UNIVERSITY OF LEICESTER

Essays on Intermediation, Advertisement and Platform Innovation

Thesis submitted for the degree of

Doctor of Philosophy at the University of Leicester

by

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March 2017

To my daughter, Charlotte Guo.

Thank you for coming into my life and giving me joy.

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Abstract

This thesis comprises three chapters in the area of intermediation, advertisement and platform innovation. Chapter 1, *Quality Uncertainty, Uninformative Advertising and Intermediation Margins*, works two contributions. First, we modify Milgrom and Roberts (1986) model of price and advertising as possible signals of an experience good quality, in such a way to rule out the possibility of price signalling alone. Second, the resulting, more tractable, model is then used to contrast the potential benefits of advertisement and intermediation, modelled as quality certification, as alternative ways to overcome information asymmetries on the quality of new experience goods. We show conditions under which intermediation can be a better way of revealing a product's true quality than advertising.

Chapter 2, *Platform innovation in a two-sided market*, studies dynamic innovation incentives in two-sided markets. We present a monopoly platform model with service quality innovations and innovation models of competing platforms. We show that, in a non-tournament duopoly model, platforms will end up in a prisoners' dilemma equilibrium where they conduct same positive R & D, even if their profit will be higher without R & D investments. This result is derived assuming perfect ex-ante symmetry of the platform, and focusing on an ex-post symmetric equilibrium. We then present three extensions with asymmetric network externalities, platform exogenous specialization in side innovations, and innovation tournament model.

Chapter 3, *A note on Armstrong (2006) monopoly platform model*, shows that the set of assumption in Armstrong (2006) monopoly platform model is not sufficient to guarantee the existence of an interior equilibrium with positive demands. We then show that the problem can be fixed by assuming that platform connections generate direct intrinsic value to the connected agents.

Acknowledgments

I would like to thank my supervisors, Prof Vincenzo Denicolò and Dr Piercarlo Zanchettin, for the patient guidance and advice they have provided throughout my time as their student. I am very grateful to Dr Subir Bose for accepting to be my second supervisor in the final period of my studies.

I send big thanks to Madeline and Jim Atkins for their motivation, trust and love. Also thanks to Geoffrey Pool and Olwen Hughes for their support and English corrections.

I must express my gratitude to my family for their unconditional love.

And, most important of all, Daisy Guo. Without Daisy not only would this thesis not have been possible, but during all the up and down moments of my research, she would either have had the answer or, if not, she would listen and always be there giving me continued support.

Finally, I thank the reader for reading my thesis.

Declaration

I declare that that the content all chapters of this thesis are entirely my own.

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CONTENTS

Two-sided markets, also called two-sided networks, differ from traditional intermediation in a fundamental way. For a traditional intermediary, one side of the market just generate costs, while revenues just arise from the other side. In two-sided networks, both costs and revenues typically arise from both sides of the market. The platform service incurs costs in serving both market sides and can collect revenue from each of them, although one side may actually be net subsidized by the other through the platform pricing choice. Furthermore, two sided markets typically show network effects between the groups of agents connected by the platform from opposite sides of the market.

In the first chapter, we present models of two-sided market platforms' innovation. We consider both models of monopoly and duopoly: a model of monopoly platform and a non-tournament model of competitive platforms with or without technology spillover effect where agents join a single platform. In the context of this chapter, innovation is defined as the increasing quality of service from platforms, i.e., the original quality of the platform is constrained to zero. We evaluate platforms' optimal pricing strategy while considering platform-side innovation competition.

In the model of monopoly platform, we use a linear Hotelling specification of Armstrong (2006) model. We modify Armstrong (2006) utility formulation by assuming that, in addition to the "network" gain arising from being connected to the other side of the market, platform connections also generate direct intrinsic value to the agents. A provider of a platform invests in innovation which increases quality of the connections. In an interior equilibrium, monopoly platform and agents are all better off with innovation. By comparing profit-maximizing outcomes and welfare-maximizing outcomes with innovation, we find that,from a social point of view, underinvestment problem exists.

In a symmetric non-tournament innovation model of competing platforms, for simplicity, we assume that R & D succeeds with exogenous probability 1. Investing on R & D is at a cost of K, which depends on the quality level. Platforms first simultaneously determine their R & D investment, and then they simultaneously set their prices on both sides of the market. They reach prisoners' dilemma equilibrium. The price and the profit don't depend on the absolutely quality level but the qualities' gap between the quality levels. The same research from both platforms will be totally unprofitable. The two platforms would like not do any innovation, but both platforms have precisely the same unilateral incentive

to increase quality because of platforms' competition. Technology spillover effect makes the innovation race less severe. The reason is the platforms perceive part of what they do is beneficial to their opponent and narrows the quality gap, which implies that platforms have lower incentives to undertake innovation.

We present three cases as extensions, which are asymmetric side-network effects in a non-tournament duopoly, exogenous specialization in innovation sides in a non-tournament duopoly and a tournament model. They are mainly as preliminary steps for future work.

In the second chapter, we show that the set of assumption in Armstrong (2006) monopoly platform model is not sufficient to guarantee the existence of an interior equilibrium with positive demands. We illustrate this problem within a linear Hotelling demand system fully consistent with Armstrong (2006) general assumptions. We show that the only possible equilibria are corner equilibria either with zero connections or with full coverage of both market sides, whereby Armstrong (2006) monopoly pricing rule does not apply. Armstrong (2006)'s price rule states that the optimal prices are related to the size of the other side group and the elasticity of this side's group's participation.

In order to get that Armstrong (2006) fundamental price rule applying in a monopoly platform's interior equilibrium, some other assumption must be made. We then show the problem can be fixed by adding an intrinsic utility value component to platform connections in Armstrong (2006) linear utility formulation.

In the third chapter, we consider a traditional intermediation relationship in the context of asymmetric information on the quality of a new experience good between producers and consumers. The intermediary which is capable of verifying and certifying the quality of a new experience product of monopolistic producer. The intermediary can then directly buy the products and sell it to consumers with quality certification. As an alternative, the monopolistic producer can signal quality through advertising. We modify Milgrom and Roberts (1986)'s signalling model to rule out the possibility that price alone can signal quality. We explore which way, intermediation or advertisement, a producer would prefer to use to reveal own product's quality to consumers.

In equilibrium, a low-quality producer would never be interested in selling products to an intermediary because there is no bargaining surplus for them to share. A high-quality producer is more willing to sell products to an intermediary, especially when the high-quality goods are expensive. Because an expensive high-quality product requires high advertising expense to signal quality, which enough high advertising expense aims to stop the low-quality producer to mimic, while intermediation requires relative less cost to signal quality.

1. PLATFORM INNOVATION IN A TWO-SIDED MARKET

Abstract. This chapter studies dynamic innovation incentives in two-sided markets. Platform growth is fundamentally driven by innovation, and platforms' innovation intensity determines the rate of market turnover. We present several models of innovation in such markets: a monopoly platform with innovation; non-tournament innovation models of competing platforms; a tournament innovation model of competing platforms. Each agent connected to the platform is assumed to join a single platform. In equilibrium, agents connected to each platform are always better off with a platform's innovation, but a platform itself is not always better off. Even so, each platform always have unilateral incentives to undertake R & D.

1.1. **Introduction.**

A two-sided market is a market within which two or more groups of agents interact with each other through the platform(s) and all parties benefit from the interaction. There are many two-sided market examples in today's society, such as Amazon, London Stock Exchange etc. The literature about two-sided markets can be considered as relatively recent, as contributions in this field only date back to early 2000s. Most of the recent analyses rely upon the seminal contributions of Caillaud and Jullien (2003), Rochet and Tirole (2003, 2006), Armstrong (2006), Hagiu (2006) and Weyl (2010). These works focus on how the platform market (i.e., monopoly v. duopoly platforms, single homed agents v. competitive bottlenecks, etc.) drives the platforms' price choices. Some prior research on innovation motivated this study was done by Delbono and Denicolo (1991) and Schmutzler (2010). This chapter combines the idea that two-sided market with innovation competition. As far as we know, our study is the first one to consider how innovation influences two-sided platform performance and whether a more competitive two-sided platform structure leads to a faster pace of innovation in a monopoly model and duopoly platform model.

In the context of this chapter, innovation improves the quality of the service offered by a platform to its connected agents. We evaluate and compare several models of innovation in two-sided markets: quality innovation by a monopoly platform; non-tournament quality innovation by duopolistic platforms; tournament quality innovation by duopolistic platforms. We assume each agent connected to the platform choose to join a single platform. We examine platforms' optimal pricing strategy while considering platform-side innovation competition. This chapter also investigates whether technology spillover can influence a platform's choices and performance.

In the model of monopoly platform, we use a linear Hotelling specification of Armstrong (2006)'s model. We modify Armstrong (2006)'s utility formulation by assuming that, in addition to the "network" gain arising from being connected to the other side of the market, platform connections also generate direct intrinsic value to the agents. A provider of a platform invests in innovation which increases the intrinsic value (quality of the connections). Innovation comes from a deterministic R & D technology whereby the quality of the connections is an increasing function of the platform's R & D investment. In an interior equilibrium, the monopoly platform is better off with innovation. Innovation leads to more agents connected to the platform and symmetrically increases the quality

of the connections of all agents on both sides of market. By comparing profit-maximizing outcomes and welfare-maximizing outcomes with innovation, from a social point of view, underinvestment problem exists.

In the model of non-tournament innovation duopoly, the R & D technology is actually the same as the monopoly platform model. Individual platform's innovation affects each platform's customers in the same way. In this symmetric model, innovation is modelled as a deterministic investment. We assume innovation of each single platform is performed to symmetrically increase the quality of the connections for all agents connected to the investing platform on both sides. Platforms first simultaneously determine their R & D investment, and then they simultaneously set their prices on both sides of the market.

We distinguish a non-tournament model between with no technology spillover effect and with technology spillover effect. We examine that how do innovation incentives change when the situations are different. In a symmetric equilibrium, only the quality gaps of two platforms impact platforms' optimal prices and profits. The demands are split equally between two platforms. The platforms will end up in the prisoners' dilemma equilibrium where they conduct positive R & D, even if their profit will be higher without R & D investment. With technology spillover effect, platforms have the less incentive to do R & D because what a platform do is beneficial to his opponent and narrows the quality gap, but technology spillover decreases the whole society's research cost.

In the last part, we present three asymmetric cases as extensions, which are asymmetric side-network effects in a non tournament duopoly without technology spillover effect, exogenous specialization in innovation sides in a non tournament duopoly without technology spillover effect and a tournament model without technology spillover effect. They are mainly as preliminary steps for future work. In the first extension, we relax the assumption to allow for different innovation and different network externalities on both sides of platforms. In the second extension, we assume platforms are specialized in different sides of market, in which one platform's innovation increases the quality of the connections on one side of the investing platform and the other platform's innovation increases the quality of the connections on the other side of the investing platform. In the last extension, endogenous probability of success is assumed to decide the investment cost. We assume the quality level can only be either high $\overline{q}$ if an R & D project succeeds or zero if an R & D project fails. Only the first successful innovator has a chance of obtaining the innovation that denies its rival the use of the new technology.

The remainder of the chapter is organized as follows. Section 2 reviews literatures. Section 3 presents monopoly platform. Section 4 presents a non-tournament

model with duopoly platforms of two-sided single-homing. Extensions are in Section 5. Section 6 concludes.

1.2. **Literature.**

The paper most relevant to this chapter is Armstrong (2006) which presents three models of two-sided markets; monopoly platform, competing platforms where agents join a single platform, competing platforms where one group of agents can join all platforms (competitive bottlenecks). The main focus of Armstrong (2006) is on the equilibrium pricing rules. In the symmetric equilibria of the two oligopoly models, the optimal price set by a platform on one side of the market depends on the cost of providing the service, the inter-side externalities effect and the elasticity of demand on that side. As a result, one side of the market can even be subsidized by the other side (i.e. negative equilibrium prices are possible) in the presence of strong inter-side externalities.

I use Armstrong's models as building blocks. We introduce dynamic investment incentives into Armstrong (2006)'s monopoly model and two-sided single-homing model. In our framework, platform growth is driven by innovation, and the platforms' innovation intensity determines social welfare.

Rochet and Tirole (2003) offer an alternative model of two-sided market. Rochet and Tirole (2003) distinguish between membership externalities occurring when additional agent participation on one side benefits the other side's agents and usage externalities when the benefit is originated by an additional transaction. They assume each agent of one side interacts with an exogenously given proportion of agents on the other side, and indicate that the volume of transactions and the profit of a platform depend not only on the total price charged to the parties to the transaction, but also on its decomposition. They present new results on two-sided markets and unveil the determinants of price allocation and end-user surplus for different governance structures.

There are many modelling differences between Armstrong (2006) and Rochet and Tirole (2003). Network externalities in Rochet and Tirole (2003) depend not only on the platforms, but also on which side of the platform the agent is on, and agents pay a per-transaction fee for each agent on the platform on the other side. In Armstrong (2006)'s analysis, network externalities vary with different sides' agent only, and agents pay a lump-sum fee to platforms for all agents on the platform from the other side. Another modelling difference is these two papers treat platforms' cost differently. The models in Rochet and Tirole (2003) and Armstrong (2006) are well suited to different markets.

Our choice of building upon Armstrong's model is essentially due to tactility as extending Rochet and Tirole (2003) modelled to dynamic innovation turned out to be intractable. Another related article is Caillaud and Jullien (2003), which studies competition among intermediaries. They propose a model of indirect network externalities across two categories of users of intermediation and price discrimination, which gives rise to a "chicken and egg" problem. They assume that each buyer has specific needs. Each of them meets an agent just once on the other side of the market. The pair is being randomly matched. Matched agents share linear trade surplus in this model. In our model, by contrast, the utility of an agent on one side depends on platform's prices and on the size of the group of agents on the other side of platform. Like Armstrong (2006), their focus is also on intermediary platform optimal pricing, while we focus on dynamic incentives to innovate the quality of the services offered by monopolistic or competing platforms.

Very few studies have considered innovation competition in a two-sided market (e.g.,Casadesus-Masanell and Llanes (2015); Bourreau and Verdier (2014)). There are very few studies that assess how innovation influences two-sided market performance. Casadesus-Masanell and Llanes (2015) investigate incentives to invest in platform quality in open-source and close-access two-sided platforms. Open-source means that the source code for that software is openly available, thus allowing for modification, and that the software may be redistributed freely. Close-access source is not distributed in the public domain. Bourreau and Verdier (2014) study how factors impact on R & D investments in a two-sided market, such as the degree of spillover, the degree of network externalities. Hui, Subramanian, Guo, and Berry (2012) work on the diffusion problem of innovation in two-sided markets where both sides choose between an incumbent technology and an innovation. Lin, Li, and Whinston (2011) examine a platform owners' optimal pricing strategy considering seller-side innovation. We differ from these papers. Our study focuses on the interaction between platform innovation which increasing the quality of connections and platform competition.

1.3. **Monopoly Platform.**

We start with modelling a monopoly platform. Three kinds of agents participate into this two-sided market: a single provider of a platform offering unit-connections to two groups of agents, agents of group 1 on one side of the platform, and agents of group 2 on the other side. Hereinafter $l = [1, 2]$ will denote

a generic group of agents. Each agent in group l, is potentially interested in establishing a single connection to the group of agents connected on the other side through the platform.

Let n_l be the numbers of agents of group l connected to the platform, and p_l be the price the platform charges any single connection on that side of the market. Connecting to the platform any agent of group l carries a connection cost $f_l \geq 0$ to the monopolistic provider of the platform. Agents of group l connected to the platform are homogeneous in their willingness to pay for platform's service. Then, the net utility any agent of group 1 would derive from connecting to the platform is specified as:

$$u_1 = V + \alpha_1 n_2 - p_1 \tag{1}$$

In equation (1), $V > 0$ is an intrinsic value that an agent of group l receives from participating in a platform. $\alpha_l > 0$ measures the intensity of the benefit each agent of group 1 would derive from accessing the other side of the market through the platform (network externalities). The gross utility the agent would gain, $\alpha_1 n_2$, linearly increases with the number of agents connected on the other side of the market, n_2. Subtracting the connection price, p_1, from the gross utility we finally obtain the agent's net utility. Similarly, for any agent of group 2 we have:

$$u_2 = V + \alpha_2 n_1 - p_2 \tag{2}$$

Each demand side of the platform market is modelled as a standard linear Hotelling model. Group l consists of a unit mass of agents distributed along a unit interval $[0, 1]$. Each agent faces the same unit-transportation cost $t > 0$ to reach the platform, which is located at point zero for each group. An agent of group l, located at distance x from the platform, will face a total transportation cost xt to reach and connect to the platform. This total transportation cost must be subtracted from the net utility u_l (as defined in equations (1) and (2), here with $l = 1, 2$) to obtain the agent's net surplus from reaching and connecting to the platform:

$$U_l = u_l - tx_l$$

Finally, the platform collects profit $p_l - f_l$ per unit of transaction on side l, where f_l is the fixed cost of each connection. Therefore, the platform's total profit is

$$\pi = n_1(p_1 - f_1) + n_2(p_2 - f_2) \tag{3}$$

1.3.1. *Equilibrium Analysis without Innovation.*

Given prices p_l, l=1,2, the marginal agent of group l, indifferent between connecting or not connecting to the platform, is identified by a location $\breve{x}_l$ such that:

$$U_l = u_l - t\breve{x}_l = 0$$

The mass of agent of group l who derive positive surplus and hence connect to the platform, n_l, is equal to $\breve{x}_l$:

$$n_l = \breve{x}_l = \frac{u_l}{t}, l = 1, 2 \tag{4}$$

We next derive the linear demand system implicitly defined by (4). Substituting (4) to (1) and (2), we get:

$$n_1 t = V + \alpha_1 n_2 - p_1$$

$$n_2 t = V + \alpha_2 n_1 - p_2$$

Solving the system in n_1 and n_2, we obtain the demand systems:

$$n_1 = \frac{V(t + \alpha_1)}{t^2 - \alpha_1\alpha_2} - \frac{tp_1 + \alpha_1 p_2}{t^2 - \alpha_1\alpha_2} \tag{5}$$

$$n_2 = \frac{V(t + \alpha_2)}{t^2 - \alpha_1\alpha_2} - \frac{tp_2 + \alpha_2 p_1}{t^2 - \alpha_1\alpha_2} \tag{6}$$

From now on, in the following monopoly platform with or without innovation, we focus on a symmetric case where $\alpha_1 = \alpha_2 = \alpha$ and $f_1 = f_2 = f$ (We consider an asymmetric case later in the extension section), the symmetric demand systems are :

$$n_1 = \frac{V(t + \alpha)}{t^2 - \alpha^2} - \frac{tp_1 + \alpha p_2}{t^2 - \alpha^2} \tag{7}$$

$$n_2 = \frac{V(t + \alpha)}{t^2 - \alpha^2} - \frac{tp_2 + \alpha p_1}{t^2 - \alpha^2} \tag{8}$$

And we mainly focus on an equilibrium where both market sides are just partially covered. The following condition (9) from second order condition guarantees the existence of an interior equilibrium of the monopoly problem [1]. Later on, we would simply consider corner solution where both market sides are fully covered.

$$\begin{cases} f < V < 2(t - \alpha) + f \\ t > \alpha \end{cases} \tag{9}$$

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[1]We discuss the condition in more detail in Chapter 2.

Using (7) and (8), we derive the platform's profit function:

$$\pi = \left(\frac{V(t+\alpha)}{t^2-\alpha^2} - \frac{tp_1+\alpha p_2}{t^2-\alpha^2}\right)(p_1-f) + \left(\frac{V(t+\alpha)}{t^2-\alpha^2} - \frac{tp_2+\alpha p_1}{t^2-\alpha^2}\right)(p_2-f) \tag{10}$$

Maximizing (10) with respect to prices, we find:

$$p_1^* = p_2^* = \frac{V+f}{2} \qquad (\equiv p^*) \tag{11}$$

and

$$u_1^* = u_2^* = \frac{t(V-f)}{2(t-\alpha)} \tag{12}$$

$$n_1^* = n_2^* = \frac{V-f}{2(t-\alpha)} \qquad (\equiv n^*) \tag{13}$$

The maximum profit of a platform is :

$$\pi^* = \frac{(V-f)^2}{2(t-\alpha)} \tag{14}$$

1.3.2. *First Best Solution without Innovation.*

We solve for the highest level of social welfare that can be obtained if a platform does not engage in innovation. Social welfare is maximized by choosing agents' net utilities, which can equivalently be re- formulated as if the social planner would choose platform's prices. Social Welfare is expressed as platform's profit and agents' surplus as follows:

$$w = \pi(u_1, u_2) + v_1(u_1) + v_2(u_2) \tag{15}$$

In words, social welfare is the sum of the monopoly platform's profit, the aggregate surplus of group 1 and the aggregate surplus of group 2. Next, $v_l(u_l)$ is given by:

$$v_l(u_l) = \int_0^{\breve{x}_l} (u_l - tx)dx$$
$$= (u_l - tx)\big|_0^{\breve{x}_l}$$
$$= u_l \breve{x}_l - \frac{t\breve{x}_l^2}{2} \tag{16}$$

Substituting now (4) into (16), we find,

$$v_l(u_l) = \frac{u_l^2}{2t} \tag{17}$$

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Using (17), we derive social welfare function:

$$w = \pi(u_1, u_2) + v_1(u_1) + v_2(u_2)$$

$$= n_1(p_1 - f) + n_2(p_2 - f) + \frac{tn_1^2}{2} + \frac{tn_2^2}{2} \tag{18}$$

$$= \frac{u_1}{t}(\alpha\frac{u_2}{t} + V - u_1 - f) + \frac{u_2}{t}(\alpha\frac{u_1}{t} + V - u_2 - f) + \frac{u_1^2}{2t} + \frac{u_2^2}{2t}$$

We mainly focus on the interior solution where both market sides are just partially covered. Later on, we would simply consider corner solution where both market sides are fully covered. The necessary and sufficient condition for the existence of an interior solution of welfare-maximizing problem is:

$$\begin{cases} f < V < t - 2\alpha + f \\ t > \alpha \end{cases} \tag{19}$$

Maximizing (18) with respect to utilities, we find:

$$u_1^{sw*} = u_2^{sw*} = \frac{t(V - f)}{t - 2\alpha} \qquad (\equiv u^{sw*}) \tag{20}$$

Substituting (20) into (1) and (2), we have the notional values of platform's prices which maximize welfare:

$$p_1^{sw*} = p_2^{sw*} = \frac{f(t - \alpha) - V\alpha}{t - 2\alpha} \qquad (\equiv p^{sw*}) \tag{21}$$

The maximum social welfare is:

$$w^{sw*} = -\frac{2(V - f)^2\alpha}{(t - 2\alpha)^2} + \frac{t(f - V)^2}{2(t - 2\alpha)^2} + \frac{t(f - V)^2}{2(t - 2\alpha)^2}$$

$$= \frac{(V - f)^2}{t - 2\alpha} \tag{22}$$

By comparing profit-maximizing outcomes and welfare-maximizing outcomes without innovation, we find that while the platform's profit is obviously positive in the profit-maximizing solution (See (14)), notional profit is negative in the welfare-maximizing solution:

$$\pi^{sw*} = -\frac{2(V - f)^2\alpha}{(t - 2\alpha)^2} \tag{23}$$

From a social perspective, lower prices than the profit-maximizing ones induce more agents of group 1 and group 2 to join the platform ($p^{sw*} < p^*$, see (21) and (11)), leading to a welfare enhancing exploitation of the network externality and higher total surplus, even at the cost of negative (notional) operating profit for the platform (i.e. prices p^{sw*} lower than the unit connection cost f).

This result is formally collected in Lemma (1) :

Lemma 1. *An interior maximum of social welfare is found in which the notional profit of the platform is negative.*

The above analysis just focuses on the interior solutions of the profit-maximizing and the welfare-maximizing problems whereby both market sides are just partially covered. As a function of parameters, when inequalities (9) and (19) are reversed, corner solutions are possible. The results are summarized as follows:

(1) When $V \geq 2(t - \alpha) + f$, there are corner solutions. The market is fully covered for both solutions. Monopoly solution and first best (FB) solution have same social welfare.

(2) When $\frac{t}{2} - \alpha + f \leq V < 2(t-\alpha)+f$, it is a fully covered market for Monopoly solution only. Social planner chooses price below marginal cost due to the externalities.

(3) When $f < V < \frac{t}{2} - \alpha + f$, it is a uncovered market for both solution. Profit-maximizing prices are higher than the welfare-maximizing prices on the interior solutions.

Figure 1 summarizes and illustrates the monopoly solution and first best solution:
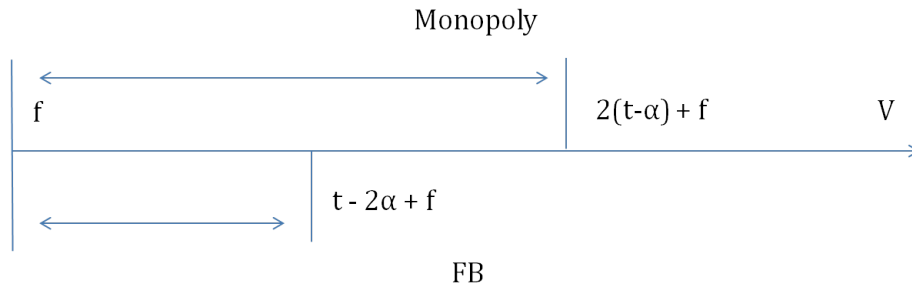
FIGURE 1. Monopoly Platform (exogenous quality)

V must be sufficiently high for an equilibrium with positive demand and output to exist, but not too high to avoid a corner equilibrium where the market is fully covered. If an intrinsic value is too low, the only possible equilibrium is at the corner where demand and output are zero. If an intrinsic value is too high, every consumer of group l has higher utility from participating in a platform than not participating. Moreover, we compare monopoly solution and first best (FB) solution. The fact that there is a gap of two conditions expressed by V suggests that there is underinvestment problem in monopoly solution.

1.3.3. *Monopoly Platform with Innovation.*

Consider now a platform that maximizes its profit by both introducing quality-increasing innovation and optimally setting prices. We assume that innovation symmetrically increases the intrinsic quality (i.e. utility) of platform connections from V to $V + q$, where $q \geq 0$. To reach quality q, the platform has to afford the R & D cost $K = \frac{1}{2}\beta q^2, \beta > 0$. First, the platform decides its R & D investment $K(q)$ which yields the associated quality increase, q with probability 1 (assuming that the R & D project is successful just with probability $p < 1$ won't change the conclusion). Then, it optimally set prices. [2]

1.3.4. *Equilibrium Analysis with Innovation.*

The platform's net profit is monopoly profit minus the cost of $R\&D$:

$$\pi = n_1(p_1 - f) + n_2(p_2 - f) - K \tag{24}$$

The utilities of agent in group l are:

$$u_1(q) = V + q + \alpha n_2 - p_1 \tag{25}$$

$$u_2(q) = V + q + \alpha n_1 - p_2 \tag{26}$$

Since $V + q$ replaces V in the net utilities, the demand systems become:

$$n_1(q) = \frac{(V + q)(t + \alpha)}{t^2 - \alpha^2} - \frac{tp_1 + \alpha p_2}{t^2 - \alpha^2} \tag{27}$$

$$n_2(q) = \frac{(V + q)(t + \alpha)}{t^2 - \alpha^2} - \frac{tp_2 + \alpha p_1}{t^2 - \alpha^2} \tag{28}$$

Substituting the demand systems (27) and (28) into (24), we derive the platform's profit function:

$$\begin{aligned}
\pi(q) = & \left(\frac{(V + q)(t + \alpha)}{t^2 - \alpha^2} + \frac{-tp_1 - \alpha p_2}{t^2 - \alpha^2}\right)(p_1 - f) \\
& + \left(\frac{(V + q)(t + \alpha)}{t^2 - \alpha^2} + \frac{-tp_2 - \alpha p_1}{t^2 - \alpha^2}\right)(p_2 - f) \\
& - \frac{1}{2}\beta q^2
\end{aligned} \tag{29}$$

When there is innovation, the profit-maximizing solutions depends on not only the optimal prices, but also optimal quality. In order to address prices, utilities, quantities and profit depending on endogenous quality of the platform connections, we will re-write all these variables as functions of q. Both groups are symmetric,so the solutions are symmetric. Imposing first order condition on p_1, p_2, the profit-maximizing prices are exactly the same formulas as in the case without

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[2]Here, we introduce the timing of the decision which is irrelevant in the case of duopoly platforms. Obviously, sequential or simultaneous of innovation and price decision in a monopoly platform are equivalent.

innovation (see (11)) but $V + q$ replacing V:

$$p_1^*(q) = p_2^*(q) = \frac{V + q + f}{2} \qquad (\equiv p^*(q)) \tag{30}$$

The associated utilities are:

$$u_1^*(q) = u_2^*(q) = \frac{t(V + q - f)}{2(t - \alpha)} \tag{31}$$

The associated quantities are:

$$n_1^*(q) = n_2^*(q) = \frac{V + q - f}{2(t - \alpha)} \qquad (\equiv n_{int}^*(q)) \tag{32}$$

The maximum profit of a platform is:

$$\pi^*(q) = \frac{(V + q - f)^2}{2(t - \alpha)} \qquad (\equiv \pi^*(q)) \tag{33}$$

We mainly focus on the interior solution where both market sides are just partially covered where $0 \leq n_{int}^*(q) \leq 1$. The necessary and sufficient condition for the existence of an interior solution of profit-maximizing problem is

$$\begin{cases} f < V < 2(t - \alpha) + f - \frac{2}{\beta} \\ t > \alpha \end{cases} \tag{34}$$

Substituting (30) into (29), by F.O.C. on q, the profit-maximizing quality is:

$$q^* = \frac{V - f}{-1 + t\beta - \alpha\beta} \qquad (\equiv q_{int}^*) \tag{35}$$

Substituting (35) into (30), we have,

$$p_1^* = p_2^* = \frac{1}{2}(V + f + \frac{V - f}{-1 + t\beta - \alpha\beta}) \tag{36}$$

Substituting (35) into (31), we have,

$$u_1^* = u_2^* = \frac{t\beta(V - f)}{2(-1 + t\beta - \alpha\beta)} \tag{37}$$

Substituting (35) into (32), we have,

$$n_1^* = n_2^* = \frac{\beta(V - f)}{2(-1 + t\beta - \alpha\beta)} \qquad (\equiv n_{int}^*) \tag{38}$$

The maximum profit of a platform is :

$$\pi^* = \frac{(V - f)^2 \beta}{-2 + 2t\beta - 2\alpha\beta} \tag{39}$$

By comparing profit-maximizing outcomes without innovation and with innovation, we find that the platform's profit with innovation is greater (See (14)) and (39)); agents of group 1 and group 2 's total surplus is greater. This result is formally collected in Lemma (2):

Lemma 2. *Suppose condition* (34) *holds, an interior equilibrium exists in which innovation makes all agents of both sides better off.*

1.3.5. *First Best Solution with Innovation.*

The social welfare with R & D is as (40):

$$
\begin{aligned}
w(q) &= n_1(p_1 - f_1) + n_2(p_2 - f_2) - \frac{1}{2}\beta q^2 + \frac{tn_1^2}{2} + \frac{tn_2^2}{2} \\
&= \frac{u_1}{t}(\alpha\frac{u_2}{t} + V + q - u_1 - f) \\
&\quad + \frac{u_2}{t}(\alpha\frac{u_1}{t} + V + q - u_2 - f) - \frac{1}{2}\beta q^2 \\
&\quad + \frac{(u_1)^2}{2t} + \frac{(u_2)^2}{2t}
\end{aligned}
\tag{40}
$$

We mainly focus on the interior solution. Later on, we would simply consider corner solution where both market sides are fully covered. The necessary and sufficient condition for the existence of an interior solution of welfare-maximizing problem is:

$$
\begin{cases}
f < V < t - 2\alpha + f - \frac{2}{\beta} \\
t > \alpha
\end{cases}
\tag{41}
$$

Maximizing (40) with respect to utilities, we find:

$$
u_1^{sw*}(q) = u_2^{sw*}(q) = \frac{t(V + q - f)}{t - 2\alpha}
\tag{42}
$$

$$
n_1^{sw*}(q) = n_2^{sw*}(q) = \frac{V + q - f}{t - 2\alpha}
\tag{43}
$$

The associated prices are:

$$
p_1^{sw*}(q) = p_2^{sw*}(q) = -\frac{(V + q)\alpha - ft + f\alpha}{t - 2\alpha}
\tag{44}
$$

Substituting (42) into (40) and imposing first order condition on q, the welfare-maximizing quality is:

$$
q^{sw*} = \frac{2(V - f)}{-2 + t\beta - 2\alpha\beta}
\tag{45}
$$

Substituting (45) into (42) and (44),
The optimal utilities satisfy:

$$
u_1^{sw*} = u_2^{sw*} = \frac{t\beta(V - f)}{-2 + t\beta - 2\alpha\beta}
\tag{46}
$$

The welfare-maximizing outcome has the optimal prices satisfying:

$$
p_1^{sw*} = p_2^{sw*} = f - \frac{\alpha\beta(V - f)}{-2 + t\beta - 2\alpha\beta}
\tag{47}
$$

The maximum social welfare is:

$$w^{sw} = \frac{(f - V)^2 \beta}{-2 + t\beta - 2\alpha\beta} \tag{48}$$

By comparing profit-maximizing outcomes and welfare-maximizing outcomes with innovation, we find that while the platform's profit is obviously positive in the profit-maximizing solution (See (39)), notional profit is negative in the welfare-maximizing solution:

$$\pi^{sw*} = -\frac{2(V - f)^2 \beta(1 + \alpha\beta)}{(2 - t\beta + 2\alpha\beta)^2} \tag{49}$$

From a social perspective, lower prices and higher quality than the profit-maximizing ones induce more agents of group 1 and group 2 to join the platform ($p^{sw*} < p^*$, see (47), (36), (45) and (35)), leading to a welfare enhancing exploitation of the network externality and higher total surplus. This result is formally collected in Lemma (3):

Lemma 3. *Suppose* (41) *holds,*

(1) *an interior maximum of social welfare with innovation is found in which the notional profit of the platform is negative.*
(2) *the welfare-maximizing quality is higher than the profit-maximizing quality.*

By comparing welfare-maximizing and profit-maximizing outcomes with innovation, we have Proposition (1):

Proposition 1. *Profit-maximizing solution shows underinvestment in R & D relative to welfare-maximizing solution.*

By comparing welfare-maximizing outcomes without innovation and with innovation, we have Proposition (2):

Proposition 2. *Suppose* (41) *holds, under the welfare-maximizing solution,*

(1) *the monopoly platform's notional profit with innovation is greater than that without innovation.*
(2) *more agents are connected to the platform with innovation than that without innovation.*

The above analysis just focuses on the interior solutions of the profit-maximizing and the welfare-maximizing problems whereby both market sides are just partially covered. We simply consider now the corner solutions.

In the profit-maximizing problem, there is one corner solution where both market sides are fully covered. The corner solution's optimal price is $V + q + \alpha - t$. We summarize the profit functions as functions of q in the interior solution and corner solution as below:

$$\pi^*(q) = \begin{cases} \dfrac{(V + q_{int} - f)^2}{2(t - \alpha)} & 0 \leq n^*_{int}(q) \leq 1 \\[3mm] 2(V + q_{cor} + \alpha - t - f) - \dfrac{1}{2}\beta q^2_{cor} & n^*_{int}(q) > 1, or, q^*_{cor} = 1 \end{cases} \tag{50}$$

When we have the optimal quality q^*_{int} in the interior equilibrium(see (35)), the optimal quality is $q^*_{cor} = \dfrac{2}{\beta}$ in the corner solution. The optimal profits are re-writed as below:

$$\pi^* = \begin{cases} \dfrac{(V-f)^2 \beta}{-2 + 2t\beta - 2\alpha\beta} & 0 \leq n^*_{int} \leq 1 \\[3mm] 2(V + \alpha - t - f) + \dfrac{2}{\beta} & n^*_{int} > 1 \end{cases} \tag{51}$$

In the welfare-maximizing problem, when inequality (40) is reversed, there is one corner solution where both market sides are fully covered, so that the social welfare is $w = 2V + 2\alpha - t - 2f + \frac{2}{\beta}$. Figure 2 summarizes and illustrates the monopoly solutions and first best solutions.
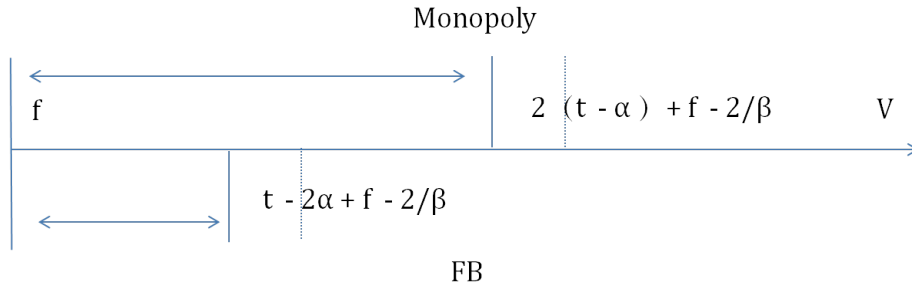
Monopoly

FIGURE 2. Monopoly Platform (endogenous quality)

By comparing Figure 1 and Figure 2, we find that, the innovation factor plays the exactly same role as an intrinsic value. In other words, an innovation factor is an example of an intrinsic value. For we model the factor of innovation as a linear function, the factor shifts the gap and the proportion of the gap is not changed.

(1) When $V \geq 2(t - \alpha) + f - \frac{2}{\beta}$, the market is fully covered. Any (symmetric) pair of prices chosen by platform will yield the same social welfare and same quality.

(2) When $t - 2\alpha + f - \frac{2}{\beta} \leq V < 2(t - \alpha) + f - \frac{2}{\beta}$, the market is fully covered for FB solution only. The monopoly solution chooses lower quality than FB solution. Underinvestment problem exists.

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(3) When $f < V < t - 2\alpha + f - \frac{2}{\beta}$, it is a uncovered market for both solutions. Optimal quality chosen by social planner is higher. Underinvestment problem exists.

1.4. a Non-tournament Duopoly Model with Single-homing Agents.

We now extend our model to duopoly competition between two platforms, but assume for exogenous reasons that each agent chooses to join a single platform. We assume there are three types of agents: two platforms, agents of groups 1 and agents of group 2. Hereinafter $h = [i, j]$ will denote a generic platform, and $l = [1, 2]$ will denote a generic group of agents. Two platforms are located at the two endpoints of a unit interval $[0, 1]$. Each platform generates revenues from fees p_1^h and p_2^h collected from group 1 and group 2 on per-transaction basis, and has to pay a per-transaction constant cost f in order to serve agents in group l.

There are n_l^h agents of group l who participate in platform h. Agents in group l are assumed to be uniformly located along a unit interval $[0, 1]$, and are homogeneous in their willingness to pay for service in the platform market. We assume the market is fully covered. Agents in group l will choose to conclude a deal with platform h, that is, $n_1^i + n_1^j = 1, n_2^i + n_2^j = 1$. The utility of agents in group 1 is :

$$u_1^i = \alpha n_2^i + q_i - p_1^i \tag{52}$$

$$u_1^j = \alpha n_2^j + q_j - p_1^j \tag{53}$$

where q_i, q_j are the quality levels of service on platform i and j. The utility of agents in group 2 is:

$$u_2^i = \alpha n_1^i + q_i - p_2^i \tag{54}$$

$$u_2^j = \alpha n_1^j + q_j - p_2^j \tag{55}$$

Consider platforms that maximizes their profits by both introducing quality-increasing innovation and optimally setting prices. We assume that innovation symmetrically increases the intrinsic quality q_h of platform connections, where $q_h \geq 0$. To reach quality q_h, the platforms have to afford the R & D cost $K = \frac{1}{2}\beta q_h{}^2, \beta > 0$.

First, platforms decide their R & D investments $K(q_h)$ which yield the associated quality increase, q_h with probability 1 (assuming that the R & D project is successful just with probability $p < 1$ won't change the conclusion). Then, it optimally set prices.

1.4.1. *Equilibrium Analysis without Technology Spillover Effect.*

In this subsection, to maintain our focus on the consequences of increasing the intrinsic quality to the two-sided model, innovation is characterized by no technology spillover effect: no exchange of ideas among platforms.

We specify each demand side of two platforms as in a standard linear Hotelling model. Group l consists of a unit mass of agents distributed along a unit interval $[0, 1]$. Each agent faces the same unit-transportation cost $t > 0$ to reach platforms. A marginal agent of group 1, indifferent between connecting to platform i or to platform j, is therefore identified by a location $\widetilde{x}_1$:

$$
\begin{aligned}
u_1^i - t\widetilde{x}_1 &= u_1^j - t(1 - \widetilde{x}_1) \\
\widetilde{x}_1 &= \frac{1}{2} + \frac{u_1^i - u_1^j}{2t}
\end{aligned}
\tag{56}
$$

Similarly, for a marginal agent of group 2 we have:

$$
\begin{aligned}
u_2^i - t\widetilde{x}_2 &= u_2^j - t(1 - \widetilde{x}_2) \\
\widetilde{x}_2 &= \frac{1}{2} + \frac{u_2^i - u_2^j}{2t}
\end{aligned}
\tag{57}
$$

The mass of agent of group l who derive positive surplus and hence connect to the platform, n_l^i, is equal to $\widetilde{x}_l$. We finally state:

$$
\begin{aligned}
n_1^i &= \int_0^{\widetilde{x}_1} 1 \, dx \\
&= \frac{1}{2} + \frac{u_1^i - u_1^j}{2t}
\end{aligned}
\tag{58}
$$

$$
\begin{aligned}
n_1^j &= \int_{\widetilde{x}_1}^1 1 \, dx \\
&= \frac{1}{2} - \frac{u_1^i - u_1^j}{2t}
\end{aligned}
\tag{59}
$$

Similarly, the demand functions of group 2 are defined as:

$$
\begin{aligned}
n_2^i &= \int_0^{\widetilde{x}_2} 1 \, dz \\
&= \frac{1}{2} + \frac{u_2^i - u_2^j}{2t}
\end{aligned}
\tag{60}
$$

$$
\begin{aligned}
n_2^j &= \int_{\widetilde{x}_2}^1 1 \, dx \\
&= \frac{1}{2} - \frac{u_2^i - u_2^j}{2t}
\end{aligned}
\tag{61}
$$

Substituting (52) and (53) into (58), we have:

$$
n_1^i = \frac{1}{2} + \frac{\alpha(n_2^i - n_2^j) + (q_i - q_j) - (p_1^i - p_1^j)}{2t}
\tag{62}
$$

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Substituting (54) and (55) into (60), we have:

$$n_2^i = \frac{1}{2} + \frac{\alpha(n_1^i - n_1^j) + (q_i - q_j) - (p_2^i - p_2^j)}{2t} \tag{63}$$

Solving the system in n_1^i and n_2^i, we finally obtain:

$$n_1^i = \frac{1}{2} + \frac{1}{2}\frac{(q_i - q_j)(t + \alpha) - t(p_1^i - p_1^j) - \alpha(p_2^i - p_2^j)}{t^2 - \alpha^2} \tag{64}$$

$$n_2^i = \frac{1}{2} + \frac{1}{2}\frac{(q_i - q_j)(t + \alpha) - \alpha(p_1^i - p_1^j) - t(p_2^i - p_2^j)}{t^2 - \alpha^2} \tag{65}$$

Using (64) and (65), we next derive platform i's profit function:

$$\begin{aligned}
\pi_{NT}^i &= (p_1^i - f)[\frac{1}{2} + \frac{1}{2}\frac{(q_i - q_j)(t + \alpha) - t(p_1^i - p_1^j) - \alpha(p_2^i - p_2^j)}{t^2 - \alpha^2}] \\
&+ (p_2^i - f)[\frac{1}{2} + \frac{1}{2}\frac{(q_i - q_j)(t + \alpha) - \alpha(p_1^i - p_1^j) - t(p_2^i - p_2^j)}{t^2 - \alpha^2}] \\
&- \frac{1}{2}\beta q_i^2
\end{aligned} \tag{66}$$

Platform j's profit can be obtained by appropriately permuting superscripts:

$$\begin{aligned}
\pi_{NT}^j &= (p_1^j - f)[\frac{1}{2} - \frac{1}{2}\frac{(q_i - q_j)(t + \alpha) - t(p_1^i - p_1^j) - \alpha(p_2^i - p_2^j)}{t^2 - \alpha^2}] \\
&+ (p_2^j - f)[\frac{1}{2} - \frac{1}{2}\frac{(q_i - q_j)(t + \alpha) - t(p_1^i - p_1^j) - \alpha(p_2^i - p_2^j)}{t^2 - \alpha^2}] \\
&- \frac{1}{2}\beta q_j^2
\end{aligned} \tag{67}$$

The necessary and sufficient condition for the existence of a market-sharing equilibrium is:

$$\begin{cases} \beta \leq \frac{2}{2t - 3\alpha + 2f} \\ t > \alpha \end{cases} \tag{68}$$

Platforms will choose $p_1^i, p_1^j, p_2^i, p_2^j$, q_i and q_j, to maximize profits (66) and (67), we are looking for an market-sharing equilibrium. From F.O.C., we find:

$$p_1^{i*}(q) = f + t - \alpha + \frac{1}{3}(q_i - q_j) \tag{69}$$

$$p_1^{j*}(q) = f + t - \alpha - \frac{1}{3}(q_i - q_j) \tag{70}$$

$$p_2^{i*}(q) = f + t - \alpha + \frac{1}{3}(q_i - q_j) \tag{71}$$

$$p_2^{j*}(q) = f + t - \alpha - \frac{1}{3}(q_i - q_j) \tag{72}$$

$$q_i* = q_j* = \frac{1}{\beta} \tag{73}$$

Substituting (69), (70), (71), (72) and (73) into (64) and (65), we have

$$n_1^{i*} = n_1^{j*} = n_2^{i*} = n_2^{j*} = \frac{1}{2} \tag{74}$$

Substituting (69), (70), (71), (72), (73) and (74) into (52), (53), (54) and (55), we have

$$u_1^{i*} = u_1^{j*} = u_2^{i*} = u_2^{j*} = \frac{1}{\beta} + \frac{3\alpha}{2} - f - t \tag{75}$$

The maximum profit of platform i and j are:

$$\pi_{NT}^{i*} = \pi_{NT}^{j*} = t - \alpha - \frac{1}{2\beta} \qquad (\equiv \pi_{NT}^{h*}) \tag{76}$$

Proposition 3. *In a symmetric equilibrium of non-tournament model with no technology spillover effect,*

(1) *only the quality gap impacts platforms' optimal prices and profits.*

(2) *investing in R & D is always a dominant strategy for each platform.*

The prisoners' dilemma helps explain why only the quality gap impacts platforms' optimal prices and profits. In equilibrium, two platforms have same quality levels, prices and profits don't depend on those quality levels. However, if one innovates, and the other doesn't, prices and profits do depend on the quality levels, and then the platform with higher quality would gain greater profit. Therefore, both the providers of platforms always have precisely the same unilateral incentive to increase quality in a two sided market. In a symmetric equilibrium, they will do exactly the same research, and then they end up in a situation in which their quality level is same. Consequently, all the investment in R & D is a pure waste from the platforms' profit point of view. Hypothetically speaking, if they reach a bound agreement in which they would better off by not doing any R & D, and then they don't coordinate with the equilibrium to get a better outcome. This result is formally collected in Proposition (4):

Proposition 4. *In the non-tournament model with no technology spillover effect, the platforms will end up in the prisoners' dilemma equilibrium where they conduct positive R & D,even if their profit will be higher without R & D investment.*

By comparing profit-maximizing outcomes without innovation and with innovation, we have the following Proposition (5):

Proposition 5. *Suppose (68) holds, under the profit-maximizing solution,*

(1) *platforms' profit with innovation is less than platforms' profit without innovation.*

(2) *agents of group l total surplus with innovation is greater than without innovation.*

(3) *social welfare with innovation is greater than that without innovation.*

Proposition (5) is derived in three steps:

Step 1: platforms' maximum profit with innovation is less than platforms' maximum profit without innovation.

Assume π^{i*} and π^{j*} are platforms' optimal profits without innovation:

$$\pi^i = (p_1^i - f)[\frac{1}{2} + \frac{1}{2}\frac{-t(p_1^i - p_1^j) - \alpha(p_2^i - p_2^j)}{t^2 - \alpha^2}]$$
$$+ (p_2^i - f)[\frac{1}{2} + \frac{1}{2}\frac{-\alpha(p_1^i - p_1^j) - t(p_2^i - p_2^j)}{t^2 - \alpha^2}] \tag{77}$$

$$\pi^j = (p_1^j - f)[\frac{1}{2} + \frac{1}{2}\frac{t(p_1^i - p_1^j) + \alpha(p_2^i - p_2^j)}{t^2 - \alpha^2}]$$
$$+ (p_2^j - f)[\frac{1}{2} + \frac{1}{2}\frac{\alpha(p_1^i - p_1^j) + t(p_2^i - p_2^j)}{t^2 - \alpha^2}] \tag{78}$$

Platforms will choose $p_1^i, p_1^j, p_2^i, p_2^j$ to maximize profits (77) and (78). From F.O.C., we get:

$$p_1^{i*} = p_1^{j*} = p_2^{i*} = p_2^{j*} = f + t - \alpha$$

$$n_1^{i*} = n_1^{j*} = n_2^{i*} = n_2^{j*} = \frac{1}{2}$$

$$u_1^{i*} = u_1^{j*} = u_2^{i*} = u_2^{j*} = \frac{3\alpha}{2} - f - t$$

Substituting these optimal prices (1.4.1) into (77) and (78), the maximum profit of platform i and j without innovation is:

$$\pi^{i*} = \pi^{j*} = t - \alpha \qquad (\equiv \pi^{h*}) \tag{79}$$

Notice that $\pi^{h*} \geq \pi_{NT}^{h*}$ (see (1.4.1) and (76)), which implies that innovation doesn't increase platforms' profit.

Step 2: agents in group l achieve greater surplus with innovation than without innovation.

Assume v_l is the aggregate surplus of agents in group l without innovation under profit-maximizing prices:

$$v_1 + v_2 = \frac{1}{2}u_1^i - \frac{t}{8} + \frac{1}{2}u_1^j - \frac{t}{8} + \frac{1}{2}u_2^i - \frac{t}{8} + \frac{1}{2}u_2^j - \frac{t}{8}$$
$$= u_1 + u_2 - \frac{t}{2} \tag{80}$$

The aggregate agent surplus of agents with innovation under profit-maximizing prices is:

$$v_{NT1} + v_{NT2} = \frac{1}{2}u_1^i(q) - \frac{t}{8} + \frac{1}{2}u_1^j(q) - \frac{t}{8} + \frac{1}{2}u_2^i(q) - \frac{t}{8} + \frac{1}{2}u_2^j(q) - \frac{t}{8}$$
$$= u_1 + u_2 + q_i + q_j - \frac{t}{2} \tag{81}$$

$v_{NT1} + v_{NT2} \geq v_1 + v_2$, which implies that innovation increases agents aggregate surplus.

Step 3: the innovation increases social welfare.

The profit-maximizing social welfare without innovation is:

$$w_{pro} = t - \alpha + u_1 + u_2 - \frac{t}{2} \tag{82}$$

The profit-maximizing social welfare with innovation is:

$$w_{sw} = t - \alpha - \frac{1}{2\beta} + u_1 + u_2 + q_i + q_j - \frac{t}{2}$$
$$= t - \alpha + u_1 + u_2 - \frac{t}{2} + \frac{3}{2\beta} \tag{83}$$

$w_{sw} > w_{pro}$

1.4.2. *Equilibrium Analysis with Technology Spillover Effect.*

In this subsection, innovation being characterized by a known spillover level $0 \leq \gamma < 1$, to reach quality q_h, the platforms have to afford the R & D cost $K_i = \frac{1}{2}\beta q_i{}^2 - \gamma q_j$, $K_j = \frac{1}{2}\beta q_j{}^2 - \gamma q_i$, $\beta > 0$.

Platform i's profit is as (84). Platform j' s profit can be obtained by appropriately permuting superscripts:

$$\pi_{SNT}^i = (p_1^i - f)[\frac{1}{2} + \frac{1}{2}\frac{(q_i - q_j)(t + \alpha) - t(p_1^i - p_1^j) - \alpha(p_2^i - p_2^j)}{t^2 - \alpha^2}]$$
$$+ (p_2^i - f)[\frac{1}{2} + \frac{1}{2}\frac{(q_i - q_j)(t + \alpha) - \alpha(p_1^i - p_1^j) - t(p_2^i - p_2^j)}{t^2 - \alpha^2}] \tag{84}$$
$$- \frac{1}{2}\beta q_i^2 + \gamma q_j$$

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Platform j's profit is

$$\pi_{SNT}^j = (p_1^j - f)[\frac{1}{2} - \frac{1}{2}\frac{(q_i - q_j)(t + \alpha) - t(p_1^i - p_1^j) - \alpha(p_2^i - p_2^j)}{t^2 - \alpha^2}]$$
$$+ (p_2^j - f)[\frac{1}{2} - \frac{1}{2}\frac{(q_i - q_j)(t + \alpha) - t(p_1^i - p_1^j) - \alpha(p_2^i - p_2^j)}{t^2 - \alpha^2}] \qquad (85)$$
$$- \frac{1}{2}\beta q_j^2 + \gamma q_i$$

Platforms will choose $p_1^i, p_1^j, p_2^i, p_2^j$, q_i and q_j, to maximize profits (84) and (85). From F.O.C., we find:

By F.O.C. on p_1^i, p_1^j, p_2^i and p_2^j, the profit-maximizing outcome has the optimal prices satisfying:

$$p_1^{i*}(q) = f + t - \alpha + \frac{1}{3}(q_i - q_j) \qquad (86)$$

$$p_1^{j*}(q) = f + t - \alpha - \frac{1}{3}(q_i - q_j) \qquad (87)$$

$$p_2^{i*}(q) = f + t - \alpha + \frac{1}{3}(q_i - q_j) \qquad (88)$$

$$p_2^{j*}(q) = f + t - \alpha - \frac{1}{3}(q_i - q_j) \qquad (89)$$

$$q_i* = q_j* = \frac{1}{\beta} \qquad (90)$$

The maximum profit of platform i and j with innovation are:

$$\pi_{SNT}^{i*} = \pi_{SNT}^{j*} = \frac{2(t - \alpha)\beta + (-1 + 4\gamma)}{2\beta}$$
$$= t - \alpha + \frac{-1 + 4\gamma}{2\beta} \qquad (\equiv \pi_{SNT}^{h*}) \qquad (91)$$

By comparing profit-maximizing outcomes without innovation and with innovation, we have Proposition (6):

Proposition 6. *Under the profit-maximizing solution,*

(1) *If $\frac{1}{4} \le \gamma < 1$, the profit of platforms in a non-tournament model with technology spillover effect is greater than the profit in a non-tournament model without innovation, which is $\pi_{SNT}^{h*} < \pi^{h*}$.*

(2) *If $0 \le \gamma < \frac{1}{4}$, the profit of platforms in a non-tournament model with technology spillover effect is less than the profit in a non-tournament model without innovation, which is $\pi_{SNT}^{h*} \ge \pi^{h*}$.*

By comparing profit-maximizing outcomes without technology spillover effect and with technology spillover effect, we have Proposition (7):

Proposition 7. *Under the profit-maximizing solution,*

(1) *platforms have the same optimal price and optimal quality under the condition without technology spillover effect or with technology spillover effect.*

(2) *each platform's profit with technology spillover effect is greater than that without technology spillover effect.*

From the platforms point of view, technology spillover effect has its advantages and disadvantages. The advantage is that the innovation with technology spillover effect is more effective way to increase quality than that with no technology spillover effect. Technology spillover effect plays as a positive externality to partially compensate the negative externality in R & D competition, which results in platforms' research cost decreasing. The investment in R & D competition are not a pure waste, but a motive to create profit. Especially, when the spillover level is high, $\frac{1}{4} \leq \gamma < 1$, platform will have positive profit. Compared with the case with no technology spillover effect, in equilibrium, they reach the same optimal quality level, but with negative profit. The disadvantages are from two sides. From one side, platforms with no technology spillover effect end up in a prisoners' dilemma equilibrium, and only the quality gap impacts platforms' optimal prices and profits. When there is technology spillover effect, the platforms also end up in a prisoners' dilemma equilibrium, and only the quality gap impacts platforms' optimal prices and profits. Furthermore, the platforms perceived part of what they do is beneficial to their opponent and narrows the quality gap. Platforms then have the less incentive to do the R & D. From the other side, platform still will end up with a negative profit when the spillover level is low, $0 \leq \gamma < \frac{1}{4}$.

1.5. Extensions.

This section presents two non-tournament duopoly models and a tournament duopoly model. in the extension 1, we relax the assumption to allow for different qualities q_l^h and different network externalities α_l on both sides of platforms. In the second extension, we present an asymmetric case in which each platform innovates in different side of the market. In the third extension, we present a tournament model. They are performed mainly as preliminary steps for our future work.

1.5.1. *Extension 1:Asymmetric Side-network Effects in a Non-tournament Duopoly without Technology Spillover Effect.*

We assume that innovation asymmetrically increases the intrinsic quality of platform connections to q_l^h, where $q_l^h \geq 0$. To reach quality q_l^h, the platform has to afford the R & D cost $K = \frac{1}{2}\beta q_{lh}^2, \beta > 0$. First, the platform decides its R & D

investment $K(q_l^h)$ which yields the associated quality increase, q_l^h with probability 1 (assuming that the R & D project is successful just with probability $p < 1$ won't change the conclusion). Then, it optimally set prices. We assume different network externalities α_l for different groups' agents.

We assume that the utility of agents in group 1 is :

$$u_1^i = \alpha_1 n_2^i + q_1^i - p_1^i \tag{92}$$

$$u_1^j = \alpha_1 n_2^j + q_1^j - p_1^j \tag{93}$$

The utility of agents in group 2 is:

$$u_2^i = \alpha_2 n_1^i + q_2^i - p_2^i \tag{94}$$

$$u_2^j = \alpha_2 n_1^j + q_2^j - p_2^j \tag{95}$$

where p_l^h is the quality level of service on the side i of platform h. In this symmetric case, platform h chooses to engage in R & D to increase different side's service level. Platforms' profit functions are :

$$\pi_{NT}^i = (p_1^i - f)n_1^i + (p_2^i - f)n_2^i - \frac{1}{2}\beta q_{i1}^2 - \frac{1}{2}\beta q_{i2}^2 \tag{96}$$

$$\pi_{NT}^j = (p_1^j - f)n_1^j + (p_2^j - f)n_2^j - \frac{1}{2}\beta q_{j1}^2 - \frac{1}{2}\beta q_{j2}^2 \tag{97}$$

Each demand side of two platforms are in a standard linear Hotelling model. Each agent faces the same unit-transportation cost $t > 0$ to reach platforms. The demand functions are defined as:

$$n_1^i = \frac{1}{2} + \frac{1}{2}\frac{t(q_1^i - q_1^j - p_1^i + p_1^j) + \alpha_1(q_2^i - q_2^j - p_2^i + p_2^j)}{t^2 - \alpha_1\alpha_2} \tag{98}$$

$$n_2^i = \frac{1}{2} + \frac{1}{2}\frac{t(q_2^i - q_2^j - p_2^i + p_2^j) + \alpha_2(q_1^i - q_1^j - p_1^i + p_1^j)}{t^2 - \alpha_1\alpha_2} \tag{99}$$

$$n_1^j = \frac{1}{2} - \frac{1}{2}\frac{t(q_1^i - q_1^j - p_1^i + p_1^j) + \alpha_1(q_2^i - q_2^j - p_2^i + p_2^j)}{t^2 - \alpha_1\alpha_2} \tag{100}$$

$$n_2^j = \frac{1}{2} - \frac{1}{2}\frac{t(q_2^i - q_2^j - p_2^i + p_2^j) + \alpha_2(q_1^i - q_1^j - p_1^i + p_1^j)}{t^2 - \alpha_1\alpha_2} \tag{101}$$

Substitute (98), (99),(100) and (101) into (96) and (97), Platforms choose $p_1^i, p_1^j, p_2^i, p_2^j$, q_1^i, q_1^j, q_2^i and q_2^j to maximize profits (96) and (97). From F.O.C., we get:

$$p_1^{i*}(q) = f + t - \alpha_2 + \frac{t(\alpha_1 - \alpha_2)(q_2^i - q_2^j) + (q_1^i - q_1^j)(3t^2 - 2\alpha_1\alpha_2 - \alpha_2^2)}{9t^2 - 2\alpha_1^2 - 5\alpha_1\alpha_2 - 2\alpha_2^2} \tag{102}$$

$$p_1^{j*}(q) = f + t - \alpha_2 - \frac{t(\alpha_1 - \alpha_2)(q_2^i - q_2^j) + (q_1^i - q_1^j)(3t^2 - 2\alpha_1\alpha_2 - \alpha_2^2)}{9t^2 - 2\alpha_1^2 - 5\alpha_1\alpha_2 - 2\alpha_2^2} \tag{103}$$

$$p_2^{i*}(q) = f + t - \alpha_1 - \frac{t(\alpha_1 - \alpha_2)(q_1^i - q_1^j) + (q_2^i - q_2^j)(3t^2 - 2\alpha_1\alpha_2 - \alpha_2^2)}{9t^2 - 2\alpha_1^2 - 5\alpha_1\alpha_2 - 2\alpha_2^2} \tag{104}$$

$$p_2^{j*}(q) = f + t - \alpha_1 + \frac{t(\alpha_1 - \alpha_2)(q_1^i - q_1^j) + (q_2^i - q_2^j)(3t^2 - 2\alpha_1\alpha_2 - \alpha_2^2)}{9t^2 - 2\alpha_1^2 - 5\alpha_1\alpha_2 - 2\alpha_2^2} \tag{105}$$

$$p_1^{i*} = f + t - \alpha_2 \tag{106}$$

$$p_1^{j*} = f + t - \alpha_2 \tag{107}$$

$$p_2^{i*} = f + t - \alpha_1 \tag{108}$$

$$p_2^{j*} = f + t - \alpha_1 \tag{109}$$

We have the optimal qualities:

$$q_l^{h*} = \frac{1}{2\beta} \tag{110}$$

The maximum profits of platforms are:

$$\pi_{NT}^{i*} = \pi_{NT}^{j*} = t - \frac{1}{2}\alpha_1 - \frac{1}{2}\alpha_2 - \frac{1}{4\beta} \tag{111}$$

When we relax the assumption to allow for different qualities and different network externalities. The asymmetric case with different parameters has almost the same conclusion with symmetric case above. We find that, in equilibrium, the price and the profit don't depend on the absolutely quality level but the qualities' gap between the quality levels. Investing in R & D is always the dominant strategy. Two platforms will end up with same quality levels and spit the demands. Tractable results show the effect of parameters in the equilibrium solutions. We find that optimal prices are highly correlated to network externalities, and there is little correlation between qualities and network externalities.

1.5.2. *Extension 2: Exogenous Specialization in Innovation Sides in a Non-tournament Duopoly without Technology Spillover Effect.*

So far, we study platforms innovate on both sides of market. However, a very interesting extension would be platforms are specialized in different sides of market. Ideally, specialization is endogenous. As a preliminary step of future more general model, specialization here is exogenous.

We assume that platform i and j can only innovate on different sides of the market. The platform i innovates one side of market that increase its service level to q_1^i, and the platform j innovates on the other side of market that increases its service level to q_2^j. They play in a non-tournament model with no technology spillover effect. Agents share same network externalities α and transportation cost t.

We assume that the utility of agents in group 1 is:

$$u_1^i = \alpha n_2^i + q_1^i - p_1^i \tag{112}$$

$$u_1^j = \alpha n_2^j - p_1^j \tag{113}$$

The utility of agents in group 2 is:

$$u_2^i = \alpha n_1^i - p_2^i \tag{114}$$

$$u_2^j = \alpha n_1^j + q_2^j - p_2^j \tag{115}$$

Platforms' profit functions are :

$$\pi_{NT}^i = (p_1^i - f)n_1^i + (p_2^i - f)n_2^i - \frac{1}{2}\beta q_{i1}^2 \tag{116}$$

$$\pi_{NT}^j = (p_1^j - f)n_1^j + (p_2^j - f)n_2^j - \frac{1}{2}\beta q_{j2}^2 \tag{117}$$

Each demand side of two platforms are in a standard linear Hotelling model. Each agent faces the same unit-transportation cost $t > 0$ to reach platforms. The demand functions are defined as:

$$n_1^i = \frac{1}{2} + \frac{1}{2}\frac{tq_1^i - \alpha q_2^j - t(p_1^i - p_1^j) - \alpha(p_2^i - p_2^j)}{t^2 - \alpha^2} \tag{118}$$

$$n_2^i = \frac{1}{2} + \frac{1}{2}\frac{\alpha q_1^i - tq_2^j - \alpha(p_1^i - p_1^j) - t(p_2^i - p_2^j)}{t^2 - \alpha^2} \tag{119}$$

$$n_1^j = \frac{1}{2} - \frac{1}{2}\frac{tq_1^i - \alpha q_2^j - t(p_1^i - p_1^j) - \alpha(p_2^i - p_2^j)}{t^2 - \alpha^2} \tag{120}$$

$$n_2^j = \frac{1}{2} - \frac{1}{2}\frac{\alpha q_1^i - tq_2^j - \alpha(p_1^i - p_1^j) - t(p_2^i - p_2^j)}{t^2 - \alpha^2} \tag{121}$$

Substituting (118), (119), (120), (121) into (116) and (117). Platforms choose $p_1^i, p_1^j, p_2^i, p_2^j, q_1^i$ and q_2^j to maximize profits. From F.O.C., we get:

$$p_1^{i*}(q) = f + t - \alpha + \frac{1}{3}q_{i1} \tag{122}$$

$$p_1^{j*}(q) = f + t - \alpha - \frac{1}{3}q_{j2} \tag{123}$$

$$p_2^{i*}(q) = f + t - \alpha - \frac{1}{3}q_{i1} \tag{124}$$

$$p_2^{j*}(q) = f + t - \alpha + \frac{1}{3}q_{j2} \tag{125}$$

The quantities demanded in equilibrium are:

$$n_1^i = \frac{1}{2} + \frac{1}{2(-1 + 9t\beta + 9\alpha\beta)} \tag{126}$$

$$n_1^j = \frac{1}{2} - \frac{1}{2(-1 + 9t\beta + 9\alpha\beta)} \tag{127}$$

$$n_2^i = \frac{1}{2} - \frac{1}{2(-1 + 9t\beta + 9\alpha\beta)} \tag{128}$$

$$n_2^j = \frac{1}{2} + \frac{1}{2(-1 + 9t\beta + 9\alpha\beta)} \tag{129}$$

The profit-maximizing qualities are:

$$q_i* = q_j* = \frac{3(t + \alpha)}{-1 + 9t\beta + 9\alpha\beta} \tag{130}$$

The maximum profits of platforms are:

$$\pi_{NT}^{i*} = \pi_{NT}^{j*} = t - \alpha + \frac{(t + \alpha)(2 - 9t\beta - 9\alpha\beta)}{2(-1 + 9t\beta + 9\alpha\beta)^2} \tag{131}$$

In previous sections, prisoners' dilemma expains the best response functions of platforms innovation, however, those findings are not unique to two-sided markets. In this extension, we specialize platforms' R & D effort on one side in order to model the parameters specially related to the two-sidedness of the market. A lot of technical work leads to some interesting results. We find that the demands are not split equally in equilibrium. Asymmetric innovation as direct effect influncing the distribution of agents in both sides is more stronger than the network externality as indirect effect when network externalities interact with innovation. If we look at one of the two platforms, we will see that the side with innovaiton attractt more consumers connections, and the other side without innovation attracts less consumers. This result is quite different from the symmetric cases' results. In symmetric cases, with the effect of network externality, more consumers in one side attract more consumers participating in the other side. In equlibrium, the demands are split equally. More analysis will be left for future work.

1.5.3. *Extension 3: a Tournament Model without Technology Spillover Effect.*

In R & D literature, studies base on non-tournament and tournament models. The last extension would be platform innovation competition in a tournament model. Hereinafter is as a preliminary step for future work.

We consider a tournament model in which duopoly competition is between two platforms with single-homing agents. Platforms innovate to increase its service quality at a cost of K, which $K = \frac{1}{2}\tau P_i^2, \tau > 0$. With endogenous probability variable P_i, a project completes successfully, and it becomes possible to reach the

certain quality level $q = \bar{q}$. With probability $1 - P_i$, the project completes unsuccessfully, and it remains its original quality level $q = 0$. To maintain our focus on the consequences of adding a quality level to the two-sided model, innovation is characterized by no technology spillover effect: only the first successful innovator obtains an patent that denies its rival the use of the new technology. First, the platform decides its R & D investment K which yields the associated possibilities of quality increase. Then, it optimally set prices.

Consider platform $i's$ profit in different states: if platform A and platform B both complete successfully with probability P_i, but only one platform wins to gain an patent. In this case, its value is π_{ss}^i; if platform A is the first innovator, it completes successfully with probability P_i, platform B completes unsuccessfully with probability $1 - P_i$, its value is π_{sf}^i; if platform A completes unsuccessfully with probability $1 - P_i$, platform B is the first innovator, it completes successfully with probability P_i, its value is π_{fs}^i; if platform A and platform B both complete unsuccessfully with probability $1 - P_i$. In this case, its value is π_{ff}^i. Platform $j's$ profit can be obtained by appropriately permuting superscripts.

$$
\begin{aligned}
\pi_{ss}^i &= \frac{1}{2}\pi_{sf}^i + \frac{1}{2}\pi_{fs}^i \\
&= \frac{1}{2}[(p_1^i - f)[\frac{1}{2} + \frac{1}{2}\frac{\bar{q}(t+\alpha) - t(p_1^i - p_1^j) - \alpha(p_2^i - p_2^j)}{t^2 - \alpha^2}] \\
&+ (p_2^i - f)[\frac{1}{2} + \frac{1}{2}\frac{\bar{q}(t+\alpha) - \alpha(p_1^i - p_1^j) - t(p_2^i - p_2^j)}{t^2 - \alpha^2}] \\
&- \frac{1}{2}\tau P_i^2] \\
&+ \frac{1}{2}[(p_1^i - f)[\frac{1}{2} + \frac{1}{2}\frac{-\bar{q}(t+\alpha) - t(p_1^i - p_1^j) - \alpha(p_2^i - p_2^j)}{t^2 - \alpha^2}] \\
&+ (p_2^i - f)[\frac{1}{2} + \frac{1}{2}\frac{-\bar{q}(t+\alpha) - \alpha(p_1^i - p_1^j) - t(p_2^i - p_2^j)}{t^2 - \alpha^2}] \\
&- \frac{1}{2}\tau P_i^2]
\end{aligned}
$$

Platforms choose $p_1^i, p_1^j, p_2^i, p_2^j$ to maximize profits above. From F.O.C., we get:
$$p_{ss1}^i = p_{ss2}^i = p_{ss1}^j = p_{ss2}^j = f + t - \alpha$$

$$
\begin{aligned}
\pi_{sf}^i &= (p_1^i - f)[\frac{1}{2} + \frac{1}{2}\frac{\bar{q}(t+\alpha) - t(p_1^i - p_1^j) - \alpha(p_2^i - p_2^j)}{t^2 - \alpha^2}] \\
&+ (p_2^i - f)[\frac{1}{2} + \frac{1}{2}\frac{\bar{q}(t+\alpha) - \alpha(p_1^i - p_1^j) - t(p_2^i - p_2^j)}{t^2 - \alpha^2}] \\
&- \frac{1}{2}\tau P_i^2
\end{aligned}
$$

The first order condition for profit maximization on p_1^i, p_1^j, p_2^i and p_2^j is then:

$$p_{sf1}^i = p_{sf2}^i = f + t - \alpha + \frac{1}{3}\overline{q}$$

$$p_{sf1}^j = p_{sf2}^j = f + t - \alpha - \frac{1}{3}\overline{q}$$

$$\pi_{fs}^i = (p_1^i - f)[\frac{1}{2} + \frac{1}{2}\frac{-\overline{q}(t + \alpha) - t(p_1^i - p_1^j) - \alpha(p_2^i - p_2^j)}{t^2 - \alpha^2}]$$
$$+ (p_2^i - f)[\frac{1}{2} + \frac{1}{2}\frac{-\overline{q}(t + \alpha) - \alpha(p_1^i - p_1^j) - t(p_2^i - p_2^j)}{t^2 - \alpha^2}]$$
$$- \frac{1}{2}\tau P_i^2$$

Platforms choose $p_1^i, p_1^j, p_2^i, p_2^j$ to maximize profits above. From F.O.C., we get:

$$p_{sf1}^i = p_{sf2}^i = f + t - \alpha - \frac{1}{3}\overline{q}$$

$$p_{sf1}^j = p_{sf2}^j = f + t - \alpha + \frac{1}{3}\overline{q}$$

$$\pi_{ff}^i = (p_1^i - f)[\frac{1}{2} + \frac{1}{2}\frac{-t(p_1^i - p_1^j) - \alpha(p_2^i - p_2^j)}{t^2 - \alpha^2}]$$
$$+ (p_2^i - f)[\frac{1}{2} + \frac{1}{2}\frac{-\alpha(p_1^i - p_1^j) - t(p_2^i - p_2^j)}{t^2 - \alpha^2}]$$
$$- \frac{1}{2}\tau P_i^2$$

Platforms choose $p_1^i, p_1^j, p_2^i, p_2^j$ to maximize profits above. From F.O.C., we get:
$p_{ff1}^i = p_{ff2}^i = p_{ff1}^j = p_{ff2}^j = f + t - \alpha$
Platform i's profit is

$$\pi_T^i = P_i P_j \pi_{ss}^i + P_i(1 - P_j)\pi_{sf}^i + (1 - P_i)P_j \pi_{fs}^i + (1 - P_i)(1 - P_j)\pi_{ff}^i \qquad (132)$$

Platform j's profit is

$$\pi_T^j = P_j P_i \pi_{ss}^j + P_j(1 - P_i)\pi_{sf}^j + (1 - P_j)P_i \pi_{fs}^j + (1 - P_j)(1 - P_i)\pi_{ff}^j \qquad (133)$$

The first order condition to equation (132) on P_i and the first order condition to equation (133) on P_j, the probability in the symmetric equilibrium satisfies:

$$P_i^* = P_j^* = \frac{\overline{q}^2 + 6\overline{q}(t - \alpha)}{\overline{q}^2 + 9\tau(t - \alpha)} \qquad (134)$$

The maximum profit of platform A and B with innovation are:

$$\pi_T^{i*} = \pi_T^{j*} = \frac{2\overline{q}^6 - 27\overline{q}^4(t - \alpha)(2t - 2\alpha - \tau) + 108q^3(t - \alpha)^2\tau + 1458(t - \alpha)^4\tau^2}{18(t - \alpha)(\overline{q}^2 + 9(t - \alpha)\tau)^2}$$
$$(135)$$

From the mechanism represented above, platforms both have the possibility to undertake or not undertake the R & D, but investing in R & D is always the dominant strategy in an asymmetric equilibrium. The incentive for investing in R & D is determined by the different expected profit if one doesn't do the R & D given the other platform does, or if one do the R & D given the other platform doesn't.

In equilibrium, platforms invest the same amount of money in R & D. They will have the same probability of success to reach a higher level of quality $\bar{q}$ and same expected equilibrium profit, but only one platform succeeds and dominates the market. Demand then won't be split equally in the end. Innovation increases agents connected to platforms total surplus.

1.6. **Conclusion.**

This chapter has asked whether platforms have incentive to innovate in a two sided market and how all agents share the benefit of innovation. We study individual values and social choice by comparing different cases.

We found that, in the context of the models used, a platform always has incentives to undertake R & D. In a symmetric equilibrium of monopoly platform, agents of group 1 & 2 and a platform are all better off from innovation, but from a social point of view, underinvestment problem exists. In a symmetric non-tournament innovation model of competing platforms, the duopolists face a prisoner's dilemma equilibrium. One platform invest more, the other platform will invest more. They end up with same quality level. Both the providers of platforms would be better off without innovation but they always have precisely the same unilateral incentive to increase quality in a two sided market because of platforms' innovation competition. Same as the case of monopoly platform, agents of group 1 & 2 are better off from innovation.

There is a limitation to the present analysis of two-sided single-homing duopoly model. The market is assumed fully covered. Therefore, innovation has no effect on the total number of agents who connect to platforms. The competition between platforms only leads to the demand from one platform to the other. If

instead, the market is not fully covered, it will be interesting to check how innovation incentives change a platform's performance.

The extension part is for future work. It would be interesting to check the models above if there is any new source of market failure when markets are two sides.

2. A NOTE ON THE ARMSTRONG (2006) MONOPOLY PLATFORM MODEL

Abstract. We show that the set of assumption in Armstrong (2006) monopoly platform model is not sufficient to guarantee the existence of an interior equilibrium with positive demands. We illustrate this problem within a linear Hotelling demand system fully consistent with Armstrong (2006) general assumptions. We show that the only possible equilibria are corner equilibria either with zero connections or with full coverage of both market sides, whereby Armstrong (2006) monopoly pricing rule does not apply. We then show that the problem can be fixed by adding an intrinsic value component of platform connections in Armstrong (2006) specification of agents' preferences.

2.1. **Introduction.**

Armstrong (2006) presented three models of two-sided markets: a model of monopoly platform, a model of competing platforms with single-homed agents only, and a model of 'competitive bottlenecks' with a group of multi-homed agents. The paper greatly improves our understanding of how the external benefit to the other side of group and the relative price elasticity of demand on the two sides of platform determine the platform optimal pricing choices.

However, we find that the set of assumption in Armstrong's monopoly platform model is not sufficient to guarantee the existence of an interior equilibrium with positive demands. To illustrate this point, we use a linear Hotelling specification of the demand system fully consistent with Armstrong (2006) general assumptions. We show that the only equilibrium with positive demands is a corner equilibrium in which the market is fully covered. This is particularly important since the main results of Armstrong's paper, and prominently the optimal pricing strategies of a monopoly platform on the two sides of its market, refers to interior equilibria with positive demands.

We then show that the problem can be fixed by assuming that, alongside the positive network effect of Armstrong (2006)'s utility formulation, platform connections also generate direct intrinsic value to the agents. More precisely, an equilibrium with positive demands become possible provided that connection's intrinsic value is great enough. Armstrong (2006) states that the profit-maximizing prices are equal to the connection cost adjusted downward by the external benefit to the other group and upward by the relative price elasticity of demand. We find that the intrinsic value doesn't change the conclusion. Clearly, the optimal price on any side of the platform is positively affected by the intrinsic value of connection to the agent connected on that side.

The rest of the paper is organized as follows. In section 2, we present Armstrong (2006) monopoly model. In section 3, we illustrate the problem. In section 4, we show extension with intrinsic value of connections. Section 5 offers some concluding remarks.

2.2. **Armstrong (2006) monopoly platform model.**

Three kinds of agents participate into this two-sided market: a single provider of a platform offering unit-connections to two groups of agents, group 1 and group 2. Hereinafter $l = [1, 2]$ will denote a generic group of agents. Connecting to the platform any agent of group l carries a connection cost $f_l \geq 0$ to the monopolistic provider of the platform.

Each agent in group l (or equivalently, each agent on the l side of the market), is potentially interested in establishing a single connection to the group of agents connected on the other side through the platform.

Following Armstrong's notation, let n_l be the numbers of agents of group l connected to the platform, and p_l be the price the platform charges any single connection on that side of the market. Then, in the model, the net utility any agent of group 1 would derive from connecting to the platform is specified as:

$$u_1 = \alpha_1 n_2 - p_1 \tag{136}$$

In equation (136), $\alpha_1 > 0$ measures the intensity of the benefit each agent of group 1 would derive from accessing the other side of the market through the platform. The gross utility the agent would gain, $\alpha_1 n_2$, linearly increases with the number of agents connected on the other side of the market, n_2. Subtracting the connection price, p_1, from the gross utility we finally obtain the agent's net utility. Similarly, for any agent of group 2 we have:

$$u_2 = \alpha_2 n_1 - p_2 \tag{137}$$

To close the model, Armstrong (2006) assumes that the number of connected agents on the l-side of the market, n_l, is a generic monotonically increasing function of net utility u_l:

Assumption 1. *(Armstrong (2006) Demand System)*

$$n_l = \phi_l(u_l), \ \ \phi_l'(u_l) > 0, \ \ l = 1, 2 \tag{A1}$$

Finally, the platform collects profit $p_l - f_l$ per unit of transaction on side l. Therefore, the platform's total profit is:

$$\pi = n_1(p_1 - f_1) + n_2(p_2 - f_2) \tag{138}$$

Armstrong (2006) finds that the profit maximizing price for one group is negatively related to the external benefit to the other group and positively related to a factor related to the elasticity of the group's participation. The profit maximizing prices satisfy:

$$\begin{cases} p_1 = f_1 - \alpha_2 n_2 + \dfrac{\phi_1(u_1)}{\phi_1'(u_1)} \\ p_2 = f_2 - \alpha_1 n_1 + \dfrac{\phi_2(u_2)}{\phi_2'(u_2)} \end{cases} \tag{139}$$

2.3. **The problem.**

In this section, we show that the set of assumption in Armstrong's monopoly platform model, as presented in the previous section, is not sufficient to guarantee

the existence of an equilibrium with positive demands. To illustrate the problem, we consider a linear specification of the demand system which is fully consistent with $(A1)$.

We specify each demand side of the platform market as in a standard linear Hotelling model. Group l consists of a unit mass of agents distributed along a unit interval $[0, 1]$. Each agent faces the same unit-transportation cost $t > 0$ to reach the platform, which is located at point zero for each group.

An agent of group l, located at distance x from the platform, will face a total transportation cost xt to reach and connect to the platform.

This total transportation cost must be subtracted from the net utility u_l (as defined in equation (136) and (137)) to obtain the agent's net surplus from reaching and connecting to the platform:

$$U_l = u_l - tx_l$$

The marginal agent of group l, indifferent between connecting or not connecting to the platform, is therefore identified by a location $\breve{x}_l$ such that:

$$U_l = u_l - t\breve{x}_l = 0$$

The mass of agent of group l who derive positive surplus and hence connect to the platform, n_l, is equal to $\breve{x}_l$. We finally state:

Assumption 2. *(Linear Specification of Armstrong (2006) Demand System)*

$$n_l = \breve{x}_l = \frac{u_l}{t}, l = 1, 2 \qquad (A2)$$

$(A2)$ is clearly fully consistent with $(A1)$. It just specifies function $\phi_l(u_l)$ as a monotonically increasing linear function of u_l. We now explicitly derive the linear demand system implicitly defined by $(A2)$, (136) and (137). Substituting (136) and (137) in $(A2)$, we get:

$$n_1 t = \alpha_1 n_2 - p_1 \qquad (140)$$

$$n_2 t = \alpha_2 n_1 - p_2 \qquad (141)$$

Solving the system in n_1 and n_2, we finally obtain:

$$n_1 = -\frac{tp_1 + p_2\alpha_1}{t^2 - \alpha_1\alpha_2} \qquad (142)$$

$$n_2 = -\frac{tp_2 + p_1\alpha_2}{t^2 - \alpha_1\alpha_2} \qquad (143)$$

Using (142) and (143), we next derive the platform's profit functions:

$$\pi = n_1(p_1 - f_1) + n_2(p_2 - f_2)$$
$$= -\frac{tp_1 + p_2\alpha_1}{t^2 - \alpha_1\alpha_2}(p_1 - f_1) - \frac{tp_2 + p_1\alpha_2}{t^2 - \alpha_1\alpha_2}(p_2 - f_2) \qquad (144)$$

The platform will choose p_1, p_2 to maximize profit (144). We are looking for an interior equilibrium (i.e., no side of the platform market is fully covered) where demands are not negative. From the F.O.C., we find:

$$p_1 = \frac{tf_2(-\alpha_1 + \alpha_2) + f_1(2t^2 - \alpha_1^2 - \alpha_1\alpha_2)}{4t^2 - (\alpha_1 + \alpha_2)^2} \tag{145}$$

$$p_2 = \frac{tf_1(\alpha_1 - \alpha_2) + f_2(2t^2 - \alpha_2^2 - \alpha_1\alpha_2)}{4t^2 - (\alpha_1 + \alpha_2)^2} \tag{146}$$

The Second Order Condition for an interior equilibrium requires[3]:

$$2t > \alpha_1 + \alpha_2 \tag{147}$$

Substituting the optimal prices (145) and (146) into the profit function (144), we get:

$$\pi = \frac{t(f_1^2 + f_2^2) + f_1f_2(\alpha_1 + \alpha_2)}{4t^2 - (\alpha_1 + \alpha_2)^2} \tag{148}$$

On the other hand, substituting (145) and (146) to (142) and (143), we have,

$$n_1 = \frac{-2tf_1 - f_2(\alpha_1 + \alpha_2)}{4t^2 - (\alpha_1 + \alpha_2)^2} \tag{149}$$

$$n_2 = \frac{-2tf_2 - f_1(\alpha_1 + \alpha_2)}{4t^2 - (\alpha_1 + \alpha_2)^2} \tag{150}$$

Second Order Condition (147) implies that $2t$ is greater than $\alpha_1 + \alpha_2$ for an interior equilibrium. Then the profit in (148) is positive, but the equilibrium quantities in (149) and (150) are negative showing that an interior equilibrium does not exist.

Especially when the inequality (147) holds, the only possible equilibrium is a corner equilibrium with no connections whatsoever to the platform and hence zero profits (the proof is in the appendix (C)). When the condition (147) is reversed, two different corner solutions are possible depending on the parameter values. We can still have the corner solution with zero connections and zero profits, to the platform, or jump to a corner solution, whereby both market sides are fully covered, so that the platform collects profit $\pi = \alpha_1 + \alpha_2 - 2t - (f_1 + f_2)$(again the proof is in the appendix (C)). Clearly, in both corner equilibria above Armstrong (2006) optimal pricing does not apply.

2.4. **Intrinsic value.**

Then we show how to fix the problem of our specification $(A2)$. We add an item $V > 0$ into utility functions (136) and (137). $V > 0$ is an intrinsic value that

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[3]The detailed calculations are in the appendix (A).

an agent of group l receives from participating in a platform. We could assume different intrinsic values in different sides of a platform, but we simplify agents' preferences by assuming a same intrinsic value, to avoid distracting us from the analysis of the equilibrium demands, the main objective of this paper. Then, the new utility functions of group l 's agent are:

$$u_1 = V + \alpha_1 n_2 - p_1 \tag{151}$$

The utility of group 2 's agent is

$$u_2 = V + \alpha_2 n_1 - p_2 \tag{152}$$

Substitute (140) to (151) and (152), we have,

$$n_1 = \frac{tV + \alpha_1 V}{t^2 - \alpha_1\alpha_2} - \frac{tp_1 + \alpha_1 p_2}{t^2 - \alpha_1\alpha_2} \tag{153}$$

$$n_2 = \frac{tV + \alpha_2 V}{t^2 - \alpha_1\alpha_2} - \frac{tp_2 + \alpha_2 p_1}{t^2 - \alpha_1\alpha_2} \tag{154}$$

Using (153) and (154), we next derive the platform's profit function:

$$\pi = n_1(p_1 - f_1) + n_2(p_2 - f_2)$$
$$= (\frac{tV + \alpha_1 V}{t^2 - \alpha_1\alpha_2} - \frac{tp_1 + \alpha_1 p_2}{t^2 - \alpha_1\alpha_2})(p_1 - f_1) + (\frac{tV + \alpha_2 V}{t^2 - \alpha_1\alpha_2} - \frac{tp_2 + \alpha_2 p_1}{t^2 - \alpha_1\alpha_2})(p_2 - f_2) \tag{155}$$

The platform will choose p_1, p_2 to maximize profit (155). First order conditions define the optimal prices for profit maximization.

$$p_1^* = \frac{V(2t^2 - t\alpha_2 + t\alpha_1 - \alpha_1\alpha_2 - \alpha_2^2) + tf_2(\alpha_2 - \alpha_1) + f_1(2t^2 - \alpha_1^2 - \alpha_1\alpha_2)}{4t^2 - (\alpha_1 + \alpha_2)^2} \tag{156}$$

$$p_2^* = \frac{V(2t^2 + t\alpha_2 - t\alpha_1 - \alpha_1\alpha_2 - \alpha_1^2) + tf_1(-\alpha_2 + \alpha_1) + f_2(2t^2 - \alpha_2^2 - \alpha_1\alpha_2)}{4t^2 - (\alpha_1 + \alpha_2)^2} \tag{157}$$

The second order condition is still required [4]:

$$2t > \alpha_1 + \alpha_2 \tag{158}$$

Substitute (156) and (157) to (153) and (154), we have,

$$n_1^* = \frac{2tV - 2tf_1 + V\alpha_1 - f_2\alpha_1 + V\alpha_2 - f_2\alpha_2}{4t^2 - (\alpha_1 + \alpha_2)^2}$$
$$= \frac{V(2t + \alpha_1 + \alpha_2)}{4t^2 - (\alpha_1 + \alpha_2)^2} + \frac{-2tf_1 - f_2\alpha_1 - f_2\alpha_2}{4t^2 - (\alpha_1 + \alpha_2)^2} \tag{159}$$

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[4]The detailed calculations are in the appendix (B).

$$n_2^* = \frac{2tV - 2tf_2 + V\alpha_1 - f_1\alpha_1 + V\alpha_2 - f_1\alpha_2}{4t^2 - (\alpha_1 + \alpha_2)^2}$$

$$= \frac{V(2t + \alpha_1 + \alpha_2)}{4t^2 - (\alpha_1 + \alpha_2)^2} + \frac{-2tf_2 - f_1\alpha_1 - f_1\alpha_2}{4t^2 - (\alpha_1 + \alpha_2)^2} \tag{160}$$

Now we can have an interior equilibrium with strictly positive demands, specifically, this is the case if and only if:

$$f_1 > f_2, \frac{2tf_1 + f_2\alpha_1 + f_2\alpha_2}{2t + \alpha_1 + \alpha_2} < V < \frac{4t^2 + 2tf_2 + f_1\alpha_1 - \alpha_1^2 + f_1\alpha_2 - 2\alpha_1\alpha_2 - \alpha_2^2}{2t + \alpha_1 + \alpha_2} \tag{161}$$

$$OR$$

$$f_1 \le f_2, \frac{2tf_2 + f_1\alpha_1 + f_1\alpha_2}{2t + \alpha_1 + \alpha_2} < V < \frac{4t^2 + 2tf_1 + f_2\alpha_1 - \alpha_1^2 + f_2\alpha_2 - 2\alpha_1\alpha_2 - \alpha_2^2}{2t + \alpha_1 + \alpha_2} \tag{162}$$

The intuition of these conditions can easily be gained by considering the symmetric costs case, $f_2 = f_1 = f$. In this case, conditions (161) and (162) are reduced to $f < V < f + 2t - \alpha_1 - \alpha_2$. V must be sufficiently high for an equilibrium with positive demand and output to exist, but not too high to avoid a corner equilibrium where the market is fully covered. If an intrinsic value is lower than the unit cost of connections, the only possible equilibrium is at the corner where demand and output are zero. If an intrinsic value is greater than a connection cost plus the sum of the utilities of these two consumers, an interior equilibrium doesn't exist, the only possible equilibrium is corner equilibrium in which the market is fully covered, and the platform collects profit $\pi = 2V + \alpha_1 + \alpha_2 - 2t - 2f$.

We transform (145) and (146) into following forms:

$$\begin{cases} p_1^* = f_1 - \alpha_2 n_2 + \dfrac{t(2tf_1 + f_2(\alpha_1 + \alpha_2))}{-4t^2 + \alpha_1^2 + 2\alpha_1\alpha_2 + \alpha_2^2} \\ p_2^* = f_2 - \alpha_1 n_1 + \dfrac{t(2tf_2 + f_1(\alpha_1 + \alpha_2))}{-4t^2 + \alpha_1^2 + 2\alpha_1\alpha_2 + \alpha_2^2} \end{cases} \tag{163}$$

We transform (156) and (157) into following forms:

$$\begin{cases} p_1^* = f_1 - \alpha_2 n_2 + \dfrac{t(2tf_1 + f_2(\alpha_1 + \alpha_2))}{-4t^2 + \alpha_1^2 + 2\alpha_1\alpha_2 + \alpha_2^2} + \dfrac{tV}{2t - \alpha_1 - \alpha_2} \\ p_2^* = f_2 - \alpha_1 n_1 + \dfrac{t(2tf_2 + f_1(\alpha_1 + \alpha_2))}{-4t^2 + \alpha_1^2 + 2\alpha_1\alpha_2 + \alpha_2^2} + \dfrac{tV}{2t - \alpha_1 - \alpha_2} \end{cases} \tag{164}$$

Armstrong (2006) shows the optimal prices in a quite general function forms as (139). When he worked with general function forms, he didn't find the optimal demands are negative. By using a linear Hotelling model, we show the optimal prices in a more specific way, that are (163) and (164). An intrinsic value is needed to be great enough to secure positive demands in equilibrium, and

the optimal prices are also impacted by an intrinsic value in positive way. Armstrong (2006) concludes that the profit-maximizing pries are equal to the connection cost adjusted downward by the external benefit to the other group, adjusted upward by the relative price elasticity of demand. We find that the intrinsic value doesn't change Armstrong (2006) conclusion, furthermore, the optimal prices are increased by a factor related to direct intrinsic value.

2.5. **Conclusion.**

In this paper, we explain that Armstrong (2006) assumption leads to negative optimal demanding quantities which not consistently satisfy a monopoly platform's demand relationship in an economic way. Over 2000 studies cited Armstrong (2006), however, they didn't find and discuss the problem in Armstrong (2006) model. Economides and Tag (2012) use similar specification as ours. They focus on studying net neutrality on the internet.

We illustrate and fix the problem by a standard linear Hotelling model which is very specific and fully consistent with Armstrong (2006) general assumptions. When we work with a specific formulation, the problem appears straightforwardly. It would be interesting to check whether the problem can be shown and solved in a general form.

3. Quality Uncertainty, Uninformative Advertising and Intermediation Margins

Abstract. This chapter works two contributions. First, we modify Milgrom and Roberts (1986) model of price and advertising as signals of quality, which allows us to focus just on equilibria with advertising signals. Second, we develop a model of intermediation as quality certification and we compare intermediation and advertisement as alternative ways to overcome informational asymmetries on the quality of new experience goods with our simplified model. We show conditions under which a high quality producer would prefer intermediation than advertisement to convey quality information to the market.

3.1. **Introduction.**

Crucial aspects of experience goods cannot be verified except through use of the product. In such circumstances, a seller's claims to be offering high quality is unverifiable before purchase and consumers face the risk of buying 'a lemon'. Intermediation may provide a solution to this problem. Thanks to their expertise and reputation in selling goods, intermediaries can credibly certify / guarantee quality to consumers. Part of the economic literature on intermediation focuses on this "screening" role of quality (e.g., Lizzeri (1999); Dasgupta and Mondria (2012)).

Alternatively, firms can signal quality through advertising or prices. The fact that money is spent to advertise an experience good could be a signal to customers that the good is of high quality, even if advertising does not by themselves have much direct informational content. Since Milgrom and Roberts (1986), research has focused on both strategic advertising and price decisions of firms as ways to signal quality and impact on customers' choices.

In this paper, we compare intermediation and advertising as alternative ways of providing the information of product quality of new experience goods. Firstly, we draw some ideas from Nelson and simplify Milgrom and Roberts's model. We present a two-period Bayesian model in which the producer of a new experience good can resort to uninformative advisement to signal quality. More specifically, the model modifies Milgrom and Roberts (1986)'s model in such a way that price alone is not efficient as a signal of quality, but prices and advertising as signals jointly can be more efficient. In this setting, we analyse the possibility of separating, pooling and hybrid equilibrium. Our focus is on the scope of intermediation so that we always select the equilibrium in the advertising model which maximizes the producer profit.

We present a model of intermediation. A producer and an intermediary play a bargaining game and reach a Nash co-operative agreement. In this setting, intermediation equilibria can conditionally improve signalling equilibria. Low cost per transaction of an intermediary plays key role in the bargaining game. Meanwhile, reputation costs create the incentive to tell the truth for an intermediary. We demonstrate that when there is a large price gap between low and high-quality good, a producer with high quality goods is more willing to sell products to an intermediary. Larger price gap requires higher advertising cost to signal quality while intermediation requires relative less cost.

In our setting, intermediation is not always a choice of separating a producer's type. A low-quality producer would never be interested in selling products to an intermediary. An intermediary is not allowed to charge a higher price than

a low-quality firm who has direct contact with consumers; therefore there is no bargaining surplus for an intermediation to join the game. Conditions are determined under which an intermediary is most likely to be active in a market to reveal the true quality of a product. Our results focus on the initial equilibrium choices of prices, advertising levels and intermediation agreements.

We contribute to both literature on signalling and intermediation. Our contribution to literature on signalling is to develop a simpler and more tractable model. In this way, we get straightforward benchmarks and less equilibria. This model is used to compare advertising with intermediation. Our contribution to literature on intermediation is to address the potential benefits of intermediation where intermediation is with quality certification. We supplement why some producers choose to introduce a new experience product by advertising, while some other producers choose to separate their types through intermediaries.

The remainder of the paper is organized as follows. Section 2 reviews literatures. Section 3 presents signalling model. Signalling equilibria are analysed in Section 4. Section 5 is devoted to analyse market equilibrium with intermediation. Comparative Statics is in section 6. Section 7 discusses an intermediary's reputation. Section 8 analyses equilibria under a different assumption. Section 9 concludes.

3.2. **Literature.**

Nelson (1970, 1974) was the first to investigate the relationship between quality and the use of non-informative advertising. He suggested that advertising may signal quality, and recognized that price signalling may also occur. However, Nelson didn't propose a formal model. Milgrom and Roberts (1986) were the first to formalize Nelson; Nelson's fundamental insight. In their model, whether advertising or price is used as signal of quality depends on how costly it is to produce quality.

Milgrom and Roberts (1986) demonstrate that, under certain circumstances, price alone is enough to signal quality. When a new high-quality product is very expensive to produce and is aimed at a limited market. A high-quality producer uses high price to signal quality, and a low-quality producer uses low price. A low-quality producer doesn't want to mimic a high quality producer because that would cause a low-quality producer to lose future demand. When a new high-quality product is cheap to produce and is aimed at a mass market, the introducing firm may set a low initial price and this by itself is a signal of high quality. If a low quality producer mimics high quality producer, a low-quality producer

whose product is expensive to produce will face a loss. In either case, no advertising is undertaken. In our framework, price alone is not efficient as a signalling instrument to separate product quality. There are two types of consumers with preferences constructed in such a way that quantity demanded changes little as price changes. If without advertising as a signal of product quality, a low quality producer would always gain more from mimicking a high quality producer unless a high-quality producer sets the intoduction price at a very low level.

Ellingsen (1997) shows a model in which price signals quality. Our model has a similar setting like his, but has the second period. He find that separating equilibria do exist, allowing some trade of high quality products even when the average quality is low. In our model, separating equilibria do exist. We rule out price signaling because price signaling alone is not efficient compared with advertising signal.

Milgrom and Roberts (1986) state that there exists a separating equilibrium under some parameter region of the model where advertising occurs. When the unit cost of high-quality good is equal or not too much greater than that of low-quality good, there is a unique separating equilibrium and no pooling equilibrium. In our framework, both separating equilibria and pooling equilibria exist. We compare those equilibria . We find that under some parameter region, separating equilibrium Pareto dominates pooling equilibrium. Under some parameter region, pooling equilibrium Pareto dominates separating equilibrium.

Among the relevant literature are articles in which economic intuition related to advertising signalling is argued,such as Bagwell and Riordan (1991) and Marvel and McCafferty (1984). They state that, without demand effects, price signalling is superior to advertising signalling. However a low-quality producer always has an interest in mimicking a high-quality producer, adding advertising to price signalling might increase profit of the signalling firm. These results correspond to some of our findings in section 4. Moraga-Gonzàlez (2000) and Bagwell and Ramey (1988) argue that when advertising occurs in equilibrium, the adverse selection problem is mitigated. Furthermore, the lower advertising costs are, the more intense the alleviation of that problem is. Schmalensee (1978), Barigozzi, Garella, and Peitz (2009), Cho/Kreps:1987 and Kaya:2009 deal with the presence of advertising as a signalling mechanism in a new product launch. How pricing mechanisms solve quality uncertainty problem is also featured in the models of Bester (1998),Bagwell (1987) and Bagwell and Riordan (1991). Empirical study has been provided by Horstmann and MacDonald (2003), Thomas, Shane, and Weigelt (1998). These analyses apply more widely.

The other strand of the literature related to our paper is the one that studies the role of intermediation in international trade. Intermediaries play a productive

role in the economy. Evidence has been provided that trade through intermediaries, like wholesalers and exporters, constitutes a significant proportion of international transactions. Intermediary firms handle about 22 percent of aggregate Chinese export sales (Ahn, Khandelwal, and Wei, 2011), 10 percent of US and Italian exports (Blum, Claro, and Horstmann, 2010), 20 percent of French exports (Crozet, Lalanne, and Poncet, 2013). Economists have begun exploring the role that intermediaries play in facilitating trade.

Trade intermediaries can mitigate credibility problem. Reputation acquisition by intermediaries can provide quality assurance. In some papers, the role of reputation acquisition is modeled in enabling an intermediary to act as a producer of credible information. Papers that theoretically study intermediaries' reputation include Dasgupta and Mondria (2012), Biglaiser (1993), Lizzeri (1999) and Chemmanur and Fulghieri (1994). When the qulity is uncertain, by screening the quality of products and then revealing it to consumers, intermediaries help to alleviate the producer asymmetric information problem. Producers exporting through intermediaries need not incur advertising cost to signal their products' quality, but effectively end up paying a certification fee. Dasgupta and Mondria (2012)'s work contributes to evaluate the effect of intermediaries on the producer's decision to export, as well as, the effect on quantity, price and average quality. Biglaiser (1993) studies whether middlemen can improve welfare. The environment he studies is different from mine. Advertising is ruled out to signal quality in his model. He allows sellers to signal quality only by waiting to trade. Other possible ways to signal are not effective than signalling through delay. Lizzeri (1999) studies how competition impacts intermediation's full revelation. Chemmanur and Fulghieri (1994) work on the role of financial intermediaries in the presence of adverse selection.

With an intermediary handling distribution, the producers' marketing expenses may be lowered. If an intermediary is more efficient at distribution than producers, this efficiency could justify the existence of the intermediary by itself. Petropoulou (2007), E. and Watson (2004),Caillaud and Jullien (2003) and Blum, Claro, and Horstmann (2010) argue that the equilibrium number of intermediaries increases as the quality of producers' own networks declines, as the cost to intermediaries of maintaining their networks falls, or as the bargaining power of intermediaries in negotiation with producers rises. In our case, we simplify that there are only two units of demand at most. An intermediary doesn't help to increase demand, but does help to decrease the marketing expense.

3.3. A Basic Model of Quality Uncertainty.

There are two kinds of agents: a producer and two consumers. A monopolistic producer introduces a new experience product. The product's quality q may be high (H) or low (L). The producer observes the true quality of a product. The producer is active for two periods and then exits the market. The cost of producing one unit of the high-quality good is $C > 0$, while the unit production cost of the low-quality good is normalized to zero. It is further assumed that (a) $H - C > L$ and (b) $L - C > 0$. [5]

We assume two types of consumers are representative of the market. For simplicity, there is one consumer of each type and they are referred to as Consumer 1 and Consumer 2. [6] The consumers' type is common knowledge. Consumers are assumed to be infinitely lived that is versus infinitely-lived intermediation. We assume that the two customers have heterogeneous valuation r for a product:

For Consumer 1,

$$r = \begin{cases} H & \text{if a high-quality good} \\ 0 & \text{if a low-quality good} \end{cases} \tag{165}$$

For Consumer 2,

$$r = \begin{cases} L & \text{if a high-quality good} \\ L & \text{if a low-quality good} \end{cases} \tag{166}$$

In words, Consumer 1 only cares about product quality while Consumer 2 doesn't care about product quality at all. On the demand side of the market, every consumer will, at most, purchase one unit of the product in every period. In any period in which each consumer purchases the good, the utility is $r - P_t(q)$. If a consumer makes no purchase, the utility is 0. Each consumer can learn true quality from a single purchase. They don't know the type of the producer but have commonly known prior beliefs: the new good can be high quality H with probability p or low quality L with probability $1 - p$. Consumers stick to these beliefs and the beliefs can only be revised only if the information set of consumers changes. Consumers don't share information with each other. [7]

3.3.1. *Strategic Variables.*

A firm's decision variables are the price, $P_t(q)$, $t = 1, 2$, and advertising level, $A(q)$. No discounting is considered. Advertising here has no direct impact on demand.

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[5]Assumption (a) means that producing the high-quality good is efficient. Assumption (b) rules out uninteresting cases by making it possible that a high-quality producer would mimic a low-quality producer.

[6]Although we could model a distribution of tastes among consumers, we chose not to as it would complicate the analysis and distract us from analysing the role of advertising and intermediation.

[7]To avoiding stating he or she each time for producers and consumers, 'he' is used for the producer and 'she' for the consumer.

3.3.2. *Timing and Strategy.*

In this chapter, our target is to study one producer's stategy of which a producer introduces his new product. This producer lives two periods. Even though comsumers and an intermediary are infinitely-lived and may also trade with other producers in this two periods or in other periods. Except this producer and this two periods, other producers and other periods are not in the field of study.

Formally, a producer play a two-period Bayesian signalling game. In the first period, the strategy of a producer is to choose a level of advertising to signal his product's quality and to announce an introduction price. Consumers don't know the type of a producer but have commonly known prior beliefs. They observe the prices charged by a producer and the advertising intensity, and then each consumer is able to make their initial purchase decisions based on their revised beliefs. After each consumer make their initial purchase decision, the consumer who buy a product can learn quality and the other consumer who doesn't buy a product sticks to the belief formed in the first period.

In the second period, a producer set the prices explored in an additional information of consumers 'preferences derived from the first period. The strategies of consumers are simply to *buy*, or *not buy* based on the new information about quality.

3.4. **Market Equilibrium with Signalling.**

There exist three logical possibilities for equilibria without intermediation which are separating equilibria, pooling equilibria and hybrid equilibria. In this section, we explore all these possible equilibria and select best equilibria by means of a Pareto dominance refinement criterion.

According to consumers' valuation function (165) and (166), a high-quality producer may set a high price at H if he prefers to sell only to the quality sensitive consumer than selling to both consumers. A high-quality producer may set a low price at L if he prefers to sell to both consumers than selling to the quality sensitive consumer only. The price of a high quality product being H or L depends on which price would bring more profit to a high-quality producer. Throughout this section, we assume a high-quality producer prefers to sell only to the quality sensitive consumer than selling to both consumers:

Assumption 3. $2(L - C) < H - C$

3.4.1. *Price Signals.* We first check whether it is possible that price signalling

alone separates product's quality. Although we could also consider mixed strategies since these have been extensively employed in the literature, we choose to restrict strategies to pure strategies for consumers as it won't compromise the basic conclusion of the chapter.

We will supplement the proof of mixed strategies in a future version. Consumer 1 mixes her strategies between buying and not buying whenever observing a price equal to H. Separating equilibria exist where type L sells to consumer 2 with probability 1 and type H sells to consumer 1 with probability less than one.

Lemma 4. *If allowing beliefs to be non-monotonic, the separating equilibria are possible and are the only possible equilibria where signalling happens through prices only.*

Proof. For separation to be successful, a producer would be to choose different prices for different product qualities. The producers' action space are two prices, H and L. With an equilibrium refinement, the prices won't be chosen outside the set. There are pure strategies' possibilities and mixed strategy:

P1: Consumers hold the following beliefs: if consumers observe $P_1 = H$, then they believe that a firm provides a high-quality good, whereas if they observe $P_1 = L$, then they believe that it is low quality. A producer has his strategy: he chooses $P_1(H) = H$ if the product is high quality, and chooses $P_1(L) = L$ if the product is low quality.

When a high-quality producer follow this strategy, the high-quality producer's first period net profit is $H - C$. Consumer 1 buys the product and make 0 net utility. In the second period, Consumer 1 is fully informed. The producer sets the price at $P_2(H) = H$. Consumer 1 buys the product again. The firm's two-period net profit is $2(H - C)$. If the high-quality producer deviates from the strategy, Consumer 1 won't buy the good and Consumer 2 buys the good instead. Then, two-period profit is $2(L - C)$. It is obvious that $2(H - C) > 2(L - C)$, therefore, the high-quality producer would stick to this strategy and choose $P_1(H) = H$ and $P_2(H) = H$ in order to maximize his profit.

When a low-quality producer follows this strategy, the low-quality producer's first period profit is L. Consumer 2 buys the product and make 0 net utility. In the second period, a low-quality producer charges $P_2(L) = L$, and his two-period profit is $2L$. If the firm deviates from this strategy, he chooses $P_1(L) = H$ in the first period, then Consumer 1 believes it is high quality and purchases the good in the first period, yielding a low-quality producer a first year profit of H. However, in the second period, a low-quality producer charges $P_2(L) = L$, Consumer 1 won't buy the product again after the quality is fully revealed, Consumer 2 buys

the product along with the price drops at L. The firm's two-period profit is $H + L$. Therefore, the low-quality producer will deviate from this strategy.

To summarize, any producer just captures one unit of demand in the first period, high and low type producers both prefer to capture the high type customer in the first period. They both prefer to price in high in the first period. In the second period, product quality is no long asymmetric information, and prices do not perform a signaling function. Therefore, price signals cannot separate a producer's type.

P2 : Assume consumers hold the following beliefs: if consumers observe $P_1 = L$, then they believe that a firm provides a high-quality good, whereas if they observe $P_1 = H$, then they believe that it is low quality. A producer has his strategy: he chooses $P_1(H) = L$ if the product is high quality, and chooses $P_1(L) = H$ if the product is low quality.

A high-quality producer would stick to this strategy in both periods. The firm's two-period profit is $2L - 2C + H - C$. He is worse off if he deviates from the strategy. If a low-quality producer sticks to this strategy, no one would buy his product. A low-quality producer would deviate from the strategy. He chooses $P_1(L) = L$ in the first period, then both consumers purchases his products in the first period, yielding a first year profit of $2L$. In the second period, he chooses $P_1(L) = L$ in the second period, Consumer 1 won't buy the product again after the quality is fully revealed, Consumer 2 buys the product. The firm's two-period profit is $3L$. A low quality producer would gain more from mimicking a high quality producer. Then, in the two periods, a producer sets the price at L whatever his product's quality is low or high. Therefore price alone can't separate a producer's type.

P3 : Assume consumers hold the following beliefs: if consumers observe $P_1 = L - \Delta$, then they believe that a firm provides a high-quality good, whereas if they observe $P_1 = L$, then they believe that it is low quality. A producer has his strategy: he chooses $P_1(H) = L - \Delta$ if the product is high quality, and chooses $P_1(L) = L$ if the product is low quality.

Suppose separating equilibria do exist. A high-quality producer would stick to this strategy. The firm's two-period profit is $2(L - \Delta - C + H - C$. The high-quality producer won't deviate if:

$2(L - \Delta - C + H - C \geq 2(L - C)$

$\Delta \leq \frac{H - C}{2}$

A low-quality producer would stick to this strategy. The firm's two-period profit is $2L$. The high-quality producer won't deviate if:

$2L \geq 2(L - \Delta) + L$

$\Delta \geq \frac{L}{2}$

Therefore, separating equilibria exist only if the following condition is satisfied:

$$\frac{L}{2} \leq \Delta \leq \frac{H - C}{2} \tag{167}$$

Given the assumption 1, any Δ satisfies condition (167) identifies a separating equilibrium. Since the condition (167) is always true, there exist an infinite number of continuous separating equilibriua. Therefore, if consumers play pure strategies, we can have seperating equilibiria. Price alone can separate a producer's type.

$\square$

The signalling model has many separating equilibria makes it difficult to make comparison. Since our focus is to find the optimal way to solve asymmetric information problem, we choose a separating equilibrium which provides the highest profit to a producer, that is, a high-quality producer chooses the lowest bound of Δ, which is $\Delta = \frac{L}{2}$, sets price at $\frac{L}{2}$. A low-quality producer sets price at L.

To sum up, I checked all pure strategy possibilities. Jullien and Mariotti2006suggested monotonic beliefs, in which there is a unique separating equilibrium but ruled out by refinement criteria. In our setting, monotonic beliefs do not support separating equilibrium. Price-only-signalling equilibrium is only supported by non-monotonic beliefs.

3.4.2. *Advertising Signals.*

Then, we choose the most efficient separating equilibria when there is an advertising signal. Firstly, a producer chooses an optimal level of advertising to signal his product's quality and announces an introduction price. After each consumer observes the advertising intensity, she makes the decision to purchase a product or not. If a consumer purchases a product, she learns true quality and revises her beliefs. If not, a consumer sticks to the beliefs formed before. Accordingly, consumers make next purchase desicions.

Lemma 5. *Under Assumption* (3), *there always exists separating equilibria.*

Proof. Suppose separating equilibria do exist. The advertising strategies of a producer to signal his quality would be to choose different advertising levels, where $A(H) \neq A(L)$. Suppose that consumers hold beliefs that there exists a cut-off

advertising level A^* such that:

$$if \begin{cases} A(q) < A^* & \text{the producer is of low quality with probability 1} \\ A(q) \geq A^* & \text{the producer is of high quality with probability 1} \end{cases} \quad (168)$$

Then, the producer's optimal strategy would be:

- to never choose $A(q) > A^*$ since $A(q) = A^*$ suffices to convince the consumers and increasing advertising expenditure further is costly;
- to choose $A(q) = A^*$ if the product is treated as high quality.
- to choose $A(q) = 0$ if the product is treated as low quality;

The consumer's beliefs in this assessment is: if a consumer observes $A(q) = A^*$, then she must believe that the firm provides a high-quality good, whereas if she observes $A(q) = 0$, then she must believe that it is low quality. Consumers stick to these beliefs. The beliefs can only be revised if consumers observe signals or consume the product.

For a high-quality producer, under Assumption 3, he sets the price and advertising pair at $(P_1(H) = H, A(H) = A^*)$. According to the level of advertising, consumers believe the new product is high quality. Consumer 1 buys the product. Consumer 1 is fully informed about the quality. Consumer 2 couldn't afford it and won't buy the product. The producer's first period net profit is $H - C - A^*$. In the second period, under Assumption 3, a high-quality producer charges $P_2(H) = H$. Consumer 1 will buy the product in the second period. Then, the firm's two-period net profit is $2(H - C) - A^*$. If the high-quality producer deviates from the strategy, which means he doesn't invest on advertising, Consumer 1 won't buy it any more no matter whether the price is set at H or L and Consumer 2 buys the good instead only if the price is L. Then, two-period profit is $2(L - C)$. So the high-quality producer will want to choose A^* and won't deviate if:

$2(H - C) - A^* \geq 2(L - C)$

$A^* \leq 2(H - L)$

For a low-quality producer, he sets the price and advertising pair at $(P_1(L) = L, A(H) = 0)$. Consumers believe it is low quality. Only Consumer 2 buys a product. The producer's first period net profit is L. In the second period, a low-quality producer charges $P_2(L) = L$. Since Consumer 1 will stick to the initial beliefs, she won't buy a product. Only Consumer 2 buys a product. Therefore, the firm's two-period profit is $2L$. If a low-quality producer deviates from this strategy, then he chooses $(P_1(L) = H, A(L) = A^*)$ in the first period. Consumer 1 believes it is high quality and purchases the good in the first period, yielding it a first year profit of $H - A^*$. Consumer 2 won't buy for she couldn't afford it. In the second period, a low-quality producer charges $P_2(L) = L$. Consumer 1 won't buy

the product again after the quality is fully revealed, Consumer 2 buys a product along with the price drops at L. The firm's two-period profit is $H - A^* + L$. So the low-quality producer won't deviate if:

$H - A^* + L \leq 2L$

$A^* \geq H - L$,

Therefore, separating equilibria exist only if the following condition is satisfied:

$$H - L \leq A^*_{SE} \leq 2(H - L) \tag{169}$$

Given the assumption 3, any advertising level satisfies condition (169) identifies a separating equilibrium. Since the condition (169) is always true, there exist an infinite number of continuous separating equilibria which the levels of advertisement are different. $\square$

The fact that this signalling model has many separating equilibria makes it difficult to use this model to make comparison with the equilibrium of intermediation. Since our focus is to find condition for intermediation to be optimal way to solve asymmetric information problem instead of advertisement, we choose a separating equilibrium which provides the highest profit to producers, that is, a producer choose the lowest bound of A^*_{SE}: $A^*_{SE} = H - L$.

Then, we consider the possibility of pooling equilibria. There always exist beliefs which can support pooling equilibria. Suppose consumers hold beliefs that there exists a cut-off advertising level A^* which is sufficient high. Consumers' supporting beliefs are: if a consumer observes A, she believe that the firm provides a high-quality good. Otherwise, she believes that the firm provides a high-quality good with probability p. :

$$if \begin{cases} A(q) \geq A^* & \text{the producer is of high quality with probability 1} \\ A(q) < A^* & \text{the producer is of high quality with probability p} \\ & \text{and of low quality with probability 1-p} \end{cases} \tag{170}$$

If a advertising level A^* is greater enough, both types of producer would prefer to choose pooling. Therefore, there always exist pooling equilibria. In order to access the scope for intermediation, we aim to select the pooling equilibria which maximizes a producer's profit. If pooling, there is a minimum level of advertising that no type of producer will send advertising signal, that is, $A(q) = 0$.

In pooling equilibria, different type of producer announces a same introduction price in the first period. A producer can either set the price at Consumer 1' s price expectation which is pH (see 201) or L. This generates sub-cases:

When pH is less than L, a high-quality producer sets the price at pH since this is the highest price Consumer 1 would accept. If he sets the price higher than

pH, Consumer 1 won't buy a product from him. A high-quality producer will lose Consumer 1 in the first period and second period. A low-quality producer mimics a high-quality producer's price strategy. Why? If a low-quality producer set the price at L, only Consumer 2 will buy a product. In this parameter's region, the profit of selling two products at price pH is greater than that of selling one product at price L.

When pH is greater than L and less than $2L - C$, a high-quality producer sets the price at L. A high-quality producer could gain more by selling Consumer 1 and Consumer 2 rather than selling to Consumer 1 only. A low-quality producer mimics a high-quality producer's price strategy. A low-quality producer set the price at L in the first period, Consumer 1 and 2 will buy 2 unit of products. A low-quality producer set the price at L in the second period, Consumer 2 will buy 1 unit of products.

When pH is greater than $2L - C$, a high-quality producer sets the price at pH. A high-quality producer could gain more by selling to Consumer 1 only. In the second period, product quality is informed. Consumers revise their beliefs. A high-quality producer sets the price at H, and a low-quality producer sets the price at L.

Lemma 6. *Under Assumption 3, if $pH > \frac{L+C}{2}$, pooling equilibria can Pareto dominate the separating equilibrium.*

The proof is provided in the appendix.

A third possible outcome is hybrid equilibrium. Different types of the producer put positive probability to send the same signal, but not all types put the same probability on all strategies. The details are left to the Appendix. This analysis is summarized in Lemma (7): .

Lemma 7. *Under Assumption 3, hybrid equilibria are always Pareto dominated.*

As stated above, we focus on the most efficient equilibrium. In this model specification, separating equilibria and pooling equilibria always exist. Firstly, we choose the most efficient separating equilibrium and the most efficient pooling equilibrium. We then select the most efficient equilibria. We find that separating equilibrium is Pareto dominant in some parameter space and pooling equilibria are Pareto dominant in some other spaces.

In the first period, if the expected price of consumer 1 is rather small, the separating equilibrium is Pareto dominant. A high-quality firm would rather spend on advertising to separate his type. The producer does so because if he does not, then the consumer believes it to be low quality, and does not find out that it might

be of high quality. A high-quality producer loses some future profit when he imitates a low-quality one. However, imitating a high-quality firm is not worthwhile for the low-quality one because the advertising cost is just too much which causes the profit of imitating less than the profit of no imitating. If the expected price of Consumer 1 is large enough, the pooling equilibrium is Pareto dominant. A producer is less motivated to introduce a high quality good through advertising. A high-quality producer would rather pool with a low-quality producer than separate his type. In the second period, the true type of a producer is revealed, consumers will have the full information about the product.

3.4.3. *The Optimal Bayesian equilibria.*

From the analysis above, we have the most efficient equilibrium when prices are as signals alone, and then we have the most efficient equilibria when advertising and prices are as signals jointly. Again, in this chapter, our focus is to select the equilibrium which maximizes the producer profit. We compare price-only-signalling equilibrium and advertising-signalling equilibria, we find that the equilibrium from price signalling is highly inefficient. The proof is left to the Appendix.

Lemma 8. *Under Assumption 3, price-only-signalling equilibria are always Pareto dominated.*

Proposition 8. *Under Assumption 3,*

(1) *if $pH \leq \frac{L+C}{2}$, a separating equilibrium is Pareto dominant which a high-quality producer sets his advertising level at $H - L$, just high enough for the low-quality producer not to mimic.*
(2) *if $pH > \frac{L+C}{2}$, pooling equilibria are Pareto-dominant.*

To sum up, in the advertising signalling model, the equilibria which maximize the producer profit have be chosen as below, we compare these results with intermediation in later section:

- If $pH \leq \frac{L+C}{2}$, pooling equilibria exist, but Pareto-dominated by separating equilibrium.

    In the first period, a high-quality producer sets the pair at $(P_1(H) = H, A(H) = H - L)$. According to the level of advertising, consumers believe the new product is high quality. Consumer 1 buys the product. Consumer 2 won't buy the product. A low-quality producer sets the pair at $(P_1(L) = L, A(L) = 0)$. For there is no advertising signal, consumers believe the new product is low quality. Consumer 1 won't buy the product

and Consumer 2 buys the product. In the second period, a high-quality producer sets $(P_2(H) = H$. Consumer 1 knows the true quality and buys it. A low-quality producer sets the pair at $P_2(L) = L$. Consumer 2 buys the product.

- If $\frac{L+C}{2} < pH \le L$, pooling equilibria exist and Pareto-dominate separating equilibrium.

  Different types of producer choose a same price in the first period. A producer sets the pair at $(P_1(H) = pH, A(H) = 0)$. Consumers don't know the quality. Both of them buy the products. In the second period, consumers know the true quality. A high-quality producer sets the price at $P_2(H) = H$, Consumer 1 buys a product. A low-quality producer sets the pair at $P_2(L) = L$. Consumer 2 buys a product.

- If $L < pH \le 2L - C$, pooling equilibria exist and Pareto-dominate separating equilibrium.

  In the first period, a producer sets the pair at $(P_1(H) = L, A(H) = 0)$. Consumers don't know the quality. Both of them buy the products. In the second period, consumers know the true quality. A high-quality producer sets the price at $P_2(H) = H$, Consumer 1 buys a product. A low-quality producer sets the pair at $P_2(L) = L$. Consumer 2 buys a product.

- If $pH > 2L - C$, pooling equilibria exist and Pareto-dominate separating equilibrium.

  In the first period, a producer sets the pair at $(P_1(H) = pH, A(H) = 0)$. Consumers don't know the quality. Consumer 1 buys a product. Consumer 2 won't buy for the price is too high. In the second period, Consumer 1 knows the true quality. A high-quality producer sets the price at $P_2(H) = H$, Consumer 1 buys a product. A low-quality producer sets the pair at $P_2(L) = L$. Consumer 2 buys a product.

3.5. **The Model with an Intermediary.**

An intermediary handles a variety of goods from several producers in a particular industry and receives no utility from consuming these goods. We assume that an intermediary is rational and infinitely lived. A discount factor is $\delta \in (0, 1)$. An intermediary has to rely on its reputation to sell goods. An intermediary could lie about its goods' quality. If an intermediary cheats, he loses sales on the defector's goods and loses sales on other products he carries.

A producer and an intermediary play a bargaining game. The bargaining parties bargain over the division of products' margin (Nash, 1950, 1953). An intermediary has no more bargaining power than ordinary buyers. An intermediary's

advantage over buyers is his ability to identify quality. He can inspect the good at a cost of $w > 0$. We assume that bargaining occurs before both the producer and the intermediary observe the quality but they can share the surplus in a way that depends on ex-post observed quality. A testing will be operated when they reach an agreement. If the bargain fails, they won't do the test.

In the beginning of the first period, a producer bargains with an intermediary. If the bargain fails, the producer goes back and introduce his new product by signalling. Consumers know the bargaining's fail, they know a new product would be introduced by a producer himself, and the product quality is still uncertain. If the bargain succeeds, a producer sells products to an intermediary at $P_I(q)$ and an intermediary sell the products to customers at $\tilde{P}_I(q)$. Then, consumers know the quality from the single purchase. After the first period, this producer doesn't need an intermediary since its quality is known to consumers. Therefore, when there is an agreement, a producer works with an intermediary one period only.

3.5.1. *The Market Equilibrium with Intermediation.*

In our model, an intermediary can verify the quality and let the consumer share the information. We make comparisons between signalling equilibria and intermediation. Proposition (9) states conditions under which a producer is better off if an intermediary is present. In this section, we assume that an intermediary always tells the truth about products' quality in order to maintain his reputation.

Proposition 9. *Under Assumption 3,*

(1) *in equilibrium, a low-quality producer always sells his good privately. Intermediation is only used by high quality producers.*
(2) *if $pH < \frac{L+C}{2}$, and $w < H - L$ holds, there exists an equilibrium where equilibrium with intermediation can Pareto-dominate signalling separating equilibrium.*
(3) *if $pH \geq \frac{L+C}{2}$, and $w < H - 2pH + C$ holds, there exist equilibria where equilibria with intermediation can Pareto-dominate pooling equilibria.*

Proposition (9) is derived in three steps:

Step 1: a low-quality producer always sells his good privately.

A producer sells products to an intermediary at $P_I(q)$. He will choose to sell products to an intermediary rather than to consumers by himself if and only if $P_I(L) \geq L$. If a low-quality producer sells his product to consumers by himself, he sells a product at the price L. Therefore, he would like to sell a product to an intermediary only if the price is not lower than L. An intermediary would like to buy the producer's good at the price $P_I(L)$ if and only if $L - P_I(L) > 0$,

which is $P_I(L) < L$. The highest price of a low-quality product is L. If an intermediary buy a product at the cost higher than L, an intermediary will face a loss. However, we find an apparent contradiction between the two conditions, which implies that the low-quality producer won't be better off to sell his products to an intermediary. Trading through an intermediary is too costly for a low-quality producer.

Step 2: If $pH < \frac{L+C}{2}$, there exists an equilibrium where intermediation can improve separating equilibrium as a way of separating a producer's type.

Assume a high-quality producer set the price at $P_I(H)$, he will choose to separate his type through an intermediary rather than through level of advertising in a separating equilibrium if and only if $P_I(H) - C > L - C$, it shows that $P_I(H)$ has to be greater than L. An intermediary accepts the price $P_I(H)$ if and only if $H - p_I(H) - w > 0$, which is $P_I(H) < H - w$. Thus, a high-quality producer sells products to an intermediary if and only if (171) is satisfied.

$$L < P_I(H) < H - w \tag{171}$$

Which means $L < H - w$, then we have $w < H - L$.

The producer and the intermediary have a common interest and reach a mutually beneficial agreement, but have a conflict of interest about which one to agree on. We assume that the producer gets utility u from the agreement, where $u = P_I(H) - C$, and the intermediary gets utility v, where $v = H - w - P_I(H)$. If there is no agreement, a producer goes back and separates his type through advertising. They get utility $u_0 = L - C, v_0 = 0$ respectively. Given these utilities, assume g(u,v) is the collective utility, which the collective utility function is given by $g(u, v) = (u - u_0)(v - v_0)$.

$$g(u, v) = (P_I(H) - L)(H - w - P_I(H)) \tag{172}$$

Nash demonstrated that there is a unique solution to a bargaining problem (Nash, 1950, 1953). The Nash bargaining solution is the value of $P_I(H)$ that maximizes 172. Differentiating with respect to $P_I(H)$ and setting the derivative equal to 0 gives:

$\frac{dg}{dP_I(H)} = H - w + L - 2P_I(H) = 0$

Solving this equation for $P_I(H)$ gives:

$$P_I^*(H) = \frac{(H - w + L)}{2} \tag{173}$$

This solution represents a situation that could not be improved on to both agents' advantage because rational participants would not accept a given agreement if some alternative agreement could make both parties be better off, or at least one

better off with the other no worse off. This is illustrated in figure 3, note that the Nash solution maximizes the area of the shaded rectangle.
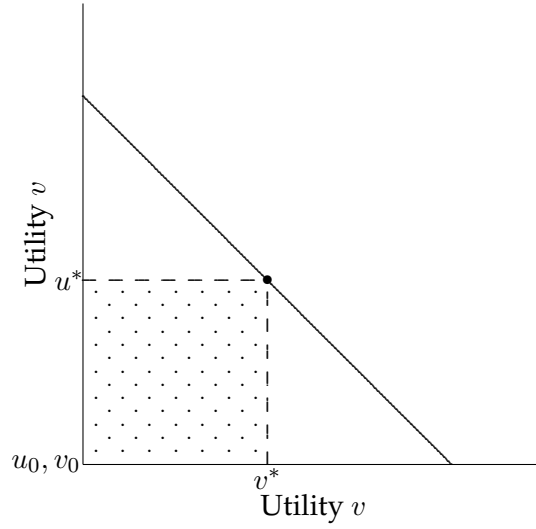

FIGURE 3. Nash Bargaining Solution

A producer compares the potential profit, and chooses the maximum. A high-quality producer trade through an intermediary at the price $\dfrac{(H - w + L)}{2}$. Consumer 1 buys the product from the intermediary at $\tilde{P}_I(H) = H$. Consumer 2 doesn't buy the product. After the first period, a producer doesn't need an intermediary once its quality is known to consumers.

Step 3: There exist equilibria where intermediation can improve pooling equilibria.

(1) If $\frac{L+C}{2} < pH \leq L$, and $w < H - 2pH + C$ holds, a high-quality producer is better off to sell a product to an intermediary at the price $\dfrac{(H + 2pH - C - w)}{2}$.

(2) If $L < pH \leq 2L - C$, and $w < H - 2L + C$ holds, a high-quality producer is better off to sell a product to an intermediary at the price $dfrac(H + 2L - C - w)2$.

(3) If $pH > 2L - C$, and $w < H - pH$ holds, a high-quality producer is better off to sell a product to an intermediary at the price $\dfrac{(H + pH - w)}{2}$.

What are the reasons behind these results? No matter how cheap the testing cost and the rent of an intermediary are, an intermediary can never sell a low-quality product higher than L because Consumer 2 never offers more than L, then there is no bargaining surplus between a low-quality producer and an intermediary. A low-quality producer is not willing to trade through an intermediary. Because of

the asymmetric information between a high-quality good and a consumer, a high-quality producer has to take a cost to reveal his type. A high-quality producer would like to choose a low-cost way to separate his type. When the testing cost of intermediation is lower enough, intermediation is always a dominating solution to introduce a new product.

3.6. **Comparative Statics.**

In this section, we demonstrate the basic analysis for the comparative statics with respect to the following parameters: p, C, H, L and w.

3.6.1. *A Change in p.*

Parameter p indicates the probability of a high-quality product. p, which is a key factor, impacts on consumers' valuations for the good and trading strategies a producer would choose. When the product quality is unknown, the higher the probability of a high-quality product is, the greater of the expected price Consumer 1 would accept. A high-quality producer would incur a lower foregone cost to separate himself from a low-quality producer.

When there is no intermediary, the higher the probability of a high-quality product is, the less motivated a high-quality producer becomes in advertising to separate himself from a low-quality producer, and the more motivated a high-quality producer becomes in pooling with a low-quality producer. When an intermediary joins the game, the higher the probability of a high-quality product is, an intermediary earn less profit from introducing a high-quality product.

3.6.2. *A Change in C, H, L.*

C indicates the cost of a high-quality product. The cost of a low-quality product is zero. Because of a high-quality good ' cost disadvantage, the greater C is, the high-quality producer has strong motivation to reveal his products' quality. Repeat purchases play a crucial role in our model. If no repeated game, With a low-quality good ' cost advantage, imitating a high-quality producer is always the best strategy for a low-quality producer, and a high-quality producer can never benefit from separating himself from a low-quality producer. a high-quality producer forego profit to attract an initial sale, in the second period, Consumer 1 know the qulity, she will buy a high-quality product again.

H indicates the valuation of a high-quality product which is the highest price Consumer 1 would accept. L indicates the valuation of a low-quality product which is the highest price Consumer 2 would accept. When there exist inefficiencies from information delays in trade between a high-quality good and a consumer, the larger of the gap between the parameters of the price H and L is,

the stronger motivated a low-quality producer becomes to imitate a high-quality producer, and a high-quality producer has to introduce his new product with a greater advertising investment or intermediation cost.

3.6.3. *A Change in w.*

For a producer, intermediation is costly. One cost is from the testing cost w which plays an vital role in the decision making on advertising signal or intermediation. The other cost is from the profit of intermediation. If higher the testing cost w is, the bargaining surplus is lower, the less motivated a high-quality producer becomes to sell products to an intermediary, and the more motivated a high-quality producer becomes to sell his product privately.

3.7. **Intermediary Reputation in Repeated Interaction.**

The model presented in the previous section assumes that the basic stage game of an intermediary is the same in every period. In the repeated game, players are all honest and they trust each other. However, in practice, it is possible that an intermediary chooses to lie. In this section, we consider optimal trust equilibria when consumers observe a defection on quality. In order to make an reputation equilibrium there must be some cost to an intermediary from deception. Suppose now that the history of past play is perfectly observed by all players. If the intermediary lies about his goods' quality, consumers will know the quality in next period and will not purchase from him for T periods.

Proposition 10. *If $T \to \infty$, there exists an equilibrium in which an intermediary won't lie about his product's quality and the intermediary's value along a 'trust' phase is always greater than the value involved in 'punishment'.*

Proof. Consider the following equilibrium in which an intermediary and consumers start in a 'trust' phase, whereby the intermediary sells high-quality goods at cost c from several producers in a particular industry, and buyers pay 1, where $0 < c < 1$. If the goods claimed to be of high-quality are proven low quality with probability ι, then the equilibrium switches to a 'punishment' phase in which buyers stop purchasing from him for T periods, upon which they revert to the 'trust' phase again.

Let E^+ be the intermediary's value along a trust phase and E^- be the intermediary's value at the start of a 'punishment' phase.

$$E^+ = 1 - c + (1 - \iota)\delta E^+ + \iota\delta E^- \tag{174}$$

$$E^- = \delta^T E^+ \tag{175}$$

Solving (174) and (175), we get

$$E^+ = \frac{1 - c}{1 - (1 - \iota)\delta - \iota\delta^{T+1}} \tag{176}$$

$$E^- = \delta^T \frac{1 - c}{1 - (1 - \iota)\delta - \iota\delta^{T+1}} \tag{177}$$

We assume all low-quality products' costs are zero. The seller's no-deviation constraint during the cooperative phase is given by the following:

$$E^+ \geq 1 + \delta E^- \tag{178}$$

Substituting (174) into (178), we have

$$E^+ - E^- \geq \frac{c}{(1 - \iota)\delta} \tag{179}$$

Substituting (176) and (177) into (179), we get

$$\delta \frac{(1 - c)(1 - \delta^T)}{1 - (1 - \iota)\delta - \iota\delta^{T+1}} > \frac{c}{1 - \iota} \tag{180}$$

Notice that (180) is not satisfied for $T = 0$ because the left hand side becomes zero. As $T \to \infty$, $\delta^T \to 0$ and $\delta^{T+1} \to 0$. Simplifying (180), we get:

$$\delta \frac{(1 - c)}{1 - (1 - \iota)\delta} > \frac{c}{1 - \iota} \tag{181}$$

Simplifying (181), we get:

$$\delta > \frac{c}{1 - \iota} \tag{182}$$

Therefore, if $\delta > \frac{c}{1-\iota}$, there exists a T such that (178) is satisfied. Specially, if $\delta > \frac{c}{1-\iota}$, from (176), E^+ is decreasing in T, then the equilibrium is the one that minimizes the value of T consistent with (182). Therefore, for the punishment cost, an intermediary is always honest and players trust each other. □

3.8. **Extension: Equilibrium under a Different Assumption.**

According to consumers' valuation function (165) and (166), a high-quality producer may set a low price at L if his target consumers are Consumer 1 and Consumer 2. Throughout this section, we assume a high-quality producer would gain more by setting the price at L. We compare the results under Assumptions (3) and (4).

Assumption 4. $2(L - C) \geq H - C$

3.8.1. *Market Equilibrium with Signalling.*

We analyse all possible equilibria when a market is without intermediation and select best equilibria by means of a Pareto dominance refinement criterion. The analysis under Assumption (4) works in the same fashion as that former analysis under Assumption (3) works.

Lemma 9. *Under Assumption 4, there always exists separating equilibria. By means of a Pareto dominance refinement criterion: a high-quality producer sets his advertising level at L, just high enough for the low-quality producer not to mimic.*

Lemma 10. *Under Assumption 4, there always exists pooling equilibria. If $pH > \frac{L}{2}$, pooling equilibria can Pareto dominate the separating equilibrium.*

Lemma 11. *Under Assumption 4, hybrid equilibria are always Pareto dominated.*

The details are left to the Appendix.

Comparing the results under Assumption 3 and 4 when there is no intermediation, we find that:

(1) The greater of the gap between the price H and L is, the greater advertising expense a producer invests on to separate his type.

(2) The greater of the gap between the price H and L is, the higher of the probability a producer is willing to separate his type through advertising.

The higher price a high-quality product is at, the higher cost a high-quality producer has to pay to stop a low-quality producer from mimicking. When the gap between the price H and L is not big, pooling equilibrium (signalling through delay) is more effective than advertising signalling.

To sum up, in the two-period Bayesian signalling model, under Assumption 4, the equilibria which maximize the producer profit have be chosen as below, we compare these results with intermediation in next subsection:

- If $pH \leq \frac{L}{2}$, pooling equilibria exist, but Pareto-dominated by separating equilibrium.

  In the first period, a high-quality producer sets the pair at $(P_1(H) = L, A(H) = L)$. According to the level of advertising, consumers believe the new product is high quality. Consumer 1 and Consumer 2 buy products. A low-quality producer sets the pair at $(P_1(L) = L, A(L) = 0)$. For there is no advertising signal, consumers believe the new product is low quality. Consumer 1 won't buy the product and Consumer 2 buys the product. In the second period, a high-quality producer sets $(P_2(H) = L$. Consumer 1 knows the true quality and buys it. Consumer 2 buys the product too.

A low-quality producer sets the pair at $P_2(L) = L$. Consumer 2 buys the product.

- $\dfrac{L}{2} < pH \leq L$, pooling equilibria exist and Pareto-dominate separating equilibrium.

  Different types of producer choose a same price in the first period. A producer sets the pair at $(P_1(H) = pH, A(H) = 0)$. Consumers don't know the quality. Both of them buy the products. In the second period, consumers know the true quality. A high-quality producer sets the price at $P_2(H) = L$, Consumer 1 and 2 buy product s. A low-quality producer sets the pair at $P_2(L) = L$. Consumer 2 buys a product.

- If $pH > L$, pooling equilibria exist and Pareto-dominate separating equilibrium.

  In the first period, a producer sets the pair at $(P_1(H) = L, A(H) = 0)$. Consumers don't know the quality. Consumer 1 and 2 buy products. In the second period, Consumer 1 knows the true quality. A high-quality producer sets the price at $P_2(H) = L$, Consumer 1 and 2 buy products. A low-quality producer sets the pair at $P_2(L) = L$. Consumer 2 buys a product.

3.8.2. *Market Equilibrium with Intermediation.*

Proposition 11. *Under Assumption 4,*

(1) *in equilibrium, a low-quality producer always sells his good privately.*
(2) *if $pH \leq \frac{L}{2}$, and $w < \dfrac{L}{2}$ holds, there exists an equilibrium where equilibrium with intermediation can Pareto-dominate signalling separating equilibrium.*
(3) *if $\frac{L}{2} < pH \leq L$, and $w < L - pH$ holds, there exist equilibria where equilibria with intermediation can Pareto-dominate pooling equilibria.*
(4) *if $pH > L$, a high-quality producer is better off to sell products privately.*

Results are derived in four steps:

Step 1: a low-quality producer always sells his good privately.

Assume a low-quality producer set the price at $P_I(L)$, he will choose to trade through an intermediary rather than to trade in a separating equilibrium if and only if $P_I(L) > 2pH$ or $P_I(L) > 2L$. An intermediary only can sell the product to Consumer 2 because he tells the truth about the products' quality. An intermediary buys the producer's good at the price $P_I(L)$ if and only if $L - P_I(L) > 0$,

which is $P_I(L) < L$. There is an apparent contradiction between the two conditions, which implies that the low-quality producer won't be better off trading through an intermediary.

Step 2: There exists an equilibrium where intermediation can improve separating equilibrium as a better way of separating a producer's type.

Assume a high-quality producer set the price at $P_I(H)$, he will choose to trade through an intermediary rather than to trade in a separating equilibrium if and only if $2(P_I(H) - C) > 2(L - C) - L$, it shows that $P_I(H)$ has to be greater than $\frac{L}{2}$. An intermediary accepts the price $P_I(H)$ if and only if $(L - P_I(H) - w) > 0$, which is $P_I(H) < L - w$. Thus, a high-quality producer sells products to an intermediary if and only if (183) is satisfied.

$$\frac{L}{2} < P_I(H) < L - w \tag{183}$$

Which means $w < \frac{L}{2}$.

We assume that the producer gets utility u from the agreement, where $u = 2(P_I(H) - C)$, and the intermediary gets utility v, where $v = 2(L - P_I(H) - w)$. If there is no agreement, they get utility $u_0 = L - 2C, v_0 = 0$ respectively. Given these utilities, a collective utility function is given by $g(u, v) = (u - u_0)(v - v_0)$.

$$g(u, v) = 2(2P_I(H) - L)(L - P_I(H) - w) \tag{184}$$

Differentiating with respect to $P_I(H)$ and setting the derivative equal to 0, solve this equation for $P_I(H)$ gives:

$$P_I^*(H) = \frac{(3L - 2w)}{4} \tag{185}$$

A producer and an intermediary bargains over the division of high-quality products' margin. A high-quality producer trades through an intermediary at the price $P_I^*(H) = \frac{(3L - 2w)}{4}$. Consumer 1 buys the product from the intermediary at $\tilde{P}_I(H) = L$. Consumer 2 buys the product from the intermediary at $\tilde{P}_I(H) = L$.

A low-quality producer trades directly, and he sets the price-advertising pair at $(L, 0)$.

Consumer 1 won't buy the product.

Consumer 2 buys the product from the producer at $P_t(L) = L$ in both periods.

Step 3: There exists an equilibrium where intermediation can improve a pooling equilibrium.

If $\frac{L}{2} < pH \le L$, and $w < L - pH$ holds, a high-quality producer is better off to sell products to an intermediary at the price $P_I^*(H) = \dfrac{(L + pH - w)}{2}$.

Step 4: If $pH > L$, a high-quality producer is better off to sell products privately.

A high-quality producer sets the price at $P_I(H)$, he will choose trade through an intermediary rather than trade in a pooling equilibrium if and only if $2(P_I(H) - C) > 2(L - C)$, which is $P_I(H) > L$. The intermediary accepts the price $P_I(H)$ if and only if $L - p_I(H) - w > 0$, which is $P_I(H) < L - w$. Thus, a high-quality producer sells products to an intermediary if (186) is satisfied:

$$L < P_I(H) < L - w \tag{186}$$

For $w > 0$, there is an apparent contradiction, which implies that a high-quality producer won't be better off to sell products to an intermediary.

Comparing the results under Assumption 3 and 4 where there is an intermediary, we find that:

(1) The greater the price H is, a producer is more willing to separate his type through intermediation rather than through advertising.
(2) For a high-quality producer, if he benefits by selling his product the same price as low quality good, pooling equilibrium is more effective than intermediation.

Expensive high-quality products face higher signalling cost to separate its type. When the testing cost is low enough, intermediation has a greater competitive advantage to dominate advertising signal. When asymmetric information is not an issue, a producer doesn't have intuition to eliminate asymmetric information. A high-quality producer doesn't need separate his type.

3.9. **Conclusion.**

In this paper, possible equilibria are compared in new product launching with quality uncertain problem. A producer may choose to introduce his product by himself, in which case a producer could signal a product's quality through levels of advertising. Or, a producer sells his goods to an intermediary who handles a variety of products. We consider Pareto efficiency selection criterion to select the best possible equilibrium. Then, we show that intermediation can be a better way of separating a producer's type.

All results of this chapter depend on the assumption that there is only one producer and two consumers. When considering demand effects, it would be

interesting to check whether the results are still robust, for e.g., it is worthwhile to check what will happen if the proportion of two types' consumers is different from one to one. A possible extension could be in which an intermediary gets more bargaining power. Furthermore, we made the convenient assumption that the highest price Consumer 1 would accept is H. By relaxing the assumption, the double marginalization will be involved. Although double marginalization effect has been extensively studied in the context of supply chain management for mature products, very limited attention has been given to newly introduced products with quality uncertainty whose demand is generated through the effect of reputation.

In this thesis, we consider platform innovation, intermediation and advertisement.

In the first chapter, we make comparisons in platform innovation. In a monopoly platform model, by comparing profit-maximizing outcomes and welfare-maximizing outcomes without and with innovation, we find that while the platform's profit is obviously positive in the profit-maximizing solution, notional profit is negative in the welfare-maximizing solution. By comparing profit-maximizing and welfare-maximizing outcomes without innovation and with innovation, we find that the platform's profit with innovation is greater; more agents are connected to the platform with innovation than that without innovation.

In non-tournament models, by comparing profit-maximizing outcomes without innovation and with innovation, we find that platforms' profit with innovation is less than platforms' profit without innovation. They end up a prisoners' dilemma equilibrium. Agents of group l total surplus with innovation is greater than without innovation. social welfare with innovation is greater than that without innovation. By comparing profit-maximizing outcomes without technology spillover effect and with technology spillover effect, we find that platforms have the same optimal price and optimal quality under the condition without technology spillover effect or with technology spillover effect. Each platform's profit with technology spillover effect is greater than that without technology spillover effect. Three extensions are model with asymmetric network externalities, platform exogenous specialization in side innovations, and innovation tournament model. We have carried out some preliminary studies which are very important start for our future work.

In the second chapter, we show a problem in Armstrong (2006) monopoly model: the set of assumption in Armstrong (2006) monopoly platform model is not sufficient to guarantee the existence of an interior equilibrium with positive demands. A linear Hotelling demand system is used to illustrate the point. If the first and second conditions hold, the only possible result is corner equilibrium where equilibrium demand quantities are zero. If the first and second conditions are reversed, the solutions are either both sides of the market are fully covered, or jump to the other corner where equilibrium demand quantities are zero. Clearly, in both corner equilibria above Armstrong (2006) optimal pricing does not apply. For future work, it will be challenging to find a way to fix the problem in a more general form.

In the third chapter, we assess the potential benefits of intermediation where intermediation is with quality certification. The benefit depends on the efficiency of equilibrium without intermediation. We develop a simpler signalling model of advertising to compare with intermediation signal. Our analysis confirms that intermediation may signal quality, but advertising may also occur, and which is used depends on the difference in costs of qualities.

The third chapter can be extended in many ways, for e.g.,demand effect could be considered. Assume there are n_1 of Consumer 1 and n_1 of Consumer 2, then all equilibria will end up with two more variables. We could then study how the conditions of which an intermediary is active in a market to reveal the true quality of a product will change when there are demand effects.

We check First and Second Order Conditions for a maximum profit of a platform when there is no V. Consider a platform who would like to maximize profit. The platform charges p_l for offering unit connection to two group of agents. Unit connection cost is $f_l \geq 0$. Unit transportation cost is $t > 0$. $\alpha_l > 0$ measures network externalities. A profit function is given by:

$$\pi = -\frac{tp_1 + p_2\alpha_1}{t^2 - \alpha_1\alpha_2}(p_1 - f_1) - \frac{tp_2 + p_1\alpha_2}{t^2 - \alpha_1\alpha_2}(p_2 - f_2)$$

The platform chooses p_l to maximize profit. A mathematical condition for optimization state that the first derivative is zero. π must satisfy:

$$\underset{p_1,p_2}{\text{maximize}} \quad \pi = -\frac{tp_1 + p_2\alpha_1}{t^2 - \alpha_1\alpha_2}(p_1 - f_1) - \frac{tp_2 + p_1\alpha_2}{t^2 - \alpha_1\alpha_2}(p_2 - f_2)$$

$$\frac{\partial \pi}{\partial p_1} = -\frac{t(-f_1 + p_1)}{t^2 - \alpha_1\alpha_2} - \frac{tp_1 + p_2\alpha_1}{t^2 - \alpha_1\alpha_2} - \frac{(-f_2 + p_2)\alpha_2}{t^2 - \alpha_1\alpha_2} = 0 \tag{187}$$

$$\frac{\partial \pi}{\partial p_2} = -\frac{t(-f_2 + p_2)}{t^2 - \alpha_1\alpha_2} - \frac{tp_2 + p_1\alpha_2}{t^2 - \alpha_1\alpha_2} - \frac{(-f_1 + p_1)\alpha_1}{t^2 - \alpha_1\alpha_2} = 0 \tag{188}$$

Solving the systems in (187) and (188), we deduce that the profit-maximizing outcome has the equilibrium prices satisfying:

$$p_1 = \frac{tf_2(-\alpha_1 + \alpha_2) + f_1(2t^2 - \alpha_1^2 - \alpha_1\alpha_2)}{4t^2 - (\alpha_1 + \alpha_2)^2}$$

$$p_2 = \frac{tf_1(\alpha_1 - \alpha_2) + f_2(2t^2 - \alpha_2^2 - \alpha_1\alpha_2)}{4t^2 - (\alpha_1 + \alpha_2)^2}$$

Substituting p_1 and p_2 upon to (142) and (143), we have,

$$n_1 = \frac{-2tf_1 - f_2(\alpha_1 + \alpha_2)}{4t^2 - (\alpha_1 + \alpha_2)^2} \tag{189}$$

$$n_2 = \frac{-2tf_2 - f_1(\alpha_1 + \alpha_2)}{4t^2 - (\alpha_1 + \alpha_2)^2} \tag{190}$$

In addition, mathematical conditions for maximization state that the second derivatives are negative. They are expressed as:

$$\frac{\partial^2 \pi}{\partial^2 p_1} = \frac{\partial^2 \pi}{\partial^2 p_2} = -\frac{2t}{t^2 - \alpha_1\alpha_2} \tag{191}$$

$$\frac{\partial^2 \pi}{\partial p_2 p_2} = \frac{\partial^2 \pi}{\partial p_2 p_2} = -\frac{\alpha_1 + \alpha_2}{t^2 - \alpha_1\alpha_2} \tag{192}$$

$$\begin{cases} \dfrac{\partial^2 \pi}{\partial^2 p_1} = -\dfrac{2t}{t^2 - \alpha_1\alpha_2} < 0 \\ \left(\dfrac{\partial^2 \pi}{\partial p_1 p_2}\right)^2 - \left(\dfrac{\partial^2 \pi}{\partial^2 p_1}\right)\left(\dfrac{\partial^2 \pi}{\partial^2 p_2}\right) = \left(-\dfrac{\alpha_1 + \alpha_2}{t^2 - \alpha_1\alpha_2}\right)^2 - \left(-\dfrac{2t}{t^2 - \alpha_1\alpha_2}\right)\left(-\dfrac{2t}{t^2 - \alpha_1\alpha_2}\right) < 0 \end{cases}$$

$$(193)$$

Solving (193), a platform would maximize profit if and only if:

$$2t > \alpha_1 + \alpha_2$$

APPENDIX B. IN THIS APPENDIX WE SUPPLY THE PROOF OF CONDITION (158)

We check First and Second Order Conditions for a maximum profit of a platform when there is V. Consider a platform who would like to maximize profit. The platform charges p_l for offering unit connection to two group of agents. Unit connection cost is $f_l \geq 0$. Unit transportation cost is $t > 0$. $\alpha_l > 0$ measures network externalities. $V > 0$ is an intrinsic value. A profit function is given by:

$$\pi = \left(\frac{tV + \alpha_1 V}{t^2 - \alpha_1\alpha_2} - \frac{tp_1 + \alpha_1 p_2}{t^2 - \alpha_1\alpha_2}\right)(p_1 - f_1) + \left(\frac{tV + \alpha_2 V}{t^2 - \alpha_1\alpha_2} - \frac{tp_2 + \alpha_2 p_1}{t^2 - \alpha_1\alpha_2}\right)(p_2 - f_2)$$

The platform chooses p_l to maximize profit. The value p_l^* of p_l that maximizes the function. A mathematical condition for optimization state that the first derivative is zero. π must satisfy:

$$\underset{p_1,p_2}{\text{maximize}} \quad \pi = \left(\frac{tV + \alpha_1 V}{t^2 - \alpha_1\alpha_2} - \frac{tp_1 + \alpha_1 p_2}{t^2 - \alpha_1\alpha_2}\right)(p_1 - f_1) + \left(\frac{tV + \alpha_2 V}{t^2 - \alpha_1\alpha_2} - \frac{tp_2 + \alpha_2 p_1}{t^2 - \alpha_1\alpha_2}\right)(p_2 - f_2)$$

$$\frac{\partial \pi}{\partial p_1} = -\frac{t(-f_1 + p_1)}{t^2 - \alpha_1\alpha_2} - \frac{tp_1 + p_2\alpha_1 - tV - \alpha_1 V}{t^2 - \alpha_1\alpha_2} - \frac{(-f_2 + p_2)\alpha_2}{t^2 - \alpha_1\alpha_2} = 0 \qquad (194)$$

$$\frac{\partial \pi}{\partial p_2} = -\frac{t(-f_2 + p_2)}{t^2 - \alpha_1\alpha_2} - \frac{tp_2 + p_1\alpha_2 - tV - \alpha_2 V}{t^2 - \alpha_1\alpha_2} - \frac{(-f_1 + p_1)\alpha_1}{t^2 - \alpha_1\alpha_2} = 0 \qquad (195)$$

Solving the systems in (194) and (195), we deduce that the profit-maximizing outcome has the equilibrium prices satisfying:

$$p_1^* = \frac{V(2t^2 - t\alpha_2 + t\alpha_1 - \alpha_1\alpha_2 - \alpha_2^2) + tf_2(\alpha_2 - \alpha_1) + f_1(2t^2 - \alpha_1^2 - \alpha_1\alpha_2)}{4t^2 - (\alpha_1 + \alpha_2)^2}$$

$$p_2^* = \frac{V(2t^2 + t\alpha_2 - t\alpha_1 - \alpha_1\alpha_2 - \alpha_1^2) + tf_1(-\alpha_2 + \alpha_1) + f_2(2t^2 - \alpha_2^2 - \alpha_1\alpha_2)}{4t^2 - (\alpha_1 + \alpha_2)^2}$$

Substitute p_1^* and p_2^* to (153) and (154), we have,

$$n_1^* = \frac{2tV - 2tf_1 + V\alpha_1 - f_2\alpha_1 + V\alpha_2 - f_2\alpha_2}{4t^2 - (\alpha_1 + \alpha_2)^2}$$

$$= \frac{V(2t + \alpha_1 + \alpha_2)}{4t^2 - (\alpha_1 + \alpha_2)^2} + \frac{-2tf_1 - f_2\alpha_1 - f_2\alpha_2}{4t^2 - (\alpha_1 + \alpha_2)^2}$$

$$n_2^* = \frac{2tV - 2tf_2 + V\alpha_1 - f_1\alpha_1 + V\alpha_2 - f_1\alpha_2}{4t^2 - (\alpha_1 + \alpha_2)^2}$$

$$= \frac{V(2t + \alpha_1 + \alpha_2)}{4t^2 - (\alpha_1 + \alpha_2)^2} + \frac{-2tf_2 - f_1\alpha_1 - f_1\alpha_2}{4t^2 - (\alpha_1 + \alpha_2)^2}$$

In addition, mathematical conditions for maximization state that the second derivative is negative. They are expressed as:

$$\begin{cases} \dfrac{\partial^2 \pi}{\partial^2 p_1} = -\dfrac{2t}{t^2 - \alpha_1\alpha_2} < 0 \\ (\dfrac{\partial^2 \pi}{\partial p_1 p_2})^2 - (\dfrac{\partial^2 \pi}{\partial^2 p_1})(\dfrac{\partial^2 \pi}{\partial^2 p_2}) = (-\dfrac{\alpha_1 + \alpha_2}{t^2 - \alpha_1\alpha_2})^2 - (-\dfrac{2t}{t^2 - \alpha_1\alpha_2})(-\dfrac{2t}{t^2 - \alpha_1\alpha_2}) < 0 \end{cases} \tag{196}$$

Solving (196), a platform would maximize profit if and only if:

$$2t > \alpha_1 + \alpha_2$$

APPENDIX C. THE PROOF OF CORNER EQUILIBRIA IN LINEAR HOTELLING DEMAND SYSTEM

From (189) and (190), when $2t > \alpha_1 + \alpha_2$, there is no an interior equilibrium with positive demands (or at least one side's demand is positive). The only equilibrium here is $n_1 = n_2 = 0$, $\pi = 0$.

We check some other possibilities to see if equilibria exist when $2t > \alpha_1 + \alpha_2$ is violated. The Equilibria can only be at corners:

- Assume $2t \leq \alpha_1 + \alpha_2 - (f_1 + f_2)$, there is an corner equilibrium at $n_1 = n_2 = 1$ (fully covered market), $\pi = \alpha_1 + \alpha_2 - 2t - (f_1 + f_2)$. The calculation is as follows:

$$n_1 t = \alpha_1 n_2 - p_1 \tag{197}$$

$$n_2 t = \alpha_2 n_1 - p_2 \tag{198}$$

$$t = \alpha_1 - p_1$$

$$t = \alpha_2 - p_2$$

$$\pi = (\alpha_1 - t - f_1) + (\alpha_2 - t - f_2) = \alpha_1 + \alpha_2 - 2t - (f_1 + f_2)$$

- Assume $\alpha_1 + \alpha_2 - (f_1 + f_2) \leq 2t \leq \alpha_1 + \alpha_2$, there is a corner equilibrium, which is $n_1 = n_2 = 0$, $\pi = 0$.
- Assume $2t \leq \alpha_1 + \alpha_2$, there is no equilibrium at corner $n_1 = 1, n_2 = 0$. The calculation is as follows:

    Substitute $n_1 = 1, n_2 = 0$ into (197) and (198), we have,

$$t = -p_1$$

$$0 = \alpha_2 - p_2$$

$$\pi = -t - f_1$$

APPENDIX D. IN THIS APPENDIX WE SUPPLY THE PROOF OF LEMMA (6)

Proof. Suppose pooling equilibria do exist. Consumers hold beliefs that there exists a critical advertising level A^* which is sufficient high.

$$if \begin{cases} A(q) \geq A^* & \text{the producer is of high quality with probability 1} \\ A(q) < A^* & \text{the producer is of high quality with probability p} \\ & \text{and of low quality with probability 1-p} \end{cases} \quad (199)$$

Since advertising expenditure is infinite, two types of a producer would rather choose the same advertising level $A(q) < A^*$ to signal themselves. The product quality cannot be distinguished on the basis. Advertising doesn't perform any signal function and is a pure waste. The optimal level of advertising in a pooling equilibrium is then $A(q) = 0$.

In a pooling equilibrium, under assumption 3, a high-quality producer sets a new product's price either at Customer 1' s price expectation (see 201) or L. If he sets the price higher than Customer 1' s price expectation, Consumer 1 won't buy a product from him, a high-quality producer will lose his current and future profit. A low-quality producer mimics a high-quality producer's strategy.

$$E(V_C) = p(H - P_1) + (1 - p)(0 - P_1) \tag{200}$$

Given the above, the expected price is:

$$P_1 = pH \tag{201}$$

In the first period, a high-quality producer sets the price at pH, he can sell products to Consumer 1 and Consumer 2. In the second period, the product quality is informed, a producer sets the price at H, he sells a product to Consumer 1 only. His two-period profit is $2(pH - C) + H - C$. We already knew maximum profit of two-period in a separating equilibrium is $H - C - (H - L) + H - C$ (more detail in Section 1.4), he won't deviate from this pooling strategy if and only if he gains more profit from pooling than separating equilibrium:

$2(pH - C) + H - C \geq H - C - (H - L) + H - C$

Thus, for PE to exist it must be the case that:

$pH \geq \frac{L+C}{2}$

For a low-quality producer, he sets the price at pH in the first period and at L in the second period. His two-period profit is $2pH + L$. His maximum profit of two-period in a separating equilibrium is $L + L$ (more detail in Section 1.4), he won't deviate from this pooling strategy if and only if he gains more profit from pooling than separating equilibrium: $2pH + L \geq L + L$.

Thus, for the PE to exist it must be the case that:

$pH \geq \frac{L}{2}$

Therefore, when pH is less than $\frac{L+C}{2}$, a high-quality producer would deviate from pooling. A high-quality producer would be more willing to advertise to separate himself from a low-quality producer. Pooling equilibrium doesn't exist. Separating equilibrium Pareto dominates the pooling equilibrium. When pH is greater than $\frac{L+C}{2}$, pooling equilibria Pareto dominates the separating equilibrium.

$\square$

APPENDIX E. IN THIS APPENDIX WE SUPPLY THE PROOF OF LEMMA (7)

Proof. Suppose Hybrid Equilibria do exist. There exists a cut-off advertising level A^*. Let us define a high-quality firm spends on advertising with the probability $\alpha = Pr(A^* \mid H)$ and the firm with low-quality products spends on advertising with the probability $\beta = Pr(A^* \mid L)$. The producer has beliefs about how a consumer will eventually play: the producer is believed to produce the high quality with probability ϖ if $A(q) = A^*$ is observed, and the producer is believed to produce the high quality with probability ς if $A(H) = 0$ is observed. A consumer

has beliefs about the type of the producer given the action she observes: these are $\rho = Pr(H \mid A(q) = A^*)$ and $\tau = Pr(H \mid A(q) = 0)$.

Hence by Bayes rule definition:

$$\begin{aligned} \rho &= \Pr(H \mid A(q) = A^*) \\ &= \frac{\Pr(A^* \mid H)\Pr(H)}{\Pr(A^* \mid H)\Pr(H) + \Pr(A^* \mid L)\Pr(L)} \\ &= \frac{\alpha p}{\alpha p + \beta(1-p)} \end{aligned} \tag{202}$$

and

$$\begin{aligned} \tau &= \Pr(H \mid A = 0) \\ &= \frac{\Pr(A(H) = 0 \mid H)\Pr(H)}{\Pr(A(H) = 0 \mid H)\Pr(H) + \Pr(A(L) = 0 \mid L)\Pr(L)} \\ &= \frac{(1-\alpha)p}{(1-\alpha)p + (1-\beta)(1-p)} \end{aligned} \tag{203}$$

Consider the situation in which a high-quality producer chooses $A(H) = A^*$ and a low-quality producer chooses $A(L) = \beta A^* + (1 - \beta)0$.

We have $\alpha = 1, \rho = \dfrac{p}{p + \beta(1-p)}$, and $p < \rho < 1$. Thus, the information set followed by $A = 0$ is reached only by a low-quality type, never by high-quality type, that is $\tau = 0, \varsigma = 0$. Given the above, the price in the Hybrid Equilibria must be as follows: If consumer observes $A = 0$, the highest price she would accept is L; If Consumer 1 observes $A = A^*$, the highest price she would accept is as (205).

$$E(V_C) = \rho(H - P_1) + (1 - \rho)(0 - P_1) \tag{204}$$

Then, we have:

$$P_1^{A^*} = \rho H \tag{205}$$

For a high-quality producer, he sets the pair either $(\rho H, A^*)$ or (L, A^*). On the basis of ASSUMPTION 3, there are four intervals:

- $\rho H \leq \frac{L+C}{2}$

  Both producers will benefit from deviating from the Hybrid Equilibria. The Hybrid Equilibria won't survive. Then, to achieve an advertising Hybrid Equilibrium, ρH must be greater than $\frac{L+C}{2}$.

- $\frac{L+C}{2} < \rho H \leq L$

  The high-quality producer sells products to Consumer 1 and 2 and sets the new product's price at ρH. The high-quality producer will want to spend $A(H) = A^*$ and won't deviate from this strategy if:

  $2(\rho H - C) - A^* + H - C \geq 2(L - C)$

  $A^* \leq (2\rho + 1)H - C - 2L$

The low-quality producer chooses $(\rho H, A^*)$ with probability β, and chooses $(L, 0)$ with probability $1 - \beta$, so its profit is $(2\rho H - A^*)\beta + L(1 - \beta)$. From the second period, the consumer is fully informed. In an equilibrium, a low-quality producer charges $P_2^L = L$; the producer's profit is L. Thus, the firm's two-period profit is $(2\rho H - A^*)\beta + L(1 - \beta) + L$. Therefore, the low- quality producer won't unilaterally deviate from the mixed strategy if:

$(2\rho H - A^*)\beta + L(1 - \beta) \geq L$

$A^* \leq 2\rho H - L$

Thus, the game has an infinite number of hybrid equilibria if (206) is satisfied:
$$\begin{cases} A^*_{SE1} \leq 2\rho H - L \\ \rho H \leq \frac{L+C}{2} \end{cases} \tag{206}$$

- $L < \rho H \leq 2L - C$

  the high-quality producer sells products Consumer 1 and 2 and sets the new product's price at L. The high-quality producer will want to spend $A(H) = A^*$ and won't deviate from this strategy if:

  $2(L - C) - A^* + H - C \geq 2(L - C)$

  $A^* \leq H - C$

  The low-quality producer chooses (L, A^*) with probability β, and chooses $(L, 0)$ with probability $1 - \beta$, so its profit is $(2L - A^*)\beta + L(1 - \beta)$. From the second period, the consumer is fully informed. In an equilibrium, a low-quality producer charges $P_2^L = L$; the producer's profit is L. Thus, the firm's two-period profit is $(2L - A^*)\beta + L(1 - \beta) + L$. Therefore, the low- quality producer won't unilaterally deviate from the mixed strategy if:

  $(2L - A^*)\beta + L(1 - \beta) \geq L$

  $A^* \leq L$

  Thus, the game has an infinite number of hybrid equilibria if (207) is satisfied:
  $$\begin{cases} A^*_{SE1} \leq L \\ L < \rho H \leq 2L - C \end{cases} \tag{207}$$

- $\rho H > 2L - C$

  The high-quality producer sells a product to Consumer 1 and sets the new product's price at ρH. The high-quality producer will want to spend $A(H) = A^*$ and won't deviate from this strategy if:

  $\rho H - C - A^* + H - C \geq 2(L - C)$

  $A^* \leq (\rho + 1)H - 2L$

The low-quality producer chooses $(\rho H, A^*)$ with probability β, and chooses $(L, 0)$ with probability $1 - \beta$, so its profit is $(\rho H - A^*)\beta + L(1 - \beta)$. From the second period, the consumer is fully informed. In an equilibrium, a low-quality producer charges $P_2^L = L$; the producer's profit is L. Thus, the firm's two-period profit is $(\rho H - A^*)\beta + L(1 - \beta) + L$. Therefore, the low- quality producer won't unilaterally deviate from the mixed strategy if:

$(\rho H - A^*)\beta + L(1 - \beta) \geq L$

$A^* \leq \rho H - L$

Thus, the game has an infinite number of hybrid equilibria if (208) is satisfied:

$$\begin{cases} A_{SE1}^* \leq \rho H - L \\ \rho H > 2L - C \end{cases} \tag{208}$$

By considering Pareto efficiency selection criterion, the producer's best response is to choose $A_{SE1}^* = 0$. This is a pooling equilibrium, therefore, hybrid equilibria are always Pareto dominated.

Consider the situation in which a high-quality producer chooses $A(H) = \alpha A^* + (1 - \alpha)0$ **and a low-quality producer chooses** $L \to A(L) = 0$**.**

We have $\beta = 0, \tau = \dfrac{(1 - \alpha)p}{(1 - \alpha)p + (1 - p)}$, and $0 < \tau < p$. Then, the information set followed by A^* is reached only by high-quality type, never by low-quality type, that is $\rho = 1, \varpi = 1$. Given the above, the price in the Hybrid equilibria must be as follows: If consumer observes $A(q) = A^*$, $P_1^{A^*} = H$, and if Consumer 1 observes $A(q) = 0$, the highest price she would accept is in which her expected utility equals zero (See (210)):

$$E(V_C) = \tau(H - P_1) + (1 - \tau)(0 - P_1) \tag{209}$$

Then, we have:

$$P_1^{A^0} = \tau H \tag{210}$$

For a high-quality producer, he sets the price at either τH or L. On the basis of ASSUMPTION 3, there are four intervals:

- $\tau H \leq \frac{L+C}{2}$
  Hybrid Equilibria won't survive.
- $\frac{L+C}{2} < \tau H \leq L$
  The high-quality producer sells products Consumer 1 and 2 and sets the new product's price at τH. If the firm chooses (H, A^*) with probability α, and chooses $(\tau H, 0)$ with probability $1 - \alpha$, its profit is $(H - C - A^*)\alpha + 2(\tau H - C)(1 - \alpha)$. Therefore, the firm's two-period profit is $(H - C -$

$A^*)\alpha + 2(\tau H - C)(1 - \alpha) + H - C$; Thus, the high-quality producer won't unilaterally deviate from the mixed strategy if:

$(H - C - A^*)\alpha + 2(\tau H - C)(1 - \alpha) \geq H - C - A^*$

$A^* \geq H - 2\tau H + C$

The low-quality producer chooses $(\tau H, 0)$. From the second period, the consumer is fully informed. In an equilibrium a low-quality producer charges $P_2^L = L$ and the consumer buys the good; Therefore, the firm's two-period profit is $2\tau H + L$; If the firm deviates from this strategy and chooses (H, A^*) in the first period, then the consumer believes it is high quality and purchases the good in the first period, yielding it a first year profit of $H - A^*$, and the firm's two-period profit with advertising signal is $H - A^* + L$.

Thus, the low-quality producer won't unilaterally deviate from the strategy if:

$2\tau H \geq H - A^*$

$A^* \geq H - 2\tau H$

Thus, the game has an infinite number of hybrid equilibria if (211) is satisfied:

$$\begin{cases} A_{SE2} \geq H - 2\tau H + C \\ \frac{L+C}{2} < \rho H \leq L \end{cases} \tag{211}$$

- $L < \tau H \leq 2L - C$

The high-quality producer sells products to Consumer 1 and 2 and sets the new product's price at L. If the firm chooses (H, A^*) with probability α, and chooses $(L, 0)$ with probability $1 - \alpha$, its profit is $(H - C - A^*)\alpha + 2(L - C)(1 - \alpha)$. Therefore, the firm's two-period profit is $(H - C - A^*)\alpha + 2(L - C)(1 - \alpha) + H - C$; Thus, the high-quality producer won't unilaterally deviate from the mixed strategy if:

$(H - C - A^*)\alpha + 2(L - C)(1 - \alpha) \geq H - C - A^*$

$A^* \geq H - 2L + C$

The low-quality producer chooses $(L, 0)$. From the second period, the consumer is fully informed. In an equilibrium a low-quality producer charges $P_2^L = L$ and the consumer buys the good; Therefore, the firm's two-period profit is $2L + L$; If the firm deviates from this strategy and chooses (H, A^*) in the first period, then the consumer believes it is high quality and purchases the good in the first period, yielding it a first year profit of $H - A^*$, and the firm's two-period profit with advertising signal is $H - A^* + L$.

Thus, the low-quality producer won't unilaterally deviate from the strategy if:

$$2L \geq H - A^*$$
$$A^* \geq H - 2L$$

Thus, the game has an infinite number of hybrid equilibria if (212) is satisfied:

$$\begin{cases} A_{SE2} \geq H - 2L + C \\ L < \rho H \leq 2L - C \end{cases} \tag{212}$$

- $\tau H > 2L - C$

The high-quality producer sells a product to Consumer 1 and sets the new product's price at τH. If the firm chooses (H, A^*) with probability α, and chooses $(\tau H, 0)$ with probability $1 - \alpha$, its profit is $(H - C - A^*)\alpha + (\tau H - C)(1 - \alpha)$. Therefore, the firm's two-period profit is $(H - C - A^*)\alpha + (\tau H - C)(1 - \alpha) + H - C$; Thus, the high-quality producer won't unilaterally deviate from the mixed strategy if:

$$(H - C - A^*)\alpha + (\tau H - C)(1 - \alpha) \geq H - C - A^*$$
$$A^* \geq H - \tau H$$

The low-quality producer chooses $(\tau H, 0)$. From the second period, the consumer is fully informed. In an equilibrium a low-quality producer charges $P_2^L = L$ and the consumer buys the good; Therefore, the firm's two-period profit is $\tau H + L$; If the firm deviates from this strategy and chooses (H, A^*) in the first period, then the consumer believes it is high quality and purchases the good in the first period, yielding it a first year profit of $H - A^*$, and the firm's two-period profit with advertising signal is $H - A^* + L$.

Thus, the low-quality producer won't unilaterally deviate from the strategy if:

$$\tau H \geq H - A^*$$
$$A^* \geq H - \tau H$$

Thus, the game has an infinite number of hybrid equilibria if (213) is satisfied:

$$\begin{cases} A_{SE2}^* \geq H - \tau H \\ \rho H > 2L - C \end{cases} \tag{213}$$

By considering Pareto efficiency selection criterion, both types of producer would be better off playing $A_{SE2}^* = 0$, which is a pooling equilibrium. Therefore, hybrid equilibria are always Pareto dominated. $\qquad \square$

Proof. In the advertising signalling model, the equilibria which maximize the producer profit are as below:

$pH \leq \frac{L+C}{2}$

A high-quality producer sets the pair at $P_1(H) = H, A(H) = H - L$ and a low-quality producer sets the pair at $P_1(L) = L, A(L) = 0$. The profit of a high-quality producer in two periods is $H - (H - L) - C + H - C$, the profit of a low-quality producer in two periods is $2L$.

$\frac{L+C}{2} < pH \leq L$,

A high-quality producer sets the pair at $P_1(H) = pH, A(H) = 0$ and a low-quality producer sets the pair at $P_1(L) = pH, A(L) = 0$. The profit of a high-quality producer in two periods is $2(pH - C) + H - C$, and the profit of a low-quality producer in two periods is 2pH+L.

$L < pH \leq 2L - C$,

A high-quality producer sets the pair at $P_1(H) = L, A(H) = 0$ and a low-quality producer sets the pair at $P_1(L) = L, A(L) = 0$. The profit of a high-quality producer in two periods is $2(L - C) + H - C$, and the profit of a low-quality producer in two periods is 2L+L.

$pH > 2L - C$,

A high-quality producer sets the pair at $P_1(H) = pH, A(H) = 0$ and a low-quality producer sets the pair at $P_1(L) = pH, A(L) = 0$. The profit of a high-quality producer in two periods is $pH - C + H - C$, and the profit of a low-quality producer in two periods is $pH + L$.

In the price signalling model, the equilibrium which maximize the producer profit is as below:

A high-quality producer sets the price at $P_1(H) = \frac{L}{2}$ and a low-quality producer sets the pair at $P_1(L) = L$. The profit of a high-quality producer in two periods is $2(\frac{L}{2} - C) + H - C$, the profit of a low-quality producer in two periods is $2L$.

The equilibria in the advertising signalling model is more efficient than the equilibrium in the price signalling model.

□

Proof. Suppose separating equilibria do exist. The advertising strategies of a producer to signal his quality would be to choose different advertising levels, where

$A(H) \neq A(L)$. Suppose that consumers hold beliefs that there exists a cut-off advertising level A^* such that:

$$if \begin{cases} A(q) < A^* & \text{the producer is of low quality with probability 1} \\ A(q) \geq A^* & \text{the producer is of high quality with probability 1} \end{cases} \quad (214)$$

For separation to be successful and a consumer's beliefs to be confirmed, the advertising signals $A(H)$ and $A(L)$ of high and low-quality types are simplified by choosing from A^* and 0. The consumer's beliefs in this assessment are: if the consumer observes $A(q) = A^*$, then she must believe that the firm provides a high-quality good, whereas if she observes $A(q) = 0$, then she must believe that it is low quality.

For the high-quality producer, he sets the pair at $(P_1(H) = L, A(H) = A^*)$; Both consumers buy the product. Consumer 1 makes $H - L$ net utility and Consumer 2 makes 0 net utility. The high-quality producer's net profit is $2(L - C) - A^*$ in the first period. In the second period, Consumer 1 is fully informed. The firm's two-period net profit is $4(L - C) - A^*$. If the high-quality producer deviates from the strategy, which means he doesn't invest on advertising, Consumer 1 won't buy it and only Consumer 2 buys the good. Then, two-period profit is $2(L - C)$. So the high-quality producer will want to choose A^* and won't deviate if:

$4(L - C) - A^* \geq 2(L - C)$

$A^* \leq 2(L - C)$

For the low-quality producer, if the firm chooses $A(L) = 0$ in the first period, then the consumer believes it is low quality, only Consumer 2 buys the product, so its profit is L. In the second period, a low-quality producer charges $P_2(L) = L$, and therefore the producer's two-period profit is $2L$. If a low-quality producer deviates from this strategy, then he chooses $(P_1(L) = L, A(L) = A^*)$ in the first period, then both consumers purchase the good in the first period, yielding it a first year profit of $2L - A^*$. Consumer 1's net utility is $-L$ and Consumer 2's net utility is 0. Consumer 1 won't buy the product again after the quality is fully revealed, only Consumer 2 buys the product with the price at L in the second period. The firm's two-period profit is $3L - A^*$. So the low-quality producer won't deviate if:

$3L - A^* \leq 2L$

$A^* \geq L$,

Thus, there exist an infinite number of separating equilibria if (215) is satisfied:

$$L \leq A_{SE} \leq 2(L - C) \quad (215)$$

Assume an $A_r \in [L, 2(L - C)]$, where:

$2(L - C) - A_{SE} < 2(L - C) - A_r$

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$L > 2L - A_r$

Then a consumer should believe $\Pr(H \mid A(H) = A_r) = 1$, but then the high-quality producer would be better off playing A_r rather than A_{SE}. Accordingly, separating equilibria with $A_{SE} > A_r$ fails to meet the Pareto efficiency. The lower bound of A_{SE} would be the most favourable solution. Therefore, there is a unique separating equilibrium which satisfies the Pareto efficiency:

$$A^*_{SE} = L \tag{216}$$

$\square$

Proof. Suppose pooling equilibria do exist. Consumers hold beliefs that there exists a critical advertising level A^* which is sufficient high.

$$if \begin{cases} A(q) \geq A^* & \text{the producer is of high quality with probability 1} \\ A(q) < A^* & \text{the producer is of high quality with probability p} \\ & \text{and of low quality with probability 1-p} \end{cases} \tag{217}$$

Since advertising expenditure is infinite, two types of a producer would rather choose the same advertising level $A(q) < A^*$ to signal themselves. The product quality cannot be distinguished on the basis. Advertising doesn't perform any signal function and is a pure waste. The optimal level of advertising in a pooling equilibrium is then $A(q) = 0$.

In a pooling equilibrium, under assumption 4, a high-quality producer sets a new product's price either at Customer 1' s price expectation (see 201) or L. If he sets the price higher than Customer 1' s price expectation, Consumer 1 won't buy a product from him, a high-quality producer will lose his current and future profit. A low-quality producer mimics a high-quality producer's strategy.

In the first period, a high-quality producer sets the price at pH, he can sell products to Consumer 1 and Consumer 2. In the second period, the product quality is informed, a producer sets the price at L, he sells products to Consumer 1 and Consumer 2. His two-period profit is $2(pH - C) + 2(L - C)$. We already knew maximum profit of two-period in a separating equilibrium is $2(L-C)-L+ 2(L - C)$ (more detail in Section 8), he won't deviate from this pooling strategy if and only if he gains more profit from pooling than separating equilibrium:

$2(pH - C) + 2(L - C) \geq 2(L - C) - L + 2(L - C)$ Thus, for PE to exist it must be the case that:

$pH \geq \frac{L}{2}$ For a low-quality producer, he sets the price at pH in the first period and at L in the second period. His two-period profit is $2pH + L$. His maximum profit of two-period in a separating equilibrium is $L + L$ (more detail in Section 8), he won't deviate from this pooling strategy if and only if he gains more profit from pooling than separating equilibrium: $2pH + L \geq L + L$. Thus, for the PE to exist it must be the case that:

$pH \geq \frac{L}{2}$

Therefore, when pH is less than $\frac{L}{2}$, a high-quality producer would deviate from pooling. A high-quality producer would be more willing to advertise to separate himself from a low-quality producer. Pooling equilibrium doesn't exist. Separating equilibrium Pareto dominates the pooling equilibrium. When pH is greater than $\frac{L}{2}$, pooling equilibria Pareto dominates the separating equilibrium.

$\square$

APPENDIX I. IN THIS APPENDIX WE SUPPLY THE PROOF OF LEMMA (11)

Proof. Suppose Hybrid equilibria do exist. There exists a cut-off advertising level A^*. Let us define a high-quality firm spends on advertising with the probability $\alpha = Pr(A^* \mid H)$ and the firm with low-quality products spends on advertising with the probability $\beta = Pr(A^* \mid L)$. The producer has beliefs about how a consumer will eventually play: the producer is believed to produce the high quality with probability ϖ if $A(q) = A^*$ is observed, and the producer is believed to produce the high quality with probability ς if $A(H) = 0$ is observed. A consumer has beliefs about the type of the producer given the action she observes: these are $\rho = Pr(H \mid A(q) = A^*)$ and $\tau = Pr(H \mid A(q) = 0)$.

Hence by Bayes rule definition:

$$
\begin{aligned}
\rho &= \Pr(H \mid A(q) = A^*) \\
&= \frac{\Pr(A^* \mid H)\Pr(H)}{\Pr(A^* \mid H)\Pr(H) + \Pr(A^* \mid L)\Pr(L)} \\
&= \frac{\alpha p}{\alpha p + \beta(1-p)}
\end{aligned}
\tag{218}
$$

and

$$
\begin{aligned}
\tau &= \Pr(H \mid A = 0) \\
&= \frac{\Pr(A(H) = 0 \mid H)\Pr(H)}{\Pr(A(H) = 0 \mid H)\Pr(H) + \Pr(A(L) = 0 \mid L)\Pr(L)} \\
&= \frac{(1-\alpha)p}{(1-\alpha)p + (1-\beta)(1-p)}
\end{aligned}
\tag{219}
$$

When a high-quality producer chooses $A(H) = A^*$ and a low-quality producer chooses $A(L) = \beta A^* + (1 - \beta)0$, we have $\alpha = 1, \rho = \dfrac{p}{p + \beta(1 - p)}$, and $p < \rho < 1$.

Thus, the information set followed by $A = 0$ is reached only by a low-quality type, never by high-quality type, that is $\tau = 0, \varsigma = 0$. Given the above, the price in the Hybrid Equilibria must be as follows: If consumer observes $A(q) = 0$, $P_1^0 = L$, and if Consumer 1 observes $A(q) = A^*$, the highest price she would accept is in which her expected utility equals zero (See (221)):

$$E(V_C) = \rho(H - P_1) + (1 - \rho)(0 - P_1) \tag{220}$$

Then, we have:

$$P_1^{A^*} = \rho H \tag{221}$$

For a high-quality producer, he sets the pair either $(\rho H, A^*)$ or (L, A^*). On the basis of ASSUMPTION 4, there are three intervals:

- $\rho H \leq \frac{L}{2}$

  Both producers will benefit from deviating from the Hybrid Equilibria. The Hybrid Equilibria won't survive. Then, to achieve an advertising Hybrid Equilibrium, ρH must be greater than $\frac{L}{2}$.

- $\frac{L}{2} < \rho H \leq L$

  The high-quality producer sells products to Consumer 1 and 2 and sets the new product's price at ρH. The high-quality producer will want to spend $A(H) = A^*$ and won't deviate from this strategy if:

  $2(\rho H - C) - A^* + 2(L - C) \geq 2(L - C)$

  $A^* \leq 2(\rho H - C)$

  The low-quality producer chooses $(\rho H, A^*)$ with probability β, and chooses $(L, 0)$ with probability $1 - \beta$, so its profit is $(2\rho H - A^*)\beta + L(1 - \beta)$. From the second period, the consumer is fully informed. In an equilibrium, a low-quality producer charges $P_2^L = L$; the producer's profit is L. Thus, the firm's two-period profit is $(2\rho H - A^*)\beta + L(1 - \beta) + L$. Therefore, the low- quality producer won't unilaterally deviate from the mixed strategy if:

  $(2\rho H - A^*)\beta + L(1 - \beta) \geq L$

  $A^* \leq 2\rho H - L$

  Thus, the game has an infinite number of hybrid equilibria if (222) is satisfied:

  $$\begin{cases} A_{SE1}^* \leq min(2(\rho H - C), 2\rho H - L) \\ \rho H \leq \frac{L}{2} \end{cases} \tag{222}$$

- $\rho H > L$

  The high-quality producer sells products to Consumer 1 and 2 and sets the new product's price at L. The high-quality producer will want to spend $A(H) = A^*$ and won't deviate from this strategy if:

  $2(L - C) - A^* + 2(L - C) \geq 2(L - C)$

  $A^* \leq 2(L - C)$

  The low-quality producer chooses (L, A^*) with probability β, and chooses $(L, 0)$ with probability $1 - \beta$, so its profit is $(2L - A^*)\beta + L(1 - \beta)$. From the second period, the consumer is fully informed. In an equilibrium, a low-quality producer charges $P_2^L = L$; the producer's profit is L. Thus, the firm's two-period profit is $(2L - A^*)\beta + L(1 - \beta) + L$. Therefore, the low-quality producer won't unilaterally deviate from the mixed strategy if:

  $(2L - A^*)\beta + L(1 - \beta) \geq L$

  $A^* \leq L$

  Thus, the game has an infinite number of hybrid equilibria if (223) is satisfied:

  $$\begin{cases} A_{SE1}^* \leq min(L, 2(L - C)) \\ \frac{L}{2} < \rho H \leq L \end{cases} \tag{223}$$

By considering Pareto efficiency selection criterion, both types of producer choose $A_{SE1}^* = 0$. This is a pooling equilibrium. Hybrid equilibria are always Pareto dominated.

Consider the situation in which a high-quality producer chooses $A(H) = \alpha A^* + (1 - \alpha)0$ and a low-quality producer chooses $L \rightarrow A(L) = 0$, we have $\beta = 0, \tau = \dfrac{(1 - \alpha)p}{(1 - \alpha)p + (1 - p)}$, and $0 < \tau < p$.

Then, the information set followed by A^* is reached only by high-quality type, never by low-quality type, that is $\rho = 1, \varpi = 1$. Given the above, the price in the Hybrid equilibria must be as follows: If consumer observes $A(q) = A^*$, $P_1^{A^*} = L$, and if Consumer 1 observes $A(q) = 0$, the highest price she would accept is in which her expected utility equals zero (See (225)):

$$E(V_C) = \tau(H - P_1) + (1 - \tau)(0 - P_1) \tag{224}$$

Then, we have:

$$P_1^{A0} = \tau H \tag{225}$$

For a high-quality producer, he sets the price at either τH or L. On the basis of ASSUMPTION 4, there are three intervals:

- $\tau H \leq \frac{L+C}{2}$

  Hybrid Equilibria won't survive.

- $\frac{L}{2} < \tau H \le L$

  The high-quality producer sells products to Consumer 1 and 2 and sets the new product's price at τH. If the firm chooses (H, A^*) with probability α, and chooses $(\tau H, 0)$ with probability $1 - \alpha$, its profit is $(H - C - A^*)\alpha + 2(\tau H - C)(1 - \alpha)$. Thus, the high-quality producer won't unilaterally deviate from the mixed strategy if:

  $(2(L - C) - A^*)\alpha + 2(\tau H - C)(1 - \alpha) \ge 2(L - C) - A^*$

  $A^* \ge 2L - 2\tau H$

  The low-quality producer chooses $(\tau H, 0)$. From the second period, the consumer is fully informed. If the firm deviates from this strategy and chooses (H, A^*) in the first period, then the consumer believes it is high quality and purchases the good in the first period, yielding it a first year profit of $H - A^*$.

  Thus, the low-quality producer won't unilaterally deviate from the strategy if:

  $2\tau H \ge H - A^*$

  $A^* \ge H - 2\tau H$

  Thus, the game has an infinite number of hybrid equilibria if (226) is satisfied:

$$\begin{cases} A_{SE2} \ge max(2L - 2\tau H, H - 2\tau H) \\ \frac{L}{2} < \tau H \le L \end{cases} \tag{226}$$

- $\tau H > L$

  The high-quality producer sells products to Consumer 1 and 2 and sets the new product's price at L. If the firm chooses (H, A^*) with probability α, and chooses $(L, 0)$ with probability $1 - \alpha$, its profit is $(H - C - A^*)\alpha + 2(L - C)(1 - \alpha)$. Thus, the high-quality producer won't unilaterally deviate from the mixed strategy if:

  $(H - C - A^*)\alpha + 2(L - C)(1 - \alpha) \ge H - C - A^*$

  $A^* \ge H - 2L + C$

  The low-quality producer chooses $(L, 0)$. From the second period, the consumer is fully informed. If the firm deviates from this strategy and chooses (H, A^*) in the first period, then the consumer believes it is high quality and purchases the good in the first period, yielding it a first year profit of $H - A^*$.

  Thus, the low-quality producer won't unilaterally deviate from the strategy if:

  $2L \ge H - A^*$

  $A^* \ge H - 2L$

Thus, the game has an infinite number of hybrid equilibria if (227) is satisfied:

$$\begin{cases} A^*_{SE2} \geq H - 2L + C \\ \tau H > L \end{cases} \tag{227}$$

By considering Pareto efficiency selection criterion, both types of producer would be better off playing $A^*_{SE2} = 0$, which is a pooling equilibrium. Therefore, hybrid equilibria are always Pareto dominated.

$\square$

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