Speculative Profits, Innovation and Growth[*]

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Abstract

When technological change affects the prices of tradeable assets, innovators can obtain speculative profits by exploiting their inside information as to the occurrence of innovations. We propose a tractable model of endogenous growth that formalizes this argument, originally due to Hirshleifer (1971). We then use the model to assess two claims advanced by Hirshleifer, namely, that speculative profits can generate excessive investment in R&D when they add to monopoly rents guaranteed by patent protection, or else even in a perfectly competitive economy. The analysis confirms the first claim, but casts doubts on the second one.

Key words: innovation, speculative profits, endogenous growth, overinvestment in R&D, asset prices

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1 Introduction

In a classic paper, Hirshleifer (1971) noted that innovations may affect the equilibrium prices of various assets that are traded in the economy. He argued that inventors are better informed than anybody else on the arrival of their own innovation, concluding that they can reap speculative profits by exploiting this inside information. These speculative profits may provide a reward for inventors even in the absence of any other appropriation mechanism, such as patents or secrecy.

To illustrate this possibility, Hirshleifer used the example of Ely Whitney, the inventor of the cotton gin:

> [t]he cotton gin had obvious speculative implications for the price of cotton, the value of slaves and of cotton-bearing land, the site value of key points in the transport network that sprang up. There were also predictable implications for competitor industries (wool) and complementary ones (textiles, machinery). It seems very likely that some forethoughted individuals reaped speculative gains on these developments, though apparently Whitney did not. And yet, he was the first in the know, the possessor of an unparalleled opportunity for speculative profit. (p.571)

Other examples of innovations affecting asset prices readily come to mind: think for instance of the effect of the invention of the combustion engine on the price of oil, or that of microchips on silicon. The secretive nature of insider trading makes it hard to find direct evidence of speculative activity carried out by innovators. However, some indirect evidence can be found, for instance, in Helfat's (1997) study on the direction of US oil firms' R&D projects after the oil shocks.[1]

In this paper, we formalize Hirshleifer's argument by developing a tractable model of endogenous growth in which the reward to inventive activity is constituted by Hish-

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[1] In particular, Helfat (1997) reports that certain firms accumulated stocks of coal as a consequence of their intensified R&D effort on coal technology (p. 345).

leiferian speculative gains rather than, or in addition to, Schumpeterian monopoly rents. We then use the model to assess certain claims made by Hirshleifer, which are still echoed in the policy debate on innovation and intellectual property.

Hirshleifer recognized that inventors can capture only a fraction of the pecuniary effects of innovations. The size of this fraction depends on the extent to which inside information can be exploited without being revealed. In a noiseless economy, for instance, under quite general conditions the fraction would vanish, as any inside information would be perfectly revealed as soon as its possessor tried to exploit it (Grossman and Stiglitz, 1980). However, a vast literature on insider trading, starting with Kyle (1985), has argued that in the presence of noise traders the insiders can obtain positive profits by hiding behind their trades.

While this literature has shown that insider trading can be an equilibrium phenomenon, it is not the aim of this paper to incorporate a fully microfounded model of insider trading into an endogenous growth framework. Rather, we simply take the fraction of the potential speculative gains that the inventor actually captures as exogenous. This reduced-form approach sidesteps many important problems related to the functioning of financial markets, but allows us to address other interesting questions, which are often discussed in the intellectual property debate (see e.g. Boldrin and Levine, 2008) and also in the long standing legal debate on insider trading and disclosure law (see e.g. Manne, 1966 and Duggan, 1995),[2] and yet have not been analyzed in formal economic models so far.

The first question is whether speculative profits alone may sustain persistent innovation and growth; whether they offer, to use Hirshleifer's words, "an appropriate inducement to invention." Our analysis shows that they may, provided that the fraction of the pecuniary effects of innovations that inventors obtain exceeds a minimum

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[2]The legal debate originates from Manne's (1966) provocative book against the legal ban on insider trading. The argument that insider trading constitutes an appropriate way of compensating innovators stands as a central proposition of the book: "Insider trading meets all the conditions for appropriately compensating entrepreneurs. It readily allows corporate entrepreneurs to market their innovations. . . . .[T]his is not a direct marketing of the idea, but rather a "sale" of information about an innovation" (p. 138). For an extensive review of this debate see, for instance, Bewaji (2012).

threshold. This threshold can be computed analytically and depends on parameters that can be assessed empirically. After solving the model, we offer a tentative, preliminary assessment which may help to get a sense of the practical relevance of Hirshleifer's mechanism.

The second question that we address is the theoretical possibility that speculative profits can create an excessive incentive to invest in research. There are two versions of this claim, both advanced by Hirshleifer. The first is that speculative profits alone can be so large as to exceed the social value of innovations. This claim is based on the observation that:

> there is no logically necessary tie between the size of the technological benefit
> on the one hand, and the amplitude of the price shifts that create speculative
> opportunities on the other. [...] A relatively minor shift in locomotive technology,
> for example, might lead railroad planners to select an entirely different route for
> a new line, with drastic upward and downward shifts of land values. (p. 572)

The second version maintains that overinvestment in R&D can occur when the innovator obtains monopoly rents in addition to speculative profits. In other words, the two versions of the claim are that speculative profits may overcompensate inventors even under perfect competition, or else only in the presence of market power.

Our analysis cast doubts on the first claim: in our model, speculative profits can never overcompensate inventors in the absence of monopoly rents. This is true even if inventors capture the pecuniary effects of their innovations fully. The intuitive reason for this is that, contrary to what Hirshleifer argued, there does exist a relation between speculative opportunities and the size of innovations. To use Hirshleifer's example, if a minor shift in locomotive technology induces railroad planners to select an alternative route, then the two routes must be close substitutes both before and after the shift. If this is so, however, then changes in land value cannot be ample.

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To be more precise, in our model tradeable assets appreciate only to the extent that innovations increase their productivity. This poses an upper bound on the size of the speculative gains. The upper bound is always lower than the social value of innovations, as the latter is given by the total increase in factor productivity and thus includes also the increase in the productivity of labour. However, the increase in labour productivity cannot be captured by speculators as claims on labour resources cannot be traded in legal markets due to laws against indentured servitude.

We then modify the perfectly competitive growth model to provide an assessment of the second version of the claim, i.e. that speculative profits may create overinvestment in R&D in the presence of patent protection and monopoly rents. We develop a model where patent protection cannot lead to overinvestment in R&D by itself. The question then is whether inventors can be overcompensated by cumulating speculative profits and monopoly rents. The answer is not obvious, as patent protection in fact crowds out speculative gains. For example, if perfectly protected patent holders could capture, by means of monopoly rents, all of the value of the innovation, then asset prices would not change at all as innovations arrive, and thus the opportunity for speculation would vanish. Therefore, overinvestment in R&D cannot occur when patent protection is perfect; nor, as we have just seen, when it is totally absent. However, we show that overinvestment may indeed arise for intermediate levels of patent protection. This validates the second, weaker version of Hirshleifer's claim.

Literature. Other papers have argued that innovation can be sustained in a perfectly competitive economy with no monopoly rents. Hellwig and Irmen (2001) and Bester and Petrakis (2003) show that persistent innovation can be driven by the inframarginal rents obtained by competitive firms in the short run. A similar mechanism has been proposed in a series of papers by Boldrin and Levine (see e.g. Boldrin and Levine 2004, 2008), who argue that innovators can profit by selling the first "copy" of their ideas. A common feature of these models is that innovative technological

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knowledge cannot be immediately used by firms other than the inventor, even in the absence of patent protection. In this sense, these models depart from the traditional assumption that innovative technological knowledge is non rival (Arrow, 1962). The mechanism proposed by Hirshleifer, in contrast, is fully consistent with that assumption. In fact, the swifter and the wider is the adoption of the new technology, the greater are changes in asset prices and hence the potential for speculative gains.

Structure of the paper. The rest of the paper is organized as follows. In section 2, we present the basic model. Section 3 derives the model equilibrium and provides conditions for sustained growth to be supported by speculative profits alone. The two versions of the over-investment hypothesis, without and with patent protection, are analyzed in sections 4 and 5, respectively. Section 6 summarizes the paper and offers some concluding remarks.

2   The baseline model

In this section, we develop a stylized general equilibrium model of endogenous growth, adapted from Acemoglu (2009). In the model, innovations affect the price of a productive asset, thereby creating an opportunity for speculative profits.

The economy is populated by identical, infinitely-lived households whose mass is normalized to one. There is a unique final good, which can be consumed or used in research. This good is taken as the numeraire. Households have additive logarithmic intertemporal preferences over consumption flows $c(t)$:

$$u(c) = \int_0^\infty \ln\left[c(t)\right] e^{-\rho t} \mathrm{d}t, \tag{1}$$

where ρ is the rate of time preference.[3] Time is continuous and is denoted by t, but to simplify the notation we shall often suppress the time index.

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[3] One can easily allow for more general preferences, such as for instance

$$u(c) = \int_0^\infty \left[\frac{c(t)^{1-\sigma} - 1}{1 - \sigma}\right] e^{-\rho t} dt,$$

where $1/\sigma$ is the intertemporal elasticity of substitution.

The final good is produced in a continuum of perfectly competitive industries indexed by $\omega \in [0,1]$. Since Grossman and Helpman (1991), the assumption of a continuum of industries has become standard in the endogenous growth literature. It guarantees that even though in each industry ω the arrival of new innovations is stochastic, by the law of large numbers there is no aggregate uncertainty. In other words, uncertainty is purely idiosyncratic and can be diversified away perfectly, implying that asset pricing is not affected by considerations of risk.

Each industry ω produces the same final good y, but using different, industry-specific, inputs: labour L_ω and an irreproducible tradeable asset T_ω. The assumption that the tradeable asset (e.g., land) is irreproducible simplifies the analysis allowing us to abstract from issues of capital accumulation. The production function is taken to be:[4]

$$y_\omega = \theta^{k(\omega)} L_\omega^\alpha T_\omega^{1-\alpha} \qquad \text{with } \theta > 1 \text{ and } 0 < \alpha < 1,$$

where α is the income share of labour, and $\theta^{k(\omega)}$ is total factor productivity if $k(\omega)$ innovations have occurred. That is, each innovation increases total factor productivity by a factor $\theta > 1$. Summing over industries, one obtains the aggregate production function:

$$y = \int_0^1 \theta^{k(\omega)} L_\omega^\alpha T_\omega^{1-\alpha} \mathrm{d}\omega \tag{2}$$

Each household inelastically supplies one unit of labour and one unit of the irreproducible asset in each industry. Thus, we have $L_\omega = 1$ and $T_\omega = 1$, which implies that at each point in time output equals total factor productivity:

$$y = \int_0^1 \theta^{k(\omega)} \mathrm{d}\omega.$$

As mentioned above, technology improves over time as a result of innovative activity. We refer to "period $k(\omega)$" as the random time interval between innovation $k(\omega)$ and innovation $k(\omega)+1$ in industry ω. (For notational simplicity, we henceforth

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[4]The Cobb-Douglas specification simplifies the calculations but is not crucial to our results.

drop the industry index ω when this does not create any confusion.) We set $k = 0$ at time zero in all industries, thereby normalizing total factor productivity at time zero to 1.

In each industry ω and each period k, there is a free-entry race for innovation $k+1$. The race starts as soon as innovation k is achieved and disclosed. A number of symmetric risk-neutral firms can participate in this race by investing the final good in independent R&D projects. We assume that there are constant returns to R&D with no R&D spillovers. This rules out congestion effects and negative or positive externalities in the research process. As is well known, these effects by themselves can cause over or underinvestment in R&D.[5] Ruling them out allows us to focus on speculative profits as the possible cause of overinvestment.

Each research firm i chooses its R&D investment $n_{i,k}$ to obtain the $k + 1$-th innovation. The R&D investment is a flow cost paid until the innovation is achieved. The R&D investment produces an instantaneous probability of success of $\lambda_k n_{i,k}$, where $\lambda_k > 0$ is the productivity of R&D. Since projects are independent, the arrival of innovation $k+1$ follows a Poisson stochastic process with a hazard rate $x_k = \lambda_k n_k$, where $n_k = \sum_i n_{i,k}$ denotes aggregate R&D investment.[6] To guarantee the existence of a steady state, we assume that $\lambda_k = \lambda\theta^{-k}$, where λ is a parameter that measures the productivity of R&D.[7]

The rate of growth of the economy is:

$$\frac{\dot{y}}{y} = \frac{\int\limits_0^1 \left[\theta^{k(\omega)+1} - \theta^{k(\omega,t)}\right] x\,\mathrm{d}\omega}{\int\limits_0^1 \theta^{k(\omega)}\mathrm{d}\omega} = (\theta - 1)x,$$

and therefore is not stochastic.

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[5] For a discussion of the possible sources of overinvestment in R&D in endogenous growth models see Denicolò and Zanchettin (2014).

[6] Firms can adjust their R&D efforts at any point in time, but with a Poisson discovery process they all will choose a constant level of R&D expenditure until someone succeeds, and the next race starts.

[7] In a steady state, the hazard rate x_k, and hence the expected duration of time periods, must be constant. Since R&D investment n_k must grow at rate θ from one period to the next, λ_k must decline at rate θ. This explains the knife-edge assumption $\lambda_k = \lambda\theta^{-k}$, which is common to all R&D-driven endogenous growth models (Barro and Sala-i-Martin, 2004, ch. 7).

3   Speculation and growth

We now determine the equilibrium of the economy under the assumption that markets are perfectly competitive and there is no patent protection (or secrecy). That is, all firms active in the product market may freely use the leading technology. Innovators are rewarded by speculative profits only. The question is whether these may suffice to sustain steady innovation and growth.

3.1   Equilibrium prices

Factor markets are perfectly competitive. Since firms can freely use the leading technology, the wage rate in period k is $w_k = \alpha\theta^k$. The income share of labour is α. The remaining share represents the rents accruing to the owners of the irreproducible asset, $R_k = (1-\alpha)\theta^k$.

The expected flow return to holding the asset is the sum of the rents R_k and any expected capital gain due to the arrival of innovation $k+1$. Let P_k denote the price of the irreproducible asset in period k. With complete information, when innovation $k+1$ arrives the price would jump to P_{k+1} and stay constant until the next innovation.[8] However, the timing of the innovation is uncertain. The innovator, being the first in the know, has for a time inside information as to the arrival of the innovation. Thus, he can anticipate the market and obtain speculative profits. This reduces, conversely, the capital gain that can be obtained by outside investors.

As mentioned in the introduction, here we do not model insider trading explicitly.[9] Rather, we simply assume that, by exploiting his superior information, the innovator captures a share γ of the change in the total value of the asset, $P_{k+1} - P_k$ (which we also refer to as the "pecuniary effects of the innovation"). We take γ as a parameter

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[8]Price increases are permanent when the asset is irreproducible. If the asset was reproducible, by contrast, any increase in its price would stimulate the accumulation of the asset until the price falls back to the asset production cost. The anticipation of this adjustment process would dampen changes in asset prices when innovations occur.

[9]At the end of this section, we briefly discuss some of the problems of explicitly modeling insider trading.

that ranges in between 0 and 1,[10] its size being ultimately an empirical question. We assume that the speculative process is instantaneous.[11]

Uninformed investors anticipate that when the next innovation arrives they will obtain only a fraction $(1 - \gamma)$ of the change in the value of the asset, as the remaining fraction γ is reaped by the inventor. This implies that their expected flow capital gain is $x_k(1 - \gamma)(P_{k+1} - P_k)$. This adds to the rents R_k, determining the total expected return from holding the asset.

Investors can perfectly diversify away risk by investing in different industries, so in equilibrium the expected return must equal the interest rate r. The asset pricing equilibrium condition then is:

$$rP_k = R_k + x_k(1 - \gamma)(P_{k+1} - P_k).$$

In a steady state, the rate of arrival of innovations x_k is constant across periods, and the asset price P_k grows by a factor θ from one period to the next. Writing $P_k = \theta^k p$, where p is the growth-adjusted asset price, the asset price equation becomes:

$$rp = (1 - \alpha) + x(1 - \gamma)(\theta - 1)p. \tag{3}$$

3.2   Equilibrium R&D investment

Consider now the equilibrium in innovation races. The prize to the winner of the $k + 1$-th race is the speculative profit $\pi_k = \gamma \left(P_{k+1} - P_k \right)$. Since the instantaneous probability of success of a generic firm i that invests $n_{i,k}$ units of the final good in that race is $\lambda_k n_{i,k}$, the firm's expected flow revenue is $\lambda_k n_{i,k} \pi_k$. On the other hand, its flow R&D cost is $n_{i,k}$. Because there is free entry, the zero-profit condition $\lambda_k n_{i,k} \pi_k - n_{i,k} \leq 0$ must hold. Furthermore, in any equilibrium with positive R&D

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[10] The reason why γ must be lower than one is that rational investors, being aware of the existence of insiders who hold superior information, will not be willing to trade derivative assets. (This would effectively amount to bet against informed bettors, which can never be profitable.) As a result, insiders can never obtain more than the total change in the value of the asset.

[11] Speculation is typically much faster than innovation. Inside information can hardly be concealed for more than a few weeks, whereas achieving an innovation may easily take many years.

investment the condition must hold as an equality, which implies:

$$\lambda_k \pi_k = 1.$$

Using the fact that $\lambda_k = \lambda\theta^{-k}$ and $P_k = \theta^k p$, the zero-profit condition becomes:[12]

$$\gamma(\theta - 1)p = \frac{1}{\lambda}. \tag{4}$$

3.3   Equilibrium growth

From the asset price equation (3) and the zero profit condition (4), one determines an increasing relationship between the interest rate and the rate of innovation:

$$r = \gamma\lambda(\theta - 1)(1 - \alpha) + (1 - \gamma)(\theta - 1)x. \tag{5}$$

The Euler equation provides another relationship:

$$r = \rho + (\theta - 1)x. \tag{6}$$

These two equations can be solved simultaneously to determine the equilibrium interest rate and the rate of innovation. This immediately leads to the following result:

Proposition 1 *The equilibrium rate of innovation x^* is positive if and only if*

$$\gamma > \underline{\gamma} = \frac{\rho}{\lambda(1 - \alpha)(\theta - 1)}. \tag{7}$$

When condition (7) holds, the equilibrium rate of innovation and rate of interest are:

$$x^* = \lambda(1 - \alpha) - \frac{\rho}{\gamma(\theta - 1)} \tag{8}$$

$$r^* = (\theta - 1)(1 - \alpha)\lambda - \left(\frac{1 - \gamma}{\gamma}\right)\rho. \tag{9}$$

Condition (7) follows from the fact that firms will invest in R&D only if, when $x = 0$, the returns exceed the cost, which is one. When the rate of innovative activity x is zero, there is no growth and hence the interest rate coincides with the rate of

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[12] With constant returns to research, the equilibrium number of research firms and individual R&D investments are indeterminate, and only aggregate R&D investment is determinate.

time preference ρ. An innovation would then increase the price of the irreproducible asset by an amount equal to $(\theta - 1)(1 - \alpha)/\rho$, of which the inventor obtains a fraction γ. Therefore, the return to R&D is $\lambda\gamma(\theta - 1)(1 - \alpha)/\rho$. Comparing it to the unit cost, condition (7) follows.

Condition (7) says that for persistent growth to be sustainable, it is necessary that inventors obtain at least a minimum fraction $\underline{\gamma}$ of the pecuniary effects of the innovation, $P_{k+1} - P_k$. The minimum threshold $\underline{\gamma}$ depends on parameters α, θ, and ρ/λ. Those are, therefore, the parameters to be gauged in order to get a sense of how large γ must be for growth to be sustainable.

Following Prescott (1986), labour's share in national income, α, may be taken to be approximately $\frac{2}{3}$ (to be precise, Prescott sets it at 0.64). As for θ, Stokey (1995) observes that if innovations occur every few years, a reasonable value for θ can be 1.05 or even less; if instead innovations occur only a couple of times per century, then reasonable values for θ can be as large as 1.25 or 1.5. We therefore let θ range in the interval $[1.05, 1.5]$. Stokey (1995) provides also a calibration of the ratio ρ/λ, suggesting for it a possible range from 0.04 to 1.[13]

Note that the minimum threshold $\underline{\gamma}$ increases with ρ/λ and decreases with θ. Therefore, using the intervals suggested by Stokey (1995), the lowest value of the threshold $\underline{\gamma}$ is 22% (to be precise, this value is obtained by setting $\alpha = 0.64$, $\theta = 1.5$ and $\rho/\lambda = 0.04$). This corresponds to top-left corner of the "reasonable" rectangle in Figure 1 below. As one moves away from that corner, however, the minimum threshold increases. In fact, the threshold exceeds one, meaning that speculative profits alone cannot sustain growth, in a large portion of the reasonable rectangle. For example, $\underline{\gamma}$ exceeds 1 when θ and ρ/λ are set at the center of the rectangle. This suggests that while it is not impossible that speculative profits alone can sustain persistent growth, the possibility may not seem very realistic.

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[13] This range is quite wide. Furthermore, it has been obtained by calibrating a standard Schumpeterian model. Therefore, it must be taken with extra caution when it is used in a model where inventors are rewarded by means of an entirely different mechanism.
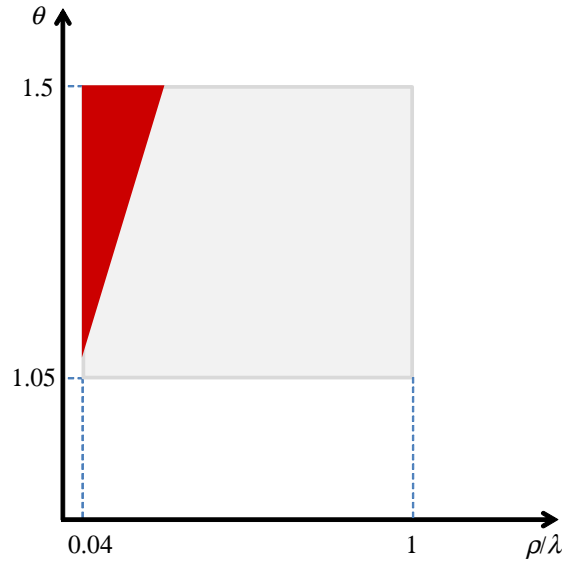
Figure 1. In the dark red region, the minimum
threshold $\underline{\gamma}$ is lower than one.

Going back to the analytical solution of the model, observe that when condition
(7) holds firms have an incentive to invest in R&D at $x = 0$ as the returns to R&D
exceed the cost. In equilibrium, the rate of innovation x must then raise to the point
where the returns become equal to the cost.[14]

From Proposition 1 several comparative statics results immediately follow. Quite
intuitively, the rate of growth increases with the inventors' ability to appropriate
speculative profits, γ. The rate of growth of the economy depends positively also on
the productivity of R&D expenditure, λ, and the size of innovations, θ, whereas it
depends negatively on the rate of time preferences ρ. These latter effects are natural
and are the same as in standard Schumpeterian models.

A novel result is that an increase in the income share of labour reduces growth.[15]
This follows from the fact that in an economy with no slavery only the fraction

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[14] An increase in x indeed reduces the return to R&D, for the following reasons. As x increases, the
interest rate r must increase in order to satisfy the Euler condition. However, there is a countervailing
effect: an increase in x increases expected future rents, raising the current price of the asset and
hence the size of the speculative profits. However, the slope of the zero-profit condition (5) is always
lower than that of the Euler condition (6), which implies that the former, negative effect must prevail
over the latter.

[15] In traditional Schumpeterian models, an increase in the income share of labour reduces the
elasticity of demand for innovative goods and hence increases the monopoly price. This allows
innovators to obtain higher monopoly rents and so stimulates growth (Aghion and Howitt, 1992).

$(1 - \alpha)$ of the increase in total factor productivity is reflected in changes in the prices of tradeable assets. This implies that the greater is the income share of labour α, the more limited are the opportunities for speculative gains.

3.4 Insider trading

While a rigorous analysis of the inventor's speculative strategy is beyond the scope of this paper, at this point it may be useful to pause in order to briefly discuss the main problems that one would have to face in developing a fully microfounded model of speculation. The first difficulty is that in a general equilibrium framework one cannot simply posit the existence of noise traders, as Kyle (1985) and many subsequent papers do, but must model their behaviour explicitly. For example, one could add a noise term to agents' endowments, creating an insurance motive for trading (as suggested by Diamond and Verrecchia, 1981). Alternatively, one could assume that different traders have different discount rates, as in De Marzo and Duffie (1999). However, these or other assumptions that may rationalize the existence of noise traders would inevitably interact in non trivial ways with the delicate structure of general equilibrium endogenous growth models.

Second, the existing literature on insider trading assumes that the inside information becomes public at some exogenously pre-specified date. In our framework, in contrast, the innovator may choose when to disclose the innovative knowledge to potential users. Indeed, it is only when the new technology is put in the public domain that the fundamental demand for the asset will change. Thus, the inventor must choose optimally not only the trading strategy but also the timing of disclosure.

These choice problems are further complicated by the fact that with free entry in the research sector, an inventor can never be sure to be the unique innovator, and hence the sole possessor of the inside information. Somebody else might have already innovated too, but might be concealing the innovation and secretly speculating.[16]

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[16]Whether this is so cannot be perfectly inferred by observing the asset price, as if the asset price was fully revealing, speculation would be impossible.

With a Poisson discovery process, this possibility becomes more and more likely as time passes.

This affects not only the trading strategy, but also the optimal strategy of investment in research. Not knowing whether anybody else has innovated yet, firms may keep investing in R&D until the innovation is disclosed. However, the reward to successful completion of the R&D project obviously depends on how many other firms have already innovated, or will innovate before the innovation is disclosed. Since this is uncertain but stochastically depends on time, the equilibrium R&D investment strategy can no longer be stationary, but must depend on the time passed since the start of the race.[17]

While these problems may not be insurmountable, they certainly complicate the analysis. Therefore, our reduced-form model may be viewed as a useful means for a preliminary analysis of important policy problems. For example, Hirshleifer claimed that speculative profits provide a reward for inventors that may exceed the social value of innovations, thereby leading to overinvestment in R&D. In the next two sections we shall assess this claim.

It is important to note that using the reduced-form model for this purpose may not entail any real loss of generality. For example, if overinvestment cannot occur when $\gamma = 1$, then it will not occur in any fully micro-founded model in which inventors can only capture a fraction of the pecuniary effects of innovations. Conversely, if in our reduced-form model overinvestment can occur as soon as $\gamma > 0$, then it should also occur in any fully micro-founded model in which insider trading is profitable.

4   The overinvestment hypothesis: perfect competition

There are in fact two versions of the overinvestment hypothesis, both advanced by Hirshleifer: that overinvestment in research may occur even under perfect competi-

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[17]For a discussion of some of the problems posed by non-stationary innovation races see e.g. Doraszelski (2003)

tion, or else that it can only in the presence of monopoly rents. In this section, we address the first version of the claim; the second will be analyzed in the next section.

Analytically, the question is whether the equilibrium rate of innovation x^* of our baseline model, given by (8), can ever be greater than the socially optimal rate of innovation. To answer this question, we must first turn to the analysis of the social optimum.

Since the only distortion in the perfectly competitive equilibrium is that the private incentives to innovate may not be perfectly aligned with the social ones,[18] the social optimum just requires that an optimal share of income is invested in research. The trade-off is that an increase in the share of income invested in research makes income grow more quickly, but reduces the share of income consumed. Since there is no capital accumulation, the social problem is stationary. Therefore, the optimal policy must be stationary. With a constant hazard rate x, total R&D expenditure is

$$n = \int_0^1 n(\omega)\mathrm{d}\omega = \frac{xy}{\lambda}.$$

We then have

$$c(t) = \left(1 - \frac{x}{\lambda}\right) y = \left(1 - \frac{x}{\lambda}\right) e^{x(\theta - 1)t}.$$

Substituting into the utility function (1) one gets

$$u = \frac{1}{\rho^2} x(\theta - 1) + \frac{1}{\rho} \ln\left(1 - \frac{x}{\lambda}\right)$$

The optimal innovation rate is then found by maximizing u and is

$$\hat{x} = \lambda - \frac{\rho}{(\theta - 1)}, \tag{10}$$

provided that $(\theta - 1) > \frac{\rho}{\lambda}$. (If this inequality is reversed, the optimal policy entails zero R&D investment, so the economy stagnates indefinitely.)

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[18] In the competitive equilibrium, there are no static distortions. In particular, while speculative profits are similar to a capital tax on tradeable assets, this "tax" is not distortionary as the assets are irreproducible.

Comparing $\hat{x}$ in equation (10) with the equilibrium rate of innovation x^* in equation (8), one immediately sees that $x^* < \hat{x}$ even when $\gamma = 1$. We may therefore conclude:

Proposition 2 *In the baseline model, there is always underinvestment in research.*

This result can be easily understood by contrasting the social and private value of innovations. The social value of innovation k is the discounted increase in total factor productivity, $(\theta^k - \theta^{k-1})/r$. The private value is the fraction γ of the increase in the price of the irreproducible asset. There are two reasons why the private value is necessarily lower than the social value. First, inventors obtain only a share γ of the increase in the price of the irreproducible asset. Second, the increase in the price of the asset is only a share $(1 - \alpha)$ of the social value of the innovation. The remaining share α translates into an increase in labour income, which cannot be captured by speculators who trade the irreproducible asset.

One may wonder that the baseline model might underestimate the potential for speculative gains. In particular, the model does not capture the redistributive effects of technical change discussed by Hirshleifer in his "locomotive technology" example. A feature of that example is that there are various assets in the same industry and innovations are asset specific, meaning that the occurrence of the innovation appreciates certain assets but depreciates others. This amplifies the opportunities for speculation.

In the Appendix we modify the baseline model so as to capture these effects, allowing for changes in *relative* asset prices within an industry. However, we show that not even in this modified model can investment in R&D be excessive when inventors are rewarded by speculative profits only. Intuitively, the reason for this is that even in the modified model asset prices are pinned down by market fundamentals. It seems that for overinvestment to be possible, asset prices must be somehow disconnected from fundamentals, as we shall discuss in the concluding section.

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5   The overinvestment hypothesis: patent protection

The second claim made by Hirshleifer is that overinvestment may occur when inventors can benefit from patent protection in addition to inside information. In particular, after noting that a perfectly discriminating patent holder can capture the entire social value of the innovation even without speculating, Hirshleifer argued that

> [...] the perfectly discriminating patent holder [...] is in a position to reap speculative profits, too; counting these as well, he would clearly be overcompensated.
>
> (p. 572)

In this section we analyze this latter claim. To this purpose, we modify the baseline model to allow for the possibility that imitation may be prevented by patent protection. In such a model, inventors obtain monopoly rents. As argued by Hirshleifer, however, that does not deprive them of the possibility of speculating, too. The issue, then, is whether inventors may be overcompensated when they cumulate monopoly rents and speculative profits.

For sake of consistency, we continue to assume that innovators do not directly engage in production. Now, however, we assume that they can license their proprietary technology to competitive firms that produce the final goods. The resulting revenue is similar in nature to monopoly rents.[19]

A standard argument, based on the Arrow replacement effect, implies that the latest innovator does not conduct any research and hence is systematically replaced by outsiders. Initially, we assume that different patents cannot be pooled together. As a result, successive innovators must compete with each other. In particular, in each period k the latest innovator faces competition from the penultimate innovator, who stands ready to license his technology at a zero royalty rate. The latest innovator,

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[19]In fact nothing changes if we assume that innovators directly engage in production, under the standard assumption of Bertrand competition among producers.

who can license the most productive technology, will then charge a royalty of:[20]

$$\varphi_k = 1 - \frac{1}{\theta} \tag{11}$$

per unit of output, obtaining an aggregate profit of $\theta^k - \theta^{k-1}$. This profit flow lasts until the next innovation arrives. Therefore, the discounted value of the rents accruing to innovator k is $\frac{\theta^k - \theta^{k-1}}{r+x}$.

In addition, the innovator can also obtain speculative profits. With patent protection, the wage rate and the rents obtained by the irreproducible asset become $w_k = \alpha \theta^{k-1}$ and $R_k = (1 - \alpha)\theta^{k-1}$, respectively. Thus, each factor is rewarded as if the technology of vintage $k - 1$, instead of the state-of-the-art technology of vintage k, were used. Relative to the baseline model, rents, and hence the equilibrium price of the asset, are scaled down by a factor $1/\theta$. Speculative gains are then scaled down by a factor $1/\theta$, too. Thus, patent protection provides monopoly rents, but crowds out speculative profits – a trade-off that we shall elaborate on later.[21]

Inventor k will then obtain speculative profits $\gamma \frac{(\theta^k - \theta^{k-1})}{\theta} p$, where the growth-adjusted asset price p is still given by the asset price equation (3). The total discounted profits accruing to innovator k are therefore

$$\pi_k = \left(\theta^k - \theta^{k-1}\right) \left(\frac{1}{r+x} + \frac{\gamma}{\theta} p\right). \tag{12}$$

In this expression, the first term captures monopoly rents and the second speculative profits.

We now show that the possibility of cumulating these two sources of reward opens up the possibility of overinvestment in R&D. The free entry condition $\lambda_k \pi_k = 1$

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[20] To understand this formula, notice that the net output of a perfectly competitive firm that licenses the state-of-the-art technology is

$$\theta^k L^\alpha T^{1-\alpha} - \left(1 - \frac{1}{\theta}\right)\theta^k L^\alpha T^{1-\alpha} = \theta^{k-1} L^\alpha T^{1-\alpha},$$

i.e., the same as if the firm used the technology of vintage $k - 1$, which is less productive but does not command any royalty.

[21] The possibility of a trade-off between patents and speculative profits was hinted at by Duggan (1995).

becomes:

$$\left(1 - \frac{1}{\theta}\right)\left(\frac{1}{r+x} + \frac{\gamma}{\theta}p\right) = \frac{1}{\lambda}. \tag{13}$$

The equilibrium rate of innovation will be positive if the left-hand side of this equation, evaluated at $x = 0$, exceeds the right-hand side. Since at $x = 0$ the asset price is $p = \frac{(1-\alpha)}{r}$ and the interest rate r coincides with the rate of time preference ρ, the equilibrium rate of innovation will be positive if

$$(\theta - 1)\left[1 + \gamma\frac{(1-\alpha)}{\theta}\right] > \frac{\rho}{\lambda}.$$

Comparing this condition with the condition for the socially optimal rate of innovation to be positive, which is

$$(\theta - 1) > \frac{\rho}{\lambda},$$

one sees immediately that as soon as $\gamma > 0$ the market equilibrium rate of innovation can be positive for parameter values for which the socially optimal rate is zero. This suffices to prove the possibility of overinvestment. We can therefore conclude:

Proposition 3 *With patent protection, as soon as inventors obtain some speculative profits, the market equilibrium rate of innovation may exceed the socially optimal rate of innovation.*

Since, as noted above, patent protection crowds out speculative gains, this result is less obvious than it might seem. Consider, for instance, Hirshleifer's argument that a perfectly discriminating patent holder can capture the entire social value of the innovation without speculating, and therefore must necessarily be overcompensated as soon as he can also obtain some speculative profits. In fact, if patent holders could capture, by means of monopoly rents, the entire productivity improvement, then asset prices would not change at all as innovations arrive, and thus the opportunity for speculation would vanish.

To better clarify this point, note that in the model analyzed so far the only limit to patent protection is that a patent holder's market power is destroyed by the

occurrence of the next innovation – in the jargon of the economics of patents, there is no *forward* patent protection. To allow for such forward protection, let us now assume that m successive patents can be consolidated into a patent pool. The patent pool licenses the patents to the competitive firms operating in the product market, and each patent holder in the pool obtains his marginal contribution to the pool's total profits. The model considered so far corresponds to the case $m = 1$. However, by varying m one can capture different degrees of patent protection. The case of perfect patent protection is obtained when m is infinite. When $m = 0$, in contrast, we are back to the perfect competition model with no patent protection at all.

The analysis is a straightforward generalization of that developed for the case $m = 1$. In period k, innovator $k - m$ has just been excluded from the patent pool and so must stand ready to license his technology at a zero royalty rate. The patent pool will then charge an aggregate royalty rate per unit of output equal to $\varphi_k = 1 - \frac{1}{\theta^m}$,[22] obtaining an aggregate profit of $\theta^k - \theta^{k-m}$.[23]

We assume that each past innovator j who still participates in the patent pool obtains a share of this aggregate profit equal to his marginal contribution, $\left(\theta^j - \theta^{j-1}\right) / \left(\theta^k - \theta^{k-m}\right)$. When a new innovation arrives, each past innovator's share in the patent pool's profits decreases, but total profits increase in such a way that individual profit stays constant. However, after m successive innovations the patent holder is excluded from the patent pool and his profits vanish. Thus, innovator k obtains monopoly rents equal to $\left(\theta^k - \theta^{k-1}\right)$ for m periods, which gives a total discounted value of $\left(\theta^k - \theta^{k-1}\right) \frac{1 - \left(\frac{x}{r+x}\right)^m}{r}$.

In addition, the innovator can also obtain speculative profits. Now, however, each factor is rewarded as if the technology of period $k - m$, instead of the state-of-the-art technology of period k, were used. Relative to the baseline model, rents, the equilibrium price of the asset, and speculative gains are all scaled down by a factor

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[22] The argument is similar to that presented in footnote 20.

[23] This formula applies as long as $m < k$. Otherwise, the royalty rate is $1 - \frac{1}{\theta^k}$, as only the initial technology, of period 0, is in the public domain.

θ^{-m}.

Thus, the total discounted profits accruing to innovator k are

$$\pi_k = \left(\theta^k - \theta^{k-1}\right) \frac{1 - \left(\frac{x}{r+x}\right)^m}{r} + \gamma \left(\theta^k - \theta^{k-1}\right) \theta^{-m} p. \tag{14}$$

Equation (14) shows that an increase in m increases monopoly rents (the first term on the right-hand side) but decreases speculative gains (the second term). The intuition is that an increase in m prolongs the expected duration of the period over which innovator k collects monopoly rents, not having been displaced by m subsequent innovations yet. However, an increase in m also implies that monopoly rents are a larger fraction of output and thus less is left to reward productive assets. Therefore, an increase in m reduces asset prices, and hence also the speculative gains that inventors can obtain.

In particular, with full patent protection ($m = \infty$), monopoly rents are maximized: each innovator k obtains a permanent flow of profits of $\left(\theta^k - \theta^{k-1}\right)$, which equals the full social value of his innovation. However, such a fully protected patent holder cannot obtain any speculative profits at all. The reason for this is that with full patent protection all the productivity gains are reaped by the patent pool. As a result, asset prices do not change when new innovations arrive. That is, speculative profits are crowded out fully. Therefore, the innovator obtains exactly the social value of his innovation: the equilibrium rate of innovation is just socially optimal, not higher.

When $m = 0$, monopoly rents (i.e., the first term in (14)) vanish and speculative profits are largest. However, in this case we are back to the baseline model, where the equilibrium rate of innovation is always lower than the socially optimal one.

As we have seen above, however, overinvestment in research may occur for intermediate levels of patent protection, such as $m = 1$. The intuition is simple. When m is finite, an inventor's profits last for m periods only. However, when x is close to zero, such "periods" are in fact very long. Thus, as x approaches zero the inventor

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can actually capture the entire social value of his innovation by means of monopoly rents. At the same time, however, should the next innovation arrive, the asset price would jump up by a discrete amount. Anticipating this, inventors who can capture a positive fraction of the change in asset price would have an excessive incentive to invest in R&D.[24]

6   Conclusion

In this paper, we have proposed a model of a perfectly competitive economy in which inventors are rewarded by speculative profits only, formalizing an insight originally due to Hirshleifer (1971). We have shown that even the steady, predictable flow of innovations that is postulated in models of equilibrium growth may create, for a range of parameter values, sufficient speculative opportunities to sustain persistent growth. Using parameter values obtained from standard calibration exercises, the possibility seems in fact rather unlikely; but this assessment is far from definitive.

We have also shown that in our model inventors rewarded by speculative gains only are necessarily undercompensated. The reason for this is that the social value of innovations is the total increase in factor productivity, which includes the increase in the productivity of labour. Since labour cannot be the object of speculation, even inventors who can exploit their inside information perfectly, capturing the pecuniary effects of their innovations fully, would obtain only a share of what they contributed to society.

Things are different in an extended version of the model in which inventors can simultaneously benefit from inside information and patent protection. The analysis of this extended model reveals a trade-off between monopoly rents and speculative gains: strengthening patent protection increases the former, but decreases the latter. As a result, the private incentive to innovate may be highest for intermediate degrees of

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[24]While this mechanism may, at first, sound reminiscent of the business stealing effect (Mankiw and Whinston, 1986), it is in fact different. Our model economy is perfectly competitive, and thus incumbents do not earn any monopoly rent that can be transferred to new entrants.

patent protection, and may then exceed the social incentive. In fact, overinvestment in R&D is possible as soon as inventors obtain a positive share of the change in asset prices that their innovations create.

This paper provides the first formal model that captures Hirshleifer's insight that innovators can be rewarded by speculative profits. Being a first attempt, the analysis is preliminary in many respects. We have already mentioned the need for a fully microfounded model of insider trading, and the issues that such a microfoundation would raise. Equally important, our asset pricing equations are based on the notion that prices are fully pinned down by market fundamentals. A recent finance-growth literature has argued that new, high-risk technologies can attract excessive investments which may amplify movements in asset prices, disconnecting them from fundamentals.

This literature has been largely inspired by prominent episodes of market exuberance, such as, for instance, the US telecom companies' development of fiber-optic lines in the mid-1990s, or the UK "railway mania" in the mid-1840s. In these episodes, it became clear *ex post* that innovation had caused excessive movements in asset prices. Various explanations have been proposed. For example, De Marzo et al. (2007) argue that investments in new technologies generate positively skewed aggregate uncertainty, which, combined with imperfect tradeability of future endowments, can generate overinvestment and hence large asset price movements. In Angeletos et al. (2012), entrepreneurs have access to imperfectly correlated private information on the uncertain returns from their investments in a new technology. Overinvestment here arises from the entrepreneurs' incentive to correlate on high investments as a signal of high expected returns.

Whatever the exact reason why asset price movements may be amplified, it is clear that this should also amplify the opportunities for speculation. Re-assessing Hirshleifer's claims in a richer framework where asset prices may be disconnected from fundamentals is an important task for future research.

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Appendix

In this appendix, we modify the baseline model so as to allow for changes in *relative* asset prices within the same industry. This amplifies the opportunities for speculation. However, we show that even in this extended model there can never be overinvestment in R&D when inventors are rewarded by speculative profits only.

1. Asset specific innovations

Assume that each industry ω comprises two sectors, indexed by $v = 1, 2$. In each sector, the final good is produced using labour and a sector-specific, irreproducible asset. We normalize the supply of both assets to one and denote their prices by $P_{v,k}$ (where k still denotes the total number of innovations in the industry). The supply of labour is fixed and equal to one in each industry. However, labour can now freely move across the two sectors of an industry.

In sector v, the production function is given by:

$$y_v = \theta^{h_v} L_v^\alpha \tag{A.1}$$

where L_v is labour input and h_v is a technological index that depends on the number of past innovations. We now specify how this technological index is determined.

In each industry, the technological frontier corresponds to a total factor productivity equal to θ^k, where k is the total number of past innovations occurred in the industry. As in the baseline model, the variable k represents the industry-wide stock of knowledge, which all subsequent innovations build on in a cumulative way. However, each innovation is now targeted to a specific asset, and hence to a specific sector of the industry. That is, innovation $k+1$ raises total factor productivity to θ^{k+1} only in the sector in which it occurs, leaving total productivity unchanged in the other sector.

With these assumptions, the two sectors never share the same technology. In the advanced sector, i.e. the sector where the latest innovation has occurred, we have

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$h_v = k$. In the less advanced sector, by contrast, h_v equals the latest period in which an innovation occurred there. The technological gap between the two sectors depends on whether sectors alternate in leading, or several innovations occur in a row in the same sector. This is determined endogenously in equilibrium, as we shall see below.

The assumption that innovations are sector specific serves to generate changes in relative asset prices. Since factor productivity increases only in the sector where the innovation has occurred, labour flows from the less productive sector to the more productive one. The rents in the advanced sector increase because of the increase in productivity, and because of the inflow of labour. The rents in the less advanced sector, by contrast, decrease because of the outflow of labour. These creates pecuniary externalities that amplify the opportunities for speculative profits as compared to the baseline model.

Like in the baseline model, we assume that the innovator can use his inside information about the arrival of the innovation to capture a share γ of the increase in the value of the irreproducible asset in the sector where the innovation occurred. Let $n_{v,k} = \sum_i n_{i,v,k}$ denote aggregate R&D investment per unit of time in period k targeted to sector v. Then, the $k+1$-th innovation occurs in sector v according to a Poisson process with a hazard rate $x_{v,k} = \lambda_k n_{v,k}$. We continue to assume that $\lambda_k = \lambda \theta^{-k}$.

2. Equilibrium

Research firms now choose both the level of the R&D investment and the sector they target. While we cannot rule out the possibility of multiple equilibria, we focus on the case where the opportunity for speculative profits are largest. Evidently, this requires that all research is directed to the less advanced sector, so that in equilibrium the sectors systematically alternate in leading.[25] The following lemma guarantees the existence of such an equilibrium:

---

[25] If in equilibrium all research was directed to the leading sector, one would effectively be back to the baseline model.

Lemma 1 *There exists an equilibrium in which all research is targeted to the less advanced sector: that is, $x_{v,k} = 0$ whenever $h_v = k$.*

Proof. If research is directed to the more advanced sector, speculative profits are a fraction γ of $(P_{2,k+1} - P_{1,k}) = (\theta p_2 - p_1)\theta^k$, where p_2 is the (growth adjusted) asset price in a sector that leads by two steps. If instead research is directed to the less advanced sector, speculative profits are a fraction γ of $(P_{1,k+1} - P_{-1,k}) = (\theta p_1 - p_{-1})\theta^k$. It follows that the incentive to invest is the less advanced sector is greater than in the advanced sector if

$$\theta p_2 - p_1 < \theta p_1 - p_{-1}. \tag{A.2}$$

Thus, (A.2) is a sufficient condition for sectors to alternate in leading. To show that this condition is indeed satisfied, we must determine p_2. This, however, may in turn depend on the asset price when a sector is leading by three, four or more steps. In general, in a steady state asset prices are determined by the following arbitrage conditions

$$
\begin{aligned}
rp_1 &= R_1 + x_1(1-\gamma)(\theta p_2 - p_1) + x_{-1}(\theta p_{-1} - p_1) \\
rp_2 &= R_2 + x_2(1-\gamma)(\theta p_3 - p_2) + x_{-2}(\theta p_{-1} - p_2) \\
&\quad\quad ... \\
rp_i &= R_i + x_i(1-\gamma)(\theta p_{i+1} - p_i) + x_{-i}(\theta p_{-1} - p_i) \\
&\quad\quad ... \\
rp_{-1} &= R_{-1} + x_{-1}(1-\gamma)(\theta p_1 - p_{-1}) + x_1(\theta p_{-2} - p_{-1}) \\
rp_{-2} &= R_{-2} + x_{-2}(1-\gamma)(\theta p_1 - p_{-2}) + x_2(\theta p_{-3} - p_{-2}) \\
&\quad\quad ... \\
rp_{-i} &= R_{-i} + x_{-i}(1-\gamma)(\theta p_1 - p_{-i}) + x_i(\theta p_{-i-1} - p_{-i}) \\
&\quad\quad ...
\end{aligned}
$$

where the index i denotes the number of innovative steps by which sector i is leading (and, conversely, sector $-i$ is lagging). The rents R_i are determined by the condition

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that the marginal productivity of labour must be equalized across sectors. This implies

$$w_{i,k} = \alpha \theta^k L_i^{\alpha-1} = \alpha \theta^{(k-i)} L_{-i}^{\alpha-1}.$$

Together with the labour market clearing condition $L_i + L_{-i} = 1$, this condition yields:

$$L_i = \frac{\eta^i}{1 + \eta^i} \qquad \text{and} \qquad L_{-i} = \frac{1}{1 + \eta^i}$$

The rents then become:

$$
\begin{aligned}
R_i &= (1-\alpha)\left(\frac{\eta^i}{1+\eta^i}\right)^\alpha \\
R_{-i} &= \frac{(1-\alpha)}{\theta^i}\left(\frac{1}{1+\eta^i}\right)^\alpha.
\end{aligned}
$$

To confirm that there is an equilibrium with $x_i = 0$, we must consider out-of-equilibrium beliefs. Assuming rational expectations, we consider a candidate equilibrium where $x_{-1}(= x) > 0$ and $x_{-2} > 0$. (Values of x_{-i} for $i > 2$ are irrelevant.) In this candidate equilibrium, the arbitrage conditions become:

$$
\begin{aligned}
rp_1 &= R_1 + x_{-1}(\theta p_{-1} - p_1) \\
rp_2 &= R_2 + x_{-2}(\theta p_{-1} - p_2) \\
rp_{-1} &= R_{-1} + x_{-1}(1-\gamma)(\theta p_1 - p_{-1}) \\
rp_{-2} &= R_{-2} + x_{-2}(1-\gamma)(\theta p_1 - p_{-2}).
\end{aligned}
\tag{A.3}
$$

Since $x_{-1} > 0$ and $x_{-2} > 0$, the corresponding zero-profit conditions must hold as equalities. Thus, we have

$$
\begin{aligned}
\gamma(\theta p_1 - p_{-1}) &= \frac{1}{\lambda} \\
\gamma(\theta p_1 - p_{-2}) &= \frac{1}{\lambda},
\end{aligned}
$$

which implies

$$p_{-1} = p_{-2}.$$

Because $R_{-2} < R_{-1}$, for p_{-1} to equal p_{-2} it must be

$$x_{-2} > x_{-1}.$$

To proceed, notice that the system (A.3) is recursive, as the asset price conditions relative to p_1 and p_{-1} are independent of the others. Notice also that p_2 is a decreasing function of x_{-2}. Since in the candidate equilibrium x_{-2} cannot be lower than x_{-1}, a sufficient condition for inequality (A.2) to hold is that the inequality be satisfied when $x_{-2} = x_{-1}$.

Assuming that $x_{-2} = x_{-1} = x$, asset prices are

$$
\begin{aligned}
p_1 &= \frac{R_1 + x\theta p_{-1}}{r + x} \\
p_2 &= \frac{R_2 + x\theta p_{-1}}{r + x}.
\end{aligned}
$$

This can be rewritten as

$$
\begin{aligned}
p_1 &= \frac{r\bar{p}_1 + x\tilde{p}_1}{r + x} \\
p_2 &= \frac{r\bar{p}_2 + x\tilde{p}_2}{r + x},
\end{aligned}
$$

where $\bar{p}_i = R_i/r$ and $\tilde{p}_i = \theta p_{-1}$. That is, p_i is a weighted average of $\bar{p}_i$ and $\tilde{p}_i$, with weights equal to r and x, respectively. Simple algebra shows that condition (A.2) is satisfied when $p_i = \bar{p}_i$, and it holds as an equality when $p_i = \tilde{p}_i$. Since p_i is a weighted average of $\bar{p}_i$ and $\tilde{p}_i$ with strictly positive weights, and the inequality is linear, (A.2) must always hold. This completes the proof of the lemma. ∎

Intuitively, the change in asset prices is largest if the innovation occurs in the less advanced sector, where innovation $k+1$ would raise total factor productivity by two steps rather than one. With constant returns to scale in research, profit-maximizing research firms will target the sector where the arrival of the innovation generates the greatest change in asset prices. Since sectors alternate in leading, we shall henceforth denote by 1 the more advanced sector and by -1 the less advanced one.

Factor markets are perfectly competitive, so the wage rate equals the marginal productivity of labour:

$$w_k = \alpha \theta^k L_1^{\alpha-1} = \alpha \theta^{(k-1)} L_{-1}^{\alpha-1}. \qquad (A.4)$$

The marginal productivity of labour is equalized across sectors at each point in time, and the allocation of labour is efficient. Together with the labour market clearing condition $L_1 + L_{-1} = 1$, equation (A.4) can be solved to yield:

$$L_1 = \frac{\eta}{1+\eta} \qquad \text{and} \qquad L_{-1} = \frac{1}{1+\eta}$$

where $\eta \equiv \theta^{\frac{1}{1-\alpha}} > 1$. Clearly, $L_1 > L_{-1}$. Since sectors alternate in leading, when a new innovation occurs labour instantaneously flows to the sector where productivity has increased.

The rents that accrue to the owners of the sector-specific assets are:

$$
\begin{aligned}
R_{1,k} &= (1-\alpha)\left(\frac{\eta}{1+\eta}\right)^\alpha \theta^k \\
R_{-1,k} &= \frac{(1-\alpha)}{\theta}\left(\frac{1}{1+\eta}\right)^\alpha \theta^k.
\end{aligned}
\qquad (A.5)
$$

It can be shown that the rate of growth of the economy is still $(\theta-1)x$.[26]

---

[26] Substituting the equilibrium labour inputs into the production function, one gets the equilibrium outputs:

$$y_{1,k} = \left(\frac{\eta}{1+\eta}\right)^\alpha \theta^k \qquad \text{and} \qquad y_{-1,k} = \left(\frac{1}{1+\eta}\right)^\alpha \theta^{k-1}.$$

In each industry, total output

$$
\begin{aligned}
y_k &= y_{1,k} + y_{-1,k} \\
&= \left(\frac{1}{1+\eta}\right)^\alpha \left(\eta^\alpha + \frac{1}{\theta}\right)\theta^k
\end{aligned}
$$

grows at rate $\theta - 1$ from one period to the next. Since there is a continuum of industries, however, aggregate variables grow smoothly. Summing across industries, aggregate output is

$$
\begin{aligned}
Y &= \int_0^1 y_k(\omega, t)\mathrm{d}\omega \\
&= \left(\frac{1}{1+\eta}\right)^\alpha \left(\eta^\alpha + \frac{1}{\theta}\right)G,
\end{aligned}
$$

where $G \equiv \int_0^1 \theta^{k(\omega)}\mathrm{d}\omega$ is an average productivity index that increases over time with technical progress. The rate of growth of output is the rate of growth of the average productivity index, G. To calculate

Like in the baseline model, the return to asset v is the sum of the rents earned by the asset plus any expected capital gain or loss. In equilibrium, the rate of return must equal the interest rate r, implying:

$$rP_{1,k} = R_{1,k} + x(P_{-1,k+1} - P_{1,k}),$$

and

$$rP_{-1,k} = R_{-1,k} + x(1 - \gamma)(P_{1,k+1} - P_{-1,k}),$$

since only a fraction $(1 - \gamma)$ of the capital gain $(P_{1,k+1} - P_{-1,k})$ accrues to outside investors; the remaining fraction γ is the reward to the innovator.

In a steady state, the asset price equations become:

$$
\begin{aligned}
rp_1 &= R_1 + x(\theta p_{-1} - p_1) \\
rp_{-1} &= R_{-1} + x(1 - \gamma)(\theta p_1 - p_{-1}),
\end{aligned}
\tag{A.6}
$$

where $P_{s,k} \equiv \theta^k p_s$ and $R_{s,k} \equiv R_s \theta^k$, where p_s and R_s are growth-adjusted prices and rents, respectively, and $s = 1, -1$. These equations can be solved to express p_1 and p_{-1} as functions of x.

For future reference, we note that the occurrence of the innovation increases the price of the asset used in the sector where productivity increases, but decreases that of the other asset.

Lemma 2 $P_{1,k+1} > P_{-1,k}$ *and* $P_{-1,k+1} < P_{1,k}$.

Proof. The first part of the lemma is obvious, so it suffices to show that $p_1 > \theta p_{-1}$. The system (A.6) is linear in p_1 and p_{-1} and the matrix of coefficients has full rank (assuming that the transversality condition $r > (\theta - 1)x$ holds, which is always true

---

it, notice that $k(\omega)$ jumps up to the next higher integer with a constant instantaneous probability x. Hence:

$$
\begin{aligned}
\dot{G} &= \int_0^1 \left[\theta^{k(\omega)+1} - \theta^{k(\omega)}\right] x \, d\omega \\
&= (\theta - 1)xG.
\end{aligned}
$$

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in equilibrium). Thus, the system implicitly defines p_1 and p_{-1} as continuous and differentiable functions of x and r. The explicit expressions are:

$$
\begin{aligned}
p_1 &= \frac{R_1\left[r + x(1 + \theta - \gamma)\right] - (R_1 - R_{-1})\theta x}{\left[r^2 + (2 - \gamma)rx - (\theta^2 - 1)(1 - \gamma)x^2\right]} \\
p_{-1} &= \frac{R_1\left[r + x(1 + \theta - \theta\gamma)\right] - (R_1 - R_{-1})(r + x)}{\left[r^2 + (2 - \gamma)rx - (\theta^2 - 1)(1 - \gamma)x^2\right]}.
\end{aligned}
\tag{A.7}
$$

Notice that p_{-1} is always increasing in x, and that θp_{-1} increases with x more rapidly than p_1. Inequality $p_1 > \theta p_{-1}$ must then hold for all values of x if it holds when x is largest, i.e. the case $\gamma = 1$. In this case, we have

$$
\begin{aligned}
p_1 &= \frac{rR_1 + x\theta R_{-1}}{r(r + x)} \\
p_{-1} &= \frac{R_{-1}}{r},
\end{aligned}
$$

so

$$
\begin{aligned}
p_1 - \theta p_{-1} &= \frac{rR_1 + x\theta R_{-1} - \theta(r + x)R_{-1}}{r(r + x)} \\
&= \frac{R_1 - \theta R_{-1}}{(r + x)} > 0,
\end{aligned}
$$

where the inequality holds as $R_1 > \theta R_{-1}$. This completes the proof of the lemma. ∎

The speculative gains accruing to the $k+1$-th inventor are $\pi_k = \gamma(P_{1,k+1} - P_{-1,k})$, or $\pi_k = \gamma(\theta p_1 - p_{-1})\theta^k$. The zero-profit condition in innovation races then becomes

$$
\gamma\lambda(\theta p_1 - p_{-1}) = 1.
\tag{A.7}
$$

Since p_1 and p_{-1} are a function of x, the zero-profit condition determines a relationship between the interest rate and the rate of innovation. Like in the baseline model, the Euler equation provides another relationship, which together with the zero-profit condition uniquely determines the equilibrium interest rate and the rate of innovation.

Like in the baseline model, speculative profits can sustain innovation and growth. The necessary and sufficient condition is that the returns to R&D when no further

innovation is anticipated exceed the unit cost of R&D. When $x = 0$, asset prices reduce to $p_1 = R_1/\rho$ and $p_{-1} = R_{-1}/\rho$. Thus, growth can be sustained by speculative profits if and only if

$$\gamma\frac{(\theta R_1 - R_{-1})}{\rho} > \frac{1}{\lambda}. \tag{A.8}$$

It is immediate to verify that condition (A.8) is weaker than condition (7), confirming that the potential for speculation is higher in the two-asset model.

3. Comparison with social optimum

Now we are in a position to compare once again the market equilibrium with the social optimum. Although the potential for speculation is higher than in the baseline model, we still have:

Proposition 4 *In the model with asset specific innovations, the market equilibrium rate of innovation is always lower than the socially optimal rate.*

Proof. It can be easily confirmed that the equilibrium rate of innovation is largest when $\gamma = 1$. In this case, the innovator captures all the increase in the value of the asset that appreciates when the innovation arrives. Asset equilibrium prices then become

$$
\begin{aligned}
p_1 &= \frac{rR_1 + x\theta R_{-1}}{r(r + x)} \\
p_{-1} &= \frac{R_{-1}}{r}.
\end{aligned}
$$

Notice that the price of the less productive asset is independent of x and always equals the discounted value of the rents R_{-1}, as all future capital gains are appropriated by the innovator. The price of the more productive asset, by contrast, decreases with x. This follows from the fact that $R_1 > \theta R_{-1}$. The intuitive reason is that holders of the more productive asset suffer a capital loss when the new innovation arrives in the other sector, causing a reallocation of labour across sectors. It follows that the incentive to innovate, $\theta p_1 - p_{-1}$, is now a decreasing function of x. Unlike the

baseline model, the zero profit condition now determines the following relationship between the interest rate r and the rate of innovation x:

$$x = r \frac{\lambda(\theta R_1 - R_{-1}) - r}{r - \lambda(\theta^2 - 1)R_{-1}}. \tag{A.9}$$

Over the relevant range, this is a decreasing function which is zero at $r = \lambda(\theta R_1 - R_{-1})$ and tends to infinity as r approaches $\lambda(\theta^2 - 1)R_{-1}$. (From (A.5), it is easy to see that $(\theta R_1 - R_{-1}) > (\theta^2 - 1)R_{-1}$.)

The equilibrium is determined by the intersection between (A.9) and the Euler equation (6). Since the Euler equation is increasing, and r^* cannot exceed $\lambda(\theta R_1 - R_{-1})$, the equilibrium rate of innovation must satisfy:

$$x^* < \lambda \frac{\theta R_1 - R_{-1}}{(\theta - 1)} - \frac{\rho}{(\theta - 1)}. \tag{A.10}$$

This provides an upper bound on the equilibrium rate of innovation.

The socially optimal rate of innovation can be calculated proceeding as in the baseline model. Optimality requires that the static allocative efficiency condition (A.4) holds. Clearly, the social planner will direct all the research to the less advanced sector, where there is more to gain from innovating. Thus, along the optimal path sectors will alternate in leading, as in the market equilibrium we have been focusing on. The optimal resolution to the dynamic trade-off between current and future consumption lead to the following optimal rate of innovation:[27]

$$\hat{x} = \lambda \frac{R_1 + R_{-1}}{(1 - \alpha)} - \frac{\rho}{(\theta - 1)}. \tag{A.11}$$

We now prove that $x^* < \hat{x}$. From (A.10) and (A.11), it follows that a sufficient condition for $x^* < \hat{x}$ is:

$$\Delta(\alpha, \theta) \equiv \theta R_1 - R_{-1} - \frac{R_1 + R_{-1}}{1 - \alpha}(\theta - 1) \leq 0.$$

---

[27] For simplicity, the calculation is based on the assumption that the initial conditions conform to the steady state properties. That is, we have assumed that initially in each industry one sector has a one-step technological lead over the other, as is always true in the steady state.

Substituting for R_1 and R_{-1}, $\Delta(\alpha, \theta)$ can be written as:

$$\Delta(\alpha, \theta) = -\left(\frac{\theta^{\frac{1}{1-\alpha}}}{1 + \theta^{\frac{1}{1-\alpha}}}\right)^{\alpha} \theta^{-\frac{1}{1-\alpha}} \left[(\theta\alpha - 1)\,\theta^{\frac{1}{1-\alpha}} - (\alpha - \theta)\right].$$

Thus, the sufficient condition becomes:

$$H(\alpha, \theta) = (\theta\alpha - 1)\,\theta^{\frac{1}{1-\alpha}} - (\alpha - \theta) \geq 0$$

Since $H(\alpha, 1) = 0$, the sufficient condition becomes $H'_\theta(\alpha, \theta) \geq 0$. We calculate:

$$H'_\theta(\alpha, \theta) = \frac{\alpha(2 - \alpha)\,\theta^{\frac{1}{1-\alpha}} - \theta^{\frac{\alpha}{1-\alpha}} + (1 - \alpha)}{1 - \alpha}.$$

This implies that a sufficient condition for $x^* < \hat{x}$ is that $K(\alpha, \theta) \equiv \left[\alpha(2 - \alpha)\,\theta^{\frac{1}{1-\alpha}} - \theta^{\frac{\alpha}{1-\alpha}} + (1 - \alpha)\right] \geq$ 0. We have $K(\alpha, 1) = \alpha(1 - \alpha) \geq 0$. Thus, the sufficient condition can be restated as $K'_\theta(\alpha, \theta) \geq 0$. Finally, we verify:

$$K'_\theta(\alpha, \theta) = \frac{\alpha\theta^{\frac{\alpha}{1-\alpha}}}{(1 - \alpha)}\left[(2 - \alpha) - \theta^{-1}\right] \geq 0$$

since $2 - \alpha \geq 1$ and $\theta^{-1} < 1$. $\blacksquare$

The intuition is as follows. The reason why there is more scope for speculation in the model with asset specific innovations is that the reallocation of labour across sectors amplifies changes in asset prices. Clearly, the effect of labour reallocation can be strong only if the income share of labour is large. However, when α is large speculative profits must be small as compared to the social value of innovations. Thus, the fact that the increase in the productivity of labour cannot be captured by speculating on the tradeable asset still produces an underinvestment result.

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