Identifying UK Aggregate Demand and Aggregate Supply Relations within the Long-Run Structural VAR Framework

Thesis submitted for the degree of
Doctor of Philosophy
at the University of Leicester

by

Dimitrios Papaikonomou, BSc (Warwick), MSc (Leicester)
Department of Economics
University of Leicester

January 2003

Identifying UK Aggregate Demand and Aggregate Supply Relations within the Long-Run Structural VAR Framework

by

Dimitrios Papaikonomou

Abstract

This thesis is inspired by the *ESRC*-Cambridge project "*Structural Modelling of the UK Economy within a VAR Framework using Quarterly and Monthly Data*" and, in particular, the studies by Garratt *et al* (1998, 2001). The primary aim is to apply the *Long-Run Structural Cointegrating VAR* approach, developed within the *ESRC*-Cambridge project, in order to empirically investigate UK Aggregate Demand and Supply. The empirical analysis is intended to complement the recently developed macro-econometric model of the UK in Garratt *et al* (1998, 2001) by *(i)* addressing the issue of structural change and *(ii)* providing an explicit model of the supply-side of the economy. The recently developed techniques in Johansen and Nielsen (1994), Hansen (2000) and Johansen, Mosconi and Nielsen (2000) are utilised in order to control for and assess the possible long-run effects of different exchange rate regimes. In the light of the well-documented finite-sample bias, statistical inference relies in large part on simulation methods along the lines of, *inter alia*, van Giersbergen (1996), Li and Maddala (1997), Harris and Judge (1998), Mantalos and Shukur (1998), Gredenhoff and Jacobson (1998), Fachin (2000), Jacobson *et al* (2001) and Greenslade *et al* (2002). A practical problem concerning the use of these methods for inference on the cointegrating parameters is identified and a solution is proposed. The Generalised Impulse Responses developed in Koop *et al* (1996) and Pesaran and Shin (1998) and the Persistence Profiles proposed by Lee *et al* (1992) and Lee and Pesaran (1993a) are used in order to illustrate the dynamic properties of the estimated systems and provide an informal comparison with the Garratt *et al* (1998, 2001) models.

Acknowledgements

I regret not being a native English speaker, so that I can fully express my gratitude to all those who have helped me through the course of this study, and first of all to Professor Kevin Lee. Professor Lee, Kevin, has been a constant source of inspiration, both as an academic and as a person. I am most grateful for his guidance, the endless hours he devoted to my (frequently silly) questions, for granting me access to data sources and computer code, presenting me with the opportunity to actively participate in leading research, for offering me a co-authorship, several pints of lager and above all, his friendship. I was also extremely privileged to have Derek Deadman as my supervisor and Dr. Ali al-Nowaihi as my thesis advisor. I want to express my deepest gratitude to Derek for his devotion to his supervising duties, his very thorough and helpful comments on the various manuscripts he received from me, for the stoical patience and cynical sense of humour with which he dealt with my incurable habit of missing the deadlines, his encouragement and faith in me. I also feel deeply indebted to Dr. al-Nowaihi for his invaluable advice, academic and non-academic, his very kind words, his encouragement and for his (largely undeserved) faith in my abilities. I am also very grateful to Professor Wojciech Charemza for sharing his academic expertise with me on several occasions, for his very constructive critique and extremely helpful suggestions, as well as for his very sharp comments during the APG/PhD seminar series. I also owe many thanks to Professor Stephen Hall for his very useful comments and especially to Professor Soren Johansen for his encouraging words and invaluable academic advice during the 5th annual conference at the University of Crete in May 2001. I would like to thank Professor Panicos Demetriades and Dr. Theofanis Mamouneas for their encouragement and friendly advice and Professor Athanasios Papadopoulos for his very kind words. I am also very grateful to Mr. Geoff Renshaw for his very high praise and Dr. Kimberley Scharf for believing in me. I would also like to say a big thank you to Professors David Fielding, Stephen Pudney, Clive Fraser and Drs. Martin Hoskins and Sarah Brown for their friendly advice, Mrs Jean Soper for her support and guidance concerning my teaching duties, Drs. Michael and Kalvinder Shields for their friendship, Dr. Alexander (Ziggy) McDonald for his friendship, unparalleled sense of humour and subtle musical taste, Dr. Monica Hernandez for having so frequently satisfied her addiction to nicotine in my company and for always carrying a spare lighter, Nikolaos Theodoropoulos for his true friendship and valiant efforts on the basketball court and Massimo Antonini and Jacinta Paola DeCampos Viera Pires for being as close as family to me. I am not prepared to forgive my parents, my brother and Sylvia for having caused me an endless amount of homesickness, and as an act of revenge I am dedicating this thesis to them, hoping that this will oblige them to go through the ordeal of reading it.

This study was in most part financed through a research studentship by the Faculty of Social Sciences of the University of Leicester, which is gratefully acknowledged and was thoroughly misspent.

Contents

Chapter 5: A Long-Run Structural VAR Model of the UK Economy

Tables to Chapter 3

Tables to Chapter 4

Tables to Chapter 5

List of Figures

Figures to Chapter 2

Figures to Chapter 3

Figures to Chapter 4

Figures to Chapter 5

List of Appendices

Introduction

Cointegration analysis can be regarded as the natural consequence of the classic study by Nelson and Plosser (1982), who demonstrated that the presence of a unit root could not be rejected for a wide range of macroeconomic time series. The unit-root reality meant that contemporary econometric analyses were subject to the spurious regression hazard. Consequently, non-stationarity was considered a nuisance, which the seminal papers of Engle and Granger (1987) and Johansen (1988) attempted to deal with. This gave rise to the development of cointegration analysis, which has been a focal aspect of macro-econometric applications over the past 15 years.

Although Johansen (1988) treated cointegration as a purely statistical phenomenon of elimination of stochastic trends, it was soon appreciated that it had great potential as a means for distinguishing between the short-run dynamics and the long-run equilibrium relations. The work of King *et al* (1991) and Mellander *et al* (1992) treated cointegrating relations as being associated with long-run equilibria derived from economic theory. This created the scope to both impose and test the structure implied by economic theory, thus, paving the way for the modelling approach that has been termed in Garratt *et al* (2000) as *Long-Run Structural Cointegrating VAR.*

This approach was in most part developed and systematically applied to UK data within the research project *Structural Modelling of the UK Economy within a VAR Framework using Quarterly and Monthly Data*, funded by the *ESRC* and the Isaac Newton Trust of Trinity College Cambridge. This project has produced a number of important developments concerning, *inter alia*, *(i)* the treatment of deterministic terms and weakly exogenous $I(1)$ variables in Pesaran, Shin and Smith (2000), *(ii)* the formulation and testing of general hypotheses on the model parameters in Pesaran and Shin (2001), and *(iii)* the investigation of the dynamic properties with the use of Persistence Profiles and Generalised Impulse Responses in Lee *et al* (1992) and Pesaran and Shin (1996, 1998). Although some of the technical aspects of *(i)* had previously been considered in the literature by, e.g., Johansen and Juselius (1990) and Johansen (1992), the project placed particular emphasis on the role of economic theory, which is described in detail in Pesaran and Smith (1998).

The primary output of this research project has been the development of a small scale macro-econometric model for the UK by Garratt *et al* (1998, 2001) that utilises most of the methodological advances mentioned above. The economic theory used in these studies as a guide in the over-identification of the cointegrating parameters consists of a general set of arbitrage and other long-run equilibrium conditions, like the Uncovered Interest Parity (UIP), the Fisher Interest Parity (FIP) and Purchasing Power Parity (PPP).

Even though economic theory can (or even should) play a key role in the development of macro-econometric cointegrating *VAR* models, a formal verdict on its empirical relevance will ultimately have to rely on statistical inference. The contributions of, *inter alia*, Johansen and Juselius (1990), Johansen (1992), Johansen and Nielsen (1994), Johansen, Mosconi and Nielsen (2000), Pesaran, Shin and Smith (2000), Hansen (2000) and Pesaran and Smith (2001) provide the statistical framework for asymptotic inference on a number of hypotheses. However, a substantial body of evidence has accumulated over the past decade, casting serious doubts on the reliability of asymptotic methods using sample sizes typically available to applied researchers. In the absence of analytical expressions for the finite-sample distributions, inference within small samples has relied on the use of correction factors and simulation methods. The dramatic increase in computing power since the early 1990s resulted in a growing amount of interest in the latter approach, as indicated by the studies of, *inter alia*, van Giersbergen (1996), Harris and Judge (1998), Mantalos and Shukur (1998), Gredenhoff and Jacobson (1998), Fachin (2000), Jacobson *et al* (2001) and Greenslade *et al* (2002).

This thesis is inspired by the *ESRC*-Cambridge project and, in particular, the Garratt *et al* (1998, 2001) papers, thanks to Professor K.C. Lee, one of the members of the project, who is based at the University of Leicester. The empirical chapters of the thesis provide, in most part, an analysis of the Garratt *et al* (1998) data set within the long-run structural cointegrating *VAR* framework. However, the structure imposed on the cointegrating relations is motivated here by a slightly different view of the UK economy, based on standard Aggregate Demand - Aggregate Supply, (AD-AS), theory. The recently developed techniques regarding the treatment of structural change in Johansen and Nielsen (1994), Hansen (2000) and Johansen, Mosconi and Nielsen (2000) are utilised in order to control and assess the possible long-run effects of different exchange rate regimes. In the light of the well-documented finite-sample bias, statistical inference on the validity of the underlying economic theory relies in large part on simulation methods. A practical problem concerning the use of these methods for inference on the cointegrating parameters is identified and a solution is proposed. Generalised Impulse Responses and Persistence Profiles are used in order to illustrate the dynamic properties of the estimated systems and provide an informal comparison with the Garratt *et al* (1998, 2001) models. The thesis is organised as follows:

Chapter 1 briefly discusses the econometric tools used in the empirical chapters. It is not intended to be a detailed literature review and simply aims at introducing the basic concepts, while providing some intuition for the relative strengths and weaknesses of the different approaches. Emphasis is placed on the treatment of the deterministic terms, the modelling conditionally on weakly exogenous variables, the application of simulation methods in cointegrating *VAR* models and the use of Impulse Responses and Persistence Profiles for the evaluation of the model's dynamic properties.

Chapter 2 is a preliminary attempt at estimating a long-run structural *VAR* model of UK Aggregate Demand. It is a variant of the Garratt *et al* (1998, 2001) papers based on IS-LM theory. The GIRs and PPs reveal many similarities in the dynamic behaviour of the estimated model with the Garratt *et al* (1998, 2001) papers, despite significant differences in model specification. However, the empirical analysis reveals certain limitations, which motivate the following chapter.

The third chapter is an attempt to improve on Chapter 2 in two directions. First, the issue of long-run structural change is addressed by utilising the techniques developed in Johansen and Nielsen (1994), Hansen (2000) and Johansen, Mosconi and Nielsen (2000). Second, a practical solution is proposed to the convergence problems that typically arise when applying simulation methods for inference on the cointegrating parameters. The suggested modifications in both areas are shown to provide meaningful improvements over the analysis in Chapter 2. Again, GIRs and PPs are used to highlight some of the similarities and differences with the Garratt *et al* (1998, 2001) papers.

While Chapters 2 and 3 focus on UK Aggregate Demand, Chapter 4 looks at Aggregate Supply by considering the behaviour of the UK labour market. A cointegrating *VAR* model is estimated with a long-run structure that is shown to be consistent with the Lee and Pesaran (1993b) view of the labour market. The identification and aggregation problems that typically arise in time series analyses of labour markets are resolved according to Lee and Papaikonomou (2002). The GIR and PP analysis demonstrates the typical sluggishness of labour markets and illustrates the relative contributions of labour demand and supply to the slow adjustment to long-run equilibrium.

Chapter 5 combines the AD and AS sub-systems estimated in Chapters 3 and 4 in order to form a cointegrating *VAR* model of the UK with a complete AD-AS long-run structure. The estimated cointegrating relations and the dynamic properties of the complete model are compared with the sub-systems in Chapters 3 and 4, as well as with the Garratt *et al* (1998, 2001) models. The empirical analysis also highlights some of the difficulties associated with statistical inference within relatively large systems, recently discussed by Greenslade *et al* (2002). It is argued that in large models, inference should be guided by the more reliable evidence obtained from smaller sub-systems and the use of economic theory, as suggested in Garratt *et al* (2000).

Chapter 1

An Introduction to Cointegrating VAR Modelling

1.1 The Unrestricted Vector Autoregression

The foundations for the Vector Autoregression (*VAR*) methodology were laid in the seminal paper by Sims (1980). Sims severely criticized the way that contemporary econometric practice was being conducted and, in particular, focused on what he termed "*incredible*" restrictions on the short-run dynamics, which were being routinely imposed in the estimation of large-scale macroeconometric models. As an alternative he proposed what is frequently termed "*a-theoretical*" Vector Autoregression.

The statistical basis of the unrestricted *VAR* modelling approach is the *Wold decomposition theorem*, Wold (1938). According to this theorem if a vector $\mathbf{z}_t = [z_{1t}, z_{2t}, ..., z_{nt}]'$ is *weakly* or *covariance stationary*, denoted $\mathbf{z}_t \sim I(0)$, it can be expressed as the sum of a deterministic component and an infinite MA process

$$\mathbf{z}_t = \mathbf{B}(L)(\mathbf{a}\psi_t + \mathbf{e}_t) \ , t = 1, 2, ..., T, \tag{1.1}$$

where ψ_t is an $n_\psi \times 1$ deterministic vector that may contain a constant, a linear term, seasonal dummies, intervention dummies or other regressors that are considered fixed and non-stochastic, $\mathbf{a}$ is an $n \times n_\psi$ coefficient matrix, $\mathbf{e}_t$ is an $n \times 1$ vector of serially uncorrelated disturbances with $E(\mathbf{e}_t) = \mathbf{0}$ and $E(\mathbf{e}_t\mathbf{e}_t') = \mathbf{\Omega}$ positive definite and $\mathbf{B}(L)$ is a polynomial matrix in the lag-operator, L, of infinite degree given by $\mathbf{B}(L) = \sum_{i=0}^{\infty} B_i L^i$, where the B_i's are $n \times n$ coefficient matrices with $B_0 = I_n$.

Provided that all roots of $\mathbf{B}(\rho)$ lie outside the unit circle, i.e. $|\mathbf{B}(\rho)| = 0$ for $|\rho| > 1$, then $\mathbf{B}(L)$ is absolutely summable and hence the process $\mathbf{B}(L)\mathbf{e}_t$ is well defined.[1] Under this

[1] For a proof see Lütkepohl (1993, Appendix C.3)

condition $\mathbf{B}(L)^{-1}$ can be approximated by a polynomial matrix $\mathbf{\Phi}(L)$ of finite degree p, where $\mathbf{\Phi}(L) \equiv \Phi_0 - \sum_{i=1}^{p} \Phi_i L^i$ and $\Phi_0 = I_n$. Pre-multiplication of expression (1.1) by $\mathbf{B}(L)^{-1}$ and application of the approximation $\mathbf{B}(L)^{-1} = \mathbf{\Phi}(L)$ yields

$$\mathbf{z}_t = \mathbf{a}\psi_t + \sum_{i=1}^{p} \Phi_i \mathbf{z}_{t-i} + \mathbf{e}_t, \tag{1.2}$$

where the Φ_i's and the B_i's are related according to $B_i = \sum_{j=1}^{i} B_{i-j}\Phi_j$ for $i > 0$.[2] Expression (1.2) is known as an unrestricted Vector Autoregression of order p, denoted $VAR(p)$. It shows that, under the assumptions mentioned above, any covariance stationary process, $\mathbf{z}_t$, can be approximately described by some finite number, p, of its own past values, a deterministic component, $\mathbf{a}\psi_t$, and some random innovation, $\mathbf{e}_t$.

1.2 Vector Autoregression and Cointegration

In the seminal work by Nelson and Plosser (1982) it was demonstrated that the null hypothesis of a unit root cannot be rejected for a wide range of macroeconomic time series. In other words, there is a wide range of variables for which $\mathbf{z}_t$ is difference stationary, denoted $\mathbf{z}_t \sim I(1)$ and therefore, the modelling problem should be re-formulated in terms of $\Delta\mathbf{z}_t$. Under the assumption that $\mathbf{z}_t \sim I(1)$, the polynomial matrix $\mathbf{\Phi}(L)$ in (1.2) is allowed to have roots that fall on, as well as outside the unit circle, i.e. $|\mathbf{\Phi}(\rho)| = 0$ for $|\rho| \geq 1$, and is commonly re-parameterized as

$$\mathbf{\Phi}(L) \equiv -\Pi L + \Gamma(L)(1-L), \tag{1.3}$$

[2]These relations are obtained from the approximation $\mathbf{B}(L)^{-1} = \mathbf{\Phi}(L)$, or equivalently $\mathbf{B}(L)\mathbf{\Phi}(L) = I_n$, by collecting terms with equal powers of L.

where $\Pi = -(I_n - \sum_{i=1}^{p} \Phi_i) = -\Phi(1)$, $\Gamma(L) = I_n - \sum_{i=1}^{p-1} \Gamma_i L^i$ and $\Gamma_i = -\sum_{j=i+1}^{p} \Phi_j$, $i = 1, 2, ..., p-1$. Therefore, the unrestricted $VAR(p)$ given by (1.2) can be expressed as

$$\Delta \mathbf{z}_t = \mathbf{a}\psi_t + \Pi \mathbf{z}_{t-1} + \sum_{i=1}^{p-1} \Gamma_i \Delta \mathbf{z}_{t-i} + \mathbf{e}_t. \tag{1.4}$$

Expression (1.4) is known as *cointegrating transformation, Johansen decomposition, Vector Error Correction Model* (*VECM*) or *Cointegrating VAR(p)*.

According to the definition of cointegration in Engle and Granger (1987), the elements of the vector $\mathbf{z}_t = [z_{1t}, z_{2t}, ..., z_{nt}]'$ are said to be cointegrated of order d, b denoted $\mathbf{z}_t \sim CI(d, b)$ with cointegrating vector β if:

1) All the elements of $\mathbf{z}_t$ are integrated of the same order, d, i.e. $\mathbf{z}_t \sim I(d)$ and

2) There is a non-zero vector $\beta = [\beta_1, \beta_2, \ldots, \beta_n]'$ such that the linear combination $\beta' \mathbf{z}_t$ is integrated of order $d - b$, i.e. $\beta' \mathbf{z}_t \sim I(d - b)$, for all d, b such that $0 < b \leq d$.

It becomes self-evident that expression (1.4) is consistent with cointegration by simply solving for $\Pi \mathbf{z}_{t-1}$ to get

$$\Pi \mathbf{z}_{t-1} = \Delta \mathbf{z}_t - \mathbf{a}\psi_t - \sum_{i=1}^{p-1} \Gamma_i \Delta \mathbf{z}_{t-i} - \mathbf{e}_t. \tag{1.5}$$

In the case that $\mathbf{z}_t$ is known to be difference stationary the right hand side of (1.5) is clearly $I(0)$, since it only involves lagged differences of $\mathbf{z}_t$, the deterministic terms $\mathbf{a}\psi_t$ and the disturbance vector $\mathbf{e}_t$, where $\mathbf{a}\psi_t$ and $\mathbf{e}_t$ are by definition $I(0)$. This implies that for a non-zero Π-matrix, the linear combinations $\Pi \mathbf{z}_{t-1}$ of the $I(1)$ elements of $\mathbf{z}_t$ are stationary, in which case according to the definition of cointegration the elements of $\mathbf{z}_t$ are cointegrated of order 1,1.

Expression (1.5) is also very useful in giving a very intuitive insight into the Johansen procedure for testing for the number of cointegrating vectors. The fact that $\mathbf{z}_t$ is assumed to be $I(1)$ places a restriction on the Π-matrix. In particular, it restricts Π to being singular, since pre-multiplication of (1.5) by Π^{-1} would lead to the contradiction that $\mathbf{z}_{t-1}$ is equal to a stationary right hand side. It is worth noting, however, that singularity of Π is merely a necessary condition for cointegration as it does not exclude the possibility $\Pi = \mathbf{0}$. In that case there would be no stationary combinations of $\mathbf{z}_t$, i.e. the elements of $\mathbf{z}_t$ would not be cointegrated and the use of a VAR in differences would be appropriate.

1.2.1 Testing for Cointegration: The Johansen Procedure

As was shown earlier with the use of (1.5), the assumption $\mathbf{z}_t \sim I(1)$ imposes a singularity restriction on Π, which from elementary matrix algebra is equivalent to rank-deficiency of Π. Thus, for any rank-deficient, non-zero Π-matrix there are linear combinations of the z_{it}'s, $i = 1, 2, \ldots, n$, which are stationary. The rank of Π, denoted $rank[\Pi]$, gives the number of linearly independent stationary combinations in $\Pi\mathbf{z}_{t-1}$, i.e. the number of cointegrating vectors.

Clearly, the question of cointegration is simply a question of $rank[\Pi]$ which by the singularity assumption has to be such that $n > rank[\Pi] \geq 0$. Finding $rank[\Pi]$ can be reduced to an eigenvalue problem. As any square matrix, Π may be expressed according to the *Jordan representation* or *Jordan canonical form* as

$$\Pi = PJP^{-1}, \tag{1.6}$$

where P is $n \times n$ full-rank and J is block-diagonal with a typical block of the form

$$J_1(\lambda) = [\lambda]\,,\; J_2(\lambda) = \begin{bmatrix} \lambda & 1 \\ 0 & \lambda \end{bmatrix},\; J_3(\lambda) = \begin{bmatrix} \lambda & 1 & 0 \\ 0 & \lambda & 1 \\ 0 & 0 & \lambda \end{bmatrix}, \ldots, \tag{1.7}$$

with λ being an eigenvalue of Π, i.e. a solution to the characteristic equation $|\lambda I_n - \Pi| = 0$. The characteristic equation is a scalar polynomial in λ of n-th degree and as such it will have n solutions which may be real, complex, distinct or repeated. The dimension of the typical block J_i of the J-matrix, indicated by the subscript "i", is equal to the multiplicity of λ. Therefore, in the case where the λ's are distinct, i.e. they all have unitary multiplicity, all J_i's are scalars and the J-matrix becomes

$$J = \begin{bmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & \lambda_n \end{bmatrix}. \tag{1.8}$$

In this special case the columns of P are eigenvectors of Π.

From the Jordan representation given by (1.6) it follows that the determinant of Π is equal to the determinant of J, since

$$|\Pi| = \left|PJP^{-1}\right| = \frac{|P|\,|J|}{|P|} = |J|\,. \tag{1.9}$$

As was shown above, in the case of distinct roots the J-matrix is diagonal which means that its determinant is given by the product of the diagonal elements[3]

$$|J| = \prod_{i=1}^{n} \lambda_i. \tag{1.10}$$

Combining (1.9) and (1.10) gives the determinant of Π as a function of Π's eigenvalues

$$|\Pi| = \prod_{i=1}^{n} \lambda_i. \tag{1.11}$$

Expression (1.11) effectively establishes the link between the eigenvalues λ_i and $rank[\Pi]$ since the dimension of the largest non-zero determinant that can be found in Π, i.e. the rank of Π, will equal the number of the non-zero λ_i's provided, of course, that the λ_i's are ordered as $|\lambda_1| > |\lambda_2| > \ldots > |\lambda_n|$. Thus, testing for the number of cointegrating vectors is equivalent to testing for the number of significantly non-zero eigenvalues of the estimated Π-matrix.

Johansen (1988) provides two test statistics for this purpose. For testing the null hypothesis of r cointegrating relations

$$H_r : rank[\Pi] = r \tag{1.12}$$

against the alternative hypothesis

$$H_{r+1} : rank[\Pi] = r + 1, \tag{1.13}$$

$r = 0, 1, ..., n - 1$, within the context of (1.4) the suggested statistic is known as *maximal*

[3] It is straight forward to show that this is a general result and is not limited to the case of distinct eigenvalues.

eigenvalue and is given by the log-likelihood ratio

$$\mathcal{LR}(H_r|H_{r+1}) = -T\log(1-\hat{\lambda}_{r+1}). \tag{1.14}$$

For testing the null hypothesis (1.12) against the alternative

$$H_n : rank[\Pi] = n \tag{1.15}$$

for $r = 0, 1, ..., n-1$ the proposed log-likelihood ratio statistic is known as $\lambda - trace$ and is given by

$$\mathcal{LR}(H_r|H_n) = -T\sum_{i=r+1}^{n}\log(1-\hat{\lambda}_i), \tag{1.16}$$

where $\hat{\lambda}_r$ in (1.14) and $\hat{\lambda}_i$, are the estimates of the r-th and i-th largest eigenvalues of the Π-matrix, respectively. Johansen (1988) proves that an estimate of the *stochastic* Π-matrix is given by

$$\mathbf{S}_{00}^{-1}\mathbf{S}_{01}\mathbf{S}_{11}^{-1}\mathbf{S}_{10}, \tag{1.17}$$

where the matrices $\mathbf{S}_{00}$, $\mathbf{S}_{01}$, $\mathbf{S}_{10}$ and $\mathbf{S}_{11}$ are defined as follows

$$\mathbf{S}_{ij} = T^{-1}\sum_{t=1}^{T}\mathbf{r}_{it}\mathbf{r}'_{jt}, \qquad i,j = 0,1, \tag{1.18}$$

with $\mathbf{r}_{0t}$ and $\mathbf{r}_{1t}$, being the residuals obtained from the OLS regressions of $\Delta\mathbf{z}_t$ and $\mathbf{z}_{t-1}$on $[\psi'_t, \Delta\mathbf{z}'_{t-1}, \Delta\mathbf{z}'_{t-2}, ..., \Delta\mathbf{z}'_{t-p+1}]'$, respectively.

Clearly, the further the estimated eigenvalues are from zero the more negative are the terms $\log(1-\hat{\lambda}_i)$ and the larger the test statistics. The critical values for the two statistics were first provided by Johansen (1988) and later by *inter alia* Johansen and Juselius (1990), Osterwald-Lenum (1992) and Pesaran, Shin and Smith (2000), hereafter PSS.

1.3 Long-Run Structural Cointegrating VARs

As shown earlier, the econometric framework of the Vector Autoregression in (1.2) allows the representation of $\mathbf{z}_t$ as a function of a finite number, p, of its own past values, a deterministic component, $\mathbf{a}\psi_t$, and some random innovation, $\mathbf{e}_t$. This approach was accused of being *a-theoretical*, as it is based solely on the statistical properties of $\mathbf{z}_t$, leaving no role for economic theory, other than perhaps the choice of the variables entering $\mathbf{z}_t$. Attempts to provide (1.2) with an economic structure have frequently been based on the fact that economic theory can quite often supply information regarding the contemporaneous relationships between the elements of $\mathbf{z}_t$. When the theoretical contemporaneous relations take the form $A_0\mathbf{z}_t$, where A_0 is a known $n \times n$ coefficient matrix, the theory-consistent, or structural VAR ($SVAR$) representation of $\mathbf{z}_t$ is given by

$$A_0\mathbf{z}_t = \mathbf{n}\psi_t + \sum_{i=1}^{p} A_i\mathbf{z}_{t-i} + \mathbf{v}_t, \tag{1.19}$$

where $\mathbf{n}$ and A_i, $i = 0, 1, ..., p$, are the structural coefficient matrices and $\mathbf{v}_t$ is the vector of structural disturbances and the structural $VECM$ ($SVECM$) representation takes the form

$$A_0\Delta\mathbf{z}_t = \mathbf{n}\psi_t + A(1)\mathbf{z}_{t-1} + \sum_{i=1}^{p-1} \Gamma_i^s\Delta\mathbf{z}_{t-i} + \mathbf{v}_t, \tag{1.20}$$

where $\Gamma_i^s = -\sum_{j=i+1}^{p} A_j$ and $A(1) = -\sum_{i=0}^{p} A_i$. Provided that $|A_0| \neq 0$, the $VAR(p)$ in (1.2) and its cointegrating re-parameterization in (1.4) can be interpreted as reduced-form versions of (1.19) and (1.20) respectively, with $\mathbf{a} = A_0^{-1}\mathbf{n}$, $\Phi_i = A_0^{-1}A_i$, $i = 1,2,\ldots,p$, $\Gamma_i = A_0^{-1}\Gamma_i^s$, $i = 1,2,\ldots,p-1$, $\Pi = A_0^{-1}A(1)$ and $\mathbf{e}_t = A_0^{-1}\mathbf{v}_t$.

Under the assumption $\mathbf{z}_t \sim I(1)$, attention is limited to the *SVECM* in (1.20) and the reduced-form *VECM* in (1.4). As illustrated in previous sections, the assumption $\mathbf{z}_t \sim I(1)$ necessarily implies singularity of $A(1)$, or equivalently $rank[A(1)] = r < n$. In the case when $r = 0$ the elements of $\mathbf{z}_t$ are not cointegrated and the use of a $VAR(p)$ in differences would be appropriate. When $0 < r < n$, i.e. $A(1)$ is a non-zero, rank-deficient matrix, it may be written as

$$A(1) = \boldsymbol{\alpha}_*\boldsymbol{\beta}', \tag{1.21}$$

or in reduced form as

$$\Pi = \boldsymbol{\alpha}\boldsymbol{\beta}', \tag{1.22}$$

where $\boldsymbol{\alpha}_*$, $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are $n \times r$, full column rank with $\boldsymbol{\alpha} = A_0^{-1}\boldsymbol{\alpha}_*$. The matrix $\boldsymbol{\beta}$, usually referred to as the *cointegrating matrix*, has columns equal to the cointegrating vectors. In other words its columns contain the parameters of the stationary combinations of the z_{it}'s. The matrices $\boldsymbol{\alpha}_*$ and $\boldsymbol{\alpha}$ contain the weights with which the cointegrating vectors enter the n equations of the structural and reduced-form models, respectively. Johansen (1995, p.71), Pesaran and Smith (1998), Pesaran and Shin (2001) and others, show that $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are not unique, since $\tilde{\boldsymbol{\alpha}}\tilde{\boldsymbol{\beta}}' = \boldsymbol{\alpha}\boldsymbol{\beta}'$ for $\tilde{\boldsymbol{\alpha}} = \boldsymbol{\alpha}Q$ and $\tilde{\boldsymbol{\beta}} = \boldsymbol{\beta}Q'^{-1}$, where Q is any non-singular $r \times r$ matrix. This illustrates the need for exactly r^2 identifying restrictions, since uniqueness of $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ requires the r^2 elements of Q to be uniquely specified.

It is, therefore, apparent that there are generally two issues of identification that arise within the modelling framework of (1.4). The first, is the traditional problem of identification of the contemporaneous coefficient matrix A_0 and the retrieval of the structural parameters and disturbances from the reduced-form model. The second issue is the identification of the cointegrating matrix, $\boldsymbol{\beta}$. Since $\boldsymbol{\beta}$ appears in both, the structural and the reduced-form models through (1.21) and (1.22) respectively, it is possible to identify the cointegrating, or long-run relations within the reduced-form model (1.4), while abstracting from the identification of the short-run dynamic coefficients, and thus, avoiding Sims' (1980) criticism of *incredible* restrictions. The term *Long Run Structural VAR* is used to describe precisely that, namely, Cointegrating VAR models that impose a structure only on $\boldsymbol{\beta}$, while leaving the short-run dynamics to be determined by the data.

1.3.1 Identification of $\boldsymbol{\beta}$ and the Role of Economic Theory: King *et al* (1991)

In the procedure proposed by Johansen (1988) the r^2 restrictions required for the exact-identification of $\boldsymbol{\beta}$ are chosen so that its columns are eigenvectors of Π with unitary length. However, this arbitrary normalising restriction has the unfortunate property that the cointegrating vectors obtained in this fashion will not have a straightforward economic interpretation. An economically more interesting choice of the exactly identifying restrictions is *via* the use of economic theory. From an economist's point of view, stationarity is linked with the concept of long-run equilibrium. Thus, if one is confronted with the problem of identifying the stationary linear combinations of economic variables it is only natural that economic theory should motivate the choice of the identifying restrictions.

However, using economic theory as a guide with respect to the nature of the cointegrating relations results quite often in more than r^2, or *over-identifying* restrictions. The work of Johansen (1991), among others, provides the statistical basis for one to impose and test restrictions over and above the r^2 just-identifying restrictions. The validity of the over-identifying restrictions can be investigated by means of the likelihood-ratio test-statistic which in this case is asymptotically distributed as a χ^2 variate with degrees of freedom equal to the number of over-identifying restrictions.[4] This provides the tool with which to directly test the empirical validity of what economic theory has to say about the long-run equilibrium.

Perhaps the most frequently cited empirical work in this vein is that of King, Plosser, Stock and Watson (1991) who, among other things, investigate the prediction of a wide class of Real Business Cycle (RBC) models, namely, that the ratios of consumption and investment relative to output known as "*the great ratios*" are constant. Provided that consumption, investment and output are $I(1)$ variables they can be modelled within the cointegrating VAR framework specified by (1.4), where $\mathbf{z}_t = [c_t, i_t, y_t]'$ and c_t, i_t, and y_t stand for the natural logarithms of consumption, investment and output respectively. RBC theory predicts that there should be two long-run relationships between these three variables, which translates into two cointegrating vectors. Furthermore, the same theory provides information on the exact values of the parameters in the cointegrating vectors. In particular, the otherwise unrestricted, long-run structural model has the following form

$$\begin{bmatrix} \Delta c_t \\ \Delta i_t \\ \Delta y_t \end{bmatrix} = \mathbf{a}_0 + \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \\ \alpha_{31} & \alpha_{32} \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} c_{t-1} \\ i_{t-1} \\ y_{t-1} \end{bmatrix} + \sum_{i=1}^{p-1} \Gamma_i \Delta \mathbf{z}_{t-i} + \mathbf{e}_t, \qquad (1.23)$$

where $\mathbf{a}_0$ is a constant.

[4] Pesaran, Shin and Smith (2000) generalise this result for systems with *weakly exogenous* $I(1)$ variables, while Pesaran and Shin (2001) consider the case of non-linear restrictions.

Since there are two cointegrating vectors, $r = 2$, there is need for $r^2 = 4$ exactly identifying restrictions. However, economic theory assigns values to all six components of the β-matrix, thus, leaving two over-identifying restrictions to be tested. Using US data and for $p = 6$, King *et al* (1991) found support for the presence of two cointegrating vectors, which were then identified by imposing the four restrictions implied by the left 2×2 block of the β'-matrix in (1.23). The remaining two over-identifying restrictions were then tested and could not be rejected at the 5% level, thus, giving support to the underlying economic theory.[5]

Clearly, the two cointegrating vectors in (1.23) have a very precise economic interpretation as stationary deviations from the long-run equilibria $c - y$ and $i - y$. Consequently, the elements α_{ij} of the α-matrix can be interpreted as speed of adjustment coefficients to departures from the long-run equilibria, which is why α is usually referred to as the *long-run* or *speed of adjustment* matrix.

1.3.2 Over-Identification of β and Unit Root Testing

The potential to impose and test over-identifying restrictions on β creates the opportunity for an interesting application. A special case of over-identifying restrictions which is of particular interest is the assignment of fixed numerical values to all the elements in a number of cointegrating vectors. This is generally expressed as $\beta = [\mathbf{b}, \phi]$, where the $n \times s$ matrix $\mathbf{b}$ is a set of s cointegrating vectors with fixed elements and the $n \times (r - s)$ matrix ϕ is the remaining set of $r - s$ cointegrating vectors to be estimated. Such a set of restrictions is especially interesting, because it allows for the formulation (and testing) of the hypothesis that a single variable or a

[5] At a further stage the authors augment the vector z_t by including real money supply, interest rates and inflation, which appear to play a very important role contrary to RBC theory.

set of variables in $\mathbf{z}_t$ is stationary. As pointed out in Johansen (1995; pp.74) "*...the question of stationarity of individual series can be formulated in a natural way in terms of parameters in the multivariate system, and is a hypothesis that is conveniently checked inside the model rather than a question that has to be determined before the analysis starts*".

The hypothesis that a variable in $\mathbf{z}_t$ is stationary is exactly equivalent to the hypothesis that this variable is the single entry in a cointegrating relationship. Thus, a test of stationarity for variable z_{it}, $i = 1, 2, \ldots, n$, is equivalent to a test of the restriction $\boldsymbol{\beta} = [\mathbf{b}, \boldsymbol{\phi}]$, where $\mathbf{b}$ is now an $n \times 1$ vector with the i-th element being the only non-zero entry. For example, if one is interested in testing the hypothesis $z_{1t} \sim I(0)$ the restricted cointegrating matrix would be

$$\boldsymbol{\beta}' = \begin{bmatrix} \mathbf{b}' \\ \boldsymbol{\phi}' \end{bmatrix} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ \phi_{11} & \phi_{21} & \cdots & \phi_{n1} \\ \vdots & \vdots & & \vdots \\ \phi_{1,r-1} & \phi_{2,r-1} & \cdots & \phi_{n,r-1} \end{bmatrix}, \tag{1.24}$$

where the non-zero element of $\mathbf{b}$ was arbitrarily set equal to unity.

As mentioned earlier, a general result concerning statistical inference in cointegrating systems is that the LR statistic for testing over-identifying restrictions on $\boldsymbol{\beta}$ is asymptotically χ^2 with degrees of freedom equal to the number of over-identifying restrictions. The set of restrictions in (1.24) can be seen as a special case of over-identification in which one cointegrating vector is fixed, i.e. subject to n restrictions, while the remaining $r-1$ vectors are each subject to r exactly identifying restrictions, giving a total of $n + r(r-1)$ restrictions. Therefore, the likelihood ratio statistic for testing the hypothesis $z_{it} \sim I(0)$, $i = 1, 2, \ldots, n$ is asymptotically chi-squared with $n + r(r-1) - r^2 = n - r$ degrees of freedom. This test is commonly referred to as *multivariate ADF*, although, contrary to the standard ADF test the null hypothesis here is that of stationarity.

This demonstrates the potential to both accommodate and test for the existence of $I(0)$ variables in $\mathbf{z}_t$ within a cointegrating VAR framework. It should be noted, however, that this result cannot be interpreted as an argument for relinquishing the use of conventional unit root tests prior to estimating the model. The reason is that the procedure described above relies on the assumption that $\mathbf{z}_t$ consists only of mixtures of $I(0)$ and $I(1)$ variables and is not applicable in the presence of variables with higher orders of integration[6]. Furthermore, this test procedure is sensitive to the choice of the number of cointegrating vectors, r, and it is widely recognised that the existing cointegrating rank tests can be quite uninformative in relatively small samples[7]. This issue, however, will be discussed in more detail in subsequent sections.

1.4 Treatment of the Deterministic Components

So far, the deterministic elements in (1.4) have been assuming the general form $\mathbf{a}\boldsymbol{\psi}_t$, where $\mathbf{a}$ and $\boldsymbol{\psi}_t$ were defined below (1.1). It is only natural that different specifications for $\mathbf{a}\boldsymbol{\psi}_t$ can have quite different implications on the deterministic behaviour of $\Delta\mathbf{z}_t$, $\mathbf{z}_t$ and the cointegrating relations $\boldsymbol{\beta}'\mathbf{z}_t$. The cointegrating VAR literature has placed considerable emphasis on these implications which are considered in detail *inter alia* by Johansen (1995, section 5.7), Pesaran and Pesaran (1997), Pesaran and Smith (1998) and PSS. When considering the case $\mathbf{z}_t \sim I(1)$,

[6]See Haldrup (1998) for a discussion on the potentially pervasive effects on the properties of the estimators in cointegrating VAR models involving mixtures of $I(1)$ and $I(2)$ variables.

[7]See, for example, Reimers(1992), Cheung and Lai (1993), van Giersbergen (1996), Harris and Judge (1998) and Mantalos and Shukur (1998).

the MA representation of $\Delta \mathbf{z}_t$ takes the form of

$$\Delta \mathbf{z}_t = \mathbf{C}(L)(\mathbf{a}\psi_t + \mathbf{e}_t), \tag{1.25}$$

where

$$\begin{aligned} \mathbf{C}(L) &\equiv \sum_{i=0}^{\infty} C_i L^i = \mathbf{C}(1) + (1-L)\mathbf{C}^*(L), \\ C_0 &= I_n,\ C_1 = \Phi_1 - I_n,\ C_i = \sum_{j=1}^{i} \Phi_j C_{i-j}, \text{ for } i > 1, \\ \mathbf{C}^*(L) &\equiv \sum_{i=0}^{\infty} C_i^* L^i,\ C_0^* = I_n - \mathbf{C}(1),\ C_i^* = C_{i-1}^* + C_i, \text{ for } i > 0. \end{aligned} \tag{1.26}$$

Using (1.26) the MA representations of $\Delta \mathbf{z}_t$ and $\mathbf{z}_t$ take the form

$$\Delta \mathbf{z}_t = \mathbf{C}(1)(\mathbf{a}\psi_t + \mathbf{e}_t) + \mathbf{C}^*(L)(\mathbf{a}\Delta\psi_t + \Delta\mathbf{e}_t), \tag{1.27}$$

$$\mathbf{z}_t = \mathbf{C}(1)\sum_{i=0}^{t}(\mathbf{a}\psi_i + \mathbf{e}_i) + \mathbf{C}^*(L)(\mathbf{a}\psi_t + \mathbf{e}_t), \tag{1.28}$$

where the last t terms of the sum $\mathbf{C}(1)\sum_{i=0}^{t}\mathbf{e}_i$ are known as the reduced-form *stochastic trends*, $\mathbf{C}(1)$ measures the cumulative effect of all past reduced-form shocks, $\mathbf{e}_t$, and according to *Granger's representation theorem* may be expressed as[8]

$$\mathbf{C}(1) = \boldsymbol{\beta}_\perp(\boldsymbol{\alpha}'_\perp \boldsymbol{\Gamma}(1)\boldsymbol{\beta}_\perp)^{-1}\boldsymbol{\alpha}'_\perp, \tag{1.29}$$

[8] For more details see Johansen (1995; Theorem 4.2; pp.49-52)

where $\boldsymbol{\alpha}_{\perp}$, $\boldsymbol{\beta}_{\perp}$ are $n \times (n-r)$, full column rank and are orthogonal complements of $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$, respectively, so that $\boldsymbol{\alpha}'\boldsymbol{\alpha}_{\perp} = \mathbf{0}$ and $\boldsymbol{\beta}'\boldsymbol{\beta}_{\perp} = \mathbf{0}$. This representation illustrates that cointegration may also be interpreted as a process of elimination of stochastic trends. The number of linearly independent stochastic trends is given by $rank[\mathbf{C}(1)]$, which by (1.29) can be shown to vary according to $n - r$. Therefore, in the case when the non-stationary elements of $\mathbf{z}_t$ do not cointegrate, i.e. $r = 0$, $\mathbf{z}_t$ is driven by n independent stochastic trends. When $r = n$ and, thus, $\mathbf{z}_t$ is stationary all stochastic trends are eliminated. This is a quite intuitive result, since stationarity of $\mathbf{z}_t$ implies that the $\mathbf{e}_t$'s will only have a temporary effect which requires that the cumulative effect $\mathbf{C}(1)$ be zero, or equivalently, $rank[\mathbf{C}(1)] = 0$.

The literature typically considers five different specifications for the deterministic components $\mathbf{a}\boldsymbol{\psi}_t$ which are available in standard econometric packages like *Microfit 4.0*. These specifications allow for $\boldsymbol{\psi}_t$ to contain, at most, a constant and a linear trend, thus, abstracting from the analysis of models with intervention or other dummies. In this section, attention is focused on the most general of the five specifications, denoted as $H(r)$ in Johansen (1995; pp.81), or Case V in Pesaran and Pesaran (1997), Pesaran and Smith (1998) and PSS.[9] The remaining four cases can be easily formulated as special cases of Case V, while the presence of intervention dummies in the deterministic vector will be considered in a separate section.

Under **Case V** the deterministic terms take the form

$$\mathbf{a}\boldsymbol{\psi}_t = \mathbf{a}_0 + \mathbf{a}_1 t, \tag{1.30}$$

where $\mathbf{a}_0$ is an n-vector of constants and $\mathbf{a}_1$ is an n-vector of coefficients to the linear trend, t.

[9]To avoid confusion, the Pesaran *et al* terminology will be adopted for the rest of this thesis.

Using this specification the MA representations (1.27) and (1.28) may be written as

$$\begin{aligned} \Delta \mathbf{z}_t &= \mathbf{b}_0 + \mathbf{b}_1 t + \mathbf{C}(L)\mathbf{e}_t, & (1.31) \\ \mathbf{z}_t &= \mathbf{z}_0 + (\mathbf{b}_0 + \frac{1}{2}\mathbf{b}_1)t + \frac{1}{2}\mathbf{b}_1 t^2 + \mathbf{C}(1)\sum_{i=1}^{t}\mathbf{e}_i + \mathbf{C}^*(L)(\mathbf{e}_t - \mathbf{e}_0), & (1.32) \end{aligned}$$

where $\mathbf{b}_0 = \mathbf{C}(1)\mathbf{a}_0 + \mathbf{C}^*(1)\mathbf{a}_1$, $\mathbf{b}_1 = \mathbf{C}(1)\mathbf{a}_1$ and $\mathbf{z}_0 = \mathbf{b}_0 + \mathbf{C}(1)\mathbf{e}_0 + \mathbf{C}^*(L)\mathbf{e}_0$. The MA representation of the cointegrating relations may be obtained through pre-multiplication of (1.32) by $\boldsymbol{\beta}'$ as

$$\boldsymbol{\beta}'\mathbf{z}_t = \boldsymbol{\beta}'\mathbf{z}_0 + \boldsymbol{\beta}'\mathbf{b}_0 t + \boldsymbol{\beta}'\mathbf{C}^*(L)(\mathbf{e}_t - \mathbf{e}_0), \qquad (1.33)$$

where the result $\boldsymbol{\beta}'\mathbf{C}(1) = \mathbf{0}$ that follows from (1.29) was applied and $\boldsymbol{\beta}'\mathbf{z}_0 = \boldsymbol{\beta}'\mathbf{b}_0 + \boldsymbol{\beta}'\mathbf{C}^*(L)\mathbf{e}_0$, $\boldsymbol{\beta}'\mathbf{b}_0 = \boldsymbol{\beta}'\mathbf{C}^*(1)\mathbf{a}_1$. It is apparent from (1.32) that under Case V, $\mathbf{z}_t$ contains both a linear and a quadratic deterministic trend, in addition to the stochastic trend $\sum_{i=1}^{t}\mathbf{e}_i$, while the cointegrating relations in (1.33) contain only a linear deterministic trend.

The remaining four cases are specified as follows:

Case I: $\mathbf{a}_0 = \mathbf{a}_1 = \mathbf{0}$. This is the case when the *VECM* has no intercepts and no deterministic trends. In terms of (1.32) and (1.33) this implies $\mathbf{b}_0 = \mathbf{b}_1 = \mathbf{0}$ and, thus, neither $\mathbf{z}_t$ nor $\boldsymbol{\beta}'\mathbf{z}_t$ contain a deterministic trend. As Pesaran and Pesaran (1997) point out "*Case I is included for completeness and is unlikely to be of relevance in economic applications*".

Case II: $\mathbf{a}_0 = -\boldsymbol{\Pi}\boldsymbol{\mu}$ and $\mathbf{a}_1 = \mathbf{0}$, where $\boldsymbol{\mu}$ is an $n \times 1$ vector of unknown coefficients. This is the case when there are no deterministic trends in the *VECM* and the intercepts are restricted so that they enter the cointegrating vectors. For any non-zero, rank-deficient $\boldsymbol{\Pi}$ this case also implies $\mathbf{b}_0 = \mathbf{b}_1 = \mathbf{0}$, since $\mathbf{b}_0 = \mathbf{C}(1)\mathbf{a}_0 = -\mathbf{C}(1)\boldsymbol{\Pi}\boldsymbol{\mu} = \mathbf{0}$ by use of (1.22) and (1.29).

Consequently, $\mathbf{z}_t$ and $\boldsymbol{\beta}'\mathbf{z}_t$ do not contain deterministic trends.

Case III: $\mathbf{a}_0 \neq \mathbf{0}$ **and** $\mathbf{a}_1 = \mathbf{0}$. In this case there are no deterministic trends in the *VECM* and the intercepts are unrestricted. The implication of this on the MA representation is that $\mathbf{b}_0 = \mathbf{C}(1)\mathbf{a}_0$ and $\mathbf{b}_1 = \mathbf{0}$. This means that quadratic trends are eliminated, while a linear trend enters the MA representation of $\mathbf{z}_t$ through the term $\mathbf{C}(1)\mathbf{a}_0 t$ but it is not present in $\boldsymbol{\beta}'\mathbf{z}_t$ since $\boldsymbol{\beta}'\mathbf{C}(1) = \mathbf{0}$ by (1.29).

Case IV: $\mathbf{a}_0 \neq \mathbf{0}$ **and** $\mathbf{a}_1 = -\Pi\boldsymbol{\gamma}$, where $\boldsymbol{\gamma}$ is an $n \times 1$ vector of unknown coefficients. This is the case when intercepts in the *VECM* are unrestricted and the trends are restricted so that they enter the cointegrating vectors. This implies $\mathbf{b}_0 = \mathbf{C}(1)\mathbf{a}_0 - \mathbf{C}^*(1)\Pi\boldsymbol{\gamma}$ and $\mathbf{b}_1 = \mathbf{0}$, since $\mathbf{b}_1 = \mathbf{C}(1)\mathbf{a}_1 = -\mathbf{C}(1)\Pi\boldsymbol{\gamma} = \mathbf{0}$ by use of (1.22) and (1.29). It can also be shown[10] that $\mathbf{C}^*(1)\Pi = -I_n$ and, thus, $\mathbf{b}_0 = \mathbf{C}(1)\mathbf{a}_0 + \boldsymbol{\gamma}$. Therefore, quadratic trends are eliminated and a linear trend is present in both $\mathbf{z}_t$ and $\boldsymbol{\beta}'\mathbf{z}_t$; in the first instance as $[\mathbf{C}(1)\mathbf{a}_0 + \boldsymbol{\gamma}]t$ and in the second as $\boldsymbol{\beta}'\boldsymbol{\gamma}t$.

A point frequently stressed in the studies by Pesaran *et al* concerns the presence of the term $\frac{1}{2}\mathbf{b}_1 t^2$ in (1.32) that arises from the inclusion of unrestricted trend coefficients in the *VECM* under Case V. The presence of these terms is argued to be undesirable in macroeconomic applications, since for any rank-deficient Π-matrix the level of $\mathbf{z}_t$ will exhibit quadratic deterministic trending behaviour, which is not characteristic of macroeconomic time series. Furthermore, the number of quadratic trends will be a function of the number of cointegrating vectors, since the number of independent quadratic trends depends on $rank[\mathbf{C}(1)] = n - r$, which varies directly with r. In the light of this, Pesaran *et al* favour the restriction of the coefficients on the linear trend in cointegrating *VAR* models according to Case IV, which as shown above, excludes the

[10] For a proof see Pesaran and Shin (1995).

possibility of quadratic trends while allowing for a linear trend in both $\mathbf{z}_t$ and $\boldsymbol{\beta}'\mathbf{z}_t$.

Statistical inference on the validity of the restrictions $\mathbf{a}_0 = -\Pi\boldsymbol{\mu}$ and $\mathbf{a}_1 = -\Pi\boldsymbol{\gamma}$ conditional on $rank[\Pi] = r$ can be made by means of the likelihood-ratio statistic which is asymptotically a chi-squared variate with $n - r$ degrees of freedom. The restriction that a linear trend is not present in the cointegrating relations, known as co-trending hypothesis [Park (1992)], takes the form $\boldsymbol{\beta}'\mathbf{C}^*(1)\mathbf{a}_1 = \mathbf{0}$ under Case V and $\boldsymbol{\beta}'\boldsymbol{\gamma} = \mathbf{0}$ under Case IV and is generally true if $\mathbf{a}_1 = \mathbf{0}$.

1.4.1 The Presence of Intervention Dummies

An interesting extension of the models considered thus far arises when the deterministic terms $\mathbf{a}\boldsymbol{\psi}_t$ are allowed to also include intervention dummies, in addition to the constant and the linear trend. This adds an important new dimension to the modelling problems that can be addressed within a cointegrating $VAR(p)$ by providing the opportunity to explicitly model structural change. The inclusion of intervention dummies has been briefly considered by Johansen (1995) and in more detail by Johansen and Nielsen (1994), Hansen (2000) and Johansen, Mosconi and Nielsen (2000). An empirical application within a small open economy VAR can be found in Jacobson *et al* (2001).

The deterministic terms will now assume the following form that will be referred to as **Case Vd:**

$$\mathbf{a}\boldsymbol{\psi}_t = \mathbf{a}_0 + \mathbf{a}_1 t + \mathbf{a}_2\mathbf{D}_t, \tag{1.34}$$

where $\mathbf{a}_2$ is an $n \times n_d$ coefficient matrix and $\mathbf{D}_t$ is an n_d vector of dummies,

$\mathbf{D}_t = [D_{1t}, D_{2t}, \ldots, D_{n_d t}]'$, with

$$D_{jt} = \begin{cases} 1, \text{ for } t_{j0} < t \leq t_{jq}, \\ 0, \text{ otherwise}, \end{cases}, j = 1, 2, \ldots, n_d.$$

The MA representations of $\Delta \mathbf{z}_t$, $\mathbf{z}_t$ and the cointegrating relations $\boldsymbol{\beta}'\mathbf{z}_t$ under (1.34) become

$$\begin{aligned} \Delta \mathbf{z}_t &= \mathbf{b}_0 + \mathbf{b}_1 t + \mathbf{b}_2 \mathbf{D}_t + \mathbf{C}^*(L)\mathbf{a}_2 \Delta \mathbf{D}_t + \mathbf{C}(L)\mathbf{e}_t, & (1.35) \\ \mathbf{z}_t &= \mathbf{z}_0 + (\mathbf{b}_0 + \frac{1}{2}\mathbf{b}_1)t + \frac{1}{2}\mathbf{b}_1 t^2 + \mathbf{b}_2 \sum_{i=1}^{t} \mathbf{D}_i + \mathbf{C}(1) \sum_{i=1}^{t} \mathbf{e}_i + \\ &\quad + \mathbf{C}^*(L)[\mathbf{e}_t - \mathbf{e}_0 + \mathbf{a}_2(\mathbf{D}_t - \mathbf{D}_0)], & (1.36) \\ \boldsymbol{\beta}'\mathbf{z}_t &= \boldsymbol{\beta}'\mathbf{z}_0 + \boldsymbol{\beta}'\mathbf{b}_0 t + \boldsymbol{\beta}'\mathbf{C}^*(L)[\mathbf{e}_t - \mathbf{e}_0 + \mathbf{a}_2(\mathbf{D}_t - \mathbf{D}_0)], & (1.37) \end{aligned}$$

where $\mathbf{b}_2 = \mathbf{C}(1)\mathbf{a}_2$, $\mathbf{z}_0 = \mathbf{b}_0 + \mathbf{b}_2\mathbf{D}_0 + \mathbf{C}(1)\mathbf{e}_0 + \mathbf{C}^*(L)(\mathbf{e}_0 + \mathbf{a}_2\mathbf{D}_0)$, and $\mathbf{b}_0$, $\mathbf{b}_1$ are defined below (1.32). Expression (1.35) reveals that $\Delta \mathbf{z}_t$ contains two sources of structural change. The first is an intercept with multiple breaks in the form of $\mathbf{b}_2\mathbf{D}_t$, and the second is a series of one-off "blips" in the form of $\mathbf{C}^*(L)\mathbf{a}_2\Delta\mathbf{D}_t$, where

$$\Delta D_{jt} = \begin{cases} 1, \text{ for } t = t_{j0} + 1, \\ -1, \text{ for } t = t_{jq} + 1, \\ 0, \text{ otherwise}, \end{cases}, j = 1, 2, \ldots, n_d.$$

The level of $\mathbf{z}_t$ in (1.36) also contains two sources of structural change. The first takes the form of the broken trends $\mathbf{b}_2\sum_{i=1}^t \mathbf{D}_i$, where $\sum_{i=1}^t \mathbf{D}_i = [\sum_{i=1}^t D_{1i}, \sum_{i=1}^t D_{2i}, \ldots, \sum_{i=1}^t D_{n_d i}]'$and

$$\sum_{i=1}^{t} D_{ji} = \begin{cases} t - t_{j0}, \text{ for } t_{j0} < t \leq t_{jq}, \\ t_{j0} - t_{jq}, \text{ for } t > t_{jq}, \\ 0, \text{ otherwise}, \end{cases} \quad , j = 1, 2, \ldots, n_d.$$

The second is a smoothed version of $\mathbf{D}_t$ in the form of

$$\mathbf{C}^*(L)\mathbf{a}_2\mathbf{D}_t = \sum_{i=0}^{t} C_i^*\mathbf{a}_2\mathbf{D}_{t-i} = \sum_{j=1}^{n_d}\sum_{i=0}^{t}(C_i^*\mathbf{a}_2)^j D_{j,t-i},$$

where

$$\sum_{i=0}^{t}(C_i^*\mathbf{a}_2)^j D_{j,t-i} = \begin{cases} \sum_{i=t-t_{jq}}^{t-t_{j0}-1}(C_i^*\mathbf{a}_2)^j, \text{ for } t_{j0} < t \leq t_{jq}, \\ 0, \text{ otherwise}, \end{cases} \quad , j = 1, 2, \ldots, n_d,$$

and $(C_i^*\mathbf{a}_2)^j$ is the j-th column of $C_i^*\mathbf{a}_2$. A smoothed version of $\mathbf{D}_t$ appears also in the cointegrating relations in (1.37) in the form of $\beta'\mathbf{C}^*(L)\mathbf{a}_2\mathbf{D}_t$.

As was discussed in the previous section, leaving the coefficient $\mathbf{a}_1$ unrestricted results in the level of $\mathbf{z}_t$ exhibiting quadratic deterministic behaviour, with the number of independent quadratic trends varying with r according to $rank[\mathbf{C}(1)] = n-r$. In order to avoid this property while still allowing for trend-stationary cointegrating vectors, it was suggested to impose the restriction $\mathbf{a}_1 = -\Pi\gamma$ according to Case IV. A similar argument can be made in favour of restricting $\mathbf{a}_2$. Leaving $\mathbf{a}_2$ unrestricted results in the level of $\mathbf{z}_t$ in (1.36) containing the broken trends $\mathbf{b}_2\sum_{i=1}^t \mathbf{D}_i$ with the number of independent broken trends varying, again, according to $rank[\mathbf{C}(1)] = n - r$.

Taking both of these considerations into account results in what will be denoted as **Case IVd**, where $\mathbf{a}_0 \neq \mathbf{0}$, $\mathbf{a}_1 = -\Pi\gamma$ and $\mathbf{a}_2 = -\Pi\delta$, where γ and δ are $n \times 1$ and $n \times n_d$, respectively, both containing unknown coefficients. This set of restrictions results in $\mathbf{b}_0 = \mathbf{C}(1)\mathbf{a}_0 + \gamma$ and $\mathbf{b}_1 = \mathbf{b}_2 = \mathbf{0}$. Therefore, as in Case IV, quadratic trends are eliminated and a linear trend is present in $\mathbf{z}_t$ as $[\mathbf{C}(1)\mathbf{a}_0+\gamma]t$ and in $\beta'\mathbf{z}_t$ as $\beta'\gamma t$. Furthermore, the broken trends are eliminated and a broken intercept is present in both $\mathbf{z}_t$ and $\beta'\mathbf{z}_t$; in the first instance as $-\mathbf{C}^*(L)\Pi\delta\mathbf{D}_t$ and in the second as $\beta'\mathbf{C}^*(L)\Pi\delta\mathbf{D}_t$.

1.4.2 Testing for Cointegrating Rank under Different Specifications for the Deterministic Terms

Different specifications for the deterministic components inevitably require different definitions for $\mathbf{r}_{0t}$ and $\mathbf{r}_{1t}$ in (1.18) when testing for $rank[\Pi]$. To illustrate this point consider the general, reduced-form cointegrating $VAR(p)$ in (1.4) which, for example, under Case IVd may be written as

$$\Delta\mathbf{z}_t = \mathbf{a}_0 + \Pi_*\mathbf{z}_{*t-1} + \sum_{i=1}^{p-1}\Gamma_i\Delta\mathbf{z}_{t-i} + \mathbf{e}_t, \tag{1.38}$$

where $\mathbf{z}_{*t-1} = [\mathbf{z}'_{t-1}, t, \mathbf{D}'_t]'$, $\Pi_* = \Pi[I_n, -\gamma, -\delta]$ and provided that $0 < r < n$, $\Pi_* = \alpha\beta'_*$ with $\beta'_* = [\beta', -\beta'\gamma, -\beta'\delta]$. Therefore, the problem of determining the number of cointegrating vectors in the context of (1.38) is a question of $rank[\Pi_*]$, where the estimate of the stochastic Π_*-matrix is now given by

$$\mathbf{S}_{*00}^{-1}\mathbf{S}_{*01}\mathbf{S}_{*11}^{-1}\mathbf{S}_{*10} \tag{1.39}$$

and the matrices $\mathbf{S}_{*00}$, $\mathbf{S}_{*01}$, $\mathbf{S}_{*10}$ and $\mathbf{S}_{*11}$ are defined as follows:

$$\mathbf{S}_{*ij} = T^{-1}\sum_{t=1}^{T}\mathbf{r}_{it}\mathbf{r}'_{jt}, \qquad i,j=0,1, \tag{1.40}$$

with $\mathbf{r}_{0t}$ and $\mathbf{r}_{1t}$, being now the residuals obtained from the OLS regressions of $\Delta\mathbf{z}_t$ and $[t, \mathbf{D}'_t, \mathbf{z}'_{t-1}]'$ on $[1, \Delta\mathbf{z}'_{t-1}, \Delta\mathbf{z}'_{t-2}, ..., \Delta\mathbf{z}'_{t-p+1}]'$, respectively.

In general, $\mathbf{r}_{0t}$ is the residual obtained from the OLS regression of $\Delta\mathbf{z}_t$ on $[\psi_t^{u\prime}, \Delta\mathbf{z}'_{t-1}, \Delta\mathbf{z}'_{t-2}, ..., \Delta\mathbf{z}'_{t-p+1}]'$, where ψ_t^u contains all unrestricted deterministic terms, i.e. those terms for which $\mathbf{a}_i \neq 0$, $i = 0, 1, 2$. The residual $\mathbf{r}_{1t}$ is obtained from the OLS regression of $[\psi_t^{r\prime}, \mathbf{z}'_{t-1}]'$ on $[\psi_t^{u\prime}, \Delta\mathbf{z}'_{t-1}, \Delta\mathbf{z}'_{t-2}, ..., \Delta\mathbf{z}'_{t-p+1}]'$, where ψ_t^r contains the deterministic terms that are restricted so that they may enter the cointegrating relations, i.e. those terms for which $\mathbf{a}_i$, $i = 0, 1, 2$ is restricted to be a multiple of Π.

Asymptotic distributions for the cointegrating rank statistics under Cases I-V have been derived and tabulated in, *inter alia*, Johansen (1995; sections 11,12) while PSS generalise the results for the case of partial systems. However, as shown in Johansen and Nielsen (1994) and Johansen, Mosconi and Nielsen (2000), the asymptotic distribution of the tests in the presence of intervention dummies is not only model but also variable-specific, as it depends on the timing of the break(s). The variety of cases that can arise in the presence of n_d intervention dummies, each with potentially different break timing is rich enough to have deterred any attempts for systematic tabulation of asymptotic percentiles. Therefore, asymptotic critical values rarely emerge in the literature, e.g. Johansen, Mosconi and Nielsen (2000) and Jacobson *et al* (2001), and when they do they are generally quite case-specific.

1.5 Partial Systems

Although, in principle, all variables in $\mathbf{z}_t$ may be treated symmetrically within (1.4), there are certain advantages to be gained in empirical applications by treating a sub-set of $\mathbf{z}_t$ as *weakly exogenous*. It is, thus, not surprising that cointegrating VAR models conditional on weakly exogenous $I(1)$ variables have attracted a considerable amount of interest. The concept of weak exogeneity within the cointegrating VAR model was first investigated by Johansen (1992) and further discussed in Johansen (1995, section 8) whereas, more recently, PSS provided an integrated statistical framework for estimation and inference.

In order to illustrate the concept as well as the implications of weak exogeneity consider the case when it is possible to partition $\mathbf{z}_t$ into a vector $\mathbf{y}_t$ of n_y endogenous variables and a vector $\mathbf{x}_t$ of n_x exogenous variables, $\mathbf{z}_t = [\mathbf{y}_t', \mathbf{x}_t']'$, and similarly the matrices $\mathbf{a} = [\mathbf{a}_y', \mathbf{a}_x']'$, $\Gamma_i = [\Gamma_{iy}', \Gamma_{ix}']'$, $i = 1, ..., p-1$, $\boldsymbol{\alpha} = [\boldsymbol{\alpha}_y', \boldsymbol{\alpha}_x']'$ and the disturbance vector $\mathbf{e}_t = [\mathbf{e}_{yt}', \mathbf{e}_{xt}']'$ with variance matrix $\boldsymbol{\Omega} = \begin{bmatrix} \Omega_{yy} & \Omega_{yx} \\ \Omega_{xy} & \Omega_{xx} \end{bmatrix}$. It is, thus, possible to re-write the general, symmetric $VECM$ given by (1.4) in partitioned form as

$$\begin{bmatrix} \Delta \mathbf{y}_t \\ \Delta \mathbf{x}_t \end{bmatrix} = \begin{bmatrix} \mathbf{a}_y \\ \mathbf{a}_x \end{bmatrix} \psi_t + \begin{bmatrix} \boldsymbol{\alpha}_y \\ \boldsymbol{\alpha}_x \end{bmatrix} \boldsymbol{\beta}' \mathbf{z}_{t-1} + \sum_{i=1}^{p-1} \begin{bmatrix} \Gamma_{iy} \\ \Gamma_{ix} \end{bmatrix} \Delta \mathbf{z}_{t-i} + \begin{bmatrix} \mathbf{e}_{yt} \\ \mathbf{e}_{xt} \end{bmatrix}. \tag{1.41}$$

Multiplication of the second row of (1.41) by $\Upsilon = \Omega_{yx}\Omega_{xx}^{-1}$ and subtraction from the first yields the conditional model for $\Delta \mathbf{y}_t$ given $\Delta \mathbf{x}_t$ as

$$\Delta \mathbf{y}_t = \mathbf{c}\psi_t + (\boldsymbol{\alpha}_y - \Upsilon \boldsymbol{\alpha}_x)\boldsymbol{\beta}' \mathbf{z}_{t-1} + \sum_{i=1}^{p-1} \Psi_i \Delta \mathbf{z}_{t-i} + \Upsilon \Delta \mathbf{x}_t + \mathbf{u}_t, \tag{1.42}$$

where $\mathbf{c} = \mathbf{a}_y - \Upsilon\mathbf{a}_x$, $\Psi_i = \Gamma_{iy} - \Upsilon\Gamma_{ix}$, $i = 1, ..., p-1$, and $\mathbf{u}_t = \mathbf{e}_{yt} - \Upsilon\mathbf{e}_{xt}$, while the marginal model of $\Delta\mathbf{x}_t$ is given by

$$\Delta\mathbf{x}_t = \mathbf{a}_x\psi_t + \boldsymbol{\alpha}_x\boldsymbol{\beta}'\mathbf{z}_{t-1} + \sum_{i=1}^{p-1}\Gamma_{ix}\Delta\mathbf{z}_{t-i} + \mathbf{e}_{xt}. \tag{1.43}$$

Despite the fact that the process $\mathbf{u}_t$ is by construction orthogonal to $\mathbf{e}_{xt}$[11], it is not possible to render the information provided by the marginal model (1.43) redundant in the efficient estimation and inference concerning the parameters of the conditional model (1.42) as they both share the common cointegrating matrix $\boldsymbol{\beta}$. As Johansen (1995; pp.122) points out "*...there may be a considerable problem as well as loss of information in the analysis of the conditional equation (.) without taking into account the second equation (.)*".

A sufficient condition for the efficient analysis of (1.42) is

$$\boldsymbol{\alpha}_x = \mathbf{0}, \tag{1.44}$$

known as *weak exogeneity condition.* Under (1.44) the conditional and the marginal models take the form

$$\Delta\mathbf{y}_t = \mathbf{c}\psi_t + \boldsymbol{\alpha}_y\boldsymbol{\beta}'\mathbf{z}_{t-1} + \sum_{i=1}^{p-1}\Psi_i\Delta\mathbf{z}_{t-i} + \Upsilon\Delta\mathbf{x}_t + \mathbf{u}_t, \tag{1.45}$$

$$\Delta\mathbf{x}_t = \mathbf{a}_x\psi_t + \sum_{i=1}^{p-1}\Gamma_{ix}\Delta\mathbf{z}_{t-i} + \mathbf{e}_{xt}, \tag{1.46}$$

where the marginal model (1.46) is now redundant for efficient conditional estimation and

[11] This is immediately verified since $E(\mathbf{e}_{xt}\mathbf{u}_t') = E[\mathbf{e}_{xt}(\mathbf{e}_{yt} - \Upsilon\mathbf{e}_{xt})'] = E(\mathbf{e}_{xt}\mathbf{e}_{yt}')$ - $E(\mathbf{e}_{xt}\mathbf{e}_{xt}'\Omega_{xx}^{-1\prime}\Omega_{yx}') = \Omega_{xy} - \Omega_{xx}\Omega_{xx}^{-1}\Omega_{xy} = \mathbf{0}$.

inference concerning both the deterministic and short-run coefficients $\mathbf{c}$, $\boldsymbol{\alpha}_y$, $\boldsymbol{\Upsilon}$, and Ψ_i, $i = 1, 2, \ldots, p-1$, as well as the long-run cointegrating coefficients $\boldsymbol{\beta}$. The obvious advantage of this in empirical applications is that, in the presence of weak exogeneity, one can escape the explicit modelling of $\mathbf{x}_t$, thus effectively reducing the dimensions of the estimated system and economising on degrees of freedom. Furthermore, the exogeneity condition (1.44) may be used to provide part of the restrictions required in order to identify the structural or policy shocks $\mathbf{v}_t$ in (1.20), although this will be discussed in more detail in subsequent sections.[12]

Pesaran and Smith (1998), among others, stress the fact that in the case of economic applications, economic theory can be a very useful guide in choosing the candidate variables to be included in the weakly exogenous vector $\mathbf{x}_t$. This is because under (1.44) the vector $\mathbf{x}_t$ has the economic interpretation of *long-run forcing* with respect to $\mathbf{y}_t$. This property is indicated by the fact that $\mathbf{x}_t$ can influence $\mathbf{y}_t$ in (1.45) both in the short run through the terms $\sum_{i=1}^{p-1} \Psi_i \Delta \mathbf{z}_{t-i}$ and $\boldsymbol{\Upsilon} \Delta \mathbf{x}_t$, as well as in the long run through the error correction terms $\boldsymbol{\alpha}_y \boldsymbol{\beta}' \mathbf{z}_{t-1}$, while $\mathbf{y}_t$ can only have a short-run effect on $\mathbf{x}_t$ *via* the terms $\sum_{i=1}^{p-1} \Gamma_{ix} \Delta \mathbf{z}_{t-i}$ in (1.46). Thus, in applications that involve the modelling of small open economies, for example, where $\mathbf{z}_t$ includes both, domestic and foreign variables, the natural candidate for $\mathbf{x}_t$ would be the set of foreign variables. However, weak exogeneity of $\mathbf{x}_t$ is clearly a hypothesis that is defined in terms of the parameters of the model and as such it may be formally tested.

[12] Perhaps one disadvantage of working with a cointegrating *VAR* model conditional on weakly exogenous $I(1)$ variables like (1.45) is the additional burden it places on computing time when engaging into simulation exercises, as will be illustrated later in this chapter.

1.5.1 Testing for Weak Exogeneity: Johansen (1992) and Pesaran, Shin and Smith (2000)

There are two main approaches to testing for weak exogeneity. The first one is due to Johansen (1992) who proposes a direct test of condition (1.44) within the symmetric system (1.41) by means of the likelihood ratio statistic which is shown to be asymptotically χ^2 with rn_x degrees of freedom. However, as mentioned earlier, the increasing interest in partial systems in macroeconomic applications arises primarily from the limited amount of available data points and the need to escape the explicit modelling of all the variables in $\mathbf{z}_t$. From this point of view, the Johansen (1992) approach has the disadvantage that it requires estimation of the symmetric, n-dimensional system.

In cases where the dimensions of the full system are prohibitive, or at least not easily handled, PSS propose an alternative approach. This is based on the implications of the weak exogeneity condition (1.44) on the marginal model. A casual inspection of (1.46) immediately reveals that the marginal model does not involve any levels' relationships. Consequently, the presence of cointegration in (1.46) would constitute a rejection of weak exogeneity of $\mathbf{x}_t$. This clearly suggests the use of the *maximum eigenvalue* and/or $\lambda - trace$ cointegration rank tests, defined in (1.14) and (1.16), respectively, within the marginal model as an alternative way of testing for weak exogeneity. However, this raises the following issue.

The Johansen (1992) test was characterised above as *direct* in the sense that the null hypothesis tested *is* the weak exogeneity condition (1.44) and is mutually exclusive to the alternative. Thus, a rejection/non-rejection of the null corresponds directly to a rejection/non-rejection of weak exogeneity. When testing for cointegration within the marginal model (1.46), however, this is no longer the case. Even though a rejection of the null of no cointegration is equivalent to rejection of weak exogeneity, failure to reject the null merely indicates absence of cointegration among the x_i's, $i = 1, 2, \ldots, n_x$. This does not necessarily verify the validity of the weak exogeneity condition (1.44), as it does not exclude the possibility of non-zero $\boldsymbol{\alpha}_x$ loadings on error-correction terms of the form $[\boldsymbol{\beta}_y', \mathbf{0}_{n_x}]\mathbf{z}_{t-1}$, where $\boldsymbol{\beta}_y'$ is $r \times n_y$ and $\mathbf{0}_{n_x}$ is an $r \times n_x$ matrix of zeros. Thus, testing for cointegrating rank in the marginal model may only provide conclusive evidence *against* weak exogeneity of $\mathbf{x}_t$ in the form of cointegrating relations among the x_i's, $i = 1, 2, \ldots, n_x$. In the inconclusive case of non cointegration in the marginal model, one possible course of action suggested in an early version of PSS is to obtain the estimates $\hat{\boldsymbol{\beta}}'\mathbf{z}_{t-1}$ using the conditional model in (1.45) and then test for their significance in the marginal model in (1.46).

1.5.2 Asymptotic Inference within Partial Systems and the Special Case in Rahbek and Mosconi (1999)

PSS provide an integrated framework for estimation and asymptotic inference within cointegrating VAR models conditional on weakly exogenous $I(1)$ variables of the type of (1.45) with deterministic components given by Cases I-V discussed earlier. In the case of partitioned systems the restrictions on the deterministic components $\mathbf{a}_0 = -\boldsymbol{\Pi}\boldsymbol{\mu}$ and $\mathbf{a}_1 = -\boldsymbol{\Pi}\boldsymbol{\gamma}$ take the form $\mathbf{c}_0 = -\boldsymbol{\Pi}_y\boldsymbol{\mu}$ and $\mathbf{c}_1 = -\boldsymbol{\Pi}_y\boldsymbol{\gamma}$, where $\boldsymbol{\Pi}_y = \boldsymbol{\alpha}_y\boldsymbol{\beta}'$ and $\mathbf{c}_i = \mathbf{a}_{iy} - \boldsymbol{\Upsilon}\mathbf{a}_{ix}$, $i = 0, 1$. PSS show that the asymptotic distribution of the LR statistic for testing $\mathbf{c}_0 = -\boldsymbol{\Pi}_y\boldsymbol{\mu}$ and $\mathbf{c}_1 = -\boldsymbol{\Pi}_y\boldsymbol{\gamma}$ conditional on $rank[\boldsymbol{\Pi}_y] = r$ is χ^2 with $n_y - r$ degrees of freedom. Under Case IV, for example, the conditional and marginal models take the form

$$\Delta \mathbf{y}_t = \mathbf{c}_0 + \Pi_{y*}\mathbf{z}_{*t-1} + \sum_{i=1}^{p-1}\Psi_i \Delta \mathbf{z}_{t-i} + \Upsilon \Delta \mathbf{x}_t + \mathbf{u}_t, \tag{1.47}$$

$$\Delta \mathbf{x}_t = \mathbf{a}_{0x} + \sum_{i=1}^{p-1}\Gamma_{ix}\Delta \mathbf{z}_{t-i} + \mathbf{e}_{xt}, \tag{1.48}$$

respectively, where $\mathbf{z}_{*t-1} = [\mathbf{z}'_{t-1}, t]'$, $\Pi_{y*} = \Pi_y[I_n, -\gamma]$ and provided that $0 < r < n$, $\Pi_{y*} = \alpha_y \beta'_*$ with $\beta'_* = [\beta', -\beta'\gamma]$. For statistical inference on cointegrating rank within (1.47) PSS present modified versions of the cointegration rank statistics (1.14) and (1.16), formulated in terms of $rank[\Pi_{y*}]$. The stochastic Π_{y*}-matrix is given by

$$\mathbf{S}_{y*00}^{-1}\mathbf{S}_{y*01}\mathbf{S}_{y*11}^{-1}\mathbf{S}_{y*10} \tag{1.49}$$

and the matrices $\mathbf{S}_{y*00}$, $\mathbf{S}_{y*01}$, $\mathbf{S}_{y*10}$ and $\mathbf{S}_{y*11}$ are defined as follows

$$\mathbf{S}_{y*ij} = T^{-1}\sum_{t=1}^{T}\mathbf{r}_{it}\mathbf{r}'_{jt}, \qquad i,j = 0,1, \tag{1.50}$$

where, in general, $\mathbf{r}_{0t}$ is the residual obtained from the OLS regression of $\Delta \mathbf{y}_t$ on $[\psi_t^{u\prime}, \Delta \mathbf{z}'_{t-1}, \Delta \mathbf{z}'_{t-2}, ..., \Delta \mathbf{z}'_{t-p+1}, \Delta \mathbf{x}'_t]'$ and ψ_t^u contains all unrestricted deterministic terms, i.e. those terms for which $\mathbf{c}_i \neq \mathbf{0}$, $i = 0, 1$. The residual $\mathbf{r}_{1t}$ is generally obtained from the OLS regressions of $[\psi_t^{r\prime}, \mathbf{z}'_{t-1}]'$ on $[\psi_t^{u\prime}, \Delta \mathbf{z}'_{t-1}, \Delta \mathbf{z}'_{t-2}, ..., \Delta \mathbf{z}'_{t-p+1}, \Delta \mathbf{x}'_t]'$, where ψ_t^r contains the deterministic terms that are restricted so that they may enter the cointegrating relations, i.e. those terms for which $\mathbf{c}_i$, $i = 0, 1$ is restricted to be a multiple of Π_y. PSS derive the limit distributions of the modified cointegration rank statistics and tabulate the asymptotic critical values for Cases

I-V.

However, there is a minor issue concerning asymptotic inference on cointegrating rank within conditional models that deserves further attention. This is a rather technical point raised in the study of Rahbek and Mosconi (1999) and revolves around the statistical concept of *"asymptotic similarity"*. A test is called asymptotically similar with respect to a parameter φ if the rejection frequency under the null is independent of φ as the sample size approaches infinity.[13] Rahbek and Mosconi (1999) demonstrate that in a model given by

$$\Delta \mathbf{y}_t = \alpha\beta' \mathbf{y}_{t-1} + \sum_{i=1}^{p-1} \Gamma_i \Delta \mathbf{y}_{t-i} + \Theta \mathbf{w}_t + \epsilon_t, \tag{1.51}$$

where $\mathbf{y}_t$ is an n_y-dimensional $I(1)$ process and $\mathbf{w}_t$ is an n_w-dimensional $I(0)$ process, the rejection frequency (and thus also the critical values) of the cointegration rank tests is not asymptotically similar with respect to Θ due to the presence of *nuisance parameters* in the asymptotic distribution of the tests. The intuition behind this result is highlighted with the use of the Granger representation of $\mathbf{y}_t$ that takes the form of

$$\mathbf{y}_t = \mathbf{C}(1) \sum_{i=1}^{t} (\epsilon_i + \Theta \mathbf{w}_i) + \mathbf{C}^*(L)(\epsilon_t + \Theta \mathbf{w}_t) + \mathbf{A}, \tag{1.52}$$

where $\mathbf{C}(1)$ and $\mathbf{C}^*(L)$ are given by (1.29) and (1.26) respectively, while $\mathbf{A}$ depends on initial values. The non-similarity of the limit distribution with respect to Θ is shown to be due to the lack of balance in the way that $\mathbf{w}_t$ appears in the stationary and non-stationary parts of $\mathbf{y}_t$. In particular, the $I(1)$ part of $\mathbf{y}_t$ contains the accumulated terms $\sum_{i=1}^{t} \mathbf{w}_i$, while the stationary

[13] For more details see also Nielsen and Rahbek (2000).

part $\beta' \mathbf{y}_t$ does not, due to (1.29).

In order to eliminate the nuisance parameters from the asymptotic distribution of the cointegration rank tests and, thus, restore asymptotic similarity, Rahbek and Mosconi propose the inclusion of $\sum_{i=1}^{t} \mathbf{w}_i$ as a regressor in (1.51) with a coefficient of the form $-\Pi\vartheta$, or equivalently $-\alpha\beta'\vartheta$, where ϑ is an $n_y \times n_w$ matrix of unknown parameters. Under this modification, the model becomes

$$\Delta \mathbf{y}_t = \alpha\beta_*' \mathbf{y}_{*t-1} + \sum_{i=1}^{p-1} \Gamma_i \Delta \mathbf{y}_{t-i} + \Theta \mathbf{w}_t + \epsilon_t, \tag{1.53}$$

where $\mathbf{y}_{*t-1} = [\mathbf{y}_{t-1}', \sum_{i=1}^{t} \mathbf{w}_i']'$ and $\beta_*' = [\beta', -\beta'\vartheta]$ which illustrates that $\sum_{i=1}^{t} \mathbf{w}_i$ now enters the cointegrating relations.

The model in (1.51) can arise as a special case of a cointegrating $VAR(p)$ conditional on weakly exogenous variables. In particular, consider the conditional model in (1.45) with the deterministic terms treated according to Case I and $p = 1$

$$\Delta \mathbf{y}_t = \alpha_y \begin{bmatrix} \beta_y' & \beta_x' \end{bmatrix} \begin{bmatrix} \mathbf{y}_{t-1} \\ \mathbf{x}_{t-1} \end{bmatrix} + \Upsilon \Delta \mathbf{x}_t + \mathbf{u}_t, \tag{1.54}$$

where β_y, β_x are $n_y \times r$ and $n_x \times r$ respectively. In the event that $\mathbf{x}_t \sim I(1)$, then for $\beta_x' = \mathbf{0}$ the model takes the form of (1.51) with $\Delta \mathbf{x}_t$ playing the role of $\mathbf{w}_t$. Although PSS [section 4.3] appreciate this possibility, they stress the fact that the inclusion of the cumulative terms in the cointegrating relations as suggested by Rahbek and Mosconi in (1.53) may lack the support of economic theory. They, therefore, suggest a test of $-\beta'\vartheta = \mathbf{0}$, which is a test of n_w over-identifying, zero-restrictions in each of the r, exactly identified cointegrating relations and is, thus, asymptotically χ^2 with rn_w degrees of freedom.

In the event that $\mathbf{x}_t \sim I(0)$ and $\boldsymbol{\beta}_x' = \mathbf{0}$ then the terms $\Delta\mathbf{x}_t$ in (1.54) follow an $I(-1)$, or over-differenced process. As a result, they cumulate to a stationary process which as explained in Nielsen and Rahbek (2000) implies that the terms $\sum_{i=1}^{t} \mathbf{w}_i$ are of smaller order of magnitude than $\sum_{i=1}^{t} \epsilon_i$ and can, thus, be ignored for asymptotic purposes. However, even with a stationary $\mathbf{x}_t$ there is still the possibility $\boldsymbol{\alpha}_y\boldsymbol{\beta}_x' = \boldsymbol{\Upsilon}$, in which case the Rahbek and Mosconi criticism applies. Again, a test of $-\boldsymbol{\beta}'\boldsymbol{\vartheta} = \mathbf{0}$ suggested by PSS will indicate whether the cumulative terms in (1.53) can be ignored.

1.6 Finite Sample Inference within Cointegrating VAR Models

As discussed in the previous sections, statistical inference within the cointegrating $VAR(p)$ framework relies almost entirely on the use of LR statistics[14]. Their asymptotic distribution may be standard chi-squared (LR tests for the choice of the intercept/trend specification, LR tests of over-identifying restrictions on the cointegrating matrix, $\boldsymbol{\beta}$) or non-standard (cointegration rank tests). In both cases a substantial body of evidence has accumulated over the past decade revealing that in finite samples the asymptotic distributions provide a very poor approximation to the actual distribution of the test statistics. Reinsel and Ahn (1988), Blangiewicz and Charemza (1989), Reimers (1992), Cheung and Lai (1993), van Giersbergen (1996), Harris and Judge (1998), Mantalos and Shukur (1998) and Greenslade *et al* (2002) among others, suggest that the asymptotic distributions of the $\lambda - trace$ and *maximal eigenvalue* cointegrating rank tests under-estimate the finite-sample critical values, thus leading to over-rejection

[14]The use of (appropriately modified) model selection criteria such as the SBC, AIC and HQC has been considered in *inter alia* Lütkepohl (1993), Pesaran and Pesaran (1997) and Pesaran and Smith (1998) as a means for joint determination of lag-length and cointegrating rank. However, as noted in Lütkepohl (1993; pp.387) "*the statistical properties of such a procedure are currently unknown in general*".

of the null. Gredenhoff and Jacobson (1998), Fachin (2000), Johansen (2000a, b) and Jacobson *et al* (2001) are examples of studies illustrating that the small-sample bias of chi-squared tests can also be quite significant. As a consequence of the evolution of this rather rich literature, the small-sample bias in asymptotic inference within multivariate cointegrating systems is considered today to be well-documented and its presence is undisputed.

Following the developments in the cointegrating VAR literature, the studies on finite-sample inference focused first on the cointegration rank tests and later on χ^2 tests. The need to control for sample size when testing for cointegration was recognised as early as Reinsel and Ahn (1988). It was both the growing amount and the compelling nature of the findings supporting the presence of a significant finite-sample bias that quickly resulted in the emphasis being placed on *how* rather than *whether* to deal with it. Two general approaches have been considered.

1.6.1 Finite Sample Correction Factors

The first approach to controlling for sample size is the traditional adjustment of the test statistics (or the asymptotic critical values) by a correction factor that takes into account the sample size, the number of estimated parameters and available degrees of freedom in the tradition of Sims' (1980) Adjusted Likelihood Ratio (ALR). The idea of adjusting the statistics, or the asymptotic critical values, through multiplication by a single scaling factor was particularly appealing in the late 80's and early 90's when the limited power of microcomputers did not allow for widespread use of simulation methods.

Reinsel and Ahn (1988, 1992) proposed a small-sample correction for symmetric systems by replacing T by $T - np$ in the calculation of the *maximal eigenvalue* and $\lambda - trace$ statistics in (1.14) and (1.16), respectively, where T, n and p are defined below (1.1). This is equivalent to scaling the statistics by a factor

$$SF = (T - np)/T, \tag{1.55}$$

or, as in Cheung and Lai (1993), adjusting the asymptotic critical values by SF^{-1}. Since both, n and p are positive integers and for $np < T$, then $SF \in (0;1)$ and $SF^{-1} \in (1;T]$ for finite values of T. This implies that in finite samples the unadjusted test statistics are too large, or equivalently, the asymptotic critical values are too small, thus leading to over-rejection of the non-cointegration null. As mentioned in Johansen (1995; pp.99) "*the theoretical justification for this result* (i.e. the scaling of the statistics by SF) *presents a very difficult mathematical problem*", which is why the studies on the validity of this approach had to rely on simulation methods. Reimers (1992) and Cheung and Lai (1993) investigated the idea of the Reinsel and Ahn scaling factor and found support in favour of the suggested correction with the use of Monte Carlo simulations.

Cheung and Lai (1993), in particular, investigate the Reinsel and Ahn correction within a symmetric system under Case III. They estimate the following response surface equation

$$\frac{CR_T}{CR_\infty} = \beta_0 + \beta_1 SF^{-1} + \text{errors}, \tag{1.56}$$

where CR_T is the simulated estimate of the finite-sample critical value and CR_∞ is the asymptotic critical value at the corresponding significance level. The ratio CR_T/CR_∞ is a measure of finite-sample bias; the further away from unity it is, the greater the extent of the bias. In the case that $\beta_0 = 0$ and $\beta_1 = 1$ the bias varies according to the Reinsel and Ahn scaling factor. The small-sample critical value CR_T was simulated for various values of T, n and p using 20,000 replications and in all cases the joint restriction $\beta_0 = 0$ and $\beta_1 = 1$ was rejected.

This suggests that the proposed correction factor does not yield unbiased estimates of the *true* finite-sample critical values and that more accurate estimates can be provided by the response surface. However, in all cases, β_1 was found to be significantly positive and close to 0.9. This indicates that the bias is a positive function of SF^{-1}, thus, verifying that in finite samples the cointegration rank tests tend to over-reject the null of no cointegration. Using a similar approach, the same authors also find that the λ-*trace* statistic is more reliable than the *maximal eigenvalue* in the absence of residual normality.

As the cointegrating VAR literature started to address the issues of the treatment of deterministic regressors and the testing of hypotheses on the cointegrating matrix, Johansen and Juselius (1990), Johansen (1991), the familiar χ^2 distribution made its debut in asymptotic inference within multivariate cointegrating systems. The timing of these developments coincided with the surging interest on bootstrap methods (discussed in the following sub-section) and as a result, the bulk of the literature investigating the finite-sample bias in χ^2 tests abstracts from the use of correction factors. A recent revival of the adjustment approach, however, can be found in Johansen (2000a, b). These two studies propose a *Bartlett* correction [see Bartlett (1937)] for the likelihood ratio test of linear over-identifying restrictions on the cointegrating matrix, $\boldsymbol{\beta}$. The first study considers the case of known adjustment coefficients, $\boldsymbol{\alpha}$, and the second generalises the results for an unknown $\boldsymbol{\alpha}$-matrix in symmetric systems like (1.4) under Cases IV and V. The proposed correction factor is rather complicated and takes the general form

$$
\begin{aligned}
SF_B &= n_v n_a + \frac{n_v n_a}{T}[\frac{1}{2}(n_v + n_a + 1) + n_d + n + n_z] + \\
&\quad + \frac{n_a}{T}[(n - n_v + n_a - 1)v(\xi) + 2(c(\xi) + c_d(\xi))],
\end{aligned} \tag{1.57}
$$

where n_v is the dimension of the restricted cointegrating vectors, n_a is the number of stochastic trends plus the number of restricted deterministic terms, n_z is the dimension of the vector of (demeaned) stacked lagged differences $[\Delta \mathbf{z}'_{t-1} - E(\Delta \mathbf{z}'_{t-1}), \ldots, \Delta \mathbf{z}'_{t-p+1} - E(\Delta \mathbf{z}'_{t-p+1})]'$, n_d is the number of unrestricted deterministic terms and $v(\xi)$, $c(\xi)$ and $c_d(\xi)$ are complicated functions of model parameters. As before, the usefulness of the scaling factor in (1.57) is demonstrated with the use of simulation methods.

Overall, the evidence on adjusted statistics indicated that they are an improvement over asymptotic inference. However, results have also suggested that the finite-sample bias cannot be adequately quantified by a simple scaling factor. Perhaps the most crucial point in studies like Cheung and Lai (1993), though, is the implicit acknowledgement of the superiority of simulation methods. After all, it was the simulated CR_T's in (1.56) that were used to produce the *true* measure of the bias by which the performance of the scaling factor was evaluated. This view, in combination with the dramatic increase in computing power, attracted considerable attention on bootstrap methods.

1.6.2 The Bootstrap Approach in the Cointegrating $VAR(p)$

The idea of the bootstrap was originally introduced by Efron (1979) and later applied to the simple regression model by Freedman (1981). The general bootstrap approach is as follows. If a distribution Ψ is expressed in terms of a parameter of interest θ and inference on θ can be based on a statistic S then it is possible to obtain an empirical approximation of the distribution of S based on a single random sample from Ψ in the following manner. If $(y_1, y_2, ..., y_q)$ is the available random sample from Ψ, generate B bootstrap samples of size l (usually chosen to be equal to q) $(y_1^j, y_2^j, ..., y_l^j)$, $j = 1, ..., B$ by drawing with replacement from $(y_1, y_2, ..., y_q)$.

For each of the bootstrap samples it is possible to compute the bootstrap statistic S^j the distribution of which is known as the bootstrapped distribution of S and can be used for making inference about θ. This particular method is valid for IID observations. Departures from the IID assumption have led to modifications, primarily, in the resampling method[15] but the underlying idea remains the same, i.e., use the single available sample to generate bootstrap samples in order to obtain an approximation of the distribution of S.

Following the dramatic increase in computing power, the bootstrap approach gained increasing popularity as a means for simulating the finite-sample distributions in multivariate cointegration analyses. Perhaps the first issue that arises when considering the application of the bootstrap in the context of a general regression model, is whether to resample the data set directly or resample the residuals first and use them in order to generate the bootstrap data sets. Li and Maddala (1997) argue that in the context of cointegrating regressions where the variables under consideration are $CI(1,1)$ the direct resampling of the data is not a valid approach for two reasons. First, resampling the data directly will not take into account the cointegrating properties of the data series, which is a piece of information that is used in the estimation of the model and, second and most important, a bootstrap sample of an $I(1)$ process is not $I(1)$ at all. Li and Maddala (1993) provide some empirical evidence on the superiority of the residual-based bootstrap over the direct resampling scheme in the context of cointegration regression models.

Thus, in the context of a cointegrating VAR model the bootstrap approach can be summarised in the following three steps:

(i) The model is estimated under the null hypothesis of interest using the available data set

[15] For an overview see Li and Maddala (1996)

of length T.

(ii) A large number of, say, B pseudo-data sets of length T are simulated using the estimates from (i) plus B *appropriately* constructed disturbance vectors $\mathbf{e}_t^j$, $j = 1, ..., B$, $t = 1, ..., T$.

(iii) The statistic of interest is computed for each of the B simulated data sets in order to obtain an empirical approximation of the finite-sample distribution of the test statistic.

The two broad methods that have been adopted for the generation of the disturbance terms to be used in step (ii) gave rise to the distinction between *parametric* and *non-parametric* bootstrap. In the parametric version the error terms used in the simulation of the pseudo-data sets are random draws from a pre-specified distribution. In the non-parametric version the error terms are obtained by drawing with replacement from either

(a) the (normalised) residuals obtained from step (i), Li and Maddala (1996, 1997), or

(b) the (normalised) residuals obtained from estimating the model under the alternative hypothesis, Fachin (2000).

As an illustration consider the process for generating bootstrap samples in a symmetric system similar to van Giersbergen (1996) and Harris and Judge (1998). The model takes the form of (1.4) with the deterministic terms being treated according to Case I and is given by

$$\Delta \mathbf{z}_t = \alpha\beta' \mathbf{z}_{t-1} + \sum_{i=1}^{p-1} \Gamma_i \Delta \mathbf{z}_{t-i} + \mathbf{e}_t, \tag{1.58}$$

where $\mathbf{e}_t \sim N_n(\mathbf{0}, \Omega)$ and $t = 1, ..., T$. Estimation of (1.58) under the null hypothesis of interest will provide estimates of $\hat{\Pi} = \hat{\alpha}\hat{\beta}'$, $\hat{\Gamma}_i$, $i = 1, ..., p-1$, $\hat{\mathbf{e}}_t$ and $\hat{\Omega}$. In the parametric case the bootstrap residuals $\mathbf{e}_t^j$, $j = 1, ..., B$ are drawn randomly from $N_n(\mathbf{0}, \hat{\Omega})$. In the non-parametric version (a) the $\mathbf{e}_t^j$'s are drawn with replacement from the (normalised) estimated residuals, $\hat{\mathbf{e}}_t$.

In the non-parametric version (b) the model is re-estimated under the alternative and gives rise to a new set of estimated residuals, say $\hat{\mathbf{e}}_{*t}$. The $\mathbf{e}_t^j$'s are now drawn with replacement from the (normalised) $\hat{\mathbf{e}}_{*t}$'s. For each of the $\mathbf{e}_t^j$, $j = 1, ..., B$ and for $t = p$, $\Delta\ \mathbf{z}_t^j$ is constructed as

$$\Delta \mathbf{z}_t^j = \hat{\alpha}\hat{\beta}' \mathbf{z}_{t-1} + \sum_{i=1}^{p-1} \hat{\Gamma}_i \Delta \mathbf{z}_{t-i} + \mathbf{e}_t^j \tag{1.59}$$

using the starting values $\mathbf{z}_0, ..., \mathbf{z}_{p-1}$ from the actual data set. The simulated data series is completed for $t > p$ as

$$\Delta \mathbf{z}_t^j = \hat{\alpha}\hat{\beta}' \mathbf{z}_{t-1}^j + \sum_{i=1}^{p-1} \hat{\Gamma}_i \Delta \mathbf{z}_{t-i}^j + \mathbf{e}_t^j. \tag{1.60}$$

Fachin (2000) argues that when bootstrapping the LR test of over-identifying restrictions on the cointegrating matrix, $\boldsymbol{\beta}$, the non-parametric bootstrap in (b) should be preferred to (a). The intuition behind this is the following. In the event that the null hypothesis under consideration is false, i.e. if the over-identifying restrictions imposed are very binding, then the cointegrating relations under the null will not be stationary which will be reflected in the presence of $I(1)$ error terms. This, however, will not be captured by the non-parametric bootstrap in (a), since drawing with replacement from an $I(1)$ process results in an $I(0)$ sample.

The problem of non-stationary error terms under the null identified in Fachin (2000) can be considered as a special case of residual autocorrelation since, by definition, an $I(1)$ process is autocorrelated of order one with unitary coefficient. As noted by Li and Maddala (1996), van Giersbergen (1996), Harris and Judge (1998) and others, the application of the non-parametric bootstrap is problematic in cases when the estimated VAR model is more generally misspecified and suffers from any form of residual autocorrelation. In such cases the resampling schemes discussed above, where the bootstrapped residuals are generated as random draws with replacement from the estimated residuals, will not maintain the serial correlation of the latter. The most popular alternative that deals with this problem is the stationary bootstrap [Politis and Romano (1994)] which resamples blocks of residuals, thus, maintaining the autocorrelation structure. However, van Giersbergen (1996) shows that the ordinary bootstrap in most cases has higher power than the stationary bootstrap. This in combination with the pervasive effects of serial correlation on the consistency of the ML estimator in cointegrating systems leads van Giersbergen (1996; pp. 401) to the conclusion that "*...it is better to have a sufficiently high order VAR model to improve the power of the test... hence, the practical use of the stationary bootstrap seems limited*".

The usefulness of the parametric bootstrap, on the other hand, is limited by the extent to which the assumptions on the pre-specified distribution of the residuals are not valid. In many applications of the parametric bootstrap the residuals are drawn from a multivariate normal distribution with variance equal to its estimate from the true data set, as for example in Garratt *et al* (1998, 2000) and Jacobson *et al* (2001). In the event that the diagnostic tests appear to reject the assumption of residual normality, parametric results may be complemented by non-parametric as in Garratt *et al* (2001), hereafter GLPS, and Lee and Papaikonomou (2002).

The application of bootstrap methods in symmetric cointegrating systems has generally been found to significantly improve on asymptotic inference concerning cointegrating rank[16], van Giersbergen (1996), as well as in the case of χ^2 tests, Gredenhoff and Jacobson (1998), Fachin (2000). The same findings are verified in Jacobson *et al* (2001) for a seven-dimensional model with intervention dummies. Studies based on small partial systems like Mantalos and Shukur

[16]Harris and Judge (1998), however, using non-parametric methods in a symmetric, three-dimensional $VAR(1)$ under Case I, find the bootstrap cointegration rank tests to have poor size properties in the presence of a single cointegrating relation.

(1998) also support strongly the use of bootstrap tests. These findings are further verified for the case of more complicated partial systems with relatively large dimensions (eight-variable models) by Greenslade *et al* (2002), although the application of the bootstrap in such systems will be discussed in more detail in the following section. In all cases, however, the bootstrap approach has confirmed the presence of a significant finite-sample bias in favour of rejection.

1.6.3 The Bootstrap Approach in Partial Systems

Early applications of the bootstrap were focused on simple symmetric systems of a relatively low dimension (typically ranging between three and five). Although the results from such studies could provide very helpful guidance, they need not be directly applicable to the more complicated systems that arose following the developments in the areas of modelling conditionally on weakly exogenous variables and the treatment of the deterministic terms. The general approach given by steps (i)-(iii) in the previous section still applies in the case of partial systems. However, the presence of weakly exogenous variables creates an additional issue with regard to the data-generating process in step (ii).

Consider, for example, the conditional and marginal models under Case IVd which take the form

$$\Delta \mathbf{y}_t = \mathbf{c}_0 + \boldsymbol{\alpha}_y \boldsymbol{\beta}_*' \mathbf{z}_{*t-1} + \sum_{i=1}^{p-1} \boldsymbol{\Psi}_i \Delta \mathbf{z}_{t-i} + \boldsymbol{\Upsilon} \Delta \mathbf{x}_t + \mathbf{u}_t, \quad (1.61)$$

$$\Delta \mathbf{x}_t = \mathbf{a}_{0x} + \sum_{i=1}^{p-1} \boldsymbol{\Gamma}_{ix} \Delta \mathbf{z}_{t-i} + \mathbf{e}_{xt}, \quad (1.62)$$

respectively, where $\mathbf{z}_t = [\mathbf{y}_t', \mathbf{x}_t']'$, $\mathbf{z}_{*t-1} = [\mathbf{z}_{t-1}', t, \mathbf{D}_t']'$, $\boldsymbol{\beta}_*' = [\boldsymbol{\beta}', -\boldsymbol{\beta}'\boldsymbol{\gamma}, -\boldsymbol{\beta}'\boldsymbol{\delta}]$, $\mathbf{u}_t \sim N_{n_y}(0, \boldsymbol{\Omega}_{uu})$,

$\mathbf{e}_{xt} \sim N_{n_x}(0, \Omega_{xx})$ and $t = 1, .., T$. As mentioned in previous sections, asymptotic critical values for the cointegration rank tests have not been systematically tabulated for models involving intervention dummies such as (1.61). As a result, a bootstrap exercise is the only available means by which to obtain small-sample critical values for these tests unless, of course, one is prepared to first simulate the asymptotic distributions and then use a scaling factor to correct for sample size. However, in the light of the empirical evidence discussed earlier on the performance of scaling factors, the latter course of action would be a less wise allocation of computing resources. The additional complication that arises when applying the bootstrap in conditional models is caused by the presence of the contemporaneous terms $\Upsilon \Delta \mathbf{x}_t$ in (1.61).

One possibility is to assume that $\mathbf{x}_t$ is fixed to its values from the original data set, as in Garratt *et al* (2000). Estimation of (1.61) under the null will give estimates of $\hat{\mathbf{c}}_0$, $\hat{\boldsymbol{\alpha}}_y$, $\hat{\boldsymbol{\beta}}_*$, $\hat{\Psi}_i$, $i = 1, ..., p-1$, $\hat{\Upsilon}$, $\hat{\mathbf{u}}_t$ and $\hat{\Omega}_{uu}$. The bootstrap disturbances $\mathbf{u}_t^j$, $j = 1, ..., B$, $t = 1, ..., T$ may be constructed according to the parametric or non-parametric schemes discussed in the previous section and for each of the $\mathbf{u}_t^j$, $j = 1, ..., B$ and for $t = p$, $\Delta\ \mathbf{y}_t^j$ is constructed as

$$\Delta \mathbf{y}_t^j = \hat{\mathbf{c}}_0 + \hat{\boldsymbol{\alpha}}_y \hat{\boldsymbol{\beta}}_*' \mathbf{z}_{*t-1} + \sum_{i=1}^{p-1} \hat{\Psi}_i \Delta \mathbf{z}_{t-i} + \hat{\Upsilon} \Delta \mathbf{x}_t + \mathbf{u}_t^j \tag{1.63}$$

using the starting values $\mathbf{z}_0, ..., \mathbf{z}_{p-1}$ from the actual data. The simulated data series $\Delta\ \mathbf{y}_t^j$ is completed for $t > p$ as

$$\Delta \mathbf{y}_t^j = \hat{\mathbf{c}}_0 + \hat{\boldsymbol{\alpha}}_y \hat{\boldsymbol{\beta}}_*' \left[\mathbf{y}_{t-1}^{j\prime}, \mathbf{x}_{t-1}', t, \mathbf{D}_t'\right]' + \sum_{i=1}^{p-1} \hat{\Psi}_i \begin{bmatrix} \Delta \mathbf{y}_{t-i}^j \\ \Delta \mathbf{x}_{t-i} \end{bmatrix} + \hat{\Upsilon} \Delta \mathbf{x}_t + \mathbf{u}_t^j, \tag{1.64}$$

where the original $\mathbf{x}_t$ series is used in each of the B simulations.

Clearly, treating the weakly exogenous vector $\mathbf{x}_t$ as fixed allows one to simulate only the $\mathbf{y}_t$ series which significantly reduces the computational burden. However, this approach is in conflict with the underlying analysis of systems with weakly exogenous variables. As shown in previous sections, a model such as (1.61) arises after conditioning $\Delta\mathbf{y}_t$ on $\Delta\mathbf{x}_t$, where $\mathbf{x}_t$ is treated as a stochastic process as indicated by the peripheral model in (1.62). Relaxing the assumption of fixed $\mathbf{x}_t$ requires the joint simulation of both, $\mathbf{y}_t$ and $\mathbf{x}_t$. There are two ways for doing so.

In the first approach (1.61) and (1.62) are estimated under the null hypothesis of interest to provide estimates of $\hat{\mathbf{c}}_0$, $\hat{\alpha}_y$, $\hat{\beta}'_*$, $\hat{\Psi}_i$, $\hat{\Upsilon}$, $\hat{\mathbf{u}}_t$, $\hat{\Omega}_{uu}$, $\hat{\mathbf{a}}_{0x}$, $\hat{\Gamma}_{ix}$, $\hat{\mathbf{e}}_{xt}$ and $\hat{\Omega}_{xx}$, $i = 1, ..., p-1$. Two sets of bootstrap residuals are required in this case, $\mathbf{u}_t^j$ and $\mathbf{e}_{xt}^j$, $j = 1, ..., B$, $t = 1, ..., T$. In the parametric version these are drawn randomly from $N_{n_y}(0, \hat{\Omega}_{uu})$ and $N_{n_x}(0, \hat{\Omega}_{xx})$, respectively. In the non-parametric version (a) the $\mathbf{u}_t^j$'s and $\mathbf{e}_{xt}^j$'s are drawn with replacement from the (normalised) estimated residuals, $\hat{\mathbf{u}}_t$ and $\hat{\mathbf{e}}_{xt}$. In the non-parametric version (b) the conditional model (1.61) is re-estimated under the alternative to produce the estimated residuals $\hat{\mathbf{u}}_{*t}$ and the $\mathbf{u}_t^j$'s are now drawn with replacement from the (normalised) $\hat{\mathbf{u}}_{*t}$.[17]

For each of the $\mathbf{e}_{xt}^j$, $j = 1, ..., B$, and for $t = p$, $\Delta\mathbf{x}_t^j$ is simulated first as

$$\Delta\mathbf{x}_t^j = \hat{\mathbf{a}}_{0x} + \sum_{i=1}^{p-1} {}_{xi}\hat{\Gamma}_{ix}\Delta\mathbf{z}_{t-i} + \mathbf{e}_{xt}^j \tag{1.65}$$

[17]The non-parametric version (b) was proposed by Fachin (2000) for the bootstrap test of over-identifying restrictions on beta. The marginal model is not affected by any hypothesis on beta and thus, does not need to be re-estimated.

and then for each of the $\mathbf{u}_t^j$, $j = 1, ..., B$, $\Delta\mathbf{y}_t^j$ is simulated as

$$\Delta\mathbf{y}_t^j = \hat{\mathbf{c}}_0 + \hat{\alpha}_y\hat{\beta}_*'\mathbf{z}_{*t-1} + \sum_{i=1}^{p-1}\hat{\Psi}_i\Delta\mathbf{z}_{t-i} + \hat{\Upsilon}\Delta\mathbf{x}_t^j + \mathbf{u}_t^j \tag{1.66}$$

using the starting values $\mathbf{z}_0, ..., \mathbf{z}_{t-1}$ from the original data set. For $p < t < 2p-1$, $\Delta\mathbf{x}_t^j$ is simulated first as

$$\Delta\mathbf{x}_t^j = \hat{\mathbf{a}}_{0x} + \sum_{k=1}^{t-p}\hat{\Gamma}_{kx}\Delta\mathbf{z}_{t-k}^j + \sum_{i=t-p+1}^{p-1}\hat{\Gamma}_{ix}\Delta\mathbf{z}_{t-i} + \mathbf{e}_{xt}^j \tag{1.67}$$

and then $\Delta\mathbf{y}_t^j$ as

$$\Delta\mathbf{y}_t^j = \hat{\mathbf{c}}_0 + \hat{\alpha}_y\hat{\beta}_*'\mathbf{z}_{*t-1} + \sum_{k=1}^{t-p}\hat{\Psi}_k\Delta\mathbf{z}_{t-k}^j + \sum_{i=t-p+1}^{p-1}\hat{\Psi}_i\Delta\mathbf{z}_{t-i} + \hat{\Upsilon}\Delta\mathbf{x}_t^j + \mathbf{u}_t^j \tag{1.68}$$

using the starting values $\mathbf{z}_{t-p}, ..., \mathbf{z}_{p-1}$. The bootstrapped data series is completed for $t \geqslant 2p-1$ as

$$\Delta\mathbf{x}_t^j = \hat{\mathbf{a}}_{0x} + \sum_{i=1}^{p-1}\hat{\Gamma}_{ix}\Delta\mathbf{z}_{t-i}^j + \mathbf{e}_{xt}^j \tag{1.69}$$

and

$$\Delta\mathbf{y}_t^j = \hat{\mathbf{c}}_0 + \hat{\alpha}_y\hat{\beta}_*'\mathbf{z}_{*t-1}^j + \sum_{i=1}^{p-1}\hat{\Psi}_i\Delta\mathbf{z}_{t-i}^j + \hat{\Upsilon}\Delta\mathbf{x}_t^j + \mathbf{u}_t^j. \tag{1.70}$$

As can be seen from (1.65)-(1.70) the contemporaneous presence of $\Delta\mathbf{x}_t$ in the conditional model forces the simulation process to iterate between the marginal and conditional models when treating $\mathbf{x}_t$ as stochastic. The first observation for $\mathbf{x}_t^j$ is generated in order to be used in the simulation of the first observation for $\mathbf{y}_t^j$, which in turn is required for the next observation of $\mathbf{x}_t^j$ and so on. Clearly, this process is considerably more time consuming than holding $\mathbf{x}_t$

fixed. An alternative data generating scheme, possibly more efficient in terms of computing time, might be the joint simulation of $\mathbf{x}_t^j$ and $\mathbf{y}_t^j$, $j = 1, ..., B$, with the use of the underlying, symmetric system implied by (1.61) and (1.62). In this case, the underlying, symmetric model (in partitioned form) is given by

$$\begin{bmatrix} \Delta\mathbf{y}_t \\ \Delta\mathbf{x}_t \end{bmatrix} = \begin{bmatrix} \mathbf{a}_{0y} \\ \mathbf{a}_{0x} \end{bmatrix} + \begin{bmatrix} \alpha_y \\ \mathbf{0} \end{bmatrix} \beta_*' \mathbf{z}_{*t-1} + \sum_{i=1}^{p-1} \begin{bmatrix} \Gamma_{iy} \\ \Gamma_{ix} \end{bmatrix} \Delta\mathbf{z}_{t-i} + \begin{bmatrix} \mathbf{e}_{yt} \\ \mathbf{e}_{xt} \end{bmatrix}, \qquad (1.71)$$

where $\mathbf{z}_t = [\mathbf{y}_t', \mathbf{x}_t']'$, $\mathbf{z}_{*t-1} = [\mathbf{z}_{t-1}', t, \mathbf{D}_t']'$, $\beta_*' = [\beta', -\beta'\gamma, -\beta'\delta]$ and $\mathbf{e}_t = [\mathbf{e}_{yt}', \mathbf{e}_{xt}']'$ with variance matrix $\Omega = \begin{bmatrix} \Omega_{yy} & \Omega_{yx} \\ \Omega_{xy} & \Omega_{xx} \end{bmatrix}$. The last n_x rows of (1.71) give the marginal model in (1.62) and the remaining terms in the first n_y rows are linked to the conditional model (1.61) through the relations

$$\mathbf{a}_{0y} = \mathbf{c}_0 + \Upsilon\mathbf{a}_{0x},\ \mathbf{e}_{yt} = \mathbf{u}_t + \Upsilon\mathbf{e}_{xt} \text{ and } \Gamma_{iy} = \Psi_i + \Upsilon\Gamma_{ix},\ i = 1, ..., p-1. \qquad (1.72)$$

Thus, it is possible to estimate the conditional and marginal models (1.61) and (1.62) to get $\hat{\mathbf{c}}_0$, $\hat{\alpha}_y$, $\hat{\beta}_*'$, $\hat{\Psi}_i$, $\hat{\Upsilon}$, $\hat{\mathbf{u}}_t$, $\hat{\Omega}_{uu}$, $\hat{\mathbf{a}}_{0x}$, $\hat{\Gamma}_{ix}$, $\hat{\mathbf{e}}_{xt}$ and $\hat{\Omega}_{xx}$, $i = 1, ..., p-1$, and then use these estimates to construct $\hat{\mathbf{a}}_{0y}$, $\hat{\mathbf{e}}_{yt}$ and $\hat{\Gamma}_{iy}$, $i = 1, ..., p-1$, according to (1.72). The bootstrap residuals $\mathbf{e}_t^j = [\mathbf{e}_{yt}^{j\prime}, \mathbf{e}_{xt}^{j\prime}]'$, $j = 1, ..., B$, $t = 1, ..., T$, may now be obtained using the parametric and non-parametric methods discussed earlier and the simulated series $\mathbf{x}_t^j$ and $\mathbf{y}_t^j$, $j = 1, ..., B$, can be generated simultaneously within (1.71). For each of the $\mathbf{e}_t^j$, $j = 1, ..., B$ and for $t = p$, $\Delta\ \mathbf{z}_t^j$ is constructed as

$$\Delta\mathbf{z}_t^j = \hat{\mathbf{a}}_0 + \begin{bmatrix} \hat{\alpha}_y \\ \mathbf{0} \end{bmatrix} \hat{\beta}_*' \mathbf{z}_{*t-1} + \sum_{i=1}^{p-1} \hat{\Gamma}_i \Delta\mathbf{z}_{t-i} + \mathbf{e}_t^j, \qquad (1.73)$$

where $\hat{\mathbf{a}}_0 = [\hat{\mathbf{a}}'_{0y}, \hat{\mathbf{a}}'_{0x}]'$ and $\hat{\Gamma}_i = [\hat{\Gamma}'_{iy}, \hat{\Gamma}'_{ix}]'$, $i = 1, ..., p-1$, and using the starting values $\mathbf{z}_0, ..., \mathbf{z}_{p-1}$ from the actual data set. The simulated data series is completed for $t > p$ as

$$\Delta \mathbf{z}_t^j = \hat{\mathbf{a}}_0 + \begin{bmatrix} \hat{\alpha}_y \\ 0 \end{bmatrix} \hat{\beta}'_* \mathbf{z}_{*t-1}^j + \sum_{i=1}^{p-1} \hat{\Gamma}_i \Delta \mathbf{z}_{t-i}^j + \mathbf{e}_t^j. \tag{1.74}$$

The performance of the bootstrap in partial systems has been investigated by Mantalos and Shukur (1998) within a small bivariate system under Case I with the weakly exogenous variable treated as a stochastic process. The evidence presented strongly suggests the use of bootstrap methods for finite-sample inference regarding cointegrating rank within samples of sizes 20, 40, 60 and 100. Greenslade *et al* (2002) consider a more realistic system in eight variables, three of which are treated as weakly exogenous. The sample size in this study is 112 and the lag-lengths considered are 2, 4, 6 and 8. The deterministic terms are specified according to Case I while the weakly exogenous vector is treated both as fixed and as a stochastic process. Again, the overall evidence supports the use of bootstrap critical values for the determination of cointegrating rank in the sample size under consideration. Treating the weakly exogenous vector as fixed or stochastic appears to have little or no impact on the performance of the bootstrap. However, the determination of the endogeneity and weak exogeneity status prior to inference on cointegrating rank is shown to significantly improve the performance of the bootstrapped cointegration rank tests.

1.6.4 Bootstrapping the LR Test for Over-Identification of β and the Use of Simulated Annealing

Although the application of the bootstrap approach (parametric or non-parametric) described above is straightforward in the case of the cointegration rank tests, there is a major practical issue that complicates its application to the tests of over-identification of the cointegrating matrix, β. The general hypothesis to be tested takes the form

$$H_0 : \beta_{EX} = \beta_{OV}, \tag{1.75}$$

where β_{EX} and β_{OV} are the exactly and over-identified cointegrating matrices, respectively, and β_{OV} can be any linear or non-linear transformation of β_{EX}. A test of (1.75) is given by comparison of the maximised log-likelihoods $LL(\hat{\beta}_{EX})$ and $LL(\hat{\beta}_{OV})$ with the statistic $LR = 2[LL(\hat{\beta}_{EX}) - LL(\hat{\beta}_{OV})]$. This test statistic is asymptotically χ^2 with degrees of freedom equal to $k - r^2$, where k stands for the total number of restrictions in β_{OV} and $k \leq nr$.[18] A bootstrap distribution of LR can be obtained in the following three steps:

(1) A large number of B data sets are simulated using the parametric or non-parametric methods discussed in previous sections.

(2) For each of the B simulated data sets the system's log-likelihood is maximised for both, the exactly and over-identified cointegrating matrices in order to obtain $LL(\hat{\beta}^j_{EX})$ and $LL(\hat{\beta}^j_{OV})$, $j = 1, ..., B$.

(3) The LR statistic $LR = 2[LL(\hat{\beta}^j_{EX}) - LL(\hat{\beta}^j_{OV})]$, $j = 1, ..., B$, is computed for each of the bootstrap samples in order to get an empirical approximation of its distribution.

[18] For more details see, for example, Pesaran and Shin (2001).

However, as noted in an early version of Garratt *et al* (2000), in the case of over-identified systems with a "large" number of cointegrating vectors and free parameters, conventional algorithms (like the modified Newton Raphson used by *Microfit 4.0*) quite frequently fail to converge to the global maximum in the absence of very accurate starting values. In order to avoid the inevitable convergence obstacles that arise when solving B such optimization problems in step (2), existing research, e.g. Garratt *et al* (1998) and Jacobson *et al* (2001), has frequently been restricting itself to the use of what will be denoted as *bootstrap 1*.

In this bootstrap experiment the over-identified cointegrating matrix is not being estimated for each bootstrap sample. Instead, it is assumed to be equal to its estimate from the original data set, $\hat{\beta}_{OV}$, and the computed statistic becomes $LR_1 = 2[LL(\hat{\beta}^j_{EX}) - LL(\hat{\beta}_{OV})], j = 1, ..., B$. In effect, this is a test of

$$H_{0,1} : \beta_{EX} = \hat{\beta}_{OV} \tag{1.76}$$

that imposes the maximum number of over-identifying restrictions $nr - r^2$. In this case the computational aspects of the bootstrap exercise are dramatically simplified as the error correction terms under the null are fixed, which allows for the over-identified model to be estimated by standard OLS. The limitation of this approach is rather obvious when β_{OV} involves free parameters, i.e. $k < nr$. In this case, the hypothesis tested by LR_1 is more restrictive than (1.75), as it imposes a specific numerical value on all the elements of the cointegrating matrix. Since $\hat{\beta}_{OV}$ is bound to be sub-optimal with respect to the free parameters in β_{OV} for all but the actual data set, the difference $LL(\hat{\beta}^j_{EX}) - LL(\hat{\beta}_{OV})$ will always be greater than $LL(\hat{\beta}^j_{EX}) - LL(\hat{\beta}^j_{OV}), j = 1, ..., B$ and, thus, the critical values obtained from bootstrap 1 will inevitably be positively biased. Although this result helps illustrate the limitations of bootstrap

1, it does not render a test of (1.76) entirely useless regarding small-sample inference on (1.75). In fact, it demonstrates that, to the extent that a bootstrap exercise is a valid approach for approximating the finite-sample distributions, these maintain the property of the asymptotic χ^2 distributions which guarantees that critical values increase with the number of over-identifying restrictions. Thus, rejection of (1.76) necessarily means rejection of (1.75) in both, infinite and finite samples.

In the event that bootstrap 1 leads to non-rejection of (1.76), though, the outcome on (1.75) is inconclusive. In the light of the practical difficulties involved in bootstrapping (1.75) directly, the two papers by Johansen (2000a, b) discussed in previous sections, have revived the scaling factor approach by proposing a Bartlett-type, small-sample correction for LR. Chapter 3 of this thesis introduces the use of the global optimization algorithm *Simulated Annealing* (SA) discussed in Goffe *et al* (1994) and adapted to *Gauss* by E.G. Tsionas (1995) as a means for overcoming the convergence problems involved in the simulation of the finite-sample distribution of LR. The main strength of SA lies on the fact that it does not depend on the parameters' initial values. The algorithm allows for downhill as well as uphill movements, which makes it possible to explore the whole surface of the likelihood function before converging to the global maximum, at a cost though, of computational speed. The user has control over the thoroughness with which the SA algorithm explores the surface of the objective function, the parameter range and the strictness of the termination criteria. This makes it possible to carry out what will be denoted as *bootstrap 2*, where $LL(\hat{\beta}^j_{OV})$, $j = 1, ..., B$, is being maximised for each of the simulated data sets with respect to the free parameters in β_{OV} with the use of the SA algorithm. This method has also been implemented in GLPS and Lee and Papaikonomou (2002).

1.7 Investigation of the Dynamic Properties in the Cointegrating $VAR(p)$

The dynamic behaviour of a general cointegrating VAR model such as (1.4) has been investigated in the literature by evaluating the response of individual variables and the cointegrating relations to both, variable-specific and system-wide shocks. The tools employed for the evaluation of the effects of such shocks are the Orthogonalised Impulse Responses (OIR) proposed by Sims (1980), the more recently developed Generalised Impulse Responses (GIR) in Koop *et al* (1996) and Pesaran and Shin (1998) and the Persistence Profiles (PP) proposed by Lee *et al* (1992), Lee and Pesaran (1993a) and Pesaran and Shin (1996). The application of these tools to partial systems such as (1.45) may be carried out within the underlying symmetric system in (1.41) subject to the weak exogeneity condition (1.44).

1.7.1 Impulse Responses

An impulse response function measures the expected effect of a hypothetical n-vector of shocks of size $\varsigma = [\varsigma_1, \ldots, \varsigma_n]'$ at a given point in time, t, on the future values of $\Delta\mathbf{z}_t$, $\mathbf{z}_t$ and $\boldsymbol{\beta}'\mathbf{z}_t$. It may, therefore, be described as a conditional expectation of $\Delta\mathbf{z}_t$, $\mathbf{z}_t$ and $\boldsymbol{\beta}'\mathbf{z}_t$, given ς and the known history of the economy

$$\mathbf{IR}_{\Delta z}(N, \varsigma, \Xi_{t-1}) = E(\Delta\mathbf{z}_{t+N}|\mathbf{e}_t = \varsigma, \Xi_{t-1}) - E(\Delta\mathbf{z}_{t+N}|\Xi_{t-1}), \tag{1.77}$$

$$\mathbf{IR}_{z}(N, \varsigma, \Xi_{t-1}) = E(\mathbf{z}_{t+N}|\mathbf{e}_t = \varsigma, \Xi_{t-1}) - E(\mathbf{z}_{t+N}|\Xi_{t-1}), \tag{1.78}$$

$$\mathbf{IR}_{\beta' z}(N, \varsigma, \Xi_{t-1}) = E(\boldsymbol{\beta}'\mathbf{z}_{t+N}|\mathbf{e}_t = \varsigma, \Xi_{t-1}) - E(\boldsymbol{\beta}'\mathbf{z}_{t+N}|\Xi_{t-1}), \tag{1.79}$$

where $N = 0, 1, 2, \ldots$, and Ξ_{t-1} stands for the information set available at time $t-1$. For the general, reduced-form model (1.4), the known history of $\Delta \mathbf{z}_t$, $\mathbf{z}_t$ and $\beta' \mathbf{z}_t$ is summarised by the MA representations

$$\Delta \mathbf{z}_t = \mathbf{C}(L)(\mathbf{a}\psi_t + \mathbf{e}_t),\ \mathbf{z}_t = \tilde{\mathbf{B}}(L)(\mathbf{a}\psi_t + \mathbf{e}_t) \text{ and } \beta' \mathbf{z}_t = \beta' \tilde{\mathbf{B}}(L)(\mathbf{a}\psi_t + \mathbf{e}_t), \tag{1.80}$$

where $t = 1, 2, ..., T$, $E(\mathbf{e}_t \mathbf{e}_t') = \mathbf{\Omega}$, $\mathbf{C}(L)$ is defined in (1.26) and

$$\tilde{\mathbf{B}}(L) = (1-L)^{-1}\mathbf{C}(L) = \sum_{i=0}^{\infty} \tilde{B}_i L^i,\ \tilde{B}_i = \sum_{j=0}^{i} C_i, \text{ for } i \geq 0. \tag{1.81}$$

Combining the definitions (1.77)-(1.79) with (1.80) results in

$$\mathbf{IR}_{\Delta z}(N, \varsigma, \Xi_{t-1}) = C_N \varsigma,\ \mathbf{IR}_{z}(N, \varsigma, \Xi_{t-1}) = \tilde{B}_N \varsigma \text{ and } \mathbf{IR}_{\beta' z}(N, \varsigma, \Xi_{t-1}) = \beta' \tilde{B}_N \varsigma, \tag{1.82}$$

where $N = 0, 1, 2, \ldots$. The matrices C_N and $\tilde{B}_N$ can be obtained from the relations

$$\begin{aligned} C_0 &= I_n,\ C_1 = \Phi_1 - I_n,\ C_N = \sum_{j=1}^{N} \Phi_j C_{N-j}, \text{ for } N > 1, \\ \tilde{B}_0 &= I_n,\ \tilde{B}_N = \sum_{j=1}^{N} \Phi_j \tilde{B}_{N-j}, \text{ for } N \geq 1, \end{aligned} \tag{1.83}$$

and (1.81) using the estimates from (1.4) and noting that

$$\Phi_1 = I_n + \Pi + \Gamma_1,\ \Phi_i = \Gamma_i - \Gamma_{i-1}, \text{ for } i = 2, \ldots, p-1 \text{ and } \Phi_p = -\Gamma_{p-1}, \tag{1.84}$$

by the definition of Π and Γ_i, $i = 1, \ldots, p-1$, in (1.3). Thus, according to (1.82), the matrices C_N, $\tilde{B}_N$ and $\beta'\tilde{B}_N$ describe the expected responses of $\Delta \mathbf{z}_{t+N}$, $\mathbf{z}_{t+N}$ and $\beta'\mathbf{z}_{t+N}$, $N = 0, 1, 2, \ldots$, respectively, to a unitary, system-wide, reduced-form impulse $\mathbf{e}_t = \varsigma = [1, \ldots, 1]'$. Over an infinite horizon the impulse responses in (1.82) converge to $\lim_{N\to\infty} \{C_N\}\varsigma = \mathbf{0}$, by stationarity of $\Delta \mathbf{z}_t$, $\lim_{N\to\infty} \{\tilde{B}_N\}\varsigma = \mathbf{C}(1)\varsigma$, by (1.81) and $\lim_{N\to\infty} \{\beta'\tilde{B}_N\}\varsigma = \beta'\mathbf{C}(1)\varsigma = \mathbf{0}$, by (1.81) and (1.29), indicating that shocks on the stationary $\Delta \mathbf{z}_t$ and $\beta'\mathbf{z}_t$ are transitory, while they have a permanent effect $\mathbf{C}(1)\varsigma$ on the $I(1)$ vector $\mathbf{z}_t$.

Although symmetric, system-wide shocks help illustrate the general dynamic behaviour of the model, they are of limited use when addressing more interesting economic questions. For the purpose of policy evaluation, for example, attention is focused on the effect of shocks in particular sectors of the economy. For this purpose the conditional expectations in (1.77)-(1.79) need to be re-defined as

$$\mathbf{IR}_{\Delta z}(N, \varsigma_i, \Xi_{t-1}) = E(\Delta \mathbf{z}_{t+N} | e_i = \varsigma_i, \Xi_{t-1}) - E(\Delta \mathbf{z}_{t+N} | \Xi_{t-1}), \tag{1.85}$$

$$\mathbf{IR}_{z}(N, \varsigma_i, \Xi_{t-1}) = E(\mathbf{z}_{t+N} | e_i = \varsigma_i, \Xi_{t-1}) - E(\mathbf{z}_{t+N} | \Xi_{t-1}), \tag{1.86}$$

$$\mathbf{IR}_{\beta' z}(N, \varsigma_i, \Xi_{t-1}) = E(\beta'\mathbf{z}_{t+N} | e_i = \varsigma_i, \Xi_{t-1}) - E(\beta'\mathbf{z}_{t+N} | \Xi_{t-1}), \tag{1.87}$$

$N = 0, 1, 2, \ldots$, and $i = 1, \ldots, n$. Combining (1.85)-(1.87) with (1.80) results in

$$\begin{aligned} \mathbf{IR}_{\Delta z}(N, \varsigma_i, \Xi_{t-1}) &= \omega_{ii}^{-1} C_N \boldsymbol{\Omega} \mathbf{s}_i \varsigma_i, \ \mathbf{IR}_{z}(N, \varsigma_i, \Xi_{t-1}) = \omega_{ii}^{-1} \tilde{B}_N \boldsymbol{\Omega} \mathbf{s}_i \varsigma_i \text{ and} \\ \mathbf{IR}_{\beta' z}(N, \varsigma_i, \Xi_{t-1}) &= \omega_{ii}^{-1} \beta' \tilde{B}_N \boldsymbol{\Omega} \mathbf{s}_i \varsigma_i, \ N = 0, 1, 2, \ldots, \ i = 1, \ldots, n, \end{aligned} \tag{1.88}$$

where ω_{ij} is the i-th element in the j-th column of $\boldsymbol{\Omega}$ and $\mathbf{s}_i$ is an $n \times 1$ selection vector with unity as its i-th element and zeros elsewhere, so that $\boldsymbol{\Omega}\mathbf{s}_i$ gives the i-th column of $\boldsymbol{\Omega}$. In the event that the e_i's, $i = 1, ..., n$, are uncorrelated and, thus, $\boldsymbol{\Omega}$ is diagonal, then $\omega_{ii}^{-1}\boldsymbol{\Omega}\mathbf{s}_i = \mathbf{s}_i$. Therefore, the effect of a unit shock in the i-th equation of the system at time t, i.e. $e_{it} = \varsigma_i = 1$, on $\Delta\mathbf{z}_{t+N}$, $\mathbf{z}_{t+N}$ and $\boldsymbol{\beta}'\mathbf{z}_{t+N}$, $N = 0, 1, 2, \ldots$, would be described by the i-th column of C_N, $\tilde{B}_N$ and $\boldsymbol{\beta}'\tilde{B}_N$, respectively. However, in the modelling framework of a cointegrating $VAR(p)$ the e_i's, $i = 1, ..., n$, are generally assumed to be correlated and, thus, $\boldsymbol{\Omega}$ is not diagonal. Several approaches have been considered in this context.[19]

1.7.2 Orthogonalised Impulse Responses

The traditional approach is due to Sims (1980) who suggested looking at the response of $\Delta\mathbf{z}_t$, $\mathbf{z}_t$ and $\boldsymbol{\beta}'\mathbf{z}_t$ to the transformed shocks

$$\mathbf{v}_t^S = S^{-1}\mathbf{e}_t, \tag{1.89}$$

where S is $n \times n$, lower triangular with unit diagonal. The transformed shocks are orthogonal since $E(\mathbf{v}_t^S\mathbf{v}_t^{S\prime}) = S^{-1}\boldsymbol{\Omega}S'^{-1} = \Sigma^S$, where Σ^S is diagonal and Sims' transformation matrix, S, may be obtained from the triangular factorisation $\boldsymbol{\Omega} = S\Sigma^S S'$. Solving (1.89) for $\mathbf{e}_t$ and substituting in the MA representations (1.80) results in

$$\Delta\mathbf{z}_t = \mathbf{C}(L)(\mathbf{a}\psi_t + S\mathbf{v}_t^S),\ \mathbf{z}_t = \tilde{\mathbf{B}}(L)(\mathbf{a}\psi_t + S\mathbf{v}_t^S) \text{ and } \boldsymbol{\beta}'\mathbf{z}_t = \boldsymbol{\beta}'\tilde{\mathbf{B}}(L)(\mathbf{a}\psi_t + S\mathbf{v}_t^S). \tag{1.90}$$

Re-defining the conditional expectations (1.85)-(1.87) in terms of the orthogonalised residuals v_{it}^S, $i = 1, \ldots, n$ and combining them with (1.90) results in Sims' orthogonalised impulse

[19] For a detailed overview see Levtchenkova *et al* (1998).

responses

$$\begin{aligned}
\mathbf{OIR}^S_{\Delta z}(N,\varsigma_i,\Xi_{t-1}) &= C_N S\mathbf{s}_i\varsigma_i,\ \mathbf{OIR}^S_z(N,\varsigma_i,\Xi_{t-1}) = \tilde{B}_N S\mathbf{s}_i\varsigma_i \text{ and} \\
\mathbf{OIR}^S_{\beta' z}(N,\varsigma_i,\Xi_{t-1}) &= \beta'\tilde{B}_N S\mathbf{s}_i\varsigma_i,\ N = 0,1,2,\ldots,\ i = 1,\ldots,n. \qquad (1.91)
\end{aligned}$$

The fact that S is lower triangular, though, has the consequence that a shock v^S_{it} has a quite different effect on $\Delta\mathbf{z}_{t+N}$, $\mathbf{z}_{t+N}$ and $\beta'\mathbf{z}_{t+N}$ depending on the ordering of the z_{it}'s in $\mathbf{z}_t$. If, for example, a unitary shock hits the n-th equation in the system at time t, then $S\mathbf{s}_n = \mathbf{s}_n$ and the responses of $\Delta\mathbf{z}_{t+N}$, $\mathbf{z}_{t+N}$ and $\beta'\mathbf{z}_{t+N}$ are given by the n-th column of C_N, $\tilde{B}_N$ and $\beta'\tilde{B}_N$, respectively. However, simply re-ordering the z_{it}'s so that z_{nt} is now the first entry in $\mathbf{z}_t$ results in the quite different responses $C_N S\mathbf{s}_1$, $\tilde{B}_N S\mathbf{s}_1$ and $\beta'\tilde{B}_N S\mathbf{s}_1$. Just as in the case of (1.82), over an infinite horizon Sims' OIR's become $\mathbf{OIR}^S_{\Delta z}(\infty,\varsigma_i,\Xi_{t-1}) = \mathbf{OIR}^S_{\beta' z}(\infty,\varsigma_i,\Xi_{t-1}) = \mathbf{0}$ and $\mathbf{OIR}^S_z(\infty,\varsigma_i,\Xi_{t-1}) = \mathbf{C}(1)S\mathbf{s}_i\varsigma_i$ by (1.81) and (1.29), thereby reflecting the fact that $\Delta\mathbf{z}_t$ and $\beta'\mathbf{z}_t$ are $I(0)$, while $\mathbf{z}_t \sim I(1)$.

Very frequently the orthogonalised impulses are alternatively computed as

$$\mathbf{v}^H_t = H^{-1}\mathbf{e}_t, \qquad (1.92)$$

where $H = S(\Sigma^S)^{1/2}$, lower triangular and is obtained from the Cholesky decomposition $\boldsymbol{\Omega} = HH'$. In this case the transformed shocks are orthogonal with $\Sigma^H = E(\mathbf{v}^H_t\mathbf{v}^{H\prime}_t) = H^{-1}\boldsymbol{\Omega}H'^{-1} = I_n$ and the OIR's take the form

$$\mathbf{OIR}^H_{\Delta z}(N,\varsigma_i,\Xi_{t-1}) = C_N H\mathbf{s}_i\varsigma_i,\ \mathbf{OIR}^H_z(N,\varsigma_i,\Xi_{t-1}) = \tilde{B}_N H\mathbf{s}_i\varsigma_i \text{ and}$$

$$\text{OIR}^{H}_{\beta' z}(N, \varsigma_i, \Xi_{t-1}) \;=\; \beta' \tilde{B}_N H \mathbf{s}_i \varsigma_i,\ N = 0, 1, 2, \ldots, i = 1, \ldots, n. \tag{1.93}$$

1.7.3 Impulse Responses and Economic Structure

Although the OIR's in (1.91) and (1.93) are routinely being used for policy evaluation, the orthogonalised disturbances $\mathbf{v}_t^S$ and $\mathbf{v}_t^H$ defined in (1.89) and (1.92), respectively, are identified merely in order to satisfy an orthogonality condition. It is for this reason that they cannot be considered *structural* in the sense of (1.19) and (1.20), as the identifying restrictions are not derived from economic theory. The structural VAR ($SVAR$) literature, on the other hand, focuses on the identification of the structural shocks

$$\mathbf{v}_t = A_0 \mathbf{e}_t \tag{1.94}$$

with variance $\boldsymbol{\Sigma} = E(\mathbf{v}_t \mathbf{v}_t') = A_0 \boldsymbol{\Omega} A_0'$. This is done by imposing restrictions on the contemporaneous loadings A_0 and $\boldsymbol{\Sigma}$ derived from relevant economic theory as in Shapiro and Watson (1988), Blanchard and Quah (1989) and Gali (1992), although, very frequently these amount to, or at least include, an orthogonality condition. In such cases theory acts as a guide regarding the ordering of the z_{it}'s in $\mathbf{z}_t$, which determines the causal chain in the system. As was hinted at in previous sections, weak exogeneity may provide part of the identifying restrictions through the relation $A_0^{-1} \boldsymbol{\alpha}_* = [\boldsymbol{\alpha}_y', \mathbf{0}]'$ that follows from (1.22) and the weak exogeneity condition (1.44).

1.7.4 Permanent-Transitory Decomposition

Another popular way of obtaining an economically meaningful A_0 is by distinguishing between $n - r$ permanent components, $\mathbf{v}_t^P = [v_{1t}^P, \ldots, v_{n-r,t}^P]'$, and r transitory components, $\mathbf{v}_t^T =$

$[v^T_{1t},\dots,v^T_{rt}]'$, in the structural shocks $\mathbf{v}_t = [\mathbf{v}^{P\prime}_t, \mathbf{v}^{T\prime}_t]'$. This is the approach taken in King *et al* (1991), Mellander *et al* (1992) and more recently in Jacobson *et al* (2001) and is based on the reduced-form, common stochastic trends representation of $\mathbf{z}_t$ in (1.28). In terms of $\mathbf{v}_t$ this takes the form

$$\mathbf{z}_t = \tilde{\mathbf{z}}_0 + \mathbf{C}(1)\sum_{i=1}^{t}(\mathbf{a}\psi_i + A_0^{-1}\begin{bmatrix} \mathbf{v}^P_i \\ \mathbf{v}^T_i \end{bmatrix}) + \mathbf{C}^*(L)(\mathbf{a}\psi_t + A_0^{-1}\begin{bmatrix} \mathbf{v}^P_t \\ \mathbf{v}^T_t \end{bmatrix}), \tag{1.95}$$

where $\tilde{\mathbf{z}}_0 = \mathbf{z}_0 - \mathbf{C}^*(L)(\mathbf{a}\psi_0 + A_0^{-1}\mathbf{v}_0)$, $\mathbf{C}^*(L)$ is defined in (1.26) and $\mathbf{C}(1)$ is given by (1.29). As discussed previously, the implication of the representation of $\mathbf{C}(1)$ according to (1.29) is that $\mathbf{z}_t$ is driven by $n-r$ independent stochastic trends $\mathbf{C}(1)A_0^{-1}\sum_{i=1}^{t}\mathbf{v}_i$. The determination of the structural loadings A_0 is based on the identification of the $n-r$ stochastic trends driving the system. The latter is achieved by first noting that the transitory nature of $\mathbf{v}^T_t$ implies that the cumulative effect $\mathbf{C}(1)A_0^{-1}$ takes the form

$$\mathbf{C}(1)A_0^{-1} = \begin{bmatrix} \mathbf{A} & \mathbf{0} \end{bmatrix}, \tag{1.96}$$

where $\mathbf{A}$ is the $n \times (n-r)$ matrix of the long-run multipliers of $\mathbf{v}^P_t$ and the $n \times r$ block of zeros is the long-run effect of $\mathbf{v}^T_t$. In the presence of more than one stochastic trends, $\mathbf{A}$ is only identified up to multiplication by a non-singular $(n-r)\times(n-r)$ matrix $\mathbf{P}$. King *et al* (1991) require that $\mathbf{A} = \tilde{\mathbf{A}}\tilde{\mathbf{P}}$ with $\tilde{\mathbf{A}}$ being an $n \times (n-r)$ matrix of known coefficients and $\tilde{\mathbf{P}}$ being an $(n-r)\times(n-r)$, lower triangular matrix with unit diagonal. The choice of $\tilde{\mathbf{A}}$ is guided by the desired economic interpretation of the v^P_{it}'s, $i = 1,\dots,n-r$, and the estimated cointegrating relations. The unknown parameters below the main diagonal of $\tilde{\mathbf{P}}$ are determined by requiring that $\mathbf{v}_t^P$ is independent of the transitory components $\mathbf{v}_t^T$, so that

$$\Sigma = E(\mathbf{v}_t\mathbf{v}_t') = \begin{bmatrix} \Sigma^P & 0 \\ 0 & \Sigma^T \end{bmatrix}, \tag{1.97}$$

where $\Sigma^T = E(\mathbf{v}_t^T\mathbf{v}_t^{T\prime})$ and the v_{it}^P's, $i = 1, \ldots, n-r$, are themselves assumed to be uncorrelated with $\Sigma^P = E(\mathbf{v}_t^P\mathbf{v}_t^{P\prime}) = I_{n-r}$. In the case of a single stochastic trend, i.e. $n-r = 1$, the matrix of long-run multipliers, $\mathbf{A}$, is an n-vector. In this case identification of the single v_{1t}^P can be achieved upon normalisation of $\mathbf{A}$ and through (1.97).

1.7.5 Generalised Impulse Responses

As illustrated above, impulse response analysis has revolved around the identification of "structural" disturbances that arise as linear transformations of the reduced form shocks $\mathbf{e}_t$.[20] In the case of Sims' (1980), the transformed residuals are identified in such a way as to capture the "*distinct patterns of movement*", while in the *SVAR* literature more emphasis is placed on the use of economic theory. Nevertheless, even when theory-consistent identification schemes are employed they usually amount to, or at least include, orthogonalisation conditions that leave the interpretation of the impulse responses depending on the ordering of the variables. The use of orthogonality conditions could be, to some extent, the result of failure on the part of economic theory to deliver the n^2 restrictions required for the unique specification of A_0 in (1.94), or even an attempt to moderate Sims' criticism of "*incredible restrictions*" on the short-run dynamics in the cases when the n^2 restrictions are indeed provided. Perhaps most importantly,

[20] This assumption is required in order to exclude non-invertible components from the model in the presence of so-called *Blaschke factors*. For more details see Warne (2000).

though, it is a reflection of the conviction that structural shocks are mutually independent.

The generalised impulse response literature, Koop *et al* (1996) and Pesaran and Shin (1998), openly challenges this view. There are many theoretical and practical reasons why structural shocks may indeed be strongly correlated and Pesaran and Smith (1998; footnote 15) provide examples from both micro and macroeconomic theory. Especially in macroeconomic applications, one practical reason why structural shocks can be expected to be correlated is the low data frequency. Even if there are no *a priori* reasons why a particular pair of structural disturbances should be correlated, there would be no realistic way of capturing their distinct effect if they occur within the same period, which typically means a quarter or even a year. Thus, rather than attempting to describe the effect of specific shocks, the GIR's focus on the effects of *realistic* shocks, that is shocks which are typical by historical standards, as described by the estimated covariance matrix $\mathbf{\Omega}$.

The GIR's take the form of (1.88), where the size of the reduced-form shock hitting sector i at time t is scaled so that $e_{it} = \varsigma_i = \omega_{ii}^{1/2}$. Thus, the scaled GIR's of $\Delta\mathbf{z}_{t+N}$, $\mathbf{z}_{t+N}$ and $\beta'\mathbf{z}_{t+N}$ are given by

$$
\begin{aligned}
\mathbf{GIR}^{e}_{\Delta\mathbf{z}}(N,\varsigma_i,\Xi_{t-1}) &= \omega_{ii}^{-1/2}C_N\mathbf{\Omega}\mathbf{s}_i,\ \mathbf{GIR}^{e}_{\mathbf{z}}(N,\varsigma_i,\Xi_{t-1}) = \omega_{ii}^{-1/2}\tilde{B}_N\mathbf{\Omega}\mathbf{s}_i \text{ and} \\
\mathbf{GIR}^{e}_{\beta'\mathbf{z}}(N,\varsigma_i,\Xi_{t-1}) &= \omega_{ii}^{-1/2}\beta'\tilde{B}_N\mathbf{\Omega}\mathbf{s}_i,\ N=0,1,2,\ldots,\ i=1,\ldots,n. \qquad (1.98)
\end{aligned}
$$

As in the case of (1.82) and Sims' OIR's above, over an infinite horizon the GIR's become $\mathbf{GIR}^{e}_{\Delta\mathbf{z}}(\infty,\varsigma_i,\Xi_{t-1}) = \mathbf{GIR}^{e}_{\beta'\mathbf{z}}(\infty,\varsigma_i,\Xi_{t-1}) = \mathbf{0}$ and $\mathbf{GIR}^{e}_{\mathbf{z}}(\infty,\varsigma_i,\Xi_{t-1}) = \omega_{ii}^{-1/2}\mathbf{C}(1)\mathbf{\Omega}\mathbf{s}_i$ by (1.81) and (1.29), thus, reflecting the stationarity of $\Delta\mathbf{z}_t$ and $\beta'\mathbf{z}_t$ and non-stationarity of $\mathbf{z}_t$.

The GIR's can also be applied in order to study the effects of the structural disturbances

$\mathbf{v}_t$ in (1.94) when $\boldsymbol{\Sigma}$ is not necessarily diagonal. In this case the GIR's to a structural shock $v_{it} = \varsigma_i = \sigma_{ii}^{1/2}$ take the form of

$$\begin{aligned}
\mathbf{GIR}^{v}_{\Delta z}(N, \varsigma_i, \Xi_{t-1}) &= \sigma_{ii}^{-1/2} C_N A_0^{-1} \Sigma \mathbf{s}_i,\ \mathbf{GIR}^{v}_{z}(N, \varsigma_i, \Xi_{t-1}) = \sigma_{ii}^{-1/2} \tilde{B}_N A_0^{-1} \Sigma \mathbf{s}_i \text{ and} \\
\mathbf{GIR}^{v}_{\beta' z}(N, \varsigma_i, \Xi_{t-1}) &= \sigma_{ii}^{-1/2} \beta' \tilde{B}_N A_0^{-1} \Sigma \mathbf{s}_i,\ N = 0, 1, 2, \ldots, i = 1, \ldots, n, \qquad (1.99)
\end{aligned}$$

with $\mathbf{GIR}^{v}_{\Delta z}(\infty, \varsigma_i, \Xi_{t-1}) = \mathbf{GIR}^{v}_{\beta' z}(\infty, \varsigma_i, \Xi_{t-1}) = \mathbf{0}$, $\mathbf{GIR}^{v}_{z}(N, \varsigma_i, \Xi_{t-1}) = \sigma_{ii}^{-1/2} \mathbf{C}(1) A_0^{-1} \Sigma \mathbf{s}_i$.

A further advantage of the GIR approach is that, unlike orthogonalised responses, the GIR's are unique and invariant to the ordering of the variables in $\mathbf{z}_t$. Pesaran and Shin (1998) prove that, apart from the obvious case when $\boldsymbol{\Omega}$ is diagonal, the OIR's and GIR's coincide only when considering a shock in the first equation. Applications of the GIR approach can be found in Pesaran and Shin (1998), Pesaran and Smith (1998), Garratt *et al* (1998, 2000, 2001) and Jacobs and Wallis (2002).

1.7.6 Persistence Profiles

Persistence Profiles (PP) were introduced by Lee *et al* (1992) and Lee and Pesaran (1993a) and are further discussed in *inter alia* Pesaran and Shin (1996) and Pesaran and Smith (1998). They measure the time profile of the effect of system-wide shocks on $\Delta\mathbf{z}_{t+N}$, $\mathbf{z}_{t+N}$ and $\beta'\mathbf{z}_{t+N}$, $N = 0, 1, 2, \ldots$. A number of equivalent definitions are discussed in Pesaran and Shin (1996) but in order to maintain a link with the previous discussion on impulse responses, the PP's will be defined here as

$$\mathbf{PP}_{\Delta z}(N) = Var\{E(\Delta\mathbf{z}_{t+N}|\Xi_t) - E(\Delta\mathbf{z}_{t+N}|\Xi_{t-1})\}, \qquad (1.100)$$

$$\begin{aligned}\mathbf{PP}_z(N) &= Var\{E(\mathbf{z}_{t+N}|\Xi_t) - E(\mathbf{z}_{t+N}|\Xi_{t-1})\}, \quad (1.101)\\ \mathbf{PP}_{\beta'z}(N) &= Var\{E(\boldsymbol{\beta}'\mathbf{z}_{t+N}|\Xi_t) - E(\boldsymbol{\beta}'\mathbf{z}_{t+N}|\Xi_{t-1})\}, \quad (1.102)\end{aligned}$$

where $N = 0, 1, 2, \ldots$, and $Var\{.\}$ denotes the variance. The persistence profiles are, therefore, measuring the variance of the revision in the N-step-ahead forecasts of $\Delta\mathbf{z}_t$, $\mathbf{z}_t$ and $\boldsymbol{\beta}'\mathbf{z}_t$. Combining (1.100)-(1.102) with the MA representations in (1.80) gives the (unscaled) persistence profiles

$$\mathbf{PP}_{\Delta z}(N) = C_N\Omega C_N', \ \mathbf{PP}_z(N) = \tilde{B}_N\Omega\tilde{B}_N' \text{ and } \mathbf{PP}_{\beta' z}(N) = \boldsymbol{\beta}'\tilde{B}_N\Omega\tilde{B}_N'\boldsymbol{\beta}, \quad (1.103)$$

where $N = 0, 1, 2, \ldots$. Just as in the case of the impulse responses discussed above, over an infinite horizon the PP's take the values $\mathbf{PP}_{\Delta z}(\infty) = \mathbf{PP}_{\beta' z}(\infty) = \mathbf{0}$ and $\mathbf{PP}_z(\infty) = \mathbf{C}(1)\Omega\mathbf{C}(1)'$ by (1.81) and (1.29), reflecting the fact that $\mathbf{z}_t$ is difference stationary and $\boldsymbol{\beta}'\mathbf{z}_t$ is $I(0)$.

In a similar manner, the (unscaled) PP's for the i-th equation $\Delta z_{i,t+N}$, $z_{i,t+N}$, $i = 1, \ldots, n$, and the j-th cointegrating relation $\boldsymbol{\beta}_j'\mathbf{z}_{t+N}$, $j = 1, \ldots, r$, can be found to be the scalars

$$\begin{aligned}PP(\Delta z_i, N) &= \mathbf{s}_i' C_N\Omega C_N'\mathbf{s}_i, \ PP(z_i, N) = \mathbf{s}_i'\tilde{B}_N\Omega\tilde{B}_N'\mathbf{s}_i \text{ and}\\ PP(\boldsymbol{\beta}_j'\mathbf{z}_t, N) &= \boldsymbol{\beta}_j'\tilde{B}_N\Omega\tilde{B}_N'\boldsymbol{\beta}_j, \ N = 0, 1, 2, \ldots, \ i = 1, \ldots, n, \ j = 1, \ldots, r, \quad (1.104)\end{aligned}$$

where $\boldsymbol{\beta}_j$ is the j-th column of $\boldsymbol{\beta}$ and $\mathbf{s}_i$ is an $n \times 1$ selection vector with the i-th element equal to unity and zeros elsewhere, so that $PP(\Delta z_i, N)$ and $PP(z_i, N)$ are the i-th element on the main diagonal of $\mathbf{PP}_{\Delta z}(N)$ and $\mathbf{PP}_z(N)$, respectively. Also, writing $\boldsymbol{\beta}_j = \boldsymbol{\beta}\mathbf{c}_j$, where $\mathbf{c}_j$ is an

$r \times 1$ selection vector with the j-th element equal to unity and zeros elsewhere, illustrates that $PP(\beta_j' \mathbf{z}_t, N) = \mathbf{c}_j' \mathbf{PP}_{\beta' z}(N) \mathbf{c}_j$, i.e. it is the j-th element on the main diagonal of $\mathbf{PP}_{\beta' z}(N)$. Over an infinite horizon the profiles in (1.104) are given by $PP(\Delta z_i, \infty) = PP(\beta_j' \mathbf{z}_t, \infty) = 0$ and $PP(z_i, \infty) = \mathbf{s}_i' \mathbf{C}(1) \mathbf{\Omega} \mathbf{C}(1)' \mathbf{s}_i$. Using the properties in (1.83) it is easily verified that the values of these profiles on impact, i.e. for $N = 0$, are given by $PP(\Delta z_i, 0) = PP(z_i, 0) = \mathbf{s}_i' \mathbf{\Omega} \mathbf{s}_i$ $= \omega_{ii}$ and $PP(\beta_j' \mathbf{z}_t, 0) = \mathbf{c}_j' \beta' \Omega \beta \mathbf{c}_j = \mathbf{c}_j' \tilde{\mathbf{\Omega}} \mathbf{c}_j = \tilde{\omega}_{jj}$. The scaled PP's are obtained by setting $PP(\Delta z_i, 0) = PP(z_i, 0) = PP(\beta_j' \mathbf{z}_t, 0) = 1$ and are, thus, given by

$$
\begin{aligned}
PP^s(\Delta z_i, N) &= \omega_{ii}^{-1} \mathbf{s}_i' C_N \mathbf{\Omega} C_N' \mathbf{s}_i, \; PP^s(z_i, N) = \omega_{ii}^{-1} \mathbf{s}_i' \tilde{B}_N \mathbf{\Omega} \tilde{B}_N' \mathbf{s}_i \text{ and} \\
PP^s(\beta_j' \mathbf{z}_t, N) &= \tilde{\omega}_{jj}^{-1} \beta_j' \tilde{B}_N \mathbf{\Omega} \tilde{B}_N' \beta_j, \qquad (1.105)
\end{aligned}
$$

where $N = 0, 1, 2, \ldots$, $i = 1, \ldots, n$, $j = 1, \ldots, r$, with $PP^s(\Delta z_i, \infty) = PP^s(\beta_j' \mathbf{z}_t, \infty) = 0$ and $PP^s(z_i, \infty) = \omega_{ii}^{-1} \mathbf{s}_i' \mathbf{C}(1) \mathbf{\Omega} \mathbf{C}(1)' \mathbf{s}_i$.

Persistence profiles, like the GIR's discussed in the previous sub-section, are unique and invariant to the ordering of the variables in $\mathbf{z}_t$. Moreover, they are invariant to whether the system-wide shock under consideration is the reduced form, $\mathbf{e}_t$, or any "structural" linear combination $\mathbf{v}_t = A_0 \mathbf{e}_t$. This is easily verified by formulating the MA representations in (1.80) in terms of $\mathbf{v}_t$ and applying the definitions in (1.100)-(1.102). For $\mathbf{PP}_{\beta' z}(N)$, for example, this yields $\beta' \tilde{B}_N A_0^{-1} \mathbf{\Sigma} (A_0^{-1})' \tilde{B}_N' \beta$, which by (1.94) is equal to $\beta' \tilde{B}_N \mathbf{\Omega} \tilde{B}_N' \beta$. Persistence profiles can, thus, be used in order to measure the speed of convergence of $\Delta \mathbf{z}_t$, $\mathbf{z}_t$ and $\beta' \mathbf{z}_t$ to their equilibrium, after-shock values by avoiding the controversies associated with the analysis of variable-specific innovations discussed above. However, the literature has been concentrated on $PP^s(\beta_j' \mathbf{z}_t, N)$ in (1.105), that measures the speed with which the j-th cointegrating relation $\beta_j' \mathbf{z}_{t+N}$, $j = 1, \ldots, r$, returns to equilibrium after a system-wide shock [see, for example, Pesaran and Shin (1996), Garratt *et al* (2000), GLPS and Lee and Papaikonomou (2002)].

Chapter 2

Identification and Testing of the IS-LM Model in the Long Run for the UK

2.1 Introduction

A recent application of the long-run structural cointegrating VAR approach to the UK economy is the development of a small-scale macroeconometric model in Garratt *et al* (1998) and its more recent version in Garratt *et al* (2001). These two studies utilise most of the econometric techniques reviewed in Chapter 1 regarding the treatment of the deterministic components, the modelling conditionally on $I(1)$ weakly exogenous variables, the use of bootstrap methods and the advances in impulse response analysis. The economic theory used in these studies as a guide in the over-identification of the cointegrating parameters consists of a general set of arbitrage and other long-run equilibrium conditions, like the Uncovered Interest Parity (UIP), the Fisher Interest Parity (FIP), Purchasing Power Parity (PPP), etc.

This chapter is inspired by the Garratt *et al* (1998, 2001) papers and uses, in most part, the data set analysed in the former study. However, the imposition of economic structure on the long-run is motivated here by a different economic framework. The basis of this is a modified version of the Mundell-Fleming model. This is, in essence, the dynamic IS-LM model developed by Turnovsky and Miller (1984) and Blanchard and Fischer (1989), reformulated within the context of a small open economy. The model gives rise to very familiar equilibrium conditions for the asset market, the goods market and the balance of payments, which are shown to impose a set of testable, over-identifying restrictions on the cointegrating parameters. The empirical analysis of the long run also considers one further cointegrating relation, namely, the modified PPP in Garratt *et al* (1998).

The aim of this chapter is not to provide a self-contained macroeconometric model of the UK. Rather than that, the main focus here is on the long-run behaviour of the demand-side of the economy, and the extent to which this can be described by the static equilibrium relations predicted by the modified IS-LM model. This is tested in the tradition of King *et al* (1991), Mellander *et al* (1992) and Garratt *et al* (1998, 2001) by assessing the significance of the over-identifying restrictions imposed by theory on the cointegrating matrix, within an otherwise unrestricted cointegrating $VAR(p)$. The short-run dynamic behaviour of the estimated, long-run structural model is evaluated with the use of Persistence Profiles and Generalised Impulse Responses. These are shown to produce very similar results with existing VAR models of the UK economy.

Chapter 2 is organised as follows. The following section derives the static, long-run equilibrium conditions to be investigated in the empirical analysis. Section 2.3 presents the data set and conducts a preliminary investigation into the order of integration of the variables. Section 2.4 illustrates how the equilibrium relationships may be embedded within a long-run structural VAR model and Section 2.5 presents the empirical findings. Most of the evidence in Section 2.5 is based on asymptotic inference and the use of small-sample scaling factors. Section 2.6 is motivated by the well-documented finite sample bias associated with asymptotic inference in cointegrating VAR applications and the limitations of scaling factors.[1] This section makes use of bootstrap techniques in order to simulate the finite-sample distributions for some key test statistics in Section 2.5. Section 2.7 looks at the short-run dynamics of the estimated model and Section 2.8 concludes.

[1] See Chapter 1, section 1.6.

2.2 The Modified IS-LM Model

In the standard IS-LM model the assets available to economic agents are distinguished between interest bearing and non-interest bearing. This section derives four long-run equilibrium conditions for a dynamic version of the open economy IS-LM similar to Turnovsky and Miller (1984) and Blanchard and Fischer (1989), in which the interest bearing assets are further distinguished between short-term (instantaneous) nominal bonds and long-term consols or perpetuities. The former pay an interest rate R_t while the latter pay the rate LI_t. The model is also extended by one further long-run relationship derived from the modified version of relative PPP introduced in Garratt *et al* (1998).

2.2.1 The LM Relation

Real demand for non-interest bearing assets is viewed as being driven by a transactions motive and a speculative motive

$$\frac{M_t^d}{P_t} = L(TRM_t, SPM_t), \tag{2.1}$$

where M_t^d denotes the level of nominal money demand, P_t is the price level, TRM_t and SPM_t stand for the transactions and speculative motive respectively and the subscript t denotes the time period.

The transactions motive represents real money demanded for transaction purposes and arises from the fact that payments and receipts are not synchronised. It has traditionally been modelled simply as a positive function of real income. However, in the light of the empirical findings in Garratt *et al* (1998), the transactions demand for money will be assumed to be also a negative function of the volume of non-cash (credit card) transactions

$$TRM_t = \ln(Y_t^{\beta_{11}} \Theta_t^{-1}), \tag{2.2}$$

where Y_t is the level of real income, β_{11} is a positive constant and Θ_t is the volume of non-cash transactions. Assuming that the volume of non-cash transactions has been increasing through time it can modelled as

$$\Theta_t = \exp\{d_{11}t\}, \tag{2.3}$$

where d_{11} is a positive constant.

The speculative motive represents the part of real money demand that is driven by the opportunity cost of holding money as opposed to short-term bonds and has been modelled as a negative function of the nominal short-term interest rate, R_t

$$SPM_t = -\beta_{12}R_t, \tag{2.4}$$

where β_{12} is a positive constant.

Real money demand is assumed to have the following functional form

$$\frac{M_t^d}{P_t} = D_{01}\exp\{TRM_t + SPM_t\}, \tag{2.5}$$

where D_{01} is a constant. Combining (2.2)-(2.5) gives

$$\frac{M_t^d}{P_t} = D_{01}Y_t^{\beta_{11}}\exp\{-\beta_{12}R_t - d_{11}t\}. \tag{2.6}$$

Equilibrium in the money market prevails when real money demand equals real money supply. Thus, in a stochastic framework the money market can be described in the long run by

$$\frac{M_t}{P_t} = \frac{M_t^d}{P_t}\exp\{\eta_{1,t}\}, \tag{2.7}$$

where M_t is the nominal money supply and $\eta_{1,t}$ is a mean zero, serially uncorrelated, covariance stationary process. Substitution of (2.6) in (2.7) gives

$$\frac{M_t}{P_t} = D_{01}Y_t^{\beta_{11}}\exp\{-\beta_{12}R_t - d_{11}t + \eta_{1,t}\}. \tag{2.8}$$

Taking the natural logarithm and rearranging gives the following log-linear expression for the temporary deviations $\eta_{1,t}$ from the long-run equilibrium

$$\eta_{1,t} = d_{01} + m_t - p_t - \beta_{11}y_t + \beta_{12}R_t + d_{11}t, \tag{2.9}$$

where lower case letters will hereafter denote the natural logarithm of the variables unless stated otherwise.

2.2.2 The IS Relation

The long-run equilibrium condition in the goods market for a small open economy is given by

$$Y_t = (C_t + IN_t + G_t + X_t - IM_t)\exp\{(1-\gamma_1)\eta_{2,t}\}, \tag{2.10}$$

where Y_t is output, C_t is planned consumption expenditure, IN_t is planned investment expenditure, G_t is planned government spending on goods and services, $X_t - M_t$ is the planned trade balance, γ_1 is a constant different from unity, $\eta_{2,t}$ is a mean zero, serially uncorrelated, covariance stationary process and all variables are in real terms. Expression (2.10) states that in the long run factor incomes are on average equal to planned aggregate spending. The stationary shocks $\eta_{2,t}$ introduce short-term deviations from the long-run equilibrium.

It is assumed that planned consumption is simply a positive function of income

$$C_t = C(Y_t), \qquad 1 > C_Y > 0. \tag{2.11}$$

Real planned investment is taken to be a negative function of the long-term interest rate, thereby reflecting the long-run financial commitments associated with real investment decisions

$$IN_t = IN(LI_t), \qquad IN_{LI} < 0. \tag{2.12}$$

Under the assumption that the Marshall-Lerner condition holds, the volume of real exports is modelled as a positive function of the real exchange rate and foreign income, while the volume of real imports is modelled as a negative function of the real exchange rate and a positive function of domestic income

$$\begin{aligned} X_t &= X(E_t P_t^*/P_t, Y_t^*), \qquad X_{EP^*/P} > 0, X_{Y^*} > 0, && (2.13) \\ IM_t &= IM(E_t P_t^*/P_t, Y_t), \qquad IM_{EP^*/P} < 0, IM_Y > 0, && (2.14) \end{aligned}$$

where E_t is the nominal effective exchange rate in units of domestic currency per unit of foreign currency and the superscript "*" denotes foreign.

Defining real private expenditure by Z_t and the ratio of planned government spending to planned private expenditure by $\omega_t \equiv G_t/Z_t$, the equilibrium condition given by (2.10) may be written as

$$Y_t = (1+\omega_t)Z_t \exp\{(1-\gamma_1)\eta_{2,t}\}, \tag{2.15}$$

where ω_t is assumed to be constant. Real private expenditure is assumed to have the following functional form

$$Z_t = D_{02}Y_t^{\gamma_1} \exp\{-\gamma_2 LI_t\} \left(\frac{E_t P_t^*}{P_t}\right)^{\gamma_3} (Y_t^*)^{\gamma_4}, \tag{2.16}$$

where D_{02} is a constant and $\gamma_i > 0$, $i = 2, 3, 4$. Substituting (2.16) in (2.15), taking logarithms and rearranging gives the following log-linear expression for the stationary shocks $\eta_{2,t}$

$$\eta_{2,t} = d_{02} + y_t + \beta_{21}LI_t - \beta_{22}(e_t - p_t + p_t^*) - \beta_{23}y_t^*, \tag{2.17}$$

where $\beta_{2i} = (1-\gamma_1)^{-1}\gamma_{i+1}$, $i = 1, 2, 3$, and $d_{02} = -(1-\gamma_1)^{-1}[\ln(D_{02}) + \ln(1+\omega_t)]$.

2.2.3 The BP Relation

The long-run equilibrium condition for the balance of payments is defined as

$$X_t - IM_t + NKI_t = \exp\{\eta_{3,t}\} - 1, \tag{2.18}$$

where $X_t - IM_t$ (current account) is as above, NKI_t stands for net capital inflows (capital account) and $\eta_{3,t}$ is a mean zero, serially uncorrelated, covariance stationary process. Expression (2.18) states that, on average, the current and capital accounts must sum to zero.

Defining the ratio of the capital account to one plus the current account by $\delta_t \equiv NKI_t/(1 + X_t - IM_t)$, allows for the equilibrium condition in (2.18) to be re-formulated as

$$(1 + X_t - IM_t)(1 + \delta_t) = \exp\{\eta_{3,t}\}. \tag{2.19}$$

It is assumed that

$$1 + X_t - IM_t = A_{03} Y_t^{-\beta_{31}} \left(\frac{E_t P_t^*}{P_t}\right)^{\beta_{32}} (Y_t^*)^{\beta_{33}}, \tag{2.20}$$

where A_{03} is a constant and $\beta_{3i} > 0$, $i = 1, 2, 3$. It is also assumed that $1 + \delta_t$ is proportional to departures from the Uncovered Interest Parity (UIP), $\eta_{UIP,t}$, and to the degree of capital mobility, K_t

$$1 + \delta_t = B_{03} \exp\{\eta_{UIP,t}\} K_t, \tag{2.21}$$

where B_{03} is a constant.

The UIP is an arbitrage condition between domestic and foreign currency denominated assets. It states that economic agents are willing to keep domestic currency denominated assets as long as the domestic (short-term) interest rate exceeds the foreign (short-term) interest rate by the amount of the expected depreciation and is assumed to hold in the following exponential form

$$\exp\{\eta_{UIP,t}\} = C_{03} \exp\{R_t - R_t^*\} \frac{E_t}{E_{t+1}^e}, \tag{2.22}$$

where C_{03} is a constant and the superscript "e" denotes expectation formed at time t.

The degree of capital mobility, K_t is modelled as the following positive exponential function of time

$$K_t = \exp\{d_{13} t\}, \tag{2.23}$$

where $d_{13} > 0$. This specification intends to capture the gradual removal of capital restrictions over the last three decades world-wide and the additional liberalisation of capital movements experienced by the UK through the process of economic integration within the European Union. Substitution of (2.20)-(2.23) in (2.19) yields

$$D_{03}Y_t^{-\beta_{31}}\left(\frac{E_tP_t^*}{P_t}\right)^{\beta_{32}}(Y_t^*)^{\beta_{33}}\exp\{R_t-R_t^*\}\frac{E_t}{E_{t+1}^e}=\exp\{\eta_{3,t}\}, \tag{2.24}$$

where $D_{03} = A_{03}B_{03}C_{03}$.

The assumption on the exchange rate expectations formation mechanism is the one adopted in Garratt *et al* (1998), namely

$$E_{t+1}^e = E_{t+1}\exp\{\eta_{e,t+1}\}, \tag{2.25}$$

where $\eta_{e,t+1}$ is a serially uncorrelated, not necessarily mean-zero, covariance stationary process. This expectations formation mechanism is consistent with Rational Expectations but it is much less restrictive as it allows for a non-zero mean in the expectational error. In other words, expression (2.25) allows for systematic over/under predictions but requires that the mean and variance of the errors is constant. Substitution of (2.25) in (2.24), taking logarithms and rearranging yields

$$\eta_{3,t}+\eta_{e,t+1}+\Delta e_{t+1}=d_{03}-\beta_{31}y_t+R_t-R_t^*+\beta_{32}(e_t-p_t+p_t^*)+\beta_{33}y_t^*+d_{13}t. \tag{2.26}$$

2.2.4 The PPP Relation

The Purchasing Power Parity (PPP) is a goods market arbitrage condition. In its absolute version it states that in a world of homogenous information sets across consumers and across countries, free trade and non-existent transport costs, market forces will equalise the prices of identical baskets of goods (expressed in domestic currency and converted by the current exchange rate) internationally

$$P_t = E_t P_t^*. \tag{2.27}$$

It is clear that absolute PPP relies on a series of strong assumptions. There is a substantial amount of literature that develops modified versions of (2.27) by relaxing its underlying assumptions. The present study adopts the modified version of relative PPP presented in Garratt *et al* (1998) given by

$$P_t = D_{04} E_t P_t^* \left(\frac{P_t^o}{P_t^*}\right)^{\theta} \exp\{-\eta_{4,t}\}, \tag{2.28}$$

where D_{04}, θ are constants, P_t^o is an oil price index and $\eta_{4,t}$ is a mean zero, serially uncorrelated, covariance stationary process. Garratt *et al* (1998) mention the possibility that $\eta_{4,t}$ follows a trend stationary process which would be consistent with the "Harrod-Balassa-Samuelson effect". This possibility arises in the presence of differential productivity growth rates in the traded and non-traded goods sectors across countries. In such a case the price of a basket of traded and non-traded goods would rise more rapidly in countries with higher productivity growth in the traded goods sector. This is a hypothesis that can and will be tested in section 2.5.6. A value of D_{04} other than unity would imply permanent deviations from absolute PPP due to, say, persisting transportation costs, trade barriers and possibly, even though less likely, persisting heterogeneity in the available information sets. The stationary shock $\eta_{4,t}$ captures transitory deviations from PPP due to, say, information heterogeneity.

Garratt *et al* (1998) justify the inclusion of relative oil prices by the fact that the discovery of oil reserves in the UK effectively classifies it as an oil producer, which means that the oil price shocks in the 1970's could have a direct effect on the real exchange rate. A non-zero value of θ would indicate such a long-run effect. They also point out that: "*...the importance of explicitly taking into account the effects of oil price changes on the dynamics of real exchange rates has been widely acknowledged in applied work; see, for example, Johansen and Juselius (1992)*".

Taking the natural logarithm of (2.28) and rearranging yields the following log-linear expression for the stationary deviations $\eta_{4,t}$ from the long-run equilibrium PPP

$$\eta_{4,t} = d_{04} + e_t - p_t + p_t^* + \theta(p_t^0 - p_t^*). \tag{2.29}$$

2.2.5 Interest Rate Arbitrage

The final building block of the model is an arbitrage condition between long and short-term real interest rates. This arises from the assumption that economic agents equalise the rates of return on both consols and short-term bonds up to a constant risk premium. This condition is assumed to hold in the following exponential form

$$\exp\{\frac{1}{P_{c,t}} + \frac{\Delta P^e_{c,t+1}}{P^e_{c,t+1}P_{c,t}}\} = D_{05}\exp\{R_t + \eta_{FIP,t}\}\frac{P_t}{P^e_{t+1}}, \tag{2.30}$$

where D_{05} is a constant, $P_{c,t}$ is the price of a consol, and $\eta_{FIP,t}$ is a mean zero, serially uncorrelated, covariance stationary process. The left-hand side of (2.30) is an exponential function of the instantaneous real rate of return of the consol which consists of the coupon payment, $1/P_{c,t}$, and the expected capital gains, $\Delta P^e_{c,t+1}/P^e_{c,t+1}P_{c,t}$.The right-hand side gives the real rate of return of short-term bonds, given by a stochastic, exponential version of the Fisher Interest Parity (FIP), weighted by the risk premium D_{05}. It states that in the long run the real short-term interest rate deviates from the short nominal rate, R_t, by the natural logarithm of expected inflation and the stationary innovation $\eta_{FIP,t}$.

Taking the natural logarithm of (2.30) and utilising the identity $LI_t \equiv 1/P_{c,t}$ gives

$$LI_t - \Delta LI^e_{t+1} = d_{05} + R_t + \eta_{FIP,t} - \Delta^e P_{t+1}. \tag{2.31}$$

Price and interest rate expectations are assumed to be formed according to

$$\begin{aligned} P^e_{t+1} &= P_{t+1}\exp\{\eta_{p,t+1}\}, && (2.32) \\ LI^e_{t+1} &= LI_{t+1} + \eta_{LI,t+1}, && (2.33) \end{aligned}$$

where $\eta_{p,t+1}$ and $\eta_{LI,t+1}$ are serially uncorrelated, covariance stationary expectational errors with possibly non-zero means. Substitution of (2.32) and (2.33) in (2.31) gives a log-linear equilibrium relation between long and short-term interest rates in terms of observables and structural shocks as

$$d_{05} + R_t - LI_t = \Delta p_{t+1} - \Delta LI_{t+1} - \eta_{FIP,t} + \eta_{p,t+1} - \eta_{LI,t+1}. \tag{2.34}$$

2.3 Data Overview

2.3.1 Definition of the Variables and the Sample Period

Section 2.2 has considered the four long-run equilibrium relationships defining the modified Mundell-Fleming model with an interest term structure and a further long-run equilibrium condition derived from the modified PPP theory. These five in total long-run relationships were formulated in terms of the ten variables in $\tilde{z}_t = [m_t, y_t, R_t, R_t^*, LI_t, e_t, p_t, p_t^*, p_t^o, y_t^*]'$. The empirical counterparts of these variables were constructed using almost exclusively the data set used in the study by Garratt *et al* (1998), which was kindly made available by Professor K. C. Lee.[2]

The data is quarterly, seasonally adjusted and extends over the period 1963q1-1998q2. Time plots of the variables are to be found in Figure 2.1. The chosen lag-lengths impose a lower bound in the sample period in order for all the regressions to be comparable, which as in Garratt *et al* (1998) is 1965q1. However, there is one potentially problematic feature associated with any attempt to estimate the five long-run relationships within the sample period 1965q1-1998q2. This has to do with the fact that it covers periods of different exchange rate regimes. There are two substantial periods in which exchange rates were not allowed to float. The first one covers the pre-1973 period when the Bretton Woods system of fixed exchange rates was still in full operation. The second extends over the eight observations 1990q4-1992q3 which signifies the UK's short membership in the European Exchange Rate Mechanism (ERM), during which the pound was allowed to fluctuate within limits of $\pm 6\%$. This may be problematic as the long-run position of the IS, LM and BP curves following exogenous or policy shocks is directly related

[2]Details on the definitions are provided in the data appendix.

to the exchange rate regime in operation. Despite this potentially problematic limitation and having in mind the quite large size of the model it was decided that all estimation be carried out over the full sample period 1965q1-1998q2.[3]

2.3.2 ADF and Phillips Perron Unit Root Tests

The empirical examination of the five equilibrium relationships within the context of the general cointegrating $VAR(p)$ in (1.4) needs to be preceded by an investigation of the order of integration of the variables in $\tilde{\mathbf{z}}_t$. As illustrated in the introductory Chapter 1, section 1.3.2, the question of stationarity of individual series can be addressed inside the model with the use of the multivariate ADF test. However, this approach is valid only under the assumption that the highest order of integration of the individual series in $\tilde{\mathbf{z}}_t$ is one. The purpose of this section is to test this assumption with the employment of Augmented Dickey-Fuller, (ADF), and Phillips-Perron (1988), (PP) tests.[4]

Tables 2.1a and 2.1b report the results of the ADF(k) tests, $k = 1, ...4$, applied to the levels and the differences of the variables, respectively. The tests appear to give consistent results as far as the variables y_t, R_t, R_t^*, LI_t, e_t and y_t^* are concerned, which are all found to be $I(1)$ irrespective of the order of augmentation, k, in the underlying Dickey-Fuller regressions. The presence of some quite large statistics in the levels of y_t^* for $k = 0$, R_t for $k = 1, 2$, and especially in the case of R_t^* for $k = 3, 4$, could raise some suspicion of stationarity for these series in the

[3]The problem of estimation over a period covering different exchange rate regimes will be addressed in the following chapters.

[4]It is a well established fact that ADF and PP tests have serious limitations and may result in ambiguous results, especially in the case of variables which are on the borderline $I(0)$-$I(1)$ or $I(1)$-$I(2)$. However, as Garratt et al (1998) point out, ambiguities of this kind may still prove very useful when being confronted with the choice of the number of cointegrating vectors, r. For example, knowing that a variable is likely to be on the borderline $I(0)$-$I(1)$ would imply that the cointegrating rank tests can be anticipated to be biased in favour of rejecting the null.

sample under consideration. Since the inclusion of $I(0)$ variables in a cointegrating $VAR(p)$ is not problematic, these results do not raise any concern and the question of stationarity of y_t^*, R_t and R_t^* is left to be determined at a later stage in a multivariate setting. In the case of p_t, p_t^* and m_t, however, the test results are alarming as they suggest that these variables could possibly be $I(2)$. The application of the Phillips-Perron tests, PP(ℓ), $\ell = 0, 5, 10, 15, 20$, reported in Tables 2.2a and 2.2b, yields similar results. The variables y_t, R_t^*, LI_t, e_t and m_t are consistently found to be $I(1)$ and there is again some indication that y_t^* and R_t could be stationary. As in the case of the ADF tests, the PP tests also suggest that p_t and p_t^* might be $I(2)$.

An important point raised in Charemza and Syczewska (1998) is that it is not possible to draw a formal conclusion concerning the order of integration of the variables based on the results presented above, without investigating the joint distribution of the ADF and PP tests. Such a task has not been undertaken here and, thus, inference will inevitably have to rely on an "eye-ball" approach. The joint application of the ADF and PP tests appears to support that y_t, R_t, R_t^*, LI_t, e_t and y_t^* are at most $I(1)$ variables and may, thus, be modelled within the general cointegrating $VAR(p)$ in (1.4). This econometric framework, however, is inconsistent with the potentially $I(2)$ variables p_t, p_t^* and m_t. A popular strategy in the presence of $I(2)$ variables is to transform time series *a priori* in order to obtain variables that are unambiguously $I(1)$ rather than dealing with mixtures of $I(1)$ and $I(2)$ variables directly.[5] Therefore, in the light of the ambiguity concerning the presence of a second unit root in p_t, p_t^* and m_t, it was decided to work with the variables $p_t - p_t^o$, $p_t^* - p_t^o$ and $m_t - p_t$ instead, which are consistently found to

[5]This strategy has been suggested *inter alia* by Alogoskoufis and Smith (1991), Boyd and Smith (1998), Haldrup (1998), Pesaran and Smith (1998) and Garratt *et al* (1998, 2001).

be $I(1)$ by both, ADF and PP tests.

The use of the transformed variables has important advantages from a statistical point of view. It resolves the problem of dealing with mixtures of $I(1)$ and $I(2)$ variables, while at the same time helps economise on degrees of freedom by effectively reducing the dimensions of the system. Nevertheless, from an economic perspective this approach has an unfortunate implication. The use of the transformed variable $p_t - p_t^o$ renders the interest rate arbitrage condition in (2.34) unidentifiable and, as a consequence, it has been dropped from the empirical analysis which will be based on $\mathbf{z}_t = [m_t - p_t, y_t, R_t, R_t^*, LI_t, e_t, p_t - p_t^o, p_t^* - p_t^o, y_t^*]'$.

2.4 Econometric Formulation of the Model

The discussion in the previous section indicated that the vector $\mathbf{z}_t = [m_t - p_t, y_t, R_t, R_t^*, LI_t, e_t, p_t - p_t^o, p_t^* - p_t^o, y_t^*]'$ is an appropriate choice for the empirical investigation of the model within a cointegrating VAR framework. For the reasons mentioned earlier the long-run arbitrage condition between short and long-term bonds has been dropped from the empirical analysis. The remaining four equilibrium conditions derived in section 2.2 can be expressed in terms of the variables in $\mathbf{z}_t$ as

$$\begin{aligned}
\varepsilon_{1,t} &= d_{01} + (m_t - p_t) - \beta_{11} y_t + \beta_{12} R_t + d_{11} t, && (2.35)\\
\varepsilon_{2,t} &= d_{02} + y_t + \beta_{21} LI_t - \beta_{22}[e_t - (p_t - p_t^o) + (p_t^* - p_t^o)] - \beta_{23} y_t^*, && (2.36)\\
\varepsilon_{3,t} &= d_{03} - \beta_{31} y_t + R_t - R_t^* + \beta_{32}[e_t - (p_t - p_t^o) + (p_t^* - p_t^o)] + \beta_{33} y_t^* + d_{13} t, && (2.37)\\
\varepsilon_{4,t} &= d_{04} + e_t - (p_t - p_t^o) + \beta_{41}(p_t^* - p_t^o), && (2.38)
\end{aligned}$$

where $\varepsilon_{i,t} = \eta_{i,t}$, $i = 1, 2, 4$, $\varepsilon_{3,t} = \eta_{3,t} + \eta_{e,t+1} + \Delta e_{t+1}$, and $\beta_{41} = 1 - \theta$.

Last period's stationary deviations $\varepsilon_{i,t-1}$, $i = 1, 2, 3, 4$, from these four equilibrium conditions may be written in matrix form as

$$\boldsymbol{\varepsilon}_{t-1} = \mathbf{d}_0 + \mathbf{d}_1(t-1) + \boldsymbol{\beta}'\mathbf{z}_{t-1}, \tag{2.39}$$

where $\boldsymbol{\varepsilon}_t = [\varepsilon_{1,t}, \varepsilon_{2,t}, \varepsilon_{3,t}, \varepsilon_{4,t}]'$, $\mathbf{d}_0 = [d_{01}, d_{02}, d_{03}, d_{04}]'$, $\mathbf{d}_1 = [d_{11}, 0, d_{13}, 0]'$ and

$$\boldsymbol{\beta}' = \begin{bmatrix} 1 & -\beta_{11} & \beta_{12} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & \beta_{21} & -\beta_{22} & \beta_{22} & -\beta_{22} & -\beta_{23} \\ 0 & -\beta_{31} & 1 & -1 & 0 & \beta_{32} & -\beta_{32} & \beta_{32} & \beta_{33} \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & \beta_{41} & 0 \end{bmatrix}. \tag{2.40}$$

These temporary disequilibria may be embedded in a $VAR(p)$ model of $\mathbf{z}_t$ as

$$\Delta\mathbf{z}_t = \mathbf{n}_0 + \boldsymbol{\alpha}\boldsymbol{\varepsilon}_{t-1} + \sum_{i=1}^{p-1}\Gamma_i\Delta\mathbf{z}_{t-i} + \mathbf{e}_t, \tag{2.41}$$

which may alternatively be written in the form of

$$\Delta\mathbf{z}_t = \mathbf{a}_0 + \mathbf{a}_1 t + \boldsymbol{\alpha}\boldsymbol{\beta}'\mathbf{z}_{t-1} + \sum_{i=1}^{p-1}\Gamma_i\Delta\mathbf{z}_{t-i} + \mathbf{e}_t, \tag{2.42}$$

where $\mathbf{a}_0 = \mathbf{n}_0 + \boldsymbol{\alpha}(\mathbf{d}_0 - \mathbf{d}_1)$ and $\mathbf{a}_1 = \boldsymbol{\alpha}\mathbf{d}_1$. Re-writing this last expression as

$$\Delta\mathbf{z}_t = \mathbf{a}_0 + \boldsymbol{\alpha}\boldsymbol{\beta}_*'\mathbf{z}_{*t-1} + \sum_{i=1}^{p-1}\Gamma_i\Delta\mathbf{z}_{t-i} + \mathbf{e}_t, \tag{2.43}$$

where $\beta'_* = [\beta', \mathbf{d}_1]$ and $\mathbf{z}_{*t-1} = [\mathbf{z}'_{t-1}, t]'$, reveals that the structural relations (2.35)-(2.38) may be studied within a cointegrating $VAR(p)$ with unrestricted intercepts $\mathbf{a}_0$ and trend coefficients restricted according to Case IV, that is $\mathbf{a}_1 = -\Pi\gamma$.[6]

Considering the size of the available data set and in the light of the small-open economy assumption, it was decided to treat the variables $p^*_t - p^o_t$ and y^*_t as weakly exogenous[7] so that $\mathbf{z}_t$ can be partitioned as $\mathbf{z}_t = [\mathbf{y}'_t, \mathbf{x}'_t]'$, where $\mathbf{y}_t = [m_t - p_t, y_t, R_t, R^*_t, LI_t, e_t, p_t - p^o_t]'$, $\mathbf{x}_t = [p^*_t - p^o_t, y^*_t]'$ and similarly the matrices $\mathbf{a}_i = [\mathbf{a}'_{iy}, \mathbf{a}'_{ix}]'$, $i = 0, 1$, $\Gamma_i = [\Gamma'_{iy}, \Gamma'_{ix}]'$, $i = 1, ..., p-1$, $\alpha = [\alpha'_y, \alpha'_x]'$ and the disturbance vector $\mathbf{e}_t = [\mathbf{e}'_{yt}, \mathbf{e}'_{xt}]'$ with variance matrix $\Omega = \begin{bmatrix} \Omega_{yy} & \Omega_{yx} \\ \Omega_{xy} & \Omega_{xx} \end{bmatrix}$. Therefore, under condition (1.44), the conditional model for $\Delta\mathbf{y}_t$ given $\Delta\mathbf{x}_t$ and the marginal model for $\Delta\mathbf{x}_t$ are given by

$$\Delta\mathbf{y}_t = \mathbf{c}_0 + \alpha_y \beta'_* \mathbf{z}_{*t-1} + \sum_{i=1}^{p-1} \Psi_i \Delta\mathbf{z}_{t-i} + \Upsilon \Delta\mathbf{x}_t + \mathbf{u}_t, \tag{2.44}$$

$$\Delta\mathbf{x}_t = \mathbf{a}_{0x} + \sum_{i=1}^{p-1} \Gamma_{ix} \Delta\mathbf{z}_{t-i} + \mathbf{e}_{xt}, \tag{2.45}$$

where $\mathbf{c}_i = \mathbf{a}_{iy} - \Upsilon\mathbf{a}_{ix}$, $i = 0, 1$, $\Psi_i = \Gamma_{iy} - \Upsilon\Gamma_{ix}$, $i = 1, ..., p-1$, $\mathbf{u}_t = \mathbf{e}_{yt} - \Upsilon\mathbf{e}_{xt}$ and $\Upsilon = \Omega_{yx}\Omega^{-1}_{xx}$. The restrictions on the trend coefficients in (2.44) take the form $\mathbf{c}_1 = -\Pi_y\gamma$, where $\Pi_y = \alpha_y\beta'$. Estimation of (2.44) will yield estimates for α_y, β, $\mathbf{d}_1$, $\mathbf{c}_0$, the short-run dynamic coefficients Ψ_i, $i = 1, ...p-1$, and the disturbances $\mathbf{u}_t$ with their associated covariance matrix Ω_{uu}. The constants $\mathbf{d}_0$ may be retrieved according to Appendix A. The long-run implications of the economic theory summarized by (2.35)-(2.38) can be tested in two

---

[6]For more details on the treatment of the deterministic components according to Cases I-V see Chapter 1, section 1.4.

[7]The choice of the weakly exogenous vector is more formally justified in subsequent sections.

broad stages within the chosen econometric framework. First, statistical inference on $rank[\Pi_y]$ will indicate the extent to which the data support the presence of four long-run relations among the variables in $\mathbf{z}_t$. Second, provided that the cointegrating rank is four, there will be the need for 16 exactly-identifying restrictions on β_*. The relations (2.35)-(2.38), however, impose a total of 29 restrictions on β_*, which leaves 13 over-identifying restrictions to be tested.

2.5 Estimation Results

Identification and testing of the long-run equilibrium relationships (2.35)-(2.38) will be carried out within the conditional model (2.44). All estimation is carried out using *Microfit 4.0* and *Gauss 386i*.

2.5.1 Determination of the Order of the *VAR*

Having decided in section 2.3.2 that the highest order of integration of the variables in $\mathbf{z}_t$ can reasonably be assumed to be one, the first issue that is addressed in this section is the choice of the lag-length of the model, p. Following Garratt *et al* (1998), Johansen (1995), Pesaran, Shin and Smith (2000) and others, this was done at a first stage within an unrestricted $VAR(4)$ in the level of $\mathbf{z}_t = [m_t - p_t,\ y_t,\ R_t,\ R_t^*,\ LI_t,\ e_t,\ p_t - p_t^o,\ p_t^* - p_t^o,\ y_t^*]'$. The maximum order 4 was chosen *a priori* bearing in mind the number of variables in $\mathbf{z}_t$, the available sample size and the quarterly nature of the data.

Table 2.3 reports the Adjusted Likelihood Ratio (ALR) statistics for testing the hypotheses $p = 0, 1, 2, 3$ as well as the values of the AIC and SBC. Pesaran and Smith (1998) point out in footnote 25 that "*...when determining the lag length in an autoregression, if the correct model is in the set being considered the AIC is inconsistent, namely as T goes to infinity it will not* *necessarily choose the correct lag length; whereas the SBC is consistent"*. The ALR tests reject the hypothesis $p = 0$ and $p = 1$ at the 5% level but provide no evidence with which to reject $p = 2$ and $p = 3$. However, the model selection criteria unanimously select $p = 1$.

At a second stage, following Pesaran and Smith (1998) a series of cointegrating VARs were estimated for $\mathbf{y}_t = [m_t - p_t, y_t, R_t, R_t^*, LI_t, e_t, p_t - p_t^o]'$ conditionally on $\mathbf{x}_t = [p_t^* - p_t^o, y_t^*]'$ for alternative values of p, r and intercept and trend specifications. Table 2.5 reports the values of the model selection criteria AIC, SBC, and HQC all of which have a maximum at $p = 1$. However, Pesaran and Smith (1998; pp. 489) highlight the fact that the use of model selection criteria in cointegrating VAR models is problematic from a statistical point of view due to the fact that they treat all the parameters symmetrically and thus, do not take into account the *super-consistency* of the ML estimates of the long-run coefficients.

Johansen (1995) advises against the inclusion of too many lags as they would very rapidly increase the number of estimated parameters in a cointegrating VAR. However, Garratt *et al* (1998) refer to Kilian (1997) for their argument that the consequences of over-estimating the order of the VAR are much less serious than under-estimating it. On the same grounds and in the light of the ALR tests in Table 2.3, it was decided to follow the Garratt *et al* (1998) approach and work with a cointegrating $VAR(2)$, instead of a $VAR(1)$ favoured by the model selection criteria. Johansen's (1995) point was also taken into consideration. However, instead of risking under-estimation of p, it was considered more sensible to reduce the number of estimated parameters by working with a cointegrating $VAR(2)$ conditional on weakly exogenous $I(1)$ variables.

2.5.2 Determination of the Weakly Exogenous Vector

In order to capitalize on the gains associated with partial systems, discussed in Chapter 1, it was decided to formulate the model conditionally on weakly exogenous $I(1)$ variables. Having in mind the evidence in Greenslade *et al* (2002)[8], the determination of the weak exogeneity status was chosen to precede inference on cointegration rank. The obvious candidates for the weakly exogenous vector are the foreign variables R_t^*, $p_t^* - p_t^o$ and y_t^*. Nevertheless, under condition (1.44) the weakly exogenous variables should not cointegrate. Following Pesaran, Shin and Smith (2000), this condition was tested within a $VAR(2)$ in the weakly exogenous variables $\mathbf{x}_t^+ = [R_t^*, p_t^* - p_t^o, y_t^*]'$ augmented by one lagged difference of the endogenous variables $\mathbf{y}_t^- = [m_t - p_t, y_t, R_t, LI_t, e_t, p_t - p_t^o]'$.[9] As can be seen from Table 2.4a, the null of no cointegration is comfortably rejected at the 95% level by both the λ-*trace* and *maximum eigenvalue* test statistics, indicating that $\mathbf{x}_t^+$ does not satisfy condition (1.44). The same hypothesis was tested for the vector $\mathbf{x}_t = [p_t^* - p_t^o, y_t^*]'$ within a $VAR(2)$ augmented by one lagged difference of the endogenous variables $\mathbf{y}_t = [m_t - p_t, y_t, R_t, R_t^*, LI_t, e_t, p_t - p_t^o]'$. As can be seen from Table 2.4b, both test statistics clearly fail to reject the null of no cointegration at the 95% and 90% levels. These results indicate that $\mathbf{x}_t$ can be treated as weakly exogenous.[10]

These results imply that the foreign interest rate will have to be treated as an endogenous variable, which even though is not problematic could be thought as counter-intuitive. Pesaran, Shin and Smith (2000) justify the treatment of the foreign interest rate as endogenous to the

[8] See Chapter 1, end of section 1.6.3.

[9] This corresponds to the marginal model for $\mathbf{x}_t^+$.

[10] It is worth noting that the presence of cointegration among the elements of $\mathbf{x}_t^+$ could be a reflection of a stationary R_t^*. The multivariate ADF tests discussed in the following section, however, appear to reject such a hypothesis for reasonable values of r. Furthermore, as will become apparent in subsequent sections, the estimated error-correction terms appear to be strongly significant in the equation for R_t^*, thus, supporting its treatment as an endogenous variable.

UK by the importance of the country's position in the world financial markets. Furthermore, as mentioned in the data appendix, the foreign interest rate variable used in this study is measured as a weighted average of the US, Japanese, German and French rates. It is not unrealistic to expect at least the German and French rates to be to some degree endogenous to the UK, in which case, the constructed variable could also be expected to satisfy this assumption. The extent to which this is true can and will be examined later by testing for joint significance of the error correction terms in the foreign interest rate equation.

2.5.3 Treatment of the Deterministic Terms

A further issue that needs to be addressed is the treatment of the deterministic components. According to the economic theory of section 2.2, the money market and balance of payments equilibrium conditions and possibly even the PPP condition can be modelled as trend-stationary processes. This implies that the trend coefficients in the empirical model would have to be restricted according to case IV, so that they may enter the cointegrating vectors. Further, as discussed in Chapter 1, section 1.4, such a restriction ensures that the levels of the variables under consideration do not exhibit quadratic trending behaviour with the number of quadratic trends varying directly with r. In the context of the conditional model (2.44) this restriction takes the form $\mathbf{c}_1 = -\Pi_y\gamma$. Pesaran, Shin and Smith (2000) show that statistical inference on the validity of this restriction, conditional on $rank(\Pi_y) = r$, can be made by means of the likelihood-ratio statistic. Asymptotically, this is a chi-squared variate with $n_y - r$ degrees of freedom, where n_y is the number of endogenous variables. This section also considers Sims' adjusted LR, (ALR), statistic in an attempt to control for sample size.

The LR and ALR statistics are reported in Table 2.6, and clearly reject the trend restrictions at the 5% level for $r = 0, ..., 5$ and $r = 0, ..., 4$, respectively. An inspection of the model selection criteria reported in Table 2.5 shows that, given the chosen value of $p = 2$, the SBC favours the restricted trends specification irrespective of the choice of r. The HQC selects the restricted model for $r = 1, 2, 3$, and 6, while for $r = 4$ it favours the unrestricted model and it is inconclusive for $r = 5$. The AIC favours the restricted trends specification only for $r = 6$. However, the reliability of the LR statistics is doubtful in the light of the relatively small size of the sample, see for example Gredenhoff and Jacobson (1998). The use of the ALR and the model selection criteria is also problematic, as they place an equal weight on all coefficients, thus, ignoring the fact that the ML estimates of the short and long-run coefficients converge to their true values at a different speed.

In the light of these mixed results and having in mind the limitations of the LR, ALR and the model selection criteria in this context, it was decided to put more weight on the theoretical priors and use the restricted trend specification, which has received solid support only from the SBC.

2.5.4 Multivariate ADF Tests

The preceding discussion on the treatment of the deterministic terms and weak exogeneity has provided reasonable evidence in favour of the specification of the empirical model according to (2.44). Based on the ADF and PP tests in section 2.3.2 it has been assumed throughout that the elements of $\mathbf{z}_t$ are at most $I(1)$. In this section, the question of stationarity of individual series in $\mathbf{z}_t$ is addressed within the multivariate setting of (2.44). As illustrated in Chapter 1, section 1.3.2, stationarity of individual elements in $\mathbf{z}_t$ is equivalent to the restriction of the cointegrating matrix β according to (1.24). Within the context of (2.44) the test of this restriction is asymptotically χ^2 with $n_* - r$ degrees of freedom, where n_* is the number of elements in $\mathbf{z}_{*t}$. The test is directly dependent on the choice of r, since the null hypothesis is formulated conditionally on the number of cointegrating vectors. For this reason the test statistics were calculated for all alternative numbers of cointegrating relations[11]. Table 2.7 summarizes the results. The null of stationarity is generally rejected for all values of r with the exception of the foreign interest rate which is found to be stationary for $r = 6$. However, the cointegration rank tests discussed in the following section suggest that this result is probably a reflection of a particularly over-estimated choice of r.

2.5.5 Determination of the Cointegrating Rank

This section investigates the number of cointegrating relationships present in the trend restricted, conditional $VAR(2)$ given by (2.44) by means of the λ-*trace* and *maximum eigenvalue* statistics. These were originally developed by Johansen (1988) and Johansen and Juselius (1990) and later adapted to the analysis of cointegrating VAR models conditional on $I(1)$ weakly exogenous variables by Pesaran, Shin and Smith (2000). In a preliminary attempt to control for the well documented small-sample bias of these tests[12], this section also makes use of the Reinsel and Ahn (1988, 1992) adjusted statistics. These were computed by scaling the λ-*trace* and *maximum eigenvalue* statistics by the factor in (1.55), where n was replaced by n_y.

The results are reported in Table 2.8a. The adjusted and unadjusted *maximum eigenvalue* favours values of r in the region 2-3, whereas the λ-*trace* and adjusted λ-*trace* indicate that r lies in the region 4-5. As illustrated previously, economic theory suggests that there are

[11] Clearly, this test is not defined in the cases $r = 0$ and $r = n_y$, since in the former the null hypothesis is true with probability 0 and in the latter with probability 1.

[12] See Chapter 1, section 1.6.

four cointegrating relationships, which is consistent with the adjusted λ-*trace*. Furthermore, Cheung and Lai (1993) argue that the λ-*trace* should be considered more reliable in the absence of residual normality, which seems to be the case in five out of the seven equations in the system. In the light of this, the theoretical prediction that $r = 4$ seems plausible. This choice is also supported by a series of cointegrating rank tests calculated for smaller VAR models, estimated separately for each equilibrium relationship and groups of two equilibrium relationships.

Nevertheless, as mentioned in Chapter 1, section 1.6.1, the Reinsel and Ahn scaling factor in (1.55) provides only a relatively crude small-sample correction. Furthermore, its application has only been considered in the context of symmetric systems and its performance within partial systems remains unknown. For these reasons the question of the cointegrating rank will be further investigated with the use of bootstrap techniques in subsequent sections.

2.5.6 Over-Identification

The inclusion of the long-run relations (2.35)-(2.38) within the cointegrating VAR framework of (2.44) imposes the following structure on the trend-augmented cointegrating matrix $\boldsymbol{\beta}_*$:

$$\boldsymbol{\beta}'_* = \begin{bmatrix} 1 & -\beta_{11} & \beta_{12} & 0 & 0 & 0 & 0 & 0 & 0 & d_{11} \\ 0 & 1 & 0 & 0 & \beta_{21} & -\beta_{22} & \beta_{22} & -\beta_{22} & -\beta_{23} & 0 \\ 0 & -\beta_{31} & 1 & -1 & 0 & \beta_{32} & -\beta_{32} & \beta_{32} & \beta_{33} & d_{13} \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & \beta_{41} & 0 & 0 \end{bmatrix}, \tag{2.46}$$

where the first row corresponds to the stationary deviations from the long-run LM equilibrium, the second row corresponds to temporary departures from the long-run IS relation, the third row represents the BP disequilibria and the fourth row is the deviation from the modified

PPP condition. This structure subjects the cointegrating matrix to a total of 29 restrictions of which $r^2 = 16$ are exactly identifying. This leaves 13 over-identifying restrictions to be tested. A subset of 12 over-identifying restrictions arises by allowing for a non-zero element in the last column of the PPP vector. This would allow for the structural shock $\eta_{4,t}$ to be trend stationary, in which case, as noted in section 2.2.4, the PPP relation would be consistent with the Harrod-Balassa-Samuelson hypothesis. This subset will be denoted as R_{OV1}, while the full set of over-identifying restrictions in (2.46) will be denoted as R_{OV2}.

Table 2.9 presents the LR and ALR statistics for testing R_{OV1} and R_{OV2}.[13] The 12 over-identifying restrictions in R_{OV1} are rejected by both test statistics at the 5% and 10% levels. The estimated trend coefficient in the PPP relation is very small (0.00023) and insignificant (t-ratio = 0.38), suggesting that the Harrod-Balassa-Samuelson hypothesis is not consistent with the data. Setting the trend coefficient in the PPP relation equal to zero results in the full set of restrictions R_{OV2}. The ALR statistic provides no evidence with which to reject the 13 over-identifying restrictions in R_{OV2} at the 5% level, even though the LR still does. In the light of a potentially substantial small-sample bias in the LR statistic and, considering the fact that the null is rejected by a relatively small margin, it seems more sensible to put more weight on the ALR. It is thus plausible to conclude that the over-identifying restrictions imposed by the theory in section 2.2 are more likely to be supported by the data. This is supported more strongly by the bootstrap experiments in subsequent sections.

The estimated long-run relations subject to R_{OV2} are reported in Table 2.10. Even though all coefficients carry the sign predicted by theory, the IS and BP relations appear to be estimated

[13]For the reasons mentioned earlier, the use of the ALR as a small-sample adjustment to the LR is not entirely appropriate in the context of cointegrating VARs. Section 7 deals more formally with the problem of small-sample bias.

relatively imprecisely. In particular, the coefficient on the real exchange rate in the IS relation appears to be strongly insignificant.

The imprecision in the estimation of the IS and the BP relations is likely to be caused by the fact that the data set covers distinct periods of fixed and flexible exchange rate regimes. As noted in section 2.3.1, the pre-1973 period and the observations between 1990q4 and 1992q3 are characterised by a fixed exchange rate regime, whereas in the remaining sample, exchange rates are floating. The standard Mundell Fleming model gives a very clear insight into the fact that the long-run position of the IS, the LM and the BP curves, following an exogenous or a policy disturbance, is directly affected by the exchange rate regime in operation. However, this is very likely to be much less problematic in the case of the BP curve as the degree of capital mobility increases. This would cause the current account to become progressively less significant compared to the capital account and, thus, eliminate the channel through which the exchange rate can affect the position of the BP curve in the R-Y plane.

As was argued in section 2.2.3 such a tendency has been observed in the case of the UK and worldwide over the period under consideration. In this study this is captured by the strongly significant linear time trend in the BP relation. This helps explain the relatively higher precision in the estimation of the BP compared to the IS. One possible way of dealing with the problem of different exchange rate regimes would be to allow for a structural break in the equilibrium relations, by allowing for 0/1 intervention dummies to enter the cointegrating vectors. This, however, will be investigated in the chapters to follow.

The LM relation has been investigated in a similar framework, *inter alia*, by Hoffman and Rasche (1991), King *et al* (1991), Stock and Watson (1993) and Garratt *et al* (1998), the general findings of which are supportive of the results presented here. However, the value of the income elasticity of the money demand has typically been found to be smaller. King et al (1991), for example, find an income elasticity of 1.2. However, this value is quite likely to be negatively biased due to the steady decline in money demand for transactions purposes, caused by the gradual increase in non-cash transactions. In this study, as in Garratt *et al* (1998), an effort was made to capture this tendency with a linear time trend, which resulted in an estimate of 2.7 for the income elasticity. A further point worth mentioning, is that the signs on the real exchange rate coefficients in both, the IS and the BP relations confirm the validity of the Marshall-Lerner condition.

2.5.7 Vector Error Correction Model

Table 2.11 summarises the estimates, the descriptive and diagnostic statistics for the estimated VECM, where the error-correction terms are identified according to R_{OV2}. All equations appear to have a reasonably good fit with the exception of Δe_t. This result is not surprising in the light of a quite voluminous literature that demonstrates the quite insignificant role of fundamentals in exchange rate determination. The exchange rate does not seem to be driven by any of the equilibrium relations under consideration and, in fact, the only significant coefficient is the one on Δe_{t-1}. Real money supply appears to respond to deviations from the LM equilibrium. Output and the domestic short-term interest rate are found to be driven by the BP and PPP equilibria. The long-term interest rate is responsive to deviations from all four equilibrium conditions, while domestic prices relative to oil prices seem to respond only to the IS relation.

However, what is of particular interest is the fact that all four error-correction terms appear to be significant in the foreign interest rate equation. The Wald statistic for the hypothesis of joint significance of the error-correction terms is 15.99 with a 95% critical value of just

9.49. This result strongly supports the decision in section 6.2 to include this variable in the endogenous vector.

One particularly problematic feature is the presence of serial correlation in the ΔR_t equation. Increasing the order of the VAR to $p = 3$ does resolve the problem in ΔR_t but creates more serious problems elsewhere, in particular in the $\Delta(p_t - p_t^o)$ equation. Inspection of the autocorrelation function and the employment of Lagrange Multiplier tests indicate serially correlated residuals of orders greater than 2, suggesting that the problem could be caused by imperfections in the seasonal adjustment of the series. Attempts to account for this using seasonal dummies proved futile. A possible cause for the presence of serial correlation in the ΔR_t equation might be the distorting effect of the fixed exchange rate periods. During such periods interest rate differentials are viable and this could have a weakening effect on the UIP condition, which is the link between ΔR_t and the BP relation.

For the remaining equations serial correlation does not appear to be an issue, however, there is a general rejection of residual normality indicating the presence of big outliers due to, mainly, the oil-price shocks within the sample period.

2.6 Bootstrapped Critical Values

The aim of the previous sections was to identify and test three long-run relationships derived from a dynamic version of the Mundell-Fleming model and a further long-run equilibrium derived from the modified version of PPP in Garratt *et al* (1998). The econometric framework that was adopted for the empirical investigation of the model allowed for a two-stage testing procedure. First, it was investigated whether the data supported the existence of four long-run equilibria and second, given the presence of the four cointegrating relations, it was tested to what extent these take the form predicted by theory. Adjusted statistics were employed in a preliminary effort to control for the well-documented small-sample bias associated with asymptotic inference.[14] However, due to the limitations associated with the use of scaling factors, identified in previous sections, it was decided to complement finite-sample inference with a bootstrap exercise. This section employs the parametric and non-parametric bootstrap methods elaborated in Chapter 1, sections 1.6.2-1.6.4, in order to generate model-specific critical values for the cointegrating rank tests and the tests of over-identifying restrictions.[15] *Gauss 386i* was used throughout. The programs utilise Y. Shin's procedures for the computation of the cointegration rank tests and modified versions of K.C. Lee's data generating routines.

2.6.1 Bootstrapped Cointegration Rank Tests

This section employs the parametric and non-parametric methods discussed in Chapter 1, section 1.6.3, in order to generate 10,000 pseudo-data sets under the null hypotheses $r = 0, ..., 6$. The weakly exogenous vector, $\mathbf{x}_t$, is treated as a stochastic process, given by the peripheral model in (2.45) and the data-generating process follows (1.65)-(1.70). The *maximum eigenvalue* and *λ-trace* statistics have been computed for each of the simulated data sets and the resulting bootstrap distributions can be found in Figures 2.3-2.6.

The corresponding 95% and 90% critical values are reported in Table 2.8b. Not surprisingly, the bootstrapped critical values are found to be higher than their asymptotic counterparts in Table 2.8a, which is in accordance with the existing literature. Both, the parametric and non-

[14] See Chapter 1, section 1.6.

[15] It is appreciated that the reliability of these methods may be compromised by the possible presence of serial correlation in the equation for ΔR_t (see Table 11).

parametric versions give very similar results, although, the critical values obtained from the latter are generally smaller. The *maximum eigenvalue* 95% critical values in both versions are found to be high enough to maintain the hypothesis of no cointegration, which can only be rejected at the 10% level. The same is also true for the hypothesis $r \leqslant 1$, but there is no evidence with which to reject $r \leqslant 2$, even at the 10% level. The bootstrapped λ-*trace* critical values, however, indicate a comfortable rejection of the null hypotheses $r = 0$, $r \leqslant 1$ and $r \leqslant 2$ at the 5% for the non-parametric version, while in the parametric version $r \leqslant 2$ is only rejected at the 10% level .

These findings clearly suggest that the earlier estimate of $r = 4$, with the use of the adjusted statistics, may have been too high and it appears that the number of long-run relations probably lies in the region 2-3, instead. This is not surprising, since the Reinsel and Ahn scaling factor is known to only partially account for the small-sample bias.[16] It should also be noted that in the general absence of residual normality, more weight should be placed on the non-parametric results and, in particular, on the λ-*trace* statistic.

2.6.2 Bootstrapped LR Tests of Over-Identifying Restrictions

As discussed previously with reference to Table 2.9, the full set of over-identifying restrictions in (2.46), denoted R_{OV2}, is asymptotically rejected at the 5% level by a relatively small margin. A preliminary effort to control for sample size with the use of the *ALR* statistic lead to non-rejection at the 5% but not at the 10% level. Considering the evidence in Gredenhoff and Jacobson (1998), Fachin (2000) and Jacobson *et al* (2001),[17] however, it seems very likely that

[16]For more details see the discussion on the findings of Cheung and Lai (1993) in Chapter 1, section 1.6.1.

[17]Based on bootstrap techniques, these studies find a very substantial small-sample bias associated with asymptotic inference on the cointegrating parameters. The extent of this bias is found to be close to, or even exceed 100%.

the *ALR* corrects only a very small part of the finite-sample bias and a bootstrap experiment is anticipated to provide much stronger support in favour of R_{OV2}.

In contrast to the earlier work of Fachin (2000) and Jacobson *et al* (2001), who only consider over-identification schemes that fully specify all cointegrating parameters, in this study, the over-identified cointegrating matrix in (2.46) involves 11 freely estimated parameters, namely β_{11}, β_{12}, d_{11}, β_{21}, β_{22}, β_{23}, β_{31}, β_{32}, β_{33}, d_{13} and β_{41}. As discussed in Chapter 1, section 1.6.4, this creates convergence problems in the maximisation of the restricted log-likelihood for each of the simulated data sets with the use of conventional optimization algorithms. In this Chapter it is not attempted to deal with this problem directly and a possible solution will be considered in the Chapters that follow. The approach taken here follows Garratt *et al* (1998) and was denoted in Chapter 1, section 1.6.4, as *bootstrap 1.*

In this case this is effectively a bootstrap test of the hypothesis

$$H_{0,1} : \boldsymbol{\beta}_*(EX) = \hat{\boldsymbol{\beta}}_*(R_{OV2}), \tag{2.47}$$

instead of the actual hypothesis of interest which is

$$H_0 : \boldsymbol{\beta}_*(EX) = \boldsymbol{\beta}_*(R_{OV2}), \tag{2.48}$$

where $\boldsymbol{\beta}_*(EX)$ is any exactly identified, trend-augmented cointegrating matrix[18], $\boldsymbol{\beta}_*(R_{OV2})$ is the over-identified cointegrating matrix in (2.46) with $\hat{\boldsymbol{\beta}}_*(R_{OV2})$ being its estimate reported in

[18]In principle, the r^2 exactly identifying restrictions in the long-run structural VAR approach are a subset of the full set of restrictions suggested by theory, in this case R_{OV2}. For the purpose of this bootstrap experiment, however, Johansen's (1988) procedure was found to be computationally more convenient. This has no real significance, as the value of the log-likelihood in the exactly identified model is the same for any set of r^2 restrictions.

Table 2.10. For the original data set, the LR statistic for testing (2.48) and (2.47) takes the value 24.93, with the asymptotic 95% critical value being 22.36 in the first instance and 36.42 in the second. As pointed out in Chapter 1, section 1.6.4, the application of bootstrap 1 can only provide conclusive results concerning (2.48) if it leads to a rejection of (2.47). In this case, this is clearly not possible, as the latter hypothesis is not rejected asymptotically. The mere purpose of this bootstrap exercise is, therefore, to simply demonstrate the magnitude of the small-sample bias in the current settings.

Ten thousand data sets were simulated under (2.47) using (1.65)-(1.70). The non-parametric version follows Fachin (2000), denoted as version (b) in Chapter 1, section 1.6.2. The statistic $LR = 2\{LL[\hat{\beta}_*^j(EX)] - LL[\hat{\beta}_*(R_{OV2})\}$, $j = 1, \ldots, 10,000$, has been computed for each of the simulated data sets and the resulting bootstrap distributions can be found in Figure 2.2. The application of the parametric bootstrap resulted in the 95% and 90% critical values of 54.86 and 49.76, while the corresponding values in the non-parametric version were found to be 62.86 and 57.96, respectively. Keeping in mind that the non-parametric results should probably be considered more reliable in the general absence of residual normality, the small-sample bias associated with the test of (2.47) appears to be approximately 73%. To the extent that the test of (2.48) is subject to an equally large bias, the 95% finite-sample critical value can be expected to be close to 38.59, which comfortably exceeds the test statistic 24.93.

2.7 Investigation of the Dynamic Properties of the Model

This section looks at the dynamic behaviour of the estimated long-run structural *VAR* model by evaluating both the response of individual variables to shocks in a given equation, as well as the dynamic adjustment of the estimated structural error-correction terms in response to system-wide shocks. The tools employed are the Generalised Impulse Responses (GIR), and the Persistence Profiles (PP) discussed in Chapter 1, section 1.7.[19] These measures have been computed for the underlying, symmetric system in (2.43). The parameters of (2.43) can be computed according to the relations below (2.45) using the estimates from the marginal and conditional models in (2.45) and (2.44), respectively.[20]

In order to examine the effects of an oil price shock similar to Garratt *et al* (1998, 2001), the model has also been estimated with oil prices as

$$\Delta \mathbf{z}_t^{+} = \mathbf{a}_0^{+} + \begin{bmatrix} \alpha_y^{+} \\ \mathbf{0}_\alpha \end{bmatrix} \begin{bmatrix} \beta' & \mathbf{0}_\beta & \mathbf{d}_1 \end{bmatrix} \begin{bmatrix} \mathbf{z}_{t-1}^{+} \\ t \end{bmatrix} + \sum_{i=1}^{p-1} \begin{bmatrix} \Gamma_{iz}^{+} \\ \mathbf{0}_\gamma \end{bmatrix} \Delta \mathbf{z}_{t-i}^{+} + \mathbf{e}_t^{+}, \qquad (2.49)$$

where $\mathbf{z}_t^{+} = [\mathbf{y}_t', \mathbf{x}_t', p_t^o]'$, Γ_{iz}^{+}, $i = 1, \ldots, p-1$, are $n \times n^{+}$ coefficient matrices, $p = 2$, $n^{+} = \dim[\mathbf{z}_t^{+}] = n + 1$, and $\mathbf{0}_\alpha$, $\mathbf{0}_\beta$ and $\mathbf{0}_\gamma$ are blocks of zeros with dimensions $(n_x + 1) \times r$, $r \times 1$ and $1 \times n^{+}$, respectively. The combination of weak exogeneity of p_t^o, indicated by $\mathbf{0}_\alpha$, with the block of zeros $\mathbf{0}_\gamma$ renders $\mathbf{z}_t$ non Granger-causal for p_t^o. The hypothesis that the terms $\Delta \mathbf{z}_{t-i}^{+}$, $i = 1, \ldots, p-1$, do not enter the last row of (2.49) has been tested, given weak exogeneity of p_t^o, within the marginal model for $[\mathbf{x}_t', p_t^o]'$. The corresponding Wald statistic was found to

[19] Confidence intervals for the PPs and the GIRs have not been computed in this study and, as a consequence, all the results must be interpreted with caution.
[20] See also Chapter 1, section 1.6.3.

be 12.9 with an asymptotic 95% critical value of 18.31, indicating that $\mathbf{z}_t$ does, indeed, not Granger-cause p_t^o.[21] The model in (2.49) allows for a direct comparison with the analysis of an oil price shock in Garratt *et al* (1998) and to a lesser extent with that in Garratt *et al* (2001), as the latter impose also a number of orthogonality restrictions.

2.7.1 Persistence Profiles

The Persistence Profiles (PP), advanced by Lee and Pesaran (1993a) and Pesaran and Shin (1996), are used to examine the effect of system-wide shocks on the cointegrating relations, thus, avoiding the controversies associated with the analysis of variable-specific innovations. Furthermore, they are unique, as they are invariant to any linear transformation $\mathbf{v}_t = A_0\mathbf{e}_t$ of the reduced-form shocks $\mathbf{e}_t$ in (2.43).[22]

The scaled PPs, given by (1.105), have been computed for the estimated LM, IS, BP and PPP relations within (2.43) and are plotted in Figure 2.7. All PPs converge towards zero, thus, confirming the stationary nature of the estimated cointegrating relations. Equilibrium appears to be restored more rapidly in the money market, as indicated by the PP for the LM relation. The rate of adjustment is initially very rapid, with 70 per cent of the adjustment process being completed within the first year. Thereafter, the pace slows down with 80 per cent of the adjustment taking place after 2.5 years and 95 per cent of the equilibrium being restored after approximately 4 years. Garratt *et al* (1998, 2001) obtain very similar results for their respective Real Money Balances (RMB) and Money Market Equilibrium (MME) relations, which are effectively LM relations with unitary income elasticity. Garratt *et al* (1998; pp.23) justify

[21] The restriction that $\mathbf{z}_t$ does not Granger-cause p_t^o is not crucial for the outcome of the GIRs. It was imposed here in order to facilitate a direct comparison with the oil price shock considered in Garratt *et al* (1998).

[22] See Chapter 1, end of section 1.7.6.

the 4-5 year long adjustment process by *"the prolonged and persistent effect of technological innovations in financial markets that have taken place, particularly over the past two decades"*.

The PP for the IS relation appears to be monotonic but the adjustment process is found to be slower than for the LM, thus, reflecting the relative rigidity in the prices of goods and services compared to the more flexible prices of financial assets. After the course of one year, equilibrium in the goods market is restored by 45 per cent, 80 per cent of the adjustment has been completed after 13 quarters and 95 per cent within approximately 6 years.

In contrast to the IS relation, long-run equilibrium in the BP and PPP relations is not achieved in a monotonic manner. During the first quarter after the disequilibriating shock, the deviation from the long-run steady state is increasing for both, BP and PPP. For the BP relation this leads to only 20 per cent of the adjustment being completed after one year. However, the rate of adjustment is fast enough for the BP to catch up with the IS, with 80 per cent of the adjustment being completed after 13 quarters and 95 per cent within 6 years.

Not surprisingly, PPP is restored at a slower rate with less than one per cent of the adjustment being completed within the first year, 80 per cent within 4 years and 95 per cent in approximately 7 years. The initial "overshooting" in the profile of the PPP is a typical finding in studies of the UK economy like Pesaran and Shin (1996), Garratt *et al* (2001) and to a lesser extent, Garratt *et al* (1998). The rate of convergence to PPP equilibrium appears to be slower than in Garratt *et al* (1998), who report the adjustment process to last approximately 5 years, instead of 7-8 years reported here. However, the kind of persistence of system-wide shocks on PPP found in this study is more consistent with Pesaran and Shin (1996), who report an adjustment process of approximately 6 years, and with the more recent evidence in Garratt *et al* (2001), who estimate that it takes approximately 8 years for a 95 per cent adjustment.[23] These variations in the estimated persistence of PPP disequilibria for the UK can be attributed to the quite wide confidence intervals that have been simulated in Pesaran and Shin (1996) and Garratt *et al* (2001). The sluggish rate of convergence towards PPP may also explain the frequent rejection of the PPP hypothesis with the use of standard regression methods.

2.7.2 Generalised Impulse Responses

The GIR approach, Koop *et al* (1996) and Pesaran and Shin (1998), was primarily developed as an alternative to the conventional Orthogonalised Impulse Responses (OIR) proposed by Sims (1980). Rather than relying on the controversial assumption of orthogonality for the identification of "structural" innovations, the GIRs can be applied to evaluate the effects of *realistic* shocks in a given equation. As discussed in more detail in Chapter 1, section 1.7.5, this is achieved by taking into account the contemporaneous correlation typically observed between the shocks of different equations, given by the estimate of the system variance matrix $\mathbf{\Omega}$.[24] This section computes the GIR functions in (1.98) in order to evaluate the effects on the levels of the variables of such a realistic shock in the foreign interest rate, R_t^* and in oil prices, p_t^o. In the first case, the GIRs have been computed for the underlying, symmetric model in (2.43), and in the second, for the oil price-augmented model in (2.49).

[23]The published version of Garratt *et al* (2001) discusses only the persistence of *specific* rather than system-wide shocks. The evidence on system-wide shocks were kindly made available by K.C. Lee.

[24]Pesaran and Shin (1998) show that OIRs and GIRs coincide only when considering shocks in the first variable or when $\mathbf{\Omega}$ is diagonal.

Foreign Interest Rate Shock

Figure 2.8 illustrates the effects of the foreign interest rate shock [a shock in the 4th equation of (2.43)], where the size of the shock is scaled to be equal to $\sqrt{\hat{\omega}_{44}} = 0.0011$. Unlike the PPs, the GIRs converge to a non-zero value, reflecting the $I(1)$ properties of $\mathbf{z}_t$. The one standard error shock translates to an increase by 48 basis points in R_t^*.[25] The foreign interest rate continues to rise, reaching a maximum at +74 basis points after 11 quarters and thereafter decreases steadily until it stabilises at +68 basis points. A similar, though, quantitatively smaller effect is observed for the domestic short and long-term interest rates, R_t and LI_t. On impact, R_t rises by 25 basis points and reaches a peak at 44 basis points in the third quarter, when it starts to steadily decline to its long-run level at approximately +12 basis points. The effect on LI_t is even smaller, with an increase of 5.2 basis points on impact, a peak at +16 basis points in the eighth quarter and a long-run effect of +1 basis point. Domestic prices relative to oil prices, $p_t - p_t^o$, are decreased on impact by 0.65 per cent and continue to fall until the third quarter, when they start to steadily climb to their long-run value at approximately +0.27 per cent. The observed gap between R_t and LI_t, combined with the overall increase in $p_t - p_t^o$, could be suggesting the presence of an interest rate arbitrage condition of the form of (2.34), although, such a relation has not been explicitly modelled here for the reasons mentioned in section 2.3.2.

The impact effect of the shock on domestic output is to increase it by 0.28 per cent. However, this positive effect becomes negative by the sixth quarter and the overall outcome is a one per cent reduction. Real money balances are also increased on impact by 0.20 per cent, but the

[25] In order to facilitate comparison with Garratt *et al* (1998), all quarterly rates have been converted to percentage annual rates through multiplication by 400.

effect immediately turns negative in the second quarter and the overall outcome is a reduction by 6.3 per cent. The shock has a positive effect on the effective exchange rate, which rises by 0.96 per cent on impact. After some oscillation during the first two years it eventually climbs to its long-run value at +2.8 per cent.

These results are quite similar to Garratt *et al* (1998) with the only marked difference being the persistent gap between R_t^* and R_t. This gap is eventually eliminated in Garratt *et al* (1998) through the UIP condition. In this study, however, UIP is not modelled as an independent cointegrating relation but it appears, instead, in a weaker form as part of the BP relation, thus, allowing for a protracted gap between domestic and foreign interest rates.

Oil Price Shock

This sub-section considers the effects of an oil price shock, that is, a (realistic) shock in the 10th equation of (2.49). As before, the size of the shock is scaled to be equal to one standard error of $\hat{e}_{10}^+$, which is equal to $\sqrt{\hat{\omega}_{10,10}^+} = 0.1646$. The respective values in Garratt *et al* (1998, 2001) are almost identical (0.1676 and 0.16485, respectively) which makes a quantitative comparison possible. The GIRs illustrating the effect of this shock on the levels of the variables are plotted in Figure 2.9. For a direct comparison with Garratt *et al* (1998) the results are also presented in the form of Figures 2.10a and 2.10b.

The one standard error shock translates to a 65.4% increase in oil prices. As illustrated in Figure 2.10a, the most striking effect of such a shock is domestic and foreign stagflation, which is consistent with the actual experience from the first oil price shock in 1973/1974. On impact domestic output falls by 1.17% and continues to drop until it stabilises at -3.5%. Domestic prices are increased by 0.8% per annum on impact and keep rising at a decreasing rate. In the long-run prices have increased by 6.9%. The implied long-run elasticities of domestic output and prices with respect to oil prices are, therefore, -3.5/65.4 = -0.05 and 6.9/65.4 = 0.11, respectively.[26] The effect on foreign output and prices appears to be quantitatively milder. Foreign output drops by 0.26% on impact and in the long-run stabilises at -1.5%. The impact effect on foreign prices is an increase of 2.45%, which is larger than the domestic price effect. However, the long-run effect turns out to be smaller with an overall increase of 6%.

The increase in oil prices is found to strengthen the pound, as it leads to a reduction in the nominal effective exchange rate, e_t. The impact effect of the shock is to reduce e_t (appreciate the pound) by 1.56%. The long-run effect is a reduction in e_t by 2.37% and it is achieved through an early overshooting in the third quarter, in the tradition of the famous Dornbusch "overshooting" model. The real exchange rate, $e_t - p_t + p_t^*$, is marginally increased on impact by 0.04%, due to the dominant impact effect on foreign prices. In the long run, however, as the effect on domestic prices exceeds that on foreign prices, the real exchange rate is reduced by 3.26% after having overshot its long-run value in the sixth quarter. As pointed out in Garratt *et al* (1998), this value is comparable with the actual depreciation of the UK real exchange rate by an average annual rate of 5.4% during the period 1974q1-1981q1.

The effects on R_t, R_t^*, Δp_t and the inverse narrow money velocity, $m_t - p_t - y_t$, are illustrated in Figure 2.10b. Although on impact, both the domestic and foreign short-term interest rates are increased on impact by 0.9 and 0.6 basis points, respectively, the long-run effect is a reduction by 1 basis point in R_t and an increase by 1.2 basis points in R_t^*. Domestic inflation rises by 0.8% on impact, reaches a peak at +0.96% in the first quarter and thereafter declines until the effect eventually dies out. On impact $m_t - p_t - y_t$ is increased (narrow money velocity is reduced) by

[26] The respective quantities in Garratt *et al* (1998) are estimated at -0.04 and 0.14.

0.38% but the effect quickly turns negative in the following quarter and continues to drop to its long-run value of -3.27%. The effect on the domestic long-term interest rate is illustrated in Figure 2.9 and indicates a positive impact effect of +0.8 basis points and a long-run effect of +2 basis points, associated with a mild overshooting in the third quarter.

Most of these effects are found to be very similar to Garratt *et al* (1998), both qualitatively and quantitatively. The only marked difference is that the model considered here allows for persistent gaps between y_t^* and y_t, on the one hand and R_t and R_t^* on the other. The first is eventually eliminated in Garratt *et al* (1998) through their "output gap" cointegrating relation, which effectively pegs domestic output to foreign output in the long run. The difference between the domestic and foreign interest rate is allowed in this model to persist due to the weaker form of UIP discussed above. The stronger version adopted by Garratt *et al* (1998) eventually eliminates the interest rate differential.

2.8 Conclusions

The aim of this chapter was to identify and test the validity of the modified version of IS-LM presented in section 2.2 within a long-run structural cointegrating VAR framework. The underlying economic theory suggested the presence of five long-run equilibrium relationships between the variables under consideration. In section 2.3.2, however, it was argued that the use of the domestic price variable might be problematic and it was decided to use the transformed variable $p_t - p_t^o$ instead. The use of this variable rendered the arbitrage condition between short and long-term interest rates intractable and as a consequence the empirical analysis focused on the remaining four equilibrium relations, LM, IS, BP and PPP.

Testing the validity of the underlying theory within the chosen econometric framework is essentially an attempt to answer the following two questions. Do the data support the presence of four cointegrating vectors, and if so, does the structure of these vectors conform to the theory? Unfortunately, in practice these questions cannot be answered in this order, nor are they independent of each other. The reason is that the cointegrating rank hypothesis is formulated conditionally on the intercept/trend specification of the model. In this case, the LM and the BP relations required that two of the cointegrating vectors be trend stationary which implied that the use of a VAR model with restricted trend coefficients would be appropriate. Clearly, the imposition of such restrictions does not only have implications on the general structure of the cointegrating vectors, but will also affect the computation of the cointegration rank tests. The LR tests in section 2.5.3 indicated a rejection of the trend restrictions in the presence of four cointegrating vectors. However, it is not straightforward to argue that these tests constitute a rejection of the theory for two reasons. First, the finite sample bias of these tests is in favour of rejection of the null and the rejection margin in this case appears to be quite small. Second, including an unrestricted deterministic trend causes the levels of the variables to exhibit quadratic deterministic trends, which is not characteristic of macroeconomic time series.

Under the restricted trend specification the cointegration rank tests were not particularly informative as to the number of cointegrating vectors supported by the data. In the light of the findings of Cheung and Lai (1993), it seemed more reasonable to rely on the λ-*trace* which indicated four or five cointegrating vectors. The Reinsel and Ahn (1992) adjusted λ-*trace* appeared to give further support to the hypothesis of four long run equilibria. However, the bootstrap exercise in section 2.6.1 indicated that r is more likely to lie in the region 2-3. In as far as this result is reliable, it signifies a rejection of the theory[27].

Given the presence of four cointegrating vectors, the restrictions imposed by theory are being rejected by a relatively small margin when the corresponding LR statistic is compared to the asymptotic 95% critical value. The bootstrap experiment in section 2.6.2, however, indicates that the finite-sample bias of these tests is substantially larger than this rejection margin. It would, therefore, appear reasonably safe to argue that, provided that the appropriate VAR model is one with restricted trends and unrestricted intercepts and, in the case that the true number of cointegrating relations is four, the structure imposed by the theoretical model can be expected to be supported by the data, having accounted for sample size. It does appear, though, that the evidence presented in this study are at best inconclusive regarding the validity of these assumptions.

The coefficients in the estimated cointegrating vectors, discussed in section 2.5.6, appear to be sensible and bear the anticipated signs. One interesting feature is the magnitude of the income elasticity of money demand which is found to be 2.71. This estimate appears to be large compared to previous work on VAR models with no trend coefficients entering the cointegrating vectors, e.g. King *et al* (1991). It is quite likely that the income elasticity of money demand in such studies may be negatively biased due to the steady decline in money demand for transactions purposes, resulting from the gradual increase in non-cash transactions. As illustrated in section 2.2.1, this study, as in Garratt *et al* (1998), attempted to capture this tendency with a linear time trend which resulted in the higher estimate of the income elasticity.

One encouraging feature, however, is that despite the uncertainty regarding the presence

[27] Section 2.6, footnote 15, pointed towards the potentially problematic application of the ordinary bootstrap in this case due to the suspected presence of serial correlation in the equation for ΔR_t.

of four cointegrating relations and the imprecision in the estimation of the IS and the BP, the dynamic behaviour of the estimated model appears to be consistent with existing research. The application of the Persistence Profiles indicated adjustment processes of reasonable duration that clearly illustrate the mean-reverting properties of the over-identified cointegrating vectors. The model also appears to capture most of the standardised effects of an oil price shock, such as domestic and foreign stagflation and depreciation of the real exchange rate. Furthermore, the magnitude of these effects is particularly similar to those reported in Garratt *et al* (1998), despite a number of significant differences in model specification, such as the number and form of the cointegrating relations and the inclusion of the additional variable, LI_t.

Based on the lessons from this first empirical exercise, the following chapter attempts to deal with the following two issues. The first issue is the presence of different exchange rate regimes and their possible implications on the long-run relations. This will be addressed by allowing for structural breaks within the cointegrating vectors with the use of restricted intervention dummies. Additionally, the bootstrap method for testing over-identifying restrictions will be developed, in order to escape the compromising solution of bootstrap1.

Chapter 3

Structural Change and Small-Sample Inference in a Long-Run Structural VAR Model of UK Aggregate Demand

3.1 Introduction

This Chapter focuses on the IS, LM and BP relations discussed in Chapter 2, section 2.2. Identification and testing of these relations is generally based on the methodology adopted in the previous chapter, but involves two innovations. First, the presence of structural breaks is allowed in the cointegrating vectors in an attempt to capture the possible long-run effects of the different exchange rate regimes covered by the sample period 1965q1-1998q2. Second, the bootstrap techniques applied to the tests of over-identifying restrictions in the cointegrating matrix, β are improved. This is done by utilising the Simulated Annealing (SA) algorithm developed in Goffe *et al* (1994) and adapted to *Gauss* by E.G. Tsionas (1995). Unlike the modified version of the Newton-Raphson algorithm utilised in Chapter 2, the SA can be easily applied in order to maximise the log-likelihood function with respect to the free parameters in the over-identified β-matrix for every bootstrap sample and may, therefore, provide more accurate finite-sample critical values for inference on the cointegrating parameters.

The possible long-run effects of the different exchange rate regimes is taken into account following Hansen (2000) and Johansen, Mosconi and Nielsen (2000), by allowing for piece-wise time-varying cointegrating vectors. The time-varying element is introduced in the intercept of the cointegrating relations with the use of two 0/1 intervention dummies, $pre73_t$ and ERM_t, which take the value of one during the fixed exchange rate periods. This allows for long-run IS, LM and BP to have different intercepts according to the exchange rate regime in operation, while all other parameters are assumed to be constant.

Even though the empirical analysis will allow for structural change in the deterministic component of all three cointegrating relations, it is anticipated that changes in the exchange rate regime are most likely to have an impact on the BP vector. As illustrated in Chapter 2, sections 2.2.3 and 2.3, the observed BP disequilibria, $\varepsilon_{3,t}$, are directly related to errors in exchange rate expectations, $\eta_{e,t+1}$, *via* the UIP condition. It is quite likely that the collapse of the Bretton Woods system in the early 1970's and the subsequent transition from fixed to flexible exchange rates had a direct effect on both the UIP and, of course, the exchange rate expectations formation. Such an effect has been reported, for example, by Johansen, Mosconi and Nielsen (2000) in their study of the UIP between the Italian lira and the Deutschemark, although, the break in that study is set in 1979.

Chapter 3 is organised as follows. The following section illustrates how the IS, LM and BP equilibrium relationships may be embedded within a long-run structural VAR model that allows for the presence of structural change of the type discussed above. Section 3.3 focuses on the empirical investigation of the IS, LM and BP relations, discusses the various choices that have been made in the specification of the econometric model and presents the estimates. Section 3.4 looks at the short-run dynamics of the estimated system by considering the Persistence Profiles of the piece-wise time-varying IS, LM and BP vectors, as well as the Generalised Impulse Responses of the variables to shocks equivalent to the ones discussed in Chapter 2. Section 3.5 summarises the results and concludes.

3.2 Econometric Formulation of the Model

In this chapter an attempt is made to account for the possibility of structural change in the long-run IS, LM and BP relations considered in Chapter 2, due to the different exchange rate regimes covered by the data set. The structural change takes the form of a time-varying intercept in the cointegrating relations, which may be modelled with the inclusion of the dummies $pre73_t$ and ERM_t. These take the value 1 during the fixed exchange rate periods. More specifically, $pre73_t$ takes the value of one for $t < 1973q1$ and zero otherwise and ERM_t takes the value of one for $1990q4 \leq t \leq 1992q3$ and zero otherwise. Therefore, the stationary deviations from the LM, IS and BP equilibria derived in Chapter 2, section 2.2, will now take the form of

$$\begin{aligned}
\varepsilon_{1,t} &= d_{01} + (m_t - p_t) - \beta_{11} y_t + \beta_{12} R_t + d_{11} t + d_{2,11}(pre73_t) + d_{2,12}(ERM_t), && (3.1)\\
\varepsilon_{2,t} &= d_{02} + y_t + \beta_{21} LI_t - \beta_{22}(e_t - p_t + p_t^*) - \beta_{23} y_t^* + d_{2,21}(pre73_t) + \\
&\quad + d_{2,22}(ERM_t), && (3.2)\\
\varepsilon_{3,t} &= d_{03} - \beta_{31} y_t + R_t - R_t^* + \beta_{32}(e_t - p_t + p_t^*) + \beta_{33} y_t^* + d_{13} t + d_{2,31}(pre73_t) + \\
&\quad + d_{2,32}(ERM_t), && (3.3)
\end{aligned}$$

where $\varepsilon_{i,t} = \eta_{i,t}$, $i = 1,2$ and $\varepsilon_{3,t} = \eta_{3,t} + \eta_{e,t+1} + \Delta e_{t+1}$.

Lagging by one period and using matrix notation results in

$$\boldsymbol{\varepsilon}_{t-1} = \mathbf{d}_0 + \mathbf{d}_1(t-1) + \mathbf{d}_2 \mathbf{D}_{t-1} + \boldsymbol{\beta}' \mathbf{z}_{t-1}, \tag{3.4}$$

where $\boldsymbol{\varepsilon}_t = [\varepsilon_{1,t},\ \varepsilon_{2,t},\ \varepsilon_{3,t}]'$, $\mathbf{d}_0 = [d_{01},\ d_{02},\ d_{03}]'$, $\mathbf{d}_1 = [d_{11},\ 0,\ d_{13}]'$, $\mathbf{d}_2 = \begin{bmatrix} d_{2,11} & d_{2,12} \\ d_{2,21} & d_{2,22} \\ d_{2,31} & d_{2,32} \end{bmatrix}$, $\mathbf{D}_t = [pre73_t,\ ERM_t]'$, $\mathbf{z}_t = [m_t - p_t,\ y_t,\ R_t,\ R_t^*,\ LI_t,\ e_t - p_t + p_t^*,\ y_t^*]'$ and

$$\beta' = \begin{bmatrix} 1 & -\beta_{11} & \beta_{12} & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & \beta_{21} & -\beta_{22} & -\beta_{23} \\ 0 & -\beta_{31} & 1 & -1 & 0 & \beta_{32} & \beta_{33} \end{bmatrix}. \tag{3.5}$$

The temporary disequilibria ε_{t-1} may be embedded in a $VAR(p)$ model of $\mathbf{z}_t$ as

$$\Delta \mathbf{z}_t = \mathbf{n}_0 + \alpha \varepsilon_{t-1} + \sum_{i=1}^{p-1} \Gamma_i \Delta \mathbf{z}_{t-i} + \mathbf{e}_t, \tag{3.6}$$

or alternatively

$$\Delta \mathbf{z}_t = \mathbf{a}_0 + \mathbf{a}_1 t + \mathbf{a}_2 \mathbf{D}_{t-1} + \alpha\beta' \mathbf{z}_{t-1} + \sum_{i=1}^{p-1} \Gamma_i \Delta \mathbf{z}_{t-i} + \mathbf{e}_t, \tag{3.7}$$

where $\mathbf{a}_0 = \mathbf{n}_0 + \alpha(\mathbf{d}_0 - \mathbf{d}_1)$ and $\mathbf{a}_i = \alpha \mathbf{d}_i$, $i = 1, 2$. Re-writing this last expression as

$$\Delta \mathbf{z}_t = \mathbf{a}_0 + \alpha\beta'_* \mathbf{z}_{*t-1} + \sum_{i=1}^{p-1} \Gamma_i \Delta \mathbf{z}_{t-i} + \mathbf{e}_t, \tag{3.8}$$

where $\beta'_* = [\beta', \mathbf{d}_1, \mathbf{d}_2]$ and $\mathbf{z}_{*t-1} = [\mathbf{z}'_{t-1}, t, \mathbf{D}'_{t-1}]'$, reveals that the structural relations (3.1)-(3.3) may be studied within a cointegrating $VAR(p)$ with deterministic components treated according to Case IVd, that is $\mathbf{a}_1 = -\Pi\gamma$ and $\mathbf{a}_2 = -\Pi\delta$.[1]

As will be argued in the empirical section, the variables $e_t - p_t + p_t^*$ and y_t^* may be treated as weakly exogenous. In this case $\mathbf{z}_t$ can be partitioned as $\mathbf{z}_t = [\mathbf{y}'_t, \mathbf{x}'_t]'$, where $\mathbf{y}_t = [m_t - p_t, y_t, R_t, R_t^*, LI_t]'$ and $\mathbf{x}_t = [e_t - p_t + p_t^*, y_t^*]'$ and similarly the matrices $\mathbf{a}_i = [\mathbf{a}'_{iy}, \mathbf{a}'_{ix}]'$, $i = 0, 1, 2$, $\Gamma_i = [\Gamma'_{iy}, \Gamma'_{ix}]'$, $i = 1, ..., p-1$, $\alpha = [\alpha'_y, \alpha'_x]'$ and the disturbance vector $\mathbf{e}_t = [\mathbf{e}'_{yt}, \mathbf{e}'_{xt}]'$ with

[1] For more details on the treatment of the deterministic components according to Case IVd see Chapter 1, section 1.4.1.

variance matrix $\boldsymbol{\Omega} = \begin{bmatrix} \Omega_{yy} & \Omega_{yx} \\ \Omega_{xy} & \Omega_{xx} \end{bmatrix}$. Therefore, under condition (1.44), the conditional model for $\Delta\mathbf{y}_t$ given $\Delta\mathbf{x}_t$ and the marginal model for $\Delta\mathbf{x}_t$ are given by

$$\Delta\mathbf{y}_t = \mathbf{c}_0 + \boldsymbol{\alpha}_y\boldsymbol{\beta}_*'\mathbf{z}_{*t-1} + \sum_{i=1}^{p-1}\Psi_i\Delta\mathbf{z}_{t-i} + \Upsilon\Delta\mathbf{x}_t + \mathbf{u}_t, \tag{3.9}$$

$$\Delta\mathbf{x}_t = \mathbf{a}_{0x} + \sum_{i=1}^{p-1}\Gamma_{ix}\Delta\mathbf{z}_{t-i} + \mathbf{e}_{xt}, \tag{3.10}$$

where $\mathbf{c}_i = \mathbf{a}_{iy} - \Upsilon\mathbf{a}_{ix}$, $i = 0,1,2$, $\Psi_i = \Gamma_{iy} - \Upsilon\Gamma_{ix}$, $i = 1,\ldots,p-1$, $\mathbf{u}_t = \mathbf{e}_{yt} - \Upsilon\mathbf{e}_{xt}$ and $\Upsilon = \Omega_{yx}\Omega_{xx}^{-1}$. The restrictions on the deterministic terms in (3.9) take the form $\mathbf{c}_1 = -\Pi_y\boldsymbol{\gamma}$ and $\mathbf{c}_2 = -\Pi_y\boldsymbol{\delta}$, where $\Pi_y = \boldsymbol{\alpha}_y\boldsymbol{\beta}'$. Estimation of (3.9) will yield estimates for $\boldsymbol{\alpha}_y$, $\boldsymbol{\beta}$, $\mathbf{d}_1$, $\mathbf{d}_2$, $\mathbf{c}_0$, the short-run dynamic coefficients Ψ_i, $i = 1,\ldots p-1$, and the disturbances $\mathbf{u}_t$ with their associated covariance matrix $\boldsymbol{\Omega}_{uu}$. The constants $\mathbf{d}_0$ may be retrieved according to Appendix A.

Statistical inference on $rank[\Pi_y]$ will indicate the extent to which the data support the presence of three long-run relations among the variables in $\mathbf{z}_t$. Provided that the cointegrating rank is three, there will be need for 9 exactly-identifying restrictions on $\boldsymbol{\beta}_*$. The relations (3.1)-(3.3), however, impose a total of 14 restrictions on $\boldsymbol{\beta}_*$, which leaves 5 over-identifying restrictions to be tested. Furthermore, a test of the restrictions $\mathbf{c}_2 = -\Pi_y\boldsymbol{\delta}$ can indicate the extent to which the different exchange rate regimes introduce a time-varying element in the long-run relations in the form of $\mathbf{D}_t$.

3.3 Estimation Results

The employment of standard ADF and Phillips-Perron (1988) unit root tests in Chapter 2, Tables 2.1 and 2.2, indicated that all variables in $\mathbf{z}_t = [m_t - p_t, y_t, R_t, R_t^*, LI_t, e_t - p_t + p_t^*, y_t^*]'$ can be reasonably assumed to be at most $I(1)$ within the sample period under consideration. These results indicate that identification and testing of the long-run equilibrium relationships (3.1)-(3.3) may safely be carried out within a cointegrating VAR model over the period 1965q1-1998q2. *Gauss 386i* is used throughout with the exception of some of the diagnostic statistics in the VECM which are obtained from *Microfit 4.0.*

3.3.1 Determination of the Order of the VAR

Following Johansen (1995), Garratt *et al* (1998) and others, the determination of the lag-length of the model, p, was investigated within an unrestricted $VAR(4)$ in the level of $\mathbf{z}_t = [m_t - p_t, y_t, R_t, R_t^*, LI_t, e_t - p_t + p_t^*, y_t^*]'$ with an intercept, a linear trend and the dummies $pre73_t$ and ERM_t. Table 3.1 reports the Adjusted Likelihood Ratio (ALR) statistics for testing the hypotheses $p = 0, 1, 2, 3$ as well as the values of the AIC and SBC. The ALR rejects $p = 0$ and $p = 1$ at the 5% level but provides no evidence with which to reject $p = 2$. The AIC also picks out $p = 2$, while the SBC selects $p = 1$. Despite the fact that the AIC is known to be inconsistent[2], it was considered more sensible to opt for the unanimous choice of the ALR and the AIC and set $p = 2$, instead of $p = 1$ favoured by the SBC. This choice is primarily driven by the argument in Kilian (1997) that the consequences of over-estimating the order of the VAR are much less serious than under-estimating it. Empirically, it was found that $p = 2$

[2]See Pesaran and Smith (1998, footnote 25) for the inconsistency of the AIC when determining the lag length.

is sufficiently long to remove any serial correlation.

3.3.2 Determination of the Weakly Exogenous Vector

Working with a cointegrating VAR conditional on weakly exogenous $I(1)$ variables is attractive on both statistical and economic grounds. The econometric framework laid out in Pesaran, Shin and Smith (2000) provides the opportunity to effectively reduce the dimensions of the system, by escaping the explicit modelling of a subset of $\mathbf{z}_t$. At the same time, economic theory gives a tempting excuse to treat the foreign variables as long-run forcing with respect to the domestic variables. This section addresses formally the question of whether a subset of $\mathbf{z}_t$ may be treated as weakly exogenous by testing condition (1.44) within both the symmetric system according to Johansen (1992) and the partial systems according to Pesaran, Shin and Smith (2000).[3]

The obvious candidate for weak exogeneity is the following vector involving all foreign variables, $\mathbf{x}_t^+ = [R_t^*, e_t - p_t + p_t^*, y_t^*]'$. However, both tests clearly reject the weak exogeneity condition (1.44) for $\mathbf{x}_t^+$. The J-test of condition (1.44) takes the value 31.2 which leads to a comfortable rejection as the asymptotic 95% critical value is 12.59. The PSS-test, reported in Table 3.2a, further supports this result as it clearly indicates the presence of a cointegrating relation in the marginal model for $\mathbf{x}_t^+$, using either cointegration rank statistic. This implies that at least one foreign variable will have to be treated as endogenous, which even though is not problematic could be thought as counter-intuitive. The evidence reported in Chapter 2, indicated that it is probably the foreign interest rate, R_t^*, which is most likely to respond to deviations from the domestic long-run equilibria. This is also in agreement with Pesaran, Shin

[3]The former will be denoted as J-test and the latter as PSS-test. The decision to employ both methods is driven by the fact the PSS approach may only provide conclusive evidence against weak exogeneity in the form of cointegrating relations in the marginal model. For more details on the two approaches see Chapter 1, section 1.5.1.

and Smith (2000), who justify their treatment of the foreign interest rate as endogenous to the UK by the importance of the country's position in the world financial markets.[4] The Johansen test statistic indeed rejects weak exogeneity of R_t^*, taking the value of 20.6 with the 95% and 90% asymptotic critical values being 7.81 and 6.25, respectively.[5]

As a consequence, the search for weak exogeneity is limited to the vector $\mathbf{x}_t = [e_t - p_t + p_t^*, y_t^*]'$. The J-test statistic for testing the weak exogeneity condition (1.44) for $\mathbf{x}_t$ is found to be 10.6, which is safely below the 95% and 90% asymptotic critical values of 12.59 and 10.65, respectively. The PSS-tests, reported in Table 3.2b, appear to produce only very weak evidence against weak exogeneity of $\mathbf{x}_t$ in the form of cointegrating relations in the marginal model for this vector at the 10% level. However, since neither cointegrating rank statistic provides any evidence with which to reject the null of no cointegration at the 5% level, and keeping in mind the plethora of evidence in the literature that these tests tend to over-reject the null in finite samples[6], the rejection of no cointegration at the 10% level does not raise any concern.

Further support for the weak exogeneity of $\mathbf{x}_t = [e_t - p_t + p_t^*, y_t^*]'$ within the PSS framework is obtained by testing for the joint significance of the estimated error correction terms $\hat{\boldsymbol{\beta}}'_*\mathbf{z}_{*t-1}$ [presented in section 3.3.6] in the marginal model of $\mathbf{x}_t$. Insignificance of these terms cannot be rejected only at the 5% level as the LR statistic is found to be 11.94 with the asymptotic 95% and 90% critical values being 12.59 and 10.65, respectively. Again, the asymptotic rejection at the 10% level does not raise any concern as the finite-sample bias of the test is in favour of rejection of the null. These results are interpreted as reasonable evidence in favour of treating

[4] See also Chapter 2, section 2.5.2.

[5] The PSS-test is not defined in this case as $\dim[\mathbf{x}_t] = 1$.

[6] See, for example, Reimers (1992), Cheung and Lai (1993), Mantalos and Shukur (1998) and Greenslade *et al* (2002).

$\mathbf{x}_t = [e_t - p_t + p_t^*, y_t^*]'$ as weakly exogenous with respect to $\mathbf{y}_t = [m_t - p_t, y_t, R_t, R_t^*, LI_t]'$.

3.3.3 Treatment of the Deterministic Terms

In section 3.1 it was shown that the piece-wise time-varying IS, LM and BP relations may be modelled within the cointegrating $VAR(p)$ given by (3.9). This modelling framework imposes a number of restrictions on both the trend coefficients and the coefficients on the dummy vector $\mathbf{D}_t$. Specifically, the trend coefficients, $\mathbf{c}_1$, are restricted according to $\mathbf{c}_1 = -\Pi_y\gamma$ and the coefficients on the dummies, $\mathbf{c}_2$, satisfy the condition $\mathbf{c}_2 = -\Pi_y\delta$. The set of restrictions on $\mathbf{c}_1$ reflects the fact that deviations from the LM and BP equilibria should be modelled as trend stationary processes according to the underlying theory in Chapter 2, section 2.2. Furthermore, as illustrated in Chapter 1, section 1.4, it ensures that the trending behaviour of $\mathbf{y}_t$ is independent of r. The restrictions on $\mathbf{c}_2$ allow for the dummy vector $\mathbf{D}_t$ to enter the cointegrating relations, thus, introducing a time-varying intercept in the IS, LM and BP equilibria.

Table 3.3 reports the LR statistics for testing the restrictions on $\mathbf{c}_1$ and $\mathbf{c}_2$, conditionally on there being three cointegrating relations,[7] as well as the small-sample critical values obtained from both a parametric and non-parametric bootstrap with 10,000 simulations.[8] Histograms of the simulated distributions can be found in Figures 3.1 and 3.2. LR_1 is the test of restricting the trend coefficients according to $\mathbf{c}_1 = -\Pi_y\gamma$ and was found to be 5.17. Provided that $r = 3$, this value is low enough to avoid asymptotic rejection at the 5% level but not at the 10% level. The small-sample critical values, however, were found to be significantly larger than their

[7] The validity of this assumption will be investigated separately in section 3.3.5.

[8] The non-parametric version should probably be considered more reliable, as residual normality is rejected in three out of the five equations in the estimated system.

asymptotic counterparts, indicating more clearly that the trend restrictions are not significant even at the 10% level. This provides clear evidence that deviations from the long-run equilibria can be modelled as trend stationary processes in accordance with the theory.

Not having been able to reject the restrictions on the deterministic trends, the next test was concerned with whether there is evidence for restricting the coefficients on the dummy variables according to $\mathbf{c}_2 = -\Pi_y\delta$, given the trend restrictions. The relevant statistic is LR_2 and was found to be 6.12. The finite-sample 95% and 90% critical values were found to be 9.37 and 5.83 in the parametric case and 9.38 and 5.84 in the non-parametric, indicating that in the presence of three trend-stationary long-run equilibria, the null of there being a structural break in the cointegrating vectors cannot be rejected.

3.3.4 Multivariate ADF Tests

The preceding discussion on weak exogeneity and the treatment of the deterministic terms provided reasonable evidence in favour of the specification of the empirical model according to (3.9). Based on the ADF and PP tests in section 2.3.2 it has been assumed throughout that the elements of $\mathbf{z}_t$ are at most $I(1)$. As in Chapter 2, the question of stationarity of the variables in $\mathbf{z}_t$ is investigated further with the application of multivariate ADF tests. These have been computed here within the conditional VAR in (3.9) for $p = 2$ and the results are summarised in Table 3.4. The null of stationarity is generally rejected for all values of r with the exception of the domestic long-term interest rate, which is found to be stationary for $r = 4$. However, the cointegrating rank tests discussed in the following section tend to give support to the fact that this result is more likely to be a reflection of an over-estimated choice of r.

3.3.5 Determination of the Cointegrating Rank

The problem of determining the number of cointegrating relations in the context of the conditional cointegrating *VAR* given by (3.9) is essentially a question of determining the rank of $\Pi_{y*} = \alpha_y \beta'_{*}$, where $\beta'_* = [\beta', \mathrm{d}_1, \mathrm{d}_2]$. As illustrated in Chapter 1, section 1.4.2, this problem can be reduced to testing for the number of significantly non-zero eigenvalues of Π_{y*}, using modified versions of the Pesaran, Shin and Smith (2000) cointegration rank statistics. However, as shown in Johansen and Nielsen (1994) and Johansen, Mosconi and Nielsen (2000), due to the presence of the 0/1 dummies the asymptotic distribution of the tests is not only model, but also variable-specific, since it depends on the timing of the breaks. Due to the computational intensity of the simulation of the asymptotic distributions and in the light of the well-documented finite-sample bias of cointegration rank tests, it was decided to move directly to the investigation of the small-sample distributions. The finite-sample distributions of the (appropriately reformulated) λ-*trace* and *maximum eigenvalue* statistics was investigated using the parametric and non-parametric bootstrap methods illustrated in Chapter 1, section 1.6.3.

Ten thousand pseudo-data sets were simulated in each version under the null hypotheses $r = 0, ..., n_y - 1$. The weakly exogenous vector, $\mathbf{x}_t$, was treated as a stochastic process, given by the marginal model in (3.10) and the data-generating process follows (1.65)-(1.70). The *maximum eigenvalue* and λ-*trace* statistics have been computed for each of the simulated data sets and the resulting bootstrap distributions can be found in Figures 3.4-3.7. The corresponding critical values are presented in Table 3.5. Both, the parametric and non-parametric versions give very similar results. The λ-*trace* statistic rejects the hypotheses $r = 0, 1, 2$, but fails to reject $r = 3$ at the 5% level, which is in agreement with the economic priors. The *maximal* *eigenvalue*, however, gives a very different picture. In both versions it only manages to reject the null of no cointegration in favour of there being one cointegrating relation but clearly fails to reject $r = 1$ in favour of $r = 2$. Being confronted with these two very contrasting results, it was decided to put more weight on the λ-*trace* statistic and choose $r = 3$. This decision was made in the light of the evidence brought forward by Cheung and Lai (1993), who argue that the λ-*trace* is more reliable than the *maximal eigenvalue* statistic in the absence of residual normality, which as will be discussed later, seems to apply in three of the five equations of the estimated system.

3.3.6 Over-Identification

As illustrated in section 3.2, the inclusion of the long-run relations (3.1)-(3.3) within the cointegrating VAR framework of (3.9) imposes the following structure on the trend and dummies-augmented cointegrating matrix $\boldsymbol{\beta}_*$:

$$\beta_*' = \begin{bmatrix} 1 & -\beta_{11} & \beta_{12} & 0 & 0 & 0 & 0 & d_{11} & d_{2,11} & d_{2,12} \\ 0 & 1 & 0 & 0 & \beta_{21} & -\beta_{22} & -\beta_{23} & 0 & d_{2,21} & d_{2,22} \\ 0 & -\beta_{31} & 1 & -1 & 0 & \beta_{32} & \beta_{33} & d_{13} & d_{2,31} & d_{2,32} \end{bmatrix}, \tag{3.11}$$

where the first row corresponds to the stationary deviations from the long-run LM equilibrium, the second row corresponds to temporary departures from the long-run IS relation and the third row represents the BP disequilibria. This structure subjects the cointegrating matrix to a total of 14 restrictions, of which $r^2 = 9$ are exactly identifying, leaving 5 over-identifying restrictions to be tested. This set of over-identifying restrictions will be denoted as R_{OV}.

As discussed in more detail in Chapter 1, section 1.6, asymptotic inference on the cointegrating parameters is generally found to be seriously biased in favour of rejection. This was also verified in Chapter 2, section 2.6.2, where the simulated finite-sample critical values were found to be approximately 73% higher than their asymptotic counterparts. However, these results were obtained using simulation methods designed for the case when the restricted cointegrating matrix does not involve free parameters. This approach was termed in Chapter 1, section 1.6.4, as "bootstrap 1" and its application in the presence of free parameters in the over-identified β-matrix will inevitably exaggerate the simulated critical values. This section introduces the use of the Simulated Annealing (SA) algorithm, discussed in Goffe *et al* (1994) and adapted to *Gauss* by E.G. Tsionas (1995), in order to carry out a *true* bootstrap test of R_{OV}, denoted "bootstrap 2".

Table 3.6 reports the LR statistic for the test of the 5 over-identifying restrictions, as well as the asymptotic and finite-sample critical values obtained from bootstrap 1 and 2. Plots of the simulated finite-sample distributions can be found in Figure 3.3. Both bootstrap experiments have been carried out using parametric and non-parametric methods and are based on 10,000 simulations. The weakly exogenous vector, $\mathbf{x}_t$, was treated as a stochastic process, given by the marginal model in (3.10) and the data-generating process follows (1.65)-(1.70). The test statistic is 18.36 and it easily rejects the null when compared with the asymptotic critical values. However, both bootstrap experiments indicate a very substantial small-sample bias, as the finite-sample critical values are found to be approximately five times greater than their asymptotic counterparts. The 95% and 90% critical values obtained from bootstrap 2 are 50.41 and 45.77 in the parametric version and 51.12 and 46.11 in the non-parametric. They all exceed very comfortably the LR statistic, indicating that when allowing for the sample size, the restriction of the cointegrating matrix according to (3.11) cannot be rejected. As anticipated, the critical values from bootstrap 2 are found to be smaller than those from bootstrap 1 which is indicative of the fact that the SA algorithm has improved on the value of the restricted likelihood function obtained by bootstrap 1. The correction of bootstrap 2 over bootstrap 1 appears to be in the region of 2-5 per cent in this case.

The estimates of the over-identified cointegrating vectors are reported in Table 3.7. All coefficients carry the sign predicted by theory and the estimates are in general agreement with the existing literature and Chapter 2. However, the cointegrating vectors, with the exception of the IS relation, appear to be estimated relatively imprecisely as indicated by the presence of some very low t-ratios. The dummy $pre73_t$ appears to be significant at the 10% level in the BP relation and ERM_t is significant again at the 10% level in the IS relation. This suggests that the long-run position of the IS and the BP is directly related to the exchange rate regime in operation.

It seems, though, that the type of structural change considered here is more profound in the BP relation. Figure 3.8 plots the estimated stationary deviations from the LM, IS and BP relations with a time-varying intercept and compares them with the estimates obtained in Chapter 2 under the assumption of constant intercepts. The LM and the IS disequilibria appear to be very similar, indicating that the type of structural change considered here is of very limited importance in these two relations. The estimated BP disequilibria, however, appear to be much less noisy than in Chapter 2. This reduction in the variance of the disequilibrium terms indicates that the introduction of a time-varying intercept clearly improves the mean-reverting properties of the BP cointegrating relation.[9] It, thus, appears that this long-run relation is indeed subject to structural change, which could have been brought about by the collapse of

[9] This is more formally illustrated in subsequent sections with the use of Persistence Profiles.

the Bretton Woods system of fixed exchange rates in 1973.

Johansen, Mosconi and Nielsen (2000) suggest two plausible interpretations for the significantly positive coefficient on $pre73_t$ in the BP relation. The first is that it may represent a reduced risk premium in the UIP condition for the pre-1973 period. The strong economic performance of the UK during the 60's, combined with the stability of exchange rates, could have reduced the risk associated with UK denominated assets, so that the deviations from the UIP condition in (2.22) are described by

$$\eta_{UIP,t} = R_t - R_t^* - E_t[\Delta e_{t+1}] - \rho_t, \tag{3.12}$$

where $\rho_t = c_{03} - d_{2,31}(pre73_t)$ is a time-varying risk premium, the value of which is lower in the pre-1973 period of exchange rate stability. Alternatively, the positive estimate of $d_{2,31}$ in the BP relation could be a reflection of the fact that the predictability of exchange rate movements before 1973 did not prepare economic agents for the sharp devaluation of the pound in the mid 1970s, illustrated in Figure 2.3. In other words, $d_{2,31}$ could represent a systematic negative bias in exchange rate expectations in (2.25), so that

$$E_t[e_{t+1}] = e_{t+1} + \eta_{e,t+1} - d_{2,31}(pre73_t). \tag{3.13}$$

This would be consistent with the view that the devaluation of the pound in the mid 70's was largely unanticipated.[10]

[10] Johansen, Mosconi and Nielsen (2000) have similar results in the pre-79 period for the UIP condition between Italy and Germany. In their case, however, the possibility of a negative risk premium is dismissed as unrealistic and the positive coefficient on the dummy for the pre-79 period is interpreted as a systematic bias in exchange rate expectations, associated with the sharp devaluation of the lira in the 70's.

3.3.7 Vector Error Correction Model

The estimates of the error correction model are reported in Table 3.8. As in Chapter 2, all error correction terms are found to be strongly significant in the equation for ΔR_t^*, indicating that the foreign interest rate should, indeed, be treated as endogenous to the UK. This reinforces the view expressed in Pesaran, Shin and Smith (2000) concerning the importance of the UK in the financial markets, which was mentioned earlier as a theoretical justification for the treatment of R_t^* as an endogenous variable. The explanatory power of all equations appears to be sensible and there are no signs of serial correlation, which does justice to the chosen lag length $p = 2$. Furthermore, this has made it possible to use the ordinary bootstrap for small-sample inference, instead of some serial correlation-consistent but low power method like the stationary bootstrap of Politis and Romano (1994). The diagnostics for normality indicate strong rejection in three out of the five equations, namely in the equations for Δy_t, ΔR_t and ΔR_t^*, indicating the presence of large outliers, possibly due to the oil-price shocks in the 1970s. In the light of the evidence presented in Cheung and Lai (1993) regarding the relative performance of the cointegrating rank statistics in the presence of skewness and excess kurtosis in the estimated residuals, this result justifies the preference for the $\lambda - trace$ over the *maximum eigenvalue* in section 3.3.5. Also, non-normality of the residuals inevitably suggests great caution when interpreting the results of the parametric bootstrap and justifies complementing in all cases the parametric findings with non-parametric results.

3.4 Investigation of the Dynamic Properties of the Model

This section looks at the dynamic behaviour of the estimated long-run structural VAR model by evaluating both the response of individual variables to shocks in a given equation, as well as the dynamic adjustment of the estimated structural error-correction terms in response to system-wide shocks. As in Chapter 2, the tools employed are the Generalised Impulse Responses (GIR), and the Persistence Profiles (PP) discussed in Chapter 1, section 1.7.[11] These measures have been computed for the underlying, symmetric system in (3.8). The parameters of (3.8) can be computed according to the relations below (3.10) using the estimates from the conditional and marginal models in (3.9) and (3.10), respectively.

In order to examine the effects of an oil price shock similar to Chapter 2 and Garratt *et al* (1998, 2001), the model has also been estimated with oil prices as

$$\Delta \mathbf{z}_t^+ = \mathbf{a}_0^+ + \begin{bmatrix} \alpha_y^+ \\ \mathbf{0}_\alpha \end{bmatrix} \begin{bmatrix} \beta' & \mathbf{0}_\beta & \mathbf{d}_1 & \mathbf{d}_2 \end{bmatrix} \begin{bmatrix} \mathbf{z}_{t-1}^+ \\ t \\ \mathbf{D}_{t-1} \end{bmatrix} + \sum_{i=1}^{p-1} \begin{bmatrix} \Gamma_{iz}^+ \\ \mathbf{0}_\gamma \end{bmatrix} \Delta \mathbf{z}_{t-i}^+ + \mathbf{e}_t^+, \qquad (3.14)$$

where $\mathbf{z}_t^+ = [\mathbf{y}_t', \mathbf{x}_t', p_t^o]'$, Γ_{iz}^+, $i = 1, \dots, p-1$, are $n \times n^+$ coefficient matrices, $p = 2$, $n^+ = \dim[\mathbf{z}_t^+] = n + 1$, and $\mathbf{0}_\alpha$, $\mathbf{0}_\beta$ and $\mathbf{0}_\gamma$ are blocks of zeros with dimensions $(n_x + 1) \times r$, $r \times 1$ and $1 \times n^+$, respectively. The combination of weak exogeneity of p_t^o, indicated by $\mathbf{0}_\alpha$, with the block of zeros $\mathbf{0}_\gamma$ renders $\mathbf{z}_t$ non Granger-causal for p_t^o. The hypothesis that the terms $\Delta \mathbf{z}_{t-i}^+$, $i = 1, \dots, p-1$, do not enter the last row of (3.14) has been tested, given weak exogeneity of p_t^o, within the marginal model for $[\mathbf{x}_t', p_t^o]'$. The corresponding Wald statistic was found to

[11] Confidence intervals for the PPs and the GIRs have not been computed in this study and, as a consequence, all the results must be interpreted with caution.

be 10.5 with an asymptotic 95% critical value of 15.51, indicating that $\mathbf{z}_t$ does, indeed, not Granger-cause p_t^o.[12] The model in (3.14) allows for a direct comparison with the analysis of an oil price shock in Chapter 2 and Garratt *et al* (1998) and to a lesser extent with that in Garratt *et al* (2001), as the latter impose also a number of orthogonality restrictions.

3.4.1 Persistence Profiles

The scaled PPs, given by (1.105), have been computed for the estimated LM, IS and BP relations within (3.8) and are plotted in Figure 3.9. All PPs converge towards zero, thus, confirming the stationary nature of the estimated cointegrating relations. The profiles for the LM and the IS relations are found to be very similar to the ones reported in Chapter 2. They indicate that 95 per cent of the adjustment process is completed, in both cases, within approximately 5 years, which is also consistent with the findings in Garratt *et al* (1998, 2001).

However, a striking feature of the PPs considered here is the marked reduction in the persistence of BP disequilibria, compared to Chapter 2. Approximately 95 per cent of the adjustment process is found here to take place in less than 10 quarters, compared to 6 years reported in Chapter 2. This dramatic reduction in the duration of BP disequilibria indicates that the introduction of a time-varying intercept in the BP relation does, indeed, improve its mean-reverting properties. This reinforces the empirical evidence discussed earlier, which indicated that the exchange rate regime in operation has a significant long-run impact on the BP equilibrium condition.

[12]The restriction that $\mathbf{z}_t$ does not Granger-cause p_t^o is not crucial for the outcome of the GIRs. It was imposed here in order to facilitate a direct comparison with the oil price shock considered in Garratt *et al* (1998).

3.4.2 Generalised Impulse Responses

This section computes the GIR functions in (1.98) in order to evaluate the effects on the levels of the variables of such a typical shock in the foreign interest rate, R_t^* and in oil prices, p_t^o. In the first case, the GIRs have been computed for the underlying, symmetric model in (3.8), and in the second, for the oil price-augmented model in (3.14).

Foreign Interest Rate Shock

Figure 3.10 illustrates the effects of the foreign interest rate shock [a shock in the 4th equation of (3.8)], where the size of the shock is scaled to be equal to $\sqrt{\hat{\omega}_{44}} = 0.00109$. Unlike the PPs, the GIRs converge to a non-zero value, reflecting the $I(1)$ properties of $\mathbf{z}_t$. A casual inspection of Figure 3.10 reveals that the responses of the variables are very similar to the GIRs reported in Chapter 2, Figure 2.20, both qualitatively and quantitatively.[13]

The one standard error shock translates to an increase by 43 basis points in R_t^*. The foreign interest rate continues to rise, reaching a maximum at +74 basis points after 11 quarters and thereafter decreases steadily until it stabilises at +70 basis points. A similar, though, quantitatively smaller effect is observed for the domestic short and long-term interest rates, R_t and LI_t. On impact, R_t rises by 25 basis points and reaches a peak at 45 basis points in the fourth quarter, when it starts to steadily decline to its long-run level at approximately +16 basis points. The impact effect on LI_t is an increase of 6.4 basis points. The long-term rate reaches its peak at +27 basis points in the tenth quarter and thereafter falls to its long-run value of +21 basis points.

[13] As in Chapter 2, all quarterly rates have been converted to percentage annual rates through multiplication by 400.

The impact effect of the shock on domestic output is to increase it by 0.41 per cent. However, this positive effect becomes negative by the sixth quarter and the overall outcome is a reduction by 1.26 per cent. Foreign output rises by 0.55 per cent on impact, reaches a peak at +0.8 per cent in the third quarter and in the long run stabilises at +0.4 per cent. Real money balances are increased on impact by 0.4 per cent, but the effect immediately turns negative in the second quarter and the overall outcome is a reduction by 5 per cent. The shock has a positive effect on the real effective exchange rate, which rises by 0.4 per cent on impact. After some rather intense oscillation during the first two years, it eventually stabilises at its long-run value at +1.1 per cent.

To the extent that the variables considered here permit a comparison with Garratt *et al* (1998), the results are roughly similar. The only marked difference is the persistent gaps observed here between R_t^* and R_t and between y_t^* and y_t. The former is eventually eliminated in Garratt *et al* (1998) through the UIP condition. In this study, however, as in Chapter 2, UIP is not modelled as an independent cointegrating relation but it appears, instead, in a weaker form as part of the BP relation, thus, allowing for a protracted gap between domestic and foreign interest rates. The gap between y_t^* and y_t is eventually eliminated in Garratt *et al* (1998) through their "output gap" cointegrating relation, which is not included in the model considered here.

Oil Price Shock

This sub-section considers the effects of an oil price shock, that is, a typical shock by historical standards in the 8th equation of (3.14). As before, the size of the shock is scaled to be equal to one standard error of $\hat{e}_8^+$, which is equal to $\sqrt{\hat{\omega}_{8,8}^+} = 0.1659$. This value is very similar to

Chapter 2 and Garratt *et al* (1998, 2001), which allows for a quantitative comparison. The GIRs illustrating the effect of this shock on the levels of the variables are plotted in Figure 3.11. There are a number of differences as well as similarities with the effects reported in Chapter 2, Figures 2.22a and 2.22b and the two studies by Garratt *et al* (1998, 2001). The similarities can be found in the general behaviour of domestic and foreign output, the real exchange rate and real money supply, while the differences concern the effects on the domestic and foreign short-term rates and the domestic long-term rate.

The one standard error shock translates to a 66.4% increase in oil prices. As illustrated in the first panel of Figure 3.11, the impact effect of such a shock is to reduce domestic and foreign output by 1.1% and 0.3%, respectively. The long-run effect, however, although still negative, is smaller in magnitude with domestic output falling by 0.4% and foreign output by 0.18%. Although, the impact effect on both these variables is comparable with the findings in Chapter 2, the long-run effect is found here to be a lot smaller. In particular, the implied long-run elasticities of domestic and foreign output with respect to oil prices are here -0.4/66.4 = -0.006 and -0.18/66.4 = -0.003, which are approximately 10 times smaller in absolute value than the respective values reported in Chapter 2 and Garratt *et al* (1998). The most likely cause for this is that in the current framework prices are not modelled explicitly, but only implicitly through the real exchange rate and the real money supply.

The effects of the oil price shock on the real exchange rate, $e_t - p_t + p_t^*$, and the real money supply, $m_t - p_t$, are illustrated in the third panel of Figure 3.11 and are found to be quite similar to Chapter 2 and Garratt *et al* (1998) in the long run. On impact, $e_t - p_t + p_t^*$ falls only marginally by 0.6%, but is more dramatically reduced over the next two quarters by 2.8%. In the long run the real exchange rate stabilises at -3%. As pointed out in Garratt *et* *al* (1998), this value is comparable with the actual depreciation of the UK real exchange rate by an average annual rate of 5.4% during the period 1974q1-1981q1. The real money supply is reduced on impact by 0.98%. In the second quarter it overshoots its long-run value by reaching -2.3% and gradually recovers until it eventually stabilises at -0.89%. Combining the effect on the real money supply with the effect on domestic output, discussed above, has the following implications on the inverse narrow money velocity, $m_t - p_t - y_t$. As in Chapter 2 and Garratt *et al* (1998), $m_t - p_t - y_t$ is increased on impact (narrow money velocity is reduced) by 0.1% but the effect quickly turns negative in the following quarter. The long-run effect is a reduction by 0.5%, which is approximately 6 times smaller than in Chapter 2 and Garratt *et al* (1998).

The effects on R_t, R_t^* and LI_t are illustrated in the second panel of Figure 3.11 and are found to be different from Chapter 2 in the long-run, although quite comparable on impact. The immediate effect of the increase in oil prices is to raise R_t, R_t^* and LI_t by 0.2, 0.12 and 0.12 basis points, respectively. This effect, though, is quickly reversed. By the third quarter R_t has been reduced by 0.64 basis points, but then starts to climb until it returns back to its impact value of +0.2 basis points. This recovery is not observed in R_t^* and LI_t, both of which are reduced in the long run by 0.48 basis points.

As in the case of the foreign interest rate shock, discussed above, the GIRs considered here also give rise to persistent gaps between y_t^* and y_t and R_t and R_t^*, in contrast to Garratt *et al* (1998). As mentioned earlier, these are caused by the absence of an output gap relation and the weaker version of UIP considered here.

3.5 Conclusions

The aims of this chapter were to test the empirical validity of the three long-run equilibria derived from an IS-LM-BP view of aggregate demand and account for the possible effects of the different exchange rate regimes on these long-run equilibria. In the light of the well-documented small-sample bias associated with asymptotic inference in a multivariate cointegration analysis, parametric and non-parametric bootstrap methods have been applied in order to simulate the finite-sample distributions of the test statistics of interest. In doing so, the use of the SA algorithm has been introduced as a means for overcoming the convergence problems typically encountered by conventional algorithms in bootstrapping the LR tests of over-identifying restrictions imposed on the cointegrating matrix.

The general findings are that, having accounted for the sample size, there is no evidence with which to reject the inclusion of the dummies $pre73_t$ and ERM_t inside the cointegrating relations, the cointegrating rank can reasonably be assumed to be three and the over-identifying restrictions suggested by theory cannot be rejected. The employment of the SA algorithm for bootstrapping the LR tests of over-identifying restrictions imposed on the cointegrating matrix does significantly improve on the estimated small-sample critical values obtained from the trivial bootstrap 1. In this case the positive bias of bootstrap 1 was found to be in the region of 2-5 per cent.

The time-varying intercept introduced in the cointegrating relations appears to have a negligible effect on the LM and the IS relations. However, as in Johansen, Mosconi and Nielsen (2000), there is evidence in favour of a significantly lower mean in the BP relation during the pre-1973 period. This may represent a reduced risk premium in the UIP condition, associated with the reduced uncertainty regarding exchange rate behaviour before 1973. Alternatively, it could be a reflection of the fact that the predictability of exchange rate movements before 1973 did not prepare economic agents for the sharp devaluation of the pound in the mid 1970s. Taking account of this time-varying intercept was found to significantly improve the mean-reverting properties of the BP relation. In particular, the Persistence Profiles indicated that deviations from the break-accommodating BP are eliminated approximately 3 times faster, with 95% of the adjustment process being completed within 10 quarters.

The application of GIRs illustrated that the estimated model possesses reasonable dynamic properties, which in many cases are similar to the model in Chapter 2 and Garratt *et al* (1998, 2001). The analysis of the oil price shock, however, revealed some differences which can probably be attributed to the fact that prices are not modelled explicitly in the current framework.

The approach taken here with regard to the different exchange rate regimes was shown to be a useful extension, especially with regard to the BP relation. However, it is in some respects over-simplified and rather limited in scope. Structural change need not be limited to a shift in the mean of the cointegrating relations and could also concern the remaining cointegrating parameters. Furthermore, the different regimes are almost certain to have an effect on the way that variables adjust to departures from the long-run equilibria, which suggests time varying long-run adjustment coefficients, α, and highlights the need for further research in this direction.

Chapter 4

A Long Run Structural VAR Model of the UK Labour Market

4.1 Introduction

The focus of the last two empirical chapters was on the identification, estimation and testing of long-run equilibrium relations derived from a Mundell-Fleming-type view of Aggregate Demand. This chapter looks at the Aggregate Supply-side of the economy by considering the behaviour of the labour market. The application of time series methods to the analysis of labour markets has a venerable history, e.g. Phillips (1958), Lipsey and Parkin (1970), Sargan (1980), Layard and Nickell (1985), while more recent examples within a multivariate framework can be found in Davidson and Hall (1991), Greenslade *et al* (2002) and Lee and Papaikonomou (2002) [hereafter LPAP].

The view of the labour market considered here is the sectoral, union-based, "competing-claims" model developed by Lee and Pesaran (1993b) [hereafter LP]. This is shown to give rise to two behavioural relations describing the forces that drive the demand and the supply for labour at the aggregate level. The first, is an "employment equation" that satisfies firms' profit maximisation condition and the second, is a "wage equation" satisfying unions' utility maximisation condition. These two relations have a natural interpretation as long-run equilibria. As such, they may be identified and tested within a long-run structural VAR, in a manner similar to Chapters 2 and 3.[1]

The following section briefly presents the LP model and derives the two behavioural relations. Section 4.3 illustrates how these may be embedded within a cointegrating VAR model and section 4.4 provides an empirical analysis based on quarterly UK data over the period

[1] A number of authors, e.g. Layard *et al* (1991), Bean (1994), and Manning (1993, 1995), have argued that, even if such behavioural relations could be successfully formulated at the aggregate level, they would not satisfy the standard rank condition for identification. LPAP, however, provide a solution to the aggregation problem and illustrate that the rank criterion is irrelevant within a long-run structural VAR framework.

1972q1 - 2000q1. As in Chapters 2 and 3, the empirical section breaks down the sequence of decisions that need to be made when working with a cointegrating VAR model and discusses them in separate, clearly defined stages. Again, particular attention is paid on the finite-sample properties of the statistics involved with the use of parametric and non-parametric bootstrap methods. The dynamic properties of the model are investigated in section 4.5 with the use of persistence profiles and GIR functions. The latter are employed in order to illustrate the effects of a positive technological (productivity) shock. Section 4.6 summarises the results and concludes.

4.2 The Lee and Pesaran (1993b) Model of the Labour Market

The theoretical underpinning of the empirical analysis to follow is the union-based "competing-claims" model developed in LP. In this model, it is assumed: *(i)* that the production technology faced by the j^{th} firm in the i^{th} industry can be characterised by the production function

$$Y_{ijt} = N_{ijt}^{\alpha_i} F(K_{ijt}, A_{it}), \qquad j = 1, ..., M_i, \qquad i = 1, ..., m, \tag{4.1}$$

where Y_{ijt}, N_{ijt} and K_{ijt} are the output, employment and capital stock in firm j, $j = 1, .., M_i$ in industry i, $i = 1, .., m$ at time t and where A_{it} represents the state of technological progress in industry i at time t; *(ii)* that there is a union that bargains with all firms in industry i and which is characterised by a utility function with a Stone-Geary functional form, so that

$$\begin{aligned} \text{utility}_{it} &= \left[\ln\left(\frac{W_{it}/P_t}{W_{it}^*/P_t}\right)\right]^{\theta_i} \left[\ln\left(\frac{N_{it}}{N_{it}^*}\right)\right]^{1-\theta_i} \\ &= (w_{it} - w_{it}^*)^{\theta_i} (n_{it} - n_{it}^*)^{1-\theta_i}, \end{aligned} \tag{4.2}$$

where W_{it} is the nominal wage rate in sector i at time t, P_t is the aggregate price deflator, W^*_{it} is the real fallback wage, and lower case letters denote the logarithm of a variable; *(iii)* that the M_i firms in the industry are identical (so that $Y_{it} = M_i Y_{ijt}$ and so on); *(iv)* that firms are risk neutral, maximise expected profits, and believe that their output decisions do not induce any change in the output decisions of others; and *(v)* that unions set wage levels and employers set employment levels unilaterally (i.e. the 'monopoly union' model).

Under assumptions *(i)*-*(v)*, there is an 'industry-wide' production function of the form

$$\begin{aligned} Y_{it} &= N_{it}^{\alpha_i} M_i^{1-\alpha_i} F\left(\frac{K_{it}}{M_i}, A_{it}\right) \\ &= N_{it}^{\alpha_i} D_{it}, \end{aligned} \tag{4.3}$$

where D_{it} is the 'demand-shift term' which captures the influence of the capital and productivity terms. Assuming that the wage and employment decisions are made before demand and productivity innovations are realised, profit maximisation by firms implies that the following relationship holds at the industry level:

$$\begin{aligned} W_{it} &= \mathrm{E}\left[P_{it}\left(1-\frac{1}{\eta_{it}}\right)\frac{\partial Y_{it}}{\partial N_{it}}\right] \\ &= \alpha_i\left(1-\frac{1}{\eta_{it}}\right)N_{it}^{\alpha_i-1}\mathrm{E}\left[P_{it}D_{it}\right], \end{aligned} \tag{4.4}$$

where P_{it} is the industry price, $\eta_{it} = \frac{\partial Y_{it}}{\partial P_{it}}\frac{P_{it}}{Y_{it}}$ is the exogenously given price elasticity of demand for the i^{th} industry's output, and $E[.]$ is the expectations operator. Under the assumption that (log) aggregate productivity shocks and unanticipated demand shocks (across firms) are normally distributed, i.e. $(p_{it}+d_{it}) \sim N(\mathrm{E}[\,p_{it}+d_{it}], \tau_i^2)$, this provides

$$n_{it} = c_{1i} - \frac{1}{1-\alpha_i}(w_{it}-p_{it}) + \frac{1}{1-\alpha_i}d_{it} + \xi_{1it}, \tag{4.5}$$

where $c_{1i} = \frac{1}{1-\alpha_i}\left(\ln(\alpha_i) + \ln(1-\frac{1}{\eta_i}) + \tau_i^2\right)$ is an industry-specific constant and where $\xi_{1it} = (p_{it}+d_{it}) - \mathrm{E}[\,p_{it}+d_{it}]$ are expectational errors. Maximisation of the utility function given in (4.2) by union i, subject to (4.5), provides the first order condition

$$w_{it} = c_{2i} + w_{it}^* + \sigma_i\frac{\theta_i}{1-\theta_i}\left(\frac{1+c_{3i}}{2}\right)(n_{it}-n_{it}^*) + \xi_{2it}, \tag{4.6}$$

where $\sigma_i = 1-\alpha_i(1-\frac{1}{\eta_i})$, c_{2i}, and c_{3i} are firm-specific parameters, and $\xi_{2it} = w_{it}^* - \mathrm{E}[w_{it}^*]$ is another expectational error.

Under the (reasonable) assumptions that $c_{3i} \approx 1$ and η_{it} is large, so that $\sigma_i(1+c_{3i})/2 \approx (1-\alpha_i)$, the behavioural relations derived in (4.5) and (4.6) become

$$\begin{aligned} n_{it} &= c_{1i} - \frac{1}{1-\alpha_i}(w_{it}-p_{it}) + \frac{1}{1-\alpha_i}d_{it} + \xi_{1it}, \\ w_{it} &= c_{2i} + w_{it}^* + \frac{(1-\alpha_i)\theta_i}{1-\theta_i}(n_{it}-n_{it}^*) + \xi_{2it}. \end{aligned} \tag{4.7}$$

The equations in (4.7) are the first order conditions from the firms' profit maximisation problem and the union's utility maximisation problem respectively. They form a simultaneous system, the joint outcome of which provides values for n_{it} and w_{it}.[2]

[2]See LPAP for a proof of the fact that this system is identifiable.

4.2.1 Identification of Aggregate Employment and Wage Equations

The analysis, so far, has been concerned with sectoral wage and employment determination, as discussed in LP. There are, of course, important reasons why a sectorally disaggregated analysis of the labour market might be preferable to an aggregate one, some of which are elaborated in LP and LPAP. However, for the purposes of this chapter interest is placed on the investigation of labour market relationships at the aggregate level only. Despite the evidence provided in LP and in Lee *et al* (1990) on the pervasiveness of slope heterogeneity in wage and employment equations for the UK, it will be assumed here that there is approximate homogeneity of parameters across sectors. As a result, the aggregate relationships that will be considered are analogous to those derived at the sectoral level; i.e. the aggregate model corresponding to (4.7) is given by

$$\begin{aligned} n_t &= c_1 - \frac{1}{1-\alpha}(w_t - p_t) + \frac{1}{1-\alpha}d_t + \xi_{1t}, \\ w_t &= c_2 + w_t^* + \frac{(1-\alpha)\theta}{1-\theta}(n_t - n_t^*) + \xi_{2t}, \end{aligned} \tag{4.8}$$

where the absence of an i subscript indicates that these variables are (logarithms) of aggregate magnitudes.

However, even if one is prepared to entertain the (possibly strong in the case of the UK) assumption of slope homogeneity across sectors, there still remains the issue of how to distinguish between the aggregate wage, w_t, and the aggregate fallback wage, w_t^*. The concept of the fallback wage represents wage opportunities available in the event of losing current employment. When working with the sectoral model in (4.7) it would, thus, be reasonable to proxy w_{it}^* by the aggregate, economy-wide wage level w_t. At the aggregate level, though, this is no longer possible since w_t^*, appearing in the second row of (4.8), would coincide with w_t, appearing in the first row of (4.7). LPAP propose the following solution to this problem.

Aggregation and the fallback wage

As discussed above, the fallback wage represents the wage opportunities available in the event of losing one's job. This event could either represent a move from one job to another with probability Π_t, or a move from employment to unemployment with probability $1-\Pi_t$. Consequently, the fallback wage may be expressed as

$$W_t^* = \Pi_t W_t^a + (1 - \Pi_t)B_t, \tag{4.9}$$

or, equivalently, taking logarithms

$$w_t^* \approx w_t^a + \ln(\Pi_t) + \frac{1 - \Pi_t}{\Pi_t} r_t, \tag{4.10}$$

where Π_t is the probability of re-employment, which is assumed to be a negative function of the general unemployment rate U_t, i.e. $\Pi_t = \Pi(U_t)$ with $\Pi_U < 0$, W_t^a is the wage available in alternative employment, B_t is the level of unemployment benefit and r_t is the (log of the) replacement ratio (B_t/W_t).

The solution to the problem of distinguishing between aggregate w_t and w_t^* proposed by LPAP rests on the following assumption. Provided that there is some economy-wide wage distribution $F_t(.)$, it is assumed that w_t^a is randomly located lower in $F_t(.)$ than w_t. Thus, the aggregate fallback wage w_t^* will not only be related to the aggregate wage level, but also to the first and second moments of the wage distribution $F_t(.)$. Using the logistic distribution as a proxy for $F_t(.)$, LPAP derive an explicit expression for w_t^*.[3] This expression has been used here for the construction of the w_t^* variable that is used in the empirical analysis.[4]

The final form of the aggregate employment and wage equations to be studied in the empirical section is obtained from (4.8) by:

a) using the assumption that the production function is Cobb-Douglas, so that $D_t = A_t K_t^{1-\alpha}$, and $\frac{1}{1-\alpha} d_t = \frac{1}{1-\alpha} a_t + k_t$, and

b) assuming further that the probability of re-employment Π_t, appearing in (4.10), has the functional form $\Pi_t = \varphi U_t^{-\beta_{22}}$, where $0 < \varphi < 1$ and $\beta_{22} > 0$.

Under these assumptions, the temporary deviations $\xi_{1,t}$, $\xi_{2,t}$ from the aggregate employment and wage equilibrium relations take the form

$$\xi_{1,t} = \tilde{c}_1 + n_t + \beta_{12}(w_t - p_t) - k_t - \beta_{12} a_t, \tag{4.11}$$

$$\xi_{2,t} = \tilde{c}_2 - \beta_{21} n_t + (w_t - p_t) - (w_t^{**} - p_t) + \beta_{21} n_t^* + \beta_{22} u_t, \tag{4.12}$$

where $w_t^{**} = w_t^* - \ln(\Pi_t)$, $\beta_{12} = \frac{1}{1-\alpha}$, $\beta_{21} = \frac{(1-\alpha)\theta}{1-\theta}$, $\tilde{c}_1 = -c_1$ and $\tilde{c}_2 = -[c_2 + \ln(\varphi)]$.

4.3 Econometric Formulation of the Model

The stationary disequilibria in (4.11) and (4.12) were formulated in terms of the variables in $\mathbf{z}_t = [n_t,\ w_t - p_t,\ w_t^{**} - p_t,\ n_t^*,\ u_t,\ k_t,\ a_t]'$ and were shown to have the interpretation of temporary deviations from the aggregate employment and wage equations, respectively. This section illustrates how these terms may be embedded within a Cointegrating $VAR(p)$ in the

[3] For more details see appendix A in LPAP.
[4] See the data appendix B.2 for details on the construction of all variables used in this chapter.

vector $\mathbf{z}_t$. The empirical counterpart of $\mathbf{z}_t$ has been constructed according to Appendix B.2 and the employment of standard ADF and multivariate ADF tests in Tables 4.1a, 4.1b and 4.2 indicate that all variables in $\mathbf{z}_t$ can reasonably be considered to be $I(1)$.[5] It is, therefore, possible to utilise the cointegrating $VAR(p)$ framework, discussed in Chapter 1. This is based on the following $VECM$ representation of $\mathbf{z}_t$

$$\Delta \mathbf{z}_t = \mathbf{n}_0 + \mathbf{a}_2 \mathbf{D}_t + \Pi \mathbf{z}_{t-1} + \sum_{i=1}^{p-1} \Gamma_i \Delta \mathbf{z}_{t-i} + \mathbf{e}_t, \tag{4.13}$$

where $\mathbf{n}_0$ is an $n \times 1$ vector of intercepts, $n = \dim[\mathbf{z}_t]$, $\mathbf{D}_t = [D_{1t}, ..., D_{n_d,t}]'$, is an $n_d \times 1$ vector of deterministic components, $\mathbf{a}_2$ is an $n \times n_d$ coefficient matrix, Γ_i, $i = 0, 1, ..., p-1$, are $n \times n$ coefficient matrices, $\mathbf{e}_t$ is an $n \times 1$ vector of *iid* disturbances with positive definite covariance matrix $\mathbf{\Omega}$ and $rank[\Pi] = r < n$.

The economic theory of the long run, summarised in (4.11) and (4.12), may be embedded within (4.13) by assuming that $\Pi \mathbf{z}_{t-1}$ takes the form

$$\Pi \mathbf{z}_{t-1} = \boldsymbol{\alpha} \boldsymbol{\xi}_{t-1} = \boldsymbol{\alpha}(\tilde{\mathbf{c}} + \boldsymbol{\beta}' \mathbf{z}_{t-1}), \tag{4.14}$$

where $\boldsymbol{\xi}_t = [\xi_{1,t}, \xi_{2,t}]'$, $\tilde{\mathbf{c}} = [\tilde{c}_1, \tilde{c}_2]'$ and

$$\boldsymbol{\beta}' = \begin{bmatrix} 1 & \beta_{12} & 0 & 0 & 0 & -1 & -\beta_{12} \\ -\beta_{21} & 1 & -1 & \beta_{21} & \beta_{22} & 0 & 0 \end{bmatrix}. \tag{4.15}$$

[5] Time plots of the variables can be found in Figure 4.1.

Thus, the $VAR(p)$ model in (4.13) becomes

$$\Delta \mathbf{z}_t = \mathbf{a}_0 + \mathbf{a}_2 \mathbf{D}_t + \boldsymbol{\alpha}\boldsymbol{\beta}' \mathbf{z}_{t-1} + \sum_{i=1}^{p-1} \boldsymbol{\Gamma}_i \Delta \mathbf{z}_{t-i} + \mathbf{e}_t, \tag{4.16}$$

where $\mathbf{a}_0 = \mathbf{n}_0 + \boldsymbol{\alpha}\tilde{\mathbf{c}}$. The model given by (4.16) maintains all the advantages of an unrestricted VAR in being able to capture complex dynamics, while at the same time incorporating a long-run structure with transparent economic interpretation, reflected in (4.15).

In order to improve the model's ability to capture the short run dynamics, the deterministic vector $\mathbf{D}_t$ in (4.16) was chosen to have the form $\mathbf{D}_t = [d74q1_t, d74q2_t]'$. The dummies $d74q1_t$, $d74q2_t$ take the value of one for the observations 1974q1 and 1974q2, respectively and zero otherwise. They aim to capture the effect of the industrial action by the miners and by power engineers in the final months of 1973, which resulted in the prohibition of space heating in industrial and commercial premises and a reduced working week through January and February of 1974. This is assumed to have a direct impact on labour dynamics in 1974q1 and, with the return to normality, in 1974q2.[6]

In principle, all variables in the system may be treated symmetrically. However, in practice there might be certain advantages when treating one or more variables as weakly exogenous.[7] Some argue that technological progress is dependent on labour market experiences (through learning-by-doing for example) and that the labour and capital input decisions are made jointly. Nevertheless, it remains the case that both k_t and a_t are regularly treated as exogenous in the

[6] It should be noted that, unlike Chapter 1, section 1.4.1 and Chapter 3, section 3.2, the vector $\mathbf{D}_t$ considered here does not contain *intervention* dummies which introduce structural change effects. The dummies considered here simply capture outliers, or one-off "blips" in the data. As a result, asymptotic inference on cointegration rank within (4.16) may be carried out using the standard critical values for models with unrestricted intercepts and no deterministic trends (Case III).

[7] See Pesaran, Shin and Smith (2000), Pesaran and Smith (1998), as well as Chapter 1 of this thesis for more details.

analysis of labour markets.[8] In this case $\mathbf{z}_t$ would be partitioned as $\mathbf{z}_t = [\mathbf{y}_t', \mathbf{x}_t']'$, where $\mathbf{y}_t = [n_t, w_t - p_t, w_t^{**} - p_t, n_t^*, u_t]'$, $\mathbf{x}_t = [k_t, a_t]'$, and similarly $\mathbf{a}_i = [\mathbf{a}_{iy}', \mathbf{a}_{ix}']'$, $i = 0, 2$, $\Gamma_i = [\Gamma_{iy}', \Gamma_{ix}']'$, $i = 1, ..., p-1$, $\boldsymbol{\alpha} = [\boldsymbol{\alpha}_y', \boldsymbol{\alpha}_x']'$ and the disturbance vector $\mathbf{e}_t = [\mathbf{e}_{yt}', \mathbf{e}_{xt}']'$ with variance matrix $\Omega = \begin{bmatrix} \Omega_{yy} & \Omega_{yx} \\ \Omega_{xy} & \Omega_{xx} \end{bmatrix}$. Under the weak exogeneity condition (1.44), the conditional model for $\Delta\mathbf{y}_t$ given $\Delta\mathbf{x}_t$ and the marginal model for $\Delta\mathbf{x}_t$ may be written as

$$\begin{aligned} \Delta\mathbf{y}_t &= \mathbf{c}_0 + \mathbf{c}_2\mathbf{D}_t - \boldsymbol{\alpha}_y\boldsymbol{\beta}'\mathbf{z}_{t-1} + \sum_{i=1}^{p-1}\Psi_i\Delta\mathbf{z}_{t-i} + \Upsilon\Delta\mathbf{x}_t + \mathbf{u}_t, \qquad (4.17) \\ \Delta\mathbf{x}_t &= \mathbf{a}_{0x} + \mathbf{a}_{2x}\mathbf{D}_t + \sum_{i=1}^{p-1}\Gamma_{ix}\Delta\mathbf{z}_{t-i} + \mathbf{e}_{xt}, \qquad (4.18) \end{aligned}$$

where $\Upsilon = \Omega_{yx}\Omega_{xx}^{-1}$, $\mathbf{c}_i = \mathbf{a}_{iy} - \Upsilon\mathbf{a}_{ix}$, $i = 0, 2$, $\Psi_i = \Gamma_{iy} - \Upsilon\Gamma_{ix}$, $i = 1, ..., p-1$, and $\mathbf{u}_t = \mathbf{e}_{yt} - \Upsilon\mathbf{e}_{xt}$.

4.4 Estimation Results

Identification and testing of the long-run equilibrium relationships (4.11) and (4.12) was carried out both jointly within the conditional model (4.17), as well as within separate sub-systems, over the period 1972q1-2000q1. The following sections concentrate on the full model, although there is some reference to indicative results regarding the sub-systems. As in the previous empirical chapters, particular attention is paid to the finite-sample properties of the statistics involved with the use of the parametric and non-parametric bootstrap methods discussed in Chapter 1, section 1.6.2.[9] All estimation was carried out in *Gauss 386i* with the exception of some of the diagnostics in the VECM which were obtained from *Microfit 4.0.*

[8] The legitimacy of treating k_t and a_t as weakly exogenous is formally tested in section 4.3.

[9] A bootstrap experiment was not carried out only in cases where there cannot be a conflict between asymptotic and finite sample results (e.g. asymptotic non-rejection when the finite-sample bias is in favour of rejection).

4.4.1 Determination of the Order of the *VAR*

Following Garratt *et al* (2001), Johansen (1995), Pesaran, Shin and Smith (2000) [hereafter PSS] and others, the lag-length, p, was determined within an unrestricted $VAR(8)$ in the level of $\mathbf{z}_t = [n_t,\ w_t - p_t,\ w_t^{**} - p_t,\ n_t^*,\ u_t,\ k_t,\ a_t]'$ with an intercept, and the deterministic vector $\mathbf{D}_t = [d74q1_t,\ d74q2_t]'$.[10] Table 4.3 reports the Adjusted Likelihood Ratio (ALR) statistics for testing the hypotheses $p = 0, 1, ..., 7$, as well as the values of the AIC and SBC. The ALR tests reject the hypothesis $p = 0$ and $p = 1$ at the 5% level but provide no evidence with which to reject $p = 2$. The AIC picks out $p = 8$, while the SBC selects $p = 1$. Taking into account Pesaran and Smith's (1998) point regarding the inconsistency of the AIC [11], more emphasis was placed on the SBC and the Adjusted LR statistics. Empirically, it was found that $p = 2$ is sufficiently long to remove any serial correlation.

4.4.2 Treatment of the Deterministic Terms

Regarding the treatment of the intercepts and trends, it was decided to follow the general-to-specific methodology and start off with a generalised version of (4.16) that includes unrestricted intercepts and unrestricted trend coefficients given by

$$\Delta\mathbf{z}_t = \mathbf{a}_0 + \mathbf{a}_1 t + \mathbf{a}_2\mathbf{D}_t + \Pi\mathbf{z}_{t-1} + \sum_{i=1}^{p-1}\Gamma_i\Delta\mathbf{z}_{t-i} + \mathbf{e}_t, \qquad p = 2. \tag{4.19}$$

As illustrated in Chapter 1, section 1.4 and PSS, this specification in not characteristic of macro-economic time series as it implies that the level of $\mathbf{z}_t$ exhibits quadratic trending behaviour.

[10] The maximum order 8 was chosen *a priori* bearing in mind the number of variables in $\mathbf{z}_t$, the available sample size and the quarterly nature of the data.

[11] See Pesaran and Smith (1998, footnote 25) for the inconsistency of the AIC when determining the lag length.

Furthermore, the trending behaviour of $\mathbf{z}_t$ would depend directly on the number of cointegrating vectors, as the number of quadratic trends varies according to $n-r$. This unsatisfactory feature may be avoided by restricting the trend coefficients according to $\mathbf{a}_1 = -\Pi\boldsymbol{\gamma}$, where $\boldsymbol{\gamma}$ is an n-vector of unknown coefficients. The LR test for these restrictions is conditional on r and is asymptotically chi-squared with $\dim[\mathbf{z}_t] - r$ degrees of freedom in the case of a symmetric model, and $\dim[\mathbf{y}_t] - r$ in the case of a model conditional on weakly exogenous $I(1)$ variables. Table 4.4 presents the results for both the symmetric $VAR(2)$ in $\mathbf{z}_t$ (upper panel) and a $VAR(2)$ in $\mathbf{y}_t = [n_t,\, w_t - p_t,\, w_t^{**} - p_t,\, n_t^*,\, u_t]'$ conditional on the weakly exogenous vector $\mathbf{x}_t = [k_t,\, a_t]'$ (lower panel). In both cases there appears to be no evidence with which to reject the trend restrictions irrespective of the value of r.[12]

The restricted trends-specification gives rise to trend-stationary cointegrating relations since the error-correction terms may now be written as $\boldsymbol{\beta}_*'\mathbf{z}_{*t-1}$ with $\boldsymbol{\beta}_*' = [\boldsymbol{\beta}', -\boldsymbol{\beta}'\boldsymbol{\gamma}]$ and $\mathbf{z}_{*t-1} = [\mathbf{z}_{t-1}', t]'$. However, as illustrated in section 4.3, embedding the two structural long-run relations (4.11) and (4.12) within the current econometric framework is consistent with $\boldsymbol{\beta}'\boldsymbol{\gamma} = \mathbf{0}$, where $\boldsymbol{\beta}'$ is given by (4.15). The restrictions $\boldsymbol{\beta}'\boldsymbol{\gamma} = \mathbf{0}$ are also known as the *co-trending hypothesis* [Park (1992)]. The LR statistic for testing the co-trending hypothesis, given (4.15), was found to be 0.30 for the symmetric model and 0.21 for the conditional model. Both these statistics are very small, since the asymptotic 95% critical value is 5.99. Recalling that the small-sample bias of the test is in favour of rejection, this result provides very solid evidence in favour of the co-trending hypothesis, given the set of restrictions in (4.15). Thus, the sequence of tests for $\mathbf{a}_2 = -\Pi\boldsymbol{\gamma}$ and $\boldsymbol{\beta}'\boldsymbol{\gamma} = \mathbf{0}$ provides ample evidence in favour of abandoning the general modelling

[12] The asymptotic non-rejection of the restrictions on the trend coefficients is a quite strong result, as these tests are known to be biased in favour of rejection in finite samples.

framework of (4.19) for the model with unrestricted intercepts and no deterministic trends given by (4.16).

Weaker evidence in favour of excluding the trends is obtained when testing $\mathbf{a}_1 = \mathbf{0}$ directly within (4.19). For $r = 2$, the relevant statistic was found to be 22.07 which leads to a rejection when compared to the asymptotic 95% critical value of 11.07. However, an approximation of the small-sample distribution with the application of a non-parametric bootstrap with 20,000 simulations provides the 95% critical value 22.26. Thus, after controlling for the well-documented small-sample bias present in LR tests within the current econometric framework, it is possible to maintain the null $\mathbf{a}_2 = \mathbf{0}$.

The empirical findings presented in this section may be interpreted as reasonable evidence in favour of the theoretical prediction of co-trending and against the inclusion of linear trends. Therefore, the use of the symmetric system given by (4.16) appears to be supported by the data.

4.4.3 Determination of the Weakly Exogenous Vector

Although, in principle, all variables in $\mathbf{z}_t$ may be treated symmetrically within (4.16), the econometric framework laid out in PSS provides the opportunity to effectively reduce the dimensions of the system, by escaping the explicit modelling of a subset of $\mathbf{z}_t$. This section addresses formally the question of whether $\mathbf{x}_t = [k_t, a_t]'$ may be treated as weakly exogenous.

A test of weak exogeneity of $\mathbf{x}_t$ is equivalent to testing condition (1.44). This may be tested directly within a symmetric framework according to Johansen (1992), although, PSS provide the statistical framework for testing the implications of (1.44) within the partial systems, in cases where the size of the symmetric model is prohibitive. The main drawback, however, of the latter approach is that it may only provide conclusive evidence *against* weak exogeneity in the form of cointegrating relations among the elements of $\mathbf{x}_t$.[13] Bearing this limitation in mind, and due to the manageable size of the symmetric system, it was decided to follow the approach in Johansen (1992) and carry out inference within the symmetric VAR given by (4.16). The statistic for testing the weak exogeneity assumption was found to be 5.20 which is safely below the asymptotic 95% and 90% critical values of 9.49 and 7.78 respectively. Recalling that the finite-sample bias of LR tests is in favour of rejection, this result renders a bootstrap experiment redundant.

4.4.4 Determination of the Cointegrating Rank

This section investigates the presence of two cointegrating relations among the variables in $\mathbf{z}_t$, as predicted by the economic theory in section 4.2. Due to the well documented small-sample bias haunting asymptotic inference, it was attempted in all cases to simulate the finite-sample distribution of the cointegration rank statistics using the parametric and non-parametric bootstrap procedures outlined in Chapter 1, section 1.6.2. The weakly exogenous variables are treated as stochastic processes and the data generating process follows (1.65)-(1.70). Twenty thousand simulations have been used throughout.

At a first stage, the two long-run equilibria were investigated separately within self-contained sub-systems. First, it was tested whether one cointegrating relation exists within the sub-system describing the demand-side of the labour market. According to the evidence presented thus far

[13] See also Chapter 1, section 1.5.1.

this sub-system is given by

$$\Delta \mathbf{y}_t^d = \mathbf{c}_0^d + \mathbf{c}_2^d \mathbf{D}_t - \boldsymbol{\alpha}^d \boldsymbol{\beta}^{d\prime} \mathbf{z}_{t-1}^d + \sum_{i=1}^{p-1} \Psi_i^d \Delta \mathbf{z}_{t-i}^d + \Upsilon^d \Delta \mathbf{x}_t + \mathbf{u}_t^d, \tag{4.20}$$

where $\mathbf{z}_t^d = [\mathbf{y}_t^{d\prime}, \mathbf{x}_t^\prime]^\prime$, $\mathbf{y}_t^d = [n_t, w_t - p_t]^\prime$ and $p = 2$. The results are reported in Table 4.5a and indicate an agreement between asymptotic and small-sample results, all of which are clearly in favour a single cointegrating relation using either statistic at the 5% level. In a similar fashion, the same hypothesis has been tested within the sub-system describing the supply-side of the labour market, given by

$$\Delta \mathbf{z}_t^s = \mathbf{a}_0^s + \mathbf{a}_2^s \mathbf{D}_t - \boldsymbol{\alpha}^s \boldsymbol{\beta}^{s\prime} \mathbf{z}_{t-1}^s + \sum_{i=1}^{p-1} \Gamma_i^s \Delta \mathbf{z}_{t-i}^s + \mathbf{e}_t^s, \tag{4.21}$$

where $\mathbf{z}_t^s = [n_t, w_t - p_t, w_t^{**} - p_t, n_t^*, u_t]^\prime$ and $p = 2$. As illustrated in Table 4.5b, again there is total agreement between asymptotic and small-sample results, all of which clearly indicate that at the 5% level $r = 1$ using either cointegrating rank statistic.[14]

Having obtained a solid body of empirical evidence supporting the presence of two cointegrating relations independently (one in each sub-system), the cointegration rank tests were next applied to the full model given by (4.17). The simulated finite-sample distributions are plotted in Figures 4.2-4.5 and the corresponding critical values are reported along with the cointegration rank statistics in Table 4.6. The test results provide further support to the theoretical prediction that the variables in $\mathbf{z}_t = [n_t, w_t - p_t, w_t^{**} - p_t, n_t^*, u_t, k_t, a_t]^\prime$ cointegrate with $r = 2$. This result is obtained from the employment of both, parametric and non-parametric bootstrap

[14]The presence of a cointegrating relation in each of the two separate sub-systems was found to be quite robust to different measures of w_t, w_t^{**}, n_t and n_t^*, all of which are described in detail in the data appendix B.2.

experiments, using either statistic at the 10% level. Asymptotically, the $\lambda - trace$ gives $r = 2$ at the 5% level, while the *maximal eigenvalue* indicates $r = 3$. Recalling that the finite-sample bias of cointegration rank tests is in favour of rejection, these results may be considered as convincing evidence in favour of $r = 2$. The asymptotic result concerning the *maximal eigenvalue* does not require any further attention for the additional reason that, according to Cheung and Lai (1993) the *maximal eigenvalue* is less reliable in the absence of residual normality, which seems to apply in two of the five equations of the estimated system.[15]

4.4.5 Over-Identification

Having found reasonable empirical support for the presence of two cointegrating relations in the previous section, it was next tested to what extent these may be identified as the aggregate employment and wage equations derived in section 4.2.1. As illustrated in section 4.3, the inclusion of the long-run relations (4.11) and (4.12) within the cointegrating VAR framework of (4.17) imposes the following structure on the cointegrating matrix $\boldsymbol{\beta}$:

$$\boldsymbol{\beta}' = \begin{bmatrix} 1 & \beta_{12} & 0 & 0 & 0 & -1 & -\beta_{12} \\ -\beta_{21} & 1 & -1 & \beta_{21} & \beta_{22} & 0 & 0 \end{bmatrix},$$

where the first row corresponds to the stationary deviations from the aggregate employment relation (4.11) and the second row corresponds to temporary departures from the aggregate wage relation (4.12). This structure subjects the cointegrating matrix to a total of 11 restrictions, of which $r^2 = 4$ are exactly identifying, leaving 7 over-identifying restrictions to be tested. One additional restriction arises from the assumption that was made in the construction of the

[15] Very similar findings are obtained with the use of a symmetric $VAR(2)$.

variable a_t[16], namely, that the structural parameter α appearing in (4.11) and (4.12) takes the value 0.56. Since $\beta_{12} = \frac{1}{1-\alpha}$, this implies a value of $\beta_{12} = 2.2727$. The set of 11 restrictions will hereafter be denoted as R_{OV1} and the full set of 12 restrictions as R_{OV2}. As in the case of the cointegrating rank, both sets of restrictions have been tested separately within the demand and supply sub-systems, and jointly within the full model. This section, however, discusses analytically only the results obtained from the full model given by (4.17).

The results are summarized in Table 4.7. Both sets of restrictions receive strong empirical support as they comfortably avoid asymptotic rejection even at the 10% level. Denoting the statistic for testing R_{OV1} by LR_1, and similarly the statistic corresponding to R_{OV2} by LR_2, then $LR_2 - LR_1$ is a test of the assumption $\alpha = 0.56$ (that implies $\beta_{12} = 2.2727$), given R_{OV1}. The value of this statistic can be found to be 0.10 which again leads to a very comfortable non-rejection, even asymptotically, since $\chi^2_{.05}[1] = 3.84$. These results constitute a particularly solid piece of evidence for the validity of R_{OV1} and R_{OV2}, bearing in mind that in finite samples the test is biased in favour of rejection.

In order to illustrate the extent of this bias, the small-sample distributions of the tests were approximated using parametric and non-parametric versions of bootstrap experiments 1 and 2 (discussed in detail in Chapter 1, section 1.6.4) with 10,000 simulations. Plots of the simulated distributions are presented in Figure 4.6, and the corresponding 95% and 90% critical values are reported in Table 4.7. As anticipated, the critical values from bootstrap2 are found to be smaller than those from bootstrap1 which is indicative of the fact that the SA algorithm has been improving on the likelihood value obtained from bootstrap1. As explained in Chapter 1, section 1.6.4, the difference between bootstrap1 and bootsrap2 results from the fact that the former

[16] See the data appendix B.2 for details.

is not maximising the restricted log-likelihood with respect to the free parameters in the over-identified β-matrix, which biases the critical values upwards.[17] However, even the application of the more accurate bootstrap 2 reveals a very significant small-sample bias ranging from 76% in the parametric version, to 99% in the non-parametric.[18] This result strongly supports the already substantial body of literature that emphasizes the need to control for sample size when using chi-squared tests within the current econometric framework.

The estimates of the over-identified cointegrating vectors under both sets of over-identifying restrictions are reported in Table 4.8. In both cases all coefficients are found to be significant and bear the sign predicted by theory. The value of β_{22} is estimated at 0.0085 under both sets of over-identifying restrictions. Recalling that $-\beta_{22}$ is the elasticity of Π_t (the probability of re-employment) with respect to unemployment, this estimate implies that a 1% rise in unemployment reduces the probability of re-employment by 0.0085% at the aggregate level. Under R_{OV1} the estimate for β_{12} is found to be equal to 2.21, which implies a value of 0.55 for the structural parameter α. This estimate justifies the very clear non-rejection of the restriction $\alpha = 0.56$ discussed above. The estimate of β_{21} is found to be -0.0126 under R_{OV1} and -0.0127 under R_{OV2} which suggests a value of approximately 0.03 for the structural parameter θ. Recalling that θ and $1-\theta$ are the weights placed by unions on the *wage* and *employment gaps* respectively, this estimate appears to be rather small, as it suggests that the ratio of the relative importance of internal and external influences on wage setting is 1/32.[19]

[17] The very small magnitude of the difference observed in this case, is the consequence of the fact that there are very few parameters to be estimated, three in R_{OV1} and two in R_{OV2}. In the case of the 16-parameters in Chapter 3 the reported bias reached 5 per cent.

[18] The non-parametric version should probably be considered more reliable, as residual normality is rejected in two out of the five estimated equations.

[19] LP and others, using more standard regression analysis of wage setting behaviour, typically find this ratio to be closer to 1/3. However, the estimate reported here was found to be very robust to different measures of w_t^{**} and n_t^*.

4.4.6 Vector Error Correction Model

The corresponding estimated error correction models of Δn_t, $\Delta(w_t - p_t)$, $\Delta(w_t^{**} - p_t)$, Δn_t^* and Δu_t under R_{OV2} are summarised in Table 4.9.[20] The coefficients on the dummies included to capture the effects of the reduced working week in the first two quarters of 1974 show that these terms are important in capturing these influences. The terms on $\xi_{1,t-1}$ and $\xi_{2,t-1}$ demonstrate that there are significant feedbacks from these disequilibrium terms to employment, the fallback level of employment, real wages, real w_t^{**} and unemployment. The explanatory power of all equations appears to be sensible and there are no signs of serial correlation[21], which does justice to the choice of the lag length $p = 2$. Furthermore, this has made it possible to use the ordinary bootstrap for small-sample inference in previous sections, instead of some serial correlation-consistent but low power method like the stationary bootstrap of Politis and Romano (1994). The diagnostics for functional form are rejected only in the equation for Δn_t and there appear to be no traces of heteroscedasticity.

The diagnostics for normality, however, indicate strong rejection in two out of the five equations, namely in the equations for Δn_t and Δn_t^*. In the light of the evidence presented in Cheung and Lai (1993) regarding the relative performance of the cointegrating rank statistics in the presence of skewness and excess kurtosis in the estimated residuals, this result justifies the preference for the $\lambda-$*trace* over the *maximum eigenvalue* in section 4.4.4. Rejection of normality, however, also causes some concern with regard to the reliability of the parametric bootstrap

[20] The results discussed in this section are not affected when the error-correction terms are identified according to R_{OV1}.

[21] Asymptotically, there seems to be a conflict between the chi-squared and the F-diagnostics in the case of Δn_t^*. The first marginaly rejects the hypothesis of serially uncorelated errors at the 5% level, while the second does not. In the light of the clear absence of serial correlation indicated by the simulated finite-sample results (Table 4.10), the marginal asymptotic rejection of the chi-squared test does not appear to be alarming.

methods employed in previous sections, which justifies the persistence in complementing the parametric findings with non-parametric results.

Most of the main findings were found to be largely unaffected by the use of different measures for n_t^* and w_t^{**}.[22] Significance of the error-correction terms and dummies, as well as partial rejection of normality are constant elements in the various VECMs that have been estimated using various definitions for the fallback level of employment and real wage.

4.5 Investigation of the Dynamic Properties of the Model

As in the previous empirical chapters, this section looks at the dynamic behaviour of the estimated long-run structural VAR model with the use of Persistence Profiles and Generalised Impulse Responses. The former are used in order to study the speed of adjustment of the estimated employment and wage equations in response to system-wide shocks, while the latter are used to illustrate the effects of a typical (by historical standards) shock in productivity.[23]

4.5.1 Persistence Profiles

The PPs for the estimated structural cointegrating relations are plotted in Figure 4.7. As anticipated, they are both found to converge to zero, thus, illustrating the temporary nature of departures from the employment and wage equations. In the case of the aggregate employment equation, the adjustment path to the long-run equilibrium is found to be almost monotonic. In response to a unitary, system-wide shock, 43% of the adjustment process has been completed within the first three quarters and approximately 94% within two years. The supply-side of

[22] Details on the construction of the different measures are found in the data appendix B.2.
[23] Confidence intervals for the PPs and the GIRs have not been computed in this study and, as a consequence, all the results must be interpreted with caution.

the labour market, however, reacts strikingly slower. For more than a year after the shock, the deviation from the estimated wage-setting equilibrium increases. The magnitude of the disequilibriating distortion reaches its peak in the sixth quarter, being 6 times higher than on impact. After the sixth quarter starts a monotonic convergence to long-run equilibrium. It takes approximately 4 and a half years for 43% of the adjustment to take place and a further 2 years for 95% of the path towards equilibrium to be completed.

These profiles are consistent with the intuitive view of a labour market in which firms are quicker and more efficient in reacting to deviations from their decision rule, as illustrated by the relatively swift and monotonic adjustment path of the first PP. Unions on the other hand, take far longer to re-establish equilibrium according to their wage-setting relation. Given that the labour market adjustment is complete only after both equilibrium relations are re-established, Figure 4.7 puts into perspective the sluggish adjustment of the labour market. It demonstrates that it is difficult to evaluate the response of the labour market to shocks until a prolonged period has elapsed (in this case approximately seven years for a 99% adjustment), so that the full implications of the shock can be observed.

4.5.2 Generalised Impulse Responses

This section uses the GIR functions in (1.98) for the analysis of the system-wide impact of a one standard error shock in the equation of Δa_t. The GIRs have been computed for the underlying, symmetric model in (4.16), the parameters of which can be recovered from the estimates of (4.17) and (4.18), using the relations below (4.18). Figure 4.8 illustrates the effects of a positive productivity (or technological) shock [a shock in the 7th equation of (4.16)], where the size of the shock is scaled to be equal to $\sqrt{\hat{\omega}_{77}} = 0.00853$. Unlike the PPs, the GIRs converge to a non-zero value, reflecting the $I(1)$ properties of $\mathbf{z}_t$.[24]

The one standard error shock translates to a 3.4% rise in a_t. The impact effect is an increase in wages and fallback wages by 1.2% and 1.25%, respectively. This is associated with a 0.4% fall in employment and a 0.25% fall in alternative employment levels, while unemployment is reduced by 1.4%. The wage variables continue to move closely together following a generally upward course in the medium run. The long-term effect on the real wage and the fallback wage is an almost identical increase by 3.07% and 3.05%, respectively. Actual employment initially picks up but eventually stabilises at -1.03%, although, alternative employment appears to be rising steadily to its long-run value at +2.47%. In the medium run unemployment appears to be falling steadily and reaches a minimum at -20.68%, however, the long-run effect, though still negative, is found to be of more moderate size at -6.76%.

The effects described in Figure 4.8 appear to be consistent with the conventional view that improvements in production technology (increases in productivity) can sustain both, higher real wages and lower levels of unemployment.

4.6 Conclusions

The primary aim of this chapter was to estimate a long-run structural VAR model of the UK labour market, in an attempt to provide an insight into some of the forces affecting the supply-side of the UK economy. The structure on the long-run behaviour of the model was provided by the union-based, competing-claims model developed by LP. This was shown to give rise to two behavioural relations describing long-run equilibrium in labour market demand and supply,

[24]The GIR for unemployment is scaled by 0.25.

which may be identified as cointegrating relations in a VAR framework.

The empirical analysis indicated that:

i) The majority of asymptotic and small-sample results appear to unanimously support the theoretical prediction that there are two cointegrating relations among the variables in $\mathbf{z}_t = [n_t, w_t - p_t, w_t^{**} - p_t, n_t^*, u_t, k_t, a_t]'$.

ii) The set of theory-imposed restrictions required for the identification of these relations as aggregate employment-setting and wage-setting long-run equilibria are insignificant, even asymptotically.

iii) The bootstrap experiments have revealed a very substantial finite-sample bias, reaching 99% in the tests of over-identification of β. This result highlights the potentially misleading nature of asymptotic inference in relatively small samples and re-enforces an already rich literature that suggests the application of bootstrap methods instead.

iv) The use of the SA algorithm and bootstrap2 in simulating the finite-sample distributions of tests of over-identification of β *does* appear to be improving on conventional bootstrap1. However, the extent of this improvement was found here to be markedly smaller than in Chapter 3, possibly due to the much smaller number of free parameters responsible for the bias in bootstrap 1.

v) Regarding the dynamics, the estimated model demonstrates that, while adjustment of employment decisions made by firms takes place relatively rapidly, wage adjustment is extremely sluggish, so that the full implications of labour market shocks can only be observed after approximately 7 years of adjustment.

With these complex interactions and dynamics in mind, it is possible to explain without recourse to more sophisticated theoretical models of the labour market or to structural breaks, the apparent anomalies of the mid-eighties, where strong real wage growth coincided with high unemployment, and the unexpectedly slow growth in real wages experienced in the early nineties. These experiences can be explained as the delayed response of firm and union behaviour to the considerable shocks to the labour market experienced during the early eighties, and the prolonged and largely continual exposure to unemployment rates which remained high by historical standards.

Chapter 5

A Long-Run Structural VAR Model of the UK Economy

5.1 Introduction

The previous empirical chapters dealt separately with the empirical investigation of aggregate demand and supply for the UK. This chapter brings together the AD and AS sub-systems estimated in Chapters 3 and 4, in order to form a cointegrating VAR model with a complete AD-AS long-run structure. The empirical analysis of the complete AD-AS system will make it possible to assess the robustness of the findings obtained from the separate sub-systems, and could provide an interesting comparison with existing long-run structural macroeconometric models of the UK.

However, as will become evident in the discussion of the empirical section, the quite large size of the complete AD-AS system appears to be undermining the performance of the bootstrap methods discussed in previous chapters. Recent evidence by Greenslade *et al* (2002), for example, suggests a negative relation between the power of the bootstrap cointegration rank tests and the dimension, n, of the VAR. Although a formal analytical relation remains to be established, the reliability of the simulation methods appears to be seriously compromised in a model of the size considered here. This inevitably reduces the extent to which the complete model may be used as a tool for assessing the findings of the separate sub-systems. In fact, in cases where the bootstrap results for the full system appear to be seriously distorted, e.g. cointegration rank tests, the evidence from the sub-systems can be a useful guide.[1]

As a result, the assessment of the findings in Chapters 3 and 4 will be limited to a comparison of the estimates for the AD and AS long-run relations and the dynamic properties with those of the complete model. A comparison of the Persistence Profiles (PPs) can, to some extent,

[1] This approach is suggested in an early version of Garratt *et al* (2000), where inference within large systems is guided by the results obtained from smaller sub-systems.

indicate how the joint determination of AD and AS affects the speed with which long-run equilibrium is restored. The dynamic behaviour of the complete model will be further compared with Chapters 3 and 4, as well as with the models developed by Garratt *et al* (1998, 2001), by considering the Generalised Impulse Responses (GIRs) of the variables to an oil price shock and a productivity shock.

The rest of this chapter is organised as follows. Section 5.2 illustrates how the long-run LM, IS and BP equilibria of Chapter 3 and the aggregate employment and wage equations of Chapter 4 may be jointly studied within a cointegrating $VAR(p)$. Section 5.3 presents the estimation results for the period 1965q3-1998q2, discussing in detail the various choices on model specification. Section 5.4 evaluates the dynamic behaviour of the estimated system with the use of PPs and GIRs and compares the findings with previous chapters and Garratt *et al* (1998, 2001). Section 5.5 summarises the results and concludes.

5.2 Econometric Formulation of the Complete AD-AS Model

The complete AD-AS model is formulated in terms of the five long-run relations (3.1)-(3.3) and (4.11), (4.12), repeated here for convenience

$$\begin{aligned}
\zeta_{1,t} &= d_{01} + (m_t - p_t) - \beta_{11} y_t + \beta_{12} R_t + d_{11} t + d_{2,11}(pre73_t) + d_{2,12}(ERM_t), \\
\zeta_{2,t} &= d_{02} + y_t + \beta_{21} LI_t - \beta_{22}(e_t - p_t + p_t^*) - \beta_{23} y_t^* + d_{2,21}(pre73_t) + \\
&\quad + d_{2,22}(ERM_t), \\
\zeta_{3,t} &= d_{03} - \beta_{31} y_t + R_t - R_t^* + \beta_{32}(e_t - p_t + p_t^*) + \beta_{33} y_t^* + d_{13} t + \qquad (5.1)
\end{aligned}$$

$$+d_{2,31}(pre73_t) + d_{2,32}(ERM_t),$$

$$\zeta_{4,t} = d_{04} + n_t + \beta_{41}(w_t - p_t) - k_t - \beta_{41}a_t,$$

$$\zeta_{5,t} = d_{05} - \beta_{51}n_t + (w_t - p_t) - (w_t^{**} - p_t) + \beta_{51}n_t^* + \beta_{52}u_t.$$

For symmetry of exposition the following notation has been introduced: $\zeta_{i,t} \equiv \varepsilon_{i,t}$, $i = 1,2,3$, $\zeta_{i,t} \equiv \xi_{j,t}$, $d_{0i} \equiv \tilde{c}_j$, $i = 4,5$, $j = 1,2$. Thus, the terms $\zeta_{i,t}$, $i = 1,2,3$, are the stationary deviations from the LM, IS and BP relations in Chapter 3, respectively, characterising long-run aggregate demand. The terms $\zeta_{i,t}$, $i = 4,5$, are the deviations from the equilibrium conditions that characterise the supply-side of the economy in Chapter 4, where the β-coefficients have been appropriately re-numbered.

These five long-run relations are formulated in terms of the 14 variables in $\mathbf{z}_t = [m_t - p_t,\ y_t,\ R_t,\ R_t^*,\ LI_t,\ n_t,\ w_t - p_t,\ w_t^{**} - p_t,\ n_t^*,\ u_t,\ e_t - p_t + p_t^*,\ y_t^*,\ k_t,\ a_t]'$, which were shown in Chapters 2, 3 and 4 to be $I(1)$ in the sample under consideration. As a result, $\mathbf{z}_t$ may be modelled within a general cointegrating $VAR(p)$ given by

$$\Delta \mathbf{z}_t = \mathbf{n}_0 + \mathbf{a}_1 t + \mathbf{a}_2 \mathbf{D}_{1,t} + \mathbf{a}_3 \mathbf{D}_{2,t} + \Pi \mathbf{z}_{t-1} + \sum_{i=1}^{p-1} \Gamma_i \Delta \mathbf{z}_{t-i} + \mathbf{e}_t, \tag{5.2}$$

where $\mathbf{n}_0$ is an $n \times 1$ vector of intercepts, $n = \dim[\mathbf{z}_t]$, t is a linear trend with $n \times 1$ coefficients $\mathbf{a}_1$, $\mathbf{D}_{1,t}$ is an $n_{d1} \times 1$ vector of intervention dummies (equivalent to $\mathbf{D}_t$ in Chapter 1, section 1.4.1 and Chapter 3, section 3.2), $\mathbf{D}_{2,t}$ is an $n_{d2} \times 1$ vector of "outlier dummies" (equivalent to $\mathbf{D}_t$ in Chapter 4, section 4.3), $\mathbf{a}_2$ and $\mathbf{a}_3$ are $n \times n_{d1}$ and $n \times n_{d2}$ coefficients, respectively, Γ_i, $i = 0, 1, ..., p-1$, are $n \times n$ coefficient matrices, $\mathbf{e}_t$ is an $n \times 1$ vector of *iid* disturbances with positive definite covariance matrix Ω and $rank[\Pi] = r < n$.

In order for the model in (5.2) to be consistent with the five long-run equilibria in (5.1) it would have to be the case that

$$\Pi \mathbf{z}_{t-1} = \boldsymbol{\alpha}\boldsymbol{\zeta}_{t-1} = \boldsymbol{\alpha}[\boldsymbol{\beta}'\mathbf{z}_{t-1} + (\mathbf{d}_0 - \mathbf{d}_1) + \mathbf{d}_1 t + \mathbf{d}_2\mathbf{D}_{1,t-1}], \tag{5.3}$$

where $\boldsymbol{\alpha}$ is an $n \times 5$, full column-rank matrix of long-run adjustment coefficients, $\boldsymbol{\zeta}_t = [\zeta_{1,t}, \zeta_{2,t}, \zeta_{3,t}, \zeta_{4,t}, \zeta_{5,t}]'$, $\mathbf{d}_0 = [d_{01}, d_{02}, d_{03}, d_{04}, d_{05}]'$, $\mathbf{d}_1 = [d_{11}, 0, d_{13}, 0, 0]'$, $\mathbf{D}_{1,t} = [pre73_t, ERM_t]'$,

$$\mathbf{d}_2 = \begin{bmatrix} d_{2,11} & d_{2,12} \\ d_{2,21} & d_{2,22} \\ d_{2,31} & d_{2,32} \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \tag{5.4}$$

and

$$\boldsymbol{\beta}' = \begin{bmatrix} 1 & -\beta_{11} & \beta_{12} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & \beta_{21} & 0 & 0 & 0 & 0 & 0 & -\beta_{22} & -\beta_{23} & 0 & 0 \\ 0 & -\beta_{31} & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & \beta_{32} & \beta_{33} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & \beta_{41} & 0 & 0 & 0 & 0 & 0 & -1 & -\beta_{41} \\ 0 & 0 & 0 & 0 & 0 & -\beta_{51} & 1 & -1 & \beta_{51} & \beta_{52} & 0 & 0 & 0 & 0 \end{bmatrix}, \tag{5.5}$$

or alternatively

$$\Pi \mathbf{z}_{t-1} = \boldsymbol{\alpha}(\mathbf{d}_0 - \mathbf{d}_1) + \boldsymbol{\alpha}\boldsymbol{\beta}_*'\mathbf{z}_{*t-1}, \tag{5.6}$$

where $\beta'_* = [\beta', \mathbf{d}_1, \mathbf{d}_2]$, $\mathbf{z}_{*t-1} = [\mathbf{z}'_{t-1}, t, \mathbf{D}'_{1,t-1}]'$. Thus, the $VAR(p)$ in (5.2) becomes

$$\Delta \mathbf{z}_t = \mathbf{a}_0 + \mathbf{a}_3 \mathbf{D}_{2,t} + \alpha \beta'_* \mathbf{z}_{*t-1} + \sum_{i=1}^{p-1} \Gamma_i \Delta \mathbf{z}_{t-i} + \mathbf{e}_t, \tag{5.7}$$

where $\mathbf{a}_0 = \mathbf{n}_0 + \alpha(\mathbf{d}_0 - \mathbf{d}_1)$ and the coefficients $\mathbf{a}_1$ and $\mathbf{a}_2$ on the linear trends and the intervention dummies are restricted according to Case IVd in Chapter 1, section 1.4.1, since $\mathbf{a}_1 = \alpha \mathbf{d}_1 = -\Pi\gamma$ and $\mathbf{a}_2 = \alpha \mathbf{d}_2 = -\Pi\delta$, where γ and δ are unknown $n \times 1$ and $n \times n_{d1}$ coefficient matrices, respectively. The symmetric model given by (5.7) maintains all the advantages of an unrestricted VAR in being able to capture complex dynamics, while at the same time incorporating a long-run structure with transparent economic interpretation, reflected in (5.6). In order to capture some extreme outliers in the data and improve the model's ability to describe the short-run dynamics, the vector $\mathbf{D}_{2,t}$ was specified in the empirical analysis as $\mathbf{D}_{2,t} = [d71q1_t, d71q2_t, d74q1_t, d74q2_t, d90q3_t]'$. These dummies take the value of one during the first and second quarters of 1971 and 1974 and the third quarter of 1990, respectively, and are equal to zero for all other observations.

The separate study of AD in Chapter 3 and AS in Chapter 4 indicated that the null of weak exogeneity cannot be rejected for specific subsets of $\mathbf{z}_t$. In the case of the AD sub-system the weakly exogenous vector was found to be $\mathbf{x}_t^D = [e_t - p_t + p_t^*, y_t^*]'$ whereas for the AS sub-system it was $\mathbf{x}_t^S = [k_t, a_t]$. In the empirical investigation of the complete AD-AS model it will be assumed that weak exogeneity holds for the joint set of $\mathbf{x}_t = [\mathbf{x}_t^{D\prime}, \mathbf{x}_t^{S\prime}]'$, although this assumption is formally tested in section 5.3.1. Under this assumption it is possible to partition $\mathbf{z}_t$ into a vector $\mathbf{y}_t$ of n_y endogenous variables and a vector $\mathbf{x}_t$ of n_x exogenous variables, $\mathbf{z}_t = [\mathbf{y}'_t, \mathbf{x}'_t]'$, where $\mathbf{y}_t = [m_t - p_t, y_t, R_t, R_t^*, LI_t, n_t, w_t - p_t, w_t^{**} - p_t, n_t^*, u_t]'$ and $\mathbf{x}_t = [e_t - p_t + p_t^*, y_t^*, k_t, a_t]'$ and similarly the matrices $\mathbf{a}_i = [\mathbf{a}'_{iy}, \mathbf{a}'_{ix}]'$, $i = 0, 1, 2, 3$, $\Gamma_i = [\Gamma'_{iy}, \Gamma'_{ix}]'$, $i = 1, ..., p-1$, $\boldsymbol{\alpha} = [\boldsymbol{\alpha}'_y, \boldsymbol{\alpha}'_x]'$ and the disturbance vector $\mathbf{e}_t = [\mathbf{e}'_{yt}, \mathbf{e}'_{xt}]'$ with variance matrix $\boldsymbol{\Omega} = \begin{bmatrix} \Omega_{yy} & \Omega_{yx} \\ \Omega_{xy} & \Omega_{xx} \end{bmatrix}$. Thus, provided that the weak exogeneity condition given by (1.44) holds, the conditional model for $\Delta\mathbf{y}_t$ given $\Delta\mathbf{x}_t$ and the marginal model for $\Delta\mathbf{x}_t$ are given by

$$\Delta\mathbf{y}_t = \mathbf{c}_0 + \mathbf{c}_3 D_{2,t} + \boldsymbol{\alpha}_y \boldsymbol{\beta}'_* \mathbf{z}_{*t-1} + \sum_{i=1}^{p-1} \Psi_i \Delta\mathbf{z}_{t-i} + \Upsilon \Delta\mathbf{x}_t + \mathbf{u}_t, \tag{5.8}$$

$$\Delta\mathbf{x}_t = \mathbf{a}_{0x} + \mathbf{a}_{3x} D_{2,t} + \sum_{i=1}^{p-1} \Gamma_{ix} \Delta\mathbf{z}_{t-i} + \mathbf{e}_{xt}, \tag{5.9}$$

where $\Upsilon = \Omega_{yx}\Omega_{xx}^{-1}$, $\mathbf{c}_i = \mathbf{a}_{iy} - \Upsilon \mathbf{a}_{ix}$, $i = 0, 1, 2, 3$, $\Psi_i = \Gamma_{iy} - \Upsilon\Gamma_{ix}$, $i = 1, ..., p-1$, $\mathbf{u}_t = \mathbf{e}_{yt} - \Upsilon\mathbf{e}_{xt}$ and the restrictions on the coefficients on the linear trend and the intervention dummy variables take the form $\mathbf{c}_1 = \boldsymbol{\alpha}_y \mathbf{d}_1 = -\boldsymbol{\alpha}_y \boldsymbol{\beta}' \boldsymbol{\gamma}$ and $\mathbf{c}_2 = \boldsymbol{\alpha}_y \mathbf{d}_2 = -\boldsymbol{\alpha}_y \boldsymbol{\beta}' \boldsymbol{\delta}$, respectively.

5.3 Estimation of the Complete AD-AS Model

The previous section illustrated that the five long-run equilibrium relations in (5.1), consistent with a simple AD-AS view of the UK economy, may be studied within the partial system given by (5.8). This model has been estimated over the period 1965q3-1998q2. As in the previous empirical chapters, statistical inference is mainly based on finite-sample results obtained with the use of the parametric and non-parametric bootstrap methods, discussed in Chapter 1, section 1.6.2.[2] However, it should be noted that the relatively large size of the model considered here may have a deteriorating effect on the performance of bootstrap tests. Although a formal

[2] All estimation was carried out in *Gauss 386i*.

relation between the performance of bootstrap tests and the dimension, n, of the VAR has not yet been established, there has been some indication in the recent study by Greenslade *et al* (2002) that in large systems, the bootstrap cointegration rank tests, for example, have a high type II error (low power). In the light of the potentially distorting effect of the quite large size of the model considered here, inference will also be guided by the results obtained from the AD and AS sub-systems in Chapters 3 and 4, respectively.

5.3.1 Determination of the Weakly Exogenous Vector

The first priority in the empirical investigation of the relations (5.1) within a cointegrating VAR framework is the choice of weakly exogenous vector.[3] It was considered crucial to determine the weakly exogenous variables at a very early stage, in order to avoid the significant difficulties that arise when working with a symmetric system of the size of (5.7). On the same grounds, it was decided to test for weak exogeneity following Pesaran, Shin and Smith (2000), (PSS), rather than Johansen (1992), as the latter requires the estimation of the symmetric system.

As mentioned in section 5.2, combining the two sets of weakly exogenous variables considered in the separate analysis of AD in Chapter 3 and AS in Chapter 4, results in the vector $\mathbf{x}_t = [e_t - p_t + p_t^*, y_t^*, k_t, a_t]'$. In the context of a complete AD-AS model of the UK economy, the treatment of the real exchange rate and foreign output as long-run forcing may be justified, as in Chapter 3, by the small-open economy assumption. Productivity (technological progress) could be regarded as weakly exogenous to the macroeconomic aggregates in $\mathbf{y}_t = [m_t - p_t, y_t, R_t, R_t^*, LI_t, n_t, w_t - p_t, w_t^{**} - p_t, n_t^*, u_t]'$ by assuming that it is primarily driven by factors

[3]The empirical analysis will abstract from an explicit discussion on the choice of the lag-length, p, which was set equal to two in accordance with Chapters 3 and 4.

like work effort, R&D expenditure and investment in human capital. There could be legitimate doubts, however, on whether the capital stock is indeed unaffected by AD-disequilibria.

Being agnostic at this stage regarding the treatment of the deterministic terms, the PSS test was applied to the general marginal model for $\mathbf{x}_t$ with no restrictions on $\mathbf{a}_{ix}$, $i = 0, 1, 2, 3$. The cointegration rank statistics are reported in Table 5.1 and clearly indicate the absence of any levels relations among the variables in $\mathbf{x}_t$. Although this result does not constitute direct evidence in favour of condition (1.44), it does not indicate that $\mathbf{x}_t$ is an inappropriate choice for the weakly exogenous vector.[4] The hypothesis that k_t may be treated as weakly exogenous has been further tested within the separate AD and AS models in Chapters 3 and 4. In both cases, the Johansen (1992) statistics indicate that this variable does not respond to the AD and AS disequilibria. In the case of the model in Chapter 3, the LR statistic was found to be 4.8 with asymptotic critical value $\chi^2_{0.05}[3] = 7.81$, while for the model in Chapter 4 the value of the test statistic was 2.6 with $\chi^2_{0.05}[2] = 5.99$. The asymptotic non-rejection in both cases renders a bootstrap experiment redundant, as the finite-sample bias is in favour of rejection.

5.3.2 Treatment of the Deterministic Terms

The previous discussion on weak exogeneity indicated that the use of a symmetric system like (5.7) may be abandoned in favour of a conditional model of the form of (5.8). However, as illustrated in section 5.2, the inclusion of the five structural relations in (5.1) within model (5.8) imposes a number of restrictions on the deterministic components. Specifically, the coefficients on the linear trend and the intervention dummies, $\mathbf{D}_{1,t}$, would have to be restricted according to Case IVd, so that $\mathbf{c}_1 = -\boldsymbol{\alpha}_y\boldsymbol{\beta}'\boldsymbol{\gamma}$ and $\mathbf{c}_2 = -\boldsymbol{\alpha}_y\boldsymbol{\beta}'\boldsymbol{\delta}$.

[4] For more details see the discussion in Chapter 1, section 1.5.1.

Table 5.2 summarises the results for testing the trend restrictions $\mathbf{c}_1 = -\boldsymbol{\alpha}_y\boldsymbol{\beta}'\boldsymbol{\gamma}$ alone. These are clearly found to be insignificant, irrespective of the choice of r, indicating that, as in Chapters 3 and 4, the presence of quadratic trends in the level of $\mathbf{y}_t$ is inconsistent with the data. Not having found evidence against the trend restrictions, it was tested next whether the coefficients on $\mathbf{D}_{1,t}$ take the form of $\mathbf{c}_2 = -\boldsymbol{\alpha}_y\boldsymbol{\beta}'\boldsymbol{\delta}$. The LR statistic for this hypothesis for $r = 5$ and given $\mathbf{c}_1 = -\boldsymbol{\alpha}_y\boldsymbol{\beta}'\boldsymbol{\gamma}$ and (5.5) was found to be 3.02, with a simulated 95% critical value of 22.01 in the parametric case and 23.22 in the non-parametric. Therefore, to the extent that there are five trend-stationary cointegrating relations with parameters satisfying (5.5), the restrictions on the intervention dummies appear to be insignificant.

5.3.3 Determination of the Cointegrating Rank

Based on the evidence presented in the subsections 5.3.1 and 5.3.2, the cointegrating rank has been investigated within the conditional model given by (5.8). As discussed in Chapter 1, section 1.4.2 and Chapter 3, section 3.3.5, the presence of the intervention dummies $\mathbf{D}_{1,t}$ renders the use of standard asymptotic critical values inappropriate in this context. For this reason it was decided to proceed directly to the simulation of the model-specific, finite-sample distributions, using the parametric and non-parametric bootstrap methods described in Chapter 1, section 1.6.3. Ten thousand pseudo-data sets were simulated in each version under the null hypotheses $r = 0, ..., n_y - 1$. The weakly exogenous vector, $\mathbf{x}_t$, was treated as a stochastic process, given by the marginal model in (5.9) and the data-generating process follows (1.65)-(1.70). The *maximum eigenvalue* and λ-*trace* statistics have been computed for each of the simulated data sets and the resulting bootstrap distributions can be found in Figures 5.1-5.4.

Table 5.3 reports the statistics along with the simulated finite-sample critical values. In the light of the evidence obtained from the analysis of the separate AD and AS sub-systems in Chapters 3 and 4, the tests appear to be under-estimating the number of cointegrating relations in the complete AD-AS model. In particular, the λ-*trace* indicates that the cointegrating rank is not higher than 2, while the *maximum eigenvalue* statistic only finds evidence in favour of $r = 1$, when one would expect to find at least five cointegrating relations. These results appear to be consistent with the evidence reported in Greenslade *et al* (2002; pp. 1524), who find that "... *the small sample correction, for a large system such as this, is clearly making too large a correction and almost finds no cointegration at all*". That is, in large systems, the bootstrap tests are most likely to have reduced power, as they do not reject a false null hypothesis frequently enough. Considering the fact that the size of the model in (5.8) is comparable to the eight-dimensional $VAR(p)$, $p = 2, 4, 6, 8$, in Greenslade *et al* (2002), the loss in power can be expected to be equally significant here.

In the light of this, it was decided to put more weight on the economic priors, supported by the combined evidence from Chapters 3 and 4, and proceed under the assumption of five cointegrating relations.

5.3.4 Over-Identification

Provided that there are five cointegrating vectors, their interpretation as the structural long-run equilibria in (5.1) requires that the trends and dummies-augmented cointegrating matrix in (5.8) is given by

$$\beta'_* = \begin{bmatrix} \beta' & \mathbf{d}_1 & \mathbf{d}_2 \end{bmatrix}, \tag{5.10}$$

where $\mathbf{d}_1 = [d_{11},\ 0,\ d_{13},\ 0,\ 0]'$ and $\mathbf{d}_2$ and $\boldsymbol{\beta}$ are given by (5.4) and (5.5), respectively. The structure of $\mathbf{d}_1$, $\mathbf{d}_2$ and $\boldsymbol{\beta}$ imposes 67 restrictions on $\boldsymbol{\beta}_*$, of which $r^2 = 25$ are exactly identifying. This leaves 42 over-identifying restrictions to be tested. The finite-sample distribution for this test has been simulated with the use of parametric and non-parametric versions of bootstrap 1, discussed in Chapter 1, section 1.6.4.[5] Ten thousand pseudo-samples were generated according to (1.65)-(1.70), and the resulting distributions can be found in Figure 5.5.

Table 5.4 summarises the statistics and the asymptotic and finite-sample critical values. The *LR* statistic for testing the over-identifying restrictions was found to be 199.7, which clearly exceeds the asymptotic critical values at the conventional 5% and 10% levels. However, the simulated critical values are found to be 3-4 times higher than their asymptotic counterparts, revealing once again a very substantial small-sample bias. Despite this quite dramatic correction, the restrictions are still found to be significant, although the rejection margin is a lot smaller in the non-parametric case which should be considered more reliable due to the widespread rejection of residual normality (in 5 out of the 10 estimated equations). Nevertheless, it should be reminded that the use of bootstrap 1 inevitably exaggerates the "true" extent of the finite-sample bias. The application of the more accurate bootstrap 2 would result in more moderate critical values and would, thus, also lead to a more comfortable rejection.[6]

On the other hand, some recent evidence reported by Greenslade *et al* (2002) suggest that the frequency of non-rejection of a true set of over-identifying restrictions (the size of the test) is negatively related to the size of the model. In particular, these authors report a size of only 4% for asymptotic inference on the cointegrating parameters at conventional significance

[5] Unfortunately, the computational demands of the more accurate bootstrap 2 within the current econometric setup exceed the computing facilities available to the author.

[6] For more details see Chapter 1, section 1.6.4, as well as the application in Chapter 3, section 3.3.6.

levels within an eight-variable symmetric system. This is shown to increase to 60% when the dimensions of the model are reduced to five, upon conditioning on three weakly exogenous variables, and reaches 80% when the test is carried out within the conditional model with simplified short-run dynamics. The extent to which these results carry over to the bootstrap tests still remains unknown however. The clear non-rejection of the over-identifying restrictions in the separate analysis of the AD and AS sub-systems in Chapters 3 and 4, could be an indication that the size of the model is negatively related to the size of the bootstrap test.

In the light of this, the results reported in Table 5.4 should probably be considered as inconclusive. As long as the relation between the performance of the bootstrap test and the size of the model remains unknown, it appears risky to reject the structure imposed on the cointegrating parameters by (5.10). For the rest of the analysis it will be assumed that this structure is valid, as indicated by the findings in Chapters 3 and 4.

The estimates of the over-identified cointegrating vectors can be found in Table 5.5. All coefficients are signed according to the underlying theory and the estimates appear to be very similar to the ones obtained in Chapters 3 and 4, although the LM and the BP relations are estimated relatively imprecisely. The coefficient on the intervention dummy $pre73_t$ is again found to be significantly positive in the BP relation, thus, verifying the finding of Chapter 3, that the disequilibrium terms from this relation have, on average, been smaller in the fixed exchange rate period before 1973. The coefficients in the aggregate wage equation are all found to be very significant and almost identical to their estimates in Chapter 4, indicating the robustness of this relation.

5.3.5 Vector Error Correction Model

The estimates of the error correction model are reported in Tables 5.6a and 5.6b. The explanatory power of all equations appears to be sensible and the system seems to be reasonably well specified, as indicated by the diagnostics in the second panel of Tables 5.6a and 5.6b and in Table 5.7. When controlling for sample size (Table 5.7) there appear to be no signs of serial correlation, with the possible exception of the equation for ΔLI_t. Despite the use of the generally significant outlier dummies in $\mathbf{D}_{2,t}$, the diagnostics for normality indicate an asymptotic rejection in five out of the ten equations, which seems to be particularly strong in the case of Δn_t and Δu_t. Non-normality of the residuals inevitably suggests great caution when interpreting the results of the parametric bootstrap and justifies the need in complementing the parametric findings with non-parametric results in all cases.

The structural error correction terms $\zeta_{i,t-1}$, $i = 1, \ldots, 5$, are generally significant, although the demand-side variables do not appear to be responding to the disequilibria in the labour market $\zeta_{4,t-1}$ and $\zeta_{5,t-1}$, with the exception of domestic output. In contrast, the long-run adjustment coefficients to the AD disequilibria are found to be significant in the supply-side variables n_t, n_t^* and u_t. Perhaps the main difference between the complete AD-AS model considered here and the separate AD sub-system, analysed in Chapter 3, is the general insignificance of the demand-side error correction terms $\zeta_{1,t-1}$, $\zeta_{2,t-1}$ and $\zeta_{3,t-1}$ in the equation for ΔR_t^*. In Chapter 3, the deviations from the domestic LM, IS and BP relations were found to be strongly significant in the ΔR_t^* equation. This was interpreted as evidence in favour of the view expressed in Pesaran, Shin and Smith (2000) concerning the treatment of R_t^* as an endogenous variable, due to the importance of the UK in financial markets. However, when taking into account the supply side of the economy, it is only the deviations from the domestic LM relation that appear to be significant in the equation for ΔR_t^* and only at the 10% level.

5.4 Investigation of the Dynamic Properties of the Model

This section illustrates the dynamic behaviour of the estimated model. As in the previous empirical Chapters, the dynamic adjustment of the estimated long-run relations in (5.1) to economy-wide shocks is examined with the use of the Persistence Profiles, while the response of individual variables to shocks in a single equation is illustrated with the use of the Generalised Impulse Responses. In order to facilitate a comparison with Chapters 2, 3 and 4, as well as with the macroeconometric models in Garratt *et al* (1998, 2001), the GIRs have been applied for the analysis of an oil price shock and a reduced-form shock in productivity, a_t. In the latter case, the GIRs have been computed for the underlying, symmetric system in (5.7), the parameters of which can be computed according to the relations below (5.9) using the estimates from the conditional and marginal models in (5.8) and (5.9), respectively.

The effects of the oil price shock have been analysed within the following model

$$\Delta \mathbf{z}_t^+ = \mathbf{a}_0^+ + \mathbf{a}_3^+ \mathbf{D}_{2,t} + \begin{bmatrix} \alpha_y^+ \\ \mathbf{0}_\alpha \end{bmatrix} \begin{bmatrix} \beta' & \mathbf{0}_\beta & \mathbf{d}_1 & \mathbf{d}_2 \end{bmatrix} \begin{bmatrix} \mathbf{z}_{t-1}^+ \\ t \\ \mathbf{D}_{1,t-1} \end{bmatrix} + \sum_{i=1}^{p-1} \Gamma_i^+ \Delta \mathbf{z}_{t-i}^+ + \mathbf{e}_t^+, \quad (5.11)$$

where $p = 2$, $\mathbf{z}_t^+ = [\mathbf{y}_t', \mathbf{x}_t', p_t^o]'$, $\mathbf{a}_3^+$ is $n^+ \times n_{d2}$, $n^+ = \dim[\mathbf{z}_t^+] = n+1$, Γ_i^+, $i = 1, \ldots, p-1$, are $n^+ \times n^+$ coefficient matrices and $\mathbf{0}_\alpha$ and $\mathbf{0}_\beta$ are blocks of zeros with dimensions $(n_x+1) \times r$ and $r \times 1$, respectively. The last row of (5.11), corresponding to the equation for Δp_t^o, was further subject to the restrictions that the coefficients on $d71q1_t$, $d71q2_t$ and $\Delta \mathbf{z}_{t-i}$, $i = 1, \ldots, p-1$, are zero. The combination of these restrictions with weak exogeneity of p_t^o, indicated by $\mathbf{0}_\alpha$, renders $\mathbf{z}_t$ non Granger-causal for p_t^o, which is modelled simply as a function of its own lagged values and the dummies $d74q1_t$, $d74q2_t$ and $d90q3_t$.

The hypothesis that $d71q1_t$, $d71q2_t$ and $\Delta \mathbf{z}_{t-i}$, $i = 1, \ldots, p-1$, do not enter the last row of (5.11) has been tested, given weak exogeneity of p_t^o, within the marginal model for $[\mathbf{x}_t', p_t^o]'$. The corresponding Wald statistic was found to be 5.28 with an asymptotic 95% critical value of 26.30, which provides very strong evidence in favour of these restrictions. The specification for the oil price equation used here is slightly different from the one in Chapters 2 and 3, and in Garratt *et al* (1998, 2001), where the equation for Δp_t^o involved only an intercept. The hypothesis that all regressors apart from the intercept can be excluded from the last row of (5.11), however, is very strongly rejected as the respective statistic is 196.5 with an asymptotic 95% critical value of 31.4. Thus, strictly speaking, the model in (5.11) does not allow for a direct comparison with the analysis of an oil price shock in Chapters 2 and 3 and Garratt *et al* (1998, 2001), although, the responses are generally found to be very similar.

5.4.1 Persistence Profiles

Figures 5.6a and 5.6b plot the PPs for the five estimated long-run relations in (5.1), illustrating the speed with which a unitary deviation from these long-run equilibria is eliminated. As anticipated, they are all found to eventually converge to zero, thus, verifying the stationary nature of the over-identified cointegrating vectors. These profiles appear to be consistent with the general findings from the separate analysis of the AD and AS sub-systems in Chapters 2, 3 and 4. In the demand side of the economy, equilibrium is restored faster in the asset markets than in the goods market, while the supply side returns to its long-run equilibrium strikingly slower, due to a profound sluggishness in wage setting.

The joint analysis of AD and AS, though, results in slightly longer adjustment periods. The LM relation is restored by 95% within approximately 5 years, which is more or less consistent with the results in Chapters 2 and 3. However, it takes approximately 5.5 years for a 95% adjustment in the BP relation, which is faster than the 6 years reported in Chapter 2, but notably slower than the 2.5 years reported in Chapter 3. For the IS relation the same adjustment is found to take place in 7.5 years, compared to 6 years in Chapter 2 and 5 years in Chapter 3. The firms' employment setting rule (employment equation) is restored by 95% within 3.5 years, compared to 2 years in Chapter 4, while for the unions' wage setting rule (wage equation) the same adjustment is found to take place after 11.5 years, compared to 6.5 years in Chapter 4.

The slower adjustment processes compared to the previous empirical Chapters, where the AD and AS long-run relations were investigated separately, could possibly be attributed to the increased complexity of the joint AD-AS model. A textbook analysis of AD and AS can provide a useful insight as to why the equilibrating process can take longer when AD and AS are jointly determined. However, given the typically substantial width of confidence intervals of Persistence Profiles,[7] it is quite possible that these differences are insignificant.

5.4.2 Generalised Impulse Responses

The GIR functions in (1.98) have been applied for the analysis of an oil price shock, similar to Chapters 2, 3 and Garratt *et al* (1998, 2001), and a reduced-form shock in productivity, a_t, as in Chapter 4. In the former case, the GIRs have been computed for the model in (5.11) and

[7] See, for example, Pesaran and Shin (1996) and Garratt *et al* (2001).

in the latter for the underlying, symmetric system in (5.7), the parameters of which can be computed according to the relations below (5.9) using the estimates from the conditional and marginal models in (5.8) and (5.9), respectively.

Oil Price Shock

This sub-section considers the effects of an oil price shock, that is, a typical shock by historical standards in the 15th equation of (5.11). The size of the shock is scaled to be equal to one standard error of $\hat{e}_{15}^{+}$, which is equal to $\sqrt{\hat{\omega}_{15,15}^{+}} = 0.1121$. Although this value is similar to the one obtained in Chapters 2 and 3 and in Garratt *et al* (1998, 2001), the nature of the shock is slightly different, due to the more dynamic specification of the oil-price equation in (5.11). On impact, the one standard error shock translates to a 44.8% increase in oil prices, which continue to rise over the next 5 quarters until they stabilise at +56.74%. The oil price equation considered in the previous chapters and the Garratt *et al* papers is a simple random walk with a drift and, thus, results in a comparable in magnitude but constant increase in oil prices over the full time horizon. Despite this difference, the responses are found to be very similar.

Figure 5.7a plots the GIRs of the variables included in the AD sub-system. The first panel illustrates a familiar negative effect on both domestic and foreign output. Both variables are steadily reduced with the long-run effect being more negative in the case of y_t at -1.33%, compared to -0.7% in the case of y_t^*. The second panel illustrates the effect on the three interest rate variables R_t, R_t^* and LI_t. All three are increased on impact with R_t rising by 15 basis points and R_t^* and LI_t by 7.6 and 4 basis points, respectively. After a short climb the domestic rate starts falling back to its pre-shock value and the long-run effect is found to be marginally negative at -1.2 basis point. The foreign rate continues to climb and eventually stabilises at

+23 basis points, while the domestic long-term rate remains fairly constant around its impact level of +4 basis points. The third panel plots the responses of the real money supply and the real exchange rate. On impact they are both reduced by approximately 0.5%. The real money supply continues to drop and stabilises at -2.74%, while the real exchange rate, having reached a low of -2.63% in the 9th quarter, starts recovering and stabilises at 0.004% above its pre-shock level.[8]

A comparison with the responses in Figures 2.9, 2.10a, 2.10b and 3.11 reveals many similarities with Chapters 2 and 3, although the effects on the interest rate variables appear to be more consistent with Chapter 2 and Garratt *et al* (1998, 2001) than with Chapter 3. As noted in Chapter 3, this is probably due to the absence of domestic prices from the AD sub-system considered in that chapter, which may, to some extent, be compensated here by the explicit modelling of the supply side. The inclusion of AS also appears to have increased (in absolute value) the long-run elasticity of y_t with respect to oil prices from -0.006 in Chapter 3 to approximately -0.03. This value is much closer to -0.05, reported in Chapter 2 and -0.04 in Garratt *et al* (1998), where prices are explicitly modelled. The persistent gaps between y_t^* and y_t and R_t and R_t^*, though, are in contrast with the two studies by Garratt *et al* (1998, 2001) and, as already mentioned in Chapters 2 and 3, are caused by the absence of an output gap relation and the weaker version of UIP considered here.

Figure 5.7b plots the GIRs for the AS variables. On impact the oil price shock reduces the wage variables and employment by approximately 0.23%. The real wage and the fallback wage are further reduced in the medium term but quickly recover in the eighth quarter. They

[8] This recovery of the real exchange rate is coincidential and should not be confused with stationarity associated with absolute PPP. Different types of shocks are more clearly found to have a persistent effect, as will be shown later.

continue to oscillate around zero and stabilise at around -0.008% and -0.010%, respectively. Employment on the other hand, despite a short-lived recovery in the second quarter, continues to drop and in the long-run it is reduced by 1.4%. Alternative employment, n_t^*, is reduced on impact by 0.08% and continues to drop to its long-run value of -2.93%. Unemployment is curiously found to fall on impact by 0.8% and continues to drop for a further three quarters, reaching a low of -5.9%. Thereafter, the negative effect is rapidly reduced, turning positive in the ninth quarter and eventually stabilising at +2.07%.

The overall effects described in Figures 5.7a and 5.7b appear to be generally consistent with a period of stagflation, characterized by reduced levels of output and employment, increased unemployment and interest rates and a temporarily reduced competitiveness. The very small negative effect on the real wage could be a reflection of the historical resistance to below-inflation adjustments in nominal wage setting, while the fall in union membership, n_t^*, appears to be consistent with the view that, in periods of reduced economic activity, workers are more willing to sacrifice the benefits of union membership in exchange for a higher probability of employment.

Productivity Shock

As in Chapter 4, this section considers the effects of a reduced form shock in productivity, i.e. a shock in the 14th equation of (5.7). The size of the shock is again scaled to be equal to $\sqrt{\tilde{\omega}_{14,14}} = 0.00821$, which is very similar to the value reported in Chapter 4 (0.00853) and, thus, makes a quantitative comparison possible.

Figure 5.8a illustrates the effects of the shock on the variables describing the AS sub-system.[9] A comparison with Figure 4.8 in Chapter 4 immediately reveals strong similarities, even quantitatively. The one standard error shock translates to a 3.3% rise in a_t. The impact effect is an increase in wages and fallback wages by 1.04% and 1.07%, respectively. This is associated with a 0.6% fall in employment and a 0.4% fall in alternative employment levels, while unemployment is reduced by 0.08%. The wage variables continue to move closely together following a generally upward course in the medium run. The long-term effect on the real wage and the fallback wage is an almost identical increase by 2.78% and 2.72%, respectively. Actual employment quickly recovers from the negative impact effect and remains quite close to its pre-shock level, with a very small long-run increase of +0.01%. Alternative employment appears to be rising rather steadily to its long-run value at +0.4%. In the medium run unemployment appears to be falling steadily and reaches a minimum at -9.5%. However, the long-run effect is found to be a bit smaller at -8.7%.

These effects appear to be very similar to the ones obtained in Chapter 4, indicating the robustness of the estimated AS sub-system. Again, the GIRs are found to be consistent with the intuitive view that improvements in production technology (increases in productivity) can sustain both higher real wages and lower levels of unemployment. Also, the rise in n_t^* appears to be consistent with the view expressed in the discussion of the oil price shock above, namely that union membership tends to be negatively related to the unemployment rate.

Figure 5.8b plots the GIRs for the variables describing the AD sub-system. In the first panel the positive productivity shock is shown to increase both domestic and foreign output. The impact effect is a rise by 2.96% for y_t and a more moderate increase of 0.56% for y_t^*, while the respective long-run effects are found to be +2.94% and +1.34%. The second panel

[9] The GIR for unemployment is scaled by 0.25.

illustrates the effects on the interest rate variables. On impact, R_t and LI_t are reduced by 2 and 6 basis points respectively, while R_t^* is marginally increased by 0.4 basis points. The foreign rate continues to rise in the medium run until it stabilises at +24 basis points. The domestic short-term rate initially drops by 16 points in the second quarter, but thereafter it starts to close the gap with the foreign rate. The effect turns positive in the fourth quarter and in the long-run it stabilises at +13 basis points. The domestic long-term rate LI_t, on the other hand, is eventually reduced by 7 basis points, having reached a minimum at -13 basis points in the second quarter. The third panel depicts the effects on the real money supply and the real effective exchange rate. On impact both these variables are increased by 1% and 1.5%, respectively. The real money supply continues to rise and stabilises at +2.14%, having overshot its long-run value in the seventh quarter. The path of the real exchange rate is found to be more dramatic. After some oscillation between +0.05% and +2% during the first year, it eventually stabilises close to its impact value at +1.15%.

The overall effects described in Figures 5.8a and 5.8b appear to be consistent with a booming economy, characterized by increased output, real wages and competitiveness and reduced unemployment and long-term interest rates.

5.5 Conclusions

This chapter attempted to carry out a joint empirical investigation of the AD and AS long-run equilibria in (5.1) within a single cointegrating $VAR(p)$, as a complement to the separate analysis of the AD and AS sub-systems in Chapters 3 and 4. The discussion in the empirical section highlighted some of the difficulties associated with statistical inference within a relatively large system. In particular, the bootstrap cointegration rank tests appeared to be making too large a correction, which resulted in a very low estimate of r. Such limitations of bootstrap methods in large systems were recently discussed in Greenslade *et al* (2002), who strongly advise the reduction of the model's dimensions at an early stage by, e.g., conditioning on weakly exogenous variables. However, it appears that even the size of the conditional model considered here is large enough to seriously distort the performance of the tests.. As a consequence, statistical inference was in large part guided by the results obtained from the sub-systems in Chapters 3 and 4.

To the extent that there are five cointegrating relations that satisfy the over-identifying restrictions imposed by the underlying theory, the estimates appear to be very similar to Chapters 3 and 4. All coefficients are found to carry the right sign, are of comparable magnitude to those in Chapters 3 and 4, and there is again evidence in favour of a reduced mean in the BP disequilibria during the pre-1973 period.

The Persistence Profiles appear to be consistent with the general findings from the separate analysis of the AD and AS sub-systems in Chapters 2, 3 and 4. In the demand side of the economy, equilibrium is restored faster in the asset markets than in the goods market, while the supply side returns to its long-run equilibrium strikingly slower, due to a profound sluggishness in wage setting. However, the adjustment processes were found here to last a bit longer, which could be due to the increased complexity of the model, or simply the consequence of the typically wide confidence bands of such profiles.

Finally, the GIR analysis illustrated that the model possesses reasonable dynamic properties. The responses of the AD variables to an oil price shock were found to be quite similar to Garratt *et al* (1998, 2001) and Chapter 2, indicating that the addition of a supply side to the AD sub-system of Chapter 3 is, to some extent, compensating for the absence of prices. The overall effects of an oil price shock were found to be generally consistent with a period of stagflation, characterized by reduced levels of output and employment, increased unemployment and interest rates and a sustained period of reduced competitiveness. The responses of the AS variables to a productivity shock were found to be very similar to Chapter 4, indicating the robustness of the AS sub-system. The overall effects of the productivity shock are consistent with a booming economy, characterized by increased output, real wages and competitiveness and reduced unemployment and long-term interest rates.

Conclusions

The primary aim of this thesis was to apply the *Long-Run Structural Cointegrating VAR* approach, developed within the *ESRC*-Cambridge research project *Structural Modelling of the UK Economy within a VAR Framework using Quarterly and Monthly Data*, in order to empirically investigate UK Aggregate Demand and Supply. This application was intended to complement the recently developed macro-econometric model of the UK in Garratt *et al* (1998, 2001), by addressing the issue of structural change and the explicit modelling of the supply-side of the economy. It was further hoped to provide a practical solution to the convergence problems, typically encountered in the application of simulation methods for inference on the cointegrating parameters. Finally, it was intended to make an informal comparison of the estimated models with those in Garratt *et al* (1998, 2001), using Persistence Profiles and Generalised Impulse Responses.

The theoretical basis for the analysis of the demand-side of the UK economy was provided by a fairly standard version of the small-open-economy IS-LM model. For the reasons discussed in Chapter 2, the empirical analysis focused primarily on the identification and testing of the IS, LM and BP long-run equilibria. The evidence presented in Chapters 2, 3 and 5 suggest that, having accounted for sample size, it can be reasonably argued that, in the long run, UK Aggregate Demand is consistent with the IS-LM-BP theory.

Furthermore, the empirical analysis in Chapters 3 and 5 indicated that these relations, and in particular the BP, may not have been stable throughout the period 1965q1-1998q2. The issue of structural change in the cointegrating relations was addressed by utilising the techniques proposed by Johansen and Nielsen (1994), Hansen (2000) and Johansen, Mosconi and Nielsen (2000). The analysis focused only on a specific type of structural change in the form of a time-varying mean in the cointegrating relations, caused by the different exchange rate regimes within the sample period. The evidence obtained from the analysis of both the AD sub-system in Chapter 3 and the complete AD-AS model in Chapter 5 revealed a significantly lower mean in the BP relation during the pre-1973 period of fixed exchange rates. Such an effect could be consistent with a reduced risk premium in the UIP condition, or/and a reduced error in exchange rate expectations, resulting from the relatively higher predictability of exchange rate movements before 1973. The importance of such an effect on the BP relation was illustrated in Chapter 3 with the use of Persistence Profiles. These indicated that taking account of this type of structural change leads to an improvement in the mean-reverting properties of the BP-relation.

The empirical analysis of the supply-side of the UK economy was based on the Lee and Pesaran (1993b) sectoral model of the labour market. The aggregation and identification problems that arise when analysing this type of model using aggregate data were overcome following the approach in Lee and Papaikonomou (2002). The evidence obtained from the AS sub-system in Chapter 4, as well as the complete AD-AS model in Chapter 5, indicated that the long-run structure suggested by the underlying theory is consistent with the data.

The empirical analysis has paid particular attention to the well-documented small-sample bias associated with asymptotic inference within the chosen econometric framework. This issue was addressed with the use of recently developed simulation methods suggested by, *inter alia*, van Giersbergen (1996), Mantalos and Shukur (1998), Gredenhoff and Jacobson (1998), Fachin (2000), Jacobson *et al* (2001) and Greenslade *et al* (2002). To the extent that these techniques are reliable, they reveal a substantial finite-sample bias comparable with existing research. This adds to the rich literature that illustrates the limitations of asymptotic inference in applied research and highlights the need for controlling for sample size.

However, the application of these methods for finite-sample inference on the cointegrating parameters has frequently been compromised by convergence problems that tend to arise when using conventional optimisation algorithms. Chapter 3 introduced the use of the global optimisation algorithm *Simulated Annealing* (SA), discussed in Goffe *et al* (1994) and adapted to *Gauss* by Tsionas (1995), as a practical means for overcoming these difficulties. The use of SA was shown to improve on the existing approach, in particular, when the over-identified cointegrating matrix involves a large number of free parameters.

The application of simulation methods to the complete AD-AS model in Chapter 5 also illustrated some of the limitations of these methods within large systems, recently discussed by Greenslade *et al* (2002). The empirical analysis indicated that inference within large systems should be guided by the more reliable evidence obtained from smaller sub-systems and the use of economic theory, as suggested in Garratt *et al* (2000).

The dynamic properties of the estimated systems were illustrated with the use of Persistence Profiles and Generalised Impulse Responses. The former verified that equilibrium is restored faster in the demand-side of the economy, as the supply-side was found to suffer from the typical sluggishness of labour markets. Adjustment in the three components IS, LM and BP of Aggregate Demand was found to last between 2 and a half and 6 years, with equilibrium being restored faster in the financial markets than in the goods market. The introduction of structural change of the type considered in Chapter 3 was shown to increase the speed with which the BP relation is restored by a factor of 3. This illustrates the important role of this modification for the BP relation, although the IS and LM relations seem to be unaffected.

In the AS sub-system, long-run equilibrium was found to be restored in approximately 7 years. This slow rate of adjustment was found to be due to the sluggish reaction of labour supply, which was reported to be approximately 3 times slower than the reaction of labour demand.

The evidence from the joint analysis of AD and AS in Chapter 5 is generally supportive of the results obtained from the separate AD and AS sub-systems. However, adjustment periods are found to be longer, with equilibrium in AD and AS being restored in approximately 7 and 11 years, respectively. This strong persistence of disequilibriating shocks suggests great caution when evaluating economic policy, especially with regard to the labour market, as the full effects of such intervention can only be appreciated after long periods of adjustment.

The GIR analysis indicated that the estimated systems possess reasonable dynamic properties, consistent with mainstream economic theory. The estimated effects of foreign interest rate and oil-price shocks were shown to be similar to those reported in Garratt *et al* (1998, 2001). These similarities were found to be stronger in Chapter 2, where prices were explicitly modelled, and Chapter 5 that possessed a complete AD-AS long-run structure. A positive oil-price shock was shown to result in stagflation, characterised by reduced levels of output and employment, increased unemployment and interest rates and a sustained period of reduced competitiveness. The analysis also considered the effects of a positive productivity shock. These were found to be consistent with a booming economy, characterised by increased output, real wages and competitiveness and reduced unemployment and long-term interest rates.

This thesis also highlighted a number of areas that need to be further investigated. Although the approach taken in Chapter 3 regarding the effects of different exchange rate regimes was shown to be a useful extension, it is in many respects over-simplified and rather limited in scope. Structural change need not be limited to a shift in the mean of the cointegrating relations and could affect all cointegrating parameters, as well as the short-run dynamics. Hansen (2000), for example, provides the framework for identifying and controlling for such effects.

The empirical analysis illustrated how simulation methods can provide a useful tool for obtaining model-specific critical values. However, several limitations were identified in the application of these methods within large systems. This indicates that they should not be treated as a panacea and suggests great caution in interpreting simulation-based results. Although Greenslade *et al* (2002), for example, present more formal evidence in favour of a negative relation between the performance of bootstrap tests and the size of the model, the details of this relation remain to be established.

Persistence Profiles and Generalised Impulse Responses were used here in order to illustrate the dynamic behaviour of the estimated systems and compare them with those of the models in Garratt *et al* (1998, 2001). However, this allowed only for an informal comparison. It would be worthwhile to pursue a more formal comparison in the future using, for example, the non-nested testing techniques in Pesaran and Weeks (2001). In addition, the use of GIRs need not be restricted to the evaluation of exogenous shocks such as increases in oil-prices. Using an appropriate economic framework they may be further applied for policy evaluation, as in Garratt *et al* (2001).

List of Tables

Table 2.1a – ADF(k) Tests Applied to the Levels of the Variables: 1965q1-1998q2

Variable	ADF(0)	ADF(1)	ADF(2)	ADF(3)	ADF(4)	95% c.v.
y_t	-2.1986^{S}	-2.2302	-2.4363	$-3.0949^{A,H}$	-3.0440	-3.4437
y_t^*	-3.0648	$-2.9324^{S,H}$	-2.9711^{A}	-3.0107	-2.9876	-3.4437
p_t	2.0075	-0.4412^{ASH}	-0.5219	-0.7384	-0.7616	-3.4437
p_t^*	1.9045	0.1536	-0.2324	-0.7178^{ASH}	-0.5884	-3.4437
p_t^o	-1.0083^{ASH}	-1.0618	-0.8915	-0.9816	-0.9483	-3.4437
$p_t - p_t^o$	-1.6707^{ASH}	-1.6719	-1.4473	-1.5179	-1.4459	-3.4437
$p_t^* - p_t^o$	-1.3520^{ASH}	-1.3731	-1.1708	-1.2281	-1.1792	-3.4437
e_t	-1.0225	-1.5910^{ASH}	-1.4627	-1.4492	-1.5305	-3.4437
m_t	-0.0702	-0.3955^{S}	-0.5416	-0.7128	$-1.0774^{A,H}$	-3.4437
LI_t	-1.3864	-1.6825^{ASH}	-1.5481	-1.6387	-1.5880	-2.8830
R_t	-2.3322^{S}	$-2.6350^{A,H}$	-2.7181	-2.4223	-2.5627	-2.8830
R_t^*	-1.3379	$-2.5241^{S,H}$	-2.3390	-2.8280^{A}	-2.8052	-2.8830
$m_t - p_t$	-1.0077	-1.1101^{S}	$-1.2140^{A,H}$	-1.2884	-1.3294	-2.8830
$e_t - p_t + p_t^*$	-1.4856^{S}	$-1.9201^{A,H}$	-1.8909	-1.9854	-2.0469	-2.8830

Notes: The *ADF* regressions include an intercept, a linear trend and k lagged first-differences of the dependent variable, with the exception of LI_t, R_t, R_t^*, $m_t - p_t$ and $e_t - p_t + p_t^*$, where the linear time trend was omitted. The superscripts *A, S* and *H* indicate the choice of the Akaike Information, the Schwarz Bayesian and the Hannan-Quinn criteria respectively.

Table 2.1b – ADF(k) Tests Applied to the Differences of the Variables: 1965q1-1998q2

Variable	ADF(0)	ADF(1)	ADF(2)	ADF(3)	ADF(4)	95% c.v.
Δy_t	$-11.649^{S,H}$	-7.7905	-5.2403^{A}	-5.0235	-4.6373	-2.8830
Δy_t^*	$-7.2058^{S,H}$	-5.2314^{A}	-4.3036	-4.0121	-4.0633	-2.8830
Δp_t	-3.2842^{ASH}	-2.9834	-2.5525	-2.4540	-2.4142	-2.8830
$\Delta^2 p_t$	-13.024^{S}	$-10.201^{A,H}$	-8.0338	-6.7859	-6.9984	-2.8830
Δp_t^*	-5.0132	-3.5513	-2.5571^{ASH}	-2.6463	-2.6948	-2.8830
$\Delta^2 p_t^*$	-16.578	-13.026^{ASH}	-8.5002	-6.8527	-6.8399	-2.8830
Δp_t^o	-11.090^{ASH}	-8.6976	-6.4450	-5.6448	-5.5210	-2.8830
$\Delta(p_t - p_t^o)$	-11.508^{S}	$-9.1265^{A,H}$	-6.7899	-6.0328	-5.9647	-2.8830
$\Delta(p_t^* - p_t^o)$	-11.337^{ASH}	-8.9827	-6.7336	-5.9049	-5.8058	-2.8830
Δe_t	-9.4357^{ASH}	-7.6057	-6.3522	-5.3408	-5.2084	-2.8830
Δm_t	-8.6712	-6.0697	-4.6937	-3.4163	-2.5439^{ASH}	-2.8830
$\Delta^2 m_t$	-18.557	-13.602	-12.473	-11.527^{ASH}	-7.3596	-2.8830
ΔLI_t	-9.5670^{ASH}	-8.0147	-6.1250	-5.5881	-5.5008	-2.8830
ΔR_t	-10.395^{ASH}	-7.6365	-7.3060	-5.8991	-6.1002	-2.8830
ΔR_t^*	-7.0240^{ASH}	-6.5233	-4.9318	-4.6830	-4.3328	-2.8830
$\Delta(m_t - p_t)$	-8.4194^{S}	$-5.7496^{A,H}$	-4.6776	-4.0858	-3.3887	-2.8830
$\Delta(e_t - p_t + p_t^*)$	-9.9002^{ASH}	-7.6715	-6.2344	-5.3740	-5.6216	-2.8830

Notes: The *ADF* regressions do not include a linear trend.

Table 2.2a - Phillips-Perron Unit Root Tests Applied to the Levels: 1965q1-1998q2

Variable	PP(0)	PP(5)	PP(10)	PP(15)	PP(20)	95% c.v.
y_t	-2.0125	-1.9115	-2.1191	-2.3504	-2.4729	-3.4437
y_t^*	-3.1487	-2.5197	-2.8776	-3.4576	-4.5355	-3.4437
p_t	2.4046	1.3177	1.2131	1.1976	1.1997	-3.4437
p_t^*	2.2908	1.2600	1.1002	1.0561	1.0584	-3.4437
p_t^o	-1.1497	-0.9209	-0.8981	-0.8913	-0.8979	-3.4437
$p_t - p_t^o$	-2.0434	-1.5433	-1.4628	-1.4333	-1.4371	-3.4437
$p_t^* - p_t^o$	-1.5269	-1.2071	-1.1782	-1.1694	-1.1795	-3.4437
e_t	-0.9967	-0.8554	-0.8014	-0.8337	-0.8701	-3.4437
m_t	-0.0731	-0.0501	-0.0442	-0.0452	-0.0492	-3.4437
LI_t	-1.0903	-1.2094	-1.3320	-1.4764	-1.6651	-2.8830
R_t	-2.4537	-2.4243	-2.6147	-3.0443	-3.1199	-2.8830
R_t^*	-1.0214	-0.9888	-1.1045	-1.2441	-1.3741	-2.8830
$m_t - p_t$	-0.9619	-0.8054	-0.8178	-0.8249	-0.8377	-2.8830
$e_t - p_t + p_t^*$	-1.3962	-1.4286	-1.5531	-1.5211	-1.5143	-2.8830

Notes: $PP(\ell)$ denotes Phillips and Perron (1988) unit root statistic based on the Bartlett window of size ℓ. The underlying *DF* regressions in the calculation of the *PP* statistics include an intercept and a linear time trend with the exception of LI_t, R_t, R_t^*, $m_t - p_t$ and $e_t - p_t + p_t^*$, where the linear time trend was omitted.

Table 2.2b - Phillips-Perron Unit Root Tests Applied to the Differences: 1965q1-1998q2

Variable	PP(0)	PP(5)	PP(10)	PP(15)	PP(20)	95% c.v.
Δy_t	-8.5836	-8.5356	-9.6319	-9.6814	-9.6328	-2.8830
Δy_t^*	-5.9349	-6.5592	-6.9122	-6.9394	-7.0686	-2.8830
Δp_t	-2.4028	-3.3850	-4.1404	-3.9750	-4.3146	-2.8830
$\Delta^2 p_t$	-10.586	-13.343	-12.747	-12.708	-12.356	-2.8830
Δp_t^*	-2.2306	-3.6747	-4.1455	-4.3850	-4.6276	-2.8830
$\Delta^2 p_t^*$	-7.0465	-13.182	-15.721	-17.107	-17.769	-2.8830
Δp_t^o	-8.6986	-8.5824	-8.4633	-8.5043	-8.8214	-2.8830
$\Delta(p_t - p_t^o)$	-8.8082	-8.7135	-8.6284	-8.6643	-8.9342	-2.8830
$\Delta(p_t^* - p_t^o)$	-8.8867	-8.6854	-8.5456	-8.5640	-8.8362	-2.8830
Δe_t	-8.8504	-12.523	-13.391	-14.496	-16.437	-2.8830
Δm_t	-6.1371	-6.0827	-5.3362	-4.9689	-4.7491	-2.8830
$\Delta^2 m_t$	-13.334	-15.177	-14.264	-14.569	-16.287	-2.8830
ΔLI_t	-7.4983	-8.3757	-8.4591	-8.9220	-9.5811	-2.8830
ΔR_t	-10.707	-13.956	-15.181	-14.383	-14.817	-2.8830
ΔR_t^*	-5.5813	-6.5222	-7.2244	-7.8973	-8.0793	-2.8830
$\Delta(m_t - p_t)$	-6.1472	-5.7794	-5.1055	-4.8621	-4.6828	-2.8830
$\Delta(e_t - p_t + p_t^*)$	-9.1007	-12.490	-12.681	-13.309	-14.837	-2.8830

Notes: The underlying *DF* regressions in the calculation of the *PP* statistics include only an intercept.

Table 2.3 – Adjusted Likelihood Ratio Tests and Model Selection Criteria for the Choice of the Lag-Length

Adjusted Likelihood Ratio Tests			
H_0	H_1	Statistic	p-value
$p=0$	$p=4$	2316.0	.000
$p=1$	$p=4$	285.15	.033
$p=2$	$p=4$	179.20	.168
$p=3$	$p=4$	86.08	.329

AIC	SBC	Order
2942.9	2916.8	$p=0$
4279.3	4135.8	$p=1^{S,A}$
4272.2	4011.4	$p=2$
4256.2	3878.0	$p=3$
4235.3	3739.7	$p=4$

Notes: Based on an unrestricted *VAR*(4) with an intercept and a linear time trend in the level of $z_t=[m_t-p_t,y_t,R_t,R_t^*,LI_t,e_t,p_t-p_t^o,p_t^*-p_t^o,y_t^*]'$. The superscripts *S* and *A* indicate the choice of the Schwarz Bayesian Criterion (*SBC*) and the Akaike Information Criterion (*AIC*) respectively.

Table 2.4a – Cointegration Rank Statistics for the Marginal Model in the Weakly Exogenous Vector

$$x_t^+=[R_t^*,p_t^*-p_t^o,y_t^*]'$$

		Trace			Max		
H_0	H_1	Statistic	95% cv	90% cv	Statistic	95% cv	90% cv
$r=0$	$r=1$	53.19	39.33	36.28	28.15	24.35	22.26
$r\le 1$	$r=2$	25.04	23.83	21.23	16.00	18.33	16.28
$r\le 2$	$r=3$	9.04	11.54	9.75	9.04	11.54	9.75

Notes: "Trace" and "Max" stand for the Pesaran Shin and Smith (2000) modified versions of Johansen's (1988) cointegrating rank statistics. Based on an unrestricted trend cointegrating VAR(2) in the vector x_t^+ augmented by one lagged difference of the vector $y_t^+=[m_t-p_t,y_t,R_t,LI_t,e_t,p_t-p_t^o]'$.

Table 2.4b – Cointegration Rank Statistics for the Marginal Model in the Weakly Exogenous Vector

$$x_t=[p_t^*-p_t^o,y_t^*]'$$

		Trace			Max		
H_0	H_1	Statistic	95% cv	90% cv	Statistic	95% cv	90% cv
$r=0$	$r=1$	18.68	23.83	21.23	12.67	18.33	16.28
$r\le 1$	$r=2$	6.01	11.54	9.75	6.01	11.54	9.75

Notes: Based on an unrestricted trend cointegrating VAR(2) in the vector x_t augmented by one lagged difference of the vector $y_t=[m_t-p_t,y_t,R_t,R_t^*,LI_t,e_t,p_t-p_t^o]'$.
See also the Notes to Table 2.4a.

Table 2.5-Model Selection Criteria for Alternative Choices of *p*, *r* and Trend/Intercept Specifications

		AIC		SBC		HQC	
		Case IV	Case V	Case IV	Case V	Case IV	Case V
$p=1$	r						
	0	3569.9	3573.8	3539.4	3533.2	3557.5	3557.3
	1	3610.7	3610.7	3557.1	3548.4	3588.9	3585.4
	2	3639.8	3640.7	3565.9	3559.5	3609.8	3607.7
	3	3660.5	3660.8	**3569.2** *	3563.7	**3623.4** *	3621.3
	4	3666.0	3666.9	3560.2	3556.8	3623.0	3622.2
	5	3667.6	3669.5	3550.3	3549.3	3619.9	3620.7
	6	3669.0	3670.4	3543.0	3542.9	3617.8	3618.5
	7	**3670.7** *	**3670.7** *	3538.8	3538.8	3617.1	3617.1
$p=2$	r						
	0	3610.7	3612.0	**3489.0**	3480.2	3561.2	3558.5
	1	3626.8	3628.0	3481.9	3474.5	3567.9	3565.6
	2	3642.2	3643.6	3477.0	3471.1	3575.1	3573.5
	3	3651.0	3653.0	3468.5	3464.6	3576.8	3576.4
	4	3657.9	3659.8	3460.9	3458.4	3577.8	3577.9
	5	3664.2	3665.4	3455.5	3453.9	3579.4	3579.4
	6	3668.7	3668.1	3451.4	3449.3	**3580.4**	3579.2
	7	**3670.1**	**3670.1**	3447.0	3447.0	3579.4	3579.4
$p=3$	r						
	0	3583.8	3584.3	3370.9	3361.2	3497.3	3493.7
	1	3613.5	3614.8	**3377.4**	3369.9	3517.6	3515.3
	2	3625.0	3627.1	3368.5	3363.4	3520.8	3519.9
	3	3637.3	3638.7	3363.4	3359.1	3526.0	3525.1
	4	3645.6	3645.9	3357.3	3353.3	3528.4	3527.0
	5	3652.5	3651.2	3352.6	3348.4	3530.6	3528.2
	6	3656.5	3656.0	3347.9	3346.0	**3531.1**	3530.0
	7	**3657.6**	**3657.6**	3343.1	3343.1	3529.8	3529.8
$p=4$	r						
	0	3576.2	3575.6	3271.9	3261.2	3452.5	3447.9
	1	3604.4	3603.3	**3276.9**	3267.1	3471.3	3466.7
	2	3615.4	3614.5	3267.6	3259.6	3474.1	3470.3
	3	3623.1	3623.0	3258.0	3252.1	3474.8	3472.3
	4	3629.8	3628.6	3250.1	3244.6	3475.5	3472.6
	5	3635.2	3633.5	3244.0	3239.4	3476.3	3473.3
	6	**3639.7**	3638.9	3239.8	3237.5	**3477.2**	3475.8
	7	3639.3	3639.3	3233.6	3233.6	3474.4	3474.4

Notes: Based on a cointegrating *VAR*(*p*) in $y_t = [m_t - p_t, y_t, R_t, R_t^*, LI_t, e_t, p_t - p_t^o]'$ conditional on the weakly exogenous vector $x_t = [p_t^* - p_t^o, y_t^*]'$. Case IV stands for the unrestricted intercepts and restricted trends specification and Case V refers to the unrestricted intercepts and trends specification. Bold face indicates a maximum given *p*, and "*" indicates global maximum.

Table 2.6 – Likelihood Ratio and Adjusted Likelihood Ratio Tests for the Restriction of the Trend Coefficients According to $\mathbf{c}_1 = -\Pi_y \gamma$

$rank(\Pi)$	LR-Statistic	ALR-Statistic	$\chi^2_{7-r,0.05}$
$r=0$	16.6 **	14.990 **	14.067
$r=1$	14.4 **	12.773 **	12.592
$r=2$	12.8 **	11.176 **	11.070
$r=3$	12.0 **	10.337 **	9.488
$r=4$	9.8 **	8.348 **	7.815
$r=5$	6.4 **	5.404	5.991
$r=6$	0.8	0.671	3.841

Notes: "**" and "*" indicate significance at the 5% and 10% levels, respectively.

Table 2.7 - Likelihood Ratio Tests for the Presence of Unitary Cointegrating Vectors

Variable	Likelihood Ratio Statistic					
$m_t - p_t$	55.32	50.00	34.03	26.92	23.64	18.11
y_t	47.42	45.28	29.96	22.71	19.79	15.28
R_t	37.70	34.31	24.94	17.18	15.39	15.38
R_t^*	32.22	29.39	23.10	17.70	12.54	*7.82*
LI_t	40.63	37.23	21.54	17.01	14.54	10.65
e_t	46.29	43.91	27.57	19.76	16.87	12.91
$p_t - p_t^o$	46.21	41.59	27.83	20.78	17.13	12.70
$p_t^* - p_t^o$	52.49	47.14	30.74	22.99	21.67	16.34
y_t^*	47.79	44.86	29.10	21.80	19.29	15.77
$rank(\Pi_y)$	$r=1$	$r=2$	$r=3$	$r=4$	$r=5$	$r=6$
95% c.v.	16.92	15.51	14.07	12.59	11.07	9.49

Notes: The null hypothesis is of stationarity and it is conditional on the number of cointegrating vectors, *r*. Statistics in *italic* signify failure to reject the null at the 5% level.

Table 2.8a – Adjusted and Non-Adjusted Cointegration Rank Statistics

		Trace			Max		
H_0	H_1	Statistic	95% cv	90% cv	Statistic	95% cv	90% cv
$r=0$	$r=1$	258.83 231.78 s	177.79	171.62	64.24 57.53 s	55.83	52.69
$r\leq 1$	$r=2$	194.58 174.25 s	141.73	136.21	58.84 52.69 s	50.10	47.08
$r\leq 2$	$r=3$	135.74 121.56 s	108.90	103.71	41.59 37.25 s	43.72	40.94
$r\leq 3$	$r=4$	94.15 84.31 s	81.20	76.68	33.79 30.26 s	37.85	35.04
$r\leq 4$	$r=5$	60.36 54.06 s	56.43	52.71	28.56 25.57 s	31.68	29.00
$r\leq 5$	$r=6$	31.81 28.48 s	35.37	32.51	21.12 18.91 s	24.88	22.53
$r\leq 6$	$r=7$	10.69 9.57 s	18.08	15.82	10.69 9.57 s	18.08	15.82

Notes: Based on a restricted trend cointegrating *VAR*(2) in $y_t = [m_t - p_t, y_t, R_t, R_t^*, LI_t, e_t, p_t - p_t^o]'$ conditional on the weakly exogenous vector $x_t = [p_t^* - p_t^o, y_t^*]'$. "s" denotes scaled according to the Reinsel and Ahn (1988, 1992) scaling factor $(T - n_y p)/T$.
See also the Notes to Table 2.5a.

Table 2.8b – Cointegration Rank Statistics and Finite-Sample Critical Values

		Trace			Max		
H_0	H_1	Statistic	95% cv	90% cv	Statistic	95% cv	90% cv
$r=0$	$r=1$	258.83	212.29 p 211.75 n	204.55 p 203.93 n	64.24	66.77 p 66.10 n	63.00 p 62.48 n
$r\leq 1$	$r=2$	194.58	175.18 p 170.19 n	167.39 p 163.73 n	58.84	61.45 p 59.43 n	57.61 p 56.11 n
$r\leq 2$	$r=3$	135.74	137.47 p 132.29 n	130.61 p 126.35 n	41.59	54.50 p 51.84 n	51.06 p 48.80 n
$r\leq 3$	$r=4$	94.15	106.61 p 99.54 n	101.03 p 94.81 n	33.79	48.39 p 43.93 n	45.13 p 41.25 n
$r\leq 4$	$r=5$	60.36	70.38 p 68.83 n	66.48 p 64.83 n	28.56	37.53 p 36.78 n	34.94 p 34.28 n
$r\leq 5$	$r=6$	31.81	42.27 p 42.48 n	39.63 p 39.48 n	21.12	29.13 p 29.07 n	26.71 p 26.71 n
$r\leq 6$	$r=7$	10.69	22.00 p 22.31 n	19.76 p 20.05 n	10.69	22.00 p 22.31 n	19.76 p 20.05 n

Notes: The critical values are based on a bootstrap with 10,000 simulations. "p" and "n" denote parametric and non-parametric bootstrap, respectively.
See also the Notes to Table 2.8a.

Table 2.9 – Likelihood Ratio and Adjusted Likelihood Ratio Tests for the Imposition of Over-Identifying Restrictions on the Cointegrating Parameters

Restriction	log-likelihood	*LR*-statistic	*ALR*-statistic	95% c.v.	90% c.v.
R_E	3793.90	–	–	–	–
R_{OV1}	3781.50	24.79 [12] **	21.54 [12] **	21.03	18.55
R_{OV2}	3781.43	24.93 [13] **	21.66 [13] *	22.36	19.81

Notes: R_E : Any set of r^2 exactly identifying restrictions.

R_{OV1} : R_{OV2} less the zero restriction on the trend coefficient in the PPP relation.

R_{OV2} : The full set of the 13 over-identifying restrictions implied by theory.

LR and *ALR* stand for Likelihood Ratio and Adjusted Likelihood Ratio respectively. Both tests are asymptotically chi-squared with degrees of freedom equal to the number of over-identifying restrictions given in square brackets. "**" and "*" indicate significance at the 5% and 10% levels, respectively.

Table 2.10 – Estimated Cointegrating Vectors under R_{OV2}

Variables	LM	IS	BP	PPP
$m_t - p_t$	1.00 [—]	—	—	—
y_t	-2.71 ** [-2.67]	1.00 [—]	-0.54 ** [-2.25]	—
R_t	28.20 ** [4.44]	—	1.00 [—]	—
R_t^*	—	—	-1.00 [—]	—
LI_t	—	9.57 * [1.90]	—	—
e_t	—	-0.09 [-0.21]	0.43 [1.52]	1.00 [—]
$p_t - p_t^o$	—	0.09 [0.21]	-0.43 [-1.52]	-1.00 [—]
$p_t^* - p_t^o$	—	-0.09 [-0.21]	0.43 [1.52]	0.94 ** [37.3]
y_t^*	—	-0.83 ** [-19.8]	0.17 [1.19]	—
t	0.016 ** [2.89]	—	0.002 ** [2.60]	—

Notes: *t*-ratios are given in square brackets. "*" and "**" indicate significance at the 10% and 5% levels respectively.

Table 2.11 – Error Correction Specification

Equation	$\Delta(m_t - p_t)$	Δy_t	ΔR_t	ΔR_t^*	ΔLI_t	Δe_t	$\Delta(p_t - p_t^o)$
$\hat{\varepsilon}_{1,t-1}$	-0.038 ** (-3.19)	-0.010 (-1.11)	-0.004 (-1.61)	0.004 ** (3.25)	0.004 ** (2.75)	-0.031 (-0.94)	-0.0001 (-0.02)
$\hat{\varepsilon}_{2,t-1}$	-0.089 (-1.42)	0.033 (0.71)	-0.011 (-0.82)	-0.019 ** (-3.09)	-0.030 ** (-4.23)	0.212 (1.20)	0.075 ** (2.07)
$\hat{\varepsilon}_{3,t-1}$	-0.341 * (-1.88)	0.428 ** (3.14)	-0.117 ** (-3.04)	-0.057 ** (-3.27)	-0.086 ** (-4.24)	0.459 (0.90)	0.069 (0.67)
$\hat{\varepsilon}_{4,t-1}$	0.147 * (1.80)	-0.152 ** (-2.49)	0.047 ** (2.73)	0.027 ** (3.50)	0.041 ** (4.55)	-0.257 (-1.12)	-0.005 (-0.10)
$\Delta(m_{t-1} - p_{t-1})$	-0.179 * (-1.95)	0.040 (0.58)	0.009 (0.47)	0.020 ** (2.32)	-0.004 (-0.34)	0.164 (0.63)	0.110 ** (2.08)
Δy_{t-1}	-0.042 (-0.34)	-0.139 (-1.51)	-0.057 ** (-2.19)	-0.001 (-0.10)	-0.022 (-1.64)	-0.546 (-1.58)	-0.030 (-0.43)
ΔR_{t-1}	0.257 (0.52)	0.225 (0.61)	0.091 (0.87)	-0.120 ** (-2.56)	0.036 (0.65)	-1.472 (-1.06)	0.371 (1.32)
ΔR_{t-1}^*	-2.388 ** (-2.41)	0.231 (0.31)	0.287 (1.37)	0.216 ** (2.28)	-0.018 (-0.16)	3.618 (1.30)	-0.358 (-0.63)
ΔLI_{t-1}	0.886 (0.94)	0.515 (0.73)	-0.085 (-0.43)	0.105 (1.16)	0.080 (0.76)	-2.788 (-1.05)	-0.122 (-0.23)
Δe_{t-1}	-0.045 (-1.10)	-0.037 (-1.21)	0.008 (0.91)	-0.001 (-0.33)	0.001 (0.24)	0.314 ** (2.73)	0.017 (0.73)
$\Delta(p_{t-1} - p_{t-1}^o)$	-0.247 (-1.48)	-0.125 (-0.99)	-0.012 (-0.35)	0.0002 (0.02)	-0.002 (-0.13)	0.218 (0.46)	0.328 ** (3.41)
$\Delta(p_t^* - p_t^o)$	0.010 (1.49)	0.016 ** (3.13)	-0.002 (-1.05)	-0.001 (-1.64)	-0.001 (-1.65)	0.029 (1.47)	1.027 ** (258.1)
$\Delta(p_{t-1}^* - p_{t-1}^o)$	0.277 (1.61)	0.121 (0.94)	0.015 (0.40)	-.000001 (0.0003)	0.004 (0.21)	-0.227 (-0.47)	-0.341 ** (-3.45)
Δy_t^*	0.422 ** (2.05)	0.709 ** (4.59)	0.070 (1.61)	0.051 ** (2.56)	0.035 (1.52)	-0.313 (-0.54)	-0.283 ** (-2.40)
Δy_{t-1}^*	0.363 (1.58)	-0.038 (-0.22)	-0.032 (-0.66)	0.050 ** (2.26)	0.043 * (1.69)	0.510 (0.79)	-0.160 (-1.21)
$\bar{R}^2$	0.446	0.306	0.168	0.377	0.224	0.021	0.998
$\hat{\sigma}$	0.012	0.009	0.002	0.001	0.001	0.033	0.007
$\chi^2_{SC}[4]$	6.28	0.40	11.96 **	5.64	3.01	0.71	2.93
$\chi^2_{FF}[1]$	2.54	0.80	0.82	2.72 *	0.15	0.45	0.10
$\chi^2_{N}[2]$	5.57 *	59.50 **	29.04 **	5.40 *	7.14 **	24.18 **	25.48 **
$\chi^2_{H}[1]$	0.01	2.08	2.49	20.17 **	0.01	0.030	0.27

Notes: *t*-ratios are given in brackets. "*" and "**" indicate significance at the 10% and 5% levels respectively. $\chi_i^2[]$, i = *SC*, *FF*, *N*, *H*, stands for the chi-squared diagnostic for serial correlation (*SC*), functional form (*FF*), normality (*N*) and heteroscedasticity (*H*) with degrees of freedom in square brackets. The estimated error correction terms $\hat{\varepsilon}_{i,t-1}$, $i = 1,\ldots,4$, correspond to LM, IS, BP, PPP, given in Table 2.10.

Table 3.1 – Adjusted Likelihood Ratio Tests and Model Selection Criteria for the Choice of the Lag-Length

Adjusted Likelihood Ratio Tests			
H_0	H_1	Statistic	p-value
$p=0$	$p=4$	1709.4	.000
$p=1$	$p=4$	177.16	.046
$p=2$	$p=4$	100.71	.406
$p=3$	$p=4$	45.68	.609

AIC	SBC	Order
2741.6	2701.1	$p=0$
3699.1	3587.6	$p=1^S$
3700.3	3517.8	$p=2^A$
3687.5	3433.9	$p=3$
3668.5	3343.9	$p=4$

Notes: Based on an unrestricted $VAR(4)$ in the level of $z_t=[m_t-p_t,y_t,R_t,R_t^*,LI_t,e_t-p_t+p_t^*,y_t^*]'$ with an intercept, a linear time trend and the dummies $pre73_t$ and ERM_t. The superscripts S and A indicate the choice of the Schwarz Bayesian Criterion (*SBC*) and the Akaike Information Criterion (*AIC*) respectively.

Table 3.2a – Cointegration Rank Statistics for the Marginal Model in the Weakly Exogenous Vector $x_t^+=[R_t^*,e_t-p_t+p_t^*,y_t^*]'$

		Trace			Max		
H_0	H_1	Statistic	95% cv	90% cv	Statistic	95% cv	90% cv
$r=0$	$r=1$	42.70	31.54	28.78	38.12	21.12	19.02
$r\leq 1$	$r=2$	4.58	17.86	15.75	4.55	14.88	12.98
$r\leq 2$	$r=3$	0.03	8.07	6.50	0.03	8.07	6.50

Notes: "Trace" and "Max" stand for the Pesaran Shin and Smith (2000) modified versions of Johansen's (1988) cointegrating rank statistics. Based on a cointegrating VAR(2) in the vector x_t^+ augmented by one lagged difference of the vector $y_t^+=[m_t-p_t,y_t,R_t,LI_t]'$ with unrestricted intercepts and no trends.

Table 3.2b – Cointegration Rank Statistics for the Marginal Model in the Weakly Exogenous Vector $x_t=[e_t-p_t+p_t^*,y_t^*]'$

		Trace			Max		
H_0	H_1	Statistic	95% cv	90% cv	Statistic	95% cv	90% cv
$r=0$	$r=1$	17.47	17.86	15.75	14.00	14.88	12.98
$r\leq 1$	$r=2$	3.47	8.07	6.50	3.47	8.07	6.50

Notes: Based on a cointegrating VAR(2) in the vector x_t augmented by one lagged difference of the vector $y_t=[m_t-p_t,y_t,R_t,R_t^*,LI_t]'$ with unrestricted intercepts and no trends.
See also the Notes to Table 3.2a.

Table 3.3 – Tests for the Trends and Dummies Specification

Test	LR-statistic	Asymptotic Critical Values		Small-Sample Critical Values	
		95% cv	90% cv	95% cv	90% cv
LR_1	5.169 [2]	5.991	4.605	8.234 p 8.320 n	6.310 p 6.493 n
LR_2	6.122	–	–	9.373 p 9.382 n	5.825 p 5.840 n

Notes: LR_1 stands for the likelihood ratio statistic for testing the trend restrictions only, while LR_2 tests the restrictions on the dummies, given the trend restrictions. Both hypotheses are formulated conditionally on $r = 3$ and $p = 2$. LR_1 is asymptotically chi-squared with $n_y - r$ degrees of freedom given in square brackets. Small-sample results are based on a bootstrap with 10,000 simulations. "p" and "n" indicate parametric and non-parametric respectively.

Table 3.4 - Likelihood Ratio Tests for the Presence of Unitary Cointegrating Vectors

Variable	Likelihood Ratio Statistic			
$m_t - p_t$	55.17	38.77	33.13	25.13
y_t	49.35	35.29	28.42	21.49
R_t	42.87	28.27	23.87	17.82
R_t^*	44.28	38.52	31.92	22.49
LI_t	42.39	24.13	18.19	*10.41*
$e_t - p_t + p_t^*$	56.77	38.52	32.83	24.55
y_t^*	49.58	35.05	28.11	22.26
$rank(\Pi_y)$	$r=1$	$r=2$	$r=3$	$r=4$
95% c.v.	16.92	15.51	14.07	12.59

Notes: The null hypothesis is of stationarity and it is conditional on the number of cointegrating vectors, r. Statistics in *italic* signify failure to reject the null at the 5% level.

Table 3.5 – Cointegration Rank Statistics and Small-Sample Critical Values

		Trace			Max		
H_0	H_1	Statistic	95% cv	90% cv	Statistic	95% cv	90% cv
$r=0$	$r=1$	189.18	149.36 p 148.72 n	142.36 p 142.56 n	63.52	57.27 p 57.28 n	53.69 p 53.78 n
$r\leq 1$	$r=2$	125.66	114.79 p 115.14 n	109.09 p 109.75 n	41.91	50.27 p 50.75 n	47.27 p 47.46 n
$r\leq 2$	$r=3$	83.75	81.47 p 81.23 n	76.65 p 76.58 n	36.18	42.82 p 43.05 n	39.91 p 39.96 n
$r\leq 3$	$r=4$	47.56	50.85 p 50.94 n	47.48 p 47.37 n	27.26	34.51 p 34.17 n	31.67 p 31.58 n
$r\leq 4$	$r=5$	20.30	24.98 p 25.77 n	22.59 p 23.28 n	20.30	24.98 p 25.77 n	22.59 p 23.28 n

Notes: Based on a cointegrating $VAR(2)$ in $y_t = [m_t - p_t, y_t, R_t, R_t^*, LI_t]'$ with restricted trends and dummies conditional on the weakly exogenous vector $x_t = [e_t + p_t^* - p_t, y_t^*]'$. Small-sample results are based on a bootstrap with 10,000 simulations. "p" and "n" indicate parametric and non-parametric respectively.

Table 3.6 – Small-Sample Critical Values for the Test of Over-Identifying Restrictions

Restrictions	*LL*	*LR*-statistic	Asymptotic		Bootstrap 1		Bootstrap 2	
			95% cv	90% cv	95% cv	90% cv	95% cv	90% cv
R_E	2979.5	–						
R_{OV}	2970.4	18.36 [5]	11.07	9.24	52.27 p 52.28 n	47.86 p 47.14 n	50.41 p 51.12 n	45.77 p 46.11 n

Notes: R_E is a set of r^2 exactly identifying restrictions, R_{OV} is the set of r^2+5 over-identifying restrictions given by (3.11) in section 3.3.6, *LL* is the value of the maximised log-likelihood, *LR* stands for the Likelihood Ratio test statistic, which is asymptotically chi-squared with degrees of freedom equal to the number of over-identifying restrictions given in square brackets. Bootstrap 1 and Bootstrap 2 are defined in Chapter 1, section 1.6.4 and are based on 10,000 simulations. "p" and "n" indicate parametric and non-parametric respectively.

Table 3.7 – Estimates of the Over-Identified Cointegrating Vectors

Variables	$\hat{\varepsilon}_{1,t}$	$\hat{\varepsilon}_{2,t}$	$\hat{\varepsilon}_{3,t}$
$m_t - p_t$	1.00 [—]	—	—
y_t	-1.16 [-1.19]	1.00 [—]	-0.36 ** [-2.06]
R_t	22.61 ** [4.60]	—	1.00 [—]
R_t^*	—	—	-1.00 [—]
LI_t	—	8.73 ** [4.51]	—
$e_t - p_t + p_t^*$	—	-0.22 ** [-3.12]	0.067 [1.60]
y_t^*	—	-0.81 ** [-16.8]	0.034 [0.45]
t	0.006 [1.05]	—	0.002 * [1.73]
$pre73_t$	-0.12 [-1.17]	0.026 [0.93]	0.02 * [1.80]
ERM_t	0.12 [1.16]	0.055 * [1.85]	-0.0056 [-0.58]

Notes: $\hat{\varepsilon}_{1,t}$, $\hat{\varepsilon}_{2,t}$, $\hat{\varepsilon}_{3,t}$ correspond to the LM, IS and BP relations respectively. *t*-ratios are given in square brackets. "*" and "**" indicate significance at the 10% and 5% levels respectively.

Table 3.8 – Estimated Vector Error Correction Model

Equation	$\Delta(m_t - p_t)$	Δy_t	ΔR_t	ΔR_t^*	ΔLI_t
$\hat{\varepsilon}_{1,t-1}$	-0.053 ** (-4.30)	-0.020 (-0.22)	-0.002 (-0.83)	0.004 ** (3.94)	0.004 ** (3.72)
$\hat{\varepsilon}_{2,t-1}$	-0.0028 (-0.05)	-0.061 (-1.50)	-0.017 (-1.57)	-0.019 ** (-3.61)	-0.026 ** (-4.99)
$\hat{\varepsilon}_{3,t-1}$	-0.366 ** (-2.18)	0.325 ** (2.75)	-0.125 ** (-4.09)	-0.034 ** (-2.28)	-0.057 ** (-3.70)
$\Delta(m_{t-1} - p_{t-1})$	-0.060 (-0.71)	0.054 (0.90)	0.003 (0.19)	0.019 ** (2.48)	-0.006 (-0.72)
Δy_{t-1}	0.037 (.30)	-0.083 (-0.94)	-0.031 (-1.34)	0.0002 (0.02)	-0.010 (-0.86)
ΔR_{t-1}	0.278 (0.54)	0.012 (0.03)	0.166 (1.77)	-0.104 ** (-2.25)	0.093 ** (1.98)
ΔR_{t-1}^*	-1.574 * (-1.62)	0.228 (0.33)	0.191 (1.08)	0.232 ** (2.65)	-0.031 (-0.35)
ΔLI_{t-1}	0.097 (0.10)	0.983 (1.45)	0.027 (0.15)	0.116 (1.34)	0.117 (1.32)
$\Delta(e_{t-1} - p_{t-1} + p_{t-1}^*)$	-0.019 (-0.46)	-0.029 (-1.02)	-0.005 (-0.71)	-0.005 (-1.29)	-0.007 ** (-1.95)
Δy_{t-1}^*	0.393 * (1.65)	-0.169 (-1.01)	-0.045 (-1.02)	0.048 ** (2.22)	0.031 (1.43)
$\Delta(e_t - p_t + p_t^*)$	0.025 (0.73)	0.067 ** (2.77)	0.028 ** (4.47)	0.001 (0.36)	0.018 ** (5.83)
Δy_t^*	0.413 ** (1.92)	0.733 ** (4.82)	0.091 ** (2.32)	0.059 ** (3.02)	0.045 ** (2.29)
$\bar{R}^2$	0.38	0.32	0.31	0.39	0.42
$\hat{\sigma}$	0.012	0.009	0.002	0.001	0.001
$\chi^2_{SC}[4]$	2.378	1.443	4.389	2.951	5.922
$\chi^2_{FF}[1]$	0.008	1.954	0.155	0.189	7.848 **
$\chi^2_{N}[2]$	0.446	29.70 **	11.11 **	6.46 **	1.070
$\chi^2_{H}[1]$	1.996	3.598 *	1.811	7.54 **	1.263

Notes: t-ratios are given in brackets. $\chi^2_i[]$, $i = SC, FF, N, H$, stands for the chi-squared diagnostic for serial correlation (*SC*), functional form (*FF*), normality (*N*) and heteroscedasticity (*H*) with degrees of freedom in square brackets. The estimated error correction terms $\hat{\varepsilon}_{i,t-1}$, $i = 1,2,3$, are given in Table 3.7. "*" and "**" indicate significance at the 10% and 5% levels respectively.

Table 4.1a – ADF(*k*) Tests Applied to the Levels of the Variables: 1972q1-2000q1

Variable	ADF(0)	ADF(1)	ADF(2)	ADF(3)	ADF(4)	95% c.v.
n_t	-1.2016^{ASH}	-1.1348	-1.4149	-1.5317	-1.2621	-2.8870
n_t^*	-4.6366	-3.3486	$-3.2183^{S,H}$	-3.2666^{A}	-3.3857	-3.4497
$w_t - p_t$	-4.6309	-3.6156	-3.5633^{S}	$-4.0819^{A,H}$	-3.9904	-3.4497
$w_t^{**} - p_t$	-4.7313	-3.6921^{S}	-3.6111	$-4.0699^{A,H}$	-3.9724	-3.4497
u_t	-1.6263	$-1.9870^{S,H}$	-2.0232	-1.8559	-1.5110^{A}	-2.8870
k_t	-3.7552	-2.7908	$-2.8477^{S,H}$	-2.8585	-3.0059^{A}	-3.4497
a_t	$-3.1176^{S,H}$	-2.5592^{A}	-2.8071	-2.3993	-2.5703	-3.4497

Notes: The *ADF* regressions include an intercept, a linear trend and *k* lagged first-differences of the dependent variable, with the exception of n_t and u_t, where the linear time trend was omitted. The superscripts *A, S* and *H* indicate the choice of the Akaike Information, the Schwarz Bayesian and the Hannan-Quinn criteria respectively.

Table 4.1b – ADF(*k*) Tests Applied to the Differences of the Variables: 1972q1-2000q1

Variable	ADF(0)	ADF(1)	ADF(2)	ADF(3)	ADF(4)	95% c.v.
Δn_t	-10.995^{ASH}	-6.6485	-5.4012	-5.5016	-4.6087	-2.8870
Δn_t^*	-5.1127	-3.2701^{S}	$-2.5922^{A,H}$	-2.2484	-2.1867	-2.8870
$\Delta(w_t - p_t)$	$-15.543^{S,H}$	-9.0320	-5.4271^{A}	-5.1086	-4.4763	-2.8870
$\Delta(w_t^{**} - p_t)$	$-15.762^{S,H}$	-9.1938	-5.5000^{A}	-5.1646	-4.4908	-2.8870
Δu_t	-3.4617^{S}	-3.2203	-3.7575	$-4.5411^{A,H}$	-4.2645	-2.8870
Δk_t	-4.2806	$-3.2526^{S,H}$	-3.0245	-2.5602^{A}	-2.4583	-2.8870
Δa_t	-13.083^{ASH}	-7.6935	-7.3216	-5.7278	-5.5954	-2.8870

Notes: The *ADF* regressions do not include a linear trend.

Table 4.2 - Likelihood Ratio Tests for the Presence of Unitary Cointegrating Vectors

Variable	Likelihood Ratio Statistic			
n_t	44.09	33.37	23.18	*5.66*
$w_t - p_t$	52.44	38.42	27.00	*5.53*
$w_t^{**} - p_t$	52.52	38.46	27.03	*5.50*
n_t^*	52.33	36.00	26.01	*5.34*
u_t	16.96	16.82	15.67	*0.94*
k_t	51.41	36.89	25.31	*5.53*
a_t	52.07	36.68	25.31	*5.44*
rank(Π_y)	$r=1$	$r=2$	$r=3$	$r=4$
95% c.v.	12.59	11.07	9.49	7.81

Notes: The null hypothesis is of stationarity and it is conditional on the number of cointegrating vectors, *r*. Statistics in *italic* signify failure to reject the null at the 5% level.

Table 4.3 – Adjusted Likelihood Ratio Tests and Model Selection Criteria for the Choice of the Lag-Length

Adjusted Likelihood Ratio Tests				AIC	SBC	Order
H_0	H_1	Statistic	p-value	1592.6	1564.0	$p=0$
$p=0$	$p=8$	1903.6	.000	3095.4	2999.9	$p=1^S$
$p=1$	$p=8$	420.4	.003	3136.3	2974.0	$p=2$
$p=2$	$p=8$	334.5	.052	3134.9	2905.8	$p=3$
$p=3$	$p=8$	289.0	.028	3160.2	2864.3	$p=4$
$p=4$	$p=8$	218.0	.135	3167.8	2805.1	$p=5$
$p=5$	$p=8$	163.9	.162	3176.4	2746.8	$p=6$
$p=6$	$p=8$	108.8	.213	3183.6	2687.3	$p=7$
$p=7$	$p=8$	55.1	.255	3192.3	2629.1	$p=8^A$

Notes: Based on an unrestricted *VAR*(8) in the level of $z_t = [n_t, w_t - p_t, w_t^{**} - p_t, n_t^*, u_t, k_t, a_t]'$ with the deterministic vector $D_t = [d74q1_t, d74q2_t]'$. The superscripts *S* and *A* indicate the choice of the Schwarz Bayesian Criterion (*SBC*) and the Akaike Information Criterion (*AIC*) respectively.

Table 4.4 – Likelihood Ratio Tests for the Intercept/Trend Specification

$rank(\Pi)$	LR-Statistic	$\chi^2_{7-r,0.05}$	$\chi^2_{7-r,0.1}$
$r=0$	11.0	14.07	12.02
$r=1$	4.4	12.59	10.64
$r=2$	4.0	11.07	9.24
$r=3$	3.8	9.49	7.78
$r=4$	2.0	7.82	6.25
$r=5$	1.8	5.99	4.61
$r=6$	1.8	3.84	2.71

Notes: The unrestricted model is a symmetric *VAR*(2) in $z_t = [n_t, w_t - p_t, w_t^{**} - p_t, n_t^*, u_t, k_t, a_t]'$ with the deterministic vector $D_t = [d74q1_t, d74q2_t]'$ plus a linear trend. In the restricted model the trend coefficients are given by $\Pi\gamma$.

$rank(\Pi_y)$	LR-Statistic	$\chi^2_{5-r,0.05}$	$\chi^2_{5-r,0.1}$
$r=0$	6.8	11.07	9.24
$r=1$	3.2	9.49	7.78
$r=2$	3.2	7.82	6.25
$r=3$	0.8	5.99	4.61
$r=4$	0.2	3.84	2.71

Notes: The unrestricted model is a *VAR*(2) in $y_t = [n_t, w_t - p_t, w_t^{**} - p_t, n_t^*, u_t]'$ conditional on the weakly exogenous vector $x_t = [k_t, a_t]'$ with the deterministic vector $D_t = [d74q1_t, d74q2_t]'$ plus a linear trend. In the restricted model the trend coefficients are given by $\Pi_y\gamma$. "**" and "*" signify rejection at the 5% and 10% levels respectively.

Table 4.5a – Cointegration Rank Statistics and Small-Sample Critical Values for the Sub-Model of the Demand Side of the Labour Market

		Trace			Max		
H_0	H_1	Statistic	95% cv	90% cv	Statistic	95% cv	90% cv
$r=0$	$r=1$	55.06	28.42 a 33.88 p 33.35 n	25.63 a 30.69 p 30.00 n	41.06	21.07 a 24.59 p 24.67 n	18.78 a 22.13 p 21.92 n
$r\le 1$	$r=2$	14.00	14.35 a 17.87 p 17.69 n	12.27 a 15.43 p 15.34 n	14.00	14.35 a 17.87 p 17.69 n	12.27 a 15.43 p 15.34 n

Notes: "Trace" and "Max" stand for the modified versions of the Pesaran Shin and Smith (2000) cointegrating rank statistics. Based on a cointegrating *VAR*(2) with unrestricted intercepts and no trends in $y_t = [n_t, w_t - p_t]'$ conditional on the weakly exogenous vector $x_t = [k_t, a_t]'$. The deterministic vector is $D_t = [d74q1_t, d74q2_t]'$. Small-sample results are based on a bootstrap with 20,000 simulations. "p" and "n" indicate parametric and non-parametric respectively, while "a" indicates asymptotic critical value.

Table 4.5b – Cointegration Rank Statistics and Small-Sample Critical Values for the Sub-Model of the Supply Side of the Labour Market

		Trace			Max		
H_0	H_1	Statistic	95% cv	90% cv	Statistic	95% cv	90% cv
$r=0$	$r=1$	97.75	70.49 a 81.69 p 81.84 n	66.23 a 76.95 p 77.22 n	52.39	33.64 a 39.63 p 39.76 n	31.02 a 36.52 p 36.56 n
$r\le 1$	$r=2$	45.36	48.88 a 60.12 p 57.77 n	45.70 a 55.92 p 53.79 n	25.43	27.42 a 34.12 p 32.75 n	24.99 a 31.20 p 30.08 n
$r\le 2$	$r=3$	19.93	31.54 a 39.18 p 36.30 n	28.78 a 35.56 p 33.19 n	13.36	21.12 a 27.67 p 25.21 n	19.02 a 24.76 p 22.77 n
$r\le 3$	$r=4$	6.57	17.86 a 19.40 p 17.37 n	15.75 a 16.93 p 15.38 n	6.56	14.88 a 17.73 p 15.52 n	12.98 a 15.46 p 13.78 n
$r\le 4$	$r=5$	0.01	8.07 a 10.72 p 4.55 n	6.50 a 8.48 p 3.22 n	0.01	8.07 a 10.72 p 4.55 n	6.50 a 8.48 p 3.22 n

Notes: Based on a cointegrating *VAR*(2) with unrestricted intercepts and no trends in $z_t = [n_t, w_t - p_t, w_t^{**} - p_t, n_t^{*}, u_t]'$. The deterministic vector is $D_t = [d74q1_t, d74q2_t]'$. Small-sample results are based on a bootstrap with 20,000 simulations. "p" and "n" indicate parametric and non-parametric respectively, while "a" indicates asymptotic critical value.

Table 4.6 – Cointegration Rank Statistics and Small-Sample Critical Values for the Conditional Model

		Trace			Max		
H_0	H_1	Statistic	95% cv	90% cv	Statistic	95% cv	90% cv
$r=0$	$r=1$	142.64	92.42 a 107.41 p 117.27 n	87.93 a 101.55 p 111.52 n	58.25	39.85 a 48.79 p 50.06 n	37.15 a 45.25 p 46.53 n
$r\leq 1$	$r=2$	84.39	68.06 a 87.63 p 87.33 n	63.57 a 82.27 p 82.25 n	41.29	33.87 a 43.76 p 42.83 n	31.30 a 40.51 p 39.74 n
$r\leq 2$	$r=3$	43.10	46.44 a 62.73 p 60.70 n	42.67 a 58.43 p 56.31 n	29.71	27.75 a 37.18 p 35.07 n	25.21 a 34.07 p 32.33 n
$r\leq 3$	$r=4$	13.39	28.42 a 39.99 p 36.74 n	25.63 a 36.52 p 33.51 n	7.09	21.07 a 29.36 p 26.48 n	18.78 a 26.52 p 24.19 n
$r\leq 4$	$r=5$	6.30	14.35 a 22.59 p 16.58 n	12.27 a 19.75 p 14.80 n	6.30	14.35 a 22.59 p 16.58 n	12.27 a 19.75 p 14.80 n

Notes: Based on a cointegrating $VAR(2)$ with unrestricted intercepts and no trends in $y_t=[n_t, w_t-p_t, w_t^{**}-p_t, n_t^*, u_t]'$ conditional on the weakly exogenous vector $x_t=[k_t, a_t]'$. The deterministic vector is $D_t=[d74q1_t, d74q2_t]'$. Small-sample results are based on a bootstrap with 20,000 simulations. "p" and "n" indicate parametric and non-parametric respectively, while "a" indicates asymptotic critical value.

Table 4.7 – Asymptotic and Small-Sample Critical Values for the Likelihood Ratio Tests of Over-Identifying Restrictions in the Conditional Model

Restrictions	LL	LR-statistic	Asymptotic		Bootstrap 1		Bootstrap 2	
			95% cv	90% cv	95% cv	90% cv	95% cv	90% cv
R_E	2198.7	–						
R_{OV1}	2192.8	11.95 [7]	14.067	12.017	27.160 p 31.075 n	23.723 p 27.339 n	27.159 p 30.820 n	23.722 p 27.178 n
R_{OV2}	2192.7	12.05 [8]	15.507	13.362	27.247 p 31.103 n	23.844 p 27.390 n	27.247 p 30.841 n	23.843 p 27.196 n

Notes: R_E is any set of r^2 exactly identifying restrictions.

$$R_{OV1}:\ \beta'=\begin{bmatrix}1 & \beta_{12} & 0 & 0 & 0 & -1 & -\beta_{12}\\ \beta_{21} & 1 & -1 & -\beta_{21} & \beta_{22} & 0 & 0\end{bmatrix}$$

$$R_{OV2}:\ \beta'=\begin{bmatrix}1 & 2.2727 & 0 & 0 & 0 & -1 & -2.2727\\ \beta_{21} & 1 & -1 & -\beta_{21} & \beta_{22} & 0 & 0\end{bmatrix}$$

Based on a cointegrating $VAR(2)$ with unrestricted intercepts and no trends in $y_t=[n_t, w_t-p_t, w_t^{**}-p_t, n_t^*, u_t]'$ conditional on the weakly exogenous vector $x_t=[k_t, a_t]'$. The deterministic vector is $D_t=[d74q1_t, d74q2_t]'$. LL is the value of the maximised log-likelihood. LR stands for the Likelihood Ratio test statistic, which is asymptotically chi-squared with degrees of freedom equal to the number of over-identifying restrictions given in square brackets. Bootstrap 1 and Bootstrap 2 are defined in Chapter 1, section 1.6.4 and are based on 10,000 simulations. "p" and "n" indicate parametric and non-parametric respectively.

Table 4.8 – Estimates of the Over-Identified Cointegrating Vectors

Restrictions:	R_{OV1}		R_{OV2}	
Cointegrating Vector:	$\hat{\xi}_{1,t}$	$\hat{\xi}_{2,t}$	$\hat{\xi}_{1,t}$	$\hat{\xi}_{2,t}$
n_t	1.00 [—]	-0.01259 ** [-4.66]	1.00 [—]	-0.01274 ** [-4.77]
$w_t - p_t$	2.21 ** [11.5]	1.00 [—]	2.27 [—]	1.00 [—]
$w_t^{**} - p_t$	—	-1.00 [—]	—	-1.00 [—]
n_t^*	—	0.01259 ** [4.66]	—	0.01274 ** [4.77]
u_t	—	0.00854 ** [2.87]	—	0.00851 ** [2.88]
k_t	-1.00 [—]	—	-1.00 [—]	—
a_t	-2.21 ** [-11.5]	—	-2.27 [—]	—

Notes: $R_{OV1}: \beta' = \begin{bmatrix} 1 & \beta_{12} & 0 & 0 & 0 & -1 & -\beta_{12} \\ \beta_{21} & 1 & -1 & -\beta_{21} & \beta_{22} & 0 & 0 \end{bmatrix}$

$R_{OV2}: \beta' = \begin{bmatrix} 1 & 2.2727 & 0 & 0 & 0 & -1 & -2.2727 \\ \beta_{21} & 1 & -1 & -\beta_{21} & \beta_{22} & 0 & 0 \end{bmatrix}$

$\hat{\xi}_{1,t}$ and $\hat{\xi}_{2,t}$, correspond to the aggregate employment and wage relations respectively. *t*-ratios are given in square brackets. "*" and "**" indicate significance at the 10% and 5% levels respectively.

Table 4.9 – Error Correction Specification for the Conditional Model under R_{OV2}

Equation	Δn_t	$\Delta(w_t - p_t)$	$\Delta(w_t^{**} - p_t)$	Δn_t^{*}	Δu_t
$\hat{\xi}_{1,t-1}$	-0.029 * (-1.86)	-0.085 ** (-3.42)	-0.083 ** (-3.35)	-0.02 (-1.60)	0.12 ** (2.09)
$\hat{\xi}_{2,t-1}$	0.81 ** (3.73)	-1.23 ** (-3.47)	-1.20 ** (-3.42)	-0.54 ** (-3.27)	-0.75 (-0.90)
Δn_{t-1}	0.20 ** (1.98)	0.18 (1.09)	0.18 (1.11)	0.01 (0.17)	-1.43 ** (-3.69)
$\Delta(w_{t-1} - p_{t-1})$	-1.54 (-1.19)	3.80 * (1.81)	3.34 (1.60)	-0.66 (-0.67)	5.09 (1.03)
$\Delta(w_{t-1}^{**} - p_{t-1})$	1.65 (1.26)	-3.86 * (-1.82)	-3.40 (-1.61)	0.62 (0.63)	-5.33 (-1.06)
Δn_{t-1}^{*}	0.22 (1.63)	-0.47 ** (-2.15)	-0.47 ** (-2.15)	0.34 ** (3.32)	0.30 (0.57)
Δu_{t-1}	-0.003 (-0.18)	0.06 ** (2.20)	0.06 ** (2.20)	0.01 (0.85)	0.58 ** (8.67)
Δk_{t-1}	-0.30 (-0.36)	2.21 (1.65)	2.23 * (1.68)	-0.95 (-1.53)	-1.86 (-0.59)
Δa_{t-1}	0.097 (1.07)	-0.17 (-1.17)	-0.15 (-1.04)	-0.004 (-0.06)	-1.20 ** (-3.46)
Δk_t	2.85 ** (3.68)	-2.09 * (-1.66)	-2.07 (-1.66)	2.04 ** (3.50)	-4.91 (-1.65)
Δa_t	-0.096 (-1.13)	0.33 ** (2.42)	0.35 ** (2.53)	-0.05 (-0.80)	-0.46 (-1.42)
$d74q1_t$	-0.06 ** (-7.92)	0.07 ** (5.36)	0.07 ** (5.44)	-0.006 (-1.02)	0.04 (1.31)
$d74q2_t$	0.07 ** (7.21)	-0.04 ** (-2.54)	-0.04 ** (-2.52)	0.005 (0.62)	-0.04 (-1.07)
$\bar{R}^2$	0.67	0.47	0.47	0.50	0.77
$\hat{\sigma}$	0.007	0.01	0.01	0.006	0.03
$\chi^2_{SC}[4]$	6.58	4.60	4.32	9.57 **	4.95
$F_{SC}[4,95]$	1.47	1.01	0.94	2.20 *	1.09
$\chi^2_{FF}[1]$	10.1 **	2.02	1.83	0.028	2.14
$F_{FF}[1,98]$	9.63 **	1.78	1.62	0.024	1.89
$\chi^2_{N}[2]$	16.3 **	1.05	1.07	78.1 **	4.52
$\chi^2_{H}[1]$	0.41	0.57	0.59	0.17	2.28
$F_{H}[1,111]$	0.40	0.57	0.58	0.16	2.28

Notes: *t*-ratios are given in brackets. $\chi_i^2[]$, $i = SC, FF, N, H$, and $F_j[]$, $j = SC, FF, H$, stand for the chi-squared and F diagnostics for serial correlation (*SC*), functional form (*FF*), normality (*N*) and heteroscedasticity (*H*) with degrees of freedom in square brackets. The estimated error correction terms $\hat{\xi}_{i,t-1}$, i = 1,2, are given in Table 4.8. "*" and "**" indicate significance at the 10% and 5% levels respectively.

Table 4.10 – Asymptotic and Small-Sample Critical Values for the Serial Correlation Diagnostics in the Conditional Model

	$\chi^2_{SC}[4]$			$F_{SC}[4,95]$		
Equation	Statistic	95% c.v.	90% c.v.	Statistic	95% c.v.	90% c.v.
Δn_t	6.58	9.49 a 10.99 p 11.13 n	7.78 a 9.00 p 9.34 n	1.47	2.47 a 2.56 p 2.59 n	2.00 a 2.06 p 2.14 n
$\Delta(w_t - p_t)$	4.60	9.49 a 10.51 p 11.22 n	7.78 a 8.78 p 9.30 n	1.01	2.47 a 2.44 p 2.62 n	2.00 a 2.00 p 2.13 n
$\Delta(w_t^{**} - p_t)$	4.32	9.49 a 11.36 p 11.22 n	7.78 a 9.39 p 9.29 n	0.94	2.47 a 2.65 p 2.62 n	2.00 a 2.15 p 2.13 n
Δn_t^*	9.57	9.49 a 11.10 p 10.91 n	7.78 a 9.26 p 9.10 n	2.20	2.47 a 2.59 p 2.54 n	2.00 a 2.12 p 2.08 n
Δu_t	4.95	9.49 a 11.19 p 11.05 n	7.78 a 9.16 p 9.12 n	1.09	2.47 a 2.61 p 2.58 n	2.00 a 2.09 p 2.09 n

Notes: $\chi^2_{SC}[]$, and $F_{SC}[]$, stand for the chi-squared and F diagnostics for Serial Correlation with degrees of freedom in square brackets. Based on a cointegrating *VAR*(2) with unrestricted intercepts and no trends in $y_t = [n_t, w_t - p_t, w_t^{**} - p_t, n_t^*, u_t]'$ conditional on the weakly exogenous vector $x_t = [k_t, a_t]'$. The deterministic vector is $D_t = [d74q1_t, d74q2_t]'$. Small-sample results are based on a bootstrap with 10,000 simulations. "p" and "n" indicate parametric and non-parametric respectively, while "a" indicates asymptotic critical value.

Table 5.1 – Cointegration Rank Tests and Small-Sample Critical Values for the Marginal Model

		Trace			Max		
H_0	H_1	Statistic	95% cv	90% cv	Statistic	95% cv	90% cv
$r=0$	$r=1$	64.41	68.77 p 68.21 n	63.57 p 62.91 n	34.39	40.56 p 41.01 n	36.98 p 36.81 n
$r\leq 1$	$r=2$	30.02	44.03 p 46.51 n	39.98 p 41.68 n	19.19	30.94 p 32.56 n	27.92 p 29.15 n
$r\leq 2$	$r=3$	10.83	23.48 p 23.82 n	20.55 p 20.57 n	10.28	20.55 p 20.99 n	18.13 p 18.07 n
$r\leq 3$	$r=4$	0.55	8.85 p 8.52 n	6.68 p 6.52 n	0.55	8.85 p 8.52 n	6.68 p 6.52 n

Notes: Based on a cointegrating $VAR(2)$ in $x_t=[e_t-p_t+p_t^*,y_t^*,k_t,a_t]'$ with an intercept, the dummy vectors $D_{1,t}=[pre73_t,ERM_t]'$ and $D_{2,t}=[d71q1_t,d71q2_t,d74q1_t,d74q2_t,d90q3_t]'$ plus a linear trend. Small-sample results are based on a bootstrap with 10,000 simulations. "p" and "n" indicate parametric and non-parametric, respectively.

Table 5.2 – Likelihood Ratio Statistics and Small-Sample Critical Values for the Restrictions on the Linear Trends

$rank(\Pi_y)$	LR-Statistic	95% c.v.	90% c.v.
$r=0$	29.02	82.00 p 84.07 n	77.43 p 79.03 n
$r=1$	17.69	25.72 p 25.99 n	22.24 p 22.52 n
$r=2$	16.40	24.65 p 24.32 n	21.08 p 21.02 n
$r=3$	14.26	22.79 p 22.20 n	19.49 p 19.24 n
$r=4$	14.26	21.16 p 20.15 n	17.97 p 17.12 n
$r=5$	14.24	18.59 p 17.67 n	15.67 p 14.96 n
$r=6$	14.15	18.02 p 15.94 n	15.05 p 13.12 n
$r=7$	10.04	16.65 p 14.73 n	13.61 p 11.97 n
$r=8$	9.31	13.68 p 11.71 n	10.67 p 9.11 n
$r=9$	6.88	9.32 p 9.81 n	7.10 p 7.17 n

Notes: The unrestricted model is a $VAR(2)$ in $z_t=[m_t-p_t,y_t,R_t,R_t^*,LI_t,n_t,w_t-p_t,w_t^{**}-p_t,n_t^*,u_t]'$ conditional on $x_t=[e_t-p_t+p_t^*,y_t^*,k_t,a_t]'$ with an intercept, the dummy vectors $D_{1,t}=[pre73_t,ERM_t]'$ and $D_{2,t}=[d71q1_t,d71q2_t,d74q1_t,d74q2_t,d90q3_t]'$ plus a linear trend. In the restricted model the trend coefficients are given by $\Pi_y\gamma$. The small-sample critical values are based on a bootstrap with 10,000 simulations. "p" and "n" indicate parametric and non-parametric, respectively.

Table 5.3 – Cointegration Rank Tests and Small-Sample Critical Values for the Conditional Model

		Trace			Max		
H_0	H_1	**Statistic**	**95% cv**	**90% cv**	**Statistic**	**95% cv**	**90% cv**
$r=0$	$r=1$	577.50	510.03 p 522.00 n	497.06 p 507.74 n	115.15	116.96 p 118.74 n	112.20 p 113.08 n
$r\leq 1$	$r=2$	462.35	449.27 p 448.51 n	437.96 p 436.66 n	102.45	111.17 p 109.44 n	106.08 p 104.62 n
$r\leq 2$	$r=3$	359.90	392.16 p 386.31 n	380.48 p 374.95 n	80.65	104.47 p 101.44 n	99.33 p 96.93 n
$r\leq 3$	$r=4$	279.25	325.68 p 320.13 n	315.53 p 310.38 n	70.01	95.04 p 92.84 n	90.60 p 88.45 n
$r\leq 4$	$r=5$	209.24	276.05 p 258.91 n	266.95 p 249.64 n	50.19	87.20 p 82.86 n	83.21 p 79.00 n
$r\leq 5$	$r=6$	159.04	217.35 p 203.29 n	208.95 p 195.29 n	47.70	75.58 p 72.70 n	72.06 p 69.40 n
$r\leq 6$	$r=7$	111.34	169.98 p 153.33 n	163.04 p 146.67 n	37.26	69.46 p 64.47 n	65.77 p 61.13 n
$r\leq 7$	$r=8$	74.09	116.19 p 111.18 n	110.75 p 105.39 n	32.29	56.22 p 54.77 n	53.07 p 51.72 n
$r\leq 8$	$r=9$	41.80	69.27 p 71.63 n	65.05 p 67.44 n	23.86	43.05 p 45.77 n	40.49 p 42.99 n
$r\leq 9$	$r=10$	17.94	29.78 p 34.58 n	27.43 p 31.96 n	17.94	29.78 p 34.58 n	27.43 p 31.96 n

Notes: Based on a cointegrating *VAR*(2) in $z_t = [m_t - p_t, y_t, R_t, R_t^*, LI_t, n_t, w_t - p_t, w_t^{**} - p_t, n_t^*, u_t]'$ conditional on $x_t = [e_t - p_t + p_t^*, y_t^*, k_t, a_t]'$ with an intercept, the dummy vectors $D_{1,t} = [pre73_t, ERM_t]'$ and $D_{2,t} = [d71q1_t, d71q2_t, d74q1_t, d74q2_t, d90q3_t]'$ plus a linear trend. The coefficients on the linear trend and the intervention dummies $D_{1,t}$ are restricted to $\Pi_y\gamma$ and $\Pi_y\delta$, respectively. Small-sample results are based on a bootstrap with 10,000 simulations. "p" and "n" indicate parametric and non-parametric, respectively.

Table 5.4 – Small-Sample Critical Values for the Test of Over-Identifying Restrictions

Restrictions	*LL*	*LR*-statistic	Asymptotic		**Bootstrap 1**	
			95% cv	90% cv	95% cv	90% cv
R_E	5852.4	–				
R_{OV}	5752.5	199.7 [42]	58.12	54.09	161.12 p 197.91 n	152.29 p 188.52 n

Notes: R_E is a set of r^2 exactly identifying restrictions, R_{OV} is the set of r^2+42 over-identifying restrictions given by (5.10) in section 5.3.4, *LL* is the value of the maximised log-likelihood, *LR* stands for the Likelihood Ratio test statistic, which is asymptotically chi-squared with degrees of freedom equal to the number of over-identifying restrictions given in square brackets. Bootstrap 1 is defined in Chapter 1, section 1.6.4 and is based on 10,000 simulations. "p" and "n" indicate parametric and non-parametric respectively.

Table 5.5 – Estimates of the Over-Identified Cointegrating Vectors in the Complete AD-AS Model

Variables	$\hat{\zeta}_{1,t}$	$\hat{\zeta}_{2,t}$	$\hat{\zeta}_{3,t}$	$\hat{\zeta}_{4,t}$	$\hat{\zeta}_{5,t}$
$m_t - p_t$	1.00 [—]	—	—	—	—
y_t	-2.23 [-0.75]	1.00 [—]	-0.35 [-0.42]	—	—
R_t	33.52 * [1.77]	—	1.00 [—]	—	—
R_t^*	—	—	-1.00 [—]	—	—
LI_t	—	17.66 ** [4.74]	—	—	—
n_t	—	—	—	1.00 [—]	-0.012 ** [-3.08]
$w_t - p_t$	—	—	—	2.27 [—]	1.00 [—]
$w_t^{**} - p_t$	—	—	—	—	-1.00 [—]
n_t^*	—	—	—	—	0.012 ** [3.08]
u_t	—	—	—	—	0.007 ** [2.73]
$e_t - p_t + p_t^*$	—	-0.61 [-1.43]	0.62 [0.87]	—	—
y_t^*	—	-0.70 ** [-3.40]	0.31 ** [2.02]	—	—
k_t	—	—	—	-1.00 [—]	—
a_t	—	—	—	-2.27 [—]	—
t	0.012 [0.72]	—	0.001 [0.26]	—	—
$pre\,73_t$	-0.035 [-0.19]	0.015 [0.16]	0.14 ** [2.33]	—	—
ERM_t	0.073 [0.43]	0.019 [0.36]	-0.10 ** [-2.05]	—	—

Notes: $\hat{\zeta}_{1,t}$, $\hat{\zeta}_{2,t}$, $\hat{\zeta}_{3,t}$ correspond to the LM, IS and BP relations respectively, while $\hat{\zeta}_{4,t}$ and $\hat{\zeta}_{5,t}$ are the temporary deviations from the equilibrium conditions derived for the firms' employment setting and the unions' wage setting decisions respectively. t-ratios are given in square brackets. "*" and "**" indicate significance at the 10% and 5% levels respectively.

Table 5.6a – Error Correction Specification for the Complete AD-AS Model (Equations 1-5)

Equation	$\Delta(m_t - p_t)$	Δy_t	ΔR_t	ΔR_t^*	ΔLI_t
$\hat{\zeta}_{1,t-1}$	-0.04 ** (-3.81)	-0.01 ** (-2.77)	-.005 ** (-2.50)	.0017 * (1.71)	.0018 * (1.69)
$\hat{\zeta}_{2,t-1}$	0.001 (0.03)	0.03 ** (2.16)	0.01 * (1.78)	-0.001 (-0.23)	-.009 ** (-2.50)
$\hat{\zeta}_{3,t-1}$	-0.07 ** (-2.09)	0.03 ** (2.13)	.002 (0.32)	.0007 (0.20)	-0.003 (-0.85)
$\hat{\zeta}_{4,t-1}$	-0.01 (-0.55)	-0.01 (-0.92)	-.001 (-0.22)	.0007 (0.28)	0.003 (1.22)
$\hat{\zeta}_{5,t-1}$	-0.49 (-1.24)	0.53 ** (3.28)	-0.08 (-0.90)	-0.05 (-1.17)	-0.005 (-0.10)
$\Delta(m_{t-1} - p_{t-1})$	-0.05 (-0.59)	-.002 (-0.04)	0.017 (0.90)	.016 * (1.69)	0.006 (0.65)
Δy_{t-1}	-0.55 * (-1.72)	-0.40 ** (-3.06)	-0.005 (-0.07)	-0.016 (-0.45)	-0.02 (-0.67)
ΔR_{t-1}	0.05 (0.10)	0.13 (0.65)	0.13 (1.17)	-0.072 (-1.37)	0.10 * (1.93)
ΔR_{t-1}^*	-1.67 ** (-1.93)	0.11 (0.32)	0.42 ** (2.21)	0.25 ** (2.69)	0.19 * (1.93)
ΔLI_{t-1}	-0.36 (-0.37)	0.32 (0.80)	-0.13 (-0.60)	0.06 (0.56)	0.01 (0.11)
Δn_{t-1}	0.41 * (1.72)	0.29 ** (2.96)	0.04 (0.73)	0.002 (0.06)	0.04 (1.59)
$\Delta(w_{t-1} - p_{t-1})$	2.26 (1.16)	-1.59 ** (-2.00)	-0.08 (-0.19)	0.16 (0.75)	0.29 (1.34)
$\Delta(w_{t-1}^{**} - p_{t-1})$	-2.19 (-1.11)	1.69 ** (2.10)	0.11 (0.25)	-0.16 (-0.77)	-0.28 (-1.30)
Δn_{t-1}^*	0.08 (0.40)	0.05 (0.59)	0.02 (0.53)	-0.027 (-1.19)	0.01 (0.55)
Δu_{t-1}	-0.04 * (-1.87)	.004 (0.44)	-.0004 (-0.07)	-.007 ** (-2.86)	0.02 (0.80)
ΔRER_{t-1}	0.03 (0.65)	0.02 (0.90)	-0.001 (-0.14)	-0.005 (-1.12)	-.01 ** (-2.04)
Δy_{t-1}^*	0.45 * (1.81)	0.06 (0.63)	-0.07 (-1.30)	0.06 ** (2.33)	0.01 (0.34)
Δk_{t-1}	0.91 (0.73)	0.47 (0.92)	-0.15 (-0.53)	-0.04 (-0.29)	-0.20 (-1.43)
Δa_{t-1}	0.55 * (1.64)	0.27 ** (1.97)	-0.06 (-0.78)	0.016 (0.43)	0.02 (0.50)
ΔRER_t	-0.02 (-0.65)	0.01 (0.73)	0.03 ** (4.11)	0.003 (0.88)	0.02 ** (5.17)
Δy_t^*	0.34 (1.53)	0.44 ** (4.83)	0.11 ** (2.32)	0.06 ** (2.49)	0.05 * (1.83)
Δk_t	-1.39 (-1.19)	0.94 ** (1.96)	0.25 (0.97)	-0.11 (-0.87)	0.02 (0.01)
Δa_t	0.25 ** (1.98)	0.83 ** (16.4)	-0.04 (-1.37)	-0.011 (-0.80)	-0.04 ** (-2.54)
$d71q1_t$	-0.02 (-1.29)	-0.03 ** (-4.78)	0.002 (0.72)	-.0003 (-0.21)	.003 ** (1.99)
$d71q2_t$	-0.02 (-1.45)	0.04 ** (5.70)	0.002 (0.52)	-.0002 (-0.14)	0.003 * (1.93)
$d74q1_t$	0.04 ** (3.34)	-0.01 ** (-2.05)	-.0003 (-0.11)	-.002 * (-1.93)	-.0001 (-0.07)
$d74q2_t$	-0.05 ** (-3.30)	0.001 (0.25)	-.001 (-0.45)	-.003 * (-1.76)	.001 (0.77)

Table 5.6a (continued) – Error Correction Specification for the Complete AD-AS Model (Equations 1-5)

$d90q3_t$	-.003 (-0.24)	-0.01 ** (-2.63)	0.001 (0.57)	-.0004 (-0.32)	.0006 (0.48)
$\bar{R}^2$	0.528	0.829	0.238	0.332	0.353
$\hat{\sigma}$	0.010	0.004	0.002	0.001	0.001
$\chi^2_{SC}[4]$ $F_{SC}[4,99]$	8.89 1.79	11.2 ** 2.30 *	11.8 ** 2.42 *	11.2 ** 2.29 *	15.4 ** 3.26 **
$\chi^2_{FF}[1]$ $F_{FF}[1,102]$	0.08 0.06	12.5 ** 10.6 **	1.16 0.91	0.01 0.01	5.14 ** 4.13 **
$\chi^2_N[2]$	0.43	15.2 **	16.5 **	9.43 **	0.51
$\chi^2_H[1]$ $F_H[1,130]$	0.27 0.26	4.09 ** 4.04 **	1.36 1.34	7.12 ** 7.02 **	1.44 1.42

Notes: *t*-ratios are given in brackets. $\chi^2_i[]$, $i = SC, FF, N, H$, and $F_j[]$, $j = SC, FF, H$, stand for the chi-squared and F diagnostics for serial correlation (*SC*), functional form (*FF*), normality (*N*) and heteroscedasticity (*H*) with degrees of freedom in square brackets. The estimated error correction terms $\hat{\zeta}_{i,t-1}$, i = 1,2,3,4,5 are given in Table 5.5. RER_t stands for the Real (effective) Exchange Rate, $e_t - p_t + p_t^*$. "*" and "**" indicate significance at the 10% and 5% levels respectively.

Table 5.6b – Error Correction Specification for the Complete AD-AS Model (Equations 6-10)

Equation	Δn_t	$\Delta(w_t - p_t)$	$\Delta(w_t^{**} - p_t)$	Δn_t^{*}	Δu_t
$\hat{\zeta}_{1,t-1}$	-0.01 * (-1.66)	0.006 (0.56)	.007 (0.62)	0.002 (0.48)	-0.006 (-0.19)
$\hat{\zeta}_{2,t-1}$	0.02 (0.91)	0.03 (0.69)	0.02 (0.60)	0.03 ** (2.51)	0.03 (0.28)
$\hat{\zeta}_{3,t-1}$	0.04 ** (2.03)	-0.03 (-0.80)	-0.03 (-0.85)	0.07 ** (5.56)	-0.19 * (-1.91)
$\hat{\zeta}_{4,t-1}$	-.0003 (-0.02)	-0.08 ** (-2.74)	-0.07 ** (-2.62)	-0.02 ** (-2.26)	0.06 (0.73)
$\hat{\zeta}_{5,t-1}$	1.03 ** (4.41)	-1.38 ** (-2.96)	-1.34 ** (-2.89)	-0.38 ** (-2.37)	-3.2 ** (-2.48)
$\Delta(m_{t-1} - p_{t-1})$	0.006 (0.12)	0.16 (1.52)	0.16 (1.54)	0.03 (0.71)	-0.52 * (-1.85)
Δy_{t-1}	-0.03 (-0.13)	-0.59 (-1.56)	-0.57 (-1.51)	0.23 * (1.80)	-0.53 (-0.51)
ΔR_{t-1}	0.30 (1.06)	-0.62 (-1.09)	-0.63 (-1.12)	-0.27 (-1.40)	-1.89 (-1.21)
ΔR_{t-1}^{*}	0.54 (1.07)	-1.21 (-1.19)	-1.16 (-1.15)	-0.26 (-0.76)	-5.16 * (-1.85)
ΔLI_{t-1}	0.20 (0.34)	0.28 (0.24)	0.24 (0.21)	0.34 (0.87)	2.14 (0.68)
Δn_{t-1}	0.26 * (1.84)	0.50 * (1.77)	0.49 * (1.76)	0.01 (0.10)	-0.95 (-1.23)
$\Delta(w_{t-1} - p_{t-1})$	-2.54 ** (-2.24)	6.38 ** (2.80)	5.91 ** (2.60)	-1.66 ** (-2.12)	9.52 (1.51)
$\Delta(w_{t-1}^{**} - p_{t-1})$	2.65 ** (2.31)	-6.54 ** (-2.83)	-6.07 ** (-2.64)	1.70 ** (2.14)	-9.35 (-1.47)
Δn_{t-1}^{*}	0.04 (0.31)	0.005 (0.02)	0.004 (0.02)	0.22 ** (2.66)	0.16 (0.24)
Δu_{t-1}	0.006 (0.49)	0.003 (0.12)	0.003 (0.11)	0.01 (1.18)	0.48 ** (6.68)
ΔRER_{t-1}	0.01 (0.41)	0.03 (0.58)	0.03 (0.55)	0.01 (0.57)	0.10 (0.76)
Δy_{t-1}^{*}	0.17 (1.19)	-0.22 (-0.76)	-0.22 (-0.77)	-0.10 (-1.05)	0.30 (0.37)
Δk_{t-1}	-0.19 (-0.26)	0.40 (0.28)	0.45 (0.31)	-1.13 ** (-2.25)	2.06 (0.51)
Δa_{t-1}	-0.06 (-0.31)	0.51 (1.31)	0.51 (1.31)	-0.38 ** (-2.81)	0.02 (0.02)
ΔRER_t	-0.04 (-0.18)	0.06 (1.59)	0.06 (1.55)	0.008 (0.58)	-0.08 (-0.77)
Δy_t^{*}	0.49 ** (3.74)	-0.10 (-0.40)	-0.10 (-0.39)	0.44 ** (4.95)	-0.92 (-1.27)
Δk_t	1.62 ** (2.36)	-1.89 (-1.37)	-1.86 (-1.35)	0.88 * (1.85)	-0.79 (-0.21)
Δa_t	-0.26 ** (-3.55)	0.29 ** (2.02)	0.30 ** (2.09)	-0.20 ** (-4.10)	0.16 (0.41)
$d71q1_t$	-0.06 ** (-7.44)	0.07 ** (4.69)	0.07 ** (4.74)	0.002 (0.30)	0.03 (0.63)
$d71q2_t$	0.07 ** (8.19)	-0.02 (-1.33)	-0.02 (-1.31)	0.004 (0.65)	-0.07 (-1.46)
$d74q1_t$	-0.01 * (-1.70)	-0.01 (-0.95)	-0.01 (-0.91)	-0.01 ** (-2.60)	-.24 ** (-6.39)
$d74q2_t$	-0.002 (-0.20)	-0.006 (-0.38)	-.006 (-0.36)	0.0006 (0.11)	0.22 ** (4.95)

Table 5.6b (continued) – Error Correction Specification for the Complete AD-AS Model (Equations 6-10)

$d90q3_t$	-0.02 ** (-3.58)	0.001 (0.11)	0.002 (0.12)	-0.03 ** (-6.48)	0.03 (0.86)
$\bar{R}^2$	0.738	0.428	0.427	0.715	0.723
$\hat{\sigma}$	0.006	0.012	0.012	0.004	0.034
$\chi^2_{SC}[4]$	10.5 **	11.1 **	10.3 **	3.92	9.99 **
$F_{SC}[4,99]$	2.15 *	2.28 *	2.32 *	0.76	2.03 *
$\chi^2_{FF}[1]$	9.37 **	0.30	0.29	0.99	0.09
$F_{FF}[1,102]$	7.79 **	0.23	0.23	0.77	0.07
$\chi^2_N[2]$	31.7 **	0.13	0.16	1.13	73.0 **
$\chi^2_H[1]$	0.29	0.26	0.26	0.04	1.07
$F_H[1,130]$	0.28	0.25	0.26	0.04	1.06

Notes: *t*-ratios are given in brackets. $\chi^2_i[\,]$, $i = SC, FF, N, H$, and $F_j[\,]$, $j = SC, FF, H$, stand for the chi-squared and F diagnostics for serial correlation (*SC*), functional form (*FF*), normality (*N*) and heteroscedasticity (*H*) with degrees of freedom in square brackets. The estimated error correction terms $\hat{\zeta}_{i,t-1}$, i = 1,2,3,4,5 are given in Table 5.5. RER_t stands for the Real (effective) Exchange Rate, $e_t - p_t + p_t^*$. "*" and "**" indicate significance at the 10% and 5% levels respectively.

Table 5.7 – Asymptotic and Small-Sample Critical Values for the Serial Correlation Diagnostics

	$\chi^2_{SC}[4]$			$F_{SC}[4,99]$		
Equation	Statistic	95% c.v.	90% c.v.	Statistic	95% c.v.	90% c.v.
$\Delta(m_t - p_t)$	8.89	9.49 a 14.73 p 14.44 n	7.78 a 12.37 p 12.01 n	1.79	2.46 a 3.11 p 3.04 n	2.00 a 2.56 p 2.48 n
Δy_t	11.2	9.49 a 13.42 p 14.59 n	7.78 a 11.03 p 12.22 n	2.30	2.46 a 2.80 p 3.07 n	2.00 a 2.56 p 2.52 n
ΔR_t	11.8	9.49 a 13.46 p 13.54 n	7.78 a 11.27 p 11.14 n	2.42	2.46 a 2.81 p 2.83 n	2.00 a 2.31 p 2.28 n
ΔR_t^*	11.2	9.49 a 13.72 p 13.96 n	7.78 a 11.39 p 11.71 n	2.29	2.46 a 2.87 p 2.93 n	2.00 a 2.34 p 2.41 n
ΔLI_t	15.4	9.49 a 13.14 p 13.73 n	7.78 a 10.79 p 11.51 n	3.26	2.46 a 2.74 p 2.87 n	2.00 a 2.20 p 2.36 n
Δn_t	10.5	9.49 a 13.48 p 14.30 n	7.78 a 11.08 p 11.88 n	2.15	2.46 a 2.82 p 3.01 n	2.00 a 2.27 p 2.45 n
$\Delta(w_t - p_t)$	11.1	9.49 a 12.82 p 13.85 n	7.78 a 10.64 p 11.60 n	2.28	2.46 a 2.66 p 2.90 n	2.00 a 2.17 p 2.38 n
$\Delta(w_t^{**} - p_t)$	10.3	9.49 a 14.10 p 13.92 n	7.78 a 11.77 p 11.59 n	2.32	2.46 a 2.96 p 2.92 n	2.00 a 2.42 p 2.38 n
Δn_t^*	3.92	9.49 a 13.55 p 14.35 n	7.78 a 11.40 p 11.88 n	0.76	2.46 a 2.83 p 3.02 n	2.00 a 2.34 p 2.45 n
Δu_t	9.99	9.49 a 13.41 p 13.91 n	7.78 a 10.92 p 11.43 n	2.03	2.46 a 2.80 p 2.91 n	2.00 a 2.23 p 2.35 n

Notes: $\chi^2_{SC}[]$, and $F_{SC}[]$, stand for the chi-squared and F diagnostics for Serial Correlation for the VECM in Table 5.6. The degrees of freedom are given in square brackets. Small-sample results are based on a bootstrap with 10,000 simulations. "p" and "n" indicate parametric and non-parametric respectively, while "a" indicates asymptotic critical value.

List of Figures

Figure 2.1: Time Plots of the Variables: 1963q1-1998q2

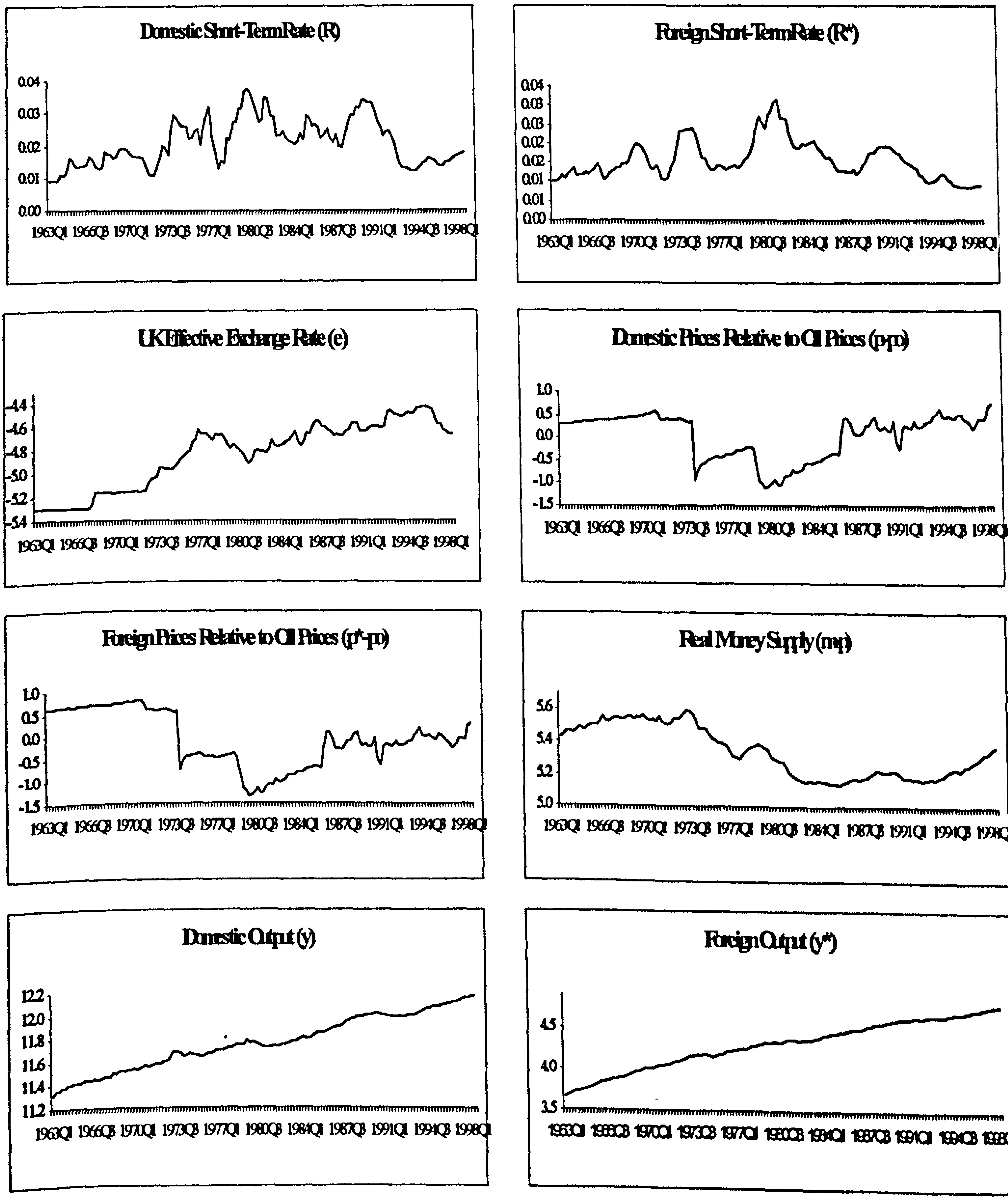


Figure 2.1 (continued): Time Plots of the Variables

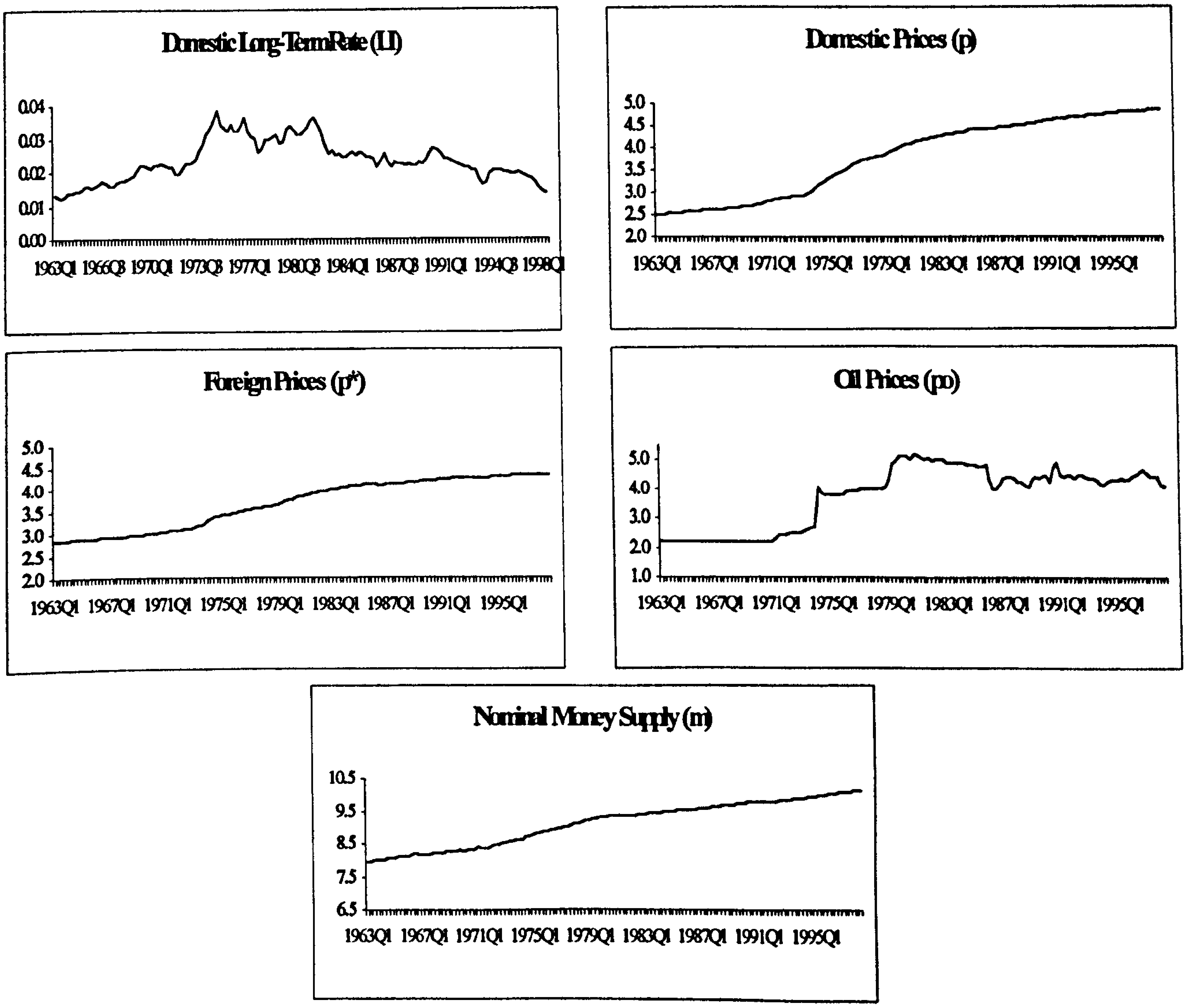


Figure 2.2: Small-Sample Distribution for the *LR* Test of Over-Identification of the Cointegrating Matrix. Based on Bootstrap 1 with 10,000 Simulations.

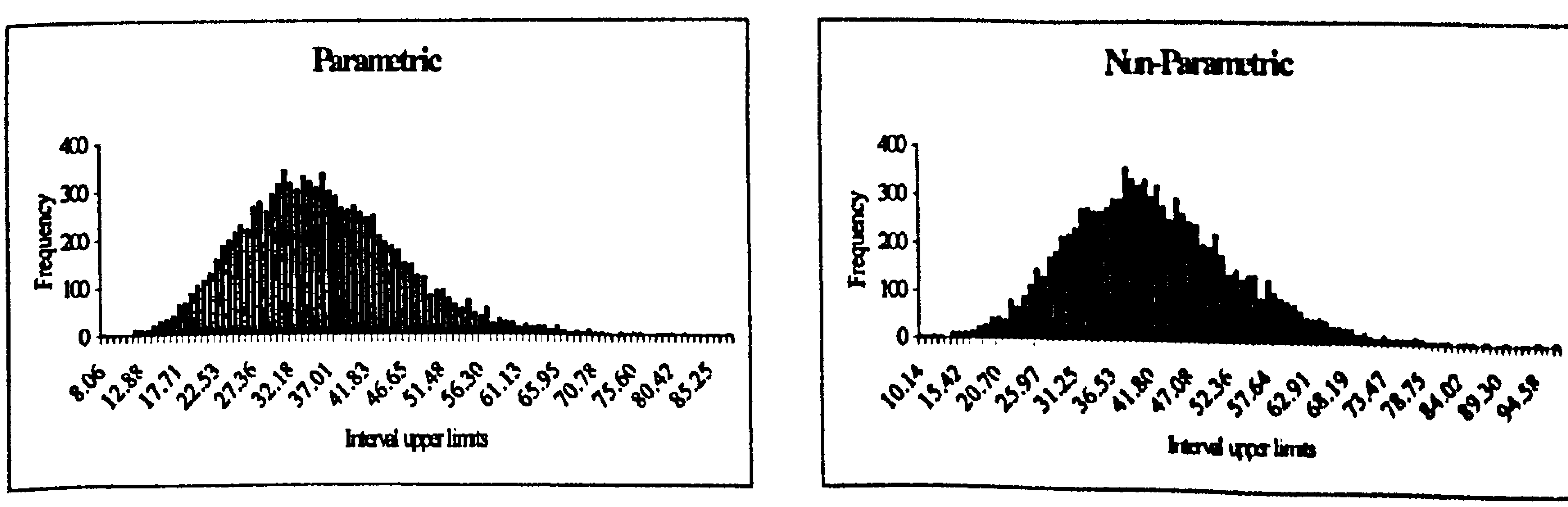


Figure 2.3: Small-Sample Distribution of the *max-eigenvalue* Cointegration Rank Statistic. Based on a Parametric Bootstrap with 10,000 Simulations.

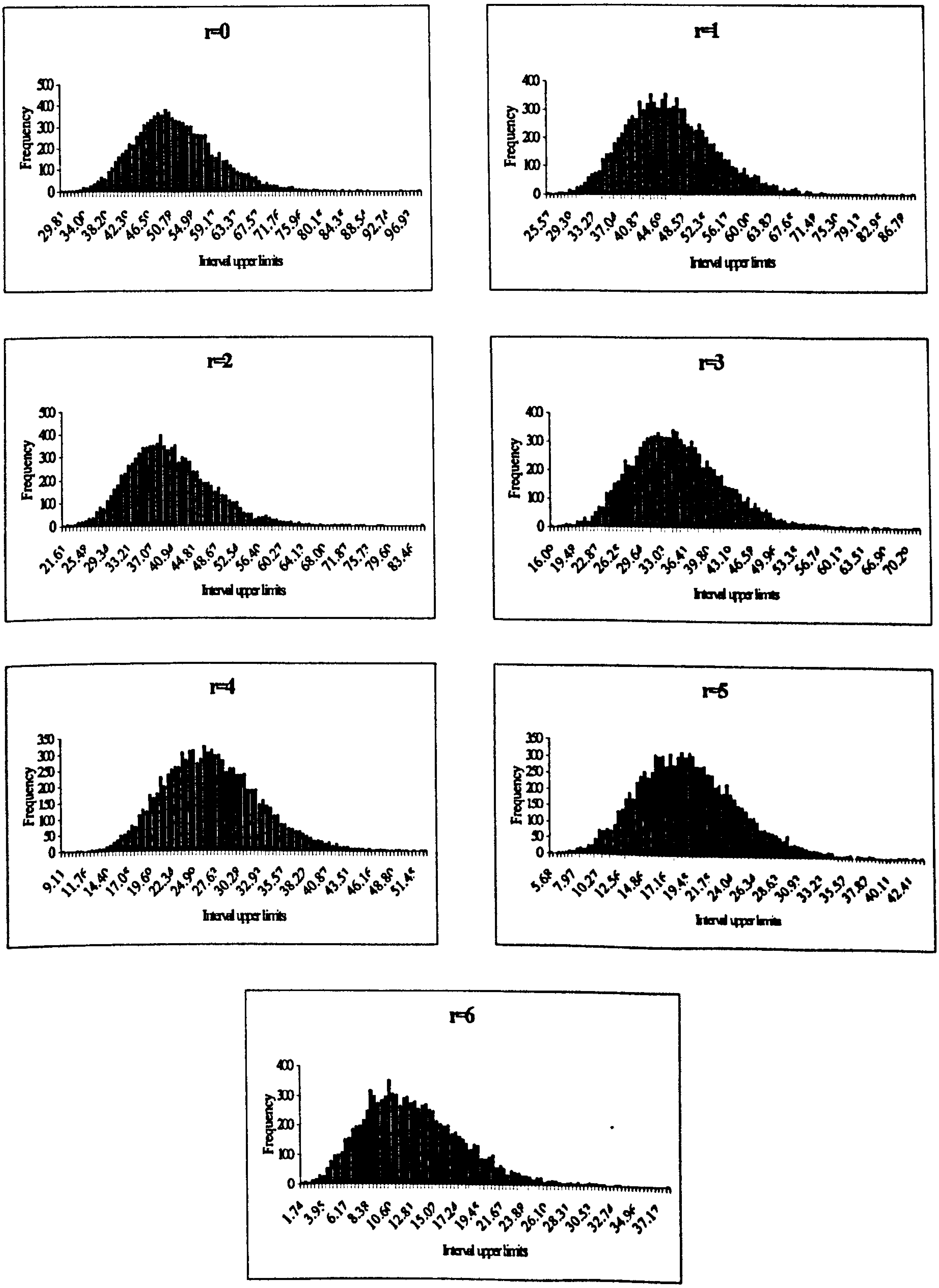


Figure 2.4: Small-Sample Distribution of the λ-*trace* Cointegration Rank Statistic. Based on a Parametric Bootstrap with 10,000 Simulations.

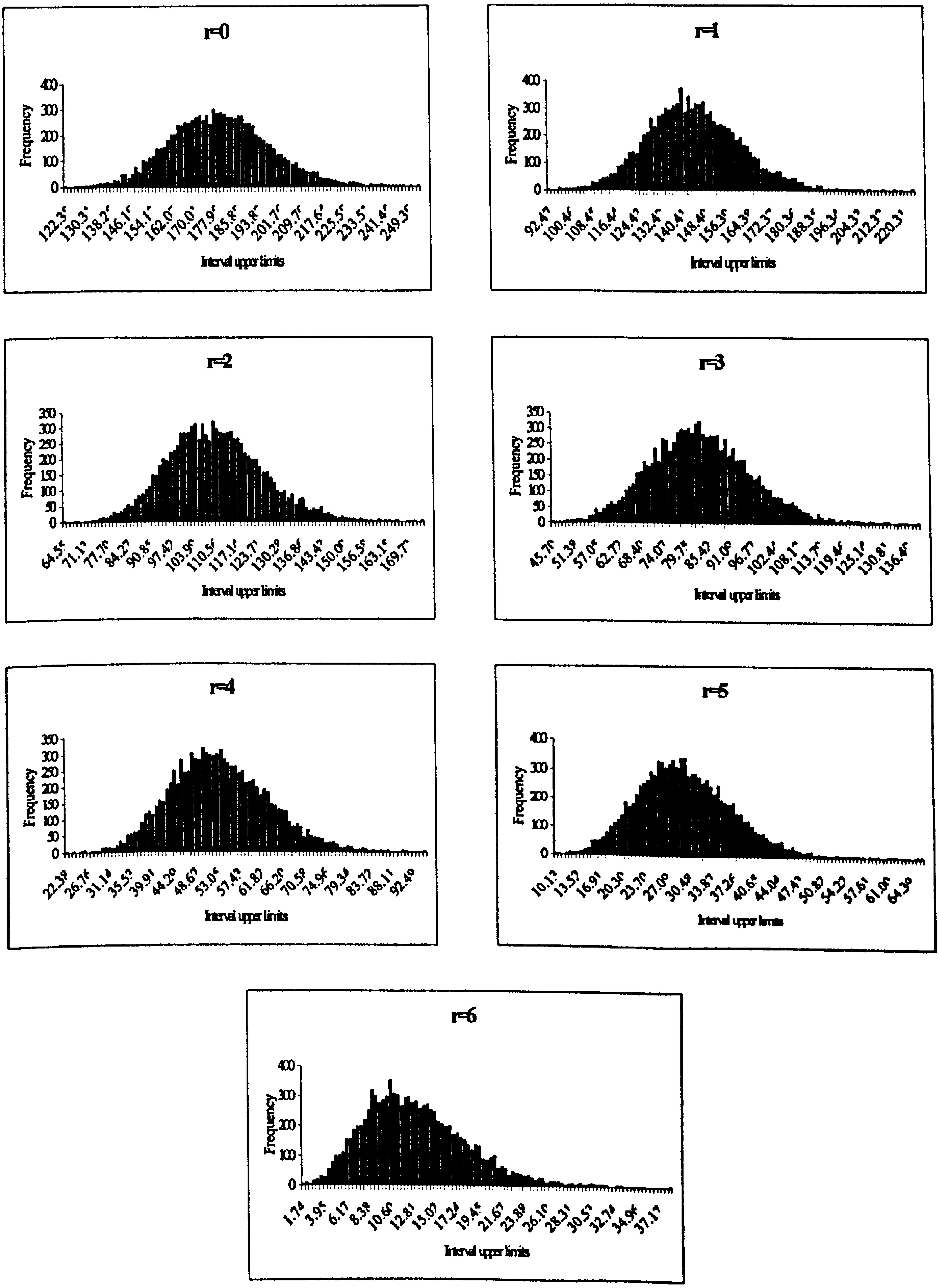


Figure 2.5: Small-Sample Distribution of the *max-eigenvalue* Cointegration Rank Statistic. Based on a Non-Parametric Bootstrap with 10,000 Simulations.

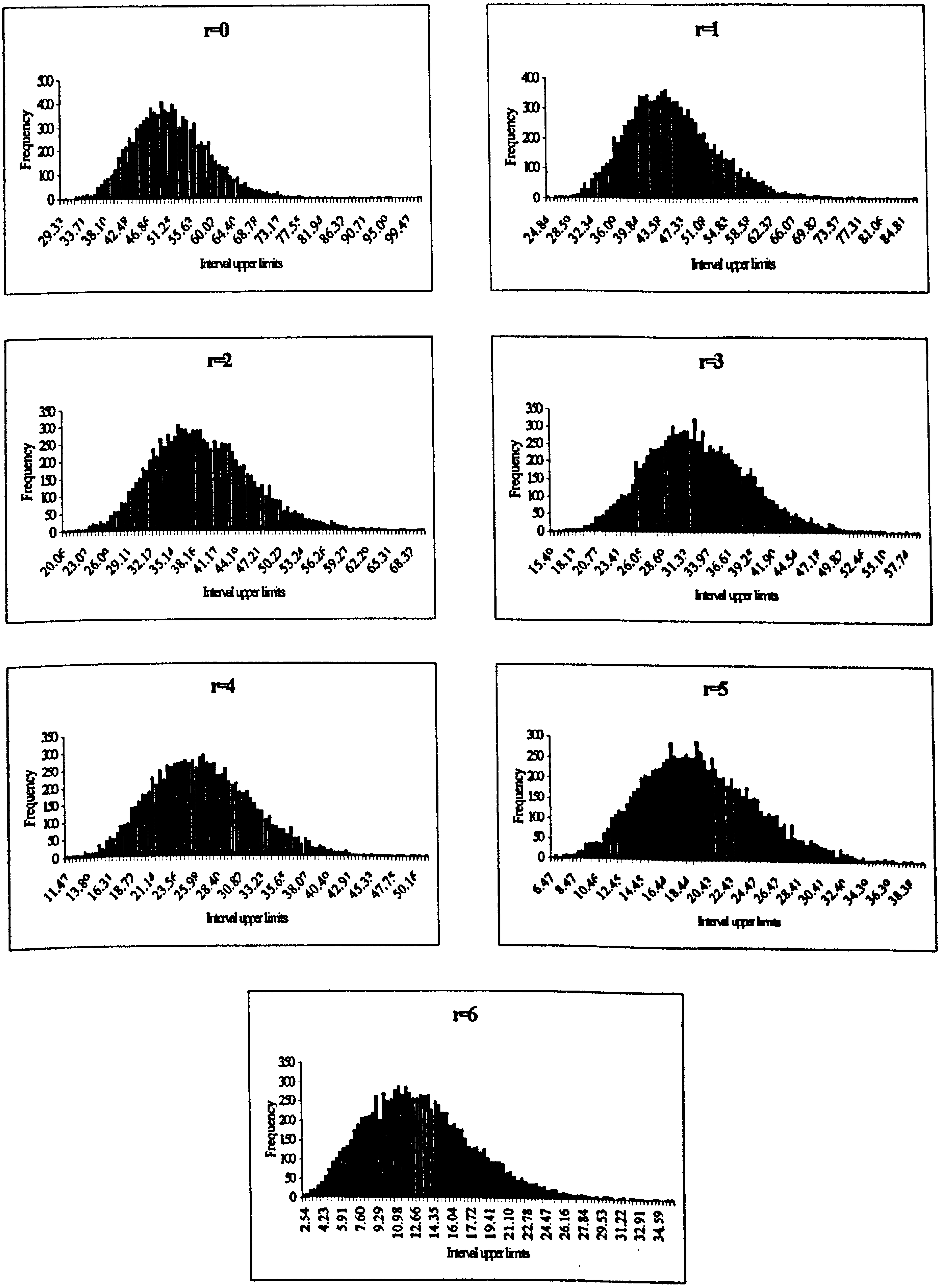


Figure 2.6: Small-Sample Distribution of the λ-*trace* Cointegration Rank Statistic. Based on a Non-Parametric Bootstrap with 10,000 Simulations.

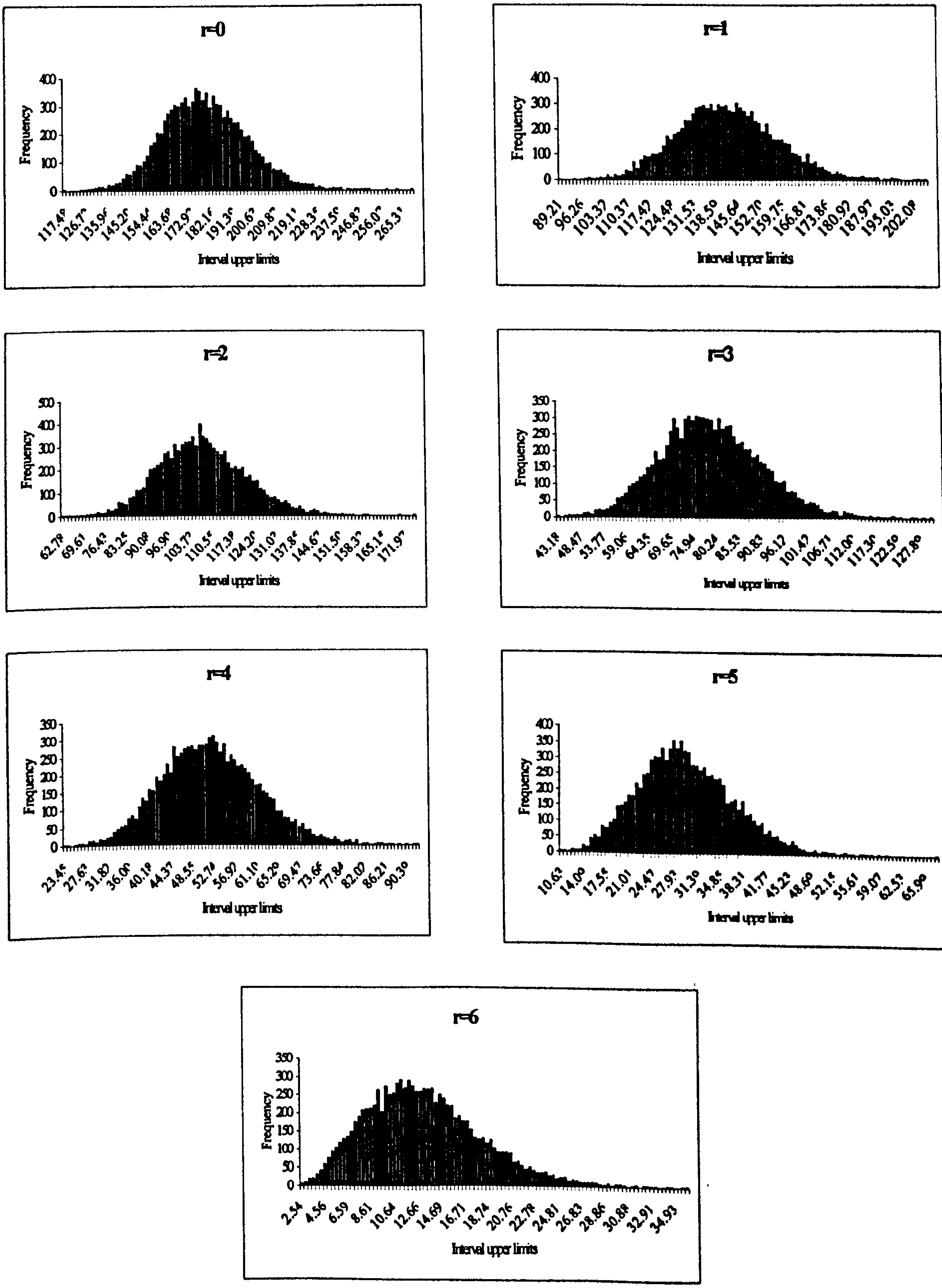


Figure 2.7: Persistence Profiles for the Estimated LM, IS, BP and PPP Relations

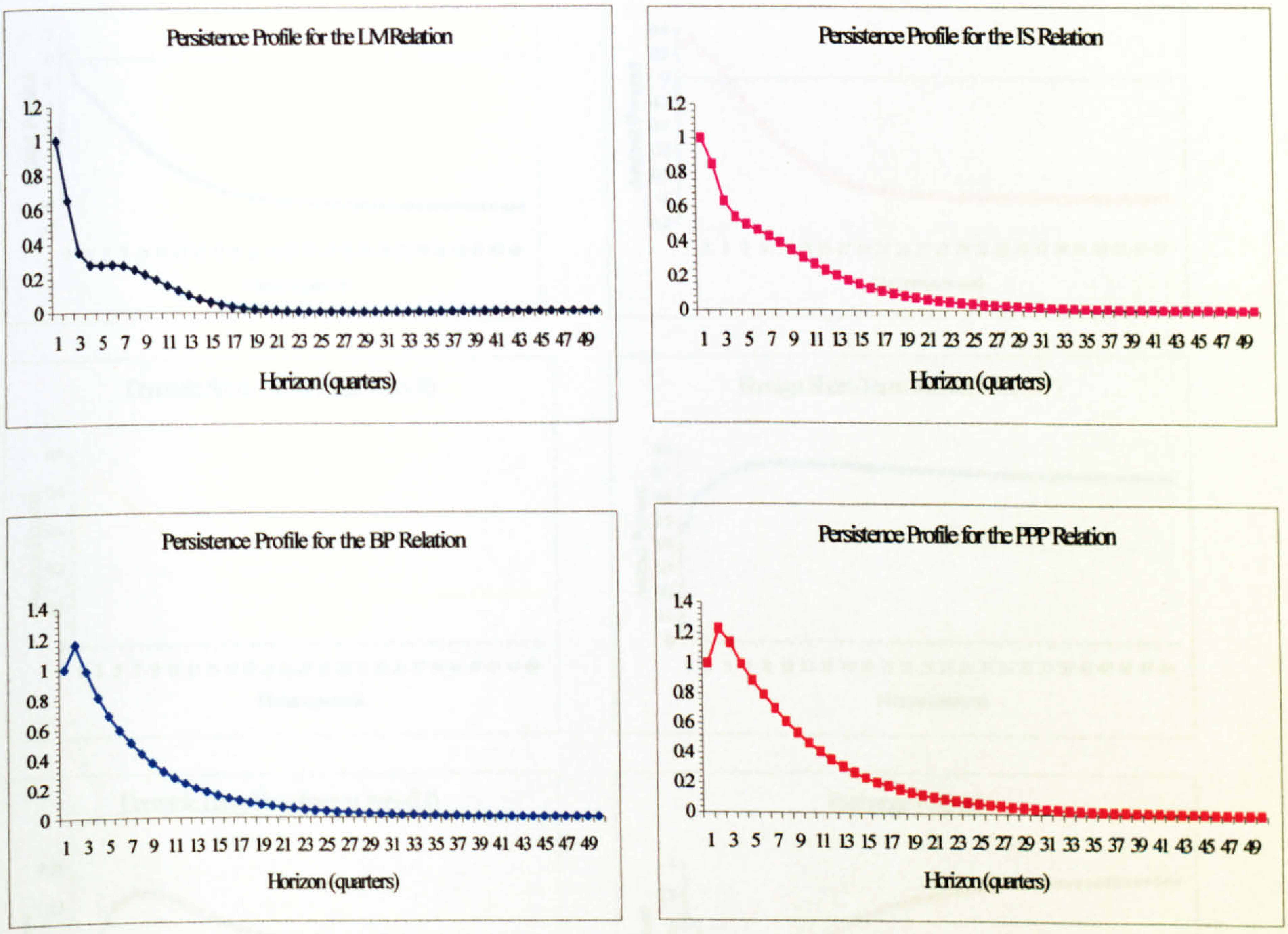


Figure 2.8: Generalised Impulse Responses to a One Standard Error Shock in the R_t^* Equation

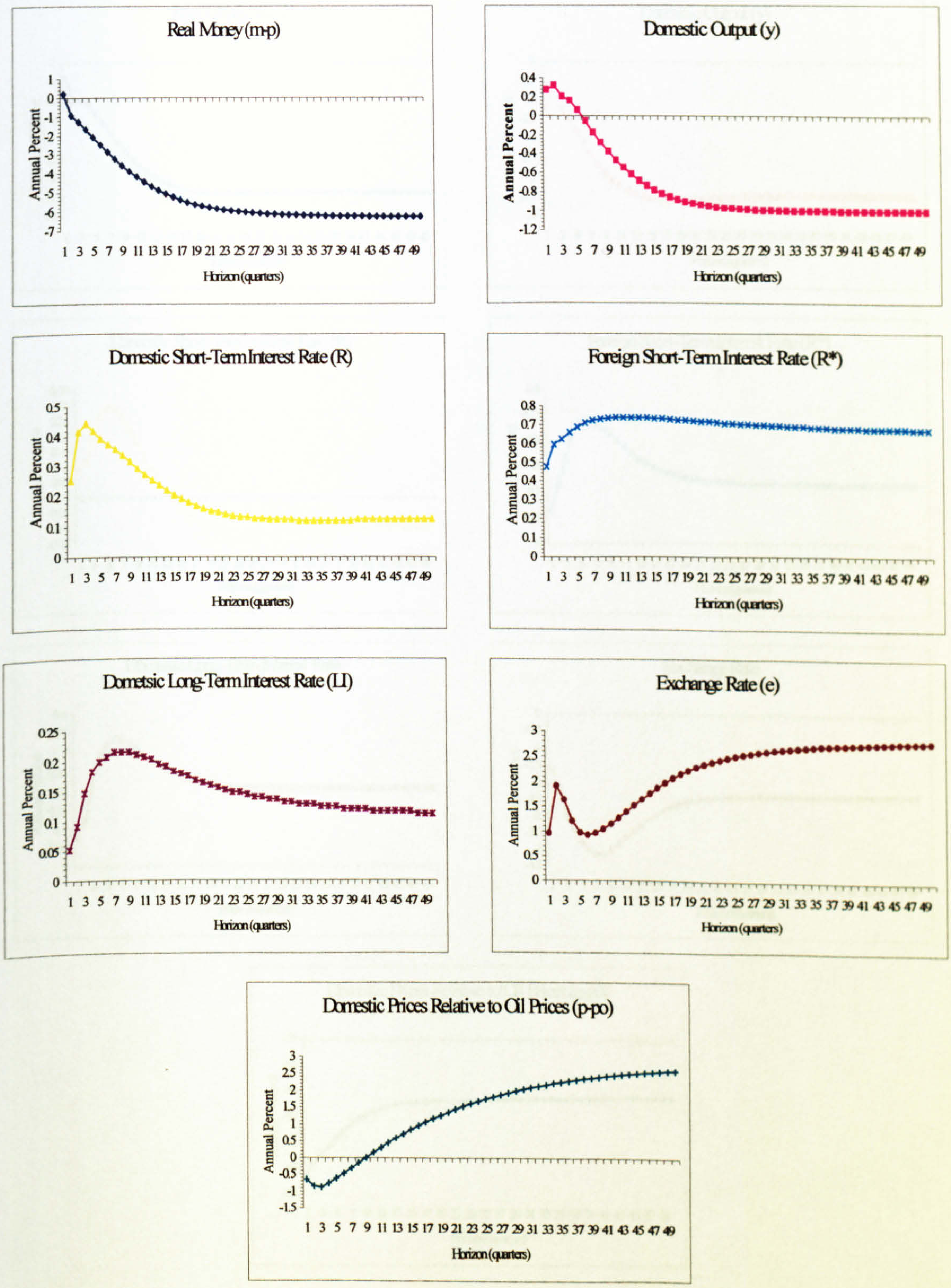


Figure 2.9: Generalised Impulse Responses to a One Standard Error Oil Price Shock

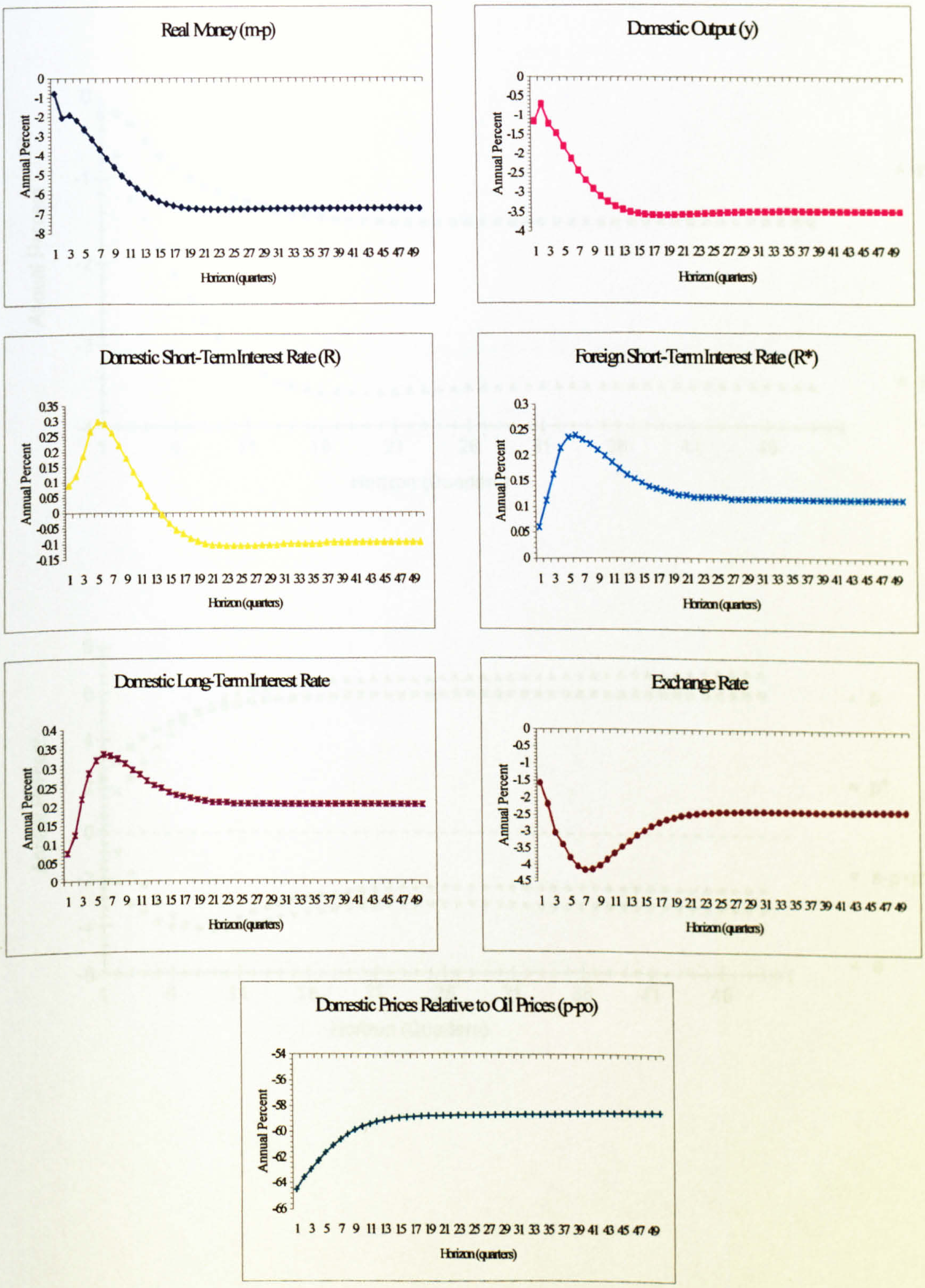
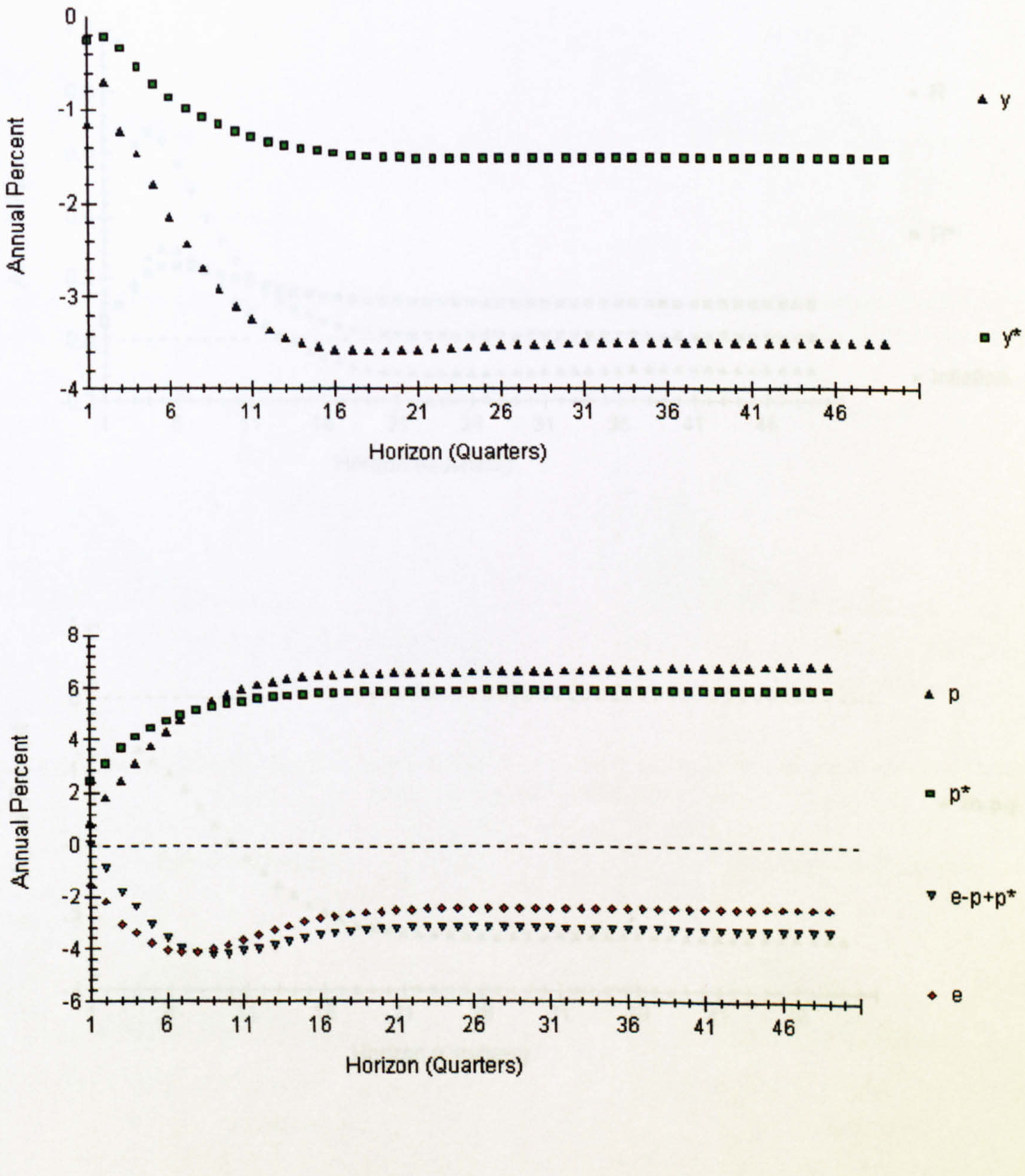
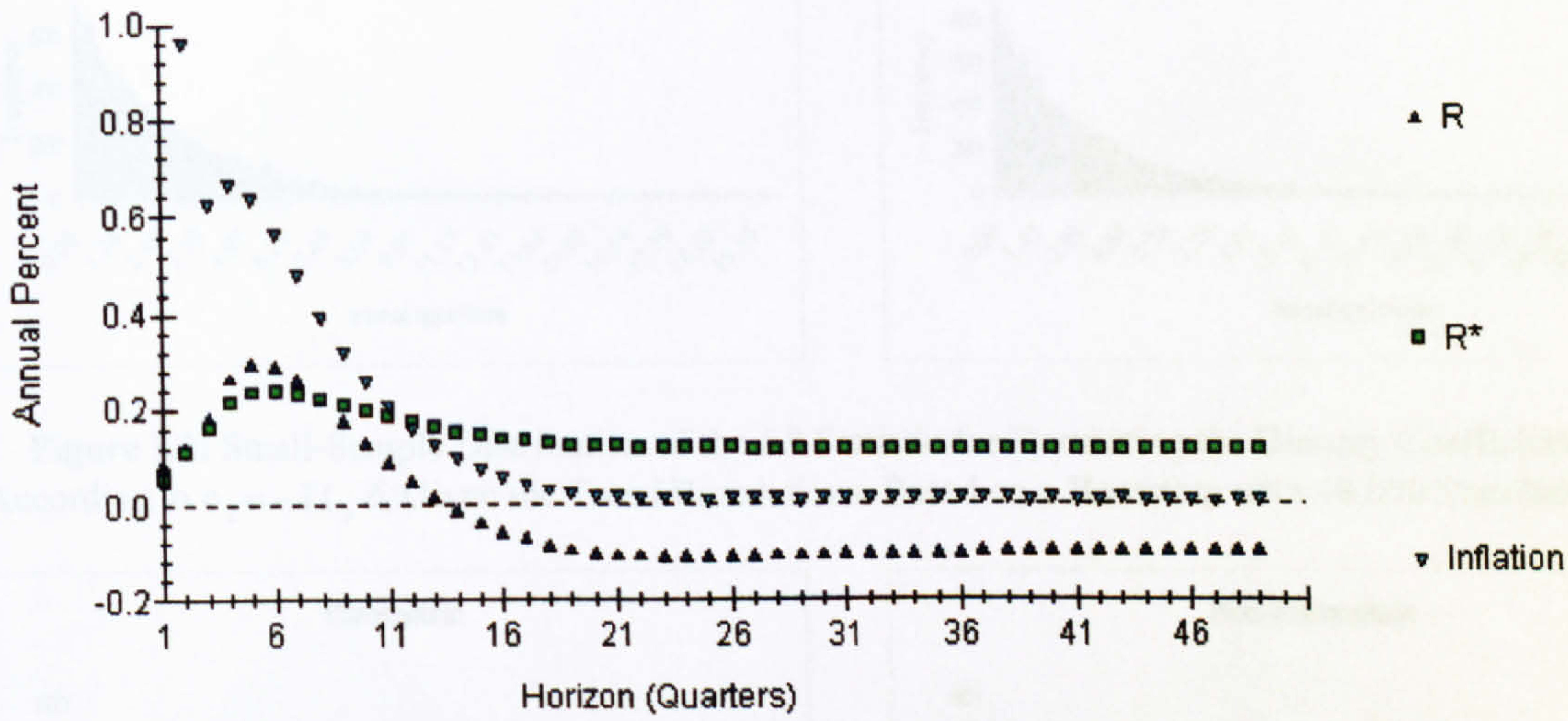


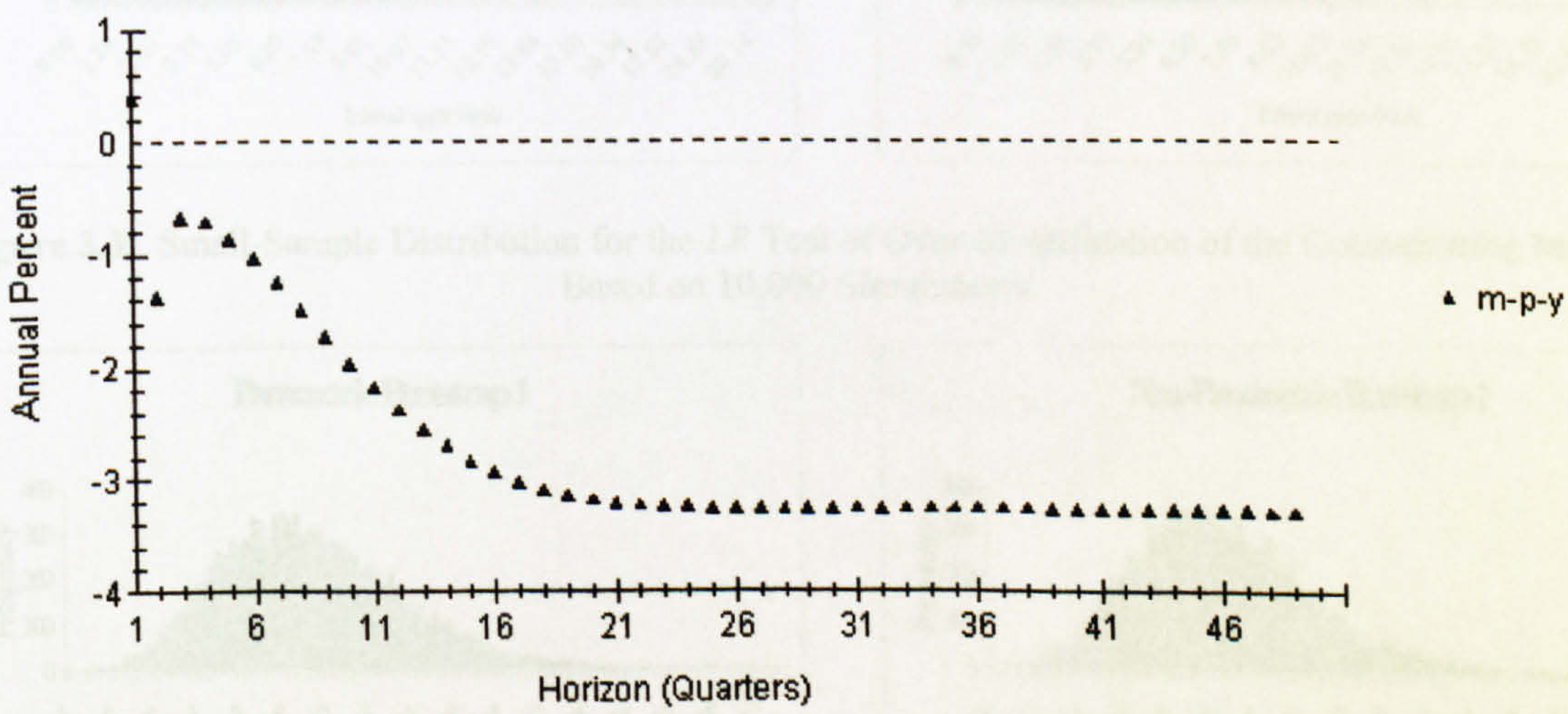
Figure 2.10a: Generalised Impulse Responses to a One Standard Error Oil Price Shock*



* Figure 2.10a is the equivalent of Figures 3 and 4 in Garratt *et al* (1998).

Figure 2.10b: Generalised Impulse Responses to a One Standard Error Oil Price Shock*





* Figure 2.10b is the equivalent of Figures 5 and 6 in Garratt *et al* (1998).

Figure 3.1: Small-Sample Distribution of the *LR* Statistic for Restricting the Trend Coefficients According to $\mathbf{c}_1 = -\Pi_y\, \gamma$. Based on a Bootstrap with 10,000 Simulations.

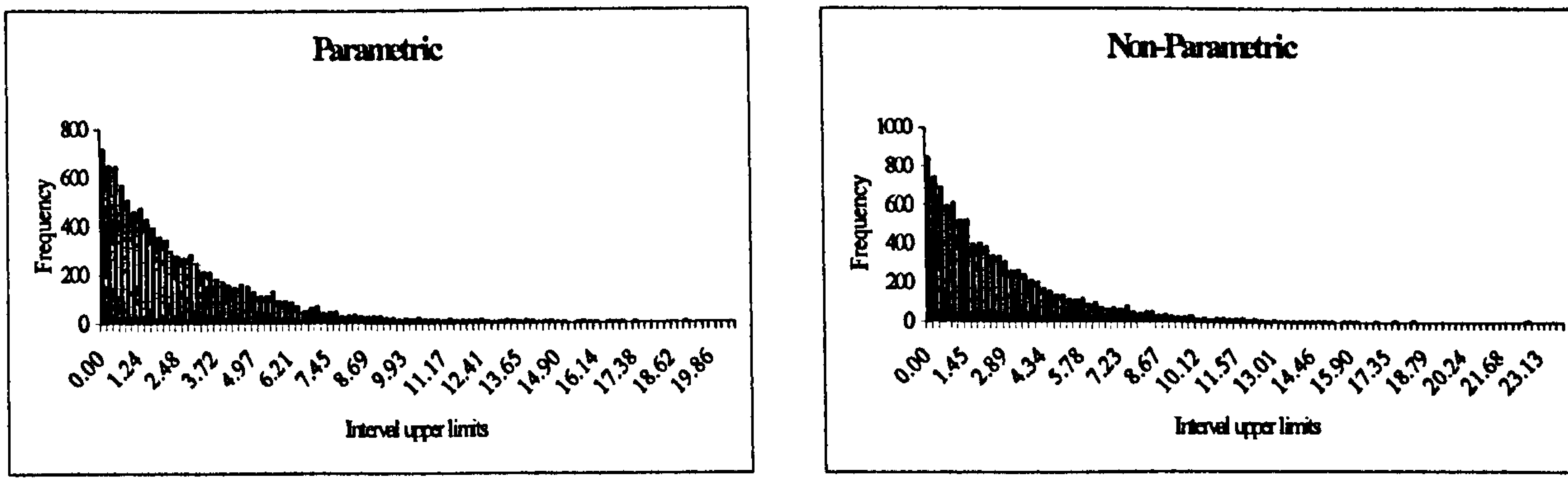


Figure 3.2: Small-Sample Distribution of the *LR* Statistic for Restricting the Dummy Coefficients According to $\mathbf{c}_2 = -\Pi_y\, \delta$, Given the Trend Restrictions. Based on a Bootstrap with 10,000 Simulations.

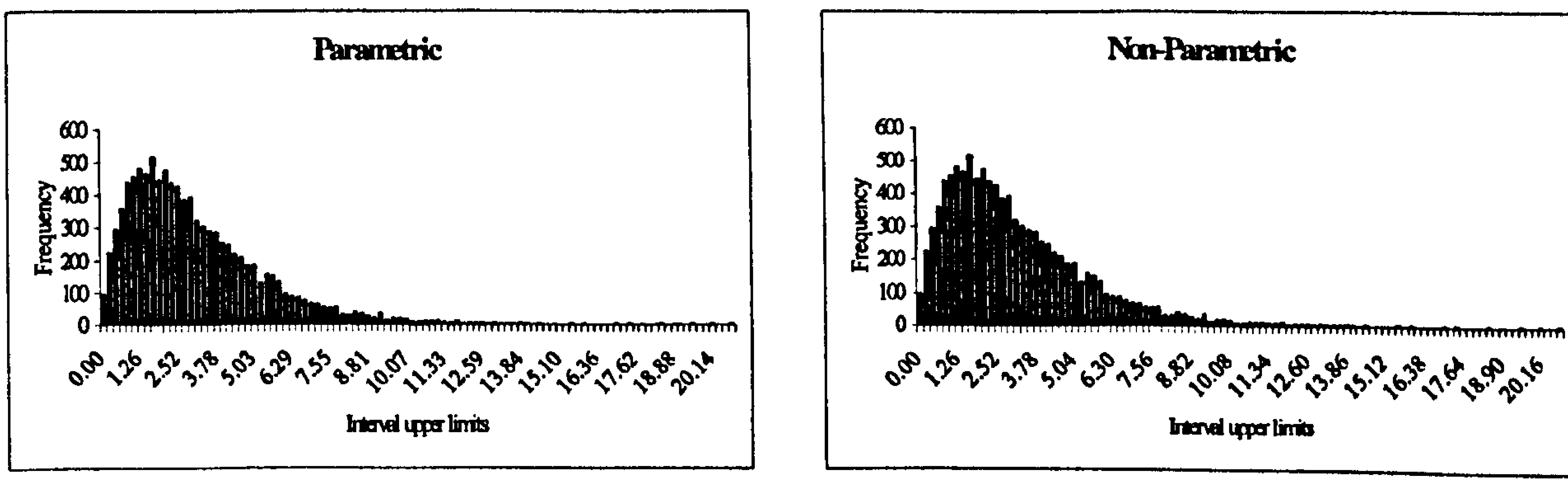


Figure 3.3: Small-Sample Distribution for the *LR* Test of Over-Identification of the Cointegrating Matrix. Based on 10,000 Simulations.

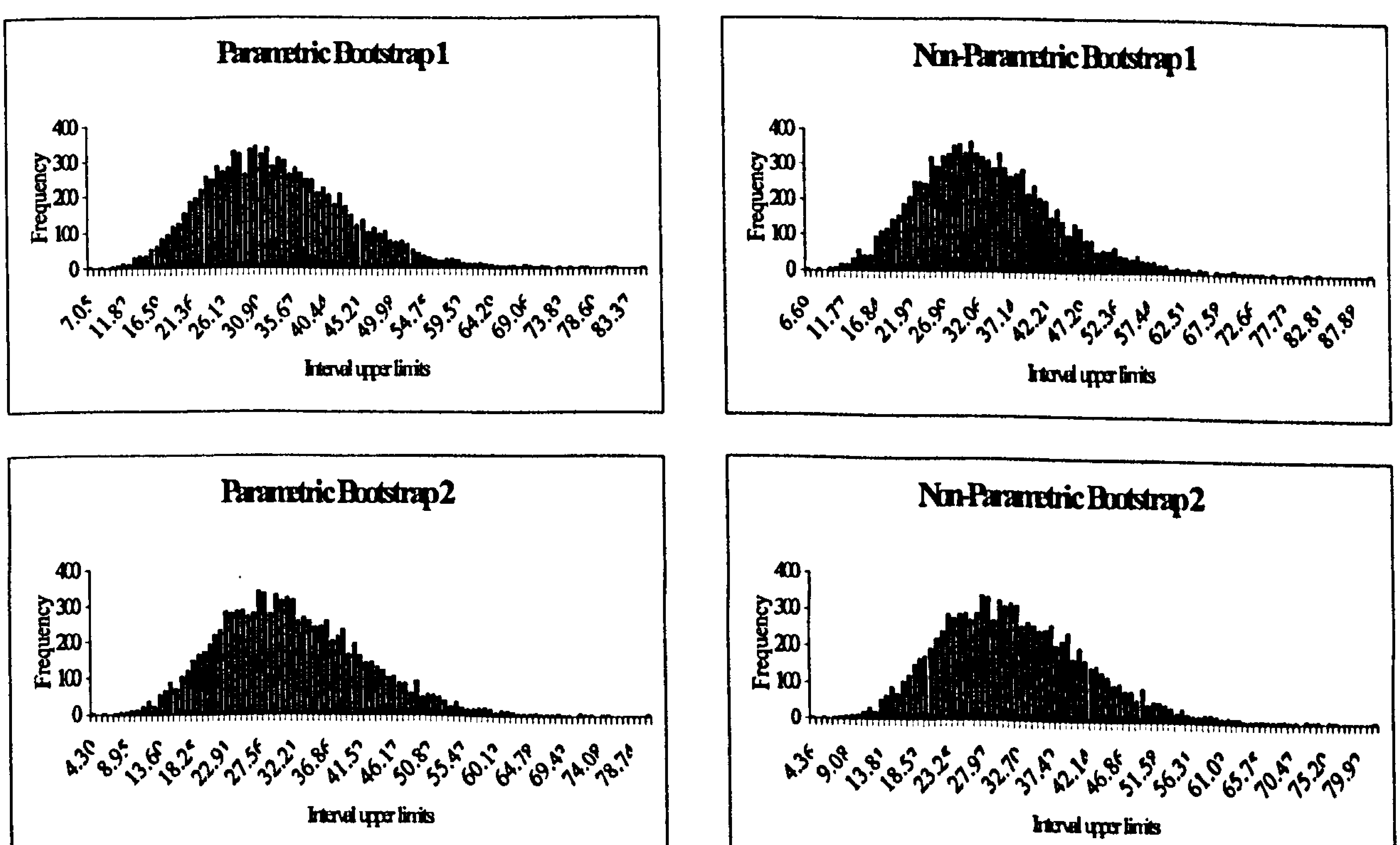


Figure 3.4: Small-Sample Distribution of the *max-eigenvalue* Cointegration Rank Statistic. Based on a Parametric Bootstrap with 10,000 Simulations.

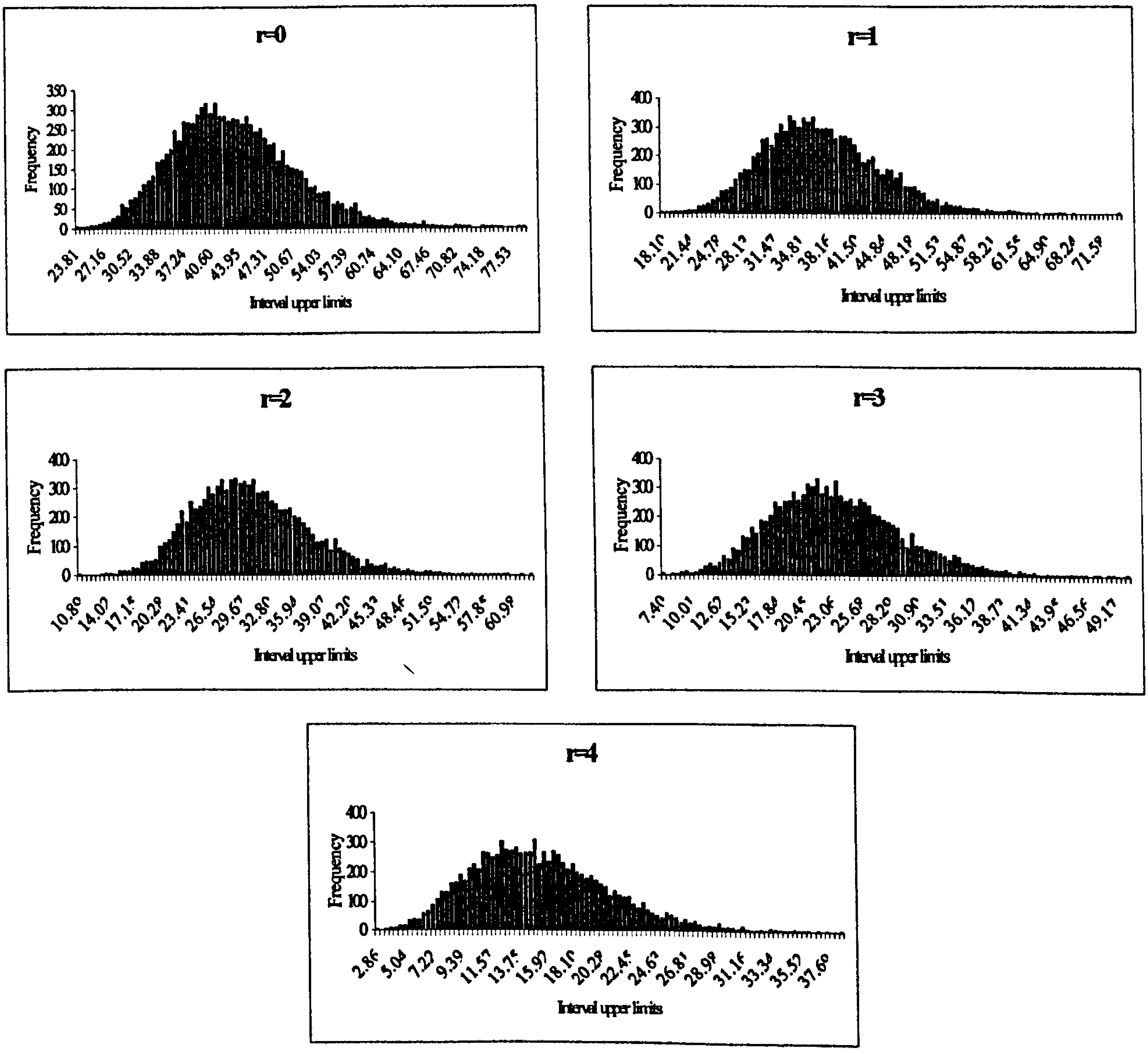


Figure 3.5: Small-Sample Distribution of the λ-*trace* Cointegration Rank Statistic. Based on a Parametric Bootstrap with 10,000 Simulations.

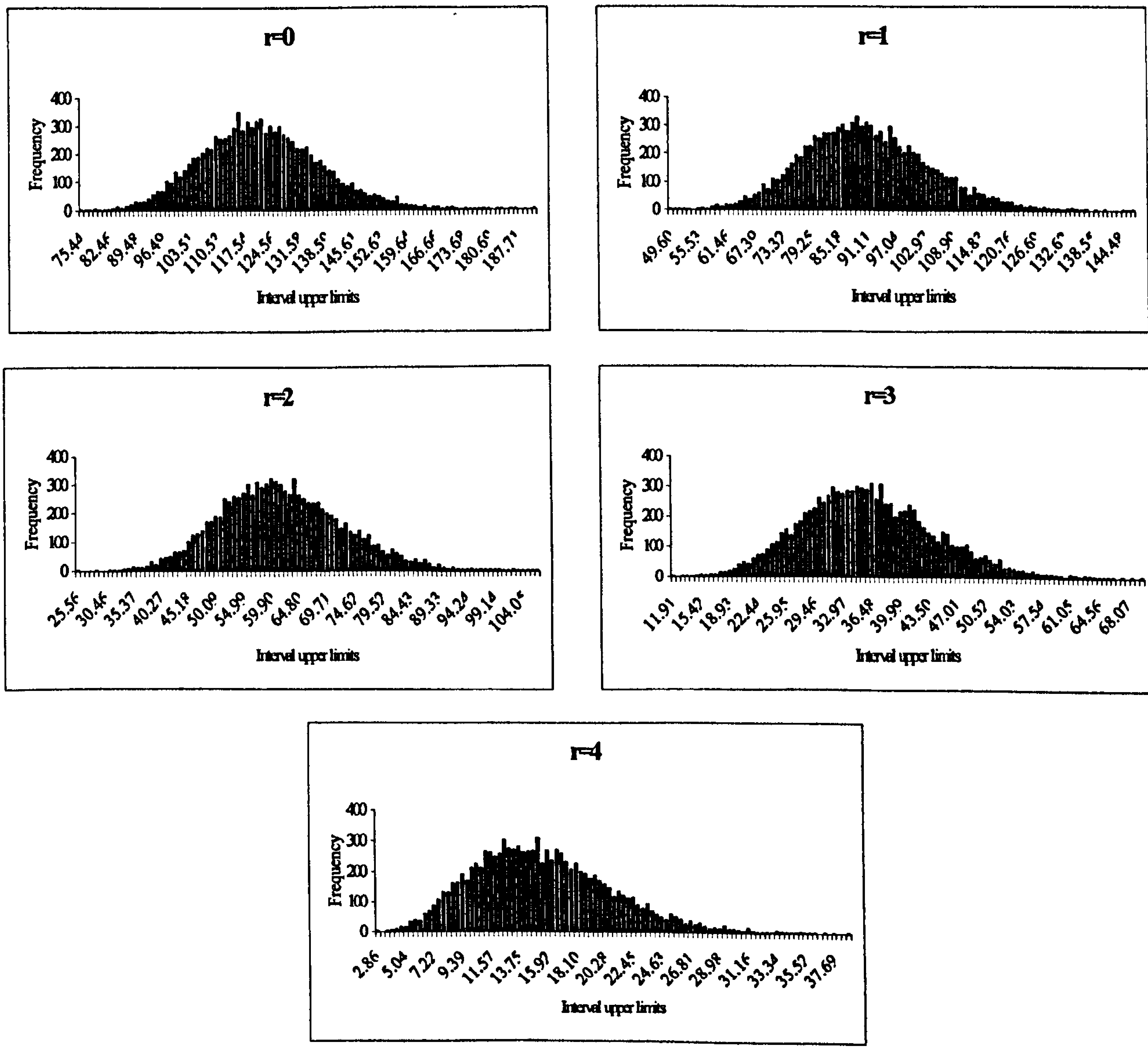


Figure 3.6: Small-Sample Distribution of the *max-eigenvalue* Cointegration Rank Statistic. Based on a Non-Parametric Bootstrap with 10,000 Simulations.

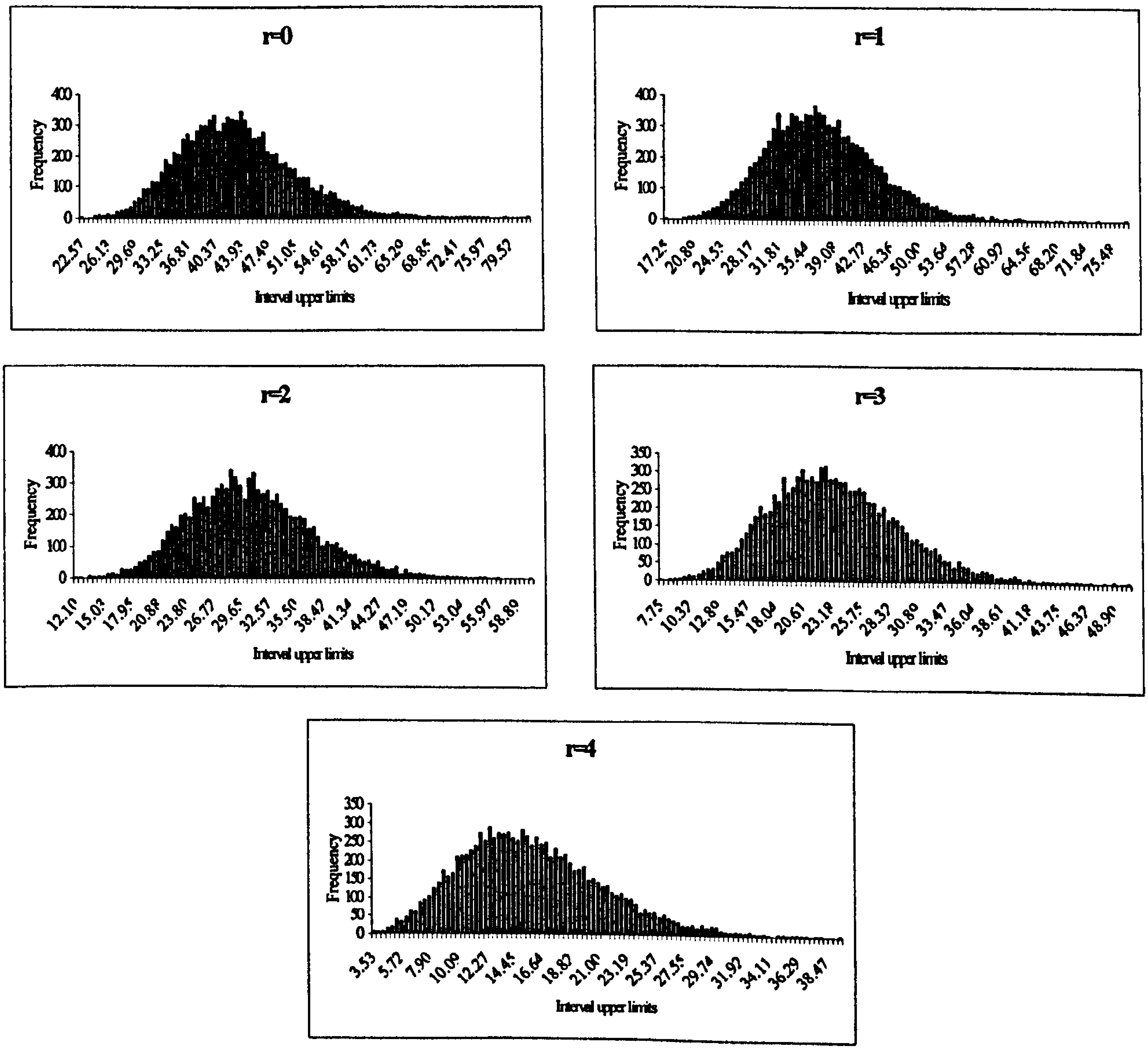


Figure 3.7: Small-Sample Distribution of the *λ-trace* Cointegration Rank Statistic. Based on a Non-Parametric Bootstrap with 10,000 Simulations.

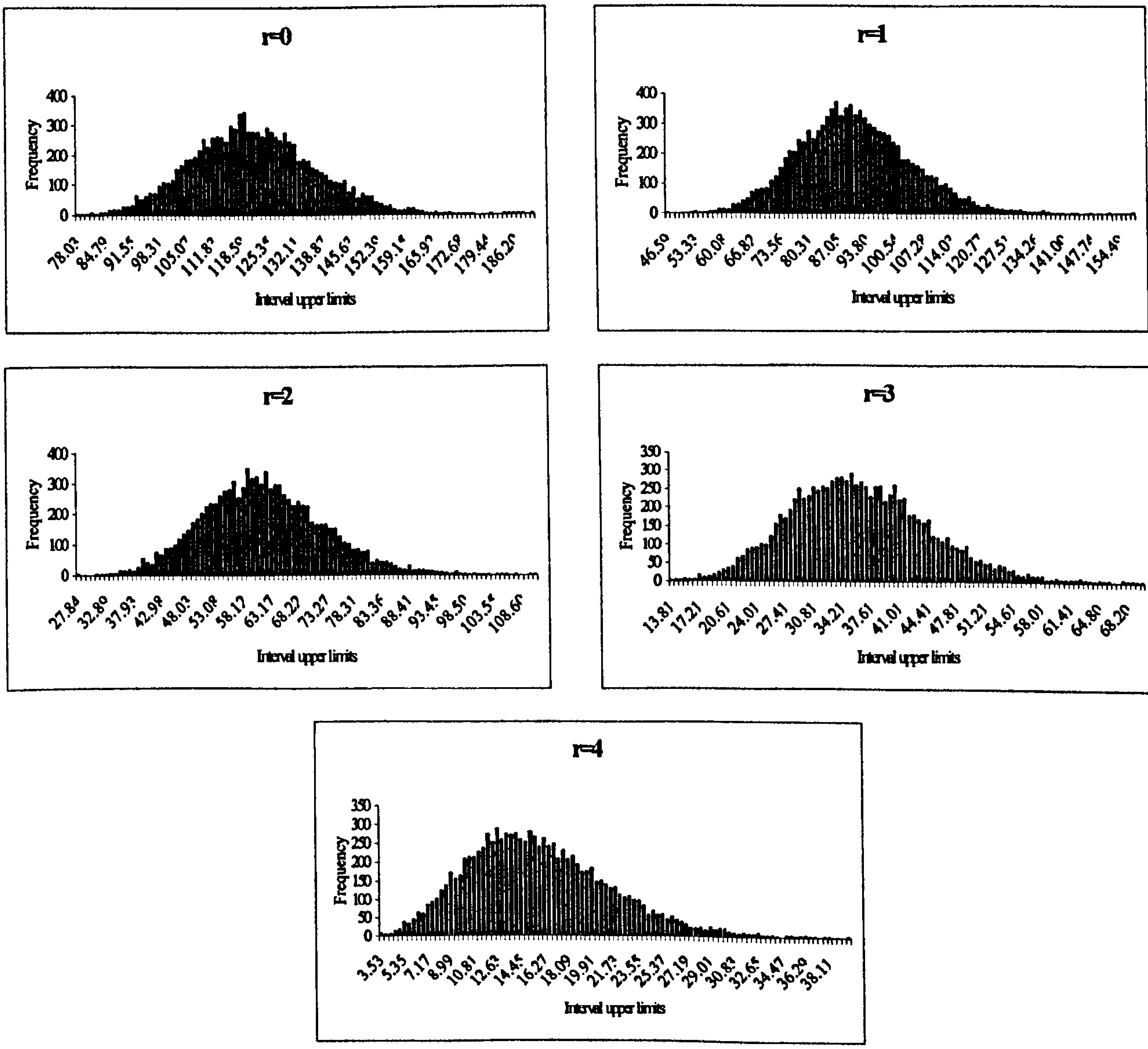


Figure 3.8: Plots of the Estimated LM, IS and BP Relations in Chapters 2 and 3

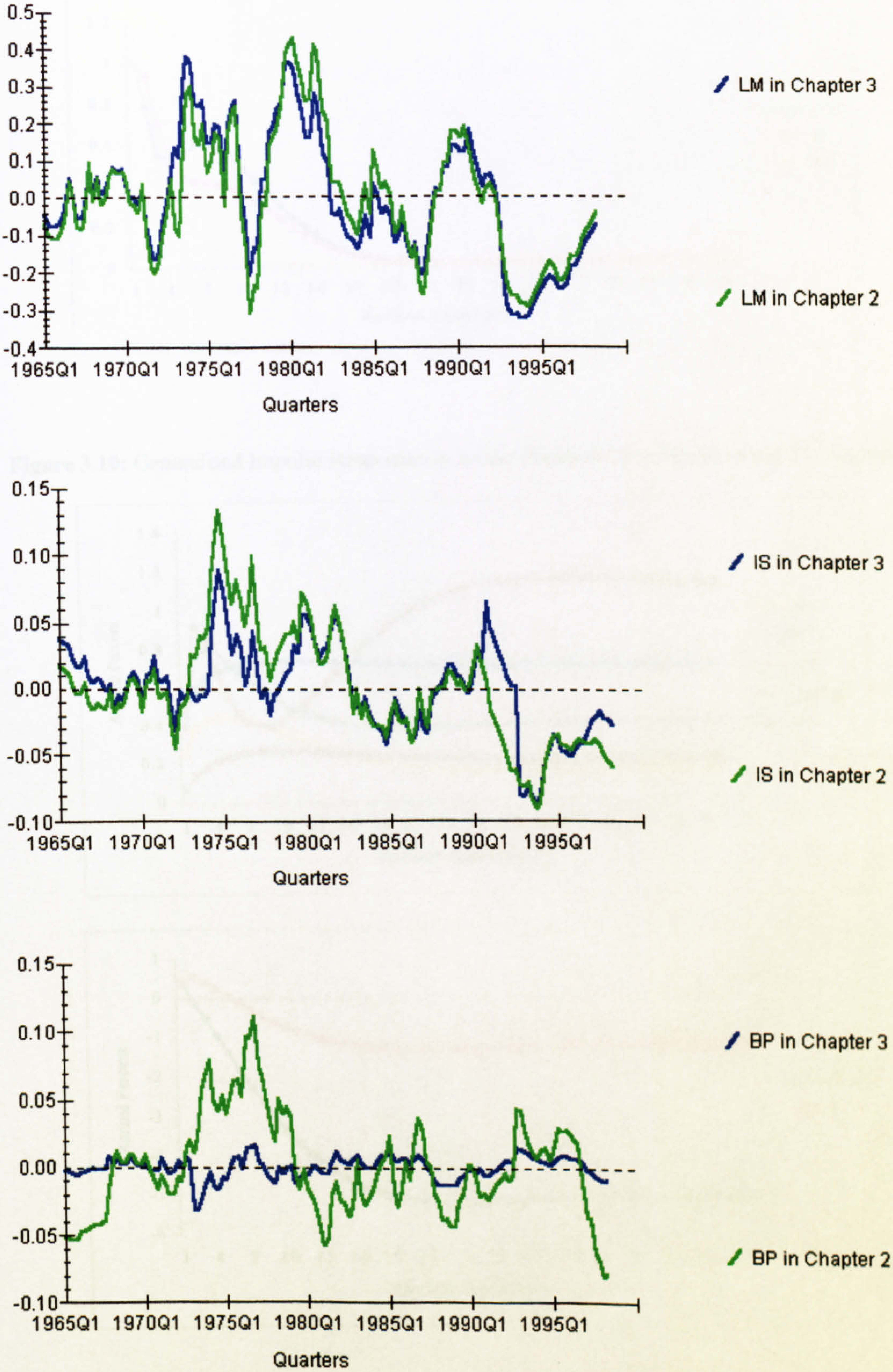


Figure 3.9: Persistence Profiles for the Estimated LM, IS and BP Relations

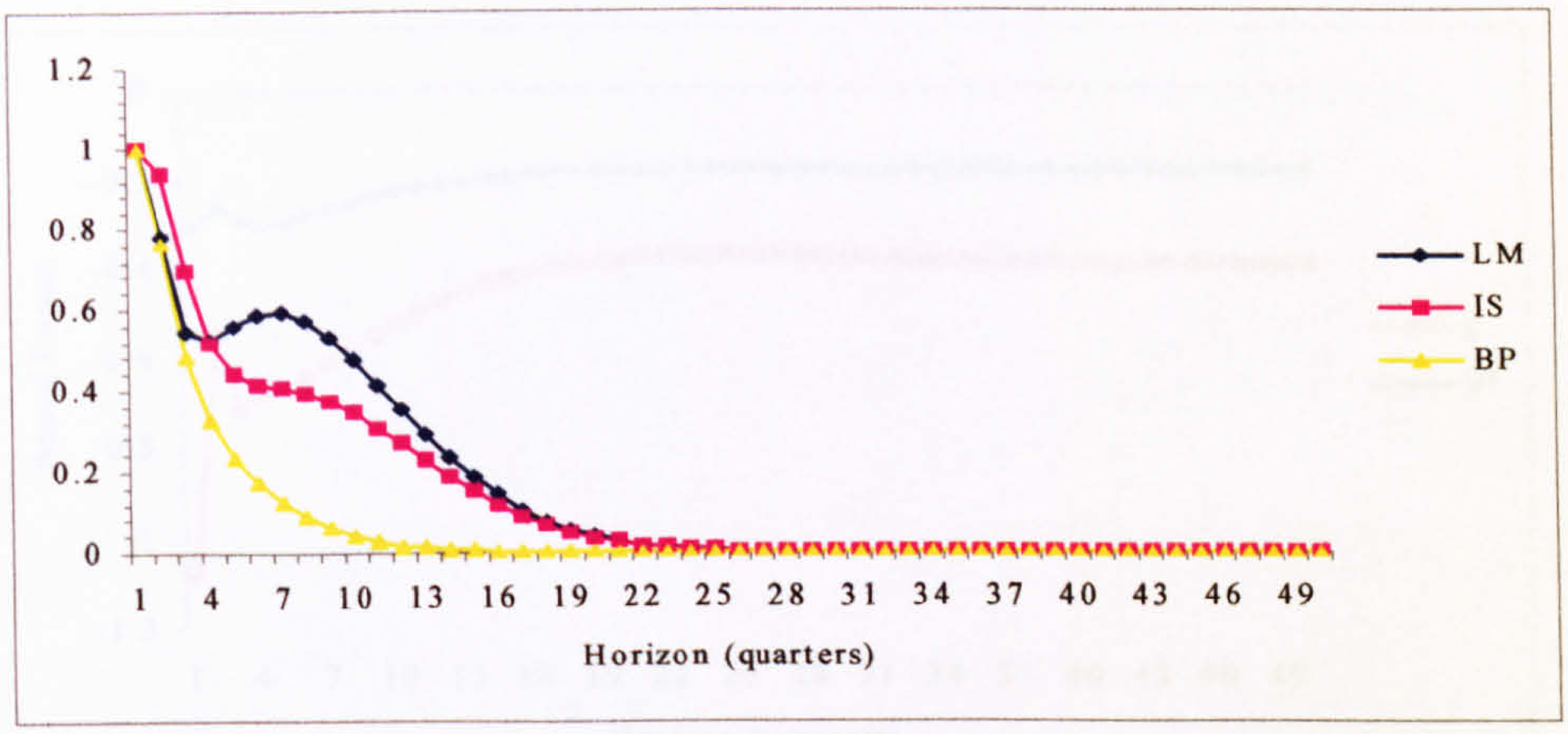
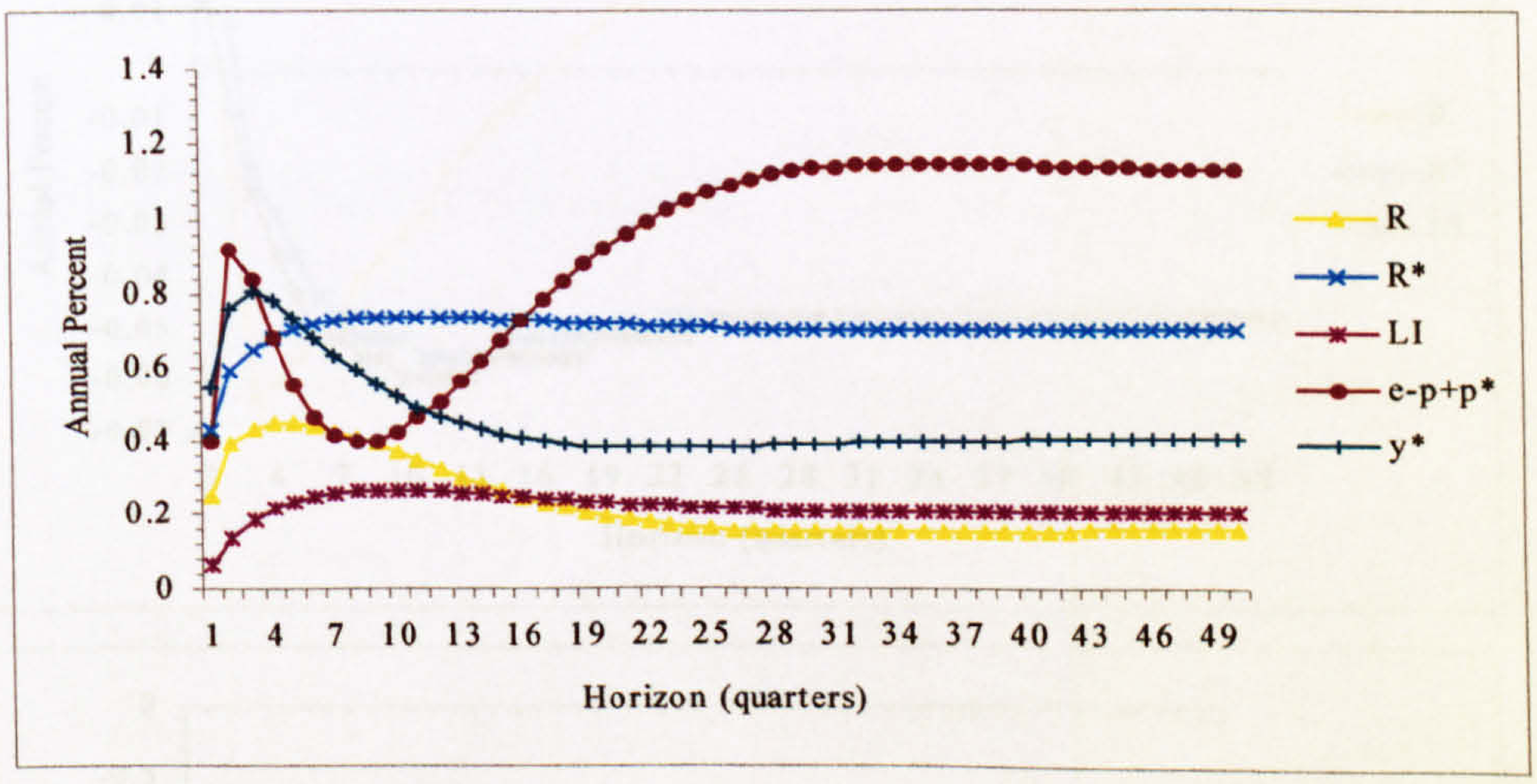


Figure 3.10: Generalised Impulse Responses to a One Standard Error Shock in the R_t^* Equation



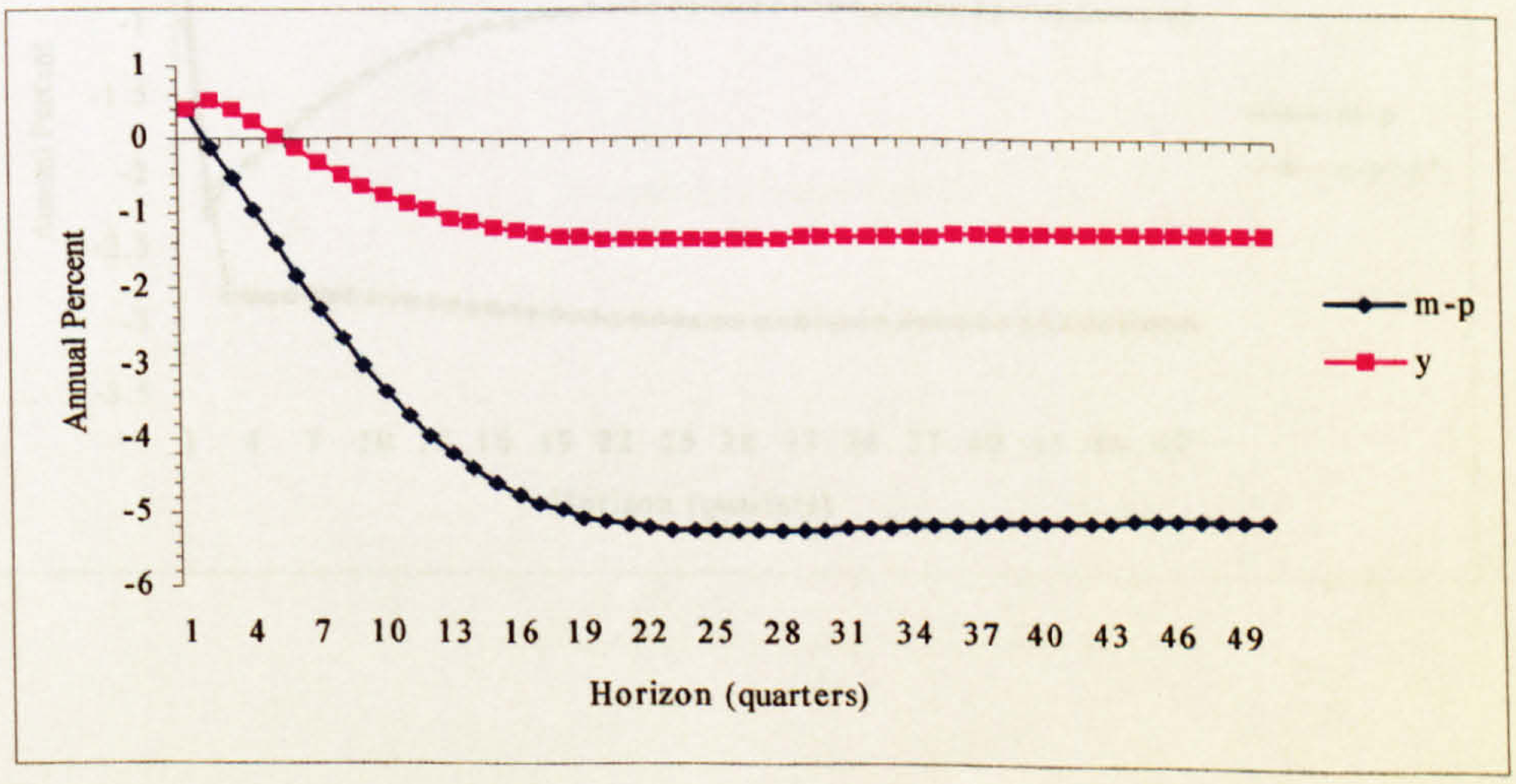


Figure 3.11: Generalised Impulse Responses to a One Standard Error Oil Price Shock

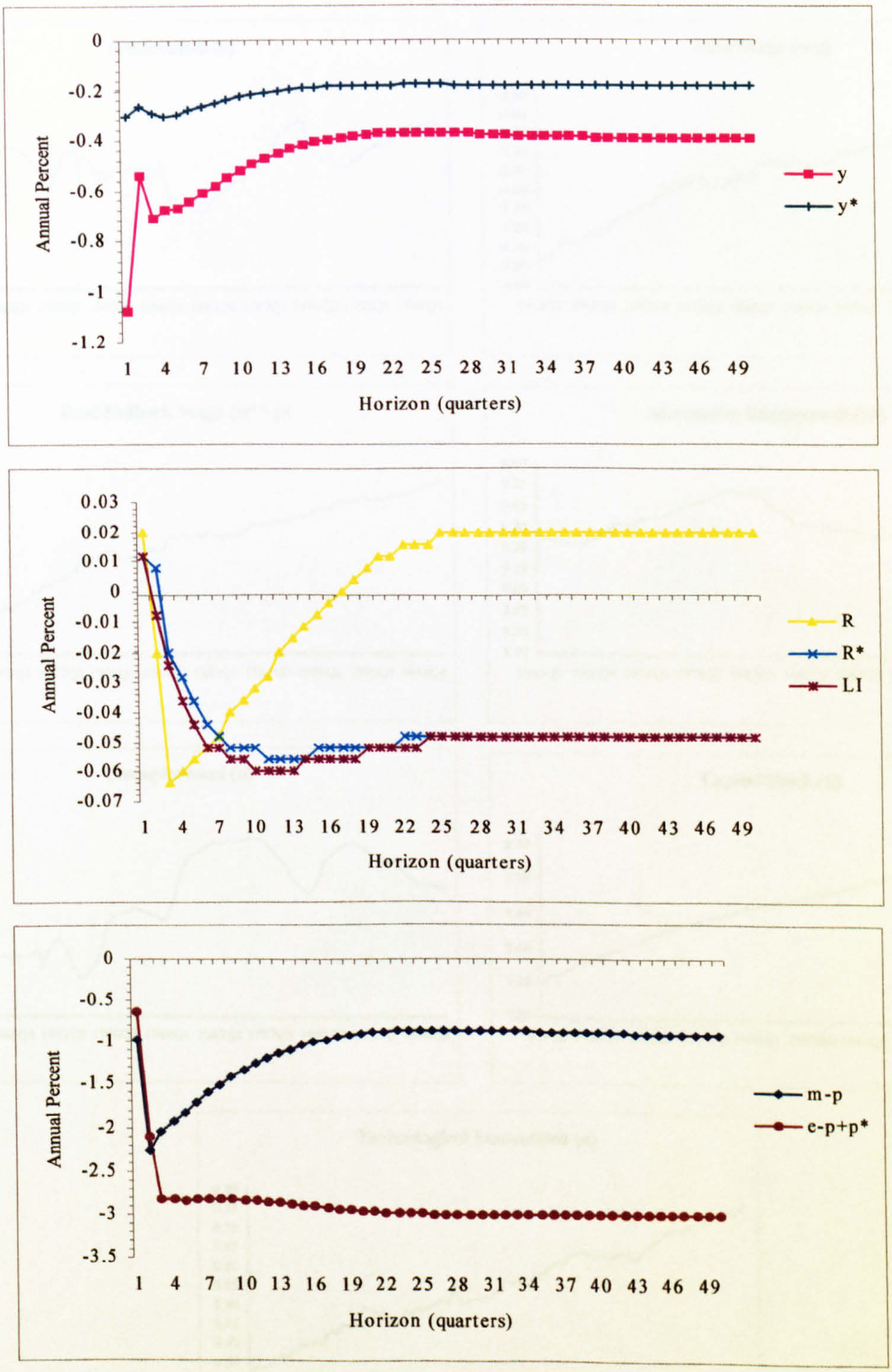


Figure 4.1: Time Plots of the Variables: 1965q1-2000q1

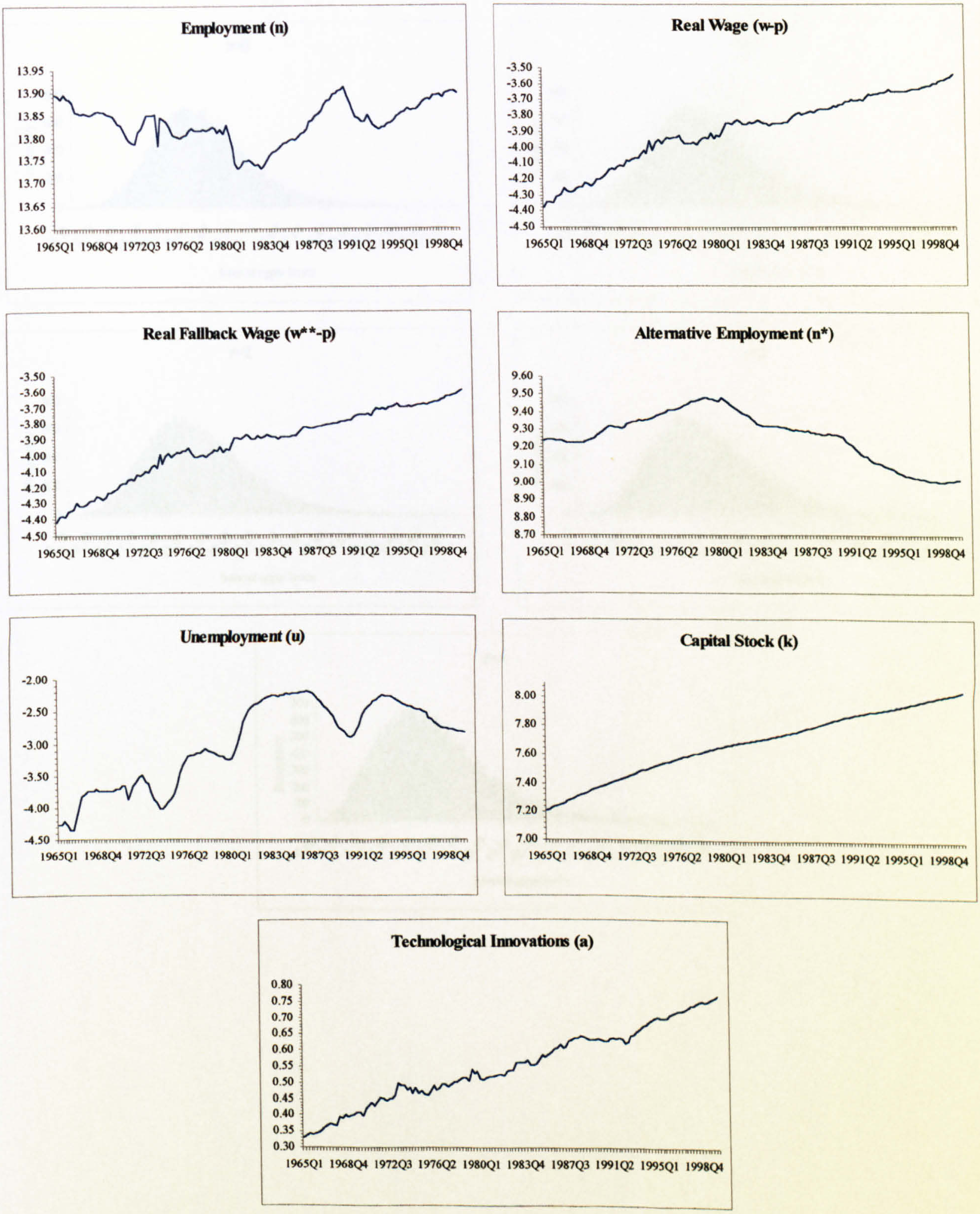


Figure 4.2: Small-Sample Distribution of the *max-eigenvalue* Cointegration Rank Statistic for the Conditional Model. Based on a Parametric Bootstrap with 20,000 Simulations.

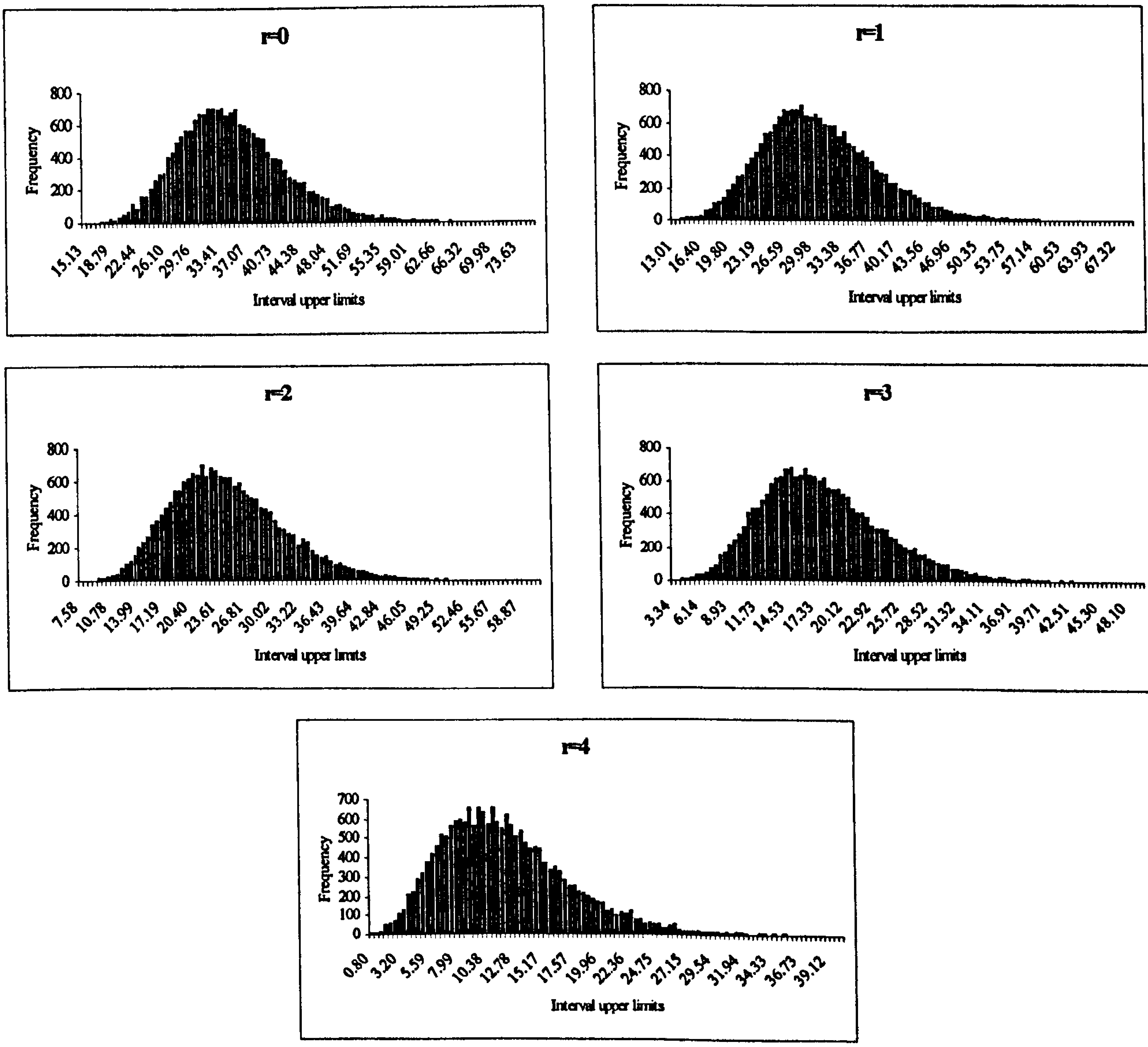


Figure 4.3: Small-Sample Distribution of the λ-*trace* Cointegration Rank Statistic for the Conditional Model. Based on a Parametric Bootstrap with 20,000 Simulations.

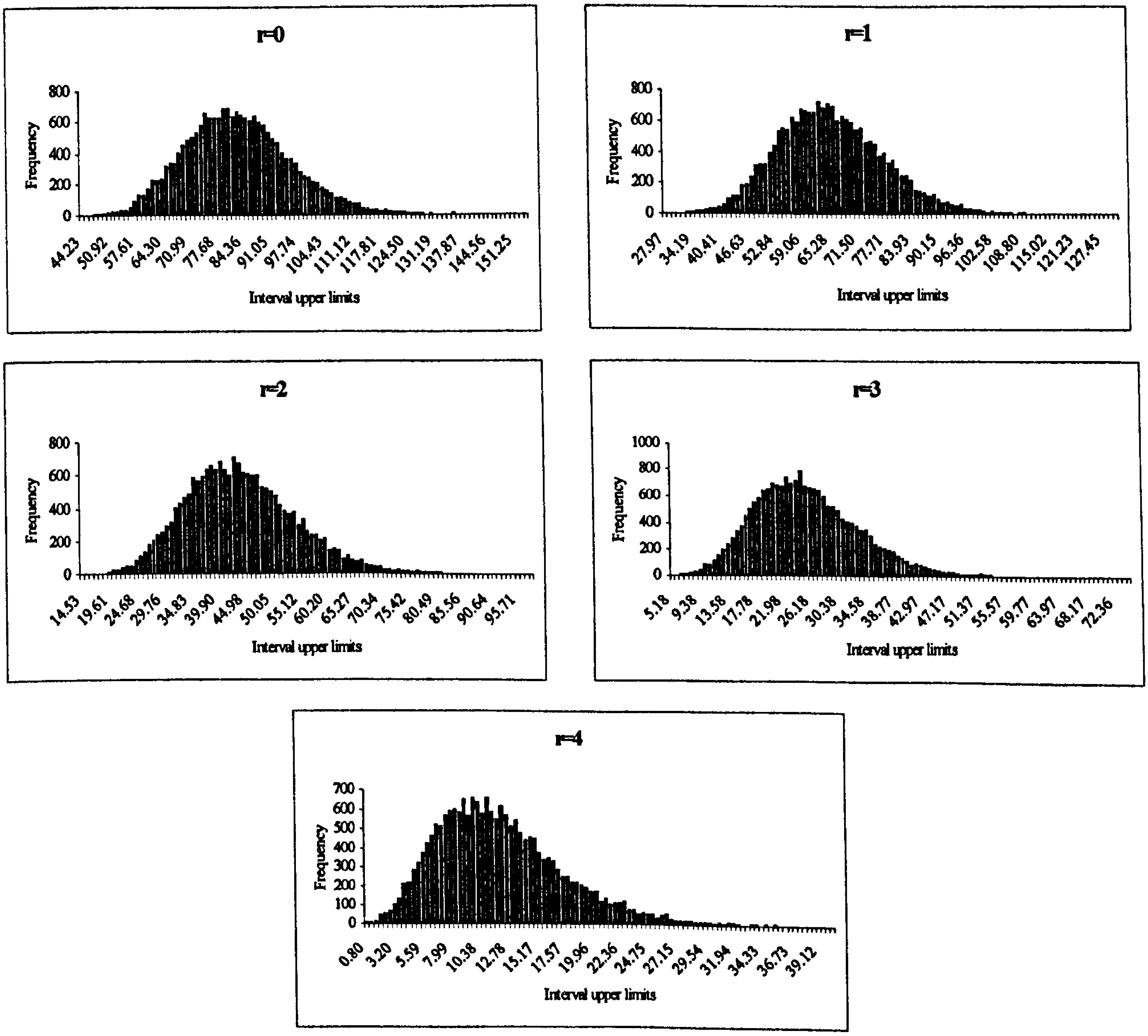


Figure 4.4: Small-Sample Distribution of the *max-eigenvalue* Cointegration Rank Statistic for the Conditional Model. Based on a Non-Parametric Bootstrap with 20,000 Simulations.

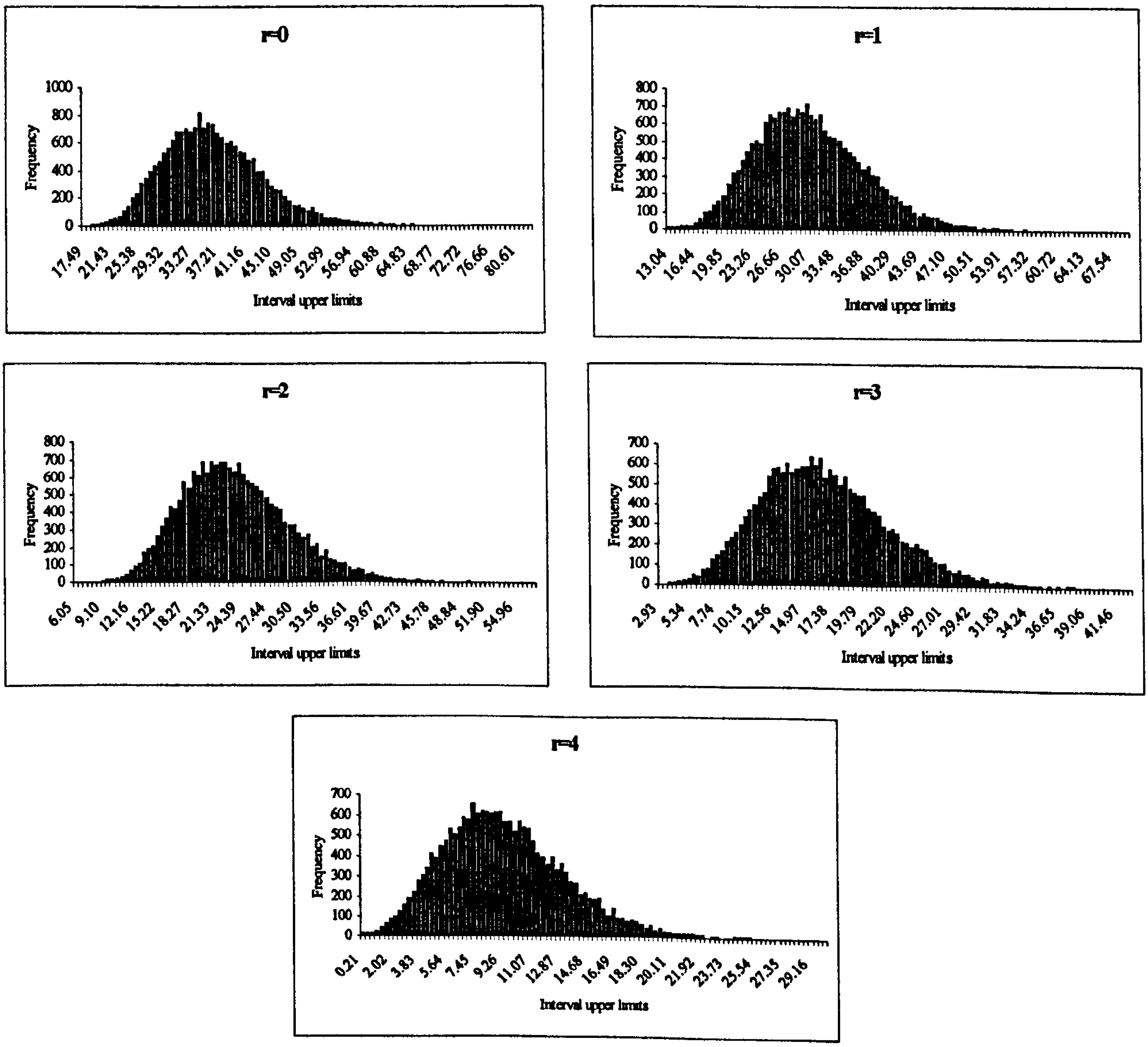


Figure 4.5: Small-Sample Distribution of the *λ-trace* Cointegration Rank Statistic for the Conditional Model. Based on a Non-Parametric Bootstrap with 20,000 Simulations.

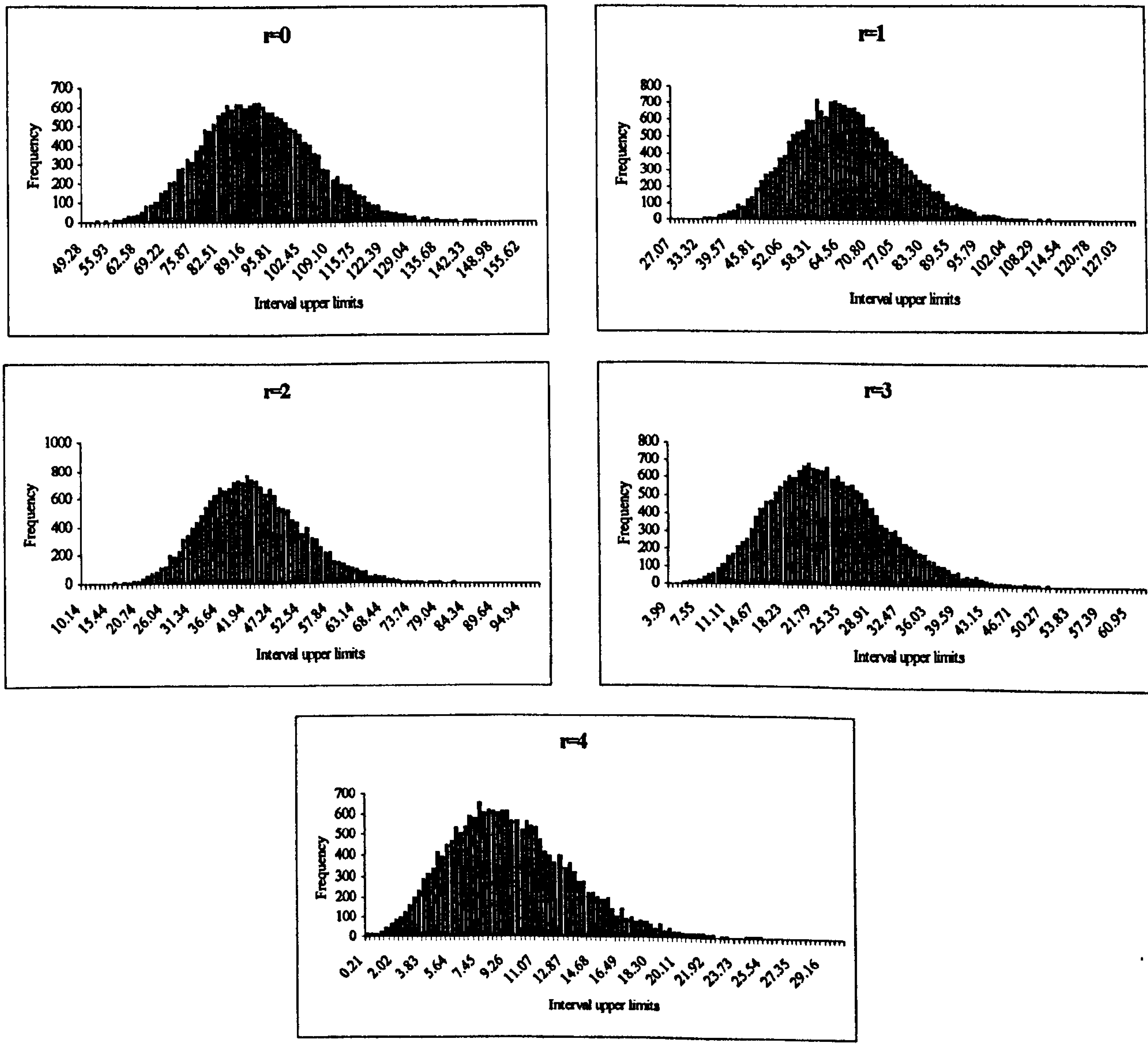


Figure 4.6: Small-Sample Distributions for the *LR* Tests of Over-Identification of the Cointegrating Matrix. Based on 10,000 Simulations.

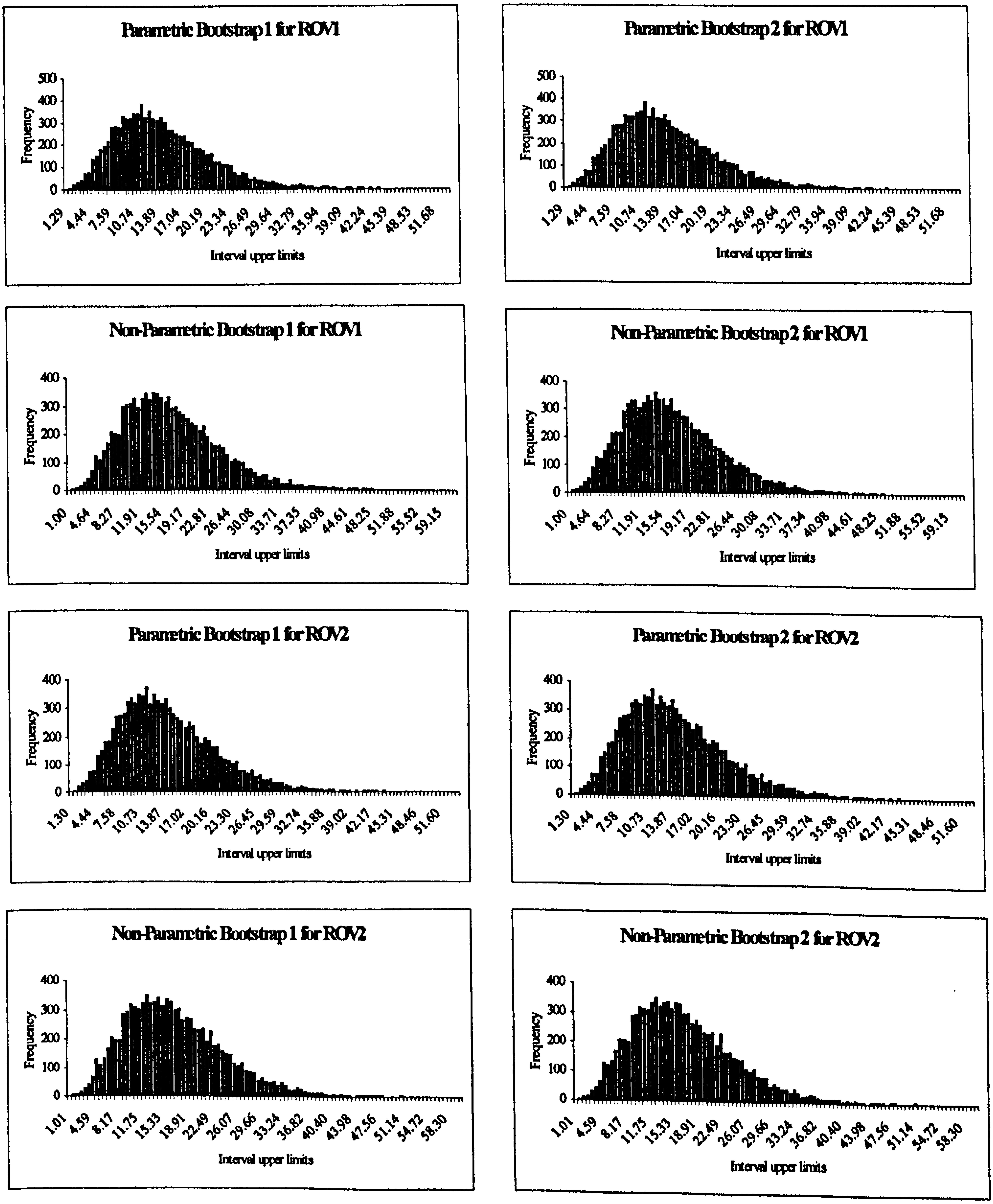


Figure 4.7: Persistence Profiles for the Estimated Employment and Wage Relations

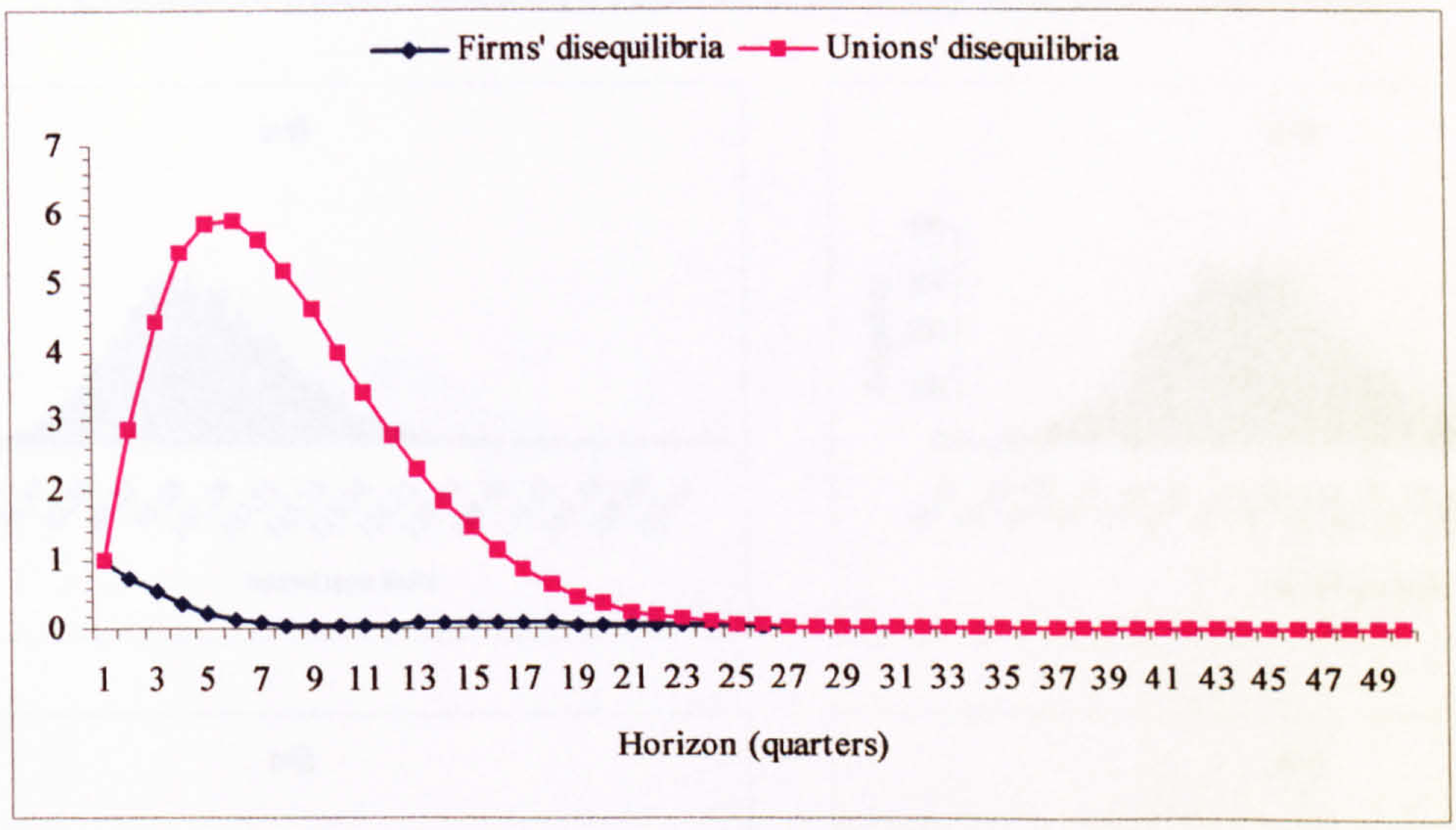


Figure 4.8: Generalised Impulse Responses to a One Standard Error Shock in Productivity

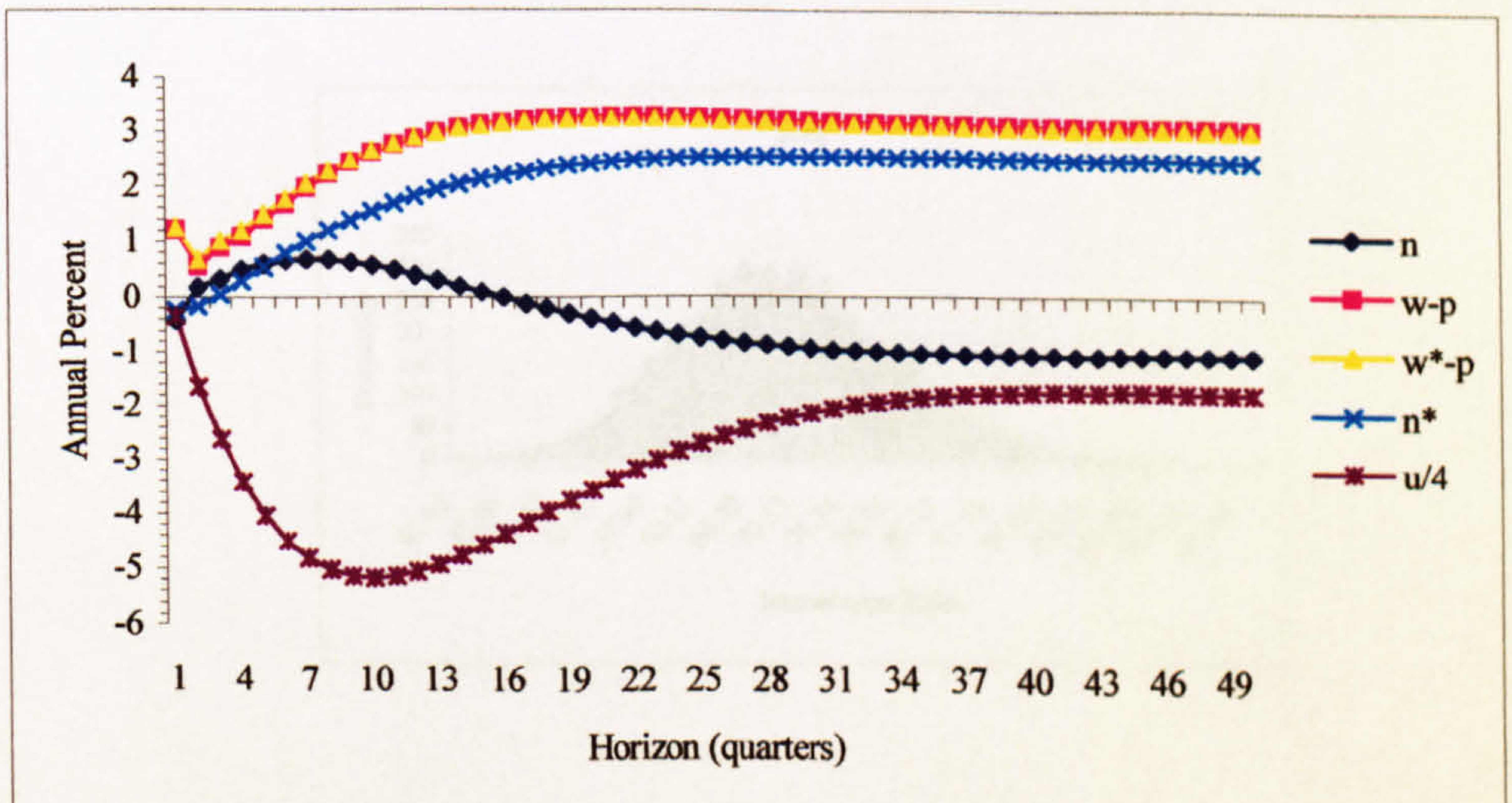


Figure 5.1: Small-Sample Distribution of the *max-eigenvalue* Cointegration Rank Statistic for the Conditional Model. Based on a Parametric Bootstrap with 10,000 Simulations.

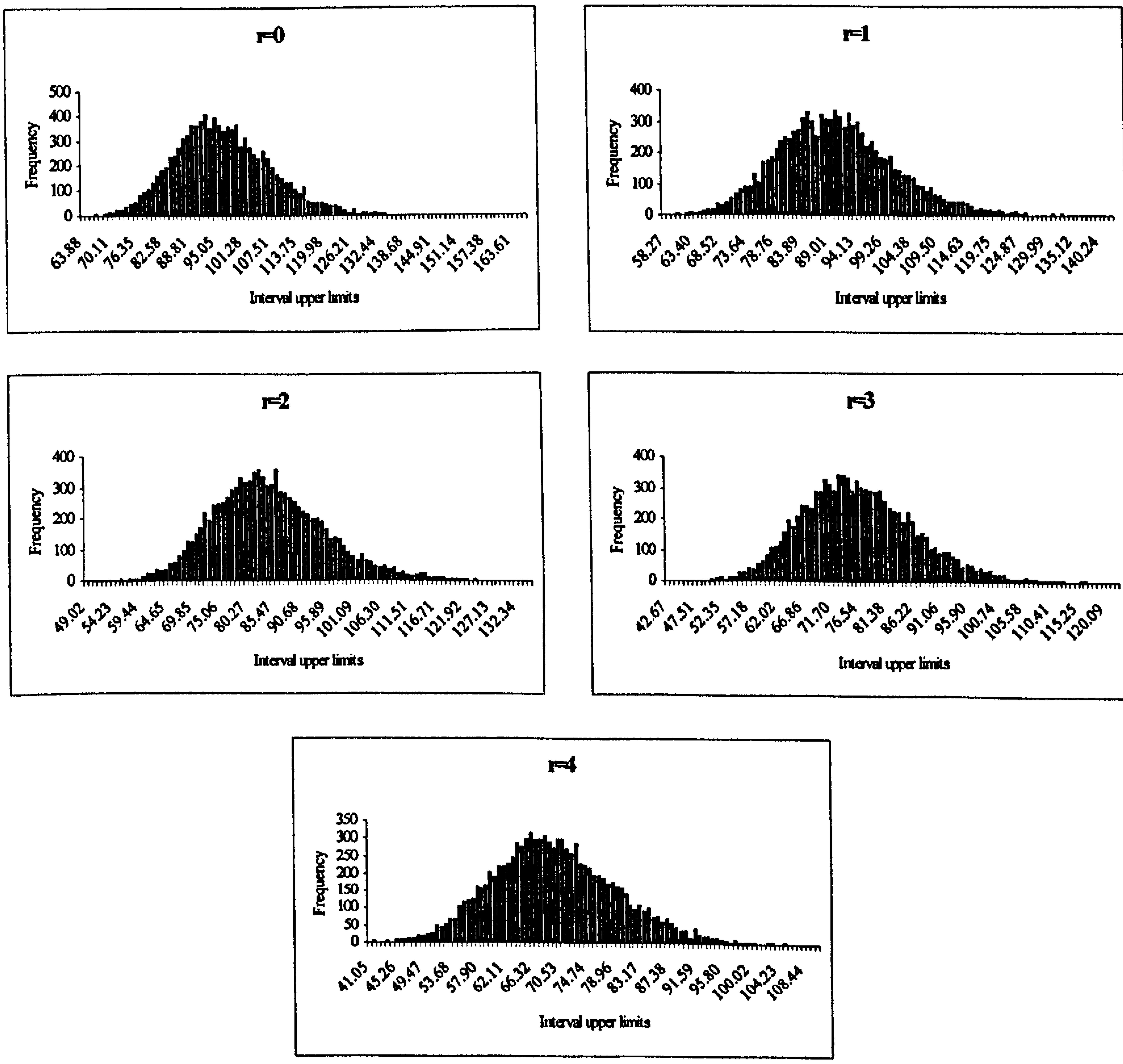


Figure 5.1 (continued): Small-Sample Distribution of the *max-eigenvalue* Cointegration Rank Statistic for the Conditional Model. Based on a Parametric Bootstrap with 10,000 Simulations.

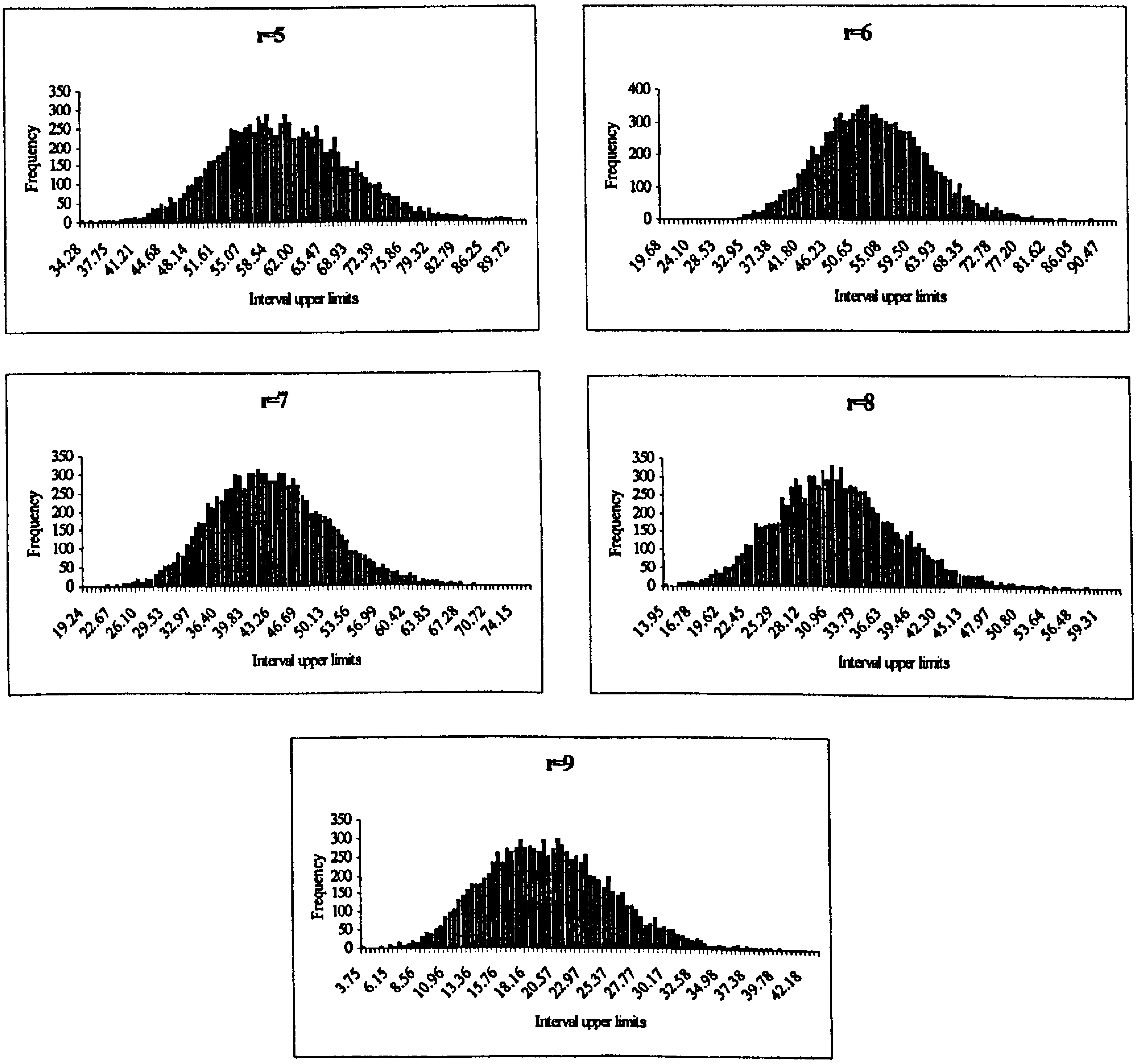


Figure 5.2: Small-Sample Distribution of the λ-*trace* Cointegration Rank Statistic for the Conditional Model. Based on a Parametric Bootstrap with 10,000 Simulations.

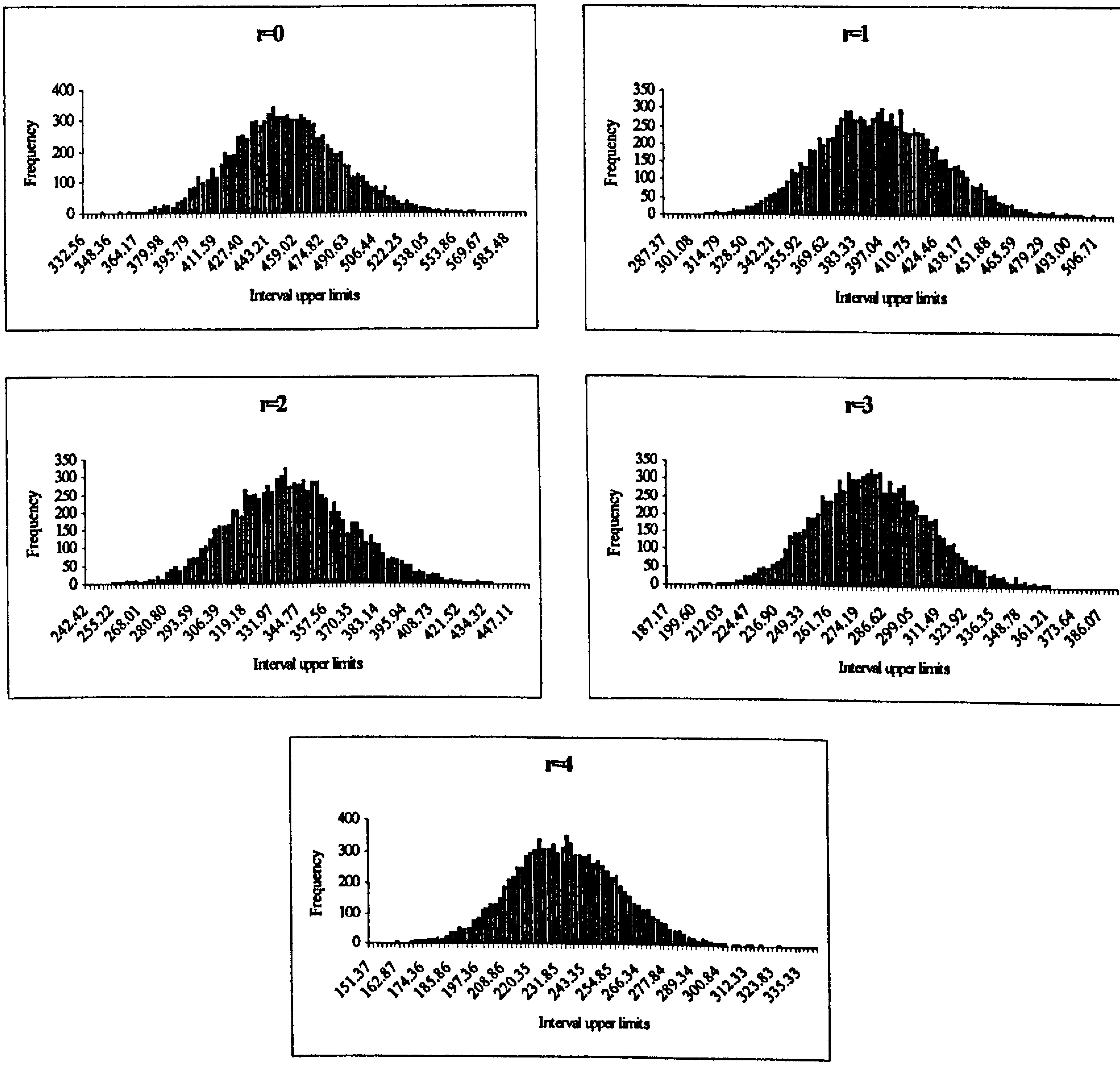


Figure 5.2 (continued): Small-Sample Distribution of the *λ-trace* Cointegration Rank Statistic for the Conditional Model. Based on a Parametric Bootstrap with 10,000 Simulations.

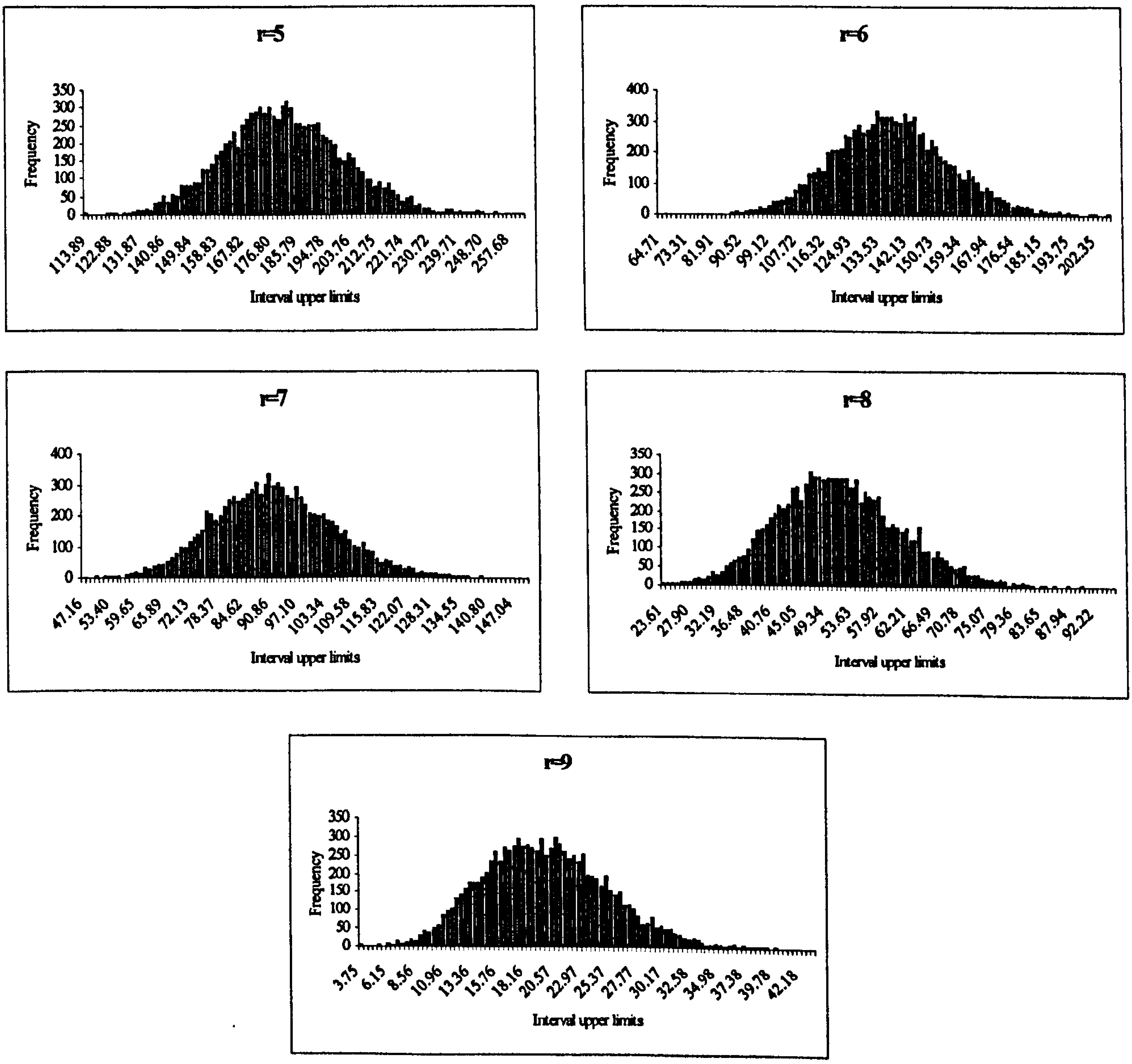


Figure 5.3: Small-Sample Distribution of the *max-eigenvalue* Cointegration Rank Statistic for the Conditional Model. Based on a Non-Parametric Bootstrap with 10,000 Simulations.

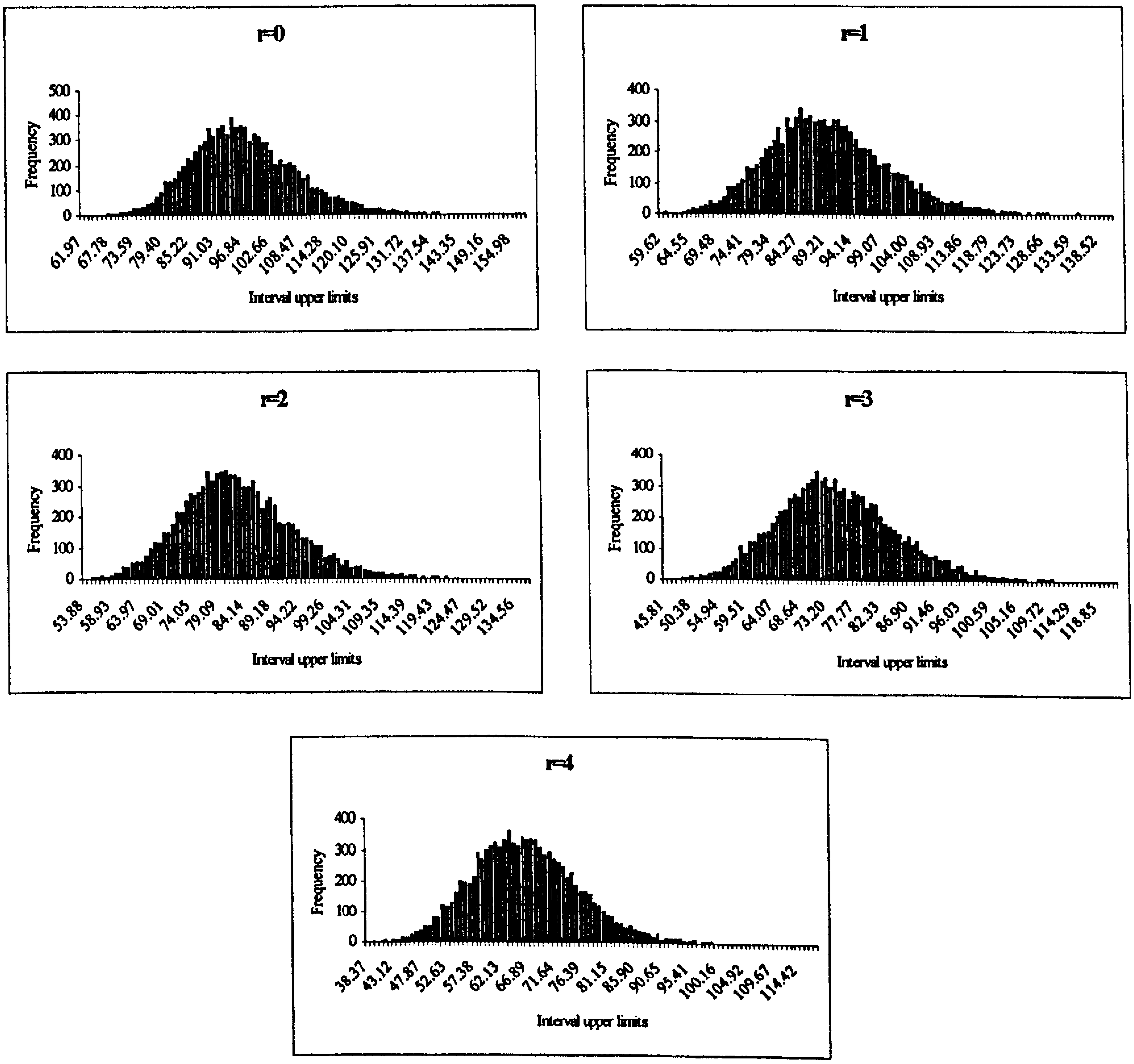


Figure 5.3 (continued): Small-Sample Distribution of the *max-eigenvalue* Cointegration Rank Statistic for the Conditional Model. Based on a Non-Parametric Bootstrap with 10,000 Simulations.

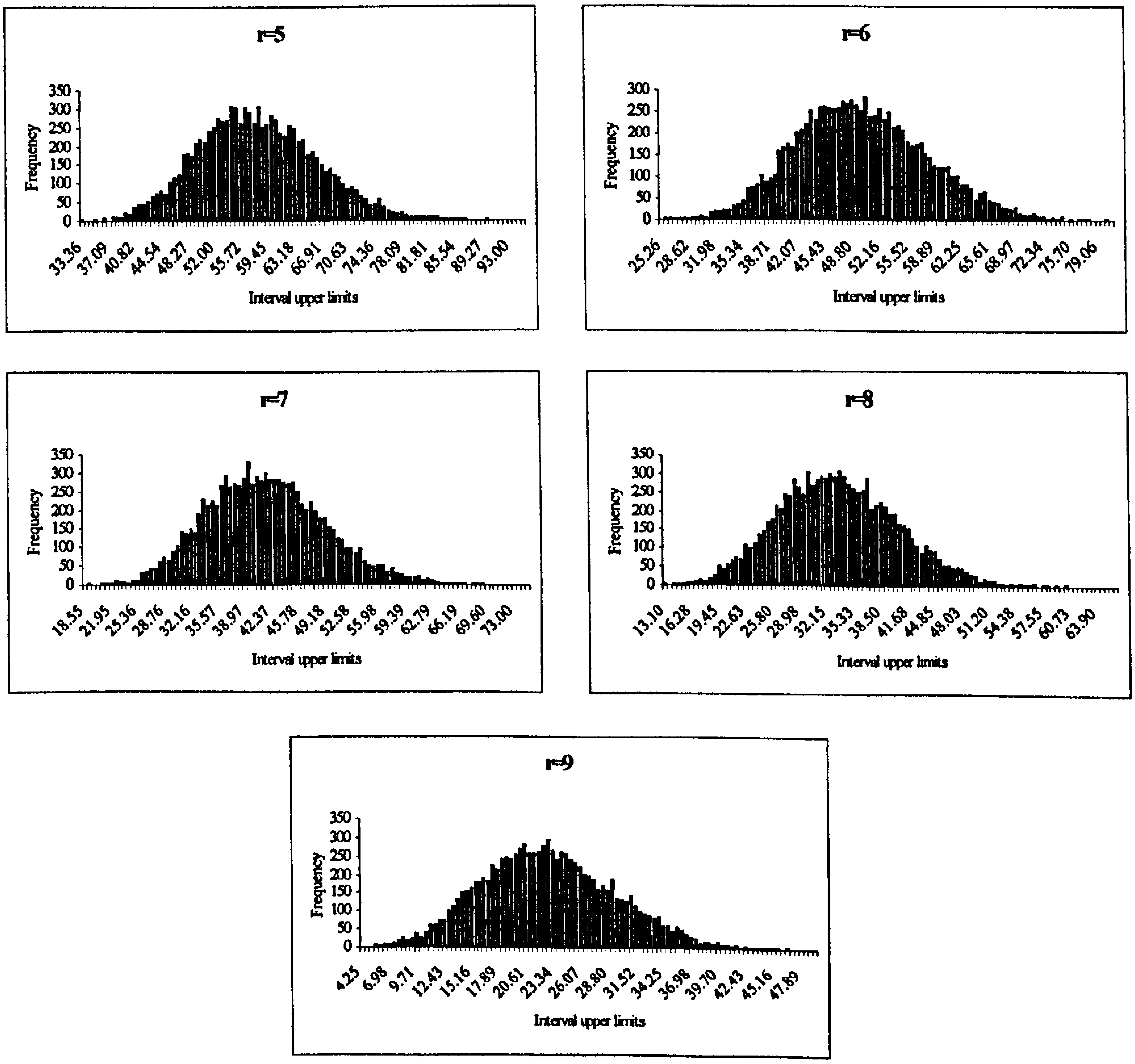


Figure 5.4: Small-Sample Distribution of the *λ-trace* Cointegration Rank Statistic for the Conditional Model. Based on a Non-Parametric Bootstrap with 10,000 Simulations.

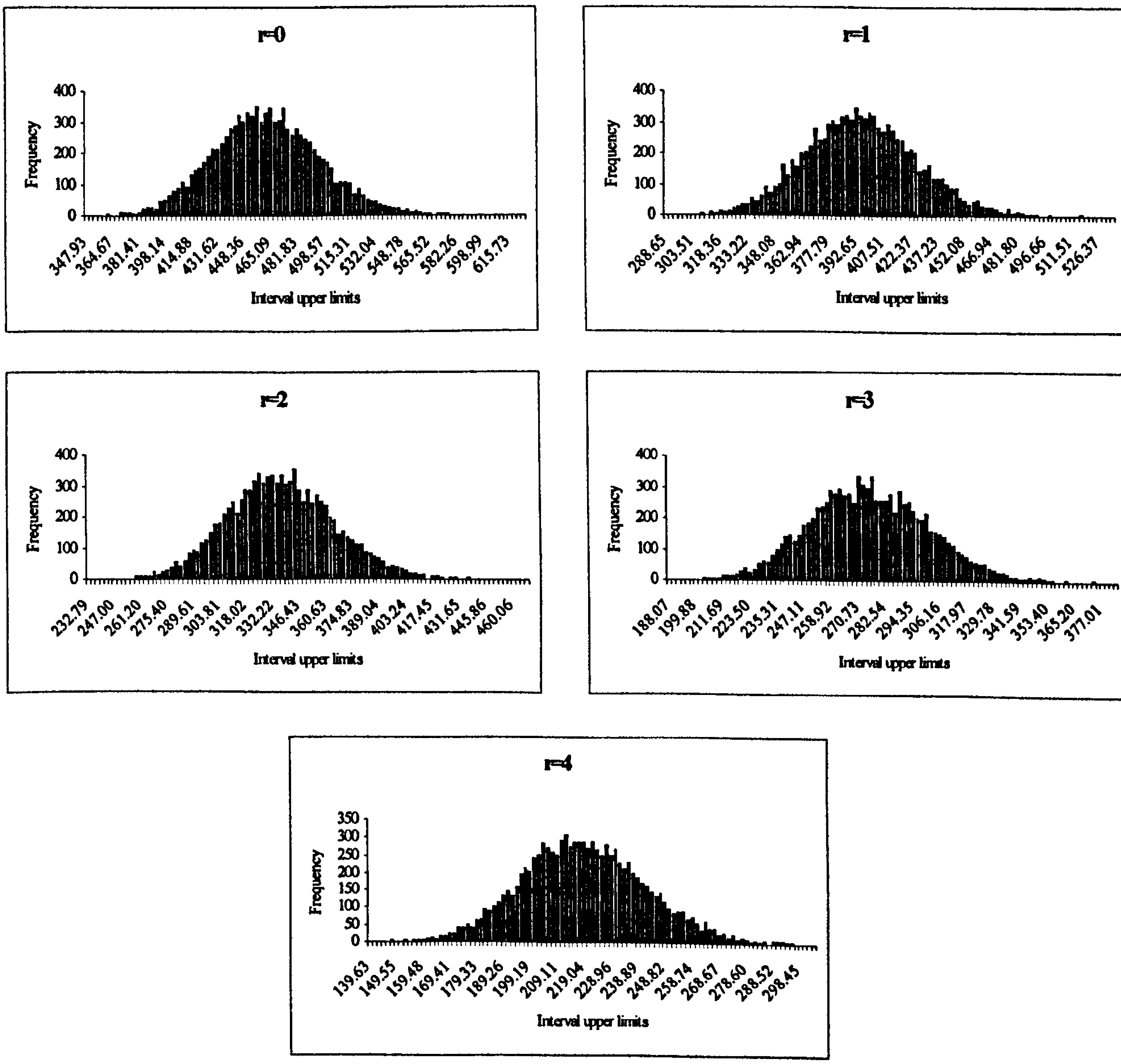
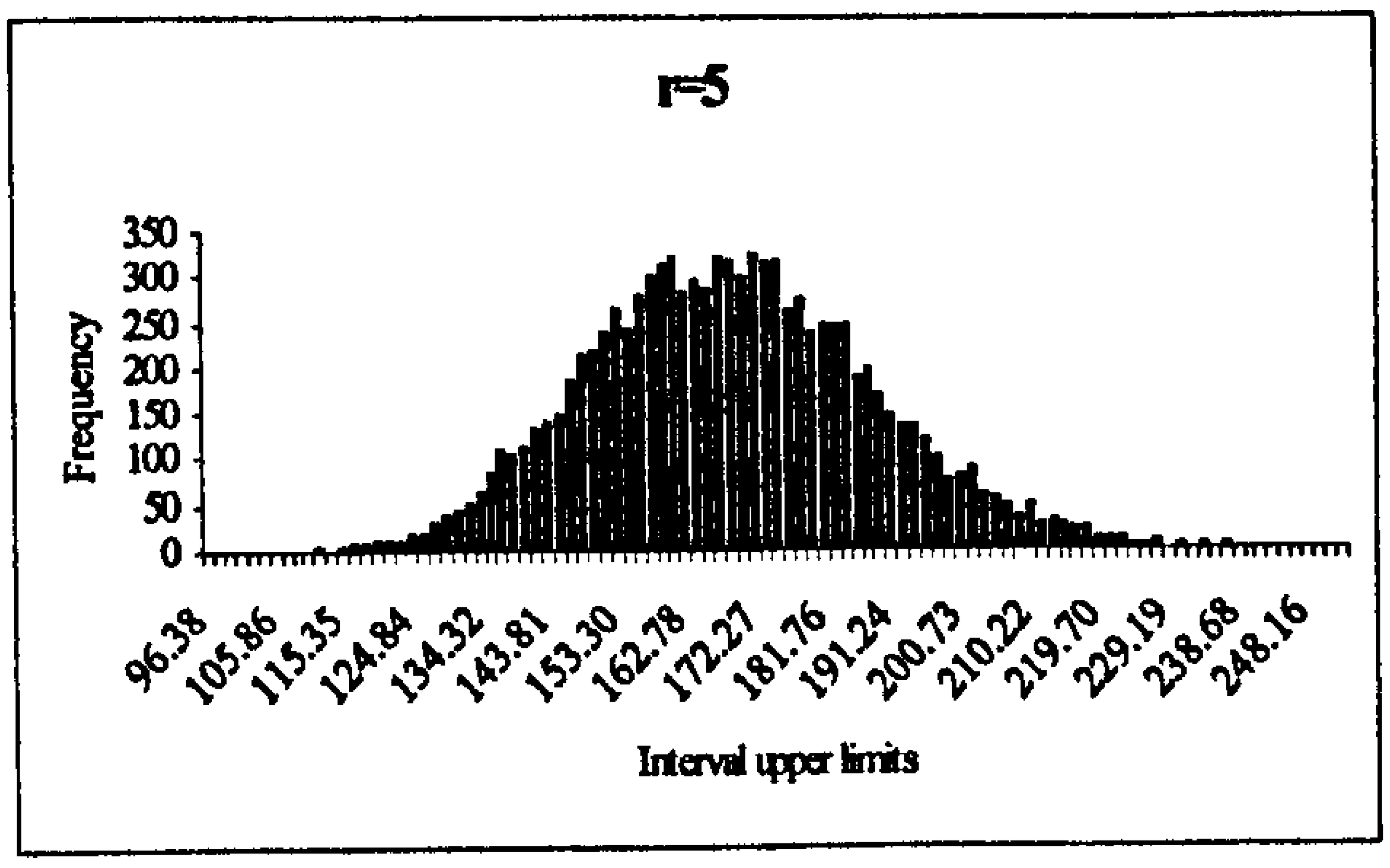
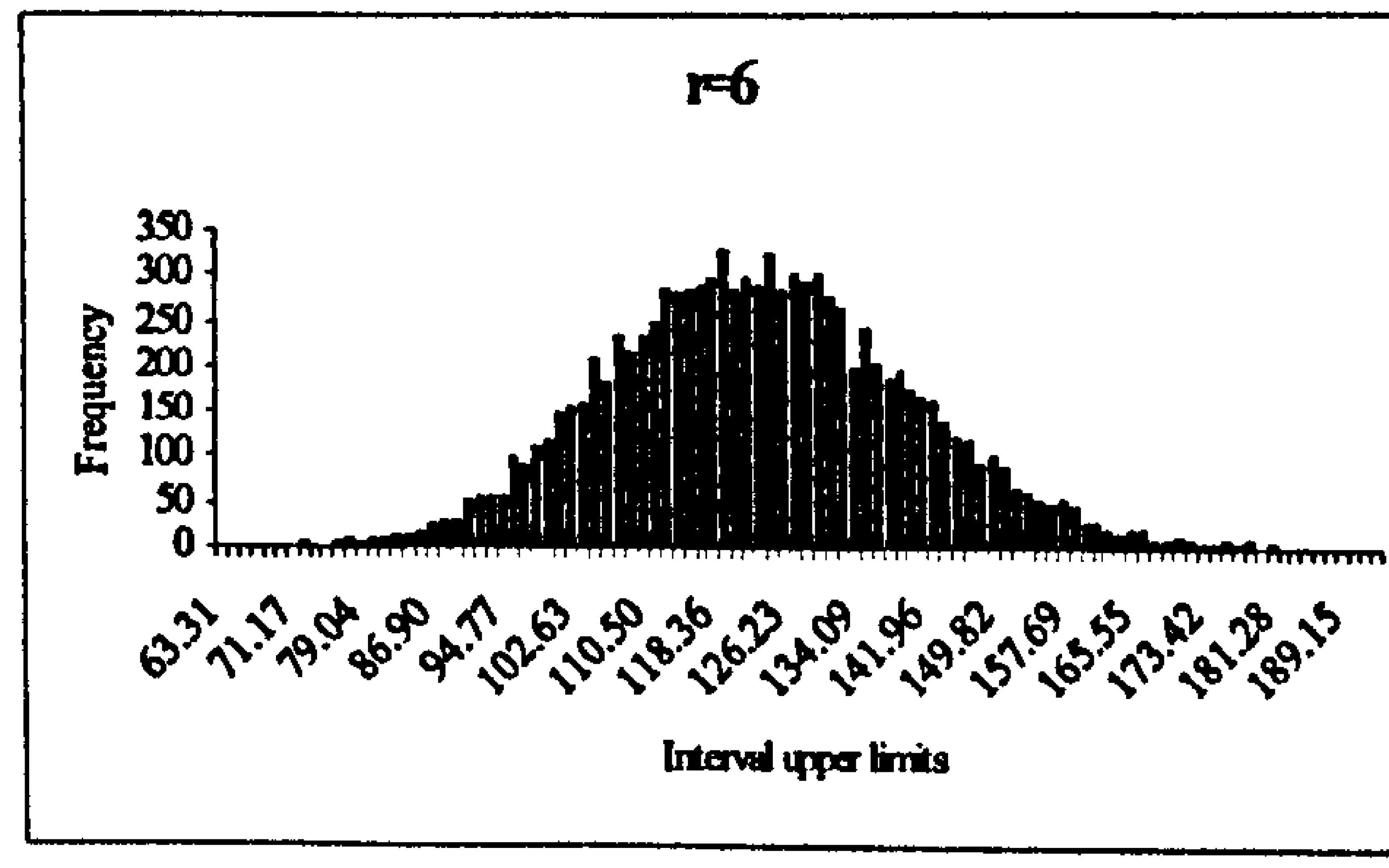
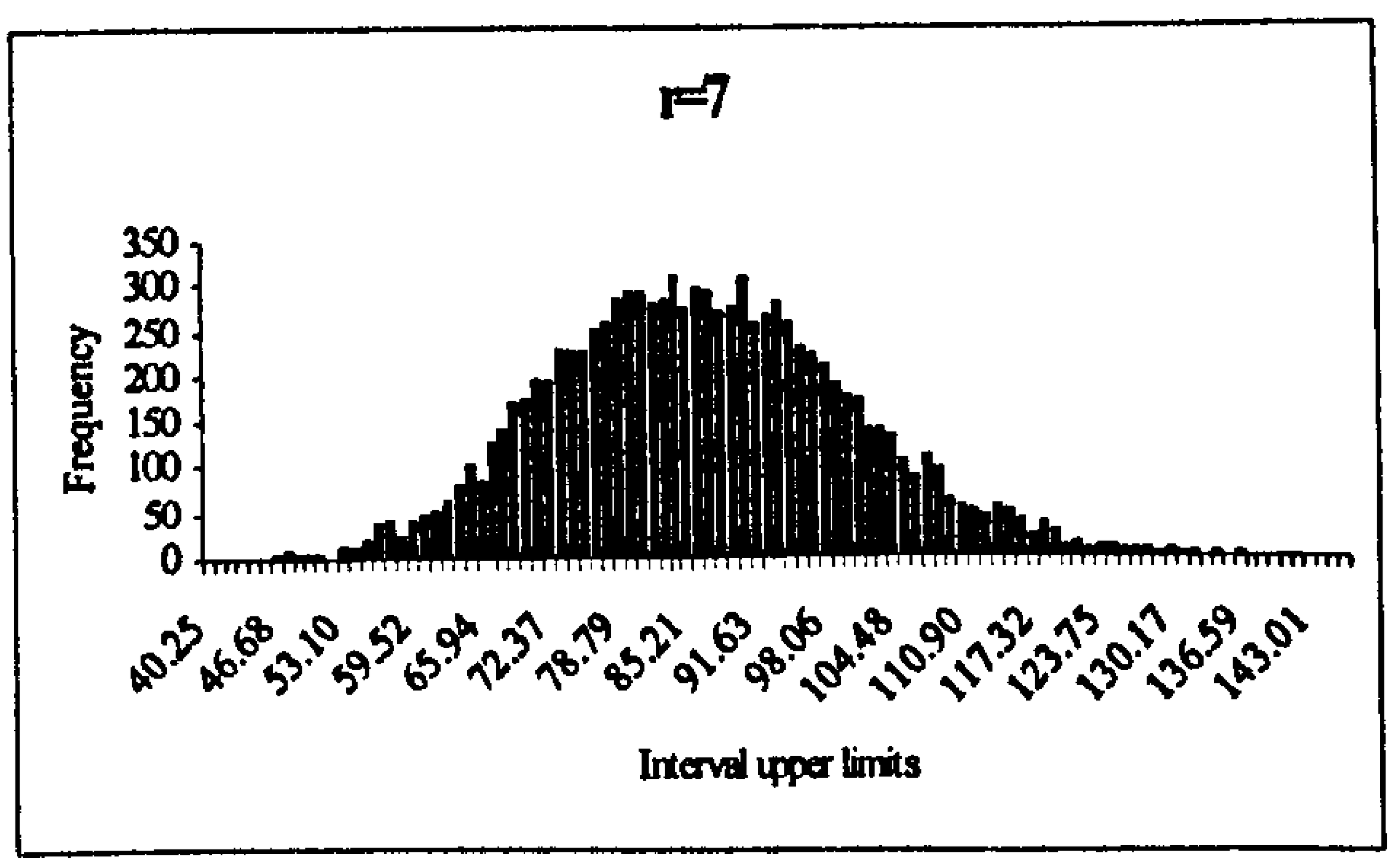
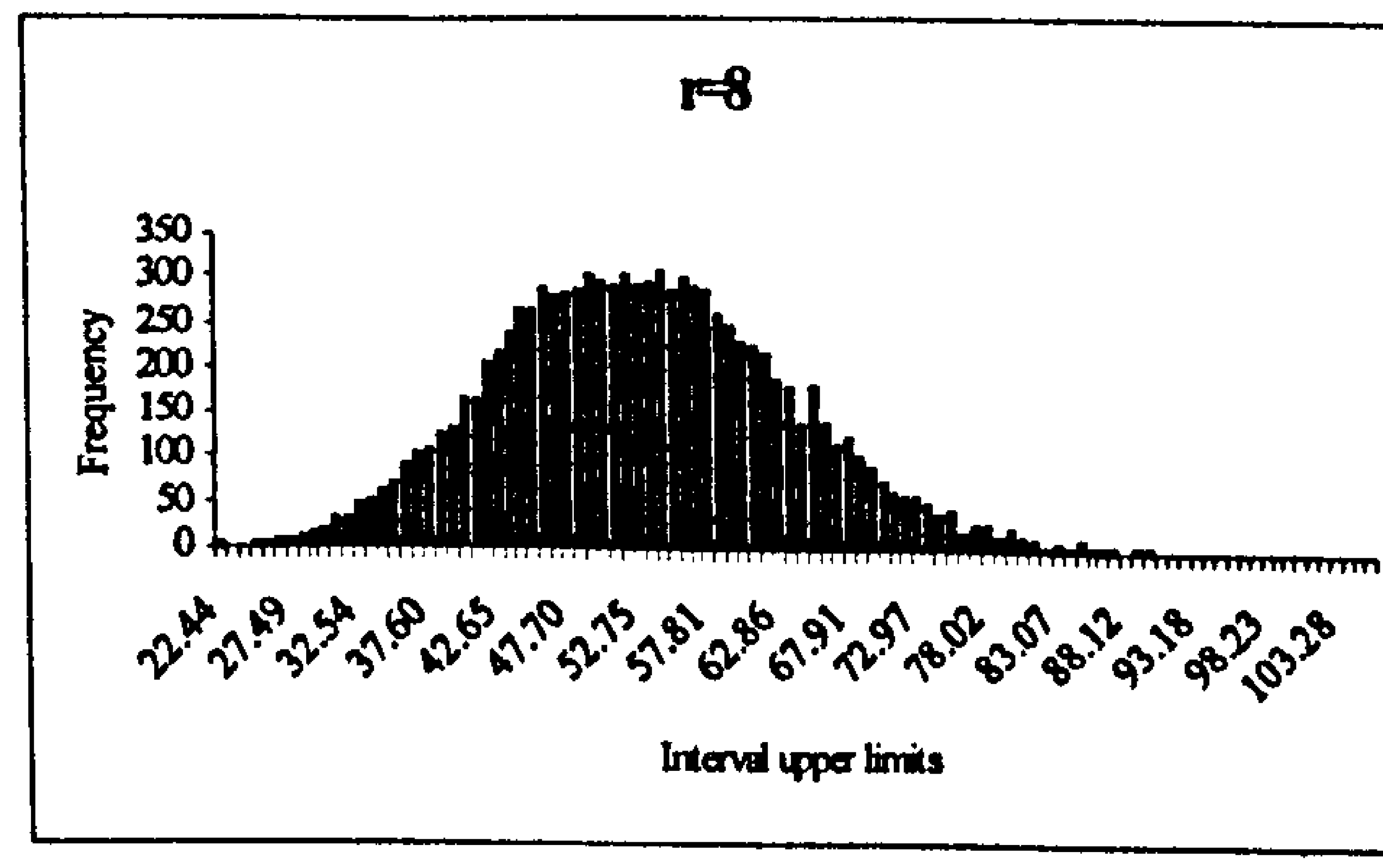


Figure 5.4 (continued): Small-Sample Distribution of the *λ-trace* Cointegration Rank Statistic for the Conditional Model. Based on a Non-Parametric Bootstrap with 10,000 Simulations.









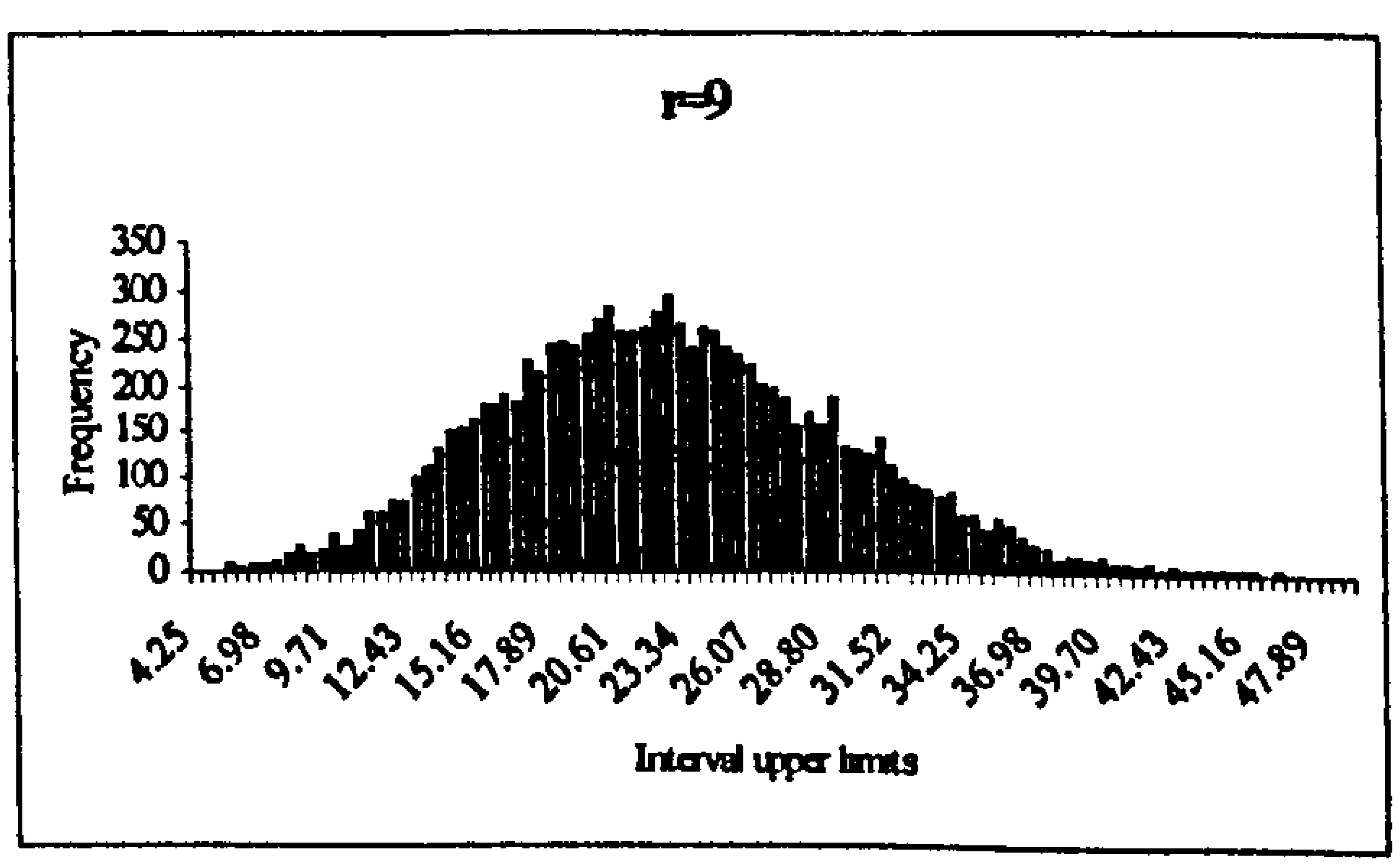


Figure 5.5: Small-Sample Distributions for the *LR* Tests of Over-Identification of the Cointegrating Matrix. Based on 10,000 Simulations.

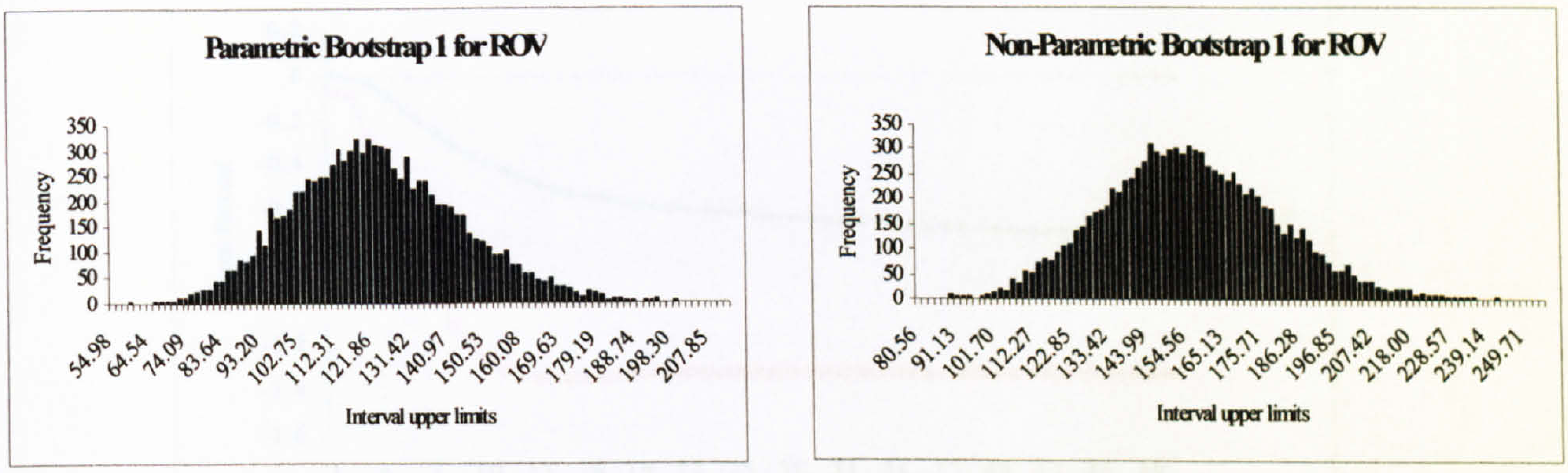


Figure 5.6a: Persistence Profiles for the Estimated LM, IS and BP Relations

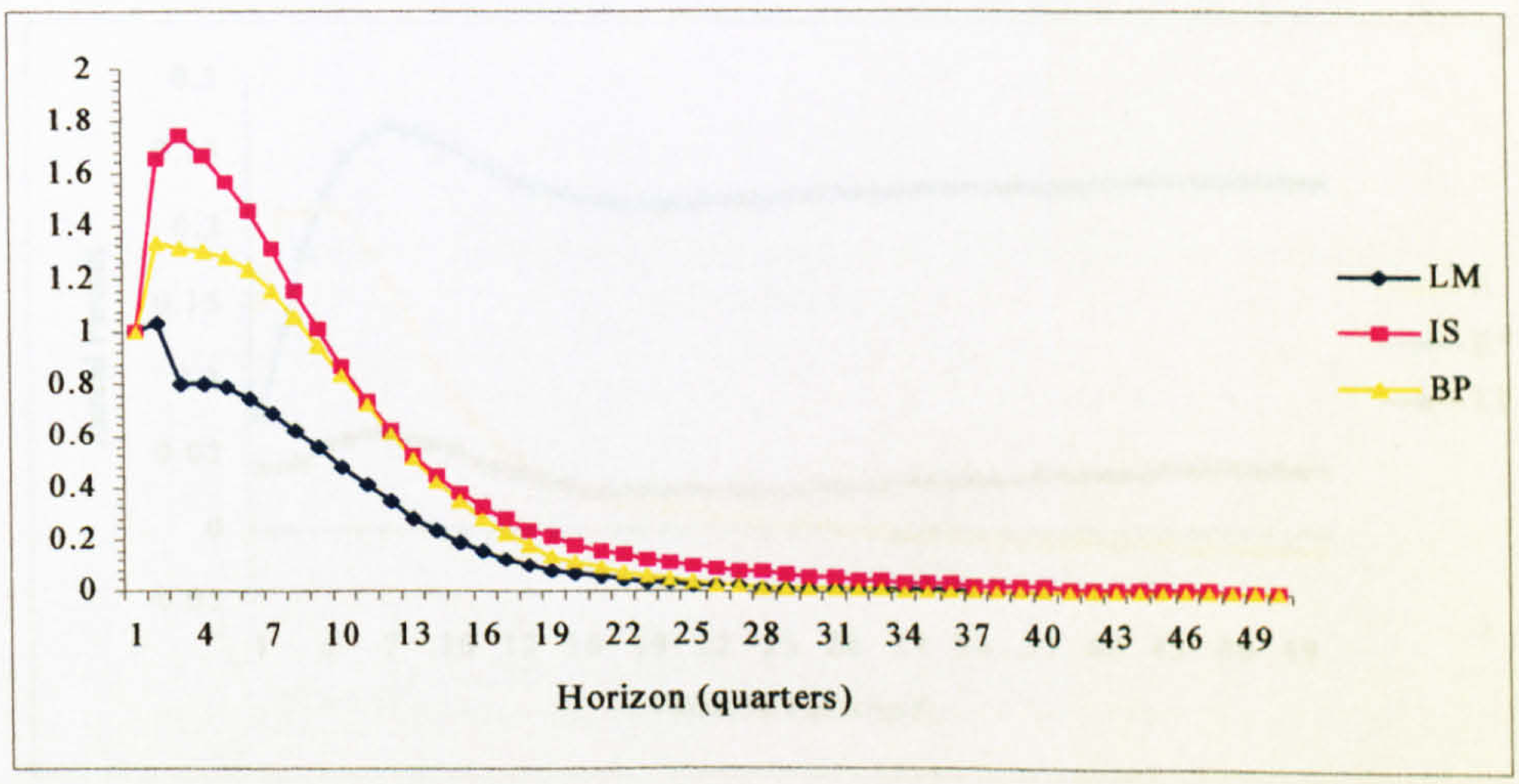


Figure 5.6b: Persistence Profiles for the Estimated Employment (n) and Wage (w) Equations

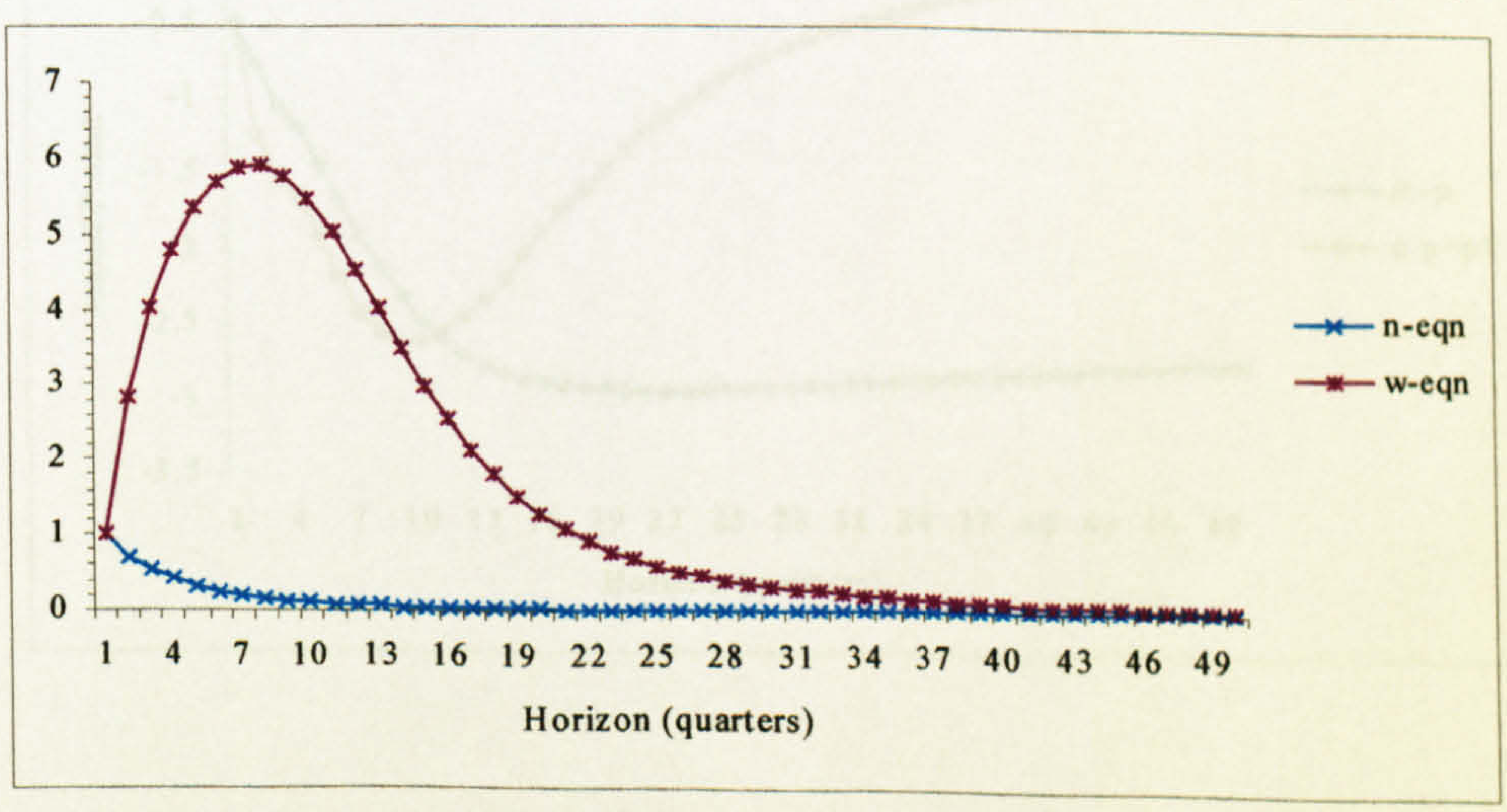
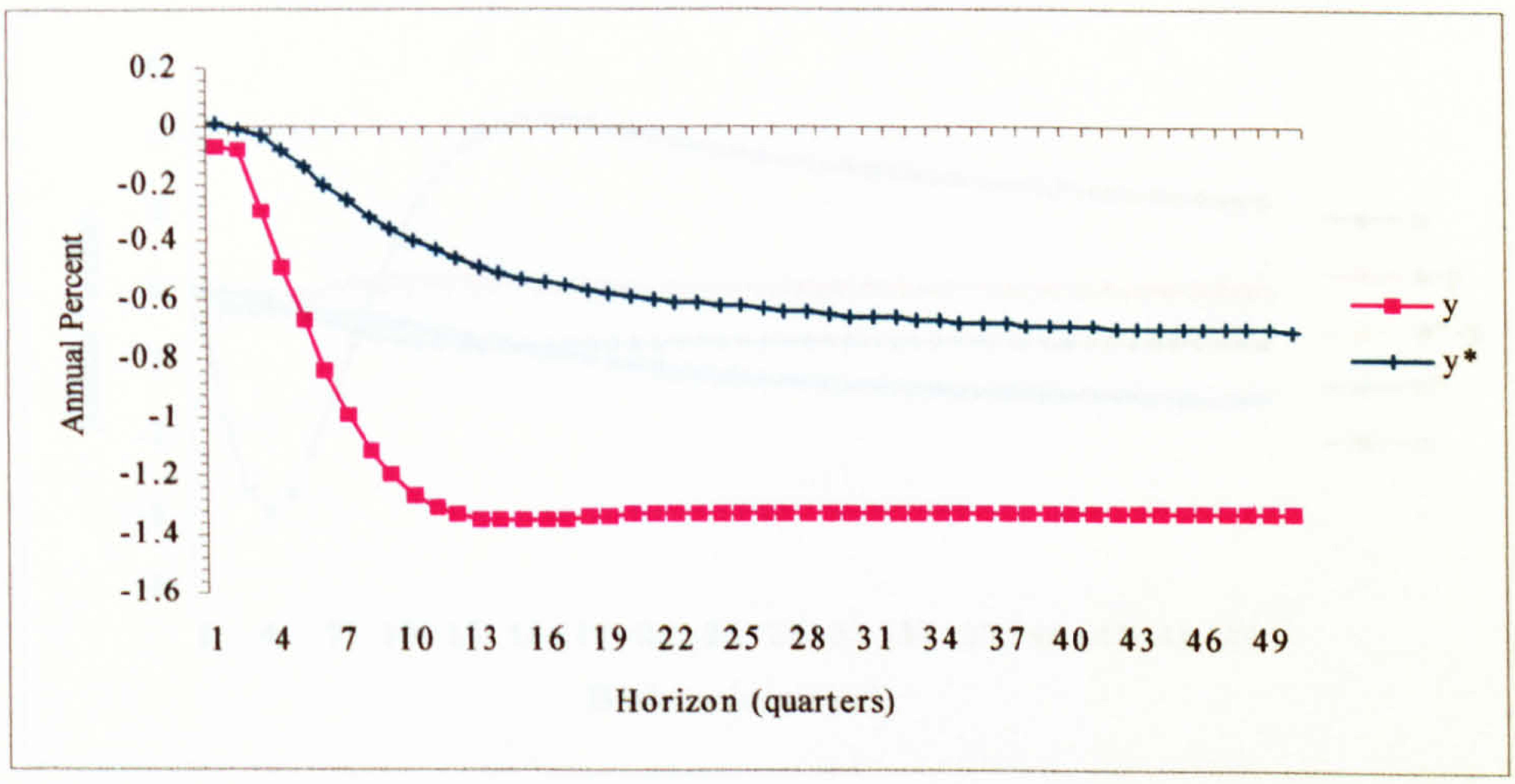
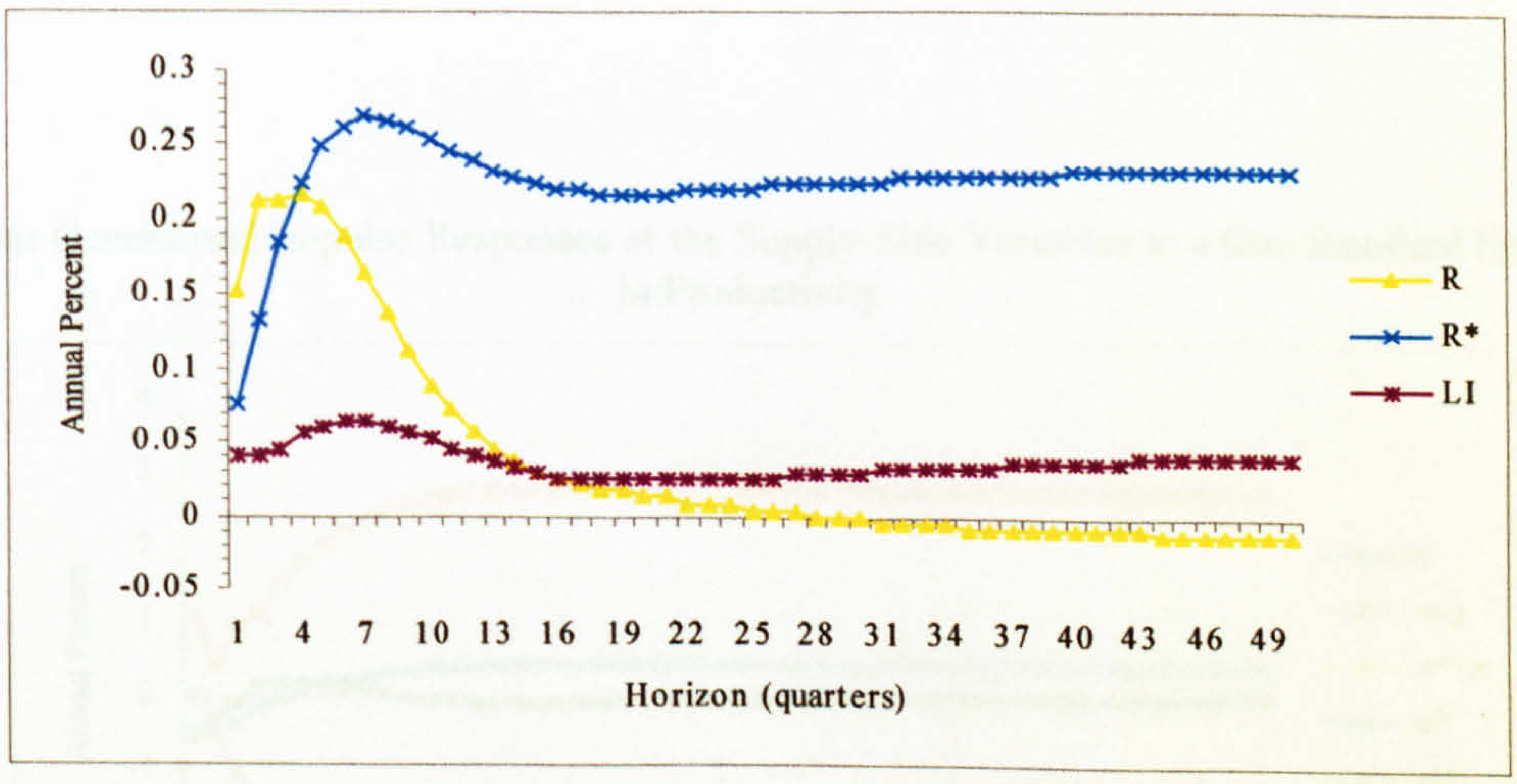


Figure 5.7a: Generalised Impulse Responses of the Demand-Side Variables to a One Standard Error Oil Price Shock





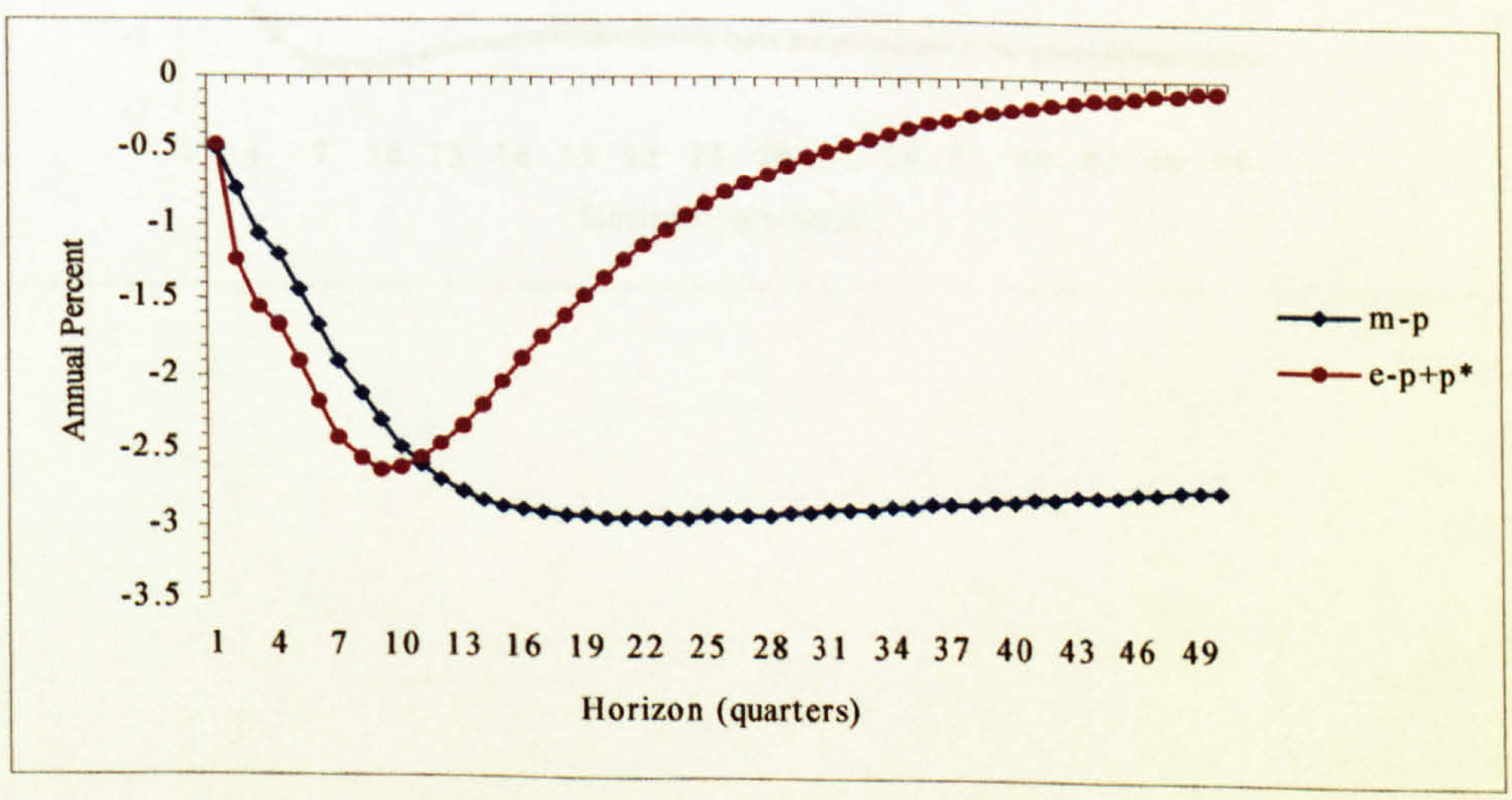


Figure 5.7b: Generalised Impulse Responses of the Supply-Side Variables to a One Standard Error Oil Price Shock

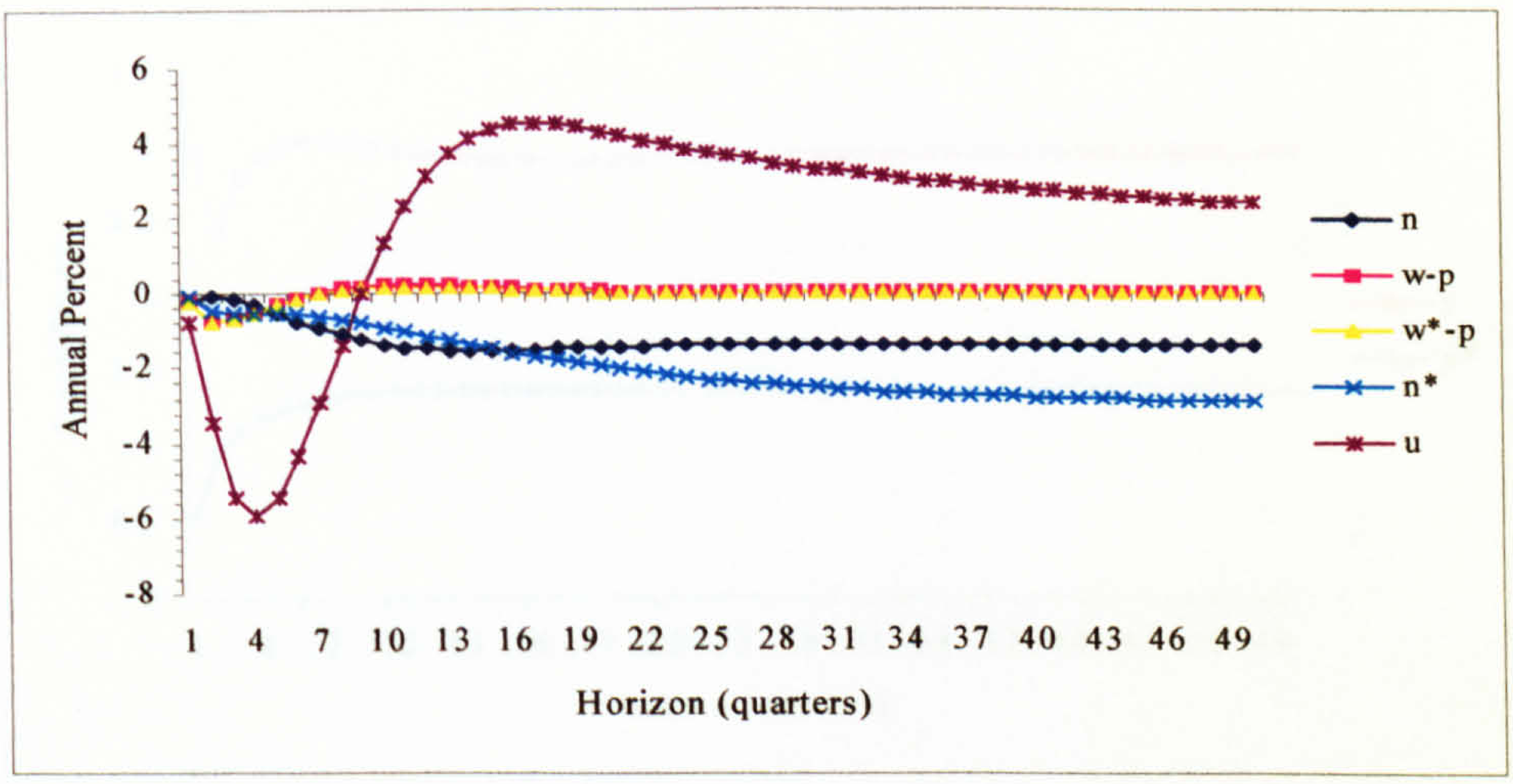


Figure 5.8a: Generalised Impulse Responses of the Supply-Side Variables to a One Standard Error Shock in Productivity

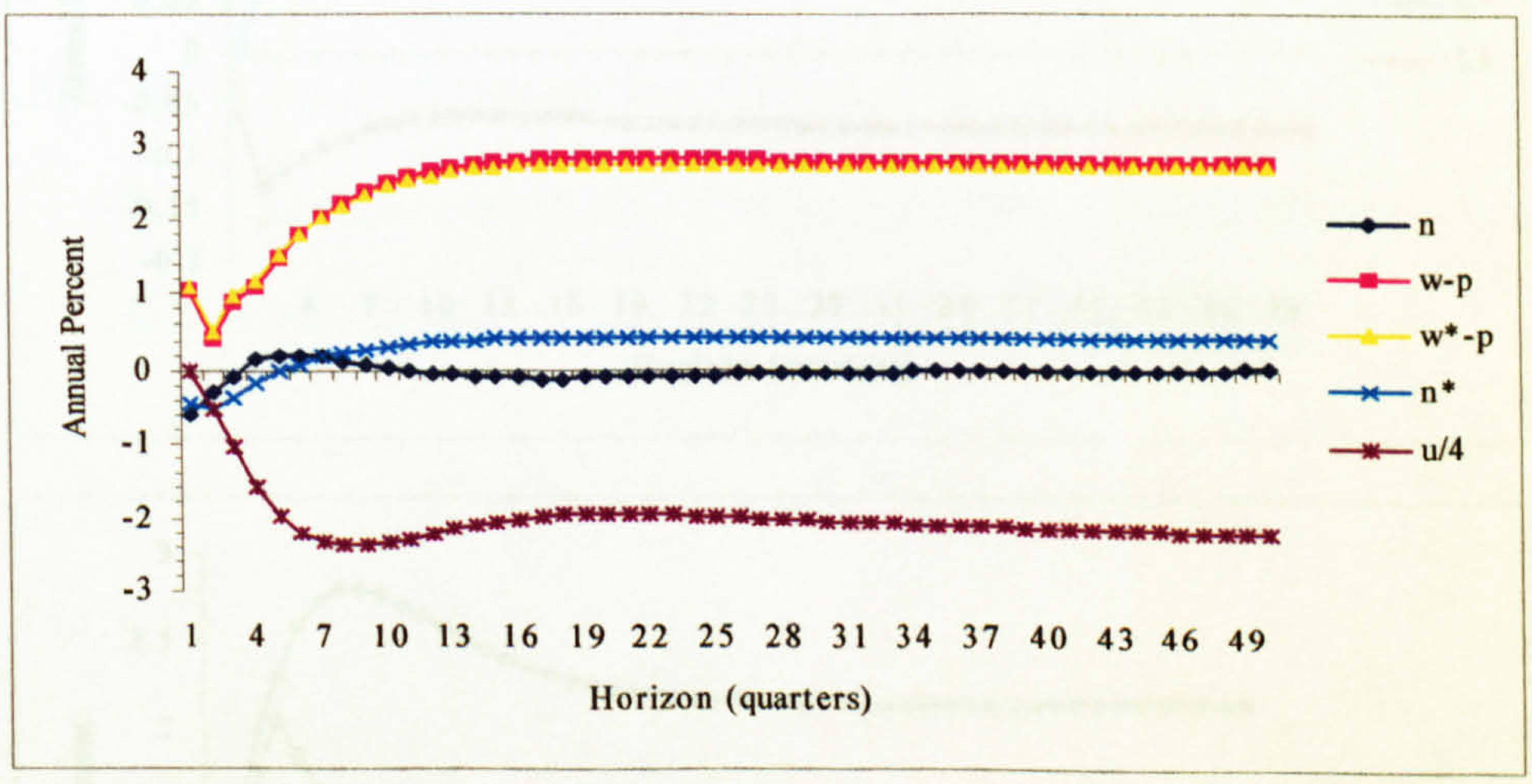
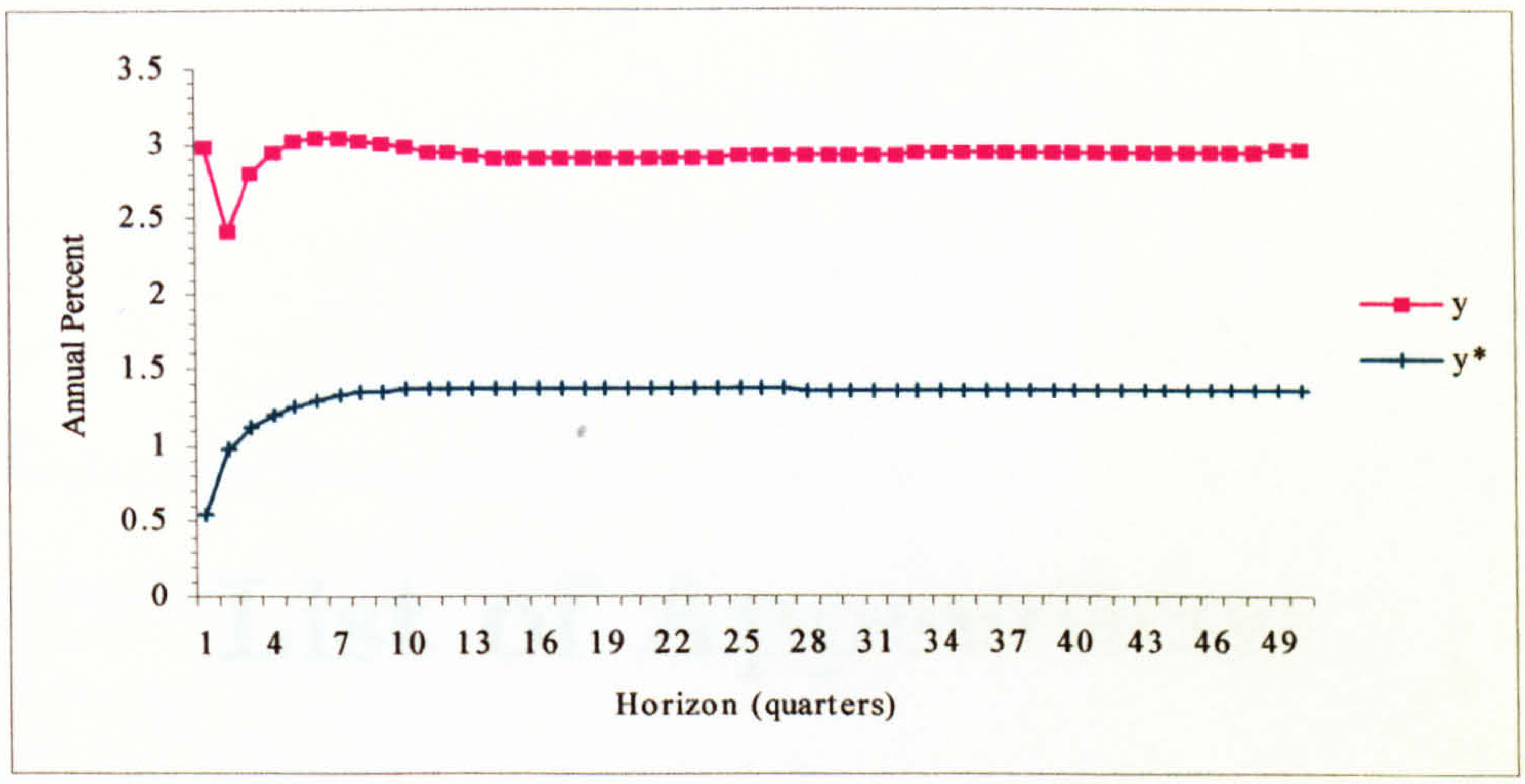
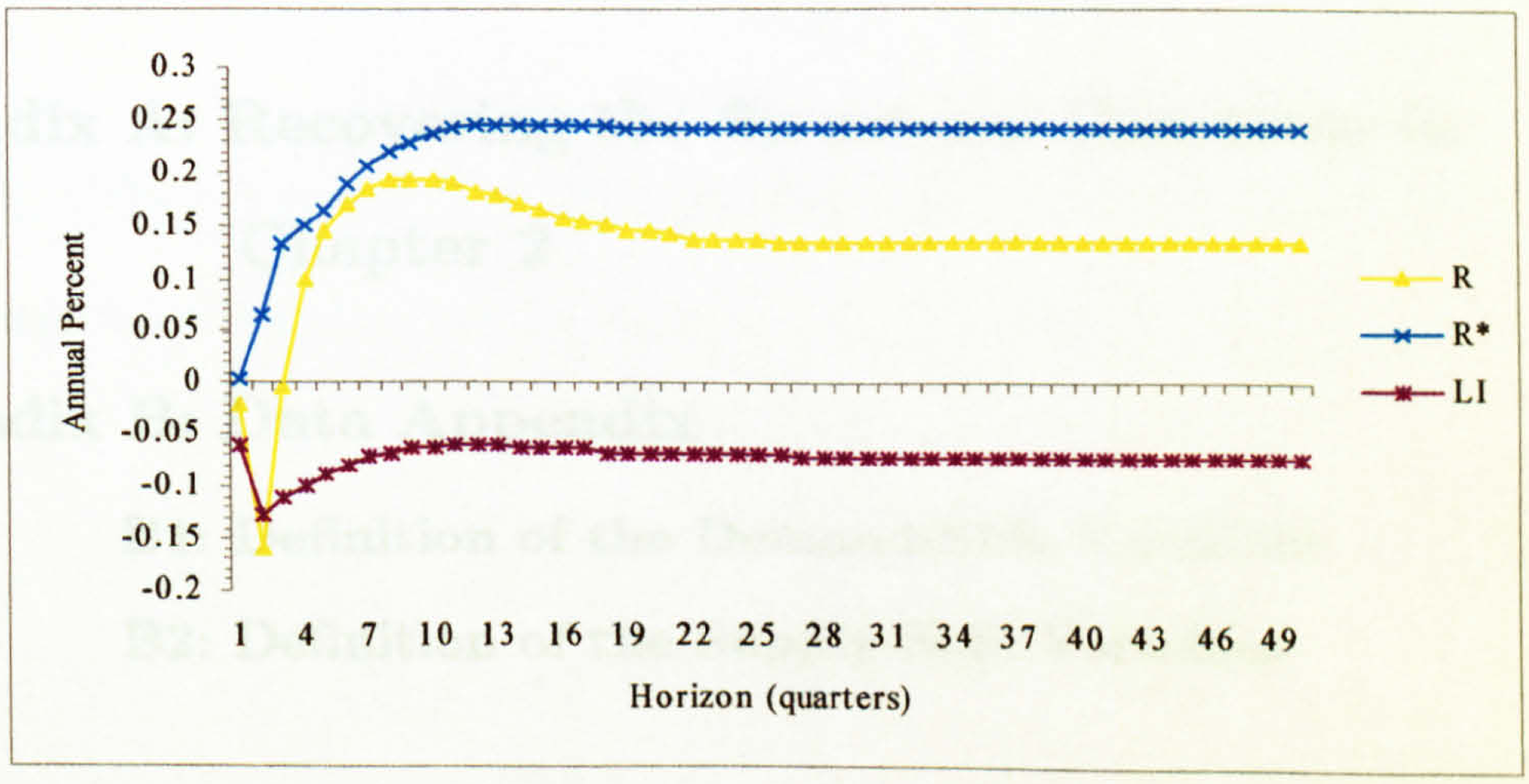
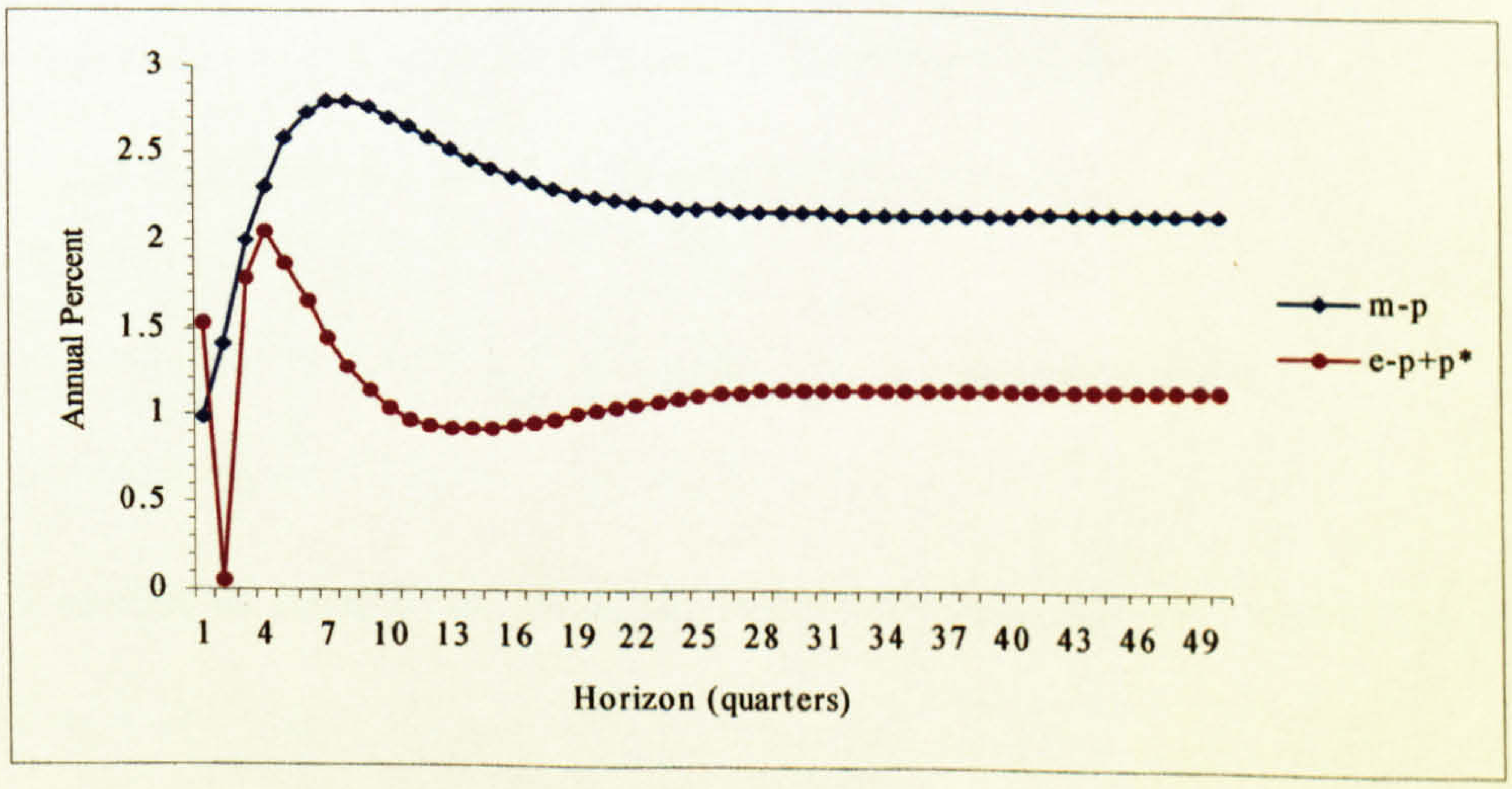


Figure 5.8b: Generalised Impulse Responses of the Demand-Side Variables to a One Standard Error Shock in Productivity







List of Appendices

Appendix A: Recovering the Structural Constants in Chapter 2

From the definitions of the structural relations (2.35)-(2.38) we have

$$\begin{aligned} \varepsilon_{i,t} &= \eta_{i,t}, \text{ for } i = 1,2,4, \text{ and} & \text{(A1)} \\ \varepsilon_{3,t} &= \eta_{3,t} + \eta_{e,t+1} + \Delta e_{t+1}, & \text{(A2)} \end{aligned}$$

where the structural disturbances $\eta_{1,t}$, $\eta_{2,t}$, $\eta_{3,t}$ and $\eta_{4,t}$ are defined in (2.7),(2.10), (2.18) and (2.28), respectively, while $\eta_{e,t+1}$ is defined in (2.25). From (2.39) we have

$$\varepsilon_{i,t} = d_{0i} + \boldsymbol{\beta}'_{*i}\mathbf{z}_{*t}, \text{ for } i = 1,2,3,4, \qquad \text{(A3)}$$

where $\boldsymbol{\beta}'_{*i}$ is the i-th row of $\boldsymbol{\beta}'_{*} = [\boldsymbol{\beta}', \mathbf{d}_1]$ and $\mathbf{z}_{*t} = [\mathbf{z}'_t, t]'$. Substituting (A3) in (A1) and (A2), taking expectations and solving for d_{0i}, $i = 1,2,3,4$, yields

$$\begin{aligned} d_{0i} &= E[\eta_{i,t}] - E[\boldsymbol{\beta}'_{*i}\mathbf{z}_{*t}], \text{ for } i = 1,2,4, \text{ and} & \text{(A4)} \\ d_{03} &= E[\eta_{3,t} + \eta_{e,t+1} + \Delta e_{t+1}] - E[\boldsymbol{\beta}'_{*3}\mathbf{z}_{*t}], & \text{(A5)} \end{aligned}$$

where $E[\eta_{i,t}] = 0$, $i = 1,2,3,4$, by definition. Therefore, using $\hat{\boldsymbol{\beta}}'_{*}$ from the estimation of (2.44) the structural constants d_{0i}, $i = 1,2,4$, may be obtained as

$$d_{0i} = -E[\hat{\boldsymbol{\beta}}'_{*i}\mathbf{z}_{*t}], \text{ for } i = 1,2,4. \qquad \text{(A6)}$$

However, as can be seen from (A5), the retrieval of d_{03} further requires prior knowledge of

$E[\eta_{e,t+1}]$. Under rational expectations $E[\eta_{e,t+1}] = 0$, in which case d_{03} may be obtained as

$$d_{03} = E[\Delta e_{t+1}] - E[\hat{\beta}'_{*3}\mathbf{z}_{*t}]. \tag{A7}$$

Appendix B: Data Appendix

B.1 Definition of the Demand-Side Variables

This appendix defines the empirical counterparts of the variables m_t, y_t, R_t, R_t^*, LI_t, e_t, p_t, p_t^*, p_t^o, y_t^*, $pre73_t$ and ERM_t. The choice of these variables is motivated by the modified IS-LM-BP model with a PPP condition outlined in section 2.2 of Chapter 2. They were constructed using the data set of Garratt *et al* (1998), which was kindly made available by Professor K. C. Lee. The data is quarterly, seasonally adjusted and extends over the period 1963q1-1998q2. Time plots of the variables are to be found in Figure 2.1. The variables are defined as follows:

m_t is the natural logarithm of the UK M0 monetary aggregate measured at the end period in million pounds,

y_t is the natural logarithm of the UK GDP measured at 1995 market prices in million pounds,

R_t is computed as $R_t = 0.25\ln[1 + (ar_t/100)]$, where ar_t is the UK 90-day Treasury Bill average discount rate *per annum*,

R_t^* is computed as $R_t^* = 0.25\ln[1 + (ar_t^*/100)]$, where ar_t^* is the weighted average of 90-day interest rates *per annum* in the United States, Germany, Japan and France. The weights are Special Drawing Right (SDR) weights of the IMF, rescaled so as to exclude the UK,

LI_t is computed as $LI_t = 0.25\ln[1 + (ali_t/100)]$, where ali_t is the annual return on long dated (20 years) UK government securities,

e_t is the natural logarithm of the UK effective exchange rate, measured as the domestic price of foreign currency,

p_t is the natural logarithm of the UK producer price index with 1990 as base year,

p_t^* is a weighted average of the natural logarithms of price indices with 1990 as base year of UK's 42 trading partners. The weights are chosen as the share of UK imports from these countries during the period 1985-1989,

p_t^o is the natural logarithm of the average crude oil price published by the IMF,

y_t^* is the natural logarithm of the total GDP volume index of all OECD member countries,

$pre73_t$ is a dummy variable taking the value of one for $t <$ 1973q1 and zero otherwise and

ERM_t is a dummy variable taking the value of one for 1990q4 $\leq t \leq$ 1992q3 and zero otherwise.

B.2 Definition of the Supply-Side Variables

This appendix defines the empirical counterparts of the variables n_t, $w_t - p_t$, $w_t^* - p_t$, n_t^*, k_t, a_t. The choice of these variables is motivated by the Lee and Papaikonomou (2002) aggregation of the sectoral labour market model in Lee and Pesaran (1993b), briefly described in section 4.2 of Chapter 4. The main data source is the ONS on-line data base accessed via the Data Archive, University of Essex, at http://www.data-archive.ac.uk/findingData/ns.asp. The data are quarterly and cover the period 1965q1-2000q1. Time plots of the variables are to be found in Figure 4.1. The series were constructed as follows:

$$n_t = ln_t = \ln[(emp_t)(ahrs_t)]$$

, where emp_t is the MGRZ series, Table FR1 from the ONS on-line database of *Labour Market*

Statistics (LMS) entitled "Employees in Employment in the UK, (ALL: aged 16+, seasonally adjusted)" and $ahrs_t$ is the YBUV series, Table FR7 from the same source, entitled "Average Actual Weekly Hours of Work, (ALL workers in main and 2nd job, seasonally adjusted)".

$$w_t - p_t = lrwe_t = \ln(wbara_t/YBGB) + t1$$

, where $wbara_t = \frac{LNMQ[1.3-0.3(nhrs_t/45)]}{nhrs_t+1.3(ahrs_t-nhrs_t)}$, for $ahrs_t \geqslant nhrs_t$. LNMQ is found in Table FR15 in the ONS on-line data base of LMS, entitled "Average Earnings Index (1995=100, whole economy, seasonally adjusted)". $nhrs_t$ is the "Normal Basic Hours in Manufacturing" series from the LMS up to 1991q1 and spliced with the "Usual Weekly Hours of Work" produced by the *Labour Force Survey* thereafter. The latter series was transformed into hours using the formula $uhrs_t = \frac{[10(YCDP)+23(YCDS)+38(YCDV)+52(YCDY)]}{(YCDP+YCDS+YCDV+YCDY)}$, where YCDP is employees working 6-15 hrs, YCDS is 16-30 hrs, YCDV=31-45 hrs and YCDY is >45 hrs. YBGB is found in Table 1.1 in the ONS on-line data base of *Economic Trends Annual Supplement* (ETAS), entitled "Gross Domestic Product Deflator, (at market prices, seasonally adjusted)". $t1$ is a measure of the *"employment tax"* borne by the firm suggested by Layard, Nickell and Jackman (1991) and is constructed as $t1 = \ln(\text{LNNL}/\text{LNNK})$, where LNNL is found in Table EG1 in the ONS on-line data base of *Employment and Earnings* (EG), entitled "Unit Labour Cost: Whole Economy, (1995=100, seasonally adjusted)" and LNNK is found in Table FR17 in the ONS on-line data base of LMS, entitled "Unit Wage Costs: Whole Economy, (1995=100, seasonally adjusted)". $ahrs_t$ is defined above.

$$w_t^{**} = lrwestar2_t = w_t^a + \frac{1-\Pi_t}{\Pi_t}(rho_t/100)$$

, where $w_t^a = lrwea_t = lrwe_t + \ln(mult_t)$ with $mult_t$ being the alternative wage scaling factor measured as $mult_t = 1 - \frac{\sqrt{3}\pi}{6}\delta_t$. This measure is based on the ideas expressed in Chapter 4, section 2 and Appendix A in Lee and Papaikonomou (2002). δ_t is the measure of sectoral wage dispersion computed as $\delta_t = \frac{S(W_{it})}{E(W_{it})}$. The quarterly data on nominal sectoral wages, W_{it}, was constructed using the three-month average of the (smoothed)[10] monthly "Average Earnings Index: All Employees: By Industry, Great Britain, 1990=100" and the data on "Average Gross Weekly Pay: by Industry, in pounds" for full-time employees on adult rates. The former was obtained from various issues of *Labour Market Trends* (formerly *Employment Gazette* and *Employment and Productivity Gazette*) as well as from the May 2000 issue of the *Monthly Digest of Statistics* (MDS). The latter was collected from Table A5 in the April 2000 issue of the *New Earnings Survey*. The data was collected for the following 16 sectors[11]: Food, Textiles, Chemicals, Metals (Metal Processing and Manufacturing), Metal Goods (Fabricated Metal Products), Mechanical Engineering (Machinery and Equipment), Electrical Engineering, Motor Vehicles (Transport Equipment), Electricity Gas & Water, Construction, Transport and Communication, Finance Insurance Banking (Financial Intermediation), Paper Printing & Publishing, Retail Trade and Repairs (Distribution), Public Administration, Agriculture[12]. Π_t, the probability of re-employment, is measured as $\Pi_t = \frac{-11.98\ln(MGSX/100)}{1-11.98\ln(MGSX/100)}$, where MGSX is found in Table FR1 in the ONS on-line data base of LMS, entitled "Unemployment Rate, (% UK), (seasonally adjusted)".[13] rho_t, the replacement ratio, is "% of Social Security Benefits out of Weekly Household Disposable Income", obtained from various issues of the *Family Expenditure*

[10] Using a 12-month Moving Average.

[11] Every effort was made to maintain a meaningful, common classification of sectors throughout the sample period.

[12] After May 1996 the AEI data on Agriculture was obtained from various issues of the *New Earnings Survey*.

[13] Over the range of values of MGSX within the sample period, this specification for Π_t is approximately equal to $0.98(MGSX)^{-0.0085}$.

Survey.

$$n_t^* = lestar_t = le_t + \ln(uden_t/100)$$

, where $le_t = \ln(emp_t)$ and $uden_t$ was made quarterly through linear interpolation of the annual series on Union Density reported in Bain and Price's (1980) *Profiles of Union Growth* up to 1974, the OECD's *Employment Outlook* up to 1989 and the "Union Density, (of employees, %)" series reported in *Labour Market Trends*, Vol. 107, July 1999, p.345 and Vol. 108, July 2000, p.333.

$$a_t = y_t - 0.44k_{t-1} - 0.56n_t$$

This measure follows Holland and Scott (1998) and assumes a Cobb-Douglas production function with constant returns to scale and an employment share $\alpha = 0.56$. y_t is measured by $y_t = \ln(\text{YBHH})$, where YBHH is "GDP at factor cost, (1995 prices, seasonally adjusted)" obtained from Table 1.2 in the ONS on-line data base of ETAS. The quarterly series for the capital stock, k_t, was constructed as in Holland and Scott (1998, p.1073) using the annual series on "Gross Capital Stock, (1995 prices, seasonally adjusted)" reported as CIXX in Table 9.10 in the ONS on-line data base of the Blue Book and the quarterly flows of "Total Gross Fixed Capital Formation, (1995 prices, seasonally adjusted)" reported as NPQT in Table 1.8 of the ONS on-line data base of ETAS. The variable u_t was measured as $u_t = \ln(\text{MGSX}/100)$.

Alternative versions of $\mathbf{z}_t = [n_t, w_t - p_t, w_t^* - p_t, n_t^*, a_t, k_t]'$ were used for robustness check, where n_t is measured as $le_t = \ln(emp_t)$ or $lh_t = \ln[(emp_t)(nhrs_t)]$. The real wage, $w_t - p_t$, has alternatively been constructed as $lrwn_t = \ln(LNMQ/YBGB) + t1$. A different measure for w_t^a is obtained as $lrwna_t = lrwn_t + \ln(mult_t)$ and a different measure for $w_t^{**} - p_t$ as $lrwnstar2_t = lrwna_t + \ln(\Pi_t) + \frac{1-\Pi_t}{\Pi_t}(rho_t/100)$. Alternative measures for the fallback level of employment, n_t^*, have been constructed as $lnstar_t = ln_t + \ln(uden_t/100)$ and $lhstar_t = lh_t + \ln(uden_t/100)$.

Bibliography

[1] Abowd, J., F. Kramarz, T. Lemieux and D. Margolis (1996), "Minimum Wages and Youth Employment in France and the United States" mimeo, Cornell University.

[2] Alogoskoufis, G. and R.P. Smith (1991), "On Error Correction Models: Specification, Interpretation and Estimation", *Journal of Economic Surveys*, 5, 97-127. Reprinted as Ch. 6 in L. Oxley et al. (eds) (1994) Survey in Econometrics, Oxford: Blackwell, 139-170.

[3] Bartlett, M.S. (1937), "Properties of Sufficiency and Statistical Tests", *Proceedings of the Royal Society of London*, Series A, 160, 268-82.

[4] Bean, C. R. (1994), "European Unemployment: A Survey", *Journal of Economic Literature*, 32, 573-619.

[5] Blanchard, O.J. and S. Fischer (1989), *Lectures on Macroeconomics*, Cambridge, MA, MIT Press.

[6] Blanchard, O.J. and D. Quah (1989), "The Dynamic Effects of Aggregate Demand and Supply Disturbances", *American Economic Review*, 79, 655-673.

[7] Blangiewicz, M. and W. Charemza (1989), "Cointegration in Small Samples: Empirical Percentiles, Drifting Moments and Customised Testing", Discussion Paper No. 106, Department of Economics, University of Leicester.

[8] Boyd, D. and R.P. Smith (1999), "Testing for Purchasing Power Parity: Econometric Issues and an Application to Developing Countries", *Manchester School*, 67(3), 287-303.

[9] Burgess, S. (1993), "Non-Linear Dynamics in a Structural Model of Employment", in M.H. Pesaran and S. Potter (eds.) *Non-Linear Dynamics, Chaos and Econometrics.*

[10] Charemza, W. and D.F. Deadman (1997), *New Directions in Econometric Practice*, Edward Elgar (2nd edition), Aldershot.

[11] Charemza, W. and E. Syczewska (1998), "Joint Application of the Dickey-Fuller and KPSS Tests", *Economics Letters*, 61, 17-21.

[12] Cheung, Y. Wong and K.S. Lai (1993), "Finite-Sample Sizes of Johansen's Likelihood Ratio Tests for Cointegration", *Oxford Bulletin of Economics and Statistics*, 55, 3, 313-328.

[13] Davidson, J. and S.G.F. Hall (1991), "Cointegration in Recursive Systems: The Structure of Wage-Price Determination in the UK", *Economic Journal*, 101, 239-151.

[14] Dickey, D.A. and W.A. Fuller (1979), "Distribution of the Estimators for Autoregressive Time Series with a Unit Root", *Journal of the American Statistical Association*, 74, 427-431.

[15] Dolado, J., F. Kramarz, S. Machin, A. Manning, B. Margolis and C. Teulings (1996), "The Economic Impact of Minimum Wages in Europe", *Economic Policy*, 23, 319-372.

[16] Driver, C. (1993), "Measuring the Pace of Structural Change", in C. Driver and P. Dunne (eds) *Structural Change in the UK Economy*, Cambridge University Press.

[17] Efron, B. (1979), "Bootstrap Methods: Another Look at the Jackknife", *The Annals of Statistics*, 7, 1-26.

[18] Engle, R. and G. Granger (1987), "Cointegration and Error-Correction: Representation, Estimation and Testing", *Econometrica*, 55, 251-76.

[19] Fachin, S. (2000), "Bootstrap and Asymptotic Tests of Long-Run Relationships in Cointegrated Systems", *Oxford Bullettin of Economics and Statistics*, 62, 4, 543-551.

[20] Freedman, D.A. (1981), "Bootstrapping Regression Models", *The Annals of Statistics*, 9, 1218-1128.

[21] Gali, J. (1992), "How well does the IS-LM Model fit Postwar US Data?", *Quarterly Journal of Economics*, 107, 709-738.

[22] van Garderen, K.J., K.C. Lee and M.H. Pesaran (2000), "Cross-sectional Aggregation of Non-Linear Models", *Journal of Econometrics*, 95, 285-331.

[23] Garratt, A., K. Lee, M.H. Pesaran and Y. Shin (1998), "A Long Run Structural Macroeconometric Model of the UK", DAE Working Paper No. 9812, University of Cambridge.

[24] Garratt, A., K. Lee, M.H. Pesaran and Y. Shin (2000), "The Structural Cointegrating VAR Approach to Macroeconometric Modelling", Chapter 5 in S. Holly and M. Weale (eds.) *Econometric Modelling; Techniques and Applications*, Cambridge University Press.

[25] Garratt, A., K. Lee, M.H. Pesaran and Y. Shin (2001), "A Long Run Structural Macroeconometric Model of the UK", *Economic Journal* (forthcoming), available at http://www.econ.cam.ac.uk/faculty/pesaran/.

[26] van Giersbergen , N.P.A. (1996), "Bootstrapping the Trace Statistic in VAR Models: Monte Carlo Results and Applications", *Oxford Bulletin of Economics and Statistics*, 58, 2, 391-408.

[27] Goffe, W.L., G.D. Ferrier and J. Rogers (1994), "Global Optimization of Statistical Functions with Simulated Annealing", *Journal of Econometrics*, 60, 65-99.

[28] Gredenhoff, M. and T. Jacobson (1998), "Bootstrap Testing and Approximate Finite Sample Distributions for Tests of Linear Restrictions on Cointegrating Vectors", unpublished paper, Department of Economic Statistics, Stockholm School of Economics.

[29] Greenslade, J.V., S.G. Hall and S.G.B. Henry (2002), "On the Identification of Cointegrated Systems in Small Samples: A Modelling Strategy with an Application to UK Wages and Prices", *Journal of Economic Dynamics & Control*, 26, 1517-37.

[30] Haldrup, N. (1998), "A Review of the Econometric Analysis of I(2) Variables", *Journal of Economic Surveys*, 12, 595-650.

[31] Hamermesh, D. (1995), "Labour Demand and the Source of Adjustment Costs", *Economic Journal*, 105, 430, 620-634.

[32] Hansen, P.R. (2000), "Structural Breaks in the Cointegrated Vector Autoregressive Model", unpublished paper, Department of Economics, University of California, San Diego.

[33] Harris, R.I.D. and G. Judge (1998), "Small Sample Testing for Cointegration using the Bootstrap Approach", *Economics Letters*, 58, 31-37.

[34] Hoffman, D.L. and R.H. Rasche (1991), "Long-Run Income and Interest Elasticities of Money Demand in the United States", *The Review of Economics and Statistics*, 73, 665-674.

[35] Holland, A. and A. Scott (1998), "The Determinants of UK Business Cycles", *Economic Journal*, 108, 1067-1091.

[36] Jacobs, J. and K.F. Wallis (2002), "Comparing SVARs and SEMs: More Shocking Stories", paper presented at the European Meeting of the Econometric Society, Venice, 25-28 August 2002 and the Money, Macro and Finance Research Group Conference, Warwick, 4-6 September 2002.

[37] Jacobson, T., P. Jansson, A. Vredin and A. Warne (2001), "Monetary Policy Analysis and Inflation Targeting in a Small Open Economy: A VAR Approach", *Journal of Applied Econometrics*, 16, 487-520.

[38] Johansen, S. (1988), "Statistical Analysis of Cointegration Vectors", *Journal of Economic Dynamics and Control*, 12, 231-254.

[39] Johansen, S. (1991), "Estimation and Hypothesis Testing of Cointegration Vectors in Gaussian Vector Autoregressive Models", *Econometrica*, 59, 1551-1580.

[40] Johansen, S. (1992), "Cointegration in Partial Systems and the Efficiency of Single Equation Analysis", *Journal of Econometrics*, 52, 389-402.

[41] Johansen, S. (1995), *Likelihood-Based Inference in Cointegrated Vector Autoregressive Models*, Oxford: Oxford University Press.

[42] Johansen, S. (2000a), "A Small-Sample Correction for Tests of Hypotheses on the Cointegrating Vectors", forthcoming in *Journal of Econometrics*, available at http://webdb.iue.it/FMPro.

[43] Johansen, S. (2000b), "A Bartlett Correction Factor for Tests on the Cointegrating Relations", *Econometric Theory* 740-778.

[44] Johansen, S. and K. Juselius (1990), "Maximum Likelihood Estimation and Inference on Cointegration: With Applications to the Demand for Money", *Oxford Bulletin of Economics and Statistics*, 52, 169-210.

[45] Johansen, S. and K. Juselius (1992), "Testing Structural Hypotheses in a Multivariate Cointegration Analysis of the PPP and UIP for UK", *Journal of Econometrics*, 53, 211-244.

[46] Johansen, S. and K. Juselius (1994), "Identification of the Long-Run and the Short-Run Structure: An Application to the IS-LM Model", *Journal of Econometrics*, 63, 7-36.

[47] Johansen, S., R. Mosconi and B.G. Nielsen (2000), "Cointegration Analysis in the Presence of Structural Breaks in the Deterministic Trend", *Econometrics Journal*, 3, 216-249.

[48] Johansen, S. and B.G. Nielsen (1994), " Asymptotics for Tests for Cointegrating Rank in the Presence of Intervention Dummies: Manual for the Simulation Program DisCo", Program and Manual at http://www.math.ku.dk/~sjo.

[49] Kilian, L. (1997), "Impulse Response Analysis in Vector Autoregressions with Unknown Lag Order", unpublished manuscript, University of Michigan.

[50] King, R.G., C.I. Plosser, J.H. Stock and M.W. Watson (1991), "Stochastic Trends and Economic Fluctuations", *American Economic Review*, 81, 819-840.

[51] Koop, G., M.H. Pesaran and S.M. Potter (1996), "Impulse Response Analysis in Nonlinear Multivariate Models", *Journal of Econometrics*, 74, 119-147.

[52] Layard, R.G.P. and S.J. Nickell (1985), "The Causes of British Unemployment", *National Institute Economic Review*, 111.

[53] Layard, R., S. Nickell and R. Jackman (1991), *Unemployment: Macroeconomic Performance and the Labour Market*, Oxford University Press.

[54] Lee, K.C. and D. Papaikonomou (2002), "A Long-Run Structural VAR Model of the UK Labour Market", paper presented at the 2002 MMF annual conference at the University of Warwick.

[55] Lee, K.C., M.H. Pesaran and R.G. Pierse, (1990), "Testing for Aggregation Bias in Linear Models", *Economic Journal*, 100, 137-150.

[56] Lee, K.C., M.H. Pesaran and R.G. Pierse, (1992), "Persistence of Shocks and its Sources in a Multisectoral Model of UK Output Growth", *Economic Journal*, 102, 342-356.

[57] Lee, K.C. and M.H. Pesaran, (1993a), "Persistence Profiles and Business Cycle Fluctuations in a Disaggregated Model of UK Output Growth", *Ricerche Economiche*, 47, 293-322.

[58] Lee, K.C. and M.H Pesaran (1993b), "The Role of Sectoral Interactions in Wage Determination in the UK Economy", *Economic Journal*, vol. 103, No. 416, 21-55.

[59] Levtchenkova, S., A.R. Pagan and J.C. Robertson (1998), "Shocking Stories", *Journal of Economic Surveys*, 12, 507-532.

[60] Li, H. and G.S. Maddala (1993), "Bootstrapping Cointegrating Regressions", paper presented at the Fourth Meeting of the European Conference Series in Quantitative Economics and Econometrics, Dec. 16-18, 1993.

[61] Li, H. and G.S. Maddala (1996), "Bootstrapping Time Series Models", *Econometric Reviews*, 15(2), 115-158.

[62] Li, H. and G.S. Maddala (1997), "Bootstrapping Cointegrating Regressions", *Journal of Econometrics*, 80, 297-318.

[63] Lipsey, R.G. and J.M. Parkin (1970), "Incomes Policy: A Reappraisal", *Economica.*

[64] Lucas, R.E. (1976), "Econometric Policy Evaluation: A Critique", in K. Brunner and A.H. Meltzer (eds) *The Phillips Curve and Labour Markets, Carneige Rochester Series on Public Policy*, Volume 1, 19-46.

[65] Lütkepohl, H. (1993), *Introduction to Multiple Time Series Analysis*, Springer-Verlag, Berlin, 2nd ed.

[66] Manning, A. (1993), "Wage Bargaining and the Phillips Curve: The Identification and Specification of Aggregate Wage Equations", *Economic Journal*, 103, 416, 98-118.

[67] Manning, A. (1995), "Developments in Labour Market Theory and their Implications for Macroeconomic Policy", *Scottish Journal of Political Economy*, 42, 3, 250-66.

[68] Mantalos, P. and G. Shukur (1998), "Size and Power of the Error Correction Model Cointegration Test: A Bootstrap Approach", *Oxford Bulletin of Economics and Statistics*, 60(2), 249-255.

[69] Mellander, E., A. Vredin and A. Warne (1992), "Stochastic Trends and Economic Fluctuations in a Small Open Economy", *Journal of Applied Econometrics*, 7, 369-394.

[70] Nelson, C.R and C.I. Plosser (1982), "Trends and Random Walks in Macro-Economic Time Series", *Journal of Monetary Economics*, 10, 139-162.

[71] Nickell, S. (1997), "Unemployment and Labour Market Rigidities: Europe versus North America", *Journal of Economic Perspectives*, 11, 3, 55-74.

[72] Ostrewald-Lenum, M. (1992), "A Note on Quantiles of the Asymptotic Distribution of the Maximum Likelihood Cointegration Rank Test Statistic", *Oxford Bulletin of Economics and Statistics*, 54, 461-472.

[73] Papaikonomou, D. (2001), "Structural Change and Small-Sample Inference in a Long-Run Structural VAR Model of UK Aggregate Demand", paper presented at the 5th Annual Conference on Macroeconomic Analysis and International Finance at the University of Crete.

[74] Park, J.Y. (1992), "Canonical Cointegrating Regressions", *Econometrica*, 60, 119-143.

[75] Patterson, K. (2000), *An Introduction to Applied Econometrics: A Time Series Approach*, MacMillan Press, London.

[76] Pesaran, M.H. (1997), "The Role of Economic Theory in Modelling the Long Run", *Economic Journal*, 107, 178-91.

[77] Pesaran, M.H. and B. Pesaran (1997), *Working with Microfit 4.0: An Interactive Introduction to Econometrics*, Oxford: Oxford University Press.

[78] Pesaran, M.H. and Y. Shin (1995), "An Autoregressive Distribution Lag Modelling Approach to Cointegration Analysis", DEA Working Paper no. 9514, Department of Applied Economics, University of Cambridge.

[79] Pesaran, M.H. and Y. Shin (1996), "Cointegration and Speed of Convergence to Equilibrium", *Journal of Econometrics*, 71, 117-143.

[80] Pesaran, M.H. and Y. Shin (1998), "Generalised Impulse Response Analysis in Linear Multivariate Models", *Economics Letters*, 58, 17-29.

[81] Pesaran, M.H. and Y. Shin (2001), "Long-Run Structural Modelling", *Econometrics Reviews* (forthcoming), available at http://www.econ.cam.ac.uk/faculty/pesaran/.

[82] Pesaran, M.H., Y. Shin and R.J. Smith (2000), "Structural Analysis of Vector Error Correction Models with Exogenous I(1) Variables", *Journal of Econometrics*, (forthcoming), available at http://www.econ.cam.ac.uk/faculty/pesaran/.

[83] Pesaran, M.H. and R.P. Smith (1995), "The Role of Theory in Econometrics", *Journal of Econometrics*, 67, 61-80.

[84] Pesaran, M.H. and R.P. Smith (1998), "Structural Analysis of Cointegrating VARs", *Journal of Economic Surveys*, 12(5), 471-505.

[85] Pesaran, M.H. and M. Weeks (2001), "Non-Nested Hypothesis Testing: An Overview", in (ed) Badi H Baltagi, *Companion to Theoretical Econometrics*, Basil Blackwell, Oxford. ISBN 0 63121 254 X.

[86] Phillips, A.W. (1958), "The Relation Between Unemployment and the Rate of Change of Money Wage Rates in the UK, 1861-1957", *Economica*, 25, 283-299.

[87] Phillips, P.C.B. and P. Perron (1988), "Testing for Unit Roots in Time Series Regressions", *Biometrica*, 75, 335-346.

[88] Politis, D.N. and J.P. Romano (1994), "The Stationary Bootstrap", *Journal of American Statistical Association*, 89, 1303-1313.

[89] Rahbek, A. and R. Mosconi (1999), "Cointegration Rank Inference with Stationary Regressors in VAR Models", *Econometrics Journal*, 2, 76-91.

[90] Reimers, H.E. (1992), "Comparisons of Tests for Multivariate Cointegration", *Statistical Papers*, 33, 335-359.

[91] Reinsel, G.C. and S.K. Ahn (1988), "Asymptotic Properties of the Likelihood Ratio Test for Cointegration in the Nonstationary Vector AR Model", Techincal Report, Department of Statistics, University of Wisconsin, Madison.

[92] Reinsel, G.C. and S.K. Ahn (1992), "Vector Autoregressive Models with Unit Roots and Reduced Rank Structure: Estimation, Likelihood Ratio Test and Forecasting", *Journal of Time Series Analysis*, 13, 353-375.

[93] Sargan, J.D. (1980), "A Model of Wage-Price Inflation", *Review of Economic Studies*, pp.97-112.

[94] Shapiro, M.D. and M. Watson (1988), "Sources of Business Cycle Fluctuations", *NBER Macroeconomics Annual*, Boston: National Bureau of Economic Research.

[95] Sims, C. (1980), "Macroeconomics and Reality", *Econometrica*, 48, 1-48.

[96] Stock, J.H. and M.W. Watson (1993), "A Simple Estimator of Cointegrating Vectors in Higher Order Integrated Systems", *Econometrica*, 61, 783-820.

[97] Stoker, T., (1993), "Empirical Approaches to the Problem of Aggregation over Individuals", *Journal of Economic Literature*, 31, 4, 1827-1874.

[98] Taylor, J. (1980), "Aggregate Dynamics and Staggered Contracts", *Journal of Political Economy*, 88, 1-23.

[99] Tsionas, E.G. (1995), *Gauss* version of the Simulated Annealing algorithm discussed in Goffe *et al* (1994), available on

http://www.american.edu/academic.depts/cas/econ/gaussres/optimize/optimize.htm

[100] Turnovsky, S.J. and M.H. Miller (1984), "The Effects of Government Expenditure on the Term Structure of Interest Rates", *Journal of Money, Credit and Banking*,16, 16-33.

[101] Warne, A. (2000), Lecture Notes on Structural Vector Autoregressions, available on http://research.texlips.org/