NON-NORMALITY AND NON-LINEARITY IN UNIVARIATE STANDARD MODELS OF INFLATION

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To my mother Maria Hristova,

to my father Dimitar Hristov,

and

to my brother Emil Hristov

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ABSTRACT

The empirical evidences presented in a vast number of recent publications gave rise to debates in the literature regarding the problem of stationarity of inflation. Sometimes considered as a unit root process and sometimes as a stationary process, in most of the studies inflationary time series are modelled assuming normality and linearity. The present thesis relaxes the frequently used assumptions of linearity in price processes and normality in distribution of inflation, and suggests two ways of modelling inflationary data. Firstly, it is assumed that distribution of inflation is a stable Paretian distribution and, under this assumption, stationarity of inflation is examined applying an appropriate test. Secondly, price time series are modelled by treating them as a unit root bilinear process, which further leads to non-normality in distribution of inflation. A recently proposed test for presence of no-bilinearity is then applied. If bilinearity is detected, the bilinear coefficient is estimated by the Kalman filter method. Subsequently, the finite sample properties of this estimator are evaluated using Monte Carlo simulation experiments. A series of Monte Carlo simulations leads to calculating the *t*-statistic critical values for testing whether the estimated bilinear coefficients significantly differ from zero.

The methodologies explained above are then applied to a large set of worldwide price and inflationary data for 107 different countries. Assuming that the distribution of inflation is a stable Paretian distribution 75% of the inflationary time series are classified as integrated of order zero. Under the assumption of normality of distribution of inflation this can be inferred for 11.11% of the inflationary time series. It has been also shown that 71.03% of the price time series exhibit unit root bilinearity. Analysis of the inflationary time series reveals the presence of bilinearity in 9.35% of them.

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CHAPTER THREE DISTRIBUTION AND STATIONARITY OF WORLDWIDE INFLATIONARY DATA: EMPIRICAL ANALYSIS

CHAPTER FOUR THE KALMAN FILTER ESTIMATION OF UNIT ROOT BILINEAR INFLATIONARY PROCESSES

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INTRODUCTION

The importance of modelling inflationary processes has been emphasised in the studies of many theoreticians and practitioners. Price stability and, therefore, low inflation rate has become a central issue of monetary and fiscal policy in many countries. In some of the countries the central bank monetary policy is oriented toward inflation targeting and is supported by a fiscal policy aiming at a balanced budget. This focuses both policymaker's interests and the economist's attention to the problem of inflationary forecasting. However, a reliable inflationary forecast rests on stationary time series, thus, highlighting the importance of correctly specifying the nature of inflationary processes regarding stationarity.

The dilemma of stationarity of inflation is a widely disputed issue in the recent literature. Many researchers treat inflation as a stationary process: Engle and Granger (1987), Clements and Mizon (1991), Johansen and Juseius (1992), Quah and Vahey (1995), etc. On the other hand, Nelson and Schwert (1977), Hall (1986), Baillie (1989), Ball and Cecchetti (1990), Johansen (1992) and Gartner and Wehinger (1998) among others have specified in their works inflation as a unit root process. Furthermore, in the studies of some researchers inflationary time series is being considered as both unit root and stationary process (*see* Engsted (1995), Barsky (1987), Mishkin (1992), Schwert (1987), etc.). Empirical evidences presented in the above publications support both suggestions regarding stationarity of inflation, as the conclusions made vary with time periods, frequency of observations and test results.

In the majority of the papers mentioned, stationarity of inflation is examined assuming normality. More precisely, testing the null hypothesis of linear unit root in prices is accompanied by the assumption that inflation, that is first difference in log of prices, is normally distributed process. However, this assumption is hypothetical and dubious one – plenty of empirical evidences demonstrate that inflationary data are far from being normally distributed. Therefore, it is reasonable to check the order of integration of inflationary processes relaxing this assumption.

Initially, inflation (and first difference of inflation) is modelled under the more realistic assumption that it distribution belongs to a broad family of distributions – stable Paretian distributions – from which the normal distribution is a special case (*see* Zolotarev (1986), Rachev and Mittnik (2000), etc.). Under the assumption that the disturbances of a random walk process are stable Paretian distributed, Rachev, Mittnik and Kim (1998) propose a unit root test conditional on the index of stability indicating the tail thickness of the distribution. Non-normality is indicated by an index of stability of magnitude smaller than the magnitude of the index of stability of the normal distribution.

Furthermore, non-normality in distribution of inflation is achieved by assuming non-linearity in prices. Imposing some restrictions, linear unit root models can be considered as a sub-class of the class of bilinear processes (*see* Granger and Andersen (1978), Subba Rao and Gabr (1974), Terdik (1999), etc.). This leads to the idea of applying bilinear processes in economic and financial time series modelling (*see* Charemza, Lifshits and Makarova (2002c)), in particular for modelling inflationary processes. Charemza, Lifshits and Makarova (2002b) propose a test for bilinear unit root conditional on the existence of linear unit root in the time series. In presence of bilinearity an issue of further investigation is evaluation of the magnitude of bilinearity in unit root bilinear processes and a way of its realization is suggested in this thesis.

Thus, the work continues with estimation of the bilinear coefficient in unit root bilinear price and inflationary processes. As bilinear processes distinguish themselves with a recursive structure, the recursive Kalman filter method (*see* Hamilton (1994), Harvey (1989), etc.) seems to be an appropriate estimation technique. Nevertheless, some of the features of this method, namely its applicability to non-stationary and non-linear processes and exact Maximum likelihood estimation make the algorithm preferable for the purposes of our analysis. It is also shown that for unit root bilinear processes with 'small' bilinearity and given sufficient number of observations the estimated innovations converge to the true values when the Kalman filter method is applied. The finite sample properties of the Kalman filter estimator are evaluated using a series of Monte Carlo simulations. Furthermore, Monte Carlo experiments are conducted for calculation of the t-statistics critical values, used later for testing if the estimated bilinear coefficients are significantly different from zero. Subsequently, the Kalman filter estimator is applied to a large selection of world-wide price and inflationary data (price and inflationary time series for 107 different countries).

In short, the rest of the thesis is outlined as follows: Chapter 1 concentrates on the problem of inflation predictability and, in this context, defines the term of core inflation (that is predictable inflation) and reviews some of the traditional ways of measuring it. It introduces a new device for measuring core inflation, called wavelets, and presents the results of measuring and forecasting inflationary processes for Poland and the United Kingdom applying different techniques. All the techniques, however, are based on the assumption of normality of inflationary distribution. Chapter 2 motivates the choice of modelling inflation under the assumption of non-normality, defines the stable Paretian distribution and the bilinear processes. It also dwells on the theoretical foundations of the tests suggested for establishing stationarity in price and inflationary processes under different assumptions, that is: the assumption of normality (classical unit root and stationarity tests), the assumption of stable Paretian distribution (unit root tests) and under the assumption of bilinearity (two-step testing procedure for presence of bilinearity). Abreast with the description of the data set (price time series for 107 different countries), Chapter 3 of this thesis discusses the test results obtained regarding stationarity of price (and inflationary) time series after the application of the tests suggested in Chapter 2. Furthermore, Chapter 4 adapts the Kalman filter for the purposes of estimating the bilinear coefficient in unit root bilinear price processes. It also presents the results of two Monte Carlo simulation experiments: (a) for evaluating the finite sample properties of the Kalman filter estimator, and (b) for calculating the t-statistics critical values for testing whether the estimated coefficients significantly differ from zero. The Kalman filter method is then applied to 108 world-wide price and inflationary data and the results obtained are discussed in Chapter 5. Finally, abreast with the concluding remarks the thesis finishes with suggestions for further research in the area.

CHAPTER ONE *PREDICTABILITY AND STATIONARITY OF INFLATION*

1. Introduction

Called by Friedman 'phenomenon', inflation is one of the main subjects of macroeconomics and is defined as the 'sustained rise in the general level of prices or a persistent fall in the value of money' (Dawson (1992)). Moreover, it is the most commonly used economic term among both economists and the general public. Robert Shiller (1996) states that the word inflation appears in 872 004 news stories in the ALLNWS (all news) section of the Nexis system[1] versus 602 885 stories for the word unemployment, and even outranks the word sex, which can be seen in only 662 920 stories only. One of the purposes of this section is to answer the question is inflation a social problem. The average person would say that inflation makes him/her poorer, since the raise in prices nullifies some of the raise in his/her salary. This is based on the assumption that in the absence of inflation people would be able to buy more goods if their salaries have risen. Although economists disagree about the size of social costs, there is a big gap between the public view and the cost of inflation identified by economists. This is supported by the curious survey results proposed by Robert Shiller in his 1997 article. He asked both the general public and economists whether their 'biggest gripe about inflation' was that 'inflation hurts my real buying power, it makes me poorer'. 77% of the public versus only 12% of economists agreed with this statement. As a result of another more curious question that Shiller asked, namely whether the people agreed with the following

[1] Nexis system is an electronic search system for English language news publications.

statement 'I think that if my pay went up I would feel more satisfaction in my job, more sense of fulfilment, even if prices went up just as much', surprisingly 49% of the public agree compared to only 8% of the economists. In short, the term inflation is widespread among economist and the general public and the view of the social cost of inflation significantly differs between these two groups of people.

What is inflation and how do economists interpret this term? To provide a macroeconomic explanation of the term inflation we must consider the following examples, demonstrating the essence of inflation. In the United States in 1970, the price of the 'New York Times' was 15 cents and the average wage in the manufacturing sector was $3.36 per hour. Twenty-seven years later, in year 1997, the price of the 'Times' was 60 cents and the same average wage had increased to $13.6 per hour.[2] If the reduction of the purchasing power of the money is so big that makes money useless and if this happens quickly, inflation is called hyperinflation. For example, in Germany, in July 1921 the purchasing power of 1 DM was equivalent to the purchasing power of 54 000 million DM in November 1923.[3] These examples focus on the importance of inflation and stress the need of its forecast as a matter of urgency, since it is important for future individual or governmental decision-making.

One of the main aims of this chapter is to provide an answer of the following question: Is inflation forecasting significant and indispensable? In short, if the level of prices is not correctly anticipated the real effect of inflation is observed, namely the difference between measured and expected level of prices, called metaphorically 'price surprise', while correctly anticipated inflationary rate would lead to correct individual and governmental decision making. As will be shown in Section 2.2, under the assumptions of RE-NRH, unanticipated inflation ('price surprise') is more costly than the anticipated inflation and, therefore, the better the inflationary forecast, the lower will be the cost of unanticipated inflation. Hence, it is important that rational inflationary expectations are formed correctly, i.e. there is a need for a reliable way of anticipated inflation forecasting.

It is, however, widely known that no single index provides a precise measurement of inflation, since price data exhibit high volatility and thus, contain substantial 'noise'.

[2] *See* Mankiw (2000).

[3] *See* Dawson (1992).

Eliminating the 'noise' part, the underlying (also called core inflation) is extracted. Although there is no explicit definition of the term core inflation, researchers try to find an accurate way of its estimation, based on different interpretations of this term. In all the concepts of core inflation, however, it is 'generally associated with expectations and demand pressure components of measured inflation and excludes supply shocks' (Roger (1998)). Thus core inflation is identified with the anticipated part of measured inflation. As unanticipated inflation is more costly than the anticipated one, it is important a reliable way of anticipated inflation forecasting to be found. Moreover, the better the forecast the lower will be the cost of unanticipated inflation.

Different methods of inflation decomposition and forecasting are known, as some of them are presented in this chapter. Chapter 1 also suggests a new technique, called wavelets, for core inflation measurement. Recently, this new device for signal decomposition has been successfully used in the economic area. As far as I know, the application of wavelets in the area of core inflation measuring is presented for first time in the present work.

The rest of the chapter is organized as follows: Section 2 states a brief overview of some inflationary indexes and discusses key measurement problems related to them. Next, the real effect of inflation is considered pointing out the importance of inflation forecasting. Further in this section a new inflationary term called core inflation is defined and an overview of different ways of measuring it is presented followed by brief theoretical explanation of three of them, as used in the empirical analysis later: Centered moving average, Exponential smoothing and Wavelets. The empirical analysis is based on inflationary data for two countries, exhibiting different patterns of inflationary processes: Poland and the United Kingdom. Further, for the purposes of inflation forecasting, the results of Exponential smoothing, Trend polynomial of order three and ARMA models are compared. However, when analysing inflationary processes a matter of primary importance is to specify correctly their order of integration. Thus, Section 3 of this chapter dwells on the often-questioned issue in the recant literature regarding the problem of stationarity of inflation, summarizes the findings related to this topic and suggests two alternative approaches of modelling inflationary processes. Finally, Section 4 concludes.

2. Inflation, predictable and core inflation

2.1 Inflation: definition and measurement

Defined by Dawson (1992) as the "sustained rise in the general level of prices or a persistent fall in the value of money", the "phenomenon" inflation can be viewed as a measure of the real changes in the average level of prices. As the main interests of the governments are directed to the establishment of economic stability and sustained economic growth, an important issue from policymakers' point of view is the availability of an accurate quantitative index for inflation. However, it is difficult to specify a single measure of this macroeconomic term: different indexes describe different sides of the inflationary process. In summary, while CPI, using fixed basket, measures the prices of certain goods and services bought by the consumer and PPI measures the producers' prices, GDP deflator is a much more complex index, that incorporates both consumption and production processes. GDP deflator measures the prices of all goods and services produced in a domestic economy (only) thus, allowing the basket of goods to change over time. However, the main disadvantages of the last measure are first, its annual basis availability only and second, it takes usually up to two or three years in order for this measure to be precisely computed. As a matter of grate concern to the policymakers in forming current policy and preparing the government budget, this measure is of a little use. Another, although more limited price measure, CPI (or similarly RPI, PPI)[4], which is available on monthly, quarterly or yearly basis, is usually used for inflationary assessment.

In 1995, the "Advisory Commission to Study the Consumer Prise Index" (widely known as the Boskin Commission) was formed and, appointed by the Senate Finance Committee in

[4] The rate of change of the prices of goods and services bought by the consumers is measured in some countries, for example UK, using Retail Price Index (RPI). According to Dawson (1992), the main difference between RPI and CPI is the measurement of the housing costs. For CPI calculation the housing costs are based on the rents, while calculating RPI, housing costs are 'taken to be the monthly mortgage repayment, on the grounds that this is the proportion of the monthly mortgage household budget that has to be allocated to housing' (Dawson (1992)).

An alternative measure of the price level is Producer Price Index (PPI), which measures the price of a typical basket of goods bought by firms rather than consumers.

order to examine the magnitude of the measurement error in the CPI in the US. The Boskin Commission's report concludes that in the United States CPI "overstated inflation by 1.1 percent per year in 1995-96" (Gordon (2000)), which significantly reflects most of the leading macroeconomic issues as "estimates of growth in output and productivity, median income, and real wages...it has major consequences for the time path of the government budget deficit and national debt; it produces misleading estimates of inflation for monetary policymakers for whom the inflation rate is a critical target" (Gordon 2000). Following Mankiw (2000) (and curiously enough), this measurement error has led to a rise in the federal government's debt by more then $1 trillion over a dozen years.

As mentioned earlier, CPI is an index that measures the overall level of prices based on the prices of goods and services included in a fixed basket. Since some of the goods are purchased more often than others, the different items included in the basket are characterised by fixed weights and the CPI is computed as a weighted average. For the case of the US, the weights for the various goods and services are derived ones per decade from the Consumer Expenditure Survey in the US. For example, those used in the Boskin Commission's Report in 1996 are based on the average of 1982-84, i.e. between 12 and 14 years out of date. The data necessary for CPI calculation are collected in the following way: first, the items are divided on broad commodity groups such as housing, food, transportation, medical care, etc. Consequently, every one of these categories is divided into subcategories, which are further subcategorised, etc. As a result, a hierarchical structure of CPI is received. Its lowest level is called an item stratum and below the strata are the entry-level items.

CPI measures the price of a fixed basket relative to the price of the same basket in a chosen base period (month, quarter, year) using the standard Laspeyers[5] formula:

$$I_{i,t} = \sum_{i=1}^{n} \left(\frac{p_{i,t}}{p_{i,0}} \right) . w_{i,0} \ ,$$

where $w_{i,0} = \dfrac{p_{i,0} . q_{i,0}}{\sum_{i=1}^{n} p_{i,0} . q_{i,0}}$ represents the weight of item i in the based period, p is the relevant price and q stands for quantities (monthly, quarterly, yearly).

[5] Paasche formula designed for a changing basket is used when GDP deflator is calculated.

According to Gordon's paper (2000) related to Boskin Commission's Report, three key measurement problems support the idea that CPI calculated using Laspeyer's formula tends to overstate inflation (Table 1.1[6]): upper and lower level of substitution and, outlet substitution. In the following lines attention is focused on the first two.[7]

Source of Bias	*Estimate (% per annum)*
Upper Level Substitution	**0.15**
Lower Level Substitution	**0.25**
Outlet Substitution	**0.1**
New Products/Quality Change	**0.6**
Total	**1.1**

Table 1.1 Boskin Comission Estimates of Bias in CPI

As for the purposes of computations a fixed basket of goods is used, one of the problems is the upward substitution bias: if consumers substitute one good for another, as a result, for example, of fall in the relative prices, it would not reflect the CPI measure. Boskin Commission's recommendation is Laspeyer's index for the upper level strata to be used and geometric weights for the most of the lower level categories to be employed when CPI is calculated. The second problem arises from the introduction of new products as well as change in the quality of the existing goods. When a new product is introduced consumers are better off since their choice is greater and the real value (i.e. purchasing power) of the money increases. Although difficult to be measure, quality improvements, such as comfort or safety, cause increase in the measured price index. For example, the price of audio-visual goods fell in real terms by 20% between 1974 and 1996 but no account of quality improvements has been made[8]. Ward and Dikhanov (1999) report in their paper an interesting example that demonstrates decrease in the speed, reliability, convenience and comfort in US airfares compared with 15-20 years ago: any flight from a major US city to an arbitrary destination in Europe is now 60-90 minutes longer; the flight service and especially the food have deteriorated; the seat space in the most widely used economy class has been significantly reduced; lots of destinations are not served now with the same

[6] *See* Gordon (2000).

[7] According to Gordon (2000) the third problem, namely outlet substitution bias seems minor. For example in US and other countries this bias is 0.1 percent per decade.

[8] Dr. Stephen Price's lectures, *Department of Economics, University of Leicester*, 2000-2001academic year.

frequency as before; schedules are less convenient; arrival and departure times are at unsatisfactory levels; the number of direct flights is significantly reduced.

The Boskin Commission divides its recommendations with respect to the three time horizons: short, medium and long run. In short run and medium run their recommendations include new data collection initiatives. According to the Boskin Commission, in short run the evaluation of the CPI has to be changed in order for this index to be more current and secondly, the cost of the living index has to be annually updated and continuously revised. In medium run the Commission suggests "reforms that are feasible in the current state of the art, ... reorganisation of activities, and/or changes in the detail of the various subindexes produced by the CPI" (Gordon (2000)). Long run recommendations however, "require additional research and attention" (Gordon (2000)).

Thus, CPI does not accurately measure inflation. One of the reasons for this is that it is usually represented as a time series of monthly or quarterly observations. Such time series are highly volatile and therefore contain a lot of 'noise'. Eliminating the 'noise' part, the underlying (core) inflation is extracted. This is relevant to one the concepts of underlying inflation, that is the view of core inflation as the persistent element[9] of the measured inflation. Prior to focus on the theory of core inflation let us briefly discuss - in the following subsection - the role of correctly formed inflationary expectations (associated with reliable inflationary forecast) by discussing the real effect of inflation.

[9] The concept of core inflation as generalised component of measured inflation is developed in Okun (1970), Flemming (1976), Fase and Folkertsma (1996).

2.2 Inflation and real effects

The present section focuses on the importance and significance of inflationary forecasting. As already mentioned, if the level of prices is not correctly anticipated the real effect of inflation is observed, known as 'price surprise'. This is simply the difference between measured and expected level of prices. The so-called 'price surprise' model is theoretically generalised by Lucas (1972) and the rest of the section presents Lucas' model supply side.

2.2.1 Lucas model

In Lucas' model suppliers are located in a large number of separated competitive markets *M*, where single good is distributed. Based on the Rational Expectations – Natural Rate Hypothesis (RE-NRH)[10] theory, the model considers an economy, in which, there is imperfect information and hence the individuals "cannot distinguish relative from general price movements" (Lucas (1973)) (similar to the labour supply example, where workers cannot distinguish nominal from real wages). Following Cuthbertson and Taylor's (1987) approach to Lucas' model and using similar notations, consider the supply side of the model.[11]

The quantity supplied by producers in market *M* at time *t*, denoted by $y_t(M)$, alters around its natural level of output $y^*(M)$:

$$y_t(M) = y^*(M) + y_t^c(M),$$

where $y^*(M)$ is a component common to all markets[12] and $y_t^c(M)$ is a cyclical component, which varies from market to market. Assume that $y^*(M)$ is constant over the time *t* and that the cyclical component $y_t^c(M)$ varies only with relative prices[13]. The latter term can be expressed as an equality of the following form:

$$y_t^c(M) = \gamma\left(p_t(M) - E^M p_t\right),$$

[10] *See* Attfield, Demery and Duck (1991), Begg (1982), Demery and Duck (1991).

[11] *See* also Turnovsky (1995), Stevenson and Muscatelli (1988), etc.

[12] In his original paper, Lucas (1972) postulates that the natural level of output $y^*(M)$ follows a time trend. For simplicity, let's assume that $y^*(M)$ i s a constant.

[13] By contrast, in Lucas' paper (1972), $y_t^c(M)$ depends on its lagged value $y_{t-1}^c(M)$.

where, for specific market M, $p_t(M)$ represents the actual local price at the market M at time t, p_t is the aggregate price level over all markets and $E^M p_t = E[p_t|\Omega_t(M)]$ is the expected price, associated with the mean general price level over all the markets, conditional on all the information available, $\Omega_t(M)$, to the economic agents in market M at time t. It is evident that the individuals, located in market M have more information about the local prices $p_t(M)$ and less information for the general price level p_t. In his model the author assumes that the price of the good in market M, $p_t(M)$ differ from the average price p_t of the good in all the markets and therefore can be viewed as:

$$p_t(M) = p_t + u_t(M), \tag{1.1}$$

where $u_t(M) \sim N(0,\sigma^2)$ independent of p_t, is a relative price shock, different for different markets and determined by events specific for the local market M.

Similarly, the average price level p_t varies from its expected value, namely:

$$p_t = Ep_t + v_t, \tag{1.2}$$

where $Ep_t = E[p_t|\Omega_t]$ is the expectation of the aggregate price level and $v_t \sim N(0,\tau^2)$ is a forecasting error. Both terms $u_t(M) = p_t(M) - p_t$ and $v_t = p_t - Ep_t$ have become known as price surprises (supply shocks), where the first error, $u_t(M)$, represents the surprise specific for market M and the second one, v_t, the average price surprise for all the economy.

Substituting then (1.2) into (1.1) yields:

$$p_t(M) = Ep_t + v_t + u_t(M).$$

For a fixed market M the price of the good is known, $p_t(M)$, and in order to form expectations about the general price level, economic agents use all the information available in their market at time t. Then, the following result can be shown algebraically:

$$y_t = y^* + \gamma(1-\beta)(p_t - Ep_t),$$

where $\beta = \dfrac{\tau^2}{\sigma^2+\tau^2}$, $0 < \beta < 1$.

The problem a supplier located in market M faces is whether a change in the local price reflects the price in all other markets. The smaller is the variance of the average price surprise shock τ^2, the bigger is β and the smaller is $(1-\beta)$, suppliers are, therefore, willing to supply more and locally the level of output will increase. Conversely, the bigger the variance τ^2 the smaller is β and the suppliers in this sector will not increase the supply in response to the price increases. The level of output then will not deviate from its natural level of output y^*.

Lucas' model of aggregate supply, therefore, can be summarised by the function:

$$y_t = y^* + \theta(p_t - Ep_t), \tag{1.3}$$

where $\theta = \gamma(1-\beta)$ $\theta > 0$. This states that the output deviates from the natural level of output only when unexpected rise in the general price level is established. Correct expectations, that is $p_t = Ep_t$, imply that the level of output y_t supplied is equal to the natural level of output y^*, consistent with long run vertical aggregate supply curve. In contrast, when expectations deviate from the actual price level, output changes in the following way: y_t increases for prise level higher than the expected one and decrease when the prise level is lower, that is the supply of output is positively related to the current price level. Actually, equality (1.3) represents the short run AS curve, where the parameter θ shows how much output responds to unexpected changes in the price level.

The analysis made above illustrates the significance of inflation forecasting. Associating the anticipated inflation with forecasted inflation, Lucas' AS model demonstrates the real inflationary effect, namely: if the actual level of prices is not correctly predicted, consistent with price surprise, the real output deviates from its natural rate, while fully forecasted price level will not affect the level of real output.

2.2.2 The Cost of Inflation

As mentioned above, inflation can be split into two parts: anticipated and unanticipated. However, distinction has to be made between the cost of perfectly anticipated and imperfectly anticipated inflation. Although costly, it can be shown that anticipated inflation is less costly than unanticipated one, which once again points out the significance and importance of inflation forecasting. The following two sections present the economists' view of the social cost of steady and predictable inflation, as well as, steady and unpredictable rate of inflation.

2.2.2.1 The Cost of Anticipated Inflation

A number of costs of the anticipated inflation have been identified by economists, e.g. shoes-leather cost, menu cost, infrequent price changes, nominal contracts, etc. Consider the leather-shoes cost of inflation. Under the assumptions of anticipated inflation, higher inflation leads to an increase in the nominal interest rate, which on the other hand, causes an increase in the cost of money holding by the people. The individuals will prefer to keep their cash holdings to the minimum required and will deposit the rest, if the interest rate is paid. Consequently, they will make more frequent trips to the bank to withdraw money and will 'attempt to synchronise cash expenditures with the receipt of cash income' (Briault (1995)). The cost of anticipated inflation is therefore associated with the loss of some of the utilities people get from the holding cash (e.g. fuel, time, unused labour, etc.). The positive relationship between the cost of inflation and the rate of inflation is consistent with the empirical results of Tobin (1972), Minford and Hilliar (1978), Fisher (1981), Lucas (1993). They report that even fully anticipated inflation has a large social effect. For example, in the US 'an extra percentage point of anticipated inflation embodied in nominal interest rates produces in principle a social cost of 2/10 of one per cent of GNP per year' (Tobin 1972). As Braiult (1995) states 'this estimates are very sensitive to the specification of the money demand function and to the chosen definition of money (in most developed countries, cash in domestic circulation is a fairly small proportion of national income).'

Menu cost arises as a result of an increase in the rate of inflation, which makes the firms to change their prices and hence, to print new menus and catalogues more frequently.[14] Since the price adjustments are costly, some of the firms will change the prices only if their desired price level is large enough to justify the costs of adjustments. The infrequent change in prices is another type of cost of inflation: higher inflation leads to higher variability in the relative prices. This is illustrated by the following example, presented by Mankiw (2000). Consider a firm that prints a new catalogue every January. If there is no inflation, firm's prices relative to the overall level of prices will remain the same over the year. However, 1% monthly inflation will lead to relatively high prices and, hence, low sales in the beginning of the year. By contrast, the relative prices will be low and the sales will be high at the end of the year. In this example, the variability in relative prices induced by inflation is demonstrated.

[14] In the German hyperinflation of 1923, for example, the prices in the pubs changed every half an hour. Under hyperinflation the individuals eating in a pub prefer to pay their bills in advance, since they expect the prices to risen at the time of their meal.

2.2.2.2 The Cost of Unanticipated Inflation

A typical example that demonstrates the cost of unexpected inflation is long-terms loans. At the time of agreement the creditor and the debtor form expectations of the future inflationary rate over the loan period. Based on their expectations, the nominal interest rate is usually fixed by a loan agreement. Assuming that inflation differs from what was expected and let us suppose rate of inflation higher than expected. An increase in the rate of inflation leads to an increase in the nominal interest rate. Since in our example the interest rate determined by the agreement remains unchanged for a fixed period, the debtor wins and the creditor loses.

The following example illustrates the impact of the unanticipated inflation on individuals with fixed pensions. Let us assume that a retired worker receives fixed nominal pension. Actually, the worker could be seen as a creditor, since during the time he/she is working, he/she provides labour services to the firm but is not fully paid. Therefore, inflation higher than the anticipated level will lower the real value of the pension that the retired worker receives. Lower inflation, on the other hand, will hurt the firm, seen as a debtor in this example.

Based on these situations, one may conclude that the more variable the rate of inflation, the greater the uncertainty that firms and individuals face. A solution in this situation is the agreement to be written in real terms rather than in nominal, using as an index more stable foreign currency. The earliest empirical results (Foster (1978), Logue and Willet (1976), Okun (1971), etc.) suggest that there is a positive relationship between the variability and the level of inflation. However, according to Briault (1995) variability and uncertainty are not the same things. 'Inflation might be highly variable, but if the process generating it were understood there might be little associated uncertainty and the costs of variable inflation will be lower if the variations are predictable' (Briault (1995)). In their empirical work Ball and Cecchetti (1990), Engle (1983), Evans (1991), Evans and Wachtei (1993) among others, try to measure uncertainty by adjusting the measures of variability and the reported results suggest a positive relationship between the rate of inflation and measured uncertainty (particularly for uncertainty over longer time horizons). Finally, according to Mankiw (2000), it is a widely documented fact that high inflation is associated with variable inflation.[15] For example, countries with high average inflation tend to have inflation rates that change greatly from year to year.

It is important, however, to point out that unanticipated inflation is more costly than the anticipated one, which emphasises once again the significance of inflation forecasting.

2.3 Core inflation

As it has been pointed out in the previous section, under the assumptions of RE-NRH, unanticipated inflation (or 'price surprise') is more costly than anticipated inflation. Thus, in order to lower the cost of unanticipated inflation it is important that the formation of rational inflationary expectations is correct. In other words, economists need a reliable way of anticipated inflation forecasting. As it has been mentioned earlier, the most commonly used measure for inflation, CPI, contains a lot of 'noise' and thus, does not evaluate precisely the 'phenomenon' inflation. In the present section a new term, named core inflation is introduced and its nature, significance and possible ways of measuring are summarized. However, before providing precise definitions of this term let us start the discussion with the relationship between anticipated and core inflation.

There are two central concepts of core inflation: first, the view of underlying[16] inflation as persistent and secondly, as generalised component of measured inflation. However, in both concepts, core inflation is 'generally associated with expectations and demand pressure components of measured inflation and excludes supply shocks' (Roger 1998). Core inflation is, therefore, associated with anticipated inflation and the difference between measured and core inflation is called unanticipated inflation ('price surprise'), or roughly speaking 'noise'. Although there is no an explicit definition, the researchers try to find a reliable estimate of the underlying inflation based on different interpretations of this term.

2.3.1 Definitions of core inflation

Although indirectly, the idea of core inflation starts from Milton Friedman (1963): he defines inflation as a "steady and sustained increase in the general price level" and also

[15] According to Briault (1995), however, 'it is difficult to reach any firm conclusion that higher rates of inflation necessary lead to greater relative price variability'.

[16] The terms core and underlying inflation are used interchangeably.

distinguishes "a steady inflation, one that proceeds at a more or less constant rate, and an intermittent inflation, one that proceeds by fits and starts". In other words, Friedman associates the 'steady' or the persistent component of inflation with the anticipated long-run inflation and the 'intermittent' inflation with unanticipated one.

The term underlying or core inflation is originally defined in the work of Eckstein (1981). He describes the underlying inflation as 'the trend increase of the cost of the factors of production'. Following Eckstein, core inflation "originates in the long-term expectations of inflation in the minds of households and businesses, in the contractual arrangements which sustain the wage-price momentum, and in the tax system". In Eckstein's definition, inflation is viewed as a product of two components, namely measured inflation, and that part of measured inflation, which results from: first, supply shocks and secondly, cyclical movements, caused by aggregate demand shocks.

Following Roger (1998) aggregate inflation can be presented as an equality of the form[17]:

$$\pi_t = \pi_t^{LR} + g_t + \zeta_t,$$

where π_t and π_t^{LR} denote the aggregate[18] and long run inflation rates, g_t measures cyclical demand shocks and ζ_t is a measure of transient disturbances (e.g. supply shocks). Then core inflation might be represented in the form:

$$\pi_t^c = (\pi_t - g_t - \zeta_t) = \pi^{LR},$$

while non-core inflation, associated with that component of inflation, which results from cyclical changes and supply shocks can be expressed as:

$$\pi^{nc} = g_t + \zeta_t$$

Hence, according to Eckstein's definition, if expectations are formed rationally, only the non-core component of inflation will show a cyclical tendency. Core inflation should demonstrate a cyclical tendency only when long run inflationary expectations are adaptive. In long run, however, prices exhibit flexibility and do not influence real output. Therefore, in the long run and following Eckstein's definition, the core component is determined as strongly output neutral. In contrast, in short run, when prices exhibit inertia and the output deviates from its natural rate, Eckstein's definition determines real output as approximately output neutral.

[17] *See* Roger (1998). The notations used are similar to those used by Roger (1998).

[18] This is the increase in the money supply.

The theory of core inflation became very popular and widely used after the publication of Quah and Vahey's (1995) paper. The authors define core inflation as 'that component of measured inflation that has no medium- to long- run impact on real output' (this definition is consistent with vertical Philips' curve). As it has been shown earlier (Section 2.2.1), based on the RE-NRH, long run vertical aggregate supply curve is consistent with correct expectations (i.e. $p_t = Ep_t$) and, therefore, in order to be output neutral in long- to medium-run, core inflation must be associated with that inflationary component, that corresponds to the rational inflationary expectations. Some of the supply shocks, however, affect permanently the level of prices, but "have no lasting impact on the rate of inflation" (Roger 1998). These shocks, therefore, are excluded from Quah and Vahey's definition. However, core inflation "does include cyclical movements in inflation associated with excess demand pressure" (Roger 1998).[19]

Using the notations made earlier, Quah and Vahey's core inflation can be represented by the following expression:

$$\pi_t^c = (\pi_t - \zeta_t) = \pi_t^{LR} + g_t \tag{1.4}$$

and the non-core inflation as:

$$\pi_t^{nc} = \zeta_t . \tag{1.5}$$

By contrast with Eckstein's definition, Quah and Vahey's underlying inflation has a strong cyclical character. Equalities (1.4) and (1.5) show that core inflation and non-core inflation correspond to anticipated and unanticipated inflation, respectively. Therefore, in long run, when prices are fully flexible, core inflation is output neutral and non-core inflation is correlated with the output. In contrast to Eckstein's definition, in short run core inflation should be correlated with output. However, in both definitions, supply shocks are considered as having only passing effect on inflation and are excluded from core inflation definition.

Another view of the term core inflation is represented by Bryan and Cecchetti (1993). Both authors consider the relationship between the persistent element of the measured price index and money growth, defining the term core inflation as "a measure of money-induced

[19] In Quah and Vahey definition the term 'cyclical component' refers to business cycles (that is short run fluctuations in output, incomes and unemployment)

inflation: that is, the component of price changes that is expected to persist over medium-run horizons of several years" (Bryan and Cecchetti (1993)).

A version of the single period model, proposed by Ball and Mankiw (1992) is presented in Bryan and Cecchetti's (1993) paper and specifies their view of the term core inflation. Consider an economy consisting of two types of price setters: ones that change their prices period after period, following changes in economy, and price setters that set their prices infrequently as a result of costly price adjustments. The latter change their prices only if their desired price change is large enough to justify the costs of adjustment, known as a "menu cost". The relevant price paths exhibit different behaviour for the different type of price setters: the firms, changing their prices often and frequently possess price paths characterised by large transitory fluctuations and as a result noise is added to the measured inflation. By contrast, the second type of price setters, forming their expectations rationally, possesses smooth price paths, associated with the desired long-run inflationary trends.

Based on the shock distribution, the response of the overall price level on the shocks that affect relative prices is demonstrated. Symmetric distribution, for example, is consistent with an increase in the prices of some of the firms, offset by price cuts made by others and therefore, the average price level remains the same. Conversely, if the shock distribution is skewed, the average price level will temporarily move up and down and hence the high readjustment costs might result in transitory movements of the headline[20] inflation from its long-run trend.

Facing the single period problem and given the assumptions of zero trend output growth, constant velocity and constant money growth *m*, Bryan and Cecchetti consider an economy consisting of a large number of firms, which face the same "menu cost" when adjusting their prices. Under these assumptions, each firm will decide to change its price by *m*, and therefore, the aggregate inflation will be equal to the monetary inflation[21]. Following both authors, the term core inflation is defined as:

$$\pi^c = m.$$

[20] The terms headline and overall inflation are used interchangeable.

[21] Aggregate measures of inflation imply a smooth, uniform rise of the general price level.

Considering a fixed firm i, let's denote by ε_i the shock experienced to either its product demand or its production costs. Hence, the firms would have changed their prices by ε_i, i.e.:

$$\pi_i = \dot{m} + \varepsilon_i$$

However, as a menu cost has to be paid, only the firms with large $|\varepsilon_i|$ will change the prices. Let's denote the critical values of ε_i with $\underline{\varepsilon_i}$ and $\overline{\varepsilon}_i$, the lower and upper limits for the shock ε_i, respectively. As one of the assumptions states that all the firms face the same menu cost, that is:

$$\underline{\varepsilon_i} < \varepsilon_i < \overline{\varepsilon}_i \qquad (1.6)$$

only the firms satisfying condition (1.6) will change their prices. The average inflation, therefore, depends of the nature of the shock distribution. Symmetrical distribution, for example, will lead to $\pi = \pi^c = m$, while skewed shock distribution will deviate from π^c in the following manner: π^c will be greater than the core inflation π^c if the distribution is positively skewed, that is the inflation of the current period will be above the core inflation, while in the period following the shock it will be below the core inflation.

Bryan and Cecchetti's definition of core inflation is based on the assumption that $\dot{m}$ and ε_i are independent, i.e. "absence of monetary response to supply shocks" occurs, which - on the other hand - is consistent with lack of monetary accommodation. In contrast to Eckstein, and similarly to Quah and Vahey's definition, Bryan and Cecchetti's core inflation is output neutral, consistent with long run vertical aggregate supply curve. "Any deviations of inflation from π^c will result in changes in real money balances and move y away from y^*" (Bryan and Cecchetti (1993)), where y and y^* denote the actual and the natural rate of output, respectively.

Finally, although slightly different, in all the definitions explained above the term core inflation is associated with the anticipated part of measured inflation. Varieties of methods for measuring and forecasting underlying inflation are known. A brief overview of selected methods is presented in the next section together with a short description of a new technique, called wavelets, which to the best of my knowledge is applied for first time in the area of inflationary measurement.

2.3.2 Core inflation measures

2.3.2.1 Overview

As mentioned, core inflation considered as the persistent element of measured inflation is simply the difference between measured inflation and its transitory element. The transitory components, associated with 'noise', present the high frequency price changes and are usually characterised by high skewness and excess kurtosis, that is the tails of the distribution are fatter than the tails of the normally distributed random variables. Limited-influence estimators, such as median and trimmed means, are possible solutions when facing such problems.[22]

The former method is developed by Bryan and Pike (1991). In order to calculate the median, the inflation values in the CPI basket are ordered in the following way: $\{p_1,..., p_n\}$, where $p_i < p_j$, $i < j$ and the associated weights are $\{w_1,..., w_n\}$, $w_i < w_j$, $i < j$ respectively. Consequently, the cumulative weights, denoted by W_i are defined as:

$$W_i = \sum_{j=1}^{i} w_j$$

and the median inflation is then determined as the inflation rate $p_k \in \{p_1, \ldots, p_n\}$ at which the cumulative weight W_k reaches 50 percent. The median inflation p_k is therefore the 50[th] percentile inflation rate at which half of the components in the CPI basket have higher inflation, and the other half less. [23]

The method of trimmed mean is introduced by Bryan and Cecchetti (1993) and involves trimming both sides of the CPI components inflation distribution by a certain percentage, which is consistent with removing the tails from the distribution. Using the notations above, α - percent trimmed mean is calculated in the following way:

$$p_\alpha = \frac{1}{1-(2\alpha)/100} \sum_{i \in I_\alpha} w_i p_i,$$

[22] Other methods are also known, e.g. excluding food and energy, elimination of extreme values, excluding the administered prices, etc.

[23] According to Hogg (1967) if the kurtosis of a data distribution is above 5.5%, the median is recommended estimator.

where $I_\alpha = \{ W_i \mid \alpha/100 < W_i < 1-\alpha/100 \}$. That is α percent trimmed mean is a weighted average of (100-α) percent observations, removing α percent of the CPI components with the smallest and largest rates of inflation.

Under the assumption of symmetrically distributed component prices in the CPI basket, there is a slight difference between the median, trimmed mean and CPI measures of core inflation. However, trimmed mean provides efficient results for leptokurtic distributions. The higher the kurtosis of a distribution, the higher the efficient trim is. This is as a result of the following: higher kurtosis means a larger number of observations located in the tails of the distributions and unrepresentative of the central tendency. The trimmed mean method described above illustrates symmetric trims and will produce a biased estimator. If, however, the distribution is skewed, the trimmed mean will be lower than the overall CPI inflation for positive skewed distribution, and larger for negative.[24] When the trimming is disproportional from the tails, the method is known as asymmetrical trimmed means and is found to be more efficient if the distribution exhibits positive or negative skewness.[25] For example, the greater the coefficient of skewness of a positively (negatively) skewed distribution, the greater percentage of trim has to be taken from the left (right) tail of the distribution.

An alternative method of core inflation measuring - output-neutral method – was proposed by Quah and Vahey and became widely used after their publication in 1995. They define the term core inflation as "that component of measured inflation that has no medium- to long- run impact on real output", consistent with a vertical long run Philips curve. The authors suggest an alternative technique for calculating and forecasting the underlying component, using Vector Autoregression (VAR)[26] model of two variables: output and measured inflation[27]. Following Quah and Vahey (1995) "the rate of the RPI is problematic as a measure of inflation, since the RPI is not designed to measure movements in the general price level". As it has been already mentioned, RPI (or CPI) is constructed based on the costs and weights of various components included in a fixed basket of consumer

[24] Wozniak (1999)

[25] Jaramillo (1998)

[26] Hamilton (1994)

[27] The models of Claus (1997), Gartner and Wehinger (1998) among others are an advance in so far as they include more variables in the VAR system

goods and services. According to Quah and Vahey (1995) "in the inflation context the weights are potentially misleading and possibly meaningless" and it is not clear whether "a given price change in a good with a higher weight" is "more inflationary than in a good with a lower one". Measured inflation is considered as influenced by two types of exogenous shocks[28]: first, the disturbances that have no effect on the real output after fixed horizon, associated with the core inflation and, secondly, the disturbances that may significantly influence output in medium to long run horizons.

Applying decomposition methods to economic time series, in particular to inflationary data, aims at subtracting the random variable of the data thus, obtaining their underlying pattern. One way of distinguishing this pattern from the 'noise' is by smoothing the observed values. The underlying pattern, known also as trend-cycle, presents the long run changes and is sometimes subdivided into trend and cyclical components. Subtracting the core inflation using time series smoothing techniques refers to Eckstein's definition of the term core inflation. The empirical analysis presented in the Section 2.3.3 is based on three smoothing techniques: Centered moving average (CMA), Single exponential smoothing and Wavelets, briefly discussed in the following sections.

2.3.2.2 Centered moving average[29]

Moving average is the simplest and oldest method of smoothing. A centered moving average technique is based on moving averages and is located at the middle of the period being averaged. When smoothing by k-centered moving average one has to distinguish between k being an odd and an even number. Consider a time series, which consists of n observations: Y_1, Y_2, ...Y_n.

If the number of the observations k being averaged is an odd number, moving average of order k and k centered moving average methods coincide. The following series can be calculated:

[28] In fact, the economy is hit by a large number of heterogeneous shocks with different effects on the measured inflation and output.

[29] *See* Markidakis (1998), Greenwald (1963).

$$\frac{Y_1+Y_2+\ldots+Y_{(k+1)/2}+\ldots+Y_k}{k},\frac{Y_2+Y_3+\ldots+Y_{(k+3)/2}+\ldots+Y_{k+1}}{k},$$
$$\ldots,\frac{Y_{n-k+1}+Y_{n-k+2}+\ldots+Y_{(2n-k+1)/2}+\ldots+Y_n}{k},$$

where $Y_{(k+1)/2},Y_{(k+3)/2},\ldots,Y_{(2n-k+1)/2}$ are the relevant centres and their number is n-$(k$-$1)$. As a result $(k-1)/2$ observations at each side of the time series are lost. In addition, for odd, k-point moving averages the k elements weighting system is $\left[\frac{1}{k},\frac{1}{k},\ldots,\frac{1}{k}\right]$.

Consider now the case when k is an even number. The moving averages are located half a period early and half a period late. For example, in a monthly inflationary time series any even moving average is located fifteen days away from the original series. Next, by averaging these two k moving average smoothers, the centered moving average is obtained. In this case the process of smoothing is equivalent to a weighted moving average of order k+1 with weights 1/2k for the first and the last observations in the average and 1/k for the rest. The relevant k+1 elements weighting system is now $\left[\frac{1}{2k},\frac{1}{k},\ldots,\frac{1}{k},\frac{1}{2k}\right]$ with n-k centres: $Y_{(k+2)/2},Y_{(k+4)/2},\ldots Y_{(2n-k)/2}$. The number of the observations lost at each side of the time series is $k/2$.

2.3.2.3 Single exponential smoothing[30]

Consider time series with length n and let's denote the observed values with Y_t and the fitted values with F_t, where t = 1, 2, ..., n. Exponential smoothing is then defined by recursion formula of the following form:

$$F_t = \alpha Y_{t-1} + (1-\alpha)\, F_{t-1}, \qquad t = 2, \ldots, n, \qquad (1.7)$$
$$\alpha \in (0,1),$$

i.e. the fitted value at the moment t is a linear combination of the most recent observation and the most recent fit with weights α and $(1-\alpha)$ respectively. Substituting consequently F_{t-1}, F_{t-2}, ... by their equal in equality (1.7) the following result is obtained:

$$F_t = \alpha Y_{t-1} + \alpha(1-\alpha)Y_{t-2} + \alpha(1-\alpha)^2Y_{t-3} +$$
$$+ \alpha(1-\alpha)^3Y_{t-4} + \ldots + \alpha(1-\alpha)^{t-2}Y_1 + \alpha(1-\alpha)^{t-1}F_1.$$

[30] *See* Markidakis (1998)

that is F_t can be seen as a weighted moving average of all the past observations with exponentially decreasing weights as the observations get older.

However, when calculating F_t, $t = 2, \ldots, n$ one faces problems of identifying α and F_1. Various methods can be used for initialising F_1. The simplest way is to equate the starting value F_1 to the first observed value Y_1. Another possibility is to initialise F_1 by simply averaging the first several observations. Other methods related to this topic have been also proposed as backcasting, decomposition, etc. Similarly, another important issue is the way of choosing the weight α, $\alpha \in (0,1)$: while choosing α closed to zero, say $\alpha = 0.1$, leads to over-smoothing of the time series, large α's value, say $\alpha = 0.9$, corresponds to very little smoothing. The parameter α is usually set to the value that yields the smallest Mean Square Error (MSE) value, i.e. the MSE is first presented as a function of the coefficient α and is later minimised with respect to this coefficient. The optimum point can be then chosen for initialising the weight α.

2.3.2.4 Wavelets [31]

Every periodical[32], square - integrable function[33, 34] has a Fourier representation[35], i.e. can be transformed into a linear combination of sines and cosines also known as 'waves'. Decomposing a function in such a way has - from practical point of view - one disadvantage: waves never tend to zero. For practical purposes we need a basis consisting of functions that tend to zero on infinity. The so-called 'wavelets'[36], usually denoted by $\psi(t)$, distinguish themselves with very fast tendency to zero. Before we look at the way of decomposing a time series into wavelets, let us focus our attention on the issue of modifying a given wavelet $\psi(t)$ in a way that it satisfies some desirable properties we would like this function to posses. Different families of wavelets are known. For the

[31] It might be interesting fact for the reader that wavelets are used by FBI for storing fingerprints.

[32] $f(t)$ is called a periodical function if the following equality holds:

$$f(t)=f(t+l), \text{ where } l \in N.$$

[33] That is $\int_{-\infty}^{+\infty} f^2(t)dt < \infty$.

[34] It is easy to show that every real function can be transformed into periodical function.

[35] *See* Harvey (1981), Granger and Hatanaka (1964).

[36] Intelligible explanation of the wavelets can be found in Chui (1992b). *See* also Daubechies (1992), Mayer (1993), Mallat (1989).

purposes of our analysis the so-called Daubechies' wavelets are used. Their advantageous are explained later.

As mentioned, wavelets exhibit fast convergence to zero. Thus, suppose the wavelet $\psi(t)$ is a function, such that $\psi(t) \to 0$ very fast. Hence, shifting $\psi(t)$ along R, $\psi(t)$ will cover the whole real line R. Therefore, it is convenient the function to be modified in the following way:

$$\psi(t-k),\ k \in Z, \tag{1.8}$$

where k shows the wavelet position.

Another feature we would like wavelets to possess is different frequencies. Let us further modify $\psi(t-k)$, or equivalently $\psi(t)$, in the following way:

$$\psi(2^{-j}\, t-k),\ j,k \in Z, \tag{1.9}$$

where 2^{-j} is the frequency of the wavelet. The inverse of the frequency is called scaling factor and the parameter j shows the relevant scale. Obviously, in (1.9) the scaling factor is 2^{j}. While the frequency shows how often a function repeats itself in a unit of time interval, the scaling factor corresponds to the wavelets' stretching or compressing.

Consider now a wavelet $\psi(t)$ with scaling factor $2^{-j} = 1$. Scaling factor greater than one corresponds to a stretched wavelet, while scaling factor smaller than one leads to a compressed wavelet. In short, the smaller is the scaling factor, the more compressed is the wavelet and the larger is the scaling factor, the more stretched it is. The wavelet $\psi(t)$ is often called in the literature *mother wavelet.* In summary, by the use of (1.8) the mother wavelet is shifted with step k, while (1.9) gives its scaled version.

While Fourier analysis decomposes a function (or time series) into waves (sines and cosines) of various frequencies, wavelet analysis breaks up a function (or time series) into wavelets $\psi(t)$ satisfying properties (1.8) and (1.9). Equality (1.10), namely:

$$f(t) = \sum_{n=-\infty}^{\infty} c_n \cdot e^{int}, \tag{1.10}$$

where c_n are constants, demonstrates that any function $f(t)$ can be presented as a linear combination of the elements of the orthonormal[37] basis $\{e^{int}\}_{n \in Z}$. Equality (1.10) is simply

[37] If all the basis elements have unit length in orthogonal basis, the basis is called orthonormal.

Fourier series presentation of $f(t)$. In a similar way and using Wavelet analysis, the aim is to find an orthonormal basis $\{\psi_{j,k}\}_{j,k\in Z}$, such that every real square-integrable function $f(t)$ can be represented as a linear combination of the basis elements, namely:

$$f(t)=\sum_{j=-\infty}^{\infty}\sum_{k=-\infty}^{\infty} D_{j,k}\psi_{j,k}(t)$$

where by definition $\psi_{j,k}(t) = 2^{-j/2}\psi(2^{-j}t-k)$. The coefficients $D_{j,k}$ can be found in the following way:

$$D_{j,k}=\frac{1}{\sqrt{2^j}}\int_{-\infty}^{+\infty} f(t)\psi_{j,k}(t)dt \qquad (1.11)$$

and characterise the wavelet $\psi(t)$ at scale j and position k.

Daubechies (1988) constructed class of *orthogonal wavelets*, called Daubechies' wavelets. They are asymmetric and every one characterises with an order N, $N = 1, 2, \ldots$ The higher the order N of the wavelet, the smoother the wavelet is. With regard to their order, this class of wavelets possess important properties, namely:

- they are compactly supported with support length $2N\text{-}1$:
  $supp\{\psi\} = [-(N-1), N]$
- they have N vanishing moments – the highest number of vanishing moments for a given support length among the other classes of wavelets

A useful device that represents the process of wavelet decomposition is the Multiresolution Analysis[38]. According to the definition of Multiresolution Analysis, there exists a function $\phi(t)$ called scaling function. It can be shown that if $\phi(t)$ has a compact support such that:

$$\phi(t)=\sum_{k=0}^{n} c_k\phi(2t-k),$$

then $\phi_{j,k}(t)=2^{j/2}\phi(2^{j}t-k)$, $j\in Z$ is an orthonormal basis in V_j.[39]. Further, it can be shown that $\psi_{j,k}(t)=2^{j/2}\psi(2^{j}t-k)$ is an orthonormal basis in the space W_j [40] as V_j and W_j are orthogonal.

[38] *See* Appendix A for Mallat's definition (1989) of Multiresolution Analysis.

[39] $(V_j)_{j\in Z}$ is a subspace in $L^2(R)$. The properties of V_j are explained in more details in Appendix A.

[40] The properties of W_j are explained in more details in the Appendix A.

Consider now time series[41] $f(t)$ of length m[42]. Let us call for simplicity the high-frequency information of $f(t)$ 'noise' (or details) and the low-frequency information approximation. Applying first, low-pass[43] and high-pass[44] filters and secondly, down sampling, our original series is split into two signals: details and approximation. Replacing then the signal with the approximated one and using the same procedure, new approximation and 'noise' are received, i.e. (Fig. 1.3):

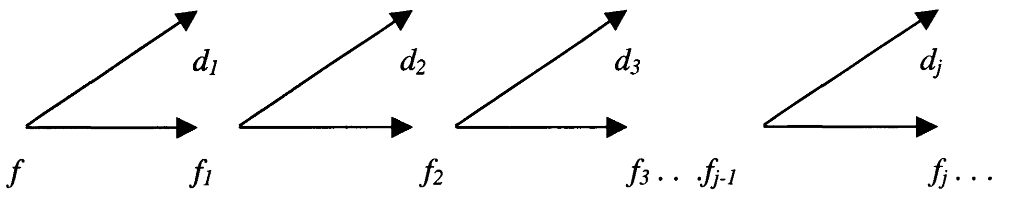

Fig. 1.3

Therefore, the decomposition of the time series $f(t)$ at level j can be represented uniquely as a sum of one approximation and j levels of noisy information:

$$f(t) = f_j(t) + \sum_{i=1}^{j} d_i(t),$$

where $f_j(t)$ and $d_i(t)$ are the approximation and the 'noise' signals respectively. This algorithm is proposed by Mallat (1987) and the relevant signals are calculated using the following two formulas:

$$f_j(t) = \sum_{k=1}^{\infty} A_{jk}\phi(2^j t - k),$$

$$d_j(t) = \sum_{k=1}^{\infty} D_{jk}\psi(2^j t - k),$$

where the coefficients A_{jk} and D_{jk} are calculated using formula (1.11). As in the real world the number of observations is limited (i.e. our data set is finite) theoretically the decomposition can continue until the approximation can be split into two parts. In practice, however, the levels of decomposition are chosen conditional on the main purpose. Following Greenblatt (1994) "different type of behaviour may become evident at different levels of resolution. We may look at trends, cycles, or extrema in the underlying data generating process".

[41] The terms time series and signal are used interchangeable.

[42] The length of the time series has to be power of two.

[43] A filter that subtract the low frequency information from the time series.

[44] A filter that subtract the high frequency information from the time series.

2.3.3 Empirical evidences

One of the main objectives of this chapter is to present the empirical results of some of the methods, theoretically presented in Chapter 2, for core inflation measuring. Further, based on data of the core inflation obtained, some traditional methods of time series forecast are used. Using monthly CPI and RPI data, the relevant analysis is done for two European countries, Poland and the United Kingdom, which exhibit different patterns of inflationary processes. Before to explain the results obtained for the measured and forecasted core inflation for both countries, let us consider the data sets used as well as the character of the time series.

2.3.2.1 Data

The data used in the present work consists of monthly RPI and CPI[45] inflationary data over the period: 05/1980 - 07/2000 (or equivalently 243 observations) for the United Kingdom and 01/93 - 01/2000 (or 85 observations) for Poland collected from "Datastream".

Further, the analysis proceeds considering lnCPI and lnRPI. The standard tests confirm that lnCPI and lnRPI can be treated as I(1) processes.

	ADF Test Statistic	*ADF Test Statistic* (1^{ST} *difference*)
POLAND	**-2.9074** (95% critical value : -3.4704)	**-10.8806** (95% critical value: -2.9012)
UK	**-1.4098** (95% critical value : -3.4303)	**-5.0213** (95% critical value: -2.8741)

Table 1.2

The results summarised in Table 1.2 lead to the conclusion that both price time series are integrated of order one. Consider for example Poland: according to the Akaike Information Criterion (AIC) the Augmented Dickey-Fuller (ADF) statistic is –2.9074, which is well bellow in absolute value than the absolute value of its 95% critical value. Hence, we do not reject the null hypothesis of a unit root in the lnCPI and we proceed with differencing the

[45] The measure of the level of prices in the United Kingdom is RPI, while Poland uses CPI.

time series in order to achieve stationarity. The ADF statistic for ΔlnCPI is –10.8806, which is in absolute value well above the absolute value of the 95% critical value –2.9012 thus, leading to the conclusion that the time series ΔlnCPI is stationary, that is lnCPI ~ I(1). By analogy, the results listed in Table 1.2. reveal that the same could be concluded for UK RPI data, i.e. ΔlnRPI ~ I(1). From now on, the analysis presented in this section is obtained based on the stationary time series ΔlnCPI and ΔlnRPI[46], that is ΔlnCPI ~ I(0) and ΔlnRPI ~ I(0).

2.3.3.2 Core inflation measurement

Several methods for core inflation measurement have been already discussed in the previous section: Centered Moving Average, Single exponential smoothing and Wavelets. In the present section the empirical results of their application for measuring core inflation for Poland and the United Kingdom are presented. Following the work of Bryan and Cecchetti (1997), Cecchetti (1998), Wozniak (1999) among others, Centered Moving Average of the measured CPI (RPI) is used as a benchmark trend. Perhaps this method does not accurately presents the "true" core inflationary process, but according to Bryan at al (1997), CMAs come very close to people beliefs about core inflation. The issue of how correctly to chose the length of the time horizon to be averaged is subject to debates. Wider horizon over which one averages corresponds to less volatile time series but leads to lost of more observations. The empirical results for the UK show that 36 CMA of monthly RPI data is the best approximation of the trend inflation among other CMAs. In the study of Poland, however, as the monthly time series of ΔlnCPI data consists of only 84 elements, 24 CMA seems the most suitable approximation for the Polish inflationary trend, as fewer observations will be lost. On the other hand, following Wozniak (1999) "in the case of transition economies (like the Polish economy during the sample period) it is reasonable to assume that the trend itself is more variable and therefore setting a narrower horizon seems desirable." The values of 36 CMA and 24 CMA inflationary trends for the United Kingdom[47] and Poland[48] are graphically represented in Fig. 1.4 and Fig. 1.5.

[46] According to the definition of inflation ΔlnCPI and ΔlnRPI are associated with inflation, since they represent the change in the level of prices.

[47] The ΔlnRPI data used are over the period: 12/1981 to 01/1999.

[48] The ΔlnCPI data used are over the period: 02/1994 to 01/1999.

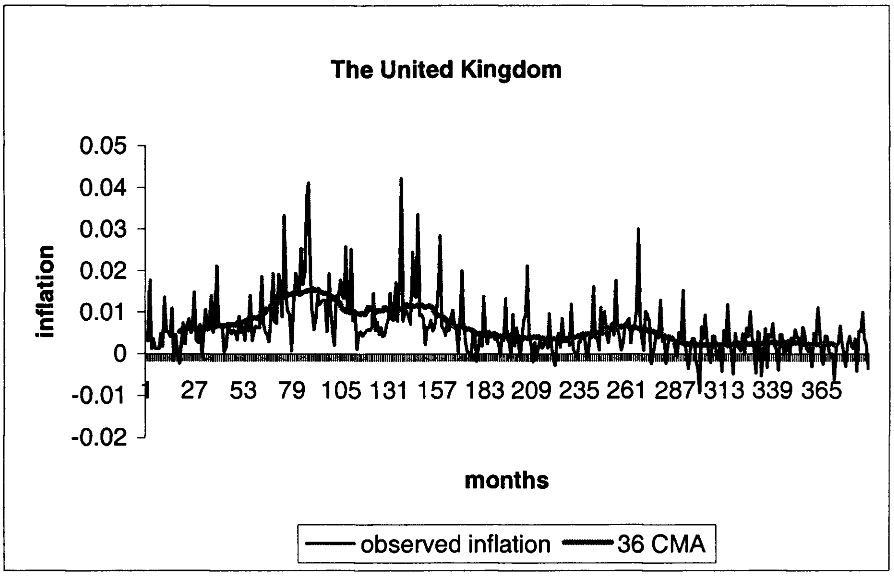

Fig. 1.4

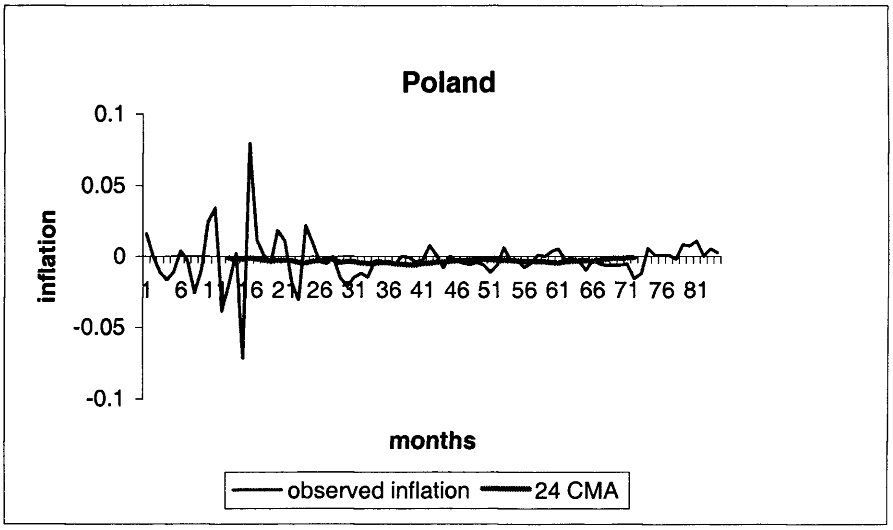

Fig. 1.5

Fig. 1.6 and Fig. 1.7 illustrate graphically the results of the exponential smoothing method for both countries.

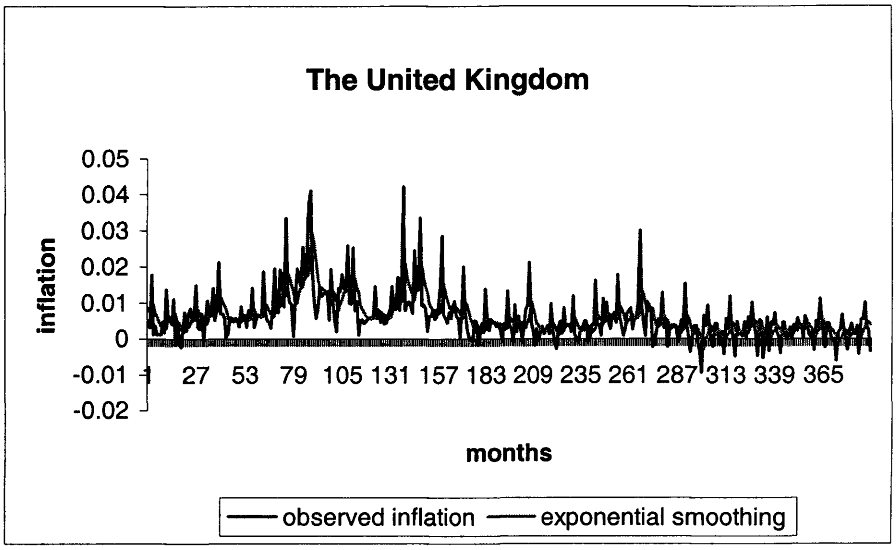

Fig. 1.6

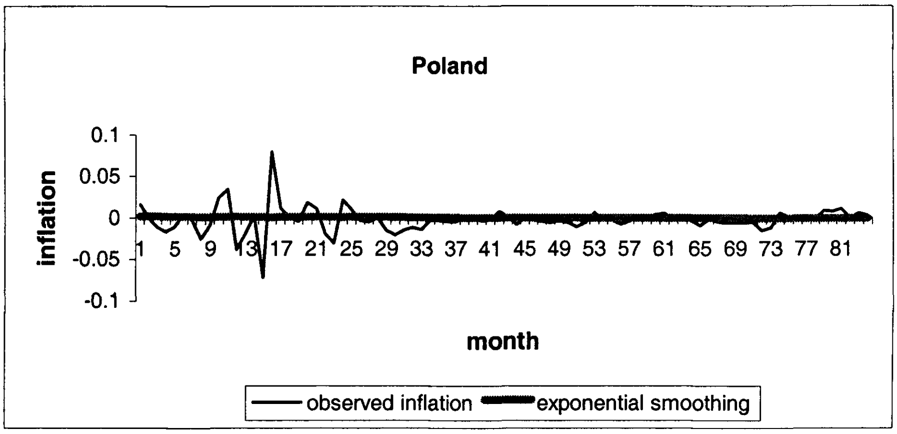

Fig. 1.7

In the previous section new method of time series decomposition was presented, namely wavelets. As far as I know this work introduces the wavelet technique for first time in the area of inflationary measuring. The empirical results for core inflation measurement obtained by means of wavelets are based on the db8 (Daubechies N8) wavelet. As it has been pointed out, when applying discrete wavelet transformation to time series the size of the data set has be to the power of two. On the other hand due to the boundary effect[49] in the wavelet algorithms, the first and the last 16 decomposed values are not accurately calculated.[50] The time series for UK and Poland consist of 242 and 84 observations

[49] *See* Daubechies (1992).

[50] The wavelet db8 has length 15 and the length of the filter is 16.

respectively. As the conditions mentioned above must be fulfilled, before proceeding with the method of time series de-noising using wavelets several operations with the data are done as follow. In order for the values at the beginning and the end of the time series to be correctly calculated, the ΔlnCPI (ΔlnRPI) series has to be continued at both sides. At the end of the UK time series 16 observations are added and the number of all the elements in the RPI series became $256 = 2^8$. Since the number of the observations for Poland CPI is only 84, 22 elements are added at the beginning and 22 elements at the end of the series and, therefore, the length of the new time series is $128 = 2^7$. How to determine the new elements? Standard techniques, when function has to be continued are extension by zero or periodization. However, these two techniques lead to discontinuity, which can be easily avoided using the following trick: an extension of the time series beyond the border by its reflection. The main advantage of this way of extending the time series is that the new and the original time series have the same properties and character (e.g. stationarity, zero mean, etc.), which significantly facilitate the following empirical work.

The extended ΔlnRPI and ΔlnCPI time series are then de-noised. The first and the last 16 observations of the de-noised UK time series as well as the first and the last 22 elements of the Poland de-noised time series are removed and the inflation trends for the United Kingdom and Poland using wavelets are shown on Fig. 1.8 and Fig. 1.9.

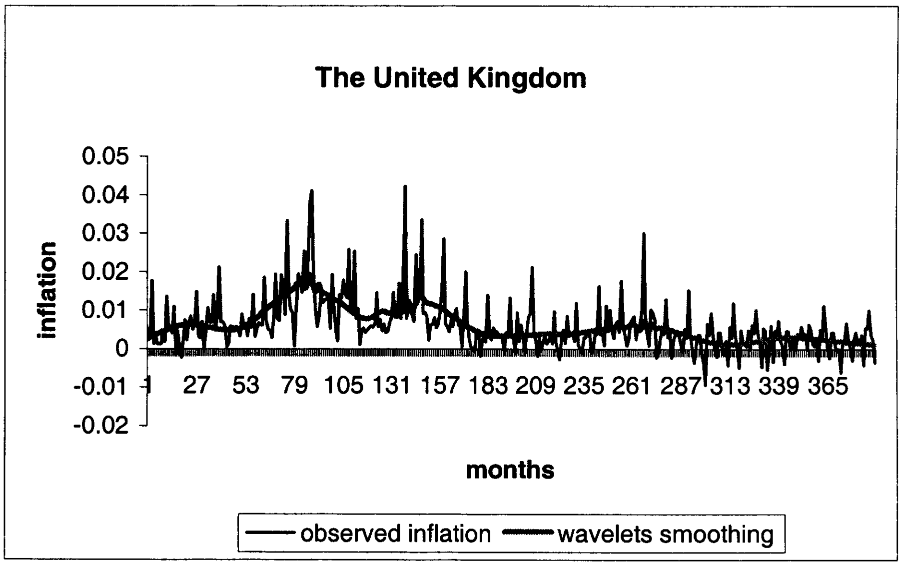

Fig. 1.8

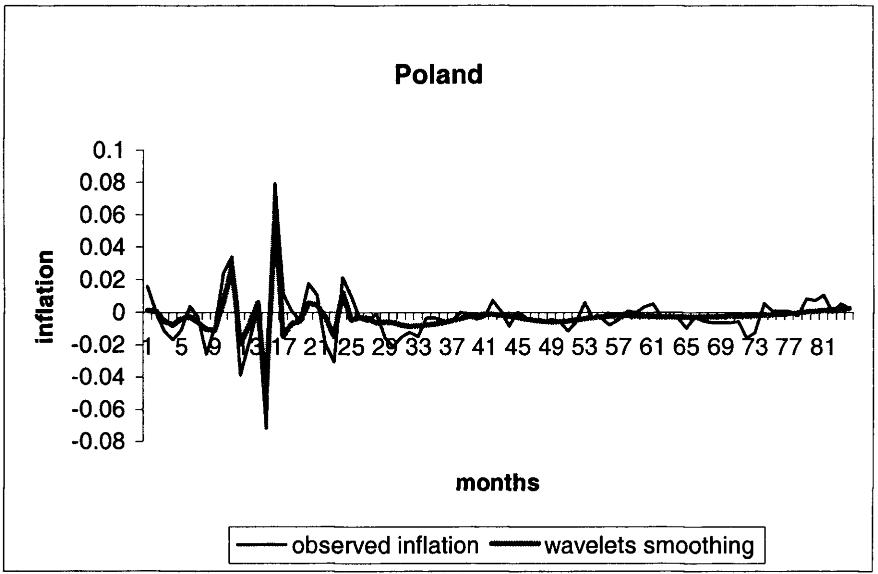

Fig. 1.9

A widespread manner used for comparison of different inflationary measurements is Root Mean Square Error (RMSE) and /or Mean Absolute Deviations (MAD). RMSEs are calculated by formula of the following form:

$$\sqrt{\frac{\sum_{i=1}^{N}\left(x_i-\hat{x}_i\right)^2}{N}}$$

where N is the number of the observations in the time series, $\hat{x}_i$, $i = 1,\dots,N$ are the values calculated by the corresponding CMA method and x_i , $i = 1,\dots,N$ are the values obtained by the use of exponential smoothing or wavelets. Using the same notations, the MADs are calculated by the formula:

$$\frac{\sum_{i=1}^{N}\left|x_i-\hat{x}_i\right|}{N}.$$

RMSE and MAD for Poland and the United Kingdom are calculated and the results obtained are summarised in Table 1.3.

Country	*Method*	*RMSE*	*MAD*
POLAND	WAVELETS – 24 CMA (%)	**1.088**	**0.464**
	EXP. SMOOTHING – 24 CMA (%)	**0.373**	**0.346**
UK	WAVELETS – 36 CMA (%)	**0.069**	**0.048**
	EXP. SMOOTHING - 36 CMA (%)	**0.147**	**0.109**

Table 1.3

The results for the United Kingdom reveal that the wavelet method performs better inflationary measurement than the exponential smoothing, as the corresponding RMSE and MAD errors are smaller. Despite the fact that RMSE and MAD related to the wavelets method are greater than those calculated for the method of exponential smoothing, one can notice from the graph at Fig. 1.9 that wavelets distinguish themselves with very good approximation properties compared to the other both methods (Fig. 1.5 and Fig. 1.7) and thus, it makes them applicable for countries in transition characterized by time varying inflationary volatility.

Fig. 1.10 and Fig. 1.11 plot the graphs of ΔlnCPI and the trend inflations obtained by the use of the three methods, namely CMA, exponential smoothing and wavelets, for the United Kingdom and Poland respectively.

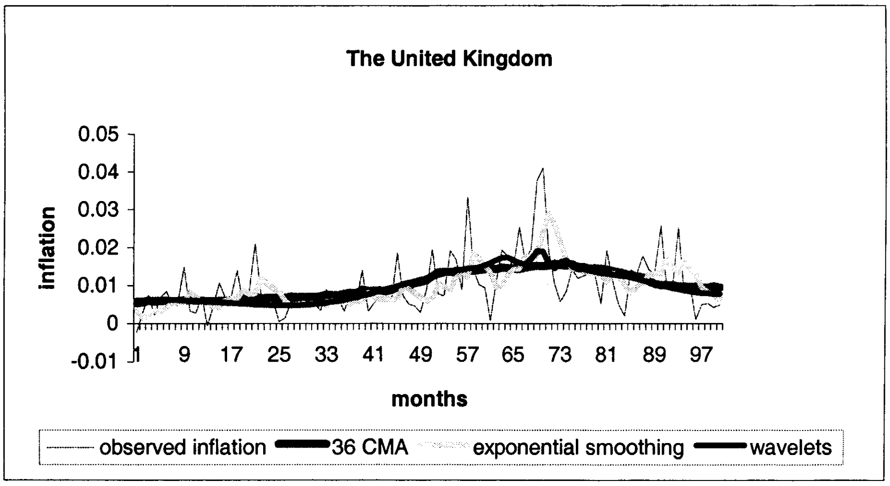

Fig. 1.10

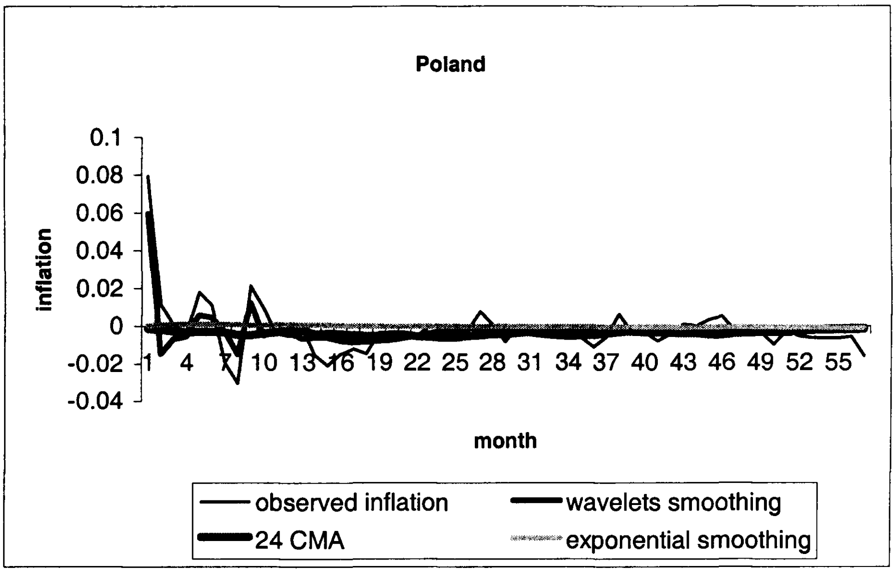

Fig. 1.11

It is evidently from both graphs that the wavelet trend line

- follows the trend line precisely, compared with CMA and exponential smoothing;
- is smoother than the trend lines obtained by CMA and exponential smoothing;
- is closer to the CMA inflation than the inflation obtained by the use of exponential smoothing.

As it was pointed out in Section 2.3.1, measured inflation π_t can be decomposed into two parts: core inflation $\pi_{t,i}^{c}$ and non-core inflation ε_t, as the latter term is associated whit 'white noise':

$$\pi_t = \pi_{t,i}^{c} + \varepsilon_t,$$

where i = {*exponential smoothing, wavelets, CMA*}. In order to evaluate the three measures of core inflation it is important, therefore, to establish whether the difference between measured inflation and core inflation is white noise. One way of checking this is by testing the null hypothesis:

$$H_0: \rho = 0$$

against the alternative one:

$$H_0: \rho \neq 0$$

in a regression equation of the form:

$$\varepsilon_t = \rho.\varepsilon_{t-1} + \xi_t,$$

where $\xi_t \sim N(0, \sigma^2)$. If α denotes the significance level of the test and t_c is the critical value such that $P[|t_{(T-2)}| > t_c] = \alpha$, then we use the test mechanism that rejects the null hypothesis, if $|t| > t_c$.

Country	*t-Ratio*	*Critical value t_c at α= 0.05*
POLAND	**Exponential smoothing: -1.0751**	**1.98**
	Wavelets: 1.5703	**1.98**
	24 CMA: -1.9097	**2.0**
UK	**Exponential smoothing: 1.3350**	**1.96**
	Wavelets: 1.4982	**1.96**
	36 CMA: 2.0000	**1.96**

Table 1.4

The results obtained are summarised in Table 1.4 and demonstrate that the null hypothesis H_0: $\rho = 0$ (that is the non-core inflation is white noise) is accepted in five out of six cases for level of significance $\alpha = 0.05$. The only exception is 36 CMA. The critical value for $t_{(205)}$ at the 0.05 level of significance is 1.96 and is less than the value of the corresponding t-ratio, namely 2.0000. The p-value, which is the probability of exceeding the computed value t= 2.0000 is p = 0.047. Thus, in this case I would reject the null hypothesis that ρ = 0 and conclude that the difference between the observed inflation and 36 CMA is not white noise at 0.05 level of significance. For α = 0.01, however, this value is smaller than the critical value 2.576 and, therefore the null hypothesis of $\rho = 0$ is accepted.

2.3.3.3 Core inflation forecast

In the previous section new method, namely wavelet smoothing was empirically used for measuring of core inflation. As a result, a long-run trends time series were obtained for the periods: (a) 02/1993 – 01/2000 (Poland) and (b) 12/1981 – 07/2000 (the United Kingdom). Further, the last 12 elements from both time series are removed and the trends are forecasted using the following two models:

- trend polynomial of order three
- suitable ARMA(p,q) model.

The 12 elements obtained from the forecast are then compared with the last 12 elements of the time series in (a) and (b). Based on the trend inflationary values obtained using wavelets, the parameters p and q of the ARMA(p,q) models for both countries are determined as follow: ARMA(2,5) for UK and ARMA (1,0) for Poland.[51] Fig. 1.12 and Fig. 1.13 graphically represent the 'true' inflation obtained by wavelets, the forecast with trend polynomial of order three and the forecast with ARMA(2,5) model for UK (Fig. 1.12) and ARMA (1,0) model for Poland (Fig. 1.13).

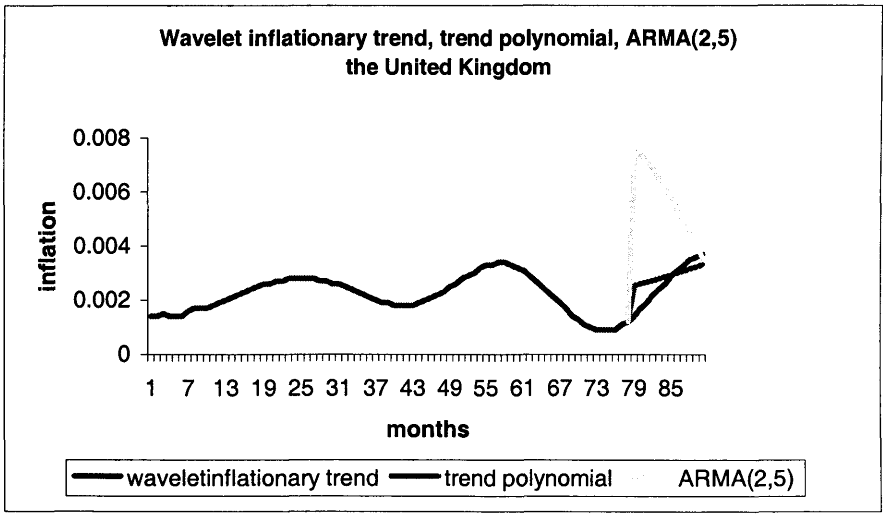

Fig. 1.12

[51] Plots of the ACF (Auto Correlation Function) and the PACF (Partial Auto Correlation Function) for the United Kingdom and Poland are presented in Appendix B.

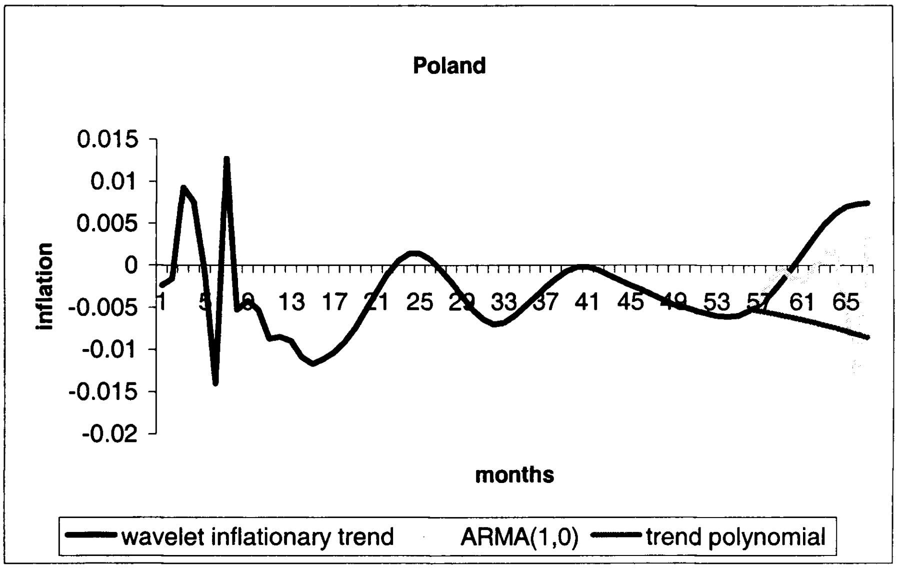

Fig. 1.13

The comparison between measured inflation using wavelets and forecasted inflation is considered in terms of RMSE and MAD, listed in Table 1.5

Country	*Method*	*RMSE (%)*	*MAD (%)*
POLAND	**ARMA(1,0)**	**0.751**	**0.483**
	TREND POLYNOMIAL	**1.026**	**0.859**
UK	**ARMA(2,5)**	**0.279**	**0.312**
	TREND POLYNOMIAL	**0.055**	**0.045**

Table 1.5

According to the results obtained for RMSE and MAD, trend polynomial of order three is more suitable than ARMA(2,5) for UK core inflation prediction, while ARMA(1,0) gives better forecasting results than trend polynomial of order three in the case of Poland.

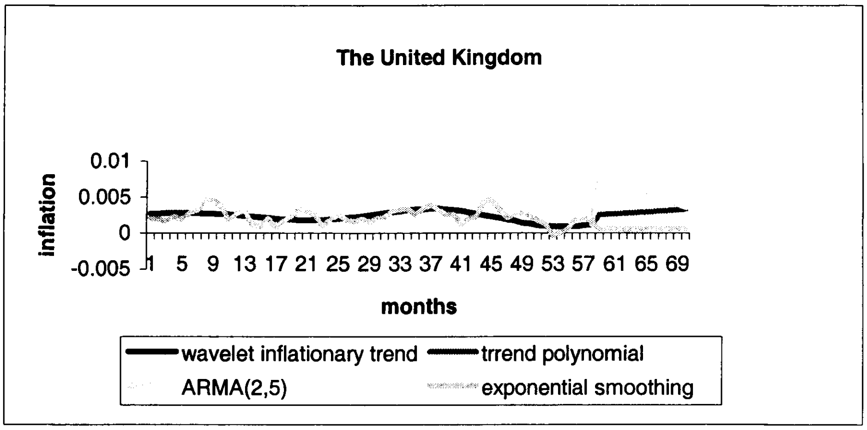

Fig. 1.14

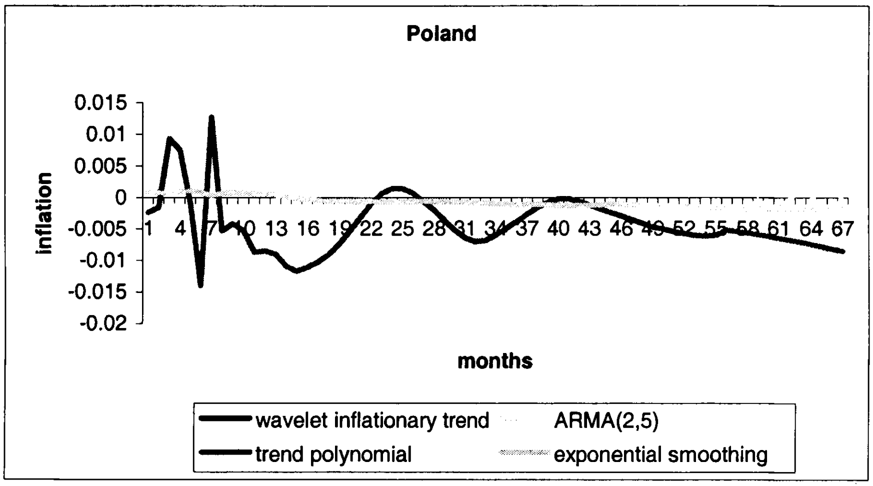

Fig. 1.15

Fig. 1.14 and Fig. 1.15 plot the forecasts based on the wavelet inflationary trend obtained by trend polynomial of order three and ARMA models and, the forecast given by the exponential smoothing method. The method of exponential smoothing I have used is a one step forecasting method, (and therefore the graph obtained is a straight line). For both countries, however, the predicted value obtained by this method is closer to the forecast obtained by the corresponding 'better' method (*see* Fig. 1.14 and Fig. 1.15).

Up to this point a main subject of discussion was the way of core inflation measurement. The Boskin Commission Report reveals that CPI overstates inflation. It is widely known as well as evident from the analysis presented that CPI contains substantial high-frequency noise, i.e. this index considerably deviates from the underlying inflation. Inflation, however, has been modelled under the assumption of normality. Following Bryan and

Cecchetti (1996), Bryan, Cecchetti and Wiggins (1997) among others (and as it will be discussed and shown later, in Chapter 2 and Chapter 3), inflationary processes distinguish themselves with substantial kurtosis and high skewness thus, leading to conclusion that these processes are not normally distributed.

As inflation is one of the leading macroeconomic factors, an issue of significant importance for both economists and policymakers is the accurate way of forecasting it. Price stability and, therefore, low inflation rate has become a central issue of monetary and fiscal policy in many countries. In some of them the central bank monetary policy is oriented toward inflation targeting and is supported by a fiscal policy aiming at a balanced budget. This focuses both policymaker's interests and the economist's attention to the problem of inflationary forecasting. The present chapter emphasises that the so-called core inflation is associated with the predictable part of the measured inflation, which stresses on the importance of precisely modelling inflationary processes. In this context, an issue of prime importance is to correctly determine whether inflation is stationary or non-stationary process. The dilemma of stationarity of inflation is a widely disputed issue in the recent literature and the following section focuses on the essence of this problem.

3. Controversies about stationarity of inflation

The empirical evidences presented in a vast number of recent publications gave rise to debates in the literature regarding the dilemma of stationariy of inflation. Occasionally considered as a unit root process and sometimes as stationary process, the empirical conclusions made vary with time periods, frequency of observations and test results.

Some researchers judge inflation as a stationary process: Engle and Granger (1987), Clements and Mizon (1991), Johansen and Juseius (1992), Quah and Vahey (1995). For example, Quah and Vahey (1995) and Engle and Granger (1987) treat monthly inflationary data for the UK and the US, respectively, as I(0) process. Clements and Mizon (1991) and Johansen and Juselius (1992) assume that the UK price is I(1) with structural changes in the deterministic trend or non-stationarity in the variance.

On the other hand, Nelson and Schwert (1977), Hall (1986), Baillie (1989), Ball and Cecchetti (1990), Johansen (1992) and Gartner and Wehinger (1998) have specified in their work inflation as a unit root process. Hence, prices are judged as time series with two unit roots, so that any shock to inflation has a permanent effect. Specifically, Gartner and Wehinger (1998) use quarterly inflationary data for nine European countries and find evidence consistent with one unit root.

Further, in the empirical study of some researchers inflationary time series is being considered as both unit root and as stationary process. Particularly, in the work of Engsted (1995), for some countries the Dickey-Fuller test results indicate that inflation is stationary, while the Johansen test clearly point out to unit root for quarterly changes in CPI and inflation is approximated as I(1) process[52]. Barsky (1987) states that inflation is stationary process until 1960 and unit root process thereafter. Similar results are presented in the study of Mishkin (1992) and Schwert (1987), where the order of integration depends on the time period being examined and on the results obtained by the use of different tests.

In order for further and precise analysis of inflationary processes to be conducted, it is important that the order of integration is correctly specified. In the majority of the papers mentioned above, stationarity of inflation is examined based on the assumption of normality. Testing the null hypothesis of linear unit root in prices is accompanied by the assumption that inflation, that is first difference in log of prices, is normally distributed process. However, this assumption is hypothetical and dubious – plenty of empirical evidences demonstrate that inflationary data are far away from being normally distributed. Therefore, it is reasonable to check the order of integration of inflationary processes ignoring this assumption.

Further, the present work suggests two alternative approaches for modelling inflationary time series regarding inflation as first, stable Paretian distributed and secondly, unit root bilinear process. As already mentioned, inflationary time series exhibit excess kurtosis (consistent with tails fatter than the tails of normally distributed process), which leads to the idea of rejecting the assumption of normality and, considered alternatively, inflation as a stable Paretian distributed (*see* Rachev, Mittnik (2000)). On the other hand, imposing some restrictions, linear unit root models can be considered as a sub-class of the class of bilinear processes (*see* Granger and Andersen (1978), Subba Rao and Gabr (1974), Terdik

[52] For the United Kingdom, for example, monthly inflation is considered as I(0) process in the work of Quah and Vahey (1995), while quarterly inflation is assumed I(1) in the study of Engsted (1995).

(1999), etc.). This leads to the idea of applying bilinear processes in economic and financial time series modelling[53], in particular for modelling inflationary processes.

4. Conclusion

In summary, Chapter 1 of this thesis concentrated on the significance of inflationary modelling and forecasting. Starting with the Boskin Commission conclusion that the most commonly used measure of inflation, CPI, tends to overstate inflation the work dwelt on the importance of subtracting the 'noisy' part of the measured inflation and thus, obtaining the so-called core or underlying inflation. Two different views of the last term were represented, namely Eckstein's, and Quah and Vahey's definitions. Under the assumptions of RE-NRH, in both definitions core inflation is associated with the anticipated part of inflation thus making the measure suitable for the purposes of inflationary forecasting. The significance of forecasting inflation follows from Lucas' 'surprise' supply model. The real inflationary effect – the difference between observed and anticipated inflation - is known as 'prise surprise' (or unanticipated inflation). The theoretical determination of the unanticipated inflation as being more costly than the anticipated one highlights the meaning of inflation forecasting and particularly forecasting its anticipated part. Thus, the researchers need a reliable way of core inflation measuring and forecasting. Abreast with the two traditional smoothing techniques, Centered moving average and Exponential smoothing, the Wavelets method of signal decomposition was also employed for the purposes of core inflation measuring. This new device became recently popular in the econometric area and as far as I know this work introduces for first time its use in the area of inflationary measurement. Chapter 1 also presented the empirical results of the application of the mentioned above smoothing methods to inflationary data for Poland and the United Kingdom. The results reveal that, according to Eckstein's definition, wavelets can be characterized as a suitable way of core inflation measurement for both countries. However, abreast with the empirical results presented in this chapter indicating presence of substantial noise in the measured inflation, plenty of empirical evidence shows that inflationary processes exhibit high levels of skewness and excess kurtosis, thus demonstrating substantial non-normality in the inflationary distribution. In the following chapter two ways of modelling inflationary processes are suggested, namely inflation as a stable Paretian distributed process and as a bilinear process.

[53] *See* Charemza, Lifshits and Makarova (2002c).

CHAPTER TWO *METHODS OF EVALUATING THE PROPERTIES OF INFLATIONARY DATA AND DISTRIBUTION*

1 Introduction

Based on the empirical evidences presented in the literature, Chapter 1, Section 3 of this thesis discussed the problem of stationarity of inflationary time series and suggested two alternative ways of their modelling. In this chapter the attention is directed to investigating inflationary stationarity, questioning the assumptions of normality and linearity. Relaxing the normality assumption distribution of inflation could be regarded as a stable Paretian distribution. On the other hand, relaxing the assumption of linearity of price time series (and, consequently normality of inflationary time series) prices could be alternatively viewed as described by bilinear processes. Thus, the present chapter concentrates on two main issues: first, the reasons of relaxing the normality and linearity assumptions when inflationary processes are considered and secondly, the theoretical foundations of the ways proposed for their modelling. Chapter 2 also gives a brief theoretical explanation of some traditional unit root tests under the assumption of normality: the Dickey-Fuller test (*see* Dickey and Fuller (1979)), the Leybourne test (*see* Leybourne (1995)) and the KPSS test (*see* Kwiatkowski *et.al* (1992)); their corresponding, recently proposed bilinear unit root tests, called *b*-tests (*see* Charemza *et al.* (2002b)), and a test for stationarity under the assumption of stable Paretian distributed processes named in this work after its authors as Rachev-Mittnik-Kim test (Rachev *et al.* (1998)).

In short, the present chapter is organized as follows: Section 2 concentrates on the features of the distribution of inflationary data followed, in Section 3, by suggestions of modelling inflation assuming non-normality in distribution. Section 4 dwells on seasonality and considers the very basic idea of the X-11 seasonal adjustment method. Next, Section 5 defines and outlines the main features of the stable Paretian distribution and exposes the McCulloch method of its parameter estimation. Sections 6 and 7 outline several unit root tests under the assumptions of normal and stable Paretian distributed processes, respectively. After the bilinear unit root processes are defined in Subsection 8.1, Subsection 8.2 presents tests for stationarity under the assumption of bilinearity. The intuitive notion of bilinear and stable processes is graphically illustrated in Section 9. Finally, Section 10 summarises.

2 Problem of Normality of Inflationary Processes

2. 1 Introduction

Chapter 1 of this thesis dwelt on the concept of core inflation and passed in review different ways of measuring it. The researchers examining underlying inflation are mainly interested in finding a robust way of its estimation. Varieties of methods are proposed in the literature but none of them seems to capture precisely this component of measured inflation. A possible reason might be that inflationary data are usually modelled under the assumption of normality. Nevertheless, many of the articles related to this topic point to substantial (empirically evident) non-normality of inflationary processes. Particularly in this section the attention is focused on the connection between inflation and its third and fourth distributional moments – skewness and kurtosis.

The often-found positive correlation between the mean and skewness of inflation (i.e. inflationary data are positively skewed) has been theoretically explained and viewed first, as a sticky-price model by Ball and Mankiw (1995) and secondly, as a flexible-price model by Balke and Wynne (1996). Bryan and Cecchetti (1996), however, doubt this result. According to their work, in the former case, "the correlation arises purely from short-run considerations", while in the latter case, "the effect need not to die out in long run". Bryan and Cecchetti (1996) examine the small sample correlation between mean and skewness of the cross sectional distribution of price changes for the US, showing existence of bias in this correlation namely, the "higher the kurtosis of the distribution, the more positive the bias" is. Following the authors, the cross-sectional distribution of inflation is leptokurtic and skewed thus, leading to a sample mean characterized with high variance. Accordingly, an obvious conclusion one can make is that price changes are not a normally distributed process in the case of the US (*see* also Bryan, Cecchetti and Wiggins (1997)). A vast number of recent publications report identical conclusions for other countries: empirical evidences supporting the hypothesis of non-normality of inflationary data are presented in the studies of Харемза, Макарова и Пархоменко (2002), Rogger (1995), Tsyplakov (2002), Wozniak (1999) among others. The authors mentioned above have examined monthly or quarterly inflationary time series for Poland, New Zealand and Russia.

In order to clarify better the issue discussed above, let us consider a few examples of distributions inflationary data. Four countries are selected – The United Kingdom, Poland, Argentina and Sierra Leone. Each of them represents one of the following groups, respectively:

- Developed Countries
- Developing Countries
  - Central and Eastern European Countries
  - Other Developing Countries
- Least Developing Countries

2. 2 Distribution of Inflation: Examples

Figures 2.1 – 2.4 displayed present histograms of monthly, seasonally adjusted inflation rates[1, 2] for four selected countries: the United Kingdom, Poland, Argentina and Sierra Leone. The data cover different time periods for the four countries as follows[3]:

- The United Kingdom: February 1956 – December 2000;
- Poland: February 1988 – December 2001;

[1] Inflation is defined as a first difference of the log of prices, i.e. $\Delta\ln(CPI)$ or $\Delta\ln(RPI)$.

[2] The procedure used for seasonal adjustment is discussed in Section 4 of this chapter.

[3] The inflationary data used for these examples are part of a large dataset containing 108 inflationary time series for different countries. The data are explained in details in Chapter 4, Section 2.

- Argentina: January 1991 – November 2001;
- Sierra Leone: October 1988 – November 2000.

and descriptive statistics for those inflationary processes are reported in Table 2.1. The tabulated results reveal that inflationary data for those particular countries are positively skewed[4]. This is also demonstrated by the histograms plotted on Fig. 2.1 – Fig. 2.4, being clearly asymmetric and right-skewed. On the other hand, the large kurtosis values graphically correspond to sharper peaks and heavy tails. Adding to these two arguments the visible large up-and-down changes in the graphs displayed (especially for the UK and Sierra Leone), lead to the conclusion that - regardless of the development status of the countries - the shape of distribution of inflation reasonably defers from the shape of the normal distribution.

Descriptive statistics for inflation							
Country	**Number of observations**	**Mean**	**Standard deviation**	**Skewness**		**Kurtosis**	
				coeff.	**p -value**	**coeff.**	**p-value**
(1)	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**	**(7)**	**(8)**
UK	540	0.005	0.005	2.057	0.000	10.174	0.000
Poland	168	0.036	0.065	4.292	0.000	22.531	0.000
Argentina	143	0.047	0.137	4.848	0.000	27.698	0.000
Sierra Leone	170	0.030	0.063	2.353	0.000	21.754	0.000

Table 2.1

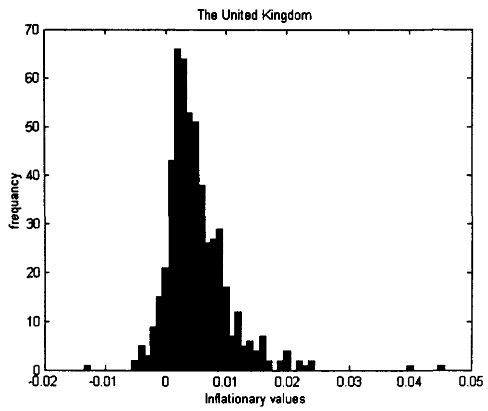

Fig. 2.1

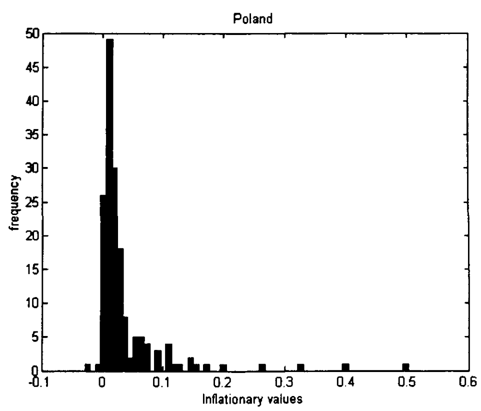

Fig. 2.2

[4] As it will be shown later in Chapter 4, Section 2 with few exceptions, the inflationary distribution is positively skewed.

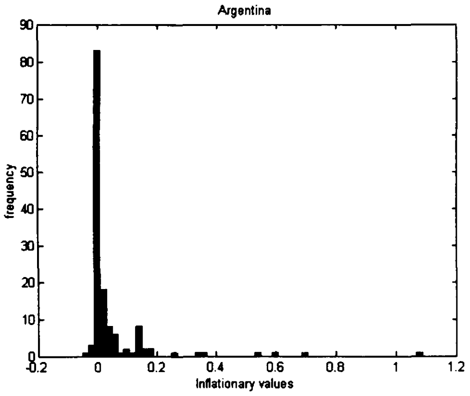

Fig. 2.3

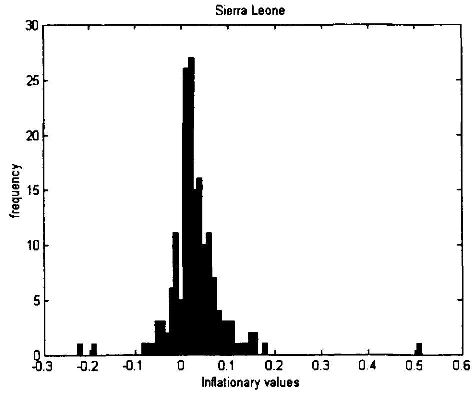

Fig. 2.4

In short, the discussion above reveals that distribution of inflation distinguishes with high levels of skewness and fat tails thus clearly demonstrating that inflationary data are far from being normally distributed.[5] Hence, in order to model accurately inflationary processes, the necessity of specifying appropriate distribution arises. The distinct non-normality of inflationary processes - evident from the graphs and supported by the numerous empirical evidences in the literature - leads to the idea of modelling these processes by relaxing the assumption of normality. Two alternative ways are proposed in the following section.

3 Bilinearity and Non-normality in a Research Hypothesis

The way of correctly modelling macroeconomic time series, in particular inflationary processes, is a subject of considerable interest for economic researchers and a matter of significant importance for politicians. Inflation is not only one of the leading macroeconomics factors but also the most commonly used economic term among both economists and the general public (*see* Shiller (1996) and Chapter 1, Section 1). Modelling it is an issue of primary interest of this research. More specifically, our attention is directed to investigating the problem of the stationarity of inflation – an issue widely disputed in recent literature. The ideas of treating inflation in the ways suggested in the present section have arisen as a result of conclusions made in a vast number of recent empirical publications that inflationary processes are non-normally distributed[6]. On the other hand,

[5] The empirical work presented later is based on a large selection of worldwide inflationary data and shows that this is valid for most of the countries examined.

[6] This problem has been discussed in more details in Section 2 of the present chapter.

most researchers consider prices and inflation as linear processes[7]. Let us clarify this issue. Prices in logarithms, p_t, are usually presumed to follow a linear process of the form:

$$p_t = p_{t-1} + \varepsilon_t ,$$

where $\varepsilon_t \sim iid \ N(0, \ \sigma_\varepsilon^2)$. Their first difference Δp_t or, in other words inflation, has the following representation:

$$\Delta p_t = \varepsilon_t ,$$

that is inflation viewed as a normally distributed process.

However, the assumptions of normality and linearity are hypothetical and dubious. The ideas suggested in the present research are examining the order of integration of inflationary processes relaxing both these assumptions. Two alternative approaches for modelling inflationary time series are proposed: (a) inflation as a stable Paretian distributed process; and, (b) prices (and inflation) as bilinear processes. In case (a) prices follow process of the form:

$$p_t = p_{t-1} + u_t ,$$

where $u_t \sim iid \ S(u; \alpha, \beta, \delta, c)$. Thus, inflation $\Delta p_t = u_t$ and is, therefore, viewed as a stable Paretian distributed process.

Regarding approach (b), prices follow bilinear process of the form:

$$p_t = p_{t-1} + bp_{t-1}e_{t-1} + e_t ,$$

where $e_t \sim iid \ N(0, \sigma^2)$. This process is a non-linear process.[8] Although the disturbances e_t are normally distributed, inflation (i.e. first difference of prices Δp_t), is a non-normally

[7] By linearity assumption it is meant that a series can be transformed to stationary by means of a linear difference operator. Let us assume that prices are described by a linear process $p_t = p_{t-1} + \varepsilon_t$, where $\varepsilon_t \sim iid \ N(0, \ \sigma_\varepsilon^2)$. If $p_t \sim I(1)$ then first difference of prices, Δp_t, is assumed to follow process of the same form, that is $\Delta p_t = \Delta p_{t-1} + \varsigma_t$, $\varsigma_t \sim iid \ N(0, \ \sigma_\varsigma^2)$ and the order of integration of Δp_t is determined, etc. The first difference operator is applied till stationarity is achieved.

[8] The process $p_t = p_{t-1} + bp_{t-1}e_{t-1} + e_t$ is a recursive process. Thus, substituting $p_{t-1}, p_{t-2}, \dots, p_0$ with their equals leads to decomposition of the form:

$$p_t = f(b, e_t, e_{t-1}, \dots, e_0) + g(b, e_t, e_{t-1}, \dots, e_0) ,$$

where $f(b, e_t, e_{t-1}, \dots, e_0)$ is a linear function, and $g(b, e_t, e_{t-1}, \dots, e_0)$ is a non-linear function of their arguments.

distributed process. This is evident from the following equality:

$$\Delta p_t = bp_{t-1}e_{t-1} + e_t .$$

The right hand side of the equality above is a non-linear function of $e_t \sim iid\ N(0, \sigma^2)$ and, therefore is a non-normally distributed process. Hence, inflationary processes are non-normally distributed, and non-linear processes.

The idea of the former concept arises from the fact already discussed that distribution of inflation is characterised by excess kurtosis - consistent with tails fatter than those of normally distributed processes – and, high levels of skewness. Thus, a straightforward conclusion one can make is that distribution of inflationary processes differs from the normal. This leads to the idea of ignoring the assumption of normality and specifying an alternative and appropriate distribution for their modelling. A family of distributions describing leptokurtic and asymmetric processes is the class of stable Paretian distributions, from which the normal distribution is a particular case. Thus – neglecting the assumption of normality - a suitable way of modelling inflationary processes seems to be their modelling under the more general assumption of stable Paretian distributed processes. Under this assumption the Rachev – Mittnik - Kim unit root test can then be empirically applied to inflationary data.

The idea of the second approach - the concept of prices and inflation as bilinear processes - results from the theoretical fact that linear unit root models can be considered as a sub-class of the class of bilinear processes. This makes bilinear processes applicable for the purposes of modelling economic and financial time series[9], in particular for modelling price and inflationary processes. Under the assumption of bilinearity, the so-called b-test for stationarity can be employed.

Prior to dwelling on the theory encompassing the suggestions for price and inflationary processes modelling above explained, this chapter continues with a brief theoretical description of the method of initial price data adjustment which is later used in our empirical work.

[9] *See* Charemza, Lifshits and Makarova (2002c).

4 Initial Data Preparation: Seasonal Adjustment

It is widely known that most of the leading macroeconomic factors, especially those analysed using time series with a monthly frequency of observations, are often affected by seasonality. Among these indicators are unemployment, income, sales, consumption, etc. and, in particular, one of both variables of interest in this research - prices. The use of seasonally adjusted data is, however, an issue broadly disputed in the literature. Sims (1974), Wallis (1974), Ericsson *et al.* (1993) among others show that seasonally adjusted series lead to distortion in the estimated dynamic relationships. Following Hylleberg (1992), p.10, "as the degree of the distortion varies the best advice for researchers is to consider both seasonally adjusted and seasonally unadjusted series" when modelling economic variables.

Shiskin *et al.* (1967) define the term 'seasonal component' as "the intrayear pattern of variation which is repeated constantly or in an evolving fashion from year to year". Fig. 2.6 gives an intuitive notion of this definition. It plots a logarithm of monthly price observations against the time in months for the UK over the periods 01/1998 – 12/1998, 01/1999 – 12/1999 and 01/2000 – 12/2000. It shows that for three consequent years the graphs of prices in logs have very close representations, showing a very similar pattern over the three periods.

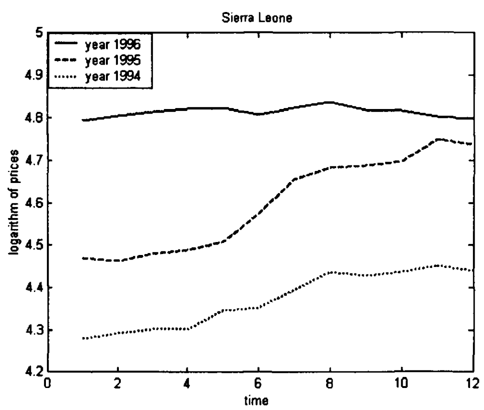

Fig. 2.5

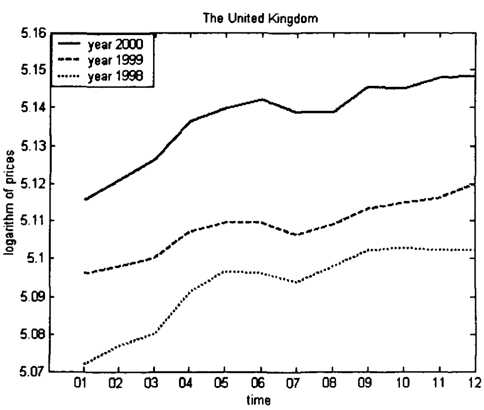

Fig. 2.6

In the case of Sierra Leone, however, the corresponding graphs follow different patterns as is evident by Fig. 2.5. The figure plots a logarithm of prices against the time in months for the periods 01/1994 – 12/1994, 01/1995 – 12/1995 and 01/1996 – 12/1996 and shows no visually distinctive seasonality. Fig. 2.5 and 2.6 simply demonstrate that adjustment for seasonality is country specific.

However, in many empirical studies investigating inflationary processes, especially those with a monthly frequency of observations, seasonality is taken into consideration (*see* Engle (1982), Quah and Vahey (1995), Fountas, Karanasos and Karanassou (2000), etc.). On the other hand, filtering for seasonality smoothes price data, thus leading to stronger conclusions regarding the non-normality of distribution of inflationary data. As the empirical analysis presented in Chapter 3 and Chapter 5 is based on a large selection of worldwide monthly price time series, all the series are initially adjusted for seasonality. On the other hand, the interest is directed to the general pattern of price processes and not to an interaction between individual price series. Thus, in the forthcoming analysis we consider a data set consisting of price time series, presuming that they are independent of each other. The latter assertion allows us to apply the most popular and broadly used X-11 seasonal adjustment procedure.

In short, price data are adjusted for seasonality using the U.S. Census Bureau X-12-ARIMA software, version 0.2.8. The method used is the Census Bureau Method II, X–11 seasonal adjustment program, as detailed in Shiskin *et al.* (1967) and Dagum (1988). For the purposes of our analysis the log-additive method is applied. It is a logarithm of the price series P_t is decomposed as follows:

$$\ln(P_t) = S_t + C_t + I_t,$$

where S_t is the seasonal component, the trend-cycle component C_t incorporates the long-term trend and the medium-to-long term movements and I_t is the irregular component (which could be further decomposed on trading days, holiday effects, etc.). The seasonally adjusted series then has the form:

$$SA_t = e^{C_t + I_t} = e^{\ln(P_t) - S_t}$$

and thus the interest is directed at estimating the seasonal component S_t.

The X-11 seasonal adjustment method consists of sequential use of appropriate moving average filters for estimating the seasonal element S_t. In addition, as the seasonal adjustment method is applied to a large selection of worldwide data characterized with different inflationary patterns, the option of controlling the extreme irregular values has been omitted and the final seasonal filter has been chosen automatically[10]. Fig. 2.7 and Fig. 2.8 plot the graphs of the seasonally adjusted (the 'thin' line) and the seasonally unadjusted (the 'thick' line) monthly price data in logarithms for Sierra Leone (January 1987 – November 2000) and the United Kingdom (January 1989 – December 2000), respectively.

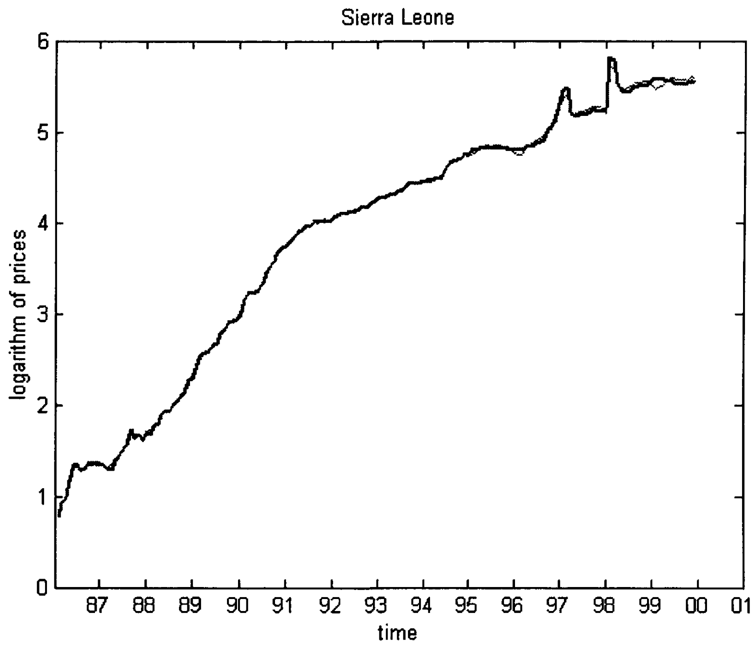

Fig. 2.7

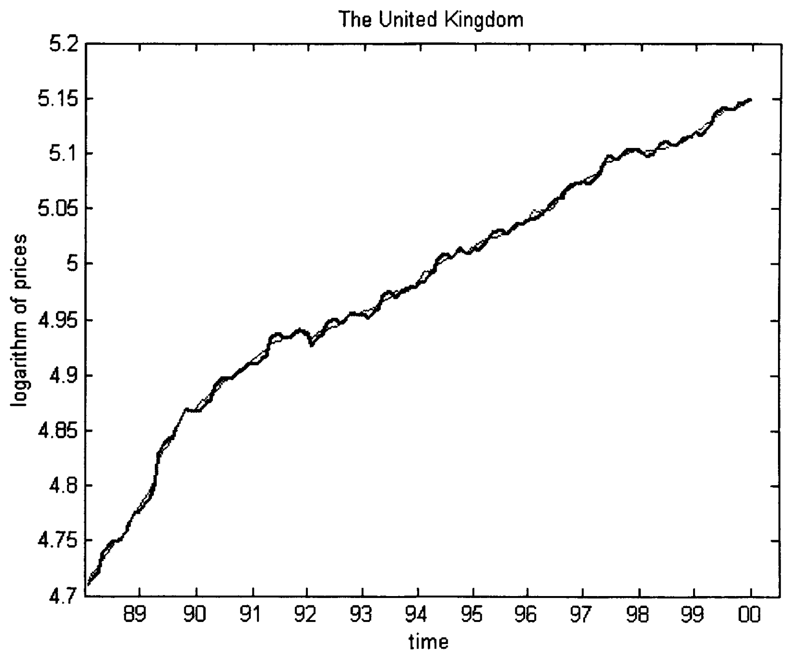

Fig. 2.8

5 Stable Paretian Distribution

It is widely accepted that traditional models fail to capture important features of the dynamic of financial and economic variables. The idea of applying stable Paretian distributions for modelling financial variables originates in the works of Mandelbrot (1962, 1963a,b, 1967) and Fama (1965) who found excess kurtosis in asset returns and strongly rejected normality as a distribution for their modelling. Their findings have attracted the attention of many financial modellers among who are Chobanov *et al.* (1996), Mittnik *et al.* (1998), Rieken *et al.* (1998), Charemza and Kominek (2000).

The present section considers the main features of the class of stable Paretian distributions and dwells on some of their properties. It is followed by an exposé of the McCulloch estimation method of stable Paretian distribution parameters (used in the empirical work presented later in this thesis).

5. 1 Definition and Properties of Stable Paretian Distributions

Following Rachev and Mittnik (2000) the distribution function *H(x)* is said to be stable (in a broad sense), if there exist sequences of normalizing constants $a_T, b_T \in R$ and $a_T > 0$, such that for each $T \in N$

$$X \overset{d}{=} a_T (X_1 + \ldots + X_T) + b_T ,$$

where $X_1, X_2, \ldots$ are *iid* random variables with the common distribution function *H(x)*.[11, 12] If in the last equality $b_T = 0$, the distribution function *H(x)* is called strictly stable. The random variable *X* is said to satisfy a stable (or, strictly stable) Paretian law.

Several parameters determine the stable Paretian distribution: index of stability, skewness, location and scale parameters. The first of them is also known in the literature as the characteristic exponent and is of main interest in our later work, when the class of stable Paretian distributions is considered. It is usually denoted by α and determines the rate at which the tails of the distribution taper off. The four parameters of the stable Paretian

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[10] *See* X-12-ARIMA Reference Manual, Version 0.2.8, 2001.

[11] The notation " $\overset{d}{=}$ " signifies an equality in distribution.

[12] According to Feller (1966), Theorem 1, p. 166 the only possible constants are the norming constants $a_T = T^{-1/\alpha}$.

distribution are specified by the characteristic function of the distribution function *H(x)*. Different ways of their parameterisation are known (e.g. Zolotarev (1957), Zolotarev (1966), Feuerverger and McDunnough (1981), etc.). A way of expressing the characteristic function is presented in the following lines, adopted from Rachev and Mittnik (2000)[13].

The distribution function *H(x)* is stable, if there exist parameters $0<\alpha\leq 2$, $-1\leq\beta\leq 1$, $c\geq 0$ and $\delta\in R$ such that the characteristic function of *H(x)* has the following form:

$$\psi_t(x)=\int_{-\infty}^{\infty}e^{itx}dH(x)=\left\{\begin{array}{l}\exp\left\{-c^{\alpha}|t|^{\alpha}\left[1-i\beta sign(t)\tan\frac{\pi\alpha}{2}\right]+i\delta t\right\},\ if\ \ \alpha\neq 1\\ \exp\left\{-c|t|\left[1+i\beta\frac{2}{\pi}sign(t)\ln|t|\right]+i\delta t\right\},\ if\ \ \alpha=1\end{array}\right\},$$

where

$$sign(t)=\left\{\begin{array}{cl}1 & if\ t>0\\ 0 & if\ t=0\\ -1 & if\ t<0\end{array}\right\}.$$

The stable Paretian (or α- stable) distribution function is usually denoted by $S(\alpha,\ \beta,\ c,\ \delta)$, where the parameter β determines the skewness of the distribution, c is a scale parameter, which compresses or extends the distribution, and δ, the location parameter, shifts the distribution on the left or on the right. Consider a stable random variable x, denoted usually by $x\sim S(x;\alpha,\ \beta,\ c,\ \delta)$. It can be shown that the standardized variable $z=(x-\delta)/c$ is $S(z;\alpha,\beta,1,0)$ distributed. Thus, the underlying parameters determining stable Paretian distributions are the characteristic exponent α and the skewness parameter β. Let us briefly outline the main features of both parameters.

Consider initially the index of stability $\alpha\in(0,2]$:

- if $0<\alpha\leq 1$ the distribution characterises with: heavy tails, infinite variance and infinite mean (i.e. the first and second moments of the distribution do not exist);[14]
- if $1<\alpha<2$ the distribution characterises with: heavy tails, infinite variance (i.e. the second moment of the distribution does not exist) and mean equal to δ;

[13] Indeed, the characteristic function is defined as in Zolotarev (1957) and this is the form employed by McCulloch for estimating the stable Paretian distribution parameters.

- if $\alpha = 2$ the distribution coincides with the normal distribution $N(\delta, 2c^2)$.

On the other hand, the skewness parameter is $\beta \in [-1, 1]$ and it exists only for $\alpha \neq 2$. According to Zolotarev (1957) the skewness of the distribution varies with the sign of β in the following way:

- if $\beta < 0$ the distribution is negatively skewed;
- if $\beta = 0$ the distribution is symmetric;
- if $\beta > 0$ the distribution is positively skewed.

Different estimation techniques for measuring the tail thickness (or more of the stable Paretian distribution parameters) are known (*see* Hill (1975), Pickands (1975), McCulloch (1986), Brorsnen and Yang (1990), Mittnik and Rachev (1996), Nolan (1997), Zolotarev (1986), etc.). The Maximum Likelihood (ML) estimators of stable Paretian models - as suggested, for example, by Brorsnen and Yang (1990) and Nolan (1997) - are based on numerical approximation and integration of stable non-Gaussian densities. Rachev and Mittnik (2000) present results from a study examining (by the use of Monte Carlo simulations) the performance of ML procedure[15] and the McCulloch (1986) quantile estimator. The researchers conclude that the ML procedure "performs accurately and possesses less dispersion than the widely used ... estimator of McCulloch". According to Rachev and Mittnik (2000), p. 135, however, the McCulloch estimator "performs remarkably well" and requires "little computational effort". Consider in brief the McCulloch estimation method, which is used in our empirical work later.

The reminder of this section focuses on the McCulloch technique for evaluating the parameters α and β, as our interest is mainly directed in estimating the stability index α. Once this parameter of the stable Paretian distribution is specified, the empirical work can proceed with application of the Rachev – Mittnik – Kim unit root test.

[14] In particular, if $\alpha = 1$ the distribution coincides with the Cauchy distribution.

[15] While Brorsnen and Yang (1990) and Nolan (1997) ML estimators are based on the Fast Fourier Transform for the parameters of stable densities as suggested by Zolotarev (1986), the results presented in Rachev and Mittnik (2000) are based on DuMouchel (1971) approach.

5. 2 McCulloch Estimation Method of Stable Paretian Distribution Parameters

Fama and Roll (1971) proposed an estimator of the characteristic exponent α and the scale parameter c of symmetric stable Paretian distributions. In short, the authors suggest estimators based on the calculation of appropriately chosen sample quantiles[16] and, evaluate their properties by employing Monte Carlo simulations. McCulloch (1986) extends Fama and Roll's work generalizing this method for the asymmetric cases. Moreover, while Fama and Roll restrict the α-values to the interval [1, 2], the McCulloch method allows these values to vary in the larger range [0.6, 2]. The parameter of interest in our work is the characteristic exponent α associated with the tail thickness of the distribution. However, the McCulloch estimation technique used later in our empirical work incorporates both the index of stability α and the skewness β. Let us consider in brief the theoretical grounds of the McCulloch method of estimation.

Let $X_1, X_2,...X_k$ are *iid* $S(x;\alpha,\beta,c,\delta)$ and lets denote by x_i the i^{th} population quantile, i.e. $i = S(x_i;\alpha,\beta,c,\delta)$. Further, denote by $\hat{x}_i$ the corresponding sample quantiles[17] and consider the following population indexes

$$g_\alpha = \frac{x_q - x_{1-q}}{x_p - x_{1-p}} \quad \text{and} \quad g_\beta = \frac{(x_q - x_{0.5}) + (x_{1-q} - x_{0.5})}{x_p - x_{1-p}}$$

and, their corresponding sample statistics

$$\hat{g}_\alpha = \frac{\hat{x}_q - \hat{x}_{1-q}}{\hat{x}_p - \hat{x}_{1-p}} \quad \text{and} \quad \hat{g}_\beta = \frac{(\hat{x}_q - \hat{x}_{0.5}) + (\hat{x}_{1-q} - \hat{x}_{0.5})}{\hat{x}_p - \hat{x}_{1-p}}.$$

According to the author $\hat{g}_\alpha$ and $\hat{g}_\beta$ are consistent estimators of g_α and g_β. However, by definition, the quantiles of a random variable depend only on the parameters α and β of its distribution function. Thus, the indexes g_α and g_β can be seen as functions of the characteristic exponent and the skewness parameter, i.e.:

[16] The q^{th} quantile (or percentile), $0 < q < 1$, of a random variable X with distribution function $F(x)$, denoted ξ_q, is defined as the smallest number ξ satisfying $F(\xi) \geq q$ (*see* Poirier (1995)).

[17] Following McCulloch (1986) these quantiles should be corrected for continuity. *See* McCulloch (1986) for more details.

$$g_{\alpha} = f_1(\alpha, \beta) \tag{2.1}$$

$$g_{\beta} = f_2(\alpha, \beta).^{18} \tag{2.2}$$

Thus, using relationships (2.1) and (2.2) the parameters α and β can be expressed as functions of g_{α} and g_{β}:

$$\alpha = z_1(g_{\alpha}, g_{\beta})$$

$$\beta = z_2(g_{\alpha}, g_{\beta})^{19}$$

The q values suggested by Fama and Roll (1971) are $q = \{0.95, 0.96, 0.97\}$. McCulloch (1986) estimates $\hat{\alpha}$ and $\hat{\beta}$ are based on the smallest q suggested by Fama and Role, i.e. 0.95, "since it reduces the sampling error of the quantiles with limited samples" and $p = 0.75$. The values of the functions $\hat{g}_{\alpha} = \hat{f}_1(\alpha, \beta)$ and $\hat{g}_{\beta} = \hat{f}_2(\alpha, \beta)$ are tabulated in McCulloch (1986) for all the possible pairs (α, β), where $\alpha = \{2, 1.9, 1.8, \ldots, 0.6, 0.5\}$ and $\beta = \{0, 0.25, 0.5, 0.75, 1\}$. Based on the estimated values $\hat{g}_{\alpha}$ and $\hat{g}_{\beta}$ the index of stability $\hat{\alpha}$ and the skewness parameter $\hat{\beta}$ can be estimated using these relationships:

$$\hat{\alpha} = z_1(\hat{g}_{\alpha}, \hat{g}_{\beta})$$

$$\hat{\beta} = z_2(\hat{g}_{\alpha}, \hat{g}_{\beta})$$

and their values are tabulated in McCulloch (1986) for fixed $\hat{g}_{\alpha}$-values in the range [2.439, 25] and fixed $\hat{g}_{\beta}$-values in the range [0, 1].[20]

In short, the present section presented the McCulloch method of estimation of the stable Paretian distribution parameters. Once the index of stability is estimated, the Rachev-Mittnik-Kim unit root test can be applied. However, before dwelling on this test (Section 7 of the present chapter), let us consider the traditional unit root and stationarity tests used in our empirical work later.

[18] Following McCulloch (1986) f_1 and f_2 are respectively even and odd functions with respect to the parameter of skewness β.

[19] Following McCulloch (1986) z_1 and z_2 are respectively even and odd functions with respect to the index g_{β}.

[20] Some exceptions might occur in the empirical work. For more details on this matter *see* McCulloch (1986).

6 Traditional Unit Root and Stationarity Tests Under the Assumption of Normality

6.1 Introduction

Although the theory of first-order autoregressive processes was developed in the 50's and early 60's years of the last century (*see* Rubin (1950), White (1958), Rao (1961)), unit root processes became more popular after the influential publications of Dickey and Fuller (1979), (1981). Subsequently, the work of Nelson and Plosser (1982) focuses the economists' attention on the significance of correct determination of the autoregressive coefficient. In their empirical analysis the authors use long historical time series for 14 US macroeconomic variables. For all of them but unemployment the Dickey-Fuller (*DF*) test fails to reject the null hypothesis of a first-order autoregressive unit root in a lagged model with intercept and deterministic trend. The development of unit root theory continues in the works of Phillips (1987), Phillips and Peron (1988) among others. In contrast with the classical unit root test theory, Kwiatkowski *et al.* (1992) developed a test with null hypothesis that of stationarity against the alternative of a unit root. Often quoted as the KPSS test[21] (named after its authors), the KPSS test is usually used for confirmatory analysis. Further development in this area has been done by Leybourne (1995) who proposed a simple modification of the *DF* test. The researcher applies the *DF* test to: first, the ordered and second, the reversed data and defines the *t*-test statistic as the maximum statistics obtained from both realisations. This test is known in the literature as either the Laybourne or the DF_{max} test. Next, Charemza and Syczewska (1998) simultaneously apply the *DF* test with null hypothesis of unit root and the KPSS test with null of stationarity and, investigate their joint distribution.

In short, a central issue in the area of first-order autoregressive processes is testing for presence of unit root. Many tests have been proposed but none of them is uniformly powerful. In this section, the attention is focused on a few of them, used in the empirical work afterward, namely the Dickey-Fuller (*DF*), the Leybourne and the KPSS tests. However, prior to outlining these tests let us briefly expose the very basic concepts of a first-order autoregressive process.

[21] *see* Kwiatkowski *et al.* (1992).

Consider a first-order autoregressive model:

$$y_t = \rho y_{t-1} + e_t,\ t = 1,2,\dots T, \tag{2.3}$$

where $e_t \sim iid\, N(0,\sigma^2)$, $\rho \in R$ and $\{y_t\}_{t=1\dots T}$ is a discrete time series. Three different states of nature can be classified in terms of the real parameter ρ:

- if $|\rho| < 1$ the process is stationary; the time series $\{y_t\}_{t=1\dots T}$ is a stationary time series since the variance of the process tends to a finite number, i.e. $\mathrm{var}(y_t) \underset{t\to\infty}{\to} \frac{\sigma^2}{1-\rho^2} < \infty$.[22]
- if $|\rho| = 1$ the process is non-stationary and is known as an autoregressive unit root process or random walk; the time series $\{y_t\}_{t=1\dots T}$ is non-stationary since the variance of the process changes stochastically, i.e. $\mathrm{var}(y_t) = t\sigma^2 \underset{t\to\infty}{\to} \infty$.[17]
- if $|\rho| > 1$ the time series is non-stationary with exponentially increasing variance as t increases[23]

As no one doubts that price processes possess one unit root, our interest is directed to investigating autoregressive unit root processes called, in short, unit root processes only. The following subsections outline the unit root and the stationarity tests used later in the empirical work, namely the Dickey-Fuller, Leybourne and KPSS tests. The first two tests consider the null hypothesis of unit root against the alternative of stationarity. However, following Kwiatkowski et al. (1992), the "standard unit root tests are not very powerful against relevant alternatives". Thus, the authors suggest test of the null hypothesis of stationarity against the alternative of a unit root. In the empirical work presented later, the last mentioned test, abbreviated hereafter as the KPSS, is used for confirmatory analysis.

[22] This formula is derived under the assumption $y_0 = 0$.

[23] For more details *see* Hendry (1995).

6. 2 The Dickey-Fuller Test[24]

Introduced by Dickey and Fuller (1979), the *DF* test is one of the most popular unit root tests. In their article the authors consider data generation process (DGP) of the form (2.3) imposing $y_0 = 0$ and $e_t \sim iid\, N(0,\sigma^2)$ and test the null hypothesis of the unit root:

$$\mathrm{H}_0:\ |\rho| = 1 \tag{2.4}$$

against the alternative of stationarity:

$$\mathrm{H}_1:\ |\rho| < 1 \tag{2.5}$$

Initially, both researchers consider the OLS estimator $\hat{\rho}$:

$$\hat{\rho} = \frac{\sum_{t=1}^{T} y_t y_{t-1}}{\sum_{t=1}^{T} y_{t-1}^2} \tag{2.6a}$$

which is a consistent estimator of ρ (*see* Rubin (1950)). Under the null hypothesis $|\rho| = 1$ it follows from (2.3) that $\Delta y_t = e_t$. Hence, the following equality holds:

$$T(\hat{\rho} - 1) = \frac{T^{-1}\sum_{t=1}^{T} y_{t-1} e_t}{T^{-2}\sum_{t=1}^{T} y_{t-1}^2}. \tag{2.6b}$$

It is shown in Maddala and Kim (1998) that the numerator and the denominator of (2.6b) converge to random variables defined as functions of Wiener processes[25] and thus the asymptotic distribution of the OLS estimator of ρ, known as Dickey-Fuller distribution, is:

$$T(\hat{\rho} - 1) \Rightarrow \frac{\int_0^1 W(r)\,dW(r)}{\int_0^1 W(r)^2\,dr}$$

as $T \to \infty$.[26]

[24] For simplicity, the present section briefly outlines the *DF* test with estimated regression of the form (2.3). Dickey and Fuller (1979) also present the relevant statistics and their limited distributions for models with an intercept and models with an intercept and linear trend.

[25] Following Maddala and Kim (1998), the Wiener process is defined by the following relation:

$$\Delta W = \varepsilon\sqrt{\Delta t}$$

(an equivalent representation in a differential equation form is $dW = \varepsilon\sqrt{dt}$), where $\varepsilon \sim IN(0,1)$ and Δt denotes the length of a small interval of time. The variable *W* is said to follow a Wiener process.

Next, both authors prove that under the null hypothesis the t - statistics $t_{\hat{\rho}}$ formulated as

$$t_{\hat{\rho}} = \frac{(\hat{\rho}-1)}{SE(\hat{\rho})} = \frac{T(\hat{\rho}-1)}{\sqrt{T(1-\hat{\rho}^2)}}$$

has an asymptotic distribution, as $T \rightarrow \infty$, given by:

$$t_{\hat{\rho}} \Rightarrow \frac{\int_0^1 W(r)dW(r)}{\sqrt{\int_0^1 W(r)^2\, dr}}.$$

However, under the null hypothesis, the asymptotic distribution of the t-test statistic and hence the conventional critical values for the standard t-distribution are not valid. Fuller (1976) tabulates the critical values for these statistics.

6. 3 The Leybourne Test[27]

Modifying the Dickey – Fuller test, Leybourne (1995) proposed a new test for unit root, known as the Leybourne or DF_{max} test. The article considers the DGP of the form (2.3) and tests the null hypothesis of unit root (2.4) against the alternative of stationarity (2.5) using regression with a constant, i.e.:

$$y_t = \alpha + \rho y_{t-1} + e_t .$$

The test introduced by Leybourne (1995) involves two main steps: first, the DF test is applied to the so-called *forward realisation*, that is $y_1, y_2, \ldots, y_T$, and secondly, the same DF test is applied to $r_1, r_2, \ldots, r_T$, where $r_1 = y_T, r_2 = y_{T-1}, \ldots, r_T = y_1$, known as a *reverse realisation*. The notations DF_f and DF_r stand for the t-test statistics of the forward and reverse realisations, respectively. Following Leybourne (1995) relationship of the following type holds:

---

[26] Originally Dickey and Fuller (1979) obtain the asymptotic distributions investigating canonical representations of the statistics. The approach presented in Maddala and Kim (1998) is by means of Wiener processes.

[27] Leybourne (1995) considers (a) DGP of the form (2.3) and an estimated regression with constant and (b) DGP with constant: $y_t = \mu + \rho y_{t-1} + e_t$ and an estimated regression with constant and linear trend $y_t = \alpha + \beta t + \rho y_{t-1} + e_t$. For simplicity, the present section dwells on the first class of models only.

$$DF_r = DF_f - \lambda, \tag{2.7}$$

where λ is a stochastic term measuring the end-effect of the difference between y_T and y_1. The author shows that the t-test statistics DF_f and DF_r will only be equal when $y_1=y_T$, specifying that in general y_1 will differ from y_T since under the null hypothesis of unit root $\{y_t\}_{t=1}^T$ is integrated of order one (or higher). The asymptotic distribution of the parameter λ under the null hypothesis is also derived, that is, as $T \rightarrow \infty$:

$$\lambda \Rightarrow \left[\int_0^1 W(r)^2 dr - \left\{\int_0^1 W(r)dr\right\}^2\right]^{-1/2} \left\{W(1)^2 - 2W(1)\int_0^1 W(r)dr\right\}$$

Next, using Phillips and Peron's (1988) result for the asymptotic distribution of the forward realisation t-test statistic DF_f and equality (2.7), Leybourne (1995) derives the asymptotic distribution of DF_r, as $T \rightarrow \infty$, namely:

$$DF_r = \left[\int_0^1 W(r)^2 dr - \left\{\int_0^1 W(r)dr\right\}^2\right]^{-1/2} \left\{-\frac{W(1)^2}{2} - \frac{1}{2} + W(1)\int_0^1 W(r)dr\right\}.$$

The DF_{max} t-test statistic is then defined as the maximum value of both statistics DF_f and DF_r. Leybourne (1995) tabulates the critical values of the DF_{max} test and shows that the DF_{max} test is "considerably more powerful" than the standard DF test.

In addition, a variation of the DF and DF_{max} tests is the augmented Dickey-Fuller (ADF) test - proposed by Dickey and Fuller (1981) - and the ADF_{max} test (*see* Leybourne (1995)) respectively. Following Dickey and Fuller (1981) and Leybourne (1995), under the null hypothesis of unit root, the t-test statistics for the ADF, ADF_f and ADF_r follow the DF distribution.

Up to this point, reviewing the DF test and its variation, the DF_{max} test, the null hypothesis of unit root was tested against the alternative of stationarity. In the following section the discussion is focused on test for stationarity as a null against the alternative of non-stationarity. Named after its authors as the *KPSS* test, this test is often referred to as a confirmatory test in conjunction with the *ADF* test.

6.4 The KPSS Test[28]

The test proposed by Kwiatkowski *et al.* (1992), usually referred to as KPSS, deviates from the classical unit root test theory: the null hypothesis is the hypothesis of stationarity against the alternative of a unit root. The researchers consider regression of the form:

$$y_t = r_t + e_t \tag{2.8}$$

where $\{e_t\}_{t=1}^{T}$ is a stationary process and and $\{r_t\}_{t=1}^{T}$ is a random walk:

$$r_t = r_{t-1} + w_t,$$

with $w_t \sim iid\,(0, \sigma_w^2)$ and a fixed initial value r_0, which is associated with the intercept of the process.

The test itself is a Lagrange Multiplier (LM) test of the null hypothesis that the random walk $\{r_t\}_{t=1}^{T}$ has a zero variance, that is, the null hypothesis of stationarity (or, in other words, the regression coefficients r_t are constants):

$$\mathrm{H_0}\colon\ \sigma_w^2 = 0$$

against the alternative of non-stationarity (or, in other words, random walk coefficients):

$$\mathrm{H_1}\colon\ \sigma_w^2 > 0.$$

Under the null hypothesis of stationarity and assuming that $w_t \sim iid\ N(0, \sigma_w^2)$ and $e_t \sim iid\ N(0, \sigma_e^2)$, the LM test statistic is defined by the authors as the following ratio:

$$LM = \frac{\sum_{t=1}^{T} S_t^2}{\hat{\sigma}_w^2}, \tag{2.9}$$

where $S_t = \sum_{i=1}^{t} w_i,\ t = 1, ..., T$ and $\hat{\sigma}_w^2$ is the estimated variance of the error term. However, following Kwiatkowski *et al.* (1992), as "the series to which the stationarity tests will be applied are typically highly dependent over time, ... the *iid* error assumption under the null is unrealistic". The authors derive the asymptotic distribution of the LM statistic under weaker assumptions defining the "long-run variance" as $\overline{\sigma}_e^2 = \lim_{T\to\infty} T^{-1}E\left(S_T^2\right)$ and construct its consistent estimator using a Bartlett window. Further, it is shown that the LM statistic specified by equality (2.9) weakly converges to the following process:

[28] Kwiatkowski *et al.* (1992) consider two classes of models: (a) models with intercept and (b) models with intercept and linear trend. For simplicity, in the present section we consider the first class of models only.

$$LM \Rightarrow \int_0^1 V(r)^2 \, dr ,$$

where $V(r) = W(r) - rW(1)$ is called a standard Brownian bridge and $W(r)$ is a Wiener process.

In the empirical analysis presented later in Chapter 4, the KPSS test is used as a confirmatory test in conjunction with the $ADF_{\max}$ test.

In short, Section 6 of this chapter has outlined several linear tests for stationarity, which are based on the assumption of normality. However, the classical theory assumptions of linearity and normality of economics time series are often dubious. As mentioned earlier, two alternative ways of modelling inflationary time series are suggested in this research: first, excluding the assumption of normality and treating them as stable Paretian distributed processes (discussed already in Section 5 of this chapter) and second, ignoring the assumption of linearity and examining inflationary processes as processes described by a non-linear class of models, known in the literature as bilinear models[29]. Prior to dwelling (in Section 8) on the definition of bilinear processes and the existing tests for the presence of bilinearity, let us briefly consider in the following section one of the unit root tests recently suggested in the literature, which assumes that the innovations are stably Paretian distributed.

7 Unit Root Tests Under the Assumption of Stable Paretian Distribution

7. 1 Introduction

Dickey and Fuller (1979) investigate the process of the form (2.3) under the assumption of normally distributed disturbances. Some of their theoretical results were outlined in Section 6.2 of this chapter. The normal distribution, however, belongs to the family of stable Paretian distributions[30], which has led Rachev, Mittnik and Kim (1998) to the idea of generalising *DF* unit root test assuming stably Paretian distributed disturbances. Before

[29] *See*, for example, Granger and Andersen (1978), Subba Rao (1981), Subba Rao and Gabr (1984), etc.

[30] *See* Section 5.1 of this chapter.

examining their recent work let us introduce some of the terms used later. We shall start with the definition of the standard Lèvy process (or Lèvy motion).

Following Rachev and Mittnik (2000), a stochastic process $L_\alpha(z)$, $\alpha \in (0, 2)$, $z \in [0, 1]$ is said to be standard Lévy process if $L_\alpha(z)$ has stationary independent increments $L_\alpha(0) = 0$ and $L_\alpha(1)$ has standard α-stable distributed increments with characteristic function:

$$Ee^{i\theta L_\alpha(1)} = e^{-|\theta|^\alpha}, \ \theta \in R.$$

Following Rachev *et al.* (1998), let u_t be *iid* symmetric, strictly stable random variables ($L_\alpha(1)$) with index of stability $\alpha \in (1, 2)$ and, such that for $\alpha = 1$, $L_1(1)$ is a symmetric stable variable. Then, the following weak convergence holds:

$$c_T^{-1} \sum_{t=1}^{[T.z]} u_t \Rightarrow L_\alpha(z), \qquad z \in [0, 1],$$

where $L_\alpha(z)$ is a Lévy process with strictly stable increments and the constant $c_T = T^{1/\alpha} l(T)$, where $l(T)$ is a slowly varying function as $T \to \infty$.

Next, Subsection 7.2 states the Rachev-Mittnik-Kim unit root test under the assumption of symmetric stable Paretian distributed innovations for the case where the estimated regression does not include intercept. The results for the cases with intercept, and intercept and linear trend as well as the related proofs, are given in Rachev *et al.* (1998).

7.2 The Rachev – Mittnik – Kim Test[31]

In brief, Rachev, Mittnik, Kim (1998) consider the first-order autoregressive process:

$$y_t = \rho^* y_{t-1} + u_t$$

with initial condition $y_0 = 0$, $t = 1, \ldots, T$ and, disturbances u_t, which are *iid*, symmetric, strictly stable random variables with index of stability $\alpha \in (1, 2)$.[32] Chan and Tran (1989) show that under the null hypothesis of unit root the asymptotic distribution of the OLS estimator $\hat{\rho}^*$ [33] as $T \to \infty$ is:

$$T(\hat{\rho}^* - 1) \Rightarrow \frac{\int_0^1 L_\alpha(s-)dL_\alpha(s)}{\int_0^1 L_\alpha^2(s)ds}.$$

Following Rachev *et al.* (1998), the *t*-test statistic $t_{\hat{\rho}^*}$ can be expressed in the following way:

$$t_{\hat{\rho}^*} = \frac{\hat{\rho}^* - 1}{SE(\hat{\rho}^*)} = (\hat{\rho}^* - 1)\sqrt{\frac{T\sum_{t=1}^{T} y_{t-1}^2}{\sum_{t=1}^{T}(y_t - \hat{\rho}^* y_{t-1})^2}}$$

and, as $T \to \infty$, its asymptotic distribution weakly converges to a function of Lèvy's processes:

$$t_{\hat{\rho}^*} \Rightarrow \frac{\int_0^1 L_\alpha(s-)dL_\alpha(s)}{\sqrt{[L_\alpha](1)\int_0^1 L_\alpha^2(s)ds}}$$ [34]

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[31] Rachev *et al.* (1998) consider two classes of models: models without and with an intercept. For simplicity, in the present section, we consider the first class of models only.

[32] According to Rachev, Mittnik, Kim (1998), 'almost all of the results can be easily extended to the case $0 < \alpha \le 1$'. However, for $\alpha \in (0,1)$, the mean of the processes does not exist and therefore has no meaning in applied work. Thus the discussion is concentrated on the case $\alpha \in (1,2)$ only, as it is relevant to our empirical work later.

[33] The OLS estimator $\hat{\rho}^*$ is given by formula (2.6a) and is irrespective of the index of stability α.

[34] "Square bracket" of the Lévy process $L_\alpha(z)$ is denoted by $[L_\alpha](z)$ and has the following presentation:

$$[L_\alpha](z) = L_\alpha^2(z) - 2\int_0^t L_\alpha(s-)dL_\alpha(s)\cdot$$

The critical values for indexes of stability $\alpha \in (1, 2)$ are tabulated in Rachev and Mittnik (2000).

Consider in the following section the stationarity tests relevant to the second approach proposed in this thesis for modelling inflationary processes, namely stationarity tests under the assumption of bilenearity. The discussion of this issue starts with a brief overview of bilinear processes before defining the unit root bilinear process.

8 Unit Root Tests Under Assumption of Bilinearity

The development of the theory of bilinear time series modelling begins in the 1970's in the works of Mohler (1973), Granger and Andersen (1978) and Subba Rao (1981) among others. The econometricians' interest toward this class of models results in the recent (for those years) popularity of non-linear models, in particular for forecasting purposes. The properties of bilinear models have received further development in the works of Subba Rao (1981), Subba Rao and Gabr (1984), Sesay and Subba Rao (1986), Kim, Billard and Basawa (1990) and Grahn (1993) among others. Following Weiss (1986), bilinearity and ARCH effects may be easily mistaken as they posses similar unconditional moments. Higgins and Bera (1988) proposed a joint test for ARCH and bilinearity in a linear model with disturbances following bilinear process. Further development in the area has been done by Peel and Davidson (1998) who have suggested a bilinear error correction mechanism which has been empirically applied for modelling first, annual real consumption data and second, monthly spot price observations for the dollar/yen exchange rate, for the UK. Next, Charemza *et al.* (2002b) define unit root bilinear processes and propose a test for presence of bilinearity. Their findings have been applied to a large data set of mature and emerging stock market indexes (*see* Charemza *et al.* (2002b), (2002c)).

8. 1 Definition of a Bilinear Unit Root

Let $\{y_t\}_{t=1...T}$ is a discrete time series. Process of the form:

$$y_t = \sum_{i=1}^{p} a_i y_{t-i} + \sum_{j=0}^{r} c_j e_{t-j} + \sum_{l=1}^{m}\sum_{l'=1}^{k} b_{ll'} y_{t-1} e_{t-l'} , \qquad (2.10)$$

where $a_i, c_j, b_{ll'} \in R$, $c_0 = 1$ and $\{e_t\}_{t=1...T}$ are *iid* $N(0,\sigma_e^2)$ is called bilinear process. Models of the form (2.9) are called bilinear time series models and are usually denoted by *BL(p, r, m, k)*. Note that the simplest process in the class of bilinear processes is the linear unit root process $BL(1, 0, 0, 0)$. Thus, linear unit root processes are a sub-class of the class of bilinear processes.

As the class of processes $BL(p, r, m, k)$ describes too broad family of bilinear processes, for the purposes of our analysis the attention is focused on the much narrower class $BL(1, 0, 1, 1)$, and more specifically on processes expressed by an equality of the form:

$$y_t = ay_{t-1} + by_{t-1}e_{t-1} + e_t . \tag{2.11}$$

The stationarity condition for these processes is $a^2 + b^2\sigma_e^2 < 1$.[35] Imposing the restriction that the coefficient $a = 1$, process (2.11) is equivalent to the following non-stationary process:

$$\Delta y_t = by_{t-1}e_{t-1} + e_t . \tag{2.12}$$

It can be shown that starting with $e_0 = y_0 = 0$, the means $E(y_t) = b\sigma_e^2(t\text{-}1)$ and $E(\Delta y_t) = b\sigma_e^2$ and hence, for most macroeconomic time series the coefficient $b \geq 0$.[36] On the other hand, the variance in returns, $var(\Delta y_t)$, changes stochastically[37], demonstrating that Δy_t presented by (2.12) is a non-stationary process only if b differs from zero, i.e. $b \neq 0$. The definition of a bilinear unit root[38] is then as follows:

Definition: A time series $\{y_t\}_{t=1 \dots T}$ is said to have a bilinear unit root if

$$\Delta y_t = by_{t-1}e_{t-1} + e_t$$

$\{e_t\}_{t=1 \dots T}$ are *iid* $N(0, \sigma_e^2)$ and the coefficient $b \neq 0$.

In the present thesis price and inflation time series will be investigated as described by the class of unit root bilinear processes. At first, however, these series need to be tested for presence of bilinearity. Charemza *et al.* (2002b) have proposed a two-step testing procedure explained in the following subsection.

[35] *See* Granger and Andersen (1978).

[36] The mean of the macroeconomic time series is usually a positive value, which, for example, is demonstrated by the empirical results presented in Chapter 5.

[37] *See* Charemza *et al.* (2002b)

[38] Note that Subba Rao (1997) defines a term 'bilinear unit root'. According to Charemza *et al.* (2002b), however, both concepts are mutually exclusive.

8. 2 Bilinear Unit Root Tests[39]

The Charemza *et al.* (2002b) method of testing for presence of bilinearity consists of a two-step conditional procedure. First, the hypothesis of a linear unit root is tested applying some of the traditional tests already outlined in Section 6. If the linear unit root is confirmed, the second step involves applying the so-called *b*-test[40] - a main issue of discussion in the present subsection.

The authors test the null hypothesis of no blinearity:

$$\mathrm{H}_0\!: b = 0,$$

against the alternative

$$\mathrm{H}_1\!: b > 0.$$

Following their work, under the null hypothesis of no bilinearity, the process Δy_t takes the form:

$$\Delta y_t = e_t.$$

Substituting $\Delta y_{t-1} = e_{t-1}$ in (2.11) yields:

$$\Delta y_t = b y^{*}_{t-1} + e_t \tag{2.13}$$

where $y^{*}_{t-1} = y_{t-1}\Delta y_{t-1}$. Hence, under the null hypothesis, the coefficient b can be estimated applying the *OLS* method to equation (2.13). The *OLS* estimate of b is given by formula of the following form:

$$\hat{b} = \frac{\sum_{t=2}^{T} y^{*}_{t-1}\Delta y_t}{\sum_{t=2}^{T} (y^{*}_{t-1})^2}. \tag{2.14}$$

Section 6 of this chapter has presented and mentioned several non-stationarity tests applicable to linear unit root processes. Charemza *et al.* (2002b) suggest modification of some of the tests, such that these tests can be applicable to unit root bilinear processes. The analogue of the Dickey – Fuller test is named the *b*-test and has a Student-*t* statistic given by:

[39] Charemza *et al.* (2002b) consider two classes of models: models with and without intercept. For simplicity, in the present section, we consider the second class of models only.

[40] For more details about the theory behind the conditional testing procedure *see* Charemza *et al.* (2002b).

$$t_{\hat{b}} = \frac{\sum_{t=2}^{T} y_{t-1}\Delta y_{t-1}\Delta y_t}{\hat{\sigma}\sqrt{\sum_{t=2}^{T} y_{t-1}^2 \Delta y_{t-1}^2}},$$

where $\hat{\sigma}$[41] is a consistent estimator of σ, which weakly converges to a process of the form:

$$t_{\hat{b}} \Rightarrow \frac{\int_0^1 W_1(t)dW_2(t)}{\sqrt{\int_0^1 W_1^2(t)dt}}$$

as $T \to \infty$, where W_1, W_2 are independent Wiener processes.

The motivation of modelling inflationary and/or price time series under the assumptions of bilinearity or stable Paretian distribution has already been discussed in Section 3 of this chapter. Nevertheless, the present chapter defined and briefly dwelt on the main properties of both classes of objects. However, before discussing the empirical results obtained for price and inflationary data, let us dwell on two simulation examples.

9 Bilinear and Stable Distributions: a Simulation Example

Consider first simulation of a symmetric stable Paretian process. Initially, samples of 500 standardised *iid* symmetric stable random variables u_α with indexes of stability $\alpha = \{1.1, 1.5, 1.9\}$ were generated following the way suggested by Fama and Roll (1967). Histograms illustrating their distribution are presented on Fig. 2.9a – 2.11a. They visually support the theoretical results that smaller values of the characteristic exponent correspond to thicker tails of the distribution (i.e. the distribution is more picked around the mean and possesses longer tails). On the other hand, Fig. 2.9 – 2.11 display the unit root processes corresponding to α = 1.1, 1.5, 1.9 which are generated using the following formula:

$$y_t = y_{t-1} + u_\alpha,$$

where the disturbances u_α are defined above. Visually, a decrease in α leads to more variable processes with processes values ranging in a larger diapason.

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[41] The standard deviation $\hat{\sigma}$ can be estimated from the regression equation $\Delta y_t = \hat{b} y_{t-1}^* + e_t$ (*see* Charemza *et al.* (2002b)).

Consider next the unit root bilinear process of the form (2.12):

$$y_t = y_{t-1} + by_{t-1}e_{t-1} + e_t,$$

where $\{e_t\}_{t=1...T}$ are *iid* $N(0, 1)$. Series of 500 observations and coefficients of bilinearity 0.044 ("high" bilinearity), 0.022 ("medium" bilinearity) and 0.011 ("low" bilinearity)[42] were generated. The corresponding graphs are plotted on Fig. 2.12 – 2.14. It is evident from the graphs that the higher the coefficient of bilinearity, the more variable is the process, with visibly large ups-and-downs and process values ranging in an increasing diapason. Further, Figs. 2.12a – 2.14a plot the distribution of the first difference of the generated bilinear processes. The graphs clearly show that an increase of the bilinear coefficient leads to distribution characterized with higher levels of skewness and kurtosis.

In short, both simulation examples intuitively illustrate the applicability of bilinear and stable Paretian distributed processes when inflationary data are modelled.

[42] Those bilinear coefficients were obtained using the findings of Charemza *et al.* (2002a), namely that the bilinear coefficient $b < \frac{1}{\sqrt{T}}$. Thus a process is considered as possessing "high" bilinearity if $b \in \left(0.5T^{-1/2}, T^{-1/2}\right)$, "medium" bilinearity if $b \in (0.25T^{-1/2}, 0.5T^{-1/2}]$ and "low" bilinearity if $b \in (0, 0.25T^{-1/2}]$.

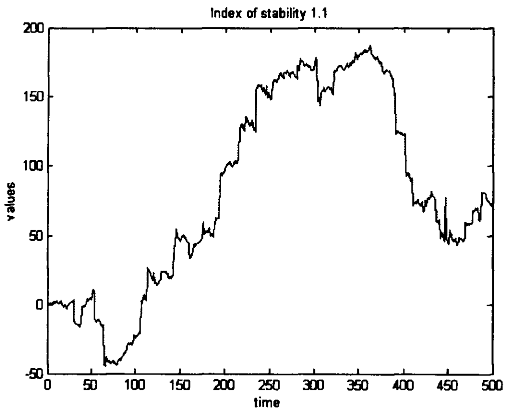

Fig. 2.9

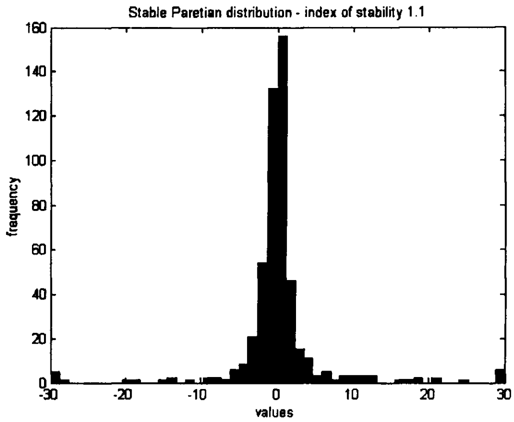

Fig. 2.9a

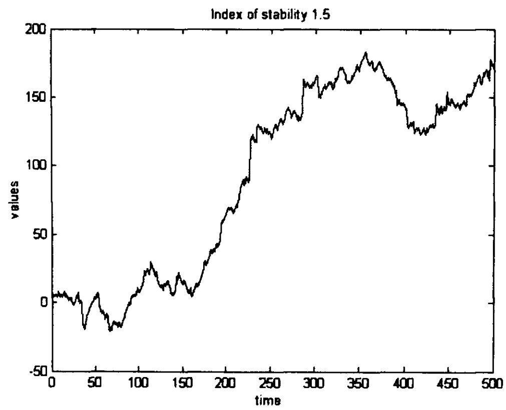

Fig. 2.10

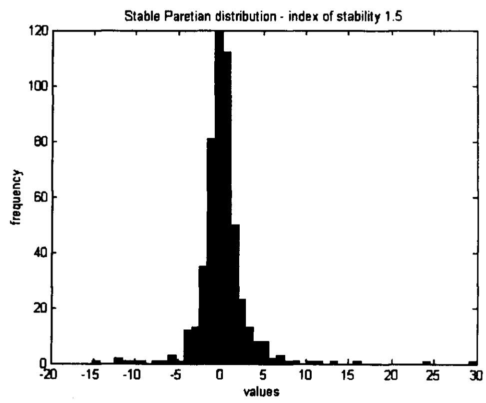

Fig. 2.10a

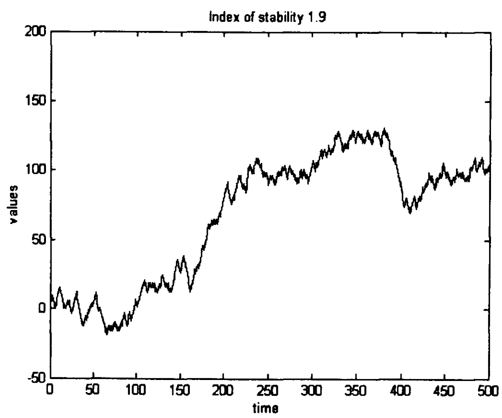

Fig. 2.11

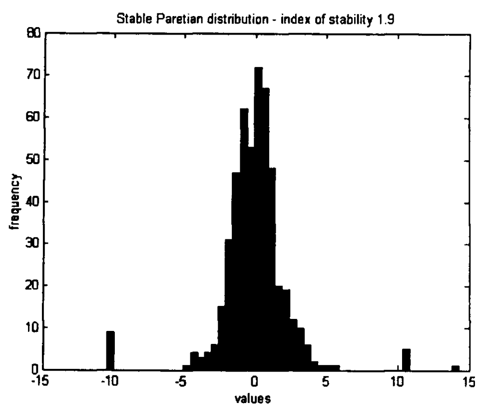

Fig. 2.11a

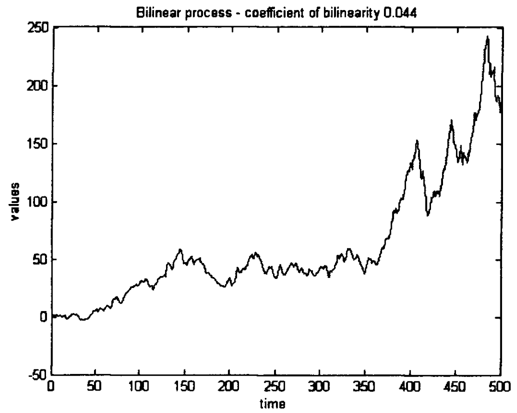

Fig. 2.12

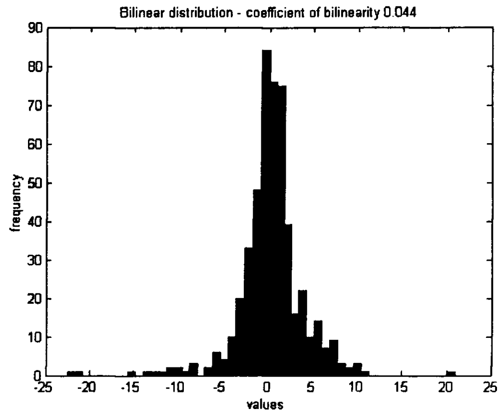

Fig. 2.12a

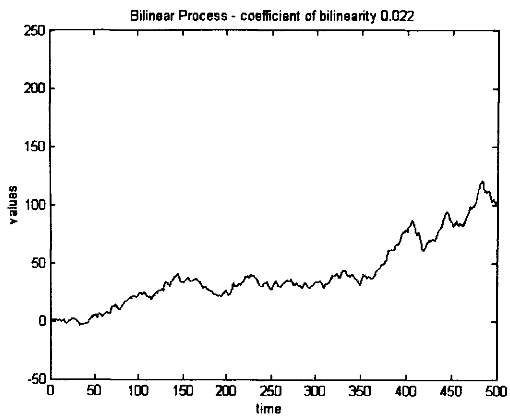

Fig. 2.13

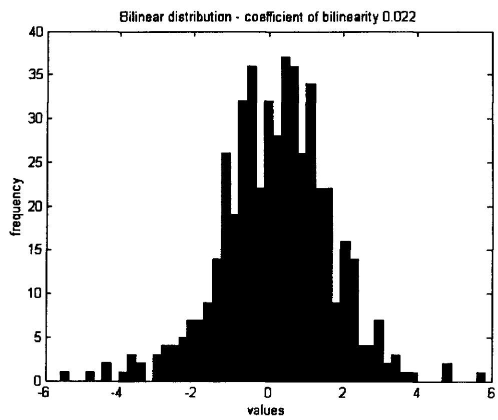

Fig. 2.13a

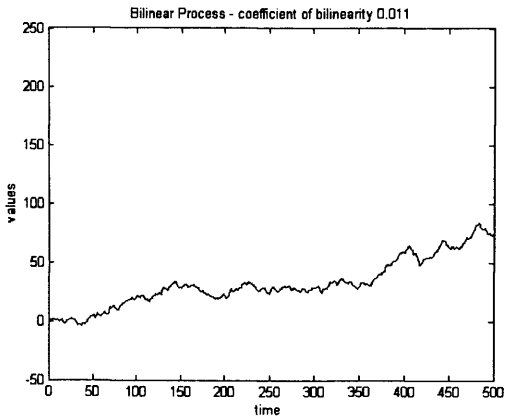

Fig. 2.14

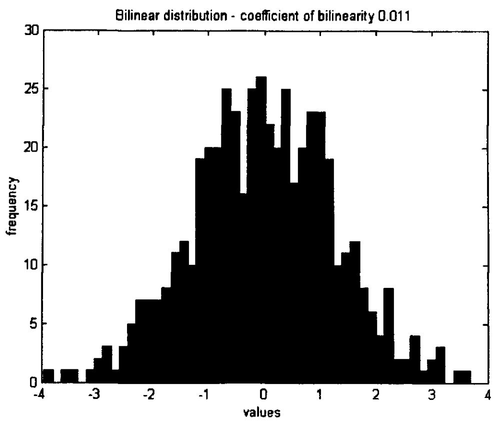

Fig. 2.14a

10 Summary

Questioning the classical assumptions of normality and linearity, Chapter 2 of the present thesis suggested two ways of modelling inflation. Firstly, it dwelt on the concept of modelling inflation under the assumption that distribution of inflation is a stable Paretian distribution. Secondly, it has suggested the view of prices as described by unit root bilinear processes, which, on the other hand leads to non-normality in distribution of inflation. Abreast with a discussion on both concepts of modelling inflation the present chapter has also outlined several unit root and stationarity tests under various assumptions: classical unit root and stationarity tests under the assumption of normality (the Dickey-Fuller test, the Leybourne test and the KPSS test), a unit root test under the assumption of stable Paretian distribution (Rachev-Mittnik-Kim test) and a unit root test under the assumption of bilinearity. The Rachev-Mittnik-Kim unit root test is conditional on the estimated values of the index of stability, which measures the tail thickness of the distribution of inflation. In order for estimating the index of stability the present chapter has suggested the use of the McCulloch quantile method.

The results of the tests application to a large selection of worldwide inflationary data are presented and discussed in the following chapter.

CHAPTER THREE *DISTRIBUTION AND STATIONARITY OF WORLD-WIDE INFLATIONARY DATA: EMPIRICAL ANALYSIS*

1 Introduction

Being one of the main macroeconomic indicators, inflation and, in particular modelling inflation, is a subject of considerable interest to economic researchers and a matter of significant importance for politicians. However, inflation has usually been modelled presuming that it is a normally distributed process. This assumption has recently been questioned in a vast number of empirical and theoretical publications. A broader discussion of this issue, supported with different examples of distributions of inflation has been presented in the previous chapter.

This chapter presents and discusses empirical results regarding the theory explained in Chapter 2. Using world-wide inflationary data, the chapter dwells on the issues of empirical investigation of first, the distribution of inflationary processes and second, the order of integration of price and inflationary processes.

Starting with a description of the data set in Section 2, the chapter proceeds in Section 3 with a brief summary of the way of initial data smoothing, namely adjustment for seasonality[1]. Subsequently, the work continues with discussion on the descriptive statistics

[1] The seasonal adjustment procedure and the motivation behind it has been discusses in Chapter 2, Section 4.

of inflationary time series presented and explained in Section 4. The distribution of inflation regarded as a stable Paretian distribution is further discussed in Section 5.

The dilemma of stationarity of inflationary processes has been comprehensively discussed in Chapter 1, Section 3, of this thesis. Attempts to answer the question 'is inflation a stationary or non-stationary process' are found in many recent studies. In the majority of them prices have been tested for a unit root assuming that inflationary processes are normally distributed and that prices follow linear processes. Subsequently, if non-stationarity of prices has been established, inflation has been tested for a unit root assuming that the first difference of inflationary processes is normally distributed and inflation follows a linear process. Although linear tests reveal non-stationarity of prices, they are weak to clearly identify the nature of inflationary processes, that is: are inflationary processes stationary or non-stationary processes. Our main aim is to further investigate this issue using non-classical and, perhaps, non-trivial techniques. The forthcoming analysis presents the empirical results of three different types of tests applied to: first, price time series and second, inflationary time series. These tests differ according to the assumptions made regarding the nature of the processes. The work begins with a discussion of the empirical results obtained from the application of classical, linear, unit root and stationarity tests, that is joint application of the ADF_{max} and the *KPSS* tests (Section 6.1). Both tests assume normality of the distribution of inflation and linearity of prices if price time series are tested, and normality of a first difference of the distribution of inflation and linearity of inflationary time series if inflation is tested. The results obtained are compared to those achieved using tests, which relax the assumptions of normality and linearity. Let us concentrate on this issue in more detail.

It has been explained in Chapter 2, Section 2 that the distribution of inflation is far from being normal. It is characterized by tails fatter than the tails of the normal distribution and, therefore, it is reasonable to consider it as a stable Paretian distributed process. Under the assumptions that prices follow a linear process and inflation is a stable Paretian distributed process, price time series can be tested for a unit root using the augmented Rachev-Mittnik-Kim (*ARMK*) test (*see* Chapter 2, Section 7.2). Subsequently, assuming that inflation follows a linear process and that the first difference of inflation is a stable Paretian distributed process, inflationary time series can be tested for a unit root applying the same test. In addition, inflation is tested for a unit root only if non-stationarity in price series has been established. The corresponding empirical results are presented in Section 6.2. Alternatively, relaxing the assumption of linearity in prices, that is presuming that they follow a unit root bilinear process with normally distributed disturbances, leads to non-normality of the inflationary distribution (*see* Chapter 2, Section 3). In this case prices can be tested for the presence of a stochastic, bilinear unit root. This can be done applying the augmented unit root bilinear (*AURB*) test, which is conditional on the confirmation of a linear unit root in prices (*see* Chapter 2, Section 8). Subsequently, for those time series only for which a unit root in prices has been established, the analysis continues with testing for a bilinear unit root in inflationary time series. In this case the *AURB* test is applied under the assumptions that inflation follows a unit root bilinear process with normally distributed disturbances. The empirical results are presented in Section 6.3. Finally, Section 7 concludes.

2 Data

The data set used consists of a large selection of world-wide price series. There are 108 CPI and RPI of monthly, not seasonally adjusted time series for 107 different countries.[2] All the data are collected from DataStream. The price series are of different length, covering various time periods between January 1950 and December 2001. The longest series contain 612 observations (Canada, Mexico, Switzerland and the US). However, the price series for the countries in transition, in particular those for the Central and Eastern European countries, are of shorter length. Earlier data for these countries are not available as a result of the recent change to a market economy. The lengths of the price time series are country specific with minimum number of observations, 120, for Estonia, Latvia and Russia.

The empirical results presented in the following sections are discussed regarding the development status of the countries. The countries are divided into three main groups: Developed, Developing and Least developing countries. Those from the second group (i.e. Developing countries) are further partitioned into (A) Central and Eastern European

[2] Two time series are available for Germany as one of them is for Former East Germany. Thus, the number of the time series is 108 and the number of the countries is 107.

Countries and (B) Other Developing Countries. The names of the countries in the different groups are:[3]

- **Developed Countries (DC)**

  Austria, Belgium, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Japan, Luxembourg, Netherlands, Portugal, Spain, Sweden, UK, Canada, Iceland, Japan, Norway, Switzerland and USA

- **Developing Countries**

  - **Central and Eastern European Countries (CEEC)**

    Albania, Bulgaria, Czech Republic, Estonia, Former East Germany (FE Germany), Hungary, Latvia, Lithuania, Poland, Romania, Russia, Slovakia and Slovenia

  - **Other Developing Countries (ODC)**

    Argentina, Aruba, Bahamas, Bahrain, Barbados, Bolivia, Botswana, Cameroon, Chile, Colombia, Costa Rica, Cyprus, Dominican Republic, Ecuador, El Salvador, Fuji, Ghana, Guatemala, Honduras, Hong Kong, India, Indonesia, Ivory Coast, Jamaica, Jordan, Kenya, Malaysia, Malta, Mauritania, Mauritius, Mexico, Morocco, Namibia, Netherlands Antilles, Nigeria, Pakistan, Paraguay, Peru, Philippines, Saudi Arabia, Singapore, South Africa, South Korea, Sri Lanka, St Kitts, St Lucia, Suriname, Swaziland, Taiwan, Thailand, Trinidad, Tunisia, Turkey, Uruguay, Venezuela, Vietnam and Zimbabwe

- **Least Developing Countries (LDC)**

  Burkina Faso, Burundi, Chad, Ethiopia, Gambia, Guinea Bissau, Haiti, Malawi, Mauritania, Myanmar, Nepal, Niger, Samoa, Senegal, Sierra Leone, Solomon Iceland, Uganda and Zambia

In addition, the data set consists of a large number of price series and some of them contain time periods, of different length for different countries, with unchanging price values. For

[3] The Least Developing Countries are specified by the United Nations. However, there is no precise definition of Developed countries and Developing countries. Following different criteria would lead to slightly different ways of their grouping. A suitable criterion, for example, is partitioning those countries according to their GNP per capita. The countries are, however, partitioned according to the author's viewpoint.

those time periods and those countries only, the data are linearly interpolated. Consider next the initial smoothing of the data.

3 Initial Smoothing of Data: Seasonal Adjustment

Although a contentious issue, adjustment for seasonality seems to be reasonable when working with monthly (or quarterly) inflationary data. The disputes in the literature, together with the motivations of filtering for seasonality regarding the current empirical study, have been discussed in more detail in Chapter 3, Section 4 of the present thesis. It is perhaps important to remember, that many researchers have inferred in their studies that seasonally adjusted series lead to distortion in the estimated dynamic relationships. Hylleberg (1992), p.10, writes: "as the degree of the distortion varies the best advice for researchers is to consider both seasonally adjusted and seasonally unadjusted series".

The present work analyses a large set of world-wide inflationary data. Precise examination for presence of seasonality is a time consuming procedure, as each series has to be analysed individually. Chapter 3, Section 4 of this thesis has graphically shown no distinctive seasonal pattern in Sri Lanka price data. This is, however, not the case with the UK (*see* Chapter 3, Section 4). A possible reason for lack of seasonality in the price data for some of the countries might be governmental control of prices. Although disputable, the present empirical work starts with initial adjustment for seasonality. However, we should bear in mind that it could lead to possible distortion in the results for some of the countries where no clear presence of seasonality is observed.

In short, all the price time series are initially adjusted for seasonality. This is done by the use of the U.S. Census Bureau X-12-ARIMA software, version 0.2.8. The method applied is Census Bureau Method II, X–11 seasonal adjustment program, as detailed in Shiskin *et al.* (1967) and Dagum (1988) and, the algorithm followed is the log-additive algorithm.

Hereafter, the term 'price time series' is used instead of the natural logarithm of seasonally adjusted price time series. Subsequently, inflation is considered as a first difference of the natural logarithm of seasonally adjusted price (i.e. CPI or RPI) time series.

4 Description of the Data

Descriptive statistics of the seasonally adjusted inflationary data are presented in Tables C3.1a-d (Appendix C)[4]. The Tables are organized in the following way: the first two columns, denoted by (1) and (2) respectively, contain the countries names and the number of observations in the corresponding price time series, followed by the mean values of inflation and their standard deviations stated in columns (3) and (4). Columns (5) and (7) tabulate the results for skewness and kurtosis of inflationary time series and their corresponding *p*-values are displayed in columns (6) and (8).

The results show that the mean values of the inflationary series are larger for the group of Central and Eastern European countries and lower for the Developed countries. The countries with minimum mean values across groups are as follows: Germany (0.002), FE Germany (0.003), Singapore (0.002) and Burkina Faso (0.003). Those with maximum mean values are Iceland (0.016), Russia (0.069), Peru (0.053) and Zambia (0.045).[5]

The results for skewness and kurtosis of inflationary time series indicate substantial non-normality. The estimated values considerably deviate from the normal distribution values of those parameters, i.e. zero for skewness and kurtosis.[6] Representatives of the countries groups with relatively ***low levels of skewness*** for the group to which they belong are as follows:

- **Developed Countries:** Luxemburg (0.503), Netherlands (0.681) and Belgium (0.779)
- **Developing Countries**
  - **Central and Eastern European Countries:** Hungary (1.036), Romania (1.901)

[4] Hereafter, the letters "a", "b", "c" and "d" after the Table number stand for the countries groups as follows: "a" – Developed Countries, "b" – Central and Eastern European Countries, "c" – Other Developing Countries, "d" – Least Developing Countries.

[5] The discussion of the mean values of the inflationary series is meaningful only if inflation is a stationary process. The empirical results regarding stationarity of inflationary processes are presented later in Section 6.1 and Section 6.2 of the present chapter.

[6] The Normal distribution is characterized by zero value for skewness and kurtosis equal to three. However, in this work the kurtosis value is scaled to zero.

- ♦ **Other Developing Countries:** Cyprus (0.197), India (0.267) and Vietnam (-0.235)
- **Least Developing Countries:** Niger (0.054), Malawi (0.115) and Samoa (0.297)

Alternatively, countries distinguished by ***high levels of skewness*** (regarding the group to which the countries belong) are:

- **Developed Countries:** Spain (6.111), Denmark (2.682) and Portugal (2.136)
- **Developing Countries**
  - ♦ **Central and Eastern European Countries:** Bulgaria (6.065) and Estonia (6.143)
  - ♦ **Other Developing Countries:** Colombia (9.558), Fuji (6.061) and Peru (7.456)
- **Least Developing Countries:** Uganda (2.844) and Sierra Leone (2.353)

The following countries possess the ***smallest kurtosis values*** in the group to which they belong:

- **Developed Countries:** Ireland (0.562) and Belgium (1.804)
- **Developing Countries**
  - ♦ **Central and Eastern European Countries:** Hungary (3.097) and Romania (4.173)
  - ♦ **Other Developing Countries:** India (1.971) and South Africa (1.878)
- **Least Developing Countries:** Ethiopia (1.788), Malawi (1.543) and Samoa (1.692)

Large values of kurtosis (with the group to which the countries belong) are observed in the following countries:

- **Developed Countries:** Spain (112.741), Denmark (20.927) and Austria (17.695)
- **Developing Countries**
  - ♦ **Central and Eastern European Countries:** Bulgaria (43.473) and Estonia (45.578)
  - ♦ **Other Developing Countries:** Colombia (117.106), Fuji (70.666) and Philippines (226.787)
- **Least Developing Countries:** Sierra Leone (21.754), Uganda (13.074) and Niger (13.719)

Moreover, for all the countries but Ghana, Philippines, Saudi Arabia, Vietnam (ODC group) and Nepal (LDC group), skewness is positive. With few exceptions, the calculated p-values show significance of the corresponding skewness and kurtosis estimates.

In short, the CEEC group is distinguished by general levels of skewness and kurtosis substantially higher than those for the rest of the groups. Kurtosis values of similar magnitude are specific for the rest of the developing countries. The general levels of skewness and kurtosis are lower for the LDC group. Remarkable examples of non-normally distributed inflationary data are the data for Spain, Estonia, Bulgaria, Philippines, and Colombia among others.

Thus, the empirical analysis of skewness and kurtosis of worldwide inflationary data reveals that the distribution of inflation significantly deviates from the normal distribution. These empirical results support the theoretical and practical findings in the literature (*see* Section 2, Chapter 2) regarding the non-normal nature of inflationary processes. In addition, the histograms of four selected countries, plotted on Fig. 2.1 – 2.4 (Chapter 2), graphically demonstrate that distribution of inflation is asymmetric and heavy-tailed. They have been discussed in more details in Section 2, Chapter 2.

Hence, we can infer that inflationary processes are far from being normally distributed. However, in the majority of studies inflation has been modelled under the assumption of normality. On the other hand, prices are usually considered as linear processes. As suggested in Chapter 2, this study examines inflation relaxing both the assumption of normality of distribution of inflation and the linearity of price processes. Two different approaches of modelling inflation have been proposed. One of them is to consider prices as a unit root bilinear process. Subsequently, as these processes belong to the class of non-linear processes, the distribution of inflation is a non-normal distribution (*see* Chapter 2, Section 3). Alternatively, inflation is viewed as a stable Paretian distributed process. It is presumed that prices are described by a linear process with disturbances following a stable Paretian law (*see* Chapter 2, Section 3). As explained in Chapter 2, Section 3, in this particular case, inflation (i.e. first difference in prices Δp_t) is associated with the disturbances of the process and, therefore, follows a stable Paretian law.

Hence, under both mentioned suggestions prices can be tested for a unit root. Assuming that they follow a unit root bilinear process the so-called bilinear unit root tests (or *b*-tests) can be applied (*see* Chapter 2, Section 8.2). Alternatively, assuming that inflation follows a stable Paretian law, the Rachev-Mittnik-Kim (*RMK*) unit root test can be applied (*see* Chapter 2, Section 7.2). Subsequently, for those time series for which the *b*-test results or the *RMK* test results, respectively, point to non-stationarity of the price processes, the *b*-test or the *RMK* test, respectively, can be further applied to inflationary processes in order to establish stationarity or non-stationarity of inflation. Thus, when applying a bilinear unit root test, inflation is assumed to follow a unit root bilinear process. Alternatively, when applying the *RMK* unit root test, first difference of inflation is assumed to follow a stable Paretian law. The *RMK* test is, however, conditional on the value of the index of stability α. Hence, prior to applying this test it is necessary to evaluate the parameter α (*see* Chapter 2, Section 5.2). Moreover, as explained in Chapter 2, Section 5.1, the characteristic component α determines the tail thickness of the stable Paretian distribution. The estimation results obtained, together with a discussion on the properties of the inflationary distribution viewed as a stable Paretian distribution, are explained in the following section.

5 Estimation of Stable Paretian Distribution Parameters

The present section examines the characteristic exponent α of the stable Paretian distribution obtained after the empirical application of the McCulloch method of estimation to inflationary data. The corresponding results are tabulated in Tables C3.2a-d (Appendix C). The table consists of four columns. After the country names and the number of price observations are stated in columns (1) and (2), columns (3) of the tables display the estimated values of the index of stability α, while column (4) presents their standard deviations. The results can be summarised in the following way:

- **Developed Countries**

$\alpha = 2$	France, Greece and Iceland
$\alpha \in (1.5, 2)$	15 countries (out of 21) among which UK, Canada and Finland
$\alpha \in (1, 1.5]$	Austria, Luxemburg and Germany

- **Developing Countries**
  - **Central and Eastern European Countries**

    $\alpha \in (1.5, 2)$ Hungary, Romania and Russia

    $\alpha \in (1, 1.5]$ 8 countries (out of 13) among which Bulgaria, Czech Republic and Slovenia

    $\alpha < 1$ Latvia and Poland
  - **Other Developing Countries**

    $\alpha = 2$ Colombia, South Africa, Turkey and Uruguay

    $\alpha \in (1.5, 2)$ 21 countries (out of 56) among which Cyprus, Namibia and Venezuela

    $\alpha \in (1, 1.5]$ 27 countries (out of 56) among which Ghana, Mexico and Singapore

    $\alpha < 1$ Argentina, Bolivia, Peru and Suriname

- **Least Developing Countries**

  $\alpha \in (1.5, 2)$ 6 countries (out of 18) among which Ethiopia, Myanmar and Uganda

  $\alpha \in (1, 1.5]$ 12 countries (out of 18) among which Burkina Faso, Gambia and Sierra Leone

Summarizing, for most of the countries, 88.78%, the index of stability α lies in the interval $(1, 2)$: for 46,73% of the countries $\alpha \in (1, 1.5]$ and for 42.05% of the countries $\alpha \in (1.5, 2)$. For seven out of 108 inflationary series the estimation results show normality of the corresponding inflationary distribution (i.e. $\alpha = 2$) and for another six inflationary time series $\alpha < 1$ meaning that the first moment of the process does not exist.

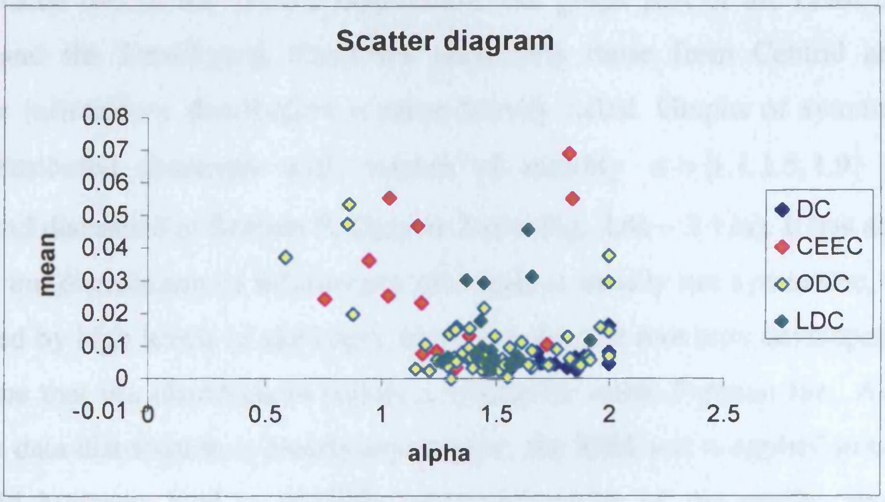

Fig. 3.1

Fig. 3.1 plots a scatter diagram of the indices of stability α (determining the thickness of the stable Paretian distribution tails) versus the mean values of the inflationary time series. Before discussing the relationship between both parameters let us summarise the results obtained regarding the values of the index of stability α. It is evident from Fig. 3.1 that the countries of the DC group, with few exceptions, possess α-values ranging in the interval [1.5, 2]. Further, while most of the α-values of the ODC group lie in the interval (1, 2], for the majority of the LDC the indices of stability are clearly smaller than 1.5. In conclusion, assuming that inflation is a stable Paretian distributed process, the distributions of inflation for the LDC group are characterised by thicker tails than the tails of the DC group under the same assumption. Finally, consider the results obtained for the CEEC group. Summarising, the distributions of inflation for these countries are clearly distinguished with tails fatter than the tails of the distributions of the countries from the rest of the groups, as the index of stability for the majority of the CEEC is between 1 and 1.5. Nevertheless those countries (with few exceptions) are distinguished by considerably larger mean values of inflation, demonstrating a negative relationship between the index of stability of the distribution of inflation and the mean values of inflation. With some exceptions, the same could be inferred for the rest of the Developing countries. In addition, the groups of the DC and the LDC (again, with few exceptions) are characterised by mean values of inflation of a small magnitude disregarding their corresponding α-values.

In summary, the estimation results for the index of stability α of the stable Paretian distribution illustrate that for the majority of the countries this parameter is considerably below the value two of the normal distribution. For grater part of the Least Developing Countries and the Developing Countries (especially those from Central and Eastern Europe) the inflationary distribution is more heavily tailed. Graphs of symmetric stably Paretian distributed processes with indexes of stability $\alpha = \{1.1, 1.5, 1.9\}$ have been presented and discussed in Section 9, Chapter 2 (*see* Fig. 2.9a – 2.11a). It has already been shown that the distribution of inflationary processes is usually not symmetric, that is it is characterized by high levels of skewness. However, the unit root tests developed (i.e. *RMK* tests) assume that the disturbances follow a symmetric stable Paretian law. Although the inflationary data distribution is clearly asymmetric, the *RMK* test is applied to our data set, which could however lead to possible misrepresentation of the results obtained. The explanation of the *RMK* test results takes part of the next section.

6 Unit Root and Stationarity Tests

This section presents the empirical results obtained after the application of several unit root and stationarity tests to price and inflationary time series. The tests used differ with the assumptions made regarding the nature of the processes of interest as follows.

Prices are tested assuming that:

- inflation is a normally distributed process (joint application of the ADF_{max} and the *KPSS* tests)
- inflation is a stable Paretian distributed process (*ARMK* tests with and without a constant)
- prices follow a unit root bilinear process and consequently inflation is a non-normally distributed process (*b*-test)

Analogously, inflation is tested assuming that:

- first difference of inflation is a normally distributed process (joint application of the ADF_{max} and the *KPSS* tests)
- first difference of inflation is a stable Paretian distributed process (*ARMK* tests with and without a constant)

- inflation follows a unit root bilinear process and consequently first difference of inflation is a non-normally distributed process (augmented *b*-test)

6. 1 Testing Under the Assumption of Normality: Analysis of Price and Inflationary Data

The main objective of this section is to present the results of the linear unit root and stationarity tests applied to first, price time series and second, inflationary time series. More specifically, the results of the joint application of the ADF_{max} and the *KPSS* tests are considered. The critical values used are those of Charemza and Syczewska (1998), calculated for the purposes of such joint testing. Both tests (ADF_{max} and the *KPSS*) differ regarding their null and alternative hypothesis. Before explaining the results, it is perhaps convenient to state the exact hypothesis tested in the empirical work. The ADF_{max} test is Leybourne style *ADF* test as explained in Leybourne (1995) (*see* also Chapter 2, Section 6.3). In short, it tests the null hypothesis of a unit root

$$H_0: |\rho| = 1$$

against the alternative of stationarity

$$H_1: |\rho| < 1$$

in a regression of the form

$$y_t = \alpha + \rho y_{t-1} + \sum_{j=1}^{k} \gamma_j \, \Delta y_{t-k} + e_t,$$

where α is an intercept, k is the number of augmentations, γ_j are the regression coefficients and $e_t \sim iid\ N(0, \sigma_e^2)$. This is known as *ADF* test. The augmented Leybourne test (or ADF_{max} test) is the *ADF* test applied to first the ordered, and second, the reversed realisation of the data. The ADF_{max} *t*-test statistic is then defined as the maximum statistics from both realisations.[7]

[7] For more details see Chapter 2, Section 6.3.

In contrast, Kwiatkowski *et al.* (1992) developed a test with null hypothesis of stationarity

$$H_0\text{: } \sigma_w^2 = 0$$

against the alternative of non-stationarity (or, in other words, random walk coefficients)

$$H_1\text{: } \sigma_w^2 > 0$$

in a regression of the form:

$$y_t = r_0 + \sum_{i=0}^{l} w_{t-l} + \varepsilon_t ,$$

where r_0 is an intercept, l is the number of autocorrelations, $w_t \sim iid(0, \sigma_w^2)$ and $\{\varepsilon_t\}_{t=1}^{T}$ is a stationary error process.[8]

The ADF_{max} test and the *KPSS* test are jointly applied to first, price data and second, inflationary data. The results obtained for price data are presented in Tables C3.3a-d (Appendix C), while those for inflationary data are displayed in Tables C3.6a-d (Appendix C). Both sets of tables are organised in an identical way: the country names are listed in columns (1), columns (2) – (4) present the results for DF_{max} test, while columns (5) – (6) tabulate those for the *KPSS* test. In both columns (2) and (5) the notations '0', '+', '++' and '+++' signify respectively: no significance of the statistics or, the statistics belongs to the 90%, 95% or 99% critical region. Columns (3) and (4) indicate the maximum significant length of augmentations in the ADF_{max} test and whether the maximum statistic is achieved in the forward or backward realisation. Finally, columns (6) tabulate the autocorrelation length for *KPSS* test.

The results in Tables C3.3a-d (Appendix C) point to clear non-stationarity of the price series for all the countries but Vietnam. This is indicated with pairs of the form (0, +++) [9] showing joint confirmation of the unit root hypothesis at the 0.01 level of significance. However, nothing can be concluded regarding the non-stationarity of the Vietnam price data. This is indicated with a pair of results of the form (+++, +++).

Next, the joint testing procedure is applied to inflationary data and the results obtained lead to different conclusions about different countries (Tables C3.6a-d, Appendix C). Joint

[8] For more details see Chapter 2, Section 6.4.

[9] Hereafter, the first and the second elements in the pairs indicate the significance of the ADF_{max} and the *KPSS* tests, respectively.

confirmation of a unit root is denoted with pairs (0, +++), (0, ++) or (0, +). The inflationary processes for the following countries are determined as non-stationary:

- **Developed Countries:** Iceland
- **Developing Countries**
  - **Central and Eastern European Countries:** Czech Republic, Estonia, FE Germany, Poland, Russia and Slovenia
  - **Other Developing Countries:** Chile, Colombia, South Korea and Uruguay
- **Least Developing Countries:** Uganda

Alternatively, joint confirmation of stationarity is denoted by (+, 0), (++, 0) or (+++, 0); the names of those countries satisfying this are:

- **Developed Countries:** none
- **Developing Countries**
  - **Central and Eastern European Countries:** none
  - **Other Developing Countries:** Ivory Coast, Malta, Philippines and Saudi Arabia
- **Least Developing Countries:** Burkina Faso, Burundi, Chad, Mauritania, Nepal and Solomon Iceland

Summarising, the joint application of the ADF_{max} and the *KPSS* tests indicate that 12 out of 107 (i.e. 10.19%)[10] inflationary time series are non-stationary and the majority of them (10 out of 12) belong to the group of the Developing countries. Moreover (in terms of percentages of the total number of countries in a group) while more than 46% of the inflationary time series of the CEEC group are non-stationary, for the ODC group of countries this percentage is only just over 7. On the other hand, stationarity of inflationary data is detected for 11 out of 107 series (i.e. 11.11%). 39% of the inflationary time series of the LDC group are non-stationary versus only 7% of the countries in the ODC group. For the rest of the countries (78.7%), however, nothing can be inferred regarding stationarity of inflationary processes.

[10] Vietnam is excluded from further analysis using the joint application of the ADF_{max} and the *KPSS* tests.

6.2 Testing Under the Assumption of Stable Paretian Distribution: Analysis of Price and Inflationary Data

The theoretical grounds of the Rachev – Mittnik – Kim (*RMK*) unit root test have been outlined in Chapter 2, Section 7.2. Before explaining the empirical results of applying this test after applying it to our data set, let us consider the corresponding estimated regression and the hypothesis tested. We are empirically using the augmented Rachev – Mittnik – Kim (*ARMK*) test or, more specifically, estimation regression of the form:

$$y_t = \alpha^* + \rho^* y_{t-1} + \sum_{j=1}^{m} \gamma_j^* \Delta y_{t-m} + u_t,$$

where the disturbances u_t are *iid*, symmetric, strictly stable random variables with index of stability $\alpha \in (1, 2)$, α^* is an intercept, m is the number of augmentations and γ_j^* are the regression coefficients. The null hypothesis tested is the hypothesis of a unit root

$$\mathrm{H_0}: \alpha^* = 0 \quad and \quad |\rho^*| = 1$$

against the alternative of stationarity

$$\mathrm{H_1}: \alpha^* \neq 0 \quad and \quad |\rho^*| < 1.$$

In addition, the results of the *ARMK* test without an intercept are also presented. For both cases, with and without an intercept, the critical values are tabulated in Rachev, Mittnik and Kim (2000). They are calculated for data generation processes with length of 500 observations. Although the lengths of the price time series used in this analysis vary between 120 and 612 observations, the Rachev *et al.* (2000) critical values are used.

The *ARMK* tests are applied first, to price data and second, to inflationary data. The *ARMK* tests assume that the disturbances of the process follow a symmetric stable Paretian law (*see* Chapter 2, Section 3). Let us explain this issue in more detail. Price time series are tested assuming that inflation is a symmetric stable Paretian distributed process, i.e.:

$$\Delta p_t = u_t(\alpha), \quad u_t(\alpha) \sim iid\ S(u; \alpha, 0, \delta, c).$$

The tests are conditional on the index of stability α. Thus, before applying the *ARMK* tests the parameter α has to be evaluated. The estimated results are presented in Tables C3.2a-d (Appendix C), columns (3) and they have been discussed in Section 5 of the present chapter.

Analogously, inflationary data are tested assuming that first difference of inflation follows a symmetric stable Paretian law, i.e.

$$\Delta\Delta p_t = \Delta u_t(\alpha) = \bar{u}_t(\bar{\alpha}), \quad \bar{u}_t(\bar{\alpha}) \sim iid\ S(\bar{u}; \bar{\alpha}, 0, \bar{\delta}, \bar{c}).$$

The symmetric stable Paretian distribution of $\Delta\Delta p_t$ is characterized by an index of stability $\bar{\alpha} \neq \alpha$ ($\bar{\alpha}$ determine the thickness of the tails of the stable Paretian distribution). Thus, prior to applying the *ARMK* tests, the parameter $\bar{\alpha}$ has to be evaluated. The estimated results are presented in Tables C3.5a-d (Appendix C) columns (3).

Let us consider the results of the empirical investigation for price and inflationary time series. The results for price data are presented in Tables C3.2a-d (Appendix C), columns (5)-(10) and those for inflationary data are stated in Tables C3.5a-d (Appendix C), columns (5)-(10). Columns (5)-(7) present the results of the *ARMK* test without a constant, while columns (8)-(10) present those with a constant. Columns (5) and (8) list the calculated *t*-statistics while columns (6) and (9) indicate the significance of the statistic obtained. The notations '0', '+', '++' and '+++' have the same meaning as explained in Section 6.1. The symbol '-' in Tables C3.2a-d (Appendix C) signifies that the unit root test has not been applied because the corresponding α value is either less than one (i.e. the mean of the process does not exist) or because it is equal to two (i.e. the inflationary distribution coincides with the normal distribution)[11]. In Tables C3.5a-d (Appendix C) this notation also shows that the *ARMK* test has rejected the null hypothesis of a unit root in price time series and therefore, further application of the *ARMK* test to inflationary data is not necessary. Finally, columns (7) and (10) tabulate the maximum significant length of augmentations in the *ARMK* tests.

The empirical results obtained for the cases with and without a constant slightly differ. For the case without a constant the null hypothesis of a unit root in prices cannot be rejected for any of the countries to which the test has been applied. However, when a constant is included, non-stationarity in prices can be inferred for all the countries to which the test has been applied, except:

[11] *RMK* test is applicable to those time series only, for which the index of stability of their distribution, α, lies in the interval $(1, 2)$. *See* also Chapter 2, Section 7.

- **Developed Countries:** Denmark, Ireland and Japan
- **Developing Countries**
  - ♦ **Central and Eastern European Countries:** Albania, Czech Republic, FE Germany, Lithuania and Slovenia
  - ♦ **Other Developing Countries:** Aruba, Bahamas, Bahrain, Barbados, Chile, Fuji, South Korea, Namibia, Netherlands Antilles, Tunisia and Vietnam
- **Least Developing Countries:** Guinea Bissau, Sierra Leone and Uganda

Subsequently, the analysis proceeds with examination for a unit root in inflationary time series. The *ARMK* test without a constant has been applied to all the inflationary series. The *ARMK* test with a constant has been applied to those series only for which the unit root in prices has been established. The null hypothesis of a unit root in inflation is not rejected for the following countries:

- **Developed Countries**
  ***ARMK* test without a constant:** Austria, Denmark, Ireland, Italy and Netherlands
  ***ARMK* test with a constant:** France and Italy
- **Developing Countries**
  - ♦ **Central and Eastern European Countries:**
    ***ARMK* test without a constant:** Hungary
    ***ARMK* test with a constant:** Hungary
  - ♦ **Other Developing Countries**
    ***ARMK* test without a constant:** Botswana, Namibia, South Africa and Turkey
    ***ARMK* test with a constant:** Mexico, South Africa and Turkey
- **Least Developing Countries**
  ***ARMK* test without a constant:** Solomon Iceland
  ***ARMK* test with a constant:** none

Stationarity is, however, identified for the rest of the countries at different levels of significance. The results reveal that for the majority of the Developing and Least Developing countries the null hypothesis of a unit root in inflationary data is strongly rejected. The significance of the test statistic is less strong for the DC group of countries.

Summarizing, the last two subsections have presented the empirical results of testing for unit root and stationarity in price and inflationary data under different assumptions. If prices are tested, inflation is assumed to be first, a normally distributed process (Section 6.1) and second, a stable Paretian distributed process (Section 6.2). Analogously, if inflation is tested for a unit root or stationarity, first difference of inflation is assumed to be first, a normally distributed process and second, a stable Paretian distributed process. A comparison of the empirical results (in percentages) of testing price and inflationary time series under both types of assumptions are presented in Table 3.1 below. All the price and inflationary time series are considered. The set of columns (1) presents a summary of the results obtained after the joint application of the ADF_{max} and the *KPSS* tests. On the other hand, the set of columns (2) presents those obtained applying the *ARMK* test with a constant.

	Assuming Normality (1)			Assuming Stable Paretian Distribution (2)		
	Stationary	**Non-stationary**	**Nothing can be inferred**	**Stationary**	**Non-stationary**	**Nothing can be inferred**
Prices	-	**99.07%**	**0.93%**	**21.29%**	**66.67%**	**12.04%**
Inflation	**11.11%**	**10.19%**	**78.7%**	**75%**	**2.78%**	**22.22%**

Table 3.1

Thus, non-stationarity can be inferred for 99.07% of the price time series under the assumption of inflation being a normally distributed process. Considering inflation as a stable Paretian distributed process this can be concluded for 66.67% of the price time series. However, nothing can be inferred for 12.04% as the *ARMK* test is applicable only to those processes, for which the index of stability $\alpha \in (1, 2)$.[12] On the other hand, the results of the *ARMK* test with a constant reveal stationarity in a vast majority (75%) of the inflationary time series, which is confirmed by the linear unit root tests for 11.11% of the series only. Non-stationarity is confirmed for 10.19% using jointly the ADF_{max} and the *KPSS* tests while only 2.78% are confirmed as non-stationary if the *ARMK* test is applied. Finally, for the majority of the inflationary time series, 78.7%, the joint testing is weak in

[12] If the index of stability $\alpha = 2$ the stable Paretian distribution coincides with the normal distribution and therefore the linear unit root and stationarity tests are applicable. If $\alpha \leq 1$ nothing can be inferred employing the tests considered in this work.

determining if inflationary data are stationary or non-stationary, while the *ARMK* test cannot identify it for 22.22% of them. In conclusion, the assumption of inflationary processes being stable Paretian distributed processes leads to a better specification of the nature of inflationary processes. Moreover, concerning the recent disputes in the literature (*see* Chapter 1, Section 3) regarding stationarity and non-stationarity of inflationary processes, the *ARMK* test results strongly endorse the viewpoint of those who have considered inflation as a stationary process.

6. 3 A Bilinear Unit Root Test: Analysis of Price and Inflationary Data

The present section discusses the results obtained after the application of the bilinear unit root test to price and inflationary data. Testing for presence of bilinearity consists of a two-step procedure as suggested by Charemza *et al.* (2002b) (*see* also Chapter 2, Section 8.2). In summary, the first step involves application of one or more linear unit root tests and if the linear unit root is confirmed, the analysis proceeds with the application of the so-called *b*-tests. In our empirical work the first step consists of joint confirmation of the linear unit root applying the ADF_{max} and the *KPSS* tests. This has already been explained in Section 6.1 of the present chapter. For those price and inflationary time series for which integration of order one has been confirmed, the analysis proceeds with the application of the augmented *b*-test. Let us consider the hypothesis tested starting with the form of the regression estimated, that is:

$$y_t = \mu + y_{t-1} + b y_{t-1} e_{t-1} + \sum_{j=1}^{n} \phi_j \, \Delta y_{t-n} + e_t \,,$$

where μ is an intercept, n is the number of augmentations, ϕ_j are the regression coefficients and $e_t \sim iid\ N(0, \sigma_e^2)$. Subsequently, the null hypothesis of no bilinearity

$$\mathrm{H_0}: b = 0$$

is tested against the alternative of bilinearity

$$\mathrm{H_1}: b > 0 .$$

The augmented *b*-test is applied first, to price data and second, to inflationary data. Price time series are tested assuming non-linearity of price processes and non-normality of inflationary processes, i.e.:

$$\Delta p_t = bp_{t-1}e_{t-1} + e_t, \quad e_t \sim iid\ N(0, \sigma_e^2).$$[13]

Subsequently, inflationary time series are tested assuming first, non-linearity of inflation, i.e. inflation follows a bilinear process of the form:

$$\Delta p_t = \Delta p_{t-1} + \bar{b}\Delta p_{t-1}\bar{e}_{t-1} + \bar{e}_t, \quad \bar{e}_t \sim iid\ N(0, \sigma_{\bar{e}}^2),$$

and second (by analogue to the case of prices), non-normality of first difference of inflation, i.e.:

$$\Delta\Delta p_t = \bar{b}\Delta p_{t-1}\bar{e}_{t-1} + \bar{e}_t.$$

In short, the bilinear unit root test is conditional on the confirmation of a linear unit root applying jointly ADF_{max} and the *KPSS* tests (*see* Chapter 2, Section 8). The tests results of the joint testing first for prices and second for inflation have been presented in Section 6.1 of the present chapter. A summary of the results for prices shows that for all the countries except Vietnam both tests confirm the unit root hypothesis. Thus, excepting Vietnam, the *b*-test can be applied to all the price series. Subsequently, the ADF_{max} and the *KPSS* tests have been jointly applied to inflationary data and the results reveal that the hypothesis of unit root in inflationary data is confirmed for 12 out of 107 time series only (Vietnam is excluded from the analysis). For those 12 inflationary series the analysis proceeds with a test for the presence of a bilinear unit root.

The results of the *b*-test for price and inflationary data are presented in Tables C3.4a-d and Table C3.7 (Appendix C) respectively. Both sets of tables are organised in the following way: the countries names are listed in columns (1), columns (2) tabulate the *t*-statistics; columns (3) indicate whether the statistic obtained is significant. The notations used are the same as in Section 6.1. The maximum significant length of augmentations in the *b*-test is displayed in columns (4).

The results obtained reveal that the null hypothesis of no bilinearity in prices has to be rejected as follows:

- **Developed Countries:** 61.9% of the countries, among which are Denmark, Japan and the US

[13] A more detailed explanation has been presented in Chapter 2, Section 3 of the present thesis.

- **Developing Countries**
  - **Central and Eastern European Countries:** 76.92% of the countries, among which are Bulgaria, Hungary and Romania
  - **Other Developing Countries:** 69.64% of the countries, among which are Argentina, Cyprus and Mexico
- **Least Developing Countries:** 72.22% of the countries, among which are Chad, Malawi and Sierra Leone

It is evident from the summary that bilinearity is present for the majority of the countries disregarding their development status. Moreover, the results in percentages for the Least developing countries and the Developing countries, especially those for the CEEC group, are much higher compare to the percentage result obtained for the Developed countries.

Bilinearity is identified, however, at different levels of significance. A summary of the results (in terms of percentages of the total number of countries in a group) follows:

- **Developed Countries**

  0.01 level of significance: 42.86%
  0.05 level of significance 14.29%
  0.1 level of significance 4.76%
- **Developing Countries**
  - **Central and Eastern European Countries**

    0.01 level of significance: 69.23%
    0.05 level of significance 7.69%
  - **Other Developing Countries**

    0.01 level of significance: 46.43%
    0.05 level of significance 17.86%
    0.1 level of significance 5.36%
- **Least Developing Countries**

  0.01 level of significance: 50.00%
  0.05 level of significance 16.67%
  0.1 level of significance 5.56%

Further, the analysis proceeds with investigating the presence of bilinearity for inflationary processes. The bilinear unit root test can be applied to those inflationary series for which non-stationarity has been jointly confirmed from the ADF_{max} and the *KPSS* tests. The names of those countries have been listed in Section 6.1. Subsequently, the null hypothesis of no bilinearity in inflation has been rejected for the following 10 countries:

- **Developed Countries:** Iceland
- **Developing Countries**
  - **Central and Eastern European Countries:** Czech Republic, FE Germany, Russia and Slovenia
  - **Other Developing Countries:** Chile, Colombia, South Africa and Uruguay
- **Least Developing Countries:** Uganda

as the levels of significance vary among countries (*see* Table C3.7, Appendix C).

A more general summary of the results shows presence of bilinearity in 71.03% of the price series and in 9.35% of the inflationary time series.

For the purposes of this analysis the set of programs 'Blini' (*see* Charemza and Makarova (2002)) together with variations of some of the programs has been used. The programs are written using the GAUSS programming language.

7 Conclusion

Based on recent findings in the literature regarding the normality of distribution of inflation, Chapter 2 of the present section introduced the idea of modelling inflationary processes relaxing both assumptions of normality of distribution of inflation and linearity of price processes. It has emphasized the non-normal nature of inflation and has considered the theoretical background of the ways proposed for modelling it. The present chapter discussed the empirical results in relation to the methods discussed in Chapter 2. The analysis is based on a large selection of world-wide price data for 107 different countries.

Initially, the discussion concentrated on examining the distribution of inflationary processes. The results reveal substantial deviation of the distribution of inflation from the normal distribution. Inflation is distinguished by high levels of skewness and kurtosis and substantial tail thickness.

Another issue of interest of this chapter is in investigating the stationarity of inflationary processes. Price and inflationary data have been tested for a unit root using conceptually different techniques. The analysis started with the joint application of: a linear unit root test (the ADF_{max} test) and stationarity test (the *KPSS* test). Both tests confirm non-stationarity in prices for all the countries but Vietnam. After applying these tests, however, it is difficult to infer whether inflationary processes are stationary or non-stationary. The results reveal that 10.19% of the inflationary time series are stationary and 11.11% are non-stationary. For the rest 78.7%, however, nothing can be concluded.

Further, the analysis has proceeded with a unit root testing under the assumptions that inflation follows a stable Paretian law if prices are tested and, first difference of inflation follows a stable Paretian law if inflation is tested. For those purposes the *ARMK* tests (with and without a constant) have been used. Both tests are applicable to symmetric data and depend on the index of stability α, which is identical for each time series. The method used for calculating it is the McCulloch (1986) quantile estimator. Although our data are clearly asymmetric the *ARMK* tests have been applied. Examination of price time series reveal non-stationarity in prices if the *ARMK* test without a constant is applied. Non-stationarity in prices can be inferred for 66.67% of the time series if the *ARMK* test with a constant is applied. Only for those countries for which prices are non-stationary processes, the analysis proceeds with testing for a unit root in inflationary processes. The results reveal that applying the *ARMK* test with a constant, stationarity can be established for 75% of all the inflationary time series, which strongly supports the standpoint of those researchers who have seen inflation as a stationary process.

Finally, the analysis ends with testing for a bilinear unit root in prices and inflation. This has been done using the two-step testing procedure proposed by Charemza *et al.* (2002b). If non-stationarity has been detected after the joint application of the ADF_{max} and the *KPSS* tests, the *b*-test has been applied. The test results reveal the presence of bilinearity in 71.03% of the price time series and in 9.35% of the inflationary time series.

In conclusion, the empirical analysis clearly shows that the often-assumed normality of the distribution of inflation should be seriously questioned. A plausible assumption when modelling these processes is inflation being a stable Paretian distributed process. Under this assumption a unit root test (the *ARMK* test) has been applied and the results reveal that most of the price time series are non-stationary. Further investigation of those time series by applying the *ARMK* test to the corresponding inflationary processes reveals that for a vast majority, 75%, of the inflationary time series in the data set considered, stationarity can be clearly inferred. The percentages results showing non-stationarity in inflationary data (and applying the *ARMK* test) are more impressive. Non-stationarity of inflation has been concluded for less than 3% of the time series. Thus, in the context of the debates in the literature regarding stationarity and non-stationarity of inflation, the results of the empirical study of a large selection of world-wide inflationary data strongly supports the standpoint of those researchers who have regarded inflation as a stationary process. On the other hand, the view of prices as non-linear processes, and consequently inflation as a non-normally distributed process, leads to the conclusion of the presence of bilinearity in the majority of the price processes considered (71.03%), which is in favour of the idea of the non-linear nature of price processes.

CHAPTER FOUR *KALMAN FILTER ESTIMATION OF UNIT ROOT BILINEAR INFLATIONARY PROCESSES*

1 Introduction

Chapter 2 of the present thesis introduced the view of price processes as non-linear processes. More specifically, prices have been considered as described by unit root bilinear processes. Subsequently, under this assumption of price behaviour, Chapter 3, Section 6.3 presented the empirical results of the two-step testing procedure for presence of bilinearity (*see* Charemza *et al.* (2002b)) applied to a large set of world-wide inflationary data. In summary, the results reveal the presence of bilinearity in the majority of the price processes considered, that is 71.03%. These results strongly support the view of price processes as non-linear or, more precisely, unit root bilinear processes. In addition, the presence of bilinearity has been established in 9.35% of the inflationary time series only.

The next step in our analysis is the estimation of the bilinear coefficient for those price and inflationary series for which presence of bilinearity has been detected. Under the null hypothesis of no bilinearity Charemza *et al.* (2002b) evaluate the bilinear parameter using the *OLS* method, which also might be a way of estimating the coefficient of bilinearity. On the other hand, as the bilinear processes posses non-linear and non-stationary nature a suitable way of evaluating the coefficients of interest is the Kalman filter method of estimation. Before proceeding with the application of this method of estimation to unit root bilinear processes, let us, for convenience, remind the definition of a unit root bilinear process. In short, the unit root bilinear processes belong to the class of non-linear models and they are defined by an equality of the following form:[1]

$$y_t = y_{t-1} + by_{t-1}\varepsilon_{t-1} + \varepsilon_t, \tag{4.1}$$

where $b \neq 0$ and $\varepsilon_t \sim iid\ N(0,\sigma^2)$ for $t = 1, 2,\ldots, T$.

According to Charemza *et al.* (2002b) if $b \approx 0$ equality (4.1) can be seen as $\Delta y_t \approx \varepsilon_t$. Substituting $\Delta y_t = \varepsilon_t$ into (4.1) gives

$$\Delta y_t = by_{t-1}\Delta y_{t-1} + \varepsilon_t \tag{4.1a}$$

and hence, the *OLS* method is applicable for estimation of the bilinear coefficient. A limitation of this method of estimation, however, is the assumption that the innovations depend on the price observations only. However, the unit root bilinear process (4.1) is an iterative process with a specific non-linear and non-stationary[2] structure. It follows from (4.1) that $\varepsilon_t = f(\varepsilon_{t-1})$ that is the disturbance ε_t at time t is a function of the disturbance ε_{t-1} at the time t-1. Consequently, $\varepsilon_{t-1} = f(\varepsilon_{t-2})$, $\varepsilon_{t-2} = f(\varepsilon_{t-3})$,..., $\varepsilon_1 = f(\varepsilon_0)$, where ε_0 is an initial value. Hence, the error term ε_t of the process (4.1) is described by a recursive equation. An often-used technique for estimating processes possessing such a structure is the Kalman filter method. In brief, the Kalman filter method allows us to generate recursively optimal, non-linear forecasts of the disturbances $\hat{\varepsilon}_1, \hat{\varepsilon}_2,\ldots,\hat{\varepsilon}_T$ based on the observations $y_1, y_2,\ldots, y_T$ and an initial value y_0. Therefore, this method should lead to higher precision in the estimated results. Abreast with its applicability to non-linear processes, two other features that are prerequisites for the Kalman filter use in our study are: its applicability to non-stationary processes and the exact maximum likelihood estimation.

[1] In this chapter the variable y_t is presumed to describe a price process and is associated with logarithm of prices (i.e. CPI or RPI) at time t. Price processes have been denoted by p_t in Chapter 2 and Chapter 3. However, in this chapter the symbol p stands for the elements of a covariance matrix, as this is a usual notation when the Kalman filter method is used.

[2] Following Granger and Andersen (1978) the stationarity condition for processes of the form $y_t = ay_{t-1} + b\varepsilon_{t-1}y_{t-1} + \varepsilon_t$ is $a^2 + b^2\sigma_e^2 < 1$. The processes of interest of this study are the last mentioned adding the condition $a = 1$, i.e. $y_t = y_{t-1} + b\varepsilon_{t-1}y_{t-1} + \varepsilon_t$ and they are, therefore, non-stationary processes.

The Kalman fiter method has been introduced by Kalman (1960). Based on the theory of conditional distributions and expectations, the Kalman's original paper presents digital filters for non-stationary processes and their application in the sphere of engineering, solving problems of practical importance. However, Kalman's work has also attracted the economists' attention. Although initially it appeared to be of a little use in the area of economics, Schweppe (1965) focused the economists' interests on the Kalman filter applicability for the purposes of maximum likelihood estimation, which is of particular interest to the present work.

In general, the Kalman filter algorithm is a recursive procedure applicable to dynamic systems presented initially in a state – space form. The development of the system over time is presented by a possibly unobserved vector α_t, known as a state vector, and described by a first-order vector autoregressive Markov-Chain process. However, as the α's might not be observed directly, their properties are specified based on knowledge of the observations $y_1, y_2, \ldots, y_T$ and by means of a state equation. Thus, the state-space form of the dynamic process y incorporates a *state vector* [3] α_t, $t = 1,\ldots,T$ representing the development of the system under study, and a *state equation*[4] depicting the relationship between the observed values y_t, $t = 1,\ldots,T$ and the α's. Subsequently, applying the Kalman filter to the state-space form updates the state vector as new observations become available, followed by forecasts of the state vector elements. Thus it allows us to obtain the best state vector estimates at any point within the sample.

Once a model is presented in a state-space form, applying the Kalman Filter enables one to obtain simultaneously estimates for both model parameters and the unobserved elements. As mentioned earlier, the main objective of this chapter is estimation of the bilinear coefficient b in a unit root bilinear process. The non-linear structure of these processes results in state-space presentations with time varying coefficient matrices[5] that is matrices with elements dependent on the observations up to time t-1. Initially the unit root bilinear process is written into a state-space form. Subsequently, applying the Kalman filter allows the exact calculation of the likelihood function, one of the parameters of which is the

[3] The state vector is also known in the literature as transition equation.

[4] The state equation is also known in the literature as observation or measurement equation.

[5] In general, the state-space form coefficients are matrices, which could be time invariant or time varying.

coefficient of bilinearity b. Next, maximizing the likelihood function leads to estimation of this parameter.

In short, Chapter 4 presents a way of unit root bilinear parameter estimation by means of the Kalman filter. It consists of four sections of which the present introduction is the first. Section 2 provides a brief overview of the Kalman filter application in the area of economics. Further, the issue of bilinear coefficient estimation by the use of the Kalman filter is discussed in Section 3 as this section contains three main subsections. The first of them, Section 3.1, outlines the state-space form used in this work. Next, Section 3.2 states the Kalman filter derivation for the relevant time varying state-space structure. Further, Section 3.3 illustrates the Kalman filter estimation of the unknown bilinear parameter using the maximum likelihood method and considers the finite sample properties of the estimator. The former requires an appropriate numerical optimisation method for maximizing the log-likelihood function. Abreast with the traditional optimisation techniques, a recently proposed algorithm called Simulated Annealing algorithm is also used in the present study. The main idea behind this optimisation method is presented in Section 3.3.1. Subsequently, a brief theoretical description of the Monte Carlo simulation method is exposed in Section 3.3.2. The Monte Carlo simulations are then employed first, for evaluating the finite sample properties of the Kalman filter estimator (using the Simulated Annealing algorithm) and second, for calculating the critical values of the Kalman filter estimates t-test statistics (using standard numerical optimisation technique). The empirical results obtained are presented in Section 3.3.3. Finally, Section 4 concludes.

2 An Overview of the Kalman Filter

Although widely used in engineering and applied statistics, the Kalman filter is a convenient and successfully used tool in the applied economics area. Before applying the Kalman filter to the model of interest, the model has to be inverted into a state-space form, and thus converted to a fully dynamic framework, providing a wide variety of possible applications. State-space models can be univariate or multivariate models and the latter include more than one dependent variable (*see* Hamilton (1994), Engle and Watson (1987), Lawson (1984), Burmeister and Wall (1982), etc.). However, as our underlying bilinear process is based on inflationary observations only, the study presented in this chapter concentrates on models with one dependent variable, i.e. the so-called univariate models.

On the other hand, state-space forms might be constant or time varying state-space forms[6]. The latter are typically used as tests for stability of a regression equation (*see* Garbade (1977), Laumas and Mehra (1976), Rauser and Laumas (1976), etc.). The Kalman filter has been successfully used in many areas of the applied economics: rational expectation models (*see* Engle and Watson (1987), Burmeister and Wall (1982), Cuthbertson (1988), etc.), regression estimation with ARMA errors (*see* Harvey and Phillips (1979)), signal extraction (*see* Pagan (1975)), seasonal adjustment (*see* Engle (1979), Burridge and Wallis (1984, 1990), Hausman and Watson (1985), etc.), forecast (*see* Engle and Watson (1987), Harvey (1984), Harvey and Todd (1983)), etc.

Among other applications, the Kalman filter method is often used as a technique for the estimation of the unknown parameters in the coefficient matrices of the likelihood function. Subsequently, one problem that has claimed attention is the problem of parameters identification.[7] Rothenberg (1971) defines locally and globally identified models and finds a necessary and sufficient condition for local identification. Two different approaches for checking local identification are presented in Hannan (1971) and Gevers and Wertz (1984), Wall (1987), respectively. Following Caines (1988), subject to certain conditions, two of which are identifiability of the model and indeterministic covariance-stationarity of the exogenous variables, the author shows that the maximum likelihood estimate of the unknown parameters is consistent and asymptotically normal. Assuming that the exogenous variables are non-stochastic, Pagan (1980) examines a class of models, presented in state-space form with coefficients varying over time, and applies the Kalman filter. Under certain conditions, the author shows that the maximum likelihood estimator of the unknown parameters is consistent and asymptotically normal[8, 9].

[6] Time varying parameter model was introduced into economics by Cooley and Prescott (1973) and Rosenberg (1972).

[7] Absence of identification occurs when more than one set of parameter values can give rise to identical value of the likelihood function.

[8] Pagan (1980), Theorem 4.

[9] *See* also Ghosh (1998), Ljung and Caines (1979).

3 Non-linear Estimation of a Unit Root Bilinear Process Parameter by the Use of the Kalman Filter

The main aim of this chapter is to introduce a way of unit root bilinear coefficient estimation. A traditional tool in the theory of parameter estimation is the likelihood function (*LF*), which for unit root bilinear processes can be formulated and calculated by a routine application of the Kalman filter. Before constructing the *LF* the underlying bilinear process has to be presented in a state-space form. However, this presentation is not unique: different state-space forms can characterise the same process. One of them, the most convenient from a researcher's viewpoint, is usually chosen for further work. Thus, the present section begins with the state-space form of the unit root bilinear process (4.1). After the Kalman filter derivation for this particular process is presented, the rest of the section continues discussing the unit root bilinear coefficient estimation by maximising the corresponding *LF*. Hamilton (1994) and Harvey (1989) describe in details the theory behind state-space models with constant parameter matrices. Following their notations, in this chapter the attention is concentrated on a class of non-linear processes (i.e. unit root bilinear processes) described by a state-space form containing a stochastically varying coefficient matrix.

3. 1 State-space Bilinear Form

In general, a state-space form of a price time series $\{y_t\}_{t=1}^{T}$ applies to multivariate time series models. However, since our attention is concentrated on price processes only, with no loss of generality we can assume that y_t is a single observation and, therefore, consider $\{y_t\}_{t=1}^{T}$ as a univariate time series. Consider now our underlying bilinear process (4.1):

$$y_t = y_{t-1} + b\varepsilon_{t-1}y_{t-1} + \varepsilon_t,$$

where $\varepsilon_t \sim iid\, N(0, \sigma^2)$, $t = 1, \ldots, T$. The state-space form of the dynamic process y consists of a system of two equations, namely: *state equation* and *state vector*. The former has a representation of the form:

$$y_t = Z_t + T_t\eta_t, \tag{4.2}$$

where y_t stands for the observed variable at time t, $Z_t = [y_{t-1}]$ and $T_t = [by_{t-1} \quad 1]$ are time-varying parameter matrices of dimension (1×1) and (2×1), respectively.[10] The price

[10] Note that the notation T denotes the sample size, while T_t is a matrix.

observation y_t is also related to an unobserved (2×1) vector η_t, known as a state vector, which is assumed to be described by process of the following form:

$$\eta_{t+1} = H\,\eta_t + \zeta_{t+1}, \tag{4.3}$$

where for the bilinear process (4.1) $\eta_t = \begin{bmatrix} \varepsilon_{t-1} \\ \varepsilon_t \end{bmatrix}$, $H = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ is a (2×2) constant parameter matrix and $\zeta_{t+1} = \begin{bmatrix} 0 \\ \varepsilon_{t+1} \end{bmatrix}$ denotes a (2×1) vector of serially uncorrelated disturbances with mean zero and (2×2) covariance matrix Q, that is:

$$E(\zeta_t \zeta_\tau') = \begin{Bmatrix} Q, & for \quad t = \tau \\ 0, & otherwise \end{Bmatrix} \tag{4.4}$$

where Q is calculated in the following way:

$$Q = E(\zeta_t \zeta_\tau') = E\left(\begin{bmatrix} 0 \\ \varepsilon_t \end{bmatrix}\begin{bmatrix} 0 & \varepsilon_t \end{bmatrix}\right) = E\begin{bmatrix} 0 & 0 \\ 0 & \varepsilon_t^2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & \sigma^2 \end{bmatrix}. \tag{4.5}$$

Summarising, one way of writing the unit root bilinear process (4.1) in a state-space bilinear form is as follow:

$$y_t = y_{t-1} + \begin{bmatrix} by_{t-1} & 1 \end{bmatrix}\begin{bmatrix} \varepsilon_{t-1} \\ \varepsilon_t \end{bmatrix} \qquad \textit{state equation}$$

$$\begin{bmatrix} \varepsilon_t \\ \varepsilon_{t+1} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \varepsilon_{t-1} \\ \varepsilon_t \end{bmatrix} + \begin{bmatrix} 0 \\ \varepsilon_{t+1} \end{bmatrix} \qquad \textit{state vector}$$

satisfying all the assumptions made above. The next section concentrates on Kalman filter derivation specified by (4.2) - (4.5).

Section 3 aims to present a way of estimating of the unknown parameter b using the Kalman filter method. In the state-space form stated above this parameter takes part of the (1,1) element of the time-varying coefficient matrix T_t.[11] The following subsection briefly considers the Kalman filter derivation for the unit root bilinear process presented in the state-space form described above under the assumption that the coefficient matrixes containing the unknown parameters b and σ^2, that is T_t, and Q, respectively, are given.

[11] A state-space representation for a given process is not unique. An alternative state-space form is stated later in this chapter (Section 3.2).

Then, the likelihood function can be constructed and the bilinear coefficient b together with the variance σ^2 can be obtained by means of the maximum likelihood method of estimation. Following the general approach of Hamilton's (1994) work the Kalman filter has been derived for the state-space bilinear form described by (4.2) - (4.5). The following section presents the main results of this derivation and an asymptotic property of the Kalman filter under the assumption of 'small' bilinearity.[12]

3. 2 The Kalman Filter Derivation

The Kalman filter algorithm is simply a recursive procedure, which generates optimal non-linear forecasts[13] of the state vector η_t, i.e. $\hat{\eta}_{1|0}, \hat{\eta}_{2|1}, \ldots, \hat{\eta}_{T|T-1}$ based on the information set $\Psi_t = \{y_t, y_{t-1}, \ldots, y_1, y_0\}$ available at time t. The derivation of the Kalman filter presented in this section is based on the assumptions that the disturbances and the initial state vector are normally distributed. The conditional distributions of the recursively-generated estimators are normal and, therefore, are completely specified by their mean and covariance matrices. Hereafter, $P_{t|t-1}$ denotes the (2×2) covariance matrix of the estimation error associated with each of these forecasts.

The derivation of the Kalman filter results in the consecutive application of the following four steps and the last three of them execute T-1 times:

step 1: initialisation of the state vector

step 2: optimal forecast of the next observation y_t based on the information available at time t

step 3: estimation of the state vector based on the information available at time t

step 4: one period ahead optimal forecast of the state vector

[12] Following Charemza *et al.* (2002b) the term 'small bilinearity ' is associated with coefficients b defined in the interval $\left(0, \frac{1}{\sqrt{T}}\right]$.

[13] If Z and T are also constant matrixes (H and Q are constants in the chosen bilinear state-space form), the forecasts of the state vector η_t are linear least square forecasts. It will be shown later that for time-varying parameter matrices the inference is non-linear function of y_{t-1}.

The recursion starts with estimation of the initial state vector $\hat{\eta}_{1|0}$, which is the forecast of η_1 based on no observations of y. The matrix H is a (2×2) constant parameter matrix with zero eigenvalues, that is the eigenvalues are inside the unit circle. Following Hamilton (1994) if the eaigenvalues are inside the unit circle, the Kalman filter iterations can be started with $\hat{\eta}_{1|0} = [0 \quad 0]'$ and covariance matrix:

$$P_{1|0} = E\left(\begin{bmatrix} \varepsilon_0 \\ \varepsilon_1 \end{bmatrix} \begin{bmatrix} \varepsilon_0 & \varepsilon_1 \end{bmatrix}\right) = E\left(\begin{bmatrix} \varepsilon_0^2 & \varepsilon_0\varepsilon_1 \\ \varepsilon_0\varepsilon_1 & \varepsilon_1^2 \end{bmatrix}\right) = \begin{bmatrix} \sigma^2 & 0 \\ 0 & \sigma^2 \end{bmatrix}.$$ [14]

Let us presume that $\hat{\eta}_{t|t-1}$ and $P_{t|t-1}$ are given for fixed t. This subsection describes the way of calculating $\hat{\eta}_{t+1|t}$ and $P_{t+1|t}$ starting with the forecast of the value y_t. The forecasted value of y_t, $\hat{y}_{t|t-1}$, is associated with the estimated expected value of y_t based on the information set $\{\Psi_{t-1}\}$ and is obtained using the state equation:

$$\hat{y}_{t|t-1} = Z_t + T_t\hat{\eta}_{t|t-1}$$

and the covariance matrix of the estimation error is $T_tP_{t|t-1}T_t'$.[15]

Then, the current value of η_t can be updated based on extended information set $\{y_t, \Psi_{t-1}\} = \{\Psi_t\}$. This updated value $\hat{\eta}_{t|t}$ is calculated as the conditional mean of η_t based on the information set $\{\Psi_t\}$ and using an updating formula.[16] It can be shown that for the state-space bilinear form this estimate has a representation of the form:

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[14] As mentioned above this state-space representation is not unique. Consider an alternative form describing the same underlying process (3.1):

$$y_t = \tilde{T}'\xi_t$$

$$\eta_{t+1} = H_t\eta_t + \zeta_{t+1},$$

where $H_t = \begin{bmatrix} 1 & by_t \\ 0 & 0 \end{bmatrix}$, $\tilde{T}' = [1 \quad 0]$, $\eta_t = \begin{bmatrix} y_t \\ \varepsilon_t \end{bmatrix}$, $\eta_{t+1} = \begin{bmatrix} \varepsilon_{t+1} \\ \varepsilon_{t+1} \end{bmatrix}$. The coefficient matrix H_t is a time varying matrix with eigenvalues one and zero. Therefore, the unitary eigenvalue lies on the unit circle thus, leading to difficulties in initialising the state vector η_t.

[15] It can be shown that $T_tP_{t|t-1}T_t' = b^2y_{t-1}^2p_t + \sigma^2$, where p_t is the (1,1) element of $P_{t|t-1}$.

[16] For the updating formula *see* Hamilton (1994), formula [4.5.30]. In our case this formula has the following form: $\hat{\eta}_{t|t} = \hat{\eta}_{t|t-1} + P_{t|t-1}T_t'(T_tP_{t|t-1}T_t')^{-1}(y_t - Z_t - T_t\hat{\eta}_{t|t-1})$.

$$\hat{\eta}_{t|t} = \begin{bmatrix} \hat{\varepsilon}_{t-1|t-1} \\ \hat{\varepsilon}_{t|t-1} \end{bmatrix} + \begin{bmatrix} by_{t-1}p_t \\ \sigma^2 \end{bmatrix} \left(b^2 y_{t-1}^2 p_t + \sigma^2\right)^{-1} \left(y_t - y_{t-1} - by_{t-1}\hat{\varepsilon}_{t-1|t-1} - \hat{\varepsilon}_{t|t-1}\right),$$

where p_t is the (1,1) element of $P_{t|t-1}$. Hence, $\hat{\eta}_{t|t}$ is a non-linear function of y_{t-1} with a covariance matrix

$$P_{t|t} = \begin{bmatrix} p_t & 0 \\ 0 & \sigma^2 \end{bmatrix} - \left(b^2 y_{t-1}^2 p_t + \sigma^2\right)^{-1} \begin{bmatrix} b^2 y_{t-1}^2 (p_t)^2 & by_{t-1}p_t\sigma^2 \\ by_{t-1}p_t\sigma^2 & \sigma^4 \end{bmatrix} \tag{4.6}$$

The last step in the Kalman filter derivation consists in forecasting η_{t+1} using the state vector equation (4.3):

$$\hat{\eta}_{t+1|t} = H\hat{\eta}_{t|t}$$

Thus, the estimated values of the state vector at time $t+1$ based on the information available at time t are:

$$\hat{\eta}_{t+1|t} = \begin{bmatrix} \hat{\varepsilon}_{t-1|t-1} \\ 0 \end{bmatrix} + \begin{bmatrix} \sigma^2 \\ 0 \end{bmatrix} \left(b^2 y_{t-1}^2 p_t + \sigma^2\right)^{-1} \left(y_t - y_{t-1} - by_{t-1}\hat{\varepsilon}_{t-1|t-1} - \hat{\varepsilon}_{t|t-1}\right)$$

Ones the forecast of the state vector is obtained, the procedure continues recursively, finding the optimal forecast of the next observation y_{t+1}.

Further, consider the covariance matrix $P_{t+1|t}$, associated with $\hat{\eta}_{t+1|t}$. It has a presentation of the form[17]:

$$P_{t+1|t} = \begin{bmatrix} p_{t|t}^{22} & 0 \\ 0 & \sigma^2 \end{bmatrix},$$

where $p_{t|t}^{ij}$ $i, j = 1,2$ is the (i, j) element of the covariance matrix $P_{t|t}$. However, $p_{t+1|t}^{11} = p_{t|t}^{22}$ is the mean square error of $\hat{\varepsilon}_{t|t}$ and let us denote this element by p_{t+1}, that is:

$$p_{t+1} = E\left(\varepsilon_t - \hat{\varepsilon}_{t|t}\right)^2 \tag{4.7}$$

As we are interested in the properties of the Kalman filter for the underlying bilinear process (4.1), consider $\lim_{t\to\infty} p_{t+1}$, where the element

$$p_{t+1} = \frac{b^2 y_{t-1}^2 p_t \sigma^2}{b^2 y_{t-1}^2 p_t + \sigma^2} \tag{4.8}$$

is obtained from formula (4.6). Starting the recursion with $p_1 = \sigma^2$ and applying formula (4.8) leads to the following result:

[17] *See* Hamilton (1994) p.380

$$p_{t+1} = \sigma^2 \frac{C_t b^{2t}}{1 + C_1 b^2 + C_2 b^4 + ... + C_t b^{2t}} \tag{4.9}$$

where $C_i = \prod_{t=1}^{i} y_{t-i}^2 \geq 0, i = 1,...,t$.

Following Charemza, Lifshits and Makarova (2002a), in the presence of 'small' bilinearity, that is $|b| < \frac{1}{\sqrt{T}}$, the underlying unit root bilinear process (4.1) has limited distribution, while, otherwise, explosion occurs. Thus first, the elements C_i , i = 1,..., t are limited and, second, as $\frac{1}{\sqrt{T}} < 1$ with no loss of generality the proceeding analysis can be reduced to the case $|b| < 1$. Consider now $\lim_{t \to \infty} p_{t+1}$. If b = 0 or C_t = 0, then p_{t+1} = 0. Otherwise, formula (4.9) can be viewed as:

$$p_{t+1} = \frac{\sigma^2}{\frac{1}{D_t b^{2t}} + \frac{1}{D_{t-1} b^{2(t-1)}} + ... + 1}$$

where $D_t = C_t$, $D_{t-j} = \frac{C_t}{C_j}$, $j = 1,...,t-1$ and thus,

$$\lim_{t \to \infty} p_{t+1} = \lim_{t \to \infty} \frac{\sigma^2}{\frac{1}{D_t b^{2t}} + \frac{1}{D_{t-1} b^{2(t-1)}} + ... + 1} = \lim_{t \to \infty} \frac{\sigma^2}{\sum_{t=1}^{\infty} \frac{1}{D_t b^{2t}}}.$$

The bilinear coefficient satisfies the inequality $|b| < \frac{1}{\sqrt{T}} < 1$. In this case the sequence $\sum_{t=1}^{\infty} \frac{1}{D_t b^{2t}}$ is divergent, i.e.:

$$\lim_{t \to \infty} \sum_{t=1}^{\infty} \frac{1}{D_t b^{2t}} = \infty,$$

and hence $\lim_{t \to \infty} p_{t+1} = 0$. Therefore, it follows from formula (4.7) that

$$\lim_{t \to \infty} p_{t+1} = \lim_{t \to \infty} E(\varepsilon_t - \hat{\varepsilon}_{t|t})^2 = 0,$$

that is $\hat{\varepsilon}_{t|t} \xrightarrow{p} \varepsilon_t$ as $t \to \infty$. In conclusion, given sufficient number of observations y_t, the disturbances $\hat{\varepsilon}_{t|t}$ of the unit root bilinear process (4.1) calculated by the Kalman filter converge to the true values ε_t.

3. 3 The Maximum Likelihood Estimation: Finite Sample Properties

In order to estimate the bilinear parameter b and the variance of the error term σ^2 in the matrices T_t and Q, the maximum likelihood method will be used. Denoting by θ the vector collecting these parameters, the log-likelihood function takes the following form:

$$\begin{aligned}\ln F(y;\theta) &= \sum_{t=1}^{T} \ln f(y_t \mid \Psi_{t-1}) = \\ &= -\frac{T^2}{2}\ln 2\pi - \frac{1}{2}\sum_{t=1}^{T}\ln\left|T_t P_{t|t-1} T_t'\right| - \\ &\quad -\frac{1}{2}\sum_{t=1}^{T}\left(y_t - Z_t - T_t\hat{\eta}_{t|t-1}\right)'\left(T_t P_{t|t-1} T_t'\right)^{-1}\left(y_t - Z_t - T_t\hat{\eta}_{t|t-1}\right)\end{aligned}$$

It can be maximised using standard numerical optimisation methods. However, their application requires determination of the starting parameter values. A way of initialising them is by first equating b's starting value, say b_0, to its *OLS* value calculated by formula (2.14) (*see* Chapter 2, Section 8.2). Second, given the values b_0 and the price time series $\{y_t\}_{t=1}^{T}$, the set of error terms $\{\varepsilon_t\}_{t=1}^{T}$ is found and its variance $\sigma_0^2 = \mathrm{var}(\varepsilon_t)$ is chosen for σ^2 starting value.

However, in contrast to the traditional numerical algorithms, a numerical optimisation technique independent of the starting values was introduced in the early 1980's. A brief expose of this method, called Simulated Annealing, is presented in the following section.

3. 3. 1 The Simulated Annealing Algorithm: Main Idea

Estimating the parameters of our model requires maximization of the log-likelihood function using numerical optimisation methods. However, some of the available algorithms (BFGS, DFP, NEWTON, BHHH, Polak-Ribiere Conjugate Gradient, etc.) often fail when trying to optimise a function. Routine problems experienced with these techniques are infinitely large parameter values, loop trough the same point over and again, slow convergence or lack of convergence, etc. The last problem seems to be the most frequent and the only way of possibly eliminating it is by trying different initial values. However, even if the algorithm eventually converges, the global maximum[18] is not guaranteed: the algorithm might stop at the first optimum encountered, or alternatively, might converge to a local rather than a global maximum. Moreover, some of the algorithms use evaluation of the derivatives of the likelihood function and thus are applicable to well-defined (smooth and continuous in the domain of interest) functions only.

The above-mentioned disadvantages of the standard numerical optimisation techniques are prerequisites for the use of the Simulated Annealing (SA) algorithm if the log-likelihood function is maximized. Initially presented as a technique for optimising functions defined in a discreet domain of interest (*see* Kirkpatric, Gelatt and Vecchi (1983)), the algorithm is later modified for continuous functions optimisation (*see* Corana, Marchesi, Martini and Ridella (1987)). Corana *et. al* (1987) show that SA overcomes the above disadvantages of the traditional methods finding the global optimum or a good, near-optimal local maximum. Next, Goffie, Ferrier and Rogers (1994) present an extension of the algorithm, checking that the global maximum is indeed achieved.

The SA algorithm outlines with fewer limitations then the traditional optimisation methods require (e.g. there is no need the function to be smooth or continues) and it is applicable to ill-conditioned functions with high number of local optima. In contrast with the conventional optimisation algorithms (moving uphill iteratively), the SA moves uphill and downhill, which leads to the following two features distinguishing this algorithm from the traditional optimisation techniques: first, the SA is independent from the initial values and second, it can escape from local maximum and proceed to find the global maximum.

Prior to explaining the essence of the SA algorithm let us first clarify the intuition behind the method. The principles of the SA can be compared with those of a physical process by which molten metal is cooled. If the process of cooling is slow (known as annealing), the metal passes gradually from high to low energy state, that is, to the global minimum energy state of the system. If, however, the metal is cooled rapidly, when fully cooled it

[18] Unless specified, the issue is discussed in terms of function maximisation as the same results hold for minimisation problems considering, however, the negative function.

might contain more energy than annealed metal, that is the system will be in a local minimum state with higher energy than the energy of a slowly cooled metal system.

Following Corana *et al.* (1987) and Goffie *et al.* (1994), let us briefly outline the main steps of the SA algorithm. The following summary demonstrates SA application if one is interested in finding the global maximum of a bounded function $L(x)$, where the vector $x \in R^n$ and n is the number of the variables. Initially, the user must set starting values as follows: vector of parameters x, temperature T and step length v. The value of the function $L(x)$ is then evaluated at the starting point x. Next, new point x_{new} is generated using formula of the form:

$$x^i_{new} = x^i + r\,v$$

where r is a uniformly distributed random number, $r \in [-1, 1]$ and i stands for the i^{th} coordinate direction, $i = 1,\ldots n$. Then, the functional value $L(x_j)$ is calculated. Based on the Metropolis criterion a decision of acceptance or rejection of the new point is taken, that is:

- if $L(x_{new}) \geq L(x)$, x_{new} is accepted, x is set to x_{new}, i.e. $x = x_{new}$ and the algorithm moves uphill
- if $L(x_{new}) < L(x)$, x_{new} is accepted with probability

$$P = e^{\frac{L(x_{new}) - L(x)}{T}} \tag{4.10}$$

If P is grater than a uniformly distributed random number P' from the interval [0,1], then x is set to x_{new}, i.e. $x = x_{new}$ and the algorithm moves downhill.

The procedure described has to be repeated N_s times[19] as this parameter is set by the user. Then, the step vector v is adjusted so that one-half of the total numbers of moves are accepted. The sequence of steps presented above has to be repeated N_T times[20] as each time the step vector length is updated. The temperature is then reduced using the following formula:

$$T_{new} = r_T T,$$

[19] N_s is a criterion for step vector adjustment.

[20] N_T is a criterion for temperature reduction set by the user.

where the parameter r_T lies in the interval [0,1]. Next, the algorithm presented above is repeated for the new temperature value, starting at the current optimum value x. It is clear from (4.10) that starting at the current maximum and at lower temperature decreases the number of downhill moves and, consequently, the length of the step vector v declines thus, concentrating the new search on the most promising area. The temperature is reduced every $N_s.N_T$ cycles of moves along every direction and after N_T step adjustments, that is, till it is low enough, such that no useful improvement can be expected from further temperature diminishing[21].

However, when using the SA algorithm, one has to choose the initial parameter values controlling it carefully. Starting values suggestions for those parameters are presented in the 1987 article of Corana *et al.* Detailed discussion related to this issue is covered in the work of Goffie *et al.* (1994), also proposing a useful way of determining the initial temperature T.

Both the Simulated Annealing algorithm and the standard numerical optimisation algorithms possess advantages and disadvantages. In the present work SA is used if we require higher precision, and the traditional numerical optimisation techniques if speed is regarded.

3. 3. 2 Principles of the Monte Carlo Methodology

Up to this point a way of estimating the unknown bilinear coefficient $b \neq 0$ in the unit root bilinear process (4.1) has been presented. Further aspects of interest are the properties of this estimator for a sample with a finite number of observations. In classical statistical inference estimator's properties are discussed in terms of their sampling distributions properties. Subsection 3.3.3 of this chapter concentrates on two finite sample criteria for assessing a single parameter: bias and Root Mean Squared Error (*RMSE*). In short, an estimator is said to be unbiased for parameter b if the mean of the sampling distribution equals b, and biased otherwise. In the latter case, the difference between b and the mean is known as bias. Although unbiasedness is a desirable property, there are biased estimators with smaller variances then the unbiased ones. As large variances may lead to estimators,

[21] The parameter ε is a small number (e.g. 10E(-M), where M is an integer number larger than 1) set by the

which are far from the true values, another property we would like our estimator to posses is a small variance. One criterion that incorporates both bias and variance is the *RMSE*. A method that allows one to examine the finite sample properties of the estimator described is the Monte Carlo simulation method, known also as resampling method, i.e. based on drawing repeated samples.

The Monte Carlo method is a numerical algorithm, providing approximate solutions to a variety of mathematical, physical, statistical, etc. problems, simulating uniformly and independently distributed random numbers and using computer sampling experiments. Substituting analytically complicated problem with equivalent stochastic, the Monte Carlo simulation method provides numerical solution for the latter. Thus, this approach can be efficiently used for solving differential and integral equations, numerical evaluation of high-dimensional integrals (*see* Fishman (1996)) and is often used in the applied statistics and econometrics area for comparison between estimators or test statistics (*see* Hendry (1984)[22], Kleijnen and Groenendaal (1992), Charemza, Lifshits and Makarova (2002b)). The Monte Carlo method is described by Hendry (1984) as an experiment that can "efficiently complement analysis to establish numerical-analytical formulae which jointly summarise the experimental findings and known analytical results in order to help interpret empirical evidence and to compute outcomes at other points within the relevant parameter space".

A fundamental issue of numerical computations is evaluating an integral of a function over a bounded region in, say, s-dimensional Euclidean space. In general, the Monte Carlo method leads to computations of expectations. The example presented bellow demonstrates that every integral can be presented as an expected value. Thus, reducing the analytical problem of integral evaluation to the stochastic one of expected value calculation allows the evaluation of integrals, which are analytically difficult to solve. Consider as an example[23] the evaluation of a s-dimensional integral:

$$\int_{D^s} g(z)\, dz = F_{D^s} \tag{4.11}$$

user. For more details about the stopping criterion *see* Corana *et. al* (1987).

[22] Hendry (1984) presents a detailed review of Monte Carlo application in the area of econometrics.

[23] This example is taken from Hendry (1984)

and notice that its calculation is equivalent to the calculation of an integral of the following form:

$$\int_{D^s}\left[\frac{g(z)}{p(z)}\right]p(z)\,dz \tag{4.12}$$

Let us consider a random variable x defined in the s-dimensional region D^s with probability density function $p(x)$ and let us set $f(x) = g(x)/p(x)$. Consequently, formula (4.12) can be seen as the mean of $f(x)$, i.e.

$$\int_{D^s} f(x)p(x)\,dx = E(f(x)).$$

Therefore, the initial deterministic problem of calculating (4.11) reduces to the stochastic one of estimating the mean $E(f(x))$.

Let us denote the s-dimensional unit cube by Q^s. With no loss of generality we can assume that $D^s \subseteq Q^s$, that is if the region D^s is of an arbitrary size applying a suitable transformation D^s can be mapped into Q^s. Although different methods exist for the evaluation of integral of the form (4.12) (e.g. quadrature formulas, equidistributed series, etc.[24]) the Monte Carlo method distinguishes with first, its applicability to bounded and unbounded functions, which satisfy the condition $\int_{Q^s} f^2(x)dx < \infty$, and second, the error of the algorithm does not depend on the continuity and variational properties of $f(x)$.

Following Fishman (1996), consider a sequence $X^{(1)}, X^{(2)}, ..., X^{(n)}$ of independent random points uniformly distributed on D^s. Then the estimator of F_{Q^s} based on a sample of size n, namely

$$\bar{F}^n_{Q^s} = \frac{1}{n}\sum_{i=1}^{n} f(X^{(i)})$$

is an unbiased estimator with standard error $\sqrt{\frac{1}{n}\left(\int_{Q^s} f^2(x)dx - F^2_{Q^s}\right)}$, i.e. assuming that $\int_{Q^s} f^2(x)dx < \infty$, this error is $O(n^{-1/2})$. Denote by σ^2 the variance of $f(x)$, i.e.

$$\sigma^2 = \int_{Q^s} f^2(x)dx - F^2_{Q^s}.$$

[24] *See* Fishman (1996) for the advantages and disadvantages of both methods.

The standard Monte Carlo approach estimates $F_{Q'}$ with variance $\operatorname{var}\overline{F}^{n}_{Q'} = \sigma^2 / n$. For the purposes of estimating $\operatorname{var}\overline{F}^{n}_{Q'}$ the following trivial formula is used:

$$\hat{\sigma}^2_n = \frac{1}{n-1}\left(\sum_{i=1}^{n} f^2\left(X^{(i)}\right) - n(\overline{F}^{n}_{Q'})^2\right)$$

where $\hat{\sigma}^2_n$ is a strongly consistent and unbiased estimator of σ^2. Under the condition $\int_{Q'} f^4(x)dx < \infty$, the following result holds:

$$\frac{(\overline{F}^{n}_{Q'} - F_{Q'})}{\sqrt{\hat{\sigma}^2_n / n}} \xrightarrow[n\to\infty]{} N(0,1).$$

Thus, based on the Central Limit Theorem, the approximation error decreases with the increase of the sample size. However, two potential sources of errors play an important role: first, the replacement of the unknown variance σ^2 with $\hat{\sigma}^2_n$ introduces a sampling error and second, if the correlation $corr(\overline{F}^{n}_{Q'}, \hat{\sigma}^2_n) > 0$, that is if $\overline{F}^{n}_{Q'}$ is much smaller than the true value $F_{Q'}$, then the estimated variance $\hat{\sigma}^2_n$ will be much smaller than the true σ^2. Consequently, for fixed sample size n the corresponding confidence interval will be shorter than the interval justified by the theory.

Every Monte Carlo experiment is based on the assumption of availability of a random source of points. In practice, however, sequences of numbers called pseudorandom are used instead, i.e. regarded as indistinguishable from sequences of truly random numbers. They are uniformly and independently distributed[25] over the interval $[0,1]$[26], simulated by means of generating algorithm, and are later converted to the distribution of interest. Pseudorandom numbers, denoted below by w_i, $j = 1,\ldots,l$, are simulated using formulas of the form:

$$w_j = r_j / q, \quad w_j \in [0,1], \quad j = 1,\ldots,l$$

$$r_{j+1} = p\, r_j (\operatorname{mod} q),$$

and they are exactly reproducible as the length of the period l depends on the choice of p and q. Therefore, the sequences of pseudorandom numbers[27] repeat themselves after a

[25] *See* Fishman (1996) and Hendry (1984) for more details about pseudorandom number generation.

[26] The interval [0,1] corresponds to the 2-dimentional unit cube.

[27] Called pseudorandom, since the generating algorithm and the "seed" r_0 are known in advance.

finite number of steps l and thus, sampling without limit does not lead to diminishing of the approximation error. Consequently, the accuracy obtained by the use of Monte Carlo simulations is limited and depends on the goodness of the generating algorithm. For example, rapid generating algorithms produce sequences whose properties depart from the desirable properties of drawing independent samples, as the error increases with the increase of the dimension s.[28]

Abreast with its application in various areas the Monte Carlo approach is often used in the applied statistics and econometrics for comparison between estimators or test statistics. An objective of the present chapter is using this technique for studying econometric model, namely unit root bilinear processes. Hereafter, the discussion of Monte Carlo simulations is concentrated on its econometric application and within the frames of the analysis undertaken. Initially, precise analysis of the econometric model should be done and then, using all the information available, the model should be framed in a Monte Carlo experiment. The former involves defining the class of processes to be investigated, called the Data Generation Process (*DGP*), assuming that it is fully known to the researcher. Once the *DGP* is specified, the numerical solution of the problem can be obtained by means of the Monte Carlo method. The method is based on drawing repeated samples of size n from the desired distribution, and using the *DGP* formulas data of size n are generated. Using those data the parameter estimates can be calculated. Subsequently, repeating this procedure a number of times, say m, m parameter estimates are obtained and their distribution gives the sampling distribution of the parameter estimates for samples of size n. In the following subsection the discussion is focused on the finite sample properties of the Kalman filter estimator conducting Monte Carlo simulations.

3. 3. 3 Properties of the Kalman Filter Estimator

The use of Monte Carlo simulations for the purposes of our analysis aims at testing the bilinear coefficient $\hat{b}$, which is estimated by means of the Kalman filter algorithm. Further in this section, the Monte Carlo method in discussed regarding the unit root bilinear process (4.1), which, for convenience, is stated once again:

$$y_t = y_{t-1} + b\varepsilon_{t-1}y_{t-1} + \varepsilon_t ,$$

where $b \neq 0$ and $\varepsilon_t \sim iid\, N(0,\sigma^2)$, $t = 1, 2,\dots,T$.

[28] *See* Fishman (1996), Chapter 7.

Since $\hat{b}$ is independent of σ^2, with no loss of generality we can set $\sigma^2 = 1$ and draw a series of size *n*, $\varepsilon_t^{(1)}$, t = 1, 2,...,T of normally distributed random numbers with mean zero and variance one, i.e. $\varepsilon_t^{(1)} \sim iid\, N(0,1)$. Assuming that $y_0^{(1)} = \varepsilon_0^{(1)} = 0$ and fixing the bilinear parameter b to b^*, a series $y_t^{(1)}$, t = 1, 2,...,T can be generated using *DGP* incorporating the unit root bilinear process:

$$y_t^{(1)} = y_{t-1}^{(1)} + b^* y_{t-1}^{(1)} \varepsilon_{t-1}^{(1)} + \varepsilon_t^{(1)}$$

with the assumptions made above.

Next, applying the Kalman filter to the generated sample values $y_t^{(1)}$, t = 1, 2,...,T, the bilinear coefficient can be estimated. Let us denote its value with $\hat{b}^{(1)}$. To reduce the noise, this procedure is repeated m times and as a result a series of m values $\hat{b}$ are obtained, namely $\hat{b}^{(1)}, \hat{b}^{(2)}, ..., \hat{b}^{(m)}$. The distribution of the estimates can be then analysed by standard descriptive statistics as for the purposes of our analysis the *BIAS* and the *RMSE* are calculated using the following formulas:

$$BIAS\left(\hat{b}\right) = \frac{1}{m}\sum_{j=1}^{m}\hat{b}^{(j)} - b^* = \bar{\hat{b}} - b^*$$

and

$$RMSE\left(\hat{b}\right) = \sqrt{\left(\frac{1}{m}\sum_{j=1}^{m}\left(\hat{b}^{(j)} - b^*\right)\right)} = var\left(\hat{b}\right) + \left(bias\left(\hat{b}\right)\right)^2$$

where $\bar{\hat{b}}$ stands for the average of $\hat{b}^{(1)}, \hat{b}^{(2)}, ..., \hat{b}^{(m)}$.

In order to evaluate the finite sample properties of the Kalman filter method of estimation Monte Carlo simulation experiments are used. The choice of the tested b's values grounds on the theoretical findings of Charemza, Lifshits and Makarova (2002a), namely: in presence of the so-called 'small' bilinearity the coefficient of interest lies in the interval (0, $T^{-1/2}$), where T denotes the sample size. Thus, the values of the bilinear parameter b are initially set to 0.009, 0.020112 and 0.02846 for samples of size 100, 500 and 1000, respectively. For each sample size the simulations are repeated 100 times and the empirical results obtained are reported in Table 4.1 bellow. The results reveal that the $\mathbf{RMSE_{KF}}$ (and consequently the variance of the estimator) is gradually and considerably decreasing as the sample size increases. On the other hand, the values calculated for BIAS show that for samples with size of 100 $\mathbf{BIAS_{KF}}$ is only 0.03% and for samples with size 500 and 1000 they are of similar magnitude being equal to 0.07% in absolute values.

	Monte Carlo Simulations on unit root bilinear coefficient estimation by means of the Kalman filter method without a constant		
Sample size	**Bilinear Coefficient**	$\mathbf{BIAS_{KF}}$	$\mathbf{RMSE_{KF}}$
100	**0.009**	**0.0003700**	**0.024720**
500	**0.020112**	**-0.0007371**	**0.006493**
1000	**0.02846**	**-0.0007295**	**0.004772**

Table 4.1

The same Monte Carlo experiments are also conducted for the finite sample properties evaluation of the *OLS* method of estimation. The corresponding results are presented in Table 4.2 bellow revealing that with the increase of the sample size the $\mathbf{BIAS_{OLS}}$ of the estimator in absolute value is considerably increasing and of a higher magnitude compared to $\mathbf{BIAS_{KF}}$. On the other hand, the $\mathbf{RMSE_{OLS}}$ is smaller than the $\mathbf{RMSE_{KF}}$ for samples of size 100 and of a similar magnitude for samples of size 500. For samples of size 1000 $\mathbf{RMSE_{OLS}}$ is slightly increasing, while $\mathbf{RMSE_{KF}}$ considerably decreases.

	Monte Carlo Simulations on unit root bilinear coefficient estimation by means of the *OLS* method without a constant		
Sample size	**Bilinear Coefficient**	$\mathbf{BIAS_{OLS}}$	$\mathbf{RMSE_{OLS}}$
100	**0.009**	**-0.001231**	**0.019899**
500	**0.020112**	**-0.002196**	**0.006449**
1000	**0.02846**	**-0.004723**	**0.006524**

Table 4.2

In short, the results of both Monte Carlo simulation experiments show that the $\mathbf{RMSE_{KF}}$ (and respectively the variance) gradually decreases with the increase of the sample size and, $\mathbf{BIAS_{KF}}$ is considerably smaller than $\mathbf{BIAS_{OLS}}$ in absolute value. Thus, we can infer that

Kalman filter method is a technique suitable to apply to samples of a large size, when the class of unit root bilinear processes is considered.

Up to this point the present section has outlined the results of the Monte Carlo experiments, which have been used for evaluating the finite sample properties of the Kalman filter method of estimation. For those purposes the log-likelihood function has been maximized applying the Simulated Annealing algorithm, which, as explained in Section 3.3, is independent of the starting values.[29] The programs used in our empirical work are written in GAUSS (for Windows) programming language, version 3.5. As mentioned earlier, SA is computationally expensive algorithm compare to the traditional numerical optimisation methods. For the calculations presented in this work a computer with CPU running at 1.9GHz has been used and the following results in seconds have been obtained: the effective computing time is 8354s, 40380s and 82359s for 100 repetitions and samples of size 100, 500 and 1000 respectively.

Based on the finite sample properties of the Kalman filter estimator, this technique seems to be a plausible way of estimating the bilinear coefficients in unit root bilinear price and inflationary processes. It has been explained earlier (*see* Chapter 2, Section 8.2 and Charemza *et al.* (2002b), Theorem 1) that under the null hypothesis $b = 0$ the asymptotic distribution of the t-test statistics based on the *OLS* estimates of b in regression models of the form (4.1) weakly converges to the normal distribution as the sample size tends to infinity. The same result has been proven for unit root bilinear regression models with a constant (*see* Charemza *et al.* (2002b), Theorem 1). However, the distribution of the t-statistics based on the Kalman filter method estimates for unit root bilinear processes (with and without a constant) is not known. On the other hand, the number of observations in a particular time series is finite. Consequently, the need of evaluating the finite sample t-statistics critical values for the Kalman filter estimates arises. Monte Carlo simulation experiments have been conducted for those purposes. Initially, 10,000 random walk processes of length n have been generated and for each of them the bilinear coefficient $\hat{b}_{KF}$ (estimated by means of the Kalman filter) and its t-statistic have been calculated. The parameter n has been set to 150, 250 and 500 and the results obtained for levels of

[29] The GAUSS program of the SA algorithm used in this work has been provided by Adriana Agapie and has been written by E.G. Tsionas.

significance 0.01, 0.05 and 0.1 are presented in Table 4.3 bellow. As the calculation of *t*-statistic requires a large number of repetitions (i.e. 10,000 in the present study), the SA algorithm is not convenient since it is computationally time consuming. Instead, the standard numerical optimisation techniques are used with initial conditions calculated as explained in Section 3.3.

Sample size	The Kalman filter *t*-statistics for an estimated regression with a constant			The Kalman filter *t*-statistics for an estimated regression without a constant		
	0.01	0.05	0.1	0.01	0.05	0.1
150	2.6858	1.8066	1.3828	2.5451	1.737	1.3527
250	2.4863	1.7170	1.3278	2.4152	1.7227	1.3343
500	2.3535	1.6478	1.2581	2.4116	1.7078	1.3037
∞ (Normal distribution)	2.3263	1.6449	1.2816	2.3263	1.6449	1.2816

Table 4.3

It is evident from Table 4.3 that the calculated *t*-statistic critical values diminish with the increase of the sample size, and those obtained for samples of size 500 approach the *t*-statistic critical values of the normal distribution. Hence, based on these empirical results we can infer that the distribution of the *t*-statistics based on the Kalman filter method estimates for unit root bilinear processes with and without a constant comes closer to the normal distribution with the increase of the sample size.

The last explained Monte Carlo experiment has been repeated for the *OLS* method. The relevant *t*-statistic critical values are listed in Table 4.4 bellow.

Sample size	The *OLS* *t*-statistics for an estimated regression with a constant			The *OLS* *t*-statistics for an estimated regression without a constant		
	0.01	0.05	0.1	0.01	0.05	0.1
150	2.3080	1.6437	1.2767	2.2795	1.6266	1.2787
250	2.2964	1.6226	1.2785	2.3135	1.6495	1.2837
500	2.3069	1.6506	1.2802	2.3309	1.6590	1.2896

Table 4.4

The t-statistic critical values calculated for the Kalman filter and the *OLS* methods of estimation are later used for testing the significance of the bilinear coefficients estimated for price and inflationary world-wide data. This issue is a main subject of the forthcoming chapter.

4 Conclusion

Abreast with the definition of unit root bilinear processes Chapter 2, Section 8.2 of the present thesis has discussed a two-step testing procedure for presence of bilinearity. If bilinearity is detected an issue of interest is finding a suitable tool for evaluating the bilinear coefficient. The present chapter has suggested the use of the Kalman filter method of estimation. Some of its features, e.g. applicability to non-linear, non-stationary processes, and the exact calculation of the maximum likelihood function make the algorithm suitable for estimating the unit root bilinear processes parameter.

The Kalman filter algorithm has been theoretically applied to unit root bilinear processes and it has been shown that under the assumption of 'small' bilinearity the estimated innovations of this process (i.e. the innovations calculated by the Kalman filter) converge to the true innovations as the time t tends to infinity. Next, the log-likelihood function has been constructed and one of the parameters of this function is the bilinear parameter. Abreast with the traditional numerical methods a non-trivial numerical optimisation technique, i.e. the Simulated Annealing algorithm, has been used for the estimation of this coefficient.

Further, in order to evaluate the properties of the Kalman filter estimator (*BIAS* and *RMSE*) a series of Monte Carlo simulation experiments has been conducted. The results of these experiments are compared to those obtained by means of the *OLS* method of estimation. They reveal that the $\mathbf{RMSE}_{KF}$ gradually decreases with the increase of the sample size, confirming consistency of the Kalman filter method, and the $\mathbf{BIAS}_{KF}$ is of much smaller magnitude than the $\mathbf{BIAS}_{OLS}$ in absolute value. In short, empirical investigation shows that the Kalman filter method of estimation is a suitable way of evaluating the bilinear coefficient in a unit root bilinear process.

Once the coefficient of bilinearity is estimated a subsequent issue of interest is testing whether or not it is significantly different from zero. The critical values for the Kalman filter and the *OLS* methods of estimation with and without constant have been tabulated using series of Monte Carlo experiments. They are used in the empirical work presented in the next chapter.

CHAPTER FIVE *KALMAN FILTER BILINEAR ESTIMATION: EMPIRICAL ANALISYS FOR WORL-WIDE INFLATIONARY DATA*

1 Introduction

Chapter 4 of this thesis has drawn the conclusion that the Kalman filter method seems to be a plausible way of estimating the bilinear coefficient in a unit root bilinear process. This has been inferred after the application of the Kalman filter method to data generated processes with desirable properties and the consequent evaluation of the finite sample properties of this estimator. A next step in our analysis is the application of the Kalman filter estimator to world-wide price and inflationary time series. The view of price processes as non-linear or, more specifically, unit root bilinear processes has been discussed in more detail in Chapter 2, Section 3 of this thesis. Subsequently, the work proceeded with an examination of world-wide price and inflationary time series for presence of a linear unit root. Confirmation of non-stationarity (applying jointly the *ADF* and *KPSS* tests in this thesis) is the first step of the two-step testing procedure for presence of bilinearity. The second step consists in testing for presence of bilinearity[1] and the way of testing it together with the results obtained for price and inflationary data has been discussed in Chapter 4, Section 6.3. The main objective of this chapter is the continuation of the analysis for price and inflationary data by evaluating the magnitude of bilinearity for those of them for which presence of bilinearity has been detected. Once the magnitude of bilinearity is specified, further aspect of interest is the establishment of possible

[1] Hereafter bilinearity and unit root bilinearity are used interchangeably.

relationship between the estimated parameters of both approaches considering distribution of inflation as a non-normal distribution that is between (a) the coefficient of bilinearity and (b) the index of stability, measuring the tail thickness of the distribution of inflation under the assumption that inflation follows a stable Paretian law. It seems also interesting to investigate a possible dependency between the magnitude of bilinearity (associated with non-normality), and the magnitude of some macroeconomic factors, namely GDP (indicating the development status of the countries) and average inflation.

In short, the present chapter is outlined as follows: Section 2 presents and discusses the results of the Kalman filter estimator applied to those price and inflationary time series of our data set, for which presence of bilinearity has been detected. The estimates of the bilinear coefficient are compared to those evaluated by means of the *OLS* estimator. Subsequently, Section 3 of this chapter focuses the attention on the establishment of an eventual relationship between unit root bilinearity in prices, and the indexes of stability. Furthermore, Section 3 examines a possible relationship between bilinearity and two macroeconomic indicators: average inflation and GDP. Finally, Section 4 concludes.

2 Estimation of Unit Root Bilinear Processes

Chapter 4 of this thesis has dwelt on two different ways of estimating the bilinear coefficient in a unit root bilinear process. Under certain conditions the *OLS* method of estimation is applicable (*see* Chapter 4, Section 1). A limitation of this technique is the assumption that the innovations of the unit root bilinear price processes depend on the price observations only, which would lead to lower precision of the estimated bilinear coefficients. Thus, the necessity of a more accurate technique for their calculation arises and such a technique seems to be the Kalman filter estimator (*see* Chapter 4). The Kalman filter method is an often-used technique when the processes of interest posses non-linear and non-stationary structure. Both non-linearity and non-stationarity are characteristics inherent to the unit root bilinear price processes, which processes are of particular interest to the present study.

It should be emphasized, however, that both the Klaman filter method and *OLS* method are applicable only to those price and inflationary time series of the data set, for which presence of bilinearity has been detected. Bearing this in mind, the main objective of the present section is to discuss the empirical results of the Kalman filter estimator (*see* Chapter 4, Section 3) applied to unit root bilinear price and inflationary processes. Secondly, the Kalman filter estimates of the bilinear coefficients are compared to those obtained by means of the *OLS* method. In both methods a constant is included in the regression equation.

The results obtained are presented in Tables D5.1a-d (Appendix D)[2] and they are organised in the following way: columns (1), as usual, list the names of the countries followed by the number of the price observations presented in columns (2). The sets of columns (3) – (6) and (7) - (10) display the results obtained by means of the *OLS* method and the Kalman filter method of estimation respectively. Columns (3) and (7) present the estimated bilinear values. As already explained the bilinear unit root in process of the form (5.1) is a stochastic unit root. Following Charemza *et al.* (2002a) the limit behaviour of the unit root bilinear model is well-defined for $b \in \left(0, \frac{1}{\sqrt{T}}\right]$ and under the assumption $y_0 = \varepsilon_0 = 0$. In addition, if b exceeds $\frac{1}{\sqrt{T}}$ the unit root bilinear process becomes explosive process. Multiplying the coefficient of bilinearity b by $\sqrt{T}$, where T is the sample size, maps the interval $\left(0, \frac{1}{\sqrt{T}}\right]$ into the interval $(0,1]$. This allows us to compare the parameters $\tilde{b} = \hat{b}\sqrt{T}$ with the unitary value, where $\hat{b}$ stands for the estimated bilinear values. The coefficients denoted by $\tilde{b}$ are called scaled bilinear coefficients and their values are tabulated in columns (4) and (8).[3] Next, columns (5) and (9) present the relevant *t*-statistics, and columns (6) and (10) indicate whether the estimated coefficients are significantly different from zero. As before, the notations '0', '+', '++' and '+++' in

[2] It has been explained in Chapter 2 that the countries are partitioned into four main groups: Developed Countries, Central and East European Countries, Other Developing Countries and Least Developing Countries. Hereafter, the letters "a", "b", "c" and "d" after the Table number stand for the countries groups as follows: "a" – Developed Countries, "b" – Central and Eastern European Countries, "c" – Other Developing Countries, "d" – Least Developing Countries.

[3] Column (4) presents the results obtained by the use of the *OLS* method, while column (8) displays those calculated by the Kalman filter method of estimation.

columns (6) and (10) stand for: no significance of the *t*-statistics or, the *t*-statistics belongs to the 90%, 95% or 99% critical region, respectively.

Initially, let us consider the results calculated for the price time series. Presence of bilinearity has been established in 71.03% (i.e. 81 out of 108) of the price time series (*see* Chapter 3, Section 6.3). For 12 of them the calculated *t*-statistic shows that the coefficients estimated by means of the Kalman filter method are not significantly different from zero. For 10 of the countries (Ethiopia, Hong Kong, Malta, Nepal, Philippines, Burundi, Guinea Bissau, Nepal, Niger and Sierra Leone) the coefficients obtained are not significantly different from zero for both methods of estimation. In the same context, there is lack of significance of the estimated coefficients for Peru and Russia, when the *OLS* method is applied and for Netherlands, Slovakia and Ivory Coast when the Kalman filter algorithm is used. For the majority of the countries the results of both the *OLS* method and the Kalman filter method reveal strong significance of the *t*-statistic, i.e. it belongs to the 99% critical region. For 21 of the time series the scaled bilinear coefficients estimated by means of the Kalman filter are significantly different from zero and smaller than one in absolute value, namely:

- **Developed Countries:** Austria and Luxembourg
- **Developing Countries**
  - ♦ **Central and Eastern European Countries:** Albania, Bulgaria, Czech Republic and FE Germany
  - ♦ **Other Developing Countries:** Bahamas, Bahrain, Barbados, Botswana, Chile, Cyprus, Peru and St Lucia
- **Least Developing Countries:** Burkina Faso, Chad, Gambia, Haiti, Malawi and Samoa

The above conclusion holds for 22 of the price processes when the *OLS* method is applied. The scaled bilinear coefficients are significantly different from zero and bigger than one in absolute value for 48 of the price time series if the OLS method is applied and for 46 of the price time series if the Kalman filter method is used. It has already been explained that under the initial condition $y_0 = \varepsilon_0 = 0$ the unit root bilinear processes are well-defined if the scaled bilinear coefficients are smaller than one in absolute value and explosion occurs otherwise. The last mentioned condition, however, is not applicable to real world data. This could be seen as one of the reasons leading to overestimation of the bilinear coefficients when the Kalman filter method and the *OLS* method are applied to price data. On the other hand, the asymptotic distribution of the estimated bilinear coefficients is known under the null hypothesis $b = 0$ only, and is not known under the alternative $b > 0$ (*see* Charemza *et al.* (2002a) and Chapter 2, Section 8). Hence although in practice the estimated scaled bilinear coefficients estimated by both methods are larger (in absolute value) than the theoretically found upper limit one, it cannot be inferred that they are insignificant. Further comparison between the results achieved by both the Kalman filter and the *OLS* methods of estimation (*see* columns (4) and (8) of Tables D5.1a-d (Appendix D)) reveals that for most of the price time series the bilinear coefficients obtained by the former method are of a smaller magnitude compared to those obtained by the *OLS* method.

Subsequently, let us consider the Kalman filter estimates of the bilinear coefficients obtained for the unit root bilinear inflationary processes. The results of the two-step testing procedure for presence of bilinearity in inflationary time series have been presented in Chapter 3, Section 6.3 and they reveal that 9.31% of them (i.e. 10 out of 108 time series) exhibit bilinearity. The Kalman filter and the *OLS* estimates of the bilinear coefficients obtained for inflationary data are presented in Table D5.2 (Appendix D). The scaled bilinear coefficient estimates obtained are of unrealistically higher magnitude compare to the unitary value, thus leading to conclusion of possible explosion in the processes considered or, perhaps, miss-specified bilinearity. Fig. 5.1 - 5.3 plot the graphs of three selected countries: Iceland, Former East Germany and South Africa. The inflationary graphs of Iceland and FE Germany distinguish with periods of high, explosive inflation followed by inflation stabilisation. A possible reason for the large magnitudes of the bilinear coefficients could be the existence of speculative bubbles in the level of prices, which leads to an exploding inflation (e.g. Fig. 5.2 - FE Germany: August 91, January 93).[4]On the other hand, the inflationary graphs of South Africa and Iceland characterise with excess variability and large up-and-down changes, similarly to the pattern of the unit

[4] Engsted (1993) examines the existence of rational bubbles in prices for Argentina, Israel and Brazil, and during hyperinflation periods in the first half of the 1980s. A weak evidence of a rational bubble has been found for Brazil only. According to the author, in the cases of Argentina and Israel the hyperinfletion is "merely monetary phenomena".

root bilinear processes. Nevertheless, presence of a structural break in September 1983 is graphically evident from Fig. 5.1.

Recent work of Charemza *et al.* (2002c) has shown that "a true bilinear unit root process might often be mistaken for the deterministic unit root process with a structural break". The authors suggest an encompassing test distinguishing bilinearity from structural breaks and apply this test to 66 stock market indexes. The results reveal that "the bilinear model encompasses the structural break model more often than the other way around" (Charemza *et al.* (2002c)). The presence of structural breaks in inflationary time series is, however, graphically evident for some of the countries for which bilinearity in inflationary time series has been detected. It seems therefore reasonable to apply the encompassing test proposed by Charemza *et al.* (2002c)[5] in order for further and more precisely to analyse the unit root bilinear inflationary processes. Applying it, however, is beyond the scope of the present work.

[5] In order to distinguish bilinearity from deterministic structural break the authors consider augmented unit root bilinear regression model with structural break:

$$\Delta y_t = \mu + b y_{t-1}\Delta y_{t-1} + \gamma BR_t + \sum_{i=1}^{k} \phi_i \Delta y_{t-i} + e_t,$$

where μ is an intercept, b denotes the bilinear coefficient, k stands for the number of augmentations, ϕ_i are regression coefficients, $e_t \sim iid\ N(0, \sigma_e^2)$ and the BR_t signifies the structural break variable. Subsequently, they propose testing consecutively the following two sets of joined hypothesis:

A) $H_0^A : b = 0 \ and \ \gamma = 0$

$H_1^A : b \neq 0 \ and/or \ \gamma \neq 0$

B) $H_0^B : b \neq 0 \ and \ \gamma = 0$

$H_1^B : b = 0 \ and \ \gamma \neq 0$

Therefore, the consecutive rejection of H_0^A and H_0^B will lead to the conclusion of existence of deterministic structural break, while rejection of H_0^A followed by non-rejection of H_0^B would mean existence of unit root bilinearity.

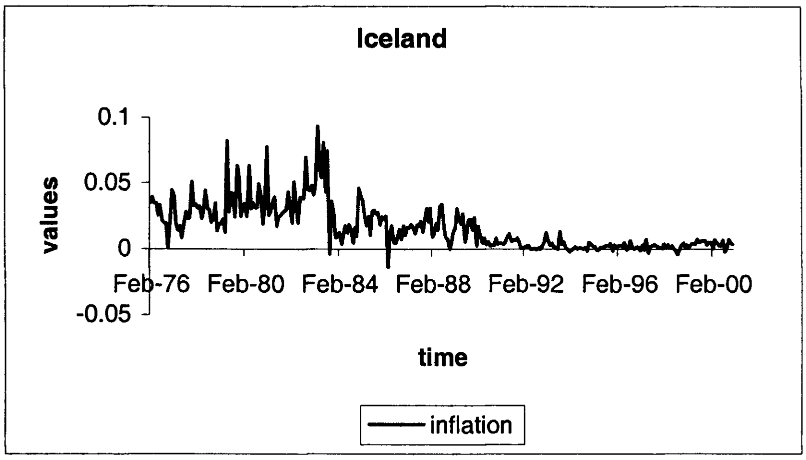

Fig 5.1

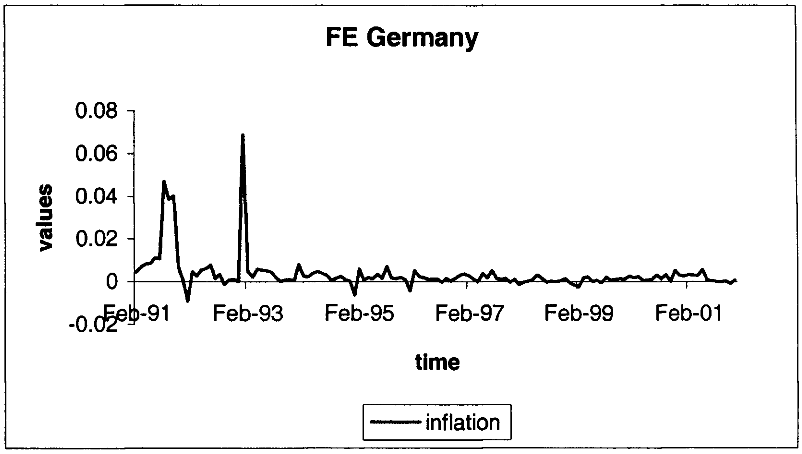

Fig. 5.2

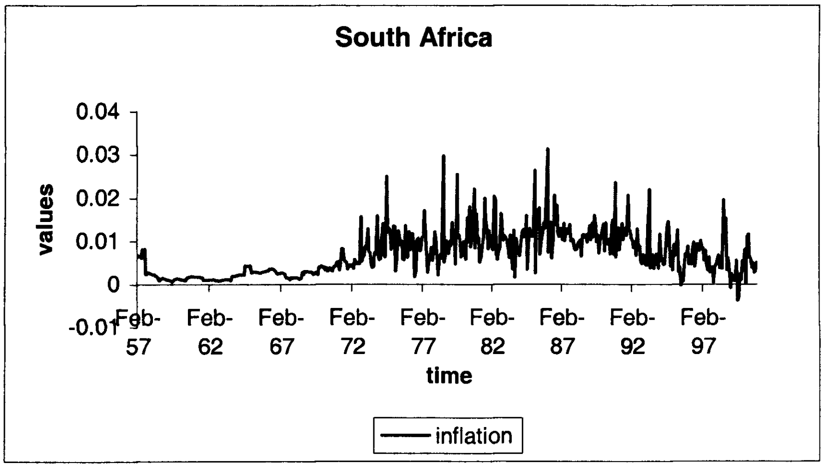

Fig. 5.3

3 Empirical Relationship Between the Parameters of a Stable Paretian Distribution and Bilinearity

Section 2 of this chapter has presented the empirical results obtained from the application of the Kalman filter method of estimation to world-wide unit root bilinear price and inflationary time series. On the other hand, Chapter 3, Section 5 discussed the estimates of the characteristic exponent α of the stable Paretian distribution obtained from the application of the McCulloch method of estimation to inflationary data. Both the bilinear coefficient and the index of stability are associated with measures of non-normality. The higher the magnitude of bilinearity the larger is the deviation of the distribution of inflation from the normal distribution. On the other hand, the distribution of inflation gets closer to the normal distribution with the increase of the index of stability. Hence, the expected relationship between both parameters is a negative dependency. The investigation of eventual dependency between the stable Paretian distribution estimates of the index of stability α of the distribution of inflation, and the estimated bilinear coefficients for price processes is a main objective of the present section

Prior to concentrate on this issue let us examine the dependency between the magnitude of bilinearity (associated with non-normality), and the magnitude of two macroeconomic indicators, that is average inflation and GDP. Consider first the relationship between the estimated bilinear coefficients in unit root bilinear price processes, and the mean values of those inflationary time series for which the coefficients of bilinearity have been estimated (Fig. 5.4). Similar discussion regarding the connection between the estimates of the indexes of stability α and the mean values of the inflationary time series has been presented in Chapter 3, Section 5. It has been inferred that regarding the group of the Developing countries (with few exceptions), the relationship between both these parameters is negative. Although weaker, similar, negative relationship between the estimated bilinear coefficients in prices and the mean values of inflationary processes can be concluded regarding the Developing countries, which is evident from the scatter diagram presented on Fig. 5.4. This, however, cannot be inferred for the groups of the Developed and the Least Developing countries.

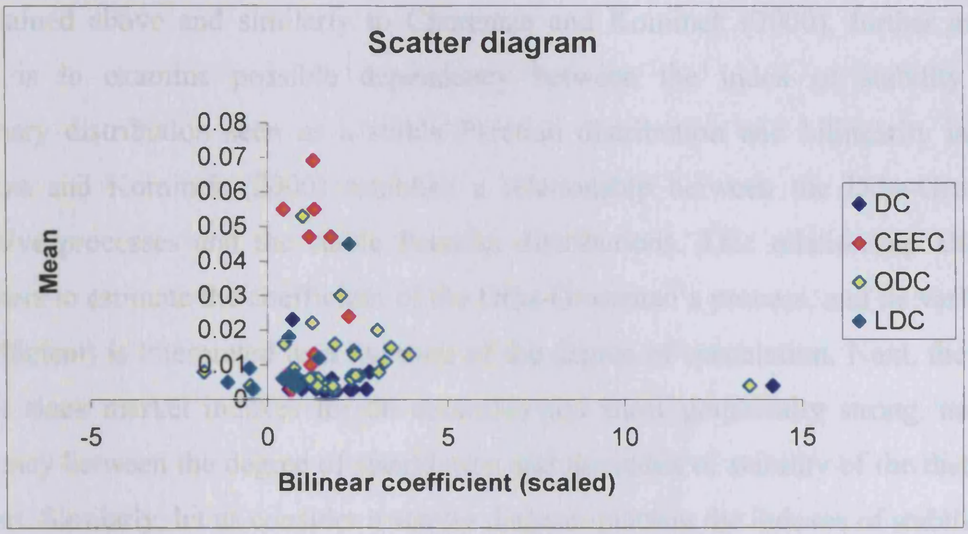

Fig. 5.4

In order to examine the relationship between the magnitude of the bilinear coefficient and the GDP indicators (regarding the development status of the countries smaller GDP values are associated with less developed countries) let us consider a scatter diagram of both sets of values (Fig. 5.5). Fig. 5.5 plots the scaled bilinear coefficients versus the per capita GDP for year 1998.[6] The diagram reveals positive relationship between the development status of the countries and the bilinear coefficient if the groups of the Least Developing Countries, the Central and Eastern European Countries and the Developed Countries are considered.

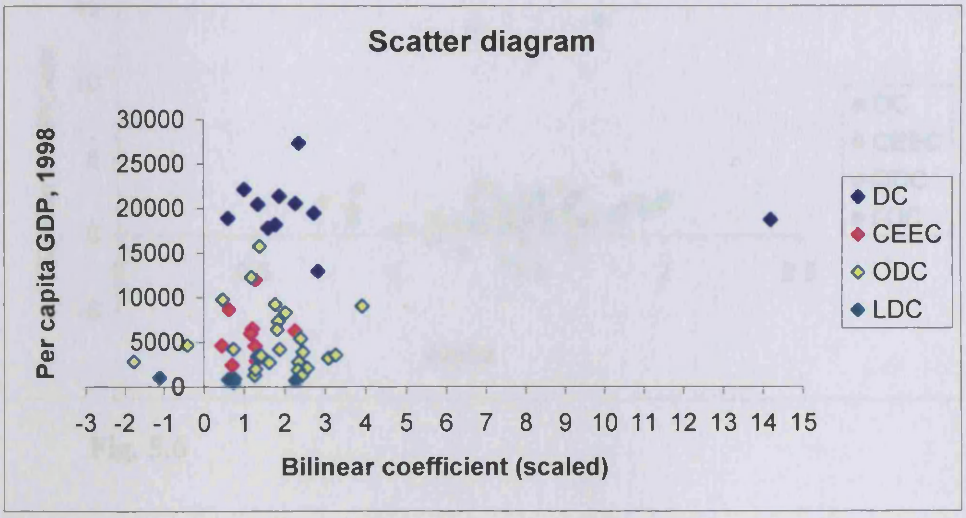

Fig. 5.5

[6] The per capita GDP data have been obtained from Maddison (2002). In addition, the scatter diagram (Fig. 5.5) presents those countries from our data set only for which the per capita GDP data are tabulated in Maddison (2002).

As explained above and similarly to Charemza and Kominek (2000), further aspect of interest is to examine possible dependency between the index of stability of the inflationary distribution seen as a stable Paretian distribution and bilinearity in prices. Charemza and Kominek (2000) establish a relationship between the Diba-Grossman's speculative processes and the stable Paretian distributions. This relationship allows the researchers to estimate the coefficient of the Diba-Grossman's process, and its variance (of the coefficient) is interpreted as a measure of the degree of speculation. Next, the authors consider stock market indexes for 66 countries and show graphically strong, non-linear dependency between the degree of speculation and the index of stability of the distribution in returns. Similarly, let us consider a scatter diagram plotting the indexes of stability α of the inflationary distribution versus the scaled bilinear coefficients evaluated for unit root bilinear price processes. This is presented on Fig. 5.6. As explained above the dependency expected between both parameters is a negative relationship. However, in contrast with the findings of Charemza and Kominek (2000), no relationship between bilinearity and the index of stability of the inflationary distribution, viewed as a stable Paretian distribution, can be graphically established.

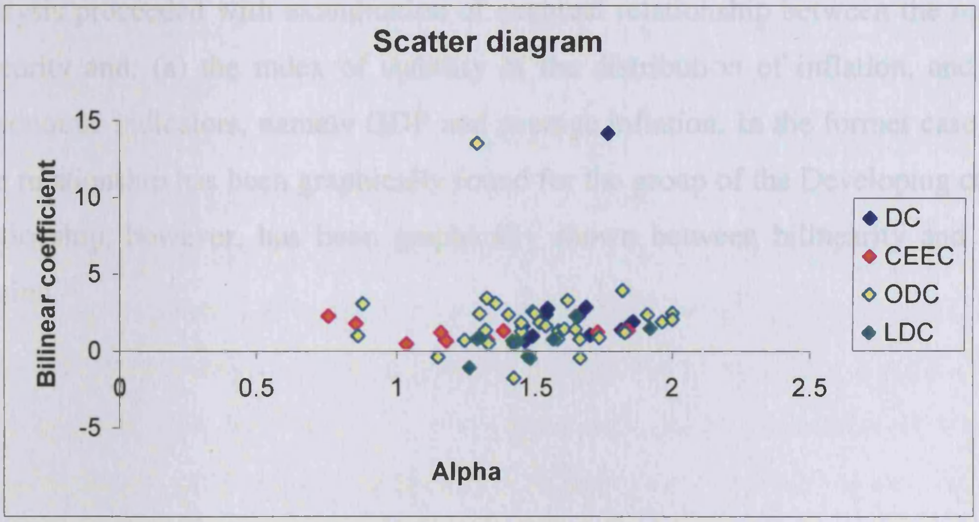

Fig. 5.6

4 Conclusion

Summarising, the present chapter has presented and discussed the results of the Kalman filter estimator applied to unit root bilinear price and inflationary processes, for which bilinearity has been detected. The Kalman filter estimates have been compared to those obtained by the use of the *OLS* method. The results reveal that the estimates of the bilinear coefficients in unit root bilinear price processes obtained by means of the Kalman filter method are of a smaller magnitude compared to those estimated by the use of the *OLS* method. For the majority of the price time series the estimates of the bilinear coefficients (applying both estimation techniques) significantly differ from zero and this significance is strong. For 21 of the price time series the scaled bilinear coefficients are significantly different from zero and smaller than one in absolute value, thus supporting the theoretical findings of Charemza *et al.* (2002a) regarding the magnitude of the scaled bilinear coefficients. Although for 46 of the price time series these estimates are bigger than one in absolute value, no inference regarding their insignificance can be done. In order to establish the significance of these coefficients further investigation regarding their asymptotic distribution under the alternative hypothesis of $b > 0$ should be done.

The analysis proceeded with examination of eventual relationship between the magnitude of bilinearity and: (a) the index of stability of the distribution of inflation, and (b) two macroeconomic indicators, namely GDP and average inflation. In the former case a weak, negative relationship has been graphically found for the group of the Developing countries. No relationship, however, has been graphically shown between bilinearity and external information.

CONCLUSIONS

One of the fundamental issues in modelling macroeconomic time series is to specify whether the series can be described by a stationary or non-stationary process. This thesis is tackling this issue in the context of modelling inflationary and price processes. Earlier attempts to answer the question 'Is inflation a stationary process?' have been made in a numeral empirical studies but none of them clarifies the nature of these processes. What these attempts have in common is the underlying assumption of normality of the inflationary data. One of the principle points of this thesis is to question this assumption. Distribution of price changes was examined for a large selection of world-wide inflationary series, and for a vast majority of them substantial deviation from the normal distribution was established. Consequently, the thesis hypothesises that non-normality is a prime feature for inflationary data. In this thesis inflation has been modelled under more realistic and general assumption that it distribution belongs to a broad family of distributions - stable Paretian distributions - from which the normal distribution is a special case. The series of statistical tests applied to the world wide data on inflation strongly support the hypothesised non-normality. Furthermore, an attempt was made to model the process generating prices, which underlines inflationary data. An often-made assumption leading to normality in distribution of inflation is that prices themselves follow a linear process. The present work relaxes the linearity assumption for price time series, and suggests the view of price processes as described by family of non-linear and non-stationary processes, that is the class of unit root bilinear processes. In this case, even if the underlying price process is constructed with the use of a normal distribution (which helps with the statistical analysis of the series), the distribution of price changes is not necessarily normal.

The idea behind the view regarding the distribution of inflation as a stable Paretian distribution relies on the empirically observed excess kurtosis, resulting in tails thicker than the tails of the normal distribution. Consequently, price and inflationary data have been then tested for stationarity applying a recently proposed unit root test under the assumption that the disturbances of these processes follow a stable Paretian law. The test is conditional on the estimates of the index of stability measuring the tail thickness of the distribution. In contrast to the classical unit root tests, which hardly determine whether inflation is a stationary or non-stationary process, the test results obtained for inflation strongly support the opinion of those researchers, who believe that inflation time series is integrated of order zero. Assuming that inflation follows a stable Paretian law 75% of the inflationary time series examined are classified as integrated of order zero. In contrast, under the assumption of normality of distribution of inflation, this can be inferred for only 11.11% of the inflationary time series.

Relaxing the classical assumption of non-linearity in price time series - which leads to possible non-normality in inflationary time series - prices were seen to follow a particular non-linear and non-stationary process, that is a unit root bilinear process. Under this assumption they were tested for the presence of unit root bilinearity by applying a new, recently proposed testing procedure. Testing for unit root bilinearity is conditional on confirmation of linear unit root in price time series. It has been shown in the present thesis that a vast majority of the price time series, 71.03%, exhibit unit root bilinearity. Analysing inflationary time series has shown the presence of bilinearity (and consequently, non-stationarity) in a minor part of the time series, that is 9.35%.

After establishing that the majority of price processes considered exhibit unit root bilinearity, question arises regarding the magnitude of the bilinearity. Consequently, the further work is on the estimation of the bilinear coefficient in unit root bilinear processes. Since the bilinear unit root process is, by definition, recursive the recursive Kalman filter algorithm was adopted for unit root bilinear processes, and it was theoretically demonstrated that under the assumption of 'small' bilinearity the disturbances of the unit root bilinear process converge to the true disturbances as the sample size tends to infinity. Evaluation of the finite sample properties of the Kalman filter estimator has shown considerably small magnitude of the estimated bias, and a root mean square error gradually decreasing with the increase of the sample size. Comparison with the finite sample results obtained for the *OLS* estimator revealed that the Kalman filter method of estimation is an appropriate technique applicable for estimating the bilinear parameter in unit root bilinear process, in particular those with a large number of observations.

In order to test whether or not the estimated bilinear coefficients are significantly different from zero the statistics suggested here is a simple t-statistic for the Kalman filter estimates. It has been shown (in a series of Monte Carlo experiments) that the distribution of this t-statistics converges to the normal distribution with the increase of the sample size. Therefore, for a finite sample, the critical values for this statistics have been calculated and subsequently used in further empirical analysis. A summary of the results for price time series (that is the analysis of 108 series) shows that the majority of the Kalman filter estimates of the bilinear coefficient in unit root bilinear price processes are of smaller magnitude compared to those obtained by the *OLS* estimator. Both estimation techniques reveal that most of the estimated bilinear coefficients are significantly different from zero and this significance is strong.

The present thesis has further examined an often disputable question in the recent literature that is whether inflationary time series are described by stationary or non-stationary process. Establishing stationarity of inflationary series is an issue of prime importance in modelling inflationary data and is closely related to the problem of inflationary forecast. A reliable inflationary forecast rests on stationary time series (i.e. rests on the so-called core inflation), which highlights the importance of correctly specifying the nature of inflationary processes regarding their stationarity.

The results presented in this thesis could be further developed in several directions. The work has shown the application of a recently proposed unit root test assuming that the distribution of inflation is a symmetric stable Paretian distribution. However, clear asymmetry regarding inflationary data has been empirically established. Transforming the data to symmetric and then testing for a unit root would lead to more reliable results. The present thesis has also shown that the estimated bilinear coefficients for unit root bilinear inflationary processes are of surprisingly high magnitude. As bilinearity could often be mistaken with structural breaks, the analysis of inflation could continue by further examining these processes applying a recently proposed set of tests, which allows one to distinguish presence of bilinearity from existence of structural breaks. The bilinear processes also posses an attractive structure in terms of applicability to time series forecast allowing further evolvement of the analysis regarding inflationary time series. On the other hand, the class of unit root bilinear processes is a relatively simple class of non-linear and non-stationary models within the family of bilinear models. In this context an extension of the analysis to broader class of bilinear processes could be another possibility for further research in the area. Finally, an attempt to find a relationship between first, unit root bilinearity and average inflation and second, unit root bilinearity and *GDP* has shown no relation of bilinearity to external information.

APPENDIX A

Definition of Multiresolution Analysis *(Malat 1989)*[1]

A *Mutliresolution analysis* of $L^2(R)$ (collection of all the real square-integrable functions) is a decomposition of the form:

$$L^2(R) = \ldots \oplus W_{-1} \oplus W_0 \oplus W_1 \oplus \ldots$$

such that $V_{j+1} = V_j \oplus W_j$, where $(V_j)_{j\in Z}$ is a sequence of closed linear subspaces of $L^2(R)$ such that:

(1) $V_j \subset V_{j+1}$, $\cap_{j\in Z} V_j = \varnothing$, $\cup_{j\in Z} V_j$ is dense in $L^2(R)$ (2)

$f(t) \in V_j \Leftrightarrow f(2t) \in V_{j+1}$

(3) $f(t) \in V_0 \Leftrightarrow f(t+1) \in V_0$

(4) $(\phi(x-k))_{k\in Z}$ is a basis in V_0

The function $\phi \in V_0$ is called scaling function for V_j. and it can be shown that if the ϕ has compact support such that:

$$\phi(t) = \sum_{k=0}^{n} c_k \phi(2t-k),$$

$\phi_{j,k}(t) = 2^{j/2}\phi(2^j t - k)$ is an orthonormal basis for V_j and $\psi_{j,k}(t) = 2^{j/2}\psi(2^j t - k)$ is an orthonormal basis for W_j.

[1] Malat's definition of Multiresolutional analysis is broadly discussed in Greenblatt (1994).

APPENDIX B

THE UNITED KINGDOM

ΔlnRPI		Error term	
ACF	PACF	ACF	PACF
0.94494	0.95219	0.33919	0.3392
0.86899	-0.31193	-0.40803	-0.59109
0.82976	0.61442	-0.28779	0.23067
0.81598	-0.17758	0.28697	0.20356
0.7826	-0.16169	0.39957	0.046097
0.72273	-0.044907	0.13124	0.26848
0.66068	-0.27171	0.012913	0.23681
0.60432	-0.26798	0.058562	0.047579
0.54774	-0.11103	0.051411	0.05222
0.49141	-0.1206	-0.0077308	-0.039021
0.4408	-0.031354	-0.0036676	-0.13633
0.39506	0.074034	0.016137	-0.18526
0.35237	0.14857	-0.0020334	-0.17201
0.31349	0.15545	-0.020351	-0.15794
0.27912	0.16034	-0.03668	-0.17301

Table B.1

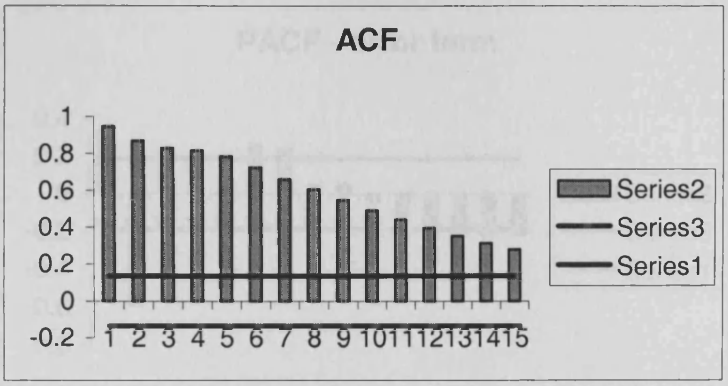
ACF
1
0.8
0.6
0.4
0.2
0
-0.2
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
Series2
Series3
Series1

Fig. B.1

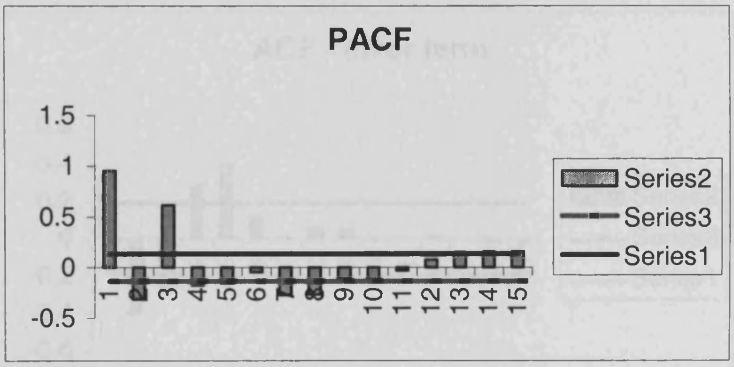
PACF
1.5
1
0.5
0
-0.5
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
Series2
Series3
Series1

Fig. B.2

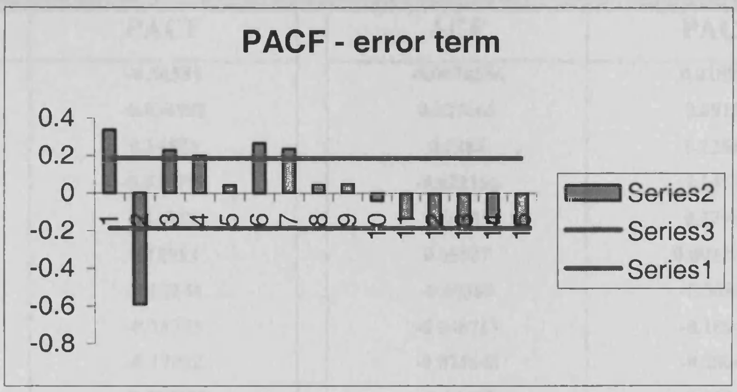
PACF - error term
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
Series2
Series3
Series1

Fig. B.3

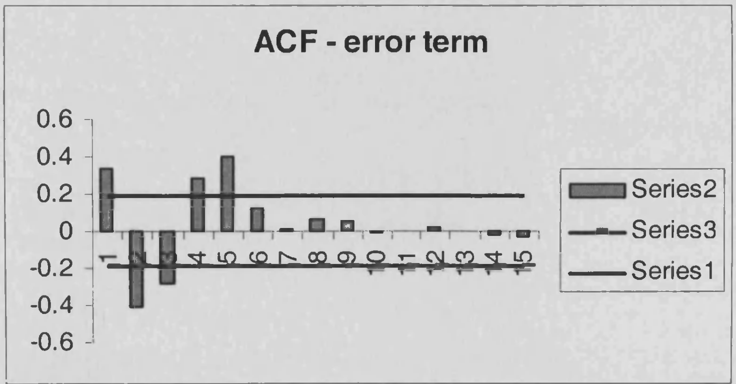
ACF - error term
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
Series2
Series3
Series1

Fig. B.4

POLAND

ΔlnCPI		Error term	
ACF	**PACF**	**ACF**	**PACF**
-0.3656	-0.36584	-0.0076556	0.018958
0.10863	-0.030902	0.027666	0.09154
0.098153	0.14873	0.1286	0.12688
-0.12055	-0.035797	-0.022164	0.14387
0.21124	0.16186	0.24252	0.27983
0.032053	0.18959	0.05537	0.0032724
-0.19163	-0.15243	-0.20369	-0.30815
0.061503	-0.15773	-0.048713	-0.18947
-0.13268	-0.17962	-0.074648	-0.28047
0.020565	-0.097895	-0.13477	-0.16577
-0.052596	-0.12821	-0.27242	-0.13811
-0.18045	-0.20426	-0.1495	-0.13402
-0.043657	-0.17172	-0.019097	-0.0054225
-0.018016	-0.030765	-0.068527	0.0060664

Table B.2

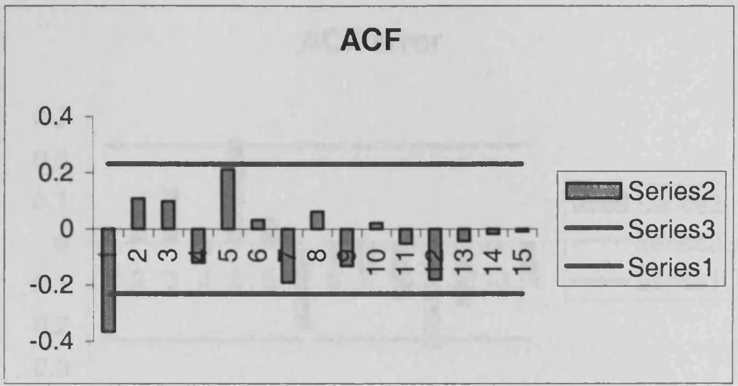
ACF
0.4
0.2
0
-0.2
-0.4
2
3
5
6
8
10
11
13
14
15
Series2
Series3
Series1

Fig. B.5

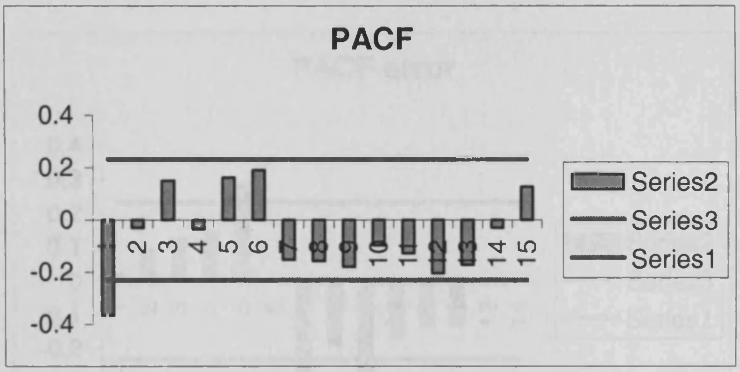
PACF
0.4
0.2
0
-0.2
-0.4
2
3
4
5
6
10
14
15
Series2
Series3
Series1

Fig. B.6

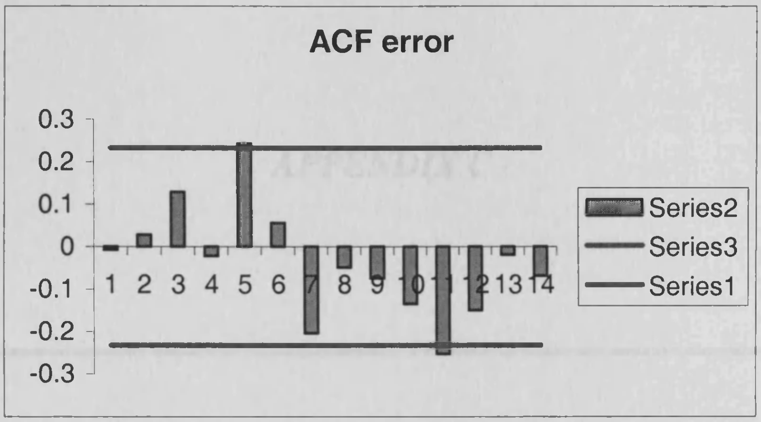
ACF error
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
1 2 3 4 5 6 7 8 9 10 11 12 13 14
Series2
Series3
Series1

Fig. B.7

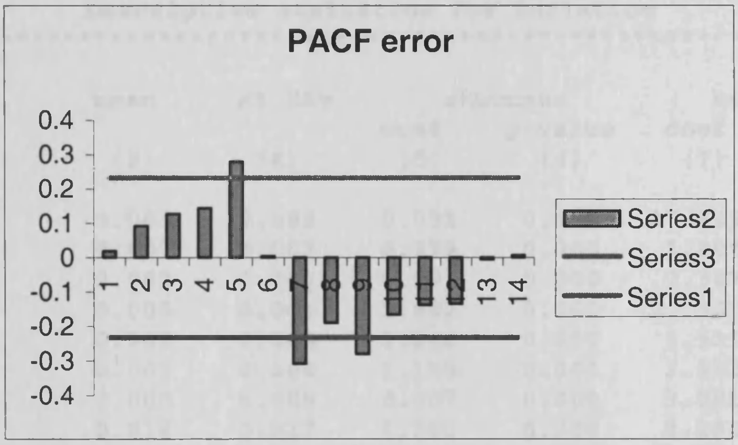
PACF error
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
1 2 3 4 5 6 7 8 9 10 11 12 13 14
Series2
Series3
Series1

Fig. B.8

APPENDIX C

Table C3.1a

****

Descriptive statistics for inflation

****

code	No.obs.	mean	st.dev	skewness		kurtosis	
				coef	p-value	coef	p-value
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Austri	528	0.003	0.005	0.931	0.000	17.695	0.000
Belgiu	528	0.003	0.003	0.779	0.000	1.804	0.000
Canada	612	0.003	0.004	0.991	0.000	2.387	0.000
Denmar	407	0.005	0.005	2.682	0.000	20.927	0.000
Finlan	528	0.005	0.005	1.246	0.000	5.531	0.000
France	528	0.005	0.004	1.198	0.000	2.982	0.000
Greece	526	0.008	0.009	0.807	0.000	3.081	0.000
Icelan	300	0.016	0.017	1.398	0.000	2.282	0.000
Irelan	385	0.006	0.006	1.115	0.000	0.562	0.024
Italy	600	0.006	0.005	1.350	0.000	3.076	0.000
Japan	526	0.003	0.006	1.631	0.000	5.453	0.000
Luxemb	527	0.003	0.004	0.503	0.000	2.812	0.000
Nether	526	0.003	0.005	0.681	0.000	13.589	0.000
Norway	528	0.004	0.004	0.952	0.000	4.622	0.000
Portug	528	0.008	0.010	2.136	0.000	12.521	0.000
Spain	528	0.007	0.012	6.111	0.000	112.741	0.000
Sweden	526	0.004	0.004	1.743	0.000	6.244	0.000
Switze	612	0.002	0.003	1.210	0.000	2.956	0.000
UK	540	0.005	0.005	2.057	0.000	10.174	0.000
US	612	0.003	0.003	1.201	0.000	2.257	0.000
German	600	0.002	0.003	1.616	0.000	9.555	0.000

Table C3.1b

****

Descriptive statistics for inflation

****

code	No.obs.	mean	st.dev	skewness		kurtosis	
				coef	p-value	coef	p-value
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Albani	131	0.023	0.047	3.992	0.000	23.383	0.000
Bulgar	132	0.055	0.134	6.065	0.000	43.473	0.000
CzechR	132	0.008	0.010	3.744	0.000	18.432	0.000
FEGerm	132	0.003	0.009	5.140	0.000	29.826	0.000
Estoni	120	0.026	0.060	6.143	0.000	45.578	0.000
Hungar	312	0.010	0.010	1.036	0.000	3.097	0.000
Latvia	120	0.024	0.056	4.089	0.000	18.703	0.000
Lithua	132	0.047	0.080	2.269	0.000	5.207	0.000
Poland	168	0.036	0.065	4.292	0.000	22.531	0.000
Romani	135	0.055	0.048	1.901	0.000	4.173	0.000
Russia	120	0.069	0.094	3.056	0.000	13.790	0.000
Slovak	131	0.009	0.010	3.915	0.000	19.174	0.000
Sloven	121	0.013	0.019	4.156	0.000	19.089	0.000

Table C3.1c

****

Descriptive statistics for inflation

****

code	No.obs.	mean	st.dev	skewness		kurtosis	
				coef	p-value	coef	p-value
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Argent	143	0.047	0.137	4.848	0.000	27.698	0.000
Aruba	177	0.003	0.002	1.807	0.000	4.827	0.000
Bahama	339	0.004	0.004	0.738	0.000	3.106	0.000
Bahrai	290	0.003	0.013	1.931	0.000	8.738	0.000
Barbad	420	0.006	0.011	0.931	0.000	10.758	0.000
Bolivi	197	0.037	0.115	5.363	0.000	35.695	0.000
Botswa	306	0.009	0.005	1.348	0.000	6.902	0.000
Camero	399	0.006	0.015	1.066	0.000	8.080	0.000
Chile	307	0.016	0.018	1.492	0.000	18.446	0.000
Colomb	528	0.015	0.019	9.558	0.000	117.106	0.000
CostRi	313	0.014	0.014	2.689	0.000	11.772	0.000
Cyprus	611	0.004	0.009	0.197	0.047	2.597	0.000
DominR	535	0.008	0.015	1.867	0.000	7.903	0.000
Ecuado	528	0.016	0.023	3.843	0.000	24.060	0.000
ElSalv	528	0.007	0.009	0.822	0.000	2.588	0.000
Fuji	384	0.006	0.010	6.061	0.000	70.666	0.000
Ghana	454	0.022	0.036	-0.539	0.000	20.432	0.000
Guatem	527	0.006	0.017	0.681	0.000	6.290	0.000
Hondur	527	0.007	0.009	0.867	0.000	2.105	0.000
HongKo	299	0.006	0.007	0.677	0.000	3.097	0.000
India	525	0.006	0.008	0.267	0.013	1.971	0.000
Indone	393	0.010	0.019	2.269	0.000	24.360	0.000
IvoryC	489	0.005	0.018	1.609	0.000	10.985	0.000
Jamaic	527	0.011	0.014	2.687	0.000	12.253	0.000
Jordan	299	0.005	0.015	1.633	0.000	8.895	0.000

Kenya	396	0.009	0.014	2.198	0.000	15.198	0.000
Korea	432	0.008	0.009	0.745	0.000	2.865	0.000
Malays	524	0.003	0.005	1.689	0.000	7.641	0.000
Malta	526	0.003	0.009	0.677	0.000	34.199	0.000
Maurit	462	0.006	0.012	3.685	0.000	32.520	0.000
Mexico	612	0.015	0.019	2.144	0.000	5.862	0.000
Morocc	525	0.004	0.007	0.323	0.003	2.298	0.000
Namibi	346	0.009	0.006	1.626	0.000	7.326	0.000
NethAn	400	0.004	0.005	1.632	0.000	5.390	0.000
Nigeri	498	0.013	0.024	0.517	0.000	32.526	0.000
Pakist	527	0.006	0.010	0.277	0.010	9.785	0.000
Paragu	526	0.010	0.014	2.859	0.000	20.295	0.000
Peru	137	0.053	0.148	7.456	0.000	68.449	0.000
Philip	526	0.008	0.039	-2.239	0.000	226.787	0.000
SaudAr	250	0.000	0.005	-1.350	0.000	38.572	0.000
Singap	480	0.002	0.007	2.464	0.000	11.607	0.000
SouthA	527	0.007	0.005	1.115	0.000	1.878	0.000
SriLan	526	0.006	0.010	1.633	0.000	10.125	0.000
StKitt	256	0.003	0.008	1.139	0.000	6.386	0.000
StLuci	429	0.005	0.010	1.320	0.000	5.942	0.000
Surina	372	0.020	0.040	3.936	0.000	20.351	0.000
Swazil	402	0.008	0.020	0.530	0.000	28.291	0.000
Taiwan	504	0.004	0.013	3.252	0.000	23.304	0.000
Thaila	432	0.004	0.006	1.609	0.000	5.111	0.000
Trinid	523	0.006	0.008	1.027	0.000	6.534	0.000
Tunisi	161	0.004	0.003	1.146	0.000	2.463	0.000
Turkey	261	0.038	0.018	3.094	0.000	22.126	0.000
Urugua	256	0.030	0.019	1.021	0.000	2.415	0.000
Venezu	526	0.013	0.019	3.573	0.000	22.756	0.000
Vietna	141	0.000	0.010	-0.235	0.257	8.809	0.000
Zimbab	264	0.015	0.015	1.377	0.000	2.889	0.000

Table C3.1d

****
Descriptive statistics for inflation
****

code	No.obs.	mean	st.dev	skewness		kurtosis	
				coef	p-value	coef	p-value
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
BurkFa	218	0.003	0.017	0.498	0.003	2.964	0.000
Burund	322	0.010	0.020	0.868	0.000	2.912	0.000
Chad	214	0.004	0.028	0.322	0.055	2.766	0.000
Ethiop	426	0.005	0.020	0.593	0.000	1.788	0.000
Gambia	476	0.007	0.018	0.958	0.000	6.774	0.000
GuinBi	175	0.029	0.049	0.577	0.002	2.359	0.000
Haiti	527	0.007	0.021	0.767	0.000	4.895	0.000
Malawi	250	0.018	0.023	0.115	0.457	1.543	0.000
Maurit	189	0.005	0.018	0.651	0.000	11.606	0.000
Myanma	368	0.012	0.024	0.826	0.000	7.266	0.000
Nepal	452	0.007	0.014	-0.632	0.000	9.309	0.000
Niger	393	0.005	0.024	0.054	0.660	13.719	0.000
Samoa	406	0.006	0.015	0.297	0.015	1.692	0.000
Senega	393	0.005	0.020	1.130	0.000	5.337	0.000
SieraL	170	0.030	0.063	2.353	0.000	21.754	0.000
Solomo	265	0.009	0.012	1.081	0.000	8.555	0.000
Uganda	237	0.031	0.054	2.844	0.000	13.074	0.000
Zambia	157	0.045	0.035	1.812	0.000	5.675	0.000

Table C3.2a

****

Stable Paretian distribution estimates for inflation and RMK test in levels

****

		distribution estimates		RMK test no costant included			RMK test costant included		
code (1)	No.obs. (2)	alpha (3)	sd(alpha) (4)	t-st. (5)	sign. (6)	max-aug. (7)	t-st. (8)	sign. (9)	max-aug. (10)
Austri	528	1.461	0.085	2.550	0	24	-2.022	0	24
Belgiu	528	1.544	0.117	1.994	0	24	-1.266	0	24
Canada	612	1.657	0.123	2.012	0	16	0.181	0	16
Denmar	407	1.699	0.154	1.126	0	20	-3.076	++	20
Finlan	528	1.707	0.136	1.744	0	24	-1.598	0	24
France	528	2.000	-	-	-	-	-	-	-
Greece	526	2.000	-	-	-	-	-	-	-
Icelan	300	2.000	-	-	-	-	-	-	-
Irelan	385	1.851	0.179	1.047	0	19	-2.765	+	19
Italy	600	1.608	0.122	1.568	0	18	-0.819	0	18
Japan	526	1.815	0.148	2.451	0	24	-2.749	+	24
Luxemb	527	1.489	0.115	2.521	0	24	-0.957	0	24
Nether	526	1.570	0.102	2.187	0	24	-1.989	0	24
Norway	528	1.740	0.138	3.041	0	24	-0.858	0	24
Portug	528	1.684	0.134	0.810	0	15	-0.857	0	15
Spain	528	1.892	0.159	1.335	0	17	-0.889	0	17
Sweden	526	1.763	0.140	1.290	0	16	-1.272	0	16
Switze	612	1.857	0.143	2.768	0	24	-0.208	0	24
UK	540	1.590	0.128	1.824	0	24	-0.923	0	24
US	612	1.539	0.117	2.936	0	24	0.043	0	24
German	600	1.453	0.099	3.993	0	16	-1.022	0	16

Table C3.2b

****

Stable Paretian distribution estimates for inflation and RMK test in levels

****

		distribution estimates		RMK test no costant included			RMK test costant included		
code (1)	No.obs. (2)	alpha (3)	sd(alpha) (4)	t-st. (5)	sign. (6)	max-aug. (7)	t-st. (8)	sign. (9)	max-aug. (10)
Albani	131	1.174	0.213	3.041	0	23	-3.379	++	24
Bulgar	132	1.035	0.182	1.832	0	1	-1.432	0	1
CzechR	132	1.178	0.180	4.219	0	20	-4.635	+++	24
EastGe	132	1.325	0.209	4.937	0	24	-3.873	++	24
Estoni	120	1.030	0.202	-0.141	0	16	-2.080	0	16
Hungar	312	1.727	0.178	2.868	0	15	-0.053	0	15
Latvia	120	0.758	0.153	-	-	-	-	-	-
Lithua	132	1.158	0.210	0.854	0	21	-13.640	+++	24
Poland	168	0.948	0.159	-	-	-	-	-	-
Romani	135	1.839	0.300	0.563	0	15	-2.341	0	2
Russia	120	1.825	0.314	1.365	0	3	-2.309	0	3
Slovak	131	1.240	0.170	4.019	0	11	-1.670	0	11
Sloven	121	1.383	0.236	2.071	0	11	-2.573	+	1

Table C3.2c

****

Stable Paretian distribution estimates for inflation and RMK test in levels

****

		distribution estimates		RMK test					
				no costant included			costant included		
code (1)	No.obs. (2)	alpha (3)	sd(alpha) (4)	t-st. (5)	sign. (6)	max-aug. (7)	t-st. (8)	sign. (9)	max-aug. (10)
Argent	143	0.855	0.156	-	-	-	-	-	-
Aruba	177	1.535	0.218	2.235	0	7	-2.576	+	13
Bahama	339	1.733	0.171	3.119	0	24	-4.067	+++	24
Bahrai	290	1.148	0.103	1.067	0	24	-4.559	++	24
Barbad	420	1.243	0.099	3.671	0	9	-4.107	++	24
Bolivi	197	0.589	0.122	-	-	-	-	-	-
Botswa	306	1.662	0.149	3.153	0	24	-1.435	0	24
Camero	399	1.530	0.114	3.845	0	24	-2.252	0	24
Chile	307	1.329	0.151	1.760	0	21	-3.148	++	7
Colomb	528	2.000	-	-	-	-	-	-	-
CostRi	313	1.297	0.131	1.604	0	17	-1.183	0	17
Cyprus	611	1.663	0.102	3.097	0	24	0.108	0	24
DominR	535	1.355	0.116	2.556	0	15	0.771	0	15
Ecuado	528	1.963	0.170	2.075	0	15	3.543	0	15
ElSalv	528	1.909	0.162	1.769	0	24	0.263	0	24
Fuji	384	1.385	0.105	6.555	0	24	-7.024	+++	24
Ghana	454	1.449	0.125	0.937	0	3	0.520	0	3
Guatem	527	1.318	0.079	3.366	0	24	1.113	0	24
Hondur	527	1.490	0.106	3.523	0	23	2.990	0	24
HongKo	299	1.918	0.210	2.235	0	24	-2.627	+	24
India	525	1.400	0.082	5.753	0	24	1.130	0	24
Indone	393	1.190	0.102	3.288	0	23	-0.687	0	23
IvoryC	489	1.330	0.085	5.252	0	12	-0.129	0	12
Jamaic	527	1.617	0.131	1.374	0	19	0.908	0	19
Jordan	299	1.327	0.115	4.225	0	24	-2.213	0	24
Kenya	396	1.383	0.108	3.532	0	24	0.202	0	24
Korea	432	1.435	0.126	2.316	0	24	-3.086	++	24
Malays	524	1.515	0.099	3.544	0	24	0.414	0	24
Malta	526	1.437	0.083	3.493	0	24	-0.311	0	24
Maurit	462	1.392	0.093	3.320	0	24	-0.322	0	24
Mexico	612	1.323	0.107	0.957	0	12	0.876	0	12
Morocc	525	1.538	0.096	3.265	0	15	-0.248	0	17
Namibi	346	1.771	0.165	2.987	0	21	-2.961	++	21
NethAn	400	1.289	0.125	2.184	0	24	-2.973	+	24
Nigeri	498	1.498	0.128	1.477	0	16	1.198	0	15
Pakist	527	1.584	0.101	4.492	0	16	0.524	0	16
Paragu	526	1.562	0.128	3.110	0	24	1.670	0	24
Peru	137	0.862	0.161	-	-	-	-	-	-
Philip	526	1.449	0.115	8.751	0	24	0.931	0	24
SaudAr	250	1.553	0.136	1.110	0	0	-0.767	0	15
Singap	480	1.209	0.086	2.413	0	24	-1.068	0	24
SouthA	527	2.000	-	-	-	-	-	-	-
SriLan	526	1.605	0.119	4.988	0	23	2.408	0	23
StKitt	256	1.398	0.148	4.647	0	12	-1.023	0	12
StLuci	429	1.582	0.114	2.467	0	24	-2.139	0	24
Surina	372	0.879	0.099	-	-	-	-	-	-
Swazil	402	1.420	0.109	4.667	0	24	-0.396	0	24
Taiwan	504	1.645	0.128	2.337	0	19	-0.998	0	14
Thaila	432	1.447	0.110	3.206	0	24	-1.030	0	24

Trinid	523	1.828	0.150	1.612	0	15	-0.321	0	15
Tunisi	161	1.875	0.284	1.466	0	17	-7.279	+++	21
Turkey	261	2.000	-	-	-	-	-	-	-
Urugua	256	2.000	-	-	-	-	-	-	-
Venezu	526	1.818	0.148	1.302	0	24	1.517	0	15
Vietna	141	1.303	0.160	-0.638	0	24	-5.235	+++	22
Zimbab	264	1.385	0.164	5.138	0	17	3.518	0	17

Table C3.2d

****

Stable Paretian distribution estimates for inflation and RMK test in levels

****

		distribution estimates		RMK test no costant included			costant included		
code	No.obs.	alpha	sd(alpha)	t-st.	sign.	max-aug.	t-st.	sign.	max-aug.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
BurkFa	218	1.472	0.135	1.621	0	24	-0.366	0	24
Burund	322	1.439	0.123	5.189	0	24	0.620	0	24
Chad	214	1.565	0.150	1.857	0	23	0.188	0	23
Ethiop	426	1.586	0.113	4.340	0	24	-1.269	0	24
Gambia	476	1.288	0.082	2.844	0	19	-0.033	0	19
GuinBi	175	1.449	0.155	0.439	0	15	-3.283	++	24
Haiti	527	1.416	0.086	5.678	0	24	3.005	0	24
Malawi	250	1.434	0.122	8.094	0	22	3.191	0	22
Maurit	189	1.261	0.136	6.170	0	17	-0.490	0	17
Myanma	368	1.919	0.196	2.888	0	24	0.518	0	24
Nepal	452	1.598	0.115	7.119	0	24	0.752	0	24
Niger	393	1.468	0.100	2.319	0	14	-2.210	0	14
Samoa	406	1.420	0.095	2.841	0	24	-1.773	0	24
Senega	393	1.494	0.107	4.475	0	20	-2.496	0	12
SieraL	170	1.372	0.150	2.431	0	13	-4.624	+++	24
Solomo	265	1.481	0.124	5.817	0	14	-1.193	0	12
Uganda	237	1.663	0.199	-0.329	0	24	-3.795	+++	24
Zambia	157	1.646	0.243	0.098	0	17	-1.793	0	17

Table C3.3a

****

Joint confirmation of the unit root tests in levels

****

constant in DF_{max}, constant in KPSS

	Leybourne DF_{max}			KPSS	
code	signif.	max.aug.	B/F	signif.	AC length
(1)	(2)	(3)	(4)	(5)	(6)
Aust	0	21	back	+++	0
Belg	0	24	back	+++	0
Cana	0	16	forw	+++	0
Denm	0	20	back	+++	0
Finl	0	24	back	+++	0
Fran	0	24	back	+++	0
Gree	0	22	forw	+++	0
Icel	0	13	back	+++	0
Irel	0	22	back	+++	0
Ital	0	18	forw	+++	0
Japa	0	24	back	+++	0
Luxe	0	24	back	+++	0
Neth	0	24	back	+++	0
Norw	0	24	back	+++	0
Port	0	15	forw	+++	0
Spai	0	17	back	+++	0
Swed	0	16	back	+++	0
Swit	0	24	forw	+++	0
UK	0	24	back	+++	0
US	0	24	forw	+++	0
Germ	0	24	back	+++	0

Table C3.3b

****

Joint confirmation of the unit root tests in levels

****

constant in DF_{max}, constant in KPSS

	Leybourne DF_{max}			KPSS	
code	signif.	max.aug.	B/F	signif.	AC length
(1)	(2)	(3)	(4)	(5)	(6)
Alba	0	24	back	+++	0
Bulg	0	1	back	+++	0
Czec	0	24	back	+++	0
FEGe	0	24	back	+++	0
Esto	0	20	back	+++	0
Hung	0	15	forw	+++	0
Latv	0	24	back	+++	0
Lith	0	24	back	+++	0
Pola	0	23	back	+++	0
Roma	0	18	back	+++	0
Russ	0	18	back	+++	0
Slov	0	22	back	+++	0
Slov	0	19	back	+++	0

Table C3.3c

****

Joint confirmation of the unit root tests in levels

****

constant in DF_{max}, constant in KPSS

	Leybourne DF_{max}			KPSS	
code	signif.	max.aug.	B/F	signif.	AC length
(1)	(2)	(3)	(4)	(5)	(6)
Arge	0	24	back	+++	0
Arub	0	2	back	+++	0
Baha	0	24	back	+++	0
Bahr	0	24	back	+++	0
Barb	0	24	back	+++	0
Boli	0	24	back	+++	0
Bots	0	24	back	+++	0
Came	0	24	back	+++	0
Chil	0	7	back	+++	0
Colo	0	14	forw	+++	0
Cost	0	17	back	+++	0
Cypr	0	24	forw	+++	0
Domi	0	15	forw	+++	0
Ecua	0	15	forw	+++	0
ElSa	0	24	forw	+++	0
Fuji	0	24	back	+++	0
Ghan	0	3	forw	+++	0
Guat	0	24	forw	+++	0
Hond	0	24	forw	+++	0
Hong	0	24	back	+++	0
Indi	0	24	forw	+++	0
Indo	0	23	back	+++	0
Ivor	0	12	forw	+++	0
Jama	0	19	forw	+++	0
Jord	0	24	back	+++	0
Keny	0	24	forw	+++	0
Kore	0	24	back	+++	0
Mala	0	24	forw	+++	0
Malt	0	24	forw	+++	0
Maur	0	24	forw	+++	0
Mexi	0	12	forw	+++	0
Moro	0	17	forw	+++	0
Nami	0	21	back	+++	0
Neth	0	24	back	+++	0
Nige	0	15	forw	+++	0
Paki	0	16	forw	+++	0
Para	0	24	forw	+++	0
Peru	0	24	back	+++	0
Phil	0	24	forw	+++	0
Saud	0	15	forw	+++	0
Sing	0	24	back	+++	0
Sout	0	21	forw	+++	0
SriL	0	23	forw	+++	0
StKi	0	12	back	+++	0
StLu	0	24	back	+++	0
Suri	0	22	forw	+++	0
Swaz	0	24	forw	+++	0
Taiw	0	12	back	+++	0
Thai	0	24	back	+++	0
Trin	0	15	forw	+++	0
Tuni	0	21	back	+++	0

Turk	0	9	forw	+++	0
Urug	0	24	back	+++	0
Vene	0	15	forw	+++	0
Viet	+++	18	back	+++	0
Zimb	0	17	forw	+++	0

Table C3.3d

****

Joint confirmation of the unit root tests in levels

****

constant in DF_{max}, constant in KPSS

	Leybourne DF_{max}			KPSS	
code	signif.	max.aug.	B/F	signif.	AC length
(1)	(2)	(3)	(4)	(5)	(6)
Burk	0	24	forw	+++	0
Buru	0	24	forw	+++	0
Chad	0	23	forw	+++	0
Ethi	0	24	back	+++	0
Gamb	0	19	forw	+++	0
Guin	0	24	back	+++	0
Hait	0	24	forw	+++	0
Mala	0	22	forw	+++	0
Maur	0	17	back	+++	0
Myan	0	24	forw	+++	0
Nepa	0	24	forw	+++	0
Nige	0	14	back	+++	0
Samo	0	24	back	+++	0
Sene	0	12	back	+++	0
Sier	0	24	back	+++	0
Solo	0	12	back	+++	0
Ugan	0	23	back	+++	0
Zamb	0	17	back	+++	0

Table C3.4a

```
**************************************
  Bilinear unit root test in levels
**************************************
    constant in B test
```

code (1)	B test signif. (2)	max.aug. (3)
Aust	++	24
Belg	+	24
Cana	++	16
Denm	+++	20
Finl	0	24
Fran	0	19
Gree	0	22
Icel	0	11
Irel	+++	22
Ital	0	18
Japa	+++	24
Luxe	+++	24
Neth	+++	24
Norw	0	24
Port	++	24
Spai	0	17
Swed	+++	16
Swit	+++	24
UK	0	24
US	+++	24
Germ	+++	16

Table C3.4b

```
**************************************
  Bilinear unit root test in levels
**************************************
    constant in B test
```

code (1)	B test signif. (2)	max.aug. (3)
Alba	+++	22
Bulg	+++	1
Czec	+	5
FEGe	++	24
Esto	0	20
Hung	+++	24
Latv	+++	23
Lith	+++	24
Pola	0	21
Roma	+++	24
Russ	+++	1
Slov	+++	6
Slov	++	18

Table C3.4c

```
**************************************
  Bilinear unit root test in levels
**************************************
    constant in B test
```

	B test	
code	signif.	max.aug.
(1)	(2)	(3)
Arge	+++	24
Arub	+++	13
Baha	+++	24
Bahr	+++	24
Barb	++	7
Boli	0	10
Bots	++	24
Came	0	24
Chil	+++	7
Colo	0	14
Cost	+++	17
Cypr	+++	24
Domi	+++	15
Ecua	+++	15
ElSa	+++	24
Fuji	0	24
Ghan	+++	3
Guat	+++	24
Hond	+++	23
Hong	++	24
Indi	+++	24
Indo	0	23
Ivor	+++	12
Jama	++	19
Jord	0	24
Keny	0	24
Kore	0	24
Mala	+	24
Malt	+++	24
Maur	0	24
Mexi	++	12
Moro	+++	17
Nami	0	21
Neth	+++	24
Nige	+++	4
Paki	+++	16
Para	0	24
Peru	+++	24
Phil	++	24
Saud	0	0
Sing	0	24
Sout	+	21
SriL	+++	23
StKi	0	12
StLu	+++	24
Suri	++	22
Swaz	+++	24
Taiw	++	14
Thai	++	24
Trin	+	15
Tuni	0	17

Turk	0	9
Urug	++	24
Vene	+++	24
Zimb	+++	17

Table C3.4d

```
*************************************
  Bilinear unit root test in levels
*************************************
    constant in B test
```

	B test	
code	signif.	max.aug.
(1)	(2)	(3)
Burk	0	24
Buru	0	24
Chad	+++	23
Ethi	+	24
Gamb	+++	19
Guin	0	24
Hait	++	11
Mala	+++	22
Maur	+++	17
Myan	+++	24
Nepa	0	24
Nige	+++	14
Samo	+++	24
Sene	+++	20
Sier	+++	24
Solo	++	12
Ugan	++	24
Zamb	0	17

Table C3.5a

****

Stable Paretian distribution estimates for first difference of inflation and RMK test in returns

****

		distribution estimates		RMK test					
				no costant included			costant included		
code	No.obs.	alpha	sd(alpha)	t-st.	sign.	max-aug.	t-st.	sign.	max-aug.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Austri	527	1.278	0.074	-0.980	0	24	-3.616	++	24
Belgiu	527	1.431	0.084	-1.598	+	23	-3.252	++	23
Canada	611	1.265	0.068	-1.940	++	13	-3.067	++	13
Denmar	406	1.452	0.100	-1.376	0	18	-	-	-
Finlan	527	1.235	0.074	-1.773	++	23	-3.109	++	23
France	527	1.384	0.083	-1.502	+	24	-2.223	0	24
Greece	525	0.931	0.062	-	-	-	-	-	-
Icelan	299	1.171	0.093	-1.777	++	16	-2.781	+	13
Irelan	384	1.149	0.084	-1.361	0	21	-	-	-
Italy	599	1.275	0.069	-1.369	0	24	-2.370	24	-
Japan	525	1.557	0.094	-2.173	++	8	-	-	-
Luxemb	526	1.212	0.071	-1.674	+	23	-3.549	++	23
Nether	525	1.217	0.071	-0.881	0	21	-3.523	++	21
Norway	527	1.403	0.084	-1.530	+	24	-3.749	++	24
Portug	527	1.082	0.069	-1.914	++	24	-3.121	++	24
Spain	527	1.318	0.077	-1.555	+	14	-3.209	++	8
Sweden	525	0.743	0.052	-	-	-	-	-	-
Switze	611	1.322	0.072	-2.401	++	23	-4.246	++	23
UK	539	1.486	0.085	-1.988	++	21	-3.400	++	21
US	611	1.330	0.071	-1.715	+	22	-3.412	++	23
German	599	1.232	0.067	-1.584	+	9	-3.545	++	23

Table C3.5b

****

Stable Paretian distribution estimates for first difference of inflation and RMK test in returns

****

		distribution estimates		RMK test					
				no costant included			costant included		
code	No.obs.	alpha	sd(alpha)	t-st.	sign.	max-aug.	t-st.	sign.	max-aug.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Albani	130	1.116	0.138	-4.463	+++	22	-	-	-
Bulgar	131	1.343	0.162	-8.365	+++	0	-8.844	+++	0
CzechR	131	1.114	0.160	-3.575	+++	5	-	-	-
FEGerm	131	1.241	0.146	-6.226	+++	24	-	-	-
Estoni	119	0.921	0.138	-	-	-	-	-	-
Hungar	311	1.175	0.093	-1.250	0	7	-2.429	0	7
Latvia	119	0.964	0.149	-	-	-	-	-	-
Lithua	131	0.812	0.107	-	-	-	-	-	-
Poland	167	0.767	0.099	-	-	-	-	-	-
Romani	134	1.341	0.162	-2.331	++	24	-3.148	++	24
Russia	119	1.090	0.154	-2.327	++	2	-2.747	+	2
Slovak	130	1.119	0.140	-2.692	+++	6	-11.372	+++	0
Sloven	120	1.363	0.184	-1.866	++	10	-	-	-

Table C3.5c

****

Stable Paretian distribution estimates for first difference of inflation and RMK test in returns

****

		distribution estimates		RMK test					
				no costant included			costant included		
code (1)	No.obs. (2)	alpha (3)	sd(alpha) (4)	t-st. (5)	sign. (6)	max-aug. (7)	t-st. (8)	sign. (9)	max-aug. (10)
Argent	142	0.617	0.107	-	-	-	-	-	-
Aruba	176	0.800	0.105	-	-	-	-	-	-
Bahama	338	1.342	0.100	-2.180	++	6	-	-	-
Bahrai	289	1.136	0.093	-4.041	+++	14	-	-	-
Barbad	419	1.235	0.084	-2.502	+++	8	-	-	-
Bolivi	196	0.900	0.097	-	-	-	-	-	-
Botswa	305	1.327	0.102	-0.901	0	23	-14.172	+++	23
Camero	398	1.340	0.091	-4.185	+++	15	-18.365	+++	0
Chile	306	1.013	0.083	-2.531	+++	6	-	-	-
Colomb	527	1.177	0.072	-1.458	+	22	-6.348	+++	22
CostRi	312	1.155	0.089	-1.857	++	16	-5.528	+++	16
Cyprus	610	1.354	0.074	-2.808	+++	9	-5.250	+++	23
DominR	534	0.894	0.056	-	-	-	-	-	-
Ecuado	527	0.977	0.062	-	-	-	-	-	-
ElSalv	527	0.964	0.061	-	-	-	-	-	-
Fuji	383	1.319	0.091	-2.432	++	23	-	-	-
Ghana	453	0.997	0.069	-	-	-	-	-	-
Guatem	526	1.291	0.074	-3.307	+++	23	-5.005	+++	16
Hondur	526	1.164	0.070	-2.418	++	22	-3.840	++	22
HongKo	298	1.379	0.111	-1.651	+	23	-	-	-
India	524	1.391	0.083	-3.072	+++	24	-11.714	+++	23
Indone	392	1.311	0.088	-2.644	+++	13	-10.799	+++	22
IvoryC	488	1.079	0.068	-2.634	+++	16	-11.568	+++	2
Jamaic	526	1.195	0.070	-2.365	++	16	-4.554	++	13
Jordan	298	1.341	0.104	-3.882	+++	23	-5.471	+++	23
Kenya	395	1.020	0.073	-2.217	++	14	-7.414	+++	23
Korea	431	1.275	0.081	-1.920	++	24	-	-	-
Malays	523	1.161	0.070	-3.054	+++	24	-5.361	+++	24
Malta	525	1.445	0.083	-3.729	+++	13	-6.277	+++	23
Maurit	461	1.403	0.090	-2.998	+++	14	-8.305	+++	13
Mexico	611	1.276	0.068	-1.718	+	24	-2.226	0	24
Morocc	524	1.508	0.089	-2.460	++	14	-4.458	+++	14
Namibi	345	1.491	0.106	-0.621	0	18	-	-	-
NethAn	399	1.287	0.086	-2.720	+++	23	-	-	-
Nigeri	497	0.874	0.058	-	-	-	-	-	-
Pakist	526	1.383	0.081	-1.935	++	16	-7.777	+++	15
Paragu	525	1.009	0.064	-2.338	++	21	-4.638	++	21
Peru	136	0.838	0.116	-	-	-	-	-	-
Philip	525	1.110	0.068	-10.796	+++	23	-10.907	+++	23
SaudAr	249	1.342	0.118	-6.116	+++	14	-6.128	+++	14
Singap	479	1.232	0.078	-4.537	+++	20	-6.533	+++	20
SouthA	526	1.049	0.064	-0.583	0	17	-1.698	0	17
SriLan	525	1.282	0.074	-1.846	++	24	-5.535	+++	24
StKitt	255	1.170	0.101	-3.175	+++	15	-15.833	+++	15
StLuci	428	1.336	0.088	-3.275	+++	19	-5.145	+++	23
Surina	371								
Swazil	401	1.173	0.080	-2.423	++	23	-25.535	+++	23
Taiwan	503	1.432	0.086	-5.192	+++	13	-7.690	+++	3
Thaila	431	1.283	0.084	-2.605	+++	20	-5.860	+++	23

Trinid	522	1.075	0.067	-1.460	+	14	-3.122	++	14
Tunisi	160	1.609	0.194	-1.679	+	13	-	-	-
Turkey	260	1.112	0.101	-0.615	0	8	-2.341	0	7
Urugua	255	0.951	0.089	-	-	-	-	-	-
Venezu	525	0.927	0.058	-	-	-	-	-	-
Vietna	140	1.287	0.151	-16.680	+++	24	-	-	-
Zimbab	263	1.303	0.106	-0.670	0	16	-4.273	++	4

Table C3.5d

****

Stable Paretian distribution estimates for first difference of inflation and RMK test in returns

****

		distribution estimates		RMK test no costant included			costant included		
code (1)	No.obs. (2)	alpha (3)	sd(alpha) (4)	t-st. (5)	sign. (6)	max-aug. (7)	t-st. (8)	sign. (9)	max-aug. (10)
BurkFa	217	1.492	0.145	-6.760	+++	23	-15.581	+++	23
Burund	321	1.435	0.108	-3.605	+++	13	-16.440	+++	9
Chad	213	1.301	0.123	-11.592	+++	0	-10.974	+++	6
Ethiop	425	1.414	0.091	-6.453	+++	4	-18.768	+++	20
Gambia	475	1.155	0.073	-4.670	+++	23	-6.321	+++	23
GuinBi	174	1.361	0.139	-1.753	+	14	-	-	-
Haiti	526	1.079	0.067	-2.413	++	15	-5.270	+++	21
Malawi	249	1.103	0.100	-2.247	++	4	-7.155	+++	1
Maurit	188	1.097	0.117	-2.919	+++	19	-18.674	+++	11
Myanma	367	1.317	0.101	-2.545	+++	23	-5.195	+++	23
Nepal	451	1.214	0.078	-3.125	+++	18	-20.212	+++	23
Niger	392	1.412	0.095	-5.156	+++	13	-19.320	+++	3
Samoa	405	1.364	0.090	-3.716	+++	24	-6.478	+++	12
Senega	392	1.367	0.095	-5.091	+++	12	-12.506	+++	22
SieraL	169	1.361	0.148	-1.757	+	23	-	-	-
Solomo	264	1.463	0.121	-1.309	0	13	-17.755	+++	0
Uganda	236	1.248	0.109	-1.591	+	24	-	-	-
Zambia	156	1.634	0.200	-1.869	++	24	-5.198	+++	18

Table C3.6a

****

Joint confirmation of the unit root tests in returns

****

constant in DF_{max}, constant in KPSS

code	Leybourne DF_{max} signif.	max.aug.	B/F	KPSS signif.	AC length
(1)	(2)	(3)	(4)	(5)	(6)
Aust	+	24	back	+++	0
Belg	+++	23	forw	+++	0
Cana	+++	13	back	+++	0
Denm	+	18	forw	+++	0
Finl	+++	23	forw	+++	0
Fran	+	24	forw	+++	0
Gree	++	8	forw	+++	0
Icel	0	15	back	+++	0
Irel	+	21	forw	+++	0
Ital	+	24	forw	+++	0
Japa	+++	23	forw	+++	0
Luxe	+++	23	back	+++	0
Neth	+++	21	forw	+++	0
Norw	+++	24	forw	+++	0
Port	+++	24	back	+++	0
Spai	+++	8	forw	+++	2
Swed	++	15	forw	+++	0
Swit	+++	23	forw	+++	0
UK	+++	21	forw	+++	0
US	+++	23	forw	+++	0
Germ	+++	23	forw	+++	0

Table C3.6b

****

Joint confirmation of the unit root tests in returns

****

constant in DF_{max}, constant in KPSS

code	Leybourne DF_{max} signif.	max.aug.	B/F	KPSS signif.	AC length
(1)	(2)	(3)	(4)	(5)	(6)
Alba	+++	22	back	+++	0
Bulg	+++	0	back	++	0
Czec	0	19	back	+++	0
FEGe	0	24	back	+++	0
Esto	0	24	back	+++	0
Hung	+	7	back	+++	0
Latv	+	22	back	+++	0
Lith	+++	24	back	+++	0
Pola	0	22	back	+++	0
Roma	++	24	back	+++	0
Russ	0	0	back	+++	0
Slov	++	18	back	+++	0
Slov	0	24	back	+++	0

Table C3.6c

****

Joint confirmation of the unit root tests in returns

****

constant in DF_{max}, constant in KPSS

	Leybourne DF_{max}			KPSS	
code	signif.	max.aug.	B/F	signif.	AC length
(1)	(2)	(3)	(4)	(5)	(6)
Arge	+++	24	back	+++	0
Arub	+++	10	back	+++	0
Baha	+++	23	forw	+++	0
Bahr	++	6	back	+++	1
Barb	+++	23	back	+++	0
Boli	+++	24	back	+++	0
Bots	+++	23	back	+++	0
Came	+++	0	forw	++	0
Chil	0	6	back	+++	0
Colo	0	2	back	+++	0
Cost	+++	16	forw	+++	0
Cypr	+++	23	forw	+	2
Domi	+++	24	forw	+++	0
Ecua	+++	15	forw	+++	0
ElSa	+++	23	forw	+++	0
Fuji	+++	23	forw	+++	0
Ghan	+++	16	back	+++	0
Guat	+++	16	back	+++	0
Hond	+++	22	back	+++	0
Hong	++	23	forw	+++	0
Indi	+++	23	forw	+++	0
Indo	+++	22	back	++	0
Ivor	+++	2	back	0	1
Jama	+++	13	forw	+++	0
Jord	+++	23	forw	++	1
Keny	+++	23	back	+++	0
Kore	+++	24	forw	+++	0
Mala	+++	24	back	+++	0
Malt	+++	23	forw	0	3
Maur	+++	23	back	+++	0
Mexi	+	24	forw	+++	0
Moro	+++	14	forw	+++	0
Nami	+++	0	forw	++	0
Neth	+++	23	forw	+++	0
Nige	+++	3	forw	+++	0
Paki	+++	15	back	+++	0
Para	+++	21	forw	+++	0
Peru	+++	19	back	+++	0
Phil	+++	23	back	0	24
Saud	+++	14	back	0	0
Sing	+++	20	forw	++	0
Sout	0	17	forw	+++	0
SriL	+++	24	back	+++	0
StKi	+++	5	back	+++	0
StLu	+++	23	back	+++	0
Suri	+++	24	forw	+++	0
Swaz	+++	23	forw	++	3
Taiw	+++	3	forw	+++	0
Thai	+++	23	forw	+++	0
Trin	+++	14	forw	+++	0
Tuni	+++	13	forw	+++	0

Turk	+	7	back	+++	0
Urug	0	24	forw	+++	0
Vene	+++	23	back	+++	0
Zimb	+++	4	forw	+++	0

Table C3.6d

****

Joint confirmation of the unit root tests in returns

****

constant in DF_{max}, constant in KPSS

code	Leybourne DF_{max} signif.	max.aug.	B/F	KPSS signif.	AC length
(1)	(2)	(3)	(4)	(5)	(6)
Burk	+++	23	forw	0	2
Buru	+++	9	forw	0	0
Chad	+++	6	forw	0	0
Ethi	+++	20	forw	0	0
Gamb	+++	23	forw	+++	0
Guin	+++	0	forw	+++	1
Hait	+++	21	back	+++	0
Mala	+++	1	forw	+++	0
Maur	+++	11	forw	0	24
Myan	+++	23	forw	+++	0
Nepa	+++	23	back	0	1
Nige	+++	3	forw	++	3
Samo	+++	15	back	+++	0
Sene	+++	22	forw	+	1
Sier	+++	10	back	+++	0
Solo	+++	0	forw	0	24
Ugan	0	24	back	+++	0
Zamb	+++	18	forw	+++	0

Table C3.7

****

Bilinear unit root test in returns

****

constant in B test

code	Leybourne B test signif.	max.aug.
(1)	(2)	(3)
Icel	++	17
Czec	+	5
FEGe	++	24
Russ	+++	1
Slov	++	18
Chil	+++	7
Colo	+++	13
Sout	+++	17
Urug	++	23
Ugan	+++	24

APPENDIX D

Table D5.1a

****
**

OLS and Kalman filter estimation results for prices

****
**

		The OLS method with constant				The Kalman filter method with constant			
country (1)	No.obs. (2)	b (3)	b-scaled (4)	t (5)	sign. (6)	b (7)	b-scaled (8)	t (9)	sign (10)
Austria	528	0.028	0.639	2.180	++	0.0267	0.6156	2.178	++
Belgium	528	0.150	3.448	15.929	+++	0.1194	2.7453	14.372	+++
Canada	612	0.140	3.453	14.937	+++	0.0927	2.2938	11.110	+++
Denmark	407	0.065	1.309	4.667	+++	0.0510	1.0294	4.618	+++
Ireland	385	0.140	2.750	24.985	+++	0.0915	1.7949	17.152	+++
Japan	526	0.085	1.957	7.626	+++	0.0594	1.3621	6.351	+++
Luxembou	527	0.064	1.470	6.045	+++	0.0418	0.9598	4.872	+++
Netherla	526	0.020	0.465	1.603	+	0.0169	0.3873	1.257	0
Portugal	528	0.157	3.600	9.882	+++	0.1236	2.8406	8.006	+++
Sweden	526	0.143	3.276	15.465	+++	0.6184	14.1832	12.160	+++
Switzerl	612	0.105	2.602	11.136	+++	0.0763	1.8881	9.414	+++
US	612	0.140	3.466	21.928	+++	0.0960	2.3761	15.712	+++
Germany	600	0.086	2.098	8.476	+++	0.0659	1.6136	7.116	+++

Table D5.1b

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**

OLS and Kalman filter estimation results for prices

****
**

country (1)	No.obs. (2)	b (3)	b-scaled (4)	t (5)	sign. (6)	b (7)	b-scaled (8)	t (9)	sign (10)
		The OLS method with constant				The Kalman filter method with constant			
Albania	131	0.086	0.983	3.880	+++	0.0621	0.7109	3.508	+++
Bulgaria	132	0.045	0.518	3.903	+++	0.0397	0.4567	3.485	+++
CzechRep	132	0.083	0.953	4.412	+++	0.0559	0.6427	3.081	+++
FEGermany	132	0.082	0.937	4.426	+++	0.0607	0.6969	3.877	+++
Hungary	312	0.109	1.932	8.030	+++	0.0689	1.2162	5.245	+++
Latvia	120	0.233	2.550	17.716	+++	0.2069	2.2673	60.43	+++
Lithuani	132	0.157	1.799	12.190	+++	0.1029	1.1817	13.34	+++
Romania	135	0.083	0.966	3.690	+++	0.1129	1.3114	7.251	+++
Russia	120	-0.030	-0.330	-0.997	0	0.1169	1.2804	4.783	+++
Slovakia	131	0.023	0.268	1.315	+	0.0137	0.1564	0.737	0
Slovenia	121	0.192	2.107	18.327	+++	0.1183	1.3015	7.941	+++

Table D5.1c

****
**

OLS and Kalman filter estimation results for prices

****
**

country (1)	No.obs. (2)	b (3)	b-scaled (4)	t (5)	sign. (6)	b (7)	b-scaled (8)	t (9)	sign (10)
		The OLS method with constant				The Kalman filter method with constant			
Argentin	143	0.082	0.982	1.567	+	0.1488	1.7799	3.501	+++
Aruba	177	0.106	1.416	7.140	+++				
Bahamas	339	0.070	1.292	5.295	+++	0.0467	0.8600	4.619	+++
Bahrain	290	-0.029	-0.498	-2.236	++	-0.0239	-0.4077	-1.885	++
Barbados	420	0.044	0.893	3.253	+++	0.0338	0.6923	3.076	+++
Botswana	306	0.035	0.610	2.231	++	0.0428	0.7488	2.948	+++
Chile	307	-0.074	-1.301	-3.029	+++	0.0275	0.4813	1.541	+
CostRica	313	0.180	3.191	10.051	+++	0.1358	2.4018	10.985	+++
Cyprus	611	-0.019	-0.477	-1.594	+	-0.0189	-0.4693	-2.514	+++
DominRep	535	0.190	4.404	17.289	+++	0.1338	3.0947	14.753	+++
Ecuador	528	0.114	2.627	10.784	+++	0.0820	1.8845	6.008	+++
ElSalv	528	0.156	3.591	13.478	+++	0.1024	2.3534	10.355	+++
Ghana	454	0.087	1.847	5.559	+++	0.0593	1.2627	4.148	+++
Guatemal	527	0.082	1.880	4.914	+++	0.0582	1.3356	3.543	+++
Honduras	527	0.151	3.473	15.397	+++	0.1115	2.5588	11.455	+++
HongKong	299	0.013	0.229	0.856	0	0.0145	0.2515	1.257	0
India	525	0.123	2.812	10.451	+++	0.1021	2.3403	10.021	+++
IvoryCoa	489	-0.018	-0.407	-1.318	+	-0.0163	-0.3598	-1.256	0
Jamaica	527	0.193	4.432	18.295	+++	0.1423	3.2657	14.337	+++
Malaysia	524	0.100	2.294	9.692	+++	0.0808	1.8491	8.578	+++
Malta	526	-0.009	-0.215	-0.804	0	-0.0079	-0.1815	-0.995	0

Mexico	612	0.175	4.336	25.316	+++	0.1386	3.4287	50.074	+++
Morocco	525	0.069	1.574	5.814	+++	0.0708	1.6224	6.162	+++
NethAnti	400	0.123	2.462	11.224	+++	0.6763	13.5260	9.588	+++
Nigeria	498	0.163	3.630	10.294	+++	0.1089	2.4296	8.606	+++
Pakistan	527	0.077	1.759	5.240	+++	0.0563	1.2915	4.262	+++
Peru	137	0.013	0.153	0.317	0	0.0844	0.9877	2.151	++
Philippi	526	0.004	0.103	0.436	0	0.0043	0.0976	0.374	0
SouthAfr	527	0.150	3.447	15.714	+++	0.1069	2.4547	10.925	+++
SriLanka	526	0.072	1.644	6.367	+++	0.0621	1.4241	6.016	+++
StLucia	429	0.044	0.920	3.525	+++	0.0374	0.7742	3.327	+++
Suriname	372	0.187	3.614	19.633	+++	0.1587	3.0603	20.409	+++
Swazilan	402	-0.072	-1.447	-5.513	+++	-0.0883	-1.7702	-7.001	+++
Taiwan	504	0.060	1.355	4.909	+++	0.0623	1.3995	5.184	+++
Thailand	432	0.114	2.361	9.646	+++	0.0879	1.8285	7.906	+++
Trinidad	523	0.069	1.572	4.944	+++	0.0521	1.1922	3.870	+++
Uruguay	256	0.080	1.273	4.986	+++	0.1273	2.0367	7.677	+++
Venezuel	526	0.183	4.203	23.342	+++	0.1711	3.9253	132.372	+++
Zimbabwe	264	0.086	1.400	7.314	+++	0.0664	1.0786	5.424	+++

Table D5.1d

****

OLS and Kalman filter estimation results for prices

****

			The OLS method with constant			The Kalman filter method with constant			
country (1)	No.obs. (2)	b (3)	b-scaled (4)	t (5)	sign. (6)	b (7)	b-scaled (8)	t (9)	sign (10)
BurkFaso	218	-0.028	-0.416	-1.826	++	-0.0260	-0.3846	-1.778	++
Burundi	322	0.011	0.194	0.800	0	0.0103	0.1844	0.889	0
Chad	214	0.046	0.668	2.929	+++	0.0510	0.7466	3.238	+++
Ethiopia	426	0.016	0.333	1.254	0	0.0142	0.2935	0.885	0
Gambia	476	0.046	1.013	3.017	+++	0.0355	0.7753	2.665	+++
GuinBis	175	-0.012	-0.165	-0.587	0	0.0044	0.0578	0.234	0
Haiti	527	0.040	0.925	2.716	+++	0.0276	0.6343	1.772	++
Malawi	250	0.067	1.062	4.471	+++	0.0391	0.6177	2.908	+++
Mauritan	189	-0.081	-1.113	-5.528	+++	-0.0802	-1.1019	-5.720	+++
Myanmar	368	0.077	1.475	4.966	+++	0.0754	1.4467	4.617	+++
Nepal	452	0.008	0.178	0.533	0	0.0068	0.1448	0.606	0
Niger	393	-0.001	-0.014	-0.058	0	0.0002	0.0034	-0.015	0
Samoa	406	0.018	0.354	1.421	+	0.0218	0.4389	1.778	++
Senegal	393	0.002	0.036	0.131	0				
SieraLeo	170	0.003	0.035	0.160	0	0.0194	0.2527	0.816	0
SolomonI	265	-0.026	-0.420	-1.640	+	-0.0302	-0.4922	-1.887	++
Uganda	237	0.048	0.744	1.818	++	0.0898	1.3833	4.176	+++
Zambia	157	0.110	1.373	4.986	+++	0.1814	2.2728	13.458	+++

Table D5.2

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OLS and Kalman filter estimation results for inflation

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			The OLS method with constant				The Kalman filter method with constant		
country (1)	No.obs. (2)	b (3)	b-scaled (4)	t (5)	sign. (6)	b (7)	b-scaled (8)	t (9)	sign (10)
Iceland	299	-10.27	-177.58	-9.53	+++	-12.59	-217.80	-11.59	+++
CzechRep	131	-11.34	-129.86	-6.77	+++	-11.79	-134.95	-6.87	+++
FEGerm	131	-12.52	-143.37	-7.70	+++	-13.62	-155.93	-7.93	+++
Russia	119	0.05	0.59	0.15	0	-3.98	-43.46	-13.59	+++
Slovenia	121	0.19	2.10	18.32	+++	-49.03	-537.17	-8.92	+++
Chile	306	-0.77	-13.61	-1.66	++	15.14	265.00	67.51	+++
Colombia	527	4.89	112.39	8.66	+++	-11.96	-274.75	-30.97	+++
SouthAfr	526	-36.25	-831.49	-14.19	+++	-42.90	-984.00	-18.78	+++
Uruguay	255	-8.30	-132.54	-11.45	+++	-164.4	-65.99	-10.13	+++
Uganda	236	-3.70	-56.87	-12.35	+++	-4.29	-65.993	-10.13	+++

REFERENCES

Attfield, C., Demery, D. and Duck, N. (1991), *Rational Expectations in Macroeconomics*, Blackwell.

Baillie, R.T. (1989), 'Commodity prices and aggregate inflation: would commodity price rule be worthwhile?' *Carnegie Rochester Conference Series on Public Policy*, 31, 185 - 240.

Balke, N.S. and Wynne, M.A. (1996), 'An Equilibrium Analysis of Relative Price Changes and Aggregate Inflation', Research Department Working Paper 96-09, Federal Reserve Bank of Dallas, 1996.

Ball, L. and Cecchetti, S.G. (1990), 'Inflation and uncertainty at short and long horizons', *Brooking Papers on Economic Activity*, 1, 215 – 54.

Ball, L. and Cecchetti, S.G. (1990), 'Inflation and uncertainty at short and long horizons', *Brookings Papers no Economic Activity*, 215-254.

Ball, L. and Mankiw, G. (1995), 'Relative Price Changes as Aggregate Supply Shocks', *Quarterly Journal of Economics*, 110, 161 – 193.

Barsky, R.B. (1987), 'The Fisher hypothesis and the forecastibility and persistence in inflation', *Journal of Monetary Economics*, 19, 3-24.

Begg, D.K.H. (1982), *The Rational Expectations Revolution in Macroeconomics*, Oxford: Philip Allan.

Bera, A.K. and Higgins, M.L. (1997), 'ARCHand bilinearity as competing models for non-linear dipendence', *Journal of Business and Economic Statistics*, 15, 43-50.

Bollerslev, T. (1986), 'Generalized Autoregressive Conditional Heteroskedasticity', *Journal of Econometrics*, 31, 307-327.

Briault, C. (1995), 'The cost of Inflation', *Bank of England Bulletin*, February 1995.

Brorsnen, W.B. and Yang, S.R. (1990), 'Maximum Likelihood Estimates of Symmetric Stable Distribution Parameters, *Comunications in Statistics-Simulation*, 19(4), 1459-1464.

Bryan, M.F. and Cecchetti, S.G. (1993), 'Measuring core inflation', NBER Working Paper, March.

Bryan, M.F. and Cecchetti, S.G. (1996), 'Inflation and the distribution of price changes', NBER Working Paper 5793, October.

Bryan, M.F. and Pike, C.J. (1991), 'Median Price Changes: An Alternative Approach to Measuring Current Monetary Inflation', *Federal Reserve Bank of Cleveland Economic Commentary,* December.

Bryan, M.F., Cecchetti, S.G. and Wiggins, R.L. (1997), 'Efficient Inflation Estimation', NBER Working Paper 6183, September.

Burmeister, E. and Wall, K.D. (1982), "Kalman Filtering Estimation of Unobserved Rational Expectations with an Application to the German Hyperinflation', *Journal of Econometrics,* 20, 255-284.

Burridge, P. and Wallis, K.F. (1984), 'Unobserved-Components Models for Seasonal Adjustment Filters', *Journal of Business and Economic Statistics*, 2, 350-359.

Burridge, P. and Wallis, K.F. (1985), 'Calculating the Variance of Seasonally Adjusted Series', *Journal of the American Statistical Assosiation*, 80, 391, 541-552.

Caines, P.E. (1988), *Linear Stochastic Systems*, New York, Wiley.

Cecchetti, S.G. (1996), 'Measuring Short-Run Inflation for Central Bankers', *Economic Review of the Federal Reserve Bank of St. Louis,* NBER Working Paper 5786.

Chan, N.H. and Tran, L.T. (1989), 'On the First-Order Autoregressive Process with Infinite Variance', *Econometric Theory*, 5, 354-362.

Charemza, W.W. and Kominek, Z. (2000), 'Thick tails and speculative processes: a simulation and empirical analysis', University of Leicester, mimeo.

Charemza, W.W. and Makarova, S. (2002), 'Blini, version 1.03: Collection of GAUSS procedures for linear and bilinear unit root analysis', European University at St. Petersburg, mimeo.

Charemza, W.W. and Syczewska, E.M. (1998), 'Joint application of the Dickey-Fuller and KPSS tests', *Econometrics Letters*, 61, 17-21.

Charemza, W.W., Lifshits, M. and Makarova, S. (2002a), 'Behaviour of statistical aggregates under small bilinearity', State University of St. Petersburg, mimeo.

Charemza, W.W., Lifshits, M. and Makarova, S. (2002b), 'A simple test for unit root bilinearity', *to be presented on the 57th Econometric Society European Meeting.*

Charemza, W.W., Lifshits, M. and Makarova, S. (2002c), 'The problem of year 2000 on the financial markets: break or bilinearity?', University of Leicester, mimeo.

Chobanov, G., Mateev, P., Mittnik, S. and Rachev, S.T. (1996), 'Modeling the Distribution of Highly Volatile Exchange-Rate Time Series', *Time Series Analysis*, Eds.

Robinson, P. and Rosenblatt, M., Athens Conference on Applied Probability and Time Series, Springer, New York.

Chui, C.K. (1992a), *Wavelets: A Tutorial in Theory and Applications*, Academic Press, San Diego.

Chui, C.K. (1992b), *An Introduction to Wavelets*, Academic Press, New York.

Claus, I. (1997), 'A measure of underlying inflation in the United States', Bank of Canada Working Paper, 97-120.

Clements, M.P. and Mizon, G.E. (1991), 'Empirical Analysis of Macroeconomic Time Series', *European Economic Review*, 35, 887-932.

Cooley, T.F. and Prescott, E.C. (1973), 'An Adaptive Regression Model', *International Economic Review*, 14, 364-371.

Corana, A., Marchesi, M., Martini, C. and Ridella, S. (1987), 'Minimizing Multimodal Functions of Continuous Variables with the "Simulated Annealing" Algorithm', *ACM Transactions on Mathematical Software,* 35, 262-280.

Culver, S.E. and Papell, D.H. (1997), 'Is there a unit root in the inflation rate? Evidence from sequential break and panel data models', *Journal of Applied Econometrics,* 12, 435-444.

Cuthbertson, K. (1988), 'Expectations, Learning and the Kalman Filter', *The Manchester School,* LVI, 3, 223-246.

Cuthbertson, K. and Taylor, M. (1987), *Macroeconomic systems*, Oxford: Basil Blackwell.

Dagum, E.B. (1988), X-11-ARIMA /88 Seasonal Adjustment Method - Foundations and Users Manual, 12-564E, Statistics Canada Catalogue.

Daubechies, I. (1992), *Ten Lectures on Wavelets*, Rutgers University and AT&T Bell Laboratories, Society for Industrial and Applied Mathematics, Philadelphia, Pennsylvania.

Dawson, G. (1992), *Inflation and Unemployment: Causes, Consequences and Cures,* Edward Elgar.

Demery, D., Duck N., Sumner M., Thomas R. and Thompson W., (1984), *Macroeconomics*, Longman.

Dickey, D.A. and Fuller, W.A. (1979), 'Distribution of the Estimators for Autoregressive Time Series with a Unit Root', *Journal of the American Statistical Assosiation,* 74, 427- 431.

Dickey, D.A. and Fuller, W.A. (1981), 'A Likelihood Ratio Statistics for Autoregressive Time Series with a Unit Root', *Econometrica*, 49, 4, 1057-1072.

DuMouchel, W. (1971), 'Stable Distributions in Statistical Inference', *PhD thesis*, University of Ann Arbor, Ml.

Eckstein, O. (1981), *Core Inflation,* Prentice-Hall, Engelewood Cliffs, N.J.

Engle, R.F. (1979), 'Estimating Structural Models of Seasonality', *Seasonal Analysis of Economics Time Series,* Ed. A. Zellner, US Bureau of Census, Washington, DC.

Engle, R.F. (1982), 'Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation', *Econometrica*, 50, 4, 978-1008.

Engle, R.F. (1983), 'Estimates of the variance of US inflation based on the ARCH model', *Journal of Money, Credit and Banking,* 15, 286 – 301.

Engle, R.F. and Granger, C.W.J. (1987), 'Co-Integration and Error Correction: Representation, Estimation, and Testing', *Econometrica,* 55, 251-276.

Engle, R.F. and Watson, M.W. (1987), *Advances in Econometrics: Fifth World Congress,* Vol. 1, Cambridge University Press.

Engsted, T. (1995), 'Does the Long-Term Interest Rate Predict Future Inflation? A Multi-Country Analysis', *The Review of Economics and Statistics,* 77, 42-54.

Ericsson, N.R., Hendry, D.F. and Tran, H.A. (1993), 'Cointegration, Seasonality, Encompassing, and the Demand for Money in the United Kingdom', International Finance Discussion Paper, 457.

Evans, M. (1991), 'Discovering the link between inflation rates and inflation uncertainty', *Journal of Money, Credit and Banking,* 23(2), 169 - 184.

Evans, M. and Wachtei, P. (1993), 'Inflation regimes and the sourses of inflation uncertainty', *Journal of Money, Credit and Banking,* 25(3), 475 – 511.

Fama, E. and Roll, R. (1971), 'Parameter Estimates for Symmetric Stable Distributions', *Journal of the American Statistical Association*, 66, 331-338.

Fase, M. and Folkertsma, C. (1996), 'Measuring inflation: an attempt to operationalize Carl Menger's concept of the inner value of money', De Nederlandsche Bank Working Paper.

Feller, W. (1966), *An Introduction to Probability Theory and Its Applications*, John Wiley & Sons, New York.

Feuerverger, A. and McDunnough, P. (1981), 'On Efficient Inference in Symmetric Stable Laws and Processes', *Statistics and Related Topics*, Eds. Gsörgö, M. et al., Amsterdam, North-Holland.

Fisher, S. (1977), 'Long-Term Contracts, Rational Expectations, and the Optimal Money Supply Rule', *Journal of Political Economy,* 85, 191 – 205.

Fisher, S. (1981), 'Towards an understanding of the cost of inflation', in *The cost and consequences of inflation*, Eds. Brunner, K and Meltzer, A, Carnegie-Rochester Conference Series on Public Policy, vol. 15, Amsterdam: North Holland.

Fishman, G. (1996), *Monte Carlo: Concepts, Algorithms and Applications*, Springer, New York.

Flemming, J. (1976), *Inflation*, Oxford University Press (London).

Foster, E. (1978),'The variability of inflation', *Review of Economics and Statistics'*, 60, 364-340.

Fountas, S., Karanasos, M. and Karanassou, M. (2000), 'A GARCH Model of Inflation Uncertainty with Simultaneous Feedback', Department of Economics Paper 414, Queen Mary and Westfield College, London.

Friedman, M. (1968), 'The Role of Monetary Policy', *American Economic Review*, 58, 1 – 17.

Fuller, W.A. (1976), *Introduction to Statistical Time Series,* New York, Wiley.

Garbade, K. (1977), 'Two methods of examining the stability of regression coefficients', *Journal of the American Statistical Association,* 72, 54-63.

Gartner, C. and Wehinger, D. (1998), 'Core Inflation in Selected European Union Countries', Oesterreichische Nationalbank, Working paper 33.

Gevers, M. and Wertz, V. (1984), 'Uniquely identifiable State-Space and ARMA Parameterisations for Multivariable linear Systems', *Automatica*, 20, 333-347.

Ghosh, D. (1998), 'Maximum Likelihood Estimation of the Dynamic Shock-Error Model', *Journal of Econometrics*, 41, 121-143.

Goffie, W.L., Ferrier, G.D. and Rogers, J. (1994), 'Global optimisation of statistical functions with simulated annealing', *Journal of Econometrics*, 60, 65-99.

Gordon, R., 'The Boskin Commission Report and its Aftermath', in Silver, M. and Fenwick, D. (20000), *Proceedings of the Measurement of Inflation Conference*, Cardiff University.

Grahn, T. (1993), 'A Conditional Least Square Approach to Bilinear Time Series Estimation', *Journal of Time Series Analysis*, 16, 5, 509-529.

Granger, C. and Hatanaka, M. (1964), *Spectral Analysis of economic time series*, Princeton University Press, Princeton, New Jersey.

Granger, C.W.J. and Andersen, A.P. (1978), *An introduction to bilinear time series models*, Vandenhoeck and Ruprecht, Gottingen.

Gray, A. (1976), 'Wage Indexation: A Macroeconomic Approach', *Journal of Monetary Economics*, 2, 221-235.

Greenblatt, S. (1994), 'Wavelets in Econometrics: An Application to Outlier Testing', Centre for Quantitative Economics and Computing, University of Reading.

Greenwald, W. (1963), *Statistics for Economics*, Charles E. Merrill Books, Inc., Columbus, Ohio.

Hall, S.G. (1986), 'An application of the Granger and Engle Two-Step Estimation Procedure to United Kingdom Aggregate Wage Data', *Oxford Bulletin of Economics and Statistics*, 48, 229-239.

Hamilton, J.D. (1994), *Time Series Analysis*, Princeton University Press, New Jersey.

Hannan, J.D. (1971), 'The identification Problem for Multiple Equation Systems with Moving Average Errors', *Econometrica*, 39, 751-765.

Harvey, A.C. (1981), *Time Series Models*, Philip Allan.

Harvey, A.C. (1984), 'A unified view of statistical forecasting procedures', *Journal of forecasting*, 3, 245-283.

Harvey, A.C. (1989), *Forecasting, structural time series models and the Kalman filter*, Cambrage University Press.

Harvey, A.C. and Todd, P.H.J. (1983), 'Forecasting economic time series with structural and Box-Jenkins models', *Journal of Business and Economic Statistics*, 1, 299-315.

Harvey, A.C., and Phillips, G.D.A. (1979), 'The estimation of regression models with autoregressive-moving average disturbances', *Biometrica*, 66, 49-58.

Hausman, J.A. and Watson, M.W. (1985), 'Errors in variablesand seasonal adjustment procedures', *Journal of the American Statistical Association*, 80, 541-552.

Hendry, D.F. (1984), 'Monte Carlo Experimentation in Econometrics', *Handbook of*

Econometrics, Volume 2, Eds. Z. Griliches and M.D. Intriligator, Elsevier Science Publishers B.V.

Hendry, D.F. (1995), *Dynamic econometrics*, Oxford University Press.

Higgins, M.L. and Bera, A.K. (1988), 'A Joint Test for ARCH Bilinearity In the Regression Model', *Econometric Reviews*, 7(2), 171-181.

Hill, B.M. (1975), 'A Simple Approach to Inference about the Tail of a Distribution', *Annals of Statistics*, 3, 1163-1174.

Hogg, R. (1967), 'Some observations on Robust Estimation', *Journal of the American Statistical Association*, 62, N320, 1179-1186, December.

Hylleberg, S. (1992), *Modelling Seasonality*, Oxford University Press, New York.

Jaramillo, C.F. (1998), 'Measuring Inflation Using Asymmetric Means', Banco de la Republica Colombia.

Johansen, S. (1992), 'Testing weak exogeneity and the order of cointegration in UK money demand data', *Journal of Policy Modeling,* 14, 3, 313-334.

Johansen, S. and Juselius, K. (1992), 'Some Structural Hypotheses in a Multivariate Cointegration Analysis of the Purchasing Power Parity and the Uncovered Interes Parity for UK', *Journal of Econometrics,* 53, 211-244.

Kalman, R.E. (1960), 'A New Approach to Linear Filtering and Prediction Problems', *Journal of Basic Engineering, Transactions of the ASME,* Series D, 4, 35-45.

Kim, W.K., Billard, L. and Basawa, I.V. (1990), 'Estimation for the First -order Diagonal Bilinear Time Series Model', *Journal of Time Series Analysis*, 11, 3, 215-229.

Kirkpatrick, S., Gelatt C.D. and Vecchi, M.P. (1983), 'Optimization by simulated annealing', *Science 220*, 4598, 671-680.

Kwiatkowski, D., P.C.B. Phillips, P. Schmidt and Y. Shin (1992), 'Testing the null hypothesis of stationarity against the alternative of a unit root', *Journal of Econometrics,* 54, 159-178.

Laumas, G.S. and Mehra, Y.P. (1976), 'The Stability of the Demand for Money Function: The Evidence from Quarterly Data', *Review of Economics and Statistics,* 58, 464-468.

Lawson, T. (1984), 'Generalized Adaptive Expectations', *Mathematical Methods in Economics*, ed. F. van der Ploeg, New York, J.Wiley and Sons.

Lee, K. *Lectures in Advanced Macroeconomics*, Department of Economics, University of Leicester (1999-2000).

Leybourne, S.J. (1995), 'Testing for unit roots using forward and reverse Dickey – Fuller regressions', *Oxford Bulletin of Economics and Statistics',* 57, 559-571.

Ljung, L. and Caines, P.E. (1979), 'Asymptotic normality and prediction error estimators for approximate system models', *Stochastics*, 3, 29-46.

Logue, D. and Willet, T. (1976), 'A nite on the relationship between the rate and variability of inflation', *Economica*, 43, 526 – 534.

Lucas, R.E. (1972), 'Expectations and the neutrality of money', *Journal of Economic theory,* 4, 103 – 124.

Lucas, R.E. (1973), 'Some international evidence on output-inflation tradeoffs', *American Economic Review*, 63, 326-34.

Lucas, R.E. (1977), 'Understanding Business Cycles', *Stabilisation of the Domestik and International Economy*, vol. 5 of Carnegie – Rochester Conference on Public Policy, Amsterdam: North – Holland.

Lucas, R.E. (1993), 'On the welfare cost of inflation', Hitotsubashi International Symposium on Financial Markets in the Changing World, March.

Maddala, G.S. and Kim, I.M. (1998), *Unit Roots, Cointegration and Structural Change,* Cambridge University Press.

Maddison A., (2002), *The world Economy: a millennial perspective*, Development Centre Studies, OECD.

Mallat, S. (1989), *Multiresolution approximation and wavelets*, Trans. Amer. Math., Soc.

Mankiw, G. (2000), *Macroeconomics*, fourth edition, Harvard University, Worth Publishers.

Markidakis, S., Wheelwright, S. and Hyndman, R. (1998), *Forecasting: Methods and Applications,* 3rd ed., John Wiley & Sons.

Mayer, Y. (1993), *Wavelets: Algorithms and Applications,* Society for Industrial and Applied Mathematics, Philadelphia.

McCulloch, J.H (1986), 'Simple Consistent Estimators of Stable Distribution Parameters', *Comunications in Statistics-Computation and Simulation,* 15, 1109-1136.

Minford, A.P.L. and Hilliard, G.W. (1978), 'The cost of Variable Inflation', in M. Artist and A. R. Nobay (eds.), *Contemporary Economic Analysis,* London: Croom Helm.

Mishkin F.S. (1992), 'Is the Fisher effect for real? A reexamination of the relationship between inflation and interest rates', *Journal of monetary economics*, Nov 1992, 30, No.2, 195- 215.

Mittnik, S. and Rachev, S.T. (1996), 'Tail Estimation of the Stable Index α ', *Applied*

Mathematics Letters, 9, 53-56.

Mittnik, S., Paolella, M.S. and Rachev, S.T. (1998), 'Unconditional and Conditional Distributional Models for the Nikkei's Index', *Asia-Pacific Financial Markets*, 5, 99-128.

Mohler, R.R. (1973), *Bilinear Control Processes*, Academic Press, New York.

Nelson, C.R. and Plosser, C.I. (1982), 'Trends and Random Walks in Macroeconomic Time Series', *Journal of Monetary Economics*, 10, 139-162.

Nelson, C.R. and Schwert, G.W. (1977), 'Short-term interest rates as predictors of inflation: on testing the hypothesis that the real rate of interest is constant', *American Economic Review*, 67, 478-486.

Nolan, J.P. (1997), 'Maximum Likelihood Estimation of Stable Parameters', *Unpublished manuscript.*

Okun, A. (1970), 'Potential GNP: its measurement and significance', Appendix in *Political Economy of Prosperity*, Washington, D.C.: Brookings Institution.

Okun, A. (1971), 'The mirage of steady inflation', *Brookings Paper Economic Analysis*, 2, 435 – 498.

Pagan, A. (1980), 'Some Identification and Estimation Results for Regression Models with Stochastically Varying Coefficients', *Journal of Econometrics*, 13, 341-363.

Pagan, A.R. (1975), 'A Note on the Extraction of Components from Time Series', *Econometrica*, 43, 163-168.

Peel, D. and Davidson, J. (1998), 'A non-linear error correction mechanism based on the bilinear model', *Econometric Letters*, 58, 165-170.

Phillips, P.C.B. (1987), 'Time Series Regression with a Unit Root', *Econometrica*, 55, 277-301.

Phillips, P.C.B. and Perron, P. (1988), 'Testing for a Unit Root in Time Series Regression', *Biometrika*, 75, 335-346.

Pickands, J. (1975), 'Statistical Inference Using Extreme Order Statistics', *The Annals of Statistics*, 3, 119-131.

Poirier, D.J. (1995), *Intermediate Statistics and Econometrics*, The MIT Press, Cambridge, Massachusetts.

Price, S. *Lectures in Modern Financial System*, Department of Economics, University of Leicester (1999-2000).

Quah, D. and Vahey, S.P. (1995), 'Measuring Core Inflation, *The Economic Journal*, 105, 1130-1144.

Rachev, S. and Mittnik, S. (2000), *Stable Paretian Models in Finance*, John Wiley & Sons, Chichester.

Rachev, S.T., Mittnik, S. and Kim, J.-R. (1998), 'Time series with unit roots and infinite-variance disturbances', *Appl. Math. Lett.,* 11, 69-74.

Rao, M.M. (1961), 'Consistency of Limiting Distributions of Estimators of Paramethers in Explosive Stochastic Difference Equations', *Annals of Mathematical Statistics,*32, 195-218.

Rausser, G.C. and Laumas, P.S. (1976), 'The Stability of the Demand for Money in Canada', *Journal of Monetary Economics,* 2, 367-380.

Rieken, S., Mittnik, S. and Rachev, S.T. (1998), 'Modelling and Forecasting the DAX-Option Market', *Technical Report*, University of Karlsruhe.

Roger, S. (1995), 'Measures of underlying inflation in New Zealand, 1981-95', Reserve Bank of New Zealand Discussion Paper G95/5.

Roger, S. (1998), 'Core inflation: concepts, uses and measurement', Reserve Bank of New Zealand Discussion Paper G98/9.

Romer, D. (1995), *Advanced Macroeconomics*, McGraw Hill – Brace.

Rosenberg, B. (1972), 'The Estimation of Stationary Stochastic Regression Parameters Re-examined', *Journal of the American Statistical Association*, 67, 650-654.

Rotemberg, J. (1982), 'Monopolistic Price Adjustment and Aggregate Output', *Review of Economic Studies,* 49, 517 – 531.

Rothenberg, T.J. (1971), 'Identification in Parametric Models', *Econometrica*, 39, 577-91.

Rubin, H. (1950), 'Consistency of maximum likelihood estimatesin the explosive case', in T.C. Koopmans (ed.) *Statistical Inference in Dynamic Economic Models,* Wiley, New York, 356-364.

Schweppe, F. (1965), 'Evaluation of likelihood functions for Gaussian signals', *IEEE Transactions on Information Theory*, 11, 61-70.

Schwert, G.W. (1987), 'Effects of model specification on tests for unit roots in macroeconomic data', *Journal of monetary economics*, 20, No.1, 73-103.

Sesay, S.A.O. and Subba Rao, T. (1986), 'Yule-Walker type difference Equations for higher-order moments and cumulants for bilinear time series models', *Journal of Time Series Analysis*, 9, 4, 385-401.

Shiller, R., (1996), 'Why Do People Dislike Inflation?', NBER, Working Paper 5539, April.

Shiskin, J., Young, A.H. and Musgrave, J.C. (1967), *The X-11 Variant of the Census*

Method II Seasonal Adjustment Prigram, Bureau of the Census Technical Paper No. 15, Washington D. C.: U.S. Department of Commerce.

Shwert, G.W., (1987), *'Effects of model specification on tests for unit roots in macroeconomic data'*, Journal of Monetary Economics, 20, 73-103.

Sims, C.A. (1974), 'Seasonality in regression', *Journal of the American Statistical Association,* 69, 618-627.

Stevenson, A., Muscateli, V. and Gregory, M. (1988), *Macroeconomic Theory and Stabilisation Policy*, Philip Allan.

Subba Rao, T. (1981), 'On the theory of Bilinear Time Series Models', *J. R. Statist. Soc.*, 43, No2, 244-255.

Subba Rao, T. (1997), 'Statistical analysis on nonlinear and non Gaussian time series', *Stochastic differential and difference equations*, Eds. Ciszar, I. And Gy Michaletzky, Birkhauser, Boston.

Subba Rao, T. and Gabr, M.M. (1974), *An introduction to bispectral analysis and bilinear time series models*, Springer, New York.

Subba Rao, T. and Gabr, M.M. (1984), *An introduction to bispectral analysis and bilinear time series models,* Series *Lecture Notes in Statistics* vol.24, Springer, New York.

Terdik, G. (1999), *Bilinear stochastic models and related problems of nonlinear time series analysis : a frequency domain approach*, Springer, New York.

Tobin, J. (1972), 'Inflation and Unemployment', *American Economic Review,* 62, 1-18.

Tsyplakov, A. (2002), 'Constructing Core Inflation Index for Russia', *Interim Report,* Economics Education and Research Consortium – Russia and CIS.

Turnovsky, J. (1995), *Methods of Microeconomic Dynamic,* 2nd ed., MIT Press.

Wall, K.D. (1987), 'Identification Theory for Varying Coefficient Regression Models', *Journal of Time Series Analysis*, 8, 359-71.

Wallis, K.F. (1974), 'Seasonal Adjustment and Relations Between Variables', *Journal of the American Statistical Association,* 69, Issue 345, 18-31.

Wallis, K.F. (1982), 'Seasonal Adjustment and Revision of Current Data: Linear Filters for the X-11 Method', *J. R. Statist. Soc.,* 145, 74-85.

Ward, M. and Dikhanov, Y. (1999), 'What is Inflation?', World Bank, mimeo.

Weiss, A.A (1986), 'ARCH and bilinear time series models: comparison and combination', *Journal of Business and Economic Statistics,* 4, 59-70.

White, J.S. (1958), 'The Limiting Distributionof the Serial Correlation Coefficient in the Explosive Case', *Annals of Mathematical Statistics,* 29, 1188-1197.

Wozniak, P. (1999),'Various Measures of Underlying Inflation in Poland 1995-1998', Working Paper Series 25, Centre for Social and Economic Research, Warsaw.

Zolotarev, V.M. (1957), 'Mellin-Stieltjes transforms in probability theory', *Theory of Probability and Its Application*, 2, 433-460.

Zolotarev, V.M. (1966), 'On Representation of the Stable Laws by Integrals', Selected Translations on Mathematical Statistics and Probability, 6, 84-88, American Mathematical Society, Povidence, Rhode Island.

Zolotarev, V.M. (1986), *One-dimentional Stable Distributions*, Translations of Mathematical Monographs, 65, American Mathematical Society, Providence.

Харемза В., Макарова С. и Пархоменко, В. (2002), 'Базовая инфляция и методы её прогнозирования', *Економические исследования: теория и приложения*, Выпуск 2, С.-Петербург.