Unverifiable Education Quality Under Labour Market Imperfection

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To my parents.

In the memory of my grandfather and great-grandmother.

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Abstract

This thesis consists of three essays that are concerned with unverifiable education quality under labour market imperfection. In all three essays, we consider a labour market where the monopsonist firm is looking to hire skilled labour. Education/training is the only channel to obtain the skill. The firm faces two-dimensional information asymmetry: the exogenous innate ability (type), and the endogenous choice of quality of education. The contract offered by the firm contains a requirement of presenting a degree as proof of skill acquisition and the wage payment, which consists of a fixed rate and a bonus.

The first essay studies the labour market inefficiency caused by the imperfect competition and the presence of fake degrees. Fake degrees add no value to the worker's productivity, and they are the extreme form of low-quality education. We show that with imperfect competition, the firm makes full use of the fixed wage to extract more rents. As a result, some types are incentivized to buy fake degrees. Once we switch to Bertrand-type competition, the equilibrium contract requires the firms to set a zero fixed rate and give all surplus to the worker. Fake degrees cease to exist in equilibrium, and the distortion in production that was present under monopsony disappears.

The second essay adds in-house training as an instrument for the firm's rent-seeking in addition to the fixed wage. In-house training enables the firm to be assured of the skill acquisition of the worker, but the usage is restricted to its cost. When its cost is weakly less than the cost of a genuine degree, the firm offers only in-house training to extract the full surplus. As the cost of in-house training increases and becomes higher than the cost of a genuine degree, the firm faces a trade-off between using the costly in-house training and using the fixed wage which means giving up a certain rent to higher types. We find that when the cost of in-house training is less than a certain value, the firm prefers to have higher types presenting a degree and relatively lower types trained in-house. When the cost exceeds the certain value, the firm offers contracts such that no type has the incentive to get in-house training.

The third essay extends the first essay by generalizing the low-quality education. Instead of having fake degrees, we consider two levels of education quality, high and low. Low-quality education adds a positive value to productivity but less than the value of high-quality education. We focus on the setting where the social optimum suggests higher types to choose high-quality education and middle types to choose the low-quality education. We find that the labour market imperfection and the information asymmetry cause more types choose no education and fewer types choose the high-quality education compared to the social optimum.

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Declaration

I declare that Chapter II titled, "Fake degrees in the Imperfect Labour Market" has been presented at the 2017 RES Ph.D. Meetings in London.

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Chapter I

Introduction and Literature Review

1 Introduction

This thesis studies an imperfectly competitive labour market where the firm hires a worker with multi-dimension private information to undertake a skilled job. Two novel ingredients of this thesis that worth highlighting are monopsony power of the firm and the worker's choice on quality of education. The former provides the firm with the power in setting the wage, and the latter enriches the dimension of the worker's private information. Education/training is required to do the skilled job, and we assume in this thesis that education/training is the only channel to get the required skill. The skill acquisition is a necessary condition for the worker to be successful in doing the job. Without the skill, the worker cannot complete the task. The firm faces two-dimensional information asymmetry: the exogenous innate ability(type), and the endogenous choice of quality of education.

Since education itself is not observable, it is common to see that a job post requires the applicant to present a degree certificate while applying for the job. For instance, when applying for a doctor or nurse job, the person has to go through the required medical training to be eligible. As a requirement, the employer asks for a certificate as a proof of completion of the training. The problem arises when the employer cannot tell the quality of training the applicant has received by looking at the certificate. As a consequence, the employer may end up hiring a poorly trained worker. In an extreme case, the applicant may not have been trained at all, the case of fake degrees.

There is an alternative illegal way of getting a degree certificate, buying from the diploma mills. The existence of fake degrees adversely affects legitimate universities, consumers, and the broader economy. From the economic point of view, Grolleau et al. (2008) pointed out that for legitimate universities, the existence of fake degrees causes a reduction in market share of the genuine degrees and ruins the reputation of degrees. Getting a fake degree is less costly compared to getting a genuine degree. Apart from the monetary payment, obtaining a genuine degree requires a lot of time and effort. On the other hand, getting a fake degree is much easier. Once the buyer makes the payment, a degree of any level on any subject can be produced within a short time. For the legitimate degree holders, they are forced to compete with applicants that hold fake degrees which affect the probability of hunting a job negatively. The effects on the broader economy lie on two aspects. First, it is costly to investigate and take action against degree mills so that people can protect their intellectual property rights. Second, the firms may end up with hiring unskilled workers when skilled workers were required. For instance, fake physicians were found in the United States[1], the United Kingdom[2], and India[3]. In this case, not only the economy of a country but also the safety and health of its people are at risk. People may lose their confidence in those socially constructive occupations.

In 2015, the New York Times revealed that a Pakistan-based software company, Axact, ran more than 370 websites for "*145 university sites, 41 high schools, and 18 fake accreditation body websites, as well as 121 degree portals*" and sold fake degrees worldwide (Walsh, 2015). The official investigation undertook by Pakistan's Federal Investigation Agency after the exposure showed that Axact received money from over 215,000 people in 197 countries. Similar investigations have been undertaken in Italy, United States, and Vietnam.[4] As the value of the sales reached hundreds of millions of dollars, this was reported to be the largest operation on fake degrees.

Fake degrees have become a major crisis in higher education. It may not be easy to cheat if the degree is purchased and used within one country. However, if one buys a fake British degree and uses in some other countries, the chance of being caught is much smaller. Ezell & Bear (2005) reports that no nation is exempt from diploma mills. In many jurisdictions, selling and buying fake degrees have been defined as criminal activities. However, the attempts to close diploma mills

[1] http://www.nytimes.com/1984/07/13/us/6-arrested-for-fake-medical-degrees-including-3-known-as-doctors.html

[2] http://www.bbc.co.uk/news/uk-40861475

[3] http://www.thehindu.com/news/cities/mumbai/fake-doctor-arrested-in-koparkhairne/article19105671.ece

[4] See Reuters Health (2003) ("*Phony dentists a major problem in Italy*". Northwest Community Healthcare, Reuters Health, March 20, 2003. http://www.nch.org/index.html), Cramer (2004), and The Asian Reporter (2005) ("*More than 1,700 police found using fake degrees*". The Asian Reporter, January 04, 2005).

have not entirely been successful. The significant profit in this industry is one of the reasons that support the existence. "*One international diploma mill, with offices in Europe and the Middle East and mailing addresses in the UK, run by Americans, has sold more than* $450,000$ *degrees—bachelor's, master's, doctorates, medicine, and law—to clients worldwide, who did nothing more than write a check. Their revenues exceeded* $\$450,000,000$" (Ezell & Bear, 2005).

This thesis considers two levels of quality of education, high-quality and low-quality. In Chapter II and III, we take the extreme form of low-quality education, which will be represented by fake degrees. Correspondingly, we call high-quality education degree a genuine degree. Fake degrees add zero value to the worker's productivity and act as an entry ticket to fulfil the firm's requirement on presenting a degree when applying for the job. In Chapter IV, we come back to the more general form of low-quality education which carries a positive value.

Throughout the thesis, we assume that innate ability and quality of education together determine the probability that a worker succeeds in a given job-task. Thereby, creating a given social value attached to the successful completion of the job. Any effect from variable worker's effort in executing the task is, on the contrary, ruled out by assuming that effort at work is exogenously given.[5]

On the demand side of the labour market, before the potential worker has taken her educational decision (on whether to get a degree and of which quality), the firm offers a binding employment contract to execute the job-task mentioned above. The contract comprises of a fixed wage and a bonus, which is conditional upon completing the task. As effort is exogenously given, this structure of worker's compensation plays a different role in our setting than the standard incentivization of effort. Specifically, while the bonus component acts as a powerful tool to incentivize high ability types to choose genuine (high-quality) education, the fixed wage components gives the firm the ability to reduce its expected wage payment, and increase rent extraction and profits.

We now briefly summarize the findings of three chapters of this thesis. In Chapter II, we study the underinvestment in education caused by labour market imperfection and presence of fake degrees. As an extreme form of low-quality education, fake degrees add no value to the worker's productivity. The monopsony power enables the firm to make full use of the fixed wage to extract more rents. However, the uncertainty on the quality of education subtracts some rent from the firm. If the firm can distinguish between a genuine and a fake degree, even the

[5]While endogenous effort might be incorporated in the analysis at the cost of unnecessary complexity, our focus on ability and education quality does not seem inappropriate for highly skilled jobs which require high qualification and ability.

firm does not know the type of the worker, the firm can extract the full surplus by setting the fixed wage equal to the cost of a genuine degree. When the firm is unable to verify the quality of education, the worker takes away some information rent, and the inefficiency in the labour market is generated.

The trade-off faced by the firm while using the fixed wage as a powerful tool to extract rent is as follows. If the firm sets a higher fixed wage, then expected wage payment could be reduced. The firm is interested in incentivizing some high ability types to get a genuine degree. Those types have a higher probability of getting the bonus. Thus, setting a high fixed wage can bring down the expected wage payment through setting a lower corresponding bonus. However, if the high fixed wage exceeds the cost of a fake degree, then all types are "subsidized" with a positive utility. Given that the cost of a fake degree is less than the fixed wage, all types prefer a fake degree over no education as buying a fake degree gives the worker a positive utility. Now if the firm wants to incentivize higher types to get a genuine degree, it has to offer a higher expected payment to exceed the positive utility from buying a fake degree.

Therefore, with known cost of a fake degree, the firm offers the fixed wage equal to the cost of a fake degree. It is the highest level of fixed wage which does not give any type incentive to buy a fake degree. With unknown cost of a fake degree, the firm balances the trade-off concerning its belief of the cost of a fake degree. In a case where the firm assigns a sufficiently low probability of zero cost of a fake degree, the firm sets fixed wage positive. If the realization of the cost is below the fixed wage, then the firm may end up with hiring the worker with a fake degree. If the cost is no less than the fixed wage, then the firm is able to reduce the expected payment without the fear of hiring the worker of a fake degree.

We then introduce competition to the labour market and consider the Bertrand-type competition. The equilibrium contract requires the firms to set a zero fixed rate and give all surplus to the worker. When we switch from imperfect to perfect competitive labour market, fake degrees cease to exist in equilibrium, and the efficiency in the labour market is restored. It further confirms that the main source of the underinvestment in education and existence of fake degrees in equilibrium is the monopsony power of the firm, through which it can set the fixed wage freely to extract more rent.

Chapter III is based on the same model with monopsony as described in Chapter II but adds in-house training as an additional instrument for the firm to extract more rent. Training the worker in-house gives the firm accurate information about the quality of education or the skill acquisition of the worker. However, whether the in-house training is an efficient tool depends on its cost. When its cost is weakly less than the cost of a genuine degree, the firm can extract the full surplus by offering only in-house training and covering the cost. When the cost of in-house training exceeds the cost of a genuine degree, the firm faces a trade-off between using the costly in-house training and the fixed wage to extract more rent. The latter is used in the same way as we discussed in Chapter II that the firm has to balance between the expected payment and the risk of hiring the worker with a fake degree. On the other hand, offering in-house training is more costly, but the firm is able to extract all the remaining rent from the worker. We find that when the cost of in-house training is less than a certain value, the firm prefers to have higher types presenting a degree and relatively lower types trained in-house. By doing so, the set of higher types, to who the firm has to give information rent, is kept small. When the cost exceeds the certain value, it is no longer beneficial for the firm to use in-house training. The firm then offers contracts such that no type has the incentive to get in-house training.

When the cost of in-house training is sufficiently low, offering in-house training not only helps the firm to extract more rent but also improves the underinvestment in education. Again, the firm is able to use the in-house training and the fixed wage because of the monopsony power, without which the perfectly competitive market will lead us to the first best outcome. In that case, the firm is left with zero rent.

Chapter IV extends Chapter II by switching back to the more generalized form of low-quality education. Instead of having fake degrees, which carries no productive value, we consider two levels of education quality, high and low. Both high and low-quality education add positive value to productivity, but the high-quality education has more value than the low-quality education. We focus on the setting where the social optimum suggests higher types to choose high-quality education and middle types to choose the low-quality education. Since low-quality education now carries a positive value, and it is socially optimal to have low-quality education in equilibrium, the use of fixed wage is ambiguous. The firm is no longer facing the same trade-off as discussed in Chapter II and III while setting the fixed wage. We find that the labour market imperfection and the information asymmetry cause more types are incentivized to choose no education, and fewer types choose the high-quality education compared to the social optimum.

2 Related Literature

In this section, we first review the literature on the imperfect labour market and then show some related literature on the economics of education. We also discuss the theoretical and empirical literature on and economics of fake degrees. However, the accessibility to the market of fake degrees is limited due to its clandestine nature. Moreover, the phenomenon of having fake degree holders around does not receive enough attention for many reasons including lack of evidence. Hence, the literature on fake degrees is rather small.

2.1 Literature on Monopsonistic Competition in Labour Market

Traditionally, the labour market in economics was considered perfectly competitive. The interest in monopsony in labour markets has revived with the review by Boal and Ransom (1997) (a static approach) and Manning (2003) (a dynamic approach) since it was first mentioned in Robinson (1969). In contrast to the perfect competitive labour market where the labour supply curve is perfectly elastic, the upward-sloping labour supply curve in an imperfectly competitive labour market provides the monopsony power for the firm to set the wage. The sources of monopsony power come from fewer competitors (Beck, 1995; Yett, 1970), classic differentiation (Diamond, 1971), moving cost (Ioannides and Pissarides, 1985; Black and Loewenstein, 1991; Ransom, 1993), or job search (Albrecht and Axell, 1984; Burdett and Mortensen, 1989).

Many puzzles in the labour market literature can be explained by the monopsony power. Wage dispersion (Idson and Oi, 1999; Bhaskar and To, 2001; Blanchflower et al., 1996), racial pay gap (Altonji and Blank, 1999; Becker, 1957), and provision of general training (Steve, 1994; Acemoglu and Pischke, 1999). In Becker (1964) and Mincer (1974)'s standard theory, general training is considered to be paid by workers. The reason is that employees' benefit of having general training is the improvement in productivity for various job opportunities. Hence, in a competitive labour market, it is natural for the worker to expect higher earnings after the training. Firm-specific training does not raise wages for workers elsewhere, and hence, the cost should be covered by the firm. However, some theoretical findings project that when facing imperfect competition in labour market with compressed wage structure, firms may pay for the general training as well (Acemoglu and Pischke 1998 and 1999, Katz and Ziderman, 1990 and Stevens, 1994). In a monopsonistic labour market, workers may not be given enough incentives to invest in general training as their wage is below their marginal product.

In this case, employers may have incentives to finance the general training since employers can reap positive returns from their investment in worker's training (Bhaskar et al., 2002).

All three essays in this thesis consider monopsony in labour market and notice the same outcome that the firm pays for the cost of general training. In addition to the phenomena mentioned above which can be explained by monopsony power, in this thesis, we also discovered that the existence of fake degrees in equilibrium in the labour market is highly related to the firm's monopsony power in setting the wage.

2.2 Literature on Economics of Education

There is a theoretical debate between the human capital theories (Becker, 1962; Becker, 1994; Schultz, 1961) and credentialism (Berg, 1970; Arrow, 1973; Collins, 1979) of education. The essential assumption of the human capital theory of education is that the rising demand for skilled labour is the driving force of increasing demand for schooling. Unless the cost of getting educated is greater than the benefit, pursuing education will always be needed. Credentialist, on the contrary, points out that more lucrative jobs are allocated to people who received more years of schooling, but not inevitably as a consequence of their skills or higher productivity. As a result, the desire for better jobs promotes the demand for education, which has little connection with skill requirements. This thesis employs both the assumption from the human capital theories of education, as well as the view of credentialism that education degrees are acting as a signal. We assume that education equips employees with a particular skill (skills), which is necessary to be successful in the job. To ensure that the job applicant is equipped with the required skill, the employer requests the applicant to present the educational degree as proof.

The positive relationship between education and productivity indicates that highly educated workers are more likely to get higher wages (Becker, 1962 & 1975, Schultz, 1972). Apart from the impact on productivity, Spence (1973) argues that education also acts as a signal to earn higher payments. Workers signal their abilities by earning degrees or diplomas, and employers require degrees from workers as a proof of the productivity or social status (see also Solnick and Hemenway, 1998, Fershtman, 2008, and Marginson, 2004). In pure signalling games, education is used only for signalling purposes, and there is no productivity-enhancing effect. What we add to this stream of literature, the presence of fake degrees, however, may adversely affect the signalling role of education. The reason is that the cost of education would not prevent low types from acquiring a fake degree and the firm cannot distinguish from a genuine one.

In the theory of incentives, as stated in Laffont and Martimort (2002), there are three types of private information: (i) hidden knowledge or adverse selection, where the agent privately knows her cost or valuation (Akerlof, 1970); (ii) hidden action or moral hazard, where the agent takes an action which is unobservable by the principal (Arrow, 1963); or (iii) nonverifiability, where both the agent and the principal know the ex-post information, but no third party can observe (Grossman and Hart, 1986 and Hart and Moore, 1988). The problem studied in this thesis does not fit in any of those standard categories. It may look like a mixed case, where we have both hidden knowledge (the innate ability of the worker) and action (quality of education). As mentioned before, we do not consider the effort level as one of the determinants of worker's productivity.

2.3 Literature on Fake Degrees

The theoretical literature on fake degrees is almost nonexistent. Grolleau et al. (2008) apply Akerlof (1970)'s adverse selection model to explain the existence of fake degrees. There are three categories of education, high quality, low quality, and fake degree. Fake degrees compete directly with the low-quality education but not with the high-quality education. The cost of getting the low-quality education is higher compared with the cost of a fake degree due to the investment in reputation to distinguish between low-quality education and fake degrees. Since the employer cannot differentiate between the low-quality degrees and fake degrees, it offers the average wage to the applicant. The average wage lowers the wage for the low-quality education and increases the wage for fake degree holders. When degrees are indistinguishable, the equilibrium wage drives out the low-quality education from the market.

Due to unavailability of the data on diploma mills and fake degrees, the empirical literature is limited. Attewell and Domina (2010) used the National Education Longitudinal Study (NELS)'s transcript data to check the differences between self-reported educational attainment from the students and transcript verified degrees from the data. Fake is categorized when the students failed to show transcript or other verification measures. They found that the incidence of fake degrees is 6% and 35% for Bachelor's and Associate's degrees respectively, and the percent of fake vocational certificates is 73%.

Chapter II

Fake Degrees in the Imperfect Labour Market

1 Introduction

This chapter is concerned with underinvestment in education and the existence of fake degrees in equilibrium in an imperfectly competitive labour market. Fake degrees are the extreme form of low-quality education which is used as an entry ticket to the labour market but adds no value to the productivity. We consider a theoretical framework with two-dimensional information asymmetry. One dimension is the exogenous private information presenting the innate ability of the worker, and the other is the endogenous private information presenting the worker's education quality. The wage offered by the firm consists of a fixed wage and a bonus, and the bonus is paid based on the worker's performance. By signing the contract, the firm agrees on paying a fixed wage to the worker. If the worker completes the job, then the worker gets the bonus. Otherwise, the fixed wage is the only payment. Since fake degrees have no productive value, a worker with a fake degree has zero probability of getting the bonus. Hence, the value of the fixed wage is key to the existence of a fake degree in equilibrium.

The firm faces a trade-off while choosing the fixed wage to extract more rent. On the one hand, a higher fixed wage helps in lowering the expected wage bill. Since the firm would like to have relatively higher types employed with genuine degrees and these types have a greater probability of getting the bonus, setting a higher fixed wage helps in lowering the bonus as well as the expected wage bill. However, a higher fixed wage also implies a higher probability of hiring the worker with a fake degree. If the fixed wage is set above the cost of a fake degree, then all types prefer a fake degree over no education. The firm "subsidies" all types with a positive wage bill. Moreover, to have some types be incentivized to get a genuine degree, the firm has to offer a higher wage as they now get a positive utility from buying a fake degree.

Labour market imperfection is the source of power that the firm has over the fixed wage. Under monopsony, the firm has the full power to use the preferred instrument, fixed wage, to make a higher profit. When the quality of education is verifiable, the firm sets the fixed wage up to the cost of a genuine degree to extract the full surplus. When the firm is unable to distinguish between a genuine and a fake degree but knows the cost of a fake degree, the fixed wage is set equal to the cost of a fake degree, giving no incentive to any type to buy a fake degree. When the cost of a fake degree is privately known to the worker, the firm sets a positive fixed wage if it believes that the probability of zero cost of a fake degree is sufficiently low. In this case, the information asymmetry leads to a result that the contract may incentivize some lower types to buy a fake degree, giving rise to the existence of fake degrees in equilibrium.

Under Bertrand-type competition, the competition takes away the firm's power over wage setting. As competition is introduced, firms offer a zero fixed wage, and hence, fake degrees cease to exist. Moreover, the distortion in production that was present under monopsony vanishes as we move to Bertrand-type competition. This coincides with the literature of the monopoly distortion that as competition intensifies, the distortion is restored (Boal and Ransom, 1997).

The rest of this chapter is as follows. We set up the model and discuss the social planner's problem in Section 2. Section 3 considers the monopsonist's profit maximization problem under both verifiable and unverifiable education degree scenarios. Then in Section 4, we switch from monopsony to Bertrand-type competition. Section 5 concludes the chapter.

2 Model

Consider a labour market where a firm(s) is looking to hire a worker with a certain skill. The worker can complete the task and generate a gross return V to the firm with a certain probability, which is determined by the acquisition of the certain skill and her innate ability. If the worker fails to complete the job, then it gets zero. Education is the only channel to obtain the required skill for the job. After getting the genuine education, the worker will be awarded a genuine degree. Let e be the educational choice. The choice set includes no degree, n, a fake degree, k, and a genuine degree, g, $e \in \{n, k, g\}$. The cost of a genuine degree, c, is higher than the cost of a fake degree, λc, where $0 \leqslant \lambda < 1$. Getting a fake degree does not equip the worker with the certain skill. Hence, the probability of being successful of a fake degree holder is the same as an uneducated worker, zero. Only a genuinely educated worker may have a positive probability of being successful in doing the job, and the probability is determined by the worker's innate ability. Nature decides the innate ability of the worker which we refer to as type in the model. Designate the type of the worker by θ. For an educated worker, the higher the type, the higher the probability of being successful. For simplicity, let the probability of being successful also be θ. Generalization is possible but does not affect quantitative results provided.

3 The Social Planner's Problem

Given that the worker has productivity that is determined both exogenously (type) and endogenously (education), we now describe the social planner's problem. The social planner chooses the education for each type, $e(\theta)$, and then decides the employment status by the allocation function, $x(\theta, e(\theta))$.

$$\max_{e(\theta), x(\theta, e(\theta))} SW = x(\theta, e(\theta))[\theta V - c]$$

The allocation function takes a value between $[0, 1]$. 1 represents that the type θ is employed by the firm, and 0 means unemployed. Since getting a fake degree generates a zero or negative social surplus, the planner will not choose a fake degree for any type. Let θ^* be the type that contributes a zero social surplus after getting a genuine degree and it is defined by $\theta^* V - c = 0$. We assume $c < V$, which implies that there exists a set of types for whom genuine education has a social value, $\theta^* < 1$. The monotonicity of θ implies that if a type is greater than θ^*, getting a genuine degree helps her to provide a positive surplus and vice versa. We assume that the planner allocates the type of worker to have a genuine degree if the type supplies a zero social surplus. By solving the social welfare maximization problem, we get the first best allocation for education.

$$e(\theta) = \begin{cases} g & \text{if } \theta \geqslant \theta^* \\ n & \text{if } \theta < \theta^* \end{cases}$$

Denote the first best allocation for employment status as $x^{FB}(\theta, e)$, and we have

$$x^{FB}(\theta, e) = \begin{cases} 1 & \text{if } \theta \geqslant \theta^* \text{ and } e = g \\ 0 & \text{if } \theta < \theta^* \text{ and } e = n \end{cases}$$

In words, the efficient allocation is that a type which is greater than or equal to θ^* will be allocated to have a genuine degree, and will be employed by the firm. If the type is below θ^*, then this worker will be allocated to have no education and stay unemployed.

We now look at two alternative labour markets, monopsony and Bertrand-type competition.

4 Monopsony

Let us now discuss the firm's problem under imperfect competition in the labour market. The monopsonist does not know the type of the worker, but it knows that the type is distributed between $[0, 1]$ with a cumulative distribution function $F(\theta)$ and a density $f(\theta)$. One way to avoid hiring unskilled labour is to request the worker to present a degree certificate to show her educational status. If the worker fails to provide any degree, she will stay unemployed. The worker has three options: undertake costly genuine education, remain uneducated but buy a fake degree, or remain uneducated without any degree. The employer cannot distinguish between a genuinely skilled/educated and the worker with a fake degree.

We consider a setting where the firm announces a take-it-or-leave-it contract ex-ante so that the worker is well informed about the wage payment before she makes a choice. The contract includes a requirement for the educational degree and the wage payment. The degree requirement states that if the worker cannot present a degree, then she will not be employed. The worker will be employed with certainty if she can present a degree.[6] The worker observes the contract and then chooses her educational status, to get a genuine degree, a fake degree, or stay with no degree. The firm has to honor the contract and hire the worker if she shows up with any degree. If hired, the employee works on the project, and then the outcomes are realized. The worker generates V when successful (with probability θ) and zero otherwise (with probability $1 - \theta$). After the outcomes are

[6]Without loss of generality, we only discuss the case that the firm recruits the worker with probability 1 if a degree is presented. For the cases where the probability is below 1, the firm can compensate through wages which will have the same effect as we assumed that probability is equal to 1.

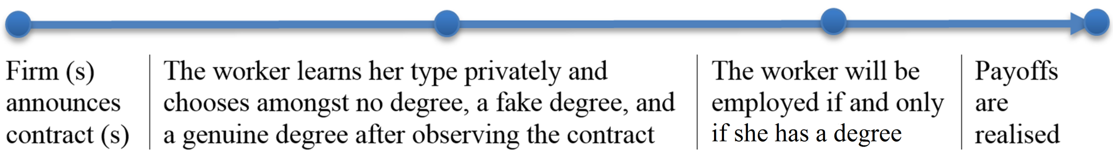

Figure 1: Timing

realized, the firm makes the payment according to the contract. The timing of the game is presented in Figure 1. Note that this timing also works for the next section when we consider the Bertrand-type competition.

The Firm

The firm offers a wage contract (w_S, w_F). If a type θ worker accomplishes the job successfully, then she gets w_S. Otherwise, w_F is the payment. Note that w_F is the fixed wage, and $w_S - w_F$ is the bonus. The expected profit from hiring a type θ worker is

$$\begin{array}{ll} \pi(g,\theta) = \theta V - \theta w_S - (1-\theta) w_F & \text{if } e = g \\ \pi(k,\theta) = -w_F & \text{if } e = k \end{array}$$

If the type θ worker holds a genuine degree, with probability θ, the firm gets V and pays w_S, and with probability $(1-\theta)$, the firm gets 0 and pays w_F. If this worker holds a fake degree, the probability of being successful is zero. Then the firm gets 0 and pays w_F with probability 1.

The expected profit $\pi(g,\theta)$ can be rewritten as

$$\pi(g,\theta) = -w_F + \theta V - \theta (w_S - w_F) \tag{1}$$

The interpretation of expression (1) is that by signing the contract, the firm is agreeing to pay a fixed wage w_F to the worker. If the type θ worker succeeds in accomplishing the job (with probability θ), the firm then gets V and rewards the worker a bonus equals to $(w_S - w_F)$.

The Worker

The reservation utility for all types is normalized to zero. If the worker chooses to have no degree, she stays with the reservation utility. If a type θ worker decides to have either a fake degree or a genuine degree, given a contract (w_S, w_F), the expected utilities, $u(e,\theta)$, are given by,

$$\begin{array}{ll} u(g,\theta) = \theta w_S + (1-\theta) w_F - c & \text{if } e = g \\ u(k,\theta) = w_F - \lambda c & \text{if } e = k \end{array}$$

If a type θ worker chooses to pay a cost c to get the genuine degree, then with probability θ, she completes the job successfully and gets w_S. With probability $(1-\theta)$, she fails and gets w_F. If this type θ worker chooses to buy a fake degree, then the probability of being successful is zero. She needs to pay λc for the fake degree and gets w_F from the employer.

Similarly, the wage scheme $\theta w_S + (1-\theta) w_F$ is equivalent to a fixed wage, w_F, plus a possible bonus, $(w_S - w_F)$. All types get the fixed wage while showing up with any degree. However, only educated types get the bonus with a probability θ. Hence, the expected utility for a type θ worker who has a genuine degree is,

$$u(g,\theta) = \theta(w_S - w_F) + w_F - c \tag{2}$$

According to the monotonicity of the probability, the expected utility of a type θ worker is monotonically increasing (decreasing) in θ if $w_S - w_F > 0$ ($w_S - w_F < 0$).

We now define the firm's problem. The firm chooses (w_S, w_F) to induce the worker to choose e so that the expected profit is maximized. Let Θ^g be the set of types that prefer a genuine degree, and Θ^k be the set of types that choose a fake degree. The firm's problem can be written as

$$\max_{(w_S,w_F)} \left[\begin{array}{l} \int_{\Theta^g} (\theta V - (\theta w_S + (1-\theta) w_F)) f(\theta) d\theta \\ -\int_{\Theta^k} w_F f(\theta) d\theta \end{array} \right]$$

subject to IR, the individual-rationality, and IC, the incentive-compatibility constraints.

$$\begin{array}{ll} \text{For } \theta \in \Theta^g & \left\{ \begin{array}{ll} IR & u(g,\theta) \geqslant 0 \\ IC & u(g,\theta) \geqslant \max\{u(k,\theta), u(n,\theta)\} \end{array} \right. \\ \text{For } \theta \in \Theta^k & \left\{ \begin{array}{ll} IR & u(k,\theta) \geqslant 0 \\ IC & u(k,\theta) \geqslant \max\{u(g,\theta), u(n,\theta)\} \end{array} \right. \end{array}$$

We now present a benchmark model where education is verifiable. We show that when the monopsonist can distinguish between a genuine and fake degree at zero cost of verification, the firm is able to extract the full surplus. Before we proceed further, let us first define *Full Surplus Extraction*.

Definition II.1 *(full surplus extraction) The firm extracts the full surplus if and only if:*
(a) A worker of type θ is educated if $\theta \geqslant \theta^*$ and is uneducated if $\theta < \theta^*$.

$$e(\theta) = \left\{ \begin{array}{ll} g & \textit{if } \theta \geqslant \theta^* \\ n & \textit{if } \theta < \theta^* \end{array} \right.$$

$$\textit{where } \theta^* V - c = 0$$

(b) The allocation is at the first best level.

$$x(\theta, e) = \begin{cases} 1 & \text{if } \theta \geqslant \theta^* \text{ and } e = g \\ 0 & \text{if } \theta < \theta^* \text{ and } e = n \end{cases}$$

(c) For all $\theta > \theta^*$, $u(g, \theta) = 0$.

The firm extracts the full surplus if the allocation of all types is the same as the first best level and no type gets a positive rent.

Proposition II.1 (full surplus extraction) *With zero cost of verification, there exists an equilibrium that the firm extracts the full surplus with a contract (w_S, w_F) where $w_S = w_F = c$.*

Proof. *Since the firm verifies the degrees at zero cost, the worker without a degree or with a fake degree will not be employed. If a type chooses a fake degree, then the utility is $u(k, \theta) = -\lambda c \leqslant 0$. If a type θ worker chooses to have a genuine degree, then the expected utility is given by*

$$u(g, \theta) = \theta c + (1 - \theta) c - c = 0$$

All types get the same utility, zero, from either no degree or a genuine degree. If all types follow the first best allocation, then the firm extracts the full surplus. ■

With the contract $w_S = w_F = c$, all types are indifferent between a genuine degree and no degree. We have infinitely many equilibria, and one of the equilibria generates the full surplus to the firm when all types follow the first best allocation. For the firm to achieve full surplus extraction, we assume that if a type is indifferent between a genuine degree and no degree, then she chooses a genuine degree if $\theta \geqslant \theta^*$. Otherwise, no degree.[7]

We have shown that even the firm does not know the type of the worker, if education is verified at zero cost, the firm can extract the full surplus. However, when the firm is unable to verify degrees, full surplus extraction is no longer possible. Let us now move to the main focus of this chapter and discuss what happens when the firm cannot verify the degrees.

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[7] Alternatively, consider a contract $w_S = c + \left(\frac{1-\theta^*}{\theta^*}\right)\varepsilon$ and $w_F = c - \varepsilon$. Type θ^* is indifferent between a genuine degree and no degree as $u(g, \theta^*) = \theta^* \left(c + \left(\frac{1-\theta^*}{\theta^*}\right)\varepsilon - (c - \varepsilon)\right) + c - \varepsilon - c = 0$, but all types above θ^* strictly prefer a genuine degree. For an arbitrarily small ε, the firm can approximately extract the full surplus.

4.1 Unverifiable Educational Degrees

When education is unverifiable, with the contract $w_S = w_F = c$, all types would like to get a fake degree and enjoy a positive utility by pretending to be a genuine degree holder. We now check how does the firm solve the profit maximization problem by choosing w_S and w_F. In the following, we first introduce the case where the cost of a fake degree is known. Then we proceed with the unknown cost of a fake degree. We assume that in the case of a tie, the worker always prefers a genuine degree over the other two, and prefer no education over a fake degree.

Furthermore, we claim that an optimal contract requires $w_S > w_F$. The intuition is as follows, suppose that the firm sets $w_S \leqslant w_F \leqslant 0$, then no type has an incentive to apply for this job as the reservation utility (zero) is no less than the expected utility of applying for the job. If the firm sets $w_S \leqslant w_F$ and $w_F > 0$, all types prefer a fake degree over a genuine degree because the expected utility is higher when they buy fake degrees, $\theta w_S + (1-\theta) w_F - c < w_F - \lambda c$. In this case, the firm earns a negative profit, $\pi(k, \theta) = -w_F < 0$. Hence, only $w_S > w_F$ may give a positive expected profit to the firm.

4.1.1 Cost of a fake degree known

In this case, the firm knows about the cost of a fake degree, λc, where $0 \leqslant \lambda < 1$. Let θ_λ be the cutoff type which is implicitly defined by

$$\theta_\lambda w_S + (1-\theta_\lambda) w_F - c = \max\{w_F - \lambda c, 0\} \tag{3}$$

The left-hand side of equation (3) is the expected utility of type θ_λ from choosing a genuine degree, and the right-hand side is the maximum utility between getting a fake degree and remain with no degree. All types greater than or equal to θ_λ prefer a genuine degree over the other two options given the monotonicity of the probability θ and $w_S > w_F$. Types below θ_λ choose no degree if $w_F - \lambda c \leqslant 0$, and a fake degree if $w_F - \lambda c > 0$. With $w_F - \lambda c > 0$, the firm not only attracts lower types to choose a fake degree but also increases the utility of the outside options than a genuine degree for all types.

The firm's problem is to solve

$$\max_{w_S, w_F} \left[\begin{array}{c} \underbrace{-\int_0^{\theta_\lambda} (w_F) f(\theta) \, d\theta}_{\text{fake degrees}} \\ \underbrace{+\int_{\theta_\lambda}^1 (\theta V - \theta w_S - (1-\theta) w_F) f(\theta) \, d\theta}_{\text{genuine degrees}} \end{array} \right] \qquad \text{(if } w_F > \lambda c\text{)}$$

or

$$\max_{w_S, w_F} \left[\int_{\theta_\lambda}^1 (\theta V - \theta w_S - (1-\theta) w_F) f(\theta) \, d\theta \right] \qquad \text{(if } w_F \leqslant \lambda c\text{)}$$

With the contract (w_S, w_F), which gives a cutoff type θ_λ, the firm's expected revenue is $\int_{\theta_\lambda}^1 (\theta V) f(\theta) \, d\theta$. The expected payments of employing a type θ worker are given by

$$\begin{aligned} E(w_F > \lambda c) &= F(\theta_\lambda) w_F + (1 - F(\theta_\lambda)) (\theta w_S + (1-\theta) w_F) \\ E(w_F \leqslant \lambda c) &= (1 - F(\theta_\lambda)) (\theta w_S + (1-\theta) w_F) \end{aligned}$$

With probability $F(\theta_\lambda)$, θ is below the cutoff type θ_λ. Then the worker chooses a fake degree if $w_F > \lambda c$, and no education if $w_F \leqslant \lambda c$. With probability $(1 - F(\theta_\lambda))$, θ is greater than or equal to θ_λ, and the type θ chooses a genuine degree.

Proposition II.2 *Suppose education is unverifiable, but the cost of a fake degree is known by the monopsonist. Then the monopsonist maximizes expected profit by setting $w_F^* = \lambda c$.*
Proof. *See Appendix* 1.*A*. ■

When w_F is not above λc, no type has an incentive to buy a fake degree. Types greater than the cut-off type prefer a genuine degree, and these types have a high probability of being successful. Hence, the higher the w_S, the higher the expected payment the firm needs to pay. w_S is a more costly instrument for the firm to extract rents. The firm sets a high w_F so that the corresponding w_S can be low to minimize the expected payment. However, if the firm increases w_F above λc, then the contract attracts lower types to get a fake degree, which makes the firm worse-off. In addition, to attract higher types to get a genuine degree, the firm has to offer a higher wage as those types can also buy a fake degree to get a positive utility rather than stay with the zero reservation utility. Hence, between lowering the expected payment and hiring a fake degree holder, the firm sets the highest possible w_F, $w_F^* = \lambda c$, which does not give incentive to any type to get a fake degree.

Proposition II.3 (underinvestment in education) *The monopsonist's profit maximizing cut-off type, θ_λ^*, is greater than the social optimal cut-off type θ^*.*
Proof. *Given that $w_F^* = \lambda c$, the firm's problem now becomes,*

$$\max_{w_S}\left[\int_{\theta_\lambda}^{1}\left(\theta\left(V-\left(w_S-\lambda c\right)\right)-\lambda c\right)f\left(\theta\right)d\theta\right]$$
$$s.t.\ \theta_\lambda\left(w_S-\lambda c\right)+\lambda c-c=0$$

$\theta_\lambda\left(w_S-\lambda c\right)+\lambda c-c=0$ yields $w_S = \frac{c-\lambda c}{\theta_\lambda}+\lambda c$. Substitute away w_S to get the firm's problem as

$$\begin{aligned}&\max_{\theta_\lambda}\left[\int_{\theta_\lambda}^{1}\left(\theta\left(V-\left(\frac{c-\lambda c}{\theta_\lambda}+\lambda c-\lambda c\right)\right)-\lambda c\right)f\left(\theta\right)d\theta\right]\\ =\ &\max_{\theta_\lambda}\left[\int_{\theta_\lambda}^{1}\left(\theta\left(V-\frac{c-\lambda c}{\theta_\lambda}\right)-\lambda c\right)f\left(\theta\right)d\theta\right]\end{aligned}$$

Apply the first order condition w.r.t. θ_λ to get

$$-\left(\theta_\lambda^* V-c\right)\underbrace{f\left(\theta_\lambda^*\right)}_{Positive}+\underbrace{\int_{\theta_\lambda^*}^{1}\left(\frac{\left(c-\lambda c\right)\theta}{\theta_\lambda^{*2}}\right)f\left(\theta\right)d\theta}_{Positive}=0 \tag{4}$$

The necessary condition for equation (4) to hold is $\theta_\lambda^* V-c>0$. As $\theta^* V-c=0$, we get $\theta_\lambda^*>\theta^*$. ■

While facing the trade-off between efficiency and information rent, the monopsonist in the labour market offers a contract that generates a distortion in production as $\theta_\lambda^*>\theta^*$. It implies that with unverifiable education, the monopolist tends not to employ some lower types $\theta\in(\theta^*,\theta_\lambda^*)$, which were socially efficient to be educated. The reason behind the underinvestment in education is that the firm is forced to give some rents to some higher types due to information asymmetry. The higher the type, the greater the rent it receives. Hence, the firm chooses not to employ a set of types, $[\theta^*.\theta_\lambda^*)$, to save some rent given to the worker.

The value of λ determines the level of underinvestment in education. We now show that the lower the cost of a fake degree, the higher the underinvestment in education.

From equation (4), we know that

$$\lim_{\lambda \to 1} \int_{\theta_\lambda^*}^{1} \left(\frac{(c - \lambda c)\,\theta}{\theta_\lambda^{*2}} \right) f(\theta)\, d\theta = 0$$

It implies that for equation (4) to hold as equality, we must have $\theta_\lambda^* V - c = 0$, $\theta_\lambda^* = \frac{c}{V} = \theta^*$. When $\lambda = 0$, the optimal cutoff type, $\theta_{\lambda=0}^*$, is given by

$$-\left(\theta_{\lambda=0}^* V - c\right) f\left(\theta_{\lambda=0}^*\right) + \int_{\theta_{\lambda=0}^*}^{1} \left(\frac{c\theta}{\theta_{\lambda=0}^{*2}} \right) f(\theta)\, d\theta \;= 0$$

Proposition II.4 (underinvestment in education and λ) *Let $f'(\theta) > 0$ for all $\theta \in [0,1]$. As λ decreases from 1 to 0, the optimal cutoff type θ_λ^* increases from θ^* to $\theta_{\lambda=0}^*$.*
Proof. *Let us now compute $\frac{d\theta_\lambda^*}{d\lambda}$ by implicit differentiation to see the comparative statics for $\lambda \in [0.1]$. Let $\psi(\theta_\lambda^*, \lambda)$ be the first order condition we obtained in (4),*

$$\psi\left(\theta_\lambda^*, \lambda\right) = -\left(\theta_\lambda^* V - c\right) f\left(\theta_\lambda^*\right) + \int_{\theta_\lambda^*}^{1} \left(\frac{(c - \lambda c)\,\theta}{\theta_\lambda^{*2}} \right) f(\theta)\, d\theta = 0$$

Then we can write

$$\begin{aligned} & \frac{d\theta_\lambda^*}{d\lambda} \\ = & -\frac{d\psi\left(\theta_\lambda^*, \lambda\right)/d\lambda}{d\psi\left(\theta_\lambda^*, \lambda\right)/d\theta_\lambda^*} \\ = & -\frac{-\int_{\theta_\lambda^*}^{1}\left(\frac{\theta}{\theta_\lambda^{*2}}\right) f(\theta)\, d\theta}{-f\left(\theta_\lambda^*\right) - \left(\theta_\lambda^* V - c\right) f'\left(\theta_\lambda^*\right) - \left(\frac{c - \lambda c}{\theta_\lambda^*}\right) f\left(\theta_\lambda^*\right) - 2\int_{\theta_\lambda^*}^{1}\left(\frac{(c-\lambda c)\theta}{\theta_\lambda^{*3}}\right) f(\theta)\, d\theta} < 0 \end{aligned}$$

which shows that as λ decreases from 1 to 0, the optimal cutoff type θ_λ^* increases from θ^* to $\theta_{\lambda=0}^*$. ■

Note that $f'(\theta) > 0$ for all $\theta \in [0,1]$ is a sufficient but not necessary condition. Proposition II.4 indicates that as the cost of a fake degree rises, the effectiveness of using fake degrees to exploit the market falls. As shown in Proposition II.2, the firm maximizes profit by setting the highest possible w_F which attracts no fake degrees, $w_F^* = \lambda c$. Given that $\lambda < 1$, the worker's incentive of getting a fake degree restricts the firm's optimal w_F to be below c. The corresponding w_S, a more costly instrument for the firm, has to rise above c to compensate for the lower w_F so that the contract still incentivizes some higher types to get a genuine degree.

As discussed in Proposition II.3, the firm now offers positive information rent to some higher types given that the bonus $(w_S - w_F)$ is positive. The inefficiency occurred while the firm minimizes the information rent by choosing not to employ some lower types. The smaller the value of λ, the greater the bonus $(w_S - w_F)$. It implies that higher types receive a greater information rent as λ increases. As a consequence, the desire to minimize the information rent leads the firm to exclude more types it wants to employ. That is, the higher the cost of a fake degree, the lower the underinvestment in education. When $\lambda = 0$, the firm has to offer the highest w_S to cover the cost of a genuine education for a certain set of types. Then, the labour market faces a highest inefficiency level where the cutoff type is given by $\theta^*_{\lambda=0}$. As the cost of a fake degree increases, the inefficiency restores. We move towards the full efficiency as λ approaching 1.

In this case with known cost of a fake degree, we have shown that the firm offers w_F exactly equal to the cost of a fake degree to avoid hiring a fake degree holder. We now investigate the unknown cost of a fake degree case to see if there exist fake degrees in the equilibrium.

4.1.2 Cost of a fake degree unknown

We now consider the case where the cost of a fake degree λc is unknown by the firm. Specifically, assume that the firm assigns probability μ for $\lambda = 0$, and probability $(1-\mu)$ for $\bar{\lambda}$. From the known cost of a fake degree case, we have learned that the firm sets w_F equal to the cost of a fake degree to maximize the expected profit, as well as excluding the fake degrees. Using the same argument, we get that [1] when the firm assigns probability 1 to zero cost of a fake degree, then it maximizes profit by setting $w_F^{\mu=1} = 0$; [2] when the firm assigns probability 0 to zero-fake degree cost, setting $w_F^{\mu=0} = \bar{\lambda}c$ maximizes firm's profit. In both cases, the firm does not expect to see a fake degree holder because the wage provides no incentive for any type to buy a fake degree. Hence, in these two extreme cases, $\mu = 0$ and $\mu = 1$, the fake degree does not exist in the equilibrium.

From Proposition II.2, we learned that the firm's expected payment decreases in w_F, and the firm does not offer w_F too high to avoid free riding. Given that the range of the cost of a fake degree is $[0, \bar{\lambda}c]$, w_F must lie between $[0, \bar{\lambda}c]$ in this unknown cost of a fake degree case. If the firm sets $w_F = 0$, then no type has an incentive to get a fake degree. The reason is that if fake degrees are costly ($\lambda > 0$), then there is no incentive for any type to get one. If fake degrees are free of charge ($\lambda = 0$), then choosing no degree or a fake degree gives the same utility. As we assumed above, no type will go for a fake degree if she gets zero utility. Hence, to examine the existence of fake degrees, we check for the possibility that the optimal w_F is positive when the firm holds a positive belief about the cost of a fake degree not being zero, $\mu > 0$.

For any contract (w_S, w_F) with $w_F \in (0, \bar{\lambda}c]$, a type θ worker prefers a genuine degree over the other two alternatives (fake degree and no degree) if

$$\theta w_S + (1-\theta)\, w_F - c \geqslant \max\{0, w_F - \lambda c\}$$

As we have argued $w_F \in \left[0, \bar{\lambda}c\right]$, then $\max\{0, w_F - \lambda c\} = w_F$ with probability μ, and $\max\{0, w_F - \lambda c\} = 0$ with probability $(1-\mu)$. Let us now define two cutoff types, θ_l and θ_h for the following analysis. θ_l is the cutoff type which is indifferent between a genuine degree and a fake degree when λ is low, $\lambda = 0$. θ_h is the cutoff type which is indifferent between a genuine degree and no degree when λ is high, $\lambda = \bar{\lambda}$. We have $\theta_l w_S + (1-\theta_l)\, w_F - c = w_F$, which gives

$$\theta_l = \frac{c}{w_S - w_F} \tag{5}$$

and $\theta_h w_S + (1-\theta_h)\, w_F - c = 0$, which can be rearranged as

$$\theta_h = \frac{c - w_F}{w_S - w_F} \tag{6}$$

In each case (either θ_l or θ_h), all types greater than the cut-off type prefer a genuine degree as we stated above that $w_S > w_F$.

Suppose that $w_F \in (0, \bar{\lambda}c]$, then the firm's expected profit can be written as

$$\begin{aligned}\pi = \mu &\left(-\int_0^{\theta_l} w_F f(\theta)\, d\theta + \int_{\theta_l}^1 (\theta V - \theta (w_S - w_F) - w_F)\, f(\theta)\, d\theta \right) \\ &+ (1-\mu) \left(\int_{\theta_h}^1 (\theta V - \theta (w_S - w_F) - w_F)\, f(\theta)\, d\theta \right)\end{aligned} \tag{7}$$

With probability μ, fake degrees are at no cost. Types below θ_l have incentives to get fake degrees. With probability $(1-\mu)$, the cost of a fake degree is $\bar{\lambda}c$, then types below θ_h have no incentive to buy a fake degree as the cost is no less than the return.

We show in the following discussion that if μ is positive but sufficiently small, the optimal contract offered by the firm requires $w_F > 0$, giving rise to existence of fake degrees in equilibrium.

Proposition II.5 (existence of fake degrees) *If $0 < \mu < \hat{\mu}$, where $\hat{\mu}$ is given as $\hat{\mu} = \frac{\int_{\theta_l^*}^1 \left(\frac{\theta}{\theta_l^*} - 1\right) f(\theta) d\theta}{1 + \int_{\theta_l^*}^1 \left(\frac{\theta}{\theta_l^*} - 1\right) f(\theta) d\theta}$, then the firm maximizes profit by setting $w_F^* > 0$ and fake degrees exist in the equilibrium. The optimal cutoff types satisfy $\theta_l^* > \theta_h^* > \theta^*$. If*

$\hat{\mu} \leqslant \mu < 1$, then the firm sets $w_F^* = 0$, which results in $\theta_l^* = \theta_h^* > \theta^*$. No fake degree exists in the equilibrium.

Proof. *See Appendix $1.B$.* ■

When the firm assigns a sufficiently small probability to zero cost of a fake degree, the firm is better off offering a positive w_F, even though offering $w_F = 0$ can help eliminating the existence of fake degrees in equilibrium. If the cost of a fake degree is realized to be greater than w_F, then the firm is better off as the firm faces a lower wage bill with a positive w_F. If the cost of a fake degree turns out to be lower than w_F, then all types prefer a fake degree over no education. The firm pays a positive wage to the worker for showing up with a fake degree. Moreover, to incentivize the worker to get a genuine degree, the bonus has to be set high enough so that the expected utility from getting a genuine degree is greater than the positive utility received from buying a fake degree. In balancing this trade-off, the firm finds it optimal to offer a positive w_F when it believes that the probability of having zero cost of a fake degree is sufficiently low. Although there is a positive probability that the cost of a fake degree is zero, and the firm may end up with hiring some fake degree holders, the firm still finds it optimal to take the risk.

We have shown the presence of fake degrees in the equilibrium with labour market imperfection. Below, we switch to the Bertrand-type competition to check the effect of market structure on the existence of fake degrees.

4.2 Bertrand-type Competition

We now consider a different structure of the labour market where there are two firms, i and j, competing for hiring a worker. The production technology is the same as described above. The timing is slightly different. Two firms simultaneously announce their contract at the first stage of the game. The worker observes the contracts and then decides amongst no education, a fake degree, and a genuine degree. After getting a degree, the worker presents the degree to the preferred firm and gets employed. The job starts, and the firm makes the payment as agreed to the wage offer. The main difference between the game described here and the standard Bertrand competition is that firms compete in choosing two wages, w_S and w_F, with the presence of fake degrees. With all the complications we have so far in this model, we restrict our attention to symmetric equilibria and known cost of a fake degree.

We show that under Bertrand-type competition, both firms offering the same contract $(w_S = V, w_F = 0)$ is the unique symmetric equilibrium. Below we first argue that $(w_S = V, w_F = 0)$ is indeed an equilibrium, and then establish its uniqueness.

We assume that if a type gets the same expected utility from both firms' offer, each firm has an equal probability of hiring this type of worker. When both firms offer $(w_S = V, w_F = 0)$, no type chooses a fake degree since the expected payment received after buying a fake degree is no greater than the cost, $w_F \leqslant \lambda c$. Given the contract $(w_S = V, w_F = 0)$, the cutoff type, which gets the same expected utility from selecting a genuine degree and no degree, is the same as the social optimal cutoff type θ^* where $\theta^* V - c = 0$. The monotonicity of θ shows that all types greater than θ^* choose a genuine degree as $u(g,\theta) > 0\ \forall \theta > \theta^*$, and all types below θ^* would like to stay with no degree as $u(g,\theta) < 0\ \forall \theta < \theta^*$. Since firms get zero expected profits from hiring any type lies between $[\theta^*, 1]$, $\pi(g,\theta) = \theta V - [\theta V + (1-\theta)\times 0] = 0$, the expected overall profit of each firm is also zero,

$$\pi = \frac{1}{2}\int_{\theta^*}^{1} (\theta V - \theta V) f(\theta)\, d\theta = 0$$

Below we show that if one of the firms offers $(w_S = V, w_F = 0)$, then the other firm has no better option than offering $(w_S = V, w_F = 0)$. For the following discussion, note that only a genuine degree holder can generate a positive expected profit for firm i and j. The reason is that a type θ will choose a fake degree only if $u(k,\theta) = w_F - \lambda c > 0$, which implies that $w_F > \lambda c \geqslant 0$. Regardless of the type of the worker, by hiring a fake degree holder, firms get no benefit but pays w_F, $\pi(k,\theta) = -w_F < 0$. Hence, a necessary condition for the firm to make a non-negative expected profit is to employ some types with a genuine degree.

Proposition II.6 *It is an equilibrium for both firms to offer $(w_S = V, w_F = 0)$ under Bertrand-type competition.*

Proof. *In equilibrium. firm i believes that firm j offers $(w_S = V, w_F = 0)$. Suppose firm i deviates and offers a contract $(w'_S, w'_F) \neq (V, 0)$.*

(i) If the contract (w'_S, w'_F) attracts a type θ between $[\theta^*, 1]$ to get a genuine degree, then it must be the case that the type θ gets a higher utility from getting a genuine degree and join firm i, $u_i(g,\theta) > \max\{u_j(g,\theta), u_i(k,\theta)\}$. It can be further written as

$$\theta w'_S + (1-\theta) w'_F - c > \max\{\theta V - c, w'_F - \lambda c\} \tag{8}$$

According to (8), $\theta w'_S + (1-\theta) w'_F - c > \theta V - c$ will always hold for all $\theta \in [\theta^*, 1]$. It implies that the expected payment of offering (w'_S, w'_F), $\theta w'_S + (1-\theta) w'_F$, is greater than the expected revenue, θV. Hence, the expected profit of the deviating firm from attracting any type between $[\theta^*, 1]$ to get a genuine degree is negative.

(ii) If the contract (w'_S, w'_F) attracts a type θ between $[0, \theta^*)$ to get a genuine degree, we must have $u_i(g, \theta) > \max\{u(n, \theta), u_i(k, \theta)\}$, that is,

$$\theta w'_S + (1-\theta) w'_F - c > \max\{0, w'_F - \lambda c\} \tag{9}$$

According to (9), $\theta w'_S + (1-\theta) w'_F - c > 0$ will always hold for all $\theta \in [0, \theta^*)$. Hence, the expected profit from hiring a type below θ^* is also negative as $\theta V - [\theta w'_S + (1-\theta) w'_F] < \theta V - c$ and $\theta V - c < 0 \; \forall \; \theta \in [0, \theta^*)$. The expected profit of the deviating firm from attracting any type between $\theta \in [0, \theta^*)$ to get a genuine degree is also negative.
Therefore, the maximum expected profit from deviating is zero since attracting any type between $[0, 1]$ give no positive expected profit. Firm i cannot profitably deviate to a contract which is not $(V, 0)$. ■

If the deviating firm seeks to attract any type greater than θ^*, since the rival is offering the highest rent and making zero profit, providing more than its rival will make the deviating firm worse off. If the deviating firm tries to attract a type below θ^* to get a genuine degree, then it has to provide the expected payment greater than or equal to the cost of a genuine degree, $\theta w_S + (1-\theta) w_F \geqslant c$, for this type. However, for all types below θ^*, $\theta V - c < 0$. It implies that the expected revenue, θV, is less than the expected payment, $\theta w_S + (1-\theta) w_F$. The deviating firm makes a negative expected profit from hiring any type below θ^*. Therefore, if the rival firm offers $(w_S = V, w_F = 0)$, then attracting any type with certainty will give a negative expected profit to the deviating firm, and there is no way it can profitably deviate to a contract that achieves a positive surplus.

4.2.1 The Uniqueness of the Symmetric Equilibrium

We have shown that both firms offer $(w_S = V, w_F = 0)$ is an equilibrium. To establish the uniqueness of this equilibrium, we show below that when firms are offering a contract that is not $(V, 0)$, any firm can find a profitable deviation. Since w_F is the key that determines whether the worker has an incentive to buy a fake degree or not, we look into the following four cases based on the value of w_F. Since firms get negative profits by offering $w_F = 0$ and $w_S > V$, we do not need to consider this case further. Hence, regardless whether the cost of a fake degree is known, we argue the uniqueness of the equilibrium by presenting profitable deviations when: (i) $w_F > \bar{\lambda} c$; (ii) $w_F < 0$; and (iii) $0 < w_F \leqslant \bar{\lambda} c$; (iv) $w_F = 0$ and $w_S < V$.

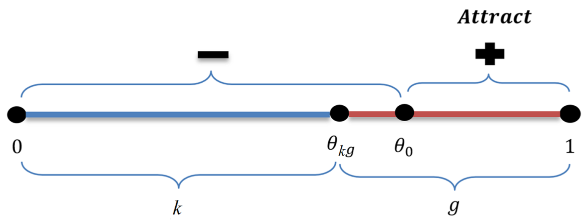

Figure 2: Deviation when $w_F > \bar{\lambda}c$ and $\theta_0 > \theta_{fg}$

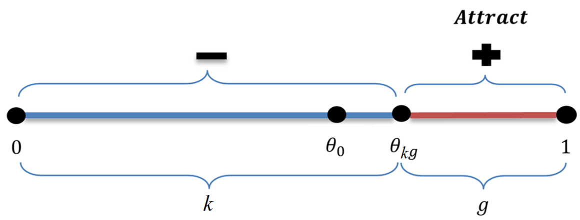

Figure 3: Deviation when $w_F > \bar{\lambda}c$ and $\theta_0 \leqslant \theta_{fg}$

Proposition II.7 *Under Bertrand-type competition, both firms offering the contract* $(w_S = V, w_F = 0)$ *is the unique symmetric equilibrium.*
Proof. *See Appendix* $1.C$. ■

The main argument behind Proposition II.7 is that if both firms offer a contract that is not $(w_S = V, w_F = 0)$, it is always profitable for one of the firms to either offer a higher wage or adjust the wage to all types or a set of types. More precisely, there are two types of profitable deviation. First, if both firms offer the same contract where all types generate positive profits (for example, when $w_F \leqslant 0$ and $w_S < V$. See Figure 4),the deviating firm can increase w_S by a small amount so that all types now prefer the new contract. The deviating firm then extracts all the rent without sharing with the rival. Second, if the contract offered by both firms results in only a set of types that generate positive profits, then the deviating firm can adjust the wages to attract only those types that generate positive profits and leave all other types to the rival. To attract relatively higher types, the deviating firm increases w_S and lowers w_F (see Figure **??**, 3, 6, and 7). Similarly, if the firm

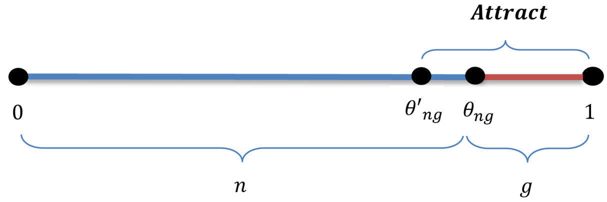

Figure 4: Deviation when $w_F < 0$ and $w_S \leqslant V$

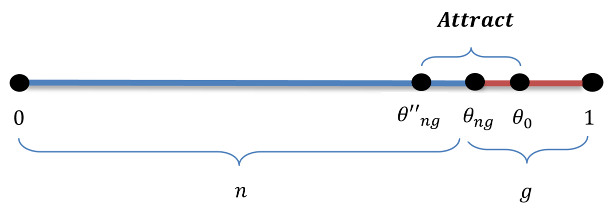

Figure 5: Deviation when $w_F < 0$ and $w_S > V$

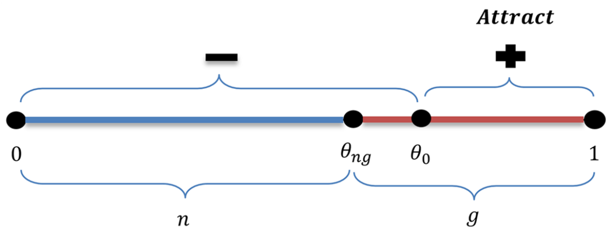

Figure 6: Deviation when $w_F \leqslant \lambda c$ and $\theta_0 > \theta_{ng}$

targets some relatively lower types, it can profitably reduce w_S and raise w_F (see Figure 5). For the full discussion, see Appendix $1.C$.

When the employer loses its monopsony power, it can no longer use the instrument, w_F, to extract more rents. We have shown that with Bertrand-type competition, the employers are forced to offer $w_F = 0$ and $w_S = V$, and make zero profits in equilibrium. No fake degree exists in equilibrium, and the competition also vanishes the trade-off the firm faces when it is a monopsonist.

5 Conclusion

In this chapter, we focus on the extreme kind of low-quality education, fake degrees, which have no productive value. It implies that the worker with a fake degree has no chance of completing the job-task and getting the bonus. The fixed wage, w_F, therefore, is the only payment for the worker with a fake degree. Hence, the key to the existence of a fake degree is the value of w_F.

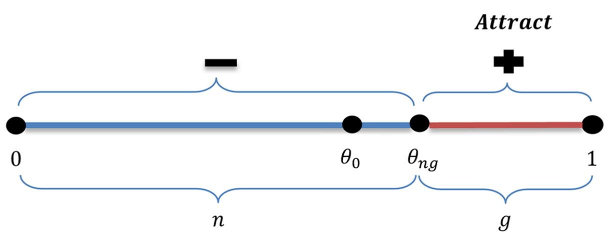

Figure 7: Deviation when $w_F \leqslant \lambda c$ and $\theta_0 \leqslant \theta_{ng}$

The information asymmetry forces the firm to give some rent to a set of higher types. A higher w_F enables the firm to reduce the expected payment to the worker through lowering the corresponding bonus, $(w_S - w_F)$. Note that $(w_S - w_F)$ is the compensation only for a genuine degree holder, and the monotonicity of the probability of being successful implies that higher types have a higher probability of getting the bonus. Since the firm employs relatively higher types with a genuine degree, a smaller bonus means a lower expected payment. However, a higher w_F means a higher probability of employing the worker with a fake degree. Once w_F exceeds the cost of a fake degree, the firm is committed to paying all types a fixed amount w_F for showing up with a degree. The firm offers all types a positive utility. As a result, the firm has to increase the wage so that those higher types will have an incentive to get a genuine degree.

However, the use of w_F is restricted to the firm's monopsony power. In the imperfectly competitive labour market, when the quality of education is verifiable, the firm sets the fixed wage up to the cost of a genuine degree to extract the full surplus. When the firm is unable to distinguish between a genuine and a fake degree but knows the cost of a fake degree, the fixed wage is set equal to the cost of a fake degree, giving no incentive to any type to buy a fake degree. The monopsony power and the uncertainty on quality of education results in underinvestment in education. When the cost of a fake degree is privately known to the worker, the firm sets a positive fixed wage if it believes that the probability of zero cost of a fake degree is sufficiently low. In this case, the additional information asymmetry leads to a result that the contract may incentivize some lower types to buy a fake degree, giving rise to the existence of fake degrees in equilibrium.

Under Bertrand-type competition, the unique symmetric optimal contract requires the firms to set $w_F = 0$ and $w_S = V$, which results in zero profits for the firms. The intense competition weakens firms' power of using the fixed wage. Competition helps in restoring the efficiency of the market, as well as getting rid of fake degrees in the equilibrium.

Chapter III

In-house Training and Fake degrees

1 Introduction

If verifying a degree from an outside institute is problematic for the firm, then offering the training by the firm itself may help. The benefit of offering in-house training is being assured of the worker's acquisition of the required skill. It helps the firm to get rid of the exogenous private information faced in Chapter II. In this chapter, we allow the firm to use in-house training for the firm's rent-seeking in addition to the use of fixed wage as discussed in Chapter II. The monopsony power is again the source which enables the firm to use these instruments. Hence, we consider only labour market with imperfect competition, more precisely, monopsony.

The use of in-house training is determined by the cost of providing the training. When the cost of in-house training is no greater than the cost of a genuine degree, it is beneficial for the firm to offer only in-house training contract. The firm does not face the degree verification problem and training the worker is also cheaper in-house. When the cost of in-house training exceeds the cost of a genuine degree, the firm faces a trade-off between offering the more costly in-house training and using the fixed wage to extract more rent. Offering in-house training incurs a higher cost, but the firm can extract all the rent from the worker.

On the other hand, to get more rent from hiring the worker with a degree from outside, the firm needs to balance the expected payment and the risk of meeting fake degrees. Setting a higher fixed wage helps to lower the expected payment, but increases the probability of hiring a fake degree. While using the fixed wage, the information asymmetry forced the firm to give away some information rent to higher types. Hence, as the cost of in-house training arises, the firm substitutes in-house training with a genuine degree for some higher types to keep the total size of information rent small. If the cost of providing in-house training is sufficiently low, then the profit-maximizing firm finds it optimal to train some middle types in-house due to the presence of fake degrees, and hire the higher types with a genuine degree. Along with the increase in the cost of in-house training, the firm chooses to have fewer types trained in-house. We get back to the case as discussed in Chapter II when the cost of training the worker in-house exceeds a certain value. In this case, the firm prefers to have no type choosing in-house training. Offering in-house training, when the cost is sufficiently low, not only helps the firm to extract more rent, but also helps in improving the problem of underinvestment in education.

The structure of this chapter is as follows. Section 2 sets up the production technology, and we show the social planner's problem in Section 3. Then in Section 4, we describe and solve the firm's profit-maximizing problem. Section 5 gives the conclusion of this chapter.

2 Model

Consider a labour market where a monopsonist is looking to hire a trained worker to undertake a job. The worker has a positive probability of generating a gross return V for the firm only if she is trained. The probability of being successful in doing the job is determined by the acquisition of the training and the type of the worker. To acquire the training, the worker has two options: getting trained in-house or from an outside institute. The in-house training is provided by the firm, and it costs the firm c^I for training the worker in-house. The outside institute provides the same training (adds the same productivity to the worker) and charges her c for undertaking the training. After completing the training, the institute awards a degree to the worker. Since the firm is looking for skilled labour, if the worker is not trained in-house, then the firm asks for a degree as a proof of skill acquisition. If, however, the worker decides not to be trained, then she can either purchase a fake degree without going through any training at a price λc, where $\lambda \in [0, 1)$, or remain untrained without a fake degree. Let us denote by e, $e \in \{n, k, g, I\}$, the worker's educational choice ($n-$ no training, $k-$ a fake degree, $g-$ a genuine degree, and $I-$ in-house training). The firm cannot distinguish between a genuine and a fake degree. The genuine training (g or I) is the key for the worker to be successful in doing the job. More precisely, a trained ($e = g$ or I) worker completes the job with a probability which is determined by her type. Let θ be the type of the worker. The higher type she has, the higher probability of being successful she obtains. For simplicity, let the probability of being successful also be denoted by θ. Fake degrees do not have any real educational value. An untrained ($e = n$ or k) worker has zero probability of being successful in doing the job regardless of her type.

3 The Social Planner's Problem

Given the above production technology, the only difference between this chapter and Chapter II is that we now have an additional option for the social planner to choice from, the in-house training. Without in-house training, all types that contribute non-negative surplus were allocated to have a genuine degree. Recall θ^* the first best cutoff type from Chapter II and it was defined by $\theta^* V - c = 0$. Types that lie between $[\theta^*, 1]$ were allocated to have a genuine degree and types below θ^* were allocated to no education. By adding in-house training, the social planner allocates the types that provide non-negative surpluses to the cheaper training as both the in-house training and the genuine degree add the same productivity to the worker. Let θ^{I*} be defined by $\theta^{I*} V - c^I = 0$, and it is the cutoff type that supplies a zero surplus after getting the in-house training. The first best allocation can be written as

$$\begin{array}{ll} \text{When } c^I > c & \begin{cases} e = g & \text{if } \theta \geqslant \theta^* \\ e = n & \text{if } \theta < \theta^* \end{cases} \\ \text{When } c^I < c & \begin{cases} e = I & \text{if } \theta \geqslant \theta^{I*} \\ e = n & \text{if } \theta < \theta^{I*} \end{cases} \\ \text{When } c^I = c & \begin{cases} e = I \text{ or } g & \text{if } \theta \geqslant \theta^{I*} = \theta^* \\ e = n & \text{if } \theta < \theta^{I*} = \theta^* \end{cases} \end{array} \tag{10}$$

which gives us the optimal social welfare

$$\begin{cases} \int_{\theta^*}^{1} (\theta V - c) f(\theta) \, d\theta & \text{if } c^I > c \\ \int_{\theta^{I*}}^{1} \left(\theta V - c^I\right) f(\theta) \, d\theta & \text{if } c^I < c \\ \int_{\theta^*}^{1} (\theta V - c) f(\theta) \, d\theta = \int_{\theta^{I*}}^{1} \left(\theta V - c^I\right) f(\theta) \, d\theta & \text{if } c^I = c \end{cases} \tag{11}$$

4 The Firm's Problem

We proceed with a labour market where the firm hires a worker through a contract $M = \{\left(w_S^I, w_F^I\right), (w_S, w_F)\}$, where $\left(w_S^I, w_F^I\right)$ is the wage payment for the worker with in-house training, and (w_S, w_F) is for the worker who presents a degree. The firm pays w_S^I or w_S when the worker completes the job successfully, otherwise, w_F^I or w_F. Alternatively, we can interpret the contract as the firm pays a fixed wage, w_F^I or w_F, to the worker by signing the contract. If the worker completes the work successfully, then she gets a bonus, $\left(w_S^I - w_F^I\right)$ or $(w_S - w_F)$. Otherwise, the fixed wage will be the only payment.

A worker without a degree and no in-house training will not be employed. The type of the worker is private information. However, the firm knows that θ is continuously distributed over a unit interval, $[0,1]$, with a cumulative distribution function $F(\theta)$ and a density function $f(\theta)$. As mentioned above, the firm bears a cost c^I for providing the in-house training. If the worker chooses in-house training, then for simplicity we assume that the firm charges the worker c^I for the training. Hence, eventually, the worker pays for the cost of in-house training. We can also have a case where the firm charges the worker a different value than c^I for the in-house training. However, any level of payment made by the worker for the in-house training has to be taken into account when the firm offers the wage compensation. If the firm charges a high amount, then the wage has to be large enough to cover the cost so that the worker has the incentive to apply. Any level of charge for in-house training will lead to the same final results. On the other hand, if the worker chooses to present a degree, then no charge takes place from the firm's side.

The expected profit, $\pi(e,\theta)$, of the firm from hiring a type θ worker with educational choice e is

$$\pi(e,\theta) = \begin{cases} \theta V - \theta w_S^I - (1-\theta)\, w_F^I & \text{if } e = I \\ \theta V - \theta w_S - (1-\theta)\, w_F & \text{if } e = g \\ -w_F & \text{if } e = k \end{cases} \tag{12}$$

With probability θ, the firm gets V and pays the worker w_S^I if the worker is trained in-house, and w_S if the worker has a genuine degree. With probability $(1-\theta)$, the firm gets zero return from employing the worker and pays either w_F^I or w_F depending on the training. By hiring a fake degree holder, the firm gets zero but pays w_F to the worker.

After observing the contract announced by the firm, M, the worker decides on educational choice. We assume that all types have the same reservation utility and we normalize it to zero. The expected utility, $u(e,\theta)$, of a type θ worker with an educational choice, e, is given by

$$u(e,\theta) = \begin{cases} \theta w_S^I + (1-\theta) w_F^I - c^I & \text{if } e = I \\ \theta w_S + (1-\theta) w_F - c & \text{if } e = g \\ w_F - \lambda c & \text{if } e = k \\ 0 & \text{if } e = n \end{cases} \tag{13}$$

The worker pays the cost of education which depends on the choice she has made. If she chooses the in-house training, then she pays c^I for receiving the training and will be employed with the contract $\left(w_S^I, w_F^I\right)$. She receives w_S^I with probability θ and w_F^I with probability $(1-\theta)$. Similarly, if she decides to get a genuine degree, then she pays c and will be offered the contract (w_S, w_F). With a probability θ, she does the job successfully and gets w_S, and with a probability $(1-\theta)$, she fails and gets w_F. On the other hand, if the worker chooses to get a fake degree, then she pays λc and will be paid with (w_S, w_F) as well. Since the fake degree has no productive value, the worker fails to complete the job and gets w_F with certainty.

The firm's problem is to find the optimal $M = \{\left(w_S^I, w_F^I\right), (w_S, w_F)\}$ to maximize profit. Let us denote the set of types which prefer in-house training as Θ^I, a genuine degree as Θ^g, and a fake degree as Θ^k. The firm's problem can be written as

$$\max_{\left(w_S^I, w_F^I\right),(w_S,w_F)} \left[\begin{array}{l} \int_{\Theta^I} \left(\theta V - \left(\theta w_S^I + (1-\theta) w_F^I\right)\right) f(\theta)\, d\theta \\ + \int_{\Theta^g} \left(\theta V - \left(\theta w_S + (1-\theta) w_F\right)\right) f(\theta)\, d\theta \\ - \int_{\Theta^k} (w_F) f(\theta)\, d\theta \end{array} \right]$$

subject to

$$\begin{array}{ll} \text{For } \theta \in \Theta^I & \begin{cases} IR & u(I,\theta) \geqslant 0 \\ IC & u(I,\theta) \geqslant \max\{u(g,\theta), u(k,\theta), u(n,\theta)\} \end{cases} \\ \text{For } \theta \in \Theta^g & \begin{cases} IR & u(g,\theta) \geqslant 0 \\ IC & u(g,\theta) \geqslant \max\{u(I,\theta), u(k,\theta), u(n,\theta)\} \end{cases} \\ \text{For } \theta \in \Theta^k & \begin{cases} IR & u(k,\theta) \geqslant 0 \\ IC & u(k,\theta) \geqslant \max\{u(I,\theta), u(g,\theta), u(n,\theta)\} \end{cases} \end{array}$$

IR represents the individual-rationality or participation constraint, and IC is the incentive-compatibility constraint.

4.1 Implementing the First Best

Before we proceed further, let us first briefly introduce the possible ways of implementing the social optimum. This part of discussion provides implications for later analysis.

Proposition III.1 *The first best outcomes are implemented when the social planner enforces the contract* $M_a = \{(w_S^I = V, w_F^I = 0), (w_S = V, w_F = 0)\}$ *for all* c^I *and* c*;*
Proof. *With* $M_a = \{(w_S^I = V, w_F^I = 0), (w_S = V, w_F = 0)\}$*, the expected utility of a type* θ *worker is given by*

$$\begin{aligned} u(I,\theta) &= \theta V - c^I \\ u(g,\theta) &= \theta V - c \\ u(k,\theta) &= -\lambda c \end{aligned}$$

All types choose whichever is less costly between the in-house training and a genuine degree, and no type would like to buy a fake degree. Then the allocation of types will be the same as the first best allocation stated in (10). ■

In a perfectly competitive labour market, the contract M_a is an equilibrium contract without the enforcement of the social planner. The contract M_a requests the firm to give all the rent to the worker and make a zero profit. Full efficiency is generated, and there exists no fake degree in equilibrium.

When facing a monopsonist in the labour market, an alternative way to reach the first best is that the firm offers $M_b = \{(w_S^I = c^I, w_F^I = c^I), (w_S = 0, w_F = 0)\}$ when $c^I \leqslant c$.

Proposition III.2 *If* $c^I \leqslant c$*, then there exists an equilibrium where the firm extracts the full surplus with contract* $M_b = \{(w_S^I = c^I, w_F^I = c^I), (w_S = 0, w_F = 0)\}$.
Proof. *When* $c^I \leqslant c$*, with* $M_b = \{(w_S^I = c^I, w_F^I = c^I), (w_S = 0, w_F = 0)\}$*, the expected utility for a type* θ *between* $[0,1]$ *is given by*

$$\begin{aligned} u(I,\theta) &= \theta c^I + (1-\theta) c^I - c^I = 0 \\ u(g,\theta) &= -c \\ u(k,\theta) &= -\lambda c \end{aligned}$$

which means that all types are indifferent between in-house training and no education. If we assume that types between $[\theta^{I*}, 1]$ *choose in-house training, and types between* $[0, \theta^{I*})$ *choose no education, then the firm extracts the full surplus, and no type has a profitable deviation.* ■

This contract can also be viewed as the firm offers only in-house training. The maximum profit is achieved when all efficient types are trained in-house, and the wage covers just the cost of training. Since $c^I \leqslant c$, to hire a worker with a genuine degree, the expected payment has to be no less than c. Even we do not take the possibility of hiring a fake degree holder into consideration, the maximum profit the firm can get from employing a worker with a genuine degree is less than that from employing the worker with in-house training. Hence, it is better for the firm to offer only in-house training.

The full surplus extraction is restricted to the condition that $c^I \leqslant c$. In the next section, we investigate the firm's profit maximization contracts in the situation where offering in-house training is more expensive than the cost of a genuine degree.

4.2 Profit Maximization When $c^I > c$

With $c^I > c$, the firm faces the following trade-off: (i) if the firm employs a type with in-house training, then it bears a higher cost c^I (compared with c) for providing the training; (ii) if the firm employs a type with a degree from outside the firm, then it either allows a positive chance of employing a fake degree holder or reduces w_F to avoid hiring a fake degree holder which increases firm's expected wage payments and lowers the profit. The first best cannot be implemented in this case due to the labour market imperfection and the information asymmetry. Below, we show that with $c^I > c$, the firm offers in-house training and accepts a degree from the outside at the same time when c^I is less than a certain value. Once c^I exceeds the certain value, the firm prefers not to offer in-house training as it is too costly to do so.

We have shown above that when $c^I \leqslant c$, the firm would like to hire the worker with only in-house training. As the cost of in-house training goes up, our conjecture here is that the firm may want to drop in-house training and hire the worker with a genuine degree. We now look for an equilibrium where the firm offers $M = \{(w_S^I, w_F^I), (w_S, w_F)\}$ such that the firm's profit is maximized with both in-house training and genuine degrees. Given the monotonicity of probability θ and the existence of two non-empty sets of types with different preferences over in-house training and a genuine degree, there must exist a type that is indifferent between in-house training and a genuine degree. Moreover, we claim that this type is also the unique indifferent type. Let θ_{Ig} be the cutoff type that is indifferent between the genuine degree and in-house training. We can write

$$\theta_{Ig}\left(w_S^I - w_F^I\right) + w_F^I - c^I = \theta_{Ig}\left(w_S - w_F\right) + w_F - c \tag{14}$$

Lemma III.1 *There exists a unique type that is indifferent between a genuine degree and in-house training if sets of types choosing in-house training and presenting a degree are both non-empty.*

Proof. *If $w_S^I - w_F^I > w_S - w_F$, then the monotonicity of θ implies that all types greater than θ_{Ig} strictly prefer in-house training over a genuine degree, and all types below strictly prefer a genuine degree over in-house training. If $w_S^I - w_F^I < w_S - w_F$, then all types greater than θ_{Ig} strictly prefer a genuine degree over in-house training, and all types below strictly prefer in-house training over a genuine degree. In both cases, θ_{Ig} is the unique cutoff type that is indifferent between in-house training and a genuine degree.*

If $w_S^I - w_F^I = w_S - w_F$, then all types have the same preference as the part of wage that is type dependent is the same from in-house training or a genuine degree. Hence, either all types prefer in-house training over a genuine degree, or all types prefer a genuine degree over in-house training.

Therefore, there is a unique cutoff type that is indifferent between in-house training and a genuine degree. ■

The proof for Lemma III.1 means that if the bonuses offered in both contracts, $\left(w_S^I - w_F^I\right)$ and $(w_S - w_F)$, are different, given the monotonicity of the probability, there exists only one type that is indifferent between in-house training and a genuine degree. If the bonuses are the same, then all types have the same preference as the expected utility is no longer type dependent.

Lemma III.1 further implies that there are the only two possible cases where the sets of types choosing in-house training and presenting a degree are non-empty. In the first case, we assume that types greater than θ_{Ig} prefer in-house training over genuine degrees, and types below θ_{Ig} favour genuine degrees compared with in-house training. In the second case, types greater than θ_{Ig} prefer genuine degrees over in-house training, and types below θ_{Ig} prefer in-house training over genuine degrees.

Before we continue with these two cases, for simplicity, we assume that obtaining a fake degree is costless, i.e., $\lambda = 0$, for the following analysis. A costless fake degree implies that if the firm offers $w_F > 0$, then all types prefer fake degrees over no degree and if $w_F \leqslant 0$, then no type has the incentive to buy a fake degree. For a positive λ, the analysis will focus on comparing w_F with λc, which generates similar results.

We now define another cutoff type, θ_g, which is indifferent between a genuine degree and no training (if $w_F \leqslant 0$) or a fake degree (if $w_F > 0$). Formally, θ_g is

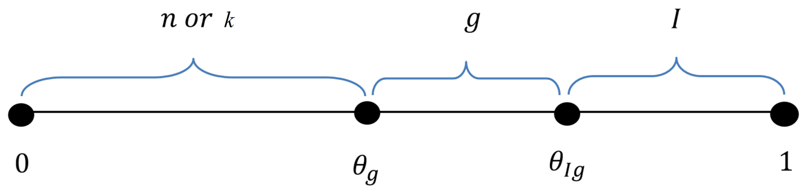

Figure 8: $0 < \theta_g < \theta_{Ig} < 1$

given by

$$\max\{0, w_F\} = \theta_g w_S + (1 - \theta_g) w_F - c \tag{15}$$

In the case (gI), types between $[\theta_{Ig}, 1]$ choose in-house training, and types between $[\theta_g, \theta_{Ig})$ choose a genuine degree, where $0 \leqslant \theta_g < \theta_{Ig} < 1$. Figure 8 shows when $0 < \theta_g < \theta_{Ig} < 1$. The firm's problem is to solve

$$\max_{(w_S, w_F), \left(w_S^I, w_F^I\right)} \begin{bmatrix} -\int_0^{\theta_g} \max\{0, w_F\} f(\theta)\, d\theta \\ +\int_{\theta_g}^{\theta_{Ig}} (\theta V - \theta (w_S - w_F) - w_F) f(\theta)\, d\theta \\ +\int_{\theta_{Ig}}^{1} \left(\theta V - \theta \left(w_S^I - w_F^I\right) - w_F^I\right) f(\theta)\, d\theta \end{bmatrix}$$

subject to

$$\begin{array}{ll} \text{For } \theta \in [0, \theta_g) & \left\{ \begin{array}{ll} IR & \max\{u(k,\theta), u(n,\theta)\} \geqslant 0 \\ IC & \max\{u(k,\theta), u(n,\theta)\} \geqslant \max\{u(I,\theta), u(g,\theta), u(n,\theta)\} \end{array} \right. \\ \text{For } \theta \in [\theta_g, \theta_{Ig}] & \left\{ \begin{array}{ll} IR & u(g,\theta) \geqslant 0 \\ IC & u(g,\theta) \geqslant \max\{u(I,\theta), u(k,\theta), u(n,\theta)\} \end{array} \right. \\ \text{For } \theta \in (\theta_{Ig}, 1] & \left\{ \begin{array}{ll} IR & u(I,\theta) \geqslant 0 \\ IC & u(I,\theta) \geqslant \max\{u(g,\theta), u(k,\theta), u(n,\theta)\} \end{array} \right. \end{array}$$

Note that all types produce the same revenue, θV, to the firm after getting trained. Hence, in this case, the expected revenue of the firm is given by

$$\int_{\theta_g}^{1} (\theta V) f(\theta)\, d\theta \tag{16}$$

Lemma III.2 *When $c^I > c$, there does not exist a profit maximizing contract such that the outcome is where types greater than θ_{Ig} prefer in-house training over genuine degrees, and types below θ_{Ig} favor genuine degrees compared with in-house training.*
Proof. *Suppose the firm offers a contract $M = \left\{\left(w_S^I, w_F^I\right), (w_S, w_F)\right\}$ which results in $0 \leqslant \theta_g < \theta_{Ig} < 1$. The necessary conditions for holding $0 \leqslant \theta_g < \theta_{Ig} < 1$*

are $\left(w_S^I - w_F^I\right) > (w_S - w_F)$*,* $w_F - c > w_F^I - c^I$[8]*, and* $w_S^I > w_S + c^I - c$[9]*. Now consider another contract* $M' = \left\{\left(w_S^{I\prime}, w_F^I\right), (w_S, w_F)\right\}$ *where* $w_S^I > w_S^{I\prime} > \max\{w_S + c^I - c, w_S + w_F^I - w_F\}$*. The cutoff type* θ_g *remains the same since* (w_S, w_F) *has not changed and the in-house training is less attractive as the new contract offers a lower* $w_S^{I\prime}$*. Hence, the firm's expected revenue, as stated in the expression (16), is the same from both contracts, but the expected payments are different. The contract* $\left(w_S^{I\prime}, w_F^I\right)$ *leads to a new cutoff type that is indifferent between in-house training and a genuine degree. Let the new cutoff type be* θ'_{Ig}*.* $w_S^{I\prime} > w_S + c^I - c$ *gives that* $\theta'_{Ig} < 1$*, and* $w_S^{I\prime} > w_S + w_F^I - w_F$ *implies that* $\theta'_{Ig} > 0$*. Moreover, since* $w_S^I > w_S^{I\prime}$*, we have*

$$\theta'_{Ig} = \frac{w_F - c - \left(w_F^I - c^I\right)}{w_S^{I\prime} - w_F^I - (w_S - w_F)} > \frac{w_F - c - \left(w_F^I - c^I\right)}{w_S^I - w_F^I - (w_S - w_F)} = \theta_{Ig}$$

With $\left(w_S^{I\prime}, w_F^I\right)$*, types between* $[\theta_{Ig}, \theta'_{Ig})$ *strictly prefer a genuine degree over in-house training.*

For the cutoff type θ_{Ig}*, with the contract* M*, we have* $u\left(I, \theta_{Ig}\right) = u\left(g, \theta_{Ig}\right)$*,*

$$\theta_{Ig}\left(w_S^I - w_F^I\right) + w_F^I - c^I = \theta_{Ig}\left(w_S - w_F\right) + w_F - c$$

It implies that the expected payment for employing the type θ_{Ig} *with in-house training is higher compared with hiring the type* θ_{Ig} *with a genuine degree because*

$$\underbrace{\theta_{Ig}\left(w_S^I - w_F^I\right) + w_F^I}_{\text{expected payment with in-house training}} - \underbrace{\left(\theta_{Ig}\left(w_S - w_F\right) + w_F\right)}_{\text{expected payment with a genuine degree}} = c^I - c > 0$$

Given that $\left(w_S^I - w_F^I\right) > (w_S - w_F)$*, it is also true for types between* $[\theta_{Ig}, \theta'_{Ig})$ *that in-house training contract incurs a higher payment. Since types between* $[\theta_{Ig}, \theta'_{Ig})$ *strictly prefer a genuine degree over in-house training with the contract* M'*, the overall expected payment must be lower compared with* M*. It contradicts that offering the contract* M *maximizes the firm's profit. As long as* $\theta_{Ig} < 1$*, it is always profitable for the firm to reduce* w_S^I *for a small amount such that a set of types switch from in-house training to genuine degrees.* ■

When the contract M yields $0 \leqslant \theta_g < \theta_{Ig} < 1$, the firm can always profitably

[8]Consider two types $\theta_a \in (\theta_g, \theta_{Ig})$, and $\theta_b \in (\theta_{Ig}, 1)$. we have $\theta_a\left(w_S^I - w_F^I\right) + w_F^I - c^I < \theta_a\left(w_S - w_F\right) + w_F - c$ and $\theta_b\left(w_S^I - w_F^I\right) + w_F^I - c^I > \theta_b\left(w_S - w_F\right) + w_F - c$. It further implies that $(\theta_b - \theta_a)\left(w_S^I - w_F^I\right) > (\theta_b - \theta_a)\left(w_S - w_F\right)$. that is $\left(w_S^I - w_F^I\right) > (w_S - w_F)$. Furthermore, the necessary condition to hold $\theta_a\left(w_S^I - w_F^I\right) + w_F^I - c^I < \theta_a\left(w_S - w_F\right) + w_F - c$ given that $\left(w_S^I - w_F^I\right) > (w_S - w_F)$ is $w_F^I - c^I < w_F - c$.

[9]$\theta_{Ig} < 1$ implies that $\frac{w_F - c - \left(w_F^I - c^I\right)}{w_S^I - w_F^I - (w_S - w_F)} < 1$, which gives $w_S^I > w_S + c^I - c$.

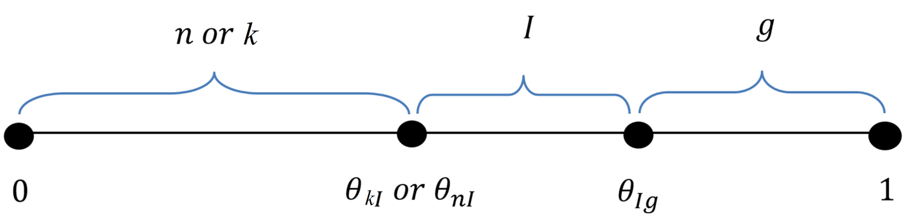

Figure 9: $0 < \theta_{fI}$ $(or\ \theta_{nI}) < \theta_{Ig} < 1$

deviate and offer a slightly smaller w_S^I to reduce the expected payment without changing the expected revenue. That is, if $\theta_{Ig} < 1$, then the firm can always change the in-house training contract such that the cutoff type θ_{Ig} shifts towards 1 to make a higher profit. In other words, if a contract results in the top types prefer in-house training and relatively lower types choose genuine degrees, then there is always a profitable deviation that the firm reduces w_S^I, so that in-house training is no longer attractive.

Remark III.1 *The firm's profit is not maximized when both individual rationality and incentive compatibility constraints are slack. As $\theta_{Ig} > \theta_g$, types between $(\theta_{Ig}, 1]$ get positive expected utility, which implies that IR constraints for those types are slack. Moreover, these types strictly prefer in-house training over a genuine degree. Hence, their IC constraints are also slack (see Laffont and Martimort, 2002).*

We now proceed with the remaining case that middle types choose in-house training, and higher types choose genuine degrees. If the firm offers $w_F \leqslant 0$, then no type has the incentive to buy a fake degree. Let θ_{nI} be a cutoff type which is indifferent between no training and in-house training, and it is given by

$$0 = \theta_{nI} w_S^I + (1 - \theta_{nI})\, w_F^I - c^I \tag{17}$$

If the firm offers $w_F > 0$, then all types prefer a fake degree over no training. Let us denote the type which is indifferent between a fake degree and in-house training as θ_{kI}, and it is defined as

$$w_F = \theta_{kI} w_S^I + (1 - \theta_{kI})\, w_F^I - c^I \tag{18}$$

We now investigate the case when $0 \leqslant \theta_{kI}$ $(or\ \theta_{nI}) < \theta_{Ig} < 1$. θ_{Ig} is the same as defined in equation (14). Figure 9 shows when $0 < \theta_{kI}$ $(or\ \theta_{nI}) < \theta_{Ig} < 1$. We get the firm's profit-maximizing contract by first showing that the firm maximizes profit by setting $w_S^I = w_F^I$ for the in-house training and then ruling out the possibility that w_F being negative and positive. We assume that if a type θ

worker is indifferent between in-house training and no training, then she chooses in-house training if $\theta \geqslant \theta^{I*}$, and chooses no training if $\theta < \theta^{I*}$.

Lemma III.3 *The contract* $M^* = \left\{\left(w_S^{I*} = c^I, w_F^{I*} = c^I\right), \left(w_S^*, w_F^* = 0\right)\right\}$ *where*

$$w_S^* \in \arg\max_{w_S} \int_{\frac{c^I}{V}}^{\frac{c}{w_S}} \left(\theta V - c^I\right) f(\theta)\, d\theta + \int_{\frac{c}{w_S}}^{1} \left(\theta V - \theta \frac{c}{w_S}\right) f(\theta)\, d\theta \tag{19}$$

maximizes the firm's profit.

Proof. *See Appendix* $2.A$. ■

The firm extracts all the surpluses generated by types between $[\theta_{nI}, \theta_{Ig})$ by offering exactly the cost of in-house training to these types. Hence, we have $\left(w_S^{I*} = c^I, w_F^{I*} = c^I\right)$ in the firm's optimal contract. While increasing w_F from negative to zero, the expected payment to types between $[\theta_{Ig}, 1]$ becomes less. By increasing w_F, the firm can lower the corresponding w_S offered to all types between $[\theta_{Ig}, 1]$. Since higher types have a higher probability of getting w_S, a lower w_S implies a lower expected payment. Therefore, we rule out $w_F < 0$ in the firm's profit maximizing solution. When $w_F > 0$, the firm provides a fixed positive payment to all types. It is equivalent to that the firm increases the utility to all types of being uneducated. While attracting any type to choose either in-house training or a genuine degree, the compensation has to be high enough to offer an incentive. Hence, it is not profitable for the firm to offer a positive w_F. It is difficult to get a simple form of optimal w_S^*. Alternatively, we examine if there exist two optimal cutoff types, θ_{nI}^* and θ_{Ig}^*, such that the firm's profit maximizing solution results in middle types choose in-house training, and higher types choose genuine degrees, $0 < \theta_{nI}^* < \theta_{Ig}^* < 1$.

We have $\theta_{nI}^* = \frac{c^I}{V}$ given that $w_S^{I*} = w_F^{I*} = c^I$. By substituting $w_S = \frac{c}{\theta_{Ig}}$ into the firm's problem stated in (19), we can rewrite the firm's problem only in terms of θ_{Ig} as

$$\max_{\theta_{Ig}} \pi\left(\theta_{Ig}\right) = \max_{\theta_{Ig}} \int_{\frac{c^I}{V}}^{\theta_{Ig}} \left(\theta V - c^I\right) f(\theta)\, d\theta + \int_{\theta_{Ig}}^{1} \left(\theta V - \theta \frac{c}{\theta_{Ig}}\right) f(\theta)\, d\theta \tag{20}$$

Taking the derivative of $\pi\left(\theta_{Ig}\right)$ w.r.t. θ_{Ig},

$$\frac{d\pi\left(\theta_{Ig}\right)}{d\theta_{Ig}} = -\left(c^I - c\right) f\left(\theta_{Ig}\right) + \int_{\theta_{Ig}}^{1} \left(\theta \frac{c}{\theta_{Ig}^2}\right) f(\theta)\, d\theta \tag{21}$$

When $c^I = c$, the first term on the R.H.S. of (21) equals to zero. Hence, the firm must offer a contract such that $\theta_{Ig}^* = 1$ for the first order condition to hold.

It implies that the firm is better off offering only in-house training contract as the firm now verifies the acquisition of the skill at no additional cost. This result coincides with the discussion we had before when $c^I \leqslant c$.

When $c^I > c$, to have the case where middle types choose in-house training, and higher types choose genuine degrees, we need the optimal cutoff type θ^*_{Ig} lies between $\left(\theta^*_{nI} = \frac{c^I}{V}, 1\right)$.

Lemma III.4 *Let $f'(\theta) > 0$ for all $\theta \in [0,1]$. As c^I increases from c, the optimal cutoff type θ^*_{Ig} decreases from 1 to 0.*

Proof. *Let us now compute $\frac{d\theta^*_{Ig}}{dc^I}$ by implicit differentiation. Denote the F.O.C. w.r.t. θ_{Ig} as $\psi\left(\theta^*_{Ig}, c^I\right) = -\left(c^I - c\right) f\left(\theta^*_{Ig}\right) + \int_{\theta^*_{Ig}}^{1} \left(\theta \frac{c}{\left(\theta^*_{Ig}\right)^2}\right) f(\theta)\, d\theta = 0$. We get*

$$\begin{aligned}
\frac{d\theta^*_{Ig}}{dc^I} &= -\frac{d\psi\left(\theta^*_{Ig}, c^I\right)/dc^I}{d\psi\left(\theta^*_{Ig}, c^I\right)/d\theta^*_{Ig}} \\
&= -\frac{-f\left(\theta^*_{Ig}\right)}{-\left(c^I - c\right) f'\left(\theta^*_{Ig}\right) - \left(\frac{c}{\theta^*_{Ig}}\right) f\left(\theta^*_{Ig}\right) - 2\int_{\theta^*_{Ig}}^{1} \left(\theta \frac{c}{\left(\theta^*_{Ig}\right)^3}\right) f(\theta)\, d\theta} < 0
\end{aligned}$$

which implies that as c^I increases, the optimal θ^*_{Ig} decreases. ■

Note that $f'(\theta) > 0$ for all $\theta \in [0,1]$ is a sufficient but not necessary condition. When $c^I = c$, we have $\theta^*_{Ig} = 1$ and $\theta^*_{nI} < 1$. Lemma III.4 shows that an increase in c^I shifts θ^*_{Ig} to the left. As c^I increases, the optimal cutoff type which is indifferent between no training and in-house training, $\theta^*_{nI} = \frac{c^I}{V} < 1$, increases and shifts to the right. As c^I increases from c, there must exist an optimal cutoff type θ^*_{Ig} such that $\theta^*_{nI} < \theta^*_{Ig} < 1$. It implies that there exists a set of types, $\left[\theta^*_{Ig}, 1\right]$, which prefers a genuine degree and a set of types, $[\theta^*_{nI}, \theta^*_{Ig})$ which prefers in-house training. Moreover, there also exists a value of c^I such that $\theta^*_{nI} = \theta^*_{Ig}$. Let us denote the cost of in-house training when $\theta^*_{nI} = \theta^*_{Ig}$ as $\hat{c}^I$, and $\hat{c}^I$ satisfies the F.O.C.

$$-\left(\hat{c}^I - c\right) f\left(\frac{\hat{c}^I}{V}\right) + \int_{\frac{\hat{c}^I}{V}}^{1} \left(\theta \frac{c}{\left(\frac{\hat{c}^I}{V}\right)^2}\right) f(\theta)\, d\theta = 0 \tag{22}$$

The upper bound of c^I that the firm is willing to employ the worker with in-house training is $\hat{c}^I$. Once the cost of in-house training exceeds $\hat{c}^I$, it is not profitable for the firm to provide in-house training. For $c^I < \hat{c}^I$, the firm has two sources of saving the information rent. One way is to use the fixed wage up to the cost of a fake degree so that the corresponding bonus can be kept low and no type is incentivized to get a fake degree. This method is the same as we discussed in Chapter II. The contribution of this chapter is that the firm now has another way to save the information rent, offering in-house training. Although in-house training is more costly than a genuine degree, the firm can exact all the rent from the worker trained in-house by covering just the cost of the training.

From Lemma III.1 to Lemma III.4, we can conclude that

Proposition III.3 *When $c < c^I < \hat{c}^I$, there exists a profit maximizing contract M^* where the firm prefers to employ higher types with a genuine degree and relatively lower types with in-house training.*

Offering in-house training incurs a fixed higher cost (since $w_S^{I*} = w_F^{I*} = c^I > c$) while hiring the worker with a degree from the outside requires the firm to give some rents to the worker due to information asymmetry. The higher the type is, the more rents this type can extract from the firm. The monotonicity of θ implies that if a type θ extracts a positive information rent, then any type greater than θ can get more rents. Hence, as the cost of in-house training increases from c, the firm finds it optimal to substitute in-house training with a genuine degree for some higher types. By doing so, the firm keeps the size of the set of types which get information rent as small as possible. If the firm replaces the in-house training with a genuine degree for some middle types, the firm has to offer a high wage to attract those higher types to choose in-house training since those types can get a positive rent from getting a genuine degree. Figure 10 shows the pattern when c^I increases from c.

We now link the results we have so far to what we had in Chapter II when there was no in-house training. We claim that when $c^I = \hat{c}^I$, we face the same situation as discussed in Chapter II where in-house training was not available. In other words, even the in-house training is available; the firm would like not to offer in-house training when it is too costly.

Recall the optimal solution from Chapter II where in-house training is not available. The optimal cutoff type, θ_λ^*, which is indifferent towards no education and a genuine degree when $\lambda = 0$, satisfies

$$-\left(\theta_\lambda^* V - c\right) f\left(\theta_\lambda^*\right) + \int_{\theta_\lambda^*}^{1} \left(\theta \frac{c}{\left(\theta_\lambda^*\right)^2}\right) f\left(\theta\right) d\theta = 0 \tag{23}$$

Since $\theta_{nI}^* \left(c^I = \hat{c}^I\right) = \theta_{Ig}^* \left(c^I = \hat{c}^I\right) = \frac{\hat{c}^I}{V}$, we can rearrange (22) to get

$$-\left(\theta_{nI}^* V - c\right) f\left(\theta_{nI}^*\right) + \int_{\theta_{nI}^*}^{1} \left(\theta \frac{c}{\left(\theta_{nI}^*\right)^2}\right) f\left(\theta\right) d\theta = 0 \tag{24}$$

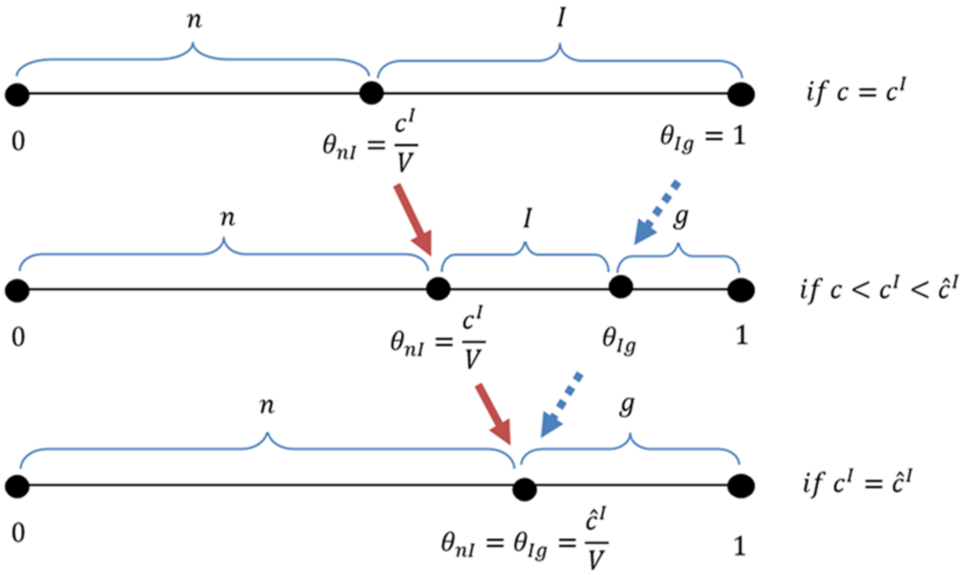

Figure 10: The patten when c^I increases from c.

By comparing (24) and (23), we get that $\theta^*_{nI}\left(c^I = \hat{c}^I\right) = \theta^*_\lambda$, which implies that all three cutoff types are the same at $c^I = \hat{c}^I$,

$$\theta^*_{nI}\left(c^I = \hat{c}^I\right) = \theta^*_{Ig}\left(c^I = \hat{c}^I\right) = \theta^*_\lambda = \frac{\hat{c}^I}{V}$$

We now can conclude that

Proposition III.4 (underinvestment in education) *When $c^I \geqslant \hat{c}^I > c$, the firm employs the worker only if the worker has a genuine degree. We face underinvestment in education as $\theta^*_{nI}\left(c^I = \hat{c}^I\right) = \theta^*_{Ig}\left(c^I = \hat{c}^I\right) = \theta^*_\lambda = \frac{\hat{c}^I}{V} > \frac{c}{V}$. When $c < c^I < \hat{c}^I$, the firm hires the worker with either in-house training or a genuine degree. The level of underinvestment in education is lower compared to the case when $c^I \geqslant \hat{c}^I$ as $\frac{c}{V} < \theta^*_{nI}\big|_{c<c^I<\hat{c}^I} = \frac{c^I}{V} < \theta^*_{Ig}\big|_{c<c^I<\hat{c}^I} < 1$.*

Proposition III.4 implies that when the cost of in-house training is sufficiently low, the firm is willing to hire the worker with in-house training. Introducing in-house training helps to restore the efficiency, but it does not lead us to the full efficiency. The lower the cost of in-house training, the more efficiency can be restored.

5 Conclusion

The monopsony power gives rise to rent-seeking which leads to a net loss in social surplus. In this chapter, we introduce another instrument for the firm to extract more rent, in-house training, in addition to the fixed wage, which was discussed in Chapter II. We have shown in Chapter II that the labour market imperfection causes a social surplus loss due to the use of the fixed wage by the monopsonist with the presence of fake degrees.

Allowing the firm to provide in-house training has mainly two effects. First, when the cost of in-house training is less than the cost of a genuine degree, offering in-house training helps in restoring the full efficiency. In this case, the monopsonist extracts the full surplus by offering the wage that covers just the cost of in-house training. Second, when the cost of in-house training is greater than the cost of a genuine degree, but less than the critical value, $\hat{c}^I$, offering in-house training helps in recovering part of the underinvestment in education caused by the labour market imperfection and the presence of fake degrees. Once the cost of in-house training exceeds the critical value, it is no longer profitable for the firm to offer in-house training.

Both in-house training and the use of fixed wage are restricted to the firm's monopsony power. In a perfectly competitive labour market, the competition leads to an equilibrium where firms offer the contract M_a, as stated in Proposition III.1. Hence, the labour market imperfection is still the main force that causes the inefficiency in the labour market, which discourages training the worker.

Chapter IV

Unverifiable Education Quality

1 Introduction

This chapter extends the model discussed in Chapter II by generalizing the value of low-quality education. While Chapter II considered the extreme case of zero value of low-quality education, where we referred to as fake degrees, we now examine the case for positive values of low-quality education.

Consider that the outside institutes offer two levels of quality of education, high-quality and low-quality. Again, we consider a labour market where a monopsonist hires a worker with two-dimensional private information through a contract. The worker holds the exogenous private information on the innate ability of the worker (type), and the endogenous private information about the quality of education the worker has chosen. We focus our interest in the setting where the social optimum suggests higher types to choose high-quality education and middle types to choose the low-quality education.

There are two main findings of this chapter. First, the labour market imperfection and information asymmetry cause underinvestment in education (high and low-quality education as a whole). Second, the monopsony power and the uncertainty about the quality of education further cause underinvestment in high-quality education. Different than Chapter II where the firm tries to avoid hiring the worker with a fake degree, low-quality education in this chapter carries positive value. Hence, we do not see the result as stated in Chapter II that the firm sets the fixed wage at the certain level. Instead, we present the range of the optimal fixed wage.

This chapter is organized as follows. We first set up the model in Section 2, and solve the social planner's problem in Section 3 with two restrictions on the parameters to get the desired case we are interested in. In Section 4, we analyze the firm's profit-maximizing problem with unverifiable education quality. Section 5 concludes this chapter.

2 Model

We extend the model described in Chapter II by considering two different levels of education quality. Chapter II took the extreme case of low-quality education, the fake degree, where getting the degree adds no value to the worker's productivity. In this chapter, we generalize the model by allowing positive value from low-quality education. We apply the same setting that a monopsonist is looking to hire a worker to undertake a skilled job in a labour market. If the worker completes the job, the firm receives a gross return V. The worker completes the job with a probability which is determined by her type and the acquisition of the skill. Type of the worker is denoted by θ and is privately known to the worker. The employer only knows that θ is continuously distributed over $[0, 1]$, with a density function $f(\theta)$ and a cumulative distribution function $F(\theta)$. The difference arises from this chapter is that the outside institutes now provide different quality of education. We consider two levels, high-quality (h) and low-quality (l). There is no in-house training in this chapter. Let e be the worker's educational choice, and we have $e \in \{n, h, l\}$ where n denotes no education. The cost of the high-quality education is $c(h) = c$, and the cost of the low-quality education is $c(l) = \lambda c$, $0 \leqslant \lambda < 1$. The high-quality education provides a type θ, $\theta \in (0, 1]$, a higher probability of being successful in doing the job, $p(\theta, h) = \beta\theta$, while the low-quality education gives a lower probability, $p(\theta, l) = \alpha\theta$, where $0 < \alpha < \beta < 1$. Once the training is completed, the worker receives a degree certificate, which does not state the level of education quality.

3 The Social Planner's Problem

The social planner chooses the education quality for each type, $e(\theta)$, and then decides the employment status by the allocation function, $x(\theta, e(\theta))$.

$$\max_{e(\theta), x(\theta, e(\theta))} SW = x(\theta, e(\theta))\left[p(\theta, e(\theta))V - c(e(\theta))\right]$$

where $x(\theta, e(\theta))$ takes a value between $[0, 1]$. 1 represents that a type is employed and 0 means unemployed. Let θ^*_α and θ^*_β be the social optimal cutoff types, which

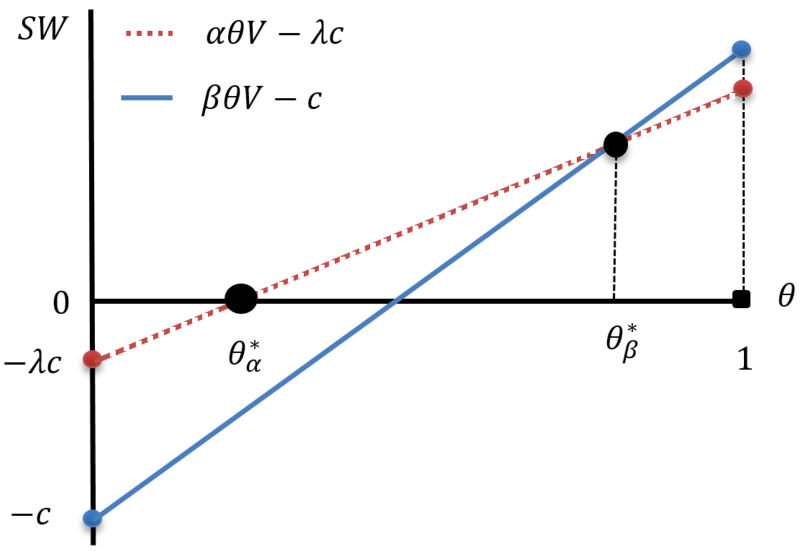

Figure 11: Social planner's problem

are implicitly defined by $\alpha\theta_\alpha^* V - \lambda c = 0$ and $\beta\theta_\beta^* V - c = \alpha\theta_\beta^* V - \lambda c$. We can write

$$\theta_\alpha^* = \frac{\lambda c}{\alpha V} \tag{25}$$

and

$$\theta_\beta^* = \frac{(1-\lambda)\,c}{(\beta-\alpha)\,V} \tag{26}$$

With different values of the exogenous variables c, λ, V, α, and β, we have various first-best allocations. We are interested in a case where both high and low-quality education are selected by the social planner. Hence, we place two restrictions on the parameters for the rest of the paper such that the lower types are allocated to have no education, middle types are assigned to the low-quality education, and the higher types are assigned to the high-quality education.

Assumption VI(a) $\alpha > \lambda\beta$

Assumption VI(b) $\alpha V - \lambda c < \beta V - c$.

The Assumption VI(a) guarantees that $\theta_\beta^* > \theta_\alpha^*$, while the Assumption VI(b) ensures $\theta_\beta^* < 1$. Let us now plot the social planner's problem as shown in Figure 11. Note that the social welfare generated by a type θ is given by $SW(\theta, e) = p(\theta, e)\,V - c(e)$. $\lambda < 1$ implies that when $\theta = 0$, $SW(0, h) < SW(0, l)$. That is, the starting point for the social welfare of high-quality education is underneath the point of departure of the low-quality education. Moreover, $\alpha < \beta$ gives that $SW(\theta, h)$ is steeper than $SW(\theta, l)$. By solving the social planner's problem, we

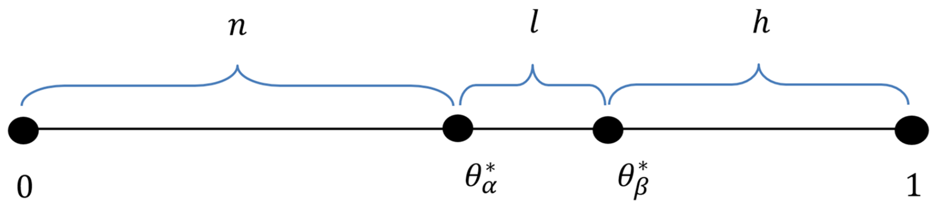

Figure 12: $0 < \theta_\alpha^* < \theta_\beta^* < 1$

get the first best allocation for education as

$$e(\theta) = \begin{cases} h & \text{if } \theta \geqslant \theta_\beta^* \\ l & \text{if } \theta_\alpha^* \leqslant \theta < \theta_\beta^* \\ n & \text{if } \theta < \theta_\alpha^* \end{cases}$$

and the first best allocation for employment status is

$$x^{FB}(\theta, e(\theta)) = \begin{cases} 1 & \begin{cases} \text{if } \theta_\alpha^* \leqslant \theta < \theta_\beta^* \text{ and } e = l \\ \text{if } \theta \geqslant \theta_\beta^* \text{ and } e = h \end{cases} \\ 0 & \quad \text{if } \theta < \theta_\alpha^* \text{ and } e = n \end{cases}$$

The social optimal cutoff types are given as $0 < \theta_\alpha^* < \theta_\beta^* < 1$ (as shown in Figure 12). That is, types between $[0, \theta_\alpha^*)$ are allocated to have no education, types between $[\theta_\alpha^*, \theta_\beta^*)$ are allocated to have the low-quality education, and types between $\left[\theta_\beta^*, 1\right]$ are allocated to have the high-quality education. The optimal social welfare is given by

$$\int_{\theta_\alpha^*}^{\theta_\beta^*} (\alpha\theta V - \lambda c) f(\theta)\, d\theta + \int_{\theta_\beta^*}^{1} (\beta\theta V - c) f(\theta)\, d\theta$$

4 The Firm's Problem

Since receiving the education is the only channel for the worker to have a positive probability of being successful, the firm employs the worker only if she is educated. As education by itself is not observable, the firm requests the job applicant to present a degree certificate as proof. If the worker fails to show a degree, then she will not be employed. However, the firm cannot distinguish between the low and high-quality education through the degree demonstrated by the worker. The firm maximizes profit by offering a contract (w_S, w_F) before the worker chooses the education. w_S is the wage for the worker when she completes the job, and w_F

if she failed. Again, we can view w_F as the fixed wage, and $(w_S - w_F)$ the bonus.

The expected profit, $\pi(\theta, e)$, of the firm from hiring a type θ worker with educational choice e can be written as

$$\pi(\theta, e) = \begin{cases} \beta\theta V - \beta\theta w_S - (1-\beta\theta) w_F & \text{if } e = h \\ \alpha\theta V - \alpha\theta w_S - (1-\alpha\theta) w_F & \text{if } e = l \end{cases}$$

If the worker is employed with high-quality education (low-quality education), then with probability $\beta\theta$ ($\alpha\theta$), the firm gets V and pays w_S to the worker; with probability $1-\beta\theta$ ($1-\alpha\theta$), the firm gets zero and pays w_F to the worker.

The worker decides on educational choice after observing (w_S, w_F) announced by the firm. We treat the reservation utility of all types as exogenous and normalize it to zero. The expected utility, $u(\theta, e)$, of a type θ worker with an educational choice, e, is given by

$$u(\theta, e) = \begin{cases} \beta\theta w_S + (1-\beta\theta) w_F - c & \text{if } e = h \\ \alpha\theta w_S + (1-\alpha\theta) w_F - \lambda c & \text{if } e = l \\ 0 & \text{if } e = n \end{cases}$$

If a type θ worker chooses the high-quality education, then she pays c and gets the expected payment equals to $\beta\theta w_S + (1-\beta\theta) w_F$. If the worker chooses the low-quality education, then she pays λc and receives the expected payment as $\alpha\theta w_S + (1-\alpha\theta) w_F$. If she chooses no education, then she will not be employed and stay with the reservation utility.

Let us now define the firm's problem. Denote the set of types which chooses high-quality education as Θ^β and low-quality education as Θ^α. The firm's problem is to solve

$$\max_{(w_S, w_F)} \begin{bmatrix} \int_{\Theta^\beta} (\beta\theta V - (\beta\theta w_S + (1-\beta\theta) w_F)) f(\theta)\, d\theta \\ + \int_{\Theta^\alpha} (\alpha\theta V - (\alpha\theta w_S + (1-\alpha\theta) w_F)) f(\theta)\, d\theta \end{bmatrix}$$

subject to

$$\text{For } \theta \in \Theta^\beta \begin{cases} IR & u(h, \theta) \geqslant 0 \\ IC & u(h, \theta) \geqslant \max\{u(l, \theta), 0\} \end{cases}$$
$$\text{For } \theta \in \Theta^\alpha \begin{cases} IR & u(l, \theta) \geqslant 0 \\ IC & u(l, \theta) \geqslant \max\{u(h, \theta), 0\} \end{cases}$$

IR is the individual-rationality or participation constraint, and IC is the incentive-compatibility constraint of the worker.

We now show that if the social planner enforces a contract $(w_S = V, w_F = 0)$, then the worker gets all the surplus and chooses efficiently.

Proposition IV.1 *Suppose the social planner enforces a contract* $(w_S = V, w_F = 0)$*, then the worker gets the full rent.*
Proof. *With* $w_S = V$ *and* $w_F = 0$*, the expected utilities of a type* θ *is given by*

$$u(\theta, e) = \begin{cases} \beta\theta V - c & \text{if } e = h \\ \alpha\theta V - \lambda c & \text{if } e = l \\ 0 & \text{if } e = n \end{cases}$$

Given the monotonicity of θ*, the definition of* θ^*_α*, as stated in equation (25), implies that types below* θ^*_α *prefer no education and types greater than* θ^*_α *prefer the low-quality education. Similarly, the definition of* θ^*_β*, as stated in equation (26), suggests that types below* θ^*_β *prefer the low-quality education over the high-quality education, and types greater than* θ^*_β *prefer the high-quality education. Together with the Assumption VI*(a) *and VI*(b)*, the allocation of all types matches with the first best allocation.* ■

The contract $(w_S = V, w_F = 0)$ is also an equilibrium contract in a perfectly competitive labour market. We can borrow the same argument from Chapter II. Consider a Bertrand-type competition, if one of the firms believes that in equilibrium the rival offers $(w_S = V, w_F = 0)$, then this firm cannot profitably deviate to another contract and make a higher profit.

Now stay with the monopsony case, if the firm is able to distinguish between high and low-quality education, then the firm practices wage discrimination to extract the full surplus. Let the contract be $\left(w^\beta_S, w^\beta_F\right)$ for the high-quality education and (w^α_S, w^α_F) for the low-quality education. We assume that in the case of a tie amongst all three choices, $\{n, l, h\}$, types between $[0, \theta^*_\alpha)$ choose no education, types between $[\theta^*_\alpha, \theta^*_\beta)$ choose the low-quality education, and types between $\left[\theta^*_\beta, 1\right]$ choose high-quality education. This assumption guarantees that when the worker is indifferent amongst all three options, $\{n, l, h\}$, the worker follows the first best allocation.

Proposition IV.2 *Suppose the firm can distinguish between high and low-quality education, then there exists an equilibrium where the firm extracts the full surplus by offering* $w^\beta_S = w^\beta_F = c$ *and* $w^\alpha_S = w^\alpha_F = \lambda c$*.*
Proof. *With* $w^\beta_S = w^\beta_F = c$ *and* $w^\alpha_S = w^\alpha_F = \lambda c$*, the expected utility for a type* θ *is*

given by

$$
\begin{aligned}
u(\theta,h) &= \beta\theta c + (1-\beta\theta)c - c = 0 \\
u(\theta,l) &= \alpha\theta\lambda c + (1-\alpha\theta)\lambda c - \lambda c = 0
\end{aligned}
$$

The worker is indifferent amongst all three options. As assumed above, types between $[0, \theta_\alpha^*)$ *choose no education, types between* $[\theta_\alpha^*, \theta_\beta^*)$ *go for the low-quality education, and types between* $\left[\theta_\beta^*, 1\right]$ *get the high-quality education. It coincides with the first best allocation. Any deviation from this strategy results in a zero utility.* ■

Since the quality of education is verifiable, the firm can offer two different contracts to different quality of education. Hence, the firm offers the lowest possible wage, which covers just the cost of education to each level of quality. The optimal contracts maximize the profit with binding participation and incentive compatibility constraints, leaving no rent to the worker.

In the case where the firm is unable to distinguish between high and low-quality education, if the firm again offers $w_S^\beta = w_F^\beta = c$ and $w_S^\alpha = w_F^\alpha = \lambda c$, then all types will get a low-quality education and pretend to be high-quality educated to get more rent. We now check how does the firm maximize profit when it does not know the education quality of the worker.

4.1 Profit Maximizing with Unverifiable Education Quality

The firm chooses one contract, (w_S, w_F), and offers to all types. Given this contract (w_S, w_F), the expected payment of a type θ worker can be written as

$$p(\theta,e)(w_S - w_F) + w_F \tag{27}$$

The value of $(w_S - w_F)$ is the key that determines the set of types, Θ^α and Θ^β. We consider the following three cases $w_S < w_F$, $w_S = w_F$, and $w_S > w_F$ to find the firm's optimal contract. In the following discussion, we will rule out the first two scenarios in equilibrium. Before we proceed further, let us first define two cutoff types. Let θ_α be the type which gets zero expected utility from choosing low-quality education, and it is given by $\alpha\theta_\alpha w_S + (1-\alpha\theta_\alpha)w_F - \lambda c = 0$. We can rewrite and get

$$\theta_\alpha = \frac{\lambda c - w_F}{\alpha(w_S - w_F)} \tag{28}$$

θ_α is indifferent between no education and low-quality education.

Let θ_β be the type which is indifferent between a low-quality education and high-quality education, $\beta\theta_\beta w_S + (1-\beta\theta_\beta) w_F - c = \alpha\theta_\beta w_S + (1-\alpha\theta_\beta) w_F - \lambda c$. Rearrange to get

$$\theta_\beta = \frac{(1-\lambda) c}{(\beta-\alpha)(w_S - w_F)} \tag{29}$$

The superscript M will be used to represent the optimal solutions for the monopsonist.

Proposition IV.3 *The profit-maximizing contract satisfies that* $\lambda c - \frac{\alpha(1-\lambda)c}{\beta-\alpha} \leqslant w_F^M \leqslant \lambda c$ *and* $w_S^M > w_F^M$*, and it results in* $0 \leqslant \theta_\alpha^M \leqslant \theta_\beta^M$.
Proof. *See Appendix* 3.*A*. ■

For $w_S < w_F$, if the firm sets $w_S < w_F \leqslant \lambda c$, then no type has an incentive to apply for this job since the expected utility of applying for the job is no greater than the reservation utility (zero). If the firm sets $w_S < w_F$ and $w_F > \lambda c$, then completing the job means the employed worker will be charged by $|w_S - w_F|$. Hence, all types prefer the low-quality education over high-quality education. Moreover, the higher the type, the lower the expected utility. Then only types below θ_α choose the low-quality education and types above prefer no education. In this case, the firm can always profitably reduce w_F and increase w_S by a small amount to bring down the expected payment without changing the expected revenue.

For $w_S = w_F$, if $w_S = w_F < \lambda c$, no type will choose to be educated since the utility after getting the education is less than the reservation utility. If $w_S = w_F > \lambda c$, all types choose the low-quality education. In this case, the firm is always better-off cutting both w_S and w_F by a small amount. By doing so, all types will still choose low-quality education, but the firm's payment will drop. We are now left with $w_S = w_F = \lambda c$. In this case, all types are indifferent between no education and the low-quality education. The maximum profit the firm gets from the contract $w_S = w_F = \lambda c$ is when all types greater than θ_α^* choose low-quality education. By increasing a small amount in w_S and reducing w_F, the firm now attracts a set of higher types choosing high-quality education which gives the firm a higher profit.

For $w_S > w_F$, based on the definition of the cutoff types stated in (28) and (29), we face the following three scenarios: (i) If $w_F > \lambda c$, then all types would like to be educated, $\theta_\alpha = 0$. In this case, the firm can reduce both w_S and w_F at the same rate, keeping all types employed but paying less. (ii) If $w_F < \lambda c - \frac{\alpha(1-\lambda)c}{\beta-\alpha} < 0$, then we get $\theta_\alpha > \theta_\beta$[10] and all types choose between no education and high-quality

[10] Compare θ_α and θ_β, by subtracting $\frac{(1-\lambda)c}{(\beta-\alpha)(w_S-w_F)}$ from $\frac{\lambda c - w_F}{\alpha(w_S-w_F)}$. Given that $w_F < \lambda c - \frac{\alpha(1-\lambda)c}{\beta-\alpha}$, we can rearrange to get $(\lambda c - w_F)(\beta-\alpha) > \alpha(1-\lambda)c$. Hence, we can write $\frac{\lambda c - w_F}{\alpha(w_S-w_F)} - \frac{(1-\lambda)c}{(\beta-\alpha)(w_S-w_F)} = \frac{(\beta-\alpha)(\lambda c - w_F) - \alpha(1-\lambda)c}{\alpha(\beta-\alpha)(w_S-w_F)} > 0$, which implies that $\theta_\alpha > \theta_\beta$.

education. The firm can make a higher profit by increasing w_F and lowering w_S to reduce the expected payment while keeping the same revenue.

Hence, we are left with $w_S^M > w_F^M$ and $\lambda c - \frac{\alpha(1-\lambda)c}{\beta-\alpha} \leqslant w_F^M \leqslant \lambda c$, which gives us the allocation of types as $0 \leqslant \theta_\alpha^M \leqslant \theta_\beta^M$. We now can write the firm's problem as

$$\max_{(w_S,w_F)} \left[\begin{array}{c} \int_{\theta_\alpha}^{\theta_\beta} \left(\alpha\theta V - \alpha\theta\left(w_S - w_F\right) - w_F\right) f\left(\theta\right) d\theta \\ + \int_{\theta_\beta}^{1} \left(\beta\theta V - \beta\theta\left(w_S - w_F\right) - w_F\right) f\left(\theta\right) d\theta \end{array} \right] \tag{30}$$

To make the firm's problem more manageable, instead of choosing wages, let the firm choose the cutoff types to solve the profit-maximizing problem. From (29), we can write

$$w_S - w_F = \frac{(1-\lambda)\,c}{(\beta-\alpha)\,\theta_\beta} \tag{31}$$

Substitute (31) into (28) to get

$$w_F = \lambda c - \frac{\alpha\,(1-\lambda)\,c}{(\beta-\alpha)}\frac{\theta_\alpha}{\theta_\beta} \tag{32}$$

Now substitute both (31) and (32) into (30) to get the firm's problem written in terms of only θ_α and θ_β,

$$\max_{\theta_\alpha,\theta_\beta}\pi = \max_{\theta_\alpha,\theta_\beta} \left[\begin{array}{c} \int_{\theta_\alpha}^{\theta_\beta} \left(\alpha\theta V - \alpha\theta\frac{(1-\lambda)c}{(\beta-\alpha)\theta_\beta} - \left(\lambda c - \frac{\alpha(1-\lambda)c}{(\beta-\alpha)}\frac{\theta_\alpha}{\theta_\beta}\right)\right) f\left(\theta\right) d\theta \\ + \int_{\theta_\beta}^{1} \left(\beta\theta V - \beta\theta\frac{(1-\lambda)c}{(\beta-\alpha)\theta_\beta} - \left(\lambda c - \frac{\alpha(1-\lambda)c}{(\beta-\alpha)}\frac{\theta_\alpha}{\theta_\beta}\right)\right) f\left(\theta\right) d\theta \end{array} \right]$$

Proposition IV.4 (underinvestment in education) *The firm maximizes expected profit by choosing a contract such that the optimal cutoff types, θ_α^M and θ_β^M, satisfy $1 > \theta_\alpha^M > \theta_\alpha^*$ and $\theta_\beta^M > \theta_\beta^*$. The optimal wages satisfy $\lambda c - \frac{\alpha(1-\lambda)c}{\beta-\alpha} \leqslant w_F^M < \lambda c$ and $0 < w_S^M - w_F^M < V$.*

Proof. *The F.O.C. w.r.t. θ_α gives*

$$\frac{d\pi}{d\theta_\alpha} = -\left(\alpha\theta_\alpha^M V - \lambda c\right) f\left(\theta_\alpha^M\right) + \left(\frac{\alpha\,(1-\lambda)\,c}{(\beta-\alpha)}\frac{1}{\theta_\beta^M}\right)\int_{\theta_\alpha^M}^{1} f\left(\theta\right) d\theta = 0 \tag{33}$$

and the F.O.C. w.r.t. θ_β gives

$$\frac{d\pi}{d\theta_\beta} = (\beta-\alpha)\left(-\theta_\beta^M V + \frac{(1-\lambda)\,c}{\beta-\alpha}\right) f\left(\theta_\beta^M\right) \tag{34}$$
$$+\frac{(1-\lambda)\,c}{(\beta-\alpha)\left(\theta_\beta^M\right)^2}\left(\int_{\theta_\alpha^M}^{\theta_\beta^M}\left(\alpha\theta - \alpha\theta_\alpha^M\right) f\left(\theta\right) d\theta + \int_{\theta_\beta^M}^{1}\left(\beta\theta - \alpha\theta_\alpha^M\right) f\left(\theta\right) d\theta\right) = 0$$

Evaluating $\frac{d\pi}{d\theta_\alpha}$ *at* $\theta_\alpha^M = 1$ *gives*

$$\left.\frac{d\pi}{d\theta_\alpha}\right|_{\theta_\alpha^M=1} = -(\alpha V - \lambda c) f(1) < 0$$

which implies that $\theta_\alpha^M < 1$. *Since* $\theta_\alpha^M < 1$, *we get* $\left(\frac{\alpha(1-\lambda)c}{(\beta-\alpha)}\frac{1}{\theta_\beta^M}\right)\int_{\theta_\alpha^M}^1 f(\theta)\,d\theta > 0$. *Then the necessary condition for the F.O.C. w.r.t.* θ_α *to hold is* $\alpha\theta_\alpha^M V - \lambda c > 0$. *Hence, we get*

$$\theta_\alpha^M > \frac{\lambda c}{\alpha V} = \theta_\alpha^*$$

Evaluating $\frac{d\pi}{d\theta_\beta}$ *at* $\theta_\beta^M = 1$ *gives*

$$\left.\frac{d\pi}{d\theta_\beta}\right|_{\theta_\beta^M=1} = (\beta-\alpha)\left(-V + \frac{(1-\lambda)c}{\beta-\alpha}\right) f(1) + \frac{(1-\lambda)c}{(\beta-\alpha)}\left(\int_{\theta_\alpha^M}^1 \left(\alpha\theta - \alpha\theta_\alpha^M\right) f(\theta)\,d\theta\right) \tag{35}$$

Since $\theta_\beta^* < 1$, *we get* $\frac{(1-\lambda)c}{(\beta-\alpha)} < V$. *Then the first term on the R.H.S. of (35) is negative, and the second term is positive. Hence, we cannot be assured whether* $\left.\frac{d\pi}{d\theta_\beta}\right|_{\theta_\beta^M=1}$ *is positive or not. From (34), given that* $\alpha\theta - \alpha\theta_\alpha^M > 0 \; \forall\, \theta \in [\theta_\alpha^M, \theta_\beta^M)$, $\beta\theta - \alpha\theta_\alpha^M > 0 \; \forall\, \theta \in \left[\theta_\beta^M, 1\right]$, *and* $\theta_\alpha^M < 1$, *we get* $\int_{\theta_\alpha^M}^{\theta_\beta^M}\left(\alpha\theta - \alpha\theta_\alpha^M\right) f(\theta)\,d\theta + \int_{\theta_\beta^M}^1 \left(\beta\theta - \alpha\theta_\alpha^M\right) f(\theta)\,d\theta > 0$. *Then the necessary condition for the F.O.C. w.r.t.* θ_β *to hold is* $-\theta_\beta^M V + \frac{(1-\lambda)c}{\beta-\alpha} < 0$. *It yields*

$$\theta_\beta^M > \frac{(1-\lambda)c}{(\beta-\alpha)V} = \theta_\beta^*$$

We know that w_F *lies between* $\left[\lambda c - \frac{\alpha(1-\lambda)c}{\beta-\alpha}, \lambda c\right]$. *If* $w_F = \lambda c$, *given that* $w_S > w_F$, *all types will choose to be educated, that is,* $\theta_\alpha = 0$. *However, we have* $\theta_\alpha^M > \theta_\alpha^* > 0$, *which implies that we can rule out the possibility of* $w_F = \lambda c$. *Moreover, from* $\theta_\beta^M > \theta_\beta^*$, *we also get that* $w_S^M - w_F^M < V$. ■

Proposition IV.4 states that compared with the social optimum, the contract that maximizes the firm's profit results in underinvestment in education. First of all, types between $[\theta_\alpha^*, \theta_\alpha^M)$ now choose no education. Moreover, fewer types choose high-quality education as $\theta_\beta^M > \theta_\beta^*$. Yet it is not sure whether the set of types that choose low-quality education has expanded or shrunk. It is possible to have an empty set of types which choose high-quality education as θ_β^M can increase to 1, and it is also possible to have no type choose low-quality education when $\theta_\alpha^M = \theta_\beta^M$. Hence, the result can be summarized in either

$$0 < \theta_\alpha^* < \theta_\alpha^M < \theta_\beta^* < \theta_\beta^M \leqslant 1$$

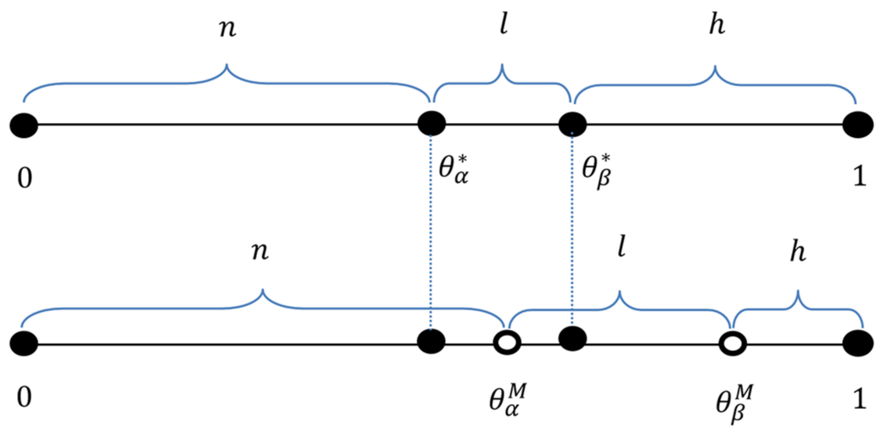

Figure 13: $0 < \theta_\alpha^* < \theta_\alpha^M < \theta_\beta^* < \theta_\beta^M < 1$

or

$$0 < \theta_\alpha^* < \theta_\beta^* < \theta_\alpha^M \leqslant \theta_\beta^M \leqslant 1$$

Figure 13 shows one possibility where $0 < \theta_\alpha^* < \theta_\alpha^M < \theta_\beta^* < \theta_\beta^M < 1$.

5 Conclusion

In this chapter, we consider a more general form of low-quality education where the value of the low-quality education is positive. The value added to the productivity is smaller when the worker is with low-quality education compared with the high-quality education. In a perfectly competitive labour market, the firm gives all the surplus to the worker, and the first best allocation is met. In an imperfectly competitive labour market, with verifiable education, even the firm does not know the type of the worker; wage discrimination based on the cost of each level of education helps the firm to extract the full rent. With unverifiable education, the optimal contract results in a reduction in the total size of educated types, including the shrink in the set that choose the high-quality education.

This chapter extends the model we have in Chapter II by allowing a positive value of α. If we take $\alpha = 0$ and $\beta = 1$, then we get back to the setting as Chapter II applies. In Chapter II, we have shown that the firm prefers to use the fixed wage to extract more rents and the monopsony power is the key that determines whether the firm can freely use the instrument or not. The next step we need to check for this chapter with two levels of education quality is to find out whether the fixed wage is still a powerful instrument for the firm's rent-seeking.

As discussed above, with perfect competition, there exists an equilibrium contract $(w_S = V, w_F = 0)$. One of our conjectures here is that if the optimal w_F turns out to be positive, then we can state that the firm again prefers to set a higher fixed wage to get more rent. We can further compare the results from the case where the firm is free to choose w_S and w_F, with the case where the firm is only allowed to choose w_S while fixing $w_F = 0$. The comparison then will show us what the consequences are, caused by the labour market imperfection through the firm's use of the fixed wage.

Another possibility of addressing the problem discussed in this chapter is to consider a richer set of contracts. While facing the information asymmetry, let the firm offer a menu of contracts for the worker to choose from, instead of one single contract. A menu of contracts may lead to a separating equilibrium where certain types choose the certain quality education. Inefficiency may arise when the firm tries to avoid the worker to make a "wrong" choice. However, considering multiple contracts requires a different setup. Hence, we do not proceed further on this.

Appendix

1 Appendix to Chapter II

1.A Proof of Proposition II.2

(I) Eliminating $w_F > \lambda c$

Proof. If $w_F > \lambda c$, then types below θ_λ choose to have a fake degree. For the cutoff type θ_λ, we can write $\theta_\lambda w_S + (1-\theta_\lambda) w_F - c = w_F - \lambda c > 0$. Compare two contracts, $(w_S, w_F > \lambda c)$ and $(w'_S = w_S - \varepsilon, w'_F = w_F - \varepsilon)$, where $w'_F > \lambda c$. Both contracts result in the same cutoff type θ_λ since we have $\theta_\lambda w'_S + (1-\theta_\lambda) w'_F = w'_F - \lambda c$. Hence, the expected revenue of these two contracts is the same. We now compare the expected payment of these two contracts.

The expected payment with w'_F is

$$E(w'_F = w_F - \varepsilon) = F(\theta_\lambda) w'_F + (1 - F(\theta_\lambda))(\theta w'_S + (1-\theta) w'_F)$$

We now compare two expected payments of these two contracts,

$$\begin{aligned} & E(w_F > \lambda c) - E(w'_F = w_F - \varepsilon > \lambda c) \\ = \; & F(\theta_\lambda) w_F + (1 - F(\theta_\lambda))(\theta w_S + (1-\theta) w_F) \\ & - [F(\theta_\lambda)(w_F - \varepsilon) + (1 - F(\theta_\lambda))(\theta(w_S - \varepsilon) + (1-\theta)(w_F - \varepsilon))] \\ = \; & \varepsilon > 0 \end{aligned}$$

Hence, $E(w_F > \lambda c) > E(w'_F = w_F - \varepsilon)$. The firm gets a higher expected profit with the contract (w'_S, w'_F). ∎

(II) profit maximizing when $w_F \leqslant \lambda c$

Proof. With $w_F \leqslant \lambda c$, no type would like to choose a fake degree. Consider two contracts $(w_S, w_F < \lambda c)$ and $(w'_S, w'_F = w_F + \varepsilon < \lambda c)$. Suppose both contracts give the same cutoff type θ_λ. Hence, the expected revenue is the same from both contracts. For θ_λ, we can write

$$\theta_\lambda w_S + (1-\theta_\lambda) w_F - c = \theta_\lambda w'_S + (1-\theta_\lambda)(w_F + \varepsilon) - c$$

which yields $w'_S = w_S - \frac{1-\theta_\lambda}{\theta_\lambda}\varepsilon$. Compare the expected payment from two contracts,

$(w_S, w_F < \lambda c)$ and $(w'_S, w'_F = w_F - \varepsilon)$,

$$\begin{aligned}
& E\left(w_S, w_F < \lambda c\right) - E\left(w'_S, w'_F = w_F - \varepsilon\right) \\
= & \left(1 - F\left(\theta_\lambda\right)\right)\left[\theta w_S + \left(1-\theta\right) w_F\right] - \left(1 - F\left(\theta_\lambda\right)\right)\left[\theta w'_S + \left(1-\theta\right) w'_F\right] \\
= & \left(1 - F\left(\theta_\lambda\right)\right)\left[\begin{array}{c} \theta w_S + \left(1-\theta\right) w_F - \theta\left(w_S - \frac{1-\theta_\lambda}{\theta_\lambda}\varepsilon\right) \\ -\left(1-\theta\right)\left(w_F + \varepsilon\right) \end{array}\right] \qquad (36) \\
= & \left(1 - F\left(\theta_\lambda\right)\right)\left(\frac{\theta}{\theta_\lambda} - 1\right)\varepsilon
\end{aligned}$$

(36): substitute $w'_S = w_S - \frac{1-\theta_\lambda}{\theta_\lambda}\varepsilon$.
Since $\frac{\theta}{\theta_\lambda} - 1 > 0 \;\forall\; \theta \in [\theta_\lambda, 1]$, we get $E\left(w_S, w_F < \lambda c\right) > E\left(w'_S, w'_F = w_F - \varepsilon\right)$. It implies that $\min E\left(w_F\right) = E\left(w_F = \lambda c\right)$. The firm maximizes the expected profit by setting $w_F^* = \lambda c$.

■

1.B Proof of Proposition II.5

Proof. The two definitions of the cut-off types, equation (5) and (6), implies that instead of choosing w_S and w_F, the firm can also choose w_F and θ_h to maximize the expected profit. Substitute $w_S - w_F = \frac{c-w_F}{\theta_h}$ and $\theta_l = \frac{c\theta_h}{c-w_F}$ into the firm's problem stated in (7). Then the firm's problem can be written as

$$\max_{\theta_h, w_F} \pi = \max_{\theta_h, w_F}\left[\begin{array}{c} \mu\left(\int_{\theta_l(w_F,\theta_h)}^1\left(\theta V - \theta\frac{c-w_F}{\theta_h}\right) f\left(\theta\right) d\theta - w_F\right) \\ +\left(1-\mu\right)\left(\int_{\theta_h}^1\left(\theta V - \theta\frac{c-w_F}{\theta_h} - w_F\right) f\left(\theta\right) d\theta\right) \end{array}\right]$$

The derivative of the expected profit with respect to w_F is

$$\begin{aligned}
\frac{d\pi}{dw_F} = & \mu\left[\begin{array}{c} -\left(\theta_l V - \theta_l\frac{c-w_F}{\theta_h}\right) f\left(\theta_l\right)\frac{d\theta_l}{dw_F} \\ +\int_{\theta_l(w_F,\theta_h)}^1 \frac{\theta}{\theta_h} f\left(\theta\right) d\theta - 1 \end{array}\right] \\
& +\left(1-\mu\right)\int_{\theta_h}^1\left(\frac{\theta}{\theta_h} - 1\right) f\left(\theta\right) d\theta
\end{aligned}$$

Since $\theta_l = \frac{c\theta_h}{c-w_F}$, we can write

$$\frac{d\theta_l}{dw_F} = \frac{c\theta_h}{\left(c - w_F\right)^2}$$

Replace $\frac{d\theta_l}{dw_F}$ with $\frac{c\theta_h}{(c-w_F)^2}$ and substitute $\theta_l = \frac{c\theta_h}{c-w_F}$ to rewrite the derivative of the expected profit with respect to w_F,

$$
\begin{aligned}
\frac{d\pi}{dw_F} &= \mu \left[\begin{array}{c} -\left(\frac{c}{c-w_F}\theta_h V - \frac{c}{c-w_F}\theta_h \frac{c-w_F}{\theta_h}\right) f\left(\theta_l\right) \frac{c\theta_h}{(c-w_F)^2} \\ + \int_{\theta_l(w_F,\theta_h)}^{1} \frac{\theta}{\theta_h} f\left(\theta\right) d\theta - 1 \end{array} \right] \\
&\quad + (1-\mu) \int_{\theta_h}^{1} \left(\frac{\theta}{\theta_h} - 1\right) f\left(\theta\right) d\theta \\
&= \mu \left[\begin{array}{c} -\left(\frac{c}{c-w_F}\theta_h V - c\right) f\left(\theta_l\right) \frac{c\theta_h}{(c-w_F)^2} \\ + \int_{\theta_l(w_F,\theta_h)}^{1} \frac{\theta}{\theta_h} f\left(\theta\right) d\theta - 1 \end{array} \right] \\
&\quad + (1-\mu) \left[\int_{\theta_h}^{1} \left(\frac{\theta}{\theta_h} - 1\right) f\left(\theta\right) d\theta \right]
\end{aligned} \tag{37}
$$

Similarly, the derivative of the expected profit with respect to θ_h is

$$
\begin{aligned}
\frac{d\pi}{d\theta_h} &= \mu \left[\begin{array}{c} -\left(\theta_l V - \theta_l \frac{c-w_F}{\alpha(\theta_h)}\right) f\left(\theta_l\right) \frac{d\theta_l}{d\theta_h} \\ + \left(c - w_F\right) \frac{1}{\theta_h} \int_{\theta_l(w_F,\theta_h)}^{1} \frac{\theta}{\theta_h} f\left(\theta\right) d\theta \end{array} \right] \\
&\quad + (1-\mu) \left[\begin{array}{c} -\left(\theta_h V - c\right) f\left(\theta_h\right) \\ + \left(c - w_F\right) \frac{1}{\theta_h} \int_{\theta_h}^{1} \frac{\theta}{\theta_h} f\left(\theta\right) d\theta \end{array} \right]
\end{aligned}
$$

We have

$$
\frac{d\theta_l}{d\theta_h} = \frac{c}{c - w_F} \tag{38}
$$

Substituting (38) into the derivative of the expected profit with respect to θ_h,

$$
\begin{aligned}
\frac{d\pi}{d\theta_h} &= \mu \left[\begin{array}{c} -\left(\frac{c}{c-w_F}\theta_h V - c\right) f\left(\theta_l\right) \frac{c}{c-w_F} \\ + \left(c - w_F\right) \frac{1}{\theta_h} \int_{\theta_l(w_F,\theta_h)}^{1} \frac{\theta}{\theta_h} f\left(\theta\right) d\theta \end{array} \right] \\
&\quad + (1-\mu) \left[\begin{array}{c} -\left(\theta_h V - c\right) f\left(\theta_h\right) \\ + \left(c - w_F\right) \frac{1}{\theta_h} \int_{\theta_h}^{1} \frac{\theta}{\theta_h} f\left(\theta\right) d\theta \end{array} \right]
\end{aligned} \tag{39}
$$

Suppose $\mu \in (0, 1)$ and the firm sets $w_F = 0$. With $w_F = 0$, no type will buy a fake degree at any cost level, and we have $\theta_l = \theta_h$. The F.O.C. with respect to θ_h now becomes

$$\begin{aligned}
&\left.\frac{d\pi}{d\theta_h}\right|_{w_F=0} \\
= \;& \mu\left[\begin{array}{c} -\left(\theta_l^* V-c\right) f\left(\theta_l^*\right) \\ +\frac{c}{\theta_l^*}\int_{\theta_l^*}^1 \frac{\theta}{\theta_l^*} f(\theta)\, d\theta \end{array}\right]+(1-\mu)\left[\begin{array}{c} -\left(\theta_l^* V-c\right) f\left(\theta_l^*\right) \\ +\frac{c}{\theta_l^*}\int_{\theta_l^*}^1 \frac{\theta}{\theta_l^*} f(\theta)\, d\theta \end{array}\right] \\
= \;& -\left(\theta_l^* V-c\right) f\left(\theta_l^*\right)+\frac{c}{\theta_l^*}\int_{\theta_l^*}^1 \frac{\theta}{\theta_l^*} f(\theta)\, d\theta=0 \qquad (40)
\end{aligned}$$

The F.O.C. (40) shows that the optimal cutoff type is independent from μ and is equal to the optimal cutoff type in equation (4) from the case when the cost of a fake degree is known as 0. Substituting (40) into (37) to get

$$\begin{aligned}
&\left.\frac{d\pi}{dw_F}\right|_{w_F=0} \\
= \;& \mu\left(\begin{array}{c} -\left(\theta_l^* V-c\right) f\left(\theta_l^*\right)\frac{\theta_l^*}{c} \\ +\int_{\theta_l^*}^1 \frac{\theta}{\theta_l^*} f(\theta)\, d\theta-1 \end{array}\right)+(1-\mu)\left(\int_{\theta_l^*}^1\left(\frac{\theta}{\theta_l^*}-1\right) f(\theta)\, d\theta\right) \\
= \;& -\mu+(1-\mu)\int_{\theta_l^*}^1\left(\frac{\theta}{\theta_l^*}-1\right) f(\theta)\, d\theta
\end{aligned}$$

Let $\frac{\int_{\theta_l^*}^1\left(\frac{\theta}{\theta_l^*}-1\right) f(\theta)d\theta}{1+\int_{\theta_l^*}^1\left(\frac{\theta}{\theta_l^*}-1\right) f(\theta)d\theta}=\hat{\mu}$, $0<\hat{\mu}<1$. For $0<\mu<\hat{\mu}$, we have $\frac{d\pi}{dw_F}\left(w_F=0\right)>0$, which is a contradiction to that the firm sets $w_F=0$. Hence, setting $w_F=0$ cannot maximize profit when $\mu\in(0,\hat{\mu})$, and the firm sets $w_F^*>0$.
Denote the optimal wages as w_S^* and w_F^*, where $w_S^*-w_F^*>0$. According to (5) and (6), we can write the optimal cutoff types as

$$\theta_l^*=\frac{c}{w_S^*-w_F^*} \text{ and } \theta_h=\frac{c}{w_S^*-w_F^*}-\frac{w_F^*}{w_S^*-w_F^*}$$

If $0<\mu<\hat{\mu}$, the firm sets $w_F^*>0$, then we get $\theta_l^*>\theta_h^*>\theta^*$. If $\hat{\mu}<\mu<1$, the firm sets $w_F^*=0$, then we have $\theta_l^*=\theta_h^*>\theta^*$. ■

1.C Proof of Proposition II.7

1.C.1 Eliminating $w_F>\bar{\lambda}c$

With $w_F>\bar{\lambda}c$, all types prefer a fake degree over no degree since $u(k,\theta)=w_F-\lambda c>u(n,\theta)=0$ for all θ and λ. Denote the type which gets the same expected utility after getting a fake degree and a genuine degree as θ_{kg}. According to the monotonicity of probability of being successful θ and $w_S > w_F$, types between $[0, \theta_{kg})$ get a higher expected utility from buying a fake degree, and types between $(\theta_{kg}, 1]$ get a higher expected utility from getting a genuine degree. Hence, the expected profit of each firm when both firms offering (w_S, w_F) is

$$\begin{aligned}\pi_{kg} &= \frac{1}{2}\left(-\int_0^{\theta_{kg}} w_F f(\theta)\, d\theta + \int_{\theta_{kg}}^1 (\theta V - \theta w_S - (1-\theta) w_F) f(\theta)\, d\theta\right) \\ &= \frac{1}{2}\left(\underbrace{-w_F}_{\text{negative}} + \underbrace{(V - w_S + w_F)\int_{\theta_{kg}}^1 \theta f(\theta)\, d\theta}_{\text{positive}}\right) \end{aligned} \tag{41}$$

$$\text{where } \theta_{kg} w_S + (1-\theta_{kg}) w_F - c = w_F - \bar{\lambda} c,\ \theta_{kg} = \frac{c - \bar{\lambda} c}{w_S - w_F}$$

Since $c - \bar{\lambda} c > 0$ and $w_S - w_F > 0$, we have $\theta_{kg} > 0$. If the firm hires only fake degree holders, the expected overall profit must be negative. Hence, firms make a nonnegative expected profit only if $\theta_{kg} < 1$, which means firms will set $w_S - w_F > c - \bar{\lambda} c$. Denote the type which makes zero profits for firms as θ_0, and it is implicitly defined by $0 = \theta_0 V - [\theta_0 w_S + (1-\theta_0) w_F]$. Rearrange to get

$$\theta_0 = \frac{w_F}{V - (w_S - w_F)}$$

Given that $w_F > 0$, firms cannot make positive expected profits by setting $w_S > V$. Hence, with $w_F > 0$ and $w_S < V$, we get $V - w_S + w_F > 0$ and $\theta_0 < 1$. From (41), the expected profits from hiring a type θ are increasing in the probability θ. Hence, only types greater than θ_0 make positive profits for firms after getting a genuine degree. If $\theta_0 < \theta_{kg}$, then all types that prefer a genuine degree make positive expected profits. However, if $\theta_0 > \theta_{kg}$, then only a subset of those types which prefer a genuine degree make positive expected profits for firms.

If both firms offer the same contract where $w_F > \bar{\lambda} c$, a firm can deviate and offer a contract that attracts only those types which make a positive expected profit. Since in this case, higher types make higher expected profits for firms, the deviating firm can offer a higher w_S and a lower w_F to attract these relatively higher types. Let us now show it formally.

Proof. In equilibrium, firm i believes that firm j offers (w_S, w_F) where $w_F > \bar{\lambda} c$.

(i) If $\theta_0 > \theta_{kg}$, only types between $(\theta_0, 1]$ make positive expected profits for firms. Suppose firm i deviates and offers $\left(w_S + \frac{V - w_S}{w_F}\varepsilon, w_F - \varepsilon\right)$ where $\varepsilon > 0$ and $w_F - \varepsilon >$

$\bar{\lambda}c$. Type θ_0 is indifferent between two contracts as

$$\theta_0\left(w_S+\frac{V-w_S}{w_F}\varepsilon\right)+(1-\theta_0)(w_F-\varepsilon)-c=\theta_0 w_S+(1-\theta_0)w_F-c$$

Because $w_S+\frac{V-w_S}{w_F}\varepsilon>w_S$ and $w_F-\varepsilon<w_F$, based on the monotonicity of probability θ, firm $i's$ offer is more attractive for all types greater than θ_0, and all types below θ_0 prefer firm j. Hence, the expected profit of firm i becomes,

$$\pi_i'=\int_{\theta_0}^{1}\left(\theta V-\theta\left(w_S+\frac{V-w_S}{w_F}\varepsilon\right)-(1-\theta)(w_F-\varepsilon)\right)f(\theta)\,d\theta$$

The deviation is profitable if $\pi_i'-\pi_{kg}>0$, that is,

$$\begin{aligned}
&\pi_i'-\pi_{kg}\\
=&\int_{\theta_0}^{1}\left(\theta V-\theta\left(w_S+\frac{V-w_S}{w_F}\varepsilon\right)-(1-\theta)(w_F-\varepsilon)\right)f(\theta)\,d\theta\\
&-\frac{1}{2}\left(-\int_0^{\theta_{kg}}w_F f(\theta)\,d\theta+\int_{\theta_{kg}}^{1}(\theta V-\theta w_S-(1-\theta)w_F)f(\theta)\,d\theta\right)\\
=&\int_{\theta_0}^{1}(\theta V-\theta w_S-(1-\theta)w_F)f(\theta)\,d\theta-\int_{\theta_0}^{1}\left(\theta\frac{V-w_S}{w_F}\varepsilon-(1-\theta)\varepsilon\right)f(\theta)\,d\theta\\
&+\frac{1}{2}\int_0^{\theta_{kg}}w_F f(\theta)\,d\theta-\frac{1}{2}\int_{\theta_{kg}}^{1}(\theta V-\theta w_S-(1-\theta)w_F)f(\theta)\,d\theta\\
=&\int_{\theta_0}^{1}(\theta V-\theta w_S-(1-\theta)w_F)f(\theta)\,d\theta-\frac{1}{2}\int_{\theta_{kg}}^{1}(\theta V-\theta w_S-(1-\theta)w_F)f(\theta)\,d\theta\\
&+\frac{1}{2}\int_0^{\theta_{kg}}w_F f(\theta)\,d\theta-\varepsilon\int_{\theta_0}^{1}\left(\frac{\theta(V-w_S+w_F)-w_F}{w_F}\right)f(\theta)\,d\theta>0
\end{aligned}$$

For $\theta>\theta_0$, $\theta(V-w_S+w_F)-w_F>0$, hence, $\int_{\theta_0}^{1}\left(\frac{\theta(V-w_S+w_F)-w_F}{w_F}\right)f(\theta)\,d\theta>0$. Because types between $[\theta_{kg},\theta_0]$ provide negative profits, we have

$$\begin{aligned}
&\int_{\theta_0}^{1}(\theta V-\theta w_S-(1-\theta)w_F)f(\theta)\,d\theta-\frac{1}{2}\int_{\theta_{kg}}^{1}(\theta V-\theta w_S-(1-\theta)w_F)f(\theta)\,d\theta\\
=&\frac{1}{2}\int_{\theta_0}^{1}(\theta V-\theta w_S-(1-\theta)w_F)f(\theta)\,d\theta-\frac{1}{2}\int_{\theta_{kg}}^{\theta_0}(\theta V-\theta w_S-(1-\theta)w_F)f(\theta)\,d\theta\\
>&\ 0
\end{aligned}$$

Hence, to hold $\pi_i' > \pi_{kg}$, we need

$$0 < \varepsilon < \min\left\{ w_F - \bar{\lambda}c, \frac{\left(\begin{array}{c} \int_{\theta_0}^1 (\theta V - \theta w_S - (1-\theta)\, w_F)\, f(\theta)\, d\theta \\ -\frac{1}{2}\int_{\theta_{kg}}^1 (\theta V - \theta w_S - (1-\theta)\, w_F)\, f(\theta)\, d\theta \\ +\frac{1}{2}\int_0^{\theta_{kg}} w_F f(\theta)\, d\theta \end{array}\right)}{\int_{\theta_0}^1 \left(\frac{\theta(V - w_S + w_F) - w_F}{w_F}\right) f(\theta)\, d\theta} \right\}$$

where $\frac{\left(\int_{\theta_0}^1 (\theta V - \theta w_S - (1-\theta) w_F) f(\theta) d\theta - \frac{1}{2}\int_{\theta_{kg}}^1 (\theta V - \theta w_S - (1-\theta) w_F) f(\theta) d\theta + \frac{1}{2}\int_0^{\theta_{kg}} w_F f(\theta) d\theta\right)}{\int_{\theta_0}^1 \left(\frac{\theta(V - w_S + w_F) - w_F}{w_F}\right) f(\theta) d\theta} > 0$.

(ii) If $\theta_0 \leqslant \theta_{kg}$, types between $(\theta_{kg}, 1]$ make positive expected profits for firms. Suppose firm i offers $\left(w_S + \frac{w_S - w_F - (c - \bar{\lambda}c)}{c - \bar{\lambda}c}\varepsilon, w_F - \varepsilon\right)$, where $\varepsilon > 0$ and $w_F - \varepsilon > \bar{\lambda}c$. $w_S - w_F - (c - \bar{\lambda}c) > 0$ as $\theta_{kg} < 1$. The type θ_{kg} is indifferent between two contracts since

$$\theta_{kg}\left(w_S + \frac{w_S - w_F - (c - \bar{\lambda}c)}{c - \bar{\lambda}c}\varepsilon\right) + (1 - \theta_{kg})\,(w_F - \varepsilon) - c = \theta_{kg} w_S + (1 - \theta_{kg})\, w_F - c$$

Now only types greater than θ_{kg} will choose firm i because $w_S + \frac{w_S - w_F - (c - \bar{\lambda}c)}{c - \bar{\lambda}c}\varepsilon > w_S$ and $w_F - \varepsilon < w_F$. The expected profit of firm i becomes

$$\begin{aligned} \pi_i' &= \int_{\theta_{kg}}^1 \left(\theta V - \theta\left(w_S + \frac{w_S - w_F - (c - \bar{\lambda}c)}{c - \bar{\lambda}c}\varepsilon\right) - (1-\theta)\,(w_F - \varepsilon)\right) f(\theta)\, d\theta \\ &= \int_{\theta_{kg}}^1 (\theta V - \theta w_S - (1-\theta)\, w_F)\, f(\theta)\, d\theta - \varepsilon \int_{\theta_{kg}}^1 \left(\theta \frac{w_S - w_F}{c - \bar{\lambda}c} - 1\right) f(\theta)\, d\theta \end{aligned}$$

The deviation is profitable if $\pi_i' > \pi_{kg}$, that is,

$$\left(\begin{array}{c} \int_{\theta_{kg}}^1 (\theta V - \theta w_S - (1-\theta)\, w_F)\, f(\theta)\, d\theta - \varepsilon\int_{\theta_{kg}}^1 \left(\theta\frac{w_S - w_F}{c - \bar{\lambda}c} - 1\right) f(\theta)\, d\theta \\ +\frac{1}{2}\int_0^{\theta_{kg}} w_F f(\theta)\, d\theta - \frac{1}{2}\int_{\theta_{kg}}^1 (\theta V - \theta w_S - (1-\theta)\, w_F)\, f(\theta)\, d\theta \end{array}\right) > 0$$

$$\text{or} \left(\begin{array}{c} \frac{1}{2}\int_{\theta_{kg}}^1 (\theta V - \theta w_S - (1-\theta)\, w_F)\, f(\theta)\, d\theta + \frac{1}{2}\int_0^{\theta_{kg}} w_F f(\theta)\, d\theta \\ -\varepsilon\int_{\theta_{kg}}^1 \left(\theta\frac{w_S - w_F}{c - \bar{\lambda}c} - 1\right) f(\theta)\, d\theta \end{array}\right) > 0$$

Since $\theta\frac{w_S - w_F}{c - \bar{\lambda}c} - 1 \geqslant 0\ \forall\theta \in [\theta_{kg}, 1]$, $\int_{\theta_{kg}}^1 \left(\theta\frac{w_S - w_F}{c - \bar{\lambda}c} - 1\right) f(\theta)\, d\theta > 0$. Hence, there must exist an ε where

$$0<\varepsilon<\min\left\{w_F-\bar{\lambda}c,\ \frac{\frac{1}{2}\int_0^{\theta_{kg}} w_F f(\theta)\,d\theta+\frac{1}{2}\int_{\theta_{kg}}^1\begin{pmatrix}\theta V-\theta w_S\\ -(1-\theta)w_F\end{pmatrix} f(\theta)\,d\theta}{\int_{\theta_{kg}}^1\left(\theta\frac{w_S-w_F}{c-\bar{\lambda}c}-1\right)f(\theta)\,d\theta}\right\}$$

and the deviation is profitable. ■

(i) When $\theta_0>\theta_{kg}$, only a subset of the types, $\theta\in(\theta_0,1]$, make positive expected profits for firms after getting a genuine degree. Any firm can deviate and offer a contract which attracts only types greater than θ_0. The deviating firm increases w_S and lowers w_F, making the type θ_0 indifferent between two contracts. By doing so, all types greater than θ_0 strictly prefer the deviating contract since they have a higher probability of getting the wage for success (see Figure **??**). The deviation is profitable since it only attracts types which make positive expected profits without sharing with the rival, and throws all the types which provide a negative expected profit off.

(ii) When $\theta_0\leqslant\theta_{kg,}$ all types which prefer a genuine degree make positive expected profits. Any firm can deviate and offer a contract that attracts all types greater than θ_{kg}. The deviating firm again increases w_S, decreases w_F, and makes type θ_{kg} indifferent between two contracts. Then the monotonicity of probability θ implies that all types greater than θ_{kg} strictly prefer the deviating firm as they have a higher chance of getting w_S (see Figure 3). This deviation is also profitable for the same reason as stated in the previous scenario.

For $w_F>\bar{\lambda}c$, the deviating strategy is to take all "good" types (make a positive expected profit) and leave all "bad" types (make a negative expected profit) to the rival. In both scenarios, the deviation targets at some relatively higher types. Hence, the deviating firm rewards more for success and pays less when the worker fails, so that only these higher types get attracted to the deviating firm. The deviation converts to $(V,0)$ as the deviating firm is increasing w_S and decreasing w_F.

1.C.2 Eliminating $w_F<0$

When both firms offer (w_S,w_F) and $w_F<0$, no type has an incentive to buy a fake degree. Let θ_{ng} be the type which gets zero expected utility after pursuing a genuine degree. According to the monotonicity of probability θ, types between $[0,\theta_{ng})$ prefer no degree and types between $[\theta_{ng},1]$ choose a genuine degree. The expected profit for each firm is

$$\pi_{ng} = \frac{1}{2}\int_{\theta_{ng}}^{1} \left(\theta V - \theta w_S - (1-\theta)\, w_F\right) f(\theta)\, d\theta$$
$$\text{where } \theta_{ng} w_S + (1-\theta_{ng})\, w_F - c = 0,\ \theta_{ng} = \frac{c - w_F}{w_S - w_F}$$

Since $c - w_F > 0$ and $w_S - w_F > 0$, we get $\theta_{ng} > 0$. To guarantee that the expected overall profits are non-negative, we must have $w_S > c$ for $\theta_{ng} < 1$. Given that $w_F < 0$, firms can either set $w_S \leqslant V$ or $w_S > V$ to make non-negative profits. If firms set $w_F < 0$ and $w_S \leqslant V$, all types that get a genuine degree provide firms positive expected profits. But if firms set $w_F < 0$ and $w_S > V$, firms may hire some types which make negative expected profits. In this case, as $V - (w_S - w_F) < 0$, the expected profits are decreasing with the probability θ. Let the type which makes a zero expected profit again be θ_0, $\theta_0 = \frac{w_F}{V-(w_S-w_F)}$, with contract (w_S, w_F) and $\theta_0 \in (0,1)$. Types which provide positive expected profits for firms are types below θ_0. The type θ_{ng} must make positive expected profits for firms so that the expected overall profits are non-negative. Hence, for firms to make non-negative expected profits, we must have $\theta_{ng} < \theta_0$. $\theta_{ng} < \theta_0$ holds as long as $\theta^* < \theta_{ng}$[11], and types below θ^* cannot make positive expected profits after getting a genuine degree. Hence, we get $\theta^* < \theta_{ng} < \theta_0$.

If both firms offer the same contract where $w_F < 0$, then there always exists a profitable deviation in each case: when $w_S \leqslant V$ and $w_S > V$.

Proof. In equilibrium, firm i believes that firm j offers (w_S, w_F) where $w_F < 0$.
(i) $w_F < 0$ and $w_S \leqslant V$. Suppose firm i offers $(w_S, w_F + \varepsilon)$ where $\varepsilon > 0$ and $w_F + \varepsilon < 0$. Given both contracts, all types are better off choosing firm i. Denote the type which gets zero expected utility from choosing firm i with a genuine degree as θ'_{ng}, $\theta'_{ng} = \frac{c-w_F-\varepsilon}{w_S-w_F-\varepsilon}$. Since $\varepsilon > 0$, we get $\theta'_{ng} < \theta_{ng}$ and all types greater than θ'_{ng} make a positive profit for firm i. The expected profit of firm i becomes,

$$\begin{aligned}\pi'_i &= \int_{\theta'_{ng}}^{1} \left(\theta V - \theta w_S - (1-\theta)(w_F + \varepsilon)\right) f(\theta)\, d\theta \\ &= \int_{\theta'_{ng}}^{1} \left(\theta V - \theta w_S - (1-\theta)\, w_F\right) f(\theta)\, d\theta - \varepsilon \int_{\theta'_{ng}}^{1} (1-\theta)\, f(\theta)\, d\theta\end{aligned}$$

[11]As $\theta_{ng} = \frac{c-w_F}{w_S-w_F} < \frac{w_F}{V-(w_S-w_F)} = \theta_0$, we have $(c - w_F)[V - (w_S - w_F)] > w_F (w_S - w_F)$, $(c - w_F) V - c (w_S - w_F) > 0$, $\frac{c-w_F}{w_S-w_F} > \frac{c}{V}$, $\theta_{ng} > \theta^*$.

The deviation is profitable if $\pi_i' - \pi_{ng} > 0$, that is,

$$\begin{pmatrix} \int_{\theta_{ng}'}^{1} (\theta V - \theta w_S - (1-\theta) w_F) f(\theta) d\theta - \varepsilon \int_{\theta_{ng}'}^{1} (1-\theta) f(\theta) d\theta \\ -\frac{1}{2} \int_{\theta_{ng}}^{1} (\theta V - \theta w_S - (1-\theta) w_F) f(\theta) d\theta \end{pmatrix} > 0$$

There must exist a ε where

$$0 < \varepsilon < \min \left\{ -w_F, \frac{\begin{pmatrix} \int_{\theta_{ng}'}^{1} (\theta V - \theta w_S - (1-\theta) w_F) f(\theta) d\theta \\ -\frac{1}{2} \int_{\theta_{ng}}^{1} (\theta V - \theta w_S - (1-\theta) w_F) f(\theta) d\theta \end{pmatrix}}{\int_{\theta_{ng}'}^{1} (1-\theta) f(\theta) d\theta} \right\}$$

and firm i makes a profitable deviation.

(ii) $w_F < 0$ and $w_S > V$. In this case, $V - w_S + w_F < 0$, which implies that the lower the type is, the higher the expected profits it can provide to firms after getting a genuine degree. Suppose firm i offers $\left(w_S - \frac{V-w_S}{w_F}\varepsilon, w_F + \varepsilon\right)$ where $w_F + \varepsilon < 0$. Type θ_0 is indifferent between two contracts since

$$\theta_0 \left(w_S - \frac{V - w_S}{w_F}\varepsilon\right) + (1-\theta_0)(w_F + \varepsilon) - c = \theta_0 w_S + (1-\theta_0) w_F - c$$

Because $w_S - \frac{V-w_S}{w_F}\varepsilon < w_S$ and $w_F + \varepsilon > w_F$, all types greater than θ_0 prefer firm $j's$ offer. For types below θ_0, denote the type which gets the same expected utility from choosing no degree and a genuine degree as θ_{ng}'',

$$\begin{aligned} \theta_{ng}'' \left(w_S - \frac{V - w_S}{w_F}\varepsilon\right) + \left(1 - \theta_{ng}''\right)(w_F + \varepsilon) - c &= 0 \\ \theta_{ng}'' &= \frac{c - w_F - \varepsilon}{w_S - w_F - \left(1 + \frac{V-w_S}{w_F}\right)\varepsilon} \end{aligned}$$

$\theta_{ng}'' < \theta_{ng}$ because

$$\begin{aligned} \theta_{ng}'' - \theta_{ng} &= \frac{c - w_F - \varepsilon}{w_S - w_F - \left(1 + \frac{V-w_S}{w_F}\right)\varepsilon} - \frac{c - w_F}{w_S - w_F} \\ &= \frac{\varepsilon}{w_S - w_F} \frac{V w_F - c w_F + c w_S - V c}{V\varepsilon + w_F^2 + \varepsilon w_F - \varepsilon w_S - w_F w_S} \\ &= \frac{\varepsilon}{w_S - w_F} \frac{c(w_S - w_F) - V(c - w_F)}{\varepsilon(V - w_S + w_F) - w_F(w_S - w_F)} \\ &= \frac{\varepsilon}{w_S - w_F} \frac{c(w_S - w_F) - V(c - w_F)}{\varepsilon V - (\varepsilon + w_F)(w_S - w_F)} < 0 \end{aligned}$$

Note that because $\theta^* < \theta_{ng}$, $\frac{c}{V} < \frac{c-w_F}{w_S-w_F}$ and $c(w_S - w_F) - V(c - w_F) < 0$. As $\varepsilon + w_F < 0$, $\varepsilon V - (\varepsilon + w_F)(w_S - w_F) > 0$, and hence $\theta''_{ng} - \theta_{ng} < 0$.
As argued above, all types below θ_0 make positive expected profits with a genuine degree. Hence, all types between $[\theta''_{ng}, \theta_0)$ make a positive expected profit for firm i. The expected profit for firm i becomes

$$\begin{aligned}
\pi''_i &= \int_{\theta''_{ng}}^{\theta_0} \left(\theta V - \theta\left(w_S - \frac{V - w_S}{w_F}\varepsilon\right) - (1-\theta)(w_F + \varepsilon)\right) f(\theta)\, d\theta \\
&= \int_{\theta''_{ng}}^{\theta_0} (\theta V - \theta w_S - (1-\theta) w_F) f(\theta)\, d\theta + \varepsilon \int_{\theta''_{ng}}^{\theta_0} \left(\theta \frac{V - w_S}{w_F} - (1-\theta)\right) f(\theta)\, d\theta \\
&= \int_{\theta''_{ng}}^{\theta_0} (\theta V - \theta w_S - (1-\theta) w_F) f(\theta)\, d\theta - \varepsilon \int_{\theta''_{ng}}^{\theta_0} \left(1 - \theta \frac{V - w_S + w_F}{w_F}\right) f(\theta)\, d\theta
\end{aligned}$$

The deviation is profitable if $\pi''_i - \pi_{ng} > 0$, that is,

$$\left(\begin{array}{c} \int_{\theta''_{ng}}^{\theta_0} (\theta V - \theta w_S - (1-\theta) w_F) f(\theta)\, d\theta - \varepsilon \int_{\theta''_{ng}}^{\theta_0} \left(1 - \theta \frac{V - w_S + w_F}{w_F}\right) f(\theta)\, d\theta \\ -\frac{1}{2}\int_{\theta_{ng}}^{1} (\theta V - \theta w_S - (1-\theta) w_F) f(\theta)\, d\theta \end{array}\right) > 0$$

$$\frac{\int_{\theta''_{ng}}^{\theta_0} (\theta V - \theta w_S - (1-\theta) w_F) f(\theta)\, d\theta - \frac{1}{2}\int_{\theta_{ng}}^{1} (\theta V - \theta w_S - (1-\theta) w_F) f(\theta)\, d\theta}{\int_{\theta''_{ng}}^{\theta_0} \left(1 - \theta \frac{V - w_S + w_F}{w_F}\right) f(\theta)\, d\theta} > \varepsilon$$

For all $\theta \in [\theta''_{ng}, \theta_0)$, $1 - \theta\frac{V - w_S + w_F}{w_F} > 0$, that is, $\int_{\theta''_{ng}}^{\theta_0} \left(1 - \theta \frac{V - w_S + w_F}{w_F}\right) f(\theta)\, d\theta > 0$. All types greater than θ_0 make negative expected profits, hence, we get

$$\int_{\theta''_{ng}}^{\theta_0} (\theta V - \theta w_S - (1-\theta) w_F) f(\theta)\, d\theta - \frac{1}{2}\int_{\theta_{ng}}^{1} (\theta V - \theta w_S - (1-\theta) w_F) f(\theta)\, d\theta > 0$$

Therefore, there must exist a ε where

$$0 < \varepsilon < \min \left\{ -w_F, \frac{\left(\begin{array}{c} \int_{\theta''_{ng}}^{\theta_0} (\theta V - \theta w_S - (1-\theta) w_F) f(\theta)\, d\theta \\ -\frac{1}{2}\int_{\theta_{ng}}^{1} (\theta V - \theta w_S - (1-\theta) w_F) f(\theta)\, d\theta \end{array}\right)}{\int_{\theta''_{ng}}^{\theta_0} \left(1 - \theta \frac{V - w_S + w_F}{w_F}\right) f(\theta)\, d\theta} \right\}$$

and the deviation is profitable. ■

(i) If firms set $w_F < 0$ and $w_S \leqslant V$, then all employed types make positive profits for firms, and the expected overall profits are positive. In this case, the deviating firm can offer a slightly higher w_F but keep the same w_S. By doing so, more types will be attracted to get a genuine degree, and all these types strictly prefer the deviating firm's offer (see Figure 4). The firm profitably deviates to a contract that allows it to extract a higher profit without sharing with the rival.

(ii) If firms set $w_F < 0$ and $w_S > V$, only types below θ_0 make positive expected profits for firms. The deviating firm offers a contract which has a lower w_S and a higher w_F, making type θ_0 indifferent between two contracts. Types below θ_0 strictly prefer the deviating contract since lower types have a higher probability of getting w_F. By offering a lower w_S and a higher w_F, a set of lower types are given incentives to get a genuine degree, and all these types make a positive expected profit (see Figure 5). The deviation is profitable since the deviating firm attracts more types to get a genuine degree and it hires only those types which make a positive expected profit. Again, with $w_F < 0$, the deviating firm can offer a contract which gets closer to $(V, 0)$ to make a higher profit.

1.C.3 Eliminating $0 < w_F \leqslant \bar{\lambda}c$

Given that $0 < w_F \leqslant \bar{\lambda}c$, firms make non-negative expected profits only if $w_S < V$. Hence, we get $V - (w_S - w_F) > 0$, which implies that the higher the type is, the higher the expected profit it generates. In this case, the cost of a fake degree affects the worker's choice. If $w_F > \lambda c$, then the worker chooses between a fake degree and a genuine degree. For this scenario, we can apply the same argument from the case where $w_F > \bar{\lambda}c$ to show the deviation. If $w_F \leqslant \lambda c$, then the worker chooses between no degree and a genuine degree. Then we apply a similar argument from the case where $w_F < 0$ to this scenario, and make a small change to get the profitable deviation.

Proof. In equilibrium, firm i believes that firm j offers (w_S, w_F) where $0 < w_F \leqslant \bar{\lambda}c$.

For $0 < w_F \leqslant \lambda c$ and $w_S < V$, we have $V - w_S + w_F > 0$, which implies that the expected profits are increasing in probability θ. Apply the same θ_0, the type that generates zero expected profit for the firm, as defined above.

(i) If $\theta_0 > \theta_{ng}$, only types between $(\theta_0, 1]$ make positive expected profits for firms. Suppose firm i deviates and offers $\left(w_S + \frac{V - w_S}{w_F}\varepsilon, w_F - \varepsilon\right)$ where $\varepsilon > 0$ and $w_F - \varepsilon > 0$. Type θ_0 is indifferent between two contracts as

$$\theta_0 \left(w_S + \frac{V - w_S}{w_F}\varepsilon\right) + (1 - \theta_0)(w_F - \varepsilon) - c = \theta_0 w_S + (1 - \theta_0) w_F - c$$

Because $w_S + \frac{V - w_S}{w_F}\varepsilon > w_S$ and $w_F - \varepsilon < w_F$, based on the monotonicity of probability θ, firm $i's$ offer is more attractive for all types greater than θ_0, and all types below θ_0 prefer firm j. Hence, the expected profit of firm i becomes,

$$\begin{aligned}\pi_i' &= \int_{\theta_0}^{1}\left(\theta V-\theta\left(w_S+\frac{V-w_S}{w_F}\varepsilon\right)-(1-\theta)\left(w_F-\varepsilon\right)\right)f(\theta)\,d\theta \\ &= \int_{\theta_0}^{1}\left(\theta V-\theta w_S-(1-\theta)\,w_F\right)f(\theta)\,d\theta-\int_{\theta_0}^{1}\left(\theta\frac{V-w_S}{w_F}\varepsilon-(1-\theta)\,\varepsilon\right)f(\theta)\,d\theta\end{aligned}$$

The deviation is profitable if $\pi_i'-\pi_{ng}>0$, that is,

$$\left(\begin{array}{c}\int_{\theta_0}^{1}\left(\theta V-\theta w_S-(1-\theta)\,w_F\right)f(\theta)\,d\theta-\int_{\theta_0}^{1}\left(\theta\frac{V-w_S}{w_F}\varepsilon-(1-\theta)\,\varepsilon\right)f(\theta)\,d\theta \\ -\frac{1}{2}\int_{\theta_{ng}}^{1}\left(\theta V-\theta w_S-(1-\theta)\,w_F\right)f(\theta)\,d\theta\end{array}\right)>0$$

It can be rewritten as

$$\left(\begin{array}{c}\int_{\theta_0}^{1}\left(\theta V-\theta w_S-(1-\theta)\,w_F\right)f(\theta)\,d\theta \\ -\frac{1}{2}\int_{\theta_{ng}}^{1}\left(\theta V-\theta w_S-(1-\theta)\,w_F\right)f(\theta)\,d\theta \\ -\varepsilon\int_{\theta_0}^{1}\left(\frac{\theta(V-w_S+w_F)-w_F}{w_F}\right)f(\theta)\,d\theta\end{array}\right)>0$$

For $\theta>\theta_0$, $\theta\left(V-w_S+w_F\right)-w_F>0$. Hence, $\int_{\theta_0}^{1}\left(\frac{\theta(V-w_S+w_F)-w_F}{w_F}\right)f(\theta)\,d\theta>0$. Because types between $[\theta_{ng},\theta_0]$ provide negative profits, we must have

$$\int_{\theta_0}^{1}\left(\theta V-\theta w_S-(1-\theta)\,w_F\right)f(\theta)\,d\theta-\frac{1}{2}\int_{\theta_{ng}}^{1}\left(\theta V-\theta w_S-(1-\theta)\,w_F\right)f(\theta)\,d\theta>0$$

Hence, there exists a positive ε which satisfies

$$0<\varepsilon<\min\left\{w_F,\frac{\left(\begin{array}{c}\int_{\theta_0}^{1}\left(\theta V-\theta w_S-(1-\theta)\,w_F\right)f(\theta)\,d\theta \\ -\frac{1}{2}\int_{\theta_{ng}}^{1}\left(\theta V-\theta w_S-(1-\theta)\,w_F\right)f(\theta)\,d\theta\end{array}\right)}{\int_{\theta_0}^{1}\left(\frac{\theta(V-w_S+w_F)-w_F}{w_F}\right)f(\theta)\,d\theta}\right\}$$

such that $\pi_i'-\pi_{ng}>0$ and the deviation is profitable.
(ii) If $\theta_0\leqslant\theta_{ng}$, all types between $(\theta_{ng},1]$ make positive expected profits for firms. Suppose firm i offers $\left(w_S+\frac{w_S-c}{c-w_F}\varepsilon,w_F-\varepsilon\right)$, where $\varepsilon>0$ and $w_F-\varepsilon>0$. Type θ_{ng} is indifferent between two contracts since

$$\theta_{ng}\left(w_S+\frac{w_S-c}{c-w_F}\varepsilon\right)+(1-\theta_{ng})\left(w_F-\varepsilon\right)-c=\theta_{ng}w_S+(1-\theta_{ng})\,w_F-c$$

The monotonicity of θ implies that all types greater than θ_{ng} will choose firm i

because $w_S + \frac{w_S - c}{c - w_F}\varepsilon > w_S$ and $w_F - \varepsilon < w_F$. The expected profit of firm i becomes

$$\begin{aligned}\pi_i' &= \int_{\theta_{ng}}^{1} \left(\theta V - \theta \left(w_S + \frac{w_S - c}{c - w_F}\varepsilon\right) - (1-\theta)(w_F - \varepsilon)\right) f(\theta)\, d\theta \\ &= \int_{\theta_{ng}}^{1} (\theta V - \theta w_S - (1-\theta) w_F) f(\theta)\, d\theta - \varepsilon \int_{\theta_{ng}}^{1} \left(\theta \frac{w_S - w_F}{c - w_F} - 1\right) f(\theta)\, d\theta\end{aligned}$$

The deviation is profitable if $\pi_i' > \pi_{ng}$, that is,

$$\begin{pmatrix} \int_{\theta_{ng}}^{1} (\theta V - \theta w_S - (1-\theta) w_F) f(\theta)\, d\theta - \varepsilon \int_{\theta_{ng}}^{1} \left(\theta \frac{w_S - w_F}{c - w_F} - 1\right) f(\theta)\, d\theta \\ -\frac{1}{2} \int_{\theta_{ng}}^{1} (\theta V - \theta w_S - (1-\theta) w_F) f(\theta)\, d\theta \end{pmatrix} > 0$$

which can be rewritten as

$$\left(\frac{1}{2} \int_{\theta_{ng}}^{1} (\theta V - \theta w_S - (1-\theta) w_F) f(\theta)\, d\theta - \varepsilon \int_{\theta_{ng}}^{1} \left(\theta \frac{w_S - w_F}{c - w_F} - 1\right) f(\theta)\, d\theta\right) > 0$$

Since $\theta \frac{w_S - w_F}{c - w_F} - 1 \geqslant 0\ \forall \theta \in [\theta_{ng}, 1]$, we get $\int_{\theta_{ng}}^{1} \left(\theta \frac{w_S - w_F}{c - w_F} - 1\right) f(\theta)\, d\theta > 0$. Hence, there must exist an ε where

$$0 < \varepsilon < \min \left\{ w_F, \frac{\frac{1}{2} \int_{\theta_{ng}}^{1} (\theta V - \theta w_S - (1-\theta) w_F) f(\theta)\, d\theta}{\int_{\theta_{kg}}^{1} \left(\theta \frac{w_S - w_F}{c - \lambda c} - 1\right) f(\theta)\, d\theta} \right\}$$

such that the deviation is profitable. ■

The intuition here is similar to the previous cases. For example, with $w_F \leqslant \lambda c$:

(1) when $\theta_0 > \theta_{ng}$, only a subset of the types, $\theta \in (\theta_0, 1]$, make positive expected profits for firms after getting a genuine degree. Any firm can deviate and offer a different contract which attracts only types greater than θ_0. The deviating firm increases w_S and lowers w_F, making the type θ_0 indifferent between two contracts. By doing so, all types greater than θ_0 strictly prefer the deviating contract since they have a higher probability of getting the wage for success (see Figure 6). The deviation is profitable since it only attracts all types which make a positive expected profit without sharing with the rival and leaves all the types which provide a negative expected profit.

(2) when $\theta_0 \leqslant \theta_{ng}$, all types which prefer a genuine degree make positive expected profits. Any firm can deviate and offer a contract that attracts all types greater than θ_{ng}. The deviating firm can again increase w_S and decrease w_F, and make type θ_{ng} indifferent between two contracts. Based on the monotonicity of probability θ, all types greater than θ_{ng} strictly prefer the deviating firm as they

have a higher chance of getting w_S (see Figure 7). Again, this deviation is profitable as it only attracts all types which make a positive expected profit.

1.C.4 Eliminating $w_F = 0$ and $w_S < V$

Given all the discussion above, this part is now very straightforward. If firms offer a contract where $w_F = 0$ and $w_S < V$, then any type they hire provides a positive profit to firms. Hence, one of the firms can profitably deviate to a contract which offers a higher w_S but less than V. By doing so, the deviating firm pays a little bit more to all types but captures the whole market without sharing with the rival.

Proof. Firm i believes that firm j offers (w_S, w_F) where $w_F = 0$ and $w_S < V$. Suppose firm i offers $(w_S + \varepsilon, w_F)$ where $\varepsilon > 0$ and $w_S + \varepsilon < V$. Given both contracts, all types which prefer a genuine degree are better off from choosing firm i. Denote the cut-off type which gets zero expected utility from a genuine degree with contract $(w_S + \varepsilon, w_F)$ as θ'_{ng}, $\theta'_{ng} = \frac{c - w_F}{w_S + \varepsilon - w_F}$. Since $w_S + \varepsilon < V$, we get $\theta'_{ng} < \theta_{ng}$ and all types greater than θ'_{ng} make a positive profit for firm i. Firm $i's$ expected profit becomes,

$$\begin{aligned} \pi'_i &= \int_{\theta'_{ng}}^{1} \left(\theta V - \theta\left(w_S + \varepsilon\right) - (1-\theta)\, w_F\right) f(\theta)\, d\theta \\ &= \int_{\theta'_{ng}}^{1} \left(\theta V - \theta w_S - (1-\theta)\, w_F\right) f(\theta)\, d\theta - \varepsilon \int_{\theta'_{ng}}^{1} \theta f(\theta)\, d\theta \end{aligned}$$

The deviation is profitable if $\pi'_i - \pi_{ng} > 0$, that is,

$$\begin{pmatrix} \int_{\theta'_{ng}}^{1} \left(\theta V - \theta w_S - (1-\theta)\, w_F\right) f(\theta)\, d\theta - \varepsilon \int_{\theta'_{ng}}^{1} \theta f(\theta)\, d\theta \\ -\frac{1}{2} \int_{\theta_{ng}}^{1} \left(\theta V - \theta w_S - (1-\theta)\, w_F\right) f(\theta)\, d\theta \end{pmatrix} > 0$$

Then, there must exist a ε where

$$0 < \varepsilon < \min \left\{ V - w_S, \frac{\begin{pmatrix} \int_{\theta'_{ng}}^{1} \left(\theta V - \theta w_S - (1-\theta)\, w_F\right) f(\theta)\, d\theta \\ -\frac{1}{2} \int_{\theta_{ng}}^{1} \left(\theta V - \theta w_S - (1-\theta)\, w_F\right) f(\theta)\, d\theta \end{pmatrix}}{\int_{\theta'_{ng}}^{1} \theta f(\theta)\, d\theta} \right\}$$

and firm i makes a profitable deviation.

■

If both firms offer a contract $(w_S < V, w_F = 0)$, one of the firms can increase w_S by a tiny amount and attract more types to get a genuine degree. This firm then profitably deviates to a new contract that takes all the profit without sharing with the other firm (see Figure 4).

From the above analysis, all symmetric contracts except $(w_S = V, w_F = 0)$ have been eliminated from being an equilibrium.

2 Appendix to Chapter III

2.A Proof for Lemma III.3

We first argue for the in-house training contract that the firm sets $w_S^I = w_F^I$. Then we rule out the possibility that w_F being negative and positive.

(I) **For employing a type θ worker with in-house training, the firm minimizes the payment by setting $w_S^I = w_F^I$.**

Proof. Consider $M = \{(w_S^I, w_F^I), (w_S, w_F)\}$ and $\hat{M} = \{(\hat{w}_S^I, \hat{w}_F^I), (w_S, w_F)\}$ where $w_S^I = w_F^I$ and $\hat{w}_S^I > \hat{w}_F^I$, and both result in the same cutoff types $0 \leqslant \theta_{kI} < \theta_{Ig} < 1$. Here we do not consider $w_S^I < w_F^I$. If $w_S^I < w_F^I$, then we have types below θ_{kI}, which means the firm hires types that provides negative surplus.

For the cutoff type θ_{kI}, we have

$$\theta_{kI}\left(w_S^I - w_F^I\right) + w_F^I - c^I = \theta_{kI}\left(\hat{w}_S^I - \hat{w}_F^I\right) + \hat{w}_F^I - c^I = \underbrace{w_F}_{u(k,\theta_{kI})}$$

which implies that for employing the type θ_{kI}, the expected payment is the same from both contracts which is equal to $w_F + c^I$.

For types between $[\theta_{kI}, \theta_{Ig})$, the expected payment with contract M is the same as $w_F + c^I$, and with contract M', the expected payment is

$$\theta\left(\hat{w}_S^I - \hat{w}_F^I\right) + \hat{w}_F^I > \theta_{kI}\left(\hat{w}_S^I - \hat{w}_F^I\right) + \hat{w}_F^I = w_F + c^I \qquad \forall\, \theta > \theta_{kI}$$

Hence, the firm is better-off offering the contract M. Similarly, when $w_F \leqslant 0$, we can apply the same argument and get the same result. ■

(II) There does not exist a profit maximizing contract where $w_F < 0$.

Proof. If $w_F < 0$, then the firm's expected profit is given by

$$\begin{aligned}\pi|_{w_F<0} &= \int_{\theta_{nI}}^{\theta_{Ig}} \left(\theta V - \theta\left(w_S^I - w_F^I\right) - w_F^I\right) f(\theta)\, d\theta \\ &\quad + \int_{\theta_{Ig}}^{1} \left(\theta V - \theta\left(w_S - w_F\right) - w_F\right) f(\theta)\, d\theta\end{aligned}$$

Now we compare two contracts, one with $w_F < 0$ and the other has $w_F = 0$. We show that the firm is better-off offering $w_F = 0$ compared with $w_F < 0$. Consider $M_{w_F<0} = \{(w_S^I, w_F^I), (w_S, w_F < 0)\}$ and $M_{\hat{w}_F=0} = \{(w_S^I, w_F^I), (\hat{w}_S, \hat{w}_F = 0)\}$ and both contracts result in the same cutoff types $0 \leqslant \theta_{nI} < \theta_{Ig} < 1$. θ_{nI} is the same for both contracts as the in-house training contracts are the same. Hence, the firm's expected revenue is the same with both $w_F < 0$ and $w_F = 0$, and it is given by

$$\int_{\theta_{nI}}^{1} (\theta V) f(\theta) d\theta$$

Moreover, the expected payment for employing types between $[\theta_{nI}, \theta_{Ig})$ with in-house training is also the same from two contracts. We now compare the expected payments for employing a type θ, $\theta \in [\theta_{Ig}, 1]$, with a genuine degree when $w_F = 0$ and $w_F < 0$. We have

$$\begin{array}{ll} EP_g|_{w_F<0} = \theta (w_S - w_F) + w_F & \text{when } w_F < 0 \\ EP_g|_{w_F=0} = \theta \hat{w}_S & \text{when } \hat{w}_F = 0 \end{array}$$

For the firm to keep the same θ_{Ig}, the contract $(\hat{w}_S, \hat{w}_F = 0)$ must satisfy

$$\theta_{Ig}\hat{w}_S - c = \theta_{Ig} (w_S - w_F) + w_F - c$$

Rearrange to get the following expression for $\hat{w}_S$,

$$\hat{w}_S = w_S - w_F + \frac{w_F}{\theta_{Ig}} \tag{42}$$

Subtract $EP_g|_{w_F<0}$ from $EP_g|_{w_F=0}$ gives

$$\begin{aligned} & EP_g|_{w_F=0} - EP_g|_{w_F<0} \\ = \; & \theta \hat{w}_S - \theta (w_S - w_F) - w_F \\ = \; & \theta (w_S - w_F) + \theta \frac{w_F}{\theta_{Ig}} - \theta (w_S - w_F) - w_F \\ = \; & \left(\frac{\theta}{\theta_{Ig}} - 1 \right) w_F \end{aligned} \tag{43}$$

(43): Substitute (42) to replace $\hat{w}_S$.
Since $\frac{\theta}{\theta_{Ig}} - 1 > 0$ for $\theta \in [\theta_{Ig}, 1]$, and $w_F < 0$, we get $EP_g|_{w_F=0} < EP_g|_{w_F<0}$. It implies that the expected payment is higher when $w_F < 0$ as compared with $w_F = 0$. Hence, the firm is better off offering $w_F = 0$ compared with $w_F < 0$. ■

(III) There does not exist a profit-maximizing contract where $w_F > 0$.
Proof. With $w_F > 0$, all types prefer a fake degree over no education. The expected profit of the firm now becomes

$$\begin{aligned} \pi|_{w_F>0} = & -\int_0^{\theta_{kI}} w_F f(\theta) d\theta + \int_{\theta_{kI}}^{\theta_{Ig}} \left(\theta V - \theta \left(w_S^I - w_F^I\right) - w_F^I\right) f(\theta) d\theta \\ & + \int_{\theta_{Ig}}^{1} (\theta V - \theta (w_S - w_F) - w_F) f(\theta) d\theta \end{aligned}$$

Consider $\{(w_S, w_F > 0), (w_S^I, w_F^I)\}$ and $\{(\hat{w}_S, \hat{w}_F = 0), (\hat{w}_S^I, \hat{w}_F^I)\}$ where both contracts give the same cutoff types $\theta_{kI} = \theta_{nI}$ and θ_{Ig}. The expected revenue is the same due to the same cutoff types. Thus, we only look at the expected payments of these contracts. A type θ lies between $[0, \theta_{kI})$ with a probability equals to $F(\theta_{kI})$, lies between $[\theta_{kI}, \theta_{Ig})$ with a probability $[F(\theta_{Ig}) - F(\theta_{kI})]$, and lies between $[\theta_{Ig}, 1]$ with a probability $[1 - F(\theta_{Ig})]$. Hence, the expected payment of employing a type θ when $w_F > 0$ is

$$\begin{aligned} EP|_{w_F>0} &= F(\theta_{kI}) w_F + [F(\theta_{Ig}) - F(\theta_{kI})] \left[\theta\left(w_S^I - w_F^I\right) + w_F^I\right] \\ &\quad + [1 - F(\theta_{Ig})] [\theta(w_S - w_F) + w_F] \end{aligned} \tag{44}$$

and the expected payment when $w_F = 0$ is

$$\begin{aligned} EP|_{w_F=0} &= [F(\theta_{Ig}) - F(\theta_{kI})] \left[\theta\left(\hat{w}_S^I - \hat{w}_F^I\right) + \hat{w}_F^I\right] \\ &\quad + [1 - F(\theta_{Ig})] [\theta \hat{w}_S] \end{aligned} \tag{45}$$

As discussed above, we have $w_S^I = w_F^I = w_F + c^I$ (when $w_F > 0$) and $\hat{w}_S^I = \hat{w}_F^I = c^I$ (when $w_F = 0$). Since both contracts result in the same θ_{Ig}, we can write

$$\begin{array}{ll} w_F^I - c^I = \theta_{Ig}(w_S - w_F) + w_F - c & \text{when } w_F > 0 \\ \hat{w}_F^I - c^I = \theta_{Ig}\hat{w}_S - c & \text{when } w_F = 0 \end{array}$$

which gives the following expression

$$\hat{w}_S = w_S - w_F \tag{46}$$

Compare two expected payments stated in (44) and (45), we get

$$\begin{aligned} & EP|_{w_F>0} - EP|_{w_F=0} \\ = \; & F(\theta_{kI}) w_F + [F(\theta_{Ig}) - F(\theta_{kI})] \left[\theta\left(w_S^I - w_F^I\right) + w_F^I\right] \\ & + [1 - F(\theta_{Ig})] [\theta(w_S - w_F) + w_F] \\ & - [F(\theta_{Ig}) - F(\theta_{kI})] \left[\theta\left(\hat{w}_S^I - \hat{w}_F^I\right) + \hat{w}_F^I\right] - [1 - F(\theta_{Ig})] [\theta \hat{w}_S] \end{aligned}$$

Substituting (46) yields

$$\begin{aligned} & EP|_{w_F>0} - EP|_{w_F=0} \\ = \; & F(\theta_{kI}) w_F + [F(\theta_{Ig}) - F(\theta_{kI})] [w_F] + [1 - F(\theta_{Ig})] [w_F] \\ = \; & w_F \end{aligned}$$

Since $w_F > 0$, we get $EP|_{w_F>0} > EP|_{w_F=0}$. It implies that when $w_F > 0$, the firm pays w_F more to all types between $[0, 1]$. Therefore, we are left with $w_F = 0$ in the profit-maximizing contract, and the corresponding in-house training contract is $w_S^I = w_F^I = c^I$. ■

3 Appendix to Chapter IV

3.A Proof of Proposition IV.3

We first eliminate $w_S < w_F$ and $w_S = w_F$ in the profit-maximizing contract. Then we proceed with $w_S > w_F$ and further check the range for w_F.

(I) **Eliminating** $w_S < w_F$

Proof. If the firm sets $w_S < w_F \leqslant \lambda c$, then no type has an incentive to apply for this job since the expected utility of applying for the job is no greater than the reservation utility (zero). If the firm sets $w_S < w_F$ and $w_F > \lambda c$, then completing the job means the employed worker will be charged by $|w_S - w_F|$. Since the cost of the low-quality education is lower compared with the high-quality education, and the probability of being charged is smaller with low-quality education, we get $\beta\theta\left(w_S - w_F\right) + w_F - c < \alpha\theta\left(w_S - w_F\right) + w_F - \lambda c$ for all θ. That is, all types prefer the low-quality education over high-quality education. Moreover, with $w_S < w_F$ and $w_F > \lambda c$, the expected utility decreases as the type increases. Hence, types below θ_α prefer the low-quality education and types above prefer no education. The firm's problem is to solve

$$\max_{(w_S, w_F)} \int_0^{\theta_\alpha} \left(\alpha\theta V - \left(\alpha\theta w_S + \left(1 - \alpha\theta\right) w_F\right)\right) f\left(\theta\right) d\theta \tag{47}$$

Consider two contracts (w_S, w_F), where $w_S < w_F$, and (w'_S, w'_F) where $w'_F = w_F - \varepsilon > \lambda c$ and $w'_S = w_S + \frac{1-\alpha\theta_\alpha}{\alpha\theta_\alpha}\varepsilon$. ε is positive and it satisfies that

$$w'_S - w'_F = w_S - w_F + \frac{1}{\alpha\theta_\alpha}\varepsilon < 0$$

The contract (w'_S, w'_F) gives the expected utility for the type θ_α as

$$\begin{aligned} & \alpha\theta_\alpha\left(w_S + \frac{1-\alpha\theta_\alpha}{\alpha\theta_\alpha}\varepsilon\right) + \left(1 - \alpha\theta_\alpha\right)\left(w_F - \varepsilon\right) - \lambda c \\ = \; & \alpha\theta_\alpha w_S + \left(1 - \alpha\theta_\alpha\right) w_F - \lambda c \end{aligned}$$

which implies that both contracts, (w_S, w_F) and (w'_S, w'_F), result in the same cutoff type θ_α. With the same cutoff type θ_α, the firm's expected revenue from both contracts are the same, and it is given by

$$\int_0^{\theta_\alpha} \left(\alpha\theta V\right) f\left(\theta\right) d\theta$$

We now compare two expected payments from employing a type θ worker, $\theta \in [0, \theta_\alpha]$,

$$\begin{aligned}
& EP|_{(w_S, w_F)} - EP|_{\left(w'_S, w'_F\right)} \\
= \;& \alpha\theta\left(w_S - w_F\right) + w_F - \left(\alpha\theta\left(w'_S - w'_F\right) + w'_F\right) \\
= \;& \alpha\theta\left(\left(w_S - w_F\right) - \left(w'_S - w'_F\right)\right) + w_F - w'_F \\
= \;& \alpha\theta\left(-\frac{1}{\alpha\theta_\alpha}\varepsilon\right) + \varepsilon \\
= \;& \left(1 - \frac{\theta}{\theta_\alpha}\right)\varepsilon
\end{aligned}$$

For $\theta \in [0, \theta_\alpha]$, we have $1 - \frac{\theta}{\theta_\alpha} > 0$, and hence, $EP|_{(w_S, w_F)} > EP|_{\left(w'_S, w'_F\right)}$. Therefore, the firm's profit is higher when it offers a smaller w_F and a greater w_S when $w_S < w_F$ and $w_F > \lambda c$. ■

As long as $w_S < w_F$ and $w_F > \lambda c$, the firm can always profitably reduce w_F and increase w_S by a small amount to lower the expected payment without changing the expected revenue. Hence, we rule out $w_S < w_F$ in the firm's profit-maximizing solution. We now turn to the case where $w_S = w_F$.

(I) **Eliminating** $w_S = w_F$

Proof. With a contract where $w_S = w_F$, any educated type will be employed at a constant wage. The utility of an educated type θ now becomes

$$\begin{aligned}
u(\theta, h) &= w_F - c \\
\text{or } u(\theta, l) &= w_F - \lambda c
\end{aligned}$$

Since $\lambda c < c$, all types prefer the low-quality education. If $w_S = w_F < \lambda c$, no type will choose to be educated since the utility after getting the education is less than the reservation utility. If $w_S = w_F > \lambda c$, all types choose the low-quality education. In this case, the firm is always better-off cutting both w_S and w_F by a small amount. By doing so, all types will still choose low-quality education, but the firm's payment will drop. We are now left with $w_S = w_F = \lambda c$. The highest profit the firm can make with $w_S = w_F = \lambda c$ is when all types greater than θ^*_α choose low-quality education.

$$\pi|_{w_S = w_F = \lambda c} = \int_{\theta^*_\alpha}^{1} \left(\alpha\theta V - \lambda c\right) f(\theta)\, d\theta \tag{48}$$

The firm extracts all the surpluses from types between $[\theta^*_\alpha, 1]$. Now we show that there exist a profitable deviation for the firm when $w_S = w_F = \lambda c$.

Consider a contract (w'_S, w'_F) where $w'_S = \lambda c + \varepsilon$, $w'_F = \lambda c - \frac{\lambda c}{V - \lambda c}\varepsilon$, and $V - \lambda c > \varepsilon > 0$. Contract (w'_S, w'_F) results in two cutoff types θ_α and θ_β such that

$$\theta_\alpha = \frac{\lambda c - \left(\lambda c - \frac{\lambda c}{V - \lambda c}\varepsilon\right)}{\alpha\left(\lambda c + \varepsilon - \left(\lambda c - \frac{\lambda c}{V - \lambda c}\varepsilon\right)\right)} = \frac{\lambda c}{\alpha V} = \theta^*_\alpha$$

and

$$\begin{aligned}\theta_\beta &= \frac{(1-\lambda)\,c}{(\beta - \alpha)\left(\lambda c + \varepsilon - \left(\lambda c - \frac{\lambda c}{V - \lambda c}\varepsilon\right)\right)} \\ &= \frac{(1-\lambda)\,c}{(\beta - \alpha)\,V}\frac{V - \lambda c}{\varepsilon} > \theta^*_\beta\end{aligned}$$

The firm's expected profit with contract (w'_S, w'_F) is

$$\begin{aligned}&\pi|_{(w'_S, w'_F)} \\ =& \int_{\theta^*_\alpha}^{\theta_\beta} (\alpha\theta V - (\alpha\theta w'_S + (1 - \alpha\theta)\,w'_F))\,f(\theta)\,d\theta \\ &+ \int_{\theta_\beta}^{1} (\beta\theta V - (\beta\theta w'_S + (1 - \beta\theta)\,w'_F))\,f(\theta)\,d\theta \\ =& \left(1 - \frac{\varepsilon}{V - \lambda c}\right)\left(\int_{\theta^*_\alpha}^{\theta_\beta} (\alpha\theta V - \lambda c)\,f(\theta)\,d\theta + \int_{\theta_\beta}^{1} (\beta\theta V - \lambda c)\,f(\theta)\,d\theta\right)\end{aligned}$$

Compare the profits from contract (w'_S, w'_F) in (**??**) and $(w_S = w_F = \lambda c)$ in (**??**),

$$\begin{aligned}&\pi|_{(w'_S, w'_F)} - \pi|_{(w_S, w_F)} \\ =& \left(1 - \frac{\varepsilon}{V - \lambda c}\right)\left(\int_{\theta^*_\alpha}^{\theta_\beta} ((\alpha\theta V - \lambda c))\,f(\theta)\,d\theta + \int_{\theta_\beta}^{1} ((\beta\theta V - \lambda c))\,f(\theta)\,d\theta\right) \\ &- \int_{\theta^*_\alpha}^{1} (\alpha\theta V - \lambda c)\,f(\theta)\,d\theta \\ =& \int_{\theta_\beta}^{1} ((\beta - \alpha)\,\theta V)\,f(\theta)\,d\theta - \frac{\varepsilon}{V - \lambda c}\left(\begin{array}{c}\int_{\theta^*_\alpha}^{\theta_\beta} ((\alpha\theta V - \lambda c))\,f(\theta)\,d\theta \\ + \int_{\theta_\beta}^{1} ((\beta\theta V - \lambda c))\,f(\theta)\,d\theta\end{array}\right)\end{aligned}$$

Let us denote $\int_{\theta^*_\beta}^{\theta_\beta} ((\alpha\theta V - \lambda c))\,f(\theta)\,d\theta + \int_{\theta_\beta}^{1} ((\beta\theta V - \lambda c))\,f(\theta)\,d\theta$ as A, and let B represent $\int_{\theta_\beta}^{1} ((\beta - \alpha)\,\theta V)\,f(\theta)\,d\theta$. Both A and B are positive. If $0 < \varepsilon < \frac{A}{B}(V - \lambda c)$, we get $\pi|_{(w'_S, w'_F)} > \pi|_{(w_S, w_F)}$. That is, when $w_S = w_F = \lambda c$, there exists a positive ε such that the firm can increase w_S by ε and decrease w_F by $\frac{\lambda c}{V - \lambda c}\varepsilon$ to make a higher profit.

■

(III) **Firm's profit-maximizing when** $w_S > w_F$

Based on the definition of the cutoff types stated in (28) and (29), when $w_S > w_F$, we face the following three scenarios. (I) If $w_F > \lambda c$, then all types would like to be educated, $\theta_\alpha = 0$. (II) If $w_F < \lambda c - \frac{\alpha(1-\lambda)c}{\beta-\alpha} < 0$, then we get $\theta_\alpha > \theta_\beta$[12], and all types choose between no education and high-quality education. (III) If $\lambda c - \frac{\alpha(1-\lambda)c}{\beta-\alpha} \leqslant w_F \leqslant \lambda c$, then we have $0 \leqslant \theta_\alpha \leqslant \theta_\beta$ which may lead us to a case that is similar to the first best allocation (low types choose no education, middle types choose low-quality education, and high types choose high-quality education). We now rule out the first two possibilities $w_F > \lambda c$ and $w_F < \lambda c - \frac{\alpha(1-\lambda)c}{\beta-\alpha}$ from the firm's profit-maximizing problem.

(i) Eliminating $w_F > \lambda c$

Proof. Consider a contract (w_S, w_F) where $w_S > w_F > \lambda c$, then all types prefer low-quality education over no education. According to (27), $w_S > w_F$ implies that all types greater than θ_β prefer high-quality education. Hence, the firm's expected profit is

$$\begin{aligned} \pi &= \int_0^{\theta_\beta} \left(\alpha\theta V - \left(\alpha\theta w_S + (1-\alpha\theta)\, w_F\right)\right) f(\theta)\, d\theta \\ &\quad + \int_{\theta_\beta}^1 \left(\beta\theta V - \left(\beta\theta w_S + (1-\beta\theta)\, w_F\right)\right) f(\theta)\, d\theta \end{aligned}$$

Consider another contract (w'_S, w'_F) where $w'_S = w_S - \varepsilon$ and $w'_F = w_F - \varepsilon$, $\varepsilon > 0$ and $w'_S > w'_F > \lambda c$. The cutoff type θ_β remains the same in both contracts (w'_S, w'_F) and (w_S, w_F) as

$$\theta_\beta = \frac{(1-\lambda)\, c}{(\beta-\alpha)\left(w'_S - w'_F\right)} = \frac{(1-\lambda)\, c}{(\beta-\alpha)\left(w_S - w_F\right)}$$

which implies that the expected revenue is the same from both contracts. However, the expected payment is lower with contract (w'_S, w'_F) than (w_S, w_F). Thus, the firm is better-off reducing both w_S and w_F at the same rate such that the expected revenue remains the same, but the expected payment drops. ■

(ii) Eliminating $w_F < \lambda c - \frac{\alpha(1-\lambda)c}{\beta-\alpha} < 0$

Proof. When $w_F < \lambda c - \frac{\alpha(1-\lambda)c}{\beta-\alpha} < 0$, no type would like to choose the low-quality education as $\theta_\alpha > \theta_\beta$. Let the type which gets zero expected utility with high-quality education be θ_h, we have $\beta\theta_h w_S + (1-\beta\theta_h)\, w_F - c = 0$. It can be written as

$$\theta_h = \frac{c - w_F}{\beta\left(w_S - w_F\right)} \tag{49}$$

[12] $\theta_\alpha > \theta_\beta$ implies that $\frac{\lambda c - w_F}{\alpha(w_S - w_F)} > \frac{(1-\lambda)c}{(\beta-\alpha)(w_S - w_F)}$, which can be simplified to $w_F < \lambda c - \frac{\alpha(1-\lambda)c}{\beta-\alpha}$.

θ_h is indifferent between no education and high-quality education. Hence, we now consider the case where all types greater than θ_h prefer high-quality education and types below prefer no education. The firm's expected profit with contract (w_S, w_F) is given by

$$\int_{\theta_h}^{1} (\beta\theta V - \beta\theta (w_S - w_F) - w_F) f(\theta) d\theta$$

where θ_h is defined in (29). Consider a contract (w'_S, w'_F) where $w'_F = w_F + \varepsilon$ and $w'_S = w_S - \left(\frac{1}{\beta\theta_h} - 1\right)\varepsilon$. We have $\varepsilon > 0$ and $w'_F < \lambda c - \frac{\alpha(1-\lambda)c}{\beta-\alpha}$. For θ_h, we can write

$$\begin{aligned}
& \beta\theta_h w'_S + (1-\beta\theta_h) w'_F - c \\
= \ & \beta\theta_h \left(w_S - \left(\frac{1}{\beta\theta_h} - 1\right)\varepsilon\right) + (1-\beta\theta_h)(w_F + \varepsilon) - c \\
= \ & \beta\theta_h w_S + (1-\beta\theta_h) w_F - c
\end{aligned}$$

which implies that both (w_S, w_F) and (w'_S, w'_F) give the same cutoff type θ_h. The expected revenue of the firm is identical for two contracts as they share the same θ_h. Hence, we compare the expected payments from employing a type $\theta \in [\theta_h, 1]$ with these two contracts to find the better option.

$$\begin{aligned}
& EP|_{(w_S,w_F)} - EP|_{(w'_S,w'_F)} \\
= \ & [\beta\theta (w_S - w_F) + w_F] - [\beta\theta (w'_S - w'_F) + w'_F] \\
= \ & \beta\theta (w_S - w'_S - w_F + w'_F) + w_F - w'_F \\
= \ & \beta\theta \left(\left(\frac{1}{\beta\theta_h} - 1\right)\varepsilon + \varepsilon\right) - \varepsilon \\
= \ & \left(\frac{\theta}{\theta_h} - 1\right)\varepsilon
\end{aligned}$$

Since $\frac{\theta}{\theta_h} - 1 > 0$ for all $\theta \in [\theta_h, 1]$, we get $EP|_{(w_S,w_F)} > EP|_{(w'_S,w'_F)}$. When $w_F < \lambda c - \frac{\alpha(1-\lambda)c}{\beta-\alpha}$, the firm can make a higher profit by increasing w_F and lowering w_S to reduce the expected payment while keeping the same revenue. ■

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