PORTFOLIO ALLOCATION PROBLEM AND QUANTITY CONSTRAINTS: AN ANALYSIS OF THE WARSAW STOCK EXCHANGE

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by

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To my mother Krystyna,

my brother Adam

and

In memory of my father Romuald

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ABSTRACT

After the political changes in 1989 the economy in Eastern and Central Europe turned from a centrally planned system to market-based one. The transformation program required substantial institutional reforms, one of the results being a new security law which in 1991 established the Warsaw Stock Exchange (WSE). In this thesis I analyse rates of returns on the WSE and try to examine the existence of the optimal portfolio on an emerging stock market with quantity constraints.

On the WSE, according to regulation, there exists a one-period returns limit, so prevailing models used for finding the level of assets risk seem to be inappropriate. The risk reduction effect of lower or upper limits institutionally imposed on stock exchange price movements are analysed. As the result of the maximisation of traders' utility function subject to expected quantity constraints, a new empirical model similar to the capital asset pricing model (CAPM) is developed, where the observed returns are corrected for the appearance of quantity constraints for the securities which constitute the market portfolio. An empirical analysis of returns from twenty-one securities traded on the Warsaw Stock Exchange has been carried out. The model with uncorrected returns has been estimated by the two-limit Tobit model and compared with the results for the corrected returns, as obtained by the unconstrained maximum likelihood method. The proposed model is tested using a second-pass cross-section regression and stronger tests based on the derivation of the security market line (SML). Results show that the imposition of trade barriers tends to increase rather than decrease the portfolio risk and it is therefore suggested that such barriers should be abolished.

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CHAPTER 1

INTRODUCTION

1.1 Introduction

The year 1989 was a year of political changes in Europe. The communist system collapsed and most of the Central and Eastern European countries began reforms to change the Soviet-type central planning system into free market economy. After over fifty years of planned economy the market was distorted by high inflation, foreign debt, social ownership and bureaucratic restrictions. The transformation program of adapting the old system to new circumstances was aimed generally at macroeconomic stabilisation, microeconomic liberalisation and fundamental institutional restructuring (see Balcerowicz 1995). Most of the Central and Eastern European countries have carried out a comprehensive liberalisation of prices, foreign trade and currency arrangements. Many small-scale state enterprises have been privatised. The progress with large-scale enterprises privatisation, restructurisation, financial sector reforms and other structural changes varies between countries (see Stern 1998). Generally, the speed of structural changes in recent years has been slower than in the period 1990-93. The reason for this is mainly due to the tasks undertaken in the beginning of the transformation being the easier ones with the most difficult tasks being left until later.

The most advanced countries in market-oriented transition included Czech Republic, Estonia, Hungary and Poland. Among them Poland become a leader in transformation. Poland is one of the biggest countries in Central and Eastern Europe with area of 312 677 square kilometres and a population of 38 mln (see Central

Europe On-line 1999). The Polish strategy, known as the Balcerowicz plan, was developed in three steps: identifying the main problems, defining what kind of economy should be reached after the transition process and specifying types of economic policy measures. The first economic reforms began in 1990. Price setting has been freed from administrative interference and foreign trade has been opened. Inflation was reduced relatively quickly from 639.6% in 1989 to 249.3% in the following year and to 60.4% in 1991[1]. During the period 1990-93 2097 state enterprises were privatised[2].

The transformation process required creating the capital market. The institutional reforms have begun in 1991. Firstly, nine state-owned commercial banks (formed from the National Polish Bank NBP (*Narodowy Bank Polski*)) were transformed into Treasure-owned joint-stock companies. As a consequence of financial reforms, the Warsaw Stock Exchange (WSE) was reopened and grew rapidly. WSE had an important role in the privatisation process of setting the market value of privatised companies. Offering shares of medium and large companies was a route to privatisation. The capital market development stimulated the government to issue the bonds directly to the public. This success of financial activities depended mainly on experience which is being acquired gradually.

In this thesis I analyse the stock exchange in Poland, namely the Warsaw Stock Exchange. I am interested in this particular case since WSE is the biggest stock

[1] CPI inflation rate was calculated from December to December each year. Source: Poland, Policies for Growth with Equity 1994, table 1.1, p. 99.
[2] See Poland, Policies for Growth with Equity 1994, table 3.1, p. 64.

exchange in Central Europe. The daily turnover per session on the WSE is approximately 50 mln USD, while, for example, on the Prague Stock Exchange (Czech Republic) it is approximately 28 mln USD, on the Bratislava Stock Exchange (Slovak Republic) is 1.15 mln USD and on the Bulgarian Stock Exchange is 0.3 mln USD. The capitalisation of the WSE at the end of the fourth quarter of 1998 was 20201 mln USD[3].

According to regulation there are quantity constraints imposed on the WSE. The constraints take the form of price limits. It is interesting to examine the relationship between risk and return in such a case. The regulator imposed the price limits having in mind a risk reduction and speculation counteraction. This thesis proposes the optimal portfolio allocation model which is developed for the analysis of the constrained returns. The model allows one to find risk levels of such assets and to identify if there are risk reduction effects in the case of imposed limits. As similar regulations are imposed on different stock exchanges (e.g. Lithuania, Turkey, China), the developed model can be applied for analysis of returns in other countries.

Recently, on 12 March 1999 Poland, as one of the most advanced countries in term of reformation within Central and East European countries joined NATO and shortly, Poland is due to be incorporated into the European Union. Obviously, the WSE will have to adapt to the European Union's (EU) standards. In spite of price limits imposed on the stock exchange market it can be shown that the returns on such a created emerging market tend to converge in the structure of behaviour of returns

[3] All data was recalculated from the data presented on the official stock exchanges web pages, see references for details.

across members of the EU (see Morley 1999). Therefore it is worthwhile to analyse the returns on the WSE and examine if the behaviour of returns tends to the behaviour of returns on mature markets. The academic challenge is to pay attention to the nontrivial organisation of the WSE caused by the price regulation. Non-standard dynamics poses additionally some new problems. As the prices of shares are limited, the distributions of returns are doubly truncated. In this thesis I examine if the non-normal distributions are only caused by the price limits.

1.2 Main contribution

In the main part of this thesis I propose a model of the portfolio allocation in case where some prices in the market are regulated (disequilibrium) prices. The regulation takes the form of an imposition of price barriers. My model is developed from the Sharpe-Lintner version of the capital asset pricing model. I proceed by developing a theoretical model based on the Benassy framework of disequilibrium trading.

The theoretical background is followed by the application of the developed model. The Warsaw Stock Exchange is an example of a market where quantity constraints are imposed, so an empirical analysis of shares of twenty-one of the longest established companies from the WSE is carried out. The analysis aims 1012 data points, daily observations from 4.01.1994 to 17.04.1998.

It is interesting to find the behaviour of returns should there be no constraints on the market. In order to find the hypothetical returns, i.e. returns in the case of no quantity constraints, the correction factor is introduced. The correction factor is based on the time varying probabilities of hitting the limits of prices.

Using the correction factor the corrected returns are then obtained. The model has been tested by a two-pass procedure and some extended tests. In order to find the level of risk of assets the model has been estimated, known as the first-pass testing. The theoretical model is estimated as a two-limit Tobit model for uncorrected returns and by maximum likelihood method for corrected returns. For comparison the model has been estimated by the OLS method for both kinds of returns.

The validity of the optimal portfolio allocation model with quantity constraints is examined by the weak second-pass testing which is the derivation of the security market line (SML). The SMLs are derived for corrected and uncorrected returns. Then some stronger tests are applied. The multivariate SMLs, known as a stronger second-pass testing, are derived for both kinds of returns. Finally, testing is extended to analysis of the Sharpe ratio indicators and Gibbons' test.

Finally, there are shown the findings of the effect of disequilibrium on the relative level of risk of the analysed shares and prospects of the further development of the WSE, especially in light of the incorporation of Poland to European Union.

1.3 Outline of chapters

Chapter 2 gives the background of the Warsaw Stock Exchange. The chapter is divided into three main parts. The first part includes the short history of the Polish capital market which gave rise to the current stock exchange. The second part introduces three trading systems operating on the WSE: a single-price auction, continuous trading and a block trade. The last part provides an overview of the organisation framework of the stock exchange. The conditions of trading for all kinds of securities on the WSE are given. Then the structure of the stock exchange is explained and some dynamics are presented. The description of the main market indices, such as WIG, WIG20, WIRR and MIDWIG is given.

The theoretical framework of the disequilibrium trading and quota signals market are included in chapter 3. The first part presents the traditional equilibrium theory. I then show that the market equilibrium conditions do not hold on the WSE; an explanation of the disequilibrium theory together with the price-setting process is then proposed. In the next part I extend single market analysis to that of a multi-market. I introduce the multi market constraints that might possibly exist, such as manipulation, stochastic and deterministic constraints and quantity constraints. Then, following Benassy (1982), I explain the concept of effective demand in the Walrasian sense, multiplier effect of constraints and two kinds of spillover effects: intertemporal and cross-sectional spillovers. Finally, I conclude with defining the properties of the WSE in the case the above of explained theoretical framework.

Based on the theoretical framework in chapter 4, I describe the standard version of the Sharpe-Lintner capital asset pricing model (CAPM) that is a background of the later developed optimum allocation model. Firstly, I give the assumptions underlying the model. Secondly, the model equation and its properties are described. Thirdly, I show three alternative methods of deriving the standard model, each based on the different assumptions important for the interpretation of empirical results. Then present a criticism of the standard model with hidden assumptions. Finally, I show some further developments together with various non-standard versions of the CAPM model.

The Benassy's framework and the standard portfolio allocation model give rise to develop a model for the WSE. In chapter 5, I present my CAPM-like model which is an extended version of the CAPM in case of disequilibrium trading, which takes the form of price constraints. I explain the main assumption and give an outline of derivation of the model.

The theoretical model developed in chapter 5 has been applied to the market. In chapter 6, I show the application of the optimal portfolio allocation model with market constraints on the Warsaw Stock Exchange. This chapter is divided into six main sections. The first section gives a critical review of earlier empirical researches on the WSE. The second section includes the formulation of the empirical model. Then an analysis of the returns from twenty-one companies from the WSE is undertaken. The following section includes an evaluation of the censored returns and the first-pass testing of the model. Then the estimation methods are explained. They include the two-limit Tobit model for uncorrected returns and the maximum likelihood method for corrected returns. In the last section I present the estimation results. I show a simple numerical experiment for simulating the efficiency frontiers for hypothetical portfolios with and without market constraints. The second-pass testing is presented in the following chapter.

Chapter 7 considers testing the optimal portfolio allocation model of disequilibrium trading. Firstly, the model is examined by the weak second-pass cross-section regression. In order to prove the validity of the model the stronger second-pass cross-section regression is applied followed by various other tests, namely the analysis of the Sharpe ratio indicators and Gibbons' test.

Finally, chapter 8 summarises the work presented in the thesis. This chapter includes the main findings of this research and suggestions for future research.

CHAPTER 2

DESCRIPTION OF THE WARSAW STOCK EXCHANGE

2.1 Introduction

For over forty years, since the Second World War, Poland was under the influence of the Soviet Union. In 1989, with determination the new leaders started political and economical reforms. From the oppressive regime of the communist system, the orientation turned into democracy. Consequently, the political changes caused economic changes. The economic situation which Poland inherited was really poor (for details see, for example, Ebril *et al.* (1994), Poland, Policies for Growth...(1994)). The transformation program designed to operate in three stages (see Balcerowicz 1995):

- an analysis of the initial conditions, for example, the macroeconomic situation, the structure of the real economy, the net foreign debt etc.,
- the transformation strategy like identifying the main problems and determining what should be reached
- the conditions prevailing during the process

The situation required institutional reforms. In 1991 a new security law, a new liberal foreign investment law and income tax law were accepted. As a natural consequence the Warsaw Stock Exchange (WSE) was opened in April 1991. This chapter presents the stock exchange operating in Poland before 1991 and describes the actual trade and price setting system. The chapter is divided into three main sections. The first section is an overview of the history of the stock exchange in Poland which gave rise to the current Stock Exchange. The second section introduces three trading systems operating on the WSE: a single-price auction, continuous trading and block trade and the procedure of trading there. The last section includes the organisational framework and the basic indicators of the Warsaw Stock Exchange. The description of the main market indices and information of the structure of the WSE is also presented.

2.2 History of the Polish capital market

The history of the capital market[1] in Poland goes back over 180 years, with a 51 year break. The first stock exchange was opened on 12 May 1817 in Warsaw. Trade was carried daily between 12.00 and 13.00. The main objects of exchange during the nineteenth century were bonds and other debt instruments. Prices of bills were set on Mondays and Thursdays, prices of commodities only on Wednesdays. The exchange was financed by the city council's budget.

The stock exchange gained legal status appeared in 1908 (see Grabowski (1996)) based on Napoleon's code. In the same year the exchange market in the Duchy of Warsaw was created. It was a mixture of securities and commodities markets. Initially there were six brokers working on the exchange and after 1921 the number increased to twelve. In

[1] According to the Dictionary of Finance and Banking (1997) by the capital market we understand 'a market in which long-term capital is raised by industry and commerce, the government, and local authorities. The money comes from private investors, insurance companies, pension funds, and banks and is usually arranged by using houses and merchant banks'.

1872 the security stock exchange separated into an independent institution and in the second half of the twentieth century equities also appeared on the market.

The stock exchange in Warsaw, known as the Warsaw Stock Exchange (WSE) since the First World War operated until the end of the Second World War (see Rozlucki 1998). Just before the Second World War there existed seven stock exchanges in Poland, in: Katowice, Krakow, Lwow, Lodz, Poznan, Warszawa and Wilno. The Warsaw Stock Exchange was the most important exchange since more than 90 % of the total trading was concentrated there[2]. 130 securities were traded in 1938, mainly as shares but also as municipal, corporate and government bonds.

Because of the political and the system rule changes that took place in Poland after the war the stock exchanges could not be re-opened. In that time Poland was a socialist country and a had non-market economic system, dominated by the state sector. The prices and exchange rates were controlled and the currency was not convertible. The macroeconomy was in a dramatic state with a high level of foreign debt (mainly in 1970s; for details see Balcerowicz 1995). Economic poverty and political repression led to social protest in 1956, 1968, 1970 and 1980. The independent trade union 'Solidarity' was dissolved in 1981 after 15 months of existance but negotiations with the government in the spring of 1989, known as the 'Round Table', led to re-legalisation of the 'Solidarity' and a partialy free election. From that time Poland was the first country in Central and Eastern

[2] Some sources notify that even more than 97 % of the national stock exchange's turnover were on the WSE at that time (see, for example, Grabowski 1996).

Europe to initiate political and economic reforms recognising the need to change the old communist rules.

The first non-communist government aimed to create a capital structure with the priorities of decentralisation, stabilisation of the economy and fundamental institutional transformation. The radical economic program was ready in January 1990 intendend to cut as a form of 'shock therapy' (see Gregory and Stuart 1995). The program aimed at stabilisation, liberalisation, changes in the tax system performing the role of a the social safety net. The institutional transformation included microeconomic liberalisation and fundamental institutional restructuring. In February 1990 the privatisation law was accepted and then the law of the central bank and financial institution was accepted in the first half of 1990. The next step in the new born system was establishing the institutional reforms with the result of the security law being passed on the beginning of 1991. Based on the Act on Public Trading in Securities and Trust Found from March 1991, the Warsaw Stock Exchange was established by the State Treasury in April 1991. At the same time they created the Polish Securities Commission, with a chairman appointed by the Prime Minister. The system of work was and still is based on French experience (Lyon Stock Exchange).

The first session took place on **16 April 1991** with 5 companies and the co-operation of seven brokerage houses. 112 orders were placed and total turnover was 1990 PLN. The official opening was on 2 July 1991.

WSE is a self-regulatory organisation. This is a modern security market[3], with centralised, fully computerised paperless trading (see Rozlucki 1998). All rules and main decisions concerning the WSE require approval of the Polish Security Commission.

In October 1994 the WSE was accepted as a full member into the International Federation of the Stock Exchanges (FIBV) and in May 1997 received the status of a designated offshore securities market from the US Securities and Exchange Commission.

The new Securities Act from 4 January 1998 introduced changes in the capital market, for example, the necessity to adapt the regulations to the OECD and European Union Rules, to introduce securities lending and borrowing mechanisms and to define rules of underwriting.

2.3 Trading systems and prices of shares

Transactions on the WSE are concluded in the order-driven system. Investors, after buying and selling securities receive only account statements from their brokers. Each security is registered in Central Depository for Securities.

[3] Security market is a part of the capital market which provides a market for shares and the represented capital has been raised there (see Dictionary of Finance and Banking 1997).

All intentions to buy or sell are deposit in licensed stockbrokers and the brokerage house is responsible for ordering securities and the correct execution of the order.

The orders can have two forms:

- with a price limit;
- at the market price PKD (*Polish: Po Kursie Dnia*).

The trader can also stress an additional condition WAN (*Wszystko Albo Nic*) meaning that he wishes to buy (or sell) everything that he ordered or nothing (see Lazor and Tryuk 1995). The WAN option excludes the possibility of a buy or sell reduction in realisation of a single order.

There are three possible systems of transactions on the WSE: **a single-price auction**, **continuous trading** and **block trades**.

The single-price auction is known as the call market, French *par casier* or German *Einheitskurs*. It means that the session price of the security is calculated based on submitted orders where each trader making the order through the brokerage house defines the quantities and prices of securities that are to be bought or sold. He can indicate the limits of prices or choose the option 'at the market price'. The order is valid maximum to the end of the next month.

The prices of securities on the WSE cannot change freely from session to session. The day price cannot be higher or lower by more than 10% than the previous session's price. The maximum price changes are called **upper** and **lower limitations**. The maximum admissible price change for bonds is 5%.

The prices of all shares, subscription rights and National Investment Fund (NIF) certificates are given in Polish zlotys (PLN). Prices of bonds are given in percentages of the nominal values and the settlement prices for bonds are calculated by adding accrued interest to the market price (see WSE web page 1998).

The most important role during the session is played by the brokerage houses operating on the WSE as specialists, nominated by the issuer of securities. The session price is established by specialists brokers who represent specialists on the trading floor. When the session is open the specialist broker obtains a list of orders for a given security, and having veryfied it establishes the price in accordance with the rules (see WSE web page 1998):

- to maximise turnover of that security;
- to reach the smallest possible difference between demand and supply at a given price;
- to minimise the difference in price between that of the current session and the previous one.

The above sequence plays an important role, for example, if two prices maximise turnover then a day price is a price that gives the smallest difference between demand and supply. If two prices give the same difference between demand and supply then the price nearest to the previous one is chosen. The established price is valid during the whole session.

The established price should be consistent with several principals (see Lazor and Tryuk 1995):

- all orders are 'at a market price' and orders to buy with limits higher than established price and orders to sell with limits lower than that price must be executed;
- orders to buy or sell with limits equal to the settled price can be executed, executed proportionally or not at all;
- orders to buy with lower limit and sell with higher limits than the established price are not executed.

Regulation on the WSE fixes the rounding-off the prices. For the different price intervals there are different settings, so:

- up to 2.50 PLN the minimal fluctuation is 0.01 PLN (prices is rounded up to 0.01 PLN);
- from 2.50 PLN to 10.00 PLN the minimal fluctuation is 0.05 PLN;
- from 10.00 PLN to 50.00 PLN the minimal fluctuation is 0.10 PLN;
- for more than 50.00 PLN the minimal fluctuation is 0.50 PLN.

For example, if the brokerage house receives an order with a given price limit of 58.25 PLN such an order will be disqualified with a note that the limit is not consistent with the price fluctuation. In this case a limit of 58.00 or 58.50 PLN could be accepted.

If all competitive orders (PKC), such as all buy orders with price limit higher than session price and all sell orders with a price limit lower than established prices, are executed at the given particular price, then this price is called the **equilibrium price** and the market is a **balanced market**. On this market only the orders with price limits equal to the session price may be executed partially (in the same proportion) or not executed at all. The partial execution is called a **buy** or **sell surplus** and is indicated by the symbol **nk** (Polish: *nadwyzka kupna)* or **ns** *(nadwyzka sprzedazy).*

Occasionally the price that maximises turnover and minimises the difference between demand and supply is higher or lower than upper or lower limit, yet according to the regulation that price cannot be accepted as a session price. In this case the price is established as a maximum (minimum) possible price, on the level ± 10% of the previous session price. This price **is not** the equilibrium price and the market is called **unbalanced**. For this market is necessary to calculate the ratio of demand to supply or vice versa. If the ratio is greater than 5:1 transactions are suspended. When there is an excess of orders to buy the non-transactional price is published on the stock exchange list with the symbol **ok** *(oferta kupna).* If there is a domination of sell orders then the symbol is **os** *(oferta sprzedazy).* For the ratio smaller than 5:1 the 'heavier side' at the market is proportionally reduced. Stock exchange information is marked by **rk** *(redukcja kupna)* in case of buy orders reduction or by **rs** *(redukcja sprzedazy)* if sell orders are reduced.

There is also a possibility of intervention by the specialist to reduce the market imbalance. The specialist broker can buy or sell securities during the session at a given price from a personal inventory.

The next phase in a market trade is the phase of **balancing**. The size of a market imbalance is determined after establishing the price. The initial imbalance from the order book is automatically reduced by the counterbalancing non-competitive orders (PCR). If PCR orders can satisfy demand and supply the crossing phase for security is active, if not, the intervention of specialist is needed. He has to balance the market or reduce the market imbalance or finally announce a **post-auction offer**. The post-auction offer is, for example, an invitation for other brokers to introduce additional orders. The additional orders are defined as orders W (*zlecenia W – zlecenia warunkowe*) and have no influence for the day-price of the assets (see Socha 1992), since only counterbalancing orders are accepted. If the balance is achieved by 12.00 (midday) the crossing phase for that security begins, if the balance has not been achieved by this time there will be no crossing phase.

The crossing phase is active only if the security was imbalanced, then all PCR's are revealed. Also, additional orders may be entered, depending on the side of imbalance. The orders are satisfied at the price established initially and in this case the specialist can also order securities. Both phases, balancing and crossing are known as **post-auction trading**. The member of the WSE can also ask a special order called order L (*zlecenie L – zlecenie w celu likwidacji*) in the case of liquidation mistakes appearing on the market. This order has to be realised first and as a whole (see Socha 1992).

The **continuous trading system** used by the Warsaw Stock Exchange was developed in 1992. Initially the system was used for Treasury bonds and then from July 1996 for the first five shares of listed companies. From August 1996 share certificates of National Investment Funds (NIF) have been traded in this system. The one of the differences between the single price system and the continuous trading system is that in the latter case the transaction units consist of round lots. The size of a round lot depends on security, but is approximately 10000 PLN (~4000 USD).

The session starts at 11 a.m. and between 9 a.m. and 12.30 p.m. only limit orders (with specified maximum or minimum price) are accepted. The orders are valid for a longer duration but no longer than the end of the following week (in a single price system it was to the end of the following month). At 12.30 p.m. the WSE stops taking orders and the opening price is announced, and continuous trading begins.

The opening price is set in the same way as for a single-price auction. Again, the opening price cannot be different from the reference price by more than 5% for bonds and 10% for shares and NIF certificates. If a security is traded in a continuous system the reference price is the price of the last transaction from the previous continuous trading session, if not, the reference price is the price from the previous session's single-price auction. In a situation when specialist cannot establish the opening price by 12.30 p.m. the first transaction's price becomes the opening price. The session starts with selecting orders with the same price limit and the time of their placement. Orders are accepted only if the limit order on the opposite side of the market is waiting in the system. The session continuous until 2.30 p.m.

The first five companies began to trade continuously on 8 July 1996 with the selection criterion acting as the highest liquidity. From the end of January 1997, 21 companies have traded in a continuous system. Still the single-price auction is a major system of trading on the WSE. On 15 July 1996 NIF certificates started on single-price auction system and from 12 August 1996 they have been traded continuously.

The last trade system have been introduced onto the WSE is the **block trade**. Block trades are called large blocks of securities that can be traded off-session which take place when buy and sell orders are announced for the same number of securities at the same price. The securities are accepted as a block trade only if their number is at least equal to the average number of sold securities during last three sessions. For admitted securities (which have not yet been introduced to trading) the block must be at least 2% of the number of securities admitted.

If the securities are traded in blocks their prices can be different from the session price for securities. If the number of securities in the block is less than 5% of securities admitted to trading then the block price can differ from the last session's price by up to 15%. If the block is larger than 5% then price difference from the last session has a 30% limit. The smaller fee is charged for block trades compared to the standard charges for the same security during the session.

There are three systems of trading on the WSE, but the most important role is still played by the single-market auction. This situation exists because continuous trading was only recently introduced and is still limited by the low liquidity of the market.

Table 2.1 Shares indicators on the WSE for different system of trading.

Shares indicator							
	1991	**1992**	**1993**	**1994**	**1995**	**1996**	**1997**
SINGLE MARKET AUCTION							
Number of orders per session	1423	3119	17323	52974	26475	25704	30106
Number of transactions per session	877	1233	9832	24594	7164	8074	9891
Average value of transaction(PLN)	926	1852	5180	4895	6814	12688	14633
Turnover value per session (PLN mil.)	0.8	2.3	51.0	120.4	49.0	102.4	144.7
Total turnover value (PLN mil.)	0.03	0.23	7.75	22.64	12.20	25.61	36.04
CONTINUOUS TRADING							
No of transactions (single counted)	-	-	-	-	-	31	460
Total turnover double-counted (PLN mil.)	-	-	-	-	-	251.32	3338.2
BLOCK TRADES (TOTAL)							
Aver. value of transactions (PLN)	-	-	NA	-	64	149	426
Total turnover double-counted (PLN mil.)	-	-	3.0	-	388.1	1574.96	4332.25

Source: WSE web page (1998).

The basic yearly information about trade in each system is given in table 2.1. As we see all indicators show a tendency to increase with the highest growth taking place in 1994. Compared to the previous year the number of orders rose by 305.8% and the number of transactions by 250.1%. Turnover value for shares rose from 0.04% of GDP in 1991 to 11.1% of GDP in 1994 (see Poland Country Profile 1996). This occured since many new companies entered the WSE in 1994. Their shares showed profit as a result of the large differences between the prices of subscribed shares and the prices of these shares in the first sessions. It caused a substantial interest in investing on the WSE and many new small investors appeared. This new way of increasing wealth was speculative and generally misunderstood by the broad population and by 1995 many investors lost money on the market and withdrew from the stock exchange.

2.4 Organisation, structure and dynamics of the WSE

2.4.1 Organisation of the WSE

The Warsaw Stock Exchange was created by the State Treasury and by regulation was intended as a non-profit joint-stock company. The share capital at present is approximately 6 million PLN held as 60000 registered shares[4] (see WSE web page (1998)). Shares of the WSE can be held only by banks, brokerage houses and the State

[4] The data are taken for a day 22.05.1998.

Treasury.

The Act of Investment Funds was adopted on 21 February 1998 and created a new type of investment vehicle. Also in January 1999 the establishment of pension funds began. An increase of exchange capitalization is expected in the future as a consequence of the privatization of the largest Polish companies representing the oil industry, the power grid, copper mining and processing and the State-owned banks.

Initially only two sessions a week took a place in 1991 and 1992, then a third session a week was introduced from the beginning of 1993, and since the end of 1994 five (daily) sessions have been taking place.

Sessions on the WSE take place from Monday to Friday between 11 a.m. and 2.30 p.m. On the Warsaw Stock Exchange there are traded shares of listed companies (with subscription rights), shares of National Investment Funds, Treasury bonds and National Investment Funds certificates. Trade takes place on three markets: on the main, parallel and free markets. The main market is aimed at the bigger more established companies. The parallel market is intended for medium-sized companies with shorter track records and a free market is intended for smaller companies (see Rozlucki 1998).

Shares can be traded on each of these three markets depending on several criteria, such as the size of company, its history etc. Most of shares and subscription rights are quoted in the single-price auction. From July 1996 most liquid stocks have been traded continuously and transactions are nominated in round lots of a size around 10000 PLN (~4000 USD). The **subscription rights** for new shares are traded automatically in the single-price system. From July 1997 the shares of National Investment have traded continuously.

Bonds are the only debt instruments on the WSE. They are traded continuously and the transaction unit is a lot size of 10000 PLN (~4000 USD). Some of Treasury bonds are still traded in the single-price auction. On the WSE there are two types of Treasury bonds: Fixed interest bonds with maturity of two and five years and bonds with floating interest rates, with one-, three- and ten-year maturity dates. **Fixed interest bonds** appeared on the market on 4 May 1994 with a nominal value for each bond of 1000 PLN and a transaction unit in lots of 10 bonds. Their interest rate varies from 12% to 18%. **Floating interest bonds** with one- and three-year maturity dates have a nominal value of 100 PLN, the ten-year maturity date bonds of 1000 PLN. The transaction units of lots are 100 and 10 respectively. For one-year bonds inflation is taken into account with an interest rate of 5% above the inflation. For three-year bonds the interest rate is 10% above the average earnings of 13-week Treasury bills and the interest rate for the last group of bonds is calculated based on the average real yield of 52-week Treasury bills plus 1%.

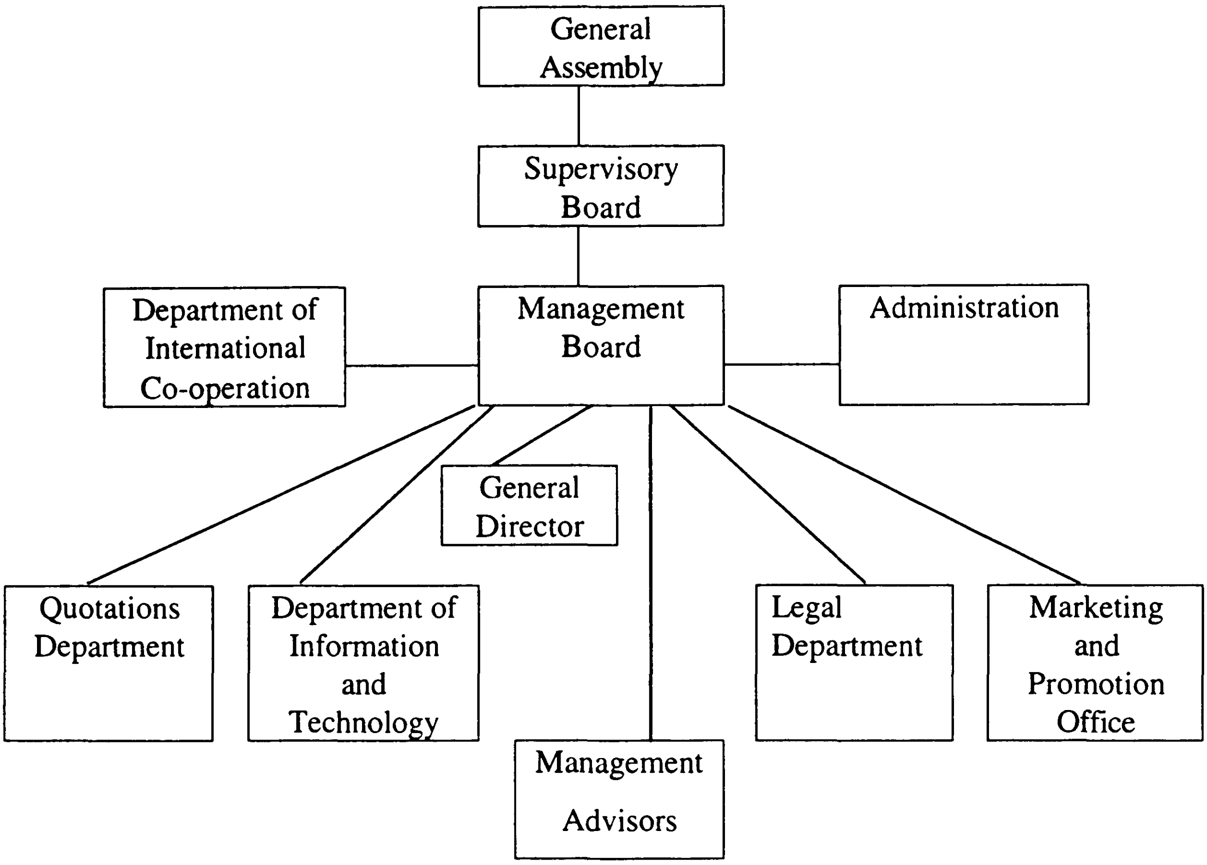
National Investment Funds certificates first time appeared on the Warsaw Stock Exchange on 15 July 1996. They were traded in the single-price auction and since 12

August 1996 they have been traded continuously. The certificates were issued by the Ministry of Privatisation in the framework of the Mass Privatization Programme. Each Polish citizen in age over 18 could buy one certificate for 20 PLN (~8 USD). Up to the end of 1997 NIF certificates were exchanged for shares in the National Investment Funds, which have been traded on the WSE since 12 June 1997. For the one certificate 15 shares, one of each Fund were given.

The Polish Securities Commission is the central body of the government administration concerning trading of securities and includes a supervisor, two vice-supervisors and six members. The main duty of the Commission is to control that the rules of honest trade and competition in the trade of securities are abided by (see Czerniawski 1994), and without their permission it is not impossible to admit securities onto the WSE market.

2.4.2 Structure of the WSE

Figure 2.1 Structure of the Warsaw Stock Exchange



Source: Lazor and Tryuk (1995)

Figure 2.1 shows the structure of the WSE. The highest decision body of the WSE is the General Assembly. The meeting takes place once a year. It has the right to make changes in Statutes and Rules. The Supervisory Board, including 14 members appointed by the General Assembly, controls the overall operation of the exchange. The day-to-day operations are co-ordinated by the Management Board. This Board consists of three people: the chairman and two other members.

The decision to admit a security is made by The Supervisory Board of the Exchange upon a motion of the Management Board of The Exchange. According to the Act on Public Trading in Securities and other Rules of the WSE the securities can be admitted for trading on the main market if they are admitted for public trading. Also they are required to be transferable without limitation. The value of the shares to be admitted must be at least 24 million PLN and the value of other securities 12 million PLN. For other securities (other than shares) at least 20% of those issued must be available on public offer. The value of the securities presented to public offer should be at least 6.2 million PLN. The company whose shares are to be admitted must have a share capital of at least 7 million PLN. According to Exchange regulations the company has to provide information concerning the financial standing and development prospects, losses and profits of its organisation.

For the parallel market and the free market conditions are similar. The main difference being in the values of limits and data to be presented in the admission prospectus.

If the issuer whose shares are on the market wishes to introduce bonds it is enough that the value of the bond issue is at least 1 million PLN.

The Exchange Supervisory Board considering the application also has to analyze (see WSE web page 1998):

- the issuer's financial standing and forecast, including profitability and liquidity;
- the prospects of the issue's development with plans, sources of financing etc.;
- the experience and competence of members of the issuer's managing and supervisory bodies;
- the compliance of conditions on which the securities were issued with the nature of exchanging trading;
- the interest and safety of market participants.

2.4.3 Dynamics of the WSE

The official Warsaw Stock Exchange share index is known as **WIG** (*Warszawski Indeks Gieldowy*). The index represents a total return, including dividends and pre-emptive rights[5] (subscription rights). The WIG includes all companies on the main market except those of foreign companies and investment funds. The value of the index is calculated once daily and is based on the fixing price. The base value of the index for the first session on 16 April 1991 was 1000.

The main formula for WIG index is:

$$WIG(t)=\frac{M(t)}{M(0)}\cdot K(t)\cdot 10$$

where $M(t)$ is a capitalisation of index portfolio on session t, $M(0)$ is a capitalisation of index portfolio on the base data that is on the first session on 16 April 1991 and $K(t)$ is a

[5] By pre-emption rights I understand rights of shareholders to be offered the new shares in the same proportion to their holding of shares (see *Dictionary of Business* 1996).

chain index factor for session t. The last chain factor was introduced in order to avoid non-market changes in the index portfolio capitalisation (i.e. changes caused by splits and issue of new securities). The value of this factor for the first session $K(0)$ was one. Then the formula for the next values of the factor is:

$$K(t+1)=\frac{M(t)}{M(t)-D(t)-V(t)+Q(t)}\cdot K(t)$$

where:

$$V(t)=\sum_i \frac{P(i,t)-P(i,em)}{S(i)}\cdot N(i).$$

The meaning of the symbols are as follows:

$D(t)$ - value of dividends from shares, which were traded on the session t with dividend,

$V(t)$ - theoretical value of subscription rights from shares, which were traded on the session t with rights[6] (only for positive values of rights),

$Q(t)$ - market value of shares included or excluded (+ or -) from the index portfolio after the session t

$P(i,t)$ - price of shares i at the session t

$P(i,em)$ - price of shares of new issue with pre-emptive rights

$S(i)$ - number of rights required to receive one share of new issue

$N(i)$ - current number of shares in the index portfolio

[6] A rights issue, according to the *Dictionary of Business* (1996), is 'a method by which quoted companies on a stock exchange raise new capital in exchange for new shares'.

If there has not been any corporate action then the formula for WIG index is easier:

$$WIG(t)=\frac{M(t)}{M(t-1)}\cdot WIG(t-1)$$

where $M(t-1)$ and $WIG(t-1)$ are capitalisation of index portfolio and value of the WIG index from the session $t-1$.

The index portfolio is adjusted at the end of each quarter to include shares of new companies, which have appeared on the market during the last quarter. During that time each company's participation is cut up to 10% and the individual sector up to 30%.

There is also calculated on the WSE a price index for 20 companies chosen from the main market known as **WIG20**, an index created on 16 April 1994. The initial value of the index was set at 1000. The company is represented in WIG20 index if the relevant shares are quoted in several lines in the main market. If the quotation line is not determined then the selection criterion is a turnover. The index is calculated based on fixed prices every 5 minutes during continuous trading.

The **MIDWIG** index is the price index for the medium companies quoted on the main market. The index includes 40 companies. WIG20 and MIDWIG cover over 90% of the trading value and market capitalisation of the WSE. The index was introduced on 31 December 1997 with the base value 1000. For continuous trading the index is calculated every 2 minutes.

The formula of MIDWIG is:

$$MIDWIG(t) = \frac{MID(t)}{MID(0)*W(t)}*1000$$

where $MID(t)$ is the current $MIDWIG$ portfolio capitalisation, $MID(0)$ is base $MIDWIG$ portfolio capitalisation and $W(t)$ is the comparability index factor.

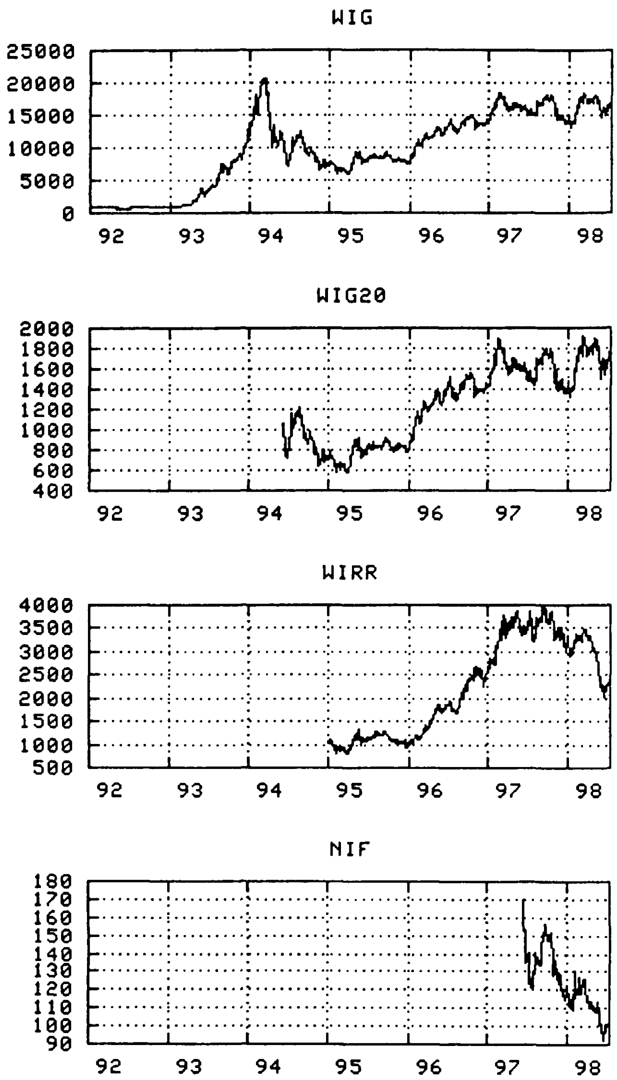
The next index, which is calculated on the Warsaw Stock Exchange, is **WIRR** (*Warszawski Indeks Rynku Rownoleglego*). This is a return index which includes dividends and pre-emptive rights of all companies listed on the parallel market. The WIRR index was created in 31 December 1994 with a base value 1000. The index is calculated once a day during the session and is based on fixed prices.

The market index for investment founds, called the **NIF index**, was introduced on 12 June 1997. This is a weighted index based on prices with a base value of 160. Recently the index included 15 funds[7].

As shown in figure 2.2 the behaviour of the market indices were similar during all the years of the WSE operations.

[7] The information for 23 July 1998.

Figure 2.2 Dynamics of the market indices



Source: PAN web page (1998)

The main yearly indicators concerning sessions and behaviours of the shares of the listed companies in 1991-1997 as presented in table 2.2.

Table 2.2 Trade on the WSE

Shares indicator							
	1991	**1992**	**1993**	**1994**	**1995**	**1996**	**1997**
Number of sessions	36	100	152	188	249	250	249
Number of listed companies (end of period)	9	16	21	36	53	66	96
Average capitalisation (PLN mil.)	79	307	1962	8928	9030	19351	33601
WIG (end of period)	919.1	1040.7	12439.0	7473.1	7585.9	14342.8	14668.0
WIG20 (end of period)	-	-	-	732.0	791.9	1441.8	1457.8
PLN return on the WIG index(%)	-	13.2	1095.3	-39.9	1.5	91.9	2.3
USD return on the WIG index(%)	-	-20.5	791.3	-47.8	0.0	62.0	-16.2

Source: WSE web page (1998).

The first major change on the WSE took place in 1993. By the end of 1993 prices rose by 12 times on average while the weekly trade turnover increased by a factor of 40 (see Bolt and Milobedzki 1994). Again the turning point for the WSE was in 1996. The average market capitalisation doubled as a result of increasing prices and the growing number of listed companies (see Rozlucki 1998). Market capitalisation rose from 0.2% of GDP in 1991 to 3.5% of GDP in 1994 and reached 4.5 billion USD in 1995 (see Poland Country Profile 1996). The tendency for main indicators is still present on the WSE.

Figure 2.3 Prices of assets of *Tonsil*

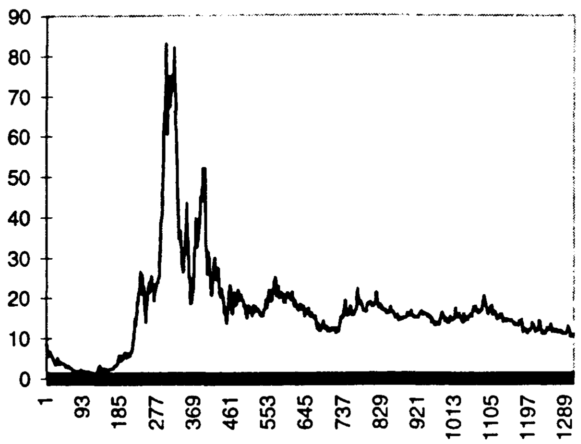


Figure 2.4 Standard deviations of the moving average of WIG

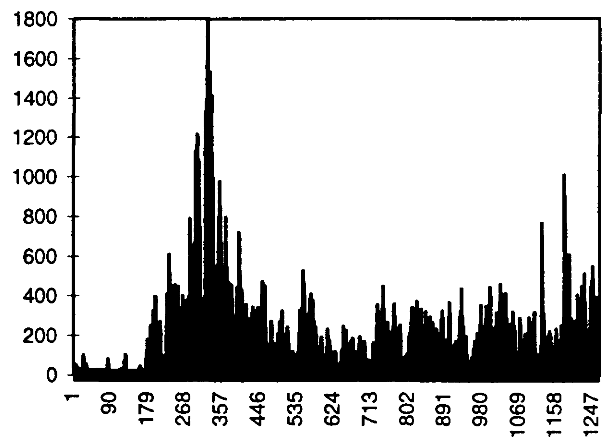


Figure 2.3 shows the behaviour of daily prices of a typical company *Tonsil* which has been on the market since the first day of operation the WSE in 16.04.1991. The horizontal axis indicates the number of the session. The standard deviations calculated based on the five-period moving average for the WIG index, a measure of dynamics, are shown in figure 2.4. During the first sessions of operating of the WSE prices varied

widely. The first five sessions indicated decreasing prices, the next few sessions price increased, and then again decreased and increased. This process continued to the end of 1991. The number of price changes by a maximum possible of 10% was large. Then the year 1992 showed a rapid decrease of prices. From December 1992 (session 133) to March 1993 there were changes up and down again. From 6.04.1993 (session 176) prices started to increase. In almost every session the market noted the maximum admissible price changes. For example, for *Tonsil*, the high price jump was noticed from 2.08.1993 (session 224) to 26.08.1993 (session 235). During the 9 sessions the price increased by 200%. Then the prices decreased marginally and then the highest increase was from the end of November 1993 to the end of January 1994. The prices of the *Tonsil* company increased by about 420% from session 270 to 299. The next noticeable price increase was in March 1994 (around session 320) and then again in August 1994. From the end of 1994 prices started to stabilise. The market does not show the calendar effects such as day-of-the-week effects (the measurement method is given by Taylor 1986). Recently the changes are more stable and the number of prices reaching the maximum price limit is much lower. Recently the WSE started to reflect the more important events on the world stock exchanges[8].

[8] For example, the crash in Hong Kong financial market and recession in Russia in September 1998 caused price decreasing in the WSE.

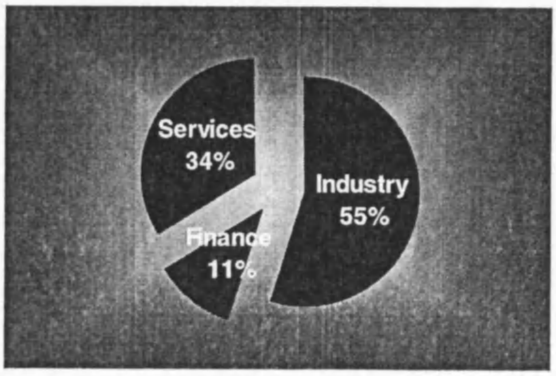
Table 2.3 Composition of listed companies on the WSE.

Sectoral composition of listed companies *)		
Macrosector	**Sector**	**Number of companies**
INDUSTRY	Food	20
	Light industry	14
	Chemicals	21
	Electromechanical	14
	Metals	8
	Wood & Other	9
FINANCE	Banking	15
	Insurance & Other	3
SERVICES	Construction	26
	Conglomerates	10
	Information technology	6
	Other	11

*) data for the day 22.05.1998

Source: WSE web page (1998).

Figure 2.5 Proportions of macrosectors in total number of listed companies.



The sectoral composition of listed companies, given in the table 2.3 and figure 2.5, shows that the shares on the WSE represent mainly industrial companies (exactly 86 companies which give 55% of total companies). Services are represent by 53 companies. The lowest proportion belongs to finance institutions (18 companies with proportion of 11%).

CHAPTER 3

THEORETICAL FOUNDATIONS: DISEQUILIBRIUM TRADING

3.1 Introduction

The organisation of the Warsaw Stock Exchange, described in the second chapter, showed that there are price limits institutionally imposed on the market. These limits can lead to market disequilibrium, so the WSE market is treated as a market in disequilibrium. Therefore it is necessary to explain the general idea of the disequilibrium. In this chapter I describe the WSE as a market in disequilibrium. There are different types of market constraints which might possibly exist. I intend to describe them and explain how they are related with the WSE. There are some important conclusions which build into a theoretical framework (the main model assumptions) for deriving the optimum portfolio allocation model for this particular market.

This chapter is divided into four sections. In order to understand concepts of disequilibrium I start by presenting an equilibrium concept. I introduce concepts of disequilibria. The price-setting process, known as the tatonnement process, which was adapted to the WSE, is explained in the second section. The next section contains the main definitions and the analysis of multi-market in case of absence limitations and with market constraints, which can lead to spillover effects. Finally, conclusions are proposed concerning the WSE arising from theoretical foundations.

3.2 Equilibrium trading

3.2.1 Equilibrium concept

In order to define a proper terminology for analysing the market equilibrium let me first examine the different concepts of equilibrium.

The equilibrium concept was first used in economics by James Steuart in 1769 (see Milgate 1987). From that time equilibrium analysis has been the basis of economic theory. Currently there are two meanings of equilibrium used by economists (see Hicks 1961). The first, used by Marshall and Walras (see Benassy 1982 or Grayson 1965), concerns market equilibrium and assumes that the market is in equilibrium when demand and supply are equal. The second, more general, definition as given by Machlup (1958), says that equilibrium is '...a constellation of selected interrelated variables, so adjusted to one another that no inherent tendency to change prevails in the model which they constitute'. From his point of view it is impossible to exclude terms 'equilibrium' and 'disequilibrium'. Machlup (1958) proposes a four step model:

'Step 1: *Initial Position*: 'equilibrium', i.e., 'Everything could go on as it is'.

Step 2: *Disequilibrating Change*: 'new datum', i.e., 'Something happens'.

Step 3: *Adjusting Changes*: 'reaction', i.e., 'Things must adjust themselves'.

Step 4: *Final Position*: 'new equilibrium', i.e., 'The situation calls for no future adjustment''.

Bannock, Baxter and Davis (1992) define equilibrium as 'a situation in which the forces that determine the behaviour of some variable are in balance and thus exert no pressure on that variable to change. It is a situation in which the actions of all economic agents are mutually consistent'. According to their understanding of equilibrium, price is established in a process in which price is increased when demand is in excess and is cut when supply is in excess. The short-run equilibrium can occur when some quickly adjusting processes are in balance. For instance, if the company maximises profit on the perfect competition, market equilibrium price could be equal to the marginal cost (see Grayson 1965). Generally there are three possible equilibria under pure competition. The trader may operate at a profit or at a loss. In the case of trading on the stock market, it is assumed in each model that the trader is maximising his utility function.

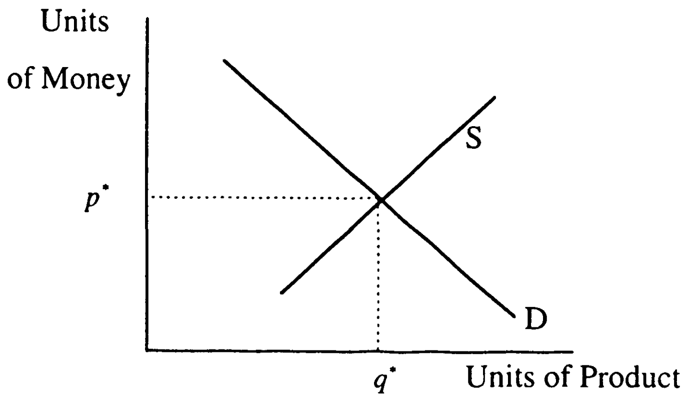
Another definition of equilibrium proposed by Blad and Keiding (1990) describes it as: 'a situation where each of economic agents (...) has made a choice under the given institutional constraints, and where choices are mutually consistent'. A particular example of a general equilibrium is market equilibrium.

According to the above definitions equilibrium and disequilibrium are conventional notions. Some economists define disequilibrium as a particular case of market equilibrium. For my purpose I assume that according to the definition given by Bannock and others that equilibrium occurs when some of the variables are in balance. The next section introduces individual market equilibrium.

3.2.2 Single market analysis

Let me analyse one hypothetical market. p is the price of particular security on this market. Based on Benassy's (1982) model and using his notation, the demanders and suppliers are denoted by $i = 1,...,n$ and their respective numbers are identical. Demand of an individual i'th consumer $d_i(p)$, which is a basic notion of the equilibrium analysis, depends on four conditions: desire, purchasing power, substitutes and expectations. According to Grayson (1965) demand for security means the actual amounts that will be purchased at the price p at a given time. Supply $s_i(p)$ represents the amount of the securities that will be supplied during a given time period at the price p. As we see both demand and supply are functions of the price p. Their graphic representations known as demand and supply curves, in the absence of any limitations, are shown in figure 3.1.

Figure 3.1 Equilibrium of demand and supply



If the curves are put on the same graph the point where they intersect is the point of equilibrium. It means that at the price p^* the quantity q^* is demanded and supplied. Where the price is higher then p^* there will be an excess of supply, at the lower price there will be an excess of demand.

Indexing all traders on the market by $i = 1,...,n$ assumes that the trader i has an initial quantity of money $\overline{m}_i \geq 0$. Symbol $\omega_i \in R_+$, represents an initial endowment of the security (R_+ is a set of all nonnegative relations). The final quantity of money that he will hold is $m_i \geq 0$ and the final quantity of securities is represented by $x_i \in R_+$. Let z_i denotes the volume of agent i's net transaction of a good, elementary transaction exchanging one unit of goods against p units of money. The sign of z_i is positive in the case of a purchase, negative for a sale and $z_i = d_i - s_i$. There could be written also that a net transaction for agent i $z_i \in R_+$ and that the price $p \in R_+$.

The relations of final holdings of securities x_i and money m_i are as follows:

$$\begin{aligned} x_i &= \omega_i + z_i \\ m_i &= \overline{m}_i - pz_i \end{aligned} \tag{3.1}$$

The **Walrasian** equilibrium is based on maximisation of the individual utility function of the investor. It is assumed that the trader i ranks his consumption vector and money holdings according to the utility function $U_i(x_i, m_i)$. Then the solution of the problem:

$$\max \mathrm{U}_i(x_i, m_i) \text{ subject to } \begin{aligned} x_i &= \omega_i + z_i \geq 0 \\ m_i &= \overline{m}_i - pz_i \geq 0 \end{aligned} \tag{3.2}$$

gives an individual vector net demand function $z_i(p)$ by giving solution in z_i.

In this case there is no demand of money.

The price that optimises trade on the capital market is also established under the assumption that the agents can purchase and sell as much as they want in the absence of any limits. That would be the case on the WSE if there were no limits. Then the optimum price (p^*) is derived from an aggregate function based on individual functions of demand and supply:

$$D(p)=\sum_{i=1}^{n} d_i(p) \quad \text{and} \quad S(p)=\sum_{i=1}^{n} s_i(p)$$

conditionally to the equality:

$$D(p^*)=S(p^*). \tag{3.3}$$

Realised transactions will be equal to demands and supplies at the equilibrium prices $d_i(p^*)$ and $s_i(p^*)$.

The main assumption of the market equilibrium is that of continuous pricing. The price is not constrained and can adjust to the changing demand and supply, with here free entry into the market. Demand is represented by the quantity that buyers wish to buy at each price. The lower the price, the higher the quantity demanded. Supply is a quantity of the securities sellers wish to sell at each price, so the higher the price, the higher the quantity. Demand curves slope downwards, supply upwards. Each trader assumes that his trading activity does not effect the price on the market. The trader can clearly define his preferences. For example, if he prefers to buy an asset A than an asset B and B than C it implies that he prefers A than C and the preference curve is concave. The other things such as technology, the price of inputs and the degree of government regulations are assumed to be constant along a given supply curve. An improvement in technology or a reduction in input prices will increase the quantity supplied at each price. The factor inducing an increase in demand shifts the demand curve to the right, thus increasing equilibrium price and equilibrium quantity (see Begg *et al.* 1994).

The main equilibrium assumptions (see, for example, Arrow and Hahn 1971) very often are not relevant on real markets. There are many reasons for this. Following Benassy (1982) the most important being:

1. Some prices can be institutionally constrained, for example price constraints imposed by government (maximum price or guaranteed minimum price). Also in the case of planned economies prices are usually fixed.
2. Price adjustment can be slow in imperfect competition, for instance sales promotions will replace competition.
3. The nature of some goods lead to the impossibility of price adjustment to the required supply and demand, i.e. on labour market where wages are influenced by other forces.

3.2.3 Trade quantities and price behaviour

The behaviour of the trade on the market can be characterised by various dynamics. One example is sequential trading. For example, on the WSE traders can sell or buy assets sequentially[1]. Another aspect is trade of small and large quantities.

A model of trade quantities in the case of sequential trade with distinguishing for sale of small and large quantities is developed by Easley and O'Hara (1987). Their

[1] By sequential trading I understand trading across time. After executing an order the trader can lodge another order there, or just join the market in any convinient time.

model is based on the Glosten and Milgrom model (1985). They assume that traders trade an asset with competitive, risk neutral market makers[2]. These market makers quote bid and ask prices and adjust quotes across time. It is additionally assumed, compared to previous known models (see, for example, Glosten and Milgrom 1985) that the size of trade could be different at each transaction (from small to large quantities). The model is based on information uncertainty and the appearance of new information is not assumed. Instead the nature of the game[3] has two moves in deciding if new information is introduced and if it is so then what this information will be.

An information event is defined as the occurrence of signals about the value of the asset denoted by s with probability α. It is assumed that if the signal occurs it happens on the day before trading begins. There are two possible values of signals, low and high. If the signal does not occur traders are **uninformed**, but if it occurs some fraction μ of traders receive it. An example of it is the private information on the market. If such situation exists it is common to stop trading up to the time when information will be known to all.

As stated above, trade occurs sequentially with different-sized orders. Traders are chosen the population with given probabilities. They could buy small or large quantities of assets, denoted by B_1 or B_2, or sell respectively A_1 or A_2 and they are allowed not to trade at all. In each case the price is set by the market maker.

[2] Market makers buy and sell shares to service the public's demand to trade. There is an important role of them on many equity markets, they stabilize prices and play a role of auctioneer (see Schwartz 1987).

[3] By the game Easley and O'Hara call all buying and selling activities (see O'Hara 1997).

The new assumption is that traders are risk neutral and they trade to maximise their expected profit. It is also important to note that the informed trader always prefers to trade larger quantities, in order to maximise profit, while the uninformed trader is unaffected by the size of trade. Therefore during the session a rational market maker interprets the presence of large orders as a signal of new information and adjusts prices accordingly.

It is worth to point out the competitive behaviour of trades. This behaviour is due to the simplification of the informed traders decision problems. Without this competition and with multiple informed traders, the market equilibrium would be difficult or even impossible to describe. According to Easley and O'Hara, two types of equilibrium are demonstrated on the market (the model is described in Easley and O'Hara 1987 and O'Hara 1997). The first type is when all informed traders choose large quantities while uninformed traders trade small quantities. This form of trade is called **separated equilibrium**. The second, called **pooled,** is when the traders can order both quantities, large and small. In early years the trade on the WSE took place only in small quantities. Recently, since the stock exchange has included more companies and capitalisation has been relatively higher, investors have traded large quantities of securities.

Assuming that traders trade only large quantities, the problem of the market maker's decision is solved. The same problem is solved in the case when both quantities can be bought or sold. Finally the model has two outputs. Both solutions apply to the situation in which equilibrium occurs.

If trade of the informed traders is made only in **large quantities** then the market maker's pricing policy is in the separating equilibrium. This policy has several properties. One of them is that there is no spread for traders trading small quantities. If there is no small trade then there is no reason for market maker to protect himself by setting a spread for this kind of trade. Suppose that the informed investors know that the true value of the security will be either low ($\underline{\mu}$) or high ($\overline{\mu}$). Prices for large quantities show a spread and are given by the following formulae (see O'Hara 1997):

$$b^* = \mu^* - \frac{\sigma_\mu^2}{\overline{\mu} - \underline{\mu}}\left[\frac{\alpha\upsilon}{w_A^2(1-\alpha\upsilon)+\omega\alpha\upsilon}\right], \qquad (3.4)$$

$$a^* = \mu^* - \frac{\sigma_\mu^2}{\overline{\mu} - \underline{\mu}}\left[\frac{\alpha\upsilon}{w_B^2(1-\alpha\upsilon)+\alpha\upsilon(1-\omega)}\right]. \qquad (3.5)$$

In the above equations μ^* is the expected value of μ where $\mu \in [\underline{\mu}, \overline{\mu}]$, w is the fraction of uninformed traders who trade large quantities, subscripts A and B represent sale or buy respectively, ω is the probability that $\mu = \underline{\mu}$, σ_μ^2 is the prior variance of μ, $\alpha\upsilon$ is the probability of informed trading. This second probability is a function of probability of information event and the fraction of traders who know about this event.

If there is no spread, traders can trade **large or small quantities** and market equilibrium is required, then prices (3.4) and (3.5) have to satisfy the conditions:

$$\frac{A^2}{A^1} \geq 1 + \frac{\alpha\upsilon\omega}{w_A^2(1-\alpha\upsilon)} \qquad (3.6)$$

$$\frac{B^2}{B^1} \geq 1 + \frac{\alpha\upsilon(1-\omega)}{w_B^2(1-\alpha\upsilon)}$$

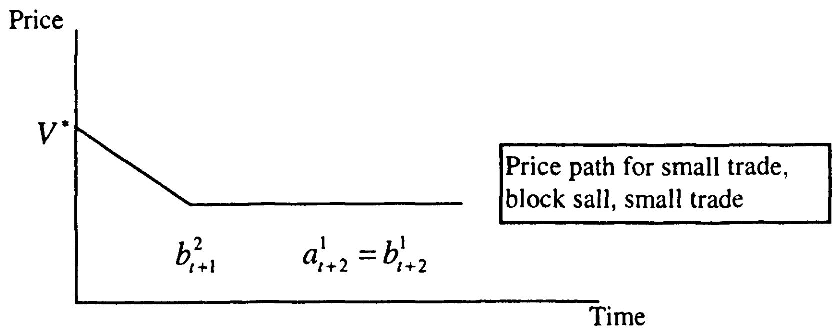
where A^2, B^2 are the large sell and buy quantities and A^1, B^1 are the small sell and buy quantities respectively. If conditions (3.6) are satisfied higher profit is guaranteed to the informed traders who trade large quantities at the lower price than to agents trading smaller quantities at the higher price. If the market does not satisfy required conditions separating equilibrium does not exist. Then pooling equilibrium may occur and optimal strategy can be determined again. The problem should be solved again for informed traders trading large and small quantities and the price should be set for both size quantities. The results are similar in both cases.

A particular behaviour of the market maker attracts interest. For example, if a market is in a separating equilibrium and during a particular day a sale is **large**, then the following day price is set at a large order, conditionally to expectation. Two-day large trade might be thought to be advertised information and the price for the next day is set below the current large sale price.

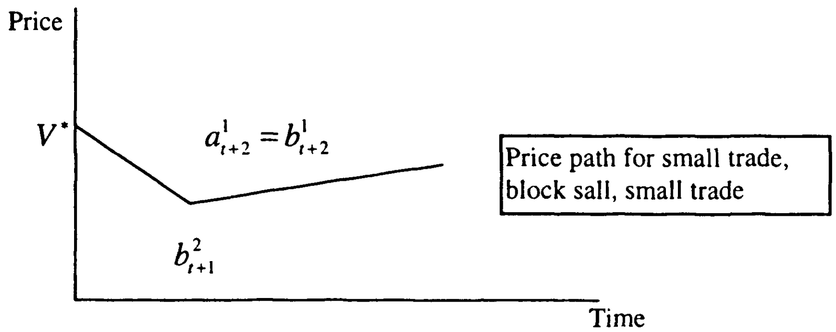
There is a different situation when the trade is **small**. According to the previous explanation the market maker knows that in this case all traders are uninformed and will trade only small quantities. But trades can provide information of the trade taking place. They have information even if the new information not exists. Then if there has been no information event then the probability of small trade arises. That is because of non-existing informed traders. Behaviours of the prices in the case of no information-event uncertainty and information-event uncertainty were shown in figure 3.2.

Figure 3.2 Time path of prices for a market in a separating equilibrium

a) when there is no information-event uncertainty



b) when there is information-event uncertainty



In a case of sequential trading, as takes place on the WSE, Easley and O'Hara propose the model to adjust the prices to information. It allows for the description of prices on a trade-by-trade basis, without looking at the dynamic relationship between trades and information. The advantage of this model is that it shows the possibility of an important distinction between quotes and prices. It is also possible to demonstrate that prices do indeed converge to full-information values, but this convergence takes place only at the limits (for details see O'Hara 1997).

However these models do not fully account for the price-setting process on the WSE. This is mainly because the model assumes that there are no price limits, but such limits exist on the WSE. The WSE market is not consistent with the assumption that information comes only between trading sessions. Also the prices are set in tatonnement process (this process will be described later). However, the modification of the Easley and O'Hara model in the case of price constraints could be imposed on the WSE. Some of aspects of this model will be discussed later. More appropriate for this purpose seems to be to analyse models in disequilibrium. In the following section I introduce and define disequilibrium concepts.

3.3 Disequilibrium trading

3.3.1 Concept of disequilibrium

The consequences of omitting the assumption of market equilibrium apply to disequilibrium trading, in particular (see Benassy 1987):

1. Transactions on the market cannot all be equal to demand and supply expressed on it.
2. Quantity of trade must be modified taking into account quantity signals.
3. Theory of price make agents responsible for price making.
4. Expectations should include not only price signals but also quantity signal expectations.

Disequilibrium is defined as 'a state in which the forces influencing a system are not in balance and there is a tendency for one or more variables in the system to change' (see Bannock *et al.* 1992). Variables can have different values depending on differing situations, for example, the quantity of a good dependant on its price. According to this definition disequilibrium is a special case of equilibrium.

In the first section I introduced notional demands and supplies (d_i and s_i) in the case of equilibrium in the absence of any market constraints. Now it is necessary to introduce $\tilde{d}_i$ and $\tilde{s}_i$ which are **effective demand** and **effective supply** of the trader i in a Benassy sense (see Benassy 1982). Looking at the market in disequilibrium there is no reason to assume that they are in balance for all investors indexed $i = 1,...,n$. So:

$$\tilde{D} = \sum_{i=1}^{n} \tilde{d}_i \neq \sum_{i=1}^{n} \tilde{s}_i = \tilde{S} \tag{3.7}$$

where $\tilde{D}$ and $\tilde{S}$ are total demand and supply on the market. Symbols $\tilde{d}_i$ and $\tilde{s}_i$ mean the demand and supply which appear on the market as a disequilibrium system. The formal definitions of effective demand and supply will be given later.

For these inconsistent values the market generates a set of realised purchases d_i^* and sales s_i^* which represent actual exchanges. The consistent process gives:

$$D^* = \sum_{i=1}^{n} d_i^* = \sum_{i=1}^{n} s_i^* = S^*. \tag{3.8}$$

Obviously some demands and supplies will not be satisfied. Each exchange process could have the mathematical representation called **rationing scheme** (which will be explained later).

Apart from price signals there is an important role for quantity signals in defining final demand and supply by an individual agent. Very often quantity constraints take the form of price constraints. They appear as an upper or lower limit on purchases or sales. As introduced in the previous chapter on the WSE a day price of an asset cannot be higher or lower by more than 10% of the previous day-price. If the quantity signals were imposed in the past, and still exist, it is assumed that they will influence trade in future. Also, constraints on the one market cause changes of the demand and supply on another one; for example constraints on the labour market will affect the effective demand for goods of households. The same rule of quantity signals work on the multimarket scheme. These additional signals (price or quantity) have often been interpreted as a measure of disequilibrium trading (see Benassy 1982).

3.3.2 Tatonnement process

The first person who proposed a 'tantonnement' set of prices adjustment equations formulating the base for later development, was Samuelson (1941, 1947). His formalisation was ground on 'perfect stability' (called so by Hicks 1961).

The 'tatonnement', process also called a 'groping' process, was proposed by Walras as an illustration of equilibrium in perfect competition. Currently the process is used in the theory of general economic equilibrium to denote a simplifying assumption of no actual transactions. It implies that there is no production or consumption in a disequilibrium market when prices are changed in accordance with the law of supply and demand. According to Walras (see Negishi 1987) there are at least three methods of tatonnement. The first method assumes that price-taking traders lodge the prices with the auctioneer and reveal their plans of demand and supply before the price is established. They do not trade until the equilibrium price is set by the auctioneer. The second method states that traders can assume to make trade contracts but re-contract is always possible and the contract can be cancelled. The last method assumes that the effect of past contracts can be changed by offering new demands and supplies, so trade is carried out at the current prices.

The traditional meaning of the tatonnement process is that the market can fetch a particular set of prices no matter what the original disequilibrium position of the market and the route by which prices move before reaching equilibrium (Bannock *et al.* 1992). Initially buyers and sellers know the prices. In the next route traders increase published prices where there is excess demand or reduce them where there is a shortfall in demand or keep them on the same level if demand and supply are in balance. The process continues until the moment when balance is reached. Also up to this moment no trade takes place. This method of price setting is used on the WSE.

The general form of price adjustment (see Fisher 1983) adopted for the security market is:

$$\dot{P}_k = F^k[Z_k(P_v)] \qquad \text{unless } P_k = 0 \text{ and } \dot{P}_k = F^k[Z_k(P_v)] \tag{3.9}$$

$$\text{in which case } \dot{P} = 0$$

where P_k is the price of security k, Z_k is total excess demand for the commodity taken as a continuous function of the prices, $F^k(\cdot)$ is a continuous, sign-preserving function of prices. This function is bounded away from zero, except as Z_k goes to zero. Symbol $\dot{P}_k$ denotes time derivative of P_k and P_v a vector of prices. This formula, proposed by Hicks, holds for each kind of good. For my purpose the formula is adapted for assets and is used to adjust prices on the WSE.

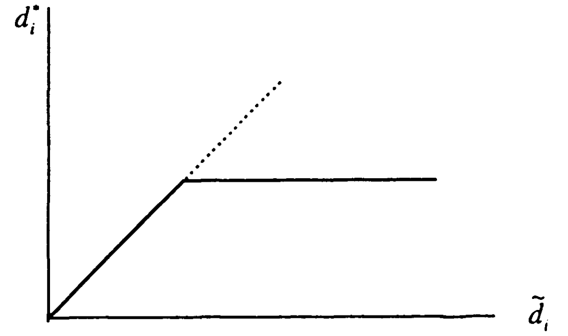
3.4 Multi-market disequilibrium and market constraints

3.4.1 Multi-market constraints

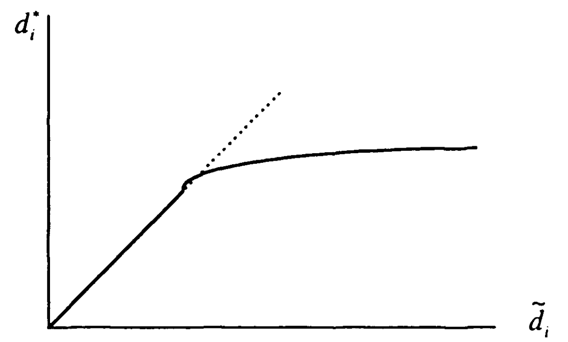
On the Warsaw Stock Exchange there exist market constraints. It seems appropriate to identify which kinds of constraints apply. One of the classifications allows for the distinction between deterministic and stochastic constraints. Before distinguishing them it is important to distinguish **nonmanipulable** and **manipulable** rationing schemes. The simplest way of explaining their difference is given in figure 3.3. This figure shows an example of purchase d_i^* and demand $\tilde{d}_i$ of the i'th trader. Demands and supplies of other traders are constant.

Figure 3.3 Demand of an agent in nonmanipulable and manipulable cases

a) nonmanipulable



a) manipulable



Source: Benassy (1982)

In the manipulable case, market trader i continues his trade by quoting higher demands 'manipulating' the outcome while in the nonmanipulable case his transactions depend upon the demands and supplies of the other traders (which he cannot manipulate). Denoting by $\bar{d}_i$ and $\bar{s}_i$ the bounds on purchase and sale, trade are described as:

$$d_i^* = \min\left(\tilde{d}_i, \bar{d}_i\right), \tag{3.10}$$

$$s_i^* = \min\left(\tilde{s}_i, \bar{s}_i\right).$$

To further extend the notation[4] previously in the case of a single market for many markets, let me assume that there are r markets and r goods on the markets. There are also n traders indexed $i = 1,...,n$. Symbol h is a number of the market. Assume that an agent i is trading on a sequence of markets, labelled by $h = 1,...,r$, visiting them one by one. The prices on them are given or expected with certainty, p_h is a price vector of good h and p is a price vector. Also:

$\overline{m}_i$ - initial quantity of money hold by agent i,

ω_{ih} - endowment of good h,

z_{ih} - net trade of agent i on market h,

z_i - vector of trades of agent i,

x_{ih} - final holding of good h by agent i,

x_i - vector of final holding by agent i,

m_i - final holding of money after trading on the r markets agent i.

Vector $\omega_i \in R^r_+$ includes components $\omega_{ih} \geq 0$, vector $x_i \in R^r_+$ has components $x_{ih} \geq 0$ and $\overline{m}_i \geq 0$. Let z_{ih} be the volume of agent i's net transaction of a good h, elementary transaction exchanging one security h against p_h units of money. The sign of z_{ih} is positive in case of purchase, negative for a sale. Calling d_{ih} and s_{ih} the demand and supply for a security h by agent i, $z_{ih} = d_{ih} - s_{ih}$. There could also be written a vector of these net transactions for agent i $z_i \in R^r_+$ and a vector of prices $p \in R^r_+$.

[4] For comparison I use the same Benassy's (1982) notation.

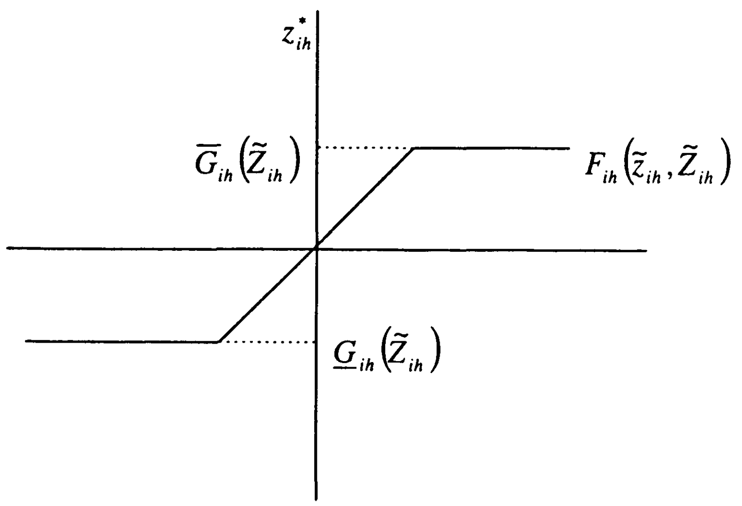
Then the basing relations between introduced symbols, as extended (3.1) are:

$$\begin{aligned} x_{ih} &= \omega_{ih} + z_{ih} \\ m_i &= \overline{m}_i - pz_i \end{aligned} \tag{3.11}$$

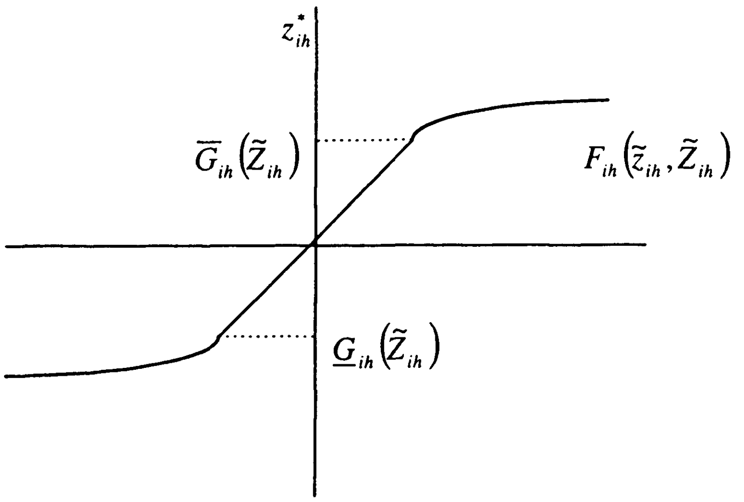
Extending it for multi-market, following Benassy (1982) it could be said that 'a scheme is nonmanipulable if each trader faces in his trades upper and lower bounds that he cannot manipulate. A scheme is manipulable if a trader can, even if he is rationed, increase his transaction by increasing his demand'. The difference between the nonmanipulable and manipulable case is shown in figure 3.4. $\overline{G}_{ih}$ and $\underline{G}_{ih}$ denote upper and lower bounds of the net demands that investor i can reach trading on the market h.

Figure 3.4 The relationship between the net transaction of an agent and his demand in case of rationing scheme

a) nonmanipulable



b) manipulable



Source: Benassy (1982)

Both $\overline{G}_{ih}$ and $\underline{G}_{ih}$ are the functions of the net demands represented by other individuals on the market $\tilde{Z}_{ih}$, so:

$$\overline{G}_{ih}(\tilde{Z}_{ih}) = \max\{\tilde{z}_{ih} \mid F_{ih}(\tilde{z}_{ih},\tilde{Z}_{ih}) = \tilde{z}_{ih}\}, \tag{3.12}$$

$$\underline{G}_{ih}(\tilde{Z}_{ih}) = \min\{\tilde{z}_{ih} \mid F_{ih}(\tilde{z}_{ih},\tilde{Z}_{ih}) = \tilde{z}_{ih}\}.$$

Because $F_{ih}(0,\tilde{Z}_{ih}) = 0$ I have:

$$\overline{G}_{ih}(\tilde{Z}_{ih}) \geq 0, \quad \underline{G}_{ih}(\tilde{Z}_{ih}) \leq 0.$$

The rationing scheme on market h is **nonmanipulable** if:

$$F_{ih}(\tilde{z}_{ih},\tilde{Z}_{ih}) = \begin{cases} \min\left[\tilde{z}_{ih},\overline{G}_{ih}(\tilde{Z}_{ih})\right] & \text{if } \tilde{z}_{ih} \geq 0 \\ \max\left[\tilde{z}_{ih},\underline{G}_{ih}(\tilde{Z}_{ih})\right] & \text{if } \tilde{z}_{ih} \leq 0 \end{cases} \quad \text{for all } i = 1,\dots,n, \tag{3.13}$$

which also could be written as:

$$F_{ih}(\tilde{z}_{ih},\tilde{Z}_{ih}) = \min\{\overline{G}_{ih}(\tilde{Z}_{ih}), \max[\underline{G}_{ih}(\tilde{Z}_{ih}), \tilde{z}_{ih}]\}. \tag{3.14}$$

In all other cases the scheme is **manipulable**.

For the nonmanipulable scheme the net transaction of investor i on the market h could be written as:

$$z_{ih}^{*} = \begin{cases} \min\left[\tilde{z}_{ih}, \bar{z}_{ih}\right] & \text{if } \tilde{z}_{ih} \geq 0 \\ \max\left[\tilde{z}_{ih}, \underline{z}_{ih}\right] & \text{if } \tilde{z}_{ih} \leq 0 \end{cases} \qquad (3.15)$$

or

$$z_{ih}^{*} = \min\left[\bar{z}_{ih}, \max\left(\underline{z}_{ih}, \tilde{z}_{ih}\right)\right]$$

where $\bar{z}_{ih}$ and $\underline{z}_{ih}$ are the perceived constraints of agent i and they are functions of demands of the other agents:

$$\bar{z}_{ih} = \overline{G}_{ih}\left(\tilde{Z}_{ih}\right) \geq 0, \quad \underline{z}_{ih} = \underline{G}_{ih}\left(\tilde{Z}_{ih}\right) \leq 0 .$$

It is worth to know that in the nonmanipulable case since the relationship (3.14) is true the functions $\overline{G}_{ih}\left(\tilde{Z}_{ih}\right)$ and $\underline{G}_{ih}\left(\tilde{Z}_{ih}\right)$ are continuous in their arguments if

and (3.14) is true the functions $\overline{G}_{ih}\left(\tilde{Z}_{ih}\right)$ and $\underline{G}_{ih}\left(\tilde{Z}_{ih}\right)$ are continuous in their arguments if

and only if the rationing function $F_{ih}\left(\tilde{z}_{ih}, \tilde{Z}_{ih}\right)$ is continuous in its arguments, so:

$$\bar{z}_{i} = \overline{G}_{i}\left(\tilde{Z}_{i}\right) \text{ and } \underline{z}_{i} = \underline{G}_{i}\left(\tilde{Z}_{i}\right).$$

According to the notions introduced above the trading system on the Warsaw Stock Exchange could be described as a manipulable one. The traders realise that the prices of assets are constrained. They also know that in case of the excess demand (or supply), when the market is unbalanced, equilibrium price cannot be established and the ratio of demand to supply (or vice versa) is smaller than 5:1, the demands and supplies are proportionally reduced. So to be sure that they will be able to buy or sell required quantities of assets they bid and ask much more of them than actually desired. The rationing scheme is then manipulable. In the case of a ratio higher than 5:1 the transactions are suspended, in which case the market can be described as nonmanipulable.

Another constraint is that of access to information. Easley and O'Hara developed a model describing the effect of information on the price-trade size relationship (for details see the second part of the chapter). They consider a model in which potential buyers and sellers trade assets with market makers. The value of the asset is represented by the random variable and depending on the occurrence of signals this variable can take one of two values, low or high. They assume that an information event appears only between trading days. All traders agree about the expectation of the asset and they behave competitively. For simplicity interpretation of the signals is consistent. However, the model is highly structured and is based on many strict assumptions, thus it could be used to analyse the influence of information on the trade size on the Warsaw Stock Exchange.

3.4.2 Effective demand

The Walrasian equilibrium as showed above is a solution of the problem (3.2) in case (3.11) and gives an individual vector net demand function $z_i(p)$. The **effective demand (supply)** in the Walrasian sense is the demand (supply) which optimises the utility function in conditions where demand and supply are equal. So, in case of price flexibility, the demand and supply of investors are obtained from maximisation of utilities subject only to budget constraints.

In this case there is no demand for money. The short-run Walrasian equilibrium assumes that the prices are settled conditionally to net demands equal to zero on all markets:

$$\sum_{i=1}^{n} z_{ih}(p) = 0 \quad \text{for} \quad h = 1, \ldots, r.$$

It is important to distinguish between demands and transactions. Let z_{ih}^{*} denote the net transaction of investor i on market h and $\tilde{z}_{ih}$ his net effective demand. The net transactions must balance as an identity on each market:

$$\sum_{i=1}^{n} z_{ih}^{*} = 0 \quad \text{for all } h$$

while effective demands do not need balance on a market. If they do not balance I have:

$$\sum_{i=1}^{n} \tilde{z}_{ih} \neq 0$$

where $\tilde{z}_{ih} = \tilde{d}_{ih} - \tilde{s}_{ih}$.

There is a particular organisation on each market, which could be represented by a rationing scheme, through which it is possible to convert an inconsistent demand and supply into a consistent one. A set of n functions relating the transactions of each trader to the effective demands of all traders may be used. They can be denoted by:

$$z^*_{ih} = F_{ih}(\tilde{z}_{1h},\dots,\tilde{z}_{nh}), \quad i = 1,\dots,n$$

or

$$z^*_{ih} = F_{ih}(\tilde{z}_{ih},\dots,\tilde{Z}_{ih}) \quad \text{where} \quad \tilde{Z}_{ih} = \{\tilde{z}_{1h},\dots,\tilde{z}_{i-1,h},\tilde{z}_{i+1,h},\dots,\tilde{z}_{nh}\}. \tag{3.16}$$

The fundamental property of functions F_{ih} is:

$$\sum_{i=1}^{n} F_{ih}(\tilde{z}_{1h},\dots,\tilde{z}_{nh}) = 0 \quad \text{for all} \quad \tilde{z}_{1h},\dots,\tilde{z}_{nh}. \tag{3.17}$$

The form of the rationing functions depends on the exchange process on market h.

Usually an investor coming into the market has realised transactions on previous markets. With this experience constraints on the current and future markets are expected. If the constraints are **deterministic** then the **effective demand** is defined as 'the trade that maximises the decision criterion of the agent, subject to the usual budget or technological constraints and also taking into account the given past transactions and the expected constraints on future markets' (see Benassy 1982). According to this definition the current constraints are omitted. This is consistent with Clower's (1965) construction of the effective demand. Dreze (1975) proposes to derive demands maximising the utility function subject to the budget constraints and all existing quantity restrictions (see Quandt 1988). In case of **stochastic** constraints effective demand takes into account computed *ex ante* probabilities of demand rationing associated with the target supply.

Now I will show how deterministic and stochastic constraints influence net trade in a sequence of markets. Use the same notation as above, where $h = 1,\dots,r$ and vector $z_i = [z_{i1},\dots,z_{ir}]$ which must belong to a compact convex set K_i. This value represents constraints caused by sequential trading and the positivity constraints for x_{ih} and m_i. An individual agent i tries to maximise utility function $U_i(x_i, m_i)$, given in (3.2), which could be written as $V_i(z_i)$:

$$V_i(z_i) = U_i(\omega_i + z_i, \overline{m}_i - pz_i). \tag{3.18}$$

Solving the problem

$$\max V_i(z_i) \text{ subject to } z_i \in K_i \tag{3.19}$$

I have a required value of z_i.

Assuming that the agent i have visited k markets where $k < h$ and realised z_{ih}^* transactions. He also expects deterministic constraints $\underline{z}_{ik}$, $\overline{z}_{ik}$ on the markets h where $k > h$. Optimal trade values from the point of view of the i'th investor should be found following the same procedure as before by maximising his utility function having in mind his already completed transactions and expected constraints; therefore the maximisation problem is defined as:

$$\max V_i(z_i) \qquad \text{subject to } z_i \in K_i, \tag{3.20}$$

$$z_{ik} = z_{ik}^* \quad \text{for } k < h,$$
$$\underline{z}_{ik} \le z_{ik} \le \overline{z}_{ik} \quad \text{for } k > h.$$

The solution will give me required value of effective demand $\tilde{z}_{ih}$.

The more realistic situation is where the future constraints are not certain. So the **stochastic** constraints effect the demand and supply with a probability. In order to find a future possible supply it is necessary to estimate ex ante probability of demand associated with supply.

If the market quantity constraints are stochastic then *ex ante* cumulative probability distribution for these constraints on each market is defined as:

$$\psi_{ih}(\bar{z}_{ih}, \underline{z}_{ih}) \text{ for } h = 1,..r. \quad (3.21)$$

For independent markets expected utility maximising problem of investor i is given by:

$$\max E[V_i(z_{i1},...,z_{ik})] \quad (3.22)$$

where utility function are based on a stream of transactions $z_{i1},...,z_{ir}$. Treating a problem as a typical dynamic-programming (see Bellman 1957) the optimal actions are taken in the future, so the expected utility results $V_{ih}(z_{i1},...,z_{ih})$ are the same as original functions:

$$V_{ir}(z_{i1},...,z_{ir}) \equiv V_i(z_{i1},...,z_{ir}).$$

For $h < r$ the function V_{ih} is determined by relation:

$$V_{ih-1}(z_{i1},...,z_{ih-1}) = \max_{z_{ih} \in K_{ih}} \int V_{ih}(z_{i1},...,z_{ih-1}, \min[\bar{z}_{ih}, \max(\underline{z}_{ih}, \tilde{z}_{ih})]) d\psi_{ih}(\bar{z}_{ih}, \underline{z}_{ih}). (3.23)$$

Each time set K_{ih} is created as a function $z_{i1},...,z_{ih-1}$:

$$K_{ih} = K_{ih}(z_{i1},...,z_{ih-1}).$$

Finally maximising expectations of the utility function the solution is an effective demand $\tilde{z}_{ih}$.

However in case of the WSE, due to the given maximum change on the price of the day, we can consider the constraints to be stochastic. This is because even if the constraints are given there is no certainty that they will be reached on the particular session.

3.4.3 Multiplier effect of constraints

As shown the quantity constraints on the market very often cause changes in demand and supply in the same or on other markets. This leads us to the definition of a **multiplier chain**. To be more specific, if k is a set of traders on the disaggregate level and $i_1,\dots,i_k$ are numbers of traders, $h_1,\dots,h_k$ numbers of securities, the markets are defined as:

$$i_1 \text{ is } \begin{cases} \text{constr. in his supply of } h_1 \\ \text{unconstr. in his supply of } h_2 \end{cases}$$

$$i_2 \text{ is } \begin{cases} \text{constr. in his supply of } h_2 \\ \text{unconstr. in his supply of } h_3 \end{cases}$$

...

$$i_k \text{ is } \begin{cases} \text{constr. in his supply of } h_k \\ \text{unconstr. in his supply of } h_1 \end{cases}$$

The chain can represent k traders on the WSE and securities traded on it. The same constraints influence all markets with the same sign, for example, all types of securities. If we treat a demand of the one type of securities as a one market, it is known as a demand chain. Finally, the same limitation returns to the initial market. There could be many different chains existing on markets simultaneously. Consequently, supply multiplier chains could be defined as chains aimed at all goods and at all traders' demands.

3.4.4 Spillover effects

Spillover effects, which exist on the Warsaw Stock Exchange, are also known as 'externalies', 'third party effects', 'neighbourhood effects' or, as named by Marshall, 'external economies' or 'external diseconomies'. Following Hardwick, Khan and Langmead (1982) '**externalies** are those gains or loses which are sustained by others as a result of action initiated by producers or consumers or both and for which no compensation is paid'. The trade spillover in the case of two people and two-securities economy can be explained by utility function of trader i (U_i):

$$U_i = f(a_1, a_2, ..., a_n; k) \tag{3.24}$$

where $a_1, a_2, ..., a_n$ are activities of trader i and k denotes activities of the trader k. It means that the utility of the investor does not depend only on his own activities but also on activities of other individuals.

The spillover effect was shown by Quandt (1988), following Ito (1980). He assumed that there are three goods on the market: a quantity of the first type assets (x_i), a quantity of the second type assets (x_j) and the real money balance (x_m). Symbols p_i and p_j denote prices of x_i and x_j in time 0 respectively. Total amount of time available is T and the problem is to maximise the Cobb-Douglas utility function subject to budget constraint, that is:

$$\max U = x_i^{\alpha}\left(T - x_j\right)^{\beta} x_m^{\gamma} \tag{3.25}$$

subject to $p_j x_j = p_i x_m + p_i x_j$.

Solving the problem using the Lagrangian multiplier I obtain Walrasian functions:

$$\tilde{x}_i = \frac{\alpha p_j T}{p_i(\alpha + \beta + \gamma)} \tag{3.26}$$

$$\tilde{x}_j = \frac{(\alpha + \gamma)T}{(\alpha + \beta + \gamma)} \tag{3.27}$$

$$\tilde{x}_m = \frac{\gamma p_j T}{p_i(\alpha + \beta + \gamma)}. \tag{3.28}$$

Assume now that the consumer knows that the quantity of the second type asset is to be rationed to amount $\hat{x}_j$ where $\hat{x}_j < \tilde{x}_j$. His optimising problem is then:

$$\max U = x_i^{\alpha} \left(T - \hat{x}_j\right)^{\beta} x_m^{\gamma} \tag{3.29}$$

subject to $p_j \hat{x}_j = p_i x_m + p_i x_j$.

The solution is:

$$x_i^e = \frac{p_j \hat{x}_j \alpha}{p_i(\alpha + \gamma)} \tag{3.30}$$

$$x_m^e = \frac{\gamma p_j \hat{x}_j \gamma}{(\alpha + \gamma)}. \tag{3.31}$$

Quandt denotes x_i^e as effective demand for the security i. From (3.26) and (3.30) I could write:

$$x_i^e = \tilde{x}_i + \frac{\alpha}{\alpha + \gamma} \frac{p_j}{p_i} \left(\hat{x}_j - \tilde{x}_j\right),$$

so in accordance with Quandt effective demand for a good is equal to the Walrasian demand plus a **spillover term** which is proportional to the difference between the Walrasian supply of the second type of assets and the amount of the first type of assets.

Two slightly different models of effective demands and supplies including the spillover term are proposed by Ito (1980) and Benassy (1975). Both of them (see Quandt (1988)) focus on two markets. In Ito's model the effective demand for a security is equal to the Walrasian demand plus a spillover term. In Benassy's version the spillovers are measured as differences between realised transactions and effective demands and supplies (not Walrasian ones).

Finally, it could be said that quantity signals give rise to spillover effects and shocks, and the combination of spillover effects on many markets could cause multiplier effects. In relation to Warsaw Stock Exchange two types of spillovers, cross-spillover and intertempotal spillover can be considered. Cross-spillover occurs when part of a transaction is not realised and excess demand is transferred to other assets, while intertemporal spillover is when a part of demand that cannot be realised during the session is shifted to the next session. If we analyse, for example, the demand for the asset where there is the reduction in sell, part of the demand can be moved as a demand for another asset. In this case we talk about cross-spillover. If this unsatisfied part of the demand is realised on the next day (during the next session) for the same asset, we have intertemporal spillover.

The econometric models in the case of excess demand are showed, for example, by Charemza (1989) and Quandt (1989). The intertemporal and cross-spillover effects can be measured based on the optimisation of the trader's utility function with financial constraint (see Charemza *et al* 1997). A model of disequilibrium trading system in the case of price constraints is proposed by Charemza, Shields and Zalewska-Mitura (1997).

3.5 Conclusions

This chapter examines the characteristics of the market in equilibrium and disequilibrium and the different kinds of market constraints which may appear under the circumstances. According to the Benassy's (1982) definition of the effective demand and supply, if there are quantity constraints on the market the market is in disequilibrium. On the Warsaw Stock Exchange there are quantity constraints, so it is a market in disequilibrium. However, it is not possible to measure the market constraints, so constraints take the form of price limits. Generally, the trade on the WSE takes place sequentially. The prices for assets are set in a process similar to the tatonnement process. The market can fetch a particular set of prices in the original disequilibrium position of the market or through a route by which prices move before reaching equilibrium. The single price is established in that process, where offers to buy and sell are logged with the 'auctioneer' before the price is established. According to the above mentioned regulation price constraints are imposed on the WSE. If the single price evaluated, as described above, is greater or lower by more than 10% than the last session price, it is artificially reduced and kept at a level which exactly 10% above or below the last session price. Normally trading takes place at such regulated price leaving a part of demand or supply in excess. These constraints are due to the manipulable rationing scheme. The trader, knowing that the demand (or supply) can be reduced, can bid or ask for a higher quantity of assets. Occasionally, if demand is greater than supply (or opposite) by more than fivefold, the transactions may be suspended altogether. In this case the rationing scheme is manipulable; but for our purpose, for the sake of simplicity we treat the WSE market as a nonmanipulable case.

The market can be also called as a stochastic one. However, even the imposition of price constraints as a rule on the WSE does not mean that on each session the limit will be reached. Also intertemporal and cross-spillover effects appear on the market but for our future study, for simplicity they are ignored herein.

The next chapter presents the standard version of the portfolio allocation model which provides the background to develope the model for the Warsaw Stock Exchange, including that of quantity constraints.

CHAPTER 4

THE STANDARD VERSION OF THE PORTFOLIO ALLOCATION MODEL

4.1 Introduction

The Capital Asset Pricing Model (CAPM) is one of the models based on equilibrium on the stock market, which describes the relationship for asset returns. The CAPM model, called also as a standard CAPM, was developed by Sharp (1964), Lintner (1965) and Mossin (1966), who were working independently with problems concerned on equilibrium on capital market. Their works were based on the contributions of Markowitz (1952), (1958). This model is widely used in the finance literature and its application can be found on many markets, for example, see Sharpe and Cooper (1972), Black, Jensen and Scholes (1972), Fama and MacBeth (1973). The standard CAPM model from the point of view of derivation, interpretation and rationality of assumptions gives a foundation of deriving the optimum portfolio allocation model with market constraints for the Warsaw Stock Exchange. It therefore it seems necessary initially to explain the general idea of the standard model. To better understand the concept of the optimal portfolio allocation model this chapter examines the standard model with it's assumptions. Then, variations of the model developed by economists aiming for consistency with the real markets are presented.

The chapter is divided into five sections. The first section contains the assumptions underlying the standard CAPM model. The second describes the model equation and it's main properties. The model has been derived in several forms involving different degrees of rigor and mathematical tools. Three different ways of deriving the model are shown in the third section of the chapter.

The fourth section of the chapter includes a critique of the standard model and various versions of the model (called as non-standard models) are described in the final part of the chapter. Versions of the model adapted to other markets such as CAPM for bonds and equities etc., are shown.

4.2 Model assumptions

In the development of CAPM a number of assumptions are made. They can be divided into two groups, investors, individuals who decide to buy or sell assets on the market, and the financial market.

The general assumptions of the efficient market hypothesis hold and, in particular, that (see, for example, Malkiel 1992, Cuthbertson 1996, Blake 1990):

1. There are no taxes;
2. There are no transaction costs and other imperfections;
3. Investors can borrow and lend at risk-free rate an unlimited amount at the same time;
4. The market is not dominated by any individual investor;
5. There is a fixed number of assets on the market and they are available to all investors;
6. There is freedom of entry and exit for both buyers and sellers.

The above assumptions imply normality of distribution of the rates of return. Assuming that the portfolio is fully described by mean and variance of returns, the distribution is described as symmetric (lack of skewness) and has a kurtosis value equal to 3 (see Hakansson 1977).

Regarding the investors, the following assumptions are made (see, for example, Varian 1992,1996):

1. Investors behave rationally, they are risk-averse and maximise expected utility of wealth;
2. They are interested only in two features of security: expected returns and risk, the latter expressed by variance of returns;
3. All investors have identical perceptions of each security;
4. Each investor defines a period of time as an investing horizon and these periods are not identical;
5. They have full access to information about the market;
6. Investors are price takers; the assumption being that their own buying and selling activity will not affect asset prices;
7. All investors can lend and borrow without any limitations at a risk-free rate r at the same time.

The separation principle holds, in that the investors take homogenous decisions regarding the composition of the risk portfolio at the efficient frontier, and then decide, individually, according to the degree of the particular investor's risk aversion, on the composition of the risk portfolio and the riskless assets.

Rational behaviour means that the investor of more and less risky assets always chooses the assets that give a higher expected rate of return. Risk-averse means the rejection of all unsure assets. It is not rejection of a risk at all. The investor is disposed to take it in exchange for a higher expected rate of return. Increase of capital by each additional amount of money leads to higher value of utility function. So by maximising of utility function every activity is understood which can bring about the possibility of highest wealth.

The assumption about an investing horizon is connected with the fact that everyone invests capital for of a different reason. Some individuals, anticipate a quick profit (due to pure speculation), others take into consideration long term investment.

Investors have freely obtained and the full available information regarding the market.

It is worth mentioning that Berk (1997) showed the conditions that provide the CAPM and derived preferences over mean and variance to be equivalent.

However, the discussion about a reality of the standard CAPM model assumptions will be presented later (see the model critiques and the non-standard models).

4.3 The standard CAPM model description

The one-period CAPM model shows the relationship for asset return in market equilibrium and is based on several assumptions. Under these assumptions the market portfolio is the only portfolio of risky assets that any investor will have. The investor adjusts the risk of the market portfolio[1] to his preferred risk-return combination by combining the market portfolio with lending or borrowing at the riskless rate. It leads to the state that investors construct an optimum portfolio by combining a market fund with the riskless assets. The optimum portfolio, which everyone wishes to hold is called market a portfolio. So the investors hold portfolios along a line connecting r_f, a rate of return of the risk-free assets, with the rate of return of the market portfolio r_M.

The CAPM model equation is:

$$r_j = r_f + \beta_{jM}(r_M - r_f)$$

where r_j is the rate of return of the risky j'th asset and r_M is the return of the market portfolio. Parameter $\beta_{jM} = \dfrac{\operatorname{cov}(r_j r_M)}{\sigma_M^2}$, calculated as a covariance between the j'th asset and the market portfolio divided by the variance of the market portfolio, is a measure of the risk of the j'th asset[2].

[1] The foundation of risk and it's measures showed i.e. Modigliani and Pogue (1974a)

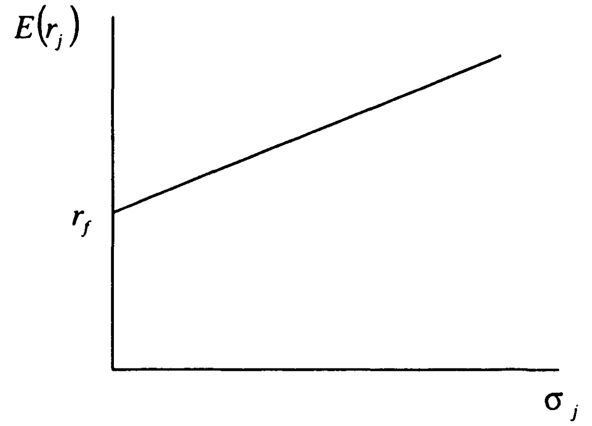
[2] The model can be shown in the 'risk premium' form by subtracting the risk-free rate from the rates of return (see Modigliani and Pogue 1974b):

$$R_j = r_j - r_f \quad \text{and} \quad R_M = r_M - r_f$$

then: $R_j = \beta_{jM} R_M$.

There are two types of line that characterise portfolios being along with the CAPM model. One of them is a capital market line (CML) presented in figure 4.1. CML shows the effective portfolios[3] in expected return and standard deviation space. The second, known as a security market line (SML), represents relationships between expected returns and *beta* parameters for portfolios or particular assets.

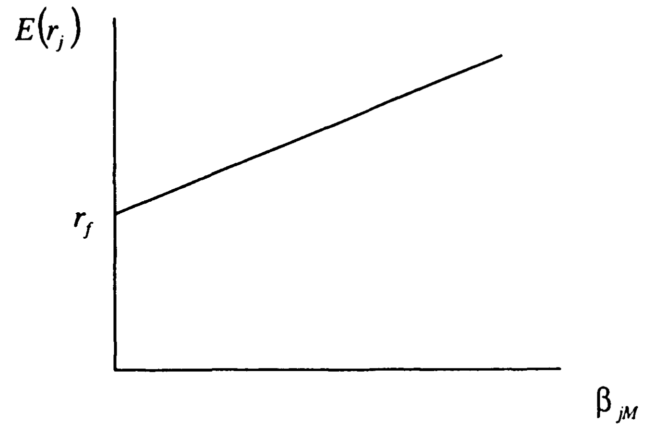
Figure 4.1 The capital market line (CML)



The slope of the SML is calculated from the formula $\frac{r_M - r_f}{\beta_{jM}}$ and is positive (see figure 4.2). This is mainly due to investors being risk averse and maximising their expected returns (with given budget constraints). However Roll and Ross (1994) find that this measure is sensitive to the choice of indices and in certain circumstances can even be negative.

[3] By the effective portfolio I understand the portfolo which minimise standard deviation for a given level of expected return or miximises expected return for a given level of risk.

Figure 4.2 The security market line (SML)



The market portfolio has beta equal to 1. For the individual asset if the value of beta is 1, then the asset could be described as a neutral or medium risky. If β_{jM} is greater than 1, then the asset is called as an aggressive or high risky and if it is smaller than 1 then it is called defensive or low risky.

4.4 Derivation of the standard model

The standard CAPM model can be derived in several forms. Each method involves different degrees of vigour and mathematical complexity. This part of the chapter includes three methods of derivation. The first method is based on the marginal rates of substitution, the second is based on other marginal measures and the third, which requires more mathematical tools, is based on the risk premium.

4.4.1 Derivation based on the marginal rates of substitution

One of the methods of obtaining the standard CAPM model, as described by Ross (1992), is based on the marginal rate of substitution (see appendix 4A for details). Assume that the investor dispose the capital K and invests it all in risk free assets (k_f), a particular type of asset j (k_j) and a portfolio of risky assets (k_p). Then:

$$K = k_f + k_j + k_p \tag{4.1}$$

or:

$$1 = w_f + w_j + w_p$$

where w_f, w_j, w_p denote proportions of the capital invested in particular types of assets (as above).

Let me now assume that the investor decides to buy more j type risky assets. Then the part of his capital which he proposes to spend for risk-free assets is spent for the next j'th asset. Denoting this part of money by Δk_j, total capital is now disposed as:

$$K = k_f - \Delta k_j + k_j + \Delta k_j + k_p . \tag{4.2}$$

According to definition the rate of return of the portfolio before making the decision of buying the next risky asset (r_{p1}) is:

$$r_{p1} = w_f r_f + w_j r_j + w_p r_p \tag{4.3}$$

and after buying the next risky assets (r_{p2}):

$$r_{p2} = r_{p1} + \Delta w_j \left(r_j - r_f\right) \tag{4.4}$$

where Δw_j is a change of the proportion of the capital kept in j'th type assets. Increase of the return (Δr) caused 'moving' a part of money, calculated as a $\Delta r = r_{p2} - r_{p1}$ is then:

$$\Delta r = \left(r_j - r_f\right)\Delta w_j . \tag{4.5}$$

Denoting by σ_p^2 a variance of the portfolio before the change, a variance of the portfolio after change is defined as:

$$\mathrm{var}(p_2) = \sigma_p^2 + \Delta w_j^2 \,\mathrm{var}(r_j) + 2\Delta w_j \,\mathrm{cov}(r_p r_j) \tag{4.6}$$

so the change in variance ($\Delta\sigma_p^2$), assuming that Δw_j is small and a variance of the risk-free assets is equal to zero, is:

$$\Delta\sigma_p^2 \approx 2\Delta w_j \,\mathrm{cov}(r_p r_j) \tag{4.7}$$

where $\mathrm{var}(r_j)$ is a variance of the rates of return of the j'th asset and $\mathrm{cov}(r_j r_p)$ is a covariance between rates of return j'th asset and a portfolio of risky assets as defined previously. So the increase of variance caused by a change of portfolio towards the risky assets depends on the change of proportion of these risky assets in portfolio and a covariance between the return of this asset and portfolio.

The marginal rate of transformation of the expected return of the capital invested in a portfolio depends on the level of risk (described as a change of variance of the portfolio ($\Delta\sigma_p^2$)) and a change of the return (Δr):

$$MRT = \frac{\Delta\sigma_p^2}{\Delta r} \tag{4.8}$$

and then from (3.5) and (3.7) I have:

$$MRT = \frac{2\operatorname{cov}(r_p r_j)}{r_j - r_f}. \tag{4.9}$$

Alternatively, the portfolio can be changed by the capital kept in risky assets as a whole. Let me analyse the following situation. As before the investor disposes of a capital K and a part of it being in risk free assets (k_f), and a rest in risky assets (k_p). It could be written as:

$$K = k_f + k_p \tag{4.10}$$

Now, if he decides to buy more risky assets a part of money (Δk_p) is 'moved', so the capital is then disposed as:

$$K = k_f - \Delta k_p + k_p + \Delta k_p. \tag{4.11}$$

The rate of return of the portfolio before the change (r_{p1}) is then:

$$r_{p1} = w_f r_f + w_p r_p \tag{4.12}$$

and after the change (r_{p2}):

$$r_{p2} = r_{p1} + (r_p - r_f)\Delta w_p. \tag{4.13}$$

The marginal rate of substitution is defined as an increase of the variance of the portfolio ($\Delta\sigma_p^2$) divided by a change of the rate of return (Δr) both a cause of change in the portfolio:

$$MRS = \frac{\Delta\sigma_p^2}{\Delta r_p}. \tag{4.14}$$

Then in this case I have:

$$MRS = \frac{2\sigma_p^2}{r_p - r_f} \tag{4.15}$$

In market equilibrium the marginal rate of transformation must be equal to the marginal rate of substitution:

$$MRT = MRS.$$

In this case from (4.9) and (4.15) I have:

$$\frac{2\operatorname{cov}(r_p r_j)}{r_j - r_f} = \frac{2\sigma_p^2}{r_p - r_f}. \tag{4.16}$$

Substituting $\beta_{jp} = \frac{\operatorname{cov}(r_j r_p)}{\sigma_p^2}$ I have the equation $r_j - r_f = (r_p - r_f)\beta_{jp}$. Parameter p concerns the optimal portfolio. Every investor would like to keep such a portfolio which then becomes the market portfolio. The rate of return of the portfolio r_p is equal to the rate of return of the market portfolio r_M. Then for each j'th asset I get a model called a standard capital asset pricing model:

$$r_j = r_f + (r_M - r_f)\beta_{jM}. \tag{4.17}$$

4.4.2 Derivation based on marginal measures

Another method of derivation of the standard CAPM model is also based on the marginal measures. It is assumed that in equilibrium a single risky portfolio has to include all risky assets and each asset is weighted by the proportion of the value of the asset in the total value of all assets (see Sibert 1992). Then the exchange between a risk and expected return takes a place by borrowing or buying assets, which must be the same for each if the equilibrium condition holds.

Let me denote by r_s the rate of return of the s'th asset, r_M the return of the market portfolio, and by r_f the rate of return of the risk-free asset. The rate of return of the portfolio on the capital market (r_p) and a rate of return of the portfolio which is a combination of the market portfolio and risk free assets (r) can be described as follows:

$$r_p = wr_s + (1-w)r_M, \tag{4.18}$$

$$r = wr_f + (1-w)r_M, \tag{4.19}$$

where w is the proportion of the values of the particular assets in a value of all assets on the market.

In equilibrium $w = 0$ the exchange between a risk and the expected return for the portfolio, including the s'th asset, and a market portfolio is the same. So the marginal rate of transformation for the asset s must be the same as for the whole market, which is:

$$\left.\frac{\partial \sigma_p}{\partial \mu_p}\right|_{w=0} = \left.\frac{\partial \sigma}{\partial \mu}\right|_{w=0} \tag{4.20}$$

Symbols σ_p and μ_p denote the standard deviation and the expected value of the portfolio, σ and μ standard deviation and the expected value of the free portfolio respectively.

From the equation (4.18) the first derivative in point $w = 0$ is (see appendix 4B for details):

$$\left.\frac{\partial\sigma_p}{\partial\mu_p}\right|_{w=0} = \left.\frac{\partial\sigma_p}{\partial w}\frac{\partial w}{\partial\mu_p}\right|_{w=0} = \frac{-\sigma_M^2+\sigma_{sM}}{\sigma_M(\mu_s-\mu_M)}. \tag{4.21}$$

Analysing the portfolio, as a combination of the risk-free assets and market portfolio, from (3.19) the derivative is:

$$\left.\frac{\partial\sigma}{\partial\mu}\right|_{w=0} = \left.\frac{\partial\sigma}{\partial w}\frac{\partial w}{\partial\mu}\right|_{w=0} = \frac{\sigma_M}{\mu_M-r_f} \tag{4.22}$$

Then including equation (4.21) and (4.22) in condition (4.20) and transforming it I obtain:

$$\mu_s - r_f = \frac{\sigma_{sM}}{\sigma_M^2}(\mu_M - r_f).$$

Finally substituting $\beta_{sM} = \frac{\sigma_{sM}}{\sigma_M^2}$ I have a standard CAPM model:

$$r_s = r_f + \beta_{sM}(r_M - r_f).$$

4.4.3 Derivation of the CAPM based on the risk premium

The starting point of deriving the CAPM is the assumption that the short sale is allowed and investors can lend and borrow unlimited amounts at the riskless rate of interest (see Elton and Gruber 1991). They try to maximise returns from the invested capital in portfolio with the lowest risk. The relationship between the rate of return and a risk measured by the standard deviation is the problem of maximising the function:

$$\theta = \frac{r_p - r_f}{\sigma_p} \tag{4.23}$$

which is a slope of a straight line passing through the riskless rate of interest on the vertical axes and the portfolio itself. Symbol r_f denotes, as before, the rate of return of the risk-free asset, r_p and σ_p the rate of return and the standard deviation of the portfolio respectively.

The only one numerical restriction is that the sum of weights of assets in portfolio is equal to 1, so:

$$\sum_{i=1}^{n} w_i = 1.$$

In case of modelling short sales it should be noted that traders have a fixed sum of money to invest. The total funds the trader invests short, plus the funds invested long, must add to the original investment. So the trader borrows assets and sells them assuming that if they have to be returned their price will be lower, but the price of which in additional profit is predicted. During the same time he buys these assets, in his opinion, are going to increase. His turnover then includes more assets than he could afford, so it is possible that $w_i < 0$ (see Elton and Gruber 1991). Because some of the assets have to be returned, I could write that

$$\sum_{i=1}^{n} |w_i| = 1. \tag{4.24}$$

The risk premium defined as a difference between rates of return of the risky asset and a risk-free asset is (see appendix 4C for details):

$$r_p - r_f = \sum_{i=1}^{n} w_i (r_i - r_f)$$

where n is a number of assets in portfolio.

The standard deviation of the portfolio is defined as:

$$\sigma_p = \sqrt{\sum_{i=1}^{n} w_i^2 \sigma_i^2 + \sum_{i=1}^{n} \sum_{\substack{j=1 \\ j \neq i}}^{n} w_i w_j \sigma_{ij}}$$

where σ_{ij} is a covariance between rates of return of the assets i and j.

The first derivative of the θ function with the respect to all assets in the portfolio gives us a set of simultaneous equations of the following form:

$$r_i - r_f = \lambda w_1 \sigma_{1i} + \lambda w_2 \sigma_{2i} + \ldots + \lambda w_{i-1} \sigma_{i-1,i} + \lambda w_i \sigma_i^2 + \lambda w_{i+1} \sigma_{i+1,i} + \ldots + \lambda w_n \sigma_{ni}$$

where

$$\lambda = \frac{\sum_{i=1}^{n} w_i (r_i - r_f)}{\sum_{i=1}^{n} w_i^2 \sigma_i^2 + \sum_{i=1}^{n} \sum_{\substack{j=1 \\ j \neq i}}^{n} w_i w_j \sigma_{ij}} \tag{4.25}$$

and $\sigma_{1i}, \sigma_{2i}, \ldots, \sigma_{ni}$ are the covariances between i'th asset and assets $1,2,\ldots,n$ respectively.

If the expectations of all assets are the same then all traders must select the same optimum portfolio. So in equilibrium if all of them choose the same portfolio then all assets must be held is the same percentage that they represent on the market. It is necessary to define the market rate of return:

$$r_M = \sum_{i=1}^{n} r_i w_i$$

and the covariance between the rates of return of the k'th asset and market portfolio:

$$\text{cov}(r_k r_M) = E[(r_k - \mu_k)(r_M - \mu_M)]$$

where:

μ_k - expected rate of return of the k'th asset,

μ_M - expected rate of return of the market portfolio.

After some transformations and multiplying by λ I have:

$$\lambda \operatorname{cov}(r_k r_M) = r_k - r_f \tag{4.26}$$

For the market portfolio $\operatorname{cov}(r_M r_M) = \sigma_M^2$, so:

$$\lambda \sigma_M^2 = r_M - r_f . \tag{4.27}$$

Substituting λ from (4.26) into (4.27) results in:

$$r_k = r_f + \frac{\operatorname{cov}(r_k r_M)}{\sigma_M^2}(r_M - r_f),$$

or the standard CAPM model:

$$r_k = r_f + (r_M - r_f)\beta_{kM}$$

where

$$\beta_{kM} = \frac{\operatorname{cov}(r_k r_M)}{\sigma_M^2}.$$

The comparison of all three methods of deriving the standard CAPM model and conclusions are given later.

4.5 Hidden assumptions and critique

The main conclusions concerning to the one-period CAPM model, according to the Cuthbertson (1996) are:

- The risky assets are held by investors with in the same proportions. The market portfolio includes these optimal proportions.

- Investor's preferences between risk and expected return are treated as a second stage in decision making. The more important is the risk aversion. The more risk averse is the investor, the less risky assets he will have in the portfolio.
- The constant for assets are the beta parameters, calculated as $\beta_{jM} = \frac{\mathrm{cov}(r_j r_M)}{\sigma_M^2}$, so the excess return of the single risky asset $r_j - r_f$ is proportional to the excess return on the market portfolio $r_M - r_f$.
- In equilibrium the CAPM does not necessary imply constant returns. The covariance $\mathrm{cov}(r_j r_M)$ changes over time.

Some of the assumptions of the standard CAPM model seems to be consistent with the intuition. Doubts appear when analysing a few of them, for example, the assumption that the investor measures a risk of an asset by analysing a standard deviation. Usually this measure is not calculated. A decision about buying assets is made based mainly upon the information of activities of companies, their conditions, plans or advice from specialists presented in the mass media. This describes the decision making process of small investors.

The assumptions about lack of taxes and transaction costs do not always hold true. The dividend and the profit from the capital are included in a price of an asset, so it is a special type of tax. A cost of transaction is carried additionally, which the investor pays his broker.

Generally, despite the weaknesses of the CAPM model it could be said that this is useful when examining assets when considering risks and expected returns. The model shows this relationship but it is not known precisely how to measure any of the inputs required for the CAPM. They should be *ex ante* data, but they are available only *ex post*, so the estimates are found with potentially large errors (see Brigham and Grapenski 1993). Also the way of thinking of maximising expected return in conditions of minimising risk seems to be rational, but it is assumes that individuals have all necessary data when making the decisions.

Because of such doubts economists have tried to develop new versions of the capital asset pricing model, which could solve such problems, raised by the standard model. The next part of the chapter analyses some of the versions known as non-standard CAPM models. However, it is important to recognise problems and limitations of the CAPM before applying it to real financial markets, such as WSE.

4.6 Further developments; non-standard models

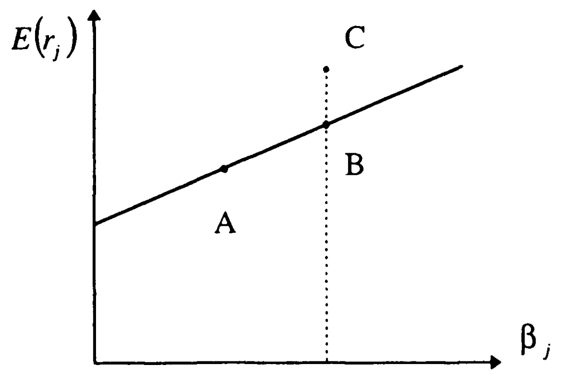
The CAPM model could describe the behaviour of capital markets providing that all assumptions are held. It is known that not all assumptions of the standard CAPM hold at all times and on all markets, so economists try to derive or describe the alternative versions of the standard model which could be used on each particular markets. One of the reasons for examining other equilibrium models is that it allows for the formulation of alternative explanations of equilibrium returns. Furthermore there are various applications of the CAPM model to the other capital markets.

One of the assumptions of deriving the CAPM model, used in deriving the standard model based on the risk premium (see part 4.4.3), is that the **short sale** is allowed. In this sense the investor can sell any asset, his or borrowed, and buy any other asset, an assumption convenient for the mathematical derivation of the CAPM. According to Elton and Gruber (1991) this is not a necessary assumption. Following them Lintner (1971) showed that the same result would have been obtained if short sales had been disallowed.

The assumption of **lending** and **borrowing** unlimited sums of money at the **riskless rate** of interest seems to be realistic in the case of lending but not necessarily so of borrowing. Lending could be equivalent to buying government securities equal in maturity to their single-period horizon, which exist and are riskless. In the case of no riskless lending or borrowing the *zero-beta* model was proposed as a version of the CAPM. Because this model is very often used, we show two ways of deriving the *zero-beta* CAPM. The first way stresses economic rationale.

As described above, the CAPM model shows that combinations of two risky portfolios lie on a straight line connecting them in expected return beta space. So the combinations of portfolios A and B lie on the line AB (see figure 4.3).

Figure 4.3 Portfolios in expected return beta space



All combinations of portfolios lie on the same line. The portfolio C has the same systematic risk[4] as portfolio B but C gives higher return. Then the investor would prefer portfolio C than B and could purchase C and sell B short, having an asset with positive expected return and no systematic risk. However, in equilibrium such an opportunity cannot exist because all portfolios and assets must be along a straight line. Also the market portfolio lies along this line. Two points describe the line. One of them is market portfolio, second, for example, a portfolio where the straight line cuts the vertical axis, where beta is equal to zero.

The straight line could be described by two points. The convenient points are the market portfolio, with *beta* equal to one, and the point where *beta* is equal to zero.

The straight line equation is:

$$\text{Expected return} = a + b(beta) \qquad \text{or} \qquad \mathrm{E}(\mathrm{r}_j) = a + b(\beta_j)$$

For zero-beta, where μ_z is the expected return on this portfolio I have:

$$\mu_z = a + b(0) \qquad \text{or} \qquad \mu_z = a, \tag{4.28}$$

[4] Systematic risk describes 'the proportion of an investment's total risk that cannot be avoided by combining it with other investements in a diversified portfolio' (Jennings 1992).

and for the market portfolio, where μ_M is the expected return of the market portfolio:

$$\mu_M = \mu_z + b(1) \qquad \text{or} \qquad b = \mu_M - \mu_z . \tag{4.29}$$

Together, from (4.28) and (4.29), if μ_j and β_{jM} are the expected rate of return and beta of the asset or portfolio, I get:

$$\mu_j = \mu_z + (\mu_M - \mu_z)\beta_{jM}$$

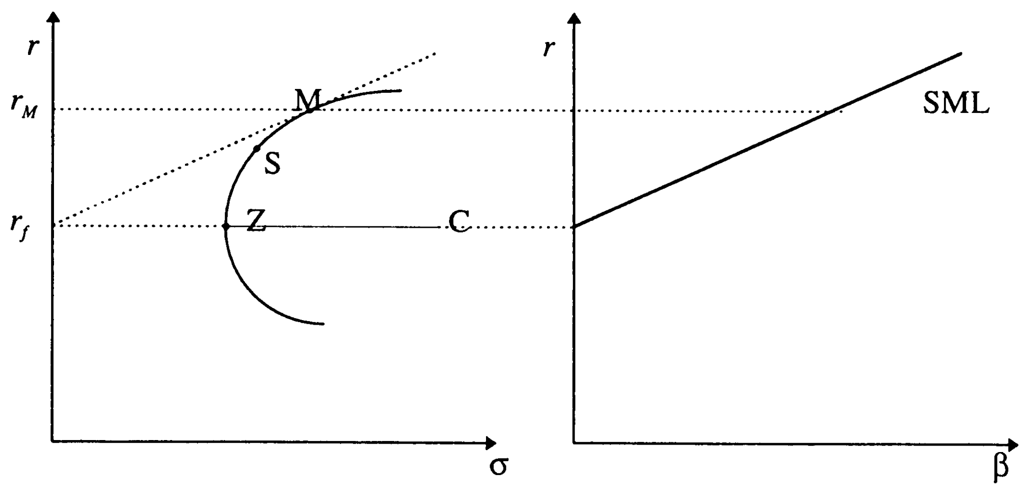
and for the rate of return:

$$r_j = r_z + (r_M - r_z)\beta_{jM} \tag{4.30}$$

which is the zero-beta version of the CAPM model.

The same zero-beta CAPM model can be derived by a more rigorous method. Assume that the investor can lend and borrow assets at the risk-free rate r_f (figure 4.4). The portfolios between Z and C are not correlated with the market portfolio.

Figure 4.4 The portfolios with return r_f



Source: Haugen (1996).

The optimal proportions of assets in portfolio can be found by solving a set of simultaneous equations directly, analogous to equation (4.25), where one of them is:

$$r_i - r_f = \lambda w_1 \sigma_{1i} + \lambda w_2 \sigma_{2i} + \ldots + \lambda w_{i-1} \sigma_{i-1,i} + \lambda w_i \sigma_i^2 + \lambda w_{i+1} \sigma_{i+1,i} + \ldots + \lambda w_n \sigma_{ni}$$

which leads to the standard CAPM model.

Assume now that the riskless assets do not exist. Then we can find a number of assets, which give a return r_f. They are located on the line R_fC (see figure 4.4). In the standard model equation for the asset with the rate of return equal to r_f *beta* is equal to zero. It implies zero covariance between this asset and the market portfolio. Then if Z denotes portfolio minimising the variance of the zero-beta we get for any asset j:

$$r_j = r_z + (r_M - r_z)\beta_{jM}$$

which is a zero-beta version of the CAPM (4.30). To show this assume that S describes the portfolio with the smallest possible variance. Then the portfolio defined as a combination of the market portfolio and the zero-beta portfolio is:

$$\sigma_s^2 = w_z^2 \sigma_z^2 + (1 - w_z)^2 \sigma_M^2$$

where the covariance between these two assets is zero.

To find the weights of the optimal portfolio is necessary to minimise variance, so the first derivative with respect to the weight is:

$$\frac{\partial \sigma_s^2}{\partial w_z} = 2 w_z \sigma_z^2 - 2\sigma_M^2 + 2 w_z \sigma_M^2,$$

which leads to the solution:

$$w_z = \frac{\sigma_M^2}{\sigma_M^2 + \sigma_z^2}.$$

Both variances, for Z and M are positive. The smallest possible variance will give positive weights. For $r_z < r_M$ portfolios Z and M (with positive weights) must have higher expected returns than Z. Then the minimum variance portfolio has higher return and smaller variance than Z, so Z cannot be the most efficient portfolio. Let us locate portfolios Z, M and S on the minimum variance frontier for all portfolios (figure 4.3). It is known that with homogenous expectations all investors face the same efficient frontier so if the short sales allow all combinations of any two minimum variance portfolios have minimum variance. It means that each investor's portfolio is efficient and if the market return is an average of the returns on the portfolios of the investors, the market portfolio has a minimum variance and is efficient.

It is known the Roll's (1977) critique of the test of the CAPM. Based on the security market line (SML) it is shown that for any efficient ex post portfolio in a sample of data there is a linear relationship between the mean rate of return and the *beta* parameter. This means that the market portfolio is mean-variance efficient, so SML must hold in the data sample. Roll concludes that if the elements of the market portfolio are not known, it is not possible to test the CAPM.

If the **riskless lending** is allowed with **no riskless borrowing** then the market portfolio is still an efficient portfolio and for all securities contained in market portfolio the rate of return is described by the zero-beta model (4.30). It is worth noting that is this case all investors no longer hold the same portfolios in equilibrium. Investors still hold most assets as long and short, with many assets short. The risk is measured by beta. The difference is only in the intercept point and slope of the line. The analysis of the capital market with restricted borrowing showed Black (1972).

In case of including **personal taxes** and **transactional costs** the return on any asset is given by the equation:

$$r_j = r_f + \beta_{jM}\left[\left(r_M - r_f\right) - \tau\left(d_M - r_f\right)\right] + \tau\left(d_j - r_f\right)$$

where d_M is a dividend yield of the market portfolio calculated as dividend divided by price, d_j is the dividend yield for the security j and τ is a tax factor. This factor measures the relevant market tax rate on capital gains and income.

A slightly more complicate situation appears when analysing a portfolio of **nonmarketable** and **marketable assets**. The example of a nonmarketable asset is human capital. The model of the portfolio in case of the portfolio included these types of assets as shown Mayers (see Elton and Gruber 1991). His version of the model is:

$$r_j = r_f + \frac{r_M - r_f}{\sigma^2_M + \frac{V_{NO}}{V_{MA}}\operatorname{cov}\left(r_M r_{NO}\right)}\left[\operatorname{cov}\left(r_j r_M\right) + \frac{V_{NO}}{V_{MA}}\operatorname{cov}\left(r_j r_{NO}\right)\right]$$

where the symbols are defined as before and r_{NO} is the one period rate of return of nonmarketable assets, V_{NO} is the total value of all nonmarketable assets and V_{MA} is the total value of all marketable assets. In this case the definition of the risk of the asset is changed. The risk is now a function of the covariance of the asset with the total set of nonmarketable assets as well as with all marketable assets. Then the risk of any asset is positively correlated with nonmarketable assets and is higher than for the standard CAPM model.

An interesting version of the model is that of **the multi-beta CAPM**, developed by Friend, Landskroner and Losq (1976). Their CAPM model, which includes inflation under certain assumptions is:

$$r_j - r_f = \sigma_{jI} + \frac{\left(r_M - r_f - \sigma_{MI}\right)}{\sigma_M^2 - \sigma_{MI}/\alpha}\left[\sigma_{jM} - \frac{\sigma_{jI}}{\alpha}\right]$$

where α is the ratio of nominal risky assets to total nominal value of all assets, risky and non-risky, σ_{jI} is the covariance of the rate of return of the j'th asset and inflation (I) and σ_{MI} the covariance of the market return with inflation.

Merton (1973) was the first to describe *an intertemporal* CAPM (ICAPM) which included a number of sources of uncertainty. The model of inflation, which is a form of *multi-beta CAPM* is formulated as:

$$r_j - r_f = \beta_{jM}\left(r_M - r_f\right) + \beta_{jI}\left(r_I - r_f\right).$$

This model is different from the standard CAPM due to the additional factor, of a new beta and the price of inflation risk r_I. The new beta measures the sensitivity of the asset to the portfolio for the inflation.

The *multi-beta* CAPM model is:

$$r_j - r_f = \beta_{jM}\left(r_M - r_f\right) + \beta_{jI1}\left(r_{I1} - r_f\right) + \beta_{jI2}\left(r_{I2} - r_f\right) + \ldots$$

The elements r_{I_i}'s represents expected return on a set of portfolios allowing the investor to hedge a set of risks.

Economists also analysed and attempted to build versions of CAPM for

heterogenous expectations of the investors, **non-price-taking behaviour, the consumption-oriented** model and many others. Some of them tried to adapt the CAPM model to other markets (see Elton and Gruber 1991). For example, Thomas and Wickens (1992) developed an international CAPM for bonds and equities. Slade and Thill (1994) showed the Hotelling capital asset pricing model (HCAPM) for cash flows from mining. Bayesian inference in asset pricing tests was shown by Knight and Satchell (1997). Recently Cable and Holland (1998) tested the linear market model in preference to the CAPM.

Finally, it could be said that relaxing some of the assumptions of the CAPM model does not necessary lead to discarding the model. If there is no riskless lending or borrowing, the rate of return of the risky asset depends linearly on weight average of the returns of the two risky portfolios: the efficient portfolio and the other risky portfolio which has a zero covariance, known as a zero-beta portfolio. Allowing for inflation, taxes or transactional costs the model has the additional factor, which appeared as a result of solving optimisation problem with additional restriction. All the models shown above were built to increase realism in modelling the rate of returns of the risky assets in real world scenarios.

4.7 Conclusions

In this chapter I examined the background of the standard portfolio allocation model and further development of the model. I presented the main assumptions of the standard CAPM model and three different methods of its derivation. Different assumptions lead to the different conclusions. The first method, based on the marginal substitution, assumes that the investor buys the combination of the risk free and risky assets and then 'moves' a part of money from risk free to risky assets. The second, also based on the marginal measures, shows that we can obtain the same model assuming that the investor disposes only risky assets. Then an exchange between risk and expected return takes place by borrowing or buying assets. The third method, based on the risk premium, assumes riskfree lending and borrowing of an unlimited amount of money. Maximising the slope of the straight line passing through the riskless rate of return by solving the set of simultaneous equations we find the composition of the portfolio. Under the assumptions of the CAPM I conclude that the only portfolio of risky assets is the market portfolio. An investor adjusts the risk of the market portfolio to his own risk and return combination. It leads to the theorem that all investors construct their optimum portfolios by combining a market fund with the riskless assets.

It is important to mention that the standard CAPM analysis is limited by equilibrium trading. As it was earlier explained the Warsaw Stock Exchange is a market in disequilibrium, caused mainly by imposed price limits. In the case of disequilibrium trading the standard model is not relevant when analysing the returns. If a price hits a barrier then disequilibrium occurs and the price is distorted. Even if the prices are not constrained but other prices are constrained spillover effects might also occur (see Charemza, Shields and Zalewska-Mitura 1997), that is both cross-sectional and intertemporal spillovers occur. Price limits cause problems with estimations. The Ordinary Least Square (OLS) method cannot be used to estimate the model because of inconsistence with the main OLS assumption of the normality of the returns. This problem will be explained later (in chapter 6). Using the OLS method the calculation of the *beta* parameter, according to the formula $\beta_{jM} = \frac{\sigma_{jM}}{\sigma^2_M}$, where σ_{jM} is the covariance of the j'th asset and the market portfolio and σ^2_M is the variance of the market portfolio, is effected at disurbance of variance. The variance σ^2_M is distorted by artificial reduction and consequently, the *beta* parameter is biased.

Although some of tight assumptions of the standard model have been relaxed, such as riskless lending and borrowing, short sales, price-taking behaviour and heterogenous expectations, any of the known models seem appropriate to measure a level of risk of assets on the Warsaw Stock Exchange.

In the next chapter I propose the optimal portfolio allocation model with market constraints. The model is based on the assumptions of the standard CAPM and includes institutionally imposed price limits.

CHAPTER 5

CAPM AND QUANTITY CONSTRAINTS

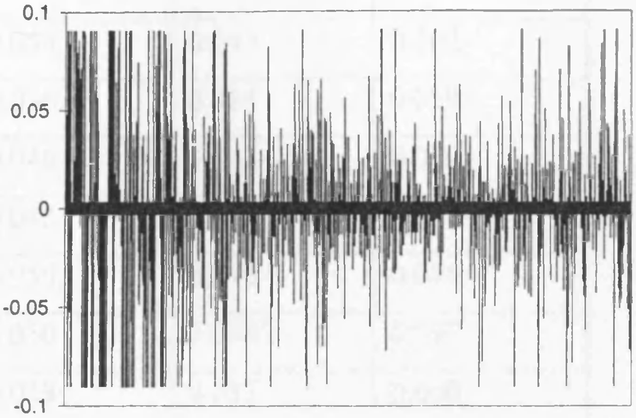
5.1 Extension of CAPM in the case of disequilibrium trading[1]

The standard CAPM model, as it is described in the previous chapter, is based on several assumptions. Evidently, no empirical market, let alone an emerging market, is fully consistent with these assumptions. One of the assumptions is that rates of return of the assets have to be unlimited. It implies no restrictions and limitations concerning price movements on the market. Although this is a case for many contemporary markets, nevertheless in some stock markets the price of an asset is regulated in such a way that it cannot move by more than a fixed percentage above or below that of the last session price. Such regulations have been applied in numerous emerging markets (e.g. in China, Lithuania, Poland, Turkey) and also in some mature ones (e.g. France). Evidently if the price is not allowed to settle at its equilibrium level because of the presence institutional constraints, demand may not match supply (or *vice versa*) and disequilibrium occurs.

On the Warsaw Stock Exchange (see part 2.3) the maximum admissible day price change is ±10% of the previous day price. Figure 5.1 shows the daily rates of return of the exemplary company *Universal* for analysed period, from 4.01.1994 to 17.03.1998.

[1] This is extended and updated version of the paper 'Regulation of the Warsaw Stock Exchange: The portfolio allocation problem', prepared jointly with Prof. Wojciech W. Charemza. The paper has been accepted by the *Journal of Banking and Finance*.

Figure 5.1 The rates of return from *Universal*



As we see there the number of rates of return which reach the maximum or minimum level is high during the analysing period. However, the number of hits against the boundaries were bigger in the early years of operating the WSE and it also happened recently. According to this fact it seems to be appropriate to take into account imposed market constraints in market analysis. In table 5.1 the frequencies of limit hits (censoring) for the data used for estimations and testing in the next two chapter are given.

Table 5.1: Frequencies of limit hits (censoring)

	lower	upper	total
BRE	0.033	0.020	0.052
EFE	0.058	0.043	0.102
ELE	0.036	0.024	0.059
EXB	0.046	0.042	0.089
IRE	0.042	0.040	0.081
KAB	0.055	0.040	0.095
KRO	0.050	0.046	0.097
MSE	0.029	0.032	0.060
MSW	0.043	0.036	0.079
OKO	0.058	0.044	0.103
POL	0.038	0.027	0.066
PRO	0.049	0.030	0.079
SOK	0.046	0.042	0.088
SWA	0.050	0.040	0.090
TON	0.062	0.047	0.110
UNI	0.061	0.045	0.107
VIS	0.047	0.036	0.083
WBK	0.040	0.031	0.070
WED	0.042	0.030	0.071
WOL	0.038	0.032	0.069
ZYW	0.034	0.028	0.061

It is generally agreed that, in the absence of price limits, emerging market anomalies causing lack of efficiency (or more precisely, predictability of returns) are no more severe than those of mature markets (see e.g. Claessens, Dasgupta and Glen 1995, Richards 1996). In particular, Buckberg (1995) and Harvey (1995a) found emerging market behaviour to be consistent with the CAPM model. It is also argued that market inefficiencies tend to evolve (diminish) over time and it is possible to capture the convergence towards market efficiencies (see Harvey 1995b, Emerson, Hall and Zalewska-Mitura 1997)[2].

Various economists have attempted to develop models for emerging markets. Hwang and Satchell (1998) built an asset pricing model for emerging markets using higher moments. A censored-GARCH model of assets returns with price limits was shown by Wei (1998), who proposed a Bayesian approach to develop the model. The censored-GARCH model, introduced by Wei, implies a set of linear constraints on the unobserved equilibrium returns required by price limits. Recently, Gouriéroux and Jouneau (1999) proposed a mean variance analysis of the portfolio choice under constraints. They showed that the portfolio under constraint can consistently be estimated and used to assess the performance of the portfolio management.

I propose the optimal portfolio allocation model based on the Sharp-Lintner version of CAPM. The model assumes that some prices in the market are regulated (disequilibrium) prices. It is assumed that the market in which the portfolio allocation decisions are made might be inefficient, but the only form of inefficiency which might possibly exist is that caused by price regulation (appearance of price constraints). Charemza, Shields and Zalewska-Mitura (1997) have shown that with the appearance of such constraints, even if they are not expected to be binding, the general

[2] It is also argued that the CAPM approach is inconsistent with the theory of efficient markets (see Reingaum 1992). Discussion of this is, however, outside the scope of this thesis.

assumptions of the efficient market still hold. The next part of the chapter examines the derivation and description of such a model.

5.2 Derivation of the model with quantity constraints

Let me introduce the following basic notation:

m - the number of investors, $i = 1,2,\dots,m$;

n - the number of assets, $j = 1,2,\dots,n$;

$\tilde{p}_j$ - expected price of asset j at the end of period, defined as the expected value of the price of the j'th asset conditional on information available at the beginning of the period;

p_j^0 - initial price of asset j;

$\text{cov}(p_{j,k})$ - covariance between prices of j and k at the end of period, that is

$$\text{cov}(p_{j,k}) = E\left[(p_j - \tilde{p}_j)(p_k - \tilde{p}_k)\right].$$

The Sharp-Lintner CAPM model could be used to describe equilibrium in term of either return or price. Let the expected value of the portfolio at the end of period for the i'th investor be μ_i and the variance σ_i^2. The utility function of the i'th investor is $V_i(\mu_i, \sigma_i^2)$, with the usual assumptions:

$$V_{i1} = \frac{\partial V_i}{\partial \mu_i} > 0 \quad , \quad V_{i2} = \frac{\partial V_i}{\partial \sigma_i^2} < 0 \quad .$$

If the investor is not constrained on any of the prices (more precisely, if the quantities traded are not constrained due to the fact that the price of any asset reaches its upper or lower limit) the problem of her/his decision-making is the maximisation of investor's utility function with respect to w_{ij} in proportion to the wealth held in asset j by the i'th investor:

$$\max_{w_{ij}} V_i(\mu_i, \sigma_i^2) \quad .$$

Let the expected value of the portfolio i be described as (see Brennan *et al.* 1992):

$$\mu_i = \sum_{j=1}^{n} w_{ij} \tilde{p}_j - R \sum_{j=1}^{n} (w_{ij} - \overline{w}_{ij}) p_j^0 \quad ,$$

where $R = r + 1$ (r is a risk free rate) and $\overline{w}_{ij}$ is an endowed fraction of asset j in the i'th portfolio. The variance of the return of i'th investor is defined as:

$$\sigma_i^2 = \sum_{j=1}^{n} \sum_{k=1}^{n} w_{ij} w_{ik} \operatorname{cov}(p_{j,k}) \quad .$$

The sum of wealth proportions in a portfolio for each investor is equal to one so that:

$$w_{in} = 1 - \sum_{j=1}^{n-1} w_{ij} \quad .$$

In order to find the optimal value of the portfolio, it is necessary to calculate the derivation of the utility function with respect to each asset. Let me do it with respect to the first asset:

$$\frac{\partial V_i}{\partial w_{i1}} = \frac{\partial V_i}{\partial \mu_i} \frac{\partial \mu_i}{\partial w_{i1}} + \frac{\partial V_i}{\partial \sigma_i^2} \frac{\partial \sigma_i^2}{\partial w_{i1}} \quad . \tag{5.1}$$

The first order condition is:

$$V_{i1}(\tilde{p}_1 - \tilde{p}_n - Rp_1^0 + Rp_n^0) + V_{i2}\left[2\sum_{k=1}^{n} w_{ik}\,\mathrm{cov}(p_{1,k})\right] = 0. \tag{5.2}$$

So far the result is consistent with the Sharp-Lintner model. Let me now assume that the price of the n'th asset is constrained as[3]:

$$\tilde{p}_n = \begin{cases} (1+\delta)p_n^0 & \text{if} \quad (\tilde{p}_n - p_n^0)/p_n^0 \geq \delta \\ p_n^* & \text{if} \quad -\delta < (\tilde{p}_n - p_n^0)/p_n^0 < \delta \\ (1-\delta)p_n^0 & \text{if} \quad (\tilde{p}_n - p_n^0)/p_n^0 \leq -\delta \end{cases}, \tag{5.3}$$

where p_n^* is an expected price of asset n in the equilibrium situation and δ is the maximum admissible (by the regulator) price movement. In particular, if the price constraints are not binding, the price dynamic is described by a martingale process $p_n^* = p_n^0$ (see Charemza, Shields and Zalewska-Mitura 1997).

Assume now that the price 'hits a boundary' (lower or upper limit) with probability ω. Then:

$$\tilde{p}_n = \begin{cases} (1 \pm \delta)p_n^0 & \text{with prob.} \quad \omega \\ p_n^* & \text{with prob.} \quad 1-\omega \end{cases}, \tag{5.4}$$

which gives:

$$E(\tilde{p}_n) = \omega(1 \pm \delta)p_n^0 + (1-\omega)p_n^* \quad . \tag{5.5}$$

[3] Often price constraints are defined as a percentage of changes of the initial price, where δ indicates the maximum admissible price change. For instance, for the Warsaw Stock Exchange δ is equal to 10% and is identical for all assets.

Let $r_{ik} = w_{ik} \cdot p_k^0$ be the amount invested in asset k, for $k = 1,2,\ldots,n$, $\mu_j = \dfrac{\tilde{p}_j - p_j^0}{p_j^0}$ is the expected rate of return of asset j, and σ_{jk} is the covariance of the rates of returns between assets j and k. It can be shown that:

$$\sigma_{jk} = \text{cov}(r_j r_k) \equiv E[(r_j - \mu_j)(r_k - \mu_k)] = E\left[\left(\frac{p_j - p_j^0}{p_j^0} - \frac{\tilde{p}_j - p_j^0}{p_j^0}\right)\left(\frac{p_k - p_k^0}{p_k^0} - \frac{\tilde{p}_k - p_k^0}{p_k^0}\right)\right]$$

$$= E\left[\left(\frac{p_j}{p_j^0} - \frac{\tilde{p}_j}{p_j^0}\right)\left(\frac{p_k}{p_k^0} - \frac{\tilde{p}_k}{p_k^0}\right)\right] \equiv \frac{1}{p_j^0 p_k^0}\text{cov}(p_{j,k}) \quad .$$

Substituting $\theta_i^{-1} = -\dfrac{V_{i1}}{2V_{i2}}$ as the measure of the investor's risk tolerance (see Brennan *et al.* 1992), for the constrained price I obtain:

$$\mu_1 - r + (r - \delta)\frac{p_n^0}{p_1^0} = \theta_M \sum_{k=1}^{n} r_{ik}\sigma_{1k} \quad , \text{ where } \theta_M = (\sum_{i=1}^{n} \theta_i^{-1})^{-1} \quad . \qquad (5.6)$$

For the aggregate market I have:

$$\mu - rI + \left[(r - \delta)\frac{p_n^0}{p_1^0}\right] I = \theta_M (r\Omega) \quad , \qquad (5.7)$$

where Ω is a variance and covariance matrix and I is a vector of units.

Solving equation (5.6) by substituting $\theta_M = \dfrac{\mu_M - r}{\sigma_M^2}$, where μ_M and σ_M^2 are respectively the expected rates of returns and the variance return of the market portfolio, I get (see Appendix 5 for details):

$$r_1 = r + \beta_1(r_M - r) - (r - \delta)\frac{p_n^0}{p_1^0} \quad , \quad \text{where } \beta_1 = \frac{\sigma_{1M}}{\sigma_M^2} \quad . \qquad (5.8)$$

For an individual asset j if the price of n'th is constrained, I have:

$$r_j = r_f + \beta_j (r_M - r_f) - (r_f - \delta) \frac{p_n^0}{p_j^0} \quad \text{, where } \beta_j = \frac{\sigma_{jM}}{\sigma_M^2} \quad , \tag{5.9}$$

which leads to a solution:

$$r_j = \begin{cases} r_f + \beta_j (r_M - r_f) & \text{with prob. } 1-\omega \\ r_f + \beta_j (r_M - r_f) - (r_f - \delta) \frac{p_n^0}{p_j^0} & \text{with prob. } \omega \end{cases} \quad \text{, where } \beta_j = \frac{\sigma_{jM}}{\sigma_M^2} . \tag{5.10}$$

The expected value of the j'th return is then:

$$E(r_j) = r + \beta_j (r_M - r) - \omega (r - \delta) \frac{p_n^0}{p_j^0} \quad . \tag{5.11}$$

As a result I have a model of the rate of return of an individual asset as a function of the rate of return of a risk-free asset (r_f), the level of systematic risk and the initial prices of assets j'th and n'th, where the price of the n'th asset is limited, and standard *CAPM* where this n'th price is not limited.

In the next chapter I estimate the model using the time series of twenty-one companies from the Warsaw Stock Exchange.

CHAPTER 6

THE EMPIRICAL PORTFOLIO ALLOCATION MODEL: ESTIMATION

6.1 Introduction

The optimal portfolio allocation model with quantity constraints, in the form of price limits was proposed earlier. The model can be applied on the markets where such limits are imposed.

The direct test of the optimal portfolio allocation model is known as a two-stage procedure (see Cuthberston 1996). A **first-pass time series regression**, assuming that *beta* parameters are constant for each security $j = 1,\ldots,n$ is:

$$r_{jt} - r_{ft} = \alpha_j + \beta_j\left(r_t^M - r_{ft}\right) + \xi_{jt},$$

where r_{jt} is the rate of return of the security j in time t, r_{ft} is the rate of return of the risk-free asset in time t, r_t^M is a market return and ξ_{jt} is the error term.

For each security the value of α is expected to be equal to zero. The value of *beta* calculated for each security can be used as a **second-pass cross-section regression**. If the rate of return of the asset j over time is denoted by $\bar{r}_j$ and ν_j the error term, then for all securities the second-pass regression is:

$$\bar{r}_j = \gamma_0 + \gamma_1\hat{\beta}_j + \nu_j \qquad \text{for } j = 1,\ldots,n,$$

which is the security market line introduced in chapter 4.

In this chapter I examine the first-pass cross-section regression on the Warsaw Stock Exchange where price limits exist. I analyse twenty-one longest established companies during the period 4.01.1994-17.04.1998. The second-pass cross-section regression will be presented in chapter 7.

The chapter is divided into six main sections. The first section gives a critical review of earlier empirical research on the WSE. Secondly, the formulation of the empirical model is presented. In the following section an analysis of the returns of companies from the WSE is undertaken. An evaluation of the censored returns and the first-pass testing of the model are given. Then the estimation methods are explained, two-limit Tobit model for uncorrected returns and maximum likelihood method for corrected returns. Finally, estimation results are presented. In order to explain obtained results a simple numerical experiment of simulating the efficiency frontiers for hypothetical portfolios with market constraints and without constraints is shown.

6.2 Empirical research on the Warsaw Stock Exchange

The empirical research on the WSE is relatively rare compared to that of other developed stock markets. Predominant research undertaken when the WSE began operationing was carried out by Osinska and Romanski (1993) and Bolt and Milobedzki (1993,1994a,b). Gordon and Rittenberg (1995) provide a simple statistical analysis of the share prices on the WSE and investigate the behavioural tendencies of investors. The following section reviews exant econometric research concerning the WSE.

The paper by Osinska and Romanski (1993) presents their analysis rates of returns on the WSE. In it, they recognise that the types of shocks or 'impulses' affecting the WSE are diverse in nature and different econometric methodologies are applied to detect the behaviour of investors once an impulse occurs. Two types of impulses are distinguished:

1. a 'pulse' function, which includes an event such as the introduction of new company shares into public trade;
2. a 'step' function, where the impulse has occurred at once, does not disappear but increases or decreases over time, for example the declining re-finance rate of the Central Bank.

Authors try to find out if there is any causal relationship between indicated impulses and prices (or returns) of shares and what the time delay is between impulse and response in price. The investigation it they makes use of Granger Causality methodology. Finally, they show that some of the hypothetical impulses has no significant impact on the rates of return, for example the time distance between particular sessions. On the other hand, other impulses, such as the primary share issue of WBK, Sokolow and Vistula, had a strong affect on the WSE. For them anticipated and lagged as well as duration effects were observed. Also ARCH and GARCH effects were tested. They also observed the tendency of clustering manifestation in trading volume.

Bolt and Milobedzki (1993) test whether the Efficient Market Hypothesis (EMH) holds in the weak form on the WSE, e.g. whether the price of an asset is generated by the random walk process. They conclude that the hypothesis of nonstationary price series of firms cannot be rejected, with the form of the models for the price series differing depending on the firm. In their second paper (1994b) they used statistical methods to show the complications in the econometric analysis of distributional deficiencies of the rates of return on shares quoted on the WSE. Thus, in a sense, they showed that the results from their earlier paper are not valid, at the same time implying that any results regarding price changes on the WSE from standard econometric tests and from the application of the methodology of capital market analysis based on the traditional mean-variance approach to investors' preference ordering, should be treated with the greatest of caution.

The major criticism of all the above studies is that they did not take into account the censored distribution of prices that characterises the price setting process of the WSE, e.g. $\pm 10\%$ price limit for the change in prices between sessions. This has implications for empirical distributions upon which the test results are based.

More recently, Charemza, Shields and Zalewska-Mitura (1997) analysed the predictability of the six main time series of returns on the WSE. They proposed modelling the predictability of returns with disequilibrium trading. Their work showed that it is possible to evaluate probabilities of reaching a disequilibrium state under the null hypothesis of non-predictability, and then to correct computed t-statistics using these probabilities. Neale, Wheeler, Kowalski and Letza (1998) tested the change in the pricing efficiency of the WSE over its limited life. The preliminary results of estimating the model with market constraints for six long established companies are presented by Charemza and Majerowska (1999).

6.3 Formulation of the model with disequilibrium trading

Equation (5.11) gives rise to a formulation of a simple CAPM-like empirical model. Ignoring, for high-frequency data (session-to-session returns) the effect of riskless assets and allowing for constant transactional costs, the model can be formulated as:

$$r_t^* = \alpha + \beta r_t^m + \varepsilon_t \quad , \tag{6.1}$$

where r_t^m denotes the session-to-session returns from the market portfolio in time (session) t, and r_t^* is the return from an individual security corrected by the censored prices. Suppose that there are $N + 1$ securities included in the market portfolio, and that they are ordered in such a way that the security investigated in (6.1) is the last, (N + 1)'th one. Hence a generalisation of (5.10) gives r_t^* being defined as:

$$r_t^* = r_t - \delta \cdot cf_t \quad ,$$

where the correction factor cf_t is

$$cf_t = \frac{\sum_{i=1}^{N} (\omega_i^+ z_{it}^+ - \omega_i^- z_{it}^-) p_t^i w^i}{p_t^{N+1}} \quad , \tag{6.2}$$

and where r_t is the observed, possibly censored, return of the $N + 1$ security in time t, δ is the relative constraint on price movements (fraction of the last period price which creates the upper or lower limit for returns, see formula (5.3)), p_t^i , $i = 1, 2, \dots , N + 1$, is the price of i'th security in time t, ω_i^+ and ω_i^- are the probabilities of hitting the upper and lower barrier by the i'th price, w^i is a weight denoting market share of i'th security, z_{it}^+ and z_{it}^- are the selector variables:

$$z_{it}^{+}=\begin{cases}1 & \text{if} \quad \frac{p_t^i - p_{t-1}^i}{p_{t-1}^i}=\delta \\ 0 & \text{otherwise}\end{cases} \quad \text{and} \quad z_{it}^{-}=\begin{cases}1 & \text{if} \quad \frac{p_t^i - p_{t-1}^i}{p_{t-1}^i}=-\delta \\ 0 & \text{otherwise}\end{cases}.$$

As the empirical observations imply (see also figure 5.1 in the text), the frequencies of hits change over time. In particular, after 1995, the frequencies of hits become noticeably smaller compared to the earlier years. Hence, a sensible estimate for ω_i^+ and ω_i^- seems to be the empirical frequencies of lower and upper hits within the sample computed in a recursive manner, that is taking into account only information available up to time t. For the first one hundred observations the values of ω_i^+ and ω_i^- are held constant and equal to the empirical frequencies of hits for this period. After the 100^{th} observation the frequencies have been updated recursively. Finally, the value of parameter δ is given by market regulations and w^i are the weights of the stocks in the market index.

6.4 Description of data

Model (6.1) has been estimated by ordinary least squares for the twenty-one longest established securities trading on the Warsaw Stock Exchange. The Warsaw Stock Exchange was established on the 16th of April 1991, initially with two sessions a week in 1991 and 1992, three sessions a week from the beginning of 1993 until the end of 1994 and five (daily) sessions since then. Detailed characteristics of the Warsaw Stock Exchange are given in chapter 2 and descriptive and econometric analyses of the

WSE can be found in chapter 2, and also in Gordon and Rittenberg (1995), Bolt and Milobedzki (1994a,b) and Shields (1997a,b). For our purpose it is important to recall that, on trading sessions, transactions are made at a single price, established by the regulator at a level which maximises demand and supply. This single price is established in a *tatonnement* process, where offers to sell and buy are lodged with the 'auctioneer' before the price is established (see parts 2.3 and 3.3.2 for details). If, however, the single price evaluated in this way is greater or lower than the last session price by more than 10%, it is artificially reduced and kept at a level which is exactly 10% above or below the last session price. Normally trade takes place at such aregulated price as to leave a part of demand or supply in excess[1]. Hence, for the Warsaw Stock Exchange the parameter δ introduced in equation (6.2) above is equal to 10%.

The twenty one companies selected for estimation are: *Bank Rozwoju Eksportu* (BRE, bank) *Efekt* (EFE, economic corporation), *Elektrim* (ELE, conglomerate services), *Exbud* (EXB, construction services), *Irena* (IRE, glass factory), *Kable* (KAB, cable factory), *Krosno* (KRO, glass factory), *Mostostal Export* (MSE, conglomerate services), *Mostostal Warszawa* (MSW, construction services), *Okocim* (OKO, brewery), *Polifarb Cieszyn* (POL, chemicals factory), *Prochnik* (PRO, light industry), *Sokolow* (SOK, food industry), *Swarzedz* (SWA, furniture factory), *Tonsil* (TON, electronics company), *Universal* (UNI, conglomerate services), *Vistula* (VIS,

[1] Occasionally, if demand is greater than supply (or the opposite) by more than five-fold, the transactions might be suspended altogether. Also, there are special regulations which allow some of the securities to be traded at a freely negotiated prices (during *extra time* trading). For simplicity, these are ignored herein.

light industry), *Wielkopolski Bank Kredytowy* (WBK, bank), *Wedel* (WED, food industry), *Wolczanka* (WOL, light industry), and *Zywiec* (ZYW, brewery). These twenty one companies are the longest established on the market and, at the beginning of the existence of the Warsaw Stock Exchange, represented the majority of trade. Over time the number of companies listed on the stock exchange has grown rapidly. Nevertheless, these twenty-one well-established 'mature' companies are still regarded as representative of the entire market[2]. Other major companies have usually been introduced to the market at a later date and their inclusion would reduce the data sample significantly. Our sample contains 1012 data points from the 289th session (3 January 1994) until 17 April 1998. Direct quantitative information concerning the identification of the disequlibrium trading sessions has not been used. Instead, I have assumed that if returns were closer than 0.05% to its upper or lower boundary (that is, if the published price was equal or higher than 1.095 times the previous session price, or 0.905 or lower than the previous session price), the upper (lower) boundary was hit. This 0.05% tolerance limit allows for accountancy of rounding errors of published prices. The data source was detailed information published in *Gazeta Bankowa* (daily) and *Rzeczpospolita* (weekly)[3] and in a few instances missing observations were interpolated. For sessions where trading was suspended and the stock market statistics denoted zero returns, returns were randomised by inserting a random number equal to the maximum of 10% of the standard deviation of returns. Simple autocorrelation analysis of returns does not reveal any substantial autocorrelation in the series, with the

[2] Except Wedel company, which withdrew shares from the WSE in April 1998.

[3] Data was collected and made available to us by the Macroeconomic and Financial Data Centre at the University of Gdańsk. Its assistance is greatefully acknowledged.

largest coefficients being approximately 0.249[4]. The lack of substantial autocorrelation supports the rationale for adopted method of interpolation of zero returns. In some other cases, where prices were allowed to go beyond that limit due to its occasional suspension, we censored the data as if the upper or lower limit was hit.

Tables 6.1 - 6.2 briefly summarise the descriptive characteristics of the series. In table 6.1 the descriptive measures for the series of all returns are represented, together with Doornik and Hansens' (1994) modification of the Bowman and Shenton (1975) test of normality. Under the null hypothesis of normality the statistic has $\chi^2(2)$ distribution. Table 6.2 gives analogous characteristics computed for the 'equilibrium' returns only, that is for the case where the lower or upper barriers were not hit.

[4] The greatest number were obtained for the Wedel company.

Table 6.1 Descriptive statistics and normality tests, for all returns

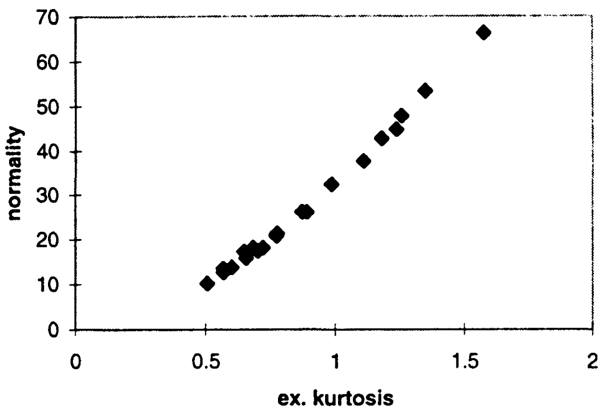
	No of obs.	mean	st. dev.	skewness	ex.kurtosis	normality
BRE	1012	0.0013	0.0328	-0.3602	1.8039	78.249
EFE	1012	0.0003	0.0408	-0.0636	0.5387	11.995
ELE	1012	0.0006	0.0346	-0.1659	1.3000	51.944
EXB	1012	0.0007	0.0398	-0.0502	0.5797	13.522
IRE	1012	-0.0003	0.0374	-0.0087	0.9474	31.452
KAB	1012	0.0002	0.0401	-0.0576	0.6419	16.139
KRO	1012	0.0008	0.0405	-0.0922	0.6362	16.060
MSE	1012	0.0008	0.0350	0.0505	1.1945	46.463
MSW	1012	0.0010	0.0387	-0.0840	0.6701	17.469
OKO	1012	-0.00001	0.0417	-0.0242	0.3594	5.8453
POL	1012	0.0005	0.0369	-0.0911	0.8106	24.079
PRO	1012	-0.0018	0.0394	-0.1449	0.4948	11.433
SOK	1012	0.0001	0.0403	0.0548	0.5123	10.965
SWA	1012	-0.0015	0.0406	-0.0078	0.4000	6.9996
TON	1012	-0.00004	0.0409	-0.1351	0.5399	12.784
UNI	1012	0.0003	0.0417	-0.0242	0.4185	7.6159
VIS	1012	-0.0007	0.0366	-0.1114	1.2234	47.807
WBK	1012	0.0005	0.0374	-0.0472	0.8512	26.164
WED	1012	0.0005	0.0352	-0.2432	1.4167	57.653
WOL	1012	-0.0007	0.0365	-0.0788	1.1292	42.116
ZYW	1012	-0.0005	0.0340	-0.1421	1.4947	65.842

Table 6.2 Descriptive statistics and normality tests, for equilibrium returns

	No of obs.	mean	st. dev.	skewness	ex.kurtosis	normality
BRE	958	0.0027	0.0258	-0.2376	1.5813	66.276
EFE	908	0.0018	0.0300	0.1610	0.7749	20.702
ELE	951	0.0017	0.0272	0.0314	0.9900	32.378
EXB	921	0.0012	0.0304	0.0060	0.7021	17.438
IRE	929	-0.0002	0.0280	-0.0027	0.7753	20.859
KAB	915	0.0018	0.0300	0.1923	0.8933	26.103
KRO	913	0.0013	0.0303	-0.0904	1.1110	37.648
MSE	950	0.0005	0.0276	0.0850	0.8757	26.190
MSW	931	0.0017	0.0302	0.0169	0.6594	15.793
OKO	907	0.0014	0.0312	0.1822	0.5702	13.435
POL	944	0.0014	0.0294	0.0376	0.6043	13.773
PRO	931	0.0001	0.0311	-0.0828	0.5719	12.589
SOK	922	0.0008	0.0313	0.2185	0.6852	18.135
SWA	920	-0.0005	0.0318	0.0694	0.5081	10.155
TON	900	0.0016	0.0291	-0.0809	0.7237	18.101
UNI	903	0.0019	0.0307	0.2506	0.6504	17.348
VIS	927	0.0006	0.0271	0.0448	1.3519	53.302
WBK	940	0.0015	0.0294	0.1121	0.7778	21.300
WED	939	0.0017	0.0265	-0.2039	1.2392	44.714
WOL	941	-0.0002	0.0288	-0.0732	1.2583	47.768
ZYW	949	0.0002	0.0263	-0.1361	1.1817	42.749

The statistics confirm the relative homogeneity of the sample. For all the series the characteristics are of a similar magnitude, and the distributions of the returns are close to being symmetric. The source of non-normality seems to be excess kurtosis, presumably caused by a concentration of randomised returns around zero for the days where trading was suspended and zero returns recorded. Figure 6.1 shows the strong positive relationship between excess kurtosis and normality of the returns for the analysed companies.

Figure 6.1 Relationship between excess kurtosis and normality of the returns



To confirm the intuition of the excess kurtosis causing non-normality of the returns I performed the following computation. Returns were randomised again, this time changing the arbitral randomisation from 0.1 of the standard deviation of the returns to 0.25 of the standard deviation. The results, given in table 6.3, confirm the expected results. This time the values of the calculated Chi-squared statistics are much smaller and the returns of two companies, *Prochnik* and *Swarzedz* are normally distributed at 1% significance level[5].

[5] The critical value is $\chi^2(2) = 9.21$.

Table 6.3 Normality tests, for equilibrium returns for randomised zero returns in case of 0.25 of standard deviations.

	No of obs.	ex.kurtosis	normality
BRE	958	1.4302	56.615
EFE	908	0.6550	16.015
ELE	951	0.8750	26.227
EXB	921	0.6033	13.394
IRE	929	0.6572	15.665
KAB	915	0.7676	20.791
KRO	913	0.9723	30.167
MSE	950	0.7751	21.324
MSW	931	0.5470	11.403
OKO	907	0.4539	10.273
POL	944	0.4824	9.3048
PRO	931	0.4589	8.8551
SOK	922	0.5849	14.833
SWA	920	0.3752	6.2538
TON	900	0.6228	14.112
UNI	903	0.4995	12.743
VIS	927	1.1828	42.713
WBK	940	0.6607	16.323
WED	939	1.0975	36.935
WOL	941	1.1216	39.402
ZYW	949	1.0103	32.985

6.5 Evaluation of the censored returns

Corrected returns can vaguely be interpreted as returns which would have happened if the price limits were not binding. Figures 6.2, 6.3 and 6.4 respectively show the series of uncorrected returns, corrected returns and the correction factor computed for one of the companies analysed, *Tonsil.* Figures 6.5 and 6.6 show distributions of the original and corrected returns. The figures indicate that most of the corrections occured in the early years of operation of the Warsaw Stock Exchange, where hits of the barriers were frequent. Distribution of the corrected returns exhibit much smaller unconditional variancesand greater concentration than that of the original returns.

Figure 6.2 Original returns from *Tonsil*

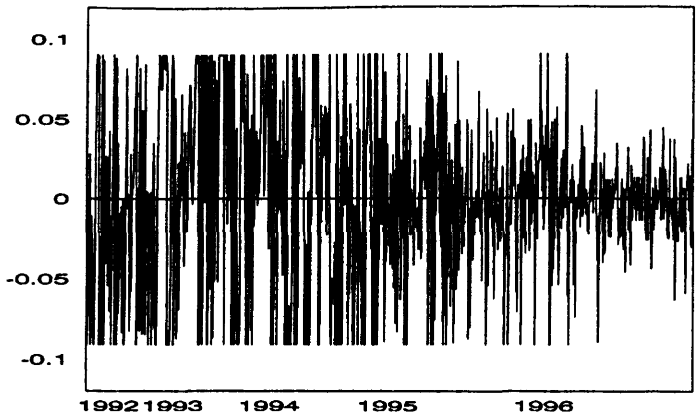


Figure 6.3 Corrected returns from *Tonsil*

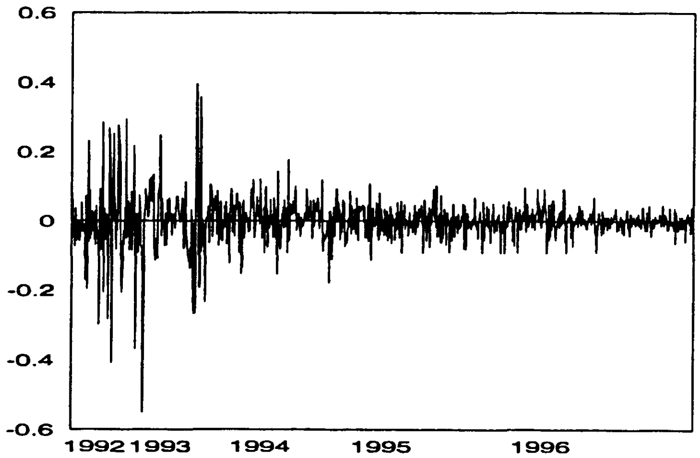


Figure 6.4 Correction factor

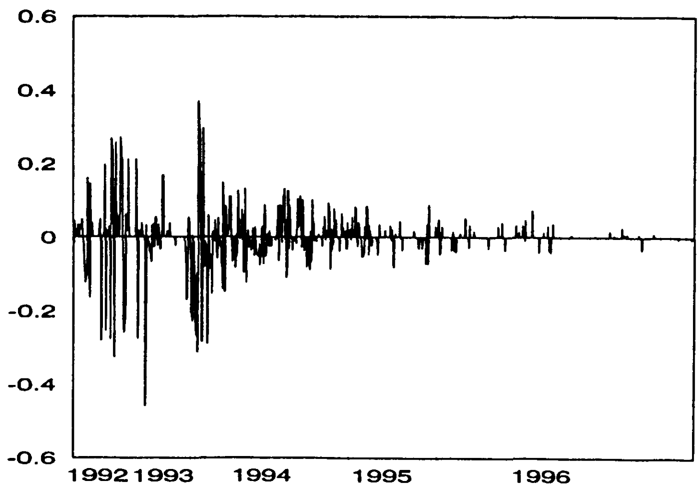


Figure 6.5 Distribution of original returns, *Tonsil*

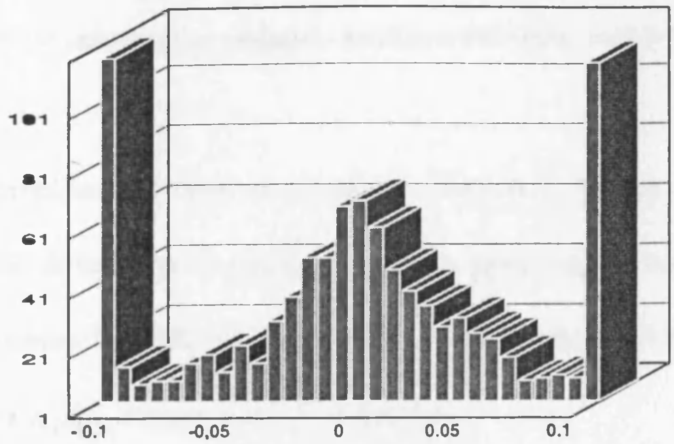


Figure 6.6 Distribution of corrected returns, *Tonsil*

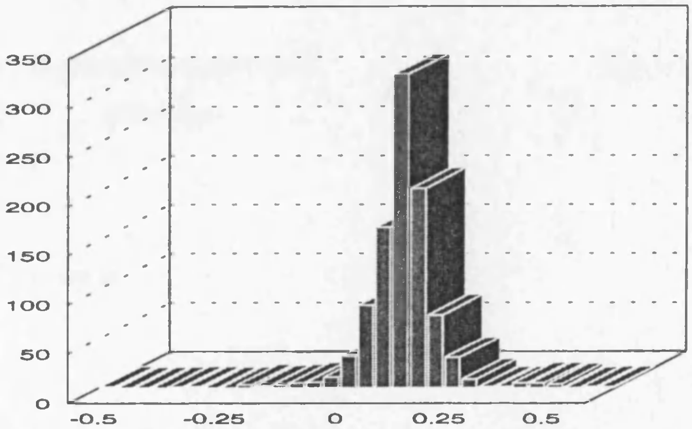


Figure 6.2 presents the behaviour of the rates of returns of the exemplary company *Tonsil*. Looking at the analysed time series it can be seen that the number of returns hitting the lower and upper limits (±10%) is relatively high. Also, it may be observed that the number of hits decreases in time which suggests that the variance of series decreases as well. This may cause addressing two problems. The first of the problems is mutual dependence of hits. That is that the probability of the price hitting the limit in time $t+k$ where $k \geq 1$, depends on the fact that the price limit in time t was hit. More generally it might be interesting to find out whether the returns are

predictable with the use of information regarding the previous hits. The second problem is whether the fact that the variance decreases involves market efficiency.

The first problem has been analysed by Charemza, Shields and Zalewska-Mitura (1997), who developed the tests based on a linear regression of returns on lagged dummy variables. In particular, they analysed two following linear regressions:

$$r_t = \lambda_1 d_{t-1}^{+} + \lambda_2 d_{t-1}^{-} + error \qquad \text{(Test I)} \qquad (6.3)$$

where:

$$d_t^{+} = \begin{cases} 1 & \text{if price hits upper limit} \\ 0 & \text{otherwise} \end{cases} \quad \text{and} \quad d_t^{-} = \begin{cases} 1 & \text{if price hits lower limit} \\ 0 & \text{otherwise} \end{cases}$$

and

$$r_t = \lambda d_{t-1}^{\pm} + error \qquad \text{(Test II)} \qquad (6.4)$$

where:

$$d_t^{\pm} = \begin{cases} 1 & \text{if price hits upper limit} \\ -1 & \text{if price hits lower limit} \\ 0 & \text{otherwise} \end{cases}$$

and r_t is the rate of return of the particular asset in time t. The models have been estimated by the corrected OLS method (for details see Charemza, Shields and Zalewska-Mitura 1997) and corrected Student-t statistics have been analysed in order to find if the relationships exist and if they are significant. Then the models have been applied to the returns from six companies traded on the Warsaw Stock Exchange from the first day of trading in April 1991 to the end of 1993. Results show that the information of previous hits may be successfully used for the prediction on returns. It implies an existence of the positive relationship between hits but this relationship cannot be considered as being a strong one. The series analysis imply some properties of the prices and returns. For unconstrained returns normally it is assumed that prices are I(1) and returns are stationary and normally distributed. For the WSE due to these truncations it is not possible to regard the series as a straightforward I(1) series. In particular, it is not possible to apply tests such as Dickey-Fuller test since the null hypothesis are not known (for the attempt see Yau 1996). It might be possible to recreate the 'unconstrained' price series from the cumulative returns defined as:

$$r_t^* = r_{t-1}^* + \delta cf_t \tag{6.5}$$

where r_t^* is the corrected return in time t, r_{t-1}^* is the corrected return in time $t-1$, δ is equal to 10% and cf_t is the correction factor, or in terms of prices:

$$p_t^* = \frac{(p_{t-1}^*)^2}{p_{t-2}^*} + \delta cf_t p_{t-1}^* \tag{6.6}$$

where $p_t^*, p_{t-1}^*, p_{t-2}^*$ are the prices of assets in time $t, t-1, t-2$ respectively. Unfortunately, since values of the 'unconstrained' returns depend very much on the starting values, the empirical application to the analysed returns from the WSE was unsuccessful. Calculated returns showed very high volatility. Some of the returns, especially in the early years of operating the WSE, had enormous values, even greater than 100%.

Evaluation of efficiency of returns was tested by Zalewska-Mitura (1998). In her work she analysed the evolving efficiency of emerging stock exchanges, in particular, the efficiency under price limits on the example of the WSE. Two types of share price series were compared, recorded and filtered by Kalman Filter (the description of the Kalman filter algorithm can be found, for example, in Hamilton 1994). The test included the analysis of the influence of the truncation in time t on the price in time $t+k$, for $k \geq 2$ (k never greater than 7). The test was evaluated through Monte Carlo simulations. The empirical analysis aimed five companies from April 1991 to June 1996. The results showed that the efficiency of analysed series tend to increase as time proceeds, so the WSE market shows the tendency to mature.

According to above results it can be concluded that the mutual dependence of hits of analysed returns from the Warsaw Stock Exchange should be taken into consideration in analysis. This dependence allows probabilities of hits being changed in time. It resulted in the procedure of constructing the correction factor used in the model of the optimal portfolio allocation. Recalling the formula (6.2) the return from an individual security corrected by the censored prices r_t^* is defined as follows:

$$r_t^* = r_t - \delta \cdot cf_t \quad .$$

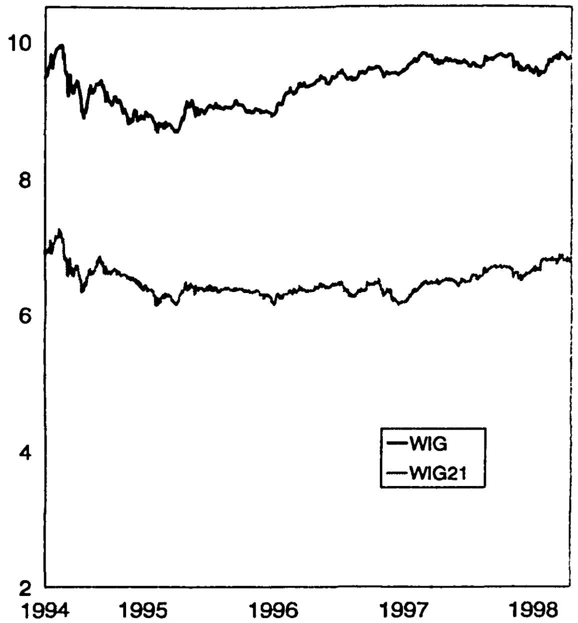
where r_t is the observed, censored, return of the N + 1 security in time t and δ is the maximum admissible price movement. The correction factor cf_t is

$$cf_t = \frac{\sum_{i=1}^{N} (\omega_i^+ z_{it}^+ - \omega_i^- z_{it}^-) p_t^i w^i}{p_t^{N+1}} \quad ,$$

where, p_t^i , i = 1, 2, ... , N + 1, is the price of i'th security in time t, w^i is a weight denoting market share of i'th security, z_{it}^+ and z_{it}^- are the selector variables equal to one if the price reaches limit (upper or lower respectively) and zero otherwise. The factor includes the probabilities ω_i^+ and ω_i^- which are the probabilities of hitting the upper and lower barrier by the i'th price. These probabilities have been approximated by the frequencies of hits. As it was showed the frequencies of hits become noticeably smaller after 1995 comparing to the earlier years so this fact was taken into consideration while constructing the correction factor. Hence, for the first one hundred observations the values of ω_i^+ and ω_i^- are held constant. Their values are equal to the empirical frequencies of hits for this period. After the 100th observation the frequencies have been updated recursively, that is taking into account only the information available up to time t. It can be seen that if the limits are not hit then the correction factor is equal to zero and the corrected return is equal to the observed return.

Two alternative ways of identifying the market returns variable r_t^m were used. Firstly, the published official Warsaw Stock Exchange share index, *WIG*, computed for all the shares traded on the market was used as the r_t^m variable. Alternatively, it was assumed that the entire market consisted of only twenty-one securities and an artificial Laspeyres type index for these twenty one companies alone was constructed (*WIG21*). For each of the twenty-one securities the r_t^m variable has been constructed by adjusting the *WIG21* index by the exclusion of price and quantity weight information on the modelled security. In another words, r_t^m represents the returns from the twenty remaining shares. The comparison of the original *WIG* and *WIG21* is given in figure 6.7. It shows that at the beginning of 1994 the development of both indices was almost identical. With the increased number of securities traded at the Warsaw Stock Exchange the dynamics of both indices started to differ and prices of new securities, other than those included in *WIG21*, were rising faster than prices of the twenty-one securities analysed herein.

Figure 6.7 *WIG* and *WIG21* (logarithmic scale)



6.6 Estimation methods

6.6.1 Maximum likelihood method

The maximum likelihood method is used for estimating models in which the right-hand side of the equation is always positive. The idea behind the method is to find a set of parameters estimates maximising the likelihood of having obtained the actual sample (see e.g. Davidson and MacKinnon 1993, Hamilton 1994). The joint probability density for the estimated model is evaluated at the observed values of the dependent variable. This is referred to the likelihood function L.

If r is the vector of dependent variables for all observations $j = 1,\ldots,n$ then the likelihood function is:

$$L(r,\theta) = \prod_{t=1}^{n} L_t(r_t,\theta) \tag{6.7}$$

where θ is a vector of population parameters, or

$$L(r) = L_1(r_1)L_2(r_2|r_1)L_3(r_3|r_2,r_1)\ldots L_n(r_n|r_{n-1},\ldots,r_1)$$
$$= \prod_{t=1}^{n} L_t(r_t|r_{t-1},\ldots,r_1)$$

The maximising joint likelihood function is known as the full-information maximum likelihood estimation (FIML).

6.6.2 Two-limit Tobit model

The model for the double truncated censored data is well established in literature (see e.g. Rosett and Nelson 1975, Nakamura and Nakamura 1983) and is based on the model developed by Tobin (1958)[6]. If r_j denotes the observed dependent rate of return and r_j^* the expected (unconstrained) return, then in the case of double truncation (see Maddala 1985, pp. 161-162):

$$r_j = \begin{cases} r^l & \text{if } r_j^* \le r^l \\ r_j^* & \text{if } r^l < r_j^* < r^u \\ r^u & \text{if } r_j^* \ge r^u \end{cases} \tag{6.8}$$

where r^u and r^l are the upper and lower limits of the returns. Assume that

[6] Tobit model is widely used in economics (see Amemiya 1984). For example, Keeley, Robins and West (1978) show the relationship between hours worked after a negative income tax program and pre-programs hours worked, Reece (1979) explains charitable contributions by the price of contributions and income. Adams (1980) examines how inheritance depends on income, marital status and number of children. Number of arrests per month after realise from prison explained by accumulated work release funds, age, race and drug use is a subject of Witte (1980). Wiggins (1981) analyses annual marketing of new chemical entities related with research expenditure.

$$\Phi(r) = \int_{-\infty}^{r} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{u^2}{2}\right) du$$

and

$$\phi(r) = \frac{\partial \Phi(r)}{\partial r} = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{r^2}{2}\right) \qquad (6.9)$$

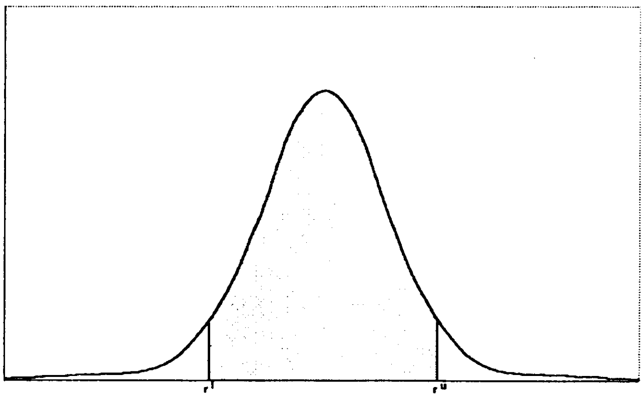
where $\Phi(r)$ and $\phi(r)$ denote the cumulative density function and density function for distributions other than the standard normal distribution. If the distribution is normal with mean μ and variance σ^2, the functions (6.9) become $\Phi\left(\frac{r-\mu}{\sigma}\right)$ and $\frac{1}{\sigma}\phi\left(\frac{r-\mu}{\sigma}\right)$ respectively.

There are many discussions regarding the development of a proper estimator for distribution of censored data[7]. An interesting concept of partial probability weighted moments which can be used to estimate a distribution from censored samples is shown by Wang (1990). He argues that censored sample quantile estimates are almost as effiicient as those obtained from uncensored data. Various Monte Carlo simulations have been performed to explain the propreties of estimation. Another example, proposed by Kaplan and Meier, for estimating a distribution function for truncated or censored data by the product-limit estimator was criticised by Lai and Ying (1991). They use martingale integral representations and empirical process to show usefulness of the minor modification of the product-limit estimator. Seo and Youm (1993) show three approximate estimators of the mean for grouped and

[7] The problem of censored distribution is analysed also in a pure mathematics (see, for example, Hahn, Kuelbs and Weiner 1990).

censored data: two approximate maximum likelihood estimators and the mid-point estimator. Two approximate estimators are described as resonable substitutes for the maximum likelihood estimator unless the probabilities of censoring and the number of inspections are too small. Their study was supported by some Monte Carlo experiments in term of the existence of each estimator.

Figure 6.7 Double truncated normal distribution



Assuming that $\varepsilon_t \sim IID\, N(0, \sigma_\varepsilon^2)$ the general likelihood function for double truncated variable is (see Davidson and MacKinnon 1993, p. 541):

$$L = \sum_{r^l < r_t^* < r^u} \ln\left[\frac{1}{\sigma_\varepsilon}\phi\left(\frac{r_t^* - \alpha - \beta r_t^m}{\sigma_\varepsilon}\right)\right] + \sum_{r_t^* = r^l} \ln\left[\Phi\left(\frac{r^l - \alpha - \beta r_t^m}{\sigma_\varepsilon}\right)\right] + \sum_{r_t^* = r^u} \ln\left[1 - \Phi\left(\frac{r^u - \alpha - \beta r_t^m}{\sigma_\varepsilon}\right)\right], \tag{6.10}$$

where, as above, $\phi(\bullet)$, $\Phi(\bullet)$ denote respectively the density and cumulative density functions of the standard normal distribution, r^u and r^l are the upper and lower limits for the returns.

The model (6.1) has been estimated by the full-information maximum likelihood method where the correction factor was used and, for comparison, by the two-limit Tobit model, using the function (6.10) in case where the returns were left uncorrected (that is, assuming $cf = 0$). Strictly speaking, in both cases the same general likelihood function has been used. The difference between the two is that for the corrected returns the limits imposed are very wide, so that the probability of reaching them is practically equal to zero, while for the uncorrected returns the limits are equal to that imposed on the Warsaw Stock Exchange (±10%). For the corrected returns, computationally this is relatively expensive way of finding the maximum likelihood estimates. Nevertheless, the fact remainsthat identical methods applied in both cases increases comparability of the results[8].

The model (6.1) has been estimated also by the Ordinary Least Square method (OLS) in case of corrected and uncorrected rates of return of assets. However, according to Cuthberston (1996), where of estimating CAPM by the OLS standard errors are incorrect, we used this method to compare estimates.

[8] All computations were conducted with the use of the *GAUSS* package and the *CML* library. The program for estimation of the two-limit Tobit model is attached.

6.7 Estimation results and first-pass testing

The first-pass testing, as it was explained in the first part of the chapter, is estimating the *beta* parameters. The estimation results are given in tables 6.4 - 6.45 (see the end of the chapter). The tables give the estimated parameters and their standard errors together with the basic characteristics: log-likelihood function, standard deviation of residuals, Durbin-Watson statistics, F-form statistic for testing the join hypothesis of residual autocorrelation up to 12th order (denoted as F(12)), and the augmented Dickey-Fuller statistic with 12 augmentations (ADF(12)) for testing the hypothesis of a unit root in the residuals. It is important to note that, for the maximum likelihood residuals, the autocorrelation and unit root (cointegration) statistics have to be treated with caution and be regarded as simply a crude indication of the residuals properties. They indicate stationarity of residuals for all estimates, but exhibit some moderate autocorrelation (normally of an order greater than one) for most of the series.

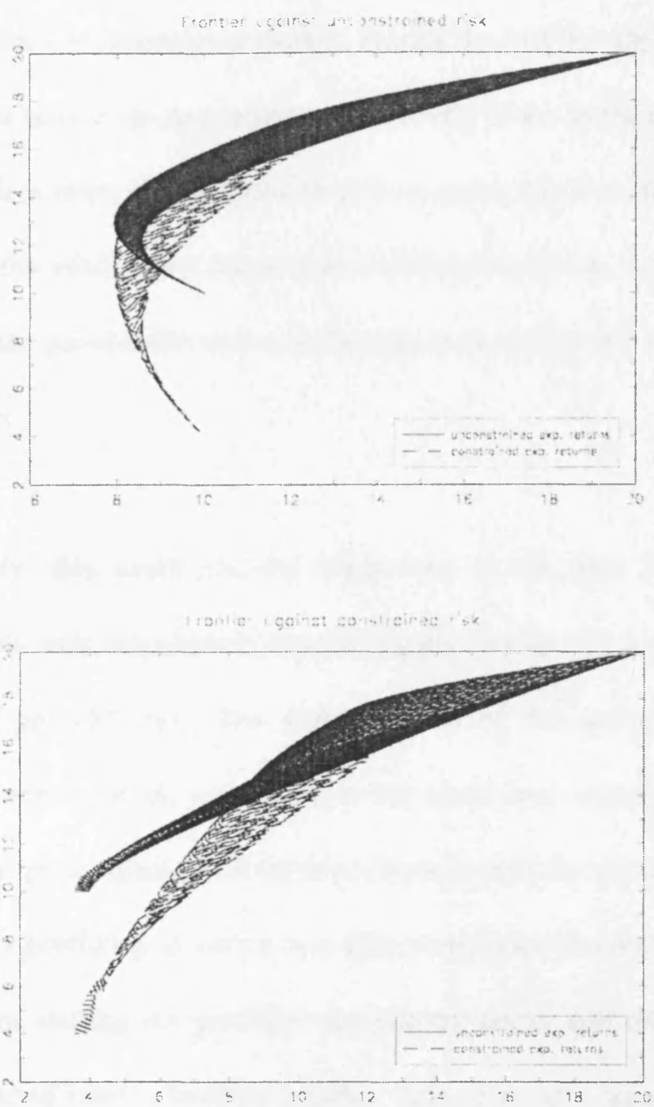
The results show the intercepts α being close to zero and 'insignificant' (that is, with relatively large standard errors). This is generally in line with the *non-zero beta CAPM* theory and confirms the decision to omit the low-variation riskless asset from the model. It also indicates that the securities were not, on average, systematically underpriced or overpriced. The two-limit Tobit estimates of the β's are visibly higher than the corresponding ordinary least squares (*OLS*) estimates. This is not surprising as, due to the nature of censoring, the unconditional (unscaled) *OLS* estimates or regression parameters are normally below of these of the Tobit, since the latter estimates have their probability mass censored. It is interesting to note that the estimates of relative risk are consistently, and markedly, higher for the corrected rather than for the uncorrected returns.

For a better interpretation of this result it is convenient to recall the concept of the efficiency frontier (see e.g. Cuthbertson 1996). The efficiency frontier shows the relationship between expected returns and risk (measured by the standard deviation of the portfolio) for all possible market portfolios for which the risk for a given level of the return is minimised.

In the case of market constraints it is possible that part of the money cannot be spent on constrained assets and is effectively withdrawn. Since the efficient market portfolio includes returns from all other (unconstrained) assets in optimal possible proportions, the efficiency frontier is altered in such a way that lower expected returns correspond to a given level of risk. This is shown by a simple **numerical experiment**, where the efficiency frontier is computed for a portfolio of three exemplary types of assets[9] (see figure 6.8). The proportion of each type of assets in a portfolio changes by 0.01 in [0,1] interval, resulting in over 5,000 portfolios. Two types of experiments have been conducted. In the first, no constraints are imposed on the market. In the second experiment I have assumed there are market constraints imposed on one of the assets in such a way that the proportion of wealth kept in it cannot exceed 0.4. Proportions of the other assets remain unconstrained. Figure 6.8 confirms that, for the constrained efficient portfolios, expected returns are lower for a given risk.

[9] The program of computation of the efficiency frontiers is attached.

Figure 6.8 Efficiency frontiers



It appears that the result of a lower risk associated with the uncorrected returns can have at least two, mutually consistent, explanations. As the definition of *beta* implies, ($\beta_i = \sigma_{iM} / \sigma_M^2$), it is equal to the ratio of the covariance of the i-th asset examined with the market portfolio to the variance of this portfolio. For the restricted market, the covariance is likely to be relatively high, due to the possible quantity spillover effect from the restricted to unrestricted markets (see Charemza, Shields and

Zalewska-Mitura 1997). It is worth noting that the market price of risk, often denoted as the *lambda* coefficient (see Cuthbertson 1996, pp. 38-41), may still be lower for the restricted than for the unrestricted market, despite the fact that the *beta* coefficients are generally higher since expected returns are normally lower in the case of the restricted market. Since it is often hypothesised that in emerging markets investor's behaviour is determined by the relative risk rather than market price of risk, the interpretation given above places into question the rationale for regulated trading and suggests the abolition of price barriers.

Secondly, this result can be interpreted in the light of the micro-market structure models with exogenously random supply (see Brown and Jennings 1989 and O'Hara 1995, pp. 157-160). The disappearance of the quantity restriction signal increases the variance of the return but, at the same time, increases the informational efficiency of the price signal. With no restrictions in portfolio allocation, the traders are able to diversify portfolios in such a way that would minimise the correlation between the asset returns shifting the portfolio opportunity set so that risks corresponding to particular expected returns become smaller. This again acts towards the decrease of correlation of assets returns with portfolio returns, resulting in smaller *beta* values.

Generally, stocks on the Warsaw market can be classified as medium to high risk. For those stocks where the β's are greater than one (indicating a risky asset), namely for *Efekt, Exbud, Irena, Kable, Krosno, Mostostal Warszawa, Sokolow, Swarzedz, Tonsil, Uniwersal, Vistula, Wielkopolski Bank Kredytowy, Wolczanka*, the β's for the corrected returns are much smaller, in the ranges for the low-risk assets.

This seems to confirm the conclusion that much of the risk comes to the market through the existence of the price regulation. Smaller *beta* values for the two-limit Tobit and maximum likelihood results are obtained for *WIG* than for *WIG21*. Generally the *WIG21* portfolio is generally more risky than the *WIG* portfolio.
Similar results have been obtained based on maximum likelihood method and OLS method for corrected returns.

It is important to note that by estimating *beta* coefficients only a one-pass test of allocative market efficiency has been completed. The second pass would be to estimate the security market line by regressing the expected returns on estimated β's which also tests the Sharpe-Lintner CAPM model (which will be presented in the next chapter).

Generally, it does not seem that the regulation of the Warsaw Stock Exchange through the imposition of price limits is effective. It is expensive, increases market inefficiency and, as the history of the Warsaw Stock Exchange reveals, does not shelter the market from the boom-bust events. At the same time, it does not seem to reduce relative risk in allocative portfolios. Portfolios would be more efficient if the correlation between them was allowed to decrease, requiring the abolishment of price barriers.

Tables 6.4-6.5 BANK ROZWOJU EKSPORTU

Table 6.4 Estimation based on *WIG21*

Method	constant (stand. error)	β param. (stand. error)	Characteristics	
Two-limit Tobit and uncorrected returns	0.0013 (0.0009)	0.8156 (0.0527)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	5.4717 0.0279 2.04 1.8170 -10.9860
Max. likelihood and corrected returns	0.0014 (0.0009)	0.7828 (0.0475)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	5.9459 0.0278 2.03 1.8651 -10.9980
OLS and uncorrected returns	0.0014 (0.0009)	0.7840 (0.0397)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	NA 0.0279 2.03 1.8727 -10.9980
OLS and corrected returns	0.0014 (0.0009)	0.7828 (0.0396)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	NA 0.0278 2.03 1.8651 -10.9980

Table 6.5 Estimation based on *WIG*

Method	constant (stand. error)	β param. (stand. error)	Characteristics	
Two-limit Tobit and uncorrected returns	0.0011 (0.0009)	0.5598 (0.0511)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	5.3469 0.0297 2.17 4.4589 -11.3980
Max. likelihood and corrected returns	0.0012 (0.0009)	0.5331 (0.0468)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	5.8172 0.0297 2.16 4.3352 -11.370
OLS and uncorrected returns	0.0012 (0.0009)	0.5340 (0.0359)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	NA 0.0297 2.16 4.3445 -11.369
OLS and corrected returns	0.0012 (0.0009)	0.5331 (0.0359)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	NA 0.0297 2.16 4.3352 -11.370

Tables 6.6-6.7 EFEKT

Table 6.6 Estimation based on *WIG21*

Method	constant (stand. error)	β param. (stand. error)	Characteristics	
Two-limit Tobit and uncorrected returns	-0.0005 (301.3452)	1.2137 (278.2832)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	4.7366 0.0335 2.02 1.2870 -10.6630
Max. likelihood and corrected returns	0.0003 (0.0010)	1.0085 (0.04579)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	5.6621 0.0320 2.01 1.2551 -10.4180
OLS and uncorrected returns	0.0002 (0.0010)	1.0582 (0.0472)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	NA 0.0333 1.99 1.3145 -10.6260
OLS and corrected returns	0.0003 (0.0010)	1.0085 (0.0454)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	NA 0.0320 2.01 1.2551 -10.4180

Table 6.7 Estimation based on *WIG*

Method	constant (stand. error)	β param. (stand. error)	Characteristics	
Two-limit Tobit and uncorrected returns	-0.0001 (0.0015)	0.6085 (0.0719)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	4.4737 0.0383 2.10 2.3135 -10.6900
Max. likelihood and corrected returns	0.0003 (0.0012)	0.5056 (0.0589)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	5.3847 0.0369 2.07 1.9219 -10.3630
OLS and uncorrected returns	0.0001 (0.0012)	0.5414 (0.0462)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	NA 0.0383 2.06 1.9524 -10.5540
OLS and corrected returns	0.0003 (0.0012)	0.5056 (0.0445)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	NA 0.0369 2.07 1.9219 -10.3630

Tables 6.8-6.9 ELEKTRIM

Table 6.8 Estimation based on *WIG21*

Method	constant (stand. error)	β param. (stand. error)	Characteristics	
Two-limit Tobit and uncorrected returns	0.0003 (0.0009)	0.9485 (0.0498)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	5.4201 0.0279 2.23 2.1159 -9.2870
Max. likelihood and corrected returns	0.0005 (0.0009)	0.8912 (0.0418)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	5.9554 0.0277 2.21 2.0107 -9.21108
OLS and uncorrected returns	0.0004 (0.0009)	0.8999 (0.0386)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	NA 0.0279 2.22 2.0287 -9.2108
OLS and corrected returns	0.0005 (0.0009)	0.8912 (0.0384)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	NA 0.0277 2.21 2.0107 -9.21108

Table 6.9 Estimation based on *WIG*

Method	constant (stand. error)	β param. (stand. error)	Characteristics	
Two-limit Tobit and uncorrected returns	0.0002 (0.0010)	0.6352 (0.0612)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	5.2211 0.0309 2.34 4.2161 -9.0893
Max. likelihood and corrected returns	0.0004 (0.0010)	0.5957 (0.0543)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	5.7559 0.0306 2.31 3.8051 -9.0391
OLS and uncorrected returns	0.0004 (0.0010)	0.6012 (0.0372)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	NA 0.0308 2.32 3.8558 -9.0206
OLS and corrected returns	0.0004 (0.0010)	0.5957 (0.0370)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	NA 0.0306 2.31 3.8051 -9.0391

Tables 6.10-6.11 EXBUD

Table 6.10 Estimation based on *WIG21*

Method	constant (stand. error)	β param. (stand. error)	Characteristics	
Two-limit Tobit and uncorrected returns	0.0003 (0.0011)	1.0736 (0.0029)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	4.8464 0.0332 2.13 1.8464 -10.2390
Max. likelihood and corrected returns	0.0007 (0.0010)	0.9517 (0.0547)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	5.6492 0.0323 2.11 1.6240 -10.0310
OLS and uncorrected returns	0.0006 (0.0010)	0.9891 (0.0470)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	NA 0.0332 2.11 1.7690 -10.1970
OLS and corrected returns	0.0007 (0.0010)	0.9517 (0.0457)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	NA 0.0323 2.11 1.6240 -10.0310

Table 6.11 Estimation based on *WIG*

Method	constant (stand. error)	β param. (stand. error)	Characteristics	
Two-limit Tobit and uncorrected returns	0.0004 (0.0011)	0.6202 (0.0673)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	4.6351 0.0371 2.15 3.0828 -10.1730
Max. likelihood and corrected returns	0.0006 (0.0011)	0.5248 (0.0532)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	5.4257 0.0361 2.12 2.5397 -9.9324
OLS and uncorrected returns	0.0006 (0.0012)	0.5561 (0.0448)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	NA 0.0371 2.12 2.7036 -10.0780
OLS and corrected returns	0.0006 (0.0011)	0.5248 (0.0436)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	NA 0.0361 2.12 2.5397 -9.9324

Tables 6.12-6.13 IRENA

Table 6.12 Estimation based on *WIG21*

Method	constant (stand. error)	β param. (stand. error)	Characteristics	
Two-limit Tobit and uncorrected returns	-0.0006 (0.0009)	1.1159 (0.0646)	Log-likelihood	5.1402
			Resid. stand. dev.	0.0294
			DW	2.13
			Autocorr. F(12)	1.4452
			ADF (12)	-9.2695
Max. likelihood and corrected returns	-0.0004 (0.0009)	0.9955 (0.0456)	Log-likelihood	5.8708
			Resid. stand. dev.	0.0288
			DW	2.10
			Autocorr. F(12)	1.2283
			ADF (12)	-9.1731
OLS and uncorrected returns	-0.0004 (0.0009)	1.0310 (0.0414)	Log-likelihood	NA
			Resid. stand. dev.	0.0294
			DW	2.10
			Autocorr. F(12)	1.3371
			ADF (12)	-9.3047
OLS and corrected returns	-0.0004 (0.0009)	0.9955 (0.0406)	Log-likelihood	NA
			Resid. stand. dev.	0.0288
			DW	2.10
			Autocorr. F(12)	1.2283
			ADF (12)	-9.1731

Table 6.13 Estimation based on *WIG*

Method	constant (stand. error)	β param. (stand. error)	Characteristics	
Two-limit Tobit and uncorrected returns	-0.0006 (0.0011)	0.6551 (0.0697)	Log-likelihood	4.8593
			Resid. stand. dev.	0.0340
			DW	2.15
			Autocorr. F(12)	1.1608
			ADF (12)	-9.2962
Max. likelihood and corrected returns	-0.0005 (0.0010)	0.5820 (0.0594)	Log-likelihood	5.5934
			Resid. stand. dev.	0.0332
			DW	2.12
			Autocorr. F(12)	0.8647
			ADF (12)	-9.1864
OLS and uncorrected returns	-0.0005 (0.0011)	0.6047 (0.0410)	Log-likelihood	NA
			Resid. stand. dev.	0.0339
			DW	2.12
			Autocorr. F(12)	0.9354
			ADF (12)	-9.3124
OLS and corrected returns	-0.0005 (0.0010)	0.5820 (0.0401)	Log-likelihood	NA
			Resid. stand. dev.	0.0332
			DW	2.12
			Autocorr. F(12)	0.8647
			ADF (12)	-9.1864

Tables 6.14-6.15 KABLE

Table 6.14 Estimation based on *WIG21*

Method	constant (stand. error)	β param. (stand. error)	Characteristics	
Two-limit Tobit and uncorrected returns	-0.0002 (0.0010)	1.2617 (0.0681)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	4.9333 0.0312 1.96 2.6364 -9.3882
Max. likelihood and corrected returns	0.0002 (0.0009)	1.0907 (0.0521)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	5.7834 0.0301 1.95 2.5117 -9.1997
OLS and uncorrected returns	0.0001 (0.0010)	1.1435 (0.0443)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	NA 0.0311 1.94 2.5681 -9.4226
OLS and corrected returns	0.0002 (0.0009)	1.0907 (0.0429)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	NA 0.0301 1.95 2.5117 -9.1997

Table 6.15 Estimation based on *WIG*

Method	constant (stand. error)	β param. (stand. error)	Characteristics	
Two-limit Tobit and uncorrected returns	-0.0003 (0.0012)	0.7131 (0.0710)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	4.6169 0.0366 2.13 2.7034 -9.3252
Max. likelihood and corrected returns	0.0002 (0.0011)	0.6052 (0.0549)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	5.4697 0.0353 2.09 2.2902 -9.1320
OLS and uncorrected returns	0.0000 (0.0011)	0.6374 (0.0442)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	NA 0.0366 2.09 2.3243 -9.3204
OLS and corrected returns	0.0002 (0.0011)	0.6052 (0.0427)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	NA 0.0353 2.09 2.2902 -9.1320

Tables 6.16-6.17 KROSNO

Table 6.16 Estimation based on *WIG21*

Method	constant (stand. error)	β param. (stand. error)	Characteristics	
Two-limit Tobit and uncorrected returns	0.0006 (0.0011)	1.2011 (0.0591)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	4.8245 0.0327 2.00 0.7931 -9.5919
Max. likelihood and corrected returns	0.0009 (0.0010)	1.0255 (0.0441)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	5.7067 0.0313 2.01 0.6910 -9.4313
OLS and uncorrected returns	0.0007 (0.0010)	1.0924 (0.0468)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	NA 0.0326 1.99 0.6100 -9.5259
OLS and corrected returns	0.0009 (0.0010)	1.0255 (0.0450)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	NA 0.0313 2.01 0.6910 -9.4313

Table 6.17 Estimation based on *WIG*

Method	constant (stand. error)	β param. (stand. error)	Characteristics	
Two-limit Tobit and uncorrected returns	0.0005 (0.0012)	0.6971 (0.0669)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	4.5720 0.0373 2.18 1.8447 -9.3771
Max. likelihood and corrected returns	0.0008 (0.0010)	0.5594 (0.0512)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	5.4446 0.0358 2.14 1.4049 -9.2179
OLS and uncorrected returns	0.0006 (0.0012)	0.6124 (0.0450)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	NA 0.0372 2.14 1.3866 -9.3042
OLS and corrected returns	0.0008 (0.0011)	0.5594 (0.0432)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	NA 0.0358 2.14 1.4049 -9.2179

Tables 6.18-6.19 MOSTOSTAL EXPORT

Table 6.18 Estimation based on *WIG21*

Method	constant (stand. error)	β param. (stand. error)	Characteristics	
Two-limit Tobit and uncorrected returns	0.0007 (0.0010)	0.8328 (0.0524)	Log-likelihood	5.2715
			Resid. stand. dev.	0.0300
			DW	2.22
			Autocorr. F(12)	2.2731
			ADF (12)	-9.6443
Max. likelihood and corrected returns	0.0007 (0.0009)	0.7784 (0.0451)	Log-likelihood	5.8133
			Resid. stand. dev.	0.0297
			DW	2.28
			Autocorr. F(12)	2.172224
			ADF (12)	
OLS and uncorrected returns	0.0007 (0.0009)	0.7889 (0.0413)	Log-likelihood	NA
			Resid. stand. dev.	0.0299
			DW	2.27
			Autocorr. F(12)	2.1680
			ADF (12)	-9.6417
OLS and corrected returns	0.0007 (0.0009)	0.7784 (0.0410)	Log-likelihood	NA
			Resid. stand. dev.	0.0297
			DW	2.28
			Autocorr. F(12)	2.1722
			ADF (12)	-9.6524

Table 6.19 Estimation based on *WIG*

Method	constant (stand. error)	β param. (stand. error)	Characteristics	
Two-limit Tobit and uncorrected returns	0.0006 (0.0010)	0.5888 (0.0543)	Log-likelihood	5.1499
			Resid. stand. dev.	0.0319
			DW	2.31
			Autocorr. F(12)	3.4634
			ADF (12)	-10.0440
Max. likelihood and corrected returns	0.0006 (0.0010)	0.5394 (0.0468)	Log-likelihood	5.6878
			Resid. stand. dev.	0.0317
			DW	2.30
			Autocorr. F(12)	3.1489
			ADF (12)	-10.0080
OLS and uncorrected returns	0.0006 (0.0010)	0.5482 (0.0386)	Log-likelihood	NA
			Resid. stand. dev.	0.0319
			DW	2.30
			Autocorr. F(12)	3.1728
			ADF (12)	-9.9959
OLS and corrected returns	0.0006 (0.0010)	0.5394 (0.0383)	Log-likelihood	NA
			Resid. stand. dev.	0.0317
			DW	2.30
			Autocorr. F(12)	3.1489
			ADF (12)	-10.0080

Tables 6.20-6.21 MOSTOSTAL WARSZAWA

Table 6.20 Estimation based on *WIG21*

Method	constant (stand. error)	β param. (stand. error)	Characteristics	
Two-limit Tobit and uncorrected returns	0.0006 (0.0010)	1.0821 (0.0599)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	5.0393 0.0312 2.13 1.0362 -9.6318
Max. likelihood and corrected returns	0.0008 (0.0010)	0.9884 (0.0491)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	5.7272 0.0308 2.12 0.9034 -9.5479
OLS and uncorrected returns	0.0008 (0.0010)	1.0046 (0.0434)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	NA 0.0311 2.11 0.9351 -9.5953
OLS and corrected returns	0.0008 (0.0010)	0.9884 (0.0429)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	NA 0.0308 2.12 0.9034 -9.5479

Table 6.21 Estimation based on *WIG*

Method	constant (stand. error)	β param. (stand. error)	Characteristics	
Two-limit Tobit and uncorrected returns	0.0006 (0.0012)	0.6107 (0.0717)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	4.7711 0.0359 2.22 2.0745 -9.5656
Max. likelihood and corrected returns	0.0007 (0.0011)	0.5487 (0.0612)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	5.4548 0.0355 2.20 1.7430 -9.4825
OLS and uncorrected returns	0.0007 (0.0011)	0.5603 (0.0435)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	NA 0.0359 2.20 1.7657 -9.5074
OLS and corrected returns	0.0007 (0.0011)	0.5487 (0.0430)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	NA 0.0355 2.20 1.7430 -9.4825

Tables 6.22-6.23 OKOCIM

Table 6.22 Estimation based on *WIG21*

Method	constant (stand. error)	β param. (stand. error)	Characteristics	
Two-limit Tobit and uncorrected returns	-0.0003 (0.0012)	0.9554 (0.0626)	Log-likelihood	4.5493
			Resid. stand. dev.	0.0367
			DW	2.11
			Autocorr. F(12)	1.5167
			ADF (12)	-9.9838
Max. likelihood and corrected returns	-0.0001 (0.0011)	0.8573 (0.0508)	Log-likelihood	5.4246
			Resid. stand. dev.	0.0361
			DW	2.10
			Autocorr. F(12)	1.3449
			ADF (12)	-9.8878
OLS and uncorrected returns	-0.0001 (0.0011)	0.8756 (0.0510)	Log-likelihood	NA
			Resid. stand. dev.	0.0367
			DW	2.09
			Autocorr. F(12)	1.3482
			ADF (12)	-9.8838
OLS and corrected returns	-0.0001 (0.0011)	0.8573 (0.0502)	Log-likelihood	NA
			Resid. stand. dev.	0.0361
			DW	2.10
			Autocorr. F(12)	1.3449
			ADF (12)	-9.8878

Table 6.23 Estimation based on *WIG*

Method	constant (stand. error)	β param. (stand. error)	Characteristics	
Two-limit Tobit and uncorrected returns	-0.0004 (0.0013)	0.6794 (0.0635)	Log-likelihood	4.4548
			Resid. stand. dev.	0.0387
			DW	2.16
			Autocorr. F(12)	1.4977
			ADF (12)	-9.6937
Max. likelihood and corrected returns	-0.0001 (0.0012)	0.5881 (0.0508)	Log-likelihood	5.3212
			Resid. stand. dev.	0.0381
			DW	2.14
			Autocorr. F(12)	1.2627
			ADF (12)	-9.5975
OLS and uncorrected returns	-0.0002 (0.0012)	0.6041 (0.0466)	Log-likelihood	NA
			Resid. stand. dev.	0.0386
			DW	2.14
			Autocorr. F(12)	1.2420
			ADF (12)	-9.5912
OLS and corrected returns	-0.0001 (0.0012)	0.5881 (0.0406)	Log-likelihood	NA
			Resid. stand. dev.	0.0381
			DW	2.14
			Autocorr. F(12)	1.2627
			ADF (12)	-9.5975

Tables 6.24-6.25 POLIFARB CIESZYN

Table 6.24 Estimation based on *WIG21*

Method	constant (stand. error)	β param. (stand. error)	Characteristics	
Two-limit Tobit and uncorrected returns	0.0003 (0.0010)	0.9290 (0.0616)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	5.1573 0.0312 2.16 2.6512 -10.2970
Max. likelihood and corrected returns	0.0004 (0.0010)	0.8572 (0.0525)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	5.7328 0.0310 2.16 2.4473 -10.1470
OLS and uncorrected returns	0.0004 (0.0010)	0.8676 (0.0428)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	NA 0.0311 2.15 2.4567 -10.168
OLS and corrected returns	0.0004 (0.0010)	0.8572 (0.0425)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	NA 0.0310 2.16 2.4473 -10.1470

Table 6.25 Estimation based on *WIG*

Method	constant (stand. error)	β param. (stand. error)	Characteristics	
Two-limit Tobit and uncorrected returns	0.0002 (0.0011)	0.6460 (0.0574)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	5.0136 0.0335 2.19 4.9011 -10.0400
Max. likelihood and corrected returns	0.0004 (0.0010)	0.5936 (0.0490)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	5.5907 0.0333 2.17 4.4510 -9.9035
OLS and uncorrected returns	0.0003 (0.0010)	0.6012 (0.0404)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	NA 0.0335 2.17 4.4945 -9.9299
OLS and corrected returns	0.0004 (0.0010)	0.5936 (0.0402)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	NA 0.0333 2.17 4.4510 -9.9035

Tables 6.26-6.27 PROCHNIK

Table 6.26 Estimation based on *WIG21*

Method	constant (stand. error)	β param. (stand. error)	Characteristics	
Two-limit Tobit and uncorrected returns	-0.0022 (0.0011)	1.0091 (0.0595)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	4.9250 0.0333 2.39 4.4209 -10.2110
Max. likelihood and corrected returns	-0.0018 (0.0010)	0.9183 (0.0487)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	5.6083 0.0329 2.38 4.1740 -10.2100
OLS and uncorrected returns	-0.0019 (0.0010)	0.9358 (0.0465)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	NA 0.0333 2.38 4.1958 -10.2250
OLS and corrected returns	-0.0018 (0.0010)	0.9183 (0.0460)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	NA 0.0329 2.38 4.1740 -10.2100

Table 6.27 Estimation based on *WIG*

Method	constant (stand. error)	β param. (stand. error)	Characteristics	
Two-limit Tobit and uncorrected returns	-0.0022 (0.0012)	0.6415 (0.0605)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	4.7531 0.0364 2.32 3.9244 -10.4690
Max. likelihood and corrected returns	-0.0018 (0.0011)	0.5704 (0.0500)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	5.4331 0.0360 2.31 3.5458 -10.3990
OLS and uncorrected returns	-0.0019 (0.0011)	0.5831 (0.0440)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	NA 0.0364 2.30 3.5742 -10.4150
OLS and corrected returns	-0.0018 (0.0011)	0.5704 (0.0435)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	NA 0.0360 2.31 3.5458 -10.3990

Tables 6.28-6.29 SOKOLOW

Table 6.28 Estimation based on *WIG21*

Method	constant (stand. error)	β param. (stand. error)	Characteristics	
Two-limit Tobit and uncorrected returns	-0.0001 (0.0007)	1.2389 (0.0638)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	4.9962 0.0311 2.10 2.3029 -9.6145
Max. likelihood and corrected returns	0.0002 (0.0010)	1.1143 (0.0511)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	5.7383 0.0307 2.06 1.9976 -9.6469
OLS and uncorrected returns	0.0001 (0.0010)	1.1266 (0.0428)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	NA 0.0310 2.06 2.0412 -9.6353
OLS and corrected returns	0.0002 (0.0010)	1.1143 (0.0425)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	NA 0.0307 2.06 1.9976 -9.6469

Table 6.29 Estimation based on *WIG*

Method	constant (stand. error)	β param. (stand. error)	Characteristics	
Two-limit Tobit and uncorrected returns	-0.0000 (0.0012)	0.6813 (0.0728)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	4.6446 0.0372 2.12 1.7757 -9.7156
Max. likelihood and corrected returns	0.0001 (0.0012)	0.5953 (0.0601)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	5.3805 0.0368 2.07 1.4455 -9.6809
OLS and uncorrected returns	0.0001 (0.0012)	0.6045 (0.0450)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	NA 0.0371 2.07 1.4583 -9.6780
OLS and corrected returns	0.0001 (0.0012)	0.5953 (0.0446)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	NA 0.0368 2.07 1.4455 -9.6809

Tables 6.30-6.31 SWARZEDZ

Table 6.30 Estimation based on *WIG21*

Method	constant (stand. error)	β param. (stand. error)	Characteristics	
Two-limit Tobit and uncorrected returns	-0.0018 (0.0012)	1.1691 (0.0593)	Log-likelihood	4.8625
			Resid. stand. dev.	0.0326
			DW	2.23
			Autocorr. F(12)	3.2629
			ADF (12)	-10.6940
Max. likelihood and corrected returns	-0.0015 (0.0010)	1.0509 (0.0477)	Log-likelihood	5.6236
			Resid. stand. dev.	0.0322
			DW	2.22
			Autocorr. F(12)	3.0609
			ADF (12)	-10.6920
OLS and uncorrected returns	-0.0016 (0.0010)	1.0664 (0.0459)	Log-likelihood	NA
			Resid. stand. dev.	0.0325
			DW	2.21
			Autocorr. F(12)	3.0892
			ADF (12)	-10.7180
OLS and corrected returns	-0.0015 (0.0010)	1.0509 (0.0455)	Log-likelihood	NA
			Resid. stand. dev.	0.0322
			DW	2.22
			Autocorr. F(12)	3.0609
			ADF (12)	-10.6920

Table 6.31 Estimation based on *WIG*

Method	constant (stand. error)	β param. (stand. error)	Characteristics	
Two-limit Tobit and uncorrected returns	-0.0019 (0.0012)	0.7558 (0.0638)	Log-likelihood	4.6431
			Resid. stand. dev.	0.0367
			DW	2.22
			Autocorr. F(12)	4.7006
			ADF (12)	-10.322
Max. likelihood and corrected returns	-0.0016 (0.0011)	0.6567 (0.0508)	Log-likelihood	5.3959
			Resid. stand. dev.	0.0363
			DW	2.19
			Autocorr. F(12)	4.0556
			ADF (12)	-10.304
OLS and uncorrected returns	-0.0016 (0.0012)	0.6687 (0.0447)	Log-likelihood	NA
			Resid. stand. dev.	0.0367
			DW	2.19
			Autocorr. F(12)	4.1107
			ADF (12)	-10.3300
OLS and corrected returns	-0.0016 (0.0011)	0.6567 (0.0443)	Log-likelihood	NA
			Resid. stand. dev.	0.0363
			DW	2.19
			Autocorr. F(12)	4.0556
			ADF (12)	-10.304

Tables 6.32-6.33 TONSIL

Table 6.32 Estimation based on *WIG21*

Method	constant (stand. error)	β param. (stand. error)	Characteristics	
Two-limit Tobit and uncorrected returns	-0.0001 (1449.4773)	1.1883 (1.7678)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	4.7037 0.0329 2.06 1.8093 -10.8830
Max. likelihood and corrected returns	0.0001 (0.0010)	1.0452 (0.0591)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	5.6662 0.0320 2.04 1.5350 -10.6710
OLS and uncorrected returns	-0.0000 (0.0010)	1.0807 (0.0459)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	NA 0.0329 2.04 1.6206 -10.7780
OLS and corrected returns	0.0001 (0.0010)	1.0452 (0.0447)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	NA 0.0320 2.04 1.5350 -10.6710

Table 6.33 Estimation based on *WIG*

Method	constant (stand. error)	β param. (stand. error)	Characteristics	
Two-limit Tobit and uncorrected returns	-0.0003 (0.0013)	0.5681 (0.0800)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	4.3865 0.0387 2.12 1.8651 -10.2800
Max. likelihood and corrected returns	0.0001 (0.0012)	0.4941 (0.0670)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	5.3443 0.0376 2.09 1.5619 -10.1230
OLS and uncorrected returns	-0.0001 (0.0012)	0.5134 (0.0467)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	NA 0.0387 2.09 1.6394 -10.1930
OLS and corrected returns	0.0001 (0.0012)	0.4941 (0.0454)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	NA 0.0376 2.09 1.5619 -10.1230

Tables 6.34-6.35 UNIVERSAL

Table 6.34 Estimation based on *WIG21*

Method	constant (stand. error)	β param. (stand. error)	Characteristics	
Two-limit Tobit and uncorrected returns	-0.0002 (0.0000)	1.2959 (0.0583)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	4.7621 0.0326 1.96 1.1999 -8.6659
Max. likelihood and corrected returns	0.0003 (0.0010)	1.1195 (0.0498)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	5.6802 0.0317 1.90 1.3311 -8.6483
OLS and uncorrected returns	0.0002 (0.0010)	1.1580 (0.0453)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	NA 0.0324 1.89 1.4275 -8.7205
OLS and corrected returns	0.0003 (0.0010)	1.1195 (0.0443)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	NA 0.0317 1.90 1.3311 -8.6483

Table 6.35 Estimation based on *WIG*

Method	constant (stand. error)	β param. (stand. error)	Characteristics	
Two-limit Tobit and uncorrected returns	-0.0001 (0.0014)	0.6927 (0.0723)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	4.4246 0.0386 2.09 1.2188 -8.8800
Max. likelihood and corrected returns	0.0002 (0.0012)	0.5846 (0.0569)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	5.3414 0.0376 2.04 1.0377 -8.8504
OLS and uncorrected returns	0.0001 (0.0012)	0.6086 (0.0466)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	NA 0.0385 2.03 1.0977 -8.8833
OLS and corrected returns	0.0002 (0.0012)	0.5846 (0.0455)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	NA 0.0376 2.04 1.0377 -8.8504

Tables 6.36-6.37 VISTULA

Table 6.36 Estimation based on *WIG21*

Method	constant (stand. error)	β param. (stand. error)	Characteristics	
Two-limit Tobit and uncorrected returns	-0.0009 (0.0011)	1.0383 (0.0591)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	5.0198 0.0303 2.30 2.9004 -10.0930
Max. likelihood and corrected returns	-0.0003 (0.0009)	0.8868 (0.0451)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	5.8459 0.0286 2.29 2.6853 -10.0330
OLS and uncorrected returns	-0.0007 (0.0010)	0.9699 (0.0463)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	NA 0.0303 2.29 2.7281 -9.9626
OLS and corrected returns	-0.0003 (0.0009)	0.8868 (0.0438)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	NA 0.0286 2.29 2.6853 -10.0330

Table 6.37 Estimation based on *WIG*

Method	constant (stand. error)	β param. (stand. error)	Characteristics	
Two-limit Tobit and uncorrected returns	-0.0008 (0.0011)	0.5545 (0.0629)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	4.8038 0.0342 2.26 3.7091 -9.6736
Max. likelihood and corrected returns	-0.0003 (0.0010)	0.4455 (0.0484)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	5.6242 0.0322 2.24 3.1094 -9.5607
OLS and uncorrected returns	-0.0007 (0.0011)	0.5037 (0.0418)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	NA 0.0342 2.24 3.3675 -9.5588
OLS and corrected returns	-0.0003 (0.0010)	0.4455 (0.0395)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	NA 0.0322 2.24 3.1094 -9.5607

Tables 6.38-6.39 WIELKOPOLSKI BANK KREDYTOWY

Table 6.38 Estimation based on *WIG21*

Method	constant (stand. error)	β param. (stand. error)	Characteristics	
Two-limit Tobit and uncorrected returns	0.0003 (0.0009)	1.2072 (0.0518)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	5.3820 0.0274 2.24 3.3785 -9.0395
Max. likelihood and corrected returns	0.0005 (0.0008)	1.1052 (0.0406)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	5.9959 0.0271 2.22 3.1362 -9.0659
OLS and uncorrected returns	0.0005 (0.0009)	1.1209 (0.0379)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	NA 0.0273 2.21 3.1593 -9.1065
OLS and corrected returns	0.0005 (0.0009)	1.1052 (0.0376)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	NA 0.0271 2.22 3.1362 -9.0659

Table 6.39 Estimation based on *WIG*

Method	constant (stand. error)	β param. (stand. error)	Characteristics	
Two-limit Tobit and uncorrected returns	0.0003 (0.0011)	0.7044 (0.0599)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	4.9870 0.0335 2.23 4.8519 -10.072
Max. likelihood and corrected returns	0.0004 (0.0010)	0.6260 (0.0493)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	5.5948 0.0332 2.19 4.0818 -9.9175
OLS and uncorrected returns	0.0004 (0.0010)	0.6380 (0.0404)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	NA 0.0335 2.19 4.1395 -9.9537
OLS and corrected returns	0.0004 (0.0010)	0.6260 (0.0401)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	NA 0.0332 2.19 4.0818 -9.9175

Tables 6.40-6.41 WEDEL

Table 6.40 Estimation based on *WIG21*

Method	constant (stand. error)	β param. (stand. error)	Characteristics	
Two-limit Tobit and uncorrected returns	0.0004 (0.0010)	0.7866 (0.0588)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	5.0141 0.0323 2.44 4.2068 -9.2660
Max. likelihood and corrected returns	0.0010 (0.0009)	0.6320 (0.0592)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	5.8271 0.0293 2.40 3.5780 -9.2377
OLS and uncorrected returns	0.0005 (0.0010)	0.7441 (0.0550)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	NA 0.0323 2.44 4.2402 -9.2741
OLS and corrected returns	0.0010 (0.0009)	0.6320 (0.0499)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	NA 0.0293 2.40 3.5780 -9.2377

Table 6.41 Estimation based on *WIG*

Method	constant (stand. error)	β param. (stand. error)	Characteristics	
Two-limit Tobit and uncorrected returns	0.0003 (0.0011)	0.4007 (0.0618)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	4.9295 0.0338 2.49 5.6884 -9.4510
Max. likelihood and corrected returns	0.0009 (0.0010)	0.3045 (0.0457)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	5.7440 0.0306 2.44 4.7022 -9.34484
OLS and uncorrected returns	0.0004 (0.0011)	0.3744 (0.0411)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	NA 0.0338 2.49 5.4973 -9.4399
OLS and corrected returns	0.0009 (0.0010)	0.3045 (0.0372)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	NA 0.0306 2.44 4.7022 -9.34484

Tables 6.42-6.43 WOLCZANKA

Table 6.42 Estimation based on *WIG21*

Method	constant (stand. error)	β param. (stand. error)	Characteristics	
Two-limit Tobit and uncorrected returns	-0.0009 (0.0009)	1.0182 (0.0580)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	5.1733 0.0300 2.26 2.0108 -9.5192
Max. likelihood and corrected returns	-0.0007 (0.0009)	0.9091 (0.0509)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	5.8219 0.0291 2.22 1.5490 -9.4528
OLS and uncorrected returns	-0.0008 (0.0010)	0.9521 (0.0441)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	NA 0.0300 2.24 1.7453 -9.5746
OLS and corrected returns	-0.0007 (0.0009)	0.9091 (0.0429)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	NA 0.0291 2.22 1.5490 -9.4528

Table 6.43 Estimation based on *WIG*

Method	constant (stand. error)	β param. (stand. error)	Characteristics	
Two-limit Tobit and uncorrected returns	-0.0010 (0.0011)	0.5576 (0.0639)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	4.9390 0.0340 2.26 2.4921 -10.0190
Max. likelihood and corrected returns	-0.0007 (0.0010)	0.4842 (0.0542)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	5.5884 0.0330 2.22 1.8927 -9.8525
OLS and uncorrected returns	-0.0008 (0.0011)	0.5125 (0.0414)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	NA 0.0340 2.23 2.0855 -9.9736
OLS and corrected returns	-0.0007 (0.0010)	0.4842 (0.0402)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	NA 0.0330 2.22 1.8927 -9.8525

Tables 6.44-6.45 ZYWIEC

Table 6.44 Estimation based on *WIG21*

Method	constant (stand. error)	β param. (stand. error)	Characteristics	
Two-limit Tobit and uncorrected returns	-0.0005 (0.0010)	0.6996 (0.0664)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	5.1679 0.0312 2.37 4.8623 -9.4723
Max. likelihood and corrected returns	-0.0003 (0.0009)	0.5605 (0.0503)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	5.8873 0.0285 2.33 4.0196 -9.4288
OLS and uncorrected returns	-0.0005 (0.0010)	0.6659 (0.0488)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	NA 0.0312 2.37 4.7198 -9.4400
OLS and corrected returns	-0.0003 (0.0009)	0.5605 (0.0446)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	NA 0.0285 2.33 4.0196 -9.4288

Table 6.45 Estimation based on *WIG*

Method	constant (stand. error)	β param. (stand. error)	Characteristics	
Two-limit Tobit and uncorrected returns	-0.0005 (0.0010)	0.4607 (0.0583)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	5.1176 0.0321 2.40 5.5937 -9.2496
Max. likelihood and corrected returns	-0.0004 (0.0009)	0.3521 (0.0434)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	5.8349 0.0293 2.34 4.3753 -9.3293
OLS and uncorrected returns	-0.0005 (0.0010)	0.4344 (0.0389)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	NA 0.0321 2.39 5.3561 -9.2311
OLS and corrected returns	-0.0004 (0.0009)	0.3521 (0.0355)	Log-likelihood Resid. stand. dev. DW Autocorr. F(12) ADF (12)	NA 0.0293 2.34 4.3753 -9.3293

CHAPTER 7

SECOND-PASS TESTING AND FURTHER EXTENSIONS

7.1 Introduction

In the previous chapter an empirical analysis of the optimal portfolio allocation model with quantity constraints was presented. The results obtained in the first-pass testing, estimated beta parameters for various methods of estimation are satisfying, therefore they can be used to examine the validity of the portfolio allocation model on the Warsaw Stock Exchange. The testing of the validity of the model is regarded to be the confirmation of the hypothesis of the existence of the optimal portfolio allocation.

There are different methods of providing the validity of the optimal portfolio allocation model known in literature. The most often used method is second-pass testing utilising estimates of the security market line (SML), (see chapter 4 for the theoretical description). According to Tucker, Becker and Isimbabi (1994) the model is positively verified for the existence of the optimal portfolio allocation scheme if:

1. The expected rate of return is positively related to the *beta* for the asset and there is a linear cross-sectional relationship between expected return and *beta*.
2. Parameters *beta* are measures of the risk of the asset.
3. The intercept of the security market line is equal to the risk-free rate.

In this chapter I examine the first of the above conditions assuming that the second and the third conditions hold. In the first part of the chapter I derive the SML based on the *beta* parameters (estimated in the previous chapter) for chosen assets from the WSE. This procedure I call herein a weak second-pass cross-section regression. The derivation of the SML is presented in the first part of this chapter. However, it is possible that the SML is not in line with expected properties. The critique presented by Roll and Ross (1994), (see chapter 4 for details) explains the reasons for such behaviour of the line. Therefore the next section of the chapter includes various stronger tests, the stronger second-pass regression and two extended tests, namely Sharpe ratio analysis and Gibbons' test. The stronger second-pass cross-section regressions, known as the multivariate versions of the SML, relax the tight assumption of linearity of the SML.

7.2 Estimation of the security market line (SML)

Testing the hypothesis of the optimal portfolio allocation model is two-staged. A **first-pass time series regression**, which was introduced in the previous chapter, involves estimating the *beta* parameters of the model (6.1) for each company. Recalling the results and notation from previous chapter the *betas* are estimated from:

$$r_{jt}^{*} = \alpha_j + \beta_j r_t^m + \xi_{jt} \qquad \text{for } j = 1, \dots, n,$$

which is a formula (6.1). Symbol r_t^m denotes the session-to-session returns from the market portfolio in time t, r_{jt}^{*} is the return from an individual security corrected by censored price and ξ_{jt} is the error term. This work analyses twenty-one companies from the Warsaw Stock Exchange, so in this case $n = 21$. The estimates of *beta*, for each security are used in **weak second-pass cross-section regression**. The formula of the security market line, introduced in chapter 4, is:

$$E(r_j) = \gamma_0 + \gamma_1\beta_j + \nu_j$$

where $E(r_j)$ is the expected return of the j'th security, β_j is the level of the risk of j'th security and ν_j the error term. Denoting by $\bar{r}_j$ the sample average rate of return which represents the expected return of the security and by ν_j the error term the second-pass regression is:

$$\bar{r}_j = \gamma_0 + \gamma_1\hat{\beta}_j + \nu_j \qquad \text{for } j = 1,\ldots,n. \qquad (7.1)$$

$\hat{\beta}_j$ denotes estimated by the first-pass time series regressions beta parameter for each company. The security market line can be represented by the plot of the expected return $E(r_j)$ as a function of corresponding *beta* parameter for all analysed assets. In this case the expected return is substituted by the average observed return. Since no riskless asset is assumed, it is expected, if the hypothesis of the optimal portfolio allocation holds, that $\gamma_0 = 0$ and $\gamma_1 = \bar{r}^M$.

Tables 7.1 and 7.2 show the results of estimation of the security market lines based on the *beta* parameters which are evaluated by different methods, namely by Two-limit Tobit model and OLS method for uncorrected returns and by maximum likelihood method for corrected returns and some characteristics of the estimated models, as described in chapter 6. The tables present standard errors of estimated parameters (in parentheses), determinations coefficient (R^2) for each SML, standard error of regression (Se), Durbin-Watson autocorrelation statistic (DW), Fisher-Snedecor statistic of significance (F), Chi-square tests of normality ($\chi^2_{Norm}(2)$) and heteroscedascity of residuals ($\chi^2_{Het}(1)$). All results confirm the insignificance of the γ_0

parameter, which is consistent with the assumption that $\gamma_0 = 0$. The values and signs of estimated γ_1 parameters differs. Hence, the estimates of the γ_1 parameters do not support the existence of the security market line. The values of γ_1, expected according to the evaluated hypothesis of the optimal portfolio allocation, should be equal to the mean market return, which is 0.000277 for *WIG* and -0.000047 for *WIG21*. However, according to Roll and Ross (1994) the cross-sectional OLS relation is highly sensitive to the choice of measure representing the optimal market portfolio. Roll and Ross measure the return of the optimal market portfolio by the stock market return indices. They show that 'any indices can be quite close to each other and to the mean-variance frontier and yet still produce significantly different cross-sectional slopes, positive, negative, or zero'. They explain that the main reason for this is that the market indices are not the appropriate approximations of the true market portfolio. It seems to be that both market indices *WIG* and *WIG21* used in this analysis are unperfected in terms of portfolio allocation. In all cases γ_1 parameters are insignificant on the 5% significance level. This could be explain in two ways. Firstly, probable it may be that the situation described by Roll and Ross exists here. Both measures, *WIG* and *WIG21* are based on constant weights. The *WIG* index is calculated based on the market capitalisation which changes every three month (see chapter 2 for detailed description of the index), *WIG21* does not include new companies introduced into the market after 4.01.1994. The number of companies in recent years increased rapidly (see chapter 2 for details) and their effect is not reflected in the created market return. Therefore the real optimal market portfolio differs from these represented measures. Secondly, the reason for such results can be that the $\bar{r}_j$ which represents the expected return for the j'th company $E(r_j)$ is not in fact a good approximation of the expected returns. In the case of price constraints imposed on the WSE the expected return is distorted by these price limits. In further analysis it is assumed that the SML exists even if the statistics do not support it.

Other statistics presented in tables 7.1 and 7.2 show properties of the security market lines. Empirically, the results show that the residuals for corrected returns are normally distributed with the constant variance and are not correlated[1]. This finding is in line with the efficient market hypothesis. For uncorrected returns the statistics are also consistent but cannot be interpreted because of the truncated distributions.

Comparing results for both applied market indices it can be seen that the *WIG21* index better represents behaviour of the real market portfolio than *WIG*. This is mainly because estimated values of γ_1 parameters, are relatively closer to their required values $\gamma_1 = \bar{r}^M$ for *WIG21* rather than for *WIG* index. Also results closer to their required values obtained for corrected returns confirm the rationality of introducing the correction factor into the model. This is also confirmed by the estimated values of γ_1 parameters.

[1] The critical values for 19 degrees of freedom at the 5% significance level are: $F(1,19)=4.38$, for Durbin-Watson test 1.221 and 1.420, for normality test 5.99 and heteroscedasity 3.84.

Table 7.1 Results of deriving the security market lines based on *WIG21*

		model and standard errors	R^2	Se of regr.	DW	F-stat.	$\chi^2_{Norm}(2)$	$\chi^2_{Het}(1)$
Two-limit Tobit and uncorrected returns	r_j uncorr.	$\bar{r}_j = \underset{(0.0015477)}{-0.000345} + \underset{(0.0014538)}{0.00008504}\,\hat{\beta}_j + \hat{v}_j$	0.0002	0.0011	2.0471	0.0034	0.33743	0.011667
	r_j correct	$\bar{r}_j = \underset{(0.0010807)}{0.0005765} - \underset{(0.0010151)}{0.0004314}\,\hat{\beta}_j + \hat{v}_j$	0.0094	0.0008	1.9452	0.1806	3.5982	0.01538
Max.likelihood and corrected returns	r_j uncorr.	$\bar{r}_j = \underset{(0.0015433)}{-0.0004693} + \underset{(0.0016355)}{0.0002293}\,\hat{\beta}_j + \hat{v}_j$	0.0010	0.0011	1.8236	0.0197	0.32584	0.058685
	r_j correct	$\bar{r}_j = \underset{(0.0010811)}{0.0004141} - \underset{(0.0011457)}{0.0003125}\,\hat{\beta}_j + \hat{v}_j$	0.0039	0.0008	1.9283	0.0744	3.2740	0.45519
OLS and uncorrected returns	r_j uncorr.	$\bar{r}_j = \underset{(0.0017231)}{-0.0005297} + \underset{(0.0017600)}{0.0002828}\,\hat{\beta}_j + \hat{v}_j$	0.0014	0.0011	2.0546	0.0258	0.33216	0.038394
	r_j correct	$\bar{r}_j = \underset{(0.0012046)}{0.0006004} - \underset{(0.0012303)}{0.0004926}\,\hat{\beta}_j + \hat{v}_j$	0.0084	0.0008	1.9453	0.1603	3.5249	0.007742

Table 7.2 Results of deriving the security market lines based on the original *WIG*

		model and standard errors	R^2	Se of regr.	DW	F-stat.	$\chi^2_{Norm}(2)$	$\chi^2_{Het}(1)$
Two-limit Tobit and uncorrected returns	r_j uncorr.	$\bar{r}_j = \underset{(0.0018092)}{0.0009943} - \underset{(0.0028898)}{0.0020144}\hat{\beta}_j + \hat{v}_j$	0.0249	0.0011	2.0207	0.4859	0.35161	0.000761
	r_j correct	$\bar{r}_j = \underset{(0.0012747)}{0.0008326} - \underset{(0.0020362)}{0.0011437}\hat{\beta}_j + \hat{v}_j$	0.0163	0.0008	1.8913	0.3155	2..6898	0.25345
Max.likelihood and corrected returns	r_j uncorr.	$\bar{r}_j = \underset{(0.0015609)}{0.0005948} - \underset{(0.0028689)}{0.0015819}\hat{\beta}_j + \hat{v}_j$	0.0157	0.0011	2.0434	0.3040	0.3957	0.044519
	r_j correct	$\bar{r}_j = \underset{(0.0011024)}{0.0003542} - \underset{(0.0020262)}{0.0004303}\hat{\beta}_j + \hat{v}_j$	0.0024	0.0008	1.8958	0.0451	2.8693	0.64863
OLS and uncorrected returns	r_j uncorr.	$\bar{r}_j = -\underset{(0.001353)}{0.000299} + \underset{(0.0024088)}{0.00007864}\hat{\beta}_j + \hat{v}_j$	0.0001	0.0011	2.0457	0.0011	0.33227	0.12239
	r_j correct	$\bar{r}_j = \underset{(0.0009492)}{0.00009177} + \underset{(0.0016898)}{0.00005636}\hat{\beta}_j + \hat{v}_j$	0.0001	0.0008	1.9074	0.0011	3.2833	0.53204

7.3 Some stronger tests

Previously the results of testing the weak second-pass cross-section has been shown. Although, some of the results confirm the existence of the security market line and some of them do not allow to identify the slope of the SML, the optimal portfolio allocation model with quantity constraints is generally in line with the efficient market hypothesis. It seems to be rational to apply some stronger tests to put more light on the hypothesis of optimal portfolio allocation in case of constraints and test the linearity of the SML.

A straightforward analysis of the rates of return is based on the capital market line[2] (CML). The classical indicator of efficiency based on the CML is the Sharpe ratio (see Sharpe 1966). The value of the ratio is calculated from the formula:

$$S_{pj} = \frac{\bar{r}_j}{\hat{\sigma}_j} \tag{7.2}$$

where $\hat{\sigma}_j$ is the standard deviation of the returns of the security j and $\bar{r}_j$ is the expected rate of return of the security approximated by the mean observed return. The value of S_{pj} is interpreted as the risk premium for a unit of risk.

It is worth noticing that for each portfolio on the CML the values of the S_{pj} are identical and equal to the value of the Sharpe ratio computed for the entire market portfolio. So the value of S_{pj}, for which the relative risk is equal to that of the market

[2] According to the Dictionary of Finance and Banking (1997), the capital market line is the graph showing 'the combinations of risk and return, resulting from investing fixed sums in the market portfolio'.

portfolio, is equal to the value of the Sharpe ratio calculated for the market portfolio. If the Sharpe ratio for a company is greater (or lower) than that for the market portfolio it indicates that the risk premium for the unit of risk is greater (or lower) than for the market portfolio.

Table 7.3 Sharpe ratio for companies

	Sharpe ratio		Proportion of the Sharp ratios
	uncorrected ret.	corrected ret.	uncorr/corr
BRE	-0.0006	0.0267	-0.0225
EFE	-0.0080	0.0056	-1.4286
ELE	-0.0098	0.0122	-0.8033
EXB	0.0073	0.0132	0.5530
IRE	0.0138	-0.0060	-2.3000
KAB	-0.0177	0.0021	-8.4286
KRO	-0.0094	0.0098	-0.9592
MSE	0.0318	0.0179	1.7765
MSW	0.0013	0.0188	0.0691
OKO	-0.0167	-0.0007	23.8571
POL	-0.0200	0.0091	-2.1978
PRO	-0.0390	-0.0280	1.3929
SOK	0.0267	0.0045	5.9333
SWA	-0.0305	-0.0258	1.1822
TON	-0.0265	0.0017	-15.5882
UNI	0.0032	0.0051	0.6275
VIS	0.0170	-0.0058	-2.9310
WBK	0.0116	0.0150	0.7733
WED	0.0049	0.0143	0.3427
WOL	-0.0025	-0.0102	0.2451
ZYW	-0.0143	-0.0038	3.7632

Tables 7.3 and 7.4 show the values of the Sharpe ratio calculated for each company and its components of the market indices respectively.

Table 7.4 Characteristics of the market indices

	WIG	WIG21
mean	0.000277	-0.000047
stand. dev.	0.025971	0.027140
Sharpe ratio	0.010664	-0.001749

The comparison of results from tables 7.3 and 7.4 shows that the corrected returns have greater Sharpe ratios. Values of the Sharpe ratios obtained for all analysed companies are relatively close to the Sharpe ratios calculated for both market indices. This is in line with the Sharpe idea of the index, so the risk premia for all companies are close to the risk premium of the market portfolio.

As it is presented in formula (7.2) Sharpe ratios are calculated from the expected rate of return of the security and the standard deviation of that security. The variance of the security in case of price constraints is affected by the disturbance of variance of market portfolio (see chapter 4 for theoretical explanation). The variance is distorted by artificial reduction, so the standard deviation is smaller for uncorrected returns. Consequently, Sharpe ratios for uncorrected returns are greater than for corrected returns. It confirms proposal that were it not for quantity constraints, the results would be closer to the ideal values (values for the market indices).

It is interesting to examine if the value of the Sharpe ratio for the company depends on other factors such as time of trade on the WSE market, size of the firm etc. I can test the hypothesis that there is a negative relationship between the Sharpe ratio and number of months of a company trading on the market. The hypothesis is that the risk premium of new companies is higher than that of the long established well-known companies. In order to prove this I attempt to estimate this relationship.

Figure 7.1 Sharpe ratios for uncorrected returns

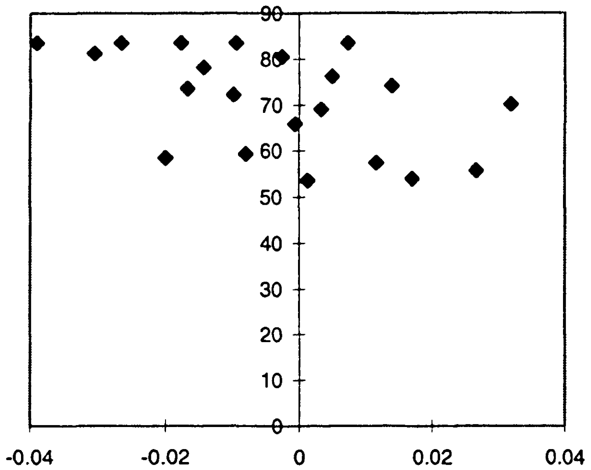
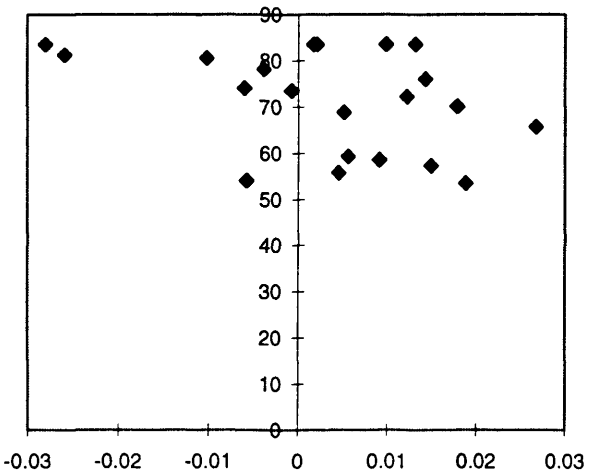


Figure 7.2 Sharpe ratios for corrected returns



Figures 7.1 and 7.2 present the relationship between Sharpe ratio for each analysed company and the number of months of trading on the WSE for uncorrected and corrected returns respectively. The negative correlation between these two values is noticeable, mainly for corrected returns. The correlation represents the estimated model (standard errors in parentheses):

$$S_j = \underset{(0.0189)}{0.0392} - \underset{(0.0003)}{0.0005} m_j + \hat{\varsigma}_j \qquad R^2 = 0.16 \qquad DW = 1.67$$

$$\chi^2_{Norm}(2) = 0.95 \qquad \chi^2_{Het}(1) = 0.67$$

where S_j is the Sharpe ratio for j'th security, m_j is the time (in months) of trading of the j'th security on the WSE and $\hat{\varsigma}_j$ is the residual. The m_j variable is statistically significant on 7.3% of significance level, and the residuals are normally distributed with the constant variance. Therefore, analysis has shown that the longer a company exists on the market, the lower the value of the ratio; so consequently, the longer performance of the company on the market the lower is the risk premium undertaken. It can be concluded that the risk of investing will decrease and converge in time.

Additional analysis shows that there is no relationship between the Sharpe ratio and the percentage of the shares of companies in the whole stock market capitalisation.

Obtained results confirm the inconclusiveness of the estimation of the SML. Roll and Ross (1994) critique allows me to extend an analysis to multivariate SML.

Stronger second-pass cross-section test of the optimal portfolio allocation, proposed by Fama and MacBeth (1974), shows that if *beta* is estimated by an unbiased estimation method then only this parameter should influence r_j. In particular, non-linearity and variance should not affect expected return of j'th asset. In order to prove unbiaseness of the estimates of the first-pass cross-section regression, I examine the relationship:

$$\bar{r}_j = \gamma_0 + \gamma_1 \hat{\beta}_j + \gamma_2 \hat{\beta}_j^2 + \gamma_3 \hat{\sigma}_{\xi j}^2 + \nu_j \tag{7.3}$$

where $\hat{\sigma}_{\xi j}^2$ is an unbiased estimate of the residual variance of the security j. The hypotheses, that should be tested, are:

$$\begin{aligned} &H_0: \ \gamma_2 = \gamma_3 = 0 \\ &H_1: \ \gamma_2 \neq 0 \ \ \vee \ \ \gamma_3 \neq 0 \end{aligned} \tag{7.4}$$

If $\gamma_2 \neq 0$ then we say that the security market line is not linear, in case of $\gamma_3 \neq 0$ the diversifable risk effects the expected rate of return.

Another study proposes estimating the equation (see Levy 1978):

$$\bar{r}_j = \gamma_0 + \gamma_1 \hat{\beta}_j + \gamma_3 \hat{\sigma}_{\xi j}^2 + \nu_j \tag{7.5}$$

where, as above, $\hat{\sigma}_{\xi j}^2$ is the residual variance around time-series regression (6.1). It is also suggested that if the optimal portfolio allocation model is correct then γ_3 should be found to be equal to zero and the contribution of $\hat{\sigma}_{\xi j}^2$ to the coefficient of correlation is more important than the contribution of the systematic risk $\hat{\beta}_j$.

The above hypothesis of rationale for introducing variables $\hat{\beta}_j^2$ and $\hat{\sigma}_{\xi j}^2$ or only $\hat{\sigma}_{\xi j}^2$ into the equation (7.1) can be tested by the F test for restrictions (see, for example, Greene 1997, pp. 343-4):

$$F_{rest} = \frac{\left(R^2 - R_{rest}^2\right)/m}{\left(1 - R^2\right)/\left(n-k\right)} \qquad \sim F\left(m, n-k\right) \tag{7.6}$$

where R^2 and R_{rest}^2 are the determination coefficients for unrestricted and restricted models respectively, m is the number of restrictions imposed, n is the number of observations and k is the number of parameters estimated in the unrestricted regression. Assuming that the error term is normally distributed the F_{rest} statistic has the F distribution with m and $(n-k)$ degrees of freedom.

Table 7.5 Results of deriving the multivariate security market lines based on the *WIG21*

		model and standard errors	R^2	Se of regr.	DW	F-stat.	$\chi^2_{Norm}(2)$	$\chi^2_{Het}(1)$
Two-limit Tobit and uncorrected returns	r_j uncorr.	$\bar{r}_j = \underset{(0.0086902)}{-0.00152} + \underset{(0.0176682)}{0.0092499}\hat{\beta}_j - \underset{(0.00855787)}{0.00286}\hat{\beta}_j^2 - \underset{(1.9173)}{3.5841}\hat{\sigma}_{\xi j}^2 + \hat{v}_j$	0.1723	0.0011	2.3154	1.1795	0.68543	0.22563
	r_j correct	$\bar{r}_j = \underset{(0.0063058)}{0.008468} - \underset{(0.012509)}{0.015461}\hat{\beta}_j + \underset{(0.0061425)}{0.00738}\hat{\beta}_j^2 - \underset{(0.076181)}{0.12595}\hat{\sigma}_{\xi j}^2 + \hat{v}_j$	0.1769	0.0007	1.8049	1.2181	2.6352	1.4637
Max.likelihood and corrected returns	r_j uncorr.	$\bar{r}_j = \underset{(0.0068366)}{0.0010} + \underset{(0.015576)}{0.003415}\hat{\beta}_j - \underset{(0.0089626)}{0.000204}\hat{\beta}_j^2 - \underset{(1.7325)}{2.9302}\hat{\sigma}_{\xi j}^2 + \hat{v}_j$	0.1462	0.0011	2.2744	0.9704	0.61532	1.0231
	r_j correct	$\bar{r}_j = \underset{(0.0049544)}{0.002709} - \underset{(0.011384)}{0.004499}\hat{\beta}_j + \underset{(0.0065068)}{0.002269}\hat{\beta}_j^2 - \underset{(0.07937)}{0.11503}\hat{\sigma}_{\xi j}^2 + \hat{v}_j$	0.1137	0.0008	1.8639	0.7266	3.8712	1.6870
OLS and uncorrected returns	r_j uncorr.	$\bar{r}_j = \underset{(0.010548)}{-0.00207} + \underset{(0.023184)}{0.010192}\hat{\beta}_j - \underset{(0.012288)}{0.003404}\hat{\beta}_j^2 + \underset{(1.7841)}{3.295}\hat{\sigma}_{\xi j}^2 + \hat{v}_j$	0.1693	0.0011	2.3351	1.1548	0.65057	0.35671
	r_j correct	$\bar{r}_j = \underset{(0.0077608)}{0.00949} - \underset{(0.016707)}{0.018937}\hat{\beta}_j + \underset{(0.00893)}{0.009855}\hat{\beta}_j^2 - \underset{(0.077406)}{0.12745}\hat{\sigma}_{\xi j}^2 + \hat{v}_j$	0.1655	0.0008	1.7609	1.1238	2.7241	1.5486

Table 7.6 Results of deriving the multivariate security market lines based on the original *WIG*

		model and standard errors	R^2	Se of regr.	DW	F-stat.	$\chi^2_{Norm}(2)$	$\chi^2_{Het}(1)$
Two-limit Tobit and uncorrected returns	r_j uncorr.	$\bar{r}_j = \underset{(0.0090539)}{-0.00626} + \underset{(0.030559)}{0.027029}\hat{\beta}_j - \underset{(0.026186)}{0.023508}\hat{\beta}_j^2 - \underset{(1.5669)}{1.0646}\hat{\sigma}^2_{\xi j} + \hat{v}_j$	0.1006	0.0011	1.9421	0.6338	0.18447	0.55957
	r_j correct	$\bar{r}_j = -\underset{(0.0060767)}{0.00304} + \underset{(0.020856)}{0.014136}\hat{\beta}_j - \underset{(0.017756)}{0.013324}\hat{\beta}_j^2 - \underset{(0.075886)}{0.10752}\hat{\sigma}^2_{\xi j} + \hat{v}_j$	0.1493	0.0008	2.0125	0.9947	3.2559	0.35053
Max.likelihood and corrected returns	r_j uncorr.	$\bar{r}_j = -\underset{(0.0062713)}{0.00433} + \underset{(0.026424)}{0.026258}\hat{\beta}_j - \underset{(0.027122)}{0.02749}\hat{\beta}_j^2 - \underset{(1.3543)}{1.3067}\hat{\sigma}^2_{\xi j} + \hat{v}_j$	0.1120	0.0011	1.9574	0.7149	0.21159	0.16951
	r_j correct	$\bar{r}_j = -\underset{(0.004214)}{0.00309} + \underset{(0.017882)}{0.017732}\hat{\beta}_j - \underset{(0.018505)}{0.019785}\hat{\beta}_j^2 - \underset{(0.07802)}{0.12777}\hat{\sigma}^2_{\xi j} + \hat{v}_j$	0.1702	0.0007	2.1342	1.1621	3.0897	0.82998
OLS and uncorrected returns	r_j uncorr.	$\bar{r}_j = \underset{(0.0032596)}{-0.00344} + \underset{(0.013419)}{0.024436}\hat{\beta}_j - \underset{(0.015085)}{0.025919}\hat{\beta}_j^2 - \underset{(1.3363)}{1.4665}\hat{\sigma}^2_{\xi j} + \hat{v}_j$	0.2163	0.0010	2.1296	1.5639	0.00214	0.89578
	r_j correct	$\bar{r}_j = -\underset{(0.0024668)}{0.00068} + \underset{(0.010571)}{0.007102}\hat{\beta}_j - \underset{(0.011583)}{0.008785}\hat{\beta}_j^2 - \underset{(0.084826)}{0.09699}\hat{\sigma}^2_{\xi j} + \hat{v}_j$	0.1353	0.0008	2.0065	0.8864	3.2935	1.5955

Table 7.7 Results of deriving the Levy version of the security market lines based on the *WIG21*

		model and standard errors	R^2	Se of regr.	DW	F-stat.	$\chi^2_{Norm}(2)$	$\chi^2_{Het}(1)$
Two-limit Tobit and uncorrected returns	r_j uncorr.	$\bar{r}_j = \underset{(0.0016959)}{0.001319} + \underset{(0.0022177)}{0.0034043}\hat{\beta}_j - \underset{(1.8646)}{3.5385}\hat{\sigma}^2_{\xi j} + \hat{v}_j$	0.1669	0.0010	2.2621	1.8027	0.79151	0.08986
	r_j correct	$\bar{r}_j = \underset{(0.0010976)}{0.0010} - \underset{(0.00099)}{0.0004785}\hat{\beta}_j - \underset{(0.10542)}{0.10542}\hat{\sigma}^2_{\xi j} + \hat{v}_j$	0.1070	0.0008	1.9758	1.0788	5.1260	1.9279
Max.likelihood and corrected returns	r_j uncorr.	$\bar{r}_j = \underset{(0.001737)}{0.00116} + \underset{(0.002245)}{0.0030645}\hat{\beta}_j - \underset{(1.6771)}{2.9337}\hat{\sigma}^2_{\xi j} + \hat{v}_j$	0.1462	0.0011	2.2734	1.5409	0.62217	0.99343
	r_j correct	$\bar{r}_j = \underset{(0.0011347)}{0.00103} - \underset{(0.0011265)}{0.00055}\hat{\beta}_j - \underset{(0.075911)}{0.10962}\hat{\sigma}^2_{\xi j} + \hat{v}_j$	0.1073	0.0008	1.9765	1.0819	4.6941	2.0338
OLS and uncorrected returns	r_j uncorr.	$\bar{r}_j = \underset{(0.001767)}{0.0008} + \underset{(0.0024986)}{0.003809}\hat{\beta}_j - \underset{(1.7342)}{3.2636}\hat{\sigma}^2_{\xi j} + \hat{v}_j$	0.1655	0.0010	2.2997	1.7855	0.74685	0.2219
	r_j correct	$\bar{r}_j = \underset{(0.0012146)}{0.00103} - \underset{(0.001201)}{0.000545}\hat{\beta}_j - \underset{(0.075198)}{0.10525}\hat{\sigma}^2_{\xi j} + \hat{v}_j$	0.1057	0.0008	1.9757	1.0637	5.0574	2.071

Table 7.8 Results of deriving the Levy version of the security market lines based on the *WIG*

		model and standard errors	R^2	Se of regr.	DW	F-stat.	$\chi^2_{Norm}(2)$	$\chi^2_{Het}(1)$
Two-limit Tobit and uncorrected returns	r_j uncorr.	$\bar{r}_j = \underset{(0.0020099)}{0.00166} - \underset{(0.0037053)}{0.00020}\hat{\beta}_j - \underset{(1.5477)}{1.2294}\hat{\sigma}^2_{\xi j} + \hat{\nu}_j$	0.0580	0.0011	2.0394	0.5537	0.32272	0.07907
	r_j correct	$\bar{r}_j = \underset{(0.0012996)}{0.00141} - \underset{(0.0019878)}{0.001441}\hat{\beta}_j - \underset{(0.074902)}{0.10974}\hat{\sigma}^2_{\xi j} + \hat{\nu}_j$	0.1211	0.0008	1.9072	1.2405	3.0856	1.2687
Max.likelihood and corrected returns	r_j uncorr.	$\bar{r}_j = \underset{(0.0019897)}{0.0017} - \underset{(0.0031998)}{0.000329}\hat{\beta}_j - \underset{(1.3526)}{1.3526}\hat{\sigma}^2_{\xi j} + \hat{\nu}_j$	0.0584	0.0011	2.0376	0.5578	0.32304	0.04517
	r_j correct	$\bar{r}_j = \underset{(0.0012128)}{0.00122} - \underset{(0.0020376)}{0.001263}\hat{\beta}_j - \underset{(0.077702)}{0.11724}\hat{\sigma}^2_{\xi j} + \hat{\nu}_j$	0.1144	0.0008	1.9074	1.1624	3.0706	1.8691
OLS and uncorrected returns	r_j uncorr.	$\bar{r}_j = \underset{(0.0018378)}{0.001285} + \underset{(0.0027514)}{0.00182}\hat{\beta}_j - \underset{(1.3962)}{1.7485}\hat{\sigma}^2_{\xi j} + \hat{\nu}_j$	0.0802	0.0011	2.1425	0.7847	0.28139	0.7136
	r_j correct	$\bar{r}_j = \underset{(0.0011069)}{0.00099} - \underset{(0.0017439)}{0.0008}\hat{\beta}_j - \underset{(0.079832)}{0.1166}\hat{\sigma}^2_{\xi j} + \hat{\nu}_j$	0.1060	0.0008	1.8826	1.0672	3.6545	2.7009

The results of estimation of the models (7.3) and (7.5) for the WSE are given in tables 7.5 - 7.8 together with its characteristic, standard errors of estimated parameters (in parentheses), determinations coefficient (R^2), standard error of regression (Se), Durbin-Watson autocorrelation statistic (DW), Fisher-Snedecor statistic of joint significance (F), Chi-square tests of normality ($\chi^2_{Norm}(2)$) and heteroscedascity ($\chi^2_{Het}(1)$). As it was shown in the previous section, the models (7.3) and (7.5) have been estimated using various estimates of *beta* (obtained in the first-pass testing, see chapter 6), mean of corrected and uncorrected returns and mean market returns for *WIG* and *WIG21* have been used. All the estimates of parameters $\gamma_0, \gamma_1, \gamma_2, \gamma_3$ in equations (7.3) and (7.5) are insignificant. This fact does not confirm non-linearity of the security market line and the diversifiable risk influence of the expected rate of return. Insignificance of $\hat{\beta}^2_j$ and $\hat{\sigma}^2_{\xi j}$ confirms linearity of the SML and that the variance $\hat{\sigma}^2_{\xi j}$ does not influence the expected return of the j'th asset. The other statistics show that residuals for corrected returns might be regarded as independent and normally distributed which confirms that the portfolio allocation model is the correct one to describe the behaviour of expected returns[3].

Testing the hypothesis (7.4) using F test (7.6) the null hypothesis cannot be rejected, so it can be concluded that there is no rationale to introduce new variables to the SML[4].

[3] The critical values for 17 degrees of freedom at the 5% significance level are: F(3,17)=3.197, for Durbin-Watson test 1.026 and 1.669, for 18 degrees of freedom: F(2,18)=3.555, for Durbin-Watson test 1.125 and 1.538. The critical value for normality test is 5.99 and heteroscedasity 3.84.

[4] For example, for the estimates for Tobit model and uncorrected returns $F_{rest} = 1.7674$ and the critical value is $F(m, n-k) = F(2,17) = 3.592$, so the null hypothesis cannot be rejected.

However, obtained results confirm the rationale of applying the optimal portfolio allocation model with quantity constraints on the WSE. It seems to be interesting to introduce another test, which can confirm the decision of applying the model.

Another methodology of testing the model is proposed by Gibbons (1982). He uses the fact that the optimal portfolio allocation model places a non-linear restriction on a set of regression equations, one for each security:

$$r_{jt}^{*} = \alpha_j + \beta_j r_t^m + \varepsilon_{jt}.$$

where, as above, r_t^m denotes the session-to-session returns from the market portfolio in time t, r_{jt}^{*} the return from j'th security corrected by censored price and ε_{jt} is the error term. Assuming, that the market model and the optimal portfolio allocation hold simultaneously, we have:

$$r_{jt}^{*} = \kappa\left(1 - \beta_j\right) + \beta_j r_t^m + \varepsilon_{jt}, \qquad (7.7)$$

or

$$\alpha_j = \kappa\left(1 - \beta_j\right).$$

where κ parameter is constant for all securities. In case of the standard optimal portfolio allocation model κ should be equal to the rate of return of the risk-free assets. In this case it is assumed that this rate of return is equal to 0, so that parameter should be insignificantly different from zero.

Table 7.9 Parameters κ based on WIG and WIG21.

		κ	
		WIG	WIG21
Two-limit Tobit and uncorrected returns	r_j uncorr.	-0.00125	0.00027
	r_j correct	-0.00033	0.00011
Max.likelihood and corrected returns	r_j uncorr.	-0.00106	-0.00072
	r_j correct	-0.00031	0.00069
OLS and uncorrected returns	r_j uncorr.	-0.00113	-0.00059
	r_j correct	-0.00032	0.00073

The values of the parameters κ estimated for each equation from the previous chapter shows table 7.9. The values of the parameters are very small and close to zero. In most of the cases they are insignificant. It confirms the decision of rationality of applying the optimal portfolio allocation model with quantity constraints on the WSE.

7.4 Conclusions

The optimal portfolio allocation model has been examined using the two-pass cross-section regression and some extended tests, such as multivatiate security market lines and Sharpe ratio.

Although some of the results of derivation of the security market line confirm existence of the security market line, some of them do not allow for identification of the slope of the SML. It can be concluded that generally the SML is in line with the second-pass testing of the optimal portfolio allocation model in the case of quantity constraints. Econometrically, all estimated SMLs and multivariate SMLs show that residuals for corrected returns might be regarded as independent and normally distributed which confirms the rationale of applying the portfolio allocation model. The reason for inconsistency of values could be explained by Roll and Ross' (1994) critique of the accuracy of SML. According to their explanations the indices *WIG* and *WIG21* used are probably not the best approximations of the market portfolio. Roll and Ross argue that the cross-section OLS relation is very sensitive to the choice of market index, so the real, optimal market portfolio should be represented by other measures. Both market indices are based on constant weights of companies in the calculated index (for *WIG* weights change every three month). *WIG21* does not reflect the effect of new companies entering the WSE. Another reason for the inconsistency of the parameters of the SMLs is that the sample average returns are probably not the proper representations of the expected returns.

Generally, obtained results are closer to their required values for *WIG21* rather than for *WIG* market index. Consequently, it seems that the index created by myself is in some sense a better measure of the market portfolio than the official one. I suggest that I have obtained more appropriate results for corrected returns. I conclude that the model for corrected returns gives more accurate measures of the risk of the assets if the returns are corrected by correction factor (see formula (6.2)).

The analysis of Sharpe ratios showed that the longer a company performs on the Warsaw Stock Exchange, the lower the risk premium. Consequently, it could be said that the level of risk of companies will decrease and converge in time.

In summary it can be said that the optimal portfolio allocation model is the model which could be applied for finding the level of the risk of the assets on the market where price limits are imposed, particularly on the Warsaw Stock Exchange.

CHAPTER 8

CONCLUSIONS

8.1 Summary of thesis

8.2 Main results and conclusions

8.3 Suggestions for future research

8.1 Summary of thesis

In this thesis I have identified two general problems in analysing returns from the stock exchange of the emerging market. Firstly, I address the question if it might be possible to examine the existence of an optimal portfolio on the emerging stock market. The second important question is whether the regulations of such a market, resulting in the imposition of market constraints, prevents the existence of the optimal portfolio allocation. In this thesis I have dealt mainly with the second problem, since the first one has already been well-researched (see literature review in chapter 5). Following the literature I have found, in particular, that ignoring institutional constraints, the emerging market behaviour is generally consistent with Sharpe-Lintner capital asset pricing model (CAPM). In that sense the market inefficiencies tend to diminish over time and it is possible to capture its convergence towards market efficiency. Thus, it seems to be reasonably safe to assume that the optimal portfolio allocation model can be applied.

If institutional regulations result in some type of constraints it is possible to amend the Sharpe-Lintner model in such a way that the optimal portfolio allocation problem might still be analysed conditionally on these constraints.

Given the hypothesis of the existence of the optimal portfolio allocation under quantity constraints, I concentrated on the defining the optimal portfolio. One of the important findings of chapter 2 was the non-trivial organisation of the Warsaw Stock

Exchange resulting in quantity constraints. The institutionally imposed quantity constraints implied the disequilibrium trading on the WSE market. It required development of the theoretical background of the disequilibrium trading in chapter 3. I showed that the potential quantity constraints might cause intertemporal and cross-sectional spillover effects.

In light of previous chapters 2 and 3 it was necessary to test the hypothesis of the existence of the optimal portfolio allocation with quantity constraints. I started from presenting the standard version of the Sharpe-Lintner CAPM, which would have been valid if the prices were bounded. It was necessary to perform various methods of deriving the model since each method was based on different assumptions, which led to different interpretations of results. I then showed that the standard model and other known CAPM-like further developed models cannot be applied to the market with quantity constraints since such optimal allocation depends on constraints. In chapter 5 I extended the standard model and proposed the new optimal portfolio allocation model that included the quantity constraints and resolved the optimal portfolio allocation problem conditionally on quantity constraints.

In order to test the optimum portfolio allocation model with quantity constraints, I applied it on the Warsaw Stock Exchange where such constraints are imposed. I analysed returns from twenty-one longest established companies. Empirical testing was applied in two stages. The first stage, known as first-pass testing, presented in chapter 6, included estimation of the *beta* parameters (meaning the relative risk) of the optimal portfolio allocation model with quantity constraints. It was concluded that the results of the first-pass testing were satisfying and used in the second stage, known as the second-pass testing (presented in chapter 7). This stage included the derivation of the relationship between estimated *beta* parameters and expected rate of return, known as a security market line (SML). Because of the relatively weak results of the derivation of the SML, the rationale of applying the optimum portfolio allocation model was further evaluated by applying some stronger tests, namely the multivariate SML, which tested non-linearity of the SML, Sharpe ratio analysis and Gibbons' test.

Interpretation of such results can be regarded as a non-trivial because of the constrained optimisation problem. Since the characteristics of admissible optimal portfolios with such constraints are not known I grounded the interpretation of the results in a numerical experiment, simulating the efficiency frontiers for hypothetical portfolios with market constraints and without constraints.

8.2 Main results and conclusions

The main findings of this thesis can be divided into four main groups. There are results concerning the theory of the optimal portfolio allocation, econometric findings, results concerning modelling emerging markets and finally, the results describing the analysed companies from the Warsaw Stock Exchange.

The first group of results concerns the theory of the portfolio allocation problem. In this thesis I showed that if the rates of returns of the assets are constrained, in particular if they are constrained by the imposition of quantity constraints, the problem of the optimal portfolio allocation has to be reformulated by constrained optimisation. However, the optimal portfolio exists and it is possible to develop the model that takes into account quantity constraints. The proposed new model of the optimal portfolio allocation is developed from the Sharpe-Lintner version of the capital asset pricing model (CAPM). As a result of the derivation I obtained a model of the rate of return of an individual asset as a function of the rate of return of a risk-free asset, the level of systematic risk and the initial prices of assets j'th and n'th, where the price of the n'th asset is limited, and standard CAPM where this n'th price is not limited. The new model is relatively simple to apply and needs only information about daily prices of securities and when the prices hit the limits.

The next group of findings concerns econometric problems. I showed that the empirical version of the theoretical model of the optimal portfolio allocation with quantity constraints can be achieved by introducing the correction factor. I suggested that the correction factor should be built on the weighted prices of the securities and the time varying probabilities of prices reaching the limits. In applying this correction factor it is possible to calculate the hypothetical equilibrium returns, that is the returns which would have occured if had there been no quantity constraints. These results were used to calculate corrected and uncorrected returns. In the case of uncorrected returns it was shown that the ordinary least square method (OLS) is inappropriate (inconsistent) due to doubly truncated data. Consequently, I proposed the use of two-limit Tobit model for uncorrected returns and maximum likelihood method for corrected returns.

Analysing the Sharpe ratios calculated for the chosen companies from the Warsaw Stock Exchange I hypothesised that the value of the ratio depends on how long the company has existed on the market. The WSE is an example of an emerging market and the Sharpe ratios for all companies were relatively high. However, I found that there is a significant negative relationship between the Sharpe ratio of the company and the age of the company on the stock market. This is related to the hypothesis that there is more information on the market for older companies which generally should result in lower risks. Consequently, it can be said that the longer the company exists on the market, the lower the risk premium. It can be concluded that the level of risk of all companies on immature markets is relatively high and will decrease and converge in time.

The last group of results concerns the empirical analysis of returns of twenty-one companies from the Warsaw Stock Exchange in the period 3 January 1994 to 17 April 1998. The statistics confirm the relative homogeneity of the sample. For all series the characteristics are of a similar magnitude and the distributions of the returns are close to being symmetric.

The econometric analysis of the returns from the WSE is aimed two stages known as two-pass testing. The results of the first-pass testing aimed the estimation of the proposed optimal portfolio allocation model with quantity constraints. The results show the intercepts being close to zero and insignificant. This is in line with the *non-zero beta* CAPM theory and confirms the decision to omit the low-variation riskless asset from the model. It also indicates that the analysed securities were not, on average, systematically underpriced or overpriced. The two-limit Tobit estimates of the β's are visibly higher than the corresponding OLS estimates. It is interesting to note that the estimates of relative risk are consistently, and markedly, higher for the corrected than for the uncorrected returns.

Lower risk of uncorrected returns can be explained in two ways. Firstly, the definition of *beta*, which is a parameter that measures a level of risk of security, implies, that *beta* is equal to the ratio of the covariance of the i'th asset examined with the market portfolio to the variance of this portfolio. For the restricted market, the covariance is likely to be relatively high, due to the possible quantity spillover effect from the restricted to unrestricted markets. The market price of risk, may still be lower for the restricted market than for the unrestricted market. It is often hypothesised that in emerging market's investors' behaviour is determined by the relative risk rather than market price of risk. The interpretation given above places into question the rationale for regulated trading and suggests the abolition of price barriers.

Secondly, the result of lower risk for uncorrected returns can be interpreted in light of the micro-market structure models with exogenously random supply. If there is no quantity restriction the signal increases the variance of the return and in the same time increases the informational efficiency of the price signal. Non-restricted portfolios can be diversified in order to minimise the correlation between the asset returns.

Diversification shifts the portfolio opportunity set so that risks corresponding to particular expected returns become smaller. This again acts towards the decrease of correlation of assets returns with portfolio returns, resulting in smaller level of risk.

The second-pass test aimed the derivation of the security market line by regressing the expected returns on estimated β's. It showed that although some of the results confirm the existence of the SML, some of them do not allow for the identification of its slope. Despite weak results the SML is generally in line with the second-pass testing of the optimal portfolio allocation model. One of the reasons for the inconsistency of the SML, following the Roll and Ross criticism, concerns the choice of the market share index. It is possibly the case that both applied market indices do not fully represent returns from the real market portfolio. Another reason could be an inaccurate approximation of the expected return by the average return.

The results obtained for *WIG21* are better than for *WIG* market index. It appears that the artificially created index *WIG21* is a better measure of the market portfolio than the officially published *WIG*. More satisfying results were obtained for corrected returns rather than for uncorrected returns. It can be concluded that the model for corrected returns gives more accurate measures of the risk of the assets if the returns are corrected by correction factor.

Generally, the WSE can be classified as medium to high risk. This seems to confirm the conclusion that much of the risk comes to the market through the existence of price regulation. The smaller *beta* values for the two-limit Tobit and maximum likelihood were obtained for *WIG* than for *WIG21*. The *WIG21* portfolio is more risky than the *WIG* portfolio. The same results were obtained based on maximum likelihood and OLS method for corrected returns.

Finally, it can be concluded that the optimum portfolio holds on a market with quantity constraints. However, it does not seem that the regulation of the Warsaw Stock Exchange through the imposition of limits is effective. It is expensive, increases market inefficiency and, as the history of the Warsaw Stock Exchange reveals, does not shelter the market from the boom-bust events. At the same time, it does not seem to reduce relative risk in allocative portfolios. Portfolios would be more efficient if correlation between them was allowed to decrease, and this requires the abolishment of price barriers.

8.3 Suggestions for future research

The provided analysis aims theoretical framework, description, application and testing the optimal portfolio allocation model with price constraints. However, there are some aspects that could be taken into consideration in future research.

One of these aspects is concerned with the estimation of the security market line (SML). The obtained results show insignificance of the slope of the SML. Following the Roll and Ross critique this insignificance might be caused by the choice of market share index as representing market portfolio. It would be interesting to reverse the estimation sequence of the model. I can assume the level of risk for particular assets (based on external information) and try to estimate the true value of the market portfolio.

The analysis could be further extended by introducing ARCH and GARCH processes. The model would be estimated by a double censored GARCH process. As the WSE developes it is possible to notice that the unconditional variance of returns decreases, it would be interesting to estimate the processes under the assumption of the time varying unconditional variance. I suggest the estimation of double truncated GARCH model in recursions and then analyse the changes in GARCH parameters. Moreover, with increase of maturity of the WSE the problem will change. In recent years the number of prices reaching limits has decreased, so it would be expected that the unconditional variance would diminish in time.

Despite the fact that I showed that the optimal portfolio allocation exist on the market with quantity constraints, it would be interesting to estimate the same panel of data using other models, for example arbitrage pricing theory (APT) for truncated data and then compare the results. In light of relatively weak results obtained for the optimal portfolio allocation model with quantity constraints the results from other methods might be shown to be more powerful.

At present there are 190 companies traded on the Warsaw Stock Exchange[1]. The analysis provided in this thesis was aimed at twenty-one of the longest established

[1] Source: WSE web page 18.03.1999.

companies. The future research could include more firms. For the comparison all companies should aim observations from the same period. It will automatically omit observations from the early years of working WSE.

Recently, the number of companies traded continuously grows rapidly[2]. In this thesis only returns established in a single-price method were analysed. In the next step I suggest the empirical analysis of the prices be obtained from the continuous trading. More advanced analysis of returns distribution could be provided.

[2] There were 73 companies traded continuously in 18.03.1999, see WSE web page.

APPENDIX 4A

Derivation of the standard CAPM model based on the marginal rates of substitution

Investor's capital is disposed as:

$$K = k_f + k_j + k_p .$$

Dividing by K I have:

$$1 = \frac{k_f}{K} + \frac{k_j}{K} + \frac{k_p}{K},$$

which is equal to proportions:

$$1 = w_f + w_j + w_p .$$

Then after decision of buying another j'th, the capital is disposed as:

$$K = k_f - \Delta k_j + k_j + \Delta k_j + k_p$$

or:

$$1 = w_f - \frac{\Delta k_j}{K} + w_j + \frac{\Delta k_j}{K} + w_p .$$

$$1 = w_f - \Delta w_j + w_j + \Delta w_j + w_p$$

The rate of return of the portfolio before a change (r_{p1}) is:

$$r_{p1} = w_f r_f + w_j r_j + w_p r_p$$

and after the change (r_{p2}):

$$r_{p2} = \left(w_f - \Delta w_j\right) r_f + \left(w_j + \Delta w_j\right) r_j + w_p r_p$$

$$r_{p2} = w_f r_f + w_j r_j + w_p r_p + \Delta w_j \left(r_j - r_f\right)$$

$$r_{p2} = r_{p1} + \Delta w_j \left(r_j - r_f\right)$$

So an increase of rate of return cause by 'moving' a part of capital into risky asset is:

$$\Delta r = r_{p2} - r_{p1}$$

$$\Delta r = (r_j - r_f)\Delta w_j$$

The expected value of the portfolio before change (μ_{p1}) is described as:

$$\mu_{p1} = w_f r_f + w_j \mu_j + w_p \mu_p, \tag{4a.1}$$

and after a change:

$$\mu_{p2} = (w_f - \Delta w_j) r_f + (w_j + \Delta w_j) \mu_j + w_p \mu_p,$$

$$\mu_{p2} = w_f r_f + w_j \mu_j + w_p \mu_p + (\mu_j - r_f)\Delta w_j,$$

$$\mu_{p2} = \mu_{p1} + (\mu_j - r_f)\Delta w_j. \tag{4a.2}$$

Let σ_p^2 be the variance of the portfolio before the change. Then the variance of the portfolio after the change is:

$$\operatorname{var}(p_2) = \sigma_p^2 + \Delta w_j^2 \operatorname{var}(r_j) + 2\Delta w_j \operatorname{cov}(r_p r_j),$$

assuming that the variance of the risk-free asset is equal to zero.

Then the increase of the variance caused by the change of portfolio is:

$$\operatorname{var}(p_2) - \sigma_p^2 = \Delta w_j^2 \operatorname{var}(r_j) + 2\Delta w_j \operatorname{cov}(r_p r_j),$$

or:

$$\Delta\sigma_p^2 = \Delta w_j^2 \operatorname{var}(r_j) + 2\Delta w_j \operatorname{cov}(r_p r_j).$$

For the small change of Δw_j I can assume that:

$$\Delta\sigma_p^2 \approx 2\Delta w_j \operatorname{cov}(r_p r_j). \tag{4a.3}$$

The marginal rate of transformation of the expected return is:

$$MRT = \frac{\Delta\sigma_p^2}{\Delta r}, \tag{4a.4}$$

so from (4a.1), (4a.2) and (4a.3) I have:

$$MRT = \frac{2\Delta w_i \operatorname{cov}(r_p r_j)}{(r_j - r_f)\Delta w_j},$$

$$MRT = \frac{2\operatorname{cov}(r_p r_j)}{r_j - r_f}. \tag{4a.5}$$

From another point of view, capital is divided as:

$$K = k_f + k_p,$$

or in proportions:

$$1 = w_f + w_p.$$

For the part of money 'moved' from risk-free to risky assets it becomes:

$$K = k_f - \Delta k_p + k_p + \Delta k_p,$$

or:

$$1 = w_f - \frac{\Delta k_p}{K} + w_p + \frac{\Delta k_p}{K},$$

which gives:

$$1 = w_f - \Delta w_p + w_p + \Delta w_p.$$

The portfolio before a change can be described as follows:

* rate of return of the portfolio:

$$r_{p1} = w_f r_f + w_p r_p; \tag{4a.6}$$

* expected return μ_{p1} :

$$\mu_{p1} = w_f r_f + w_p \mu_p; \tag{4a.7}$$

* variance of the portfolio:

$$\sigma_{p1}^2 = w_p^2 \sigma_p^2, \tag{4a.8}$$

because the variance of the risk-free asset is equal to zero, which is $\sigma_f^2 = 0$.

The portfolio after a change is described as follows:

* rate of return of the portfolio:

$$r_{p2} = \left(w_f - \Delta w_p\right) r_f + \left(w_p + \Delta w_p\right) r_p,$$

$$r_{p2} = r_{p1} + \left(r_p - r_f\right) \Delta w_p; \tag{4a.9}$$

* expected return:

$$\mu_{p2} = \left(w_f - \Delta w_p\right) r_f + \left(w_p + \Delta w_p\right) \mu_p,$$

$$\mu_{p2} = w_f r_f + w_p \mu_p + \left(\mu_p - r_f\right) \Delta w_p,$$

$$\mu_{p2} = \mu_{p1} + \left(\mu_j - r_f\right)\Delta w_p; \tag{4a.10}$$

* variance of the portfolio:

$$\mathrm{var}(p_2) = \mathrm{var}(p_1) + \Delta w_p^2 \,\mathrm{var}(p) + 2\Delta w_p \,\mathrm{var}(p),$$

$$\sigma_{p2}^2 = w_p^2\sigma_p^2 + \Delta w_p^2 w_p^2 \sigma_p^2 + 2\Delta w_p w_p^2 \sigma_p^2. \tag{4a.11}$$

The marginal rate of substitution is:

$$MRS = \frac{\Delta\sigma_p^2}{\Delta r_p}, \tag{4a.12}$$

so from (4a.6), (4a.8), (4a.9) and (4a.11) I get:

$$MRS = \frac{\sigma_{p2}^2 - \sigma_{p1}^2}{r_{p2} - r_{p1}},$$

$$MRS = \frac{\Delta w_p^2 w_p^2 \sigma_p^2 + 2\Delta w_p (w_p^2\sigma_p^2)}{(r_p - r_f)\Delta w_p},$$

and in case of a small marginal change of w_p:

$$\Delta\sigma_p^2 \approx 2\Delta w_p\left(w_p^2\sigma_p^2\right),$$

which gives:

$$MRS = \frac{2\Delta w_p (w_p^2\sigma_p^2)}{(r_p - r_f)\Delta w_p},$$

$$MRS = \frac{2w_p^2\sigma_p^2}{r_p - r_f},$$

$$MRS = \frac{2\sigma_p^2}{r_p - r_f}. \tag{4a.13}$$

In equilibrium:

$$MRT = MRS,$$

(4a.4) has to be equal to (4a.12), so in our case, from (4a.5) and (4a.13):

$$\frac{2\,\mathrm{cov}(r_p r_j)}{r_j - r_f} = \frac{2\sigma_p^2}{r_p - r_f}.$$

Transforming the equation I have:

$$r_j - r_f = (r_p - r_f)\frac{\mathrm{cov}(r_p r_j)}{\sigma_p^2}.$$

If $\beta_{jp} = \frac{\text{cov}(r_j r_p)}{\sigma_p^2}$ then I have:

$$r_j - r_f = (r_p - r_f)\beta_{jp}.$$

Substituting r_p by the rate of return of the market portfolio r_M I get a standard Capital Asset Pricing Model:

$$r_j = r_f + (r_M - r_f)\beta_{jM}.$$

APPENDIX 4B

Derivation of the CAPM based on the marginal measures

The rate of return of the portfolio on the capital market (r_p) is:

$$r_p = wr_s + (1-w)r_M$$

and a rate of return of the portfolio which is a combination of the market portfolio and a risk free assets (r) is:

$$r = wr_f + (1-w)r_M .$$

In equilibrium $w = 0$ and:

$$\left.\frac{\partial \sigma_p}{\partial \mu_p}\right|_{w=0} = \left.\frac{\partial \sigma}{\partial \mu}\right|_{w=0} . \tag{4b.1}$$

Above condition can be divided into the factors:

$$\left.\frac{\partial \sigma_p}{\partial \mu_p}\right|_{w=0} = \left.\frac{\partial \sigma_p}{\partial w}\frac{\partial w}{\partial \mu_p}\right|_{w=0} = \left.\frac{\partial \sigma}{\partial \mu}\right|_{w=0} = \left.\frac{\partial \sigma}{\partial w}\frac{\partial w}{\partial \mu}\right|_{w=0}$$

For the free portfolio described by the rate of return:

$$r_p = wr_s + (1-w)r_M$$

the standard deviation is:

$$\sigma_p = \sqrt{w^2\sigma_s^2 + (1-w)^2\sigma_M^2 + 2w(1-w)\sigma_s\sigma_M \operatorname{cov}(r_M r_s)} ,$$

$$\sigma_p = \sqrt{w^2\sigma_s^2 + \sigma_M^2 - 2w\sigma_M^2 + w^2\sigma_M^2 + 2w\sigma_s\sigma_M \operatorname{cov}(r_M r_s) - 2w^2\sigma_s\sigma_M \operatorname{cov}(r_M r_s)} .$$

Then the first derivative of σ_p in point $w = 0$:

$$\left.\frac{\partial \sigma_p}{\partial w}\right|_{w=0} = \frac{1}{2\sqrt{\sigma_M^2}}[-2\sigma_M^2 + 2\sigma_s\sigma_M \operatorname{cov}(r_M r_s)],$$

$$\left.\frac{\partial \sigma_p}{\partial w}\right|_{w=0} = \frac{-\sigma_M^2 + \sigma_s\sigma_M \operatorname{cov}(r_M r_s)}{\sigma_M},$$

$$\left.\frac{\partial \sigma_p}{\partial w}\right|_{w=0} = \frac{-\sigma_M^2 + \sigma_{sM}}{\sigma_M} . \tag{4b.2}$$

The expected return of the free portfolio p is:

$$\mu_p = w\mu_s + (1-w)\mu_M$$

so then:

$$w\mu_M - w\mu_s = \mu_M - \mu_p$$

which gives:

$$w = \frac{\mu_M - \mu_p}{\mu_M - \mu_s}.$$

The first derivative with respect to the expected return of the portfolio is:

$$\frac{\partial w}{\partial \mu_p} = \frac{-1}{\mu_M - \mu_s}. \qquad (4b.3)$$

Then the marginal rate of transformation for the free portfolio, from (4b.2) and (4b.3) is:

$$\left.\frac{\partial \sigma_p}{\partial \mu_p}\right|_{w=0} = \left.\frac{\partial \sigma_p}{\partial w}\frac{\partial w}{\partial \mu_p}\right|_{w=0} = \frac{-\sigma_M^2 + \sigma_{sM}}{\sigma_M(\mu_s - \mu_M)} \qquad (4b.4)$$

From the other side for the portfolio of combination of risk-free assets and the market portfolio I have:

$$r = wr_f + (1-w)r_M$$

Standard deviation of the above portfolio is:

$$\sigma = \sqrt{w^2\sigma_f^2 + (1-w)^2\sigma_M^2 + 2w(1-w)\sigma_f\sigma_M \operatorname{cov}(r_M r_f)},$$

$$\sigma = \sqrt{w^2\sigma_f^2 + \sigma_M^2 - 2w\sigma_M^2 + w^2\sigma_M^2 + 2w\sigma_f\sigma_M \operatorname{cov}(r_M r_f) - 2w^2\sigma_f\sigma_M \operatorname{cov}(r_M r_f)}.$$

The variance of the risk-free assets is equal to zero, so:

$$\sigma = \sqrt{\sigma_M^2 - 2w\sigma_M^2 + w^2\sigma_M^2}.$$

Then the first derivative of the standard deviation in point $w = 0$ is:

$$\left.\frac{\partial \sigma}{\partial w}\right|_{w=0} = \frac{1}{2\sqrt{\sigma_M^2}}(-2\sigma_M^2)$$

$$\left.\frac{\partial \sigma}{\partial w}\right|_{w=0} = -\sigma_M \qquad (4b.5)$$

Expected return of the portfolio is described by:

$$\mu = wr_f + (1-w)\mu_M,$$

which gives:

$$w\mu_M - wr_f = \mu_M - \mu,$$

$$w = \frac{\mu_M - \mu}{\mu_M - r}.$$

The first derivative of the weight with respect to the expected return is:

$$\frac{\partial w}{\partial \mu} = \frac{-1}{\mu_M - r_f}. \tag{4b.6}$$

Then the marginal rate of transformation for this portfolio in point $w = 0$, from (4b.5) and (4b.6), is:

$$\left.\frac{\partial \sigma}{\partial \mu}\right|_{w=0} = \left.\frac{\partial \sigma}{\partial w}\frac{\partial w}{\partial \mu}\right|_{w=0} = \frac{\sigma_M}{\mu_M - r_f} \tag{4b.7}$$

Both marginal rates of transformations must be equal in market equilibrium, which is:

$$\left.\frac{\partial \sigma_p}{\partial \mu_p}\right|_{w=0} = \left.\frac{\partial \sigma}{\partial \mu}\right|_{w=0}.$$

In this case, from (4b.4) and (4b.7), I have:

$$\frac{-\sigma_M^2 + \sigma_{sM}}{\sigma_M(\mu_s - \mu_M)} = \frac{\sigma_M}{\mu_M - r_f},$$

$$(-\sigma_M^2 + \sigma_{sM})(\mu_M - r_f) = \sigma_M^2(\mu_s - \mu_M),$$

$$(-1 + \frac{\sigma_{sM}}{\sigma_M^2})(\mu_M - r_f) = \mu_s - \mu_M,$$

$$\frac{\sigma_{sM}}{\sigma_M^2}(\mu_M - r_f) - \mu_M + r_f = \mu_s - \mu_M.$$

It gives:

$$\mu_s - r_f = \frac{\sigma_{sM}}{\sigma_M^2}(\mu_M - r_f)$$

Then from expected returns I get rates of return:

$$r_s - r_f = \frac{\sigma_{sM}}{\sigma_M^2}(r_M - r_f).$$

Substituting $\beta_{sM} = \frac{\sigma_{sM}}{\sigma_M^2}$ I have a model:

$$r_s = r_f + \beta_{sM}(r_M - r_f).$$

APPENDIX 4C

Derivation of the CAPM based on the risk premium

The problem is in maximising the function:

$$\theta = \frac{r_p - r_f}{\sigma_p}. \tag{4c.1}$$

The numerical constraint is:

$$\sum_{i=1}^{n} w_i = 1 \ .$$

The rate of return of the risk-free asset can be written as:

$$r_f = 1 \times r_f = (\sum_{i=1}^{n} w_i) r_f = \sum_{i=1}^{n} w_i r_f \ . \tag{4c.2}$$

In case of short sales it is possible that $w_i < 0$, so the constraint with Lintner definition of short sale is:

$$\sum_{i=1}^{n} |w_i| = 1. \tag{4c.3}$$

The rate of return of the portfolio is:

$$r_p = \sum_{i=1}^{k} w_i r_i + \sum_{i=k+1}^{n} w_i (r_i - 2r_f)$$

in case where investor held asset 1 to k long and assets $k+1$ to n are sold short. Then:

$$r_p = \sum_{i=1}^{k} w_i r_i + \sum_{i=k+1}^{n} w_i r_i - \sum_{i=k+1}^{n} w_i 2r_f ,$$

$$r_p = \sum_{i=1}^{n} w_i r_i - 2 \sum_{i=k+1}^{n} w_i r_f \ . \tag{4c.4}$$

After substituting 1 by (4c.3) the rate of return of the risk-free asset is:

$$r_f = \sum_{i=1}^{n} |w_i| r_f ,$$

$$r_f = \sum_{i=1}^{k} w_i r_f - \sum_{i=k+1}^{n} w_i r_f . \qquad (4c.5)$$

The risk premium, from (4c.4) and (4c.5) is then:

$$r_p - r_f = [\sum_{i=1}^{n} w_i r_i - 2\sum_{i=k+1}^{n} w_i r_f] - [\sum_{i=1}^{k} w_i r_f - \sum_{i=k+1}^{n} w_i r_f],$$

$$r_p - r_f = \sum_{i=1}^{n} w_i r_i - 2\sum_{i=k+1}^{n} w_i r_f - \sum_{i=1}^{k} w_i r_f + \sum_{i=k+1}^{n} w_i r_f ,$$

$$r_p - r_f = \sum_{i=1}^{n} w_i r_i - \sum_{i=k+1}^{n} w_i r_f - \sum_{i=1}^{k} w_i r_f ,$$

$$r_p - r_f = \sum_{i=1}^{n} w_i r_i - \sum_{i=1}^{n} w_i r_f ,$$

$$r_p - r_f = \sum_{i=1}^{n} w_i (r_i - r_f) . \qquad (4c.6)$$

The standard deviation of the portfolio is defined as:

$$\sigma_p = \sqrt{\sum_{i=1}^{n} w_i^2 \sigma_i^2 + 2\sum_{i=1}^{n-1} \sum_{j=i+1}^{n} w_i w_j \sigma_i \sigma_j \rho_{ij}}$$

or

$$\sigma_p = \sqrt{\sum_{i=1}^{n} w_i^2 \sigma_i^2 + \sum_{i=1}^{n} \sum_{\substack{j=1 \\ j \neq i}}^{n} w_i w_j \sigma_{ij}} , \qquad (4c.7)$$

so the function θ from (4c.6) and (4c.7) substituted in (4c.1) can be written as:

$$\theta = \frac{\sum_{i=1}^{n} w_i (r_i - r_f)}{\sqrt{\sum_{i=1}^{n} w_i^2 \sigma_i^2 + \sum_{i=1}^{n} \sum_{\substack{j=1 \\ j \neq i}}^{n} w_i w_j \sigma_{ij}}}$$

or

$$\theta = [\sum_{i=1}^{n} (r_i - r_f)] \times [\sqrt{\sum_{i=1}^{n} w_i^2 \sigma_i^2 + \sum_{i=1}^{n} \sum_{\substack{j=1 \\ j \neq i}}^{n} w_i w_j \sigma_{ij}}\,]^{-\frac{1}{2}} . \qquad (4c.8)$$

The first order condition of maximising the θ function is:

$$\frac{\partial\theta}{\partial w_1}=0$$

$$\frac{\partial\theta}{\partial w_2}=0$$

....

$$\frac{\partial\theta}{\partial w_n}=0$$

which means that first derivatives of the function θ with respect to each weight w_i for $i=1,2,\ldots,n$ must be equal to zero.

The function θ is a product of two functions F_1 and F_2 where

$$F_1=\sum_{i=1}^{n} w_i(r_i-r_f),$$

$$F_2=[\sum_{i=1}^{n} w_i{}^2\sigma_i^2+\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n} w_i w_j\sigma_{ij}]^{-\frac{1}{2}}=(\sigma_p^2)^{-\frac{1}{2}},$$

so the product rule states that the derivative of the product of two functions is:

$$\frac{\partial}{\partial w}[F_1(w)F_2(w)]=F_1(w)\frac{\partial F_2(w)}{\partial w}+F_2(w)\frac{\partial F_1(w)}{\partial w}.$$

Then the derivative of θ with respect to the particular w_k is[1]:

$$\frac{\partial\theta}{\partial w_k}=[\sum_{i=1}^{n} w_i(r_i-r_f)][(-\frac{1}{2})(\sum_{i=1}^{n} w_i{}^2\sigma_i^2+\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n} w_i w_j\sigma_{ij})^{-\frac{3}{2}}\times(2w_k\sigma_k^2+2\sum_{\substack{j=1\\ j\neq k}}^{n} w_j\sigma_{jk})]+[\sum_{i=1}^{n} w_i{}^2\sigma_i^2+\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n} w_i w_j\sigma_{ij}]^{-\frac{1}{2}}[r_k-r_f]$$

which gives:

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[1]* the first derivative of the function F_1 with respect to the weight w_k is:

$$\frac{\partial F_1}{\partial w_k}=r_k-r_f$$

* the first derivative of the function F_2 with respect to the weight w_k is:

$$\frac{\partial F_2}{\partial w_k}=(-\frac{1}{2})(\sum_{i=1}^{n} w_i{}^2\sigma_i^2+\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n} w_i w_j\sigma_{ij})^{-\frac{3}{2}}\times(2w_k\sigma_k^2+2\sum_{\substack{j=1\\ j\neq k}}^{n} w_j\sigma_{jk})$$

which gives:

$$\frac{\partial F_2}{\partial w_k}=(-\frac{1}{2})(\sigma_p^2)^{-\frac{3}{2}}\times(2w_k\sigma_k^2+2\sum_{\substack{j=1\\ j\neq k}}^{n} w_j\sigma_{jk})$$

$$\frac{\partial\theta}{\partial w_k}=[\sum_{i=1}^{n}w_i(r_i-r_f)][(-\frac{1}{2})(\sigma_p^2)^{-\frac{3}{2}}\times(2w_k\sigma_k^2+2\sum_{\substack{j=1\\j\neq k}}^{n}w_j\sigma_{jk})]+[\sigma_p^2]^{-\frac{1}{2}}[r_k-r_f]$$

Then the derivative must be equal to zero. Multiplying by $(\sigma_p^2)^{-\frac{1}{2}}$ I have:

$$[\sum_{i=1}^{n}w_i(r_i-r_f)](-\frac{1}{2})[\sigma_p^2]^{-1}\times(2w_k\sigma_k^2+2\sum_{\substack{j=1\\j\neq k}}^{n}w_j\sigma_{jk})+[r_k-r_f]=0$$

and

$$-[\frac{\sum_{i=1}^{n}w_i(r_i-r_f)}{\sigma_p^2}](w_k\sigma_k^2+\sum_{\substack{j=1\\j\neq k}}^{n}w_j\sigma_{jk})+(r_k-r_f)=0,$$

$$-[\frac{\sum_{i=1}^{n}w_i(r_i-r_f)}{\sum_{i=1}^{n}w_i{}^2\sigma_i^2+\sum_{i=1}^{n}\sum_{\substack{j=1\\j\neq i}}^{n}w_iw_j\sigma_{ij}}](w_k\sigma_k^2+\sum_{\substack{j=1\\j\neq k}}^{n}w_j\sigma_{jk})+(r_k-r_f)=0. \qquad (4c.9)$$

Defining λ as:

$$\lambda=\frac{\sum_{i=1}^{n}w_i(r_i-r_f)}{\sum_{i=1}^{n}w_i{}^2\sigma_i^2+\sum_{i=1}^{n}\sum_{\substack{j=1\\j\neq i}}^{n}w_iw_j\sigma_{ij}}$$

the equation (4c.9) takes a form:

$$-\lambda(w_k\sigma_k^2+\sum_{\substack{j=1\\j\neq k}}^{n}w_j\sigma_{jk})+(r_k-r_f)=0,$$

$$-(\lambda w_k\sigma_k^2+\sum_{\substack{j=1\\j\neq k}}^{n}\lambda w_j\sigma_{jk})+(r_k-r_f)=0.$$

Then derivative of θ with respect to the weight w_i is:

$$\frac{\partial\theta}{\partial w_i}=-(\lambda w_i\sigma_i^2+\lambda w_1\sigma_{1i}+\lambda w_2\sigma_{2i}+...+\lambda w_{i-1}\sigma_{i-1,i}+\lambda w_{i+1}\sigma_{i+1,i}+...+\lambda w_n\sigma_{ni})+r_i-r_f=0,$$

or rearranging the formula:

$$r_i-r_f=\lambda w_1\sigma_{1i}+\lambda w_2\sigma_{2i}+...+\lambda w_{i-1}\sigma_{i-1,i}+\lambda w_i\sigma_i^2+\lambda w_{i+1}\sigma_{i+1,i}+...+\lambda w_n\sigma_{ni} \qquad (4c.10)$$

The rate of return of the market portfolio is defined as:

$$r_M = \sum_{i=1}^{n} r_i w_i \,. \tag{4c.11}$$

The covariance between the rates of return of the k 'th asset and market portfolio is:

$$\operatorname{cov}(r_k r_M) = E[(r_k - \mu_k)(r_M - \mu_M)] \tag{4c.12}$$

where:

μ_k - expected rate of return of the k 'th asset,

μ_M - expected rate of return of the market portfolio.

Substituting (4c.11) in (4c.12) I have:

$$\operatorname{cov}(r_k r_M) = E[(r_k - \mu_k)(\sum_{i=1}^{n} r_i w_i - \sum_{i=1}^{n} \mu_i w_i)],$$

$$\operatorname{cov}(r_k r_M) = E[(r_k - \mu_k)(\sum_{i=1}^{n} w_i (r_i - \mu_i)],$$

$$\begin{aligned}\operatorname{cov}(r_k r_M) = E[&w_1(r_k - \mu_k)(r_1 - \mu_1) + \\ &+ w_2(r_k - \mu_k)(r_2 - \mu_2) + \\ &\ldots\ldots\ldots\ldots \\ &+ w_k(r_k - \mu_k)(r_k - \mu_k) + \\ &+ w_n(r_k - \mu_k)(r_n - \mu_n)].\end{aligned}$$

Then multiplying by λ:

$$\lambda \operatorname{cov}(r_k r_M) = \lambda E[w_1(r_k - \mu_k)(r_1 - \mu_1) + \ldots\ldots + w_n(r_k - \mu_k)(r_n - \mu_n)]. \tag{4c.13}$$

From (4c.10) and (4c.13) I have:

$$\lambda \operatorname{cov}(r_k r_M) = r_k - r_f \,.$$

For the market portfolio $\operatorname{cov}(r_M r_M) = \sigma_M^2$, so then:

$$\lambda \sigma_M^2 = r_M - r_f \,,$$

which gives λ:

$$\lambda = \frac{r_M - r_f}{\sigma_M^2},$$

so:

$$\frac{r_M - r_f}{\sigma_M^2} \operatorname{cov}(r_k r_M) = r_k - r_f,$$

and then finally:

$$r_k = r_f + \frac{\operatorname{cov}(r_k r_M)}{\sigma_M^2}(r_M - r_f).$$

Denoting by β_{kM} the measure of the risk of the k'th asset, where $\beta_{kM} = \frac{\operatorname{cov}(r_k r_M)}{\sigma_M^2}$ I get CAPM model:

$$r_k = r_f + (r_M - r_f)\beta_{kM} \quad .$$

APPENDIX 5

Derivation of the model with quantity constraints

The expected rate of return and the variance of market portfolio are defined as:

$$\mu_M = \sum_{j=1}^{n} w_j \mu_j \quad ,$$

$$\sigma_M^2 = \sum_{j=1}^{n} w_j^2 \sigma_j^2 + \sum_{j=1}^{n} \sum_{\substack{j=1 \\ j \neq k}}^{n} w_j w_k \sigma_{jk} \quad .$$

The problem is in maximisation of the utility function:

$$\max_{w_{ij}} V_i(\mu_i, \sigma_i^2) \quad .$$

The expected value of the i 'th portfolio is:

$$\mu_i = \sum_{j=1}^{n} w_{ij} \tilde{p}_j - R \sum_{j=1}^{n} (w_{ij} - \overline{w}_{ij}) p_j^0 \quad .$$

It is known that $\sum_{j=1}^{n} w_{ij} = 1$, then let $w_{in} = 1 - \sum_{j=1}^{n-1} w_{ij}$:

$$\mu_i = \sum_{j=1}^{n-1} w_{ij} \tilde{p}_j + (1 - \sum_{j=1}^{n-1} w_{ij}) \tilde{p}_n - R \left[\sum_{j=1}^{n} (w_{ij} - \overline{w}_{ij}) p_j^0 + (1 - \sum_{j=1}^{n-1} w_{ij}) p_n^0 \right] \quad .$$

The first derivatives of the expected value is:

$$\frac{\partial \mu_i}{\partial w_{i1}} = \tilde{p}_1 - \tilde{p}_n - R p_1^0 + R p_n^0 \quad .$$

The variance of the rate of return is given by:

$$\sigma_i^2 = \sum_{j=1}^{n} \sum_{k=1}^{n} w_{ij} w_{ik} \operatorname{cov}(p_{j,k}) \quad .$$

Because $\operatorname{cov}(p_{j,k}) = \operatorname{cov}(p_{k,j})$ the first derivative of the variance with respect to the first asset is:

$$\frac{\partial \sigma_i^2}{\partial w_{i1}} = 2 w_{i1} \operatorname{cov}(p_{1,1}) + 2 w_{i2} \operatorname{cov}(p_{1,2}) + \ldots + 2 w_{in} \operatorname{cov}(p_{1,n}) \quad ,$$

$$\frac{\partial \sigma_i^2}{\partial w_{i1}} = 2\sum_{k=1}^{n} w_{ik}\operatorname{cov}(p_{1,k}) \quad .$$

The derivative of the utility function for asset j is:

$$\frac{\partial V_i}{\partial w_{ij}} = \frac{\partial V_i}{\partial \mu_i}\frac{\partial \mu_i}{\partial w_{ij}} + \frac{\partial V_i}{\partial \sigma_i^2}\frac{\partial \sigma_i^2}{\partial w_{ij}} \quad ,$$

and, in case of the first asset:

$$\frac{\partial V_i}{\partial w_{i1}} = \frac{\partial V_i}{\partial \mu_i}\frac{\partial \mu_i}{\partial w_{i1}} + \frac{\partial V_i}{\partial \sigma_i^2}\frac{\partial \sigma_i^2}{\partial w_{i1}} \quad ,$$

$$\frac{\partial V_i}{\partial w_{i1}} = V_{i1}(\tilde{p}_1 - \tilde{p}_n - Rp_1^0 + Rp_n^0) + V_{i2}(2\sum_{k=1}^{n} w_{ik}\operatorname{cov}(p_{1,k})) \quad .$$

The first order condition of maximum of the utilitity function is given by:

$$V_{i1}(\tilde{p}_1 - \tilde{p}_n - Rp_1^0 + Rp_n^0) + V_{i2}\left[2\sum_{k=1}^{n} w_{ik}\operatorname{cov}(p_{1,k})\right] = 0 \quad .$$

Assume now that $\tilde{p}_n = (1+\delta)p_n^0$. Then:

$$V_{i1}\left[\tilde{p}_1 - (1+\delta)p_n^0 - Rp_1^0 + Rp_n^0\right] + V_{i2}(2\sum_{k=1}^{n} w_{ik}\operatorname{cov}(p_{1,k})) = 0 \quad .$$

For $R = 1 + r$, after dividing by p_1^0, I have:

$$V_{i1}\left[\frac{\tilde{p}_1 - p_1^0}{p_1^0} - r + (r-\delta)\frac{p_n^0}{p_1^0}\right] + 2V_{i2}\left[\sum_{k=1}^{n}\frac{w_{ik}}{p_1^0}\operatorname{cov}(p_{1,k})\right] = 0 \quad .$$

Making substitutions $r_{ik} = w_{ik} \cdot p_k^0$ and $\mu_1 = \dfrac{\tilde{p}_1 - p_1^0}{p_1^0}$ I obtain:

$$V_{i1}\left[\mu_1 - r + (r-\delta)\frac{p_n^0}{p_1^0}\right] + 2V_{i2}\left[\sum_{k=1}^{n}\frac{r_{ik}}{p_1^0 p_k^0}\operatorname{cov}(p_{1,k})\right] = 0.$$

Since $\sigma_{jk} = \dfrac{\operatorname{cov}(p_{j,k})}{p_j^0 p_k^0}$:

$$V_{i1}\left[\mu_1 - r + (r-\delta)\frac{p_n^0}{p_1^0}\right] = -2V_{i2}\sum_{k=1}^{n} r_{ik}\sigma_{1k} \quad .$$

If $\theta_i^{-1} = -\dfrac{V_{i1}}{2V_{i2}}$ then:

$$\theta_i^{-1}\left[\mu_1 - r + (r-\delta)\frac{p_n^0}{p_1^0}\right] = \sum_{k=1}^{n} r_{ik}\sigma_{1k} \quad .$$

If $\theta_M = (\sum_{i=1}^{n} \theta_i^{-1})^{-1}$, I also have:

$$\mu_1 - r + (r-\delta)\frac{p_n^0}{p_1^0} = \theta_M \sum_{k=1}^{n} r_{ik}\sigma_{1k} .$$

For the aggregate market I can write:

$$\mu_1 - rI + \left[(r-\delta)\frac{p_n^0}{p_1^0}\right]I = \theta_M (r\Omega),$$

where Ω is a variance-covariance matrix $[\text{cov}(j,k)]$. Ω could be substituted by $\theta_M = \frac{\mu_M - r}{\sigma_M^2}$. Then, for the the first asset the expression is:

$$\mu_1 - r + (r-\delta)\frac{p_n^0}{p_1^0} = \frac{\mu_M - r}{\sigma_M^2}\sigma_{1M} \quad ,$$

$$\mu_1 = r - (r-\delta)\frac{p_n^0}{p_1^0} + \frac{\sigma_{1M}}{\sigma_M^2}(\mu_M - r) \quad .$$

Let $\beta_1 = \frac{\sigma_{1M}}{\sigma_M^2}$. This gives:

$$r_1 = r + \beta_1 (r_M - r) - (r-\delta)\frac{p_n^0}{p_1^0} \quad .$$

For the individual asset j, by substituting r for r_f, I get:

$$r_j = r_f + \beta_j (r_M - r_f) - (r_f - \delta)\frac{p_n^0}{p_j^0} \quad , \qquad \text{where } \beta_j = \frac{\sigma_{jM}}{\sigma_M^2} \quad ,$$

which is the formula (5.9) in the main text.

GAUSS programs

1 Program for estimating *beta* parameters

```
new;
/* This program computes the uncorrected and corrected returns for
the Warsaw Stock Exchange and estimates betas using doubly
truncated Tobit and maximum likelihood method
        21 companies from 3.01.94 to 17.04.98
                November 1998                                    */

library coint, cml, pgraph;
#include pgraph.ext;
graphset;
#include cml.ext;
cmlset;


output file = c:\gauss\ewa\returns.out reset;

ntot = 1012;    @ Total no. of observations in data set @
nvar = 21;      @ No. of variables in data set @
rec0 = 100;     @ Number of first observations used for computation @
                @ of initial frequencies of hits @

/* 1 corresponds to the first session of 1994 (3 January)
  1012 corresponds to the session from 17.04.1998 */

rndseed 1964463448;
```

```
/* Data order in file data98t.asc: No., BRE, EFE, ELE, EXB, IRE, KAB, KRO, MSE,
   MSW, OKO, POL, PRO, SOK, SWA, TON, UNI, VIS, WBK, WED, WOL, ZYW.
   File wig1998d.asc contains data for WIG from the first session of 1994 until
   17.04.1998 */

load price[ntot,nvar] = c:\gauss\dat1998d.asc;
load wig[ntot,1] = c:\gauss\wig1998d.asc;
let weights = {0.047619 0.047619 0.047619 0.047619 0.047619 0.047619 0.047619
 0.047619 0.047619 0.047619 0.047619 0.047619 0.047619 0.047619 0.047619
 0.047619 0.047619 0.047619 0.047619 0.047619 0.047619};
/* weights for the 'private' index */

format 4,4;

/* price = price[., 2:(nvar+1)]; */


/*At present, weights are proportional to means of variables */
weights = weights.*meanc(price)';
weights = weights./sumc(weights');

wig21 = price*weights';          /* Private WIG for 21 companies */
wig21 = (wig21./wig21[1,1]).*1000;
pricea = price;
price = ln(price);
/* price = price[(lowp-1):rows(price),.];
pricea = pricea[(lowp-1):rows(pricea),.]; */

wig = ln(wig);
wig21 = ln(wig21);
```

```
/* Original returns */
ret1 = diff(price,1);
retw = diff(wig,1);
ret21 = diff(wig21,1);

tscale = seqa(1,1,rows(ret1));
tscale1 = seqa(1,1, rows(wig));


/*Graphs */
i = 1;
do until i > nvar;
  xy(tscale , ret1[.,i]);
  i = i + 1;
endo;

xy(tscale1, wig~wig14);

? " Wig and Wig21 series ";
wig~wig21;


/* Artifficially truncated returns */

tr=1.095;

ret=ret1.*(abs(ret1).<ln(tr))-ln(tr).*(ret1.<-ln(tr))+ln(tr).*(ret1.>ln(tr));

/* Randomisation of zero returns by 0.1 of st.dev. of particular returns */
i = 1;
do until i > nvar;
   j = 1;
```

```
  do until j > rows(ret);


    if ret[j,i] == 0;
      ret[j,i] = rndn(1,1).*0.1*stdc(ret[.,i]);
     endif;
    if price[j,i] == 0;
      price[j,i] = rndn(1,1).*0.1*stdc(ret[.,i]);
     endif;


    j = j + 1;
    endo;
 /*
 xy(tscale , ret[.,i]);
 */
 i = i + 1;
endo;

tr = ln(tr).*ones(rows(ret),cols(ret));

zup = ret.>=tr ;
zlo = ret.<=-tr;
zlo1 = -zlo;
zal = zup + zlo ;

sup = sumc(zup);
slo = sumc(zlo);
sal = sup+slo;

fhit=(sup+slo)./rows(ret);
fhit_u = sup./rows(ret);
fhit_l = slo./rows(ret);
```

```
/* Time-varying frequencies of hits */

fhit_ut = zeros(rows(ret),cols(ret));
fhit_lt = zeros(rows(ret),cols(ret));

/* Initial frequencies */
zup0 = zup[1:rec0,.];
zlo0 = zup[1:rec0,.];
fhit_u0 = sumc(zup0)./rec0;
fhit_l0 = sumc(zlo0)./rec0;

i = 1;
do until i > rec0;
 fhit_ut[i,.] = fhit_u0';
 fhit_lt[i,.] = fhit_l0';
i = i + 1;
endo;

/* Frequencies after initial */

i = rec0 + 1;
do until i > rows(ret);
  zupt = zup[1:i,.];
  zlot = zlo[1:i,.];
  fhit_uv = sumc(zupt)./i;
  fhit_lv = sumc(zlot)./i;
  fhit_ut[i,.] = fhit_uv';
  fhit_lt[i,.] = fhit_lv';
i = i + 1;
endo;

? "                    No. of upper hits ";
```

```
? "BRE EFE ELE EXB IRE KAB KRO MSE MSW OKO POL PRO SOK SWA
TON UNI VIS WBK WED WOL ZYW";
? ;
sup';
?;
? "                    No. of lower hits ";
? "BRE EFE ELE EXB IRE KAB KRO MSE MSW OKO POL PRO SOK SWA
TON UNI VIS WBK WED WOL ZYW";
? ;
slo';
?;
? "                    No. of all hits ";
? "BRE EFE ELE EXB IRE KAB KRO MSE MSW OKO POL PRO SOK SWA
TON UNI VIS WBK WED WOL ZYW";
? ;
sal';
?;
? "                    Frequency of hits ";
? "BRE EFE ELE EXB IRE KAB KRO MSE MSW OKO POL PRO SOK SWA
TON UNI VIS WBK WED WOL ZYW";
? ;
fhit';
?;
? "                  Frequency of lower hits ";
? "BRE EFE ELE EXB IRE KAB KRO MSE MSW OKO POL PRO SOK SWA
TON UNI VIS WBK WED WOL ZYW";
? ;
fhit_l';
?;
? "                   Frequency of upper hits ";
? "BRE EFE ELE EXB IRE KAB KRO MSE MSW OKO POL PRO SOK SWA
TON UNI VIS WBK WED WOL ZYW";
```

```
? ;
fhit_u';
?;
sdret1=stdc(ret1);
sdret=stdc(ret);
mret=meanc(ret);
mret1=meanc(ret1);
?;
? "                    Means of returns              ";
? "BRE EFE ELE EXB IRE KAB KRO MSE MSW OKO POL PRO SOK SWA
TON UNI VIS WBK WED WOL ZYW";
?;
mret';
?;
? "              Means of returns without outliers              ";
? "BRE EFE ELE EXB IRE KAB KRO MSE MSW OKO POL PRO SOK SWA
TON UNI VIS WBK WED WOL ZYW";
?;
mret1';
?;
? "              Standard deviation of returns              ";
? "BRE EFE ELE EXB IRE KAB KRO MSE MSW OKO POL PRO SOK SWA
TON UNI VIS WBK WED WOL ZYW";
?;
sdret1';
?;
? "          Standard deviation of returns without outl. ";
? "BRE EFE ELE EXB IRE KAB KRO MSE MSW OKO POL PRO SOK SWA
TON UNI VIS WBK WED WOL ZYW";
?;
sdret';
?;
```

```
? "                    Sharpe ratio.  ";
? "BRE EFE ELE EXB IRE KAB KRO MSE MSW OKO POL PRO SOK SWA
TON UNI VIS WBK WED WOL ZYW";
(mret./sdret1)';
?;
? "                 Sharpe ratio without outl.  ";
? "BRE EFE ELE EXB IRE KAB KRO MSE MSW OKO POL PRO SOK SWA
TON UNI VIS WBK WED WOL ZYW";
(mret1./sdret1)';

/* ? "Standard deviation of nontruncated returns:   ";; sdnt; */
?;
/*
{b,m,freq}=hist(ret,10);
*/
/*
b~m~freq; wait;
*/

/* ML, Twotobit and OLS results */
_olsres = 1;
__con = 1;

limit_u = ln(1.095);
limit_l = -ln(1.095);

limit_um = 10;      @ "Limits" for the ML estimation@
limit_lm = -10;

_cml_ParNames = "CONST" | "RetM" | "Variance";

_ww_ = { -10 10 };
```

```
_cml_Bounds = ones(3,2).*_ww_;          @ Bounds for all params @
_cml_Bounds[3,1] = .00001;          @ Variance must be positive @
_cml_MaxIters = 250;

_cml_CovPar = 3;
_cml_Delta = 1e-3;    /* floor for Hessian eigenvalues when n.p.d  */
_cml_Algorithm = 3;

res1 = zeros(ntot-1,nvar);
res2 = zeros(ntot-1,nvar);
res3 = zeros(ntot-1,nvar);
res4 = zeros(ntot-1,nvar);
res5 = zeros(ntot-1,nvar);
res6 = zeros(ntot-1,nvar);
res7 = zeros(ntot-1,nvar);
res8 = zeros(ntot-1,nvar);

price = price[2:rows(price),.];
i = 1;
do until i > 1;

 ?;
 ? "x x  x x x x x x x x x x x x x x x x x x x x x x x x x x x x x x x x  ";
 ? "
XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
XXXXXXXXXXXXXXXXX " ;
 ? "x x  x x x x x x x x x x x x x x x x x x x x x x x x x x x x x x x x  ";
   @ Aggregated market returns except for the return on the i-th asset @
    ?;
    if i == 1    ; ? " ******************** BRE ********************";
    elseif i == 2; ? " ******************* Efekt *******************";
    elseif i == 3; ? " ***************** Elektrim *****************";
```

```
elseif i == 4; ? " ******************** Exbud *******************";
elseif i == 5; ? " ******************** Irena *******************";
elseif i == 6; ? " ******************** Kable *******************";
elseif i == 7; ? " ******************** Krosno ******************";
elseif i == 8; ? " **************** Mostostal Exp ***************";
elseif i == 9; ? " **************** Mostostal Waw ***************";
elseif i == 10;? " ******************** Okocim ******************";
elseif i == 11;? " ******************* Polifarb *****************";
elseif i == 12;? " ******************* Prochnik *****************";
elseif i == 13;? " ******************* Sokolow ******************";
elseif i == 14;? " ******************* Swarzedz *****************";
elseif i == 15;? " ******************** Tonsil ******************";
elseif i == 16;? " ****************** Universal *****************";
elseif i == 17;? " ******************* Vistula ******************";
elseif i == 18;? " ********************* WBK ********************";
elseif i == 19;? " ******************** Wedel *******************";
elseif i == 20;? " ****************** Wolczanka *****************";
elseif i == 21;? " ******************** Zywiec ******************";
endif;
?;

reti = ret[.,i];        @ Original truncated returns     @
retxi = ret1[.,i];         @ Original returns             @
prii = price[.,i];

retm = ret*weights' - weights[i]*reti;

@ Correction factor @
/* zal = zup.*fhit_u' - zlo.*fhit_l'; */
zal = zup.*fhit_ut - zlo.*fhit_lt;

cfac = tr[1,1].*(exp(price[.,i]).*zal[.,i].*weights[.,i])./exp(price[.,i]);
```

```
  @ Corrected returns @
   retc = reti - cfac;

  @ Truncation of corrected returns @
tr = 1.095;
retx=retc.*(abs(retc).<ln(tr))-ln(tr).*(retc.<-ln(tr))+ln(tr).*(retc.>ln(tr));

   /*
   xy(tscale,reti~retc);
   */
/*
   if i == 9;
   /*
? " Tonsil : corrected returns, original returns, differences ";

   retc~reti~retc-reti;
   */
   ?;
   ? " Tonsil; original returns " ;
   {c,m,freqo} = hist(reti,30);
   ?;
   ? " Tonsil; corrected returns ";
   {c,m,freqc} = hist(retc,30);
   c~freqo~freqc;

   endif;
*/

xx = retxi~retm~retw;
e = (abs(xx[.,1]) .lt ln(tr));
y = selif(xx, e);
xc = retc~retm~retw;
```

```
ec = (abs(xc[.,1]) .lt ln(tr));
yc = selif(xc, ec);

xm = ones(rows(reti),1)~retm;  @ WIG21 and original returns @
zmi = reti~xm;
startmi = invpd(xm'xm)*xm'reti;
ssmi = sumc((reti - xm*startmi)^2)./(rows(reti) - cols(xm));
startmi = startmi | ssmi;

zmc = retc~xm;                  @ WIG21 and corrected returns @
startmc = invpd(xm'xm)*xm'retc;
ssmc = sumc((retc - xm*startmc)^2)./(rows(retc) - cols(xm));
startmc = startmc | ssmc;

xw = ones(rows(reti),1)~retw;  @ WIG and original returns @
zmw = reti~xw;
startwi = invpd(xw'xw)*xw'reti;
sswi = sumc((reti - xw*startwi)^2)./(rows(reti) - cols(xw));
startwi = startwi | sswi;

zwc = retc~xw;                  @ WIG and corrected returns @
startwc = invpd(xw'xw)*xw'retc;
sswc = sumc((retc - xw*startwc)^2)./(rows(retc) - cols(xw));
startwc = startwc | sswc;

?;
?;
? " xxxxxxxxxxxxxxx Estimations with WIG21 xxxxxxxxxxxxxxxxxxxxxxx ";
?;
_title = "WIG21, TOBIT and original returns";
{b,f,g,h,retcode} = cml(zmi, 0, &twotobit, startmi);
?;
```

```
?;
_title = "WIG21, TOBIT and original returns";
call cmlprt(b,f,g,h,retcode);
b = b[1:2,.];
res1[.,i] = reti - (xm*b);
?;
?;
_title = "WIG21, maximum likelihood and corrected returns";
{b,f,g,h,retcode} = cml(zmc, 0, &twot_2, startmc);
?;
?;
_title = "WIG21, maximum likelihood and corrected returns";
call cmlprt(b,f,g,h,retcode);
b = b[1:2,.];
res2[.,i] = retc - (xm*b);
?;
?;
? "    Wig21, OLS and uncorrected returns  " ;
?;
{a1,a2,a3,a4,a5,a6,a7,a8,a9,a10,a11}=ols(0,reti,retm);
res3[.,i] = reti - xm*(a3);
?;
? "    Wig21, OLS and corrected returns  " ;
?;
{a1,a2,a3,a4,a5,a6,a7,a8,a9,a10,a11}=ols(0,retc,retm);
res4[.,i] = retc - xm*(a3);
?;
? " xxxxxxxxxxxxxxx Regressions with WIG xxxxxxxxxxxxxxxxxxxxxxx ";
?;
?;
_title = "WIG, TOBIT and original returns";
{b,f,g,h,retcode} = cml(zmw, 0, &twotobit, startwi);
```

```
?;
?;
_title = "WIG, TOBIT and original returns";
call cmlprt(b,f,g,h,retcode);
b = b[1:2,.];
res5[.,i] = reti - (xw*b);
?;
?;
_title = "WIG, maximum likelihood and corrected returns";
{b,f,g,h,retcode} = cml(zwc, 0, &twot_2, startwc);
?;
?;
_title = "WIG21, maximum likelihood and corrected returns";
call cmlprt(b,f,g,h,retcode);
b = b[1:2,.];
res6[.,i] = retc - (xw*b);
?;
?;
? "    Wig, OLS and uncorrected returns  " ;
?;
{a1,a2,a3,a4,a5,a6,a7,a8,a9,a10,a11}=ols(0,reti,retw);
res7[.,i] = reti - xw*(a3);
?;
? "    Wig, OLS and corrected returns  " ;
?;
{a1,a2,a3,a4,a5,a6,a7,a8,a9,a10,a11}=ols(0,retc,retw);
res8[.,i] = retc - xw*(a3);
?;


   i = i + 1;
 endo;
print "Resisuals based on WIG21, Tobit and original returns";
```

```
? "BRE EFE ELE EXB IRE KAB KRO MSE MSW OKO POL PRO SOK SWA
TON UNI VIS WBK WED WOL ZYW";
res1;
print "Resisuals based on WIG21, Maximum likelihood and corrected returns";
? "BRE EFE ELE EXB IRE KAB KRO MSE MSW OKO POL PRO SOK SWA
TON UNI VIS WBK WED WOL ZYW";
res2;
print "Resisuals based on WIG21, OLS and original returns";
? "BRE EFE ELE EXB IRE KAB KRO MSE MSW OKO POL PRO SOK SWA
TON UNI VIS WBK WED WOL ZYW";
res3;
print "Resisuals based on WIG21, OLS and corrected returns";
? "BRE EFE ELE EXB IRE KAB KRO MSE MSW OKO POL PRO SOK SWA
TON UNI VIS WBK WED WOL ZYW";
res4;
print "Resisuals based on WIG, Tobit and original returns";
? "BRE EFE ELE EXB IRE KAB KRO MSE MSW OKO POL PRO SOK SWA
TON UNI VIS WBK WED WOL ZYW";
res5;
print "Resisuals based on WIG, Maximum likelihood and corrected returns";
? "BRE EFE ELE EXB IRE KAB KRO MSE MSW OKO POL PRO SOK SWA
TON UNI VIS WBK WED WOL ZYW";
res6;
print "Resisuals based on WIG, OLS and original returns";
? "BRE EFE ELE EXB IRE KAB KRO MSE MSW OKO POL PRO SOK SWA
TON UNI VIS WBK WED WOL ZYW";
res7;
print "Resisuals based on WIG, OLS and corrected returns";
? "BRE EFE ELE EXB IRE KAB KRO MSE MSW OKO POL PRO SOK SWA
TON UNI VIS WBK WED WOL ZYW";
res8;
end;
```

```
/*************** procedures *******************/

proc twotobit(b, x);
  local u2,h,s_hig,s_low,s_mid,fit,upl,umi;
  h = b[rows(b)];
  fit = x[.,2:cols(x)] * b[1:(rows(b)-1),.];       @ fitted values     @
  u2 = (x[.,1] - fit)^2;                           @ squared residuals @
  upl = limit_u - fit;
  umi = limit_l - fit;

  s_hig = x[.,1] .>= limit_u;
  s_low = x[.,1] .<= limit_l;
  s_mid = 1 - (s_hig + s_low);
  retp( s_mid.*(-0.5*( (u2 ./ h) + ln(2 * pi) + ln(h^2) ) ) +
    s_hig.*( ln(1 - cdfn(upl./sqrt(h)))) +
    s_low.*(ln(cdfn(umi./sqrt(h)))));

endp;

proc twot_2(b, x);
  local u2,h,s_hig,s_low,s_mid,fit,upl,umi;
  h = b[rows(b)];
  fit = x[.,2:cols(x)] * b[1:(rows(b)-1),.];       @ fitted values     @
  u2 = (x[.,1] - fit)^2;                           @ squared residuals @
  upl = limit_um - fit;
  umi = limit_lm - fit;

  s_hig = x[.,1] .>= limit_u;
  s_low = x[.,1] .<= limit_l;
  s_mid = 1 - (s_hig + s_low);
  retp( s_mid.*(-0.5*( (u2 ./ h) + ln(2 * pi) + ln(h^2) ) ) +
```

```
    s_hig.*( ln(1 - cdfn(upl./sqrt(h)))) +
    s_low.*(ln(cdfn(umi./sqrt(h)))));

endp;
```

2 Program simulating portfolios with constraints

```
new;
/*  Simple empirical experiment.
    This program simulates portfolios with all combinations of 3 types of assets
                              with one market constraint
     and calculates expected returns and standard deviations
                                      May 1998                                  */

library pgraph; graphset;
output file = f:\gauss\ewa\simul.out reset;

npoints = 100;
n=npoints;
x1v = seqa(1,1,npoints); x1v=x1v./npoints;
x1v = 0|x1v;

npoints = npoints + 1;
m1 = 10;
m2 = 20;
m3 = 16;
rho12 = -0.1;
rho13 = 0.2;
rho23 = 0.5;

s1 = sqrt(100);
s2 = sqrt(400);
s3 = sqrt(121);
constr1 = 0.4;
constr2 = 0.2;
```

```
npoints2 = (n+1)^2;
erv = zeros(npoints2,1);
ervq = zeros(npoints2,1);
spv = zeros(npoints2,1);
spvq = zeros(npoints2,1);

x1 = zeros(n+1,1);
x = zeros(n+1,1);
x2v= x1v; x2a = x2v;
i = 1;
do until i >= n+1;
  x1=x1+(1/n);
  x=x|x1;
x2v=x2v|x2a;
i = i+1;
endo;
x1v=x;



i = 1;
do until i > npoints2;

x1 = x1v[i,.];
x2 = x2v[i,.];
 if x1+x2 > 1;  x2 = 1-x1; endif;

x1q = minc(x1v[i,.]|constr1);
x2q = minc(x2v[i,.]|constr2);
 if x1q+x2 > 1; x2 = 1-x1q; endif;

x3 = 1 - x1 - x2;
x3q = 1 - x1 - x2;
```

```
er  = x1*m1 + x2*m2 + x3*m3;              /* expected returns */
erq = x1q*m1 + x2*m2 + x3q*m3;

cor12 = 2*x1*x2*rho12*s1*s2;              /* correlation coefficients */
cor13 = 2*x1*x3*rho13*s1*s3;
cor23 = 2*x2*x3*rho23*s2*s3;
cor12q = 2*x1q*x2q*rho12*s1*s2;
cor13q = 2*x1q*x3q*rho13*s1*s3;
cor23q = 2*x2q*x3q*rho23*s2*s3;

s2p = (x1^2)*(s1^2) + (x2^2)*(s2^2) + (x3^2)*(s3^2) + cor12 + cor13 + cor23;
s2pq = (x1q^2)*(s1^2)+(x2^2)*(s2^2)+(x3q^2)*(s3^2) + cor12q + cor13q + cor23q;
sp = sqrt(s2p);
spq = sqrt(s2pq);

erv[i,1] = er;
ervq[i,1] = erq;
spv[i,1] = sp;
spvq[i,1] = spq;

i = i + 1;
endo;


format 10,3;

? "     Prop.   ex.ret.u     ex.ret.c.     s.td.u     s.td.c. ";
?;

x1v~x2v~erv~ervq~spv~spvq;
wait;
```

```
_pdate = 0;
_pcolor = 1;
_plegctl = 1;
graphprt("-w=5 -c=3 -cf=gr1.plt");
title("Frontier against unconstrained risk");
_plegstr= "unconstrained exp. returns\000constrained exp. returns";
xy(spv,erv~ervq);

graphprt("-w=5 -c=3 -cf=gr2.plt");
title("Frontier against constrained risk");
_plegstr= "unconstrained exp. returns\000constrained exp. returns";

xy(spvq,erv~ervq);

 end;
```

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