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THE M-σ RELATION FOR NUCLEATED GALAXIES

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ABSTRACT

Momentum feedback from super-Eddington accretion offers a simple explanation for the observed M-σ and M-$M_{\rm spher}$ relations between supermassive black holes and the spheroids of their host galaxies. Recently Ferrarese et al. and Wehner & Harris observed analogous relations between the masses of central star clusters and their hosts. We show that stellar winds and supernovae from such nuclear clusters give similar feedback explanations for this case also, and we discuss the connection to the Faber-Jackson relation for the spheroids themselves.

Subject headings: galaxies: formation — galaxies: nuclei — galaxies: star clusters

1. INTRODUCTION

It is well known that the masses of supermassive black holes (SMBHs) in the nuclei of early-type galaxies and late-type bulges correlate tightly with the velocity dispersions of the stellar spheroids: $M_{\rm BH} \propto \sigma^x$, with $x \simeq 4.0$–4.5 (Tremaine et al. 2002; Ferrarese & Ford 2005). $M_{\rm BH}$ also increases nearly linearly with galaxy spheroid mass (e.g., Häring & Rix 2004). It is now becoming clear that many galaxies have nuclear star clusters (NCs) with masses similarly connected to the host properties.

A recent *HST* ACS survey of 100 early-type galaxies in Virgo has found that ~70%–80% of systems with $-20.5 \lesssim M_B \lesssim -15$ contain dense nuclear components that are resolved by *HST* into star clusters with luminosities $L \simeq 10^5$ to $5 \times 10^7\ L_\odot$, half-light radii of about 4 pc (possibly with a size-luminosity relation among the brighter nuclei), and colors suggesting ages of ~2–10 Gyr and metallicities [Fe/H] $\approx -0.5 \pm 1$ (Côté et al. 2006). Most spiral bulges and bulgeless disk galaxies also contain nuclear clusters with rather similar properties (Phillips et al. 1996; Carollo et al. 1998; Böker et al. 2002; Walcher et al. 2005).

All known NCs have masses less than a few $10^8\ M_\odot$, while most measured SMBHs have $M_{\rm BH} \gtrsim 10^8\ M_\odot$. The apparent paucity of low-mass SMBHs is at least partly a selection effect, but the upper limit on the nuclear clusters may well be real. Côté et al. (2006) find that NC luminosity increases with spheroid luminosity, such that $M_{\rm NC} \gtrsim 2 \times 10^8\ M_\odot$ is expected for galaxies with $M_B \lesssim -20.5$. But although they have *HST* surface photometry of the cores of *all* such galaxies in Virgo, Côté et al. find no evidence for nucleation in any of them.

Ferrarese et al. (2006) have obtained long-slit spectra for 29 of the nucleated ellipticals in Virgo. They find that the NC masses correlate well with the galaxies' velocity dispersions averaged over an effective radius (≈1–2 kpc): $M_{\rm NC} \propto \sigma^x$ with $x = 4.3 \pm 0.6$, essentially the same as for the SMBH relation. However, they also find an *offset* between the cluster and black hole scalings. Fitting power-law M-σ relations with $x \equiv 4$ to the Ferrarese et al. NC data and to SMBH data from the literature yields (L. Ferrarese 2006, private communication)

$$\log M_{\rm NC,8} = (1.25 \pm 0.55) + 4 \log \sigma_{200},$$
$$\log M_{\rm BH,8} = (0.25 \pm 0.33) + 4 \log \sigma_{200}, \qquad (1)$$

where $M_8 \equiv M/10^8\ M_\odot$ and $\sigma_{200} \equiv \sigma/(200\ {\rm km\ s^{-1}})$. A limit of $M_{\rm NC} \lesssim 2 \times 10^8\ M_\odot$ thus corresponds to $\sigma \lesssim 120\ {\rm km\ s^{-1}}$. The nuclei of spheroids with velocity dispersion less than this are dominated by stellar clusters, with the mass of any SMBH that might be present expected to be $\simeq$10 times smaller. More massive galaxies apparently always contain nuclear SMBHs but, as far as is known, not NCs.

Wehner & Harris (2006) use photometric data in the literature for about 40 dwarf elliptical nuclei to show that $M_{\rm NC}$ increases almost linearly with galaxy spheroid mass. Ferrarese et al. (2006) also find this for their nucleated Virgo galaxies with dynamical mass estimates. Moreover, both studies conclude that the $M_{\rm NC}$-$M_{\rm spher}$ and $M_{\rm BH}$-$M_{\rm spher}$ relations meet almost seamlessly at a mass scale ~$10^8\ M_\odot$, i.e., there is no large offset as in the M-σ relations. These authors therefore refer to nuclear clusters and supermassive black holes together as "central massive objects," or CMOs. We adopt this term here.

A derivation of the $M_{\rm BH}$-σ scaling has been given by King (2003, 2005; see also Fabian 1999; Murray et al. 2005; Begelman & Nath 2005). He considers super-Eddington accretion onto a seed SMBH at the center of an isothermal dark matter halo. Accretion feedback produces a momentum-driven superbubble that sweeps ambient gas into a thin shell, which expands into the galaxy. Eventually the shock cooling time becomes so long that the shell becomes energy-driven and accelerates to escape the galaxy. This truncates accretion and freezes in a relation of the form $M_{\rm BH} \propto \sigma^4$ that, with no free parameters, matches the observed one remarkably well. A roughly linear relation between $M_{\rm BH}$ and $M_{\rm spher}$ is also established as part of this process.

In this Letter, we examine the possibility that the mass of a central star cluster in a protogalaxy might be similarly self-regulated, by feedback from stellar winds and supernovae.[1]

2. THE $M_{\rm CMO}$-σ RELATION

The argument of King (2003, 2005) for the $M_{\rm BH}$-σ relation has gas in a protogalaxy flowing into a low-mass, seed black hole at super-Eddington rate. This grows the mass of the hole, but also drives an intense outflow with momentum flux given by the Eddington luminosity: $\dot{M} v_w \simeq L_{\rm Edd}/c$, independent of the actual supercritical accretion rate (King & Pounds 2003).

[1] After this paper was submitted, a preprint appeared by Li et al. (2006) that uses numerical simulations to study the development of an apparently common $M_{\rm CMO}$-$M_{\rm spher}$ relation for both NCs and SMBHs. Unlike that work, in our analytical model we consider stellar feedback explicitly to explain this result, and moreover we attempt to understand the *offset* between the M-σ scalings for NCs vs. SMBHs.

Here v_w is the outflow velocity and $L_{\rm Edd} = 4\pi G M_{\rm BH} c/\kappa$, with $\kappa = 0.398$ cm^2 g^{-1} being the electron scattering opacity.

King (2003) shows that this outflow is initially momentum-conserving, as the shocked gas cools efficiently. The ambient medium is swept up into a thin supershell, which is driven outward by the ram pressure of the SMBH wind: $\rho_w v_w^2 = \dot{M} v_w/(4\pi R^2) = G M_{\rm BH}/(\kappa R^2)$ at radius R. The dark matter halo is assumed to be an isothermal sphere and the ambient gas fraction spatially constant, so that $\rho_{\rm amb}(R) = f_g \sigma^2/(2\pi G R^2)$. Realistically, there could be a gradient in f_g due to a concentration of cool gas toward the center of the galaxy, but here we take f_g to be everywhere equal to its average over the entire halo: $f_g = \Omega_b/\Omega_m = 0.16$ (Spergel et al. 2003). If gravity is ignored, the supershell accelerates once $M_{\rm BH}$ has grown to the point that $\rho_w v_w^2 \gtrsim \rho_{\rm amb} \sigma^2$. Any ambient gas outside it is then driven out of the galaxy, stopping the growth of the SMBH. This happens when $M_{\rm BH} = f_g \kappa \sigma^4/(2\pi G^2)$.

Including the gravity of dark matter inside the superbubble alters this result by a factor of 2 (King 2005). The initial dynamical expansion of the shell then stalls at a radius $R_{\rm stall} \sim (1 - M_{\rm BH}/M_{\rm crit})^{-1}$, with

$$M_{\rm crit} = f_g \kappa \sigma^4/(\pi G^2), \tag{2}$$

where the gravity balances ram pressure. As long as $M_{\rm BH} \ll M_{\rm crit}$, this happens well inside the galaxy. More gas can then filter through to the nucleus, feeding the hole and causing the shell to reexpand (on a Salpeter timescale) to a larger $R_{\rm stall}$ appropriate to the new $M_{\rm BH}$. As $M_{\rm BH}$ approaches $M_{\rm crit}$, however, the stall radius becomes very large, and before the shell can actually reach it the gas cooling time becomes longer than the crossing time of the bubble. The shell then enters an energy-conserving snowplow phase and accelerates to escape the galaxy, leaving $M_{\rm BH} \simeq M_{\rm crit}$. As emphasized above, it is noteworthy that $M_{\rm crit}$ contains no free parameter.

Our main point is that the above argument is qualitatively unchanged if the CMO is not an SMBH but instead a *very young* star cluster where massive stars are still present. Then stellar winds and supernovae drive a superwind from the nucleus with a momentum flux that is much less than $L_{\rm Edd}/c$ but still directly proportional to it. We can thus treat the two types of CMOs simultaneously by parameterizing the wind thrust as

$$\dot{M} v_w \equiv \lambda L_{\rm Edd}/c = \lambda (4\pi G M_{\rm CMO}/\kappa). \tag{3}$$

Here λ takes a value $\simeq 1$ in the black hole case, but a value $\ll 1$ (related to the mass fraction in massive stars) for a nuclear cluster. The limiting mass in equation (2) becomes

$$M_{\rm CMO} = 3.67 \times 10^8 M_\odot \lambda^{-1} \sigma_{200}^4 (f_g/0.16), \tag{4}$$

and the offset $M_{\rm BH}$-σ and $M_{\rm NC}$-σ relations of equation (1) follow immediately if $\lambda \sim 0.1$ for a typical NC.

3. EFFICIENCY OF STELLAR FEEDBACK

To evaluate the efficiency λ of the massive-star feedback from a young nuclear cluster, we rewrite equation (3) as

$$\lambda_{\rm NC} = \frac{\dot{M} v_w}{4\pi G M_{\rm NC}/\kappa} = \frac{\dot{M} v_w}{4.2 \times 10^{27} (M_{\rm NC}/M_\odot)\ {\rm dyn}} \tag{5}$$

and estimate the separate contributions from supernovae and stellar winds.

First, the combined momentum flux from all supernovae is $2 N_{\rm SN} E_{\rm SN}/(v_{\rm SN} \tau_{\rm SN})$, where $N_{\rm SN} \approx 0.011 (M_{\rm NC}/M_\odot)$ is the number of stars with mass $>8\ M_\odot$ in a cluster with a Chabrier (2003) initial mass function (IMF), $E_{\rm SN} = 10^{51}$ ergs is the energy released per supernova, $v_{\rm SN} \approx 4000$ km s^{-1} is the typical ejecta velocity (Weiler & Sramek 1988), and $\tau_{\rm SN} \approx 2 \times 10^7$ yr is the main-sequence lifetime of an "average" SN progenitor (see Leitherer et al. 1992). Putting this into equation (5) gives $\lambda_{\rm SN} \approx 0.02$.

Second, the line-driven wind from a single hot star produces a momentum flux of $\approx (L_*/c)$ on average—somewhat less than this if there are few lines to drive the wind, but several times higher for O and Wolf-Rayet stars in which photons are multiply scattered (see, e.g., Lamers & Cassinelli 1999). Using the main-sequence mass-luminosity relation of Tout et al. (1996) to integrate (L_*/c) over all stars more massive than 5 $M_\odot$ in the IMF of Chabrier (2003), we find that the total momentum flux from stellar winds is about $1.3 \times 10^{26} (M_{\rm NC}/M_\odot)$ dyn, and thus $\lambda_{\rm winds} \approx 0.03$.

Despite its very simple derivation, our final

$$\lambda_{\rm NC} = \lambda_{\rm SN} + \lambda_{\rm winds} \approx 0.05 \tag{6}$$

is in good agreement with the values implied by the detailed calculations of Leitherer et al. (1992) for the total momentum deposition in solar-metallicity starbursts. Note also that the stellar luminosity corresponding to the limiting NC mass in equation (4) with $\lambda = 0.05$ is comparable to that derived by Murray et al. (2005) from related considerations (see their eq. [18]).

One caveat here is that, while SN momentum fluxes are insensitive to stellar metallicity, wind momenta are roughly proportional to Z (Leitherer et al. 1992). For $(Z/Z_\odot) \approx 1/3$, typical of NCs in Virgo, this might then imply a net $\lambda_{\rm NC} \sim 0.03$. However, this effect could be easily balanced by increases in both $\lambda_{\rm SN}$ and $\lambda_{\rm winds}$ if the IMF in these dense, central starbursts were slightly "top heavy," as may be the case near the center of the Milky Way (e.g., Nayakshin & Sunyaev 2005; Stolte et al. 2005). We proceed assuming the fiducial value for $\lambda_{\rm NC}$ in equation (6).

With $\lambda \approx 0.05$ for a nuclear cluster and $\lambda \simeq 1$ for a supermassive black hole, equation (4) implies an offset of a factor of 20 between the two $M_{\rm CMO}$-σ relations, while observationally it is only a factor of 10 (eq. [1]). However, population-synthesis models (e.g., Fioc & Rocca-Volmerange 1997; Bruzual & Charlot 2003) show that a star cluster more than $\sim 10^9$ yr old will have lost some 40%–50% of its initial total mass to stellar winds and supernovae, and to the conversion of massive stars into degenerate remnants. Thus, we expect the $M_{\rm NC}$-σ scaling originally to have been more offset from the SMBH correlation than it is now, by an additional factor of about 2. By the same reasoning, the fact that the two $M_{\rm CMO}$-$M_{\rm spher}$ relations currently appear to have very similar normalizations must be something of a coincidence.

4. THE $M_{\rm CMO}$-$M_{\rm spher}$ RELATION

In this feedback-regulated picture of CMO and galaxy formation, an $M_{\rm CMO}$-σ relation emerges as the primary correlation. A relation between $M_{\rm CMO}$ and spheroid mass follows by combining equation (4) with details of the cooling of the wind from the central object. The basic steps are outlined in King (2003). Knowing how the shocked gas cools, we find the cooling timescale $t_{\rm cool}$ as a function of supershell radius R and compare it to the dynamical time $t_{\rm flow} = R/v_{\rm shell}$. For small radii, $t_{\rm cool} < t_{\rm flow}$ and the outflow is momentum-driven. However, when the CMO is at about the critical mass in equation (4), the bubble is so large that $t_{\rm cool}$ exceeds $t_{\rm flow}$, the thin shell becomes energy-conserving, and the wind can escape the galaxy. We use $R_{\rm cool}$ to denote the radius at which this happens. The detailed fate of the swept-up ambient gas afterward is beyond the scope of this Letter, but in general terms it should recollapse to much smaller radii (since the CMO wind that pushed the gas to large R in the first place carries essentially no angular momentum). It will cool rapidly as it does, because $t_{\rm cool} < t_{\rm flow}$ by construction inside $R < R_{\rm cool}$, and $t_{\rm flow}$ is of order the free-fall time in the halo. We therefore expect most of the ambient gas in the supershell at the point of wind blowout to form a concentrated stellar spheroid, and thus we identify the mass of the shell at $R_{\rm cool}$ with $M_{\rm spher}$.

King (2003) shows that for a relativistic wind from an SMBH, the swept-up gas cools by Compton scattering, and ultimately,

$$\frac{M_{\rm BH}}{M_{\rm spher}} = 1.6 \times 10^{-3} b^{4/5} (c/v_w)^{8/5} (f_g/0.16)^{-3/5} M_{\rm spher,\,11}^{-1/5}. \quad (7)$$

Here $v_w \sim c$, $b \sim 1$ is an outflow collimation parameter, and $M_{\rm spher}$ is in units of $10^{11}\ M_\odot$. This agrees well with the observed SMBH-to-spheroid mass ratios in giant galaxies (Häring & Rix 2004).

If the CMO is a star cluster, equation (7) no longer applies because the wind driving the superbubble is far from relativistic, and the Compton cooling time for the shocked gas exceeds a Hubble time. The cooling in this case is by atomic transitions. However, the $M_{\rm NC}$-$M_{\rm spher}$ relation still has the basic form of equation (7), i.e., $M_{\rm CMO} \propto M_{\rm spher}^{4/5}$ in all cases.

To see this, note first that we always have $t_{\rm flow} = R/v_{\rm shell} = R/\sqrt{2}\sigma$ for the shell as it escapes. In the SMBH case, the Compton cooling time is proportional to the inverse of the radiation energy density, which is diluted by the $1/R^2$ law: $t_{\rm cool} \propto (L_{\rm Edd}/4\pi R^2 c)^{-1} \propto R^2/M_{\rm BH}$. In the NC case, the cooling time has the same dependence because the wind density *also* falls off as $1/R^2$: $\rho_w \propto M_{\rm NC}/(4\pi R^2 v_w^2)$, and then $t_{\rm cool} \propto \rho_w^{-1} \propto R^2/M_{\rm NC}$. Thus, either type of CMO has $t_{\rm cool} = t_{\rm flow}$ at a radius $R_{\rm cool} \propto M_{\rm CMO}/\sigma$, or simply $R_{\rm cool} \propto \sigma^3$ using equation (4). Finally, $M_{\rm spher} \propto \sigma^2 R_{\rm cool} \propto \sigma^5$ and hence $M_{\rm CMO} \propto M_{\rm spher}^{4/5}$.

In detail, the flow time is always (e.g., King 2003)

$$t_{\rm flow} = 6.6 \times 10^6\ {\rm yr}\ R_{\rm kpc} \sigma_{200} \lambda^{-1/2} M_8^{-1/2} (f_g/0.16)^{1/2} \quad (8)$$

for the shell radius R in units of kiloparsecs and $M_8 \equiv M_{\rm CMO}/10^8\ M_\odot$. Again, $\lambda = 1$ for an SMBH and $\lambda \approx 0.05$ for an NC. In the latter case, we further find for the radiative cooling time

$$t_{\rm cool} \simeq 1.4 \times 10^4\ {\rm yr}\ R_{\rm kpc}^2 v_{w,\,300}^{5.5} \lambda^{-1} M_8^{-1} (Z/Z_\odot)^{-0.6}, \quad (9)$$

where $v_{w,\,300}$ is the speed of the cluster superwind in units of 300 km s^{-1}. This follows from the definition $t_{\rm cool} = \mu m_{\rm H} kT/(\rho_w \Lambda_N)$, with $\mu \simeq 0.6$, the wind density ρ_w given by the continuity equation $4\pi R^2 \rho v_w^2 = \dot{M} v_w$, the shock temperature $kT/\mu m_{\rm H} = (3/16) v_w^2$, and Λ_N the normalized cooling function calculated by Sutherland & Dopita (1993). This last is approximately $\Lambda_N \simeq 3.55 \times 10^{-18} (Z/Z_\odot)^{0.6} T^{-0.75}$ ergs cm^3 s^{-1} for $(Z/Z_\odot) = 0.1$–1 and $T \simeq (0.5$–$5) \times 10^6$ K ($v_w \simeq 200$–600 km s^{-1}).

Finding the radius at which $t_{\rm cool} = t_{\rm flow}$ leads to $M_{\rm spher} = 2 f_g \sigma^2 R_{\rm cool}/G$ and combining with equation (4) gives

$$\frac{M_{\rm NC}}{M_{\rm spher}} = 2.7 \times 10^{-4} \lambda^{-1} \left(\frac{Z}{Z_\odot}\right)^{-0.48} \left(\frac{f_g}{0.16}\right)^{-3/5} v_{w,\,300}^{4.4} M_{\rm spher,\,11}^{-1/5}. \quad (10)$$

Comparing equation (7) to equation (10) with $\lambda = 0.05$, the predicted offset of the *original* $M_{\rm NC}$-$M_{\rm spher}$ relation from the corresponding SMBH relation is only a factor of $\approx$3–4 for a wind velocity near 300 km s^{-1} and slightly subsolar metallicities. Allowing for the long-term mass loss from the NC, discussed at the end of § 3, the normalizations of the present-day scalings should agree, rather fortuitively, to within a factor of 2, just as Ferrarese et al. (2006) and Wehner & Harris (2006) infer observationally.

The contrast between this and the much larger offset separating the two $M_{\rm CMO}$-σ relations is due to the fact that, for given σ, the radiative $t_{\rm cool}$ with an NC at the limiting mass of equation (4) is nearly 10 times shorter than the Compton cooling time that applies when the CMO is an SMBH (see King 2003). Thus, the final $R_{\rm cool}$ and $M_{\rm spher}$ are larger by this amount for a galaxy containing a nuclear star cluster versus one with the same σ but a central black hole.

5. THE FABER-JACKSON RELATION

For a galaxy with a supermassive black hole in its nucleus, equations (7) and (4) imply a relation between the mass and velocity dispersion of the spheroid alone (King 2005):

$$M_{\rm spher}({\rm SMBH}) = 2.8 \times 10^{11}\ M_\odot b^{-1} (c/v_w)^{-2} (f_g/0.16)^2 \sigma_{200}^5. \quad (11)$$

Perhaps despite appearances, this prediction is consistent with the well-known relation between the velocity dispersion and *luminosity* of giant ellipticals, $L \propto \sigma^4$ (Faber & Jackson 1976). This is because the mass-to-light ratio in the cores of these galaxies increases systematically as the $\simeq$0.2–0.3 power of luminosity (van der Marel 1991; Cappellari et al. 2006), so that the Faber-Jackson relation in fact implies $M_{\rm spher} \propto \sigma^5$. The normalization in equation (11) is also in remarkably good agreement with observation: from Haşegan et al. (2005)

$$M_{\rm spher}({\rm obs}) \simeq 1.93 \times 10^{11}\ M_\odot\ \sigma_{200}^{5.2} \quad (12)$$

for galaxies with $\sigma \gtrsim 100$ km s^{-1}, which, as we discussed in § 1, are the ones that contain SMBHs rather than NCs.

For smaller galaxies with nuclei dominated by star clusters, $t_{\rm cool} = t_{\rm flow}$ gives the cooling radius

$$R_{\rm cool}({\rm NC}) = 880\ {\rm kpc} (Z/Z_\odot)^{0.6} (f_g/0.16) v_{w,\,300}^{-5.5} \sigma_{200}^3. \quad (13)$$

As we mentioned above, this is nearly 10 times larger than the equivalent scale in the SMBH-dominated case, and the spheroid mass is consequently larger than in equation (11):

$$M_{\rm spher}({\rm NC}) = 2.6 \times 10^{12}\ M_\odot (Z/Z_\odot)^{0.6} (f_g/0.16)^2 v_{w,\,300}^{-5.5} \sigma_{200}^5. \quad (14)$$

In a plot of σ versus $M_{\rm spher}$, we would therefore expect low-mass, nucleated galaxies to define a locus with $\sigma \propto M_{\rm spher}^{0.2}$, parallel to the nonnucleated galaxies with SMBHs but falling below them by a factor of $\approx 10^{-0.2}$. Then, although our model does not predict a value for the final effective radius of the stellar spheroid in terms of $R_{\rm cool}$, the virial theorem requires any such scale to depend on the spheroid mass roughly as $R_{\rm eff} \propto M_{\rm spher}^{0.6}$, with nucleated galaxies lying above nonnucleated ones by a factor of about 2.5.

6. DISCUSSION

Current data probably do not rule out the idea that nuclei of galaxies with $\sigma \lesssim 120$ km s^{-1} could harbor *both* star clusters *and* SMBHs some ∼10 times less massive, and thus it will be important to ask how such galaxies might choose between SMBH and NC feedback channels in regulating their formation. For now, a possibly more straightforward question is why there are *no* nuclear clusters in larger galaxies with $\sigma \gtrsim 120$ km s^{-1} [$M_{\rm spher} \gtrsim (2\text{–}3) \times 10^{10}\ M_\odot$], which apparently all contain black holes.

For our basic scenario to be self-consistent, we evidently require that the dynamical time of the superbubble always be shorter than that of the entire halo, until the point of blowout when $t_{\rm flow} = t_{\rm cool}$. Equivalently, the radius of the shell must always be less than the halo virial radius, and in particular $R_{\rm cool} \lesssim R_{\rm vir}$ is required for the growth of a CMO (whether an SMBH or an NC) to be self-regulated, as we envision. From the relations in Bryan & Norman (1998), if $\Omega_{m,0} = 0.3$, $\Omega_\Lambda = 0.7$, and $H_0 = 70$ km s^{-1} Mpc^{-1}, then $R_{\rm vir} \approx 540$ kpc $\times\ \sigma_{200}(1 + z)^{-1.1}$ (accurate to better than 10% for $z \leq 2$). Combining this with equation (13), our picture can work for NCs forming at redshift $z_{\rm NC}$ only in halos with $\sigma \lesssim 160$ km s^{-1} $\times\ (1 + z_{\rm NC})^{-0.55}$.

If nuclear clusters are typically ∼5 Gyr old, $z_{\rm NC} \simeq 0.5$ and this upper limit becomes $\sigma \lesssim 130$ km s^{-1}. In halos with velocity dispersion higher than this, the superbubble blown by an NC reaches the halo virial radius before it becomes energy-conserving and can accelerate to escape. It is presumably then held there, or even driven into collapse, by the infall of material from beyond $R_{\rm vir}$. The growth of the central star "cluster" is never choked off, and it ultimately becomes indistinguishable from the galaxy spheroid itself. When the CMO is a black hole, the much longer Compton-cooling time found in King (2003) implies that $R_{\rm cool} \lesssim R_{\rm vir}$ for all $\sigma \lesssim 500$ km s$^{-1}(1 + z_{\rm BH})^{-0.55}$, so it is only in very massive systems indeed that the self-regulated $M_{\rm BH}$-σ relation of equation (2) breaks down.

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Note added in proof.—J. Rossa et al. (AJ, 132, 1074 [2006]) have recently analyzed a sample of nuclear clusters in 40 spiral galaxies, finding a relation between $M_{\rm NC}$ and the luminosity of the galaxies' *bulge* components, which has the same slope as the relation between SMBH mass and bulge luminosity with a comparable intercept. The situation for NCs in late-type spheroids thus appears very similar to that found by Wehner & Harris (2006) and Ferrarese et al. (2006) to hold in early-type galaxies.