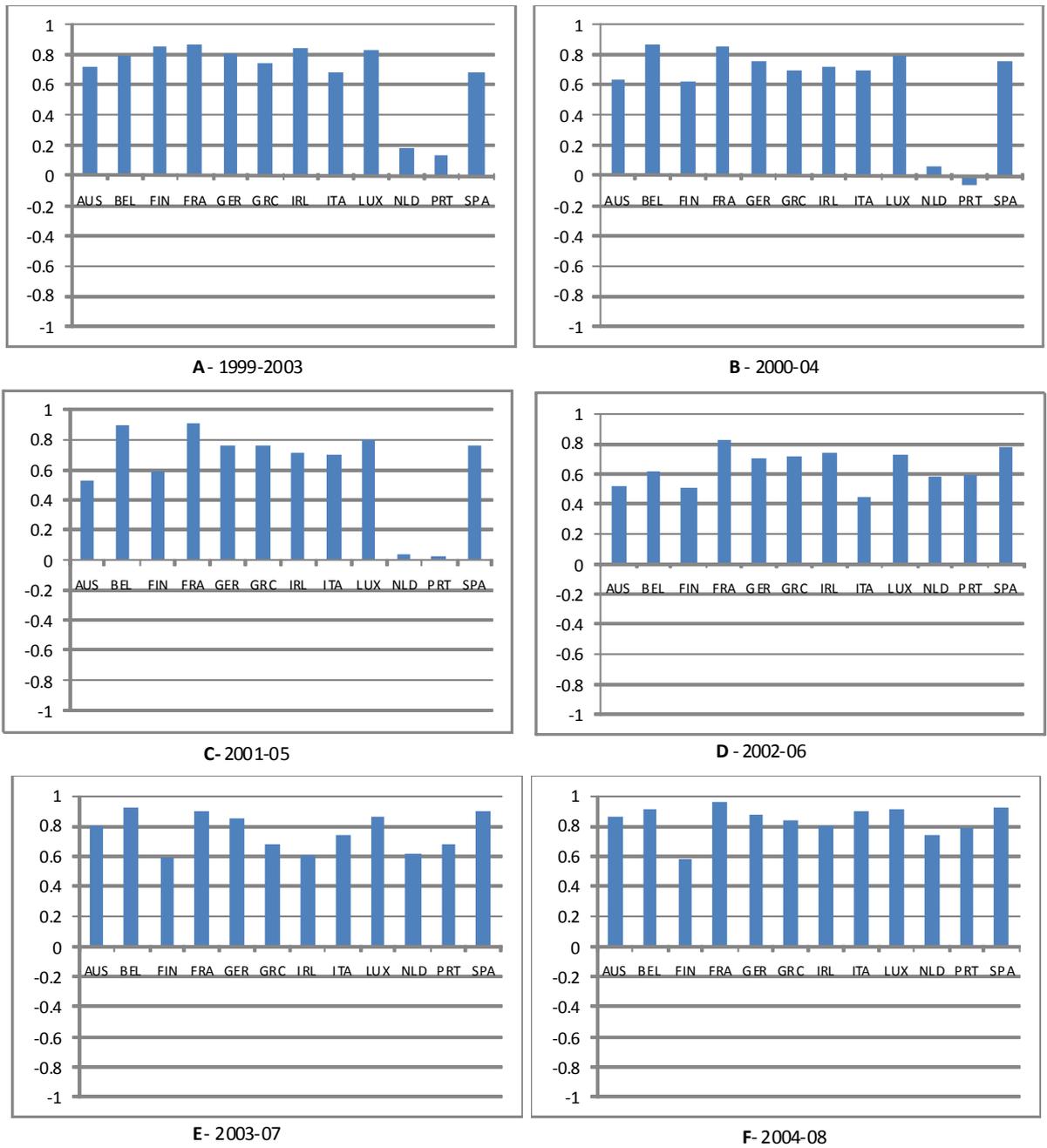


Figure 4-13: Country weights of first principal component for inflation after the introduction of the euro

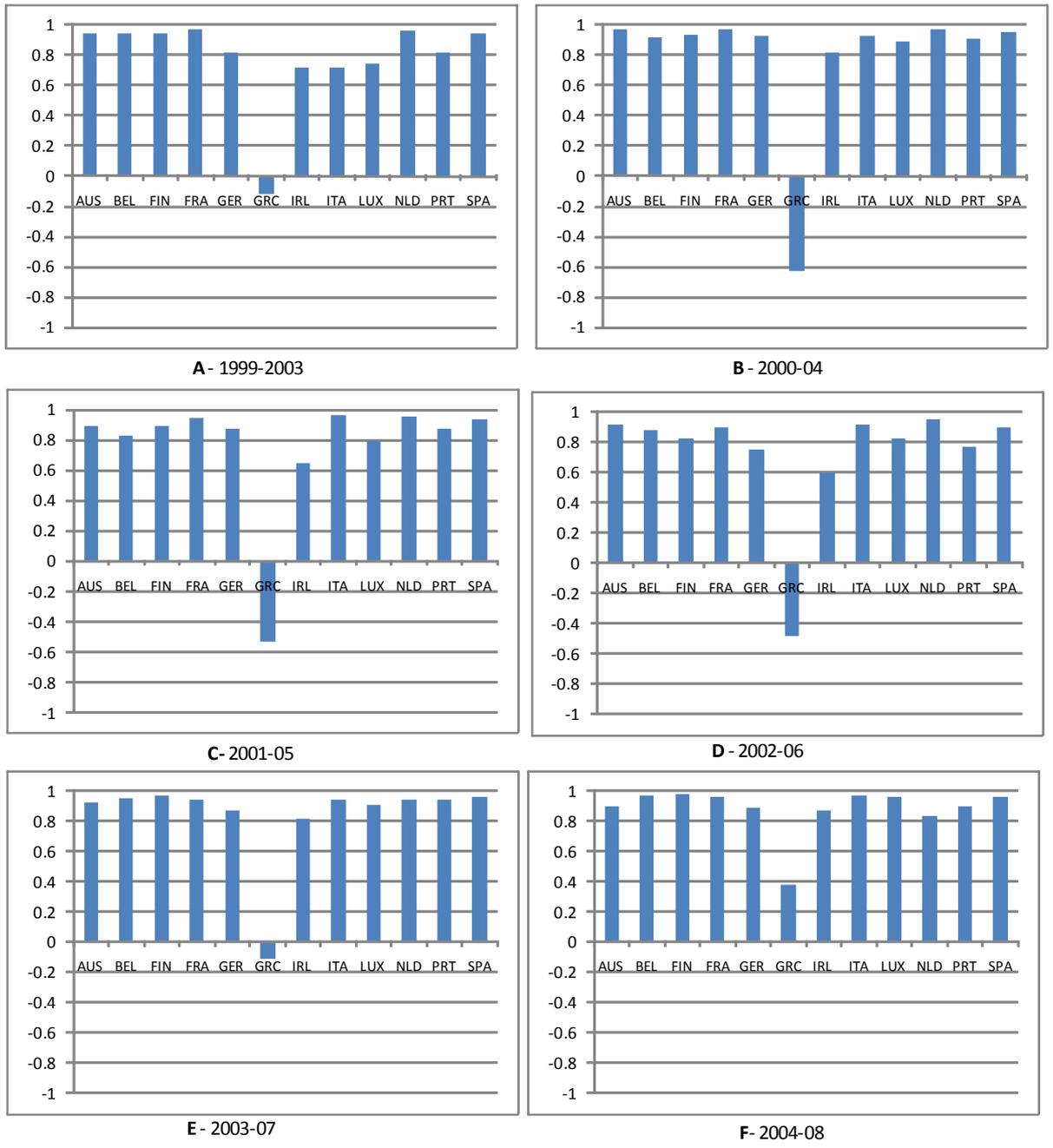


Figure 4-14: Country weights of first principal component for output gap after the introduction of the euro

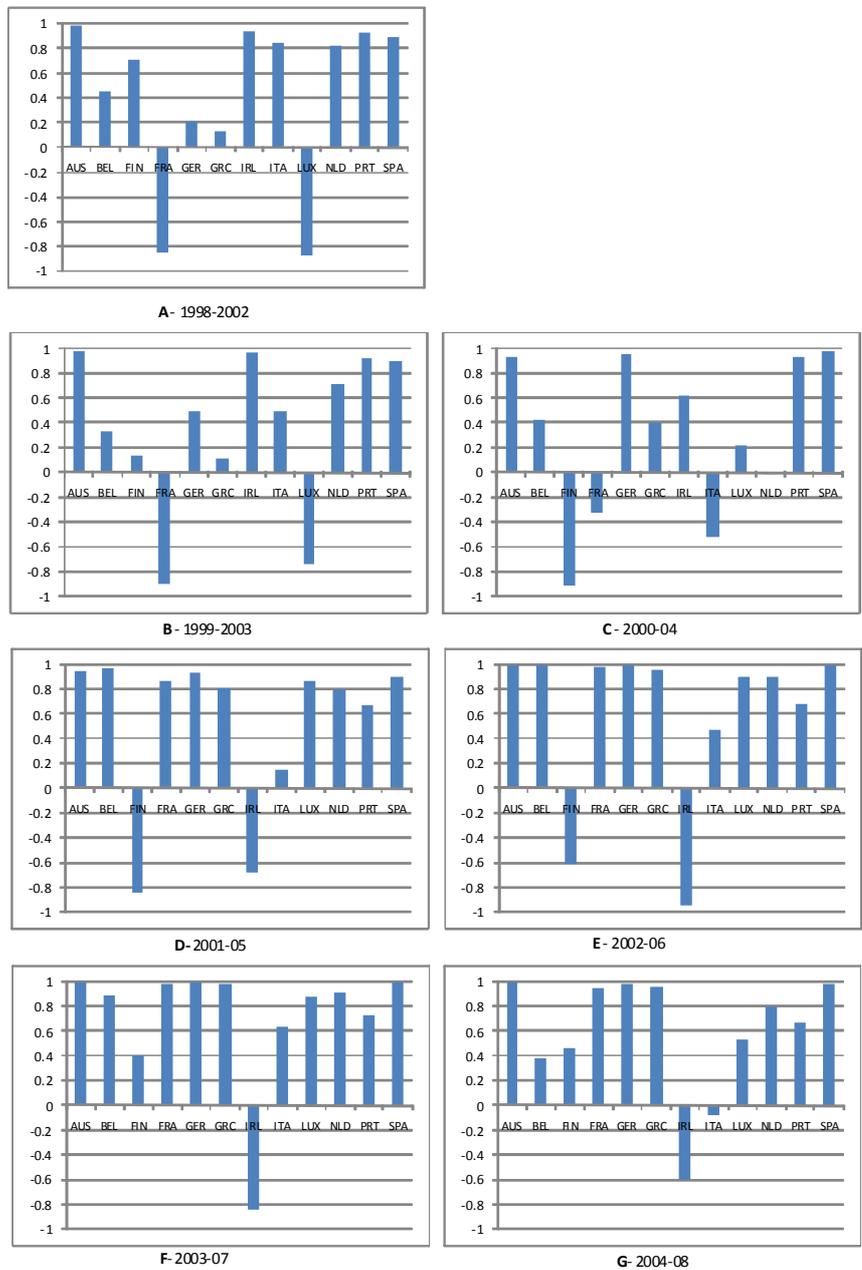


Figure 4-15: Country weights of first principal component for RULC after the introduction of the euro

Table 4.19: Determinants of inflation and inflation differentials with fixed effects

	Eq. (1)	Eq. (2)	Eq. (3)
$c$	0.00097 (0.0027)	0.000039 (0.01324)	0.0020 (0.0022)
$\pi_{i,t+1}$	0.5882** (0.2831)	1.0235*** (0.2286)	0.7520*** (0.1481)
$\pi_{i,t-1}$	0.0991 (0.0691)	-0.0334 (0.0957)	0.0288 (0.0722)
$x_{i,t}$	0.038 (0.037)	-	-
$\Delta ulc_{i,t}$	-	0.0783** (0.0382)	0.0659* (0.0346)
$\Delta e_{i,t+1}$	0.0166 (0.0640)	0.3109* (0.1869)	-
$\Delta e_{i,t}$	0.0319 (0.0960)	-0.5380 (0.3880)	-
$\Delta e_{i,t-1}$	0.0463** (0.0182)	-0.0181 (0.0561)	-
$\Delta pi_{i,t}$	-	-	0.0693 (0.0425)
$i_{i,t}$	0.00018 (0.00028)	0.000049 (0.00015)	-
$i_{i,t} \cdot D_t$	0.000055 (0.00013)	-	-
$\Delta i_{i,t}$	-	-	0.00056 (0.0012)
$pl_{i,t-1}$	-	0.0092* (0.0049)	0.0080 (0.0048)
$ulc_{i,t-1} + i_{i,t-1} - p_{i,t-1}$	-	-	0.0068* (0.0038)
$ulc_{i,t-1} + i_{i,t-1} - pl_{i,t-1}$	-	-	-0.0076* (0.0041)
Cross-section dummies	Yes	Yes	Yes
Time dummies	No	Yes	Yes
F-stat 1 <sup>st</sup> stage reg.:			
$\pi_{i,t+1}$	6.8535	15.3410	14.69
$\Delta e_{i,t+1}$	8.8733	92.54	-
J-statistic	2.3287 [0.5087]	2.8064 [0.4234]	7.2412 [0.2033]
Q (2) stat.	72.050 [0.00]	87.410 [0.00]	108.98 [0.00]

Instruments: Eq. (1): constant,  $\pi_{i,t-1}$ ,  $\pi_{i,t-2}$ ,  $x_{i,t-1}$ ,  $\Delta e_{i,t-1}$ ,  $\Delta pi_{i,t-1}$ ,  $q_{i,t-1}$ ,  $i_{i,t-1}$ ,  $pl_{i,t-1}$ ,  $p_{i,t-2}$ ,  $s_{i,t-1}$ , cross-section dummies and one dummy,  $D_t$ , that takes the value one for the period 1999Q1-2002Q4.

Eq. (2): constant,  $\pi_{i,t-2}$ ,  $x_{i,t-1}$ ,  $\Delta e_{i,t-1}$ ,  $\Delta pi_{i,t-1}$ ,  $q_{i,t-1}$ ,  $i_{i,t-1}$ ,  $pl_{i,t-1}$ ,  $p_{i,t-2}$ ,  $p_{i,t-3}$ ,  $diffp_{i,t-1}$ ,  $\Delta ulc_{i,t-1}$ , time dummies and cross-section dummies. Eq. (3): constant,  $\pi_{i,t-2}$ ,  $x_{i,t-1}$ ,  $q_{i,t-1}$ ,  $i_{i,t-1}$ ,  $\Delta i_{i,t-1}$ ,  $p_{i,t-2}$ ,  $p_{i,t-3}$ ,  $diffp_{i,t-1}$ ,  $\Delta pi_{i,t-1}$ ,  $ulc_{i,t-1} + i_{i,t-1} - p_{i,t-1}$ ,  $ulc_{i,t-1} + i_{i,t-1} - pl_{i,t-1}$ ,  $\Delta ulc_{i,t-1}$ ,  $pl_{i,t-1}$ , time dummies and cross-section dummies.

## Chapter 5

# Conclusion

The study of the supply side effect of interest rate has been done in the literature without explicitly taking into account the role of open economy variables on inflation. Against this background, in Chapter 2 the general equilibrium effects of the cost channel are studied using a small open economy New Keynesian model. The concept of cost channel is broadened, assuming that, besides wages, imports of both consumption goods and inputs have to be paid in advance.

When compared with the standard model, our model is characterised by new effects of the nominal interest rate and the terms of trade on the flexible-price equilibrium output, which occur through the price of imported inputs, CPI inflation and exports.

Likewise, the assumption that imports are paid in advance introduces new effects of the nominal interest rate on the IS curve and on the equation describing CPI inflation. Now, the IS curve depends on the expected change in the nominal interest rate, and CPI inflation equation is affected by the change in the nominal interest rate. The new configuration of the IS curve breaks the equivalence between closed and open economy representations in the domestic inflation and output gap's space, which characterises the standard open economy New Keynesian model.

In the Phillips curve the direct effect of the nominal interest rate on domestic inflation is richer than in the standard curve with a cost channel. This effect now works directly not only through the cost of labour, but also through the price of imported inputs and consumption goods, CPI inflation and the terms of trade.

Like with the standard model, the cost channel produces a trade-off between output gap and domestic inflation. Under an optimal discretion policy it is shown that the central bank allows more inflation variability when the cost channel is present.

The model was calibrated and the economy's response to a technology shock with different policy regimes was analysed. Five policy regimes were considered: domestic inflation-based Taylor rule (DITR), CPI inflation-based Taylor rule (CITR), optimal commitment, optimal discretion and exchange rate peg. Our results show that the cost channel produces a larger change in the nominal interest rate, a more volatile and positive response of the output gap, and larger initial depreciations in the terms of trade and the nominal exchange rate. In addition, there is a smaller domestic inflation rate after the shock and a smaller initial increase in CPI inflation.

It is worth noticing that after a decrease in the interest rate, domestic and CPI inflations are lower with the cost channel, confirming previous results for the closed economy of a mitigated response of inflation in the presence of the working capital channel.

Once all shocks are considered simultaneously (shocks to the domestic technology, preferences and foreign output), it is possible to conclude that for all the policy regimes considered, with the exception of the peg, the cost channel increases the volatility of domestic inflation, output gap, and the nominal interest rate. In contrast, the volatility of CPI inflation decreases in all policy regimes, except in the CITR, where it increases. The effects on the volatility of the terms of trade and nominal exchange rate are more diverse.

Also with the full set of shocks in place, the policy functions without commitment indicate that, when the cost channel is present, the central bank changes the interest rate more aggressively in response to shocks. This is explained by the fact that, in the absence of commitment, the policy maker has to change the interest rate more when the trade-off between the output gap and domestic inflation deteriorates with the introduction of the working capital channel.

While optimal commitment, optimal discretion and DITR maximize welfare without the cost channel, when that channel is present only optimal commitment maximises welfare. If full commitment is not possible, the commitment to a Taylor rule is better than an optimal discretion policy. The advantages arising from commitment also justify interest rate smoothing in the context of the Taylor rule.

In addition, the cost channel can partially justify the relatively small empirical contem-

poraneous correlation between CPI inflation and the nominal exchange rate.

Finally, the increase of imported inputs share in output, with the associated decrease in imported consumption goods share, in general, reduces macroeconomic volatility, especially under the CITR and the exchange rate peg. This occurs in these latter regimes because the effect of the terms of trade in resources allocation and inflation becomes smaller.

We have shown that many of the cost channel's implications in a closed economy are also valid in an open economy. Moreover, that channel has significant implications for the economy dynamics and monetary policy, and also contributes to explaining some relevant empirical evidence. For all of this, the cost channel deserves more research and attention from monetary authorities.

In Chapter 3 we analysed empirically the cost channel in the G7 countries using a NKPC with open economy variables. It is argued that, without such variables, the cost channel may not be correctly identified.

Our results show that open economy variables are statistically significant in explaining domestic inflation and CPI inflation dynamics, and lead to some interesting conclusions. For France, Germany, Japan and the UK, the backward component of CPI inflation is larger in the open economy Phillips curve than in the closed economy curve. The NKPC with slow exchange rate pass-through has shown to be empirically more successful than the one with immediate pass-through. This was valid both for imported inputs and imported consumption goods. The model of McCallum and Nelson (2000), where imports are solely considered as inputs in production, is rejected by the data. Instead, a model with imports as both consumption goods and inputs has a better empirical adherence.

The fact that slow exchange rate pass-through is empirically relevant and that imports should also be considered as inputs has important implications for monetary policy: apart from domestic inflation, the central bank should be concerned also with CPI inflation, especially if inflation has sufficient weight on its goal; commitment in central bank actions becomes more relevant; and the volatility of the nominal exchange rate becomes more harmful to social welfare.

In an open economy Phillips curve, we tested two ways of looking at the cost channel that may exist simultaneously. The first assumes that inputs (wages and imported intermediate goods) are paid in advance. The second version considers that trade companies pay imports

of consumption goods in advance. While in the first version the nominal interest rate affects the marginal cost, in the second it directly affects CPI inflation. In our sample, without considering import prices, there is some evidence in favour of the first concept of the cost channel in both domestic and CPI inflations. That evidence becomes weaker when import prices are added to the Phillips curve. However, there is strong evidence that the cost channel is present in imported consumption goods.

In Chapter 3 we focused our attention in the inflation processes of the G7 countries, including three euro area countries, France, Germany and Italy. When we consider the euro area as a whole, it is possible to observe periods of convergence and divergence in inflation. Taking into consideration that large diverging inflation differentials can undermine a monetary union, in Chapter 4 we study inflation convergence process and its determinants. Our major goals were to measure convergence of inflation rates and business cycles in the euro area and study the relationship between these convergence processes.

We started studying the convergence of inflation, output gap, and RULC towards Germany, using the Kalman filter to estimate an unobserved convergence component. From 1980 to 2008, inflation' differentials in the euro area have converged in expectation, despite the emergence of temporary divergence in some countries after the introduction of the euro.

The business cycles of euro area countries have also become more aligned between 1980 and 2008, and that came out clearer when using the output gap rather than the RULC.

For the countries where convergence of output gap and inflation was identified, convergence of inflation occurred at a faster rate than the convergence of output gap. Looking at the causality between the two phenomena, while output gap convergence had a positive effect on inflation convergence, even though statistically weak for the majority of countries, it seems that the causality in the opposite direction does not exist. As a result, the destabilising impact of inflation divergence is more limited.

In order to better identify sub-periods of convergence and divergence and to obtain an aggregate measure of convergence, we use the common factor approach developed by Becker and Hall (2009a). This methodology allows us to compare convergence between periods of time, also called windows. Compared with the period 1990-98, the co-movement in European inflation rates on average did not increased during the euro period. However, when looking at *sub-periods* after the introduction of the euro, we observed that the link between European

inflation rates has increased strongly in the last years studied.

Regarding business cycle convergence, results depend on the indicator used. With respect to the window of 1990-98, the co-movement between output gaps has increased for the *overall* euro period, while the convergence of RULCs has decreased. However, when looking at *sub-periods* during the euro period, the co-movement between RULCs has increased slightly from the beginning to the end of the period.

A common characteristic of the convergence processes of the three variables studied is that during the euro period, there were windows of convergence and divergence. In general, for the three variables, comparing the initial and the final five-year windows, there was an increase in co-movement, which was smaller for the RULC.

Our next step was to use panel data regressions to assess the determinants of inflation differentials in more detail. We started by estimating the NKPC for a panel of 12 euro area countries. The results were similar to those of other studies with time-series or panel data. Inflation has both forward- and backward-looking components, with the former being more important. Exchange rates also play a role in price changes, with the lagged exchange rate having a statistically significant impact. While the cost channel is present, the output gap or the RULC have a positive effect on inflation that however is not statistically significant.

Regarding inflation differentials, we observe that the expected inflation rate and exchange rate movements are important determinants of differences in inflation rates. Nevertheless, the past dynamics of inflation and exchange rate do not play very relevant roles, despite their significance in explaining national inflation rates. Finally, the usual measures of the business cycle, output gap and the RULC, are not significant in causing differences in inflation dynamics.

Observe that the expected inflation rate plays a fundamental role in the results. When it was introduced, the lagged price level and the output gap lost their statistical significance. It seems then that these variables were significant because they forecasted inflation.

In addition and even with expected inflation as a regressor, the growth of nominal ULC plays a significant role in explaining inflation differentials. Then, the business cycle affects inflation differentials when causes differences in wages evolution across countries. Inflation differences also respond to the lagged disequilibrium in the long-run relationship proposed by the Imperfect Competition Model (ICM), which involves domestic prices on the one hand,

and the ULC, nominal interest rate and imported goods prices on the other hand. We can also argue that the nominal interest rate affects inflation differentials due to its effect on the equilibrium marginal cost. Additionally, the introduction of the error correction terms proposed by the ICM reduced the coefficient of expected inflation but did not eliminate its statistical significance.

There are some interesting future extensions to our work. In Chapter 2 it would be relevant to analyse how not only the interest rate but also the amount of working capital available to firms affects the Phillips curve. In Chapter 3, an important extension would be to test empirically the implications that the cost channel associated with imported consumption goods has on the IS curve. Finally, in Chapter 4 it would be interesting to extend our analysis to Eastern European countries, where the process of inflation convergence is at an earlier stage.

# Chapter 6

## Appendices

### 6.1 Chapter 2

#### 6.1.1 Households' FOC

The household problem is

$$V(H_t, D_t) = \text{Max} \left\{ \frac{\xi_t C_t^{1-\sigma}}{1-\sigma} - \chi \frac{N_t^{1+\eta}}{1+\eta} + \beta E_t V(H_{t+1}, D_{t+1}) \right\}$$

s.t.

$$H_t + W_t N_t + D_t - E_t(\Omega_{t,t+1} D_{t+1}) - P_t^c C_t - DP_t = 0$$

, where  $H_{t+1} = W_t N_t + D_t - E_t(\Omega_{t,t+1} D_{t+1}) - P_t^c C_t + (I_t - 1)DP_t = 0$ .

The FOCs are

$$\xi_t C_t^{-\sigma} + \beta E_t V_H(H_{t+1}, D_{t+1})(-P_t^c) + \mu_t(-P_t^c) = 0 \quad (6.1)$$

$$-\chi N_t^\eta + \beta E_t V_H(H_{t+1}, D_{t+1})(W_t) + \mu_t W_t = 0 \quad (6.2)$$

$$\beta E_t V_D(H_{t+1}, D_{t+1}) + \beta E_t V_H(H_{t+1}, D_{t+1})(-E_t \Omega_{t,t+1}) + \mu_t(-E_t \Omega_{t,t+1}) = 0 \quad (6.3)$$

$$\beta E_t V_H(H_{t+1}, D_{t+1})(I_t - 1) - \mu_t(-1) = 0 \quad (6.4)$$

where  $V_X = \partial V / \partial X$  and  $X$  is any variable. Using the envelope theorem, we have

$$\begin{aligned} V_H(H_t, D_t) &= \beta E_t V_H(H_{t+1}, D_{t+1}) + \mu_t \\ V_D(H_t, D_t) &= \beta E_t V_H(H_{t+1}, D_{t+1}) + \mu_t. \end{aligned} \quad (6.5)$$

From (6.5) we obtain

$$\mu_t = V_D(H_t, D_t) - \beta E_t V_H(H_{t+1}, D_{t+1}). \quad (6.6)$$

Replacing the last expression in (6.3), yields

$$\frac{E_t V_D(H_{t+1}, D_{t+1})}{V_D(H_t, D_t)} = \frac{E_t \Omega_{t,t+1}}{\beta}. \quad (6.7)$$

From (6.3), we also have

$$\mu_t = \frac{\beta E_t V_D(H_{t+1}, D_{t+1})}{E_t \Omega_{t,t+1}} - \beta E_t V_H(H_{t+1}, D_{t+1}). \quad (6.8)$$

Using the last equation and (6.7) in (6.1), we obtain (2.8).

Using (6.6) on (6.1), yields

$$V_D(H_t, D_t) = \frac{\xi_t C_t^{-\sigma}}{P_t^c}. \quad (6.9)$$

Finally, using (6.2), (6.6) and (6.9), we get (2.9).

### 6.1.2 The real exchange rate and the terms of trade

The foreign economy is almost closed, *i.e.*, the share of imported consumption goods in the CPI is irrelevant. Consequently, it is not necessary to distinguish between domestic price index and CPI:  $P_t^{c*} = P_t^{h*}$ , where  $P_t^{h*}$  is the foreign country's domestic price index denominated in foreign currency. Thus,  $Q_t = E_t P_t^{h*} / P_t^c$ . Since all goods produced in the foreign country are exported to the home country, we have  $P_t^{h*} = P_t^{f*}$ , where  $P_t^{f*}$  is the price index of foreign goods imported by the home country (denominated in foreign currency). Altogether, leads to

$$\widehat{q}_t = \widehat{e}_t + \widehat{p}_t^{f*} - \widehat{p}_t^c. \quad (6.10)$$

Now, substituting the price of imports in domestic currency (2.13) in (2.14), we get

$$\widehat{e}_t + \widehat{p}_t^{f*} = \widehat{\delta}_t + \widehat{p}_t^h - \widehat{i}_t.$$

Plugging the last expression on the real exchange rate, we obtain

$$\widehat{q}_t = \widehat{\delta}_t + \widehat{p}_t^h - \widehat{i}_t - \widehat{p}_t^c.$$

Finally, plugging the expression for  $\widehat{p}_t^c$  from equation (2.18) into the last equation, one obtain (2.25).

### 6.1.3 Production function, prices and consumption in deviations from the steady-state

For sake of simplicity, we can write the production function in equation (2.26) as:

$$Y_t^v = \alpha_N (Z_t N_t)^v + \alpha_M (M_t)^v$$

where  $v = 1 - \frac{1}{\varepsilon}$ . In deviations from the steady-state, one have:

$$v\widehat{y}_t = -1 + \alpha_N \left[ \left( \frac{ZN}{Y} \right)^{ss} \right]^v + \alpha_M \left[ \left( \frac{M}{Y} \right)^{ss} \right]^v + \alpha_N \left[ \left( \frac{ZN}{Y} \right)^{ss} \right]^v v (\widehat{z}_t + \widehat{n}_t) + \alpha_M \left[ \left( \frac{M}{Y} \right)^{ss} \right]^v v \widehat{m}_t.$$

Since

$$-1 + \alpha_N \left[ \left( \frac{ZN}{Y} \right)^{ss} \right]^v + \alpha_M \left[ \left( \frac{M}{Y} \right)^{ss} \right]^v = 0,$$

the last expression becomes

$$\Leftrightarrow \widehat{y}_t = \alpha_N \left[ \left( \frac{ZN}{Y} \right)^{ss} \right]^v (\widehat{z}_t + \widehat{n}_t) + \alpha_M \left[ \left( \frac{M}{Y} \right)^{ss} \right]^v \widehat{m}_t. \quad (6.11)$$

Defining

$$(1 - \gamma'_i) = \alpha_N \left[ \left( \frac{ZN}{Y} \right)^{ss} \right]^v \quad \text{and} \quad \gamma'_i = \alpha_M \left[ \left( \frac{M}{Y} \right)^{ss} \right]^v,$$

the latter equation simplifies to

$$\widehat{y}_t = (1 - \gamma'_i) (\widehat{z}_t + \widehat{n}_t) + \gamma'_i \widehat{n}_t.$$

The next step is to calibrate  $\gamma'_i$ . Firstly, it is necessary to use equation (2.53) to obtain

$$\alpha_M \left( \frac{M_t}{Y_t} \right)^v = \frac{M_t}{Y_t} \frac{\delta_t}{MC_t}.$$

Taking into account that in the steady-state  $MC_t = 1/\Phi$  and  $\delta = 1$ , we get in the steady-state

$$\gamma'_i = \alpha_M \left[ \left( \frac{M\delta}{Y} \right)^{ss} \right]^v = \left( \frac{M\delta}{Y} \right)^{ss} \Phi = \frac{\gamma_i}{1 + \gamma_i} \Phi.$$

In the discussion below, we are going to express equation (2.1) in percentage deviation from the steady-state. Applying here the same manipulations done for the CES production function, we get:

$$\widehat{c}_t = (1 - \gamma)^{\frac{1}{a}} \left[ \left( \frac{C_t^h}{C_t} \right)^{ss} \right]^{\frac{a-1}{a}} \widehat{c}_t^h + \gamma^{\frac{1}{a}} \left[ \left( \frac{C_t^f}{C_t} \right)^{ss} \right]^{\frac{a-1}{a}} \widehat{c}_t^f. \quad (6.12)$$

Since in the steady-state  $P_t^h = P_t^f = P_t^c$ , from equations (2.4) and (2.5) we obtain

$$\left( \frac{C_t^h}{C_t} \right)^{ss} = (1 - \gamma) \quad \text{and} \quad \left( \frac{C_t^f}{C_t} \right)^{ss} = \gamma.$$

This implies

$$\widehat{c}_t = (1 - \gamma) \widehat{c}_t^h + \gamma \widehat{c}_t^f.$$

Next, we express equation (2.6) in percentage deviations from the steady-state. Applying the same calculations that were made for the production function, we obtain for the CPI:

$$\widehat{p}_t^c = (1 - \gamma) \left[ \left( \frac{P_t^h}{P_t^c} \right)^{ss} \right]^{1-a} \widehat{p}_t^h + \gamma \left[ \left( \frac{P_t^f}{P_t^c} \right)^{ss} \right]^{1-a} \widehat{p}_t^f.$$

Since in the steady-state  $P_t^h = P_t^f = P_t^c$ , we get:

$$\widehat{p}_t^c = (1 - \gamma)\widehat{p}_t^h + \gamma\widehat{p}_t^f.$$

#### 6.1.4 Elasticity of output with respect to labour

The elasticity of output with respect to labour is given by

$$\gamma_t = \frac{\frac{\partial Y}{\partial N}}{\frac{Y}{N}} = \frac{\alpha_N Z_t (Z_t N_t)^{-\frac{1}{\varepsilon}} \left[ \alpha_N (Z_t N_t)^{1-\frac{1}{\varepsilon}} + \alpha_M (M_t)^{1-\frac{1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}-1}}{\frac{Y_t}{N_t}}.$$

The last term in the numerator is

$$\left[ \alpha_N (Z_t N_t)^{1-\frac{1}{\varepsilon}} + \alpha_M (M_t)^{1-\frac{1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}-1} = Y_t^{\frac{1}{\varepsilon}}.$$

Therefore, we can write

$$\gamma_t = \alpha_N \left( \frac{Z_t N_t}{Y_t} \right)^{1-\frac{1}{\varepsilon}}. \quad (6.13)$$

#### 6.1.5 Real uncovered interest parity

From (2.15), we have  $E_t \widehat{\delta}_{t+1} = E_t \widehat{e}_{t+1} + E_t \widehat{i}_{t+1} + E_t \widehat{p}_{t+1}^* - E_t \widehat{p}_{t+1}^h$ . Now, add and subtract some variables to the nominal UIP as follows

$$\begin{aligned} \widehat{i}_t &= \widehat{i}_t^f + (E_t \widehat{e}_{t+1} - \widehat{e}_t) + (E_t \widehat{p}_{t+1}^* - E_t \widehat{p}_{t+1}^*) + / - \left( E_t \widehat{i}_{t+1} - \widehat{i}_t \right) \\ &\quad + \left( E_t \widehat{p}_{t+1}^h - E_t \widehat{p}_{t+1}^h \right) + (\widehat{p}_t^* - \widehat{p}_t^*) + \left( \widehat{p}_t^h - \widehat{p}_t^h \right) \\ \widehat{p}_t^h &= \widehat{p}_t^f + \left( E_t \widehat{\delta}_{t+1} - \widehat{\delta}_t \right). \end{aligned}$$

#### 6.1.6 Imports demand

From the production function, we got the conditional demand for imports, equation (2.52).

Applying the same reasoning, we obtain a similar expression for the labour demand. With

the two demand functions, we can write the total cost function as <sup>1</sup>

$$TC_t = \frac{I_t W_t}{P_t^h} N_t + \delta_t M_t = \left[ \left( \frac{(W_t I_t) / P_t^h}{\alpha_N^{\frac{\varepsilon}{\varepsilon-1}} Z_t} \right)^{1-\varepsilon} + \left( \frac{\delta_t}{\alpha_M^{\frac{\varepsilon}{\varepsilon-1}}} \right)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} Y_t$$

This means that the marginal cost is:

$$MC_t = \frac{\partial TC_t}{\partial Y_t} = \left[ \left( \frac{(W_t I_t) / P_t^h}{\alpha_N^{\frac{\varepsilon}{\varepsilon-1}} Z_t} \right)^{1-\varepsilon} + \left( \frac{\delta_t}{\alpha_M^{\frac{\varepsilon}{\varepsilon-1}}} \right)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}.$$

Finally, we can observe that in equation (2.52) the expression in square brackets is equal to

$$\left[ \left( \frac{(W_t I_t) / P_t^h}{\alpha_N^{\frac{\varepsilon}{\varepsilon-1}} Z_t} \right)^{1-\varepsilon} + \left( \frac{\delta_t}{\alpha_M^{\frac{\varepsilon}{\varepsilon-1}}} \right)^{1-\varepsilon} \right]^{\frac{\varepsilon}{1-\varepsilon}} = MC_t^\varepsilon$$

### 6.1.7 The signs of Phillips curve's coefficients

To start with, notice that we can get

$$\frac{\partial \pi_t^h}{\partial x_t} = \left( \frac{(1 - \gamma'_i) \eta}{1 + \gamma'_i \eta \varepsilon} + \frac{\sigma}{1 + w} \right) k > 0, \text{ with } \sigma a > 1.$$

On the other hand, the sign of  $\partial \pi_t^h / \partial \hat{i}_t$  depends on the sign of

$$v - \frac{h(1 - \gamma) \gamma_c (\sigma a - 1)}{w + 1} = v - \frac{h}{1 + \frac{(1 - \gamma) + \gamma \sigma a}{(1 - \gamma) \gamma_c (\sigma a - 1)}}.$$

The last expression is positive with  $\sigma a > 1$ , because

$$v - h = \frac{\gamma \sigma a}{1 - \gamma} + 1 > 0.$$

---

<sup>1</sup>Varian (1992).

### 6.1.8 Discretionary policy

The central bank's objective is to minimize the loss function,  $\frac{1}{2} [\pi_t^h{}^2 + \lambda x_t^2]$ , period by period, subject to the constraints

$$x_t = E_t x_{t+1} - \frac{1+w}{\sigma} [\hat{i}_t - E_t \pi_{t+1}^h - \hat{\rho}_t^{ho}] + \frac{(1-\gamma)\gamma_c(\sigma a - 1)}{\sigma} (\hat{i}_t - E_t \hat{i}_{t+1}),$$

$$\pi_t^h = \beta E_t \pi_{t+1}^h + u \hat{i}_t + r x_t,$$

$$\hat{\delta}_t' = \frac{\sigma}{w+1} (x_t + \hat{y}_t^o - \hat{y}_t^f) - \frac{(1-\gamma)\gamma_c(\sigma a - 1)}{w+1} \hat{i}_t - \frac{1-\gamma}{w+1} \hat{\xi}_t,$$

, with  $\hat{\rho}_t^{ho}$ ,  $\hat{y}_t^o$  and  $\hat{\delta}_t^{jo}$  exogenous.

The FOCs are

$$\begin{aligned} \lambda x_t + \phi_{1,t} + \phi_{2,t} (-r) + \phi_{3,t} \left( -\frac{\sigma}{w+1} \right) &= 0 \\ \pi_t^h + \phi_{2,t} &= 0 \\ \phi_{1,t} \left( \frac{1+w}{\sigma} - \frac{(1-\gamma)\gamma_c(\sigma a - 1)}{\sigma} \right) + \phi_{2,t} (-u) + \phi_{3,t} \frac{(1-\gamma)\gamma_c(\sigma a - 1)}{w+1} &= 0 \\ \phi_{3,t} &= 0 \end{aligned}$$

The last four equations can be easily simplified to obtain the policy rule

$$\pi_t^h = -\frac{\lambda}{r - \frac{\sigma u}{1+\gamma(\sigma a - 1)}} x_t.$$

### 6.1.9 The coefficient of output gap on the Phillips curve: the impact of imported inputs

To start with, notice that using the Phillips curve we have:

$$\frac{\partial \pi_t^h}{\partial x_t} = \left( \frac{(1-\gamma_i')\eta}{1+\gamma_i'\eta\varepsilon} + \frac{\sigma}{1+w} \right) k.$$

Then,

$$\frac{\partial \left( \frac{\partial \pi_t^h}{\partial x_t} \right)}{\partial \gamma_i} = \left( -\frac{\eta\Phi(1+\varepsilon\eta)}{[1+\gamma_i(1+\eta\varepsilon\Phi)]^2} + \frac{\sigma(\sigma a - 1)(1-\gamma)^2}{(w+1)^2} \right) k.$$

Since

$$\frac{\partial w}{\partial \gamma} = (\sigma a - 1) [(1 - \gamma \gamma_c) + (1 - \gamma) (1 + \gamma_i)] > 0,$$

and assuming  $\sigma a > 1$ , we have

$$\frac{\partial \left[ \frac{\sigma(\sigma a - 1)(1 - \gamma)^2}{(w + 1)^2} \right]}{\partial \gamma} < 0.$$

With  $\gamma = 1$ ,

$$\frac{\partial \left( \frac{\partial \pi_t^h}{\partial x_t} \right)}{\partial \gamma_i} < 0.$$

To conclude, it is possible to define  $\gamma > \bar{\gamma}$  such that

$$\frac{\partial \left( \frac{\partial \pi_t^h}{\partial x_t} \right)}{\partial \gamma_i} < 0.$$

### 6.1.10 The coefficient of interest rate on the Phillips curve: the impact of imported inputs

To begin with, define  $\partial \pi_t^h / \partial \hat{i}_t = f$ . Then, we can write

$$\frac{df}{d\gamma_i} = \frac{\partial f}{\partial \gamma_c} \frac{\partial \gamma_c}{\partial \gamma_i} + \frac{\partial f}{\partial \gamma_i}.$$

Firstly,  $\partial \gamma_c / \partial \gamma_i = -(1 - \gamma) < 0$ .

Secondly,

$$f = \left[ 1 + (\sigma a - 1) (1 - \gamma) \left( -\frac{\gamma_c}{w + 1} \right) + \frac{1}{w + 1} \left( \frac{\gamma_i' (1 + \eta \varepsilon)}{1 - \gamma_i'} [1 + (\sigma a - 1) \gamma] \right) \right] k.$$

Since with  $\sigma a > 1$ ,  $\partial w / \partial \gamma_c = (\sigma a - 1) (1 - \gamma) > 0$ , we get

$$\frac{\partial \left( \frac{1}{w + 1} \right)}{\partial \gamma_c} < 0.$$

In addition,

$$\frac{\partial \left( -\frac{\gamma_c}{w + 1} \right)}{\partial \gamma_c} = \frac{-(\sigma a - 1) \gamma - 1}{(w + 1)^2} < 0.$$

It is then possible to conclude that  $\partial f / \partial \gamma_c < 0$ .

Thirdly, with  $\gamma_i = 0$

$$f = 1 - \gamma_c(\sigma a - 1) + \frac{\gamma + w}{1 - \gamma} \frac{(1 - \gamma)\gamma_c(\sigma a - 1)}{w + 1}.$$

After some manipulations, it is possible to conclude that  $f$  with  $\gamma_i = 0$  and  $\gamma_i \neq 0$  are equal.

This means that  $\partial f / \partial \gamma_i = 0$ .

In conclusion, we can say that  $df/d\gamma_i > 0$ .

## 6.2 Chapter 3

### 6.2.1 Annex: data description

**Inflation** ( $\pi_t^c$  and  $\pi_t^h$ ): to measure inflation we used two price indexes,  $P$ : the GDP deflator and the CPI. The quarterly inflation rate was obtained as  $\pi_t = \log(P_t) - \log(P_{t-1})$ . In general, price indexes were obtained from International Financial Statistics (IFS) of IMF. The exceptions are the CPI of Canada and UK that were obtained from OECD. Also, the UK's CPI excludes mortgage interest payments. The GDP deflator was already seasonally adjusted on the source. But the CP Index was seasonally adjusted using the method Census X12, assuming multiplicative seasonality, for all countries, except the USA, where seasonality was not found significant.

**Labour income share or real unit labour cost** ( $s_t$ ) =  $\log$  (compensation of employees / nominal GDP). Compensation of employees includes wages and the social contribution paid by employers. Data from OECD National Accounts.

**Labour income share of the business sector** ( $s_t^b$ ) =  $\log$  (nominal ULC business sector / GDP deflator). The ULC of the business sector is from OECD - Main Economic Indicators. OECD defines business sector as "the institutional sector whose primary role is the production and sale of goods and services. This sector consequently corresponds to the aggregation of the corporate, quasi-corporate and unincorporated enterprises including public enterprises".

**GDP at constant prices**: from IFS/IMF, except for Canada, where data from OECD National Accounts was used. The data was already seasonally adjusted in the source.

**Treasury Bill Rate** ( $i_t$ ): from IFS/IMF.

**Import prices change** ( $\pi_t^m$ ). Import prices were measured with import prices deflator,  $P_{m,t}$ , from OECD National Accounts. In turn, import prices change was obtained as  $\pi_t^m = \log(P_{m,t}) - \log(P_{m,t-1})$ . The data was already seasonally adjusted in the source.

**Commodity Price Index**: from IFS/IMF. This index was constructed as follows. Firstly, we calculated a weighted sum of both the non-fuel primary commodities index (2000=100, US dollars) and the price of spot crude in US dollars (2000=100). The weights used to aggregate these indexes were 52.2% and 47.8% respectively. <sup>2</sup> Secondly, the commodity price index

---

<sup>2</sup>These weights are the ones used in the Index of Primary Commodity prices of IMF. However, that index attributes a weight of 47.8% to the energy index. Here, to simplify we attributed that weight only to crude,

was converted to the respective national currency, using the IFS exchange rate (quarterly average) between each national currency and the US dollar.

**Real commodity prices:** commodity price index / GDP deflator.

**Real effective exchange rate (CPI based) ( $q_t$ ):** from IFS/IMF.

**Wages in manufacturing ( $w_t$ ):** from OECD Main Economic Indicators. The data was already seasonally adjusted in the source.

**Real wage ( $\omega_t$ ):** wages in manufacturing deflated by the CPI.

### 6.2.2 Annex: some proofs

Here we show how to obtain  $\frac{N_t}{M_t} = \left(\frac{\alpha_N P_{m,t}}{\alpha_M W_t}\right)^\varepsilon$ .

The conditional factor demand functions are:

$$N_t = \frac{1}{\alpha_N^{\frac{\varepsilon}{\varepsilon-1}}} \left(\frac{W_t I_t}{\alpha_N^{\frac{\varepsilon}{\varepsilon-1}}}\right)^{-\varepsilon} \left[ \left(\frac{W_t I_t}{\alpha_N^{\frac{\varepsilon}{\varepsilon-1}} Z_t}\right)^{1-\varepsilon} + \left(\frac{P_{m,t} I_t}{\alpha_M^{\frac{\varepsilon}{\varepsilon-1}}}\right)^{1-\varepsilon} \right]^{\frac{\varepsilon}{1-\varepsilon}} Y_t$$

and

$$M_t = \frac{1}{\alpha_M^{\frac{\varepsilon}{\varepsilon-1}}} \left(\frac{P_{m,t} I_t}{\alpha_M^{\frac{\varepsilon}{\varepsilon-1}}}\right)^{-\varepsilon} \left[ \left(\frac{W_t I_t}{\alpha_N^{\frac{\varepsilon}{\varepsilon-1}} Z_t}\right)^{1-\varepsilon} + \left(\frac{P_{m,t} I_t}{\alpha_M^{\frac{\varepsilon}{\varepsilon-1}}}\right)^{1-\varepsilon} \right]^{\frac{\varepsilon}{1-\varepsilon}} Y_t.$$

Dividing the first by the second expression, we obtain

$$\frac{N_t}{M_t} = \left(\frac{\alpha_N P_{m,t}}{\alpha_M W_t}\right)^\varepsilon.$$

In what follows we show that

$$\gamma_t = 1 - \alpha_M \left(\frac{Y_t}{M_t}\right)^{\frac{1}{\varepsilon}-1}. \quad (6.14)$$

From Chapter 2, we know that

$$\gamma_t = \alpha_N (Z_t N_t)^{\frac{\varepsilon-1}{\varepsilon}} Y_t^{\frac{1-\varepsilon}{\varepsilon}}.$$

---

that is the most important sub-category of the energy index.

From the production function we have the equality

$$\alpha_N (Z_t N_t)^{\frac{\varepsilon-1}{\varepsilon}} = Y_t^{\frac{\varepsilon-1}{\varepsilon}} - \alpha_M M_t^{\frac{\varepsilon-1}{\varepsilon}}.$$

Combining the two last equations, we get equation (6.14).

## 6.3 Chapter 4

### 6.3.1 Data description

**Price Level** ( $pl_{i,t}$ ): price level index (PLI) of household final consumption expenditure from Eurostat. The PLI is obtained as the purchasing power parity of consumption over current nominal exchange rate.<sup>3</sup> The 12 initial member countries of euro area are taken as a reference (Euro 12). If for a country the PLI is larger than one means that the overall price level is larger in that country than in the Euro 12. We obtained the quarterly data interpolating the original annual data with a local quadratic polynomial.

**Inflation rate** ( $\pi_{i,t}$ ):<sup>4</sup> when available, we used the quarterly *harmonized* CPI from Eurostat. From 1998 onwards the harmonized CPI was available for all countries, and earlier for some of them. When this indicator was not available, we used the national CPI (from OECD Main Economic Indicators (MEI)). The seasonality was removed from the CPI using Census X12 multiplicative adjustment.

In general, we used the quarterly inflation annualized. First, the quarterly inflation rate is:  $\inf quarterly = \frac{p_t}{p_{t-1}} - 1$ , where  $p_t$  is the CPI. Second, the annualized quarterly inflation rate is:  $(1 + \inf quarterly)^4 - 1$ . But for the Phillips Curve estimation, quarterly inflation rate was obtained as:  $\log(p_t) - \log(p_{t-1})$ .

**Difference of CPI indexes** ( $difp_{i,t}$ ):  $\log(p_{i,t}) - \log(p_{euro12,t})$ . The CPI for each country was obtained as described above. In turn, the harmonized CPI for euro area 12 (from 1996 onwards) was obtained from Eurostat. Both indexes were seasonally adjusted and have value 100 in 2005.

**Output gap** ( $x_{i,t}$ ):<sup>5</sup> Let us start by stating the source of GDP. When possible we used data from OECD Quarterly National Accounts, to be compatible with the ULC, which are also from OECD. The data it was already seasonally adjusted. Due to the lack of data from OECD sources for Ireland, Luxembourg and Portugal, we used data from International Financial Statistics (IFS) of IMF. But even IMF does not has quarterly data for the all sample for Ireland and Luxembourg. In order to be able to apply the convergence analysis for the

---

<sup>3</sup>The concept is the same as the one used by Honohan and Lane (2003).

<sup>4</sup>This data was used for convergence, PCA, and explaining differentials.

<sup>5</sup>This data was used for factor analysis, convergence and explaining differentials.

entire sample 1979Q1-2008Q4, we decided to interpolate the annual data at the *beginning of the sample*, using a local quadratic polynomial (match sum) (see Eviews, 2007, for more details). But at the end of the sample we used non-interpolated data: from 1997Q1 onwards for Ireland and from 1996Q1 onwards for Luxembourg, we used seasonally adjusted (with Census X12 multiplicative adjustment) GDP from IFS. To extend these series backwards, we used the growth rate of the interpolated GDP data.

The *output gap* was obtained using GDP series starting in 1979Q1 or 1980Q1, depending on data availability. And this was also used in explaining inflation differentials for 1999Q1-2008Q4. Output gap for each country was calculated as the difference between the log of output and the log of output's trend. To calculate the output's trend we used the HP filter, with lambda fixed at 1600.

The **gdp for the euro area 12** (fixed composition),  $x_t^{euro}$ , was obtained from ECB and was already seasonally adjusted.

**Nominal unit labour cost** ( $ulc_t$ ) and **real unit labour cost or wage share** ( $s_t$ ): nominal unit labour cost (ULC) was obtained from MEI/OECD (ULQBBU01.IXOBTE). This data covers the entire economy. In order to obtain a larger sample and reduce the considerable volatility of the raw data, we used trend-cycle series, which include all non-seasonal and non-irregular movements in the underlying series. Compared with the data from OECD National Accounts, the MEI/OECD's has two advantages: (1) the sample available is bigger and (2) it is adjusted for self employment. Since for Portugal the ULC of the entire economy were not available, we used the ULC of the business sector. The real ULC, or wage share, was obtained dividing the nominal ULC (2005=100) by the GDP deflator (2005=100). For the GDP deflator of Ireland and Luxembourg, we interpolated the annual data to get quarterly data, as described below. From here, we observe that the real ULC takes in to account wages ( $w_t$ ) and labour productivity ( $pr_t$ ), as it can be written in logs as:  $s_t = ulc_t - pd_t = w_t - pr_t - pd_t$ .

**GDP deflator** ( $pd_{i,t}$ ) was obtained from OECD national accounts, except for Portugal, Ireland and Luxembourg, where data from IFS/IMF was used. The data from IFS/IMF was desazonalised using Census X12 multiplicative adjustment. The IMF's data was used due to the lack of OECD data for part of the sample. And even the IMF did not have quarterly GDP deflator for Luxembourg and Ireland available for the beginning of the sample. Therefore we

decided to interpolate the annual data, using quadratic local polynomials (where the annual GDP deflator matches the GDP deflator for the fourth quarter). The growth rate of the interpolated data was used to extend the actual data backwards (since 1996Q4 for Ireland and 1994Q4 for Luxembourg).

**Nominal effective exchange rate** ( $e_{i,t}$ ): from IFS/IMF, base year 2005 - NEUZF NEER FROM ULC (Units: Index Number). This measure uses weights from the trade of manufactured goods. An increase in NEER is an appreciation of the euro.<sup>6</sup> In the residuals analysis we used the level of the NEER, while in the estimation of the Phillips curve we used the *log* of the NEER's level.

**Real effective exchange rate** ( $q_{i,t}$ ): from IFS of IMF (RECZF REER based on relative Consumer Prices, 2005=100).

**Import prices** ( $pi_{i,t}$ ): imports price deflator from Quarterly National Accounts OECD (Imports of goods and services, seasonally adjusted, 2005=100). For Ireland there is only data from 2000Q1 onwards.

**Euro interest rate** ( $i_t^{euro}$ ): three month interbank rate of the euro area, from IFS of IMF (60B.ZF INTERBANK RATE (3-MONTH MATURITY) (Units: Percent per Annum)).

**Retail interest rate** ( $i_t$ ): loans to corporations up to one year from Eurostat. Before 2003q1 data is not harmonized. To accommodate this fact, we used a dummy for the period 1998Q1-2002Q4. Notice that before 2003Q1 there was no data available for Luxembourg. And before 2003Q1 for Finland we used the interest rate of loans to firms *above* one year.

**Price of oil in euros** ( $po_t$ ): the price of oil in USD was converted to euros. The price in USD was obtained from IFS/IMF: average crude price (Units: US Dollars per Barrel) 00176AAZZF (Source: World). The conversion to euros was done using period average exchange rate (first for the ecu and latter for the euro) from MEI/OECD: ECU-EUR/USD exchange rate monthly average, EMU.CCUSMA02.ST, Units: EUR/USD.

---

<sup>6</sup>We use the same measure as Honohan and Lane (2003).

### 6.3.2 Kalman filter

The model composed of equations (4.1) and (4.2) is in the state-space form, with (4.1) as the measurement or observation equation and (4.2) as the state or transition equation.

This model cannot be estimated *directly* using standard procedures, like OLS, IV or GLS. Instead, it has to be estimated applying the Kalman Filter to the state-space form equations. Firstly, this filter provides “optimal” forecasts of the unobserved component.<sup>7</sup> Then, they are used to generate series of one-step-ahead prediction errors and their variances, which contain unknown parameters to be estimated. Finally, using those series of errors and variances, standard maximum likelihood techniques can be used to estimate the unknown parameters.

The described model decomposes the difference between two series in two components: a permanent component,  $\alpha_t$ , which we interpret as a measure of convergence, and an error  $\varepsilon_t$ , which is a transitory component. What the Kalman filter actually does is to determine which part of the change in the dependent variable,  $X_t - \theta Y_t$ , can be attributed to each one of that components.

Next, we present a general formulation of the Kalman filter based on Hamilton (1994, Chapter 13) and Cuthbertson et al. (1992). To start with,  $\mathbf{y}_t$  is a  $(n \times 1)$  vector of variables observed at time  $t$ . There is also  $\boldsymbol{\xi}_t$  that is an unobservable  $(r \times 1)$  state vector. The state-space representation of the dynamics of  $\mathbf{y}_t$  comprises the state and measurement equations, respectively:

$$\boldsymbol{\xi}_{t+1} = \mathbf{F}\boldsymbol{\xi}_t + \mathbf{v}_{t+1} \quad (6.15)$$

$$\mathbf{y}_t = \mathbf{A}'\mathbf{x}_t + \mathbf{H}'\boldsymbol{\xi}_t + \mathbf{w}_t, \quad (6.16)$$

where  $\mathbf{F}$ ,  $\mathbf{A}'$ , and  $\mathbf{H}'$  are matrices of parameters of dimension  $(r \times r)$ ,  $(n \times k)$ , and  $(n \times r)$ , respectively, and  $\mathbf{x}_t$  is a  $(k \times 1)$  vector of exogenous or predetermined variables. It can be observed that the state variable follows an AR(1) process. More complex dynamic models can also be represented in the state-space form if they are properly re-parameterised. The error

---

<sup>7</sup>They are optimal in the sense that they minimize the Mean Squared Error.

terms  $\mathbf{v}_t$  ( $r \times 1$ ) and  $\mathbf{w}_t$  ( $n \times 1$ ) are white noise vectors:

$$E(\mathbf{v}_t \mathbf{v}'_\tau) = \begin{cases} \mathbf{Q} & \text{for } t = \tau \\ \mathbf{0} & \text{otherwise} \end{cases}$$

$$E(\mathbf{w}_t \mathbf{w}'_\tau) = \begin{cases} \mathbf{R} & \text{for } t = \tau \\ \mathbf{0} & \text{otherwise,} \end{cases}$$

where  $\mathbf{Q}$  and  $\mathbf{R}$  are ( $r \times r$ ) and ( $n \times n$ ) matrices, respectively. This means that both  $\mathbf{v}_t$  and  $\mathbf{w}_t$  are not serially correlated. Additionally  $\mathbf{v}_t$  and  $\mathbf{w}_t$  are uncorrelated for all lags

$$E(\mathbf{v}_t \mathbf{w}'_\tau) = \mathbf{0}, \text{ for all } t \text{ and } \tau.$$

Also,  $\mathbf{v}_t$  is uncorrelated with lagged values of  $\boldsymbol{\xi}$ , and  $\mathbf{w}_t$  is uncorrelated with all lags of  $\boldsymbol{\xi}$ .

The model just described can be extended in two ways. First, it is possible to assume correlation between the errors,  $\mathbf{v}_t$  and  $\mathbf{w}_t$ . Second, the matrices  $\mathbf{F}$ ,  $\mathbf{Q}$ ,  $\mathbf{A}$ ,  $\mathbf{H}$  and  $\mathbf{R}$  can depend on  $\mathbf{x}_t$ , as we will see further on in the discussion.

The sample available is  $\mathbf{y}_1, \mathbf{y}_2, \dots, \mathbf{y}_T, \mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_T$ . The goal is to obtain a series of linear least squares forecasts of the state vector, using the data available at each point  $t$ :

$$\widehat{\boldsymbol{\xi}}_{t+1|t} = \widehat{E}(\boldsymbol{\xi}_{t+1} | \mathbf{y}_t^*),$$

where  $\mathbf{y}_t^* \equiv (\mathbf{y}'_t, \mathbf{y}'_{t-1}, \dots, \mathbf{y}'_1, \mathbf{x}'_t, \mathbf{x}'_{t-1}, \dots, \mathbf{x}'_1)$ . For each of these forecasts there is a Mean Squared Error (MSE) ( $r \times r$ ) matrix of the forecast error:

$$\mathbf{P}_{t+1|t} = E \left[ \left( \boldsymbol{\xi}_{t+1} - \widehat{\boldsymbol{\xi}}_{t+1|t} \right) \left( \boldsymbol{\xi}_{t+1} - \widehat{\boldsymbol{\xi}}_{t+1|t} \right)' \right].$$

In order to forecast the state vector, the following recursive procedure is done. Firstly, some initial values are suggested:  $\widehat{\boldsymbol{\xi}}_{1|0}$  and  $\mathbf{P}_{1|0}$ . These values correspond to forecasts without any information on  $\mathbf{y}_t$  and  $\mathbf{x}_t$ . After, recursively for each  $t = 1, \dots, T$ , we get: (1) the forecast of  $\mathbf{y}_t$ ,  $\widehat{\mathbf{y}}_{t|t-1}$ , and obtain its  $\text{MSE}_{t|t-1}$ , using  $\widehat{\boldsymbol{\xi}}_{t|t-1}$ ,  $\mathbf{P}_{t|t-1}$ ,  $\mathbf{y}_{t-1}$  and  $\mathbf{x}_t$ ; (2) making use of  $\mathbf{y}_t$ , update the forecast of  $\boldsymbol{\xi}_t$ ,  $\widehat{\boldsymbol{\xi}}_{t|t}$  and  $\mathbf{P}_{t|t}$ ; (3) forecast  $\widehat{\boldsymbol{\xi}}_{t+1|t}$  and  $\mathbf{P}_{t+1|t}$ .

## Starting values

The starting value for the state variable is simply the unconditional mean of  $\boldsymbol{\xi}_1$ ,  $\widehat{\boldsymbol{\xi}}_{1|0} = E(\boldsymbol{\xi}_1)$ , with MSE

$$\mathbf{P}_{1|0} = E [(\boldsymbol{\xi}_1 - E(\boldsymbol{\xi}_1))(\boldsymbol{\xi}_1 - E(\boldsymbol{\xi}_1))'].$$

If the eigenvalues of  $\mathbf{F}$  are inside the unit circle, then  $\boldsymbol{\xi}_t$  is covariance-stationary and  $\widehat{\boldsymbol{\xi}}_{1|0} = 0$ , with

$$\text{vec}(\mathbf{P}_{1|0}) = [\mathbf{I}_{r^2} - (\mathbf{F} \otimes \mathbf{F})]^{-1} \text{vec}(\mathbf{Q}).$$

When the state vector is non-stationary or the model is time variant,<sup>8</sup> it is not possible to solve for the initial conditions (Harvey, 1989, p. 121). Notice that this is the situation we have in our model of convergence. And on that case, diffuse or non-informative initial values are used, as suggested by Koopman et al. (1999):  $\widehat{\boldsymbol{\xi}}_{1|0} = 0$  and  $\mathbf{P}_{1|0} = \kappa \mathbf{I}$ , where  $\kappa$  is an arbitrarily large number and  $\mathbf{I}$  is a  $(r \times r)$  matrix. The large value for  $\mathbf{P}_{1|0}$  translates the great uncertainty surrounding the initial value proposed for the state variable.

After defining the starting values, the next step is to obtain  $\widehat{\boldsymbol{\xi}}_{2|1}$  and  $\mathbf{P}_{2|1}$ . Since procedures to estimate the two former variables and the correspondent variables for  $t = 2, 3, \dots, T$  are similar, we are going to describe this step for a generic moment  $t$ .

## Forecasting $\mathbf{y}_t$

From equation (6.16), it is obtained the forecast for  $\mathbf{y}_t$

$$\widehat{\mathbf{y}}_{t|t-1} = \mathbf{A}'\mathbf{x}_t + \mathbf{H}'\widehat{\boldsymbol{\xi}}_{t|t-1}. \quad (6.17)$$

The implied forecast error is

$$\mathbf{y}_t - \widehat{\mathbf{y}}_{t|t-1} = \mathbf{H}'(\boldsymbol{\xi}_t - \widehat{\boldsymbol{\xi}}_{t|t-1}) + \mathbf{w}_t,$$

with  $\text{MSE}_{t|t-1}$

$$E \left[ (\mathbf{y}_t - \widehat{\mathbf{y}}_{t|t-1}) (\mathbf{y}_t - \widehat{\mathbf{y}}_{t|t-1})' \right] = \mathbf{H}'\mathbf{P}_{t|t-1}\mathbf{H} + \mathbf{R}. \quad (6.18)$$

---

<sup>8</sup> $\mathbf{F}$  or the variance-covariance matrices,  $\mathbf{Q}$  and  $\mathbf{R}$ , are time variant.

The MSE depends on two elements: the uncertainty in predicting  $\boldsymbol{\xi}_t$ ,  $\mathbf{P}_{t|t-1}$ , and the intrinsic uncertainty in equation (6.16), arising from the error term  $\mathbf{w}_t$ .

### Updating the forecast of $\boldsymbol{\xi}_t$

When information on  $\mathbf{y}_t$  is considered, the estimate for  $\boldsymbol{\xi}_t$  can be updated. Using the formula for updating a linear projection, it can be shown that

$$\widehat{\boldsymbol{\xi}}_{t|t} = \widehat{\boldsymbol{\xi}}_{t|t-1} + \mathbf{P}_{t|t-1} \mathbf{H} (\mathbf{H}' \mathbf{P}_{t|t-1} \mathbf{H} + \mathbf{R})^{-1} (\mathbf{y}_t - \widehat{\mathbf{y}}_{t|t-1}), \quad (6.19)$$

with its MSE given by

$$\mathbf{P}_{t|t} = \mathbf{P}_{t|t-1} - \mathbf{P}_{t|t-1} \mathbf{H} (\mathbf{H}' \mathbf{P}_{t|t-1} \mathbf{H} + \mathbf{R})^{-1} \mathbf{H}' \mathbf{P}_{t|t-1}. \quad (6.20)$$

These are called the filtered state mean and variance.

### Forecasting $\boldsymbol{\xi}_{t+1}$

Now, with the updated value for  $\boldsymbol{\xi}_t$ , a forecast of the state variable for  $t + 1$  can be produced using (6.15):

$$\widehat{\boldsymbol{\xi}}_{t+1|t} = \mathbf{F} \widehat{\boldsymbol{\xi}}_{t|t}.$$

Replacing (6.19) for  $\widehat{\boldsymbol{\xi}}_{t|t}$  in the last equation, it results

$$\widehat{\boldsymbol{\xi}}_{t+1|t} = \mathbf{F} \widehat{\boldsymbol{\xi}}_{t|t-1} + \mathbf{K}_t (\mathbf{y}_t - \mathbf{A}' \mathbf{x}_t - \mathbf{H}' \widehat{\boldsymbol{\xi}}_{t|t-1}), \quad (6.21)$$

where  $\mathbf{K}_t = \mathbf{F} \mathbf{P}_{t|t-1} \mathbf{H} (\mathbf{H}' \mathbf{P}_{t|t-1} \mathbf{H} + \mathbf{R})^{-1}$  is the Kalman gain. Since an unpredictable change in  $\mathbf{y}_t$  can be due to the permanent or the transitory component, the Kalman gain decides how much of that change is attributed to the permanent component. So, the matrix  $\mathbf{K}_t$  depends on the relative MSE. An increase in the MSE of state variables,  $\mathbf{P}_{t|t-1}$ , in relation to the MSE of the observable variables,  $\mathbf{H}' \mathbf{P}_{t|t-1} \mathbf{H} + \mathbf{R}$ , increases the Kalman gain. On other words, if the MSE of the state variables is *relatively* large, then a large part of the change in  $\mathbf{y}_t$  is explained by changes in the state variables, and  $\mathbf{K}_t$  will be large. In opposition, for a given forecast error of  $\mathbf{y}_t$ , if the MSE of the observable variables is relatively large, it means that

there is a lot of transitory noise in the data, and so the adjustment to the state variable will be small.

The MSE of that forecast is

$$\mathbf{P}_{t+1|t} = \mathbf{F}\mathbf{P}_{t|t}\mathbf{F}' + \mathbf{Q}, \quad (6.22)$$

with  $\mathbf{P}_{t|t}$  given by (6.20).

### Maximum Likelihood estimation of parameters

Up to now, we assumed to know matrices  $\mathbf{F}$ ,  $\mathbf{Q}$ ,  $\mathbf{A}$ ,  $\mathbf{H}$  and  $\mathbf{R}$ . But they may include unknown parameters, which can be estimated using maximum likelihood. In order to do that, it has to be assumed that  $\boldsymbol{\xi}_1$  and  $\{\mathbf{w}_t, \mathbf{v}_t\}_{t=1}^T$  are Gaussian, implying that

$$\mathbf{y}_t | \mathbf{x}_t, \mathbf{y}_{t-1}^* \sim N \left( \mathbf{A}'\mathbf{x}_t + \mathbf{H}'\widehat{\boldsymbol{\xi}}_{t|t-1}, \mathbf{H}'\mathbf{P}_{t|t-1}\mathbf{H} + \mathbf{R} \right),$$

where the mean is from (6.17) and the variance from (6.18). The conditional Gaussian density function of  $\mathbf{y}_t$  comes out as

$$f_{\mathbf{y}_t | \mathbf{x}_t, \mathbf{y}_{t-1}^*} = (2\pi)^{-n/2} |\mathbf{H}'\mathbf{P}_{t|t-1}\mathbf{H} + \mathbf{R}|^{-1/2} \times \exp \left\{ -\frac{1}{2} \left( \mathbf{y}_t - \mathbf{A}'\mathbf{x}_t - \mathbf{H}'\widehat{\boldsymbol{\xi}}_{t|t-1} \right)' \left( \mathbf{H}'\mathbf{P}_{t|t-1}\mathbf{H} + \mathbf{R} \right)^{-1} \left( \mathbf{y}_t - \mathbf{A}'\mathbf{x}_t - \mathbf{H}'\widehat{\boldsymbol{\xi}}_{t|t-1} \right) \right\}$$

and the sample log likelihood is

$$\sum_{t=1}^T \log f_{\mathbf{y}_t | \mathbf{x}_t, \mathbf{y}_{t-1}^*}$$

This can be maximized numerically with respect to the unknown parameters in the matrices  $\mathbf{F}$ ,  $\mathbf{Q}$ ,  $\mathbf{A}$ ,  $\mathbf{H}$  and  $\mathbf{R}$ , following these steps: 1) start with an educated guess for the unknown parameters, 2) calculate the matrices  $\mathbf{F}$ ,  $\mathbf{Q}$ ,  $\mathbf{A}$ ,  $\mathbf{H}$  and  $\mathbf{R}$ , 3) run the Kalman filter for  $t = 1, \dots, T$  and obtain  $\widehat{\boldsymbol{\xi}}_{t|t-1}$  and  $\mathbf{P}_{t|t-1}$ , 4) evaluate the log likelihood function using estimates from step 2) and 3), and 5) if a maximum was not yet attained, use numerical optimization methods to make better guesses of the unknown parameters in order to maximize the log likelihood, and go again to step 2).

## Smoothing

Above, only information up to  $t$  was used to estimate  $\boldsymbol{\xi}$ . However, the full sample can be used to estimate the state variable. In that case we have a smoothed estimate of  $\boldsymbol{\xi}$ :

$$\widehat{\boldsymbol{\xi}}_{t|T} = \widehat{E}(\boldsymbol{\xi}_t | \mathbf{y}_T^*),$$

with a MSE given by

$$\mathbf{P}_{t|T} = E \left[ \left( \boldsymbol{\xi}_t - \widehat{\boldsymbol{\xi}}_{t|T} \right) \left( \boldsymbol{\xi}_t - \widehat{\boldsymbol{\xi}}_{t|T} \right)' \right].$$

It can be shown that the updating formula for the smoothed estimate of  $\boldsymbol{\xi}$  is:

$$\widehat{E}(\boldsymbol{\xi}_t | \mathbf{y}_T^*) = \widehat{\boldsymbol{\xi}}_{t|t} + \mathbf{J}_t \left( \widehat{\boldsymbol{\xi}}_{t+1|T} - \widehat{\boldsymbol{\xi}}_{t+1|t} \right), \quad (6.23)$$

with

$$\mathbf{J}_t = \mathbf{P}_{t|t} \mathbf{F}' \mathbf{P}_{t+1|t}^{-1}. \quad (6.24)$$

This formula corrects the estimate of  $\boldsymbol{\xi}$  done with information in  $t$  using the  $\boldsymbol{\xi}$  forecast error in  $t + 1$ . This error is calculated comparing the forecast of the state variable done with information up to  $t$  with the one done with all the information in the sample,  $\widehat{\boldsymbol{\xi}}_{t+1|T}$ .

The entire sequence of smoothed estimates  $\left\{ \widehat{\boldsymbol{\xi}}_{t|T} \right\}_{t=1}^T$  is then obtained as follows. (1) Firstly, run the Kalman filter and obtain  $\widehat{\boldsymbol{\xi}}_{t|t}$ , from (6.19),  $\widehat{\boldsymbol{\xi}}_{t+1|t}$ , from (6.21),  $\mathbf{P}_{t|t}$ , from (6.20), and  $\mathbf{P}_{t+1|t}$ , from (6.22). Store the sequences  $\left\{ \widehat{\boldsymbol{\xi}}_{t|t} \right\}_{t=1}^T$ ,  $\left\{ \widehat{\boldsymbol{\xi}}_{t+1|t} \right\}_{t=0}^{T-1}$ ,  $\left\{ \mathbf{P}_{t|t} \right\}_{t=1}^T$ , and  $\left\{ \mathbf{P}_{t+1|t} \right\}_{t=0}^{T-1}$ . Notice that  $\widehat{\boldsymbol{\xi}}_{T|T}$  is just the last element of  $\left\{ \widehat{\boldsymbol{\xi}}_{t|T} \right\}_{t=1}^T$ . (2) Obtain  $\left\{ \mathbf{J}_t \right\}_{t=1}^{T-1}$  from (6.24). (3) From (6.23) we calculate for  $t = T - 1$

$$\widehat{E}(\boldsymbol{\xi}_{T-1} | \mathbf{y}_T^*) = \widehat{\boldsymbol{\xi}}_{T-1|T-1} + \mathbf{J}_{T-1} \left( \widehat{\boldsymbol{\xi}}_{T|T} - \widehat{\boldsymbol{\xi}}_{T|T-1} \right).$$

The same can be obtained for  $t = T - 2$ . And proceeding backwards in this manner, the entire series of smoothed estimates,  $\left\{ \widehat{\boldsymbol{\xi}}_{t|T} \right\}_{t=1}^T$  can be obtained.

Starting from (6.23), it can be shown that the smoothed MSE of the estimate is

$$\mathbf{P}_{t|T} = \mathbf{P}_{t|t} + \mathbf{J}_t \left( \mathbf{P}_{t+1|T} - \mathbf{P}_{t+1|t} \right) \mathbf{J}_t'.$$

Using the same procedure as for  $\widehat{\boldsymbol{\xi}}_{t|T}$ , this equation can be used to obtain  $\mathbf{P}_{t|T}$  for the entire sample, starting in  $t = T - 1$  and working backwards.

The smoothed estimates of the observation variables are easily obtained as

$$\widehat{\mathbf{y}}_{t|T} = \mathbf{A}'\mathbf{x}_t + \mathbf{H}'\widehat{\boldsymbol{\xi}}_{t|T}.$$

The smoothed error estimates can be produced using

$$\begin{aligned}\widehat{\mathbf{w}}_{t|T} &= \mathbf{y}_t - \widehat{\mathbf{y}}_{t|T} \\ \widehat{\mathbf{v}}_{t+1|T} &= \widehat{\boldsymbol{\xi}}_{t+1|T} - \mathbf{F}\widehat{\boldsymbol{\xi}}_{t|T}.\end{aligned}$$

The smoothed error variance matrices can also be estimated, yielding  $\widehat{E}(\mathbf{Q}|\mathbf{y}_T^*)$  and  $\widehat{E}(\mathbf{R}|\mathbf{y}_T^*)$ . These estimates can be time-varying, as is explained next.

### Time-varying parameters

Up to this point we assumed the matrices  $\mathbf{F}$ ,  $\mathbf{Q}$ ,  $\mathbf{A}$ ,  $\mathbf{H}$  and  $\mathbf{R}$  as constants. However, this assumption can be relaxed, with that matrices depending on  $\mathbf{x}_t$ . The vectors of errors have the same properties as before, but now they are assumed to follow a Gaussian distribution. In this context and taking in to account information on  $\mathbf{y}_t$ , it can be deducted the conditional distribution

$$\boldsymbol{\xi}_t|\mathbf{y}_t, \mathbf{x}_t, \mathbf{y}_{t-1}^* \sim N\left(\widehat{\boldsymbol{\xi}}_{t|t}, \mathbf{P}_{t|t}\right),$$

where  $\widehat{\boldsymbol{\xi}}_{t|t}$  and  $\mathbf{P}_{t|t}$  (and also  $\widehat{\boldsymbol{\xi}}_{t+1|t}$ , and  $\mathbf{P}_{t+1|t}$ ) are obtained using exactly the same formulas as before, but were the matrices  $\mathbf{F}$ ,  $\mathbf{Q}$ ,  $\mathbf{A}$ ,  $\mathbf{H}$  and  $\mathbf{R}$  are time-varying. Also as before, the log likelihood function can be used to estimate the unknown parameters.

### 6.3.3 Principal components analysis

In this annex, we are going to present a synthesis of the principal component technique applied in the study of convergence. Synthesis as this one can be found in works like Johnson and Wichern (2007) and Jolliffe (2002).

In general terms, principal components analysis is a statistical technique used to summarize a set of data. Unobservable variables, called principal components (PC), are identified as linear combinations of the original data, which are able to explain the variance on those data. Ideally, the first few PCs will be able to reproduce almost all the variability in the original variables, and can substitute them without great loss of information.

To start, we have  $n$  observations on  $p$  random variables:  $\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_n$ , where  $\mathbf{x}_j = [\mathbf{x}_{1j} \ \mathbf{x}_{2j} \ \dots \ \mathbf{x}_{pj}]$ , with  $j = 1, 2, \dots, n$ . The statistics of interest describing these data are the sample mean vector  $\bar{\mathbf{x}}$ , the  $p \times p$  sample covariance matrix  $\mathbf{S}$ , and the sample correlation matrix  $\mathbf{R}$ .

The sample principal components are defined as the linear combinations of the data which have maximum sample variance. The first PC is the one that explains the larger proportion of sample variance. The second PC is uncorrelated with the first one and explains the second largest proportion of the variance. Proceeding successively in this way, in the limit with a number of PC equal to the number of variables, the total variance in the data can be replicated.

In concrete, the first sample PC is the linear combination  $\alpha'_1 \mathbf{x}_j = \alpha_{11}x_{1j} + \alpha_{21}x_{2j} + \dots + \alpha_{p1}x_{pj}$ ,  $j = 1, \dots, n$ , which maximizes the variance of  $\alpha'_1 \mathbf{x}_j$ ,  $\alpha'_1 \mathbf{S} \alpha_1$ , subject to  $\alpha'_1 \alpha_1 = 1$ .

The second PC accounts for the maximum remaining variance on the data. It is obtained as the linear combination  $\alpha'_2 \mathbf{x}_j$  which maximizes the variance of  $\alpha'_2 \mathbf{x}_j$ , having zero covariance with  $\alpha'_1 \mathbf{x}_j$  and a vector of unit length,  $\alpha'_2 \alpha_2 = 1$ .

In general, the  $i$ th sample PC is the linear combination  $\alpha'_i \mathbf{x}_j$  that maximizes the variance of  $\alpha'_i \mathbf{x}_j$  subject to (1)  $cov(\alpha'_i \mathbf{x}_j, \alpha'_k \mathbf{x}_j) = 0$ , for  $k < i$ , and (2)  $\alpha'_i \alpha_i = 1$ .

If the eigenvalue-eigenvectors pairs of  $\mathbf{S}$  are  $(\hat{\lambda}_1, \hat{\mathbf{e}}_1), \dots, (\hat{\lambda}_p, \hat{\mathbf{e}}_p)$ , with  $\hat{\lambda}_1 \geq \hat{\lambda}_2 \geq \dots \geq \hat{\lambda}_p \geq 0$ , then it can be shown that the  $i$ th PC is:<sup>9</sup>

$$\hat{y}_i = \hat{\mathbf{e}}'_i \mathbf{x} = \hat{e}_{i1}x_1 + \hat{e}_{i2}x_2 + \dots + \hat{e}_{ip}x_p, \quad (6.25)$$

---

<sup>9</sup>To simplify the notation, we express the PC in a generic way, and not attached it to any observation  $j$ . But for each observation  $j$  of the  $p$  variables, a PC  $i$  is obtained that summarizes the variation on the  $p$  variables.

for  $i = 1, \dots, p$ . Since the PC was obtained with the restriction of a vector of unit length vector, we have for each PC  $i$ :  $\sum_{j=1}^p \hat{e}_{ji}^2 = 1$ .

The variance of the  $i$ th PC is the  $i$ th eigenvalue:  $var(\hat{y}_i) = \hat{\mathbf{e}}_i' \mathbf{S} \hat{\mathbf{e}}_i = \hat{\lambda}_i$ ,  $i = 1, \dots, p$ . By definition, the PCs are uncorrelated:  $cov(\hat{y}_i, \hat{y}_k) = 0$ ,  $k \neq i$ .

The total sample variance is matched by the sum of the eigenvalues:  $\sum_{i=1}^p var(\mathbf{x}_i) = \hat{\lambda}_1 + \hat{\lambda}_2 + \dots + \hat{\lambda}_p$ , where  $var(\mathbf{x}_i) = \mathbf{s}_{ii}$  is the variance obtained with  $n$  observations of variable  $i$ . Therefore, the proportion of the sample variance explained by the PC  $i$  is simply

$$\frac{\hat{\lambda}_i}{\hat{\lambda}_1 + \hat{\lambda}_2 + \dots + \hat{\lambda}_p}.$$

Besides the eigenvalues, the eigenvectors also deserve a careful examination. Usually, they are also called factor loadings and they represent the contribution of each variable to the PC (see equation (6.25)). Therefore, they can be used to assess the co-movement between the variables and the respective PC. Besides that, the loadings can also be used to interpret the PC. To do that, it is necessary to analyse which is the sign and size of each variable's factor loading for a given PC, and from that infer a qualitative meaning for the PC.

In a different way, each observation  $j$  can be recovered from the set of PCs using the eigenvectors:

$$\mathbf{x}_j = \hat{y}_{1j} \hat{\mathbf{e}}_1 + \hat{y}_{2j} \hat{\mathbf{e}}_2 + \dots + \hat{y}_{pj} \hat{\mathbf{e}}_p$$

,  $j = 1, 2, \dots, n$ . This means that the factor loadings can also be interpreted as the weight of each PC on a specific variable  $j$ .

Based on this discussion, it is possible to conclude that the factor loadings are related with the correlation between the PCs and the variables. Formally, we have:

$$\rho_{\hat{y}_i, x_k} = \frac{\hat{e}_{ki} \sqrt{\hat{\lambda}_i}}{\sqrt{s_{kk}}}$$

,  $i, k = 1, \dots, p$ .

One drawback of the principal components analysis occurs when the original variables are heterogenous with respect to their variances, *i.e.*, when there are differences in scale in the data set. In this case the variables with larger variance will dominate the first few PCs.

One way of overcoming this problem consist in obtaining the PCs from the standardized variables  $i$ :

$$z_{i,j} = \frac{x_{i,j} - \bar{x}_i}{\sqrt{s_{ii}}}$$

,  $j = 1, 2, \dots, n$  and  $i = 1, \dots, p$ . Due to the standardization of the variables,  $E(z_i) = 0$  and  $Var(z_i) = 1$ . In matrix notation we have:

$$\mathbf{z}_j = \mathbf{D}^{-1/2} (\mathbf{x}_j - \bar{\mathbf{x}})$$

,  $j = 1, 2, \dots, n$  and where  $\mathbf{D}$  is a diagonal matrix of the sample standard deviations. The matrix ( $p \times n$ ) with the  $n$  observations of the standardized observations is  $\mathbf{Z} = [\mathbf{z}_1, \mathbf{z}_2, \dots, \mathbf{z}_n]$ , with  $\mathbf{Z} = \mathbf{D}^{-1/2} (\mathbf{X} - \bar{\mathbf{X}})$ .

The sample covariance matrix of the standardized variables is equal to the correlation matrix of the original variables:

$$cov(\mathbf{Z}) = \mathbf{D}^{-1/2} \mathbf{S} \mathbf{D}^{-1/2} = \mathbf{R}.$$

Then, the PC of the standardized observations are obtained from the correlation matrix of the original variables,  $\mathbf{R}$ . All the results seen above hold for this case, with some slight modifications. If the  $i$ th pair of eigenvalue-eigenvector of  $\mathbf{R}$  is  $(\hat{\lambda}_i, \hat{\mathbf{e}}_i)$ , with  $\hat{\lambda}_1 \geq \hat{\lambda}_2 \geq \dots \geq \hat{\lambda}_p \geq 0$ , then the  $i$ th PC is

$$\hat{y}_i = \hat{\mathbf{e}}_i' \mathbf{z} = \hat{e}_{1i} z_1 + \hat{e}_{2i} z_2 + \dots + \hat{e}_{pi} z_p,$$

$i = 1, \dots, p$ .

The  $var(\hat{y}_i) = \hat{\lambda}_i$ ,  $i = 1, \dots, p$ , and we still have  $cov(\hat{y}_i, \hat{y}_k) = 0$ ,  $k \neq i$ . The total standardized sample variance is  $p = \hat{\lambda}_1 + \hat{\lambda}_2 + \dots + \hat{\lambda}_p$ . Consequently, the proportion of the standardized variance explained by the  $i$ th PC is  $\hat{\lambda}_i/p$ . Finally, the correlation between each variable and the PC is simplified to:  $\rho_{\hat{y}_i, z_k} = \hat{e}_{ki} \sqrt{\hat{\lambda}_i}$ ,  $i, k = 1, \dots, p$ .

The PCs are obtained through the singular value decomposition of the ( $n \times p$ )  $\mathbf{X}$  matrix

of the demeaned data (or matrix  $\mathbf{Z}$  of the standardized data). It can be shown that

$$\mathbf{X} = \mathbf{U}\mathbf{L}\mathbf{A}'$$

where  $\mathbf{U}$  and  $\mathbf{A}$  are respectively  $(n \times r)$  and  $(p \times r)$ , and they are orthogonal, such that  $\mathbf{U}'\mathbf{U} = \mathbf{I}_r$  and  $\mathbf{A}'\mathbf{A} = \mathbf{I}_r$ ;  $\mathbf{L}$  is a  $(r \times r)$  diagonal matrix; and  $r$  is the rank of  $\mathbf{X}$ . Each matrix contains important elements for the PC analysis. Matrix  $\mathbf{A}$  contains the eigenvectors of  $\mathbf{X}'\mathbf{X}$ , *i.e.*, the coefficients of the PCs. Matrix  $\mathbf{L}$  includes the square-roots of the eigenvalues of  $\mathbf{X}'\mathbf{X}$ , that are the standard deviations of the PCs. And  $\mathbf{U}$  gives a scaled version of the PC.

# Bibliography

- [1] Adolfson, M. (2001). “Monetary policy with incomplete exchange rate pass-through,” *Sveriges Riksbank Working Paper Series*, No. 127.
- [2] Anderson, G. A. and Moore, G. R. (1985). “A linear algebraic procedure for solving perfect foresight models,” *Economic Letters*, 17, 247-252.
- [3] Andrews, D. (1991). “Heteroskedasticity and autocorrelation consistent covariance matrix estimation,” *Econometrica*, 59, 817–858.
- [4] Angeloni, I. and Ehrmann, M. (2007). “Euro area inflation differentials,” *The B.E. Journal of Macroeconomics*, 7:1.
- [5] Artis, M. J. and Zhang, W. (1997). “International business cycles and the ERM: Is there a European Business cycle?,” *International Journal of Finance and Economics*, 2, 1-16.
- [6] Artis, M. J. and Zhang, W. (1999). “Further evidence on the international business cycle and the ERM: Is there a European business cycle?,” *Oxford Economic Papers*, 120-132.
- [7] Bache, W. and Naug, B. (2007). “Estimating New Keynesian import price models,” *Norges Bank Working Paper*, No. 15.
- [8] Balakrishnan, Ravi and Lopez-Salido, J. David (2002). “Understanding UK inflation: the role of openness,” *Bank of England Working Paper*.
- [9] Baltagi, B. (2008). *Econometric Analysis of Panel Data*, John Wiley & Sons, Fourth edition.

- [10] Bardsen, G., Jansen, E., Nymoen, R. (2004). "Econometric evaluation of the New Keynesian Phillips curve," *Oxford Bulletin of Economics and Statistics*, 66, 671-686.
- [11] Barro, R. and Sala-i-Martin, X. (1991). "Convergence across states and regions," *Brookings Papers on Economic Activity*, Vol. 1, 107-182.
- [12] Barth, M. and Ramey, V. (2001). "The cost channel of monetary transmission," *NBER Macroeconomic Annual*, 2001.
- [13] Batini, N.; B. Jackson and S. Nickell (2005) "An open-economy New Keynesian Phillips Curve for the UK," *Journal of Monetary Economics* 52, 1061-1071.
- [14] Beck, N. and Katz, Jonathan (1995). "What to do (and not to do) with time-series cross-section data," *American Political Science Review*, 89 (3), 634-647.
- [15] Becker, B. and Hall, S. (2009a). "A new look at economic convergence in Europe: A common factor approach," *International Journal of Finance & Economics*, 14, 85-97.
- [16] Becker, B. and Hall, S. (2009b). "How far from the Euro Area? Modelling convergence of inflation rates in Eastern Europe," *Economic Modelling*, 26, 788-798.
- [17] Bernanke, Ben S. and Gertler, Mark (1989). "Agency costs, net worth, and business fluctuations," *American Economic Review* , 79: (March 1989): 14-31.
- [18] Bernanke, Ben and Gertler, Mark (1995). "Inside the black box: The credit channel of monetary policy transmission," *Journal of Economic Perspectives*, 9 (Fall 1995).
- [19] Bernanke, B., Gertler, M. and Gilchrist, S. (1999). "The financial accelerator in a quantitative business cycle framework", in: M. Woodford and J.B. Taylor (eds.), *Handbook of Macroeconomics*, North-Holland, Amsterdam, 1341-1393.
- [20] Bernanke, B. S. and Mihov, I. (1998). "Measuring monetary policy," *Quarterly Journal of Economics*, 113 (August): 869-902.
- [21] Blinder, Alan (1997). "Credit rationing and effective supply failures," *The Economic Journal*, 97 (June): 327-352.

- [22] Binder, M. and Pesaran, M. H. (1995). “Multivariate rational expectations models: a review and some new results,” in Pesaran M. H. and Wickens M. R. (eds), *Handbook of Applied Econometrics*, Volume Macroeconomics, Blackwell Publishing, Oxford, pp. 139–187.
- [23] Bjornstad, R. and Nymoen, R. (2008). “The New Keynesian Phillips Curve tested on OECD panel data,” *Economics E-Journal*, 2.
- [24] Busetti, F., Forni, L., Harvey, A. and Venditti, F. (2006). “Inflation convergence and divergence within the European Monetary Union,” *ECB Working Paper No. 574*.
- [25] Calvo, G. A. (1983). “Staggered prices in a utility-maximizing framework,” *Journal of Monetary Economics*, 12(3), Sept., 983–998.
- [26] Campa, J. and Goldberg, L. (2002). “Exchange rate pass-through into import prices,” *The Review of Economics and Statistics*, 87 (4), pg. 679-690.
- [27] Canova, F. (2007). *Methods for Applied Macroeconomic Research*, Princeton University Press.
- [28] Castelnuovo, Efrem (2006). “Inflation dynamics and the cost channel of monetary transmission: a comment,” *University of Padua Working Papers*.
- [29] Christiano, Lawrence. J., Eichenbaum, Martin and Evans, Charles (1994). “Identification and the effects of monetary policy shocks,” *Federal Reserve Bank of Chicago*, WP-94-7, 1994.
- [30] Christiano, Lawrence. J., Eichenbaum, Martin and Evans, Charles (1997). “Sticky price and limited participation models of money: A comparison,” *European Economic Review*, 41 (June): 1201-49.
- [31] Christiano, Lawrence J., Eichenbaum, Martin, and Evans, Charles L. (2005). “Nominal rigidities and the dynamic effects of a shock to monetary policy,” *Journal of Political Economy*, vol. 113, no. 1.
- [32] Christiano, Lawrence J. and Fitzgerald, Terry J. (2003). “The Band Pass Filter,” *International Economic Review*, 44(2), 435-465.

- [33] Chowdwury, Ibrahim; Hoffmann, Mathias and Schabert, Andreas (2006). "Inflation dynamics and the cost channel of nonetary transmission," *European Economic Review*, 50, 995-1016.
- [34] Clarida, Richard; Gali, Jordi and Gertler, Mark (2001). "Optimal policy in open versus closed economies: An integrated approach," *American Economic Review*, 248- 252.
- [35] Cuthbertson, K., Hall, S.G., and Taylor, M. (1992). *Applied Econometric Techniques*, Harvester Wheatsheaf.
- [36] den Haan, W. and Levin, A. (1996). "A practitioner's guide to robust covariance matrix estimation," *University of California, San Diego, Department of Economics Discussion Paper*, 96-17.
- [37] Dullien, S. and Fritsche, U. (2008), "Does the dispersion of unit labor cost dynamics in the EMU imply long-run divergence?," *International Economics and Economic Policy*, 5, 269-295.
- [38] ECB (2003), *Inflation differentials in the Euro Area: Potential causes and policy implications*, ECB
- [39] Eviews (2007), *User's Guide*, Quantitative Micro Software, California.
- [40] Favero, C. A. (2001). *Applied Macroeconometrics*, Oxford, Oxford University Press.
- [41] Fritsche, U. and Kuzin, V. (2007). "Unit labor growth differentials in the Euro area, Germany, and the US: lessons from PANIC and cluster analysis," *German Institute for Economic Research DIW Discussion Paper No. 667*.
- [42] Fuhrer, J., and Moore, G. (1995). "Inflation persistence," *Quarterly Journal of Economics*, 110(1), 127-159.
- [43] Gaiotti, Eugenio and Secchi, Alessandro (2006). "Is there a cost channel of monetary policy transmission? An investigation into the Pricing behavior of 2000 firms," *Journal of Money, Credit and Banking*, Vol. 38, n.<sup>o</sup> 8. pp. 2013-2037.

- [44] Gagnon, E., and Khan, H. (2005). “New Phillips curve under alternative production technologies for Canada, the United States and the Euro area,” *European Economic Review*, 49 (6), 1571-1602.
- [45] Gali, J. (2008). *Monetary Policy, Inflation, and the Business Cycle*, Princeton University Press.
- [46] Gali, J. and M. Gertler (1999). “Inflation dynamics: A structural econometric analysis,” *Journal of Monetary Economics* 44, 195-222.
- [47] Gali, J., Gertler, M., and Lopez-Salido, J. D. (2001). “European inflation dynamics,” *European Economic Review*, 45-7, 1237-1270.
- [48] Gali, J., Gertler, M. and Lopez-Salido, J.D. (2005). “Robustness of the estimates of the hybrid New Keynesian Phillips Curve,” *Journal of Monetary Economics*, 52 (6), 1107-1118.
- [49] Gali, J. and Lopez-Salido, J.D. (2001). “A New Philips Curve for Spain,” *BIS Papers* No 3, p. 174-203.
- [50] Gali, J. and Monacelli, T. (2005). “Monetary policy and exchange rate volatility in a small open economy,” *Review of Economic Studies*, 2005, v72(252, Jul), 707-734.
- [51] Garratt, A., Lee, K., and Shields, K. (2009). “Measuring the natural output gap using actual and expected output gap,” *University of Leicester Department of Economics Working Papers*.
- [52] Goodfriend, M. and King, R. (1997). “The New Neoclassical synthesis and the role of monetary policy,” *NBER Macroeconomics Annual*.
- [53] Gregoriou, A., Kontonikas, A. and Montagnoli, A. (2007). “Euro area inflation differentials: Unit roots, structural breaks and non-linear adjustment,” *Department of Economics University of Glasgow Working Paper No. 13/2007*.
- [54] Hahn, J., Hausman, J., and Kuersteiner, G. (2004). “Estimation with weak instruments: Accuracy of higher-order bias and MSE approximations,” *Econometrics Journal* 7(1): 272–306.

- [55] Hall, S. G. and Robertson, D. and Wickens, M.R. (1997). "Measuring Economic Convergence," *International Journal of Finance & Economics*, 2, 131-143.
- [56] Hansen, L. (1982). "Large sample properties of the generalized method of moments estimators," *Econometrica*, 50, 1029-1053.
- [57] Hansen, L., Heaton, J., and Yaron, A. (1996). "Finite sample properties of some alternative GMM estimators," *Journal of Business and Economic Statistics*, 14(3), 262-280.
- [58] Hayashi, F. 2000. *Econometrics*. 1st ed. Princeton, NJ: Princeton University Press.
- [59] Henzel, Steffen, Hülsewig, Oliver, Mayer, Eric, and Wollmershäuser, Timo (2009). "The price puzzle revisited: can the cost channel explain a rise in inflation after a monetary policy shock?," *Journal of Macroeconomics*, 31, pp. 268-289.
- [60] Hofmann, B. and Remsperger, H. (2005). "Inflation differentials among the Euro area countries: Potential causes and consequences," *Journal of Asian Economics*, 16, 403-419.
- [61] Holmes (2002). "Panel data evidence on inflation convergence in the European Union," *Applied Economic Letters*, Vol. 9.
- [62] Honohan, P. and Lane, P. R. (2003). "Divergent inflation rates in EMU," *Economic Policy*, 357-394.
- [63] Honohan, P and Lane, P. R. (2004). "Exchange rates and inflation under EMU: An update," *IHS Discussion Paper No. 31*.
- [64] Jondeau, E. and Le Bihan., H. (2005). "Testing for the New Keynesian Phillips Curve. Additional international evidence," *Economic Modelling* 22, 521-550.
- [65] Johnson, Richard, and Wicher, Dean (2007). *Applied Multivariate Statistical Analysis*, 6th edition, Prentice Hall.
- [66] Jolliffe, I. T. (2002). *Principal Component Analysis*, second edition, Springer.
- [67] Kara, A. and Nelson, E. (2003). "The exchange rate and inflation in the UK," *Scottish Journal of Political Economy*, vol. 50(5), pages 585-608, November.

- [68] Kirsanova, T., Leith, C. and Wren-Lewis, S. (2006). “Should central banks target consumer prices or the exchange rate?,” *The Economic Journal*, 116 (June), pages F208-F231.
- [69] Koopman, Siem Jan, Neil, and Doornik, Jurgen (1999). “Statistical algorithms for models in state space using SsfPack 2.2,” *Econometrics Journal* 2(1), 107-160.
- [70] Lane, P. R. (2006). “The Real Effects of European Monetary Union,” *Journal of Economic Perspectives*, 20, 47-66.
- [71] Leith, C. and Malley, J. (2007). “Estimated open economy New Keynesian Phillips Curves for the G7,” *Open Economy Review*, 18 (4), 405-426.
- [72] Linde, J. (2005). “Estimating New-Keynesian Phillips Curves: A Full Information Maximum Likelihood Approach,” *Journal of Monetary Economics*, 52(6), 1135-1149.
- [73] Lown, C.S., and Rich, R.W. (1997). “Is there an inflation puzzle?,” *Federal Reserve Bank of New York Quarterly Review*, pp. 51-69.
- [74] Massmann, M. and Mitchell, J. (2004). “Reconsidering the evidence: are Eurozone business cycles converging?,” *Discussion Paper NIESR*.
- [75] Mavroeidis, S. (2004). “Weak identification of forward-looking models in monetary economics”, *Oxford Bulletin of Economics and Statistics*, 66, 609- 632.
- [76] Mavroeidis, S. (2005). “Identification issues in forward-looking models estimated by GMM, with an application to the Phillips Curve,” *Journal of Money, Credit and Banking*, 37, 421-448.
- [77] McCallum, Bennett T. (2001). “Inflation targeting and the liquidity trap,” *NBER Working Paper* No. 8225.
- [78] McCallum, Bennett T. and Nelson, Edward (1999). “Nominal income targeting in an open-economy optimizing market,” *Journal of Monetary Economics*, 43 (3), Pages 553-578.

- [79] McCallum, Bennet T. and Nelson, Edward (2001). "Monetary policy for an open economy: an alternative framework with optimising agents and sticky prices," *Oxford Review of Economic Policy*, Vol. 16, Winter, pp. 74-91.
- [80] Monacelli, Tommaso (2005). "Monetary Policy in a Low Pass-Through Environment," *Journal of Money, Credit and Banking*, Vol. 37, No. 6, pages 1047-1066.
- [81] Mihailov, A., Rumler, F., and Scharler, J. (2009). "The small open-economy New Keynesian Phillips Curve: Empirical evidence and implied inflation dynamics," *Open Economy Review*, July.
- [82] Newey, C. R. and West, K. D. (1987). "A simple positive definite, heteroscedasticity and autocorrelation consistent covariance matrix," *Econometrica*, 55 (3), 703-708.
- [83] Pesaran, M.H. (1987). *The Limits of Rational Expectations*. Oxford, UK: Basil Blackwell.
- [84] Rabanal, Pau (2007). "Does inflation increase after a monetary policy tightening? Answers based on an estimated DSGE model," *Journal of Economic Dynamics and Control*, 31, 906-937.
- [85] Ravenna, Federico and Walsh, Carl E. (2006). "Optimal monetary policy with the cost channel," *Journal of Monetary Economics*, Vol. 53 (2), pp.199-216, March.
- [86] Rogers, J. H. (2002). "Monetary union, price level convergence, and inflation: How close is Europe to the United States?," *International Finance Discussion Papers No. 740*, Board of Governors of the Federal Reserve System.
- [87] Rogers, J. H. (2007). "Monetary union, price level convergence, and inflation: How close is Europe to the USA?," *Journal of Monetary Economics*, 54, 785-796.
- [88] Rotemberg, J. and Woodford., M. (1997). "An Optimization-based econometric framework for the evolution of monetary policy," *NBER Macroeconomic Annual 1997*, MIT Press, pages 297-346.
- [89] Rudd, J. and Whelan, K. (2005a). "New tests of the New Keynesian Phillips Curve," *Journal of Monetary Economics*, 52 (6), 1167-1181.

- [90] Rudd, J. and Whelan, K. (2005b). "Modelling inflation dynamics: A critical review of recent research", *Finance and Economics Discussion Series*, Federal Reserve Board, n.º 66.
- [91] Rumler, F (2007). "Estimates of the open economy New Keynesian Phillips Curve for Euro Area countries," *Open Economies Review*.
- [92] Sbordone, A.M. (2002). "Prices and unit labor cost: A new test of price stickiness," *Journal of Monetary Economics* 49 (2), 265-292.
- [93] Sims, Christopher (1992). "Interpreting the macroeconomic time series facts: The effects of monetary policy," *European Economic Review*, 36: 975-1000.
- [94] Siklos, P. and Wohar, M (1997). "Convergence in interest rates and inflation rates across countries and over time," *Review of International Economics*, 5 (1).
- [95] Smets, F. and Wouters, R. (2003). "An estimated dynamic stochastic general equilibrium model of the Euro area," *Journal of the European Economic Association*, 1(5), 1123-1175.
- [96] Stiglitz, J.E. and Weiss., A. (1981). "Credit rationing in markets with imperfect information," *American Economic Review* 71, 393-410.
- [97] Stock, J. H., Wright, J. H., and Yogo, M. (2002). "A survey of weak instruments and weak identification in Generalized Method of Moments," *Journal of Business and Economic Statistics*, 20, 518-529.
- [98] Taylor, J. (1980). "Aggregate dynamics and staggered contracts," *Journal of Political Economy*, 88(1), 1-24.
- [99] Taylor, J. B. (1983), *Macroeconomic Policy in a World Economy*, New York: W. W. Norton.
- [100] Tillmann, P. (2008). "Do interest rates drive inflation dynamics? An analysis of the cost channel of monetary transmission," *Journal of Economics Dynamics and Control*, 32, 2723-2744.

- [101] Tillmann, P. (2009a). “The time-varying cost channel of monetary transmission,” *Journal of International Money and Finance*, 28, pp. 941-953.
- [102] Tillmann, P. (2009b). “Optimal monetary policy with an uncertain cost channel,” *Journal of Money, Credit and Banking*.
- [103] Tirole, J. (1988). *The Theory of Industrial Organisation*. MIT Press, Cambridge.
- [104] Varian, Hal R. (1992). *Microeconomic Analysis*, 3rd edition. WW Norton.
- [105] Vines, David, Kirsanova, Tatiana and Wren-Lewis, Simon J.Q. (2006) “Fiscal Policy and Macroeconomic Stability within a Currency Union,” *CEPR Discussion Paper* No. 5584.
- [106] Walsh, Carl E. (2003). *Monetary Theory and Policy*, MIT Press, Second Edition.
- [107] Woodford, Michael (2003). “Chapter 2 - Price-Level Determination Under Interest-Rate Rules,” in *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton University.