An integrated and quantitative approach to petrophysical heterogeneity.

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Abstract

Exploration in anything but the simplest of reservoirs is commonly more challenging because of the intrinsic variability in rock properties and geological characteristics that occur at all scales of observation and measurement. This variability, which often leads to a degree of unpredictability, is commonly referred to as "heterogeneity", but rarely is this term defined. Although it is widely stated that heterogeneities are poorly understood, researchers have started to investigate the quantification of various heterogeneities and the concept of heterogeneity as a scale-dependent descriptor in reservoir characterization.

Based on a comprehensive literature review we define "heterogeneity" as the variability of an individual or combination of properties within a specified space and / or time, and at a specified scale. When investigating variability, the type of heterogeneity should be defined in terms of grain - pore components and the presence or absence of any dominant features (including sedimentological characteristics and fractures). Hierarchies of geologic heterogeneity can be used alongside an understanding of measurement principles and volumes of investigation to ensure we understand the variability in a dataset.

Basic statistics can be used to characterise variability in a dataset, in terms of the amplitude and frequency of variations present. A better approach involves heterogeneity measures since these can provide a single value for quantifying the variability, and provide the ability to compare this variability between different datasets, tools / measurements, and reservoirs. We use synthetic and subsurface datasets to investigate the application of the Lorenz Coefficient,

Dykstra-Parsons Coefficient and the coefficient of variation to petrophysical data – testing assumptions and refining classifications of heterogeneity based on these measures.

Keywords

Heterogeneity, quantifying, reservoir, petrophysics, statistics

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Introduction

Petrophysics is the study of the (physical and chemical) rock properties and their interactions with fluids (Tiab & Donaldson 2004). We can define a number of petrophysical properties, for example porosity, saturation, and permeability, and many of these depend on the distribution of other properties such as mineralogy, pore size, or sedimentary fabric, and on the chemical and physical properties of both the solids and fluids. Consequently petrophysical properties can be fairly constant throughout a homogeneous reservoir or they can vary significantly from one location to another, in an inhomogeneous or heterogeneous reservoir. This variation would be relatively easy to describe if petrophysical analysis was only applied at a single scale and to a constant measurement volume within the reservoir. While many petrophysical measurements are typically made in the laboratory at a core plug scale (cm) or within the borehole at a log scale (m), fluid distribution is controlled at the pore scale (nm to mm) by the interaction of fluids and solids through wettability, surface tension and capillary forces, at the core scale by sedimentary facies, fabrics or texture (mm to m), and at bed-to-seismic scales by the architecture and spatial distribution of geobodies and stratigraphic elements (m to kms). Note we use the words fabric and texture here to indicate generic spatial organisation or patterns. At each scale of measurement various heterogeneities may exist, but it is important to note that a unit which appears homogeneous at one scale may be shown to be heterogeneous at a finer-scale, and vice versa. Clearly, as more detailed information is obtained, reservoir characterisation and the integration of the various data types can become increasingly complex. It is important to fully understand the variability and spatial distribution of petrophysical properties, so that we can understand whether there is any pattern to the variability, and appreciate the significance of simple averages used in geologic and simulation modelling. This is especially true in the case of complex hydrocarbon reservoirs that have considerable variability. Carbonate reservoirs often fall into this category, and the term heterogeneous is often used to describe a reservoir that is complex and evades our full understanding. Indeed, an early definition states heterogeneous as meaning extraordinary, anomalous, or abnormal (Oxford English Dictionary; Simpson & Weiner 1898).

Most, if not all, of the literature on reservoir characterisation and petrophysical analysis refers to the heterogeneous nature of the reservoir under investigation. Heterogeneity appears to be a term that is readily used to suggest the complex nature of the reservoir, and authors often assume the reader has a pre-existing knowledge and understanding of such variability. No single definition has been produced and consistently applied. Researchers have started to investigate the quantification of various heterogeneities and the concept of heterogeneity as a scale-dependent descriptor in reservoir characterization (Frykman 2001; Jennings & Lucia 2003; Pranter et al. 2005; Westphal et al. 2004).

Here we review what heterogeneity means, and how it can be described in terms of geological attributes before discussing how the scale of geological heterogeneity can be related to the measurement volumes and resolution of traditional subsurface data types. We then discuss using a variety of statistical techniques for characterising and quantifying heterogeneity, focussing on petrophysical heterogeneities. We focus here on the principles and controls on the statistics and measures, before applying these to real reservoir data in four case studies. In doing so, we consider approaches used in a range of scientific disciplines (primarily the environmental sciences and ecology) to explore definitions and methods which may be applicable to petrophysical analysis. These statistical techniques are then applied to reservoir sub-units to investigate their effectiveness for quantifying heterogeneity in reservoir datasets.

Defining Heterogeneity

Heterogeneity refers to the quality or condition of being heterogeneous, and was first defined in 1898 as difference or diversity in kind from other things, or consisting of parts or things that are very different from each other (Oxford English Dictionary; Simpson & Weiner 1989). A more modern definition is something that is diverse in character or content (Oxford Dictionaries, 2014). This broad definition is quite simple and does not comment on the spatial and temporal components of variation, nor does it include a consideration of directional dependence, often referred to as isotropy and anisotropy. Other words or terms that may be used with, or instead of, heterogeneity include; complexity, deviation from a norm, difference, discontinuity, randomness, and variability.

Nurmi et al. (1990) suggest that the distinction between homogeneous and heterogeneous is often relative, and is based on economic considerations. This highlights how heterogeneity is a somewhat variable concept which can be changed or re-defined to describe situations that arise during production from a reservoir, and is heavily biased by the analyst's experience and expectations. Li and Reynolds (1995) and Zhengquan et al. (1997) state that heterogeneity is defined as the complexity and/or variability of the system property of interest in three-dimensional space, while Frazer et al. (2005) define heterogeneity, within an ecological model, as variability in the density of discrete objects or entities in space. These definitions suggest that heterogeneity does not necessarily refer to the overall system, or individual rock/reservoir unit, but instead may be dealt with separately for individual units, properties, parameters and measurement types.

Frazer et al. (2005) commented that heterogeneity is an inherent, ubiquitous and critical property that is strongly dependent on scales of observation and the methods of measurement used. They studied forest canopy structure and stated that heterogeneity is the degree of departure from complete spatial randomness towards regularity and uniformity. This may seem, at first, counterintuitive because heterogeneity is commonly regarded as being complete spatial randomness. Here, the introduction of regular features, such as bedding in a geological context, adds to the heterogeneous nature of the formation in a structured or anisotropic manner. Nurmi et al. (1990) suggest that heterogeneity, in electrical borehole images, refers to elements that are distributed in a non-uniform manner or composed of dissimilar elements/constituents within a specific volume. Therefore, as well as looking at a specific element or property, it is also suggested that the volume of investigation influences heterogeneity, alluding to the scale-dependence of heterogeneities. Interestingly, Dutilleul (1993) comments that a shift of scale may create homogeneity out of heterogeneity, and vice-versa, and suggests that heterogeneity is the variation in density of measured points compared to the variation expected from randomly spread points. In a discussion of the relationship between scale and heterogeneity in pore size, Dullien (1979) suggests that to be a truly homogeneous system random subsamples of a population should have the same local mean values. Lake and Jensen (1991) provide a flow-based definition in their review of permeability heterogeneity modelling within the oil industry. In this latter case, heterogeneity is defined as the property of the medium that causes the flood front to distort and spread as displacement proceeds; in this context the medium refers to the rock, and fluid front is the boundary between displacing and displaced fluids. Thus many authors provide the foundation in which we begin to see that heterogeneity may be a quantifiable term.

Pure homogeneity, with regard to a reservoir rock, can be visualised in a formation that consists of (1) a single mineralogy with (2) all grains of similar shapes and sizes with (3) no spatial organization or patterns present; in this example, similar grain shapes and sizes, together with lack of spatial patterns would lead to a uniform distribution of porosity and permeability. Therefore, ignoring the scalar component of heterogeneity for a moment, there are two contrasting examples of heterogeneity in a reservoir rock (Figure 1). The first example is a formation of consistent mineralogy and grain characteristics that has various spatial patterns (for example bedding, foresets, syn-sedimentary faulting, or simply grain packing). The second example has no spatial organisation (it is massive) but has variable mineralogy and grain size and shape, i.e. it is a poorly sorted material. Both are clearly not homogeneous but which has the stronger heterogeneity? Quantifying the degree of heterogeneity would enable these two different systems to be differentiated from each other, and in turn these values may be related to other characteristics such as reservoir quality. In attempting to quantify heterogeneity we can consider several approaches. It is probably best, however, to start by defining the degree of heterogeneity in relation to the nature of the investigation; for example in a study of fluid flow, sedimentological structures may be of more importance than variation in mineralogy. In contrast in an investigation of downhole gamma ray variability the mineralogical variability (or strictly chemical variability of potassium, thorium and uranium) would be more relevant than any spatial variation.

Lake and Jensen (1991) suggest that there are five basic types of heterogeneity in earth sciences; (1) Spatial - lateral, vertical and three-dimensional, (2) Temporal - one point at different times, (3) Functional - taking correlations and flow-paths into account, (4) Structural

- either unconformities or tectonic elements, such as faults and fractures, and (5) Stratigraphic. Formations may have regular and penetrative features such as bedding and cross-bedding, or alternatively less regularly distributed features, including ripples, hummocky cross-bedding, and bioturbation. The intensity, frequency and orientation of such features may additionally reflect repetition or repetitive patterns through the succession. A heterogeneity, in terms of the grain component, may appear rhythmic or repeated, patchy, gradational / transitional, or again it may be controlled by depositional structures (Nurmi et al. 1990).

Homogeneity and heterogeneity can be considered as end members of a continuous spectrum, defining the minimum and maximum heterogeneity, with zero heterogeneity equating to homogeneity. There are a number of characteristics that occur in both end-member examples provided above (for example vertical rhythmicity in terms of bedding or grain size distribution). Neither end-member is obviously more heterogeneous than the other; there may indeed be a relative scale difference between the two examples. Some researchers may perceive a regularly structured system, for example a laminated or bedded reservoir, as homogeneous because these structures are spatially continuous and occur throughout the formation. The presence of structures within a formation is, however, more commonly interpreted as a type of heterogeneity, regardless of how regular their distribution. In this scenario, the structures represent deviation from the homogeneous mono-mineralic ‘norm’. Equally the concept of increased heterogeneity could be viewed as an increase in the random mixing of components of a formation. Here, as the formation becomes more heterogeneous there is less spatial organization present, so that the formation has the same properties in all directions, i.e., it is isotropic. Although the rock is more heterogeneous, the actual reservoir properties (such as the porosity distribution) become more homogeneous throughout the reservoir as a whole.

If grain-size alone varies, two possible extremes of heterogeneity may occur. An example where there is a complete mix of grain sizes that show no evidence of sorting would be classified as a heterogeneous mixture in terms of its components. The mixture itself would appear isotropic, however, because on a larger-scale the rock properties would be the same in all directions (in the sense of a transverse isotropic medium). If this mixture of grain sizes was completely unsorted then the grains would be completely randomly distributed and the rock would appear homogeneous at a larger scale. In another example where a formation has continuous and discontinuous layers of different grain sizes, the individual layers of similar grain size may appear homogeneous, however if looking at a contact between two layers, or the complete formation, then the heterogeneity will be much more obvious. This may be classed as a 'structural' or 'spatial' heterogeneity, again depending upon the scale of investigation.

When defining a measure of how heterogeneous a system property is, it is important to consider only those components of heterogeneity that have a significant impact on reservoir properties and production behaviour / reservoir performance. This leads to the discussion of heterogeneity as a scale-dependent descriptor in the next section.

Scale and measurement resolution

Regardless of reservoir type, geological heterogeneity exists across a gradational continuum of scales (Nichols 1999; Moore 2001). Observations from outcrop analogues have been used to characterise and quantify these features (examples for carbonate outcrops include Mutti et al. 1996; Pomar et al. 2002; Badenas et al. 2010; Cozzi et al. 2010; Koehrer et al. 2010; Palermo et al, 2010; Pierre et al. 2010; Amour et al. 2012). Hierarchies of heterogeneity are now frequently used to classify these heterogeneities over levels of decreasing magnitude within a broad stratigraphic framework. Heterogeneity hierarchies have been developed for wave-influenced shallow marine reservoirs (e.g. Kjønsvik et al. 1994; Sech et al. 2009), fluvial reservoirs (e.g. Jones et al. 1995), fluvio-deltaic reservoirs (e.g. Choi et al. 2011), and carbonate reservoirs (e.g. Jung & Aigner 2012). These hierarchies break the continuum of scales of geologic and petrophysical properties into key classes or ranges.

A single property can differ across all scales of observation. Porosity in carbonates is an example of a geological property that can exist, and vary, over multiple length-scales. In carbonate rocks pore size can be seen to vary from less than micrometre-size micro-porosity (e.g., North Sea chalks; Brasher & Vagle 1996) to millimetre-scale inter-particle and crystalline porosity (e.g., carbonate reservoirs of the Middle East, Lucia 1995, Ramamoorthy et al. 2008; offshore India, Akbar et al. 1995; and the microbialite build-ups of offshore Brazil, Rezende et al. 2013) . Vugs are commonly documented to vary in size from millimetre to tens of centimetres (e.g., Nurmi et al. 1990). Additional dissolution and erosion may create huge caves, or “mega-pores” (often being metres to kilometres in size, e.g., Akbar et al. 1995; Kennedy 2002).

In order to investigate heterogeneity at different scales and resolutions, the concept of "scale" and how it relates to different parameters is considered. Figure 2 illustrates the scales of common measurement volumes and their relationship to geological features observed in the subsurface. While geological attributes exist across the full range of length-scale (mm – km scale; e.g. van Wagoner et al. 1990; Jones et al. 1995; Kjønsvik et al. 1994; Frykman and Deutsch 2002; Sech et al. 2009; Choi et al. 2011; and Jung & Aigner 2012), subsurface measurements typically occur at specific length-scales depending upon the physics of the tool used. For example, seismic data at the kilometre scale, well logs at the centimetre to metre scale, and petrophysical core measurements at millimetre to centimetre scales. In general the insitu borehole and core measurement techniques are considered to interrogate a range of overlapping volumes, but in reality a great deal of "white space" exists between individual measurement volumes (Figure 2). How a measurement relates to the scale of the underlying geological heterogeneity will be a function (and limitation) of the resolution of the measurement device or tool used. The analyst or interpreter should ensure that appropriate assumptions are outlined and documented.

The issue of how the scale and resolution of a measurement will be impacted by heterogeneity can be represented through the concept of a Representative Elementary Volume (REV) to characterise the point when increasing the size of a data population no longer impacts the average, or upscaled, value obtained (Bear 1972, Bachmat & Bear 1987). The REV concept lends itself to an extensive discussion on upscaling and the impact of heterogeneity on flow behaviour, which are beyond the current scope of this study. Examples of previous studies into REV, sampling and permeability heterogeneity include Haldorsen (1986), Corbett et al. (1999), Nordahl & Ringrose (2008), Vik et al. (2013).

Different wireline log measurements, for example, will respond to, and may capture, the different parts or scales of geological heterogeneity (Figure 2C and 3). The geological features that exist below the resolution of tools shown in Figure 2 will in effect be averaged out in the data (Ellis & Singer 2007). Figure 3 shows how the heterogeneity of a formation can vary depending on the scale at which we sample the formation. Examples are shown for three distinct geological features; beds of varying thickness only (Figure 3A), a set of graded beds, again, of varying thicknesses (Figure 3B), and a "large" and "small" core sample for two sandstone types (Figure 3C). A quantitative assessment of whether a formation appears homogeneous or heterogeneous to the measurement tool as it travels up the borehole is possible. The degree of measured heterogeneity will also change as the measurement volume changes (e.g. Figure 3A and B); shallow measurements (e.g. bulk density or micro resistivity) will sample smaller volumes, whereas deep measurements (e.g. gamma radiation, acoustic travel time or deep resistivity) will sample large volumes.

Assessment of thinly bedded siliciclastic reservoirs highlights the issues of correlating geological-petrophysical attributes to petrophysical measurement volumes. Thin beds are defined geologically as being less than 10 cm thick (Campbell 1967), whereas a "modern" petrophysical thin bed is referred to as less than 0.6 m in thickness, and is defined to reflect the vertical resolution of most porosity and resistivity logs (Qian & Zhong 1999; Passey et al. 2006). The micro-resistivity logs (including dipmeter and borehole electrical imaging logs) have a higher vertical resolutions and so can recognise thin beds on a scale that is more consistent with the geological scale (Cheung et al. 2001; Passey et al. 2006). Figure 3 (A and B) illustrates how alternating high and low porosity thin beds, that are significantly below the resolution of typical wireline well logs, would appear as low variability within the measurement volume.

Up-scaling from core measurements to petrophysical well log calibration, and eventually to subsurface and flow simulation models of the reservoir at *circa* seismic-scale is a related topic. This process of upscaling represents a change of scale and hence properties may change from being heterogeneous at one scale to homogeneous at another scale. A discussion of up-scaling is beyond the scope of this paper.

To summarise, 'heterogeneity' may be defined as the complexity or variability of a specific system property in a particular volume of space and/or time. Effectively there is the intrinsic heterogeneity of the property itself (e.g. porosity or mineralogy) and the measured heterogeneity as described by the scale, volume and resolution of the measurement technique.

Evaluating Heterogeneity

Having defined heterogeneity, we consider a variety of statistical techniques that can be used to quantify heterogeneity. Techniques are grouped into two themes: (1) characterising the variability in a dataset and; (2) quantifying heterogeneity through heterogeneity measures. Firstly we illustrate how standard statistics can be used to characterize the variability or heterogeneity in a carbonate reservoir. Secondly we use four simple synthetic datasets to illustrate the principles of and controls on three common heterogeneity measures, before applying the heterogeneity measures to (a) the porosity data from two carbonate reservoirs, (b) a comparison of core and well log-derived porosity data in a clastic reservoir, (c) core measured grain density as a proxy for mineralogic variation in a carbonate reservoir, and (d) gamma ray log-derived bedding heterogeneities in a clastic reservoir..

Characterising the variability of the dataset

The core-calibrated well log-derived porosity data from an Eocene-Oligocene carbonate reservoir are used to illustrate the concepts for characterising heterogeneity (Figure 4). Formation A is *c.*75 m in vertical thickness, and is dominated by wackestone and packstone facies, with carbonate mudstone & grainstone interbeds. Formation B is *c.*54 m in vertical thickness, and is composed of grain-rich carbonate facies (predominantly comprising packstone to grainstone facies). Micro- and matrix-porosity dominate Formations A and B in the form of vugs, inter- and intra-granular porosity (Reddy et al. 2004; Wandrey 2004; Naik et al. 2006; Barnett et al. 2010). Metre-thick massive mudstone interbeds are observed toward the top of Formation A. The mudstone is suggested to be slightly calcareous and dolomitic in nature, with trace disseminated pyrite (Thakre et al. 1997; Estebaan 1998).

A simple glance at the wireline data for this reservoir (e.g., Figure 4) suggests Formation-A is more variable or “heterogeneous”. An early step in completing a routine petrophysical analysis is often to produce cross plots of the well log data; these give additional visual clues as to the presence of heterogeneities within the data (e.g. Figure 5). Formation-A has a diverse distribution of values across the bulk density – neutron porosity cross plot, indicating its more heterogeneous character when compared to Formation-B, which is more tightly clustered (Figure 5). The bulk density – neutron porosity cross plot reflects the varied facies and porosity systems of Formation-A, in comparison to the carbonate packstone-grainstone dominated Formation-B with a more uniform porosity system.

Basic statistics can be used to characterise the variation in distribution of values within a population of data. The basic statistics (Table 1) and histogram (Figure 6) for the values of wireline log derived porosity for Formations A and B clearly reflect different variability within the data populations. Log-derived porosity in Formation A is skewed toward lower values around a mean value of 8.5 %, with a moderate kurtosis (Figure 6, Table 1). The statistics for the log-derived porosity of Formation B records a tendency toward higher values (negatively skewed) around a mean of 21.9 % and a stronger kurtosis (Figure 6, Table 1). The standard deviation, of values around the mean, is moderate for both Formations. This suggests that values are neither tightly clustered nor widely spread around the mean, although we note that the standard deviation for Formation B is one unit lower.

These basic statistics can be used to characterise variation within a dataset, producing a suite of numerical values that describe data distributions. However, we need to complete and understand the full suite of statistical tests to achieve what is still a fairly general numerical characterisation of heterogeneity. We note that we could not use a similar suite of statistics to directly compare the variability between different data types that occur at different scales as the range of values has strong control on the outputs, for example comparing the variability in porosity (on a theoretical maximum scale of 0 to 100) with permeability (which for a conventional reservoir can vary between over several orders of magnitude, from close to 0 to 1000s mD). Thus, when using basic statistics, there is no single value to adequately define the quantitative heterogeneity of a dataset as being "x", that would enable direct comparison of different well data, formations and reservoirs. Instead, to achieve a direct heterogeneity comparison that is both robust and useful we must consider established *heterogeneity measures*.

Quantifying Heterogeneity: heterogeneity measures

Measures used in quantifying heterogeneity use geostatistical techniques to provide a single value to describe the heterogeneity in a dataset. Published *heterogeneity measures*, such as the coefficient of variation and the Lorenz Coefficient, have been in common use throughout most scientific disciplines, and are frequently used in establishing porosity and permeability models in exploration (e.g. Dykstra & Parsons 1950; Lake & Jensen 1991; Reese 1996; Jensen et al. 2000; Elkateb et al. 2003; Maschio & Schiozer 2003; Sadras & Bongiovanni 2004; Sahni et al. 2005).

Four simple synthetic datasets (Table 2) are used to illustrate the impact of common types of variability in a dataset on the heterogeneity measures. These measures are then applied to specific heterogeneities in a series of case studies. Of the synthetic datasets, Dataset (i) is homogeneous with no internal variation, Dataset (ii) is composed of two values representing a high and low setting, Dataset (iii) comprises a simple linear increase in values, and Dataset (iv) represents an exponential increase in values (Table 2).

Coefficient of Variation

The coefficient of variation (Cv) is a measure of variability relative to the mean value. The most commonly used method for calculating the coefficient of variation is shown below

(Equation 1), although numerous variations on this approach can be found in published literature. A homogeneous formation will have a coefficient of variation of zero, with the value increasing with heterogeneity in the dataset (Elkateb et al. 2003).

$$Cv = \frac{\sqrt{\sigma^2}}{\bar{x}} \qquad \text{(Equation 1)}$$

[Where: *Cv* is the coefficient of variation, $\sqrt{\sigma^2}$ is the standard deviation, and $\bar{x}$ is the mean]

For our synthetic test datasets, we see coefficient of variation increase with heterogeneity; (i) Cv = 0, (ii), Cv = 0.35, (iii) Cv = 0.55, and (iv) Cv = 2.82.

The Lorenz Coefficient

The original Lorenz technique was developed as a measure of the degree of inequality in the distribution of wealth across a population (Lorenz 1905). Schmalz and Rahme (1950) modified the Lorenz Curve for use in petroleum engineering by generating a plot of cumulative flow capacity against cumulative thickness, as functions of core measured porosity and permeability. Fitch et al. (2013) investigated the application of the Lorenz technique directly to porosity and permeability data. In our application of the Lorenz Coefficient, and to allow comparison of the heterogeneity in a single data type between the different measures, the cumulative of the property of interest (e.g., porosity), sorted from high to low values, is plotted against cumulative measured depth increment (Figure 7A; Fitch et al. 2013, and Figure 7B, the synthetic dataset considered here). In a purely homogeneous formation, the cumulative property will increase by a constant value with depth, this is known as the “line of perfect equality” (Sadras & Bongiovanni 2004). An increase in the heterogeneity of the property will cause a departure of the Lorenz Curve away from the line of perfect equality. The Lorenz Coefficient (Lc) is calculated as twice the area between the Lorenz Curve and the line of perfect equality; a pure homogeneous system will return a Lorenz Coefficient of zero, while maximum heterogeneity is shown by a Lorenz Coefficient value of one (Figure 7A).

The Lorenz Coefficients generated for our synthetic test datasets demonstrate some of the key features of the Lorenz technique; Dataset (i) matches the line of perfect equality (Figure 7B), returning an Lorenz Coefficient of zero, Datasets (ii) and (iii) return Lorenz Coefficient values of 0.16 and 0.25, respectively, and the exponential data of set (iv) returns a Lorenz Coefficient value of 0.86, and is clearly visible as the most heterogeneous data with the largest departure from the line of perfect equality (set (i)) on Figure 7B.

Dykstra-Parsons Coefficient

The Dykstra-Parsons Coefficient (V_{DP}) is commonly used in the quantification of permeability variation. A method for calculating V_{DP}, provided by Jensen et al. (2000), begins by ranking the property of interest (e.g., porosity) in order of decreasing magnitude. We have followed the method presented by Maschio and Schiozer (2003) to assign probability values; for each individual value calculate the percentage of values greater than, or the 'cumulative probability', so that the probability of X is $P(x \leq X)$. The original permeability values are then plotted on a log probability graph with the cumulative probability values (Figure 8A). The slope and intercept of a line of best fit, for all data, from this plot is then used to calculate the 50th and 84th probability percentile, which are used in Equation 3 to derive V_{DP}. Here, we assume a log-normal distribution, so that the

Log(property) value at the 84th percentile represents one standard deviation away from the 50 % probability (Machio & Schiozer 2003). As heterogeneity increases the slope of the line of best fit increases along with the difference between the 50th and 84th percentile, and subsequently the value of V_{DP} (Figure 8A).

$$V_{DP} = \frac{x_{50} - x_{84}}{x_{50}} \quad \text{(Equation 3)}$$

[Where x_{50} is the 50th property percentile, and x_{84} is the 84th property percentile]

Our synthetic datasets show significant differences in the Dykstra-Parsons plots produced (Figure 8B) and resultant Dykstra-Parsons values; set (i) $V_{DP} = 0.0$, set (ii) $V_{DP} = 0.31$, set (iii) $V_{DP} = 0.57$, and set (iv) $V_{DP} = 0.99$.

Selection of Appropriate Heterogeneity Measures

The key advantage to using a heterogeneity measure is the ability to define the heterogeneity of a dataset as a single value, allowing direct comparison between different data types, reservoir units (formations) and fields.

The coefficient of variation provides the simplest technique for generating a single value measure of heterogeneity, with no data pre-processing required. By calculating the standard deviation as a fraction of the mean value we are looking at the variability within the data distribution, removing the influence of the original scale of measurement. As such the coefficient of variation should provide a more appropriate measure of the heterogeneity of a dataset than the basic statistics (as in Table 1), that can be compared between different measurement types and scales of observation. Lake and Jensen (1991) comment that the estimate of Cv is negatively biased, suggesting that the Cv estimated from data will be smaller than the value for the true population. Sokal and Rohlf (2012) suggest that care should be used in applying the coefficient of variation to 'small samples' and provide a simple correction. In addition the coefficient of variation should only be applied to data which exist on a ratio scale with a fixed zero value, for example it is not appropriate for temperature measurement in Fahrenheit or Celsius (Sokal & Rohlf 2012). The coefficient of variation (Cv) increases with heterogeneity to infinity as no upper limit is defined in the calculation (Figure 9). Lake and Jensen (1991) suggest that this is a major advantage in use of the coefficient of variation as a heterogeneity measure, in that it can distinguish extreme variation. However, we favour a heterogeneity measure with defined upper and lower limits, allowing a clear comparison of variation in different datasets with different scales, resolutions and hypothetical end-member values across a similarly scaled range. We note that Jensen and Lake (1988) suggest that high levels of heterogeneity are compressed in the case of the Dykstra-Parsons and Lorenz Coefficients, and urge caution when using these techniques on small datasets (e.g., less than 40 samples).

The Lorenz Coefficient provides a simple graphical-based approach to visualising and quantifying heterogeneity. As heterogeneity in a dataset can only vary between zero and one, all data types can be easily compared, regardless of the scale of original measurement. This effectively removes the influence that the scale of the original data may have on magnitude of variability present, which would be described by the mean, standard deviation and other basic statistics. The Lorenz Coefficient values more accurately reflect the heterogeneity within a formation, and provide a measure that can be directly compared between different data types. Our initial work with the synthetic dataset suggests that low heterogeneity occurs around a Lorenz Coefficient of 0.16 (set ii, Figure 9), moderate linear heterogeneity is associated with a Lorenz Coefficient of 0.25 (set iii, Figure 9), and high-level exponential heterogeneity increases heterogeneity up to a Lorenz Coefficient of 0.86 (set iv, Figure 9). We have not yet been able to generate a sufficiently heterogeneous dataset to return the maximum heterogeneity of Lorenz Coefficient = 1.0. For comparison, Lake and Jensen (1991) suggest that typical Lorenz Coefficient values, for cumulative flow capacity against cumulative thickness, in carbonate reservoirs ranges from 0.3 to 0.6. Fitch et al. (2013) show that the several orders of magnitude variability in permeability measurements play a major control in the heterogeneity recorded using the traditional Lorenz technique.

The Dykstra-Parsons Coefficient may be considered as a more statistically robust technique, but it is more complex and requires additional application and understanding of mathematical and statistical methodologies (i.e., probability functions). Additionally, unlike the Lorenz plot, the Dykstra-Parsons plot does not provide a simple graphical approach for visually comparing heterogeneity between datasets. Jensen and Currie (1990) and Rashid et al. (2012) provide discussion of the weakness of using a line of best fit to calculate heterogeneity, rather than the actual “raw” data points, placing weighting on the central portion of the data and decreasing the impact of high or low extreme values. However, as long as the technique is used consistently comparisons can be made between different data types and reservoir settings. A classification scheme based on the Dykstra-Parsons value exists for permeability variation where lower values (0 – 0.5) represent small heterogeneities (zero being homogeneous), while larger values (0.7–1) indicate large to extremely large heterogeneities (Lake & Jensen 1991). Results from our initial trial using the synthetic data are comparable; with simple, small heterogeneities varying from V_{DP} values of 0.3 to 0.6, and the large exponential heterogeneity producing a V_{DP} value of 0.99 (Figure 9). Lake and Jensen (1991) comment that most reservoirs have V_{DP} values between 0.5 and 0.9.

As with any data analysis and interpretation, understanding the measurement device used and what it is actually responding to within the subsurface is key, and this can aid in understanding what heterogeneities are being described and why. This suite of techniques can be easily applied to a range of datasets at a formation scale (i.e. estimation of shale volume, water saturation, and even the original wireline log measurements), providing a comprehensive understanding of heterogeneities and underlying controls. Jensen et al. (2000) comment that heterogeneity measures are not a substitute for detailed geological study, measurements and analysis. They suggest that, at this scale, heterogeneity measures provide a simple way to begin assessing a reservoir, guiding investigations toward more detailed analysis of spatial arrangement and internal reservoir structures which may not be shown directly.

An overall summary of the heterogeneity measures and the advantages and disadvantages associated with each is provided in Figure 10 for quick reference. Each of these measures provides a quantitative estimate of the heterogeneity in a dataset. There is currently no best practice choice from these heterogeneity measures, indeed it seems that the choice of which measure one should use is based solely upon the analyst's preference, often based on experience, skills, and knowledge. The fact that all measures discussed here point toward similar numerical ranking of the heterogeneity present in the datasets investigated is reassuring. We have a preference for the Lorenz Coefficient as a heterogeneity measure. This uses a simple technique to produce both graphical and numerical indicators of heterogeneity that can be easily compared across a range of datasets, measurement, and reservoir types. In the final section of this manuscript we summarise the findings from four case studies as examples.

Jensen and Lake (1988) demonstrate that both the Dykstra-Parson and Lorenz Coefficients provide only an estimate of the true heterogeneity, depending on the population size, sampling frequency and location. Sampling frequency and location will play an impact on the measured heterogeneity in a property; this is demonstrated in Case Study 2 below. An additional issue, not addressed by the three static heterogeneity measures discussed here, is spatial organisation of the property, or the non-uniqueness of the heterogeneity measure. Figure 11 provides examples of nine 'simple' heterogeneous layered models, each is composed of two sets of fifty layers assigned a value of 1 and 100, respectively (in this case units are mD for permeability, but could represent any numerical property). The layers in model A and B are grouped into separate high and low property domains, model Q alternates high and low property layers throughout, and models C to M represent a range in spatial organisation of the layers. The standard statistics are identical for each spatial model (i.e. mean value 20.5, standard deviation of 49.75). The coefficient of variation, Lorenz Coefficient and Dykstra-Parsons Coefficient are 0.985, 0.485 and 0.856, respectively, for each of the models regardless of spatial organisation of the heterogeneity. In the case of these permeability models, each will behave significantly differently under flow simulation in terms of fluid production, breakthrough time and sweep efficiency. There is a potential for modifying existing techniques to quantify variability while maintaining the spatial organisation of heterogeneity, for example the Stratigraphic Modified Lorenz Plot (Gunter et al. 1997).

Case Studies

1) Porosity heterogeneity in a complex carbonate reservoir

The heterogeneity measures have been applied to the Eocene-Oligocene carbonate reservoir described above in terms of how standard statistics can be used to characterize variability in porosity measurements. To summarise the core-calibrated porosity log values describe Formation A as a moderate to highly variable porosity succession composed of predominantly low values around a mean value of 8.5 %, and Formation B as a less variable succession of high porosity values spread around a mean of 21.9 % (Figure 6, Table 1).

The coefficient of variation values for the porosity of Formation A is 0.532 and is reduced by *c.*70 % for Formation B (0.161; Table 3). Formation A porosity values have a Lorenz Coefficient of 0.288, and Formation B has a Lorenz Coefficient of 0.085 (Figure 12A, Table 3). The Dykstra-Parsons coefficient for the Formation A porosity values returns a V_{DP} of 0.353 and Formation B, again, has lower heterogeneity with a V_{DP} of 0.123 (Figure 12B, Table 3). As with results from the synthetic data, it is reassuring that all three heterogeneity measures provide the same relative ranking of the two formations. Differences in the measures ranges by *c.*50 % for both Formations A and B. This highlights that although we can compare heterogeneity between specific techniques, we should not attempt to compare heterogeneity values measured with the different techniques.

2) Porosity and permeability heterogeneity in a sandstone reservoir

To provide a comparison of how heterogeneity levels are captured at two scales of measurement we compare the core measured and well log-derived porosity and permeability data from a North Sea Jurassic sandstone reservoir (Fig.13a) using the Lorenz Coefficient. Permeability is clearly more heterogeneous than porosity in both measurement types (Figure 13b). This reflects the difference in scale of measurement for permeability (typically ranging from 0.1 to 1000 mD, for example) and porosity (e.g., 0 to 0.3, or 0% to 30 %). Similar observations were made by Fitch et al. (2013) with regard to carbonate rock property data.

Heterogeneity in the well log-derived data is typically lower than that of the core data (Figure 13b). This observation relates to the irregular sampling of core measurements in comparison to continuous log measurements down a borehole. Resampling the well log porosity data illustrates that measured heterogeneity depends on sampling frequency and whether sampling location captures extreme values in a population. Figure 13c illustrates that decreasing sampling frequency and altering sample locations can enhance the range of heterogeneities recorded, supporting the study by Jensen and Lake (1988). Additional work in this area has the potential of informing best practise sampling protocols in both industrial and scientific drilling (e.g., Corbett & Jensen 1992a; b).

3) Lithological heterogeneity in a carbonate reservoir

Analysis of grain density and porosity measurements from an Eocene carbonate reservoir allows for a simple comparison of the heterogeneity in grain- and pore-components of the two zones, by using grain density as a proxy for mineralogy (grain component) and porosity as a proxy for facies (pore component), alongside sedimentological descriptions of the core plugs. Reservoir zone X is calcite dominated, with a range in facies from carbonate mudstone, to wackestone and packstone. Low variability in the grain density data, and large variability in porosity with facies type is observed in the raw data (Figure 14), and is reflected in Lorenz Coefficient heterogeneities of 0.028 and 0.334, respectively. Reservoir zone Y is composed of wackestone and packstone facies, with dolomite and disseminated pyrite observed in thin section. Consequently, porosity variability appears lower with a Lorenz Coefficient of 0.198, while grain density heterogeneity is almost twice as high as that of reservoir X (Lc 0.049).

In reservoir characterisation studies, heterogeneity measures are traditionally applied to permeability and porosity data. This pilot study indicates that there is potential to apply the techniques to quantify other types of heterogeneity that are described by any numerical data. These may include other rock property data (e.g., photoelectric, nuclear magnetic resonance, or resistivity logs to investigate heterogeneity in mineralogy, pore-size distribution and fluid content), digitized sedimentological descriptions (including facies codes and point count data), and borehole image facies analysis.

4) Bedding heterogeneity in a clastic reservoir

The gamma ray log from the North Sea Jurassic sandstone reservoir outlined in Case Study 2 is used to provide an example of how heterogeneity in bedding can be investigated using the Lorenz Coefficient. Figure 15 illustrates how using different gamma ray API values can be used as thresholds to define “bed boundaries”. Different threshold values will impact not only the bed locations but also how many beds are identified and the variability in bed thickness through the succession. By converting the presence of consecutive beds into a binary code we can calculate the heterogeneity in bed thickness (in this example using the Lorenz Coefficient). As the gamma ray threshold is increased above 50 API the number of beds is decreased, but the thickness of beds is increased, reflected in a decrease in the heterogeneity level (Figure 15B). The lowest GR threshold of 40 API identifies two beds with a bedding heterogeneity of 0.14 (Figure 15A(iii)). A gamma ray threshold of 50 API generates a large number of illogically placed bed boundaries, and subsequently has a higher bedding heterogeneity of 0.34 (Figure 15A(iv)). The original gamma ray log gives a Lorenz Coefficient heterogeneity value of 0.288, which is replicated by the bedding succession identified using a threshold of 120 API (Figure 15A(i)). Visual comparison suggests that appropriate bed boundaries between mudstone and sandstone layers are picked using this simple technique, supported by a similar level of heterogeneity being captured.

Although this is a somewhat simple application, with a major assumption that the gamma ray signature is only caused by the presence of clay minerals and that bed thickness is greater than the vertical resolution of the gamma ray log, application of this type of analysis could be made to selecting appropriate grid block size in high resolution geological models and subsequent upscaling of rock properties.

Further investigations of heterogeneities that occur across a range of length scales in datasets, or with different measurement resolutions may aid our understanding of the scale of variability in reservoir heterogeneity, for example, incorporating core, image logs and numerical sedimentological observations.

Conclusions

The term "heterogeneity" can be defined as the variability of an individual or combination of properties within a known space and/or time, and at a specified scale. Heterogeneities within complex hydrocarbon reservoirs are numerous and can co-exist across a variety of length-scales, and with a number of geological origins. When investigating heterogeneity, the type of heterogeneity should be defined in terms of both grain / pore components and the presence or absence of structural features in the widest sense (including sedimentary structures, fractures and faults). Hierarchies of geological heterogeneity can be used alongside an understanding of measurement principles and volumes of investigation to ensure we understand the variability in a dataset.

Basic statistics can be used to characterise variability in a dataset, in terms of the amplitude and frequency of variations present but a better approach involves heterogeneity measures because these can provide a single value for quantifying the variability. Heterogeneity measures also provide the ability to compare this variability between different datasets, tools / measurements, and reservoirs. Three separate heterogeneity measures have been considered here:

- The coefficient of variation is a very simple technique, comparing the standard deviation of a dataset to its mean value. A value of zero represents homogeneity, but there is no maximum value associated with extreme heterogeneity (increasing to infinity). Individual measurement scales will influence the documented heterogeneity level, and therefore comparison between different datasets is limited
- The Lorenz Coefficient is a relatively simple yet robust measure that provides graphical and numerical outputs for interpretation and classification of variability in a dataset, where heterogeneity varies between zero (homogeneous) and one (maximum heterogeneity).
- The Dykstra-Parsons coefficient is a more complex technique, requiring greater understanding of statistical methods. Numerical output defines a value of heterogeneity between zero (homogeneous) and one (maximum heterogeneity).

Initial work incorporating synthetic and subsurface datasets allows the prior assumptions and classification schemes for each measure to be tested and refined. Application to a wider selection of subsurface data types, and from a range of complex reservoir types and geographic locations will enhance our understanding of the link between geological and petrophysical heterogeneity. Drawing on a larger volume of examples, this work may also indicate one heterogeneity measure to be of more use than another. At this time, the choice between heterogeneity measures ultimately depends upon the objectives of the analysis, together with the analyst's preference, often based on experience, skills, and knowledge.

Beyond the results presented here, but taking account of published research, integration of heterogeneity analysis from outcrop and subsurface examples with geocellular and simulation modelling experiments investigating the impact of geologic features on flow behaviour may help streamline both exploration and production phases by focussing attention on what it is important to capture, at what scale and which of the data types is of most use in characterising heterogeneity in petrophysical properties.

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Figure Captions

Figure 1. An illustration of how heterogeneity can be separated into two ‘end-members’ of spatial fabric and grain component.

Figure 2. Sketches illustrating how scales of geological features, wireline logs and different types of hydrocarbon reservoir data / model elements are related: Schematic illustrations of (A) key geological heterogeneities and the scales of which they exist (see van Wagoner et al. 1990), (B) measurement volume and resolution of different types of subsurface data (modified from Frykman and Deutsch 2002), and (C) different tool resolution and volume of investigation of typical wireline log measurements.

Figure 3. Schematic illustration of the influence of thin beds (A, B), grading (B) and grain size and sorting (C) on petrophysical measurement volumes. (A, B) focus on deep and shallow well log measurements, and (B) focuses on core and thin section measurements.

Figure 4. Petrophysical data for Formations A and B. Panels from left to right; (1) caliper, (2) bulk density (RHOB) & neutron porosity (NPHI), and (3) core calibrated porosity log and core measured porosity (grey circles).

Figure 5. Cross plot of bulk density and neutron porosity measurements from Formation A (black circles) and Formation B (grey circles) (Figure 4).

Figure 6. Histogram distributions of core calibrated porosity log values for Formations A and B (Figure 4).

Figure 7. (A) Schematic illustration of the Lorenz plot, and (B) Lorenz curves generated using the synthetic datasets (Table 3)

Figure 8. (A) Schematic illustration of the cross plot underlying the Dykstra-Parson coefficient, and (B) Dykstra-Parson plots generated using the synthetic datasets (Table 3).

Figure 9. The heterogeneity values obtained for the four synthetic datasets; set (i) homogeneous, set (ii) two end-member values, set (iii) a simple linear change in values, and set (iv) an exponential change in values.

Figure 10. Summary of the heterogeneity measures discussed in this paper, listing the advantages and disadvantages of each technique.

Figure 11. Nine examples of permeability models which have the same statistical characteristics and heterogeneity measures.

Figure 12. (A) Lorenz curves generated for the porosity data of Formations A and B (Table 3), and (B) Dykstra-Parson plots generated for the porosity data of Formations A and B (Table 3).

Figure 13. Core and well log calibrated measurements of porosity (A) and permeability (B) for a North Sea Jurassic sandstone reservoir. (C) provides a graphical comparison of the Lorenz Coefficient for the whole succession (Bz4) and zones A to F. (D) illustrates the spread of Lorenz Coefficient values obtained by re-sampling the well log porosity data at different locations and frequencies.

Figure 14. Special core analysis measurements of grain density (A) and porosity (B) through reservoir zones X and Y of an Eocene carbonate succession. Facies code: Mdst – carbonate mudstone, Wkst – wackestone, Pkst – packstone, and dol – dolomite.

Figure 15. Depth plots of the gamma ray log (A(ii)), and bed boundary location picked using gamma ray value thresholds of 120 API (A(i)), 40 API (A(iii)), and 50 API (A(iv)). Crossplot of the number of beds identified by gamma ray log thresholding against Lorenz Coefficient heterogeneity in bed thickness.

Table captions

Table 1. Results of statistical analysis for core calibrated porosity log values of Formation A and B (Figure 4). Statistical analysis; (a) mean, mode and median averages, (b) standard deviation and variance, (c) maximum, minimum and range between minimum and maximum, (d) skewness (measure of the asymmetry of a distribution, positive indicates lower values are

more common than higher values), and (e) kurtosis (measure of the spread of data around a mean, more positive indicates single peak around a mean with less tails, more negative indicates less of a mean peak and larger tails).

Table 2. Synthetic dataset used to investigate the impact of different styles of data variability on the heterogeneity measures. Dataset (i) homogeneous, dataset (ii) two end-member values, dataset (iii) a simple linear change in values, and dataset (iv) an exponential change in values.

Table 3. Heterogeneity measures returned for the core calibrated porosity log values of Formation A and B (Figure 4).

	Formation A (porosity, %)	Formation B (porosity %)
Mean	8.5	21.9
Median	7.6	22.2
Standard Deviation	4.5	3.5
Maximum	23.3	29.2
Minimum	0.4	4.9
Skewness	0.945	-1.037
Kurtosis	0.579	2.834

Table 1. Results of statistical analysis for core calibrated porosity log values of Formation A and B (Figure 4). Statistical analysis; (a) mean, mode and median averages, (b) standard deviation and variance, (c) maximum, minimum and range between minimum and maximum, (d) skewness (measure of the asymmetry of a distribution, positive indicates lower values are more common than higher values), and (e) kurtosis (measure of the spread of data around a mean, more positive indicates single peak around a mean with less tails, more negative indicates less of a mean peak and larger tails).

Depth (m)	Set (i)	Set (ii)	Set (iii)	Set (iv)
100.50	1	2	2	10000
101.00	1	2	1.8	1000
101.50	1	2	1.6	100
102.00	1	2	1.4	10
102.50	1	2	1.2	1
103.00	1	1	1	0.1
103.50	1	1	0.8	0.01
104.00	1	1	0.6	0.001
104.50	1	1	0.4	0.0001
105.00	1	1	0.2	0.00001

Table 2. Synthetic dataset used to investigate the impact of different styles of data variability on the heterogeneity measures. Set (i) homogeneous, set (ii) two end-member values, set (iii) a simple linear change in values, and set (iv) an exponential change in values.

	Formation A (porosity)	Formation B (porosity)
Coefficient of variation	0.532	0.161
Lorenz Coefficient	0.288	0.085
Dykstra-Parsons Coefficient	0.353	0.123

Table 3. Heterogeneity measures returned for the core calibrated porosity log values of Formation A and B (Figure 4).

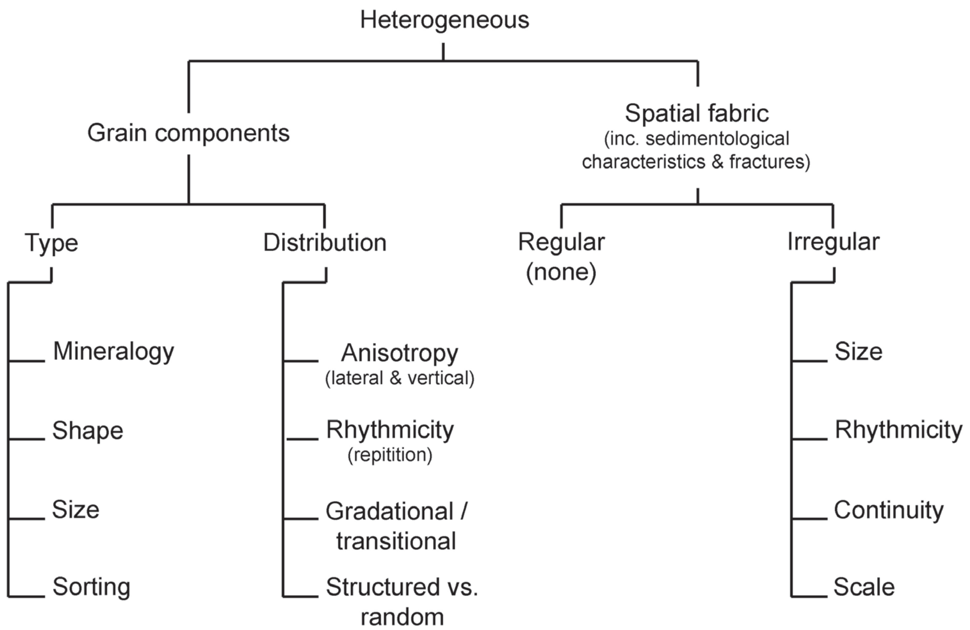


Figure 1

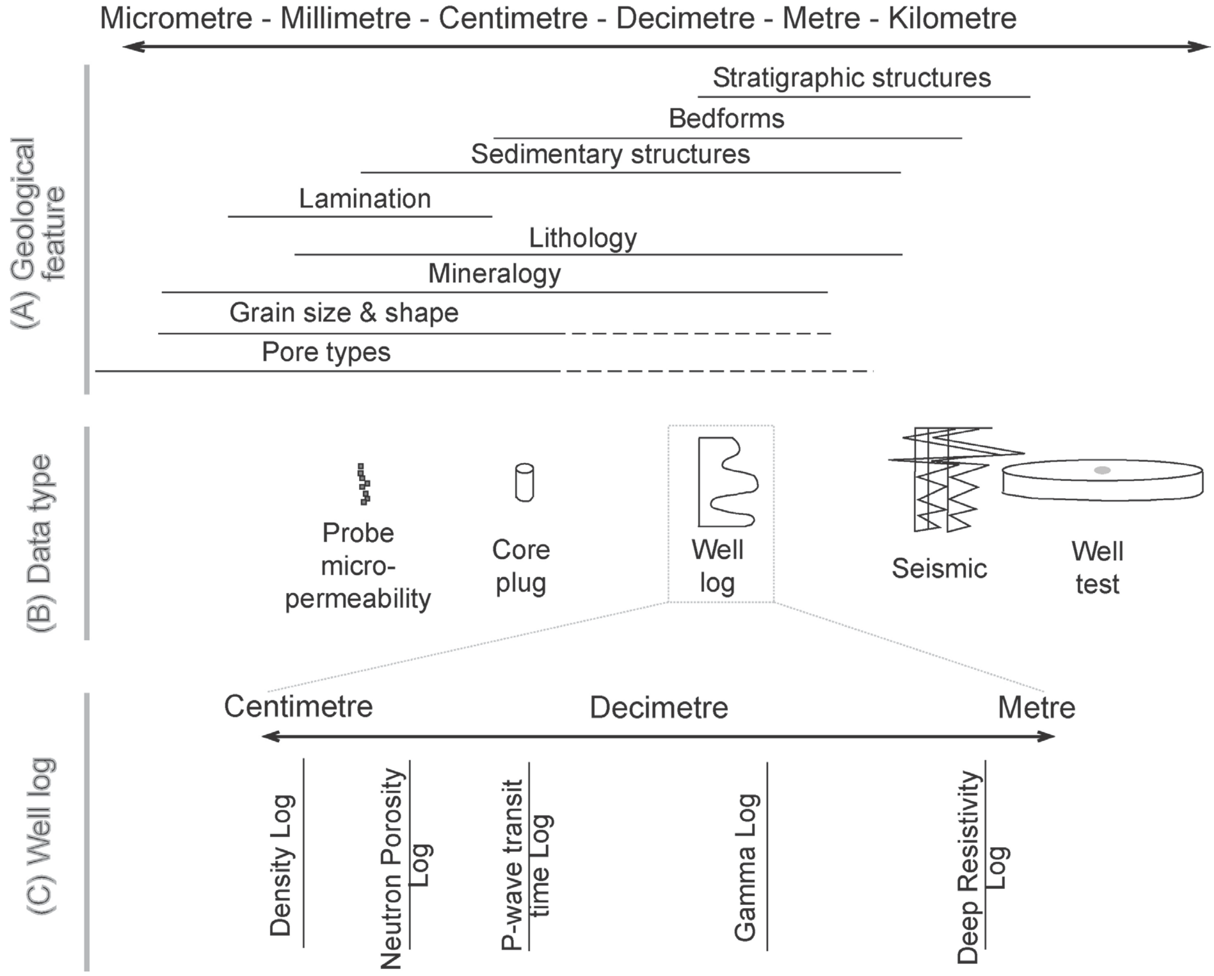

Micrometre - Millimetre - Centimetre - Decimetre - Metre - Kilometre
(A) Geological feature
Stratigraphic structures
Bedforms
Sedimentary structures
Lamination
Lithology
Mineralogy
Grain size & shape
Pore types
(B) Data type
Probe micro-permeability
Core plug
Well log
Seismic
Well test
(C) Well log
Centimetre
Decimetre
Metre
Density Log
Neutron Porosity Log
P-wave transit time Log
Gamma Log
Deep Resistivity Log

Figure 2

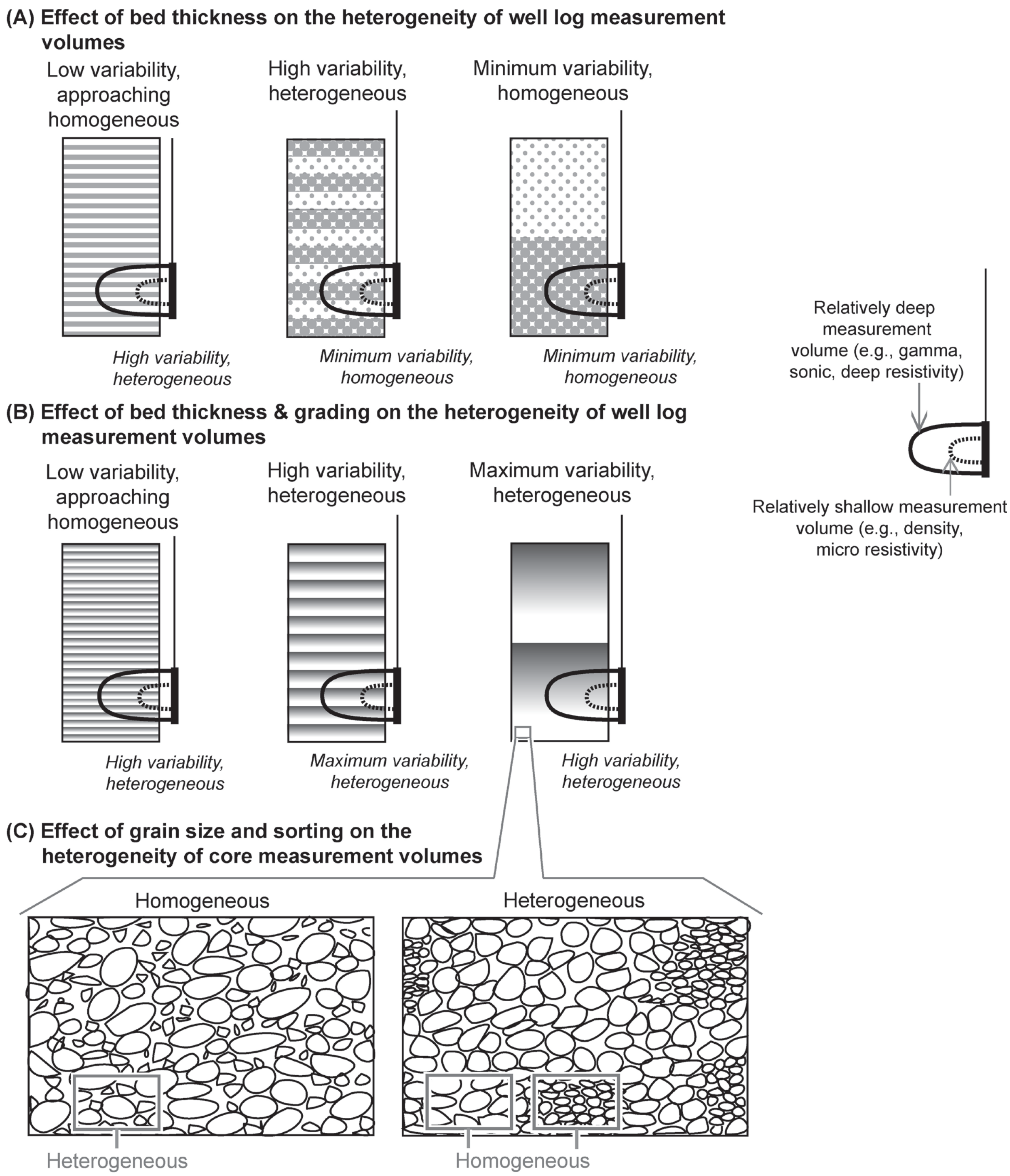
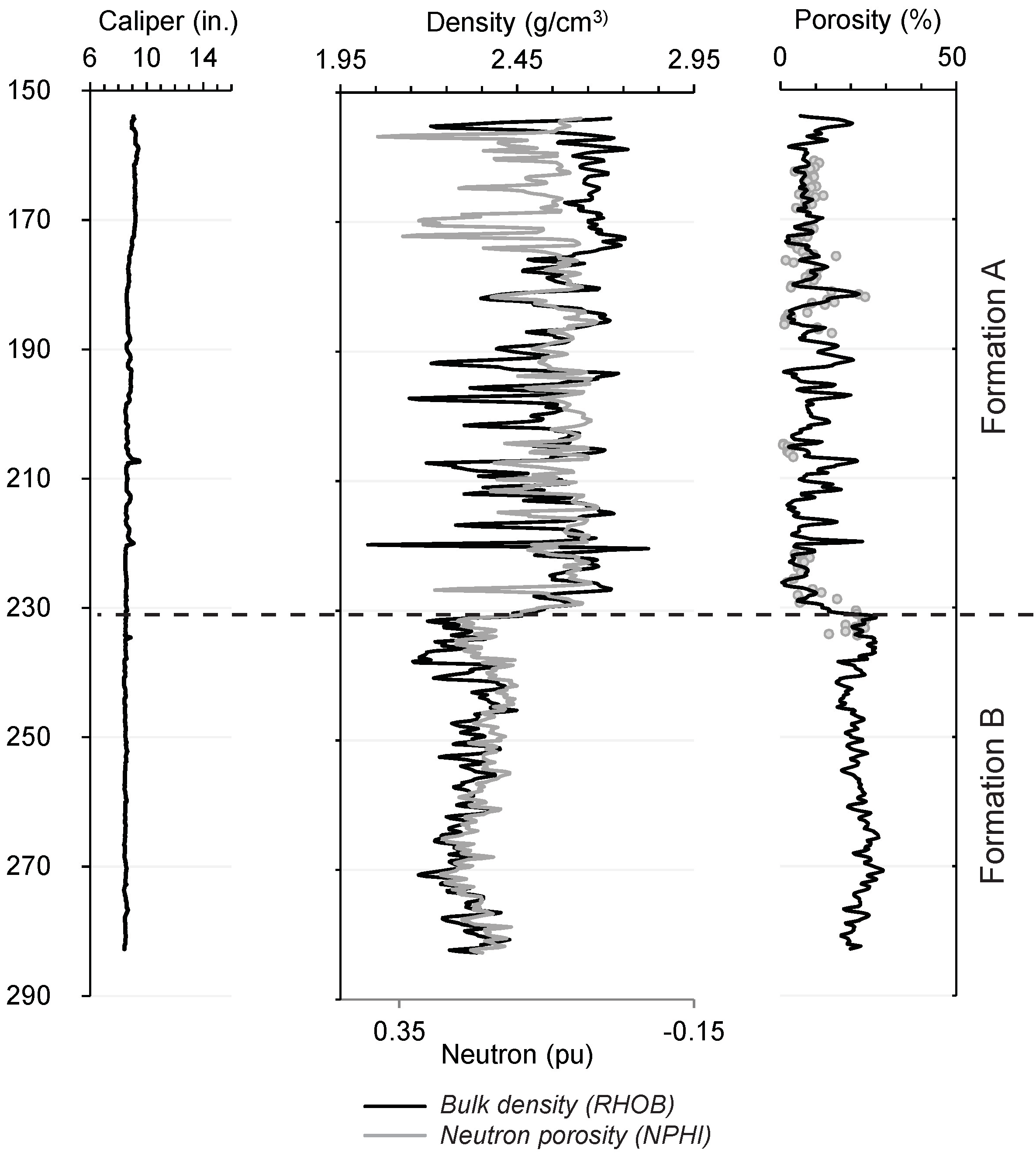

(A) Effect of bed thickness on the heterogeneity of well log measurement volumes
Low variability, approaching homogeneous
High variability, heterogeneous
Minimum variability, homogeneous
High variability, heterogeneous
Minimum variability, homogeneous
Minimum variability, homogeneous
Relatively deep measurement volume (e.g., gamma, sonic, deep resistivity)
Relatively shallow measurement volume (e.g., density, micro resistivity)
(B) Effect of bed thickness & grading on the heterogeneity of well log measurement volumes
Low variability, approaching homogeneous
High variability, heterogeneous
Maximum variability, heterogeneous
High variability, heterogeneous
Maximum variability, heterogeneous
High variability, heterogeneous
(C) Effect of grain size and sorting on the heterogeneity of core measurement volumes
Homogeneous
Heterogeneous
Heterogeneous
Homogeneous

Figure 3



Caliper (in.)
6
10
14
150
170
190
210
230
250
270
290
Density (g/cm3)
1.95
2.45
2.95
0.35
-0.15
Neutron (pu)
Bulk density (RHOB)
Neutron porosity (NPHI)
Porosity (%)
0
50
Formation A
Formation B

Figure 4

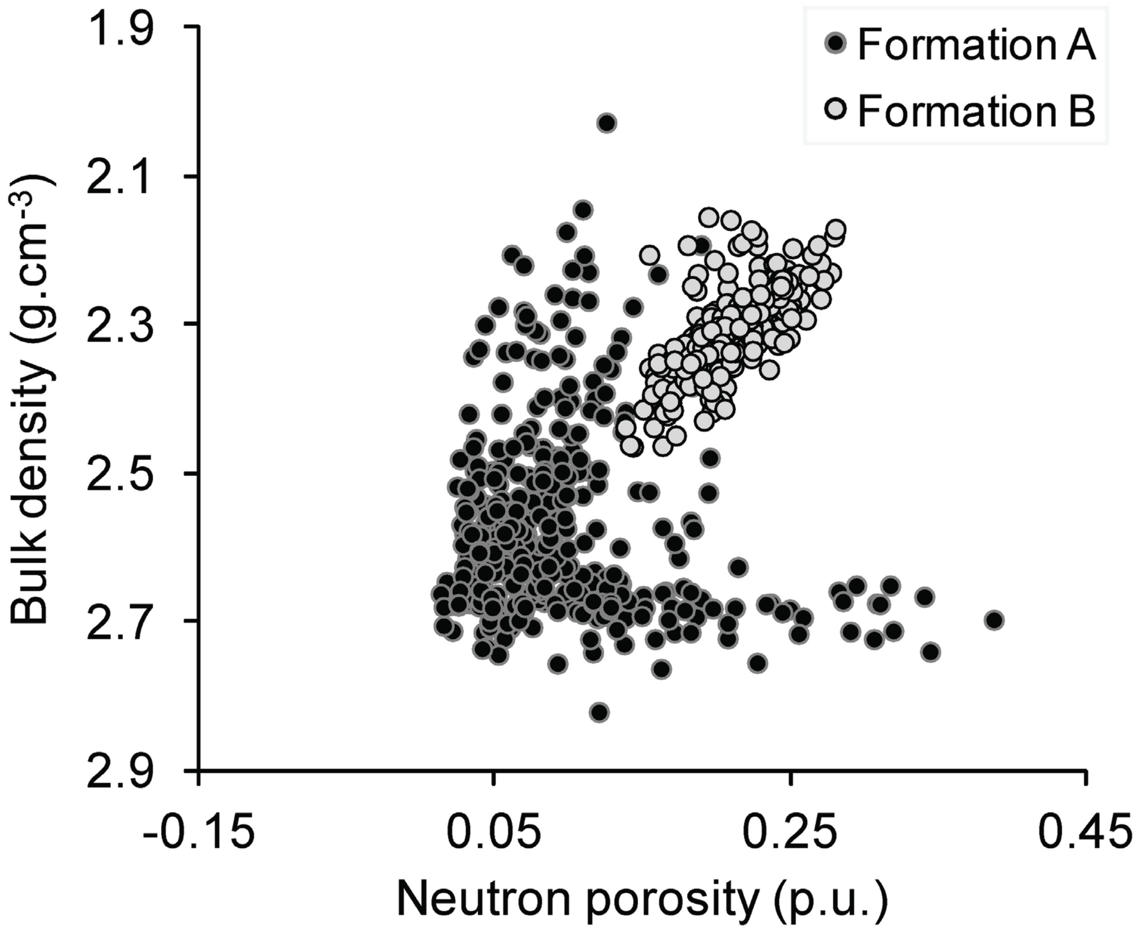

Formation A
Formation B
1.9
2.1
2.3
2.5
2.7
2.9
Bulk density (g.cm-3)
-0.15
0.05
0.25
0.45
Neutron porosity (p.u.)

Figure 5

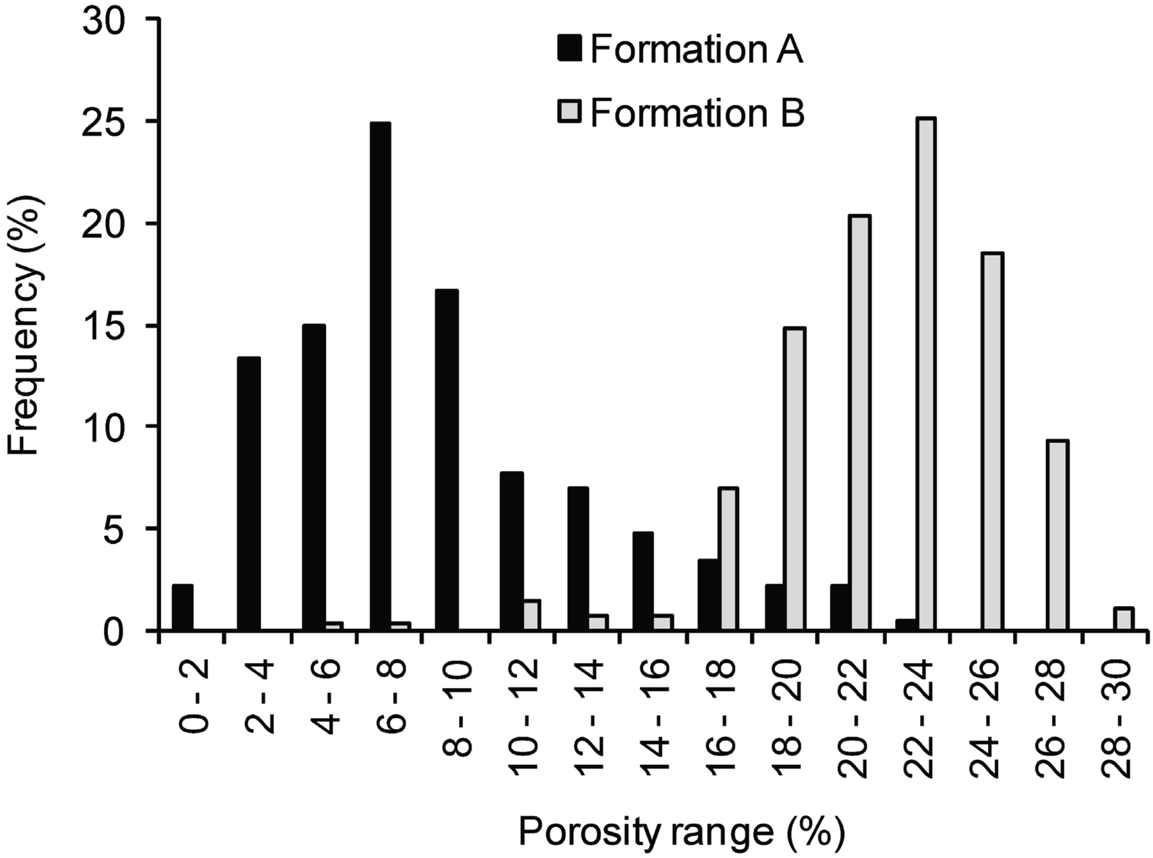

Formation A
Formation B
Frequency (%)
30
25
20
15
10
5
0
0 - 2
2 - 4
4 - 6
6 - 8
8 - 10
10 - 12
12 - 14
14 - 16
16 - 18
18 - 20
20 - 22
22 - 24
24 - 26
26 - 28
28 - 30
Porosity range (%)

Figure 6

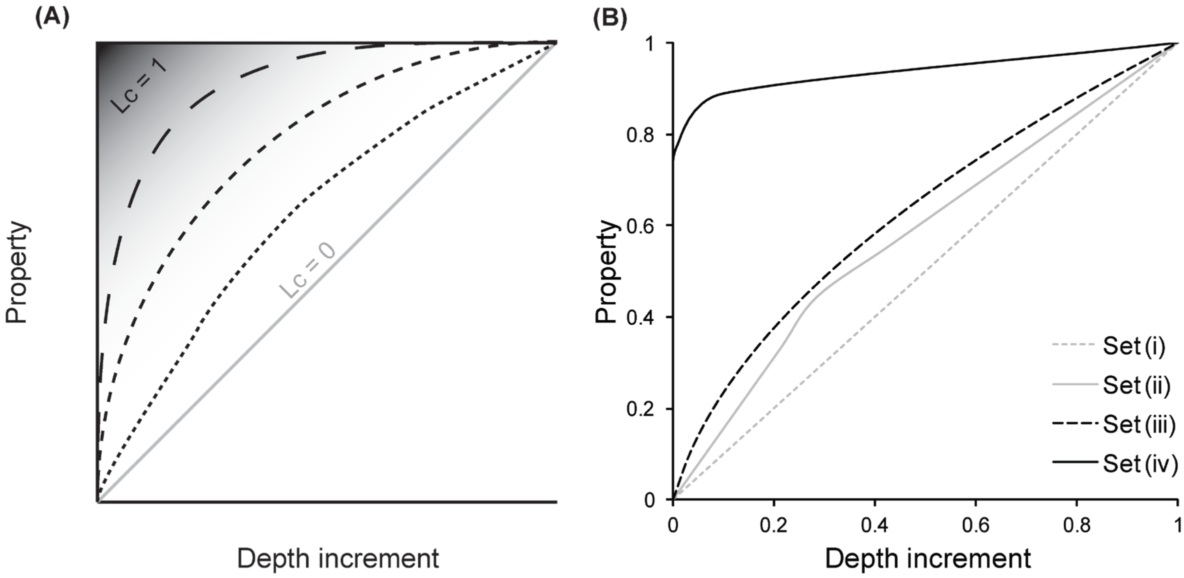


Figure 7

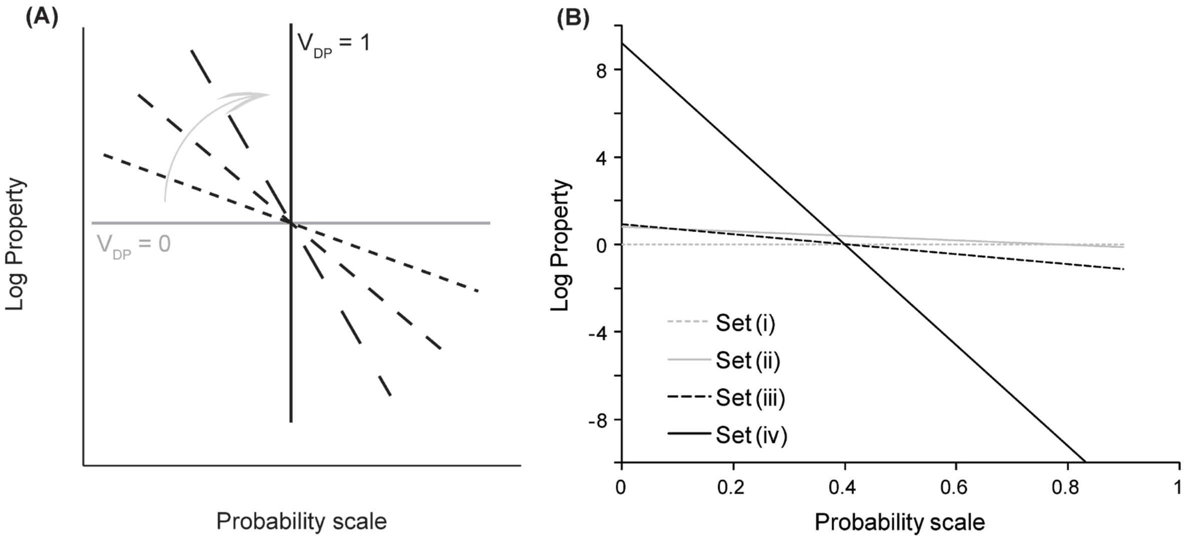


Figure 8

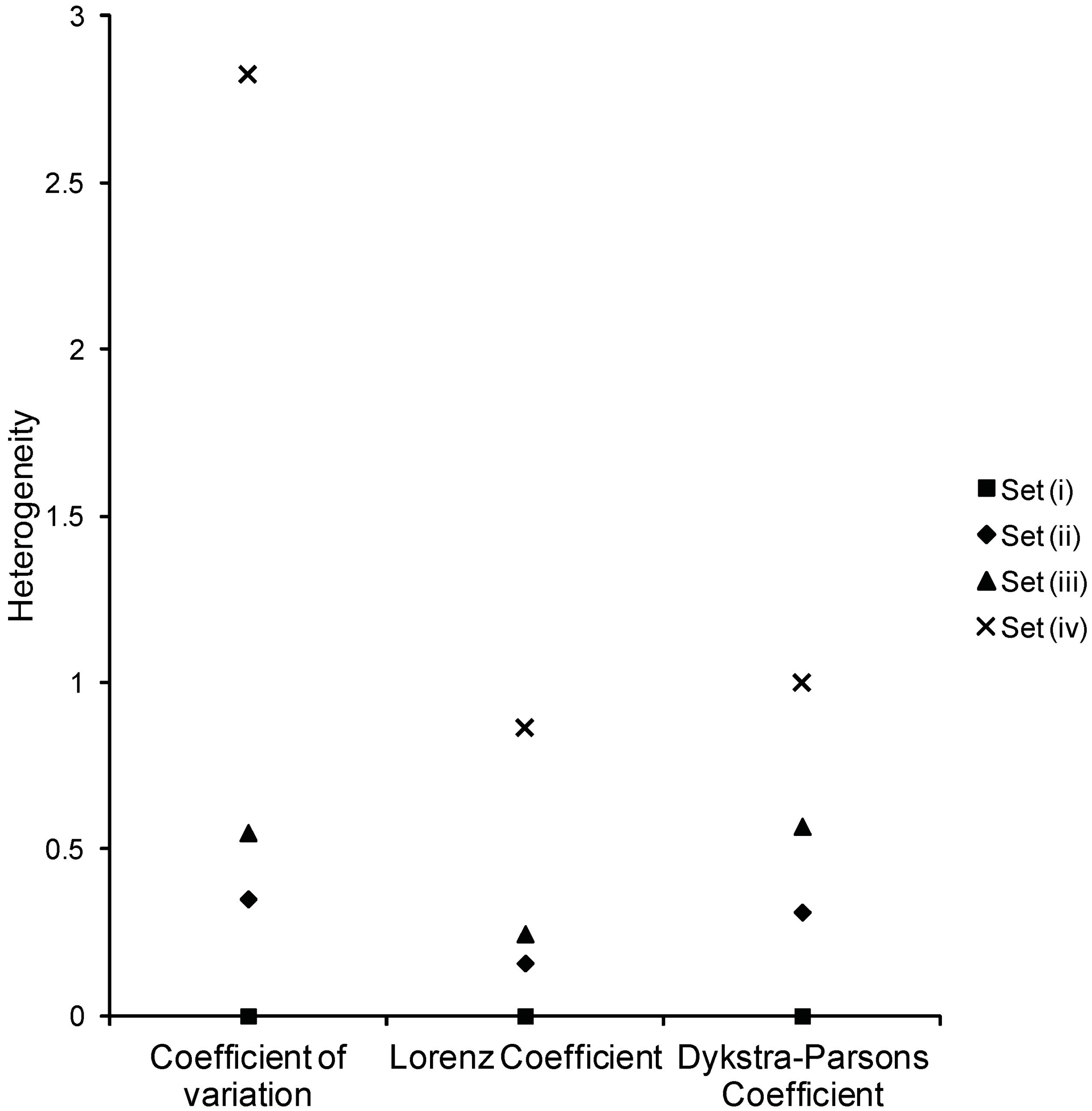

3
2.5
2
1.5
1
0.5
0
Heterogeneity
Set (i)
Set (ii)
Set (iii)
Set (iv)
Coefficient of variation
Lorenz Coefficient
Dykstra-Parsons Coefficient

Figure 9

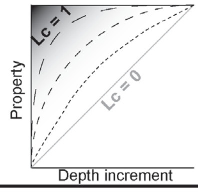
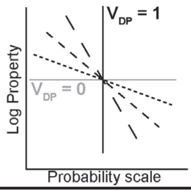
Heterogeneity measure	Summary	Advantages	Disadvantages
Coefficient of variation (Cv)	Homogeneous = 0 Heterogeneous = ∞ $$Cv = \frac{\sqrt{\sigma^2}}{\bar{x}}$$	Simple statistical technique, No pre-processing of data required. Easily applied to any data.	No maximum value, different measurement scales may influence heterogeneity results. Limited comparison between different datasets
Lorenz Coefficient (Lc)	Homogeneous = 0 Heterogeneous = 1 	Simple, Graphical plot for comparison, Easily applied to any data. Direct comparison for different tools, formations and reservoirs.	Possible user error in sorting & normalization, Negative values may complicate processing, but uncommon on well log datasets.
Dykstra-Parsons Coefficient (V_{DP})	Homogeneous = 0 Heterogeneous = 1 	Strong statistical basis, classification scheme established for interpretation. Direct comparison for different tools, formations and reservoirs.	Complicated pre-processing required (probabilities), Percentile values used in calculation are based on best fit line, rather than actual data.

Figure 10

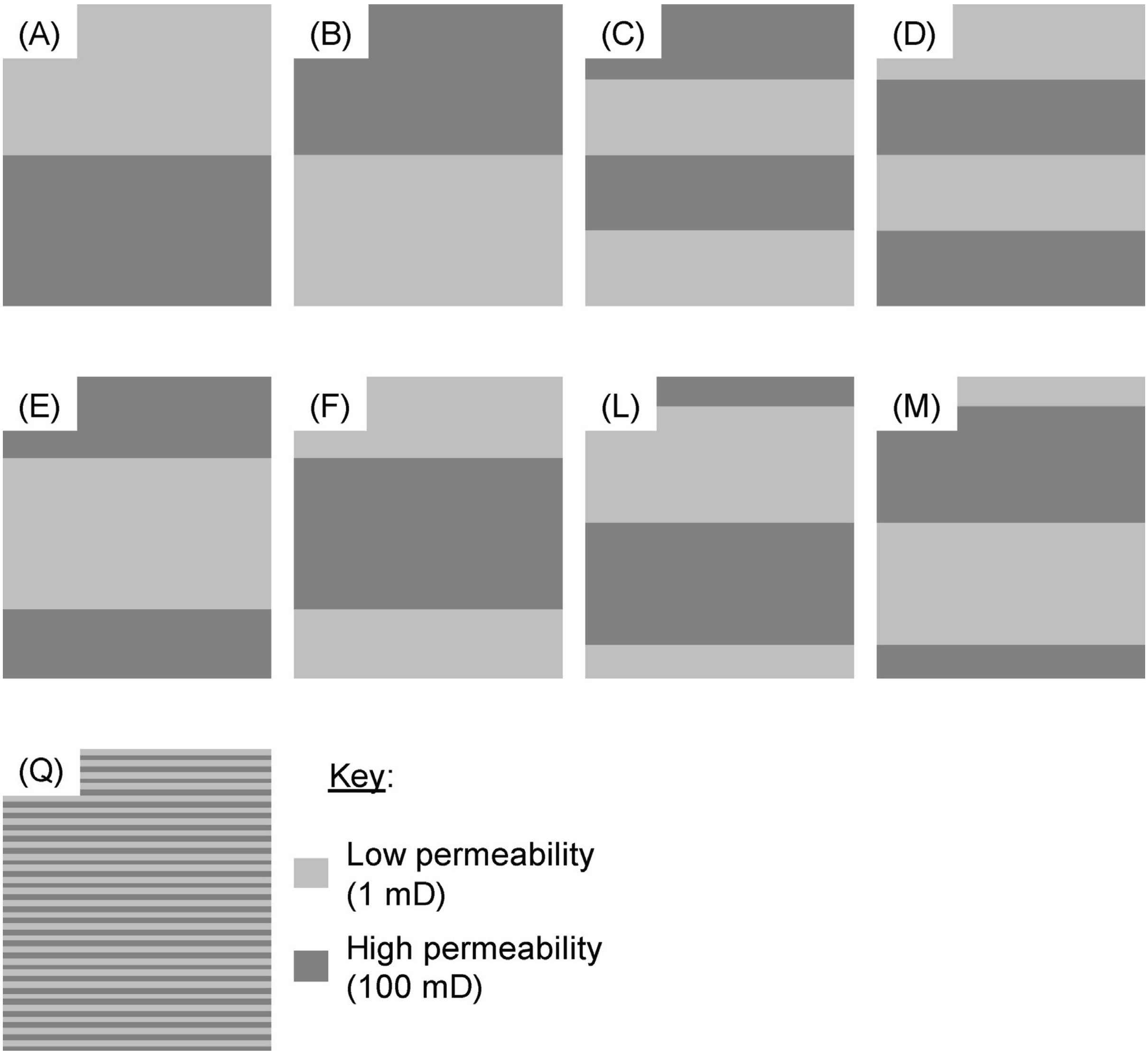

(A)
(B)
(C)
(D)
(E)
(F)
(L)
(M)
(Q)
Key:
Low permeability
(1 mD)
High permeability
(100 mD)

Figure 11

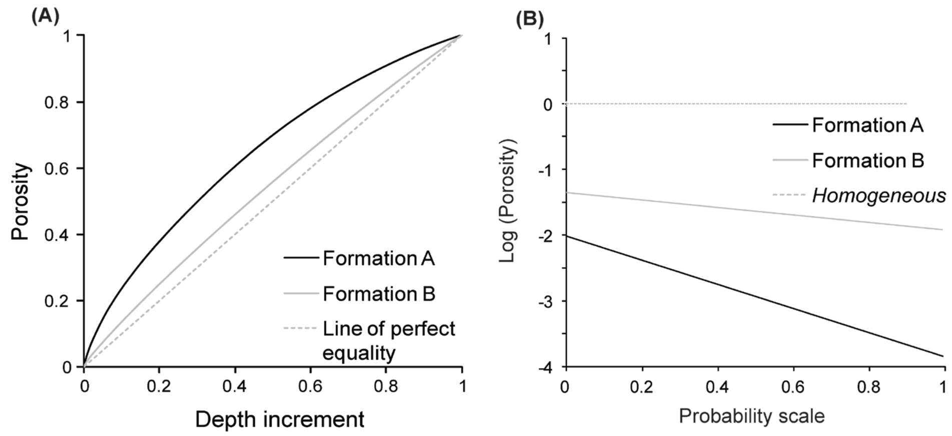
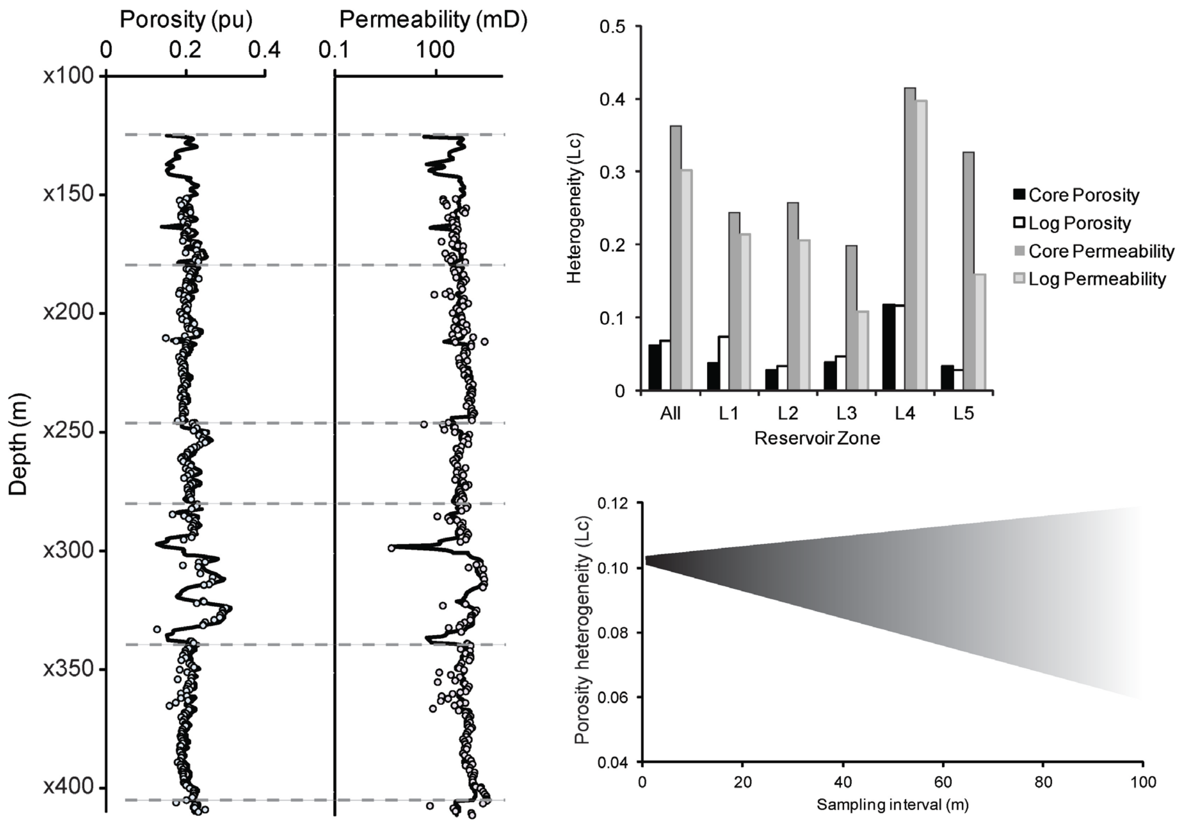

(A)
Porosity
Depth increment
Formation A
Formation B
Line of perfect equality
0
0.2
0.4
0.6
0.8
1
(B)
Log (Porosity)
Probability scale
Formation A
Formation B
Homogeneous
-4
-3
-2
-1

Figure 12



Porosity (pu)
0
0.2
0.4
Permeability (mD)
0.1
100
x100
x150
x200
x250
x300
x350
x400
Depth (m)
Heterogeneity (Lc)
0.5
0.4
0.3
0.2
0.1
0
All
L1
L2
L3
L4
L5
Reservoir Zone
Core Porosity
Log Porosity
Core Permeability
Log Permeability
Porosity heterogeneity (Lc)
0.12
0.10
0.08
0.06
0.04
0
20
40
60
80
100
Sampling interval (m)

Figure 13

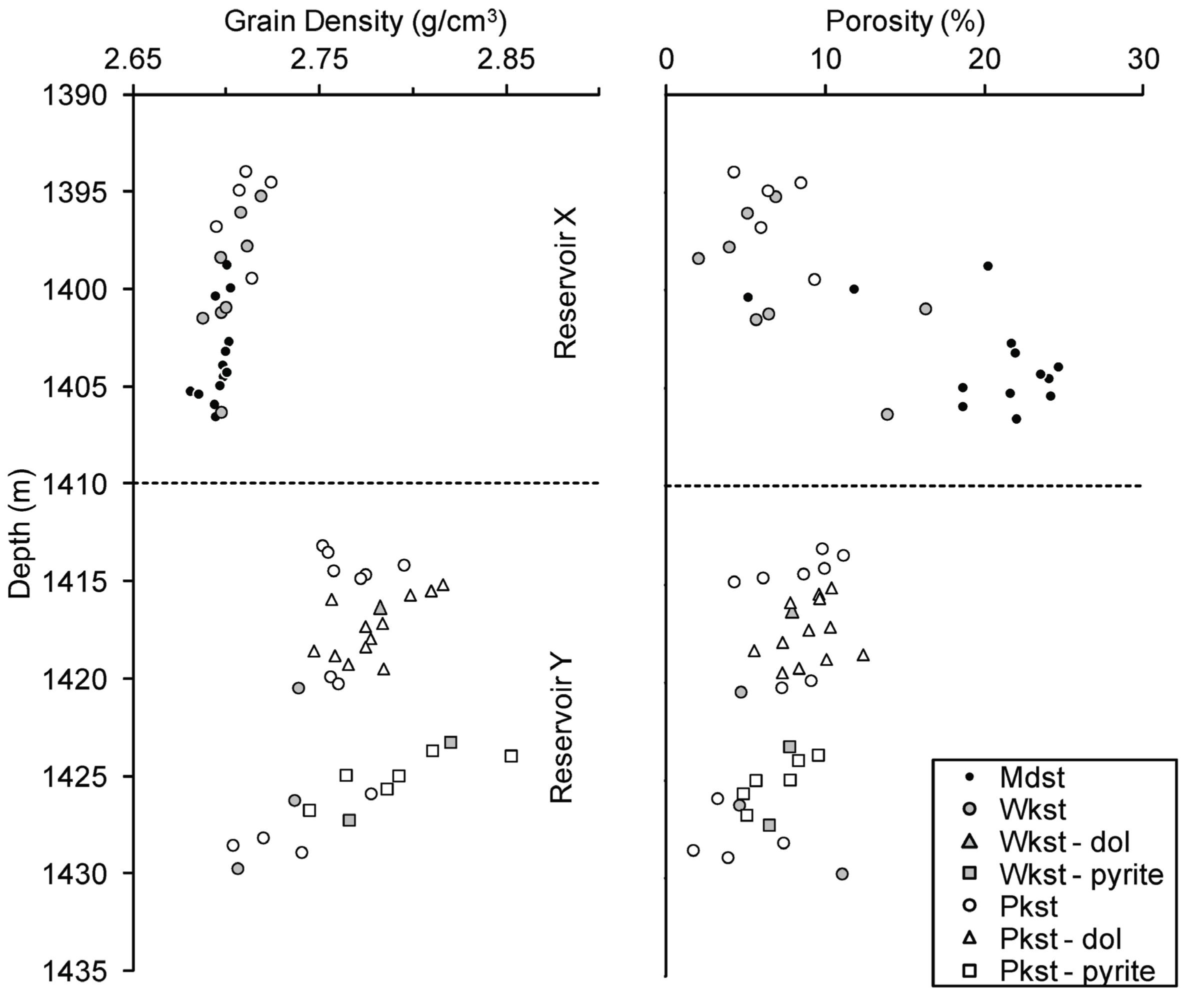

Grain Density (g/cm3)
2.65
2.75
2.85
Porosity (%)
0
10
20
30
1390
1395
1400
1405
1410
1415
1420
1425
1430
1435
Depth (m)
Reservoir X
Reservoir Y
Mdst
Wkst
Wkst - dol
Wkst - pyrite
Pkst
Pkst - dol
Pkst - pyrite

Figure 14

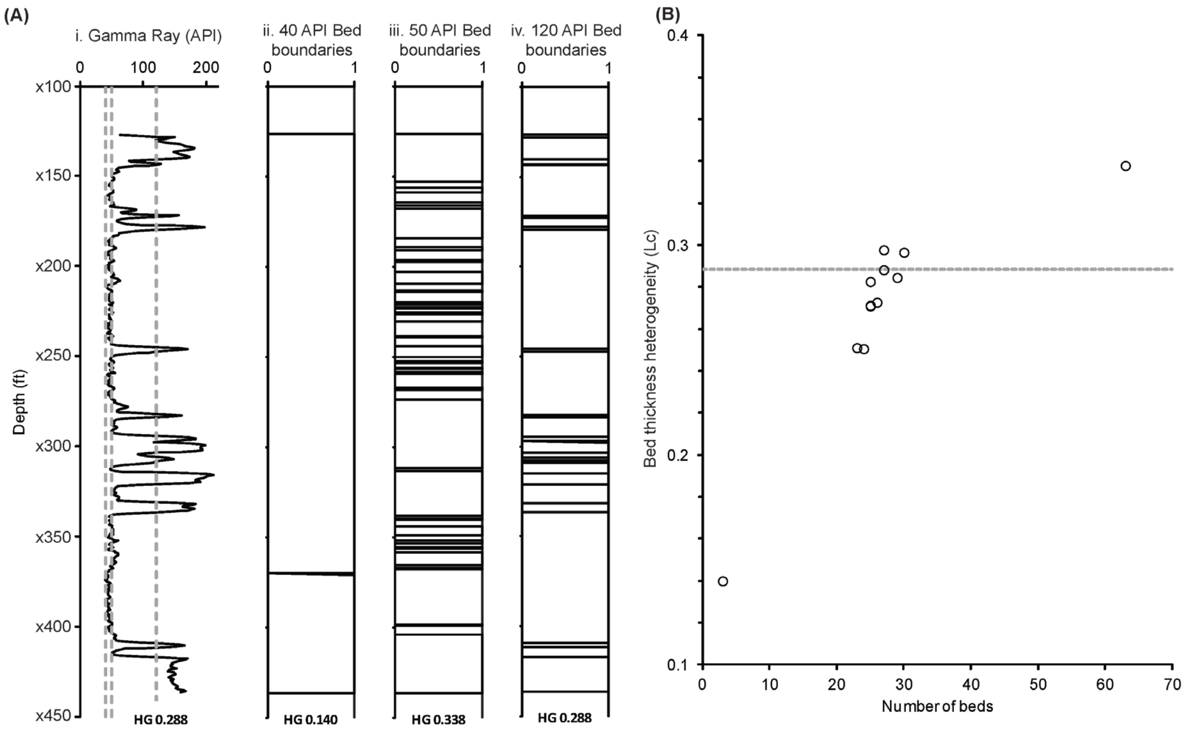


Figure 15

Title: An integrated and quantitative approach to petrophysical heterogeneity.

Authors: Fitch, P. J. R., Lovell, M. A., Davies, S. J., Pritchard, T. and Harvey, P. K.

Highlights

We explore how the term heterogeneity can be defined in earth sciences.

We show that standard statistics can be used to characterise the variability in a dataset.

We investigate the main controls on three static heterogeneity measures.

Four case studies illustrate the application of heterogeneity measures to different data types.