



# STELLAR WIND ACCRETION AND DYNAMICS IN BINARY STARS AND EXOPLANETARY SYSTEMS

**Hastyar Omar Mohammed**

Submitted in fulfilment of the requirements of the degree of  
Doctor of Philosophy

November 19, 2015

Theoretical Astrophysics Group  
Department of Physics and Astronomy  
University of Leicester

---

# Abstract

## Stellar Wind Accretion and Dynamics in Binary Stars and Exoplanetary Systems

by

**Hastyar Omar Mohammed**

This thesis work is concerned with the accretion processes and flow structures associated with stellar winds in binary systems. I study mass transfer via stellar wind capture in symbiotic recurrent nova RS Ophiuchi, using Smoothed Particle Hydrodynamics. I investigate the modes of mass transfer from the mass-losing star to the mass-accreting companion by implementing wind expansion based on the analytical Parker solution for isothermal winds. Mass capture fractions are calculated and found to be dependent on the velocity of the wind. The structure of the accretion discs formed is also investigated. The results show that all the accretion discs have radial extents larger than the predicted stability radius against thermal viscous instabilities. It is nevertheless found that the surface density profiles of the accretion discs are too low to trigger such disc outbursts. I also explore the effect of rotation on mass transfer and disc morphology.

I also study the interaction between the transiting hot Jupiter WASP-12b and its host star, using ZEUS-2D and SPH-3D to simulate the planetary magnetosphere interactions with the stellar wind self-consistently. I attempt to model NUV absorption due to enhancements in density at the bow shock ahead of the planet. The numerical results show that the bow shock is always weak and broad due to the modest wind Mach number at the planetary distance. I compute theoretical UV light-curves from the hydrodynamic models and use a grid of stellar wind, planetary magnetic field strength and wind opacity parameters to show how the UV light-curves depend on different physical model parameters. The results show consistency with the existing UV data for WASP-12b. I also model two other transiting hot Jupiters and show that additional UV observations of more massive short-orbit hot Jupiters should distinguish clearly between different models for circumplanetary absorption.

---

# Acknowledgements

I would like to acknowledge:

Dr Graham Wynn, from whom I learnt a lot and for his patience and support during his supervision on my PhD project. I am grateful for all these.

Dr Christopher Nixon, for all his kind assistance regarding the SPH code and helpful discussions.

Dr Richard Alexander and Dr Mark Wilkinson for any scientific discussions I have had with them during the past four years of my study.

All the PhD students in the Theoretical Astrophysics group during my study at the University of Leicester for any beneficial informations, especially Martin Bourne who offered to do proofreading for my thesis.

---

# Contents

|                                                                        |           |
|------------------------------------------------------------------------|-----------|
| <b>Abstract</b>                                                        | <b>i</b>  |
| <b>Acknowledgements</b>                                                | <b>ii</b> |
| <b>1 Introduction</b>                                                  | <b>1</b>  |
| 1.1 Binary Systems . . . . .                                           | 2         |
| 1.1.1 Binary Geometry . . . . .                                        | 2         |
| 1.1.2 Binary total energy and orbital angular momentum . . . . .       | 3         |
| 1.1.3 Roche Lobe Geometry . . . . .                                    | 4         |
| 1.1.4 Observational Classification of binary stars . . . . .           | 7         |
| 1.1.5 Types of interaction in binary systems . . . . .                 | 7         |
| 1.1.6 Orbital evolution due to mass transfer . . . . .                 | 8         |
| 1.1.6.1 Conservative mass transfer . . . . .                           | 8         |
| 1.1.6.2 Non-conservative mass transfer . . . . .                       | 10        |
| 1.2 Accretion . . . . .                                                | 10        |
| 1.2.1 Accretion power and Eddington Limit . . . . .                    | 11        |
| 1.2.2 Accretion Disc Theory . . . . .                                  | 12        |
| 1.2.2.1 Conservation of mass . . . . .                                 | 13        |
| 1.2.2.2 Conservation of angular momentum . . . . .                     | 14        |
| 1.2.2.3 Viscous evolution of a ring . . . . .                          | 16        |
| 1.2.3 Accretion Disc timescales . . . . .                              | 16        |
| 1.3 Stellar Wind . . . . .                                             | 17        |
| 1.4 Accretion via stellar wind capture (analytical approach) . . . . . | 21        |
| 1.5 Symbiotic stars and symbiotic novae . . . . .                      | 22        |
| 1.6 Type Ia Supernovae . . . . .                                       | 24        |
| 1.7 RS Ophiuchi . . . . .                                              | 25        |
| 1.8 Exoplanets and their detection . . . . .                           | 27        |
| 1.8.1 Methods of Exoplanet detection . . . . .                         | 27        |
| 1.8.1.1 Direct detection . . . . .                                     | 27        |
| 1.8.1.2 Radial velocity . . . . .                                      | 29        |

---

|          |                                                               |           |
|----------|---------------------------------------------------------------|-----------|
| 1.8.1.3  | Pulsar timing . . . . .                                       | 30        |
| 1.8.1.4  | Transits . . . . .                                            | 30        |
| 1.8.1.5  | Astrometry . . . . .                                          | 31        |
| 1.8.1.6  | Microlensing . . . . .                                        | 32        |
| 1.9      | This Thesis . . . . .                                         | 33        |
| <b>2</b> | <b>Theories of Fluid Dynamics</b>                             | <b>34</b> |
| 2.1      | Introduction . . . . .                                        | 35        |
| 2.1.1    | Grid-based numerical method . . . . .                         | 35        |
| 2.1.1.1  | Lagrangian grid-based method . . . . .                        | 36        |
| 2.1.1.2  | Eulerian grid-based method . . . . .                          | 36        |
| 2.1.2    | Mesh-free method . . . . .                                    | 36        |
| 2.2      | Fluid Equations . . . . .                                     | 37        |
| 2.2.1    | Conservation of mass . . . . .                                | 37        |
| 2.2.2    | Conservation of momentum . . . . .                            | 37        |
| 2.2.3    | Conservation of Energy . . . . .                              | 38        |
| 2.2.4    | Equations of state . . . . .                                  | 38        |
| 2.3      | Smoothed Particle Hydrodynamics . . . . .                     | 39        |
| 2.3.1    | Discrete approximation to a continuous Scalar Field . . . . . | 40        |
| 2.3.2    | Gradient of a scalar field . . . . .                          | 41        |
| 2.3.3    | Vector calculus . . . . .                                     | 41        |
| 2.3.4    | The smoothing Kernel . . . . .                                | 42        |
| 2.3.5    | The smoothing length . . . . .                                | 43        |
| 2.4      | SPH fluid equations . . . . .                                 | 44        |
| 2.4.1    | Conservation of mass . . . . .                                | 44        |
| 2.4.2    | Conservation of momentum . . . . .                            | 45        |
| 2.4.3    | Conservation of Energy . . . . .                              | 46        |
| 2.4.4    | Dissipative terms and viscosity switching . . . . .           | 46        |
| 2.4.5    | Time-stepping . . . . .                                       | 47        |
| 2.5      | The SPH code . . . . .                                        | 48        |
| 2.6      | SPLASH . . . . .                                              | 50        |
| 2.7      | ZEUS-2D . . . . .                                             | 52        |
| 2.7.1    | Operator split solution . . . . .                             | 53        |
| 2.7.2    | The finite-difference mesh . . . . .                          | 55        |

---

|          |                                                                  |           |
|----------|------------------------------------------------------------------|-----------|
| <b>3</b> | <b>The Recurrent Nova RS Ophiuchi</b>                            | <b>57</b> |
| 3.1      | Introduction . . . . .                                           | 58        |
| 3.2      | The recurrent outburst in RS Ophiuchi . . . . .                  | 58        |
| 3.3      | This chapter . . . . .                                           | 62        |
| 3.4      | Accretion disc stability and limit cycle . . . . .               | 62        |
| 3.5      | Computational Method . . . . .                                   | 64        |
| 3.6      | Simulation setup . . . . .                                       | 65        |
| 3.7      | Results . . . . .                                                | 67        |
| 3.7.1    | Mass capture fraction . . . . .                                  | 71        |
| 3.7.2    | Comparison of the numerical results with BHL . . . . .           | 73        |
| 3.7.3    | Mass accumulation between outbursts in RS Oph . . . . .          | 76        |
| 3.8      | Rotation of the red giant . . . . .                              | 79        |
| 3.8.1    | Effects of rotation on stellar shape and mass loss . . . . .     | 80        |
| 3.8.2    | The effects of rotation on the RG wind . . . . .                 | 81        |
| 3.8.3    | Mass transfer in RS Oph with synchronised red giant . . . . .    | 83        |
| 3.9      | Discussion . . . . .                                             | 88        |
| 3.10     | Summary . . . . .                                                | 92        |
| <b>4</b> | <b>The Transiting Hot Jupiter WASP 12b</b>                       | <b>94</b> |
| 4.1      | Introduction . . . . .                                           | 95        |
| 4.2      | This Chapter . . . . .                                           | 96        |
| 4.3      | Bow-shock formation . . . . .                                    | 97        |
| 4.4      | Two-dimensional simulations of WASP-12b . . . . .                | 99        |
| 4.4.1    | Numerical Setup . . . . .                                        | 99        |
| 4.4.2    | Stellar Wind model . . . . .                                     | 99        |
| 4.4.3    | Planet model . . . . .                                           | 101       |
| 4.5      | WASP-12b models . . . . .                                        | 103       |
| 4.5.1    | Simulations . . . . .                                            | 103       |
| 4.5.2    | Light-curves . . . . .                                           | 108       |
| 4.6      | Models of other systems . . . . .                                | 114       |
| 4.6.1    | Results . . . . .                                                | 115       |
| 4.7      | Discussion . . . . .                                             | 117       |
| 4.7.1    | Limitations . . . . .                                            | 117       |
| 4.7.2    | WASP-12 vs. WASP-18: a critical test of UV absorption theories . | 120       |
| 4.8      | WASP-12b transit light-curves in 3D . . . . .                    | 121       |

---

|          |                                                               |            |
|----------|---------------------------------------------------------------|------------|
| 4.8.1    | Results of the 3D model of WASP-12b transits . . . . .        | 121        |
| 4.8.2    | Comparison between 2D and 3D results . . . . .                | 125        |
| 4.9      | Summary . . . . .                                             | 127        |
| <b>5</b> | <b>Summary and Conclusion</b>                                 | <b>129</b> |
| 5.1      | Summary . . . . .                                             | 130        |
| 5.2      | Future work . . . . .                                         | 132        |
| <b>A</b> | <b>Appendix A</b>                                             | <b>134</b> |
| A.1      | Binary Orbits . . . . .                                       | 135        |
| A.2      | Orbital total kinetic energy and Angular momentum . . . . .   | 137        |
| A.3      | Conservation of mass in accretion discs . . . . .             | 138        |
| A.4      | Viscous torque . . . . .                                      | 140        |
| A.5      | Conservation of angular momentum in accretion discs . . . . . | 140        |
| <b>B</b> | <b>Appendix B</b>                                             | <b>143</b> |
| B.1      | Conservation equations in fluids . . . . .                    | 144        |
| B.2      | SPH formalisms . . . . .                                      | 145        |
| <b>C</b> | <b>Appendix C</b>                                             | <b>148</b> |
| C.1      | Force due to the gradient of magnetic pressure . . . . .      | 149        |
| C.2      | Planet-centred accelerations in polar co-ordinates . . . . .  | 149        |
|          | <b>References</b>                                             | <b>151</b> |

---

# List of Tables

|     |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |    |
|-----|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----|
| 3.1 | Subsonic injection simulations. In column 1, number of simulations; column 2, sound speed of the wind; column 3, critical radius of the wind in units of binary separations; column 4, injection velocity of the wind in units of the wind sound speed; column 5, numerical capture fraction and column 6, the CPU time consumption for the simulations to reach steady state in hours. It is worth mentioning that binary mass ratio for all these simulations is $q = 0.6$ . . . . .                                                                                                                                                                          | 69 |
| 3.2 | Supersonic injection simulations. The columns have the same definition as those in Table 3.1. Also, the binary mass ratio $q = 0.6$ . . . . .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   | 70 |
| 3.3 | Effect of rotation on stellar surface, mass loss and structure of the outflow. Three temperature regimes are selected, corresponding to $c_s = 7, 13, \text{ and } 19 \text{ km s}^{-1}$ . Wind parameters are then compared for rotation that is synchronous with the RS Oph orbital period and for critical stellar rotation. Here, the ratio of the stellar radius at the equator and at the pole ( $R_{eq}/R_p$ ) is calculated using (3.13), while the equivalent density ratio, $\rho_{eq}/\rho_p$ is obtained numerically. The increase factor in mass loss compared to a non-rotating RG ( $\dot{M}/\dot{M}_{no}$ ) is also measured in the simulations | 82 |
| 3.4 | Recurrence time between thermal-viscous instability-driven outbursts. Three simulations are selected, with initially subsonic winds where $c_s = 6, 10, \text{ and } 19 \text{ km s}^{-1}$ . The maximum surface density has been calculated at 3 different disc radii from (3.16). The steady state surface density is listed for each simulation alongside an upper limit for $d\Sigma/dt$ . Recurrence times have then been estimated from these values. . . . .                                                                                                                                                                                             | 90 |

---

# List of Figures

|     |                                                                                                                                                                                                                                                                                                                                                                                                                                       |    |
|-----|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----|
| 1.1 | Geometry of an ideal binary system having binary mass ratio $q = 0.6$ and eccentricity of 0.6. . . . .                                                                                                                                                                                                                                                                                                                                | 3  |
| 1.2 | Upper panel: Equipotential surfaces (contours in 2D xy plane), locations of Lagrangian points can be seen. Lower panel: The binary gravitational potential in 1D along the line where $L_1$ , $L_2$ and $L_3$ lie. The code units used here are as follows [ $G = 1$ , $M_1 + M_2 = 1$ and $a = 1$ ]. Binary mass ratio is $q = 0.6$ . . . . .                                                                                        | 6  |
| 1.3 | Types of binaries in interaction through mass transfer. Top panel: the size of the two components are less than their Roche volume and thus no mass transfer through $L_1$ . Middle panel: one of the components undergo RLO and mass transfers to the companions Roche volume. Bottom panel: both components overfill their Roche lobes and a contact binary is produced. Figure credit to Pearson Prentice Hall, Inc. 2005. . . . . | 9  |
| 1.4 | Diffusion of a ring of matter under the influence of viscosity and gravity of the accretor. $x = \frac{R}{R_o}$ , $\sigma = \frac{\pi r^2 \Sigma}{m}$ and $\tau = 12 \nu t R_o^{-2}$ . . . . .                                                                                                                                                                                                                                        | 15 |
| 1.5 | The solutions of Parker wind equation. The blue curve is the case of accelerated wind. The red line represents the supersonic set off of the wind from the beginning. The red and blue solutions are the two types of wind outflow from a simulated Red giant in RS Oph discussed in Chapter 3. . . . .                                                                                                                               | 19 |
| 1.6 | Chandra images resolves the components of Mira AB and a bridge between them as an evidence of mass transfer (left). On the right is an artists impression of the interacting system, showing evidence of mass transfer through RLO-like and wind (Karovska, 2006, and references therein). . .                                                                                                                                        | 24 |
| 1.7 | Observations of RS Oph outbursts from AAVSO database. Top panel shows the light-curve of RS Ophiuchi during outburst and quiescent phases from 1933 outburst to the quiescent phase after 1985 outburst. Bottom panel shows visual magnitude of the last outburst of RS Oph in 2006. Figures credit to AAVSO archives. . . . .                                                                                                        | 26 |

|     |                                                                                                                                                                                                                                                                                                                                                                  |    |
|-----|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----|
| 1.8 | Mass-orbital period distribution of discovered exoplanets to date by different methods. Up-to-date figure is available at ( <a href="http://exoplanetarchive.ipac.caltech.edu/">http://exoplanetarchive.ipac.caltech.edu/</a> ).                                                                                                                                 | 28 |
| 1.9 | A model light-curve of a central star with a transiting planet. The brightness of the host star changes when a transiting planet orbiting the star. Figure credit to <a href="http://www.superwasp.org/">http://www.superwasp.org/</a> .                                                                                                                         | 31 |
| 2.1 | SPLASH interactive visualisation of SPH simulations. Top, a snapshot of wind- Roche lobe overflow simulation in a binary, with an accretion disc formed (SPH particles in black with two sink particles in red). Bottom, the same snapshot in it's fluid version column density calculated with the arrows representing the velocity of the flow in the regions. | 51 |
| 2.2 | Discretisation of the two dimensional domain, showing grid-cells with the ghost cells.                                                                                                                                                                                                                                                                           | 54 |
| 2.3 | A typical grid-cell. Scalar quantities such as density and pressure are zone centered, while the vector quantities are face centered; velocity is shown as an example.                                                                                                                                                                                           | 55 |
| 3.1 | The dependence of the amount of mass required to trigger nova explosion on the mass of the White dwarf and accretion rate (Nomoto, 1982; Nomoto et al., 2007). Also shown in the figure the range of accretion rate leading to steady nuclear burning for different white dwarf masses. Figure credit to Nomoto et al. (2007).                                   | 59 |
| 3.2 | Limit cycle of an annulus of radius $R$ in an accretion disc. The upper part (CD) and the lower part (AB) of the S-curve are the stable thermal equilibria. The middle part (BC) represents the the unstable solution for the disc thermal equilibrium. Figure adapted from Frank et al. (2002).                                                                 | 63 |
| 3.3 | Radial velocity profile of an initially supersonic simulated stellar wind. The profile is consistent with the Parker solutions illustrated by the red line.                                                                                                                                                                                                      | 65 |
| 3.4 | Radial velocity profile of an initially subsonic simulated wind. The profile is consistent with the Parker solution represented by the red line.                                                                                                                                                                                                                 | 66 |

- 3.5 Example morphologies of the accretion disc around the WD, formed via mass transfer from the RG wind. From left to right, panels show morphologies for the three types of mass transfer observed in the simulations; Case A (wind RLO), Case B (RG mass loss via  $L1$  and  $L2$ ) and Case C (spherically symmetric wind outflow). These are shown for simulations with subsonic wind injection of 6, 10 and 19  $\text{km s}^{-1}$  for the left middle and right panels respectively. In the upper panels, images from the simulations are shown, face-on (x-y plane) and side-on (x-z plane), with the colour scale indicating particle column densities. The middle rows then show the one-dimensional surface density profiles of the accretion disc around the WD. Finally, the orbital velocity profiles of these discs are plotted in the lower panels. Here, velocities of individual particles are shown in blue and the Kepler profiles are plotted in red. . . . . 68
- 3.6 Time evolution of the accretion fraction for five simulations with subsonic RG wind velocities. Mass capture fraction is plotted against number of orbits for sound speeds of 5.78, 7, 9, 15 and 19  $\text{km s}^{-1}$  in the top to bottom panels respectively. In all cases the disc has reached steady state after 2–4 orbits. The discs in the upper two panels undergo Case A mass transfer, while the 3rd panel represents Case B transfer and Case C transfer is shown in the bottom 2 panels. . . . . 72
- 3.7 Mass capture fractions for simulations with subsonic wind injection. The corresponding wind temperatures are shown in the upper horizontal axis. Error bars on the y-axis indicate the variance in the mass capture fraction once steady state has been reached, as shown in figure 3.6. . . . . 74
- 3.8 Mass capture fraction for simulations with supersonic wind injection. Mass capture fraction is plotted against the ratio of injection velocity to sound speed for four different sound speeds, coloured according to the figure legend. . . . . 75
- 3.9 Comparison of the mass capture fraction results from the simulations to the BHL estimate. In the upper panel, the capture fraction is plotted against the ratio of the wind speed at the WD position to the orbital speed of the RG. This is shown for the theoretical BHL capture fraction (red points) and simulations with subsonic wind injection (blue). In the lower panels the vertical axis shows the ratio of the BHL estimate to the numerical result. . . . . 77

- 3.10 Mass accumulated between novae in units of  $M_{nov}$  versus wind sound speed. The horizontal black line represents the mass accumulation for nova every 20 years. The blue curve is the total accreted mass normalised to  $M_{nov}$  for  $M_{WD} = 1.4M_{\odot}$  and mass accretion rate  $\dot{M} = 10^{-7}M_{\odot}/\text{yr}$  from (Yaron et al., 2005). The red curve is similar but for  $\dot{M} = 10^{-8}M_{\odot}/\text{yr}$ . The shaded area infers the possible range of parameter space in the simulations for nova to happen in RS Oph; it is truncated at  $M_{acc}/M_{nov} > 2$  which lead to mass accumulation more than twice that required for nova to take place, corresponding to a recurrence time half that observed. The green line represents  $M_{acc}/M_{nov}$  for  $M_{WD} = 1.25M_{\odot}$  and  $\dot{M} = 10^{-7}M_{\odot}/\text{yr}$ , it is always below the horizontal black line meaning that a nova does not happen within 20 years. . . . . 78
- 3.11 Stellar wind morphology in the case of synchronised rotation. Images are shown in the x-y plane (upper panel) and x-z plane (lower panel) from a simulation of a mass-losing star in isolation, with initially subsonic wind particle velocities and  $c_s = 19 \text{ km s}^{-1}$ . The centre of the star is shown as a red dot. In the upper panel, the star is viewed in its rotation plane and the arrows show the direction of the wind outflow. In the lower panel, the stellar environment is imaged perpendicular to the rotation axis. Both panels are coloured according to density contrast. . . . . 82
- 3.12 Same as figure 3.11 but for the case of critical rotation. . . . . 83
- 3.13 The effect of RG rotation on mass capture fraction and disc radius for simulations with subsonic wind injection. In the upper panel, mass capture fraction  $f_{cap}$  is plotted against sound speed for prograde (red) and retrograde rotation (green) normalised to mass capture fraction corresponding to no stellar rotation. In the lower panel, the disc radius is plotted against sound speed for simulations with no RG rotation normalised to the stability radius (blue). Also disc radii for the same 2 rotation regimes normalised to the disc radius with no stellar rotation. For both rotation cases, the orbital period of rotation  $P_{rot} = P_{orb}$ . . . . . 85

|      |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |     |
|------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----|
| 3.14 | The effect of RG rotation on $f_{\text{cap}}$ and disc radius for simulations with supersonic wind injection (same as figure 3.13 but for supersonic wind). Upper panel: results for the normalised mass capture fraction. Lower panel: results for the accretion disc radius for the cases of no RG rotation normalised to stability radius (blue). Also disc radii for prograde (red) and retrograde (green) normalised to non-rotating RG disc radius. . . . .                                                                                                                                                                                                                                                                                                  | 86  |
| 3.15 | Variation of $J_r = J_{\text{acc}}/J_1$ with the sound speed in the wind (Blue). $J_r$ approaches unity where the accretion disc cannot be formed. Also shown is the circularisation radius of the accretion disc from Bondi- Hoyle estimates (red) normalised to the stability radius $2.0 \times 10^{11}$ cm . The majority of the $R_{\text{circ}}$ values are comparable to that of the stability radius according to Bondi- Hoyle calculations . . . . .                                                                                                                                                                                                                                                                                                      | 87  |
| 4.1  | Orientation of the deflection angle $\theta$ according to the ratio of the planetary orbital velocity and the components of the stellar wind velocity. Figure credit to Vidotto et al. (2010). . . . .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             | 97  |
| 4.2  | Mach number at the planets position (in blue) and the angle of deflection $\theta$ of the shock with respect to the direction of orbital motion of the planet (in red). The two vertical dotted lines determine the range of the sound speed of the stellar wind studied in this chapter. . . . .                                                                                                                                                                                                                                                                                                                                                                                                                                                                  | 98  |
| 4.3  | Velocity profiles for isothermal (Parker) winds, calculated using ZEUS-2D and scaled to the parameters of the WASP-12 system ( $M_* = 1.35M_\odot$ , $a_p = 0.0229\text{AU}$ ). Profiles, from bottom to top respectively, are plotted for sound speeds $c_s=100, 120, 140, 160, 180$ & $200$ km/sec. In each case the filled black circle denotes the location of the sonic point. . . . .                                                                                                                                                                                                                                                                                                                                                                        | 100 |
| 4.4  | Density profiles for the (Parker) wind solutions in figure 4.3. The absolute density in the Parker solution is scale-free, so I adopt arbitrary units on the vertical axis and scale the solutions to a constant wind rate $\dot{M}_w = 4\pi r^2 u \rho$ . As in figure 4.3 the profiles, from top to bottom respectively, are plotted for sound speeds $c_s=100, 120, 140, 160, 180$ & $200$ km/sec, with the sonic points marked by circles. Note that a relatively small (factor of two) decrease in the sound speed dramatically increases the density gradient $d(\log \rho)/dr$ , and consequently increases the density contrast between the inner boundary (the stellar surface) and the planet's position by more than three orders of magnitude. . . . . | 101 |

- 4.5 Steady-state density structure in the simulations with  $C_B = 0.3$ . The white line denotes the inner grid boundary at  $r_{\text{in}}=R_*$ , while the filled white circle denotes the position and radius of the planet (i.e., both the star and planet are plotted to scale). In each panel the density is normalised to the value at the planet's orbital radius,  $180^\circ$  out of phase with the planet (denoted by  $\rho_p$ ). The magnetospheric radius is approximately constant in all the models, but as the sound speed increases the cavity and wake are angled progressively more towards the star. In addition, lower sound speeds result in slightly stronger (though still weak) bow shocks, with more pronounced density enhancements ahead of the planet. . . . . 104
- 4.6 Azimuthal density profiles at the planet's orbital radius, for models with  $C_B=0.3$  and various values of the sound speed  $c_s$ . (For clarity, the models with  $c_s=100$  &  $120 \text{ km s}^{-1}$  are omitted.) As in figure 4.5, the density is normalised to the value  $180^\circ$  out of phase with the planet. The lower axis shows the azimuthal coordinate in units of orbital phase (which ranges from  $[-0.5, 0.5]$ ), while the upper axis shows the distance around the planet's orbit in units of the planet radius  $R_p$ . The size of the magnetospheric cavity ( $\approx 7.5R_p$  in radius) and the position of the bow shock (with peak density  $\approx 14R_p$  ahead of the planet) are essentially independent of the sound speed in the wind, but the density contrast in the shock decreases with increasing  $c_s$ . . . . . 106
- 4.7 As figure 4.6, but for models with  $c_s=160 \text{ km s}^{-1}$  and various values of the magnetic constant  $C_B$ . The magnetospheric cavity increases in size for stronger magnetic fields, from  $\approx 6.0R_p$  for  $C_B=0.1$  to  $\approx 9.0R_p$  for  $C_B=1.0$ . The dashed black line denotes the simulation with  $C_B=0.3$  run at twice the numerical resolution ( $N_\phi=500$ ,  $N_r=276$ ), and shows that the simulations have achieved convergence. . . . . 107
- 4.8 WASP-12b UV light curves for the fiducial model ( $c_s=160 \text{ km s}^{-1}$  and  $C_B=0.3$ ), calculated for various optical depth parameters  $\tau_0$ . For reference, the optical transit  $I_p(\phi)$  is shown as a dotted grey line. The minimum of the UV transit corresponds to the peak density in the magnetosheath, and leads the planet in orbital phase by  $\approx 12\text{--}13\%$ . Note also that lines-of-sight through the magnetospheric cavity have relatively low column densities, resulting in a normalised flux greater than unity in the post-transit region. . . . . 109

- 4.9 As figure 4.8, but for models with  $c_s=160 \text{ km s}^{-1}$ ,  $\tau_0=0.1$ , and various values of  $C_B$ . Stronger magnetic fields (i.e., larger magnetospheric cavities) increase the amplitude of the UV transit, and also result in a larger phase offset between the minima of the UV and optical transits. . . . . 110
- 4.10 As figures 4.8 & 4.9, but for models with  $C_s=0.3$ ,  $\tau_0=0.1$ , and various sound speeds. For higher sound speeds the phase offset between UV and optical transits is smaller, as the bow shock is oriented towards the star (see figure 4.5). In addition, the steep density profiles in the low- $c_s$  wind solutions (see figure 4.4) mean that for low sound speeds ( $c_s \lesssim 140 \text{ km s}^{-1}$ ) most of the absorbing column lies interior to the planet’s orbit. In this case there is little (relative) absorption in the magnetosheath, and the radially-extended magnetospheric cavity instead results in a “negative-depth” transit. 112
- 4.11 Comparison between the model ( $c_s=160 \text{ km s}^{-1}$ ,  $C_s=0.3$ ,  $\tau_0=0.1$ ) and existing near-UV observations of WASP-12b. The black and blue points respectively denote the “Visit 1” and “Visit 2” fluxes reported in Fossati et al. (2010) and Haswell et al. (2012), normalised to the final “out-of-transit” data point: vertical error bars represent the Poisson errors on the observed fluxes, while the horizontal error bars denote the duration of the individual exposures. Solid circles, open circles and crosses respectively denote the three different wavelength regions labelled NUVA ( $\approx 2540\text{--}2580\text{\AA}$ ), NUVB ( $\approx 2660\text{--}2700\text{\AA}$ ) and NUVC ( $\approx 2780\text{--}2820\text{\AA}$ ) in Haswell et al. (2012). (Note that these are approximate, as the exact wavelength ranges differed slightly between the two visits.) NUVA contains many overlapping absorption lines (primarily Mg I and Fe I), while NUVC is centred on the strong Mg II doublet at  $2798/2803\text{\AA}$ ; NUVB is a “continuum” region with no strong absorption lines. It is striking that the transit depths are similar across all three wavelength regions. The dashed red line is the same as the corresponding model in figures 4.8–4.10, while the solid curve is normalised to the post-transit region ( $\phi=[0.08,0.10]$ ) in the same manner as the data. The predicted light-curves are consistent with the observations, but the large error bars and limited phase coverage means that we cannot place useful constraints on the model parameters. . 113

- 4.12 Steady-state density structure in my simulations with  $C_B = 0.3$  and  $c_s = 160 \text{ km s}^{-1}$ , for WASP-12, WASP-18 & HD209458. As in figure 4.5, the density colour-scale is normalised to the value at  $a_p$ , and the planets and stars are plotted to scale. The flow solution around the more massive WASP-18b has no significant differences from that around WASP-12b, showing that the planet's gravity is essentially negligible. By contrast, the larger semi-major axis of HD209458b results in a different wind speed and ram pressure at the planet's orbit, leading to a larger magnetospheric cavity (relative to the star and planet) and the bow shock being angled significantly more towards the star. . . . . 115
- 4.13 Light curves for the three models shown in figure 4.12 ( $C_B = 0.3$  and  $c_s = 160 \text{ km s}^{-1}$ ); as before, the dotted lines denote the optical transits. For clarity, the curves for WASP-18b (red) and HD209458b (blue) have been offset by  $\pm 4\%$  in flux, respectively. The light-curve for the more massive WASP-18b is essentially identical to that of WASP-12b. However, the larger orbital separation of HD209458b dramatically reduces the (relative) absorption in the bow shock, and the resulting UV light-curve is almost indistinguishable from the optical transit. . . . . 116
- 4.14 Azimuthal density profiles at the planet's orbital radius for the SPH models with wind sound speed  $c_s = 160 \text{ km s}^{-1}$  and constants  $C_B = 0.05, C_B = 0.25$  and  $C_B = 0.5$ . The lower axis shows the azimuthal coordinate in units of orbital phase (which ranges from  $[-0.5, 0.5]$ ), while the upper axis shows the distance around the planet's orbit in units of the planet radius  $R_p$ . . . . 122
- 4.15 Normalised flux for the models with wind sound speed  $c_s = 160 \text{ km s}^{-1}$ ; in black is the optical light-curve. Top panel: average opacity  $\tau = 0.1$  and various constant  $C_B$  values. Bottom panel:  $C_B = 0.25$  and various values of  $\tau$  . . . . . 123
- 4.16 Normalised flux for the models with wind sound speeds  $c_s = 100, 140, 160 \text{ & } 200 \text{ km s}^{-1}$ ; the constant  $C_B = 0.5$  and opacity  $\tau = 0.1$ . The optical light-curve of the planetary transit is in black. . . . . 124
- 4.17 Comparison between the two dimensional and three dimensional results. Stronger absorption feature is seen for the 2D than the 3D results prior to the optical transit. The two results converge at post transit. . . . . 126

---

*Dedicated to my Wife, Son and my Parents*

---

# 1

## Introduction

## 1.1 Binary Systems

Stars are a conspicuous part of the observable universe and important constituents of stellar clusters and galaxies. They can exist as a single star or as part of multiple star systems. In the latter, the stars are bound together by their mutual gravitation and orbit around their common centre of mass (CM). Observations indicate that there are as many binaries as the number of single stars yet identified (Padmanabhan, 2001; Hilditch, 2001). Recently a study by Raghavan et al. (2010) on stellar families showed that for a sample of 454 solar like stars in the solar neighbourhood, only 54% of them seem to be single star systems. Binaries compose 34% of the sample and the rest (12%) are triple or higher order stellar systems.

Systems of two gravitationally bound celestial objects orbiting their mutual centre of mass are called binary systems or "Binaries", and are the focus of interest here. The structure of a model binary system is shown in figure 1.1

### 1.1.1 Binary Geometry

Consider two stars, modelled as point masses, with  $m_1$  as the primary mass and  $m_2$  as the secondary mass in a binary system, for which the mass ratio  $q = \frac{m_2}{m_1} < 1$ . They are on average  $a$  units of length apart orbiting their common centre of mass at distances of  $r_1$  and  $r_2$  respectively. They complete one orbit in a time  $P_{orb}$  which from the Newtonian form of the Kepler's third law is,

$$P_{orb}^2 = \frac{4\pi^2}{G(m_1 + m_2)} a^3, \quad (1.1)$$

where  $G$  is the universal gravitational constant and  $a$  is the semimajor axis of the relative orbit. This form does not only apply to stars orbiting each other, but also to the orbits of planets around stellar objects or moons around planets. The usefulness of equation 1.1 is that it combines the total binary mass, semimajor axis and orbital period together in one relation.

Let's assume an isolated binary, so that the only force acting on the system is the mutual gravitation between the binary components. In that case their common centre of mass has acceleration zero according to Newton's third law of gravitation, thus the centre of mass can be an inertial reference frame. According to the definition of centre of mass,

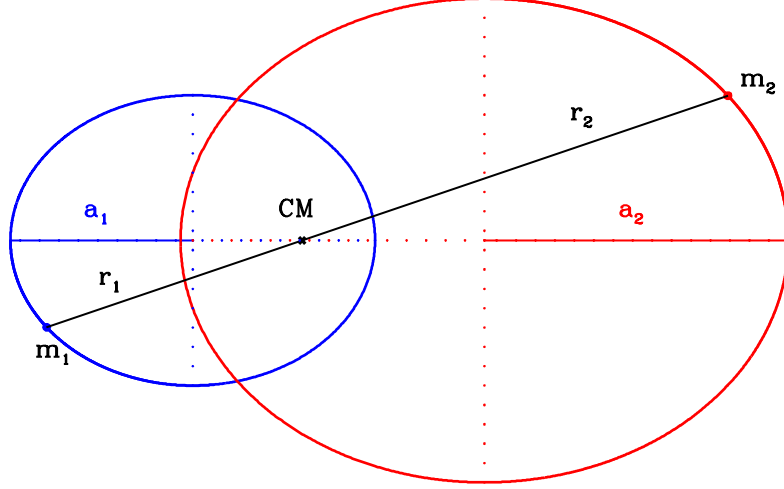


Figure 1.1: Geometry of an ideal binary system having binary mass ratio  $q = 0.6$  and eccentricity of 0.6.

$$a_1 = \frac{m_2}{m_1 + m_2}a \quad \text{and} \quad a_2 = \frac{m_1}{m_1 + m_2}a. \quad (1.2)$$

where ( $a_1$ ,  $a_2$  and  $a$ ) are the semimajor axis of the barycentric orbit of the component of mass  $m_1$  (the blue orbit in Figure 1.1), the semimajor axis of the barycentric orbit of the component of mass  $m_2$  (the red orbit in Figure 1.1) and the semimajor axis of the relative orbit of the binary components respectively (see Appendix A.1).

### 1.1.2 Binary total energy and orbital angular momentum

For the two bodies to stay bound in a binary, their total energy must be negative by definition. The total energy is the sum of kinetic energy  $E_k$  and gravitational potential energy  $E_p$  of the system. For the kinetic energy as clarified in Appendix A.2,

$$\begin{aligned} E_k &= \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{m_1m_2}{2(m_1 + m_2)}v^2 \\ &= Gm_1m_2\left[\frac{1}{r} - \frac{1}{2a}\right], \end{aligned} \quad (1.3)$$

where  $v$  is the velocity of the components in the relative orbit and  $r$  is the instantaneous binary separation (line of centres). The Gravitational potential energy may be written as

$$E_p = \int_r^\infty \frac{Gm_1m_2}{r'^2} dr' = -\frac{Gm_1m_2}{r}, \quad (1.4)$$

so the binary total energy

$$E_{tot} = E_k + E_p = -\frac{Gm_1m_2}{2a}. \quad (1.5)$$

$E_{tot}$  is negative and equivalent to the binding energy in the system. This tells us that the total energy determines the semi-major axis of the relative orbit, the larger the orbital separation, the less negative is the total energy. Similarly, the total energy has its minimum value for  $e = 0$  and is less negative (less bound) for  $e > 0$ . The binary orbital orientation is determined by the total angular momentum vector  $\mathbf{J}$ . Furthermore, for a given value of  $E_{tot}$  it follows that circular orbits ( $e = 0$ ) have the most angular momentum and as the value of  $e$  increases,  $J$  decreases such that when ( $e \rightarrow 1$ ), ( $J \rightarrow 0$ ). The magnitude of  $J$  is given by the sum of the barycentric angular momenta of the components  $J = m_1L_1 + m_2L_2$ ; after some simple derivation steps depicted in Appendix A.2 (see e.g., Hilditch, 2001) it takes the (last) form as

$$\begin{aligned} J &= m_1m_2 \left[ \frac{Ga(1-e^2)}{(m_1+m_2)} \right]^{1/2}, \\ &= \frac{2\pi a^2(1-e^2)^{1/2}m_1m_2}{P(m_1+m_2)} \end{aligned} \quad (1.6)$$

### 1.1.3 Roche Lobe Geometry

The concept of the Roche Lobe is particularly important in the study of binary systems, because, as we shall see later, it tells us about the evolution of the binary system and how the components interact. It is obvious that the gravitational potential of a point mass like simplified model of a single star is  $\Phi = \frac{-GM_*}{r}$ , which is spherically symmetric and inversely proportional to  $r = R_* + \text{height}$ . However, the potential in a binary system is quite complicated as the gravity is due to two point mass stars.

To map the gravitational potential energy, the so-called three body problem explains qualitatively the combined gravitation from the two components. We restrict ourselves to the orbital plane  $xy$  with  $z = 0$  in Cartesian coordinates and fix the common centre of mass to be at  $(0,0)$  where the  $x$ -axis lies along the line joining the two centres of the

stars. Furthermore we assume a circular orbit and consider the frame of reference that is rotating with the binary. The problem maps the gravitational potential experienced by an infinitesimally small mass test particle. The total potential at any point in the  $xy$  plane (Warner, 1995) is the sum of the gravitational potentials of the two components and the effective potential due to the fictitious centrifugal force .

$$\Phi_r = -\frac{Gm_1}{r_1} - \frac{Gm_2}{r_2} - \frac{1}{2}\omega^2 r^2 \quad (1.7)$$

In the orbital plane where the CoM is at the origin, the primary is at  $(-a_1, 0)$  and the secondary is at  $(a_2, 0)$ , recalling equation 1.1, equation 1.7 gives.

$$\Phi_{(x,y)} = -\frac{Gm_1}{\sqrt{(x+a_1)^2 + y^2}} - \frac{Gm_2}{\sqrt{(x-a_2)^2 + y^2}} - \frac{1}{2} \frac{GM}{a^3} (x^2 + y^2) \quad (1.8)$$

The result is contours of equipotential values with two minima tending to infinity at the centres of each component and five maxima (saddle points) which are the so called Lagrangian points. The first (inner) one,  $\mathbf{L}_1$ , is between the two point masses, its position depends on the binary mass ratio  $q = \frac{m_2}{m_1}$ . The second Lagrangian point,  $\mathbf{L}_2$ , lies to the far side of the less massive component and  $\mathbf{L}_3$  lies to the far side of the more massive component; all of which lie on the x-axis. The fourth and the fifth Lagrangian points are located away from the x-axis, making an equilateral triangle with the two stars. The line joining  $L_4$  and  $L_5$  to either component makes an angle  $60^\circ$  and  $-60^\circ$  with the x- axis respectively.

The equipotential curve in the  $xy$  plane is a surface in 3D which passes through  $\mathbf{L}_1$  and makes a figure of eight around the stars, is significant because it determines if the two stars can interact by mass transfer such as Roche Lobe overflow. The detailed shape of this curve again depends on the mass ratio of the binary. There have been attempts to estimate the size of each lobe (Kopal, 1978), an accurate one is given by Eggleton (1983) with

$$\frac{R_L}{a} = \frac{0.49q^{2/3}}{0.6q^{2/3} + \ln(1 + q^{1/3})} \quad (1.9)$$

If we assume only one dimensional motion of the test particle, the potential curve along the x- axis is depicted in the bottom panel of figure 1.2. Here, only  $\mathbf{L}_1, \mathbf{L}_2$  and  $\mathbf{L}_3$  appear and one can compare their values.

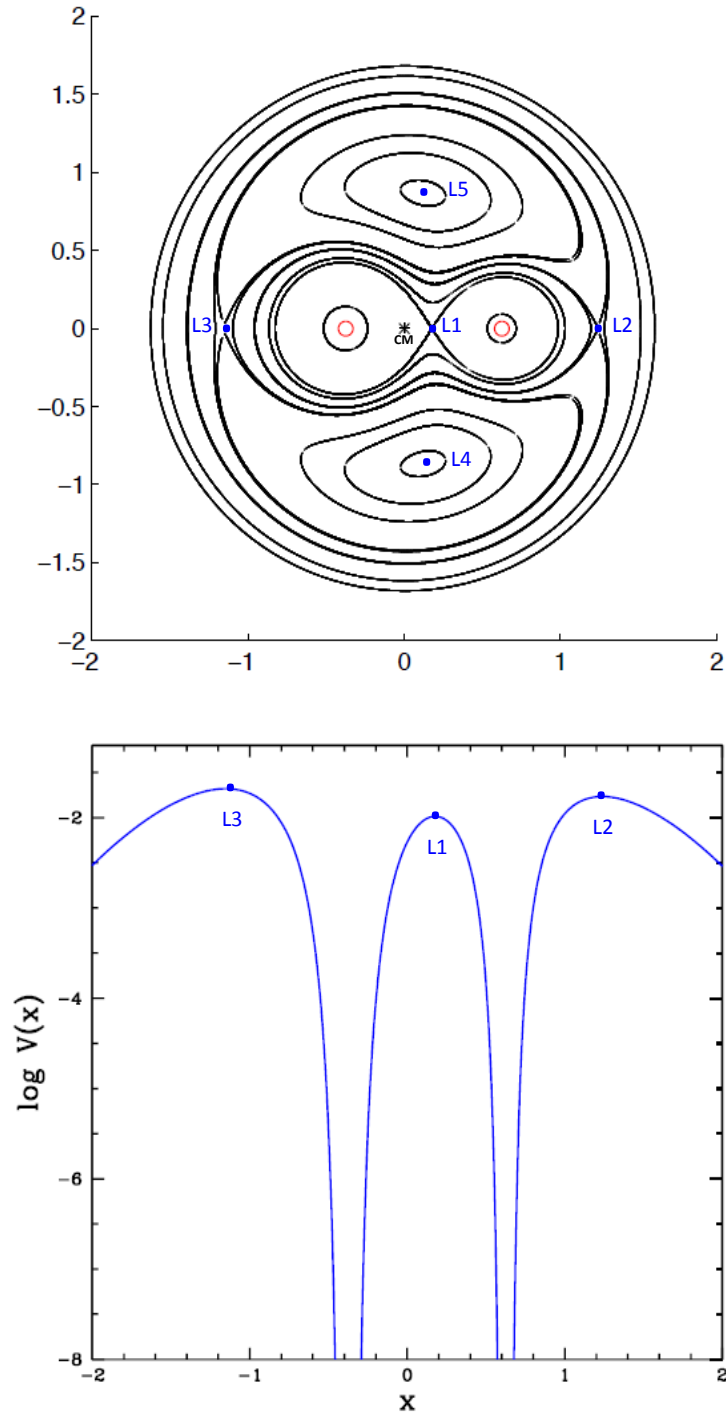


Figure 1.2: Upper panel: Equipotential surfaces (contours in 2D  $xy$  plane), locations of Lagrangian points can be seen. Lower panel: The binary gravitational potential in 1D along the line where  $L_1$ ,  $L_2$  and  $L_3$  lie. The code units used here are as follows [  $G = 1$ ,  $M_1 + M_2 = 1$  and  $a = 1$  ]. Binary mass ratio is  $q = 0.6$ .

### 1.1.4 Observational Classification of binary stars

Binary systems may be classified according to how they appear.

**Visual binaries:** In these systems the binary components can be spatially resolved. This can be achieved when the system is sufficiently close and has a sufficiently large semi-major axis. As observational techniques have developed, the resolution of such types of binaries has reached the order of milliarcseconds (see eg. Hilditch, 2001; Carroll and Ostlie, 2006).

**Astrometric binaries:** In most cases, because the system is too distant or one of the companions is much brighter or due to a very small angular separation, both components cannot be resolved. The binary nature of the system can still be inferred from the gravitational effect of the unresolved star on the bright (visual) one. These are referred to as Astrometric binaries.

**Eclipsing binaries:** many stars that show regular changes in their apparent magnitude, are often found to be binary systems whose orbital plane is edge-on to the observer. In this case the two components eclipse one another regularly when one component passes between the other one and the observer along the line of sight.

**Spectroscopic binaries:** As long as one component is not much more luminous than the other, the spectrum of the system is the superposition of the spectra of the two stars. Due to their orbital motions, they may have a velocity component relative to the observer causing a periodic Doppler-shift in the spectral lines. Even if they are face-on, the spectrum can still be interpreted as the combination of the two component spectra. If the orbital plane is not completely perpendicular to the line of sight and the orbital period is not extremely long, their spectra reveal a periodic Doppler shift. One component is blue shifted as it is approaching the observer, the other is receding and red shifted.

### 1.1.5 Types of interaction in binary systems

Binary stars do not all interact only through the gravitational interaction to maintain orbital motion, this can extend to tidal distortion, irradiation and mass transfer (Warner, 1995). The Roche model identifies two nearly spherical lobes around the stars in a binary. The size of the stars relative to their Lobes determines the way they interact through mass transfer (Kopal, 1959) .

In detached binaries, (the top panel in Figure 1.3) each star is quite spherical and the material of each component is well below the Roche volume. In this case no physical contact or mass exchange takes place (other than wind) between the two stars. Semi-

detached binaries are those in which one component swells to fill or over fill its Roche lobe while the companion's size is less than its own Roche volume. In this case mass from the Roche-filled star transfers through  $L_1$  and accretes onto the relatively compact one as depicted in the middle panel in Figure 1.3. Contact binaries are systems where both stars fill or overfill their Roche lobes and connect through  $L_1$  or they make a common envelope, it is shown in the bottom panel in Figure 1.3. If the size of this envelope reaches either  $L_2$  or  $L_3$  mass can escape the system or might produce a circumbinary disc around the system.

### 1.1.6 Orbital evolution due to mass transfer

Mass transfer in binary systems occur in various modes, for instance the mass lost by one binary component may be totally or partially gained by the companion through accretion. Though the case of no accretion is also possible where the mass lost by the mass losing star is totally lost by the binary system.

Mass transfer is associated with the transfer of angular momentum. Therefore, the binary orbital configuration changes when there is mass loss or transfer in the system. In fact the total angular momentum of a binary system is composed of the sum of the spin angular momentum of each component and their orbital angular momenta. The spin angular momentum is mostly ignored for simplicity because it is only 1 – 2% of the total orbital angular momentum (Hilditch, 2001). Equation 1.6 can be re-written as

$$a = \frac{J^2 M}{m_1^2 m_2^2 G (1 - e^2)}, \quad (1.10)$$

where  $M = m_1 + m_2$  is the total mass of the binary. Remembering Kepler's third law (equation 1.1), the logarithmic differentiation of Equation 1.10 gives

$$\frac{\dot{a}}{a} = 2 \frac{\dot{J}}{J} + \frac{\dot{M}}{M} - 2 \frac{\dot{m}_1}{m_1} - 2 \frac{\dot{m}_2}{m_2}. \quad (1.11)$$

where we have assumed zero or constant eccentricity during mass transfer.

#### 1.1.6.1 Conservative mass transfer

This may be the simplest mode of mass transfer in which all the mass lost by one component is totally captured by the companion (capture fraction  $f = 100\%$ ). So the total mass of the binary remains unchanged and that demands the conservation of the total orbital angular momentum of the binary. Therefore,  $\dot{m}_1 = -\dot{m}_2$ ,  $M = \text{constant}$  and thus

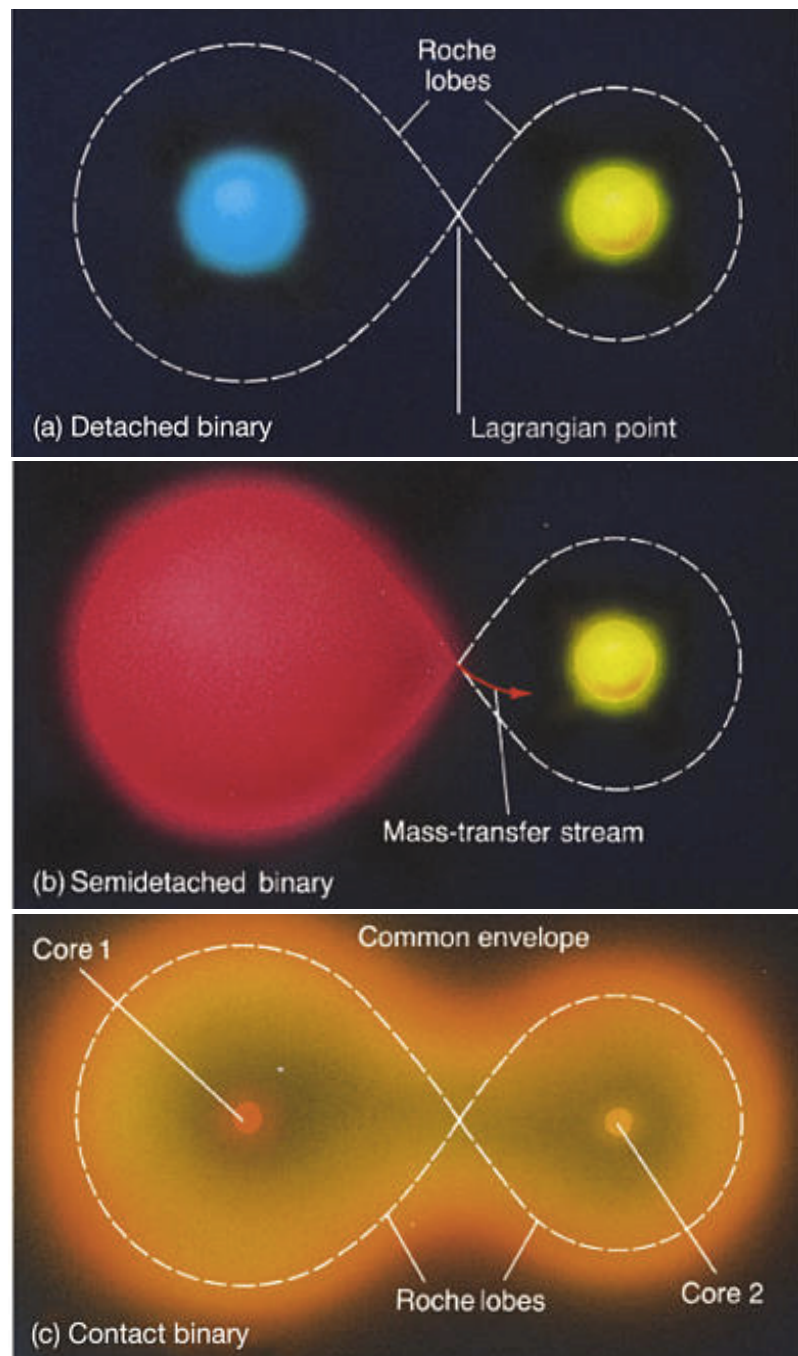


Figure 1.3: Types of binaries in interaction through mass transfer. Top panel: the size of the two components are less than their Roche volume and thus no mass transfer through L1. Middle panel: one of the components undergo RLO and mass transfers to the companions Roche volume. Bottom panel: both components overfill their Roche lobes and a contact binary is produced. Figure credit to Pearson Prentice Hall, Inc. 2005.

$J = \text{constant}$  and equation 1.11 reduces to

$$\frac{\dot{a}}{a} = \frac{2}{3} \frac{\dot{P}}{P} = \frac{2\dot{m}_1(m_1 - m_2)}{m_1 m_2}. \quad (1.12)$$

Equation 1.12 infers not only the orbital period changes but also tells us if it increases or decreases. For instance, if the initially more massive star loses mass then  $\dot{m}_1$  is negative but  $(m_1 - m_2)$  is positive, then  $\dot{P}$  is negative and the orbital period (or the binary separation  $a$ ) decreases until  $m_1 = m_2$  when the orbital period reaches its minimum. If the star continues transferring mass to the companion then  $m_1 < m_2$  which turns  $\dot{P} > 0$  then the orbit expands again.

### 1.1.6.2 Non-conservative mass transfer

When the mass gaining star only accretes a fraction of the mass lost by the companion that is when capture fraction ( $0 \leq f < 1$ ), the total mass of the binary is no longer constant. The binary orbital angular momentum is no longer constant in time because mass loss is associated with angular momentum loss. This case is referred to as non-conservative mass transfer; this is the most frequent and common mode of mass interaction for example accretion of stellar wind of the companion in a binary or catastrophic mass loss such as those of novae and supernovae. To find a formula for orbital period evolution we now assume the simplified case of zero capture fraction where  $\dot{m}_1 = \dot{M} < 0$  and  $\dot{m}_2 = 0$ . Equation 1.11 then becomes

$$\frac{\dot{a}}{a} = 2 \frac{\dot{J}}{J} - \frac{\dot{m}_1}{m_1} \left( \frac{m_1}{M} + 1 \right). \quad (1.13)$$

This is of course a special case of non-conservative mass loss; however it would be applicable for the cases we will discuss in Chapter 3, where the mass transfer is through relatively fast stellar wind and the capture fraction is only in the range of a few percent.

## 1.2 Accretion

The process in which the accumulation of matter on the surface of an accreting object takes place is called accretion. Accretion is proposed to occur in a very wide range of astronomical systems (Treves et al., 1988): from the study of the formation of our solar system planets to the circumstellar discs around newborn stars, very high luminosity quasars and accreting supermassive black holes in Active Galactic nuclei (AGN). Bright

galactic x-ray sources are also believed to be accreting neutron stars or stellar mass black holes in a binary. Novae and related activities are thought to be the result of accretion of matter by a white dwarf from its companion, this is part of the work presented in this thesis.

The first analytical derivation of accretion was by Hoyle and Lyttleton (1939). They considered the relative speed of a moving accreting body in a gas cloud. A few years later, Bondi (1952) formulated the infall rate of gas accreting onto the surface of a star at rest taking the thermal properties of the gas into account. The equation derived by Bondi is also a possible solution of simplified stellar wind outflows (opposite process). Accretion is regarded as a significant source of energy in the universe because it is associated with the emission of radiation.

### 1.2.1 Accretion power and Eddington Limit

Let's, for simplicity, assume a gas particle such as a proton falls freely from a very large distance  $r$  on to the surface of a star of radius  $R_*$ . The kinetic energy it gains on the surface of the star is related to its escape velocity. If for some reason, such as bringing it to rest, the kinetic energy can be converted to electromagnetic radiation, thus

$$\Delta E_{acc} = \frac{1}{2} m_p v_e^2 = \frac{GM_* m_p}{R_*}, \quad (1.14)$$

where  $m_p$  is the mass of the accreting proton,  $v_e$  is the escape velocity on the surface of the star,  $M_*$  and  $R_*$  are the mass and radius of the star respectively. Equation (1.14) is valid if  $R_* \ll r$ . This tells us for compact accreting bodies, the accretion energy is relatively high; for the case of a neutron star  $M_* = 1.4M_\odot$  and  $R_* = 10\text{km}$ , the gas particle arrives at the surface with  $v_e = 0.64c$  or in other words about 10% of its rest energy would be released. On the other hand, if the emitted radiation can be absorbed, the radiation force on the absorbing particles in the case of spherically symmetric accretion is

$$F_{rad} = \frac{\kappa F}{c} = \frac{\kappa L}{4\pi c r^2}, \quad (1.15)$$

where,  $\kappa$  is the gas opacity,  $F$  is the radiation flux,  $c$  is the speed of light and  $L = L_{acc}$  is the object luminosity assumed to be generated by a continuous accretion of matter with rate  $\dot{m}$ . From equation (1.14) The accretion luminosity can be expressed as (Frank et al., 2002)

$$L_{acc} = \frac{GM_*\dot{m}}{R_*}. \quad (1.16)$$

If the accretion rate reaches a certain level, one might have a balance between radiation and gravitational forces such that

$$\frac{GM_*}{r^2} = \frac{\kappa L_E}{4\pi c r^2},$$

where  $L_E$  is called the Eddington luminosity,

$$L_E = \frac{GM_*\dot{m}_E}{R_*} = 1.3 \times 10^{38} \left( \frac{M_*}{M_\odot} \right), \quad (1.17)$$

Such a luminosity is achieved when the accretion rate reaches the Eddington accretion rate

$$\dot{m}_E = \frac{4\pi c R_*}{\kappa}. \quad (1.18)$$

Accretion at a rate more than  $\dot{m}_E$  will halt accretion as the force due to radiation pressure is greater than the force due to gravity.

### 1.2.2 Accretion Disc Theory

The process of accretion mentioned so far is assumed to take place spherically on to the surface of the star. The fact that the accreting gas is likely to possess a finite amount of angular momentum, means that it cannot fall radially to the surface of the accretor. Instead the gas must adopt an orbit about the accretor. In a binary system, when this orbit does not intercept the accretor directly, it precesses and the gas flow is forced to intersect itself. The gas particles then shock with each other, during this process part of their energy is lost but the angular momentum is conserved. As a result they settle into the circular orbit in which they have the lowest energy for a certain angular momentum. The gravitationally bound particles then make a ring since they do not all have the same angular momentum. After the ring has formed, the gas particles spiral slowly inward and accrete onto the surface of the accretor under the action of viscosity. The steady state structure is termed an accretion disc. Pringle (1981), Frank et al. (2002) and Lodato (2008) have given a comprehensive review on the subject of accretion processes and accretion discs.

### 1.2.2.1 Conservation of mass

Here we consider thin discs in the orbital plane of binaries; which means  $\frac{H}{R} \ll 1$  where  $H$  is the scale height of the disc. This condition requires the disc to rotate supersonically, therefore the pressure force is unimportant. Since the disc forms inside the Roche lobe of the accretor, a negligible gravitational effect by the companion is assumed on the disc. It is also assumed that the disc self gravity is zero. We then take the cylindrical polar coordinates  $(R, \phi, z = 0$  for the orbital plane) centered on the accretor.

As described before, the matter spirals through the disc, such that each particle has an angular velocity  $\Omega$  and a much smaller radial velocity  $v_R$  relative to the accretor. The small value of the radial velocity by which accretion is achieved means that the accretion is on a timescale much longer than the dynamical timescale. Within the disc, gas moves azimuthally with the Keplerian velocity. The disc is assumed to be axisymmetric so the quantities all vary with  $R$  only.

$$v_\phi(R) = R\Omega(R) , \quad (1.19)$$

where  $\Omega(R)$  is the Keplerian angular velocity

$$\Omega(R) = (GM_*/R^3)^{1/2} . \quad (1.20)$$

An accretion disc is characterised by its surface density,  $\Sigma(R, t)$ , defined as the mass per unit area of the disc  $\Sigma(R, t) = 2 \int \rho dz$ .

Let's now take an annulus of the disc a radial distance  $R$  from the centre of the accretor, the width of the annulus is  $\Delta R$ , the mass of the matter lying in this annulus is given by

$$M = 2\pi R \Delta R \Sigma(R, t) , \quad (1.21)$$

According to the continuity equation, the temporal change in the surface density is the net flow in and out of the annulus:

$$\frac{\partial M}{\partial t} = 2\pi R \Delta R \frac{\partial \Sigma(R, t)}{\partial t} = \dot{M}_{in} - \dot{M}_{out} , \quad (1.22)$$

After a little algebra this will take the form (Appendix A.3).

$$R \frac{\partial \Sigma}{\partial t} + \frac{\partial}{\partial R} [R \Sigma v_R] = 0 . \quad (1.23)$$

### 1.2.2.2 Conservation of angular momentum

In a similar manner, conservation of angular momentum relates the rate of change of angular momentum in an annulus to the net viscous torque  $\tau_{net}$  on the annulus.

$$\frac{\partial}{\partial t} J = \dot{J}_{in} - \dot{J}_{out} + \tau_{net} \quad (1.24)$$

The angular momentum associated with the matter in the annulus of breadth  $\Delta R$  and at radius  $R$  is

$$J = 2\pi R \Delta R \Sigma(R, t) R^2 \Omega(R) . \quad (1.25)$$

The net torque is significant because it provides the channel through which gas particles lose angular momentum and accrete onto the central object by transporting angular momentum to the outer disc.

The Keplerian velocity profile causes the disc to rotate differentially. As long as the viscosity exists, the viscous force acts azimuthally on each ring in the disc, which is proportional to the rate of shear  $R \frac{d\Omega}{dR}$ . The viscous torque  $G$  is (Frank et al., 2002) (see also Appendix A.4).

$$G(R, t) = 2\pi R \nu \Sigma R^2 \frac{d\Omega}{dR} , \quad (1.26)$$

where  $\nu$  is the kinematic viscosity coefficient.

Again, in the limit of infinitesimally small breadth annulus,  $\Delta R \rightarrow 0$ , the equation of angular momentum conservation can be re-written as

$$R \frac{\partial}{\partial t} [\Sigma R^2 \Omega] + \frac{\partial}{\partial R} [R \nu_R \Sigma R^2 \Omega] = \frac{1}{2\pi} \frac{\partial G}{\partial R} . \quad (1.27)$$

Making use of equations (1.20) for a Keplerian azimuthal motion in the annulus, and equations (1.23) and (1.26), Equation (1.27) becomes (see also appendix A.5 )

$$\frac{\partial \Sigma}{\partial t} = \frac{3}{R} \frac{\partial}{\partial R} \left[ R^{1/2} \frac{\partial}{\partial R} \left( R^{1/2} \nu \Sigma \right) \right] . \quad (1.28)$$

This is the basic diffusion equation governing the time evolution of the surface density in a viscous Keplerian disc. The parameter which has to be defined is the kinematic viscosity  $\nu$  which plays an important role in angular momentum transfer. If it is assumed to be constant throughout the disc then equation (1.28) can be solved analytically. However,

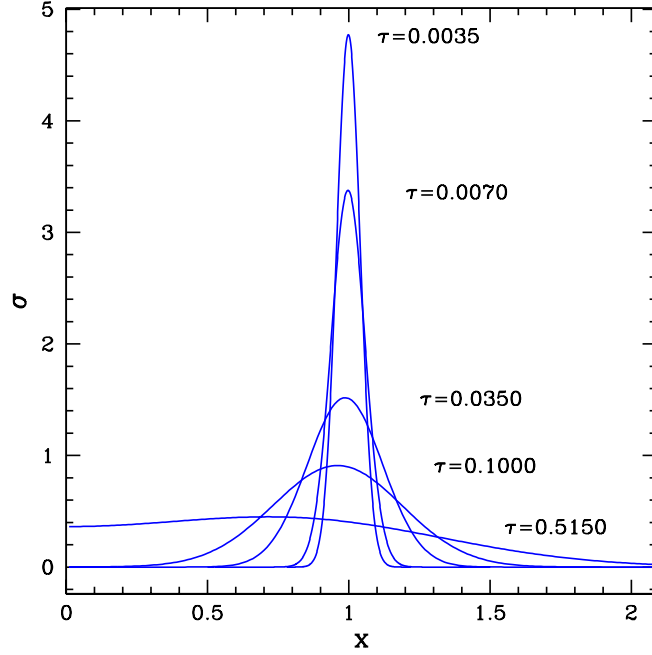


Figure 1.4: Diffusion of a ring of matter under the influence of viscosity and gravity of the accretor.  $x = \frac{R}{R_o}$ ,  $\sigma = \frac{\pi r^2 \Sigma}{m}$  and  $\tau = 12\nu t R_o^{-2}$ .

viscosity is in fact a function of the disc local variables including  $\Sigma$ . The origin of the viscosity is still unknown, it has been shown that the standard molecular viscosity cannot evolve the accretion discs in an appropriate time scale as the Reynolds number in this case for a typical accretion disc exceeds  $10^{14}$  (Frank et al., 2002) while for the viscosity to dominate the flow the Reynolds number must be  $\ll 1$ . Experimentally, it has been shown that for Reynolds numbers greater than some critical value (10 – 1000), turbulence in the fluid sets in. With no observational evidence, and lack of its origin, it is assumed that this turbulence occurs in accretion discs to generate turbulent viscosity. Shakura and Sunyaev (1973) parameterised viscosity which is known as  $\alpha$ -prescription:

$$\nu = \alpha c_s H, \quad (1.29)$$

where  $\alpha$  is a constant with values less than unity. Although this parameterisation left us with uncertainty in  $\alpha$  values which is the outcome of the unknown origin of viscosity, it allowed a great number of important insights into accretion disc physics.

### 1.2.2.3 Viscous evolution of a ring

As mentioned above, if we assume, constant  $\nu$  for simplicity, the diffusion equation 1.28 is linear in  $\Sigma$  and gives the evolution of disc surface density in a disc (Frank et al. 2002). If we start with only a ring with an initial surface density  $\Sigma_o = \Sigma(R, t_o = 0) = \frac{m}{2\pi R_o} \delta(R - R_o)$  at radial distance  $R_o$  from the accreter (centre) then

$$\Sigma(R, \tau) = \frac{m}{\pi R_o^2} \tau^{-1} x^{-1/4} \exp \left[ -\frac{(1+x^2)}{\tau} \right] I_{1/4}(2x/\tau) . \quad (1.30)$$

where  $\delta$  is the Dirac delta function,  $x = \frac{R}{R_o}$ ,  $\tau = 12\nu t R_o^{-2}$  and  $I_{1/4}(2x/\tau)$  is modified Bessel function of the first kind. As shown in figure 1.4 the equation gives a generic idea of how accretion takes place due to the spread of a ring at  $R_o$ . After the ring evolves most of the matter goes inward to accrete while a small part of it spreads outwards carrying nearly all of the angular momentum of the initial ring for the sake of conservation.

## 1.2.3 Accretion Disc timescales

The typical timescales on which accretion disc evolution takes place are:

### The viscous timescale ( $t_{visc}$ ) :

this is the timescale for the evolution of surface density in an accretion disc through the diffusion of matter under the effect of the viscous torque  $G$ .

$$t_{visc} \sim \frac{R^2}{\nu} \sim \frac{R}{v_R} .$$

### Dynamical timescale ( $t_\phi$ ) :

is the characteristic timescale on which the disc reaches a new dynamical equilibrium after some perturbation.

$$t_\phi \sim \frac{R}{v_\phi} \sim \Omega_K^{-1} .$$

**Hydrostatic timescale ( $t_z$ ) :**

is the timescale for the disc to reach hydrostatic equilibrium in the vertical direction. This is typically the sound speed crossing time.

$$t_z \sim \frac{H}{c_s} .$$

**Thermal timescale ( $t_{th}$ ) :**

is the time required for the disc to reach thermal equilibrium after a process involving heating or cooling; it is defined as the ratio between the heat content per unit disc area and dissipation rate per unit disc area.

$$t_{th} \sim \frac{c_s^2}{v_\phi^2} \frac{R^2}{\nu} \sim \mathcal{M}^{-2} t_{visc} .$$

where  $\mathcal{M}$  is Mach number. For a typical value of  $\alpha \lesssim 1$ , (Shakura and Sunyaev, 1973), the hierarchy of the timescales would be

$$t_\phi \sim t_z \lesssim t_{th} \lesssim t_{visc} . \quad (1.31)$$

For typical parameters, the dynamical ( $t_\phi$ ), hydrostatic ( $t_z$ ) and thermal ( $t_{th}$ ) timescales are of the order of minutes while the viscous timescale ( $t_{visc}$ ) is of the order of days to weeks (Frank et al., 2002, for more details).

## 1.3 Stellar Wind

Most classes of stars undergo mass loss by means of a continuous outflow of material from their surface which is called a stellar wind. The mechanisms behind this outflow and other properties of the wind depend on the spectral class of the star. Mass loss rates can be enhanced by evolutionary changes on the RGB and particularly the AGB.

A stellar wind is characterised by the mass loss rate  $\dot{M}$  and its terminal velocity  $v_\infty$ , which are important because mass loss rate determines the way the star evolves.  $\dot{M}$  and  $v_\infty$  determine the energy deposition from the star to the surrounding interstellar medium. The rate of energy deposition is  $\frac{1}{2} \dot{M} v_\infty^2$ . In the case of a constant mass loss rate and a spherically symmetric outflow the continuity equation demands that

$$\dot{M} = 4\pi r^2 \rho(r) v(r) . \quad (1.32)$$

Since the density drops with distance, the velocity increases until it reaches its nearly constant terminal velocity  $v_\infty$ . The radial velocity profile of a stellar wind is often approximated by the semi-empirical model called the  $\beta$ -law, where

$$V(r) = V_\infty \left( 1 - \frac{R_*}{r} \right)^\beta , \quad (1.33)$$

where  $R_*$  is considered to be the photospheric radius of the star,  $\beta > 0$  is a constant which determines the acceleration associated with the outflow; for example  $\beta = 0.8$  corresponds to hot star winds where rapid acceleration occurs and the wind reaches its terminal velocity in a relatively very short distance. Cool stars have slow winds thus slow accelerations corresponding to larger  $\beta$  values.

Despite the much earlier work on outflows from stars, Parker (Parker, 1958) is considered to have been the first to derive a simplified analytical solution for solar and stellar winds in which only the central gravity of the star and the force due to the thermal pressure gradient are accounted for. In a stationary wind, i.e the wind velocity at a given distance from the stellar centre does not change in time, then the equation of motion (momentum equation) is

$$v \frac{dv}{dr} + \frac{1}{\rho} \frac{dP}{dr} + \frac{GM_*}{r^2} = 0 . \quad (1.34)$$

Here, the forces are the inward gravitational (the last term on the left side) and the outward directed force due to the radial pressure gradient of the gas (the second term on the left side). If we consider an isothermal gas so the energy equation is simply  $T(r) = T = \text{constant}$ ; moreover if we assume the wind behaves like an ideal gas then

$$P = \frac{RT}{\mu} \rho , \quad (1.35)$$

where  $R$  is the gas constant,  $\mu$  is the mean atomic weight in units of  $m_H$ , for solar composition, for example  $\mu = 0.602$ . With some straight forward derivations, the momentum equation will take another form as

$$\frac{1}{v} \frac{dv}{dr} = \frac{\left[ \frac{2c_s^2}{r} - \frac{GM_*}{r^2} \right]}{[v^2 - c_s^2]} , \quad (1.36)$$

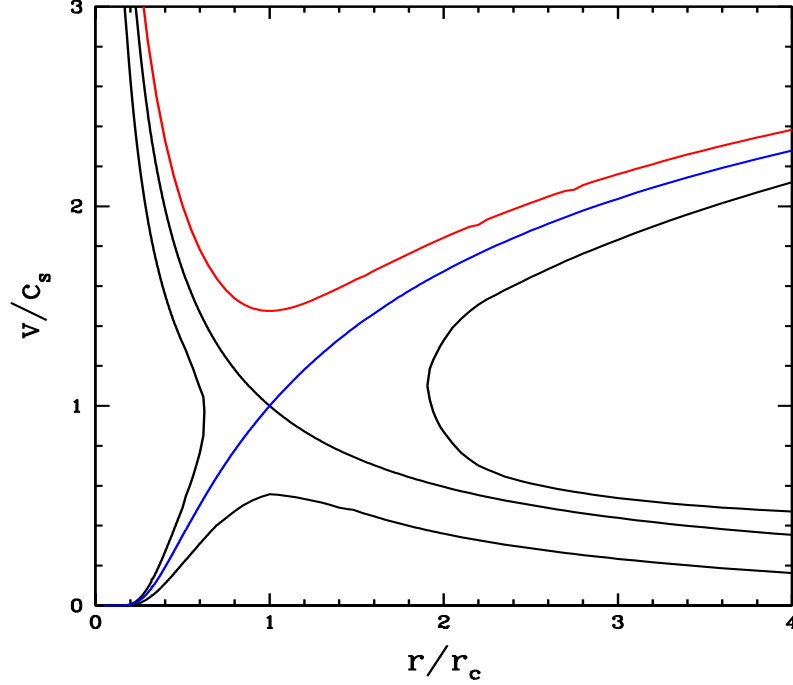


Figure 1.5: The solutions of Parker wind equation. The blue curve is the case of accelerated wind. The red line represents the supersonic set off of the wind from the beginning. The red and blue solutions are the two types of wind outflow from a simulated Red giant in RS Oph discussed in Chapter 3.

where  $c_s = (RT/\mu)^{1/2}$  is the sound speed in the isothermal gas.

The solution of the above differential equation can be given as:

$$\left(\frac{v}{c_s}\right)^2 - \ln\left(\frac{v}{c_s}\right)^2 = 4 \ln\left(\frac{r}{r_c}\right) + 4\frac{r_c}{r} + C. \quad (1.37)$$

where  $r_c = GM_*/2c_s^2$  is called the critical radius where the wind speed is equal to the sound speed in the wind medium and the constant  $C$  is the integration constant.

The value of  $C$  is crucial to determine the radial wind speed profile. When  $C = -3$  the solution is the unique positively accelerated wind starting from subsonic values, passing through the sonic point at  $r_c$  and then having supersonic values afterwards (the blue curve in Figure 1.5). When  $C > -3$ , the solution is supersonic at the beginning, decreasing its value until the sonic radius to reach its minimum value which is slightly greater than the sound speed, then a small acceleration outwards as shown in red in Figure 1.5. These are

the two possible sorts of Parker solutions used in this work. (see Lamers and Cassinelli, 1999; Parker, 1958).

For a relatively hot young star like the sun, the thermal pressure is sufficient to overcome the gravity in the corona to onset the outflow, but in the case of AGB stars, the thermal pressure is not strong enough to maintain the outward motion in the stellar atmosphere. For such a case an outward force is needed (Boffin et al., 1994; de Val-Borro et al., 2009). We shall see that this additional force is in fact the effect of radiation pressure on the wind and approximately follows as  $r^{-2}$ . The momentum equation then becomes

$$\frac{1}{v} \frac{dv}{dr} = \frac{[\frac{2c_s^2}{r} - \frac{GM_*}{r^2} + \frac{A}{r^2}]}{[v^2 - c_s^2]}, \quad (1.38)$$

or

$$\frac{1}{v} \frac{dv}{dr} = \frac{[\frac{2c_s^2}{r} - \frac{G}{r^2} M_*(1 - \Gamma)]}{[v^2 - c_s^2]}, \quad (1.39)$$

where  $A$  is a positive constant and  $\Gamma = (A/GM_*)$ . With the existence of this additional force, the critical point shifts to

$$r_c(\Gamma) = \frac{GM_*(1 - \Gamma)}{2c_s^2}. \quad (1.40)$$

The scenario of heavy mass loss via a slow wind in evolved asymptotic giant branch (AGB) stars is a two stage process. These stages are the atmospheric levitation of gas molecules by pulsation-induced shock waves to certain stellar atmospheric heights to form dust grains (at the dust condensation radius) and radiative acceleration initiated at that radius outwards (Bladh and Höfner, 2012). So the dynamics of the wind elements from the condensation radius outwards is controlled mainly by the gravitation and the radiation pressure force as the thermal pressure effect is much less effective.

$$\text{acceleration} = a_{\text{grav}} + a_{\text{rad}} = -\frac{GM_*}{r^2} + \frac{A}{r^2}, \quad (1.41)$$

$$A = \frac{\langle K \rangle_H L_*}{4\pi c}. \quad (1.42)$$

where  $G$  is the gravitational constant,  $c$  is the speed of light,  $M_*$  and  $L_*$  are the stellar mass and luminosity respectively and  $\langle K \rangle$  is the total flux-averaged dust opacity.

The winds from stars at earlier stages of their lives, like the red giant phase, are poorly understood (Espey and Crowley, 2008). The giant component in most of the symbiotic binaries including RS Oph is a Red Giant Branch (RGB) star and has no dusty atmosphere. Therefore, here we just depend on the Parker wind model as a generic description of wind

outflow.

## 1.4 Accretion via stellar wind capture (analytical approach)

The fact that the mass transfer through the wind flow strongly depends on the details of a particular stellar system (orbital parameters, wind flow etc) means that the fraction of the wind mass that is captured is going to be quantitatively uncertain. For the case of wind mass transfer in a binary system the simplest definition of the mass capture fraction can be analytically solved as the ratio of mass that is accreted on to the primary to that lost by the secondary or the ratio of their time derivatives:

$$f = \frac{\dot{M}_c}{\dot{M}_2} . \quad (1.43)$$

This ratio can be estimated by making some kind of compromise between Hoyle and Lyttleton (1939) and Bondi (1952) models of the accretion rate.

Hoyle and Lyttleton (1939) first neglected the gas pressure and looked at the accretion rate onto a star of mass  $M$  moving with a supersonic and constant velocity through the gas medium. In this model an accretion line forms behind the object as the material is gravitationally deflected around the star and collides with the material from the other side of the object. Due to loss of energy the material now has less kinetic energy than gravitational potential energy relative to the accretor, making it bound to the star and meaning it will eventually be accreted. According to Hoyle- Lyttleton if the luminosity effect of the accretor is neglected (see e.g., Fukue and Ioroi, 1999) the accretion rate is

$$\dot{M}_{HL} = \pi R_{HL}^2 \rho_{\infty} v_{\infty} , \quad (1.44)$$

where  $R_{HL} = 2GM/v_{\infty}^2$  is the characteristic radius inside which the particles are bound to the accretor,  $\rho_{\infty}$  is the density of the medium in which the star moves,  $v_{\infty}$  is the relative velocity to the medium. Bondi and Hoyle (1944) concluded that the accretion rate above  $\dot{M}_{HL}$  is the maximum and

$$\dot{M}_{HL} = 2\alpha\pi G^2 M^2 \rho_{\infty} v_{\infty}^{-3} . \quad (1.45)$$

Bondi (1952) then included the gas pressure and neglected the motion of the star, modelling spherically symmetric accretion on to a stationary point mass, as the thermal pressure is no longer negligible. Thus,

$$\dot{M}_B = 2\pi G^2 M^2 \rho_\infty c_\infty^{-3} , \quad (1.46)$$

where  $c_\infty$  is the sound speed of the medium. Finally, Bondi (1952) suggested the intermediate case of taking into account both the relative velocity of the star- gas motion and the pressure of the gas medium (see Edgar (2004) for more details). Hence the Bondi- Hoyle accretion rate is

$$\dot{M}_{BH} = 4\pi G^2 M^2 \rho_\infty (c_\infty^2 + v_\infty^2)^{-3/2} . \quad (1.47)$$

From the continuity equation, mass loss rate by the secondary is

$$\dot{M}_2 = 4\pi r^2 v_w \rho . \quad (1.48)$$

Thus the capture fraction according to Bondi- Hoyle accretion will be

$$f_{BH} = \frac{G^2 M^2}{a^2 v_w} (c_\infty^2 + v_\infty^2)^{-3/2} . \quad (1.49)$$

The application of the Bondi- Hoyle- Lyttleton (as referred to in Edgar, 2004) accretion rate (BHL hereafter) is common in the study of the mass capture fraction in binary systems to estimate the value of the accretion rate or to compare the numerical results with. However, as will be discussed in section 3.7.2, the BHL accretion rate is applicable only if the wind speed is much greater than the orbital speed of the stars.

## 1.5 Symbiotic stars and symbiotic novae

In terms of their binary properties, symbiotic stars belong to the long period interacting binaries which are made up of a compact, hot component such as a white dwarf and a cool component such as a red giant referred to as S-types, or a white dwarf with a Mira variable surrounded by an optically thick shell of dust in D-types (See Mikołajewska, 2007, 2012 for recent reviews). The S-type stands for "Stellar" which is represented by a Stellar type of the infrared (IR) continuum from a normal giant whereas D-types or "Dusty" is characterised by an additional strong IR emission from the surrounding dust. The existence of two different spectral features in one object led early researchers to conclude that a cool and a hot component exist together in a binary. This was evident from the optical absorption lines from the surface of the cool component and nebular emission lines from the hot source with the components being known as "symbiotic hot and cool components"

(Sokoloski, 2003). Figure 1.6 shows Mira AB, the nearest symbiotic binary in which the hot component (Mira B) and the cool component (Mira A) have been resolved from x-rays to radio wavelengths (Karovska, 2006). The presence of a giant star requires the systems to have large orbital separations. As a result the orbital periods for S-types are about (1-15) years, whereas for D-types they are more than 20 years, the longest orbital periods among interacting binaries (Mikołajewska, 2007).

They are interacting because the giant loses mass and a fraction of this transfers to the white dwarf via accretion. The behaviour of symbiotic systems is a consequence of this interaction. Symbiotic systems are divided into two subclasses according to their behaviour; the ordinary or classical symbiotic stars and the symbiotic novae (SNe). Classical symbiotics constitute the majority of the symbiotic stars in which the hot components luminosity remains nearly constant for months to years and is associated with up to 3 magnitude increases in the optical and UV, which is thought to be due to accretion rates higher than that of stable nuclear burning on the surface of the WD (Kilpio et al., 2011). Examples of classical symbiotic systems are Z And, CI Cyg and AX Per.

Symbiotic novae (SNe) are thought to be produced by thermonuclear runaway events on the surface of the white dwarf. It is either a slow nova where the outburst lasts for decades (the case of classical novae) or fast recurrent novae (SyRNe) with short-lasting outbursts ( $\sim$  several days) and recurrence time of several years to decades. The mass of the white dwarf determines the nova type, with relatively very massive white dwarfs in SyRNe and much less massive white dwarfs in SyNe. There is also a strong correlation between the duration of the novae and the luminosity during plateau phases, the more massive the white dwarf is the shorter the plateau phase with higher luminosity. Symbiotic novae are very rare (Mikołajewska, 2010), only nine out of 200 known symbiotic systems (AG Peg, RT Ser, V1329 Cyg and PU Vul) from S-types and (RR Tel, V2110 Oph, V1016 Cyg, HM Sge and RX Pup) from D-types belong to slow SyNe and only four belong to fast SyRNe (RS Oph, T CrB, V3890 Sgr and V745 Sco) all from S-types. In addition the most recent symbiotic nova V407 Cyg (in 2010) is the first SyRNe with a Mira companion.

Earlier researchers such as Webbink et al. (1987) concluded that RNe are the results of either thermonuclear runaway on the surface of the WD, such as the cases of (T Pyx and U Sco), or because of an accretion rate enhancement on to the WD due to instability in the atmosphere of the giant or even in the accretion rate itself as in (RS Oph and T CrB). Classical novae need tens of thousands of years to recur. That means for the novae to recur frequently (years to decades), there must be a very massive WD accreting at a very

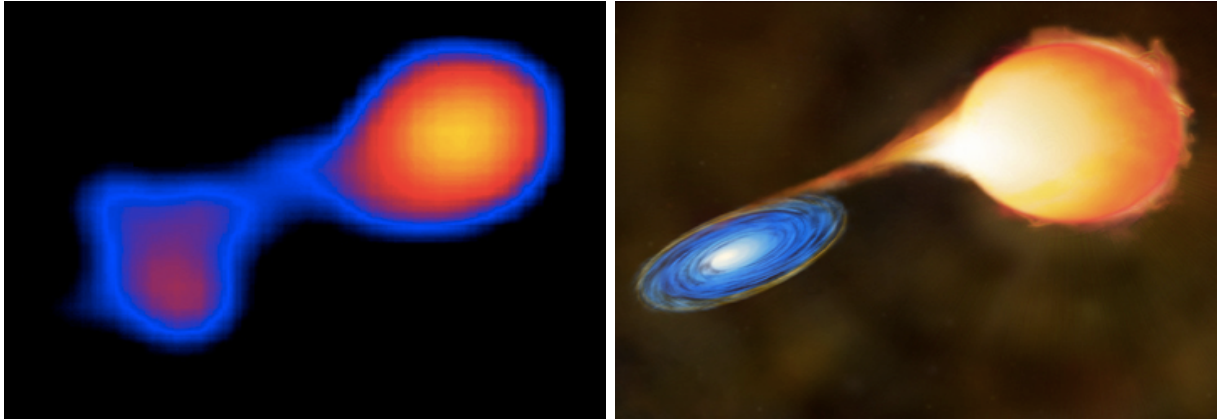


Figure 1.6: Chandra images resolves the components of Mira AB and a bridge between them as an evidence of mass transfer (left). On the right is an artists impression of the interacting system, showing evidence of mass transfer through RLO-like and wind (Karovska, 2006, and references therein).

high rate (see section 3.2). The presence of high mass transfer rates make the SyRNe stars promising candidates for SN Ia, where the massive white dwarf accretes matter from the less massive red giant to approach the Chandrasekhar Limit (see section 1.6). Nova explosions remove almost all of the matter accumulated between two successive explosions which means that the WD mass does not increase quickly. However Alexander et al. (2011) show a steady accretion rate of about  $\approx 3\text{--}8 \times 10^{-7} M_{\odot} \text{yr}^{-1}$  can sustain steady nuclear burning on the white dwarf and allow its mass to grow on a  $Myr$  timescale.

## 1.6 Type Ia Supernovae

Supernova type Ia occurs when the mass of a White dwarf reaches  $\sim 1.4 M_{\odot}$  known as the Chandrasekhar limit (CL) which is the mass above which the WD is subject to gravitational collapse. Through a catastrophic explosion, it is disrupted completely, ejecting mass  $\sim 1.4 M_{\odot}$  with kinetic energy  $\sim 10^{51}$  ergs equivalent to the binding energy of the parent star. SNe Ia are considered important sources of elements and energy in galaxy evolution as well as being standard candles and widely used as cosmological distance indicators.

When the mass of a degenerate star with a Carbon-Oxygen composition, like a white dwarf, is less than the Chandrasekhar limit, its electron degeneracy pressure counterbalances the gravitational pull toward the centre. If via accretion, it grows in mass to reach this limit, the white dwarf undergoes core-collapse giving rise to an increase in the

core temperature to trigger a Carbon-Oxygen (CO) runaway fusion reaction. The mass growth mechanism in white dwarfs is still unclear, the question here is how it reaches the Chandrasekhar limit (CL). Two scenarios so far have been suggested to determine the progenitors of SNe Ia. These are either Double degenerate (DD) or Single Degenerate (SD).

In the DD scenario, two degenerate Carbon-Oxygen white dwarfs may collide head on or they are in a binary system (review of Type Ia see Wheeler and Harkness, 1990; Maoz et al., 2014). Provided that their total mass is larger than the CL, the massive star tidally disrupts the other and accretes matter from it through an accretion disc or accretion may happen spherically, thus a high rate of accretion of CO can ignite Carbon in the core. Such high accretion rates is believed to result in off-centre ignition and eventually gravitational instability giving rise to accretion-induced collapse to produce a neutron star rather than a SN Ia event. Nevertheless, it is believed that the merger remnant rotation could slow down the large accretion and avoid the off-centre ignition.

On the other hand, some believe that the progenitor of SNe Ia could be a CO white dwarf accreting H-rich matter from a non-degenerate companion such as a red giant in cataclysmic variables in a binary system. The main issue with this scenario is that accretion would lead to nova explosions on the surface of the WD during which the majority if not all of the accreted matter will escape (Yaron et al., 2005). It is however believed that some specific systems undergo steady nuclear burning on the surface of the WD where the accreted hydrogen will convert to helium; this prevents thermonuclear runaway on the surface of the WD. As a result, the accreted matter grows the mass of the white dwarf toward the CL (Alexander et al., 2011).

## 1.7 RS Ophiuchi

The recurrent nova RS Ophiuchi (RS Oph) in the constellation Ophiuchus was first discovered by Fleming in 1901 (Kenyon, 1986; Wallerstein, 2008) when the following studies showed that the system had undergone an outburst in 1898. It is one of the best studied systems that has more than one century of observations. The recorded outbursts are 1898, 1933, 1958, 1967, 1985 and 2006, i.e. a semi-regular recurrence of outbursts on  $\approx 20$ -year timescale (see eg. Starrfield, 2008).

During its quiescent phase, the visible magnitude fluctuates between (12 – 10). It has shown similar behaviour during the outbursts (Rosino, 1987); reaching its maximum  $V_{mag} \sim 4.8$  in a few hours followed by a rapid decline in its brightness by losing 1 magni-

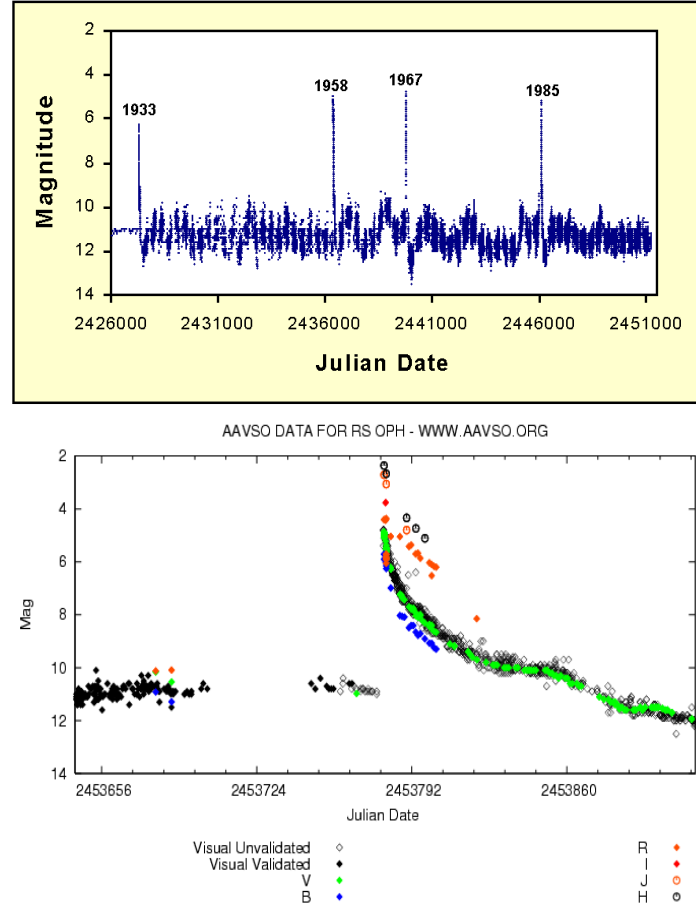


Figure 1.7: Observations of RS Oph outbursts from AAVSO database. Top panel shows the light-curve of RS Ophiuchi during outburst and quiescent phases from 1933 outburst to the quiescent phase after 1985 outburst. Bottom panel shows visual magnitude of the last outburst of RS Oph in 2006. Figures credit to AAVSO archives.

tudes in 2 days past the maximum during which broad emission bands of hydrogen, He I and Fe II are detected. A further decline is seen between day 2 and 15, reaching  $V_{mag} \sim 8.3$  followed by a shallower decline until the visual magnitude goes back to 11.5 – 13 after the outburst, when the system is considered to be in its subsequent quiescent state.

Studies during its quiescent phase estimate an orbital period of  $\sim 445$  days with an orbital inclination of  $30^\circ$  to  $40^\circ$  at a distance of  $\sim 1.6$  kpc (Sokoloski et al., 2006). The system is considered to consist of an M2 red giant in orbit with a white dwarf in a binary system.

## 1.8 Exoplanets and their detection

The curiosity of human beings to discover life beyond Earth goes back centuries. After the discovery of the other planets and their satellites around the sun, scientists attempted to reach beyond our solar system but the modest astronomical instruments at that time were the big challenge on the way towards that discovery. Advances in telescope technology during mid 20th century led researchers to claim to have detected extrasolar planets, but these were disproved by later studies. de Kamp (1950) concluded the existence of a planet around "Barnard's star" the second closest star to the sun, but soon the opposite was confirmed (e.g., Murdin, 2001; Livio et al., 2008 for reviews) In the nineties the first historically confirmed exoplanet system PSR B1257+12 was detected by Wolszczan and Frail (1992), followed by another detection of a Jupiter mass exoplanet around a solar like star known as 51 Pegasi by Mayor and Queloz (1995). To date, using various detection methods, more than 1500 confirmed extra solar planets have been discovered, with thousands (Han et al., 2014) more unconfirmed candidates also (up to date data has been taken from exoplanet data explorer at [exoplanet.org](http://exoplanet.org)).

### 1.8.1 Methods of Exoplanet detection

In searching for extra solar planets and attempting to determine their orbital properties, astronomers use a variety of observational methods. The applicability of each depends on the star-planet system properties, such as planetary orbital period and separation, mass, temperature, brightness, system distance and orientation with respect to the observer. Figure 1.8 depicts the confirmed exoplanets mass-orbital period distribution detected with variety of detection methods. This up-to-date plot is provided by NASA Exoplanet Archive at <http://exoplanetarchive.ipac.caltech.edu/>.

#### 1.8.1.1 Direct detection

Direct observation is perhaps the most difficult method in searching for extra solar planets because of the large brightness contrast between the planet and the host star. One of the possible solution to this issue is to search for planets around relatively dimmer stars such as brown and white dwarfs. Another solution is to search for planets at Infra red (IR) wavelengths where the planet has its peak emission; this can reduce the brightness contrast substantially and help to resolve the planet. In the optical region the emission from the planet is almost as much as the reflection of the starlight incident on it and is

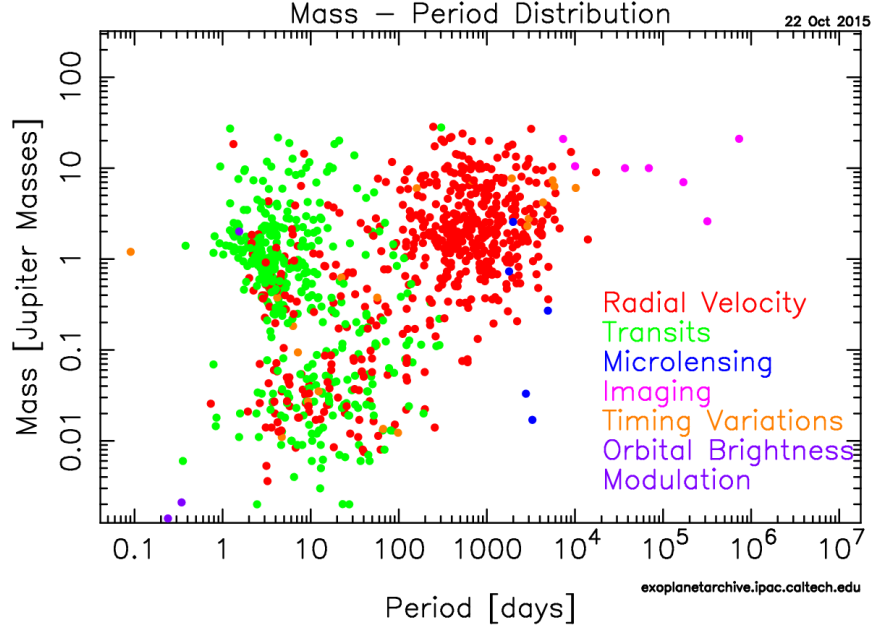


Figure 1.8: Mass-orbital period distribution of discovered exoplanets to date by different methods. Up-to-date figure is available at (<http://exoplanetarchive.ipac.caltech.edu/>).

much less than that of the central star. Since the brightness follows an inverse square law, the maximum brightness of the planet is

$$L_p = \frac{L_*}{4} \frac{R_p^2}{a^2}, \quad (1.50)$$

where  $L_p$  and  $L_*$  are the planets and the stars brightness respectively,  $R_p$  is the radius of the planet and  $a$  is the planet orbital separation. This tells us at optical wavelengths,  $L_p$  is many orders of magnitudes less than  $L_*$ . In addition to the very small angular planet-star separation, this makes the planet spatial resolution hard to detect. To overcome these difficulties several techniques are being used to improve direct imaging because it is the only way of studying the planet's diurnal rotation period and the existence of water on their surfaces. The techniques work in general to suppress the light from the star or to reduce it as much as possible without affecting the reflected light by the planet. Hence relatively few exoplanet detections have been made using this method and it is more desirable for nearby systems in which luminous giant planets are in wide orbits.

Neuhäuser et al. (2005) first detected a spatially resolved planet around the classical T tauri star GQ Lup. Deming et al. (2005) and Charbonneau et al. (2005) directly detected the light from two transiting hot planets using the facilities of Spitzer space telescope (SST), these planets are sufficiently bright in the mid infrared that the difference between

the secondary eclipse and out of eclipse in the mid IR was measurable.

### 1.8.1.2 Radial velocity

Early observations using radial velocity measurements were limited to binary companions of main sequence stars with companion masses down to sub solar masses (Perryman, 2011) that could reach the range of brown dwarf detection. Improvements in accuracy to measuring (10 – 20) m/sec allowed the detection of planetary mass companions. The discovery of a  $0.47 M_{\text{Jup}}$  within 0.05 AU orbiting a main sequence star 51 Peg by Mayor and Queloz (1995) was the first confirmed exoplanet using the radial velocity technique. Most of the early detections were made using this method; 98 out of the first 100 exoplanets were detected using this method (Perryman, 2011, and references therein).

The presence of an orbiting planet makes gravitational perturbations on the star and sets it to orbit the barycentre with a relatively small orbital radius. The star thus has different radial velocities along the line of sight at different orbital phases; this is measured using doppler spectroscopy; the stellar spectral lines shift redward and blueward as it approaches and recedes along the line of sight by the quantity (Deeg et al., 2007)

$$\frac{\Delta\lambda}{\lambda} = \frac{v}{c}, \quad (1.51)$$

where  $v$  is the maximum speed of the star around barycentre. The period of the radial velocity change in the star is equal to the planets orbital period. The amplitude of this variation gives the planet's mass function ( $m_p \sin i$ ). The stellar radial velocity varies periodically and has the radial velocity semi- amplitude

$$K = \frac{v_* \sin i}{(1 - e^2)^{1/2}}. \quad (1.52)$$

From the conservation of momentum

$$v_* = \left( \frac{m_p}{m_* + m_p} \right) v_p = \left( \frac{m_p}{m_* + m_p} \right) \left( \frac{2\pi a_p}{P_p} \right), \quad (1.53)$$

and recalling Kepler's third law (equation 1.1) , $K$  becomes

$$K = \frac{m_p (2\pi G)^{1/3} (m_* + m_p)^{-2/3}}{(1 - e^2)^{1/2}} P^{(-1/3)} \sin i. \quad (1.54)$$

For a circular orbit and assuming  $m_* \gg m_p$ ,  $K$  reduces to

$$K = 28.42(\text{ms}^{-1}) \left( \frac{P}{1\text{yr}} \right)^{-1/3} \left( \frac{m_p \sin i}{M_{\text{Jup}}} \right) \left( \frac{m_*}{M_{\odot}} \right)^{-2/3}. \quad (1.55)$$

For a Jupiter-like planet detection, a precision of  $\sim 10$  m/sec is required but an Earth-like planet detection requires better than 10 cm/sec precision.

### 1.8.1.3 Pulsar timing

The pulsar timing irregularities in the object PSR B1257 + 12 led Wolszczan and Frail (1992) to come up with the first historically proven detection of extra solar planets when they concluded that for the neutron star, the pulsar timing irregularities are caused by the existence of two planets. Again, when the star orbits the common centre of mass, its instantaneous radial distance oscillates along the line of sight and then the time taken for the pulse to reach the observer changes accordingly. If the distance between the observer and the system barycentre along the line-of-sight is  $d$  then the pulsars radial distance varies between  $(d - a_*)$  and  $(d + a_*)$  where  $a_*$  is the orbital radius on which the neutron star orbits the barycentre. This is also subject to uncertainties due to any lack of information about the orbital inclinations. The maximum amplitude of the delay time  $\tau$  for a planet in a circular orbit around a pulsar is

$$\tau = \left( \frac{a_*}{c} \right) \sin i = \left( \frac{a_p}{c} \right) \left( \frac{m_p}{m_*} \right) \sin i, \quad (1.56)$$

where  $a_p$ ,  $m_p$  and  $i$  are the orbital radius, mass and orbital inclination of the planet respectively ( $i = 90^\circ$  when edge-on);  $m_*$  is the mass of the neutron star and  $c$  is the speed of light.

### 1.8.1.4 Transits

This technique gives much more detail about the planet such as planetary size, oblateness, composition, atmosphere and temperature. Although the detailed shape of the transit lightcurves depend on the details of the planet's ingress/egress times and limb darkening in the stellar surface, transits in principle are the measurement of the stellar light blocked by the planet as the latter passes between the star and the observer (see Figure 1.9). In this case both the star and the planet appear as discs.

$$\delta = \frac{R_p^2}{R_*^2}, \quad (1.57)$$

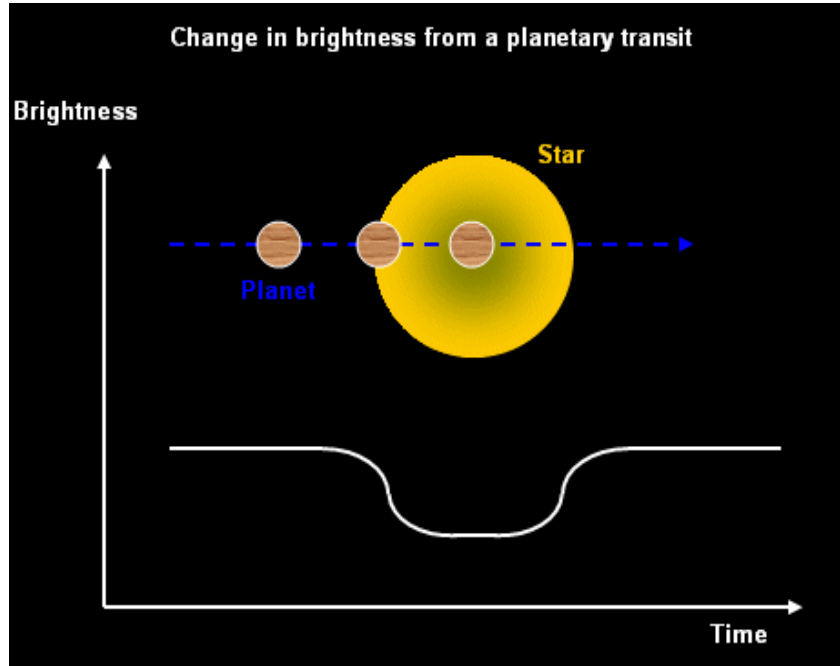


Figure 1.9: A model light-curve of a central star with a transiting planet. The brightness of the host star changes when a transiting planet orbiting the star. Figure credit to <http://www.superwasp.org/>.

where  $\delta$  is the amount of light blocked by the planet (maximum).  $R_p$  and  $R_*$  are planetary and stellar radii respectively.

For a Jupiter- Sun-like system  $\delta_J = 0.01$  while for Earth-like planet  $\delta_\oplus$  is as small as  $0.8 \times 10^{-5}$ ; therefore to detect the effect of transits, high precision in the host stars photometry is required that must be much less than  $\delta$ . The first extrasolar planet detection using the transit method was presented in the studies of Charbonneau et al. (2000) and Henry et al. (2000) when they detected transit events in the G0 dwarf HD 209458. They suggested the existence of a hot Jupiter around the solar like star in the system.

### 1.8.1.5 Astrometry

Astrometry, in fact, deals with the position and motion of astronomical systems and bodies such as galaxies, clusters, stars and solar system. As mentioned before, an orbiting star is already gravitationally perturbed by its planetary companion; similar to its radial velocity, the stars transverse motion in the plane of the sky can be measured. Taking off the stars apparent motion in the sky, the projected motion of a star in the plane of the sky around the barycentre appears as an ellipse with angular semi-major axis  $\alpha$  in arcseconds.

$$\alpha = \frac{m_p}{(m_* + m_p)} \frac{a_p}{d} \simeq \frac{m_p}{m_*} \frac{a_p}{d}, \quad (1.58)$$

where  $a_p$  is the semi-major axis of the planet's orbit and  $d$  is the distance along the line of sight.

This method can be regarded as complementary to the radial velocity measurement method. Astrometry favours detection of planets on large semi-major axis orbits (long period) while radial velocity method is sensitive to those with high velocity variation in short orbits. The precision of the current observational instruments can achieve measurements down to 1 milli-arc seconds as those in Hipparcos and Hubble space telescope (HST) fine guidance sensors which is of the order of stellar displacement in orbit with a planet (Perryman, 2011). Gaia is a European Space agency mission to discover new celestial objects based on astrometry. It was Launched in September 2013. Part of the mission is devoted to determining the distances and space motions of the known exoplanets. Perryman et al. (2014) expects Gaia to discover around 70000 extrasolar planets within its 10-year mission.

#### 1.8.1.6 Microlensing

To an observer, the received light from an astronomical object (galaxy, star, ...etc.) can be amplified by another existing astronomical object on the way between the source and the observer. In general relativity, the path of electromagnetic radiation is deflected in the presence of matter. As a result, the light from the background source can be focussed by the foreground object which acts like a lens and hence leads to an amplification in the light.

The phenomenon was first observed in galaxies but it was realised that star-planet systems can produce lensing on the background light sources. The first exoplanet detection by microlensing was made by Bond et al. (2004) when they observed a planetary microlensing event in a system known as *OGLE* 2003 – *BLG* – 235/*MOA* 2003 – *BLG* – 53. The system comprises a main sequence star with a  $1.5 M_J$  planet with an orbital radius  $\sim 3$  AU.

The principle of planetary microlensing is that the focusing of the background starlight by the foreground star is more enhanced for a few days by an orbiting planet. This amplification increases with the planets mass and its distance to the observer. Thus it favours very distant massive planets hardly observed by any other method.

## 1.9 This Thesis

In chapter 2, I review the equations governing fluid dynamics with numerical approaches to solve these equations such as Eulerian/Lagrangian grid based/mesh free approaches. Also in that chapter, the numerical hydrodynamic methods used in this thesis work are introduced, these are a 3-dimensional Smoothed Particle Hydrodynamics 3D-SPH code and a 2-dimensional ZEUS code (ZEUS-2D).

In chapter 3, I present the numerical simulations of Symbiotic recurrent nova RS Ophiuchi to investigate mass transfer, mass capture fraction by the white dwarf from the wind of the red giant companion. I also look at the accretion disc structure in terms of size and surface density. Furthermore, the effect of the red giant rotation on accretion disc and accretion rate is studied.

In chapter 4, I present the results of numerical simulations of the interaction of the transiting exoplanet WASP-12b with its host star in order to reproduce the optical and ultraviolet light-curve of that system. This is studied both in 2D and 3D using ZEUS-2D and 3D-SPH codes respectively.

Finally, in chapter 5, I summarise and conclude all of the work in chapters 3 and 4.

---

# 2

## Theories of Fluid Dynamics

## 2.1 Introduction

The importance of computational hydrodynamics in astrophysics arises from the fact that fluid dynamic processes are associated with the evolution of astrophysical systems over a large range of scales, from planetary, stellar and interstellar systems to galactic and intergalactic systems. Hydrodynamics of such systems manifests itself as a set of conservation laws in the form of integral equations or partial differential equations (PDEs), these are conservation of mass (continuity equation), momentum and energy. It is demanded that the quantity of mass, momentum and energy must be conserved before and after a process. The solution of such equations is not straight-forward and hardly achieved analytically except in a few circumstances or with some system simplifications. Numerical solutions have been introduced for this purpose, especially after the development of computational technology. Numerical simulations using computers for hydrodynamic problems involve discretisation in space and time of the simulated system and solves the three governing equations of the conservation laws with assistance from the gas equation of state and boundary/ initial conditions.

The conservation equations as will be seen in the next sections are expressed in two fundamental frames, the Eulerian description and Lagrangian description. The Eulerian method is basically represented by a finite difference method (FDM) or it is a spatial description that means fluid variables are observed on a certain position in the fluid whilst the Lagrangian method is represented by a finite element method (FEM) and is a material description where the observer is in motion with a fluid element.

There are also two main types of numerical approaches to discretise the computational domain, grid based methods and mesh free methods. In grid based methods the computational domain is made up of a mesh of grid cells, from each of which the fluid properties are locally obtained. In the mesh free method the fluid is represented by individual elements which carry the spatial and temporal properties of the fluid.

### 2.1.1 Grid-based numerical method

There are basically two grid-based numerical approaches in which the computational domain is discretised into a grid of cells. These are Lagrangian grid and Eulerian grid and their applicability depends on the problem preference in terms of their computational cost and accuracy.

### **2.1.1.1 Lagrangian grid-based method**

In this method the computational domain is split into grid cells. As per the name, it is a Lagrangian solution of the conservation laws and is represented by the Finite Element Method (FEM), so the grid cells and therefore the entire mesh move with the material constituting the simulated system. The mass within each cell is constant and the movement of each cell is responsible for mass, momentum and energy transportation and the bulk motion represents the system deformation. This method is sensitive to large deformation therefore it is designed for solving Computational Solid Mechanics (CSM) where the system does not deform as much as the flow in a fluid system.

### **2.1.1.2 Eulerian grid-based method**

Unlike the Lagrangian approach, the Eulerian grid is spatially fixed; the volume and shape of each grid cell also remains unchanged in the entire process; in other words, the material moves across the boundaries of a mesh of fixed grid cells. The flux of mass, momentum and energy across the cell boundaries are computed using a Finite Difference Method (FDM). Since the Eulerian grid is fixed in space and time, simulated system deformation, resulting from material transport, does not change the mesh structure, this makes the Eulerian approach widely used and successful in simulating fluid flows.

## **2.1.2 Mesh-free method**

Despite the popularity of the conventional grid-based codes in simulating dynamical systems, methods such as (FEM) and (FDM) suffer from some inherent difficulties that reflect their weaknesses and limit their application to some particular problems (see Liu and Liu, 2003 for details). For example, simulation of explosions or high velocity impacts results in large deformation and inhomogeneities or deformation in the boundaries. Grid based numerical methods are also not suitable for problems containing a set of discrete physical particles rather than a continuum. For such cases, mesh-free methods can be promising alternatives. This method aims to provide a stable and accurate solution for integral equations and partial differential equations with any kind of boundary conditions using a set of arbitrarily distributed nodes or particles without using any mesh connecting these elements. Smoothed Particle hydrodynamics (SPH) is an example of interest in this work which is a mesh free particle method (MPM). Details about SPH are given in the next sections.

It is evident that both grid based and mesh free methods have advantages and disadvantages. If both methods are used to study the same problem, they should in principle give the same result as they solve the same set of equations. Nevertheless the results are practically slightly different due to different resolutions and numerical uncertainties (Bodenheimer et al., 2007). Therefore each method is used for a certain kind of simulated system undergoing hydrodynamical processes which can be reproduced by the method with the least computational cost and the highest accuracies.

## 2.2 Fluid Equations

The governing equations of conservation of fluid quantities such as mass, momentum and energy with equation of state describe the fluid properties. I will briefly present here the equations of fluids in both the Lagrangian and Eulerian form. Let's begin with the assumption for simplicity that an inviscid compressible fluid has density  $\rho$ , velocity  $\mathbf{v}$ , pressure  $P$  and internal energy  $U$ .

### 2.2.1 Conservation of mass

The mass conservation in fluids is also known as the continuity equation. This states that the change in density of a certain volume of fluid is equal to the net flux of mass in to and out of that volume. The continuity equation in Eulerian form is

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \mathbf{v}) . \quad (2.1)$$

The term on the left hand side represents the temporal change of density and the right hand side term describes the net flow of mass which is called the convective term. By making use of the Lagrangian derivative (see appendix B.1) one can change the Eulerian form of the continuity equation to Lagrangian form. Hence

$$\frac{d\rho}{dt} = -\rho(\nabla \cdot \mathbf{v}) . \quad (2.2)$$

### 2.2.2 Conservation of momentum

Also called the Euler equation (Frank et al., 2002), it is simply Newton's second law of motion, its Eulerian form is

$$\rho \frac{\partial \mathbf{v}}{\partial t} + \rho(\mathbf{v} \cdot \nabla) \mathbf{v} = -\nabla P + \mathbf{f} . \quad (2.3)$$

The term  $\rho(\mathbf{v} \cdot \nabla) \mathbf{v}$  is the momentum convection in the fluid due to a velocity gradient.  $\nabla P$  is the gradient of the gas pressure.  $\mathbf{f}$  in the RHS is some kind of force, like gravity for instance  $-\rho g$  or force due to an applied magnetic field or in the existence of viscosity this can be viscous force. For some particular instances or simplicity one considers  $\mathbf{f} = 0$ . Again the Lagrangian form of equation of momentum conservation can be obtained straightforwardly, (see appendix B.1)

$$\frac{d\mathbf{v}}{dt} + \frac{\nabla P}{\rho} = 0 . \quad (2.4)$$

### 2.2.3 Conservation of Energy

The Eulerian form of the rate of change in internal energy in fluids is expressed as

$$\rho \left[ \frac{\partial u}{\partial t} + (\mathbf{v} \cdot \nabla) u \right] = -P \nabla \cdot \mathbf{v} , \quad (2.5)$$

where  $u$  is the the internal energy. In a similar way, using equation (B.1) by substituting equation (2.2), its Lagrangian form is

$$\begin{aligned} \frac{du}{dt} &= -\frac{P}{\rho} (\nabla \cdot \mathbf{v}) , \\ &= \frac{P}{\rho^2} \frac{d\rho}{dt} . \end{aligned} \quad (2.6)$$

Thus, the Lagrangian specific total energy conservation is

$$\frac{de}{dt} = \mathbf{v} \cdot \frac{d\mathbf{v}}{dt} + \frac{du}{dt} . \quad (2.7)$$

From equations 2.4 and 2.6 the conservation of total energy becomes:

$$\frac{de}{dt} = -\mathbf{v} \cdot \left( \frac{\nabla P}{\rho} \right) - \frac{P}{\rho} (\nabla \cdot \mathbf{v}) . \quad (2.8)$$

### 2.2.4 Equations of state

In astrophysics, the gas properties are often approximated to the ideal gas when modelling gas flow around stars; though it is less accurate in modelling the stellar and planetary

interiors. The ideal gas equation relates the gas pressure and density

$$P = c_s^2 \rho; \quad \text{where } (c_s = \sqrt{KT/\mu}) \text{ is the isothermal sound speed in the gas.}$$

I use the isothermal gas equation throughout this work where the gas internal energy is assumed to be constant. The ideal gas equation takes another form if the internal energy  $u$  is not constant

$$P = (\gamma - 1)u\rho ,$$

where  $\gamma = C_p/C_v$  is the adiabatic index. For isentropic gas which involves adiabatic compression or expansion, the ideal gas equation is

$$P = K\rho^\gamma ,$$

where  $K$  represents the gas entropy and the adiabatic sound speed is expressed as  $c = \sqrt{\gamma(\gamma - 1)u}$ .

## 2.3 Smoothed Particle Hydrodynamics

Smoothed particle Hydrodynamics (SPH) is a numerical method to integrate the equations of motion for fluids. It is not only widely used in the field of astrophysics (as it was born for) but is also popular in solving geophysical and engineering research problems. Nowadays, it has applications in the film and game industries (Price, 2012).

Gas dynamics play an important role in astrophysics. On the other hand, the solution of the fluid equations is not achieved analytically apart from some simplified conditions. SPH has been introduced to the community of computational programming as a popular numerical technique to solve the governing gas dynamics equations.

It was first introduced to simulate astrophysical problems by Lucy in 1977 (Lucy, 1977). In the same year, Gingold and Monaghan used SPH to study non- spherical stellar systems (Gingold and Monaghan, 1977). There have been considerable developments since then, there are a handful of review papers about the methodology and principles of SPH such as Monaghan (1992) , Rosswog (2009), Springel (2010) and most recently, Price (2012). I will here, present briefly the basic formulae associated with SPH algorithms, starting with the theory of discretisation of a continuous function.

### 2.3.1 Discrete approximation to a continuous Scalar Field

Lets consider a scalar function  $A(r)$  which is defined over the three dimensional co-ordinate system  $(r)$  in the range of some volume  $V(r)$ . Using the identity below, it can be expressed as

$$A(\mathbf{r}) = \int_V A(\mathbf{r}') \delta(\mathbf{r} - \mathbf{r}') d\mathbf{r}' , \quad (2.9)$$

where  $r'$  is another variable in  $V$  and  $\delta$  is the Dirac delta function. A smoothed approximation to this can be expressed as

$$A(\mathbf{r}) = \int_V A(\mathbf{r}') W(\mathbf{r} - \mathbf{r}', h) d\mathbf{r}' , \quad (2.10)$$

where  $W$  is an interpolant, called a smoothing kernel with a width parameterised by the smoothing length  $h$ , such that

$$\lim_{h \rightarrow 0} W(\mathbf{r} - \mathbf{r}', h) = \delta(\mathbf{r} - \mathbf{r}') ,$$

subject to the normalisation

$$\int_V W(\mathbf{r} - \mathbf{r}', h) d\mathbf{r}' = 1 ,$$

If  $\rho(\mathbf{r}')$  is a finite density value within  $V$ , then Equation (2.9) can be re- written as

$$A(\mathbf{r}) = \int_V \frac{A(\mathbf{r}')}{\rho(\mathbf{r}')} W(\mathbf{r} - \mathbf{r}', h) \rho(\mathbf{r}') d\mathbf{r}' \quad (2.11)$$

Finally, discretising the integral interpolant (equation above) to a finite set of interpolation points (particles) with mass element  $m = \rho(\mathbf{r}') d\mathbf{r}'$  and replacing the integral by a summation of mass elements, it becomes

$$A(\mathbf{r}) = A_a \approx \sum_{b=1}^{N_{neigh}} m_b \frac{A_b}{\rho_b} W(\mathbf{r} - \mathbf{r}_b, h) \quad (2.12)$$

Here, the subscript  $a$  is for the property associated with position  $r$ , similarly, subscript  $b$  for  $r'$ ; that means the particle at  $r' = r_b$  has mass  $m_b$ , the scalar function  $A(\mathbf{r}')$  is  $A_b$  and  $\rho(\mathbf{r}') = \rho_b$  at that point. Now what if we assume the scalar function  $A$  be the quantity

representing the density of the fluid, then  $A(\mathbf{r}) = A_a = \rho_a$  and  $A(\mathbf{r}') = A_b = \rho_b$  for  $\mathbf{r}$  and  $\mathbf{r}'$  respectively. We then arrive at

$$\rho_a \approx \sum_{b=1}^{N_{neigh}} m_b W(\mathbf{r}_a - \mathbf{r}_b, h) . \quad (2.13)$$

We should note that the point  $a$  is not necessarily the location of a particle, we just want to estimate the quantities associated with the fluid there. Equation (2.13) tells us that the mass of each particle is smoothed over the region of the kernel to have a smooth density distribution; in other words, the mass of each particle has its weighted effect on the density in the kernel region. This is the basis of the SPH formalism and that is why it is called "Smoothed Particle Hydrodynamics".

### 2.3.2 Gradient of a scalar field

Let's go back to equation 2.10 to investigate the gradient of the scalar function  $A(\mathbf{r})$  as follows

$$\nabla A(\mathbf{r}) = \frac{\partial}{\partial \mathbf{r}} \int_V A(\mathbf{r}') W(\mathbf{r} - \mathbf{r}', h) d\mathbf{r}' \approx \int_V A(\mathbf{r}') \nabla W(\mathbf{r} - \mathbf{r}', h) d\mathbf{r}' \quad (2.14)$$

In the same way as done before, changing the continuous integration to a discrete summation one obtains

$$\nabla A_a \approx \sum_{b=1}^{N_{neigh}} m_b \frac{A_b}{\rho_b} \nabla_a W(\mathbf{r}_a - \mathbf{r}_b, h) . \quad (2.15)$$

Assuming  $A(\mathbf{r}) = \rho_a$

$$\nabla \rho_a \approx \sum_{b=1}^{N_{neigh}} m_b \nabla_a W(\mathbf{r}_a - \mathbf{r}_b, h) . \quad (2.16)$$

### 2.3.3 Vector calculus

Now suppose we have a vector quantity  $\mathbf{A}(\mathbf{r})$  rather than a scalar one. The vector quantities also have the identity shown in equation 2.10 and can be discretised the same way as is done to the scalar field in equation 2.12. So one expects the following relations for vector quantities:

$$\mathbf{A}(\mathbf{r}) \approx \sum_{b=1}^{N_{neigh}} m_b \frac{\mathbf{A}_b}{\rho_b} W(\mathbf{r}_a - \mathbf{r}_b, h) , \quad (2.17)$$

$$\nabla \cdot \mathbf{A}_a \approx \sum_{b=1}^{N_{neigh}} m_b \frac{\mathbf{A}_b}{\rho_b} \cdot \nabla_a W(\mathbf{r}_a - \mathbf{r}_b, h) , \quad (2.18)$$

$$\nabla \times \mathbf{A}_a \approx - \sum_{b=1}^{N_{neigh}} m_b \frac{\mathbf{A}_b}{\rho_b} \times \nabla_a W(\mathbf{r}_a - \mathbf{r}_b, h) . \quad (2.19)$$

### 2.3.4 The smoothing Kernel

Despite mentioning the two conditions regarding the smoothing kernel in equation 2.10 it has not been introduced in detail in this discussion yet. The conditions are that in the limit  $h \rightarrow 0$  it tends to the Dirac delta function and that it is normalised so that the area under the curve is unity. Equation 2.13 tells us that the smoothing kernel is a significant factor in determining the density anywhere; to have an accurate density estimate, a good smoothing kernel should be chosen. Reasonably it should at least have the following properties

- \*  $W(\mathbf{r}_a - \mathbf{r}_b, h) > 0$  for all  $\mathbf{r}_b$  in the range of  $h$  and decreases monotonically from  $\mathbf{r}_a$  and have smooth first and second derivatives.
- \* Symmetry, the kernel should only depend on the separation between particles (and  $h$  as well). So  $W_{ab} = W_{ba}$ ; and
- \* Flat inner profile; slight change in position of a near neighbour does not make a significant effect on the density estimate.

A function such as

$$W(q_{ab}) = \frac{\sigma}{h^d} \exp[-q_{ab}^2] , \quad (2.20)$$

with  $q_{ab} = \frac{|\mathbf{r}_a - \mathbf{r}_b|}{h}$ ,  $d$  the number of spatial dimensions and  $\sigma$  the normalisation factor =  $[1/\sqrt{\pi}, 1/\pi, 1/\pi\sqrt{\pi}]$  in [1, 2, 3] dimensions respectively. Equation 2.20 is a Gaussian function which satisfies all of the above characteristics. However, this function will be

non- zero through the whole domain, that means for a certain position, all the particles contribute to the density estimate and thus all of the other physical quantities accordingly; as a result the numerical computation will be very expensive and will be  $O(N^2)$ .

Alternatively, a function should be implemented so that it goes to zero at some finite distance as the distant particles have negligible effects. It is reasonable to do so because at a certain location in the fluid, the distant particles have no significant contribution to the physical properties at that location.

The B- spline functions are the most commonly used smoothing kernels. Since continuity required at least at first and second derivatives, the lowest order B- spline is the  $M_{(4)}$  (cubic) truncated at  $2h$

$$W(q_{ab}) = \frac{1}{h^d} w(q) , \quad (2.21)$$

where

$$w(q) = \sigma \begin{cases} 1/4(2-q)^3 - (1-q)^3 & \text{if } 0 \leq q < 1 \\ 1/4(2-q)^3 & \text{if } 1 \leq q < 2 \\ 0 & \text{if } q \geq 2 \end{cases} \quad (2.22)$$

The normalisation constant here is  $[2/3, 10/(7\pi), 1/\pi]$  in  $[1, 2, 3]$  dimensions respectively. For more accurate computations, higher order kernels can be used such as  $M_5(\text{quartic})$  or  $M_6(\text{quintic})$ .

### 2.3.5 The smoothing length

Now that the structure of the smoothing kernel  $W$  is known, then  $\rho$  can be calculated from equation 2.13 as long as the value of the smoothing length  $h$  is determined which is an essential parameter to determine the resolution between sparse regions and where the particles cluster. Thus  $h$  is required to be variable within the system.

$$h_{(r_i)} = \eta \left( \frac{m_i}{\rho_i} \right)^{1/d}, \quad (2.23)$$

where  $\eta$  is an optional parameter specifying the smoothing length in units of mean particle spacing  $(\frac{m}{\rho})^{1/d}$ . It should be chosen such that  $\eta \gtrsim 1$  to avoid steep decreases in the kernel function before reaching the nearest neighbours. On the other hand  $\eta \gg 1$  is not desired as it will smooth out the regions that should have been resolved. The parameter  $\eta$  can be converted into an average number of neighbours by,

$$N_{neigh} \approx \frac{4}{3} \pi (\zeta \times \eta)^3, \quad (2.24)$$

where  $\zeta$  is the truncation factor and as mentioned before is equal to 2 for the  $M_4$  cubic spline truncated at  $r = 2h$ .

Now equations 2.13 and 2.23 determine density and smoothing length. To solve density and smoothing length self-consistently, the Newton- Raphson method can be used to iterate between them.

## 2.4 SPH fluid equations

Conservation of fluid quantities such as mass, momentum and energy in SPH are in their discrete form as can be seen below.

### 2.4.1 Conservation of mass

To reformulate the continuity equation in SPH, let's recall equation 2.2,

$$\frac{d\rho}{dt} = -\rho(\nabla \cdot \mathbf{v})$$

Using the identity B.2, the equation at point  $a$  can be expressed as ,

$$\left( \frac{d\rho}{dt} \right)_a = (\nabla \rho \cdot \mathbf{v})_a - (\nabla \cdot \rho \mathbf{v})_a \quad (2.25)$$

Using equations of scalar and vector calculus 2.13, 2.16, 2.17 and 2.18, then as derived in Appendix B.2, equation 2.2 becomes,

$$\frac{d\rho_a}{dt} = \sum_{b=1}^{N_{neigh}} m_b \mathbf{v}_{ab} \cdot \nabla_a W_{ab} , \quad (2.26)$$

where  $\mathbf{v}_{ab} = \mathbf{v}_a - \mathbf{v}_b$  and  $\nabla W_{ab} = \nabla W(\mathbf{r}_a - \mathbf{r}_b, h)$ .

### 2.4.2 Conservation of momentum

Now let's write the equation of conservation of momentum in its SPH form by starting with equation 2.4

$$\frac{d\mathbf{v}}{dt} = -\frac{\nabla P}{\rho} .$$

As shown in Appendix B.2, equation 2.4 can also be written as

$$\frac{d\mathbf{v}}{dt} = -\nabla\left(\frac{P}{\rho}\right) - \frac{P}{\rho^2}\nabla\rho . \quad (2.27)$$

Using equations 2.15 and 2.16 , then it becomes

$$\left(\frac{d\mathbf{v}}{dt}\right)_a = -\sum_{b=1}^{N_{neigh}} m_b \left(\frac{P_a}{\rho_a^2} + \frac{P_b}{\rho_b^2}\right) \nabla_a W_{ab} . \quad (2.28)$$

Now, to find out if the linear momentum of the whole system is conserved, one calculates the summation of the time derivative of linear momentum of each individual particle.

$$\sum_a m_a \frac{d\mathbf{v}_a}{dt} = -\sum_a \sum_b m_a m_b \left(\frac{P_a}{\rho_a^2} + \frac{P_b}{\rho_b^2}\right) \nabla_a W_{ab} . \quad (2.29)$$

The equation above equals zero since the double summations vanish due to the anti-symmetric kernel gradients (Price, 2012). Hence the total linear momentum is conserved. The rate of change of total angular momentum of the system, by definition is,

$$\frac{d\mathbf{L}}{dt} = \sum_a m_a \left(\mathbf{r}_a \times \frac{d\mathbf{v}_a}{dt}\right) = -\sum_a \sum_b m_a m_b \left(\frac{P_a}{\rho_a^2} + \frac{P_b}{\rho_b^2}\right) \mathbf{r}_a \times \nabla_a W_{ab} = 0 , \quad (2.30)$$

which is again zero because of antisymmetry in the double summation. This tells us that the total angular momentum is conserved.

### 2.4.3 Conservation of Energy

In the absence of dissipation, the time evolution of internal energy at particle  $a$  can be given by equation 2.6

$$\frac{du_a}{dt} = \frac{P_a}{\rho_a^2} \frac{d\rho_a}{dt} , \quad (2.31)$$

and introducing equation 2.26 one gets

$$\frac{du_a}{dt} = \frac{P_a}{\rho_a^2} \sum_{b=1}^{N_{neigh}} m_b \mathbf{v}_{ab} \cdot \nabla_a W_{ab} . \quad (2.32)$$

This tells us how the internal energy evolves in SPH codes. It is worth noting that the fluid regimes simulated in this thesis have adopted an isothermal equation of state which means internal energy is constant in time.

To investigate the total energy of the system. One starts with the total specific energy presented in equation 2.7 for point  $a$  as

$$\frac{de_a}{dt} = (\mathbf{v}_a \cdot \frac{d\mathbf{v}_a}{dt}) + \frac{du_a}{dt} .$$

Substituting equations 2.28 and 2.32 into that above, as shown in Appendix B.2 the time change of specific total energy at point  $a$  becomes

$$\frac{de_a}{dt} = - \sum_{b=1}^{N_{neigh}} m_b \left[ \frac{P_a}{\rho_a^2} \mathbf{v}_b + \frac{P_b}{\rho_b^2} \mathbf{v}_a \right] \cdot \nabla_a W_{ab} . \quad (2.33)$$

The total energy of the whole system is then

$$\frac{dE}{dt} = \sum_a m_a \frac{de_a}{dt} = - \sum_b \sum_a m_a m_b \left[ \frac{P_a}{\rho_a^2} \mathbf{v}_b + \frac{P_b}{\rho_b^2} \mathbf{v}_a \right] \cdot \nabla_a W_{ab} = 0 , \quad (2.34)$$

which is zero for the same reason mentioned above. That means the total energy of the system in SPH is conserved.

### 2.4.4 Dissipative terms and viscosity switching

The equations that have been derived so far do not include any source of energy dissipation. The sources of energy dissipation in gaseous systems are thermal conduction and viscosity. Though there is no thermal conductivity prescription in the scope of this thesis

work, below is a brief description of viscosity implementation in SPH.

In gas dynamics, even for perfectly smooth initial conditions, it is common that the simulated system may evolve to have steep discontinuities or "shocks". For this, there has been attempts to introduce physical viscosity to help in numerical stability and allow shocks within simulations to be resolved. It is nevertheless a small value and also hard to code efficiently. Standard SPH (artificial) viscosity as an alternative can smooth the gas particle velocities in the shocked region. It is not meant to mimic the physical viscosity, it is rather a kind of sub-grid model. Therefore, viscosity switching is required as to be effective where it should be (such as shocked regions) and to be turned-off elsewhere. The implementation is achieved by adding a new term  $\Pi_{ab}$  to the Euler equation to account for the effect of viscosity on the motion of the gas

$$\left(\frac{d\mathbf{v}}{dt}\right)_a = - \sum_{b=1}^{N_{neigh}} m_b \left( \frac{P_a}{\rho_a^2} + \frac{P_b}{\rho_b^2} + \Pi_{ab} \right) \nabla_a W_{ab}, \quad (2.35)$$

where

$$\Pi_{ab} = \begin{cases} \frac{-\alpha c_{ab} \mu_{ab} + \beta \mu_{ab}^2}{\rho_{ab}} & \text{where } \mathbf{v}_{ab} \cdot \mathbf{r}_{ab} < 0 \\ 0 & \text{otherwise} \end{cases} \quad (2.36)$$

$$\mu_{ab} = \frac{h_{ab} \mathbf{v}_{ab} \cdot \mathbf{r}_{ab}}{r_{ab}^2 + \epsilon h_{ab}^2},$$

and  $\rho_{ab} = (\rho_a + \rho_b)/2$ ,  $c_{ab} = (c_a + c_b)/2$  and  $h_{ab} = (h_a + h_b)/2$  are the average density, sound speed and smoothing length respectively taken from points  $a$  and  $b$ . In general  $\beta \geq 2\alpha$  and unless stated otherwise,  $\alpha = 1$  and  $\beta = 2$  in the SPH simulations within this thesis work. These values along with  $\epsilon = 0.01$  (which acts to soften the value of  $\mu_{ab}$  at very small  $r_{ab}$  value) are adopted from Rosswog (2009). Equation (2.36) shows the switching of viscosity such that the viscous force is active when the flow is converging that is when two particles at points  $a$  and  $b$  are approaching each other (when  $\mathbf{v}_{ab} \cdot \mathbf{r}_{ab} < 0$ ) and is zero otherwise. It is worth noting that this extra term to the equation of motion does not violate the conservation equations.

## 2.4.5 Time-stepping

The SPH equations derived so far involve time derivatives, which show the evolution of the gas in time. This is done numerically by employing time integrators in the codes

which work to split the time of evolution into small timesteps. It is of high importance to choose a proper time-step for the sake of calculation accuracy of the gas parameters (variables) and to maintain numerical stability within the code. The time stepping condition depends on the nature of the problem that is simulated in the code, the followings are two common and essential timestep conditions:

**Courant- Friedrich- Lewy (CFL) condition:** the most common time-step criterion for hydrodynamical systems which prohibits spatial transmission of information in a time-scale larger than that the sound speed takes. At any point in the simulation domain, its simple form is

$$dt_{CFL} \leq \frac{h}{c_s}, \quad (2.37)$$

where  $h$  and  $c_s$  are the local smoothing length and sound speed respectively.

**The force condition:** is another commonly used timestep condition which is related to the acceleration of the particle (total acceleration). It is possible that a particle is accelerated within the smoothing sphere and escapes before its information has been transmitted and that means its presence is shorter than the (CFL) timescale so the force condition is necessary for such cases.

$$dt_f = \left( \frac{h}{|a|} \right)^{1/2}, \quad (2.38)$$

where  $a$  is the total acceleration of the particle. The minimum of the two conditions,

$$dt = \min\{dt_{CFL}, dt_f\}, \quad (2.39)$$

which is the time step chosen to integrate the gas evolution in the simulations.

## 2.5 The SPH code

The code that has been used in Chapter 3 is a three-dimensional implementation of the SPH algorithms. It is first used by Nixon et al. (2011) to study the evolution of circumbinary gas discs around supermassive blackholes. The code had been set up with two point masses in a binary and a circumbinary disc of SPH particles. The gas disc has an initial mass and surface density profile. The code then modified to simulate mass transfer in

binary systems by continuous injection of SPH particles from an arbitrary radial distance to the point mass representing the mass-losing component, this makes a spherical outflow of the gas particles from the simulated surface of the star and the continuous mass injection determines the mass loss rate. The binary companion acts like an accreting star as it captures a fraction of the SPH particles under the action of the stellar gravitation. The gas self-gravity is neglected, this is justified by the fact that the total gas mass in the simulation is always much less than the total binary mass. The gas has standard SPH artificial viscosity described in Section 2.4.4. All the binary and SPH gas parameter values can be assigned in the code to simulate a certain system.

To check the validity of the code, I have tested the numerical solution of Parker wind in this code. Both subsonic and supersonic initial wind velocities follow the radial velocity profile of the analytical results from Parker model of an isothermal wind. Figures 3.3 and 3.4 show that the numerical solution of the wind given by the SPH code and the analytical solutions are consistent. I have also tested the effect of injection radius on the wind radial velocity profile. Results from different injection radii were identical and consistent with the analytical estimates by Parker solutions. That is provided that the injection velocity of the SPH particles is the same as that given by the analytical results i.e.  $v_{inj} = v(r_{inj})_{\text{Parker}}$  for  $r_{inj} \ll RL_2$  to  $r_{inj} \sim RL_2$  and  $r_{out} \gg RL_2$  where  $r_{inj}$  is the injection radius,  $RL_2$  is Roche radius for the red giant in RS Oph and  $r_{out}$  is the outer radius which determines the size of the simulation sphere (particles beyond  $r_{out}$  are taken out of the simulations).

Apart from some new executables in the code for desired post processing, I have made some more modifications in the code such as controlling the injection location of each SPH particle on the surface of the simulated mass-losing star. By convention in the code, x-axis and y-axis make the plane of the binary orbit, thus the z-axis is the normal to the binary orbital plane. If we call  $\theta$  the angle which determines the "latitude" on the surface of the star and  $\phi$  to be the angle which determines the "longitude". the coordinates of the injection of each particle is then determined by  $(x_{inj}, y_{inj}, z_{inj})$  which are in turn determined by each  $(r_{inj}, \theta, \phi)$ .  $r_{inj}$  is an input constant of the simulation,  $-\pi/2 \leq \theta \leq +\pi/2$  and  $0 \leq \phi \leq 2\pi$ . For each injected particle, the angles  $\theta$  and  $\phi$  were assigned by a "random number generator" which lets the injected particles have random spaces with their neighbouring particles. Now, the spacing between the neighbouring particles are equal in  $\theta$  and  $\phi$  by setting the injection points on the surface of radius  $r_{inj}$  at certain positions determined by  $d\theta$  and  $d\phi$  where  $d\theta = \pi/n_{inj}$ ,  $d\phi = 2\pi/n_{inj}$  and  $n_{inj}$  is the number of injected particles each time step. Now the user not only can switch between random and regular injection method but also can simulate particle injection at any certain

patches on the stellar surface such as injection at the upper hemisphere or cutting the injection in the polar region which are done by determining the limits  $\theta_{min} \leq \theta \leq \theta_{max}$  and  $\phi_{min} \leq \phi \leq \phi_{max}$ .

I have also given the mass-losing star a rotation which is done by giving each injected particle a tangential velocity as well as their initial radial velocities. As mentioned above, xy plane is by convention the binary orbital plane. For the star to rotate about z-axis the wind particles acquire a tangential (rotational) velocity  $v_{rot}$  as well as their radial velocity  $v_{inj} = v_{inj}(x, y, z)$  which is the injection velocity and is normal to the stellar surface while  $v_{rot} = v_{rot}(x, y)$ . It is worth noting that I have considered solid rotators in which the rotational velocity  $v_{rot}$  is not the same for all particles injected at different stellar latitude, that is to maintain constant angular velocity ( $\Omega_{star} = \text{constant}$ ). One can do the opposite in which the rotational velocity  $v_{rot}$  of the whole star is constant while the star acquires different angular velocity for different latitudes (differential rotation). Figures 3.11 and 3.12 indicate the simulation results of wind structure from a rotating star.

I also ran simulations for winds from AGB stars, although these results are not included in this thesis. To account for radiation pressure, I used the simplified model described in section 1.3. in this model the force acts on the dusty wind is simplified by an inverse square law which is similar but opposite in direction to the stellar gravitation. As a result, the force due to stellar gravity acting on the SPH particles is reduced according to equation 1.41.

## 2.6 SPLASH

Here I introduce SPLASH very briefly. It is an open source code <sup>1</sup> designed to visualise SPH output files in 1, 2 and 3 dimensions. The code software is written in Fortran 90. Having been used to solve equations of fluid dynamics, SPH output is a set of point masses representative of fluids. These points move according to the fluid flow. There have been attempts to visualise SPH outputs as scattered point masses based on their location in the domain, these points are sometimes colour coded according to e.g. their local density value. This is often done by making use of N- body visualisation sources suited for SPH data. However, SPLASH can produce rendered versions of the plot which are more appropriate to the fluid properties. Figure 2.1 shows a particle version and a density rendered version of SPH particles using one of the simulations in Chapter 3.

The success of SPLASH comes from its ability to work interactively using mouse and

---

<sup>1</sup><http://users.monash.edu.au/dprice/splash/download.html>

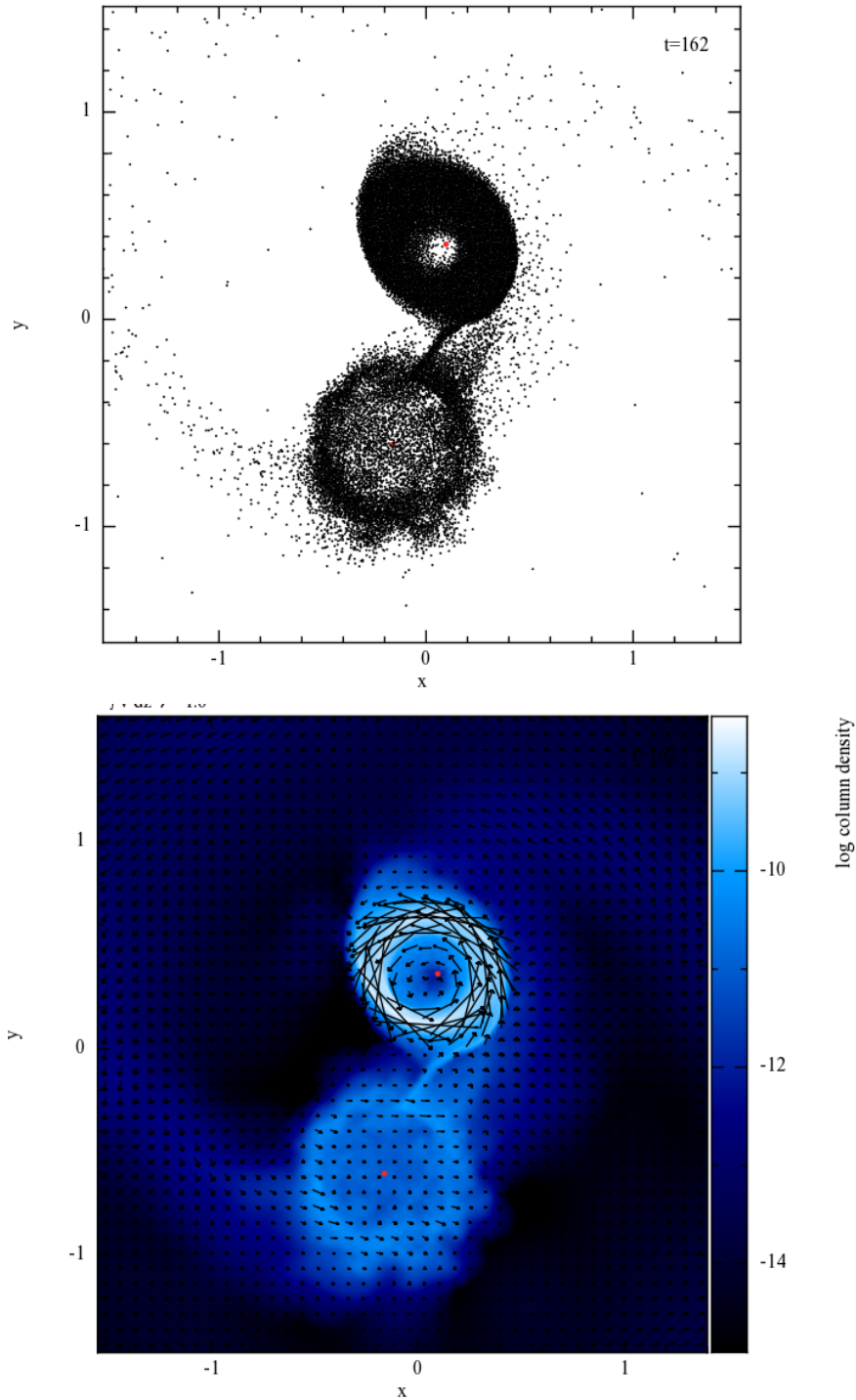


Figure 2.1: SPLASH interactive visualisation of SPH simulations. Top, a snapshot of wind- Roche lobe overflow simulation in a binary, with an accretion disc formed (SPH particles in black with two sink particles in red). Bottom, the same snapshot in it's fluid version column density calculated with the arrows representing the velocity of the flow in the regions.

keyboard for plot settings as well as non-interactively by accessing the sub menu through the main menu and setting up the variables using the command line. It can read binary dump files directly (by using **ssplash** package for example) and output files with ascii formats (such as **asplash**). Its capability of making a sequence of images with a variety of desired formats such as PS and PNG allows one to make movies of the simulations. Interactive visualisation can also be achieved by using the x-window driver so that the user can have an idea about the results. It has remote access capabilities to visualise SPH simulations on supercomputers. And last but not least, it is a free public tool to download. The rendered plots in SPLASH are produced by interpolating particle data into a 2-dimensional pixel array in the shape of a viewing surface. The principle of interpolation in SPLASH is similar to what is seen in SPH. Rendering applies to 2D and 3D simulations and does not apply in 1D. For 3D systems, the rendering is either a projection or surface rendered plot (cross-sectional or slices), for example, our simulation renderings represents density in the line of sight for each pixel (projection density which gives column densities). Similarly for 2D simulations, note that the cross-sectional view gives a 1D projection in that case.

Non-SPH particles such as N-body or sink particles can be plotted alongside; interpolation applies to the SPH particles only and the non-SPH ones are plotted as distinct points on the rendered plot. The case of our two point mass binary stars as sink particles are shown as two red points on the plots. Vector quantities can also be plotted in SPLASH, they are represented by an arrow for each particle for which the direction and the magnitude are represented by the direction and length of the arrow respectively.

## 2.7 ZEUS-2D

The grid-based hydrocode ZEUS-2D is developed to model astrophysical systems in two dimensions in which numerical algorithms are implemented to solve equations of hydrodynamics (Stone and Norman, 1992a) including the effects of magnetic fields and radiation transfer (Stone and Norman, 1992b; Stone et al., 1992). It is a time explicit Eulerian code designed for hydrodynamic computation in any orthogonal coordinate system using the finite difference method (FDM). In the following I describe very briefly the implementations of hydrodynamic algorithms in ZEUS- 2D. Much more detail can be found in Bodenheimer et al. (2007) as well as in the three papers mentioned above.

### 2.7.1 Operator split solution

ZEUS-2D solves the fluid equations using a multi-step scheme, that is splitting the operators. This way the solution of the partial differential equations is broken into parts, each of which represents a single term in the equation. The solution of each part is achieved by their successive evaluations using the results of the update preceding it. For example, if the following is a dynamical equation

$$\frac{\partial y}{\partial t} = \mathcal{L}(y) . \quad (2.40)$$

Using the operator split method, we assume

$$\mathcal{L}(y) = \mathcal{L}_1(y) + \mathcal{L}_2(y) + \dots , \quad (2.41)$$

Then, for the time step  $\Delta t$ , the operator split solution will be

$$\begin{aligned} (y^1 - y^0)/\Delta t &= L_1(y^0) , \\ (y^2 - y^1)/\Delta t &= L_2(y^1) , \\ &\dots \\ (y^{n+1} - y^n)/\Delta t &= L_{n+1}(y^n) , \end{aligned} \quad (2.42)$$

where for each  $\mathcal{L}_i$  there is a corresponding finite-difference representative  $L_i$ . The procedure is to obtain an approximate solution of the non-linear multidimensional operator  $\mathcal{L}$ . Furthermore, the individual parts of the solutions are grouped into *source* and *transport* steps. In the *source* step, an approximate finite-difference solution of the following equations are deduced,

$$\rho \frac{\partial \mathbf{v}}{\partial t} = \nabla P - \rho \nabla \Phi - \nabla \cdot \mathbf{Q} , \quad (2.43)$$

$$\frac{\partial e}{\partial t} = P \nabla \cdot \mathbf{v} - \mathbf{Q} , \quad (2.44)$$

where  $\mathbf{Q}$  is added to the Euler equations to account for the viscous stresses and dissipations due to artificial viscosity. This is to account for the real viscosity in real fluids to smooth out the flow discontinuities that may appear. The artificial viscosity terms are mostly nonlinear, designed to be strong at shock positions and weak elsewhere.

In the transport step an approximate finite-difference solution of the following integral equations is deduced which comprises of advection terms,

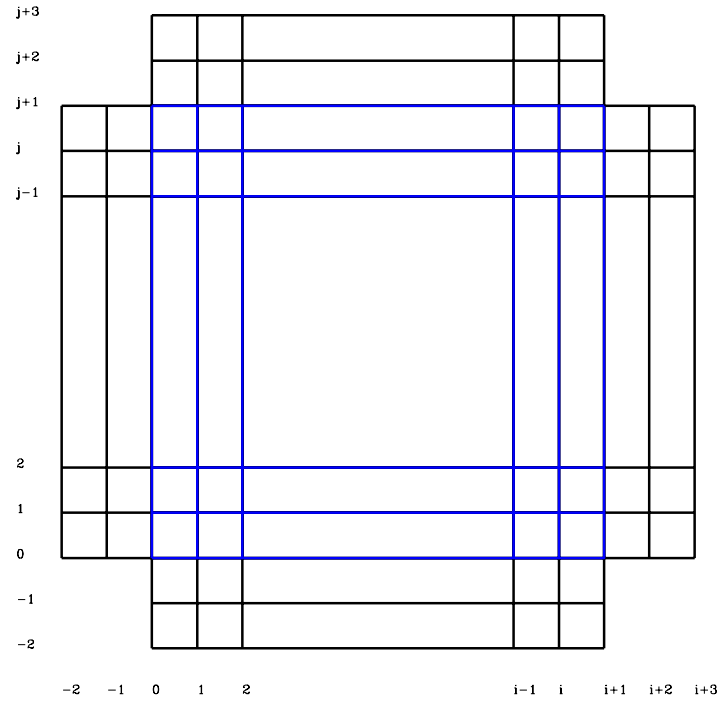


Figure 2.2: Discretisation of the two dimensional domain, showing grid-cells with the ghost cells.

$$\frac{d}{dt} \int_V \rho dV = - \int_{dV} \rho(\mathbf{v} - \mathbf{v}_g) \cdot \mathbf{dS} , \quad (2.45)$$

$$\frac{d}{dt} \int_V \rho \mathbf{v} dV = - \int_{dV} \rho \mathbf{v}(\mathbf{v} - \mathbf{v}_g) \cdot \mathbf{dS} , \quad (2.46)$$

and

$$\frac{d}{dt} \int_V e dV = - \int_{dV} e(\mathbf{v} - \mathbf{v}_g) \cdot \mathbf{dS} , \quad (2.47)$$

where  $\mathbf{v}_g$  is called the grid velocity which accounts for a moving grid.

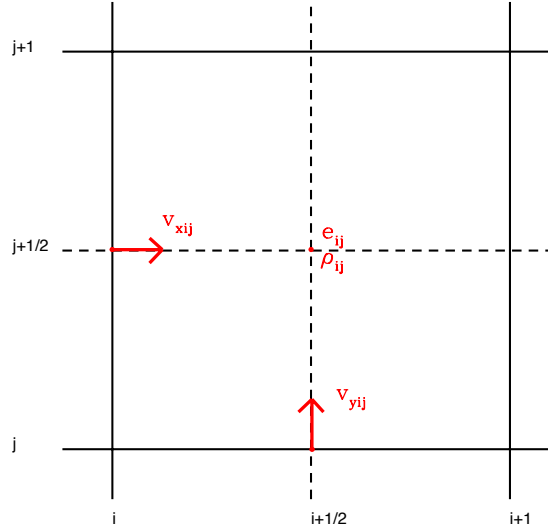


Figure 2.3: A typical grid-cell. Scalar quantities such as density and pressure are zone centered, while the vector quantities are face centered; velocity is shown as an example.

### 2.7.2 The finite-difference mesh

ZEUS uses the finite difference method in which the dependent variables in the partial differential equations to be solved are discretised over the computational domain. To simplify the discretisation method, let's consider a two dimensional Eulerian grid in the  $xy$  plane in a Cartesian coordinate system with uniform grid cells. As shown in figure (2.2), a frame of two layer ghost cells surrounds the active grid where the fluid flow evolves. The presence of such ghost cells is to provide boundary conditions to the simulated flow process.

For each individual grid cell to be identified, let them be labelled with a subscript  $i$  along the  $x$ - axis with  $i = 0$  for the cell at the inner boundary and  $i = I$  at the outer boundary; by convention the inner and outer ghost cell layers will adopt labels  $i = -2, -1$  and  $i = I + 1, I + 2$  respectively. In the same manner, the cells can be labelled with subscript  $j$  along the  $y$ - axis so that the active cells span  $j = 0$  to  $j = J$  and the ghost layers as  $j = -2, -1$  and  $j = J + 1, J + 2$ . The cell edges, where the fluxes are measured, should also be labelled as they are the boundaries of each grid cell; thus the first edge is labelled 0 which is the inner boundary between the first active cell and the neighbouring ghost cell to  $I + 1$  and  $J + 1$  for the outer boundary in the  $x$  and  $y$  axes respectively. That demands the edges of the ghost cells start from  $-2, -1$  and  $I + 2, I + 3$  along the  $x$ - axis and  $-2, -1$  and  $J + 2, J + 3$  along the  $y$ - axis.

Using the finite-difference method, ZEUS uses a staggered mesh where the discretisation of the physical quantities takes place. The scalar quantities such as density and pressure are stored at the centre of each grid cell, while the vector quantities such as velocity are stored at the edges as shown in figure 2.3. The coordinates at the cell edges will then be determined by the edge label  $x_i, y_j$  ( $0 \leq i \leq I + 1$  and  $0 \leq j \leq J + 1$ ) while those at the centres of the cells have coordinates

$$x_{i+1/2} = (x_{i+1} + x_i)/2 \text{ where } (0 \leq i \leq I)$$

$$y_{j+1/2} = (y_{j+1} + y_j)/2 \text{ where } (0 \leq j \leq J)$$

Figure (2-3) shows a typical grid cell and the locations where the scalar and vector quantities are stored. The velocity vector has two components,  $v_{xij}$  and  $v_{yij}$ .

---

# 3

## The Recurrent Nova RS Ophiuchi

### 3.1 Introduction

The recurrent novae (RNe) are a diverse set of close binary star systems that are observed to display novae-like outbursts every 10 – 80 years (Webbink et al., 1987; Anupama, 2008; Schaefer, 2010). While many interacting binaries containing white dwarfs (WDs) are thought to undergo regular novae eruptions, most are expected to recur on typical timescales of  $10^4 - 10^5$  years. Nova outbursts are believed to occur once a WD has accreted sufficient mass to trigger a thermonuclear runaway (nova) explosion on its surface (Kato, 1991; Gehrz et al., 1998; Anupama and Miłojewska, 1999; Starrfield et al., 2000). The mass required to trigger a nova event is dependent, primarily, on the WD mass and the accretion rate (see figure 3.1). The extremely short inter-novae timescales of the recurrent novae suggests that they harbour massive white dwarfs accreting at a very high rate (Prialnik and Kovetz, 1995; Hachisu and Kato, 2001). Consequently, these systems have been identified as potential progenitors to Type Ia Supernova (see e.g. Hachisu and Kato, 2001; Parthasarathy et al., 2007, and references therein), which form when a white dwarf is driven over the Chandrasekhar mass limit (Hillebrandt and Niemeyer, 2000).

### 3.2 The recurrent outburst in RS Ophiuchi

RS Ophiuchi is a recurrent nova and symbiotic star that undergoes nova eruptions every  $\sim 20$  years (Hachisu and Kato, 2000; Anupama, 2008; Wallerstein, 2008). Light-curves of the recurrent nova RS Oph is depicted in figure 1.7. It comprises a  $\sim 1.35M_{\odot}$  WD primary accreting mass from a red giant (RG) secondary, orbiting at a period of 453.6 days (Brandi et al., 2009; Schaefer, 2009).

The short inter-nova timescale of RS Oph places severe constraints on the WD mass and accretion rate. Nova detonation requires a critical pressure ( $P_{\text{crit}}$ ) at the base of the accreted hydrogen layer on the surface of the WD of  $\sim 10^{20}$  dyn cm $^{-2}$  (e.g. Bode and Evans, 2008), which can be written in terms of the WD mass ( $M_{\text{WD}}$ ), radius ( $R_{\text{WD}}$ ) and hydrogen shell mass ( $\Delta M$ ) as

$$P_{\text{crit}} = \frac{GM_{\text{WD}}\Delta M}{4\pi R_{\text{WD}}^4}. \quad (3.1)$$

Assuming a WD close to the Chandrasekhar mass limit, nova detonation in RS Oph requires a hydrogen shell of mass  $\sim 10^{-6} M_{\odot}$  as given by equation (3.1), adopting the mass-radius relation of Nauenberg (1972). Osborne et al. (2011) estimate a similar value of  $\Delta M = 4.4 \times 10^{-6} M_{\odot}$  by assuming  $R_{\text{WD}} = 1.9 \times 10^8$  cm (Starrfield et al., 1991) and Hachisu

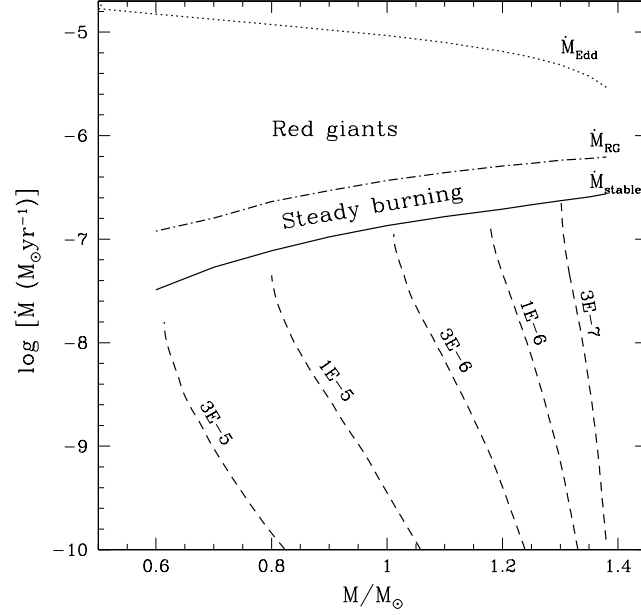


Figure 3.1: The dependence of the amount of mass required to trigger nova explosion on the mass of the White dwarf and accretion rate (Nomoto, 1982; Nomoto et al., 2007). Also shown in the figure the range of accretion rate leading to steady nuclear burning for different white dwarf masses. Figure credit to Nomoto et al. (2007).

et al. (2007) calculate  $\Delta M = 4 \times 10^{-6} M_{\odot}$ . The requirement of  $\Delta M \sim 10^{-6} M_{\odot}$  for a nova eruption, indicates a mean inter-nova accretion rate of  $\dot{M}_{\text{WD}} \sim 10^{-7} M_{\odot} \text{ yr}^{-1}$ . This accretion rate is within an order of magnitude of the Eddington limit and on the border of that expected for steady nuclear burning on the WD, as occurs in the supersoft X-ray sources ( $\sim 3.5 \times 10^{-7} M_{\odot} \text{ yr}^{-1}$ , Hachisu and Kato, 2001). In common with other novae, RS Oph is only briefly observed as a supersoft source after novae eruptions, meaning that  $\sim 10^{-7} M_{\odot} \text{ yr}^{-1}$  is at the upper limit of allowable inter-novae accretion rates.

To accrete the  $\sim 10^{-6} M_{\odot}$  required for a nova outburst, a high fraction of the mass lost from the RG must be transferred to the WD. Estimates of the mass lost from the RG via winds range from  $8 \times 10^{-8} M_{\odot} \text{ yr}^{-1}$  –  $1.8 \times 10^{-6} M_{\odot} \text{ yr}^{-1}$  (see e.g. Hjellming et al., 1986; Bohigas et al., 1989; Seaquist and Taylor, 1990). For  $\dot{M}_{\text{RG}} = X \times 10^{-7} M_{\odot} \text{ yr}^{-1}$ , the WD must be accreting a large fraction ( $f_{\text{cap}} \gtrsim 1/X$ ) of the mass lost by the giant to power the observed nova eruptions. Mass transfer from the RG to the WD in RS Oph may occur via Roche Lobe Overflow (RLO) or stellar wind capture (SWC). The fraction of the mass captured  $f_{\text{cap}}$  depends on the mode of mass transfer (RLO or SWC) which is uncertain. In the case of RLO, an accretion disc forms at the circularisation radius  $R_{\text{circ}}$ . Conservation of angular momentum of the stream implies that

$$R_{L1}^2 \Omega = (GM_{WD} R_{circ})^{1/2},$$

where  $\Omega = \frac{2\pi}{P_{orb}}$ , which gives

$$R_{circ} = \left( \frac{R_{L1}}{a} \right)^4 \left( 1 + \frac{M_{RG}}{M_{WD}} \right), \quad (3.2)$$

where  $M_{WD}$  is the WD mass,  $M_{RG}$  is the RG mass and  $R_{L1}$  is the distance of the WD from the inner Lagrange ( $L1$ ) point of the binary potential. For WD and RG masses of  $1.35M_{\odot}$  and  $0.8M_{\odot}$  (Hachisu and Kato, 2000; Brandi et al., 2009; Schaefer, 2009), the binary separation is  $\simeq 2 \times 10^{13}$  cm and  $(R_{L1}/a) \simeq 0.500 - 0.227 \log(M_{RG}/M_{WD})$  (Warner, 1995, and references therein) so that  $R_{circ} \simeq 3 \times 10^{12}$  cm. This is the minimum size of an accretion disc formed in the case of mass transfer via RLO because viscous stresses will cause the disc's outer radius to increase from this point. The disc formation radius in the case of mass transfer via stellar wind capture (SWC) depends on the mean specific angular momentum of the captured mass, which is a function of the wind speed. However, the RLO estimate provides a useful upper limit on the disc formation radius for the case of SWC.

The size of the accretion disc ultimately determines whether it will undergo dwarf-nova like outbursts driven by the thermal-viscous instability (Lasota, 2001). The maximum radius of a disc that can accrete stably at the rate  $\dot{M}_{nov}$  in the hot, viscous state (and hence not undergo dwarf-nova like outbursts) can be determined by equating the effective temperature of the disc with the hydrogen ionization temperature ( $T_H \sim 6500K$ )

$$T_{eff}^4 \simeq \frac{3GM_{WD}\dot{M}_{nov}}{8\pi\sigma R_{max}^3} \gtrsim T_H^4, \quad (3.3)$$

(see e.g. Frank et al., 2002; King et al., 2003). Adopting  $M_{WD} = 1.35M_{\odot}$  gives a maximum stable radius  $R_{max} \simeq 2 \times 10^{11}$  cm (which is  $\simeq 0.06R_{circ}$  and  $\simeq 0.01a$  in RS Oph). Any disc larger than  $R_{max}$  will be unstable to dwarf-nova type outbursts.

Alexander et al. (2011) examined the susceptibility of the disc in RS Oph to the thermal-viscous instability using a one-dimensional, time-dependent model of the binary-disc system. They conclude, in accordance with the estimate above, that any disc with radius  $\gtrsim 10^{11}$  cm must undergo dwarf-nova like thermal-viscous outbursts every 10 – 20 years, but that the disc luminosity during outburst is not sufficient to explain the observed

RNe eruptions. However, Alexander et al. (2011) also show that the thermal-viscous outbursts of large discs ( $R_D \gtrsim 10^{11}$  cm) involve mass transfer rates greater than the nuclear burning limit and are likely to involve nuclear burning at the surface of the WD. The mass of unprocessed hydrogen accreted during thermal-viscous outbursts is small ( $\lesssim 10^{-7} M_\odot$ ) and at least an order of magnitude below that required to trigger a nova via a thermonuclear runaway. They suggest that the thermal-viscous disc instability combined with nuclear burning at the WD surface may be able to explain the observed eruptions of RS Oph and that the WD dwarf mass may grow, via this route, to the Chandrasekhar mass on Myr timescales, triggering a Type Ia supernova. In contrast, thermonuclear nova explosions are thought to erode the mass of the WD over time (Nomoto, 1982; Yaron et al., 2005).

The nature of the observed eruptions every  $\sim 20$  years in RS Oph and how the WD accretes mass depends critically on the disc size ( $R_D$ ). For  $R_D > R_{\max}$  mass is most likely delivered to the WD via episodes of enhanced accretion during thermal-viscous outbursts of the accretion disc that involve nuclear burning on the surface of the WD. The mass of the WD is likely to increase over time in this case as the processed mass does not contain sufficient hydrogen to trigger a thermonuclear runaway. If  $R_D < R_{\max}$  then the disc will remain in the hot, stable state for a steady accretion rate of  $\dot{M}_{\text{nov}}$ . This is just below the nuclear burning limit and would allow enough hydrogen rich mass to be delivered to the WD to trigger novae eruptions via thermonuclear runaways. The only way to achieve such a small disc radius in RS Oph is via SWC. In general, higher wind speeds are likely to result in smaller disc radii as the specific angular momentum of the captured material decreases (Frank et al., 2002). However, the fraction of the wind mass that is captured by the WD ( $f_{\text{cap}}$ ) is also expected to decrease with increasing wind speed, which will place an upper limit on the speed because the capture fraction needs to be high enough to trigger novae on the observed timescale.

Numerical simulations of RS Oph in quiescence can help us develop an understanding of the key parameters constraining its regular outbursts;  $\dot{M}_{\text{acc}}$ ,  $f_{\text{cap}}$ ,  $M_{\text{WD}}$ ,  $R_D$  (if a disc forms) and surface density of the disc. Walder et al. (2008) ran 3D simulations of RS Oph during quiescence to find a self-consistent interpretation of the observed parameters with  $\dot{M}_{\text{RG}} = 10^{-7} M_\odot \text{ yr}^{-1}$  and a mass capture fraction of 0.1 leading to  $\dot{M}_{\text{acc}} = 10^{-8} M_\odot \text{ yr}^{-1}$ . Using the accretion rate constraints defined in Yaron et al. (2005) for a WD mass of  $\sim 1.4 M_\odot$  they show that these parameters lead to nova outbursts over a similar time range to those observed for RS Oph. However, although the set of parameters obtained by Walder et al. (2008) provide one consistent explanation for the observed behaviour of RS

Oph, they may not represent a unique solution. There may in fact be a wider range of parameter-space for which outbursts occur in the observed timeframe, depending on the properties of the RG wind and even the mass of the WD. In particular, while Walder et al. (2008) find that a  $1.4M_{\odot}$  WD accreting at a rate of  $\sim 10^{-8}M_{\odot}\text{yr}^{-1}$  can explain nova eruptions on a 20 year timescale, can a less massive WD also accrete enough mass for a nova in this time? Additionally, there remains the possibility of outbursts being driven by disc instability rather than thermonuclear runaway. All of these factors will have an impact on our understanding of the evolution of the system. It is important, therefore, that a wider parameter space be explored before the accretion flow in the system and its potential to produce a SNe Ia can be determined.

### 3.3 This chapter

Using smoothed particle hydrodynamics, I study mass accretion in the symbiotic recurrent nova RS Ophiuchi. I investigate different modes of wind particle injection from a mass donating star to the surroundings, based on the Parker solutions of isothermal winds. The test binary system is selected to have physical properties compatible with those of symbiotic binaries. I determine the circumstances under which Roche lobe overflow like mass transfer is obtained, as well as conditions that give rise to direct mass transfer via stellar wind. The structure of the accretion disc and mass capture fraction from the numerical simulation results in each case are also estimated. In all simulations studied, stable accretion discs form around the component representing the white dwarf. I relate the accretion disc size to the corresponding mass capture fraction, providing an analogue to RS Oph. The results show nearly all the accretion discs formed have a radial extent greater than the predicted stability radius. I explore the effect of rotation on the mass capture fraction and disc morphology, setting the red giant star to spin once on its axis per orbit. This includes both prograde and retrograde rotation with respect to the binary orbit.

### 3.4 Accretion disc stability and limit cycle

An accretion disc is in thermal equilibrium if the rate of cooling  $Q_-$  and the rate of heating  $Q_+$  are equal. If we consider viscous dissipation the main source of heating

$$Q_+ = \frac{9}{8}\nu\Sigma\Omega^2, \quad (3.4)$$

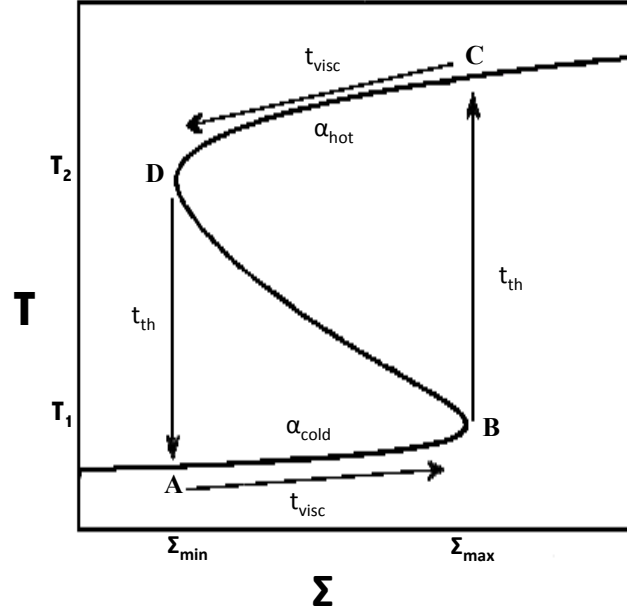


Figure 3.2: Limit cycle of an annulus of radius  $R$  in an accretion disc. The upper part (CD) and the lower part (AB) of the S-curve are the stable thermal equilibria. The middle part (BC) represents the unstable solution for the disc thermal equilibrium. Figure adapted from Frank et al. (2002).

and radiative cooling to dominate  $Q_-$  (see Frank et al. (2002) for more details) that is

$$Q_- = \frac{4\sigma}{3\tau} T_c^4 \quad (3.5)$$

where  $\nu$  is coefficient of viscosity,  $\Sigma = \Sigma(R)$  is the disc (annulus) surface density at radius  $R$ ,  $\Omega$  is the Keplerian angular velocity of the annulus of radius  $R$ ,  $\sigma$  is Stefan-Boltzmann constant,  $\tau$  is the local optical depth and  $T_c$  is the central temperature. Moreover,  $\nu = \nu(R, \Sigma)$  and  $\tau = \kappa\Sigma$ , where  $\kappa$  is gaseous disc mean opacity. The accretion disc thermal equilibrium thus can be expressed in a  $T$ - $\Sigma$  relation. This relation forms an S-curve on the  $(\Sigma, T)$  plane as shown in figure 3.2. The middle part (the path BC) is a thermally unstable branch because radiative cooling is slower than viscous heating, or

$$\frac{dQ_-}{dT_c} < \frac{dQ_+}{dT_c}. \quad (3.6)$$

Any small increase in  $T_c$  leads to further increase in the annulus temperature as cooling rate is inadequate. This is due to change in opacity in the gas of the disc when the disc temperature is close to  $10^4 K$  where hydrogen ionisation starts.

As shown in figure 3.2, the annulus surface density increases through mass flux when the disc is in cold state ( lower branch AB) meanwhile the gas temperature increases until  $(\Sigma_{max}, T_1)$  is reached, this is a slow process which takes place in viscous timescale. When point B is reached, any small perturbation results in the temperature rising as the heating rate exceeds the cooling rate, this makes the annulus even hotter to the point of hydrogen ionisation where opacity increases dramatically. The annulus has reached C, this process is quick and is done in thermal timescale. Now, the annulus is in its hot state and starts to outburst in viscous time along the CD branch which is stable. Outburst is due to the disc high viscosity and results in decrease in surface density. Transition along DA is quick as the surface density is minimum then viscosity decreases resulting in a dramatic cooling in the disc in thermal timescale (this is opposite to the BC transition). The disc is now in its cold state and ready to grow in surface density to go again along AB branch. The process repeats through the limit cycle.

### 3.5 Computational Method

To model RS Oph in quiescence, I use the Smoothed Particle Hydrodynamics (SPH) code first described in Nixon et al. (2011). This is a three-dimensional Lagrangian version of SPH (e.g. Springel and Hernquist, 2002) in which self-gravity is neglected,

The simulation follows the evolution of a binary system undergoing mass transfer between the stellar components, with each star represented by a point mass and viewed in the inertial frame. One of the binary components represents the mass losing star and SPH particles are injected isotropically into the simulation from an injection radius at an arbitrary distance from the star's centre. Changing this radius should in principle have no effect on the simulation results as the injection velocity of the particles should follow the Parker solution, so the initial wind speed changes with varying the injection radius accordingly. In Practice, I have tested radial wind profile from an isolated (the effect of the companion on the wind was turned off in the simulations) mass losing star for different injection radii and the results were the same. Initially, the system is completely devoid of stellar wind, but with time the continuous injection fills the simulated system environment with wind particles until a steady state is reached. Meanwhile, the other stellar component captures a fraction of this wind as an accretion disc begins to form around it.

Given quantities are measured in code units, so that the gravitational constant  $G = 1$ , the total binary mass  $M = M_1 + M_2 = 1$ , the binary separation  $a = 1$  and the orbital period  $P = 2\pi$ . For RS Oph, I adopt the binary mass ratio  $q = \frac{M_{RG}}{M_{WD}} = 0.6$  for all of the simulations

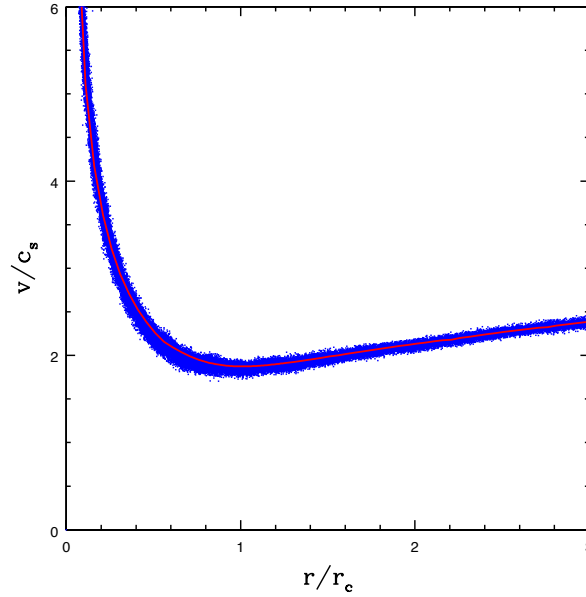


Figure 3.3: Radial velocity profile of an initially supersonic simulated stellar wind. The profile is consistent with the Parker solutions illustrated by the red line.

in this thesis (see Alexander et al., 2011, and references therein). Particles are removed from the simulation if they reach the accretion radius of either binary component and are counted towards the accreted mass. Particles are also removed if they stray too far from the binary centre of mass (typically 2 – 2.5 binary separations), since they are considered to have sufficient kinetic energy to escape the system. Removing such particles reduces the computational cost of the simulation. The self-gravity of the SPH wind particles is not considered because the mass of the gas in the simulations (the total mass of the active particles in the simulations) are much less than the total mass of the binary and the gas is isothermal throughout.

### 3.6 Simulation setup

For the simulations, I follow the Parker wind model for an isothermal gas (section 1.3) and consider both supersonic and subsonic wind injection velocities (e.g. Parker, 1958; Lamers and Cassinelli, 1999). The two implementations are described below.

1. **Supersonic injection of particles:** In this scenario, the particles are injected from the surface of the giant star with a supersonic injection velocity that exceeds the escape velocity of the potential well. Wind particles are injected at  $0.9 \times$  the stellar

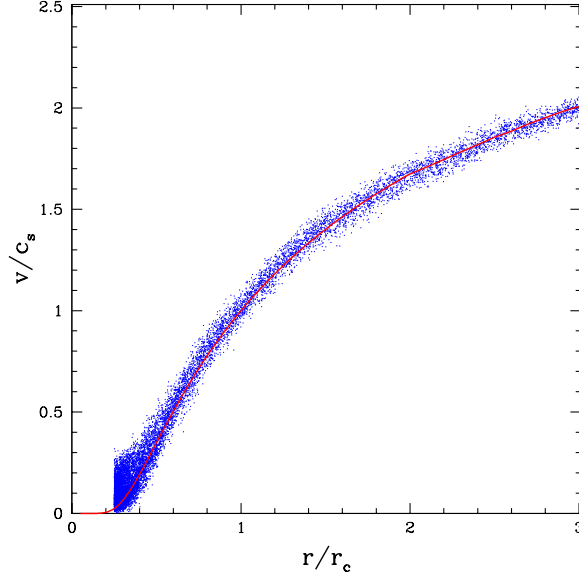


Figure 3.4: Radial velocity profile of an initially subsonic simulated wind. The profile is consistent with the Parker solution represented by the red line.

Roche Lobe radius (using the Eggleton formula. Eggleton, 1983), choosing parameters  $c_s$  and  $v_{\text{inj}}/c_s$ . Figure 3.3 plots the velocity of wind particles around an isolated RG for one of the models with supersonic wind injection. This velocity profile is consistent with the Parker solutions illustrated by the red curve in figure 1.5. The SPH particles decelerate until they reach the critical radius, where they then begin to accelerate slightly.

2. **Subsonic injection of particles:** Here, the particles are given a subsonic initial velocity, which is determined by the isothermal velocity estimate at the injection radius (the Parker solution estimate). The particles are not only able to escape the star's gravity, but also accelerate once they are released, passing through the sonic point at  $r_c$ . The only unknown parameter for this model is the sound speed,  $c_s$ , which depends on the temperature of the wind. Here, I investigate sound speeds in the range  $5 - 19 \text{ km s}^{-1}$ , corresponding to the temperature range of  $4000 - 30000 \text{ K}$ . Figure 3.4 shows the radial-velocity profile formed in the models of an isolated giant wind for this case, and is consistent with the critical solution of the Parker model illustrated by the blue curve in figure 1.5. Here, the wind starts with a close-to-zero initial velocity, passes the sonic point where  $v_{\text{wind}} = c_s$  at  $r_c$  and continues to accelerate out to greater radii.

### 3.7 Results

Accretion discs are formed around the WD in all of the simulations. The discs are generally thin, with different sizes depending on the wind speed. A bow shock is seen in front of the accretion disc in all cases, with a stream of captured wind particles trailing the shock.

The morphologies of mass transfer and accretion disc formation from three example simulations are shown in the upper panels of figure 3.5. Here, both the x-y plane, face-on to the disc, and the x-z plane, perpendicular to the disc, are plotted, coloured according to column density of the wind particles. In each model, wind injection is subsonic, with sound speed increasing from the left- to right-hand panels, at 6, 10 and 19 km s<sup>-1</sup> respectively. The resulting wind velocities produce three different accretion morphologies, which are described as Cases A, B and C below.

For Case A accretion, shown in the upper left-hand panel of figure 3.5, the low velocity of the wind means that the particles are only able to stream through the inner Lagrange point ( $L1$ ) to the Roche lobe of the WD. This wind-driven RLO results in a large disc of material forming around the WD. The middle panel of figure 3.5 shows a second type of mass transfer, Case B, where the particle velocities are higher. In this case, the particles escape the RG from the outer Lagrangian point ( $L2$ ), as well as  $L1$ . Finally, Case C type accretion (right-hand panel) occurs for wind velocities that are higher than the orbital velocity of the giant. In this third case, an almost spherically symmetric outflow of wind particles from the star's surface can be seen, with an enhanced stream through  $L1$  between the two components.

The surface density profile of the discs that form with Case A, B and C mass transfer are shown in the left, middle and right-hand columns of the second row of figure 3.5. For Case A wind RLO, the surface density reaches its peak (35 g cm<sup>-2</sup>) at  $\sim 5 \times 10^{12}$  cm from the WD and the disc extends to  $\sim 10^{13}$  cm. The peak surface density is lower and closer to the WD for Case B mass transfer. For Case C accretion, the disc has a peak density  $< 1.0$  g cm<sup>-2</sup> at  $8.0 \times 10^{11}$  cm, beyond which the density drops off sharply, with the disc extending to approximately  $6 \times 10^{12}$  cm. In the bottom panels of figure 3.5, the orbital velocity profiles of the discs are plotted. Kepler profiles are plotted in red, showing that the rotation of the discs are consistent with Keplerian motion.

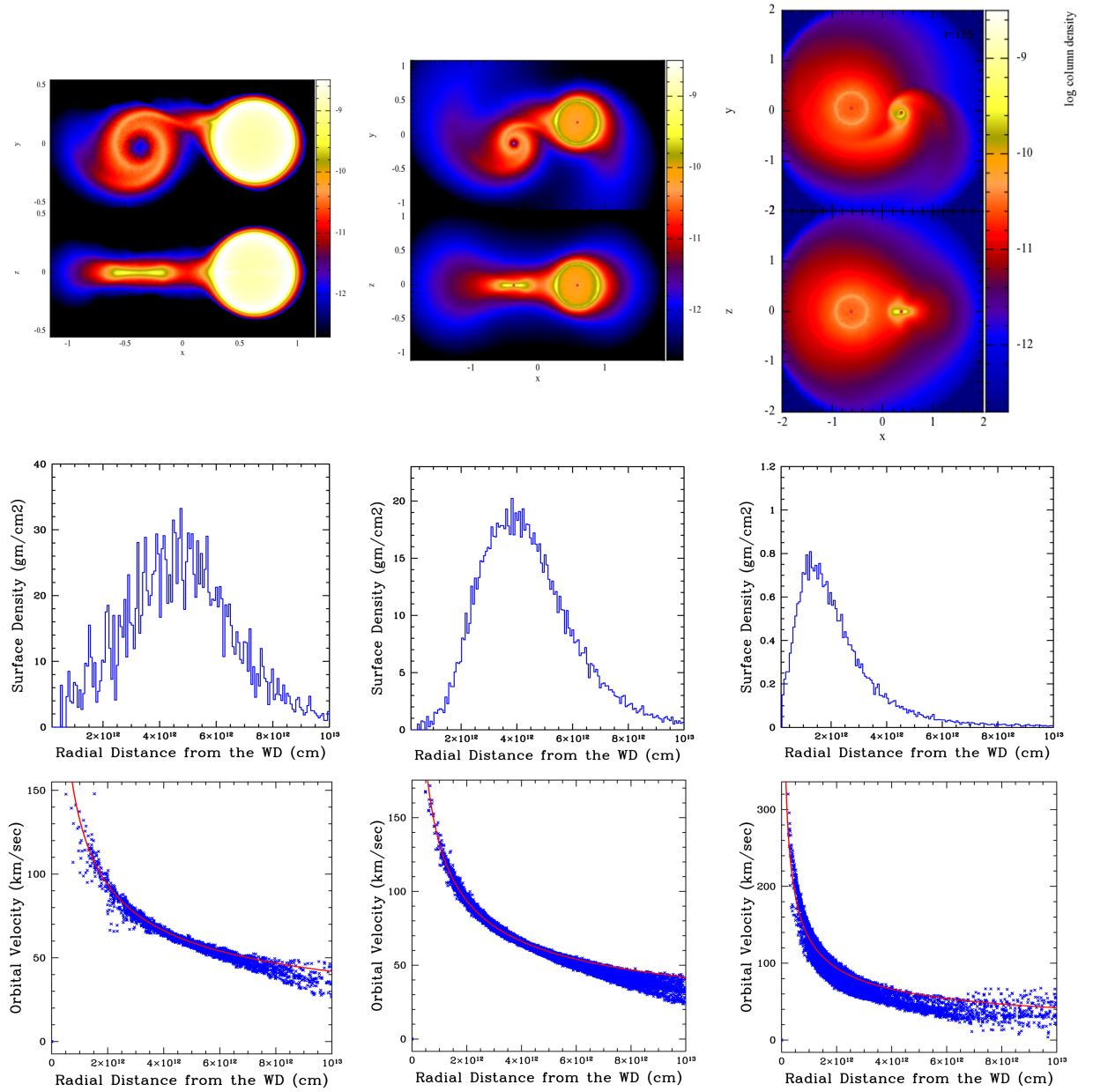


Figure 3.5: Example morphologies of the accretion disc around the WD, formed via mass transfer from the RG wind. From left to right, panels show morphologies for the three types of mass transfer observed in the simulations; Case A (wind RLO), Case B (RG mass loss via  $L1$  and  $L2$ ) and Case C (spherically symmetric wind outflow). These are shown for simulations with subsonic wind injection of 6, 10 and  $19 \text{ km s}^{-1}$  for the left middle and right panels respectively. In the upper panels, images from the simulations are shown, face-on (x-y plane) and side-on (x-z plane), with the colour scale indicating particle column densities. The middle rows then show the one-dimensional surface density profiles of the accretion disc around the WD. Finally, the orbital velocity profiles of these discs are plotted in the lower panels. Here, velocities of individual particles are shown in blue and the Kepler profiles are plotted in red.

| Simulation | $c_s \text{ kmsec}^{-1}$ | $r_c/a$ | $v_{\text{inj}}/c_s$  | $f_{\text{cap}}$ | CPU/hour |
|------------|--------------------------|---------|-----------------------|------------------|----------|
| 1          | 5                        | 9.566   | $1.1 \times 10^{-20}$ | 0.95             | 1440     |
| 2          | 5.357                    | 8.333   | $2.3 \times 10^{-18}$ | 0.95             | 1200     |
| 3          | 5.714                    | 7.324   | $4.2 \times 10^{-17}$ | 0.89             | 960      |
| 4          | 5.786                    | 7.144   | $1.1 \times 10^{-16}$ | 0.88             | 840      |
| 5          | 6                        | 6.643   | $3.5 \times 10^{-15}$ | 0.85             | 720      |
| 6          | 6.25                     | 6.122   | $6.5 \times 10^{-14}$ | 0.77             | 504      |
| 7          | 6.5                      | 5.661   | $7.9 \times 10^{-13}$ | 0.75             | 336      |
| 8          | 7                        | 4.881   | $7.1 \times 10^{-11}$ | 0.65             | 168      |
| 9          | 7.25                     | 4.55    | $5.2 \times 10^{-10}$ | 0.58             | 192      |
| 10         | 7.5                      | 4.252   | $2.9 \times 10^{-9}$  | 0.49             | 192      |
| 11         | 8                        | 3.737   | $4.8 \times 10^{-8}$  | 0.35             | 195      |
| 12         | 8.5                      | 3.31    | $5.3 \times 10^{-7}$  | 0.3              | 204      |
| 13         | 9                        | 2.953   | $4.1 \times 10^{-6}$  | 0.24             | 204      |
| 14         | 9.5                      | 2.65    | $2.2 \times 10^{-5}$  | 0.18             | 216      |
| 15         | 10                       | 2.392   | $8.8 \times 10^{-5}$  | 0.15             | 228      |
| 16         | 10.5                     | 2.169   | $2.8 \times 10^{-4}$  | 0.12             | 240      |
| 17         | 11                       | 1.977   | $7.9 \times 10^{-4}$  | 0.097            | 240      |
| 18         | 12                       | 1.661   | $3.96 \times 10^{-3}$ | 0.084            | 264      |
| 19         | 13                       | 1.415   | 0.013                 | 0.065            | 276      |
| 20         | 14                       | 1.22    | 0.033                 | 0.058            | 280      |
| 21         | 15                       | 1.063   | 0.066                 | 0.059            | 288      |
| 22         | 16                       | 0.9342  | 0.115                 | 0.057            | 312      |
| 23         | 17                       | 0.8275  | 0.176                 | 0.056            | 336      |
| 24         | 18                       | 0.7381  | 0.248                 | 0.057            | 384      |
| 25         | 19                       | 0.6625  | 0.327                 | 0.058            | 432      |

Table 3.1: Subsonic injection simulations. In column 1, number of simulations; column 2, sound speed of the wind; column 3, critical radius of the wind in units of binary separations; column 4, injection velocity of the wind in units of the wind sound speed; column 5, numerical capture fraction and column 6, the CPU time consumption for the simulations to reach steady state in hours. It is worth mentioning that binary mass ratio for all these simulations is  $q = 0.6$ .

| Simulation | $c_s \text{ kmsec}^{-1}$ | $r_c/a$ | $v_{\text{inj}}/c_s$ | $f_{\text{cap}}$ | CPU/hour |
|------------|--------------------------|---------|----------------------|------------------|----------|
| 26         | 4                        | 14.9    | 9.487                | 0.95             | 192      |
| 27         | 4                        | 14.9    | 11.16                | 0.95             | 192      |
| 28         | 4                        | 14.9    | 12.28                | 0.9              | 204      |
| 29         | 4                        | 14.9    | 13.11                | 0.52             | 240      |
| 30         | 4                        | 14.9    | 13.39                | 0.45             | 288      |
| 31         | 4                        | 14.9    | 13.73                | 0.22             | 300      |
| 32         | 4                        | 14.9    | 13.95                | 0.14             | 264      |
| 33         | 4                        | 14.9    | 14.51                | 0.09             | 216      |
| 34         | 4                        | 14.9    | 15.63                | 0.05             | 198      |
| 35         | 4                        | 14.9    | 16.74                | 0.035            | 198      |
| 36         | 5                        | 9.56    | 7.589                | 0.92             | 186      |
| 37         | 5                        | 9.56    | 8.929                | 0.94             | 186      |
| 38         | 5                        | 9.56    | 9.821                | 0.9              | 192      |
| 39         | 5                        | 9.56    | 10.71                | 0.58             | 228      |
| 40         | 5                        | 9.56    | 10.85                | 0.38             | 264      |
| 41         | 5                        | 9.56    | 10.94                | 0.36             | 288      |
| 42         | 5                        | 9.56    | 11.16                | 0.27             | 288      |
| 43         | 5                        | 9.56    | 11.61                | 0.1              | 224      |
| 44         | 5                        | 9.56    | 12.5                 | 0.04             | 198      |
| 45         | 5                        | 9.56    | 13.39                | 0.03             | 192      |
| 46         | 6                        | 6.64    | 6.324                | 0.77             | 180      |
| 47         | 6                        | 6.64    | 7.44                 | 0.73             | 180      |
| 48         | 6                        | 6.64    | 8.185                | 0.7              | 192      |
| 49         | 6                        | 6.64    | 8.557                | 0.54             | 216      |
| 50         | 6                        | 6.64    | 8.929                | 0.43             | 224      |
| 51         | 6                        | 6.64    | 9.301                | 0.31             | 288      |
| 52         | 6                        | 6.64    | 9.673                | 0.2              | 270      |
| 53         | 6                        | 6.64    | 10.42                | 0.035            | 192      |
| 54         | 6                        | 6.64    | 11.16                | 0.03             | 192      |
| 55         | 10                       | 4.25    | 3.795                | 0.15             | 192      |
| 56         | 10                       | 4.25    | 4.241                | 0.15             | 192      |
| 57         | 10                       | 4.25    | 4.911                | 0.15             | 198      |
| 58         | 10                       | 4.25    | 5.357                | 0.14             | 198      |
| 59         | 10                       | 4.25    | 5.804                | 0.08             | 240      |
| 60         | 10                       | 4.25    | 6.25                 | 0.07             | 240      |
| 61         | 10                       | 4.25    | 6.696                | 0.05             | 240      |

Table 3.2: Supersonic injection simulations. The columns have the same definition as those in Table 3.1. Also, the binary mass ratio  $q = 0.6$

### 3.7.1 Mass capture fraction

When particles reach the WD radius, they are deemed to have been accreted and are removed from the simulation. The mass capture fraction is defined as

$$f_{\text{cap}} = \frac{\dot{M}_{\text{WD}}}{\dot{M}_{\text{RG}}}, \quad (3.7)$$

where  $\dot{M}_{\text{WD}}$  is the accretion rate onto the WD and  $\dot{M}_{\text{RG}}$  is the mass loss rate from the RG. In the simulations, mass can be added to the disc ( $\dot{M}_{\text{d}}$ ), accreted onto the surface of the WD ( $\dot{M}_{\text{c}}$ ), or lost outside the boundary of the simulation ( $\dot{M}_{\text{out}}$ ), where the boundary radius is  $> 2 \times$  the binary separation. One can therefore calculate  $f_{\text{cap}}$  as

$$f_{\text{cap}} = \frac{\Delta M_{\text{WD}}}{\Delta M_{\text{out}} + \Delta M_{\text{WD}} + \Delta M_{\text{d}}} \quad (3.8)$$

Once steady state has been achieved  $\Delta M_{\text{d}} \sim 0$  so this term can be neglected. The parameter  $f_{\text{cap}}$  is plotted against number of orbits in figure 3.6 for five simulations. In each of these runs, the wind particles are injected subsonically, with sound speeds of 5.78, 7, 9, 15 and 19  $\text{kms}^{-1}$ , as plotted in rows 1 – 5 respectively. The simulations demonstrate capture fractions for Case A (upper two panels), Case B (3rd row) and Case C (bottom two panels) mass transfer. The resolution of the simulations are not the same in all simulations; for instance, the number of active particles in the simulations corresponding to wind-RLO (case A) are less than those corresponding to case B and the latter has lower resolution than the simulations with fast winds (case C). However, the constant mass loss rate from the red giant,  $\dot{M}_{\text{RG}} = 10^{-7} M_{\odot}/\text{yr}$  is done by giving the SPH particles different masses in different simulations to maintain the same mass loss rate from the giant in physical units. Since the simulations initially contain no gas particles, the early stages of accretion exist in a non-steady state, reflected by the variable mass accretion fraction for the first two or three orbits, during which time the accretion disc is forming. Once steady state has been reached, the mass capture fraction stabilises at 0.95 for  $c_s = 5.78 \text{ km s}^{-1}$ , dropping to 0.06 for  $c_s = 19 \text{ km s}^{-1}$ .

In figures 3.7 and 3.8, the steady-state mass capture fraction is plotted for supersonic and subsonic initial RG wind speeds respectively. In the subsonic injection case, the key parameter governing the mass capture fraction is the sound speed, which increases with the temperature of the wind. The upper and lower horizontal axes of figure 3.7 show both of these parameters. To calculate the temperature of the wind from the sound speed, I have used an isothermal equation of state (see section 2.2.4) and considered the mean

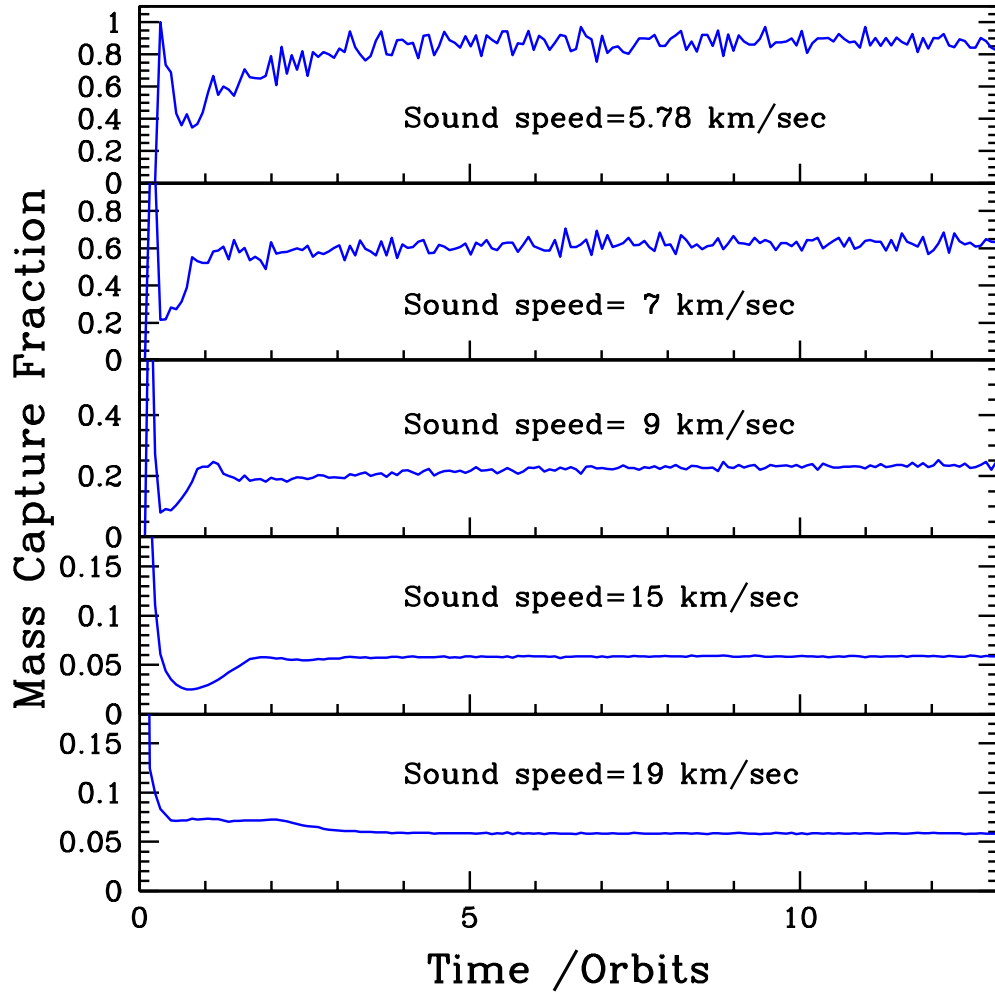


Figure 3.6: Time evolution of the accretion fraction for five simulations with subsonic RG wind velocities. Mass capture fraction is plotted against number of orbits for sound speeds of 5.78, 7, 9, 15 and 19  $\text{km s}^{-1}$  in the top to bottom panels respectively. In all cases the disc has reached steady state after 2 – 4 orbits. The discs in the upper two panels undergo Case A mass transfer, while the 3rd panel represents Case B transfer and Case C transfer is shown in the bottom 2 panels.

molecular weight  $\mu$  as a dependent variable with temperature as in Huré (2000) rather than a generic constant  $\mu$ . The y-error bars represent the maximum range over which  $f_{\text{cap}}$  varies after a steady state has been reached, as shown in figure 3.6. As the temperature and velocity of the wind particles increase, the mass capture fraction drops from  $f_{\text{cap}} \sim 1$  for  $T = 5000\text{K}$  to  $f_{\text{cap}} = 0.06$  at  $T > 20\,000\text{K}$ .

In the case of supersonic wind injection, the eventual particle velocities depend on both the initial injection velocity ( $v_{\text{inj}}$ ) and the sound speed of the wind. The former is plotted on the horizontal axis of figure 3.8 as a ratio of  $v_{\text{inj}}/v_{\text{orb}}$  while different sound speeds are plotted in separate colours, as labeled in the legend. The mass capture fraction increases as injection velocities decrease, and is also higher for lower sound speeds. The biggest range in capture fractions for different sound speeds are seen for low injection velocities; when  $v_{\text{inj}}/v_{\text{orb}} = 1.7$ ,  $f_{\text{cap}}$  ranges from 0.95 for  $c_s = 4\text{km s}^{-1}$  down to 0.15 for  $c_s = 10\text{ km s}^{-1}$ .

The range in  $f_{\text{cap}}$  can be explained by the different modes of mass transfer present dependent on wind speed. Both methods of wind injection (subsonic and supersonic) produce wind-driven RLO (Case A), for which the mass capture fractions range between 0.8 and 0.95. Case B, the intermediate situation in which gas particles manage to escape through  $L2$  as well as  $L1$ , results in a decrease of the mass capture fraction due to the two spiral tails that feed into either the accretion disc or the  $L2$  stream. Finally, the minimum mass capture fractions are obtained for Case C, where the particles are injected with sufficient energy to escape the potential well. This situation is obtained either by increasing the wind temperature in the subsonic injection case so that  $c_s > 12\text{ km s}^{-1}$ , or by further increasing the injection velocity in the supersonic case, to  $v_{\text{inj}} > 2.5 v_{\text{orb}}$ .

### 3.7.2 Comparison of the numerical results with BHL

The mass capture fraction ( $\dot{M}_c/\dot{M}_2$ ) can be estimated using the theoretical model of stellar accretion known as Bondi- Hoyle- Lyttleton (BHL) accretion described in section 1.4. The BHL accretion rate is commonly used in the study of the mass capture fraction in binary systems to estimate the value of the accretion rate, or to compare with numerical results. However, the case of mass transfer in binary systems is much more complicated than the BHL picture. It does not apply to accretion via RLO (see Edgar, 2004, and references therein), but rather to the rate of mass transfer by a stellar wind from the donor star in the binary. Moreover, accretion rates of slow winds deviate substantially from the BHL estimates. Accretion rates have been found to be an order of magnitude lower than BHL predictions for winds with velocities comparable to the orbital velocity (Theuns

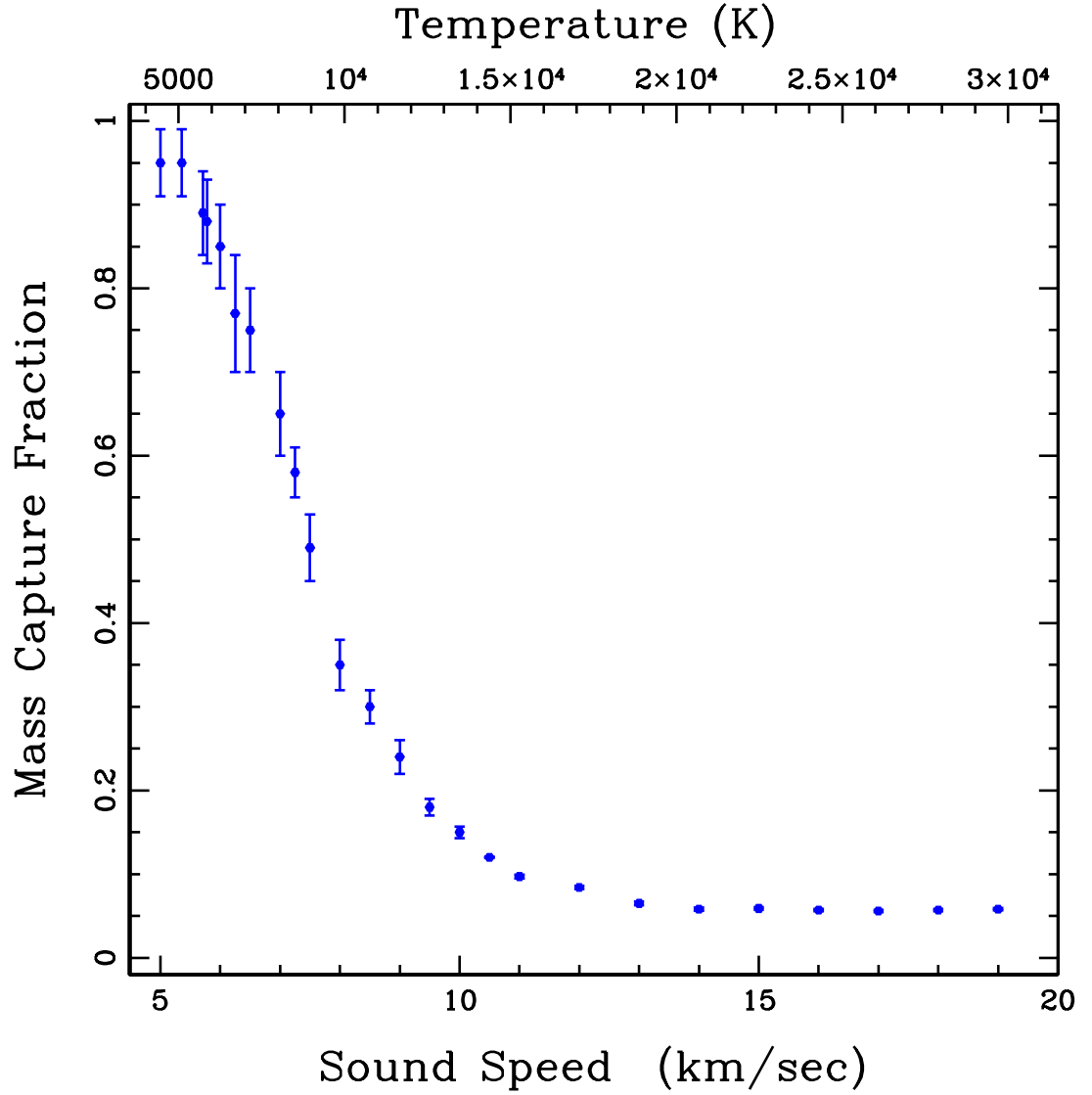


Figure 3.7: Mass capture fractions for simulations with subsonic wind injection. The corresponding wind temperatures are shown in the upper horizontal axis. Error bars on the y-axis indicate the variance in the mass capture fraction once steady state has been reached, as shown in figure 3.6.

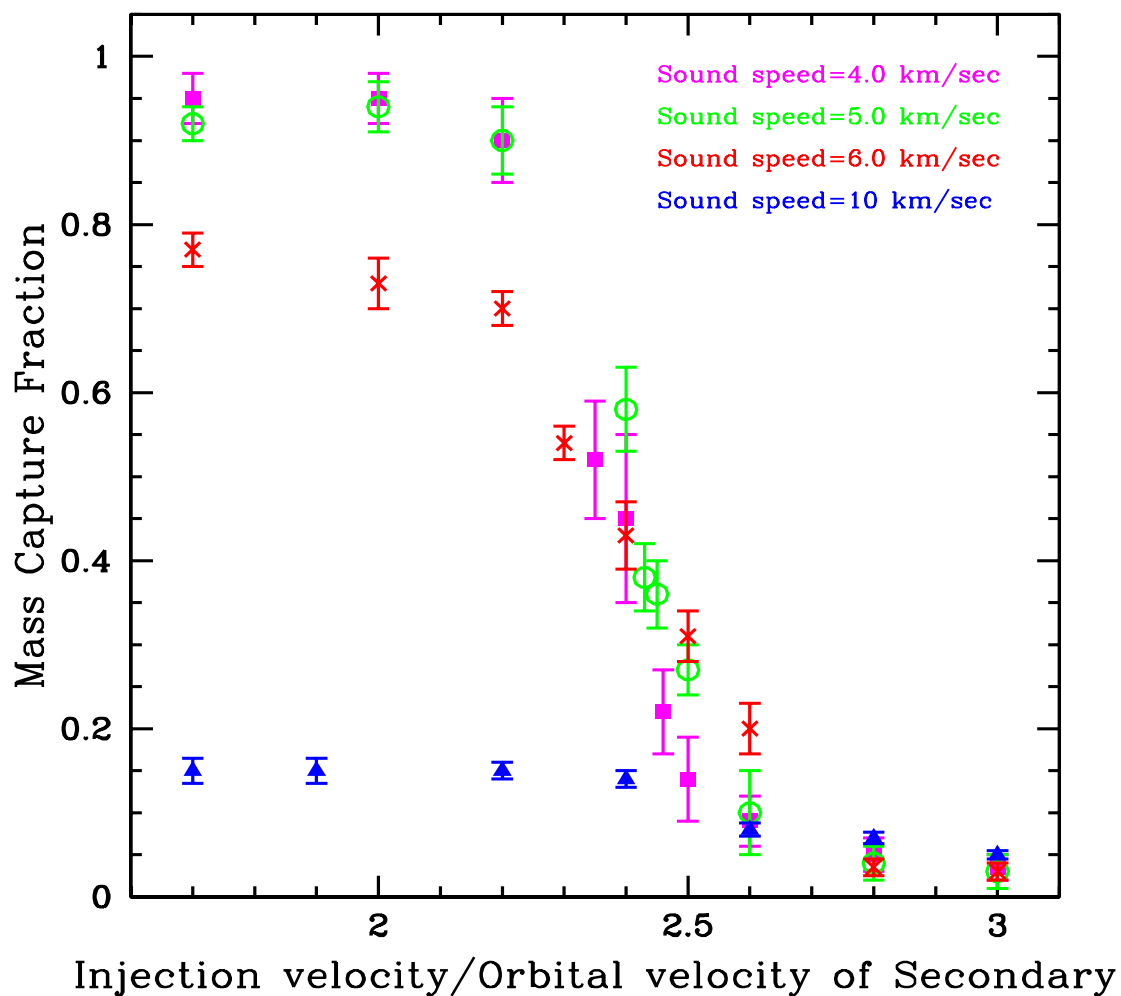


Figure 3.8: Mass capture fraction for simulations with supersonic wind injection. Mass capture fraction is plotted against the ratio of injection velocity to sound speed for four different sound speeds, coloured according to the figure legend.

et al., 1996; Edgar, 2004) and it has been suggested that the BHL accretion rate is only applicable if the wind speed is much greater than the orbital speed of the stars.

In the upper panel of figure 3.9, the mass capture fraction is plotted against the ratio of the wind speed at the WD position to the orbital speed of the RG, in blue, for simulations with subsonic wind injection. BHL estimates from equation (1.49) are then plotted alongside, in red. In the lower panel, the numerical result is compared to the BHL, by plotting their ratio. The range of wind speeds compatible with a wind temperature for a cool giant are tested here; from velocities much less than the RG orbital speed, up to ones that are comparable. This means that there are no fast winds in the range which exceed the conditions for which BHL is applicable. At the position of the accretor, the wind speed is subsonic and being accelerated, and has not yet achieved its terminal velocity. The injection speed of the SPH particles at the mass losing star's surface is much less than the star's orbital speed. The difference between the two results in terms of mass capture fraction is most prominent when the wind speed is low. When the velocity ratio is around unity, the two results begin to converge.

### 3.7.3 Mass accumulation between outbursts in RS Oph

To calculate the total mass accreted during the time between novae from the simulations, I assume,

$$M_{acc} = f_c(-\dot{m}_{RG})t_{rec} , \quad (3.9)$$

where  $0.05 \lesssim f_c \lesssim 1$  is the mass capture fraction,  $-\dot{m}_{RG} = 10^{-7} M_{\odot}/yr$  is the mass loss rate from the red giant and  $t_{rec} \simeq 20 yr$  is the recurrence time of outbursts in RS Oph.

Yaron et al. (2005) find for a typical WD temperature ( $T_{WD} = 30 \times 10^6 K$ ), the accreted mass  $M_{nov}$  required for a nova explosion on the WD surface is  $1.96 \times 10^{-6} M_{\odot}$  when  $M_{WD} = 1.25 M_{\odot}$  and accretion rate  $\dot{M} = 10^{-7} M_{\odot}/yr$ . Similarly, for  $M_{WD} = 1.4 M_{\odot}$ , the required mass  $M_{nov} = 2.02 \times 10^{-7} M_{\odot}$  and  $7.94 \times 10^{-8} M_{\odot}$  corresponding to mass accretion rates  $\dot{M} = 10^{-8} M_{\odot}/yr$  and  $\dot{M} = 10^{-7} M_{\odot}/yr$  respectively.

Figure 3.10 shows total mass accreted in the simulations for the recurrence time  $t_{rec} = 20 yr$  using equation 3.9, in units of  $M_{nov}$  as given in Yaron et al. (2005) as a function of wind sound speed. The horizontal black line represents the accreted mass required for outburst every 20 years. It is clear that for a WD of mass  $1.25 M_{\odot}$ , as indicated by the green line, it is not possible to accrete enough mass to fuel novae on the timescale of RS Oph outbursts. It is worth noting the green line in figure 3.10 is estimated using the results

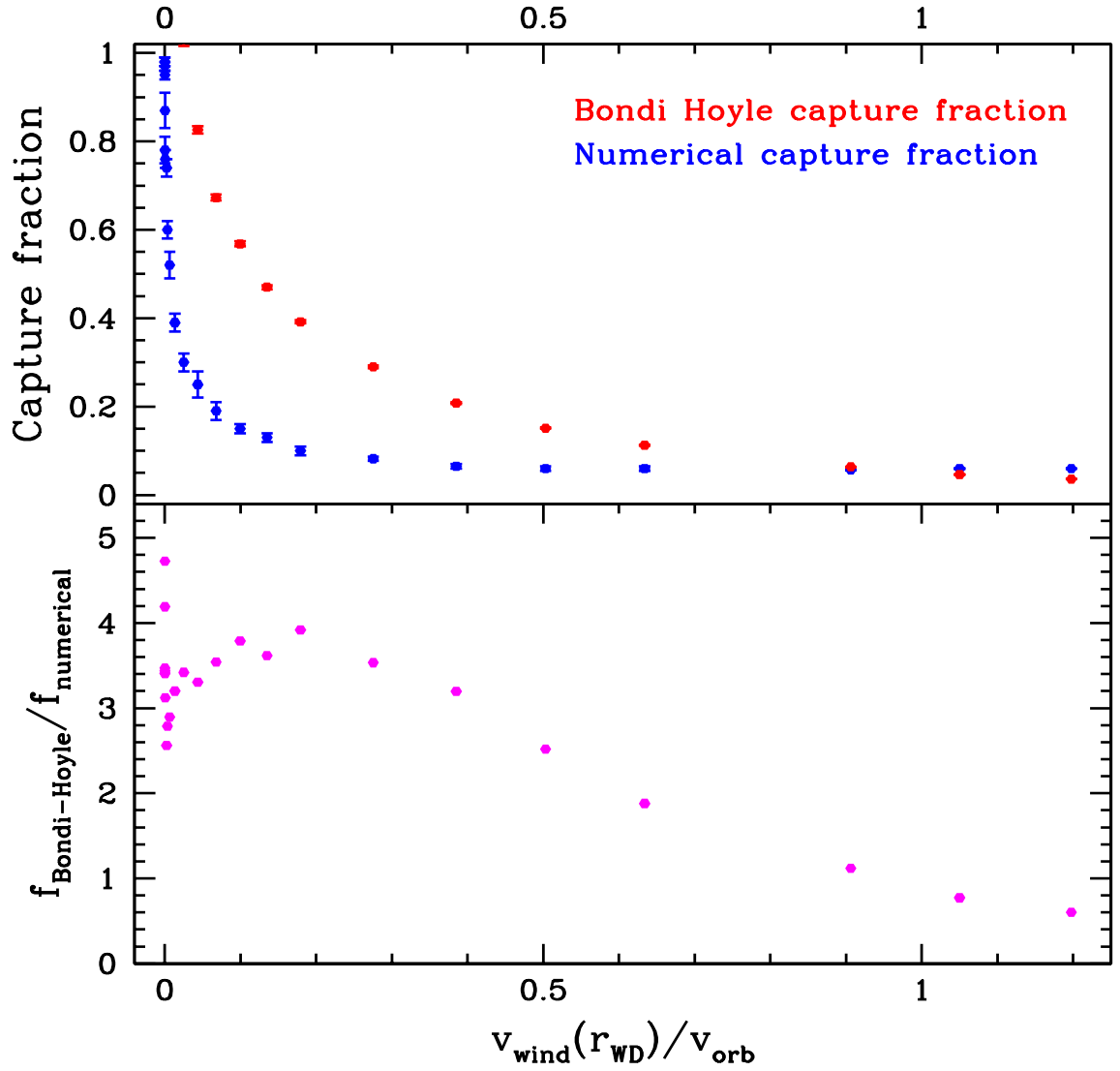


Figure 3.9: Comparison of the mass capture fraction results from the simulations to the BHL estimate. In the upper panel, the capture fraction is plotted against the ratio of the wind speed at the WD position to the orbital speed of the RG. This is shown for the theoretical BHL capture fraction (red points) and simulations with subsonic wind injection (blue). In the lower panels the vertical axis shows the ratio of the BHL estimate to the numerical result.

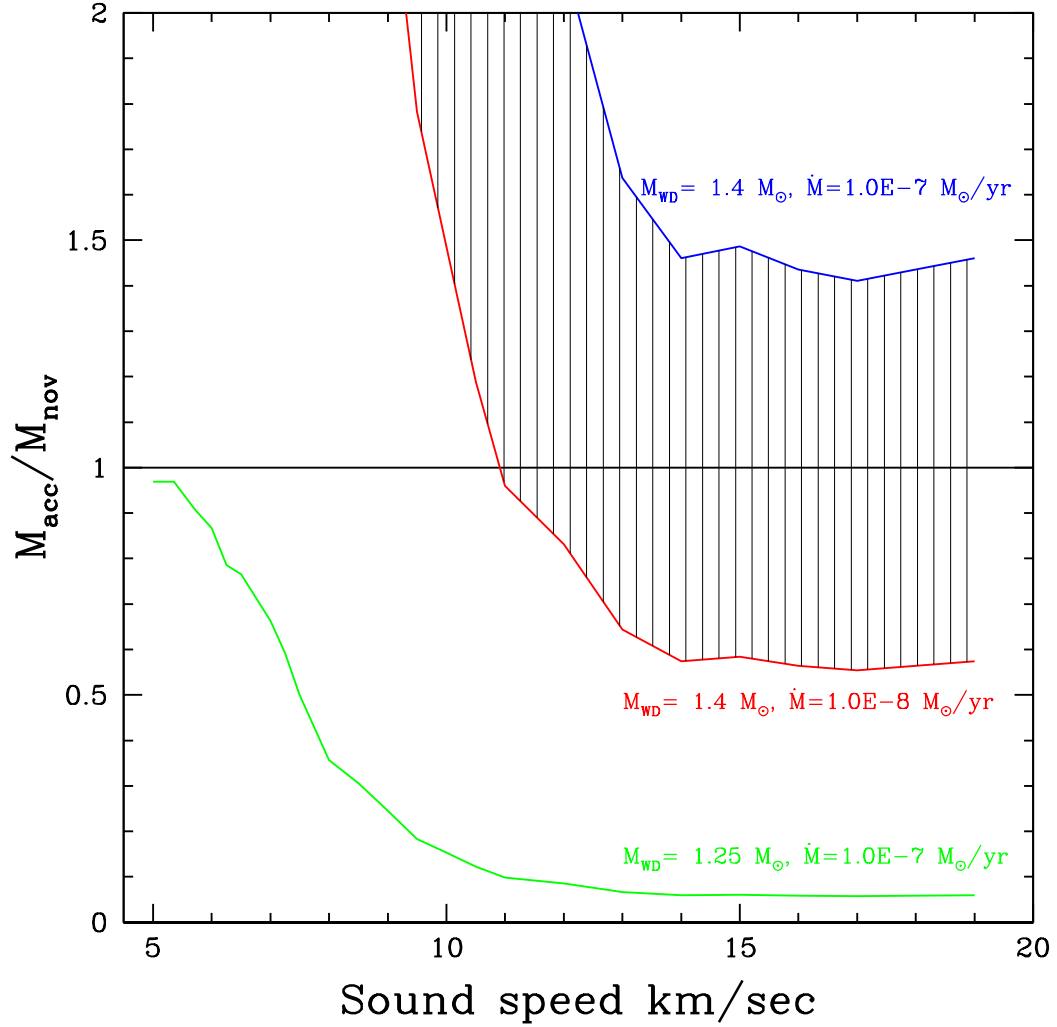


Figure 3.10: Mass accumulated between novae in units of  $M_{nov}$  versus wind sound speed. The horizontal black line represents the mass accumulation for nova every 20 years. The blue curve is the total accreted mass normalised to  $M_{nov}$  for  $M_{WD} = 1.4M_{\odot}$  and mass accretion rate  $\dot{M} = 10^{-7}M_{\odot}/yr$  from (Yaron et al., 2005). The red curve is similar but for  $\dot{M} = 10^{-8}M_{\odot}/yr$ . The shaded area infers the possible range of parameter space in the simulations for nova to happen in RS Oph; it is truncated at  $M_{acc}/M_{nov} > 2$  which lead to mass accumulation more than twice that required for nova to take place, corresponding to a recurrence time half that observed. The green line represents  $M_{acc}/M_{nov}$  for  $M_{WD} = 1.25M_{\odot}$  and  $\dot{M} = 10^{-7}M_{\odot}/yr$ , it is always below the horizontal black line meaning that a nova does not happen within 20 years.

of  $f_{\text{cap}}$  from my simulations which are for binary mass ratio  $q = 0.6$ ; it is nevertheless expected that such small change in binary mass ratio would have a negligible effect on the simulation results. However, the shaded region shows the range of possible mass accumulation to trigger thermonuclear runaway on the WD surface for a WD of mass  $1.4M_{\odot}$ . This region is surrounded by the curves of accumulated mass due to mass accretion rate of  $10^{-8}M_{\odot}/\text{yr}$  (in red) and  $10^{-7}M_{\odot}/\text{yr}$  (in blue). The values of  $M_{\text{acc}}/M_{\text{nov}} > 2$ , give rise to recurrence time less than 10 years. On the other hand, the minimum WD mass for novae in RS Oph is  $\sim 1.25M_{\odot}$  as shown in figure 3.10.

### 3.8 Rotation of the red giant

Broadening in spectral lines can give an insight into the rotation of the secondary. If the rotation axis is not completely along the line of sight, the doppler shifts from both receding and approaching sides of the star occurs. The origin of stellar rotation is thought to be inherited from the molecular cloud of which the star had been formed, but the quantity remains uncertain as the parent molecular cloud would be expected to have specific angular momentum orders of magnitude larger than what is seen in rotating stars (e.g. de Mink et al., 2013) and pre main sequence stars have rotational velocities much smaller than their critical velocity.

Theoretical estimates of cool giant rotation (Soker, 2002) show that they are likely to rotate faster in binaries than as single stars. Soker (2002) suggested several reasons for their spin-up including angular momentum accretion from the wind of the other binary component during the main sequence phase, tidal interactions, or accretion of back-flowing material from its own wind when it has gained some orbital angular momentum. These predictions were observationally confirmed by Zamanov et al. (2006), who proposed additional mechanisms for the evolution of the measured rotation speeds, such as synchronisation for short orbital period binaries, or the swallowing of planets during the star's evolution, so that the planet's angular momentum is added to that of the star. Zamanov et al. (2007, 2008) measured the projected rotational velocity of a number of cool giants and found that those in binary systems rotate on average 1.5 times faster than the field stars and that a subset of them appeared to be synchronised. As a consequence, they have suggested that the giants in symbiotic stars are likely to lose 3 – 30 times more mass than single stars of the same spectral class. Note that in RS Oph, the red giant appears to be rotating faster than its orbital period, implying that it is in a period of deceleration towards  $P_{\text{rot}} = P_{\text{orb}}$  with an expected  $\tau_{\text{syn}} \leq 5 \times 10^4$  yr (Zamanov et al., 2007).

### 3.8.1 Effects of rotation on stellar shape and mass loss

For a stellar photosphere in hydrostatic equilibrium, considering the conservation of angular momentum, the star's rotational velocity can be expressed as

$$v_\phi(r, \theta) = \Omega r \sin \theta, \quad (3.10)$$

where  $\Omega$  is the stellar angular velocity (solid rotator),  $\theta$  is the angle with respect to the axis of stellar rotation,  $r$  is the stellar radius. The effective gravity of the wind flow is

$$g_{\text{eff}} = -\frac{GM_*}{r^2} + \Omega^2 r \sin \theta, \quad (3.11)$$

where the first and second terms represent the gravitational and centrifugal ( $v_\phi^2/r \sin \theta$ ) forces respectively. Rotation induces mass flux, with higher rotational velocities resulting in more mass loss from the stellar surface.

According to the Roche model, the surface of a star is equipotential with a point mass source at the centre. By taking the gravitational potential at the poles as a boundary condition,  $\Psi(r, \theta) = \Psi(R_{\text{pole}}, 0)$ , where  $\Psi$  is the sum of the gravitational potential and the centrifugal potential  $\Psi = -\frac{GM_*}{r} - \frac{1}{2}(\Omega r \sin \theta)^2$  (Lamers and Cassinelli, 1999), then

$$\frac{GM_*}{r} + \frac{1}{2}\Omega^2 r^2 \sin^2 \theta = \frac{GM_*}{R_{\text{pole}}}, \quad (3.12)$$

where  $R_{\text{pole}}$  is the stellar radius at the pole. Equation 3.12 can be rearranged to give

$$r^3 - \frac{2GM_*}{R_{\text{pole}}\Omega^2 \sin^2 \theta} r + \frac{2GM_*}{\Omega^2 \sin^2 \theta} = 0. \quad (3.13)$$

If the mass and angular velocity of the star are known, the solution of the cubic equation above gives the equipotential surface as a function of  $\theta$ . The solution indicates that a rotating star strays from its spherical shape and tends toward oblateness; the faster it rotates, the greater the difference between  $R_{\text{equator}}$  and  $R_{\text{pole}}$ . The Keplerian  $v_\phi = \sqrt{GM_*/r}$  is the maximum possible rotational velocity a star can have at which  $R_{\text{equator}} \sim 1.5R_{\text{pole}}$ .

Rotation is likely to enhance the equatorial density of the stellar wind, but the mechanism behind this non-spherical wind structure is not yet determined (Lamers, 2004). It may be achieved through an enhancement of mass loss in the equatorial regions or due to equatorial wind compression where the wind flows toward the equator (Lamers and Cassinelli, 1999). Skopal and Cariková (2015) analytically estimated wind compression

for isolated slowly-rotating cool giants. Their results show that the wind density in the equatorial plane is up to 5 times higher than poleward and mass loss rate is enhanced by a factor 10 compared to a non-rotating isolated red giant wind when the ratio of the stellar rotation velocity to the wind terminal velocity is about  $\sim 5$ . They propose that this enhanced equatorial mass loss could result in significantly higher wind mass accretion, potentially high enough to trigger novae in some systems.

### 3.8.2 The effects of rotation on the RG wind

To study the effect of rotation on the properties of the RG wind in RS Oph, I have modelled an isolated red giant with an isothermal, initially subsonic, Parker wind. Three temperature regimes for the wind have been considered; a cool wind ( $c_s = 7 \text{ km s}^{-1}$ ) corresponding to Case A wind-RLO, a wind with Case B accretion ( $c_s = 13 \text{ km s}^{-1}$ ) and a relatively hot, fast wind with  $c_s = 19 \text{ km s}^{-1}$ , corresponding to Case C accretion. For these models, I have compared wind parameters with no stellar rotation to those with either synchronous rotation ( $P_{\text{rot}} = P_{\text{orb}}$ ) or critical RG rotation speeds. This is implemented by giving the wind particles a rotational velocity in addition to their initial radial velocities.

The results of these comparisons can be found in Table 3.3. Here, the ratio of the equatorial radius to the polar one has been calculated from (3.13), while the ratios of equatorial to polar density and mass loss rate with and without rotation are both measured from simulations of the wind. The binary orbital period of RS Oph is relatively long, so, in the case of synchronisation, the red giant rotates at much lower rate than for critical rotation. In this case, the ratio of stellar equatorial radius to the polar one is 1.061, so the stellar surface is still close to spherical. The ratio of equatorial to polar densities increases with distance from the star. The values listed in Table 3.3 are calculated at the binary separation distance from the RG.

Synchronised rotation does not have the same effect on the wind properties for different wind temperatures; sound speed is a key parameter. Slow winds are more sensitive to rotation than fast winds. For instance, in the case of simulations with high sound speed, the synchronised rotation has no significant effect on the radial density profile of the wind, which is nearly isotropic, and the mass loss rate is just 1.43 times greater than that for no rotation. Conversely for a cool wind ( $c_s = 7 \text{ km s}^{-1}$ ), synchronisation leads to 280% increase in mass loss rate with a noticeable density contrast between the equatorial and polar winds. Figure 3.11 shows the x-y view in the upper panel and x-z view in the lower panel for synchronised rotation. Note that these simulations reflect the binary param-

| $c_s$                  | Rotation     | $R_{eq}/R_p$ | $\rho_{eq}/\rho_p$ | $\dot{M}/\dot{M}_{no}$ |
|------------------------|--------------|--------------|--------------------|------------------------|
| $7 \text{ km s}^{-1}$  | No Rotation  | 1            | 1                  | 1                      |
|                        | Synchronised | 1.061        | 1.25               | 2.8                    |
|                        | Critical     | 1.5          | 31.6               | 51.7                   |
| $13 \text{ km s}^{-1}$ | No Rotation  | 1            | 1                  | 1                      |
|                        | Synchronised | 1.061        | 1.2                | 1.8                    |
|                        | Critical     | 1.5          | 28.18              | 6.35                   |
| $19 \text{ km s}^{-1}$ | No Rotation  | 1            | 1                  | 1                      |
|                        | Synchronised | 1.061        | 1.12               | 1.43                   |
|                        | Critical     | 1.5          | 14.12              | 2.41                   |

Table 3.3: Effect of rotation on stellar surface, mass loss and structure of the outflow. Three temperature regimes are selected, corresponding to  $c_s = 7, 13$ , and  $19 \text{ km s}^{-1}$ . Wind parameters are then compared for rotation that is synchronous with the RS Oph orbital period and for critical stellar rotation. Here, the ratio of the stellar radius at the equator and at the pole ( $R_{eq}/R_p$ ) is calculated using (3.13), while the equivalent density ratio,  $\rho_{eq}/\rho_p$  is obtained numerically. The increase factor in mass loss compared to a non-rotating RG ( $\dot{M}/\dot{M}_{no}$ ) is also measured in the simulations

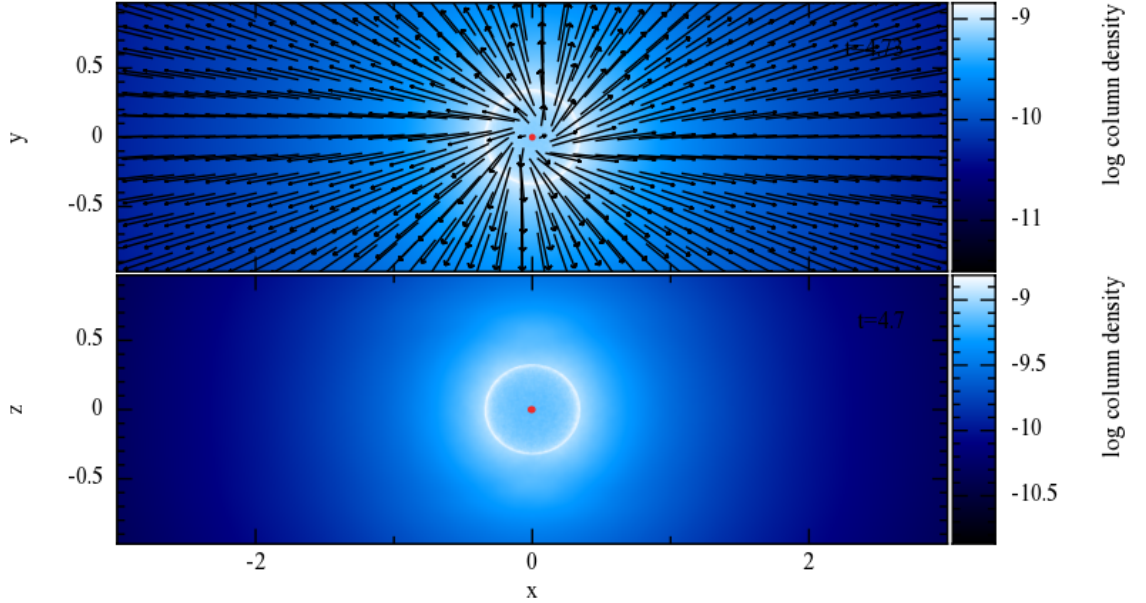


Figure 3.11: Stellar wind morphology in the case of synchronised rotation. Images are shown in the x-y plane (upper panel) and x-z plane (lower panel) from a simulation of a mass-losing star in isolation, with initially subsonic wind particle velocities and  $c_s = 19 \text{ km s}^{-1}$ . The centre of the star is shown as a red dot. In the upper panel, the star is viewed in its rotation plane and the arrows show the direction of the wind outflow. In the lower panel, the stellar environment is imaged perpendicular to the rotation axis. Both panels are coloured according to density contrast.

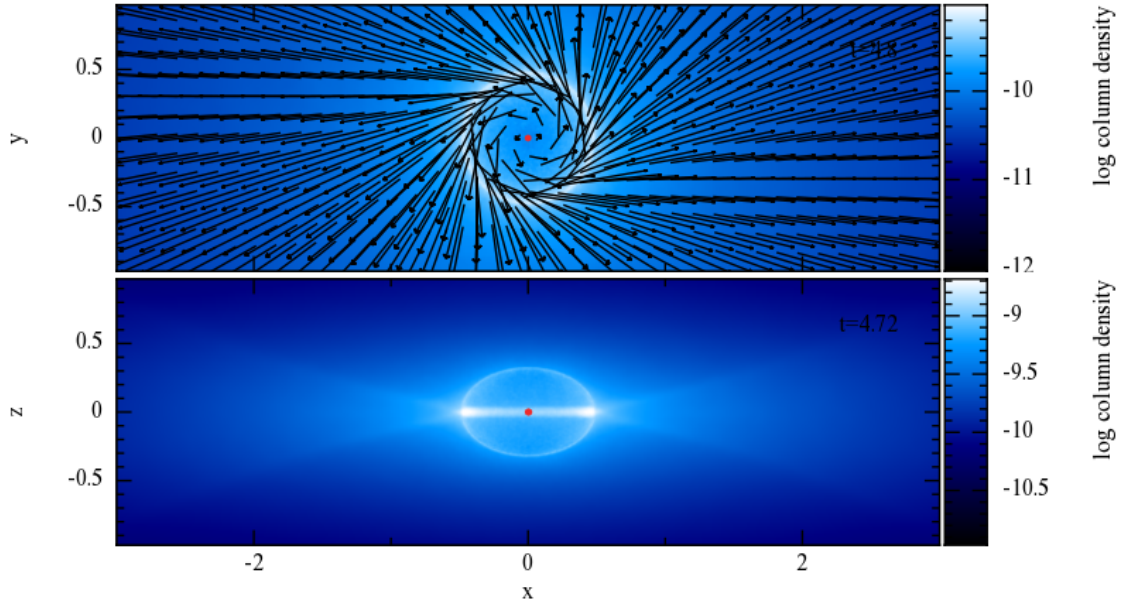


Figure 3.12: Same as figure 3.11 but for the case of critical rotation.

ters of RS Oph, which require relatively low RG rotation speeds for synchronisation. For systems with shorter orbital periods, stars will have faster rotation and a pronounced alteration in shape; this may lead to a greater increase in mass loss and equatorial-to-polar density ratios.

In the case of critical rotation ( $R_{\text{equ}}/R_{\text{pole}} = 1.5$ ), the gas possesses a large rotational velocity that cannot be suppressed, even with fast winds. The resultant morphology of the wind in this case is shown in figure 3.12, with an x-y view in upper panel and an x-z view below. The arrows in the upper panel show the direction of the wind outflow. In both panels, the planar-polar density contrast is clearly visible. The density contrast is 31.6, 28.18 and 14.12 for winds with  $c_s = 7, 13$  and  $19 \text{ km s}^{-1}$  respectively and corresponding mass loss rates increase by factors 51.17, 6.35 and 2.41 compared to cases with no rotation.

### 3.8.3 Mass transfer in RS Oph with synchronised red giant

To investigate the effect of rotation on the mass capture fraction and accretion disc formation, I have performed self-consistent simulations of mass transfer in a binary system from a synchronised red giant (as observed for many symbiotic systems; see Zamanov et al., 2008; Zamanov and Stoyanov, 2012, for details). This is achieved by giving the red giant one complete rotation per orbital period. I have investigated the results of mass transfer from prograde rotation and opposite (retrograde) to cover all the senses of rota-

tion (although the red giant would not be expected to rotate in the opposite sense with respect to the orbital motion) and compared to the cases of no rotation.

In the upper panels of figure 3.13, I show the effect of rotation on mass capture fraction for an initially subsonic wind as a function of sound speed. For both prograde and retrograde motion, the decrease in mass capture fraction as sound speed increases follows the same profile as for a non-rotating RG. At low sound speeds ( $\lesssim 7 \text{ km s}^{-1}$ ) in wind-driven RLO-cases,  $f_{\text{cap}}$  is slightly lower for retrograde motion, while at higher wind speeds ( $\gtrsim 12 \text{ km s}^{-1}$ ), the difference between simulations is negligible.

Equivalent results are plotted for a supersonic wind in the top panel of figure 3.14, with mass capture fraction compared to  $v_{\text{inj}}/c_s$ . Again, both prograde and retrograde rotation of the RG appear to have little effect on  $f_c - c_s$  relation, which is dominated by the temperature of the wind, rather than rotation effects.

In the lower panels of figures 3.13 and 3.14, the size of the accretion disc that is formed is compared for simulations with no RG rotation, to those with pro- and retrograde RG rotation synchronous to the binary orbital period, for initially subsonic and supersonic winds respectively. Both plots show that the disc size is broadly independent of the rotation of the mass losing component, with the disc size depending strongly on the temperature (and thus speed) of the wind instead. In these plots, the disc sizes have been plotted as a ratio of the maximum stable disc radius suggested by Alexander et al. (2011). In all cases, the accretion discs have larger radii than that required for stability, indicating that they are susceptible to thermal-viscous instabilities.

It is obvious that with higher velocity winds, the accretion disc size shrinks. Following Soker and Rappaport (2000); the condition for an accretion disc to form around a central star is that the specific angular momentum  $J_{\text{acc}}$  of the accreted material must be larger than the specific angular momentum  $J_1$  of a particle in a Keplerian orbit at the equator of the accreting star of radius  $R_1$ . Thus for a white dwarf of radius  $R_{WD} \sim 0.01R_{\odot}$ ,

$$J_1 = (Gm_{WD}R_{WD})^{1/2} . \quad (3.14)$$

The specific angular momentum of the accreted material having impact parameters  $b < R_{BH}$  may be expressed as

$$J_{\text{acc}} = \eta j_{BH} = \frac{\eta}{2} \left( \frac{2\pi}{P} \right) R_{BH}^2 , \quad (3.15)$$

where  $\eta$  is 0.1 and 0.3 for isothermal and adiabatic winds respectively (Livio et al., 1986);  $P$  is the orbital period and  $R_{BH}$  is the Bondi- Hoyle accretion radius.  $j_{BH} = \frac{1}{2} \left( \frac{2\pi}{P} \right) R_{BH}^2$  is

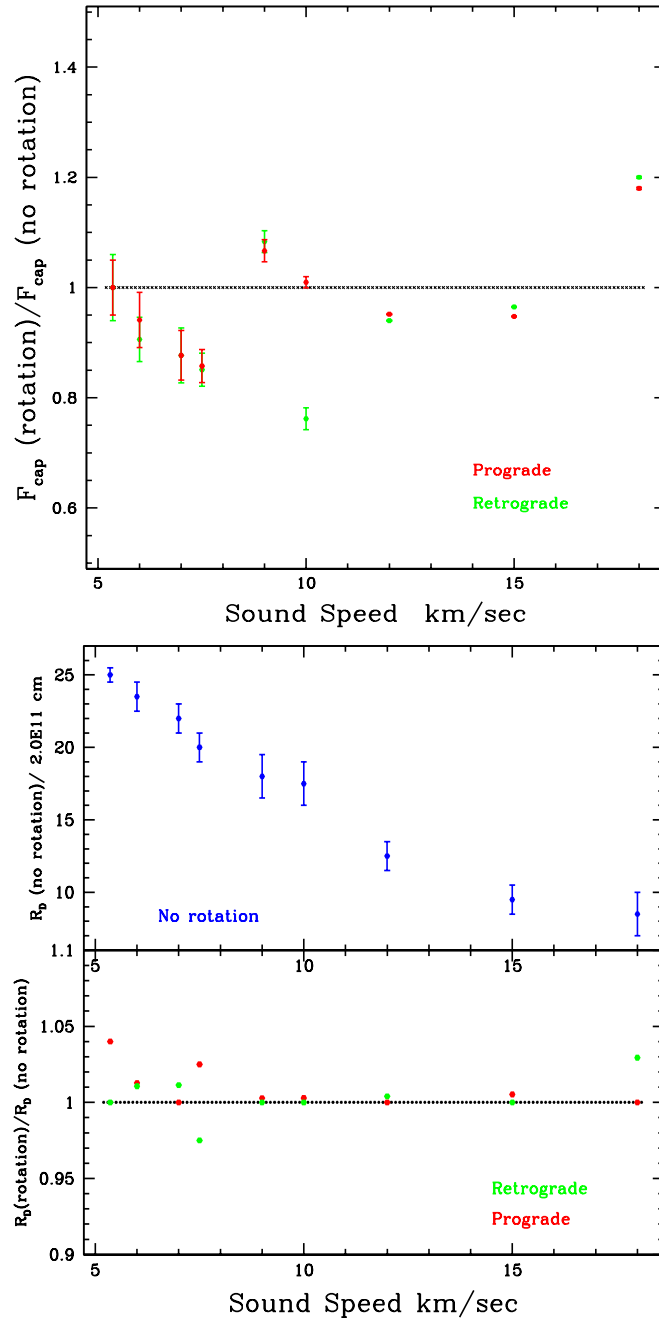


Figure 3.13: The effect of RG rotation on mass capture fraction and disc radius for simulations with subsonic wind injection. In the upper panel, mass capture fraction  $f_{\text{cap}}$  is plotted against sound speed for prograde (red) and retrograde rotation (green) normalised to mass capture fraction corresponding to no stellar rotation. In the lower panel, the disc radius is plotted against sound speed for simulations with no RG rotation normalised to the stability radius (blue). Also disc radii for the same 2 rotation regimes normalised to the disc radius with no stellar rotation. For both rotation cases, the orbital period of rotation  $P_{\text{rot}} = P_{\text{orb}}$ .

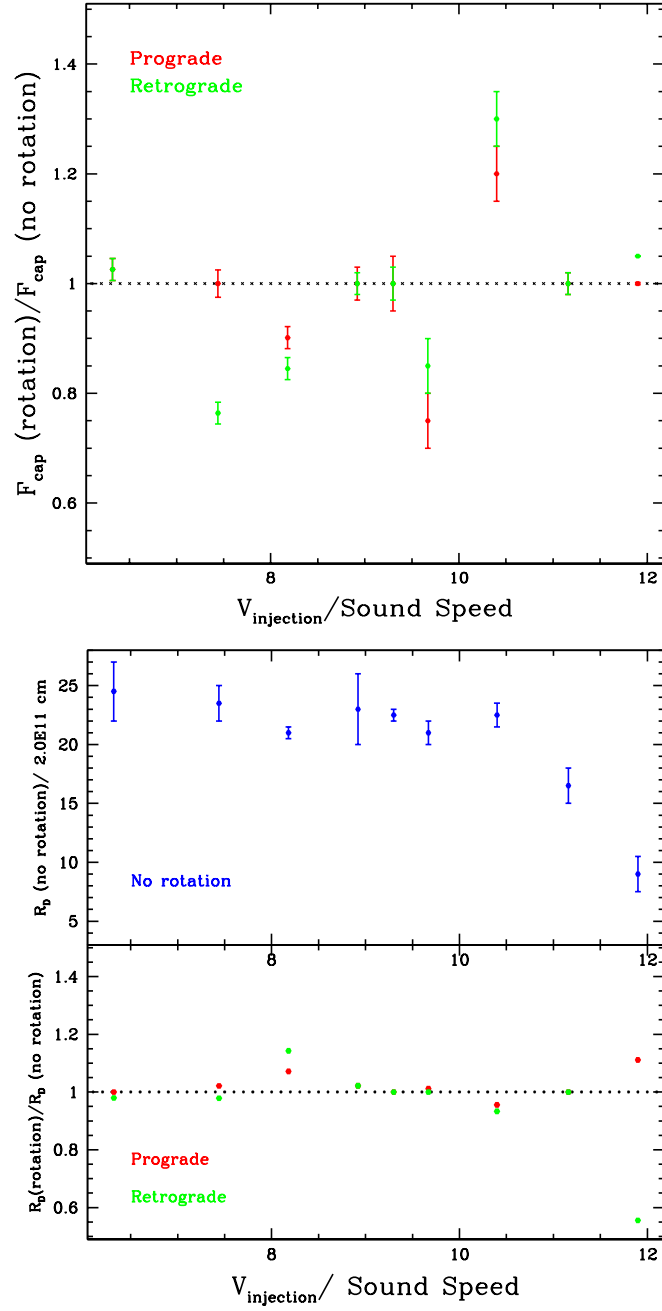


Figure 3.14: The effect of RG rotation on  $f_{\text{cap}}$  and disc radius for simulations with supersonic wind injection (same as figure 3.13 but for supersonic wind). Upper panel: results for the normalised mass capture fraction. Lower panel: results for the accretion disc radius for the cases of no RG rotation normalised to stability radius (blue). Also disc radii for prograde (red) and retrograde (green) normalised to non-rotating RG disc radius.

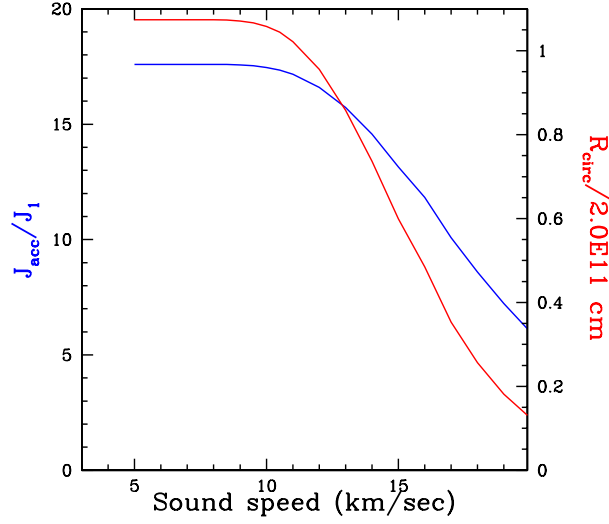


Figure 3.15: Variation of  $J_r = J_{acc}/J_1$  with the sound speed in the wind (Blue).  $J_r$  approaches unity where the accretion disc cannot be formed. Also shown is the circularisation radius of the accretion disc from Bondi- Hoyle estimates (red) normalised to the stability radius  $2.0 \times 10^{11}$  cm . The majority of the  $R_{circ}$  values are comparable to that of the stability radius according to Bondi- Hoyle calculations

the net specific angular momentum of the material entering the Bondi- Hoyle radius  $R_{BH}$  (Wang, 1981). Hence the condition requires that  $J_r = \frac{J_{acc}}{J_1} > 1$ . Figure 3.15 shows  $J_r$  versus sound speed of the wind (blue curve). The curve tends to unity for winds with higher sound speed values. The circularisation radius is also plotted in red based on the Bondi- Hoyle estimates assuming that the accreted material conserves its angular momentum. The result shows that the size of the disc predicted in this way is of order the stability radius  $2.0 \times 10^{11}$  cm which is at least one order of magnitude less than the numerical results shown in figures 3.13 and 3.14. The circularisation radius is not the upper limit of the accretion disc size as the disc will spread out  $R_{circ}$  under the action of viscosity to conserve angular momentum, so the disc radius could be larger than  $R_{circ}$  . However, it is not the reason behind the differences between the analytical estimates and the numerical results. In the numerical simulations the accretion disc size is close to its formation radius, i.e.  $R_{circ}$ . The fact that the disc predicted by Bondi Hoyle estimate is crude and are much smaller than the numerical disc show that Bondi Hoyle estimate is not a good predictor of disc size, particularly for slow winds.

Although the mass capture fraction is similar when the RG is rotating compared to when it is stationary, the mass loss rate for the RG in the case of rotation is likely to

be significantly higher, as described in Table 3.3. For a cool wind, where mass transfer occurs via RLO, the mass loss rate may increase by a factor of 2.8 (see Table 3.3), for synchronous RG rotation, rising to a factor of 51.7 in the case of critical rotation. These increases may lead to a considerable increase in the total mass accreted in each  $\sim 20$  year inter-nova period for RS Oph.

### 3.9 Discussion

The simulations have shown that the fraction of mass captured by RS Oph in quiescence varies strongly as a function of the RG wind speed, governed by the temperature of the wind. Mass capture fractions of  $0.05 \lesssim f_{\text{cap}} \lesssim 1$  are obtained for RG wind temperatures of 3000 – 30 000K, with lower temperatures (i.e. lower wind speeds) leading to higher capture fractions. Low particle velocities result in RLO-like mass transfer, with the majority of the wind passing through the  $L1$  point into the Roche sphere of the WD, whereas for higher velocity winds ( $v_{\text{wind}} \geq v_{\text{orb}}$ ), the mass outflow from the RG is almost spherically symmetric. In this latter regime, mass capture fractions are correctly predicted by the BHL estimate (equation 1.49). However, at lower wind speeds, the BHL prediction is consistently higher than the numerical results. Using 3D hydrodynamic calculations of close binary systems, Nagae et al. (2004) also found that BHL theory overestimates  $f_{\text{cap}}$  at low  $v_{\text{wind}}$ ; see Edgar (2004) for a discussion of when BHL is valid.

The range of  $f_{\text{cap}}$  values found in the models are in agreement with estimates from other works. Walder et al. (2008) carried out 3D simulations of RS Oph in quiescence, to find  $f_{\text{cap}} = 0.1$ , which is close to the lower end of my estimate. Other simulations of wind transfer in symbiotic binaries include calculations of  $f_{\text{cap}} = 0.22 - 0.25$  for Z Andromeda (Mitsumoto et al., 2005),  $f_{\text{cap}} = 0.06$  for RW Hydrae (Dumm et al., 2000) and  $f_{\text{cap}} \leq 0.18$  (Nagae et al., 2004). Mastrodemos and Morris (1998) have also estimated  $f_{\text{cap}} = 0.09 - 0.4$  from their simulations of mass-losing giant stars in detached binary systems having binary mass ratio  $q = 1.5$ .

I can now use my estimates of  $f_{\text{cap}}$  to extract details about the mass transfer in RS Oph. If I assume the RG loses mass at a rate of  $\dot{M}_{\text{RG}} \sim 3 \times 10^{-7} M_{\odot} \text{yr}^{-1}$  (Patat et al., 2011), my estimates of  $f_{\text{cap}}$  give an accretion rate onto the WD of  $1.5 \times 10^{-8} M_{\odot} \text{yr}^{-1} \lesssim \dot{M}_{\text{acc}} \lesssim 3 \times 10^{-7} M_{\odot} \text{yr}^{-1}$ . Yaron et al. (2005) have used multicycle nova outburst calculations over a wide range of parameter space to constrain the behaviour of novae as a function of the mass accretion rate, WD mass and WD temperature. Relevant to this study are their tabulated results for  $M_{\text{WD}} = 1, 1.25$  and  $1.4 M_{\odot}$ . From these tables, I find that mass ac-

cretion rates at the lower end of my estimate ( $10^{-8}M_{\odot}\text{yr}^{-1}$ ) can produce outbursts over the correct ( $\sim 20\text{yr}$ ) timescale for  $M_{\text{WD}} = 1.4M_{\odot}$ , while those at the upper end of my estimate ( $10^{-7}M_{\odot}\text{yr}^{-1}$ ) are instead consistent with a lower WD mass of  $1.25M_{\odot}$ . Hachisu et al. (2006) fit optical light curves from the 2006 outburst of RS Oph to theoretical models, to predict  $M_{\text{WD}} = 1.35 \pm 0.01M_{\odot}$ , which they later confirm by modelling the X-ray lightcurve, in Hachisu et al. (2007). These results may indicate values of  $f_{\text{cap}}$  towards the lower end of my estimates.

Note that measurements of  $\dot{M}_{\text{RG}}$  in RS Oph remain highly uncertain, with estimates from as low as  $\dot{M}_{\text{RG}} \geq 10^{-9}M_{\odot}$  (Shore, 2008) through to  $\dot{M}_{\text{RG}} \sim 1.8 \times 10^{-6}M_{\odot}\text{yr}^{-1}$  (Bohigas et al., 1989). Constraints on the outburst recurrence times, from the tabulated parameters in Yaron et al. (2005), rule out  $\dot{M}_{\text{RG}} < 10^{-8}M_{\odot}\text{yr}^{-1}$ , while a RG mass loss rate of  $10^{-6}M_{\odot}\text{yr}^{-1}$  would require  $M_{\text{WD}} \leq 1.25M_{\odot}$  or  $f_{\text{cap}} \ll 1$  to match these constraints.

Another important feature of the simulations is the effect of RG rotation on the system parameters. I have shown that  $f_{\text{cap}}$  is not significantly altered by synchronous, prograde or retrograde rotation. However, the models of an isolated rotating RG indicate that increasing the rotation speed leads to a greater mass loss rate ( $\dot{M}_{\text{RG}}$ ) from the star. I find that for synchronous rotation,  $\dot{M}_{\text{RG}}$  is enhanced by a factor of 1.43 for  $c_s = 19\text{kms}^{-1}$ , assuming a subsonic Parker wind model, increasing to 2.8 for  $c_s = 7\text{kms}^{-1}$ . Observations indicate that the rotation speed of the RG in RS Oph may be slightly faster than synchronous (Zamanov et al., 2007). As an upper limit, critical rotation (the velocity at which the RG is no longer gravitationally bound), leads to a mass loss rate increase of up to 51.7 times the non-rotating value, for the lowest wind speeds. Rotation of the RG component in RS Oph has implications for the accretion rate onto the WD, enhancing the mass accretion rate by up to an order of magnitude. This would allow for a lower range of white dwarf masses or, conversely, if  $M_{\text{WD}} = 1.35M_{\odot}$ , would indicate lower values of  $f_{\text{cap}}$ , so that  $\dot{M}_{\text{acc}}$  remains  $\sim 10^{-8}M_{\odot}\text{yr}^{-1}$ .

It is important to note that the self-consistent solution found by Walder et al. (2008) fits within the parameter range that I have described. However, I have shown that this solution is by no means unique and that a much larger range of accretion rates can be achieved, dependent on the RG wind speed, the mass of the WD,  $f_{\text{cap}}$  and the rotation rate of the RG.

Each of the simulations, for the whole range of parameters I have explored, has resulted in disc formation around the WD. Additionally, in all cases, the disc extends beyond  $R_D = 2 \times 10^{11}\text{cm}$ , the critical radius above which it is subject to thermal-viscous instability, calculated from equation 3.3. This means that the disc will undergo instability-driven

| Distance from WD (cm)                 | $5 \times 10^{11}$ | $1 \times 10^{12}$ | $5 \times 10^{12}$ |
|---------------------------------------|--------------------|--------------------|--------------------|
| $\Sigma_{\max}(\text{g/cm}^2)$        | $4.1 \times 10^4$  | $8.9 \times 10^4$  | $5.6 \times 10^5$  |
| $c_s = 6 \text{ km/s}$                |                    |                    |                    |
| $\Sigma(\text{g/cm}^2)$               | 2.3                | 5.5                | 28.0               |
| $d\Sigma/dt(\text{g/cm}^2/\text{yr})$ | 507.9              | 0.2                | 22.1               |
| $t_{\text{rec}}(\text{yr})$           | 81                 | $4.4 \times 10^5$  | $2.5 \times 10^4$  |
| $c_s = 10 \text{ km/s}$               |                    |                    |                    |
| $\Sigma(\text{g/cm}^2)$               | 0.5                | 0.7                | 14.7               |
| $d\Sigma/dt(\text{g/cm}^2/\text{yr})$ | 4.2                | 6.2                | 1.69               |
| $t_{\text{rec}}(\text{yr})$           | $9.7 \times 10^3$  | $1.4 \times 10^4$  | $3.3 \times 10^5$  |
| $c_s = 19 \text{ km/s}$               |                    |                    |                    |
| $\Sigma(\text{g/cm}^2)$               | 0.3                | 0.7                | 0.07               |
| $d\Sigma/dt(\text{g/cm}^2/\text{yr})$ | 0.4                | 0.2                | 0.06               |
| $t_{\text{rec}}(\text{yr})$           | $1.0 \times 10^5$  | $4.4 \times 10^5$  | $9.3 \times 10^6$  |

Table 3.4: Recurrence time between thermal-viscous instability-driven outbursts. Three simulations are selected, with initially subsonic winds where  $c_s = 6, 10$ , and  $19 \text{ km s}^{-1}$ . The maximum surface density has been calculated at 3 different disc radii from (3.16). The steady state surface density is listed for each simulation alongside an upper limit for  $d\Sigma/dt$ . Recurrence times have then been estimated from these values.

outbursts (see Lasota, 2001, for a review of the disc instability model). Whether it is plausible that these are responsible for the observed outbursts of RS Oph depends on the recurrence time ( $t_{\text{rec}}$ ).

For a thin disc, the critical density above which instability occurs within a particular annulus is given by (Cannizzo, 1993)

$$\Sigma_{\max} = 11.4\alpha^{-0.86} \left( \frac{M_1}{M_{\odot}} \right)^{-0.35} \left( \frac{r}{10^{10}\text{cm}} \right)^{1.05} \text{ g cm}^{-2}, \quad (3.16)$$

where  $\alpha$  is a free parameter in the Shakura and Sunyaev (1973) viscosity approximation (it is usually assumed that  $\alpha \lesssim 0.01$  during quiescence/cold state),  $M_1$  is the mass of the accretor, and  $r$  is the distance of the annulus from the centre of the disc. During an outburst, the instability will be initiated at some radius, before propagating through the disc. With such a large disc, instabilities are more likely to be of the "inside-out" variety (Lasota, 2001; Alexander et al., 2011), where the outburst is triggered in the inner disc, spreading outwards. However, for estimating  $t_{\text{rec}}$ , I consider both outer and inner trigger radii.

In Table 3.4,  $\Sigma_{\max}$  is calculated at  $5 \times 10^{11}\text{cm}$ ,  $1 \times 10^{12}\text{cm}$  and  $5 \times 10^{12}\text{cm}$  from the white dwarf of mass  $1.35M_{\odot}$ . These values are compared to the steady state surface density

of the disc in three of the simulations, with initially subsonic wind velocities and  $c_s = 6, 10,$  and  $19 \text{ km s}^{-1}$ , corresponding to the three mass transfer regimes (Cases A-C) shown in figure 3.5. The rate at which the surface density changes over time,  $d\Sigma/dt$ , is also shown. In the models, the discs reach a steady state after  $\sim 5$  orbits, after which their mass remains approximately constant. The values shown here are based on the variance of  $\Sigma$  once steady state is reached, which I treat as upper estimates of  $d\Sigma/dt$ . These values are then used to calculate  $t_{\text{rec}}$ , the time taken for the surface density of each annulus to build up to  $\Sigma_{\text{max}}$ .

Alexander et al. (2011) suggest that the observed recurrent outburst in RS Oph is due to enhancement in accretion rate on to the surface of the WD as a result of thermal-viscous instabilities in the accretion disc which happens on a  $\sim 20$  year timescale. For all three simulations shown in Table 3.4,  $t_{\text{rec}}$  is considerably longer than the  $\sim 20$  yr interval between the observed outbursts in RS Oph. Typically,  $t_{\text{rec}} > 10^4$  yr, except when  $c_s = 6 \text{ km s}^{-1}$  and  $r = 5 \times 10^{11} \text{ cm}$ , where  $t_{\text{rec}} = 81$  yr.

An alternative estimate for  $t_{\text{rec}}$  can be obtained from the definition

$$t_{\text{rec}} = \frac{\Delta M}{\dot{M}_{\text{acc}}} , \quad (3.17)$$

where  $\Delta M$  is the total mass that must be accreted and  $\dot{M}_{\text{acc}}$  is the mass accretion rate onto the disc.  $\Delta M$  can be estimated as

$$\Delta M = \int 2\pi R \Sigma_{\text{max}} dR = 1.147 \times 10^{-12} \alpha^{-0.83} \left( \frac{M_1}{M_{\odot}} \right)^{-0.35} \left( \frac{r}{10^{10}} \right)^{3.14} M_{\odot} ,$$

using (3.16). If  $\dot{M}_{\text{acc}} = 10^{-8} M_{\odot} \text{ yr}^{-1}$ ,  $t_{\text{rec}} \sim 3.3 \times 10^4 \text{ yr}$ .

RS Oph has been cited as a potential Type 1a supernova progenitor due to its high WD mass and rapid accretion rate (Sokoloski et al., 2006; Hachisu et al., 2007). In the single-degenerate model for Type 1a supernova formation, accretion onto the WD primary drives it over the Chandrasekhar mass limit (see e.g. Voss and Nelemans, 2008; Wang and Han, 2012, for observational evidence). Supernova formation via mass transfer from a RG is expected to be relatively rare (Yungelson and Livio, 1998; Wang et al., 2010) and has been ruled out for a large number of supernovae in recent surveys (Panagia et al., 2006; Hayden et al., 2010). However, several spectroscopic studies of individual supernovae (Patat et al., 2007; Sternberg et al., 2011; Patat et al., 2011) have detected circumstellar material around supernovae with comparable structure to that expected from recurrent nova outbursts. To become a supernova, the WD in RS Oph must be gaining mass. Consequently, the ejected

mass  $\Delta M_{\text{ej}}$  during its nova outbursts must be less than the accreted mass  $\Delta M_{\text{acc}}$  during quiescence. Estimates of the mass ejected during the 2006 RS Oph outburst range from  $10^{-7} M_{\odot}$  (Sokoloski et al., 2006) to  $10^{-6} M_{\odot}$  (Orlando et al., 2009). For WD growth,  $\dot{M}_{\text{acc}}$  must therefore be towards the upper end of my estimates ( $\geq 10^{-7} M_{\odot} \text{yr}^{-1}$ ), indicating high values for  $f_{\text{cap}}$  and correspondingly low wind velocities. Estimates of  $\Delta M_{\text{ej}}$  from Yaron et al. (2005) show that a  $1.4 M_{\odot}$  WD will increase in mass, provided  $\dot{M}_{\text{acc}} \geq 10^{-7} M_{\odot} \text{yr}^{-1}$  while a  $1.25 M_{\odot}$  WD will need  $\dot{M}_{\text{acc}} \geq 10^{-6} M_{\odot} \text{yr}^{-1}$ .

The high mass transfer rates required for the RS Oph WD to increase in mass are consistent with my wind-driven RLO models, and with a high mass  $M_{\text{WD}}$ , close to  $1.4 M_{\odot}$ . This high WD mass means the system is likely to be very close to the supernova threshold, leading Hachisu et al. (2007) to predict an RS Oph supernova explosion within 100 000 yr.

### 3.10 Summary

Using 3-dimensional smoothed particle hydrodynamics simulations, I have modelled the process of mass transfer via stellar wind capture in the symbiotic recurrent nova system, RS Oph. I have considered a range of physically-reasonable parameters for the RG-donor wind, as well as studying the effect of RG rotation. As a consequence, I have been able to compile a comprehensive range of binary parameters that can explain the system's  $\sim 20$  yr outbursts.

In my models, the fraction of mass carried in the stellar wind that is accreted onto the WD ranges from  $0.05 \lesssim f_{\text{cap}} \lesssim 1$ , increasing as the speed of the wind decreases. Almost all of the mass is captured by the WD when the wind velocity is low, because particles stream through the  $L1$  point into the Roche sphere of the WD, resulting in Roche lobe overflow-like behaviour. Neither prograde or retrograde rotation of the RG have a significant effect on the mass capture fraction, but synchronised rotation (where  $P_{\text{rot}} = P_{\text{orb}}$ ) results in enhanced mass loss by the RG, up to a factor of 3 for slow winds. Even faster rotation can cause  $\dot{M}_{\text{RG}}$  to increase by an order of magnitude.

These possible mass capture fractions, alongside the additional effects of rotation, lead to a large range of RS Oph parameters that are consistent with the system's observed outbursts. Potential solutions are approximately bounded by  $10^{-8} < \dot{M}_{\text{RG}} < 10^{-6} M_{\odot}/\text{yr}$ ,  $10^{-8} < \dot{M}_{\text{acc}} < 10^{-7} M_{\odot}/\text{yr}$  and  $M_{\text{WD}} \gtrsim 1 M_{\odot}$ . The self-consistent model presented by Walder et al. (2008), where  $\dot{M}_{\text{RG}} \sim 10^{-7} M_{\odot}/\text{yr}$ ,  $\dot{M}_{\text{acc}} = 10^{-8} M_{\odot}/\text{yr}$  can be found within this range of parameters. This model can correctly predict the recurrence time and additional features of the system's outburst, but is not a unique description of mass transfer in

RS Oph.

In all of my simulations, a disc forms around the WD, which is unstable to thermal-viscous outbursts, as predicted by King and Pringle (2009) and Alexander et al. (2011). However, the surface density profiles of the discs that form are too low to trigger instability in the short term. Indeed, estimates of the initial recurrence timescales suggest that these are likely to be in the range of tens of thousands rather than tens of years. The results indicate that RS Oph is likely to undergo recurrent nova on the surface of the WD while the possibility of outbursts due to accretion disc-instability would be on a longer timescale.

The high mass and rapid accretion rate of the WD in RS Oph make it a promising Type 1a supernova candidate (Sokoloski et al., 2006; Hachisu et al., 2007). During a dwarf nova outburst, significant mass is ejected from the WD, but if  $\Delta M_{\text{acc}} > \Delta M_{\text{ej}}$ , then the WD nevertheless gains in mass. Provided  $M_{\text{WD}} \sim 1.4M_{\odot}$ , I find that the WD gains mass for  $\dot{M}_{\text{acc}} \gtrsim 10^{-7}M_{\odot}/\text{yr}$ . These values can be found within the available binary parameter space I have identified in my simulations.

---

# 4

## The Transiting Hot Jupiter WASP 12b

## 4.1 Introduction

Hot Jupiters as a subset of the discovered extrasolar planets are giant gaseous planets having mass comparable to that of Jupiter (Armitage, 2009). Unlike Jupiter in the solar system, these planets orbit their parent stars in just a few days, and consequently are subject to a variety of extreme physical processes. Tidal forces from their parent stars may dissipate their orbital eccentricity and may inflate the planets' atmospheres (Rasio et al., 1996; Bodenheimer et al., 2001). Hot Jupiters are sometimes inflated by stellar irradiation (Showman and Guillot, 2002), which leads to RLO from the planet as their atmospheres can expand as they are heated (Gu et al., 2003). Their short orbital periods also make hot Jupiters, particularly those which transit their host stars, extreme systems through which we can improve our understanding of planetary physics. Close-in Jupiter-mass planets challenge planet formation theories as giant planets are unlikely to have formed in-situ (Armitage, 2009; Lin et al., 1996); One possible explanation of their presence at such short orbital radii is that they may have initially formed by gradual accretion of solids and capture of the gas further from the star and migrated inward through interactions with the circumstellar disc remnants. Their migration was then halted due to tidal interactions with the star or the fact that the inner disc was truncated by the stellar magnetosphere (Lin et al., 1996).

One of the shortest-period hot Jupiters detected to date is WASP-12b (Hebb et al., 2009). It is a transiting extrasolar planet in orbit around a late F star with orbital period  $P_{orb} = 1.09d$ . It is among the largest transiting planets with radius  $R_p = 1.79R_{Jup}$  and mass of  $M_p = 1.41M_{Jup}$ . The host star WASP-12 is a bright F9V "super Solar" metallicity star which is evolving off the zero-age main sequence with mass and radius  $M_* = 1.35M_\odot$  and  $R_* = 1.57R_\odot$ . WASP-12b is one of the exoplanet systems to have been observed in and out of transit in multiple wavebands. Fossati et al. (2010) and Haswell et al. (2012) studied two transits of the planet using near-UV observations of Cosmic Origins Spectrograph (COS) on the Hubble Space Telescope (HST). Their studies suggests the likelihood of an early ingress in the NUV of the planet compared to its optical light curve which is suggestive of absorbing material around the planet.

Several models have been suggested to explain this excess absorption in the near UV, one possible model is Roche lobe overflow from the planet since the radius of the planet is comparable to its Roche volume and the highly irradiated gas in the planet may expand beyond its Roche limit (Lai et al., 2010; Bisikalo et al., 2013); as a result, mass transfers to the star through the inner Lagrange point. Another possible model is that a bow shock

is formed due to the stellar wind and/or coronal interactions with the planetary magnetic field (Lai et al., 2010; Vidotto et al., 2010; Llama et al., 2011). Here a shock is formed around the planet as it orbits supersonically through the stellar wind plasma, producing a local density enhancement. However, to date these models have been somewhat idealised and the current observational data do not allow one to distinguish between different scenarios. More recently, Nichols et al. (2015) observed four closely spaced NUV transits of WASP-12b using HST/COS observations to have a higher phase resolution of the system NUV light curve. They found a profound NUV excess absorption during the transit that was deeper than the optical. However, they did not find evidence of an early NUV ingress, rather they suggest the presence of a highly variable NUV intensity prior to the optical transit possibly due to a variable region of NUV absorption.

## 4.2 This Chapter

Here I present hydrodynamic models of the interaction between stellar winds and the planetary magnetospheres of hot Jupiters in an attempt to model the pre-transit variable NUV absorption. I work within the existing picture of absorption in a magnetospheric bow shock (Lai et al., 2010; Vidotto et al., 2010; Llama et al., 2011) but, for the first time, build a self-consistent hydrodynamic model of both the stellar wind and the shock around the simulated planet orbiting the host star from which the wind expands by using a 2-dimensional version of ZEUS (Stone and Norman, 1992a) and the 3D SPH code described in Chapter 3. I find that the bow shock is always weak, and that the shock structure differs substantially from that assumed by Vidotto et al., 2010 and Llama et al., 2011. I use hydrodynamic models to compute theoretical UV light-curves, and investigate how the UV transit shape varies with different physical model parameters (such as the wind temperature and planetary magnetic field strength). My models are consistent with existing UV data for WASP-12b, and I show that 100% phase coverage is highly desirable if we are to use such observations to test theoretical models in detail. I also construct models for other known systems, and show that additional UV observations of more massive short-period hot Jupiters (such as WASP-18b; Hellier et al., 2009) should distinguish clearly between different models for circumplanetary absorption.

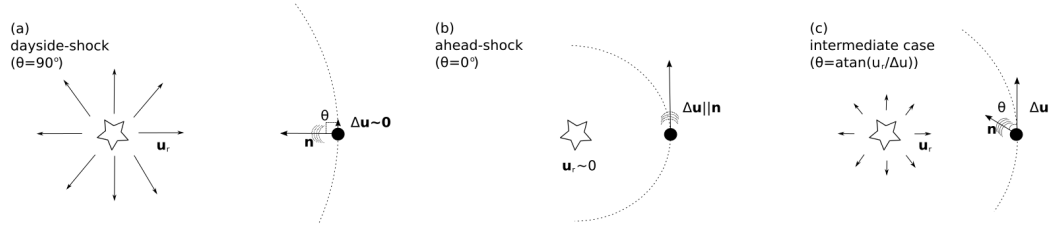


Figure 4.1: Orientation of the deflection angle  $\theta$  according to the ratio of the planetary orbital velocity and the components of the stellar wind velocity. Figure credit to Vidotto et al. (2010).

### 4.3 Bow-shock formation

The analytic derivation by Vidotto et al. (2010) aims at determining the conditions of the formation and orientation of the bow shock formed around an orbiting planet in an isothermal Parker wind blown from the host star. The shock forms when the relative velocity between the planet and the stellar wind is supersonic. Following their notations, I consider the radial component of the stellar wind  $u_r$ , and  $u_\phi$  the azimuthal (representing stellar rotation) component of the wind. The planet has a Keplerian azimuthal velocity  $u_K$  on an orbital radius  $a_p$ . As shown in figure 4.1, the angle  $\theta$  is defined as the deflection angle between the planetary orbital motion and  $\mathbf{n}$  which is a vector defining the outward direction of the shock, that is  $-\mathbf{n}$  is the direction of the velocity of the impacting wind material as seen from the planet. Thus

$$\theta = \arctan\left(\frac{u_r}{\Delta u}\right),$$

where  $\Delta u = u_K - u_\phi$  is the relative velocity of the azimuthal components.  $u_\phi$  is given by the conservation of angular momentum at  $a_p$  since the star rotates with period  $P_*$  then

$$u_\phi = \frac{2\pi R_*^2}{P_* a_p}. \quad (4.1)$$

Then the relation above becomes

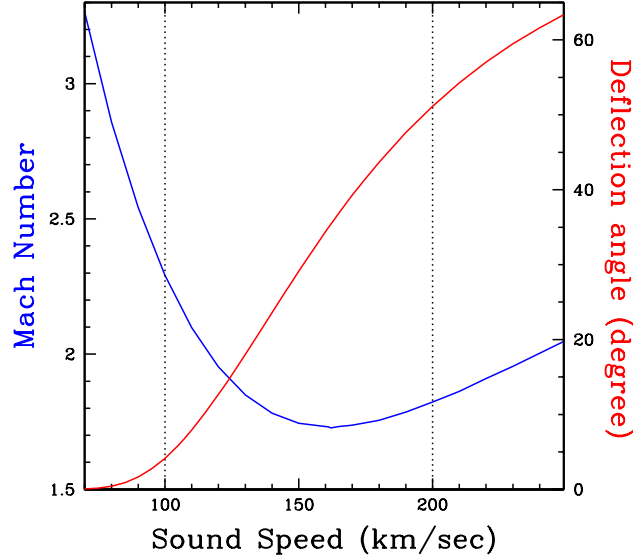


Figure 4.2: Mach number at the planets position (in blue) and the angle of deflection  $\theta$  of the shock with respect to the direction of orbital motion of the planet (in red). The two vertical dotted lines determine the range of the sound speed of the stellar wind studied in this chapter.

$$\theta = \arctan \left( \frac{u_r}{\left| \left( \frac{Gm_*}{a_p} \right)^{1/2} - \frac{2\pi R_*^2}{P_* a_p} \right|} \right). \quad (4.2)$$

Let's take two extreme examples: When the dominant flux of the impacting wind is from the radial component of the stellar wind, then  $\theta \rightarrow 90^\circ$ , this is called a "day-side shock" such as the case of the supersonic solar wind impacting the earth's magnetosphere. When the azimuthal component is dominant such as when the orbiting planet moves inside the ambient gas in hydrostatic equilibrium that is  $u_r = 0$ , or in the case of very small orbital radius  $a_p$  where  $u_K$  is highly supersonic and  $u_r$  is negligible, then  $\theta = 0^\circ$ , and is called "ahead shock". For WASP-12b as discussed in the next sections the possible range of the sound speed for the stellar wind is around 100- 200 km sec<sup>-1</sup> with the planet's orbital velocity 228.7 km sec<sup>-1</sup>. For pure radial stellar wind with no rotation, figure 4.2 shows the variation of Mach number and the shock deflection angle. It shows that the wind at the planetary distance is modestly supersonic with maximum Mach number  $\sim 2.3$  and the shock deflection angles are between the two extremes described above where  $\theta$  varies between 5 and 50 degrees.

## 4.4 Two-dimensional simulations of WASP-12b

### 4.4.1 Numerical Setup

To simulate the interaction between a stellar wind and a planetary magnetosphere, I use the 2-dimensional hydrodynamical code ZEUS-2D (Stone and Norman, 1992a) using a polar  $(r, \phi)$  grid where an infinitesimal midplane “wedge” of a 3-D spherical polar grid is considered whose volume element scales as  $\Delta(r^3/3)$ . The computational grid spans the range  $[0, 2\pi]$  in the  $\phi$ -direction, with periodic boundary conditions. The grid cells are logarithmically-spaced in  $r$  and linearly-spaced in  $\phi$ , with the numbers of radial ( $N_r$ ) and azimuthal ( $N_\phi$ ) grid cells chosen so that the grid cells are approximately square (i.e.,  $\Delta r = r\Delta\phi$ ). I adopt the van Leer (second order) interpolation scheme and the standard von Neumann & Richtmyer form for the artificial viscosity (with  $q_{\text{visc}} = 2.0$ ); tests indicate that neither of these choices has a significant influence on the results. In such a numerical domain, I simulate a planet orbiting a central star, I adopt a system of units such that the unit of mass is the stellar mass  $M_*$ , the unit of length is the planet semi-major axis  $a_p$ , and the unit of time is the planet’s orbital period  $P$ . This sets the gravitational constant  $G = 4\pi^2$  in code units, and the planet’s (Keplerian) orbital velocity  $u_p = \sqrt{GM_*/a_p} = 2\pi$ .

### 4.4.2 Stellar Wind model

In this study, a spherically-symmetric isothermal wind is set to expand from the surface of the central star. This model has the well-known analytic solution discussed earlier in section (1.3) (Parker, 1958; Cranmer, 2004) which is fully specified by the choice of sound speed  $c_s$  and passes through a sonic point at  $r_c = GM_*/2c_s^2$ . I implement this in ZEUS-2D by specifying an “inflow” inner boundary condition at the stellar radius  $r_{\text{in}} = R_*$ , with the injection velocity  $u_{\text{in}}$  equal to that of the Parker solution at that radius [i.e.,  $u_{\text{in}} \equiv u_w(r_{\text{in}})$ ]. The gas density appears in the Parker solution only as a normalisation constant, and consequently the density in the simulations is arbitrary; I normalise to the (fixed) value at the inner boundary,  $\rho_0$ .

The model is scaled to the physical units by adopting the parameters of the WASP-12b system (see section 4.1) and therefore set  $R_* = 0.319a_p$ . I place the outer boundary at 10 times the planet’s orbital radius, so the grid spans  $[r_{\text{in}}, r_{\text{out}}] = [0.319a_p, 10.0a_p]$  in the radial direction. My standard models have  $N_\phi = 250$  grid cells in the azimuthal direction, and therefore  $N_r = 139$  cells in the radial direction. I initially set the density everywhere on the grid to a small value ( $10^{-15}\rho_0$ ), and allow the wind to evolve to a steady state.

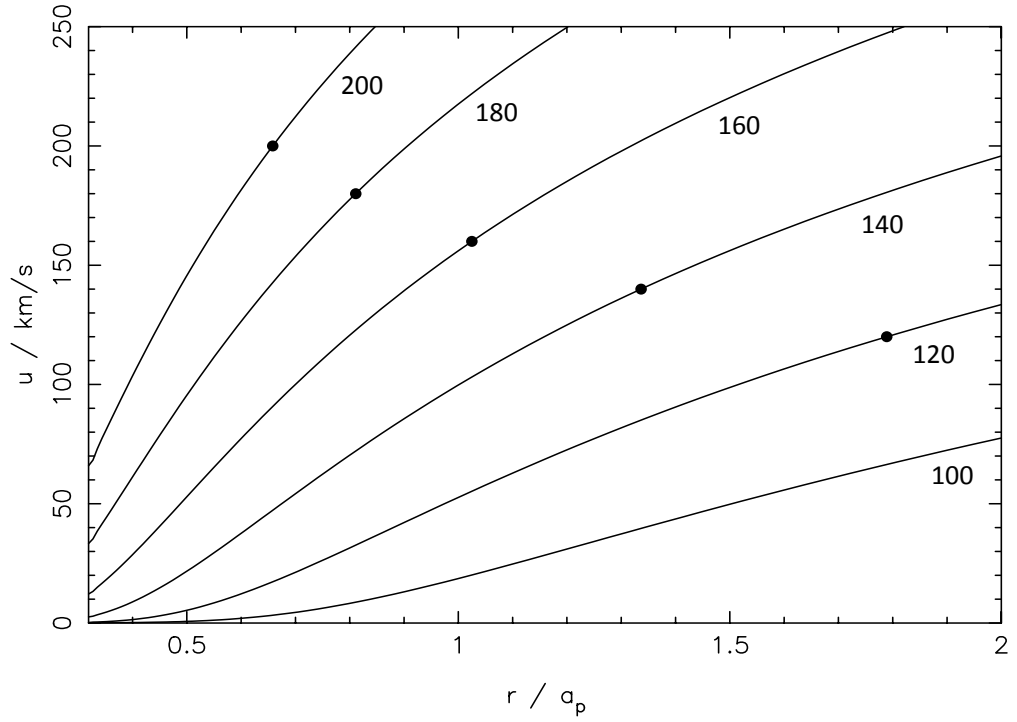


Figure 4.3: Velocity profiles for isothermal (Parker) winds, calculated using ZEUS-2D and scaled to the parameters of the WASP-12 system ( $M_* = 1.35M_\odot$ ,  $a_p = 0.0229\text{AU}$ ). Profiles, from bottom to top respectively, are plotted for sound speeds  $c_s=100, 120, 140, 160, 180$  &  $200$  km/sec. In each case the filled black circle denotes the location of the sonic point.

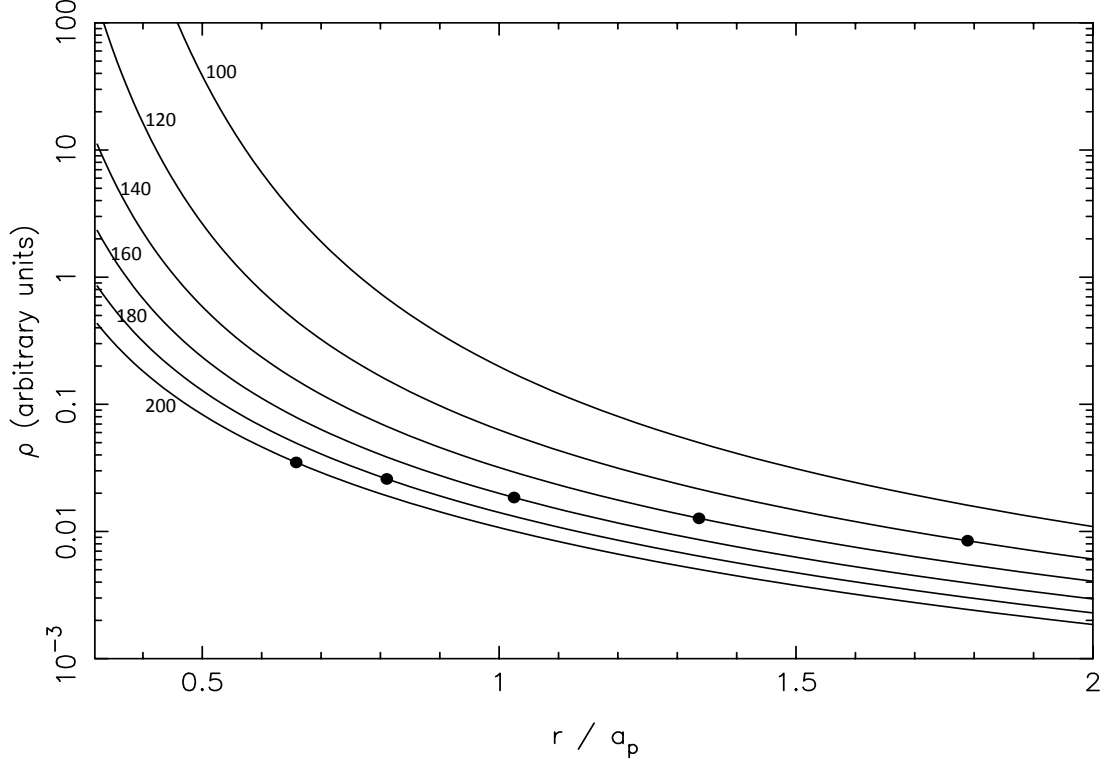


Figure 4.4: Density profiles for the (Parker) wind solutions in figure 4.3. The absolute density in the Parker solution is scale-free, so I adopt arbitrary units on the vertical axis and scale the solutions to a constant wind rate  $\dot{M}_w = 4\pi r^2 u \rho$ . As in figure 4.3 the profiles, from top to bottom respectively, are plotted for sound speeds  $c_s = 100, 120, 140, 160, 180$  &  $200$  km/sec, with the sonic points marked by circles. Note that a relatively small (factor of two) decrease in the sound speed dramatically increases the density gradient  $d(\log \rho)/dr$ , and consequently increases the density contrast between the inner boundary (the stellar surface) and the planet’s position by more than three orders of magnitude.

Slow stellar winds from Sun-like stars typically have temperatures  $\sim 10^6$  K (e.g., Lamers and Cassinelli, 1999), so I explore a range of sound speeds from  $100$ – $200$  km s $^{-1}$ . This approach reproduces the Parker wind solution to high accuracy, matching the analytic solution to within approximately 0.1%. Example velocity and density profiles (for a fixed wind rate  $\dot{M}_w$  and different choices of the sound speed  $c_s$ ) are shown in figures 4.3 & 4.4.

#### 4.4.3 Planet model

The interaction of a close-in planet with the stellar wind is simulated by adding both the planet’s gravity and the effect of its magnetic field. I assume a fixed, circular orbit for

the planet, which has mass  $M_p$ , semi-major axis  $a_p$  and orbital frequency  $\Omega$ . The planet's position at time  $t$  is therefore  $\mathbf{r}_p = (r_p, \phi_p) = (a_p, \Omega t)$ .

ZEUS-2D solves the momentum equation in the form

$$\rho \frac{d\mathbf{u}}{dt} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \rho \mathbf{g}. \quad (4.3)$$

In the absence of a planet the gravitational acceleration is simply that due to the star, so  $\mathbf{g} = -(GM_*/r^2)\hat{\mathbf{r}}$ . I neglect self-gravity, radiation hydrodynamics and magnetic fields in the wind (these modules are switched off in the code), and incorporate accelerations due to the planet by adding an additional term  $\rho \mathbf{a}$  to the right-hand-side of the momentum equation. The gravitational acceleration  $\mathbf{g}$  appears explicitly in the ZEUS-2D code as a source term, and I compute the accelerations due to the planet  $\mathbf{a}$  in a similar manner.

I assume that the planet has a dipolar magnetic field aligned with the planet's orbital angular momentum, and work in the far-field limit (i.e., I assume  $r = |\mathbf{r} - \mathbf{r}_p| \gg R_p$ , where  $R_p$  is the radius of the planet), so the magnetic field strength scales as  $B \propto r^{-3}$ . The acceleration due to the planet at an arbitrary position  $\mathbf{r}$  is therefore

$$\mathbf{a} = \left( \frac{C_B}{|\mathbf{r} - \mathbf{r}_p|^8} - \frac{GM_p}{|\mathbf{r} - \mathbf{r}_p|^3} \right) (\mathbf{r} - \mathbf{r}_p), \quad (4.4)$$

where  $C_B$  is a normalisation constant that sets the magnitude of the “magnetic” accelerations (see Appendix C.1). I adopt this formalism in order to preserve the scale-free behaviour of the Parker wind: the additional accelerations due to the planet depend only on  $M_p$ ,  $C_B$  and position, and are independent of the density normalisation. I treat  $C_B$  as an input parameter, which effectively determines the magnetospheric radius (see section 4.5.1 and figures 4.6 & 4.7). However, the magnetic field strength is not scale-free, as the relationship between  $C_B$  and  $B$  depends on the local gas density  $\rho$ . If I follow convention and specify the planetary magnetic field strength in terms of the surface field  $B_0$ , then equation 4.4 implies that  $C_B = 3B_0^2 R_p^6 / 4\pi\rho$ .

Equation 4.4 diverges as  $|\mathbf{r} - \mathbf{r}_p| \rightarrow 0$ , so I soften the potential in order to prevent numerical errors close to the planet's position. I compute the accelerations as

$$\mathbf{a} = (a_B + a_g)(\mathbf{r} - \mathbf{r}_p), \quad (4.5)$$

where  $a_B$  and  $a_g$  are the magnitudes of the magnetic and gravitational terms, respectively. I use Plummer softening for the gravitational potential

$$a_g = -\frac{GM_p}{[r^2 + (\epsilon R_p)^2]^{3/2}}, \quad (4.6)$$

and soften the magnetic potential using a Gaussian function

$$a_B = \begin{cases} 0 & \text{if } r < \delta R_p \\ \frac{C_B}{r^8} \exp\left(-\frac{[r-(1+\delta)R_p]^2}{2(\delta R_p)^2}\right) & \text{if } \delta R_p \leq r \leq (1+\delta)R_p \\ \frac{C_B}{r^8} & \text{if } r > (1+\delta)R_p. \end{cases} \quad (4.7)$$

I adopt softening parameters  $\epsilon = 0.4$  and  $\delta = 0.15$  throughout. In practice the softened potential deviates from the true potential only for  $r < 2R_p$  (i.e., within one planetary radius of the planet's surface), where the gas density is very low, and tests indicate that the exact choice of softening parameters has no significant effect on my results. I then decompose the vector  $\mathbf{a}$  into its components  $(a_r, a_\phi)$  (see Appendix C.2) and add these terms as explicit accelerations in the ZEUS-2D source step. I also impose a numerical density floor at  $10^{-15}\rho_0$ , in order to prevent the low-density region close to the planet restricting the time-step to unreasonably small values.

## 4.5 WASP-12b models

The simulations are scaled to the WASP-12b system a prototype short-period hot Jupiter and one of only two exoplanets (to date) to have been observed in transit at near-UV wavelengths (Fossati et al., 2010; Haswell et al., 2012). I adopt the parameters of the WASP-12b system derived by Hebb et al. (2009): (section 4.1). I note that with these parameters the stellar wind velocity relative to the planet  $u = \sqrt{u_p^2 + u_w^2(a_p)}$  is always supersonic as shown in figure 4.2. The orbital velocity  $u_p$  of the planet is supersonic in the range of the sound speed values studied. In the frame of the planet, the minimum Mach number at  $r = a_p$  is in fact  $\mathcal{M} = 1.73$ , which occurs for  $c_s = 161 \text{ km s}^{-1}$ . However, in all cases of interest the wind is only modestly supersonic:  $\mathcal{M} < 2.3$  for  $c_s = 100\text{--}200 \text{ km s}^{-1}$  (see figure 4.2). I therefore expect any interactions between the planet and the stellar wind in the WASP-12b system to result in weak shocks.

### 4.5.1 Simulations

When the hydrodynamic simulations are allowed to evolve the planet carves out a magnetospheric cavity in the stellar wind, and the simulations rapidly evolve into a steady state.

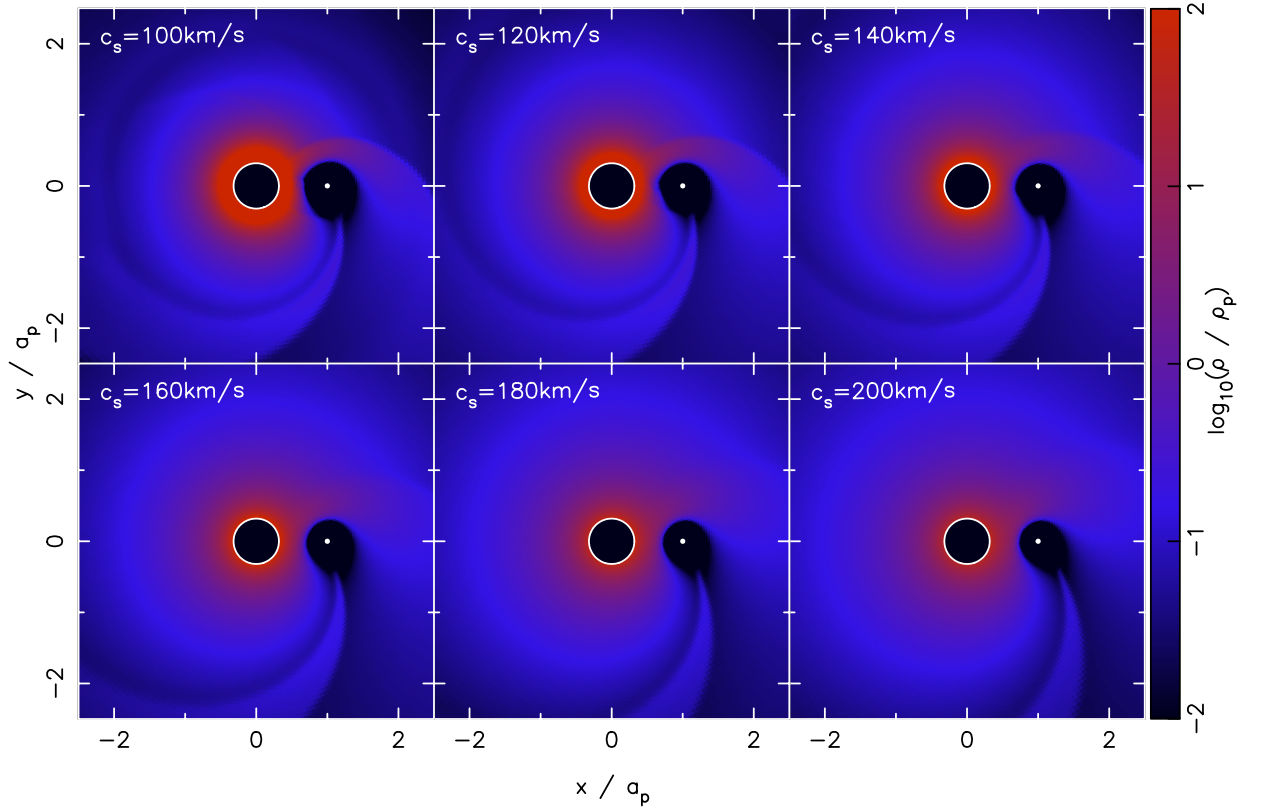


Figure 4.5: Steady-state density structure in the simulations with  $C_B = 0.3$ . The white line denotes the inner grid boundary at  $r_{\text{in}} = R_*$ , while the filled white circle denotes the position and radius of the planet (i.e., both the star and planet are plotted to scale). In each panel the density is normalised to the value at the planet’s orbital radius,  $180^\circ$  out of phase with the planet (denoted by  $\rho_p$ ). The magnetospheric radius is approximately constant in all the models, but as the sound speed increases the cavity and wake are angled progressively more towards the star. In addition, lower sound speeds result in slightly stronger (though still weak) bow shocks, with more pronounced density enhancements ahead of the planet.

This steady state is typically reached after 3–5 planetary orbits; The results are taken after 10 orbits as the final flow solutions. I computed models for sound speeds  $c_s=100, 120, 140, 160, 180$  &  $200 \text{ km s}^{-1}$ , and magnetic constants  $C_B=0.1, 0.3$  &  $1.0$ . The resulting steady-state density structures (for  $C_B=0.3$ ) are shown in figure 4.5; azimuthal density profiles  $\rho(a_p, \phi)$  are shown in figures 4.6 & 4.7 (for varying sound speed  $c_s$  and magnetic constant  $C_B$ , respectively). Figure 4.7 also shows the result of a numerical convergence test run at twice my standard numerical resolution (i.e.,  $N_\phi=500, N_r=276$ ): no significant differences are seen at higher resolution, indicating that the simulations are well-resolved and numerically robust.

In all cases qualitatively similar flow solutions are seen. The planetary magnetic field carves out a large, near-circular magnetospheric cavity, which is preceded by a near semi-circular bow shock and followed by a bifurcated wake. The geometry of the magnetosphere changes with sound speed, with the cavity and wake oriented progressively more towards the star as the sound speed increases (as suggested by Vidotto et al., 2010). For  $c_s=100 \text{ km s}^{-1}$  the bow shock is almost perpendicular to the planet’s orbit, but for sound speeds  $>160 \text{ km s}^{-1}$  the wind velocity at  $a_p$  is comparable to the planet’s orbital speed ( $228.7 \text{ km s}^{-1}$ ), and the shock is angled significantly towards the star.

As expected, in all the models the bow shock is weak. The peak Mach number in the pre-shock gas (ahead of the magnetosheath) ranges from  $\mathcal{M} \simeq 1.6$ – $1.8$  for the models with  $c_s=140$ – $200 \text{ km s}^{-1}$ . For lower sound speeds slightly stronger shocks are seen, but even for  $c_s=100 \text{ km s}^{-1}$  the peak Mach number is only  $\mathcal{M} \simeq 2.3$ . Consequently the shocks are broad: the magnetosheaths have widths comparable to the radius of the magnetospheric cavities, and have only modest density enhancements. The density contrast between the pre- and post-shock gas exceeds 2.5 only for  $c_s < 140 \text{ km s}^{-1}$ , and the shocks are never strong enough to produce a density discontinuity.

The simulations are parameterised such that the size of the magnetosphere is essentially independent of the sound speed in the gas (as can be seen in figures 4.5 & 4.6), and depends only on the magnetic constant  $C_B$ . The choice of magnetic constant therefore specifies the magnetospheric radius, with values  $C_B=0.1, 0.3$  &  $1.0$  corresponding to cavity radii of  $\simeq 6.0, 7.5$  &  $9.0 R_p$ , respectively (see figure 4.7). In the simulations (which neglect the stellar magnetic field) the magnetospheric radius is determined by the balance between ram pressure in the wind and magnetic pressure from the planet (see section 4.4.3), with

$$B_0 = \sqrt{(4/3)\pi\rho C_B} R_p^{-3}. \quad (4.8)$$

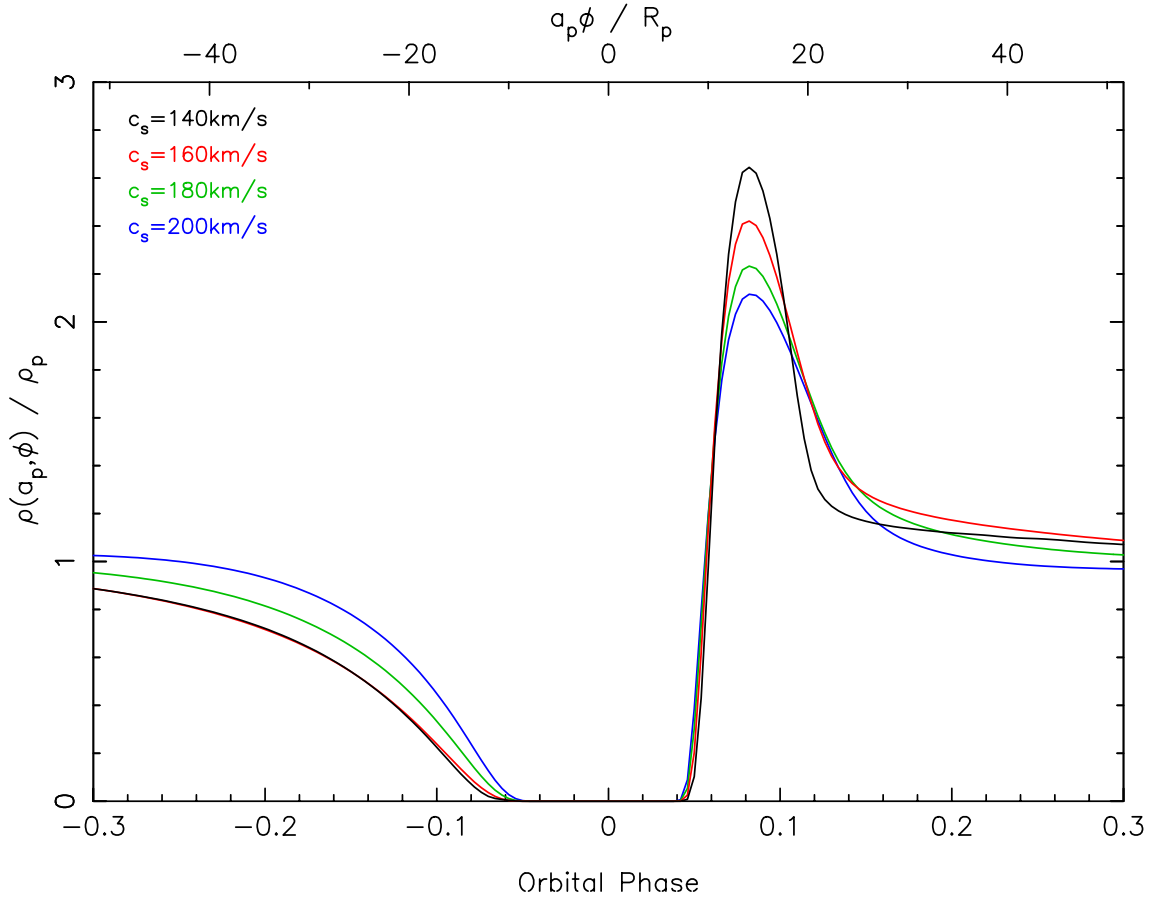


Figure 4.6: Azimuthal density profiles at the planet’s orbital radius, for models with  $C_B=0.3$  and various values of the sound speed  $c_s$ . (For clarity, the models with  $c_s=100$  &  $120 \text{ km s}^{-1}$  are omitted.) As in figure 4.5, the density is normalised to the value  $180^\circ$  out of phase with the planet. The lower axis shows the azimuthal coordinate in units of orbital phase (which ranges from  $[-0.5, 0.5]$ ), while the upper axis shows the distance around the planet’s orbit in units of the planet radius  $R_p$ . The size of the magnetospheric cavity ( $\approx 7.5R_p$  in radius) and the position of the bow shock (with peak density  $\approx 14R_p$  ahead of the planet) are essentially independent of the sound speed in the wind, but the density contrast in the shock decreases with increasing  $c_s$ .

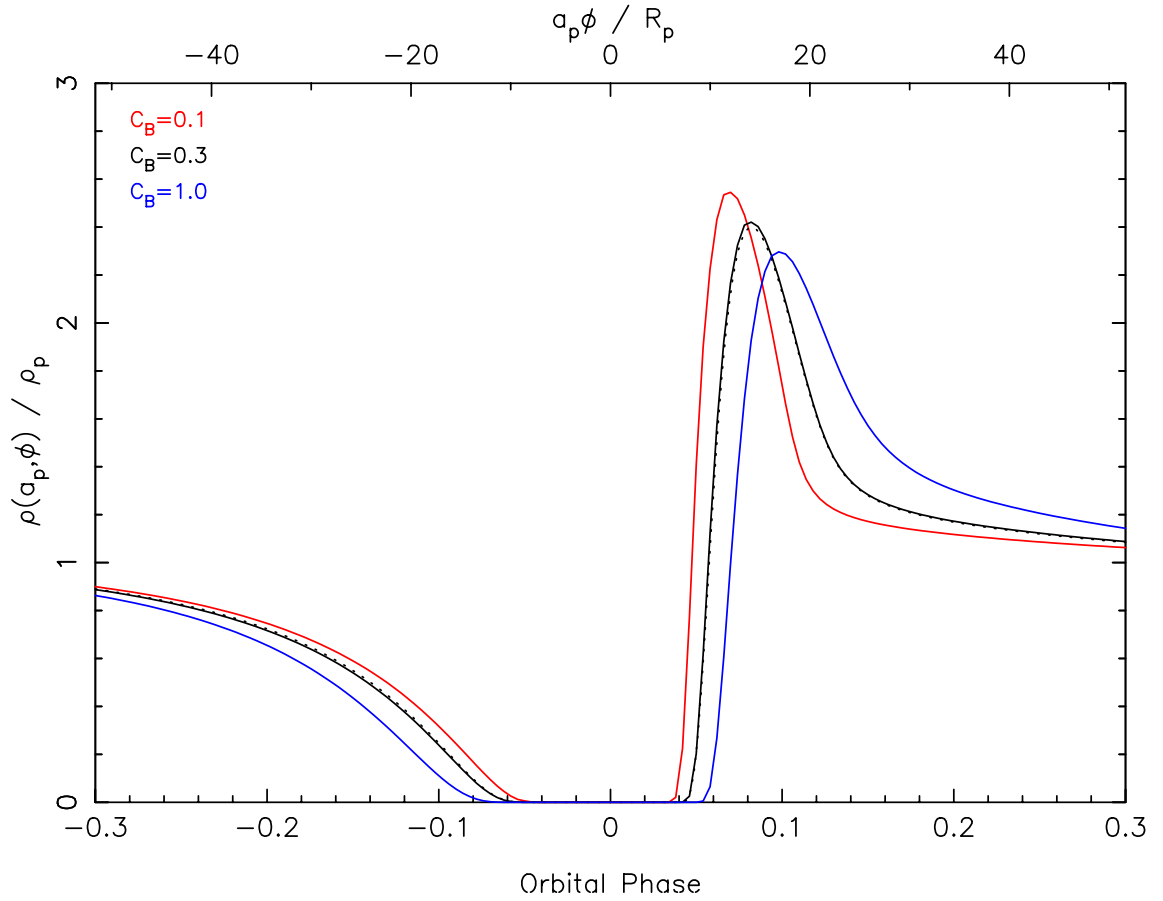


Figure 4.7: As figure 4.6, but for models with  $c_s=160\text{km s}^{-1}$  and various values of the magnetic constant  $C_B$ . The magnetospheric cavity increases in size for stronger magnetic fields, from  $\approx 6.0R_p$  for  $C_B=0.1$  to  $\approx 9.0R_p$  for  $C_B=1.0$ . The dashed black line denotes the simulation with  $C_B=0.3$  run at twice the numerical resolution ( $N_\phi=500$ ,  $N_r=276$ ), and shows that the simulations have achieved convergence.

For a fiducial stellar wind rate  $\dot{M}_w = 10^{-15} M_\odot \text{ yr}^{-1}$  and<sup>1</sup>  $c_s = 160 \text{ km s}^{-1}$ ,  $C_B = 0.3$  therefore equates to a surface magnetic field  $B_0 \sim 4 \text{ G}$ . In all cases the half-width of the magnetosheath (i.e., the distance between the outer edge of the cavity and the peak gas density) is approximately the same as the magnetospheric radius. The planetary magnetic field therefore carves out a large, extended structure, with the shock and cavity typically stretching around  $\gtrsim 30\%$  of the planet's orbit.

### 4.5.2 Light-curves

Having found steady-state flow solutions, I now compute UV light-curves for the combined transit of the planet and magnetosphere. I first calculate the gas column density  $\Sigma(\phi)$  from the steady-state solutions by integrating the gas density along the line-of-sight to the star. Using the density structures from the 2-D simulations, I compute the absorbing column at an arbitrary angle  $\phi_0$  as

$$\Sigma(\phi_0) = \frac{1}{2R_*} \int_{y=-R_*}^{+R_*} \int_{x=0}^{r_{\text{out}}} \rho(x, y) dx dy, \quad (4.9)$$

where  $x = r \cos(\phi - \phi_0)$  and  $y = r \sin(\phi - \phi_0)$ . This procedure essentially rotates the steady-state flow structure to compute the absorbing column around the orbit, and by considering values of  $\phi_0$  spanning the range  $[-\pi, \pi]$  synthetic light-curves with 100% phase coverage can be generated.

As the density in the numerical simulations is scale-free, I define a normalised column density  $N$  relative to the column density  $180^\circ$  out of phase with the planet:

$$N(\phi) = \frac{\Sigma(\phi)}{\Sigma(\phi_p + \pi)}. \quad (4.10)$$

I calculate the line-of-sight optical depth by assuming that the UV opacity of the wind gas is constant; I discuss the validity of this approximation in section 4.7.1. The UV intensity  $I(\phi)$  is therefore given by

$$I(\phi) = I_p(\phi) e^{-\tau_0 N(\phi)}. \quad (4.11)$$

Here the exponent is the line-of-sight optical depth  $\tau$ . The optical depth parameter  $\tau_0$  represents the mean optical depth of the gas in the wind (i.e.,  $\tau(\phi) = \tau_0 N(\phi)$ ), and  $I_p(\phi)$  is the light-curve of the planetary transit (i.e., the optical transit light-curve). I neglect

---

<sup>1</sup>In the planet's frame, this corresponds to a dynamic pressure at the planet's orbital radius of  $P_w = \rho(u^2 + c_s^2) \simeq 3 \times 10^{-6} \text{ g cm}^{-1} \text{ s}^{-2}$ .

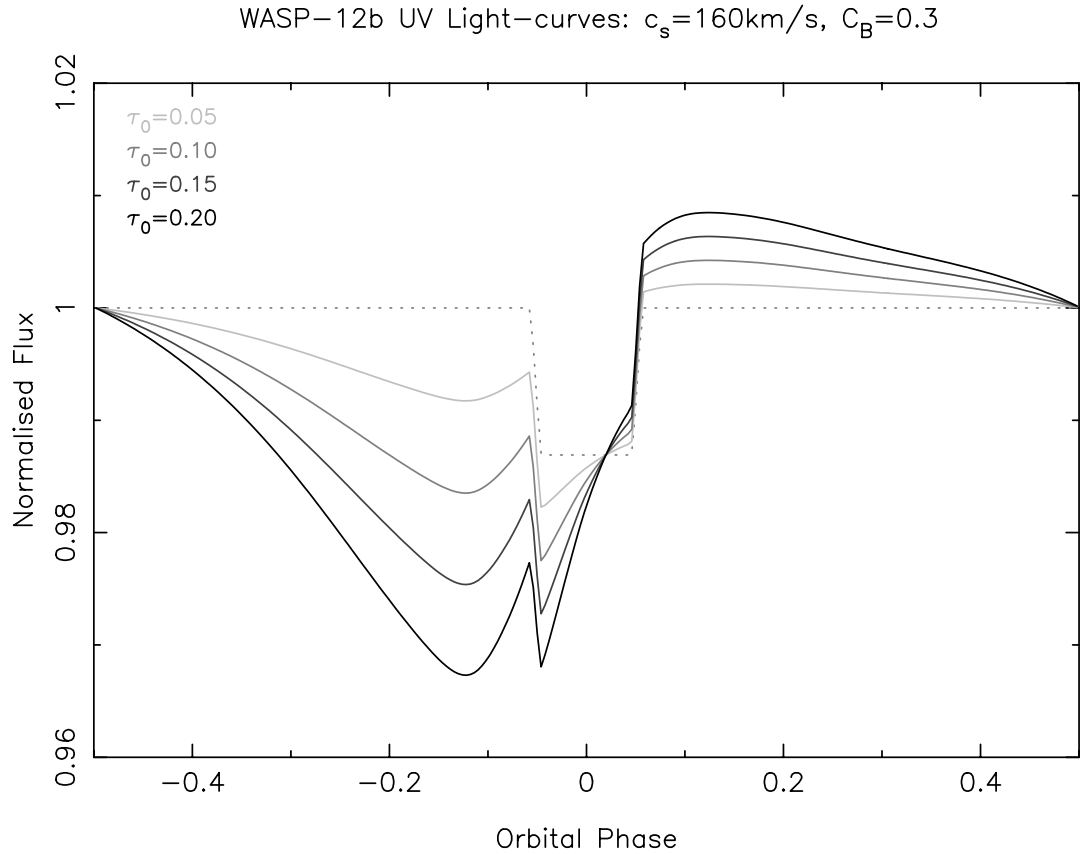


Figure 4.8: WASP-12b UV light curves for the fiducial model ( $c_s=160\text{km s}^{-1}$  and  $C_B=0.3$ ), calculated for various optical depth parameters  $\tau_0$ . For reference, the optical transit  $I_p(\phi)$  is shown as a dotted grey line. The minimum of the UV transit corresponds to the peak density in the magnetosheath, and leads the planet in orbital phase by  $\approx 12\text{--}13\%$ . Note also that lines-of-sight through the magnetospheric cavity have relatively low column densities, resulting in a normalised flux greater than unity in the post-transit region.

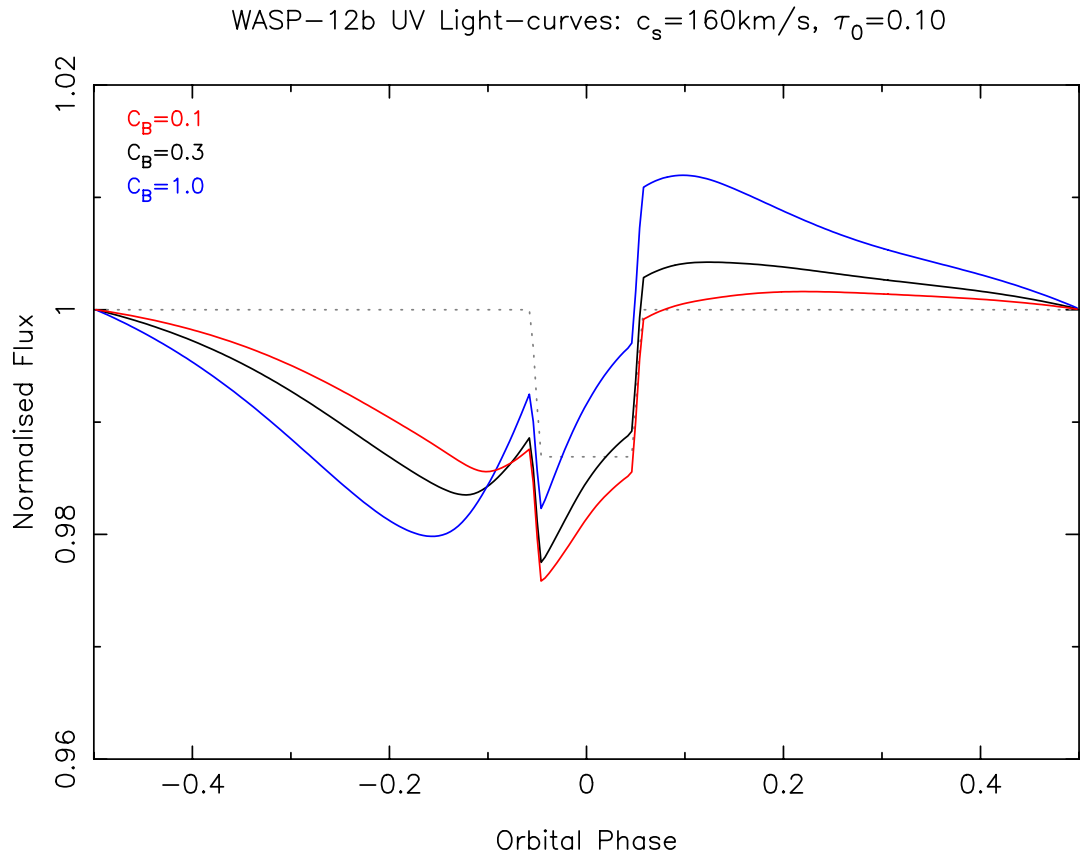


Figure 4.9: As figure 4.8, but for models with  $c_s=160\text{ km s}^{-1}$ ,  $\tau_0=0.1$ , and various values of  $C_B$ . Stronger magnetic fields (i.e., larger magnetospheric cavities) increase the amplitude of the UV transit, and also result in a larger phase offset between the minima of the UV and optical transits.

limb darkening and simply compute  $I_p(\phi)$  by assuming an opaque planet and a uniform brightness stellar disc, using equation 1 of Mandel and Agol (2002) and the parameters of WASP-12b from Hebb et al. (2009). I plot all transit light-curves in units of normalised flux, relative to the intensity  $180^\circ$  out of phase with the planet [i.e.,  $I(\phi)/I(\phi_p + \pi)$  is plotted].

Figure 4.8 shows how the UV transit varies as a function of the optical depth parameter  $\tau_0$  for a fiducial model (that with  $c_s=160 \text{ km s}^{-1}$  and  $C_B=0.3$ ). The magnetosheath provides significant absorption of the stellar UV flux ahead of the optical transit, resulting in a broad “dip” in the UV light-curve ahead of the optical transit. By contrast, the line-of-sight through the magnetospheric cavity has a lower optical depth than the out-of-transit mean, so the normalised UV flux exceeds unity (by as much as 1%) in the post-transit region. The minimum UV flux precedes the optical transit by 12–13% in phase, which corresponds to a phase offset of  $\approx 3.5$  hours for the 1.09d period of WASP-12b.

Figure 4.9 shows how the UV transit profiles change for different magnetospheric cavity sizes (i.e., different values of  $C_B$ ). Increasing the magnetic field strength results in more pronounced features in the UV transit: the higher density in the bow shock causes increased absorption ahead of the optical transit, while the larger cavity results in a larger UV flux enhancement in the post-transit region. Larger cavities also subtend larger azimuthal angles, resulting in increased phase offsets between the UV absorption in the bow shock and the optical transit. For  $C_B=0.1$  (i.e., a cavity size of  $\approx 6.0R_p$ ) the UV minimum is relatively weak, and occurs only marginally ahead of the optical transit ingress. By contrast, for  $C_B=1.0$  (i.e., a cavity size of  $\approx 9.0R_p$ ) the minimum UV flux occurs at phase  $-0.16$ , which for WASP-12b corresponds to a phase offset of more than four hours.

Similarly, figure 4.10 shows how the UV transit profiles depend on the sound speed in the wind,  $c_s$ . Here two different physical effects come into play. First, as seen in figure 4.5, the geometry of the bow shock changes significantly with sound speed, with the magnetosphere oriented progressively more towards the star as the wind speed increases. The azimuthal offset between the planet and peak density in the bow shock therefore decreases with increasing  $c_s$ , and this is reflected in the transit profiles: the phase offset of the maximum absorption in the wind decreases from  $\approx 22\%$  ( $\approx 5.8$  hours) for  $c_s=140 \text{ km s}^{-1}$  to  $\approx 8\%$  ( $\approx 2.1$  hours) for  $c_s=200 \text{ km s}^{-1}$ . Second, and more significantly, the radial density profile of the wind changes dramatically with  $c_s$  (see figure 4.4), and is much steeper for low sound speeds. For  $c_s \lesssim 140 \text{ km s}^{-1}$  most of the absorbing column is therefore interior to the planet’s orbit, and the bow shock provides relatively little absorption. In fact, for  $c_s \lesssim 120 \text{ km s}^{-1}$  the decrease in optical depth along lines-of-sight through the cavity is

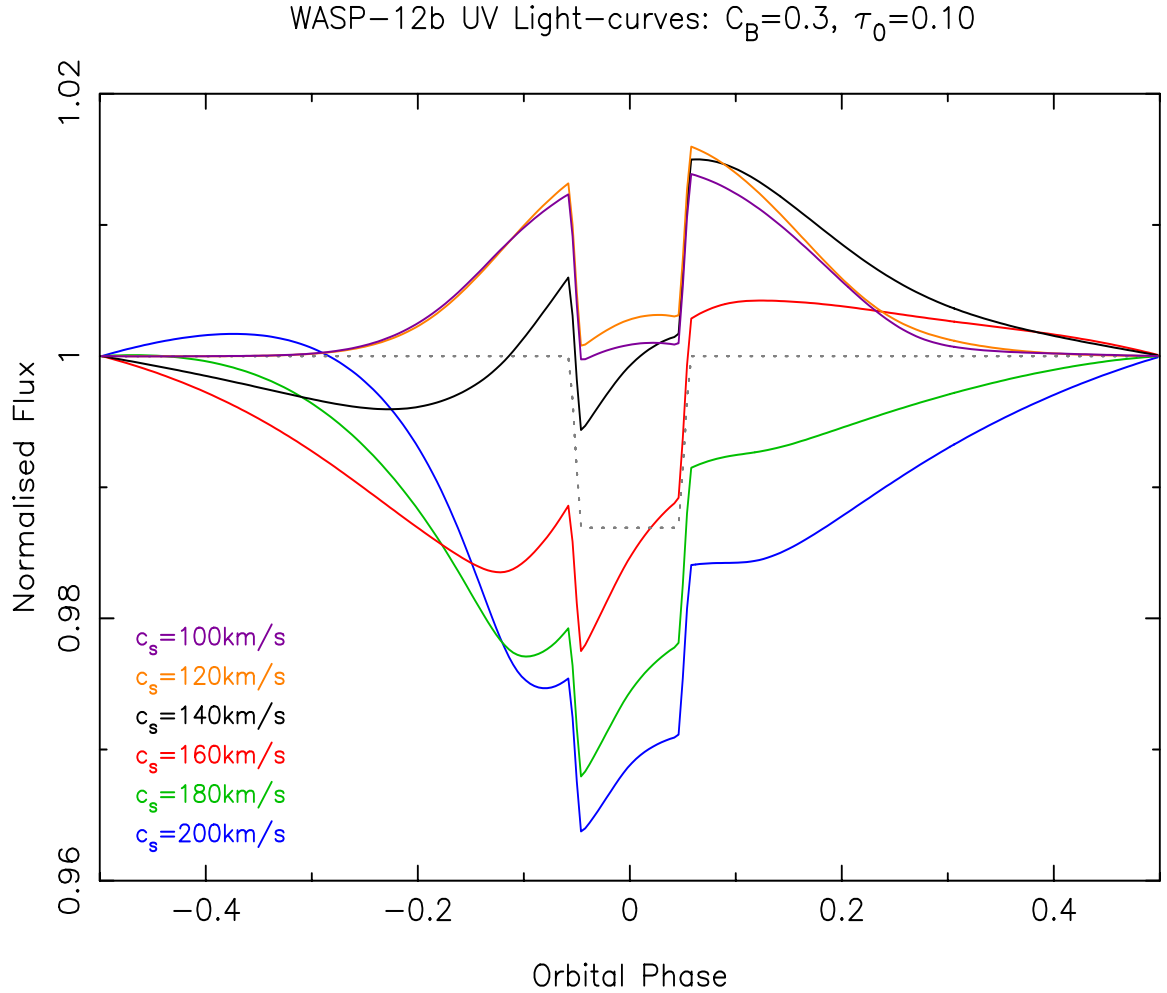


Figure 4.10: As figures 4.8 & 4.9, but for models with  $C_s=0.3$ ,  $\tau_0=0.1$ , and various sound speeds. For higher sound speeds the phase offset between UV and optical transits is smaller, as the bow shock is oriented towards the star (see figure 4.5). In addition, the steep density profiles in the low- $c_s$  wind solutions (see figure 4.4) mean that for low sound speeds ( $c_s \lesssim 140 \text{ km s}^{-1}$ ) most of the absorbing column lies interior to the planet’s orbit. In this case there is little (relative) absorption in the magnetosheath, and the radially-extended magnetospheric cavity instead results in a “negative-depth” transit.

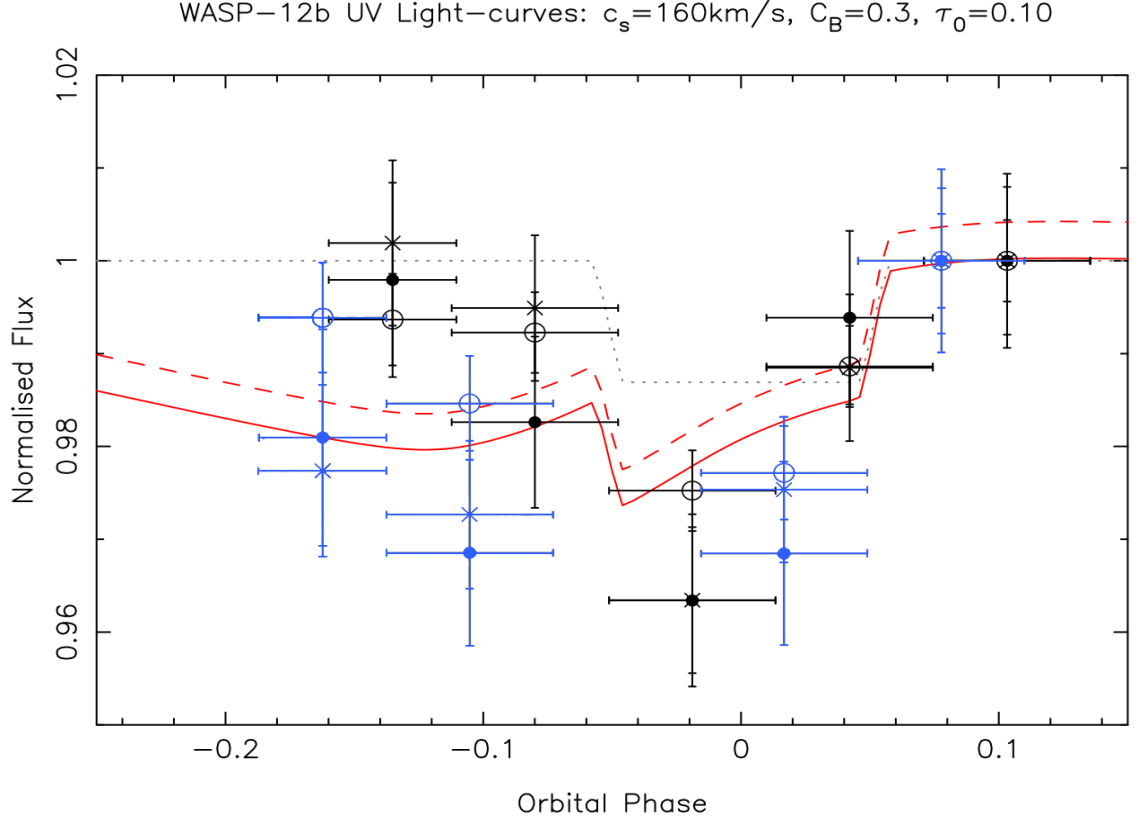


Figure 4.11: Comparison between the model ( $c_s=160\text{ km s}^{-1}$ ,  $C_s=0.3$ ,  $\tau_0=0.1$ ) and existing near-UV observations of WASP-12b. The black and blue points respectively denote the “Visit 1” and “Visit 2” fluxes reported in Fossati et al. (2010) and Haswell et al. (2012), normalised to the final “out-of-transit” data point: vertical error bars represent the Poisson errors on the observed fluxes, while the horizontal error bars denote the duration of the individual exposures. Solid circles, open circles and crosses respectively denote the three different wavelength regions labelled NUVA ( $\approx 2540\text{--}2580\text{\AA}$ ), NUVB ( $\approx 2660\text{--}2700\text{\AA}$ ) and NUVC ( $\approx 2780\text{--}2820\text{\AA}$ ) in Haswell et al. (2012). (Note that these are approximate, as the exact wavelength ranges differed slightly between the two visits.) NUVA contains many overlapping absorption lines (primarily Mg I and Fe I), while NUVC is centred on the strong Mg II doublet at  $2798/2803\text{\AA}$ ; NUVB is a “continuum” region with no strong absorption lines. It is striking that the transit depths are similar across all three wavelength regions. The dashed red line is the same as the corresponding model in figures 4.8–4.10, while the solid curve is normalised to the post-transit region ( $\phi=[0.08, 0.10]$ ) in the same manner as the data. The predicted light-curves are consistent with the observations, but the large error bars and limited phase coverage means that we cannot place useful constraints on the model parameters.

larger than the increase through the magnetosheath, resulting in an increase in the relative UV flux and a “negative-depth” transit.

The results show that the magnetosheath has a clearly identifiable signature: a broad minimum in the UV light-curve which leads the optical transit by  $\approx 10\text{--}20\%$  in orbital phase. Figure 4.11 shows a comparison between the model ( $c_s=160\text{ km s}^{-1}$ ,  $C_B=0.3$ ,  $\tau_0=0.1$ ) and the HST near-UV observations of WASP-12b by Fossati et al. (2010) and Haswell et al. (2012). Here I normalise both the data and model to the post-transit flux (at orbital phase 0.08–0.10), as the limited phase coverage of the observations prevents us from normalising the observed light-curve in the same way as in figures 4.8–4.10. The model light-curves are consistent with the observed near-UV transits of WASP-12b, but the large error bars and partial phase coverage of the observations limit our ability to discriminate between different models. Moreover, it is clear from figures 4.8–4.10 that the predicted light-curves have significant degeneracies between the model parameters (wind sound speed/temperature, magnetic field strength and optical depth). This suggests that transit light-curves obtained from broad-band UV observations alone cannot provide strong limits on these parameters. With additional constraints (such as measurements of the stellar wind temperature), wider phase coverage, and sufficiently high signal-to-noise data it may be possible to break the degeneracies and establish a unique solution, but current UV data (e.g., Fossati et al., 2010; Haswell et al., 2012; Nichols et al., 2015) do not set meaningful constraints on the model parameters.

## 4.6 Models of other systems

To investigate how the observable signatures of the planet-wind interaction vary with model parameters, I also apply the model to two other transiting hot Jupiters: WASP-18b and HD209458b. WASP-18b, a  $\approx 10 M_{\text{Jup}}$  planet with a 0.94d period (Hellier et al., 2009), is one of the most massive known close-in exoplanets, and also represents a more massive analogue of WASP-12b. By contrast HD209458b, the first transiting exoplanet to be discovered (Charbonneau et al., 2000), is a  $\approx 0.5 M_{\text{Jup}}$  planet in a 3.5d period. The adopted parameters for these systems are as follows:

### WASP-18b

I adopt the parameters from Triaud et al. (2010):  $M_*=1.24M_\odot$ ,  $a_p=0.0202\text{AU}$ ,  $P=0.94\text{d}$  (and therefore  $u_p=233.4\text{km s}^{-1}$ ),  $R_*=1.36R_\odot$  ( $=0.313a_p$ ),  $M_p=10.1M_{\text{Jup}}=7.78 \times 10^{-3}M_*$

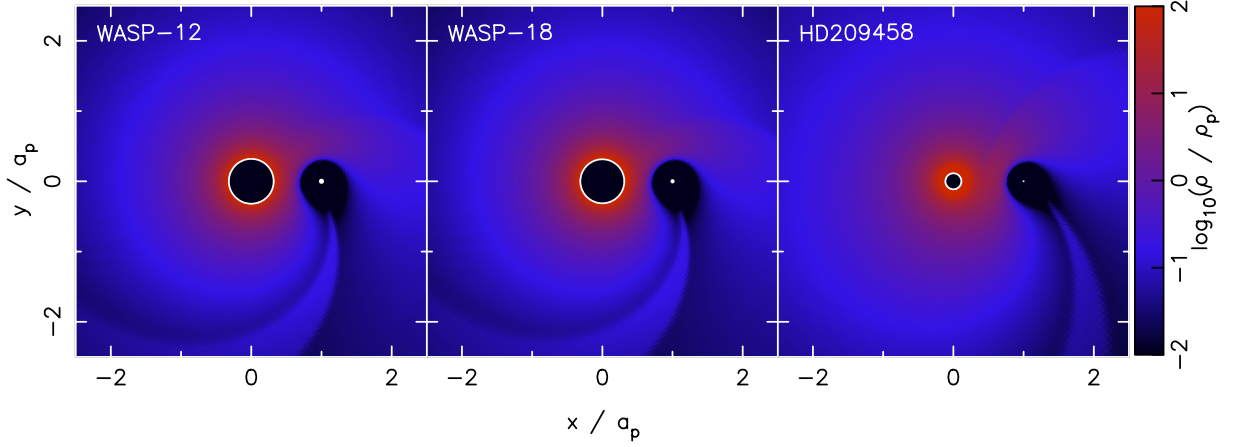


Figure 4.12: Steady-state density structure in my simulations with  $C_B = 0.3$  and  $c_s = 160 \text{ km s}^{-1}$ , for WASP-12, WASP-18 & HD209458. As in figure 4.5, the density colour-scale is normalised to the value at  $a_p$ , and the planets and stars are plotted to scale. The flow solution around the more massive WASP-18b has no significant differences from that around WASP-12b, showing that the planet’s gravity is essentially negligible. By contrast, the larger semi-major axis of HD209458b results in a different wind speed and ram pressure at the planet’s orbit, leading to a larger magnetospheric cavity (relative to the star and planet) and the bow shock being angled significantly more towards the star.

and  $R_p = 1.27 R_{\text{Jup}} = 0.0299 a_p$ . I maintain the same numerical resolution as before, with  $N_\phi = 250$ ; the computational grid spans  $[r_{\text{in}}, r_{\text{out}}] = [0.313 a_p, 10.0 a_p]$ , so  $N_r = 140$ .

### HD209458b

I adopt the parameters from Torres et al. (2008):  $M_* = 1.12 M_\odot$ ,  $a_p = 0.0471 \text{ AU}$ ,  $P = 3.52 \text{ d}$  (and therefore  $u_p = 145.2 \text{ km s}^{-1}$ ),  $R_* = 1.16 R_\odot (= 0.114 a_p)$ ,  $M_p = 0.685 M_{\text{Jup}} = 5.85 \times 10^{-4} M_*$  and  $R_p = 1.36 R_{\text{Jup}} = 0.0135 a_p$ . Here the grid spans  $[r_{\text{in}}, r_{\text{out}}] = [0.114 a_p, 10.0 a_p]$ , so with  $N_\phi = 250$ ,  $N_r = 180$  is required.

#### 4.6.1 Results

In each case I run a new version of a fiducial model, with  $c_s = 160 \text{ km s}^{-1}$  and  $C_B = 0.3$ . Figure 4.12 shows the steady-state flow solutions for both systems, as well as WASP-12b, while figure 4.13 shows the corresponding UV light-curves. In the case of WASP-18, the density structure and light-curve are both essentially indistinguishable from WASP-12. This is despite the planet:star mass ratio being a factor of 7.8 larger for WASP-18, and demonstrates that the planet’s gravity is negligible in determining the structure of the mag-

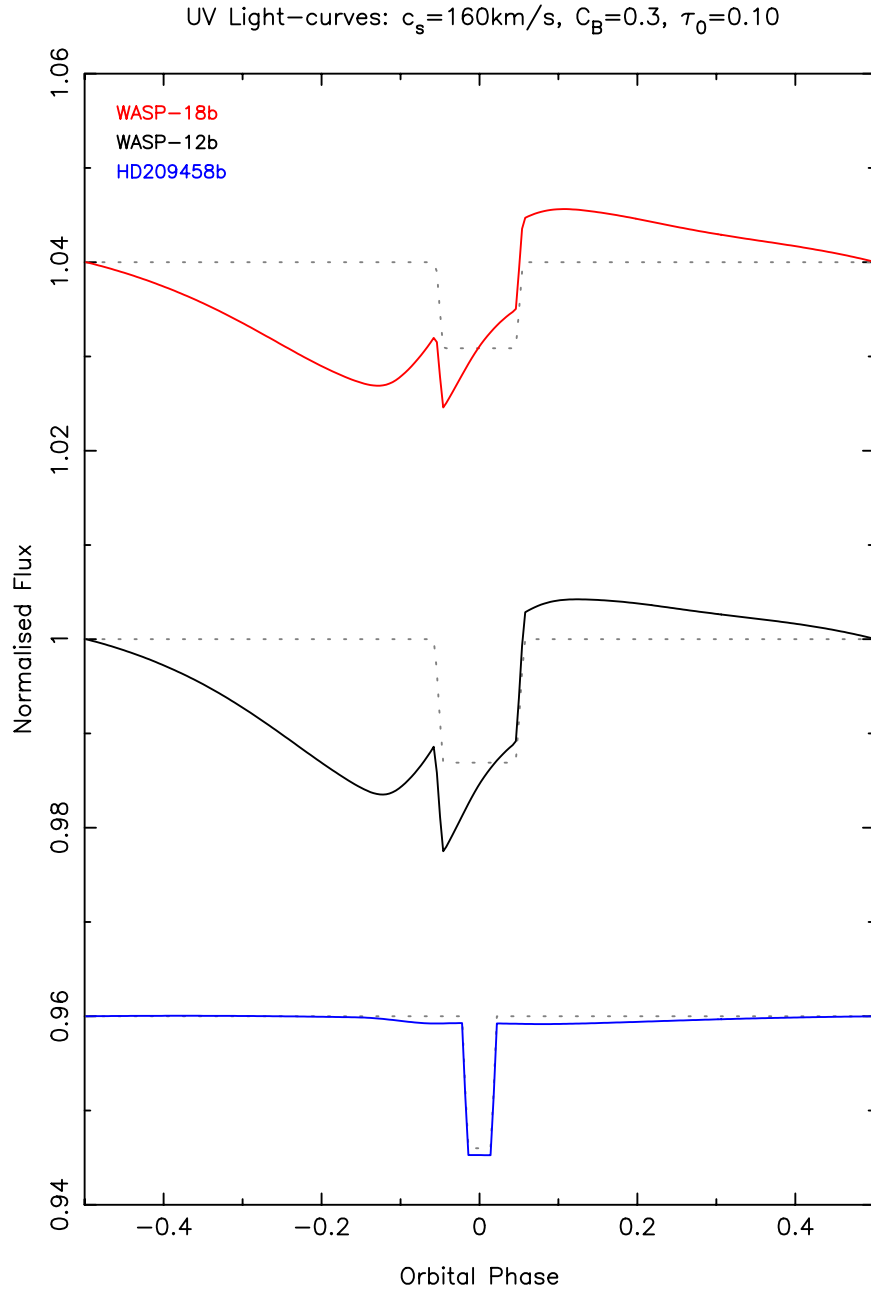


Figure 4.13: Light curves for the three models shown in figure 4.12 ( $C_B = 0.3$  and  $c_s=160 \text{ km s}^{-1}$ ); as before, the dotted lines denote the optical transits. For clarity, the curves for WASP-18b (red) and HD209458b (blue) have been offset by  $\pm 4\%$  in flux, respectively. The light-curve for the more massive WASP-18b is essentially identical to that of WASP-12b. However, the larger orbital separation of HD209458b dramatically reduces the (relative) absorption in the bow shock, and the resulting UV light-curve is almost indistinguishable from the optical transit.

netosphere in hot Jupiters. This is in stark contrast with the Roche lobe overflow model, which depends strongly on the planet’s mass; I discuss the observational consequences of this result in section 4.7.2.

HD209458, by contrast, shows pronounced differences from WASP-12 in both its flow solution and its transit light-curve. The larger semi-major axis results in a higher wind speed at the planet’s position, but also a much lower gas density. Consequently the magnetosphere is much more extended (relative to both the star and planet) than for WASP-12b or WASP-18b. The bow shock is also angled much more strongly towards the star (see figure 4.12), primarily as a result of the planet’s lower orbital velocity. However, the most significant effect of the larger orbital separation in HD209458 is that line-of-sight column density through the wind is dominated by gas well inside the planet’s orbit. The magnetosheath and cavity contribute only a very small fraction of the total column, and consequently are almost undetectable: the UV light-curve is essentially indistinguishable from the optical transit (see figure 4.13). This is consistent with existing UV observations of HD209458 (Vidal-Madjar et al., 2013), and suggests that absorption of stellar UV in the magnetosheaths of hot Jupiters will be undetectable for planets with periods  $\gtrsim 1.5$ d.

## 4.7 Discussion

### 4.7.1 Limitations

The models presented here are somewhat simplified, and neglect several potentially important issues. First, I neglect both motion and rotation of the central star. Neglecting the motion of the star by fixing the planetary orbit is a good approximation, and I expect significant “reflex” stellar motion only in the case of WASP-18 (where  $M_p/M_*=7.78\times 10^{-3}$ ). However, even in this case the star’s orbital radius (around the system barycentre) is at least an order of magnitude smaller than the magnetospheric radius, so this is unlikely to alter my results significantly. Similarly, although Vidotto et al. (2010) argue that rotation of the stellar wind may be important, I find that even very fast stellar rotation has a negligible effect on my results. For WASP-12 a stellar rotation period of 3d corresponds to a rotational velocity of  $26 \text{ km s}^{-1}$ , and if angular momentum is conserved in the wind I expect the azimuthal component of the wind velocity at  $a_p$  to be  $< 10 \text{ km s}^{-1}$ , much smaller than the planet’s orbital velocity ( $u_p=228 \text{ km s}^{-1}$ ). Even in the extreme limit of a magnetised wind which co-rotates with the star, the azimuthal component of the wind velocity at  $a_p$  is  $\simeq 80 \text{ km s}^{-1}$ , and the effective Mach number in the planet’s frame is reduced by only

10–20%. Rapid stellar rotation therefore represents only a small perturbation to the wind structure, and test calculations indicate that it has no measurable effect on my predicted light-curves.

The major simplification in my hydrodynamic calculations is the treatment of magnetic fields. As discussed in section 4.4.3, I do not run full MHD simulations, and instead model the planetary magnetic field as a spherically-symmetric acceleration that scales  $\propto |\mathbf{r} - \mathbf{r}_p|^{-7}$ . This approximation is strictly valid only for a dipole magnetic field which is aligned with the planet’s orbit. However, at the radii of interest ( $|\mathbf{r} - \mathbf{r}_p| \simeq 5\text{--}25R_p$ ) the dipole component of the field is dominant, and in this regime even a strongly inclined field will result in only a moderately elliptical magnetosphere. Moreover, the integrated column density (and therefore the predicted light-curve) is not particularly sensitive to the precise shape of the magnetospheric shock and cavity, so a more realistic treatment of the planet’s magnetic field is unlikely to change the results significantly.

A potentially more important simplification is neglecting the stellar magnetic field. In the general case, the magnetospheric radius  $r_m$  (strictly, the radius of the head of the magnetopause) is given by (e.g., Lai et al., 2010)

$$\rho u^2 + \rho c_s^2 + \frac{B_w^2}{8\pi} = P_p + \frac{B_0^2}{8\pi} \left( \frac{R_p}{r_m} \right)^6. \quad (4.12)$$

The first and second terms on the left-hand-side, respectively, are the ram and thermal pressures of the wind gas, while the final term on the right-hand-side is the pressure from the planetary magnetic field; all of these are included explicitly in my simulations.  $P_p$  is the gas pressure inside the magnetosphere; this is neglected here, but at the radii of interest ( $>5\text{--}10R_p$ ) it is unlikely to be significant unless the planet is losing significant mass via Roche lobe overflow (see section 4.7.2 below), or the magnetosphere is filled with plasma from another source (such as moons or rings). The third term on the left-hand side is the magnetic pressure in the wind, due to stellar magnetic field lines threading the wind ( $B_w$  is the magnetic field strength in the wind). The magnitude of the stellar magnetic field in hot Jupiter host stars is not well constrained, but comparison to the Solar wind suggests that the magnetic pressure in the wind could be comparable to the gas pressure (see discussion in Lai et al., 2010). I do not explicitly include this term in the models, but my adopted parameterisation (using accelerations rather than pressures) means that only the gradient of the pressure  $\partial/\partial r(B_w^2)$  is important; a constant magnetic pressure  $B_w^2/8\pi$  simply represents an offset in the scaling relation between my input parameter  $C_B$  and the planetary magnetic field  $B_0$  (equation 4.8). The magnetic pressure in the wind is unlikely

to vary dramatically across the magnetosphere, so neglecting this term has only a small effect on the magnetospheric geometries in the simulations.

My estimates of the planetary field strength  $B_0$ , however, do depend on the absolute value of  $B_w$ . If magnetic pressure in the wind is significant then equation (4.8) represents a lower limit on  $B_0$  for a given input parameter  $C_B$  (or, alternatively, a given magnetospheric radius). With a fiducial stellar wind rate of  $\dot{M}_w = 10^{-15} M_\odot \text{ yr}^{-1}$  and my adopted values of  $C_B$ , equation (4.8) gives surface planetary field strengths spanning the range  $B_0 \approx 2\text{--}25\text{G}$  (corresponding to pressures of  $0.2\text{--}2 \times 10^{-5} \text{ g cm}^{-1} \text{ s}^{-2}$ ); the true values could be higher or lower, depending on the contributions from magnetic pressure in the wind and gas pressure in the cavity. However, unless one of these terms (which I have neglected) dominates the dynamics, which seems unlikely, my simulations should give qualitatively correct structures. Moreover, my predicted light-curves depend primarily on the size of the magnetospheric cavity and the density in the bow-shock, both of which are largely insensitive to the absolute value of  $B_w$  (unless the field has a very unusual geometry, such as a steep magnetic pressure gradient across the magnetosphere).

The other obvious simplification in my approach is that the simulations are 2-D, while the transit of a magnetospheric bow shock is intrinsically a 3-D phenomenon. 3-D hydrodynamic simulations remain computationally expensive. The accuracy of the 2-D approximation is greatly increased in this case, however, because the observational constraints on the WASP-12b system require the magnetospheric radius to be comparable to the stellar radius (e.g., Lai et al., 2010; Vidotto et al., 2010). The UV transit therefore involves two objects with comparable angular sizes, and this is much less sensitive to, for example, inclination, than a transit of objects of very different sizes. Results of 3-D SPH simulations for WASP-12b will be discussed in section 4.8

Finally, the treatment of the gas opacity in my model light-curves is highly idealised. I assume a constant UV opacity  $\kappa$ , and parametrize the light-curves in terms of the mean (azimuthally-averaged) optical depth  $\tau_0$ . In reality, the continuum near-UV (2000–3000Å) opacity of  $\sim 10^6\text{K}$  gas is due to a complex blend of thousands of lines of metal ions (e.g., Iglesias and Rogers, 1996), and each individual line can be very sensitive to both density and temperature. It is relatively straightforward to compute the opacity for individual spectral lines (e.g., Lai et al., 2010), but calculating the “continuum” opacity from the blanketed, blended line forest is extremely challenging. Lai et al. (2010) and Vidotto et al. (2010) focussed on absorption by the Mg II resonance lines at 2800Å. However, it is striking that existing UV observations of WASP-12b show significant absorption of the stellar continuum across the entire near-UV waveband, with roughly constant levels

of (relative) absorption in a number of different UV spectral regions (Fossati et al. 2010; Haswell et al. 2012; Nichols et al. 2015). This suggests that (away from strong resonance lines) the integrated “continuum” opacity, which is due to many thousands of individual spectral lines, is not a strong function of wavelength or temperature. My assumption of constant opacity is therefore a reasonable first-order approximation.

I can extend this line of argument by computing the absorbing column in my models in physical units. If I adopt a wind rate of  $\dot{M}_w = 10^{-15} M_\odot \text{ yr}^{-1}$ , then for my model (with  $c_s = 160 \text{ km s}^{-1}$  and  $C_B = 0.3$ ) the number density in the (unperturbed) wind at the planet’s orbital radius is  $n(a_p) = 1.4 \times 10^7 \text{ cm}^{-3}$ . The “excess” column density along the line-of-sight through the magnetosheath (i.e., the column corresponding to the peak absorption in the light-curves) is  $6.0 \times 10^{16} \text{ cm}^{-2}$ . I require an optical depth  $\tau \sim 0.1$  in order to reproduce the observed UV transit depths, which implies a gas opacity  $\kappa \sim 10^6 \text{ cm}^2 \text{ g}^{-1}$ . Detailed computation of the near-UV opacity is subject to uncertainties, and is beyond the scope of this thesis work. Nevertheless, it is encouraging that my models give absorbing column densities that are comparable to observed estimates: this suggests that a weak magnetospheric shock in a typical stellar wind can plausibly account for the excess UV absorption seen in WASP-12b.

#### 4.7.2 WASP-12 vs. WASP-18: a critical test of UV absorption theories

At present there are two competing explanations in the literature for the excess UV absorption seen in WASP-12: Roche lobe overflow (Lai et al., 2010; Bisikalo et al., 2013) and a magnetospheric bow shock (Lai et al., 2010; Vidotto et al., 2010). As discussed above, current data do not allow us to distinguish between these models, but the simulations point to a critical test that will provide a clear answer to this question. WASP-18 has a comparable stellar mass to WASP-12, and the planets have similar radii and orbital separations. In my magnetospheric shock models the planet’s gravity is negligible, and the shape of the UV light-curve depends primarily on the sound speed in the wind and the size of the magnetosphere. My predicted light-curves for WASP-18 are therefore indistinguishable from those for WASP-12 (see figure 4.13). However, as WASP-18b is 7.2 times more massive than WASP-12b, it has a much larger Roche lobe ( $RL_1 = 3.2 R_p$ , compared to  $1.3 R_p$  for WASP-12b). In the Roche overflow model the mass-flux through the L1 point decreases exponentially with increasing Roche lobe radius, and significant mass-loss is essentially impossible for a planet as massive as WASP-18b.

Taken together, these results imply that UV observations of WASP-18 represent a straightforward test for these models. A UV transit of WASP-18b which looks similar to that of WASP-12b (i.e., showing excess UV absorption) would argue strongly against the Roche lobe overflow model, as Roche lobe overflow cannot provide significant mass-loss from WASP-18b. By contrast, the absence of excess UV absorption in WASP-18b would suggest that magnetospheric absorption is not significant. Both processes may occur simultaneously in the WASP-12 system, and independent measurements of the planetary magnetic fields are required for a definitive test of the magnetospheric bow shock hypothesis. However, a clear detection of excess UV absorption in WASP-18 would essentially rule out the Roche lobe overflow hypothesis, and point strongly towards magnetospheric absorption as the most likely explanation.

## 4.8 WASP-12b transit light-curves in 3D

One of the limitations of the 2D work was that the transit takes place in a 3D space while the light-curves obtained so far have been tested in a 2-dimensional regime. In order to check the validity of the 2D-model discussed before, I present the hydrodynamic models of WASP-12b transit light-curves in three dimensions using the Smoothed Particle Hydrodynamic code described in Chapter three. SPH is useful for 3D as the Lagrangian mesh-free nature of the code means that no symmetries are imposed on the results. This provides a useful contrast and control on the 2D results.

### 4.8.1 Results of the 3D model of WASP-12b transits

Setup and initial and boundary conditions: despite a different approach to model the transit light-curves of WASP-12b through the interaction between the stellar wind and the planetary magnetic field in three dimensions, the physical parameters and the setup of the planetary magnetic field behaviour is the same as the 2D work. The system is considered as two point masses in a circular orbit. The system parameters adopted are as mentioned before (Hebb et al., 2009). The system of units adopted is such that the unit of length is the planetary orbital radius  $a_p$  and the unit of time is chosen so that one orbital period is equal to  $2\pi$  units and the unit of mass is the total mass of the star and planet.

I again assume an isothermal Parker wind where SPH particles are injected at the surface at distance  $r_{in} = R_* = 0.319a_p$  from the point mass representing the star with an injection velocity given by the Parker analytic solution at that radial distance. Particles

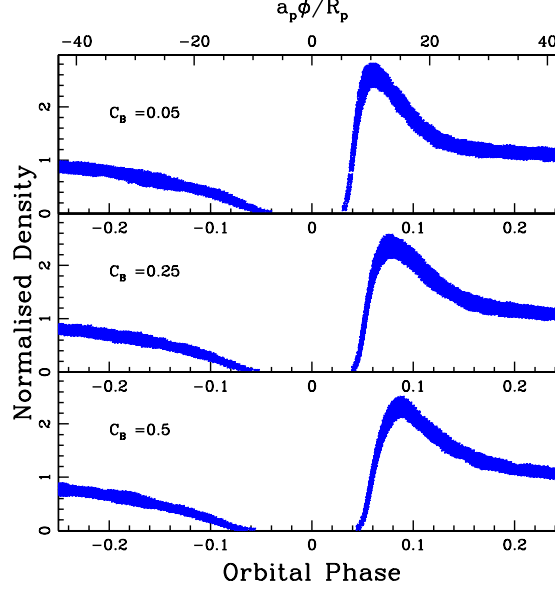


Figure 4.14: Azimuthal density profiles at the planet’s orbital radius for the SPH models with wind sound speed  $c_s = 160 \text{ km s}^{-1}$  and constants  $C_B=0.05, C_B=0.25$  and  $C_B=0.5$ . The lower axis shows the azimuthal coordinate in units of orbital phase (which ranges from  $[-0.5, 0.5]$ ), while the upper axis shows the distance around the planet’s orbit in units of the planet radius  $R_p$ .

are then subject to the resultant forces from the gravity of the star and the gradient of the thermal pressure within the wind. The outer boundary of the computational zone is set as a sphere of radius  $r_{out} = 2.5a_p$  to reduce the computational cost. Tests with larger outer boundary radii show convergent results as the gas density drops 3 orders of magnitude (see figure 4.4).

To model the planetary interaction with the stellar wind, the same assumption has been made as mentioned in section 4.4.3 where the resultant acceleration due to the planet is caused by the magnetic pressure and the planetary gravitation as seen in equation 4.4. In Cartesian coordinates the orbital plane lies on the  $xy$  plane and  $z$  – *axis* points to the direction of the orbital angular momentum of the system. This way,  $|\mathbf{r} - \mathbf{r}_p| = \sqrt{(x - x_p)^2 + (y - y_p)^2 + (z - z_p)^2}$  where  $x, y$  and  $z$  are the particles coordinates and  $x_p, y_p$  and  $z_p = 0$  are the planets coordinates. The magnetic constant  $C_B$  is again kept as an input parameter to represent the magnetic force strength.

With the setup mentioned above the hydrodynamic simulations start to evolve rapidly to steady state after 2 planetary orbits when the number of active particles reaches  $\sim 2$  million. Simulations with wind sound speeds 100, 140, 160 & 200 km/sec have been

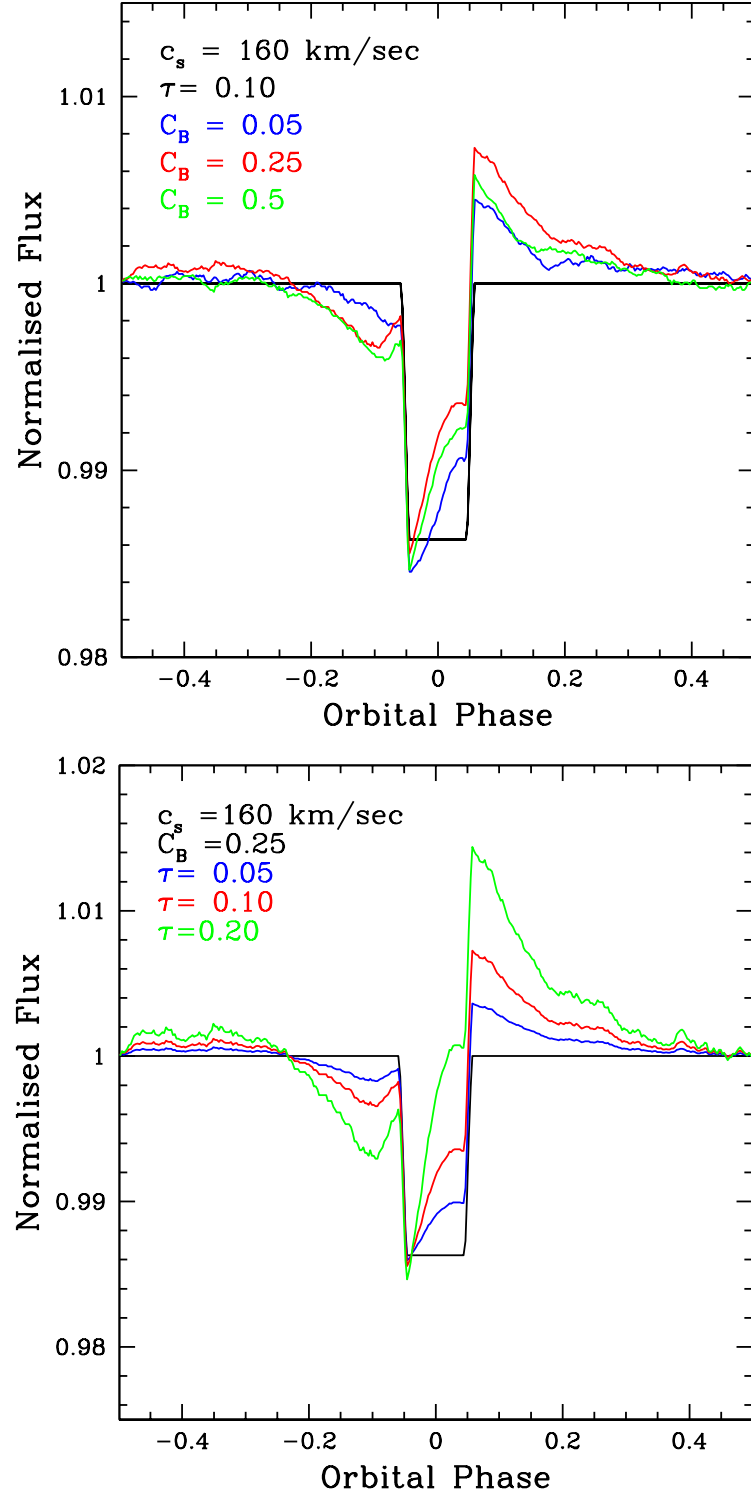


Figure 4.15: Normalised flux for the models with wind sound speed  $c_s = 160 \text{ km s}^{-1}$ ; in black is the optical light-curve. Top panel: average opacity  $\tau = 0.1$  and various constant  $C_B$  values. Bottom panel:  $C_B = 0.25$  and various values of  $\tau$ .

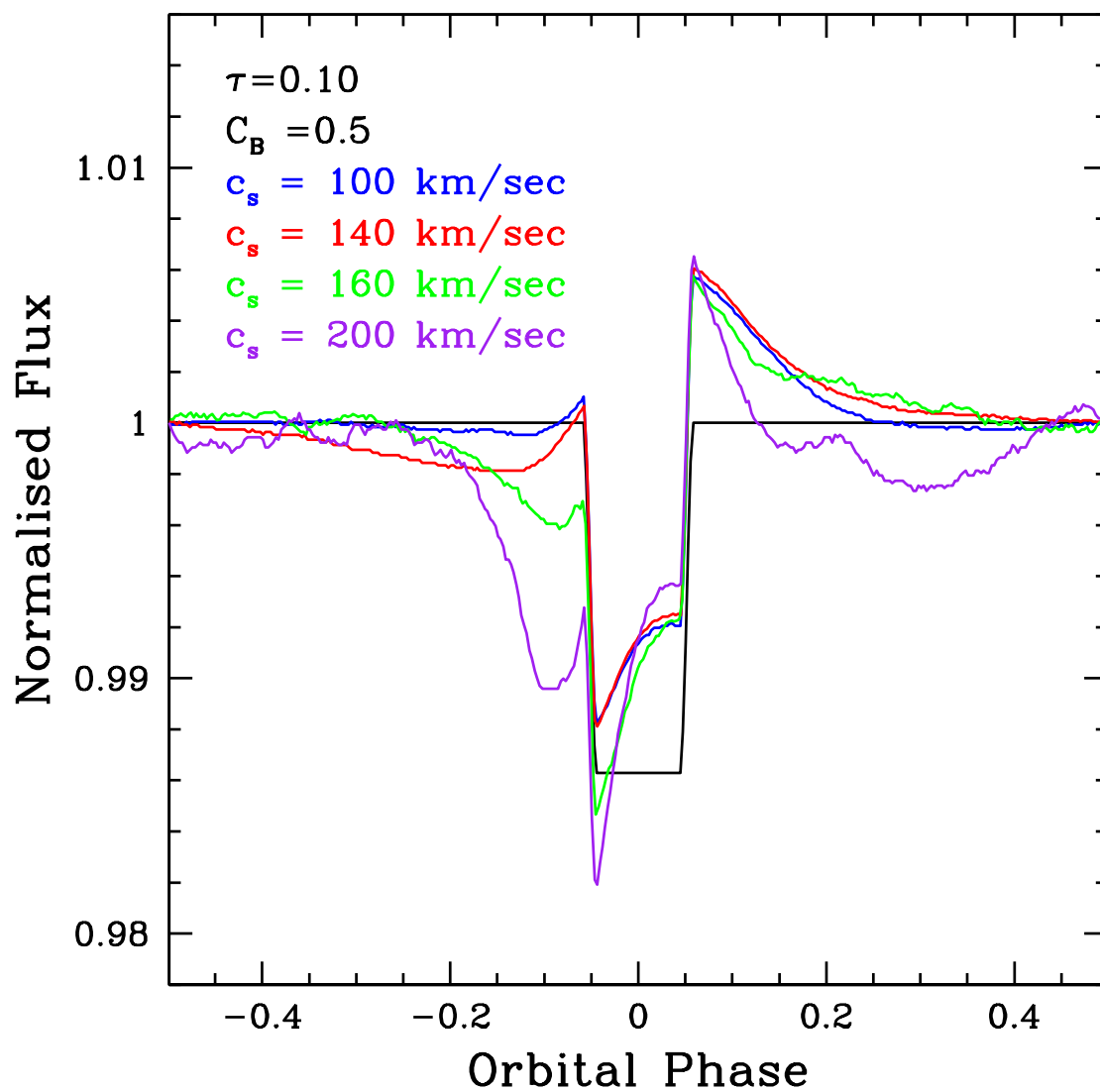


Figure 4.16: Normalised flux for the models with wind sound speeds  $c_s = 100, 140, 160$  &  $200 \text{ km s}^{-1}$ ; the constant  $C_B = 0.5$  and opacity  $\tau = 0.1$ . The optical light-curve of the planetary transit is in black.

examined with the planetary magnetic force constant  $C_B = 0.05, 0.25 \text{ \& } 0.5$  resulting in a magnetospheric cavity with radii  $5.16R_p, 6.88R_p \text{ \& } 5.16R_p$  respectively. In agreement with the 2D results, the bow shock is weak and broad and the width of the magnetosheath is comparable to the size of the magnetospheric cavity; this is due to the small local wind Mach number. Figure 4.14 shows that the density contrast is less than 3 between the shocked region and  $180^\circ$  out of phase with the planet. The constant  $C_B$  acts to determine the size of the magnetosphere and the density contrast.

The method for estimating the UV light curve of the planetary transit is in principle the same as in the 2D case, however, because of the 3D particle distribution. Calculation of the absorbing column of material is slightly different to that in equation 4.9. I instead count the number of particles along the line of sight contained in a cylinder having a circular cross-section radius  $R = R_*$ . Similar to equation 4.10, the normalised column density is calculated as the total number of particles in the cylinder in each orbital phase normalised to the unperturbed cylinder of particles  $180^\circ$  out of phase with the planet. Assuming constant UV opacity of the absorbing column of wind material I then use equation 4.11 to obtain the normalised UV intensity.

Figure 4.15 shows how the UV transit profiles change by changing the value of  $C_B$  which means the change in magnetic field strength. Increasing  $C_B$  results in enhancement in the column density of the shocked region, the size of the magnetic cavity increases too. So for high magnetic field strength, there are more absorption features in the UV when the bow shock passes across the line of sight and more UV flux when the post shock (magnetic cavity) passes and vice-versa. It also shows the effect of optical depth of the absorbing column through changing the value of the gas opacity. The higher the opacity the more absorption and vice-versa. Figure 4.16 shows that as the sound speed increases the width of the shocked region becomes narrower and the density contrast becomes higher but the magnetic cavity remains somewhat unchanged.

### 4.8.2 Comparison between 2D and 3D results

Figure 4.17 depicts the results from the 2D and 3D simulations with similar physical conditions except a slight difference in the values of the magnetic force constant. The results in general agree as there is an absorption feature prior to the planetary optical transit because of an enhancement in the column density ahead of the planet. There is also a magnetic cavity whose size depends on the planetary magnetic field strength. There is however, some difference in the details of the results; in the 2D results the column density in the shocked region is clearly stronger and wider while the shock is narrower

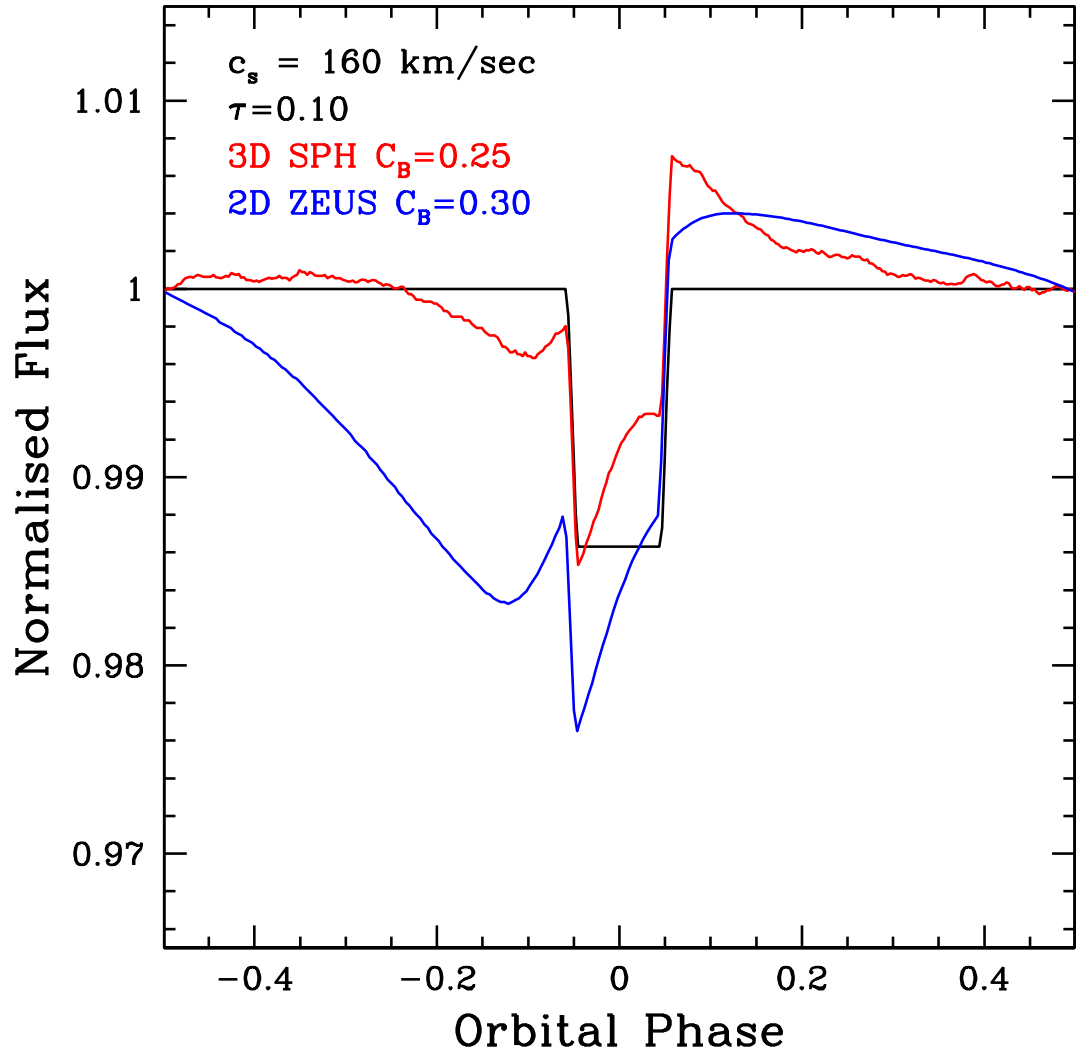


Figure 4.17: Comparison between the two dimensional and three dimensional results. Stronger absorption feature is seen for the 2D than the 3D results prior to the optical transit. The two results converge at post transit.

and weaker in the case of 3D simulations. Moreover, the effect of the magnetic cavity on decreasing the column density is bigger in the 3D results than in 2D: the column density is significantly less than that  $180^\circ$  out of phase with the planet when the magnetic cavity passes across the line of sight. It is clear that the geometry of the column density is different for the two regimes. In the simulations with ZEUS-2D the column density is calculated from the density contribution of a nearly rectangular segment (surface area of the star reduces to a curved line) while the column density calculations in the 3D-SPH simulations take into account the particles contained within the cylinder projected along the line of sight to the star. The 3D simulations smooth out the extremes of the 2D column density variations. The resulting light curves, while retaining the broad features of the 2D case, show less pre-transit absorption. In particular, the broad pre-transit absorption is narrower and less deep. Post transit agreement is much better.

## 4.9 Summary

I have presented hydrodynamic simulations of stellar wind–magnetosphere interactions in hot Jupiters such as WASP-12b. I work within an existing theoretical picture (e.g., Lai et al., 2010; Vidotto et al., 2010; Llama et al., 2011), but in this context my models are the first to use numerical hydrodynamics to compute the wind and shock structure self-consistently in two and three dimensions. I find that the structure of the magnetospheric bow shock differs substantially from that assumed by Vidotto et al. (2010, 2011) and Llama et al. (2011, 2013). For fiducial stellar wind rates I find that a planetary magnetic field of order a few G results in an extended magnetospheric cavity around the planet, typically  $6\text{--}9R_p$  in radius. In the frame of the planet the stellar wind is always supersonic, leading to a bow shock ahead of the magnetosphere, but the Mach number is modest ( $\mathcal{M} \simeq 1.6\text{--}1.8$ ) and consequently the shock is weak and broad. The planet’s magnetic field therefore creates a large perturbation to the wind, which typically extends around  $\gtrsim 30\%$  of the planet’s orbit, and the increased gas density in the shock results in increased UV absorption. I have used numerical simulations to generate synthetic transit light-curves, and find that the weak bow shock has a characteristic signature in the UV light-curve: broad excess absorption which leads the optical transit by 10–20% in orbital phase. I ran a large grid of models, and investigated how the structure of the magnetosphere and the shape of the UV light-curve vary with model parameters (sound speed, magnetic field strength, optical depth). The light-curves are consistent with existing observations of WASP-12b (e.g., Fossati et al., 2010; Haswell et al., 2012), but these data do not set useful

limits on the model parameters. The results also suggest that 100% phase coverage is highly desirable in future such observations. Finally, I have applied the model to two other hot Jupiters (WASP-18b and HD209458b), and find that UV observations of WASP-18b (which is much more massive than WASP-12b) should provide a straightforward means of distinguishing between Roche lobe overflow and a magnetospheric bow shock. The two regimes of 3-D and 2-D simulations give similar results indicating the convergence of the two dimensional results. Both results show pre-transit absorption due to enhancement in the absorbing column; that happens when the simulated bow shock passes across the line-of-sight with a magnetospheric cavity in both cases acting to reduce the column density. In general, the 2D results show that absorption happens before transit and covers  $\sim 30\%$  of the planetary orbital phase while for the 3-D results, this absorption feature lasts only for  $\sim 15\%$  in the planetary orbital phase and is weaker compared to the 2-D results. The two results are in better agreement at post-transit as both show "negative-depth" with nearly the same duration. The negative-depth is due to the extension of the effect of the magnetospheric cavity.

---

# 5

## Summary and Conclusion

## 5.1 Summary

This thesis has discussed the accretion and flow dynamics of stellar winds in two cases: mass capture by the white dwarf from the stellar wind of the companion in the Symbiotic recurrent nova RS Ophiuchi and the Planetary magnetic field interaction with the stellar wind of the host star in the transiting hot Jupiter WASP-12b.

I have studied the process of mass transfer via stellar wind in the symbiotic recurrent nova RS Ophiuchi using three dimensional Smoothed Particle Hydrodynamics. I have looked at the modes through which mass transfer takes place and estimated mass capture fractions from the red giant wind by the mass accreting white dwarf; I have also studied the structure of the accretion disc formed around the white dwarf such as its radial extent and surface density.

I have considered an isothermal wind nature for the outflow from the surface of the red giant as described by Parker (1958) and followed the two methods of subsonic and supersonic initial speed of the wind; with this prescription, I have reproduced all known modes of mass transfer from wind Roche lobe overflow (where mass loss takes place only through the inner Lagrange point) to an isotropic spherical expansion of wind from the red giant surface. These two modes are mediated by a special mode of mass transfer which is neither RLO-like nor a proper wind outflow where mass is being lost from both  $L1$  and  $L2$  to make a circumbinary gaseous medium.

The mass capture fraction is found to be sensitive to the stellar wind velocity; within the range of wind parameters tested in this work, the very low wind speeds produce wind Roche lobe overflow associated with mass capture fraction  $\sim 100\%$  and a large accretion disc size while fast winds lead to low mass capture fraction down to  $\sim 5\%$  and are characterised by a small accretion disc size.

In addition to pure radial outflow of the wind from the red giant, I have also explored the effect of rotation on the mass transfer mechanism; the two senses of rotation, prograde and retrograde, with respect to the direction of orbital motion have been studied. Neither direction of rotation seem to have a significant effect on mass capture fraction; however, rotation can enhance mass loss rate. Synchronised rotation of the red giant in RS Oph has been found to increase mass loss by a factor of three more than a non-rotating one; even faster rotations lead to up to one order of magnitude increase in mass loss rate. Consequently, mass accretion rate on to the white dwarf increases by the same factor.

All simulations reach steady state on a few orbital period timescale, leading to formation of sustainable accretion discs whose radii are beyond the stability radius and are then

likely to be susceptible to thermal-viscous instabilities (King and Pringle, 2009; Alexander et al., 2011) and undergo disc outbursts. It is nevertheless found that the surface density profile of the accretion discs are too low to trigger such disc outbursts. I conclude that the accretion discs in my simulations have not yet reached steady state and they might need longer timescales for the first outburst.

Rapid and high mass accretion rate of the white dwarf in RS Oph makes it a promising type 1a supernova progenitor (Sokoloski et al., 2006; Hachisu et al., 2007) as long as  $\Delta M_{\text{acc}} > \Delta M_{\text{ej}}$ . The results from the numerical simulations in this work suggest that accretion rate takes place with a rate  $M_{\text{acc}} \gtrsim 10^{-7} M_{\odot}/\text{yr}$ .

In the second part of my research, I look at the interaction of the transiting hot Jupiter WASP-12b with the stellar wind of its host star. I follow the scenario of NUV absorption due to the planetary magnetospheric bow shock (Lai et al., 2010; Vidotto et al., 2010; Llama et al., 2011). In order to study the pre-transit variable NUV absorption in WASP-12b, for the first time I model the stellar wind and the bow shock self consistently in two and three dimension simulations by using ZEUS-2D and 3-D SPH codes. In both numerical techniques, the star is set to emit wind with an isothermal Parker prescription (Parker, 1958) and the planetary magnetic field is assumed to be a dipolar magnetic field with full consideration of the gravity of the star and the planet.

The numerical results in this thesis show that there is a substantial difference between the structure of the magnetospheric bow shock in the simulations with those assumed by (Vidotto et al., 2010, 2011) and (Llama et al., 2011, 2013). A planetary magnetic field of order a few G results in an extended magnetospheric cavity around the planet, typically  $6-9R_p$  in radius. The stellar wind is always supersonic relative to the planet, which leads to a bow shock ahead of the magnetosphere, but the shock is broad and weak because of the modest Mach number ( $\mathcal{M} \approx 1.6-1.8$ ) of the wind.

The effect of the planet's magnetic field on the stellar wind subtends  $\gtrsim 30\%$  of the planet's orbit and the increased gas density in the shock results in increased UV absorption while the magnetospheric cavity acts to decrease UV absorption. Both the two dimensional and three dimensional simulation results consistently show UV light-curves which have excess absorption ahead of the optical transit by 10–20% in orbital phase and they are broad in nature as a result of the broad bow shock.

I have tested a large range of parameter space to see how the structure of the magnetosphere and the shape of the UV light-curve vary with the model parameters. The grid of parameters comprise the values of (sound speed, magnetic field strength, optical depth).

The simulated light-curves are consistent with existing observations of WASP-12b (e.g., Fossati et al., 2010; Haswell et al., 2012), but these data do not set useful limits on the model parameters.

Having applied the model to two other hot Jupiters (WASP-18b and HD209458b), the UV observations of WASP-18b (which is much more massive than WASP-12b) should straightforwardly discriminate between the proposed scenarios of excess UV absorption in some transiting hot Jupiters which are the Roche lobe overflow and a magnetospheric bow shock.

## 5.2 Future work

This thesis consists of the study of outbursts in recurrent nova RS Oph in chapter 3 and the study of NUV light-curves of the transiting hot Jupiter WASP-12b in chapter 4. From the light of the results I have presented, it is worth doing some future work which are related to what I have done so far. For the sake of testing my results, extending the scope of my work, better understanding and addressing the limitations, the followings below are my proposed future work.

1. The hydrodynamic simulations of RS Oph have isothermal gas properties and standard SPH viscosity. To better estimate accretion disc outburst recurrence time, accretion disc physics prescribed self-consistently in these simulations for the region of the accumulation of gas around the accretor would not be straight-forward. However, this could be more accurate than my results for calculating the accretion disc surface density.
2. Simulating nova outbursts for the cases that result in enough mass accumulation (the shaded area in figure 3.10) to look for any details of the expanding shell through the stellar wind and studying the shock structure and comparison with observations if possible.
3. It is suggested in 4.7.1 that the rotation of the central star has a negligible effect on the structure of the bow shock. This is because the azimuthal component of the wind speed at the position of the planet would be much smaller than the planetary orbital speed. Numerical simulations with rotating stars self-consistently would be helpful in showing the effect of stellar rotation on the shock structure and the light-curves. That could be implemented in the 3-D SPH code.

4. An accurate comparison between the light-curves of WASP-12b and WASP-18b infers the range of planetary mass influence on the light-curves. This might be evidence to show planetary RLO or magnetosphere is responsible for the excess NUV absorptions in such systems.
5. A complete 100% planetary orbital phase is highly demanded. Only a narrow phase coverage of pre-transit to post-transit could not tell much about the planetary interaction with the stellar wind. Out of transit NUV observations is important for information about the extent of the bow shock for instance the light-curves normalised to the flux corresponding to  $180^\circ$  out of transit would give the most realistic picture about NUV flux variations during or pre/post transit.



# Appendix A

## A.1 Binary Orbits

Consider two stars, modelled as point masses, with  $m_1$  as the primary mass and  $m_2$  as the secondary mass in a binary system, for which  $q = \frac{m_2}{m_1} < 1$ . For such a system there are three orbits

1- The barycentric orbit of the primary component of mass  $m_1$  ( The blue orbit in figure 1.1 ), where the point mass  $m_1$  orbits the binary centre of mass with the following specifications:

\* The orbital semimajor axis

$$a_1 = \left( \frac{Gm_2^3}{(m_1 + m_2)^2} \frac{P_1^2}{4\pi^2} \right)^{1/3}. \quad (\text{A.1})$$

where  $P_1$  is the primary's orbital period.

\* The instantaneous distance  $r_1$  from the binary centre of mass

$$r_{p1} \leq r_1 \leq r_{a1}. \quad (\text{A.2})$$

where  $r_{p1} = a_1(1 - e_1)$  is the Periastron distance,  $r_{a1} = a_1(1 + e_1)$  is the apastron distance and  $e_1$  is the eccentricity of the orbit.

\* The instantaneous orbital velocity ( as a function of orbital position  $r_1$  )

$$v_1 = \left[ \frac{Gm_2^3}{(m_1 + m_2)^2} \left( \frac{2}{r_1} - \frac{1}{a_1} \right) \right]^{1/2}. \quad (\text{A.3})$$

\* The specific orbital angular momentum

$$L_1 = \left[ \frac{Gm_2^3}{(m_1 + m_2)^2} a_1 (1 - e_1^2) \right]^{1/2} \quad (\text{A.4})$$

2- The barycentric orbit of the secondary component of mass  $m_2$  ( The red orbit in figure 1.1 ),

\* The orbital semimajor axis

$$a_2 = \left( \frac{Gm_1^3}{(m_1 + m_2)^2} \frac{P_2^2}{4\pi^2} \right)^{1/3}. \quad (\text{A.5})$$

\* The instantaneous distance  $r_2$  from the binary centre of mass

$$r_{p2} \leq r_2 \leq r_{a2}. \quad (\text{A.6})$$

where  $r_{p2} = a_2(1 - e_2)$  and  $r_{a2} = a_2(1 + e_2)$ .

\* The instantaneous orbital velocity ( as a function of orbital position  $r_2$  )

$$v_2 = \left[ \frac{Gm_1^3}{(m_1 + m_2)^2} \left( \frac{2}{r_2} - \frac{1}{a_2} \right) \right]^{1/2}. \quad (\text{A.7})$$

\* The specific orbital angular momentum

$$L_2 = \left[ \frac{Gm_1^3}{(m_1 + m_2)^2} a_2(1 - e_2^2) \right]^{1/2} \quad (\text{A.8})$$

3- The two barycentric orbits above are combined to make a relative orbit as a result of the relative motion of the binary components. This orbit has the following specifications

\* The orbital semimajor axis

$$a = \frac{G(m_1 + m_2)}{4\pi^2} P^2. \quad (\text{A.9})$$

\* The instantaneous relative distance  $r$  (the line of centres)

$$r_p \leq r \leq r_a. \quad (\text{A.10})$$

\* The instantaneous orbital velocity ( as a function of  $r$  )

$$v = \left[ G(m_1 + m_2) \left( \frac{2}{r} - \frac{1}{a} \right) \right]^{1/2}. \quad (\text{A.11})$$

\* The specific orbital angular momentum

$$L = G(m_1 + m_2)a(1 - e^2) \quad (\text{A.12})$$

\* Summary of the Orbital relations

$$\begin{aligned} P_1 &= P_2 = P \\ e_1 &= e_2 = e \\ a_1 &= \frac{m_2}{m_1 + m_2}a \quad \text{and} \quad a_2 = \frac{m_1}{m_1 + m_2}a \quad \text{or} \quad a = a_1 + a_2 \\ r_1 &= \frac{m_2}{m_1 + m_2}r \quad \text{and} \quad r_2 = \frac{m_1}{m_1 + m_2}r \quad \text{or} \quad r = r_1 + r_2 \\ L_1 &= \frac{m_2^2}{(m_1 + m_2)^2}L \quad \text{and} \quad L_2 = \frac{m_1^2}{(m_1 + m_2)^2}L \quad \text{or} \quad L = \frac{(m_1 + m_2)^2}{m_1^2 + m_2^2}(L_1 + L_2). \end{aligned} \quad (\text{A.13})$$

## A.2 Orbital total kinetic energy and Angular momentum

For the binary depicted in Figure 1.1, the total kinetic energy is (Equation 1.3)

$$E_k = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

Using relations in equations A.3 and A.7,

$$\begin{aligned}
E_k &= \frac{1}{2}m_1 \frac{Gm_2^3}{(m_1 + m_2)^2} \left( \frac{2}{r_1} - \frac{1}{a_1} \right) + \frac{1}{2}m_2 \frac{Gm_1^3}{(m_1 + m_2)^2} \left( \frac{2}{r_2} - \frac{1}{a_2} \right) \\
&= \frac{G}{2} \frac{m_1 m_2}{m_1 + m_2} \left[ \frac{m_2^2}{m_1 + m_2} \left( \frac{2}{r_1} - \frac{1}{a_1} \right) + \frac{m_1^2}{m_1 + m_2} \left( \frac{2}{r_2} - \frac{1}{a_2} \right) \right] \\
&= \frac{G}{2} \frac{m_1 m_2}{m_1 + m_2} \left[ m_2 \left( \frac{2}{r} - \frac{1}{a} \right) + m_1 \left( \frac{2}{r} - \frac{1}{a} \right) \right] \\
&= Gm_1 m_2 \left[ \frac{2}{r} - \frac{1}{a} \right] = \frac{m_1 m_2}{2(m_1 + m_2)} v^2
\end{aligned} \tag{A.14}$$

That is after substituting for  $r_1$ ,  $r_2$ ,  $a_1$ ,  $a_2$ , and  $v$  in equations A.13 and A.11.

Assuming the total spin angular momentum has ignorable contribution to the total orbital angular momentum, then

$$J = m_1 L_1 + m_2 L_2 \tag{A.15}$$

Similarly, substituting equations A.4, A.8 and A.13 to get

$$\begin{aligned}
J &= m_1 \left[ \frac{Gm_2^3}{(m_1 + m_2)^2} a_1 (1 - e^2) \right]^{1/2} + m_2 \left[ \frac{Gm_1^3}{(m_1 + m_2)^2} a_2 (1 - e^2) \right]^{1/2} \\
&= m_1 \left[ \frac{Gm_2^3}{(m_1 + m_2)^2} \frac{m_2}{m_1 + m_2} a (1 - e^2) \right]^{1/2} + m_2 \left[ \frac{Gm_1^3}{(m_1 + m_2)^2} \frac{m_1}{m_1 + m_2} a (1 - e^2) \right]^{1/2} \\
&= m_1 m_2 \left[ \frac{G}{(m_1 + m_2)} a (1 - e^2) \right]^{1/2}.
\end{aligned} \tag{A.16}$$

### A.3 Conservation of mass in accretion discs

Following the expressions in Section (1.2.2.1 and 1.2.2.2), consider an annulus of matter in an accretion disc, where  $R$  is the radial distance of the annulus from the centre of the accretor,  $\Delta R$  is the breadth of the annulus. The mass of the matter lying in this annulus is

$$M = 2\pi R \Delta R \Sigma(R, t)$$

The orbital motion of the matter inside the annulus is assumed to be keplerian, thus the angular velocity  $\Omega(R)$  is

$$\Omega(R) = (GM_*/R^3)^{1/2}$$

And the total angular momentum in that annulus is

$$J = 2\pi R \Delta R \Sigma(R, t) R^2 \Omega(R)$$

The continuity equation can be written as

$$\frac{\partial M}{\partial t} = 2\pi R \Delta R \frac{\partial \Sigma(R, t)}{\partial t} = \dot{M}_{in} - \dot{M}_{out} \quad (\text{A.17})$$

$$2\pi R \Delta R \frac{\partial \Sigma(R, t)}{\partial t} = -2\pi [R + \Delta R] v_R(R + \Delta R, t) \Sigma(R + \Delta R, t) + 2\pi R v_R(R, t) \Sigma(R, t)$$

$$2\pi R \Delta R \frac{\partial \Sigma(R, t)}{\partial t} = -2\pi \left[ [R + \Delta R] v_R(R + \Delta R, t) \Sigma(R + \Delta R, t) - R v_R(R, t) \Sigma(R, t) \right]$$

$$R \frac{\partial \Sigma(R, t)}{\partial t} = - \frac{[R + \Delta R] v_R(R + \Delta R, t) \Sigma(R + \Delta R, t) - R v_R(R, t) \Sigma(R, t)}{\Delta R} \quad (\text{A.18})$$

From the relation of derivatives

$$\lim_{x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} = \frac{df}{dx}$$

taking the limit  $\Delta R \rightarrow 0$ , one can obtain

$$R \frac{\partial \Sigma}{\partial t} + \frac{\partial}{\partial R} [R \Sigma v_R] = 0 \quad (\text{A.19})$$

## A.4 Viscous torque

Below is a derivation of equation 1.26 in a simple way to find an expression for the viscous torque between two neighbouring annuli in an accretion disc. The viscous force  $F_v$  is related to the area of contact interface between the two annulie and the velocity gradient  $R \frac{d\Omega}{dR}$

$$\begin{aligned} F_v &\propto 2\pi R H R \frac{d\Omega}{dR} \\ &= \mu 2\pi R H R \frac{d\Omega}{dR} \\ &= \rho \nu 2\pi R H R \frac{d\Omega}{dR} \\ &= 2\pi \Sigma \nu R^2 \frac{d\Omega}{dR} \end{aligned} \quad (\text{A.20})$$

Then the viscous torque  $G$  is

$$\begin{aligned} G &= R F_v = 2\pi R \Sigma \nu R^2 \frac{d\Omega}{dR} \\ &= 2\pi R \nu \Sigma R^2 \Omega' \end{aligned} \quad (\text{A.21})$$

where  $\mu$  is the coefficient of dynamic viscosity,  $H$  is the accretion disc scaleheight and  $\nu$  is the kinematic viscosity described in equation 1.29.

## A.5 Conservation of angular momentum in accretion discs

Recalling equation (1.24) the conservation of angular momentum can be written as

$$\frac{\partial}{\partial t} J = \dot{J}_{in} - \dot{J}_{out} + \tau_{net}$$

where  $(\tau_{net} = \frac{\partial G}{\partial R} \Delta R)$  is the the net viscous torque on the annulus. Thus

$$\begin{aligned} \frac{\partial}{\partial t} \left( 2\pi R \Delta R \Sigma(R, t) R^2 \Omega(R) \right) &= -2\pi [R + \Delta R] v_R(R + \Delta R, t) \Sigma(R + \Delta R, t) [R + \Delta R]^2 \Omega(R + \Delta R) \\ &\quad + 2\pi R v_R(R, t) \Sigma(R, t) R^2 \Omega(R) + \frac{\partial G}{\partial R} \Delta R \\ 2\pi R \Delta R \frac{\partial}{\partial t} \left( \Sigma(R, t) R^2 \Omega(R) \right) &= -2\pi \left[ [R + \Delta R] v_R(R + \Delta R, t) \Sigma(R + \Delta R, t) [R + \Delta R]^2 \Omega(R + \Delta R) \right. \\ &\quad \left. - R v_R(R, t) \Sigma(R, t) R^2 \Omega(R) \right] + \frac{\partial G}{\partial R} \Delta R \end{aligned}$$

$$R \frac{\partial (\Sigma R^2 \Omega)}{\partial t} = - \left[ \frac{R v_R \Sigma R^2 \Omega \Big|_{(R+\Delta R)} - R v_R \Sigma R^2 \Omega \Big|_R}{\Delta R} \right] + \frac{1}{2\pi} \frac{\partial G}{\partial R} \quad (\text{A.22})$$

Similar to what is done for section (A.3) if we assume  $f(x) = R v_R \Sigma R^2 \Omega$ , then

$$R \frac{\partial}{\partial t} (\Sigma R^2 \Omega) + \frac{\partial}{\partial R} (R v_R \Sigma R^2 \Omega) = \frac{1}{2\pi} \frac{\partial G}{\partial R} \quad (\text{A.23})$$

Now, for constant angular momentum in time, and using equation (A.19), the equation above becomes

$$R(R^2 \Omega) \frac{\partial \Sigma}{\partial t} + (R^2 \Omega) \frac{\partial}{\partial R} (R v_R \Sigma) + (R v_R \Sigma) \frac{\partial}{\partial R} (R^2 \Omega) = \frac{1}{2\pi} \frac{\partial G}{\partial R}$$

$$R(R^2 \Omega) \frac{\partial \Sigma}{\partial t} - R(R^2 \Omega) \frac{\partial \Sigma}{\partial t} + (R v_R \Sigma) \frac{\partial}{\partial R} (R^2 \Omega) = \frac{1}{2\pi} \frac{\partial G}{\partial R}$$

$$(R v_R \Sigma) \frac{\partial}{\partial R} (R^2 \Omega) = \frac{1}{2\pi} \frac{\partial G}{\partial R}$$

Or,

$$(R v_R \Sigma) = \left[ \frac{\frac{1}{2\pi} \frac{\partial G}{\partial R}}{\frac{\partial}{\partial R} (R^2 \Omega)} \right] \quad (\text{A.24})$$

Substituting equation (1.20) for Keplerian angular velocity

$$\begin{aligned}\frac{1}{2\pi} \frac{\partial G}{\partial R} &= \frac{1}{2\pi} \frac{\partial}{\partial R} \left( 2\pi R v \Sigma R^2 \frac{d\Omega}{dR} \right) \\ &= -\frac{3}{2} (GM_*)^{1/2} \frac{\partial}{\partial R} (R^{1/2} v \Sigma)\end{aligned}$$

$$\frac{\partial}{\partial R} (R^2 \Omega) = \left( \frac{GM_*}{4R} \right)^{1/2}$$

Then, equation (A.24) becomes

$$(R v_R \Sigma) = -3R^{1/2} \frac{\partial}{\partial R} (R^{1/2} v \Sigma)$$

Substituting this into equation (A.19), finally gives the diffusion equation

$$\frac{\partial \Sigma}{\partial t} = \frac{3}{R} \frac{\partial}{\partial R} \left[ R^{1/2} \frac{\partial}{\partial R} (R^{1/2} v \Sigma) \right] \quad (\text{A.25})$$

---

# B

## Appendix B

## B.1 Conservation equations in fluids

To derive the Lagrangian form of continuity equation from the Eulerian one, let's begin with equation 2.1

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \mathbf{v})$$

Using the lagrangian derivative

$$\frac{dF}{dt} = \frac{\partial F}{\partial t} + \mathbf{v} \cdot \nabla F \quad (\text{B.1})$$

where  $F$  is any scalar/ vector quantity associated with the fluid. The equation above is the Lagrangian rate of change  $\frac{d}{dt}$  on the LHS and on the RHS is the Eulerian rate of change  $\frac{\partial}{\partial t}$  plus the convective rate of change  $\mathbf{v} \cdot \nabla$ . Consider  $F = \rho$  and Substituting the Eulerian continuity equation into it, then

$$\begin{aligned} \frac{d\rho}{dt} &= \frac{\partial \rho}{\partial t} + \mathbf{v} \cdot \nabla \rho \\ &= -\nabla \cdot (\rho \mathbf{v}) + \mathbf{v} \cdot \nabla \rho \end{aligned}$$

And using the identity ( for any scalar quantity  $\rho$  and vector  $\mathbf{v}$ )

$$\nabla \cdot (\rho \mathbf{v}) = (\nabla \rho) \cdot \mathbf{v} + \rho (\nabla \cdot \mathbf{v}) \quad (\text{B.2})$$

Thus

$$\begin{aligned} \frac{d\rho}{dt} &= -(\nabla \rho) \cdot \mathbf{v} - \rho (\nabla \cdot \mathbf{v}) + \mathbf{v} \cdot \nabla \rho \\ \frac{d\rho}{dt} &= -\rho (\nabla \cdot \mathbf{v}) \end{aligned} \quad (\text{B.3})$$

which is a Lagrangian form of the continuity equation. Similarly, the conservation of momentum in equation 2.3

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} = -\frac{\nabla P}{\rho}$$

can be re-written in Lagrangian form by using equations B.1 and B.2, thus

$$\begin{aligned}
\frac{d\mathbf{v}}{dt} &= \frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \\
\frac{d\mathbf{v}}{dt} &= -(\mathbf{v} \cdot \nabla) \mathbf{v} - \frac{\nabla P}{\rho} + \mathbf{v} \cdot \nabla \mathbf{v} \\
\frac{d\mathbf{v}}{dt} &= -\frac{\nabla P}{\rho}
\end{aligned} \tag{B.4}$$

Now, for conservation of internal enrgy, let's substitute equation 2.5

$$\frac{\partial u}{\partial t} + (\mathbf{v} \cdot \nabla) u = -\frac{P}{\rho} \nabla \cdot \mathbf{v},$$

into equation B.1 such that ( $F = u$ ) to get

$$\begin{aligned}
\frac{du}{dt} &= \frac{\partial u}{\partial t} + \mathbf{v} \cdot \nabla u \\
&= -(\mathbf{v} \cdot \nabla) u - \frac{P}{\rho} (\nabla \cdot \mathbf{v}) + (\mathbf{v} \cdot \nabla) u \\
&= -\frac{P}{\rho} (\nabla \cdot \mathbf{v}) \\
&= \frac{P}{\rho^2} \frac{d\rho}{dt}
\end{aligned} \tag{B.5}$$

That is after using the relation in equation B.3

## B.2 SPH formalisms

To derive the SPH formalism of conservation of mass at any point  $a$  in the fluid, let's begin with the equation 2.25

$$\left(\frac{d\rho}{dt}\right)_a = (\nabla \rho \cdot \mathbf{v})_a - (\nabla \cdot \rho \mathbf{v})_a$$

Using equation 2.16, the first term on the right hand side becomes

$$(\nabla \rho \cdot \mathbf{v})_a = \nabla_a \rho_a \cdot \mathbf{v}_a = \mathbf{v}_a \cdot \sum_{b=1}^{N_{neigh}} m_b \nabla_a W(\mathbf{r}_a - \mathbf{r}_b, h)$$

If we assume  $\rho_a \mathbf{v}_a = \mathbf{A}$  in equation 2.18 , the second term on the same side becomes

$$(\nabla \cdot \rho \mathbf{v})_a = \nabla_a \cdot \rho_a \mathbf{v}_a = \sum_{b=1}^{N_{neigh}} \frac{m_b}{\rho_b} \rho_b \mathbf{v}_b \cdot \nabla_a W(\mathbf{r}_a - \mathbf{r}_b, h)$$

Substituting these into equation 2.25 we get

$$\begin{aligned} \frac{d\rho_a}{dt} &= \sum_{b=1}^{N_{neigh}} m_b (\mathbf{v}_a - \mathbf{v}_b) \cdot \nabla_a W(\mathbf{r}_a - \mathbf{r}_b, h) \\ \frac{d\rho_a}{dt} &= \sum_{b=1}^{N_{neigh}} m_b \mathbf{v}_{ab} \cdot \nabla_a W_{ab} \end{aligned} \quad (\text{B.6})$$

Now let's recall equation 2.4 to write equation of conservation of momentum in its SPH form

$$\frac{d\mathbf{v}}{dt} = -\frac{\nabla P}{\rho}$$

Both density and pressure are scalar quantities, then their divergence gives

$$\begin{aligned} \nabla \left( \frac{1}{\rho} P \right) &= \frac{1}{\rho} \nabla P + P \nabla \frac{1}{\rho} \\ &= \frac{1}{\rho} \nabla P - \frac{P}{\rho^2} \nabla \rho \end{aligned}$$

Equation 2.4 thus becomes

$$\frac{d\mathbf{v}}{dt} = -\nabla \left( \frac{P}{\rho} \right) - \frac{P}{\rho^2} \nabla \rho \quad (\text{B.7})$$

Using equations 2.15 (assuming  $A = \frac{P}{\rho}$ ) and 2.16, then equation 2.2 becomes

$$\begin{aligned} \left( \frac{d\mathbf{v}}{dt} \right)_a &= - \sum_{b=1}^{N_{neigh}} m_b \frac{(P/\rho)_b}{\rho_b} \nabla_a W(\mathbf{r}_a - \mathbf{r}_b, h) - \left( \frac{P}{\rho^2} \right)_a \sum_{b=1}^{N_{neigh}} m_b \frac{\rho_b}{\rho_b} \nabla_a W(\mathbf{r}_a - \mathbf{r}_b, h) \\ \left( \frac{d\mathbf{v}}{dt} \right)_a &= - \sum_{b=1}^{N_{neigh}} m_b \left( \frac{P_a}{\rho_a^2} + \frac{P_b}{\rho_b^2} \right) \nabla_a W_{ab} \end{aligned} \quad (\text{B.8})$$

The time derivative of the specific total energy at point  $a$  is ( See equation 2.7)

$$\frac{de_a}{dt} = (\mathbf{v}_a \cdot \frac{d\mathbf{v}_a}{dt}) + \frac{du_a}{dt}$$

Substituting equations (2.28) and (2.32) into that above, we obtain

$$\begin{aligned} \frac{de_a}{dt} &= - \sum_{b=1}^{N_{neigh}} m_b \left( \frac{P_a}{\rho_a^2} + \frac{P_b}{\rho_b^2} \right) \mathbf{v}_a \cdot \nabla_a W_{ab} + \frac{P_a}{\rho_a^2} \sum_{b=1}^{N_{neigh}} m_b (\mathbf{v}_a - \mathbf{v}_b) \cdot \nabla_a W_{ab} \\ \frac{de_a}{dt} &= - \sum_{b=1}^{N_{neigh}} m_b \left( \frac{P_a \mathbf{v}_a}{\rho_a^2} + \frac{P_b \mathbf{v}_a}{\rho_b^2} \right) \cdot \nabla_a W_{ab} + \sum_{b=1}^{N_{neigh}} m_b \left( \frac{P_a \mathbf{v}_a}{\rho_a^2} - \frac{P_a \mathbf{v}_b}{\rho_a^2} \right) \cdot \nabla_a W_{ab} \\ \frac{de_a}{dt} &= \sum_{b=1}^{N_{neigh}} m_b \left( - \frac{P_a \mathbf{v}_a}{\rho_a^2} - \frac{P_b \mathbf{v}_a}{\rho_b^2} + \frac{P_a \mathbf{v}_a}{\rho_a^2} - \frac{P_a \mathbf{v}_b}{\rho_a^2} \right) \cdot \nabla_a W_{ab} \\ \frac{de_a}{dt} &= - \sum_{b=1}^{N_{neigh}} m_b \left[ \frac{P_a}{\rho_a^2} \mathbf{v}_b + \frac{P_b}{\rho_b^2} \mathbf{v}_a \right] \cdot \nabla_a W_{ab} \end{aligned} \tag{B.9}$$

---

# C

## Appendix C

## C.1 Force due to the gradient of magnetic pressure

We assume that the planet has a dipolar magnetic field aligned with the planet's orbital angular momentum, and work in the far-field limit (i.e., we assume  $r = |\mathbf{r} - \mathbf{r}_p| \gg R_p$ , where  $R_p$  is the radius of the planet), so the magnetic field strength scales as  $B \propto r^{-3}$ . if we normalise  $B(r)$  to the surface field, then

$$\frac{B(r)}{B_o} = \frac{R_p^3}{r^3} \quad (\text{C.1})$$

The magnetic pressure  $P_B = B^2/8\pi$  (Wang, 1981), and the acceleration due to the magnetic pressure is

$$\begin{aligned} a_B &= \frac{-1}{\rho} \frac{\partial P_B}{\partial r} \\ &= \frac{3}{4} \frac{B_o^2 R_p^6}{\pi \rho r^7} \\ &= \frac{C_B}{r^7}. \end{aligned} \quad (\text{C.2})$$

where the force constant  $C_B = 3B_o^2 R_p^6 / 4\pi\rho$ .

## C.2 Planet-centred accelerations in polar co-ordinates

The accelerations due to the planet are evaluated in polar co-ordinates as follows. The acceleration due to the gravitational and magnetic potential of a planet at position  $\mathbf{r}_p = (r_p, \phi_p)$  at an arbitrary position  $\mathbf{r} = (r, \phi)$  is (Equation 4.4):

$$\mathbf{a} = \left( \frac{C_B}{|\mathbf{r} - \mathbf{r}_p|^8} - \frac{GM_p}{|\mathbf{r} - \mathbf{r}_p|^3} \right) (\mathbf{r} - \mathbf{r}_p). \quad (\text{C.3})$$

The relative position vector can be written as

$$\mathbf{r} - \mathbf{r}_p = (r \cos \phi - r_p \cos \phi_p) \hat{\mathbf{x}} + (r \sin \phi - r_p \sin \phi_p) \hat{\mathbf{y}}, \quad (\text{C.4})$$

where  $\hat{\mathbf{x}}$  &  $\hat{\mathbf{y}}$  are the usual Cartesian unit vectors, and its magnitude is therefore

$$|\mathbf{r} - \mathbf{r}_p| = \left( r^2 + r_p^2 - 2rr_p \cos(\phi - \phi_p) \right)^{1/2}. \quad (\text{C.5})$$

We write equation (C.3) as

$$\mathbf{a} = A(\mathbf{r} - \mathbf{r}_p), \quad (\text{C.6})$$

where the normalisation factor  $A(=a_B+a_g)$  is given by

$$A = \frac{C_B}{|\mathbf{r} - \mathbf{r}_1|^8} - \frac{GM_p}{|\mathbf{r} - \mathbf{r}_1|^3} . \quad (\text{C.7})$$

To implement these accelerations in polar co-ordinates we must express  $\mathbf{a}$  as a vector  $(a_r, a_\phi)$ . This is done by taking the scalar products of  $\mathbf{a}$  with the unit vectors in the  $r$  &  $\phi$  directions, respectively. The unit vectors here are

$$\hat{\mathbf{r}} = \cos \phi \hat{\mathbf{x}} + \sin \phi \hat{\mathbf{y}} , \quad (\text{C.8})$$

$$\hat{\phi} = -\sin \phi \hat{\mathbf{x}} + \cos \phi \hat{\mathbf{y}} . \quad (\text{C.9})$$

We then take scalar products of equation (C.6) with equations (C.8) & (C.9) and rearrange to find

$$a_r = A \left( r - r_p \cos(\phi - \phi_p) \right) , \quad (\text{C.10})$$

$$a_\phi = A \left( r_p \sin(\phi - \phi_p) \right) . \quad (\text{C.11})$$

These two terms are then added as explicit accelerations in the source step of the ZEUS-2D code.

---

## References

- Alexander, R. D., Wynn, G. A., King, A. R., and Pringle, J. E.: 2011, *MNRAS* **418**, 2576
- Anupama, G. C.: 2008, in A. Evans, M. F. Bode, T. J. O’Brien, and M. J. Darnley (eds.), *RS Ophiuchi (2006) and the Recurrent Nova Phenomenon*, Vol. 401 of *Astronomical Society of the Pacific Conference Series*, p. 31
- Anupama, G. C. and Mikołajewska, J.: 1999, *A&A* **344**, 177
- Armitage, P. J.: 2009, *Astrophysics of Planet Formation*
- Bisikalo, D., Kaygorodov, P., Ionov, D., et al.: 2013, *ApJ* **764**, 19
- Bladh, S. and Höfner, S.: 2012, *A&A* **546**, A76
- Bode, M. F. and Evans, A.: 2008, *Classical Novae*
- Bodenheimer, P., Laughlin, G. P., Różyczka, M., and Yorke, H. W. (eds.): 2007, *Numerical Methods in Astrophysics: An Introduction*
- Bodenheimer, P., Lin, D. N. C., and Mardling, R. A.: 2001, *ApJ* **548**, 466
- Boffin, H. M. J., Theuns, T., and Jorissen, A.: 1994, *Mem. Soc. Astron. Italiana* **65**, 1199
- Bohigas, J., Echevarria, J., Diego, F., and Sarmiento, J. A.: 1989, *MNRAS* **238**, 1395
- Bond, I. A., Udalski, A., Jaroszyński, M., et al.: 2004, *ApJ* **606**, L155
- Bondi, H.: 1952, *MNRAS* **112**, 195
- Bondi, H. and Hoyle, F.: 1944, *MNRAS* **104**, 273
- Brandi, E., Quiroga, C., Mikołajewska, J., Ferrer, O. E., and García, L. G.: 2009, *A&A* **497**, 815
- Cannizzo, J. K.: 1993, *ApJ* **419**, 318

- 
- Carroll, B. W. and Ostlie, D. A.: 2006, *An introduction to modern astrophysics and cosmology*
- Charbonneau, D., Allen, L. E., Megeath, S. T., et al.: 2005, *ApJ* **626**, 523
- Charbonneau, D., Brown, T. M., Latham, D. W., and Mayor, M.: 2000, *ApJ* **529**, L45
- Cranmer, S. R.: 2004, *American Journal of Physics* **72**, 1397
- de Mink, S. E., Langer, N., Izzard, R. G., Sana, H., and de Koter, A.: 2013, *ApJ* **764**, 166
- de Val-Borro, M., Karovska, M., and Sasselov, D.: 2009, *ApJ* **700**, 1148
- Deeg, H., Belmonte, J. A., and Aparicio, A.: 2007, *Extrasolar Planets*
- Deming, D., Seager, S., Richardson, L. J., and Harrington, J.: 2005, *Nature* **434**, 740
- Dumm, T., Folini, D., Nussbaumer, H., et al.: 2000, *A&A* **354**, 1014
- Edgar, R.: 2004, **48**, 843
- Eggleton, P. P.: 1983, *ApJ* **268**, 368
- Espey, B. R. and Crowley, C.: 2008, in A. Evans, M. F. Bode, T. J. O'Brien, and M. J. Darnley (eds.), *RS Ophiuchi (2006) and the Recurrent Nova Phenomenon*, Vol. 401 of *Astronomical Society of the Pacific Conference Series*, p. 166
- Fossati, L., Haswell, C. A., Froning, C. S., et al.: 2010, *ApJ* **714**, L222
- Frank, J., King, A., and Raine, D. J.: 2002, *Accretion Power in Astrophysics: Third Edition*
- Fukue, J. and Ioroi, M.: 1999, *PASJ* **51**, 151
- Gehrz, R. D., Truran, J. W., Williams, R. E., and Starrfield, S.: 1998, *PASP* **110**, 3
- Gingold, R. A. and Monaghan, J. J.: 1977, *MNRAS* **181**, 375
- Gu, P.-G., Lin, D. N. C., and Bodenheimer, P. H.: 2003, *ApJ* **588**, 509
- Hachisu, I. and Kato, M.: 2000, *ApJ* **536**, L93
- Hachisu, I. and Kato, M.: 2001, *ApJ* **558**, 323
- Hachisu, I., Kato, M., Kiyota, S., et al.: 2006, *ApJ* **651**, L141
-

- 
- Hachisu, I., Kato, M., and Luna, G. J. M.: 2007, *ApJ* **659**, L153
- Han, E., Wang, S. X., Wright, J. T., et al.: 2014, *PASP* **126**, 827
- Haswell, C. A., Fossati, L., Ayres, T., et al.: 2012, *ApJ* **760**, 79
- Hayden, B. T., Garnavich, P. M., Kasen, D., et al.: 2010, *ApJ* **722**, 1691
- Hebb, L., Collier-Cameron, A., Loeillet, B., et al.: 2009, *ApJ* **693**, 1920
- Hellier, C., Anderson, D. R., Collier Cameron, A., et al.: 2009, *Nature* **460**, 1098
- Henry, G. W., Marcy, G. W., Butler, R. P., and Vogt, S. S.: 2000, *ApJ* **529**, L41
- Hilditch, R. W.: 2001, *An Introduction to Close Binary Stars*
- Hillebrandt, W. and Niemeyer, J. C.: 2000, *ARA&A* **38**, 191
- Hjellming, R. M., van Gorkom, J. H., Taylor, A. R., et al.: 1986, *ApJ* **305**, L71
- Hoyle, F. and Lyttleton, R. A.: 1939, *Proceedings of the Cambridge Philosophical Society* **35**, 405
- Hur , J.-M.: 2000, *A&A* **358**, 378
- Iglesias, C. A. and Rogers, F. J.: 1996, *ApJ* **464**, 943
- Karovska, M.: 2006, in A. Wilson (ed.), *The X-ray Universe 2005*, Vol. 604 of *ESA Special Publication*, p. 183
- Kato, M.: 1991, *ApJ* **369**, 471
- Kenyon, S. J.: 1986, *The symbiotic stars*
- Kilpio, E., Bisikalo, D., Tomov, N., and Tomova, M.: 2011, *Ap&SS* **335**, 155
- King, A. R. and Pringle, J. E.: 2009, *MNRAS* **397**, L51
- King, A. R., Rolfe, D. J., and Schenker, K.: 2003, *MNRAS* **341**, L35
- Kopal, Z.: 1959, *Leaflet of the Astronomical Society of the Pacific* **8**, 81
- Kopal, Z. (ed.): 1978, *Dynamics of close binary systems*, Vol. 68 of *Astrophysics and Space Science Library*
-

- 
- Lai, D., Helling, C., and van den Heuvel, E. P. J.: 2010, *ApJ* **721**, 923
- Lamers, H. J. G. L. M.: 2004, in A. Maeder and P. Eenens (eds.), *Stellar Rotation*, Vol. 215 of *IAU Symposium*, p. 479
- Lamers, H. J. G. L. M. and Cassinelli, J. P.: 1999, *Introduction to Stellar Winds*
- Lasota, J.-P.: 2001, **45**, 449
- Lin, D. N. C., Bodenheimer, P., and Richardson, D. C.: 1996, *Nature* **380**, 606
- Liu, G. R. and Liu, M. B.: 2003, *Smoothed Particle Hydrodynamics: A Meshfree Particle Method*
- Livio, M., Sahu, K., and Valenti, J.: 2008, *A Decade of Extrasolar Planets around Normal Stars*
- Livio, M., Soker, N., de Kool, M., and Savonije, G. J.: 1986, *MNRAS* **222**, 235
- Llama, J., Vidotto, A. A., Jardine, M., et al.: 2013, *MNRAS* **436**, 2179
- Llama, J., Wood, K., Jardine, M., et al.: 2011, *MNRAS* **416**, L41
- Lodato, G.: 2008, **52**, 21
- Lucy, L. B.: 1977, *AJ* **82**, 1013
- Mandel, K. and Agol, E.: 2002, *ApJ* **580**, L171
- Maoz, D., Mannucci, F., and Nelemans, G.: 2014, *ARA&A* **52**, 107
- Mastrodemos, N. and Morris, M.: 1998, *ApJ* **497**, 303
- Mayor, M. and Queloz, D.: 1995, *Nature* **378**, 355
- Mikołajewska, J.: 2007, *Baltic Astronomy* **16**, 1
- Mikołajewska, J.: 2010, *ArXiv e-prints*
- Mikołajewska, J.: 2012, *Baltic Astronomy* **21**, 5
- Mitsumoto, M., Jahanara, B., Matsuda, T., et al.: 2005, *Astronomy Reports* **49**, 884
- Monaghan, J. J.: 1992, *ARA&A* **30**, 543

- 
- Murdin, P.: 2001, *Encyclopedia of astronomy and astrophysics*
- Nagae, T., Oka, K., Matsuda, T., et al.: 2004, *A&A* **419**, 335
- Nauenberg, M.: 1972, *ApJ* **175**, 417
- Neuhäuser, R., Guenther, E. W., Wuchterl, G., et al.: 2005, *A&A* **435**, L13
- Nichols, J. D., Wynn, G. A., Goad, M., et al.: 2015, *ApJ* **803**, 9
- Nixon, C. J., Cossins, P. J., King, A. R., and Pringle, J. E.: 2011, *MNRAS* **412**, 1591
- Nomoto, K.: 1982, *ApJ* **253**, 798
- Nomoto, K., Saio, H., Kato, M., and Hachisu, I.: 2007, *ApJ* **663**, 1269
- Orlando, S., Drake, J. J., and Laming, J. M.: 2009, *A&A* **493**, 1049
- Osborne, J. P., Page, K. L., Beardmore, A. P., et al.: 2011, *ApJ* **727**, 124
- Padmanabhan, T.: 2001, *Theoretical Astrophysics - Volume 2, Stars and Stellar Systems*
- Panagia, N., Van Dyk, S. D., Weiler, K. W., et al.: 2006, *ApJ* **646**, 369
- Parker, E. N.: 1958, *ApJ* **128**, 664
- Parthasarathy, M., Branch, D., Jeffery, D. J., and Baron, E.: 2007, **51**, 524
- Patat, F., Chandra, P., Chevalier, R., et al.: 2007, *Science* **317**, 924
- Patat, F., Chugai, N. N., Podsiadlowski, P., et al.: 2011, *A&A* **530**, A63
- Perryman, M.: 2011, *The Exoplanet Handbook*
- Perryman, M., Hartman, J., Bakos, G. Á., and Lindegren, L.: 2014, *ApJ* **797**, 14
- Prialnik, D. and Kovetz, A.: 1995, *ApJ* **445**, 789
- Price, D. J.: 2012, *Journal of Computational Physics* **231**, 759
- Pringle, J. E.: 1981, *ARA&A* **19**, 137
- Raghavan, D., McAlister, H. A., Henry, T. J., et al.: 2010, *ApJS* **190**, 1
- Rasio, F. A., Tout, C. A., Lubow, S. H., and Livio, M.: 1996, *ApJ* **470**, 1187

- 
- Rosino, L.: 1987, in M. F. Bode (ed.), *RS Ophiuchi (1985) and the Recurrent Nova Phenomenon*, p. 1
- Rosswog, S.: 2009, **53**, 78
- Schaefer, B. E.: 2009, *ApJ* **697**, 721
- Schaefer, B. E.: 2010, *ApJS* **187**, 275
- Sequist, E. R. and Taylor, A. R.: 1990, *ApJ* **349**, 313
- Shakura, N. I. and Sunyaev, R. A.: 1973, *A&A* **24**, 337
- Shore, S. N.: 2008, in A. Evans, M. F. Bode, T. J. O'Brien, and M. J. Darnley (eds.), *RS Ophiuchi (2006) and the Recurrent Nova Phenomenon*, Vol. 401 of *Astronomical Society of the Pacific Conference Series*, p. 19
- Showman, A. P. and Guillot, T.: 2002, *A&A* **385**, 166
- Skopal, A. and Cariková, Z.: 2015, *A&A* **573**, A8
- Soker, N.: 2002, *MNRAS* **337**, 1038
- Soker, N. and Rappaport, S.: 2000, *ApJ* **538**, 241
- Sokoloski, J. L.: 2003, *Journal of the American Association of Variable Star Observers (JAAVSO)* **31**, 89
- Sokoloski, J. L., Luna, G. J. M., Mukai, K., and Kenyon, S. J.: 2006, *Nature* **442**, 276
- Springel, V.: 2010, *ARA&A* **48**, 391
- Springel, V. and Hernquist, L.: 2002, *MNRAS* **333**, 649
- Starrfield, S.: 2008, in A. Evans, M. F. Bode, T. J. O'Brien, and M. J. Darnley (eds.), *RS Ophiuchi (2006) and the Recurrent Nova Phenomenon*, Vol. 401 of *Astronomical Society of the Pacific Conference Series*, p. 4
- Starrfield, S., Truran, J. W., and Sparks, W. M.: 2000, **44**, 81
- Starrfield, S., Truran, J. W., Sparks, W. M., and Krautter, J.: 1991, in R. F. Malina and S. Bowyer (eds.), *Extreme Ultraviolet Astronomy*, p. 168
- Sternberg, A., Gal-Yam, A., Simon, J. D., et al.: 2011, *Science* **333**, 856
-

- 
- Stone, J. M., Mihalas, D., and Norman, M. L.: 1992, *ApJS* **80**, 819
- Stone, J. M. and Norman, M. L.: 1992a, *ApJS* **80**, 753
- Stone, J. M. and Norman, M. L.: 1992b, *ApJS* **80**, 791
- Theuns, T., Boffin, H. M. J., and Jorissen, A.: 1996, *MNRAS* **280**, 1264
- Torres, G., Winn, J. N., and Holman, M. J.: 2008, *ApJ* **677**, 1324
- Treves, A., Maraschi, L., and Abramowicz, M.: 1988, *PASP* **100**, 427
- Triaud, A. H. M. J., Collier Cameron, A., Queloz, D., et al.: 2010, *A&A* **524**, A25
- Vidal-Madjar, A., Huitson, C. M., Bourrier, V., et al.: 2013, *A&A* **560**, A54
- Vidotto, A. A., Jardine, M., and Helling, C.: 2010, *ApJ* **722**, L168
- Vidotto, A. A., Jardine, M., and Helling, C.: 2011, *MNRAS* **411**, L46
- Voss, R. and Nelemans, G.: 2008, *Nature* **451**, 802
- Walder, R., Folini, D., and Shore, S. N.: 2008, *A&A* **484**, L9
- Wallerstein, G.: 2008, in A. Evans, M. F. Bode, T. J. O'Brien, and M. J. Darnley (eds.), *RS Ophiuchi (2006) and the Recurrent Nova Phenomenon*, Vol. 401 of *Astronomical Society of the Pacific Conference Series*, p. 14
- Wang, B. and Han, Z.: 2012, **56**, 122
- Wang, B., Li, X.-D., and Han, Z.-W.: 2010, *MNRAS* **401**, 2729
- Wang, Y.-M.: 1981, *A&A* **102**, 36
- Warner, B.: 1995, *Cambridge Astrophysics Series* 28
- Webbink, R. F., Livio, M., Truran, J. W., and Orio, M.: 1987, *ApJ* **314**, 653
- Wheeler, J. C. and Harkness, R. P.: 1990, *Reports on Progress in Physics* **53**, 1467
- Wolszczan, A. and Frail, D. A.: 1992, *Nature* **355**, 145
- Yaron, O., Prialnik, D., Shara, M. M., and Kovetz, A.: 2005, *ApJ* **623**, 398
- Yungelson, L. and Livio, M.: 1998, *ApJ* **497**, 168
-

---

Zamanov, R. K., Bode, M. F., Melo, C. H. F., et al.: 2007, *MNRAS* **380**, 1053

Zamanov, R. K., Bode, M. F., Melo, C. H. F., et al.: 2006, *MNRAS* **365**, 1215

Zamanov, R. K., Bode, M. F., Melo, C. H. F., et al.: 2008, *MNRAS* **390**, 377

Zamanov, R. K. and Stoyanov, K. A.: 2012, *Bulgarian Astronomical Journal* **18(3)**, 41