Essays on the impact of pollution externalities on economic activity

Alessandro Vescia

Department of Economics
University of Leicester

A thesis submitted for the degree of
Doctor of Philosophy at the University of Leicester.

March 2016

To your smile, which is the most precious thing I have

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by

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Abstract

This thesis considers how pollution affects economic activities. Chapters 2 considers effects of pollution on humans' health. In an overlapping generation model with capital accumulation, agents' status is negatively affected by pollution. Individuals may invest in private health to reduce the burden of the environmental degradation, but this reduces the aggregate savings. Lower savings reduce the capital accumulation dynamic, reducing the optimal growth of the economy. The government can intervene to improve agents' health with public health spending, which crowds out private health investment and is complementary to savings. This work shows that, according to the initial level of capital of the economy and to the "net dirtiness", i.e. the difference of the pollution elasticity with respect to output minus the public health spending elasticity, the economy experiences different long-run growth equilibriums. Chapter 3 evaluates and compare the capacity of an emission tax and of free issued (non-auctioned) permits in terms of the incentives in investing in emission abatement research and in the social welfare. In the model, firms compete *à la* Cournot with knowledge spillovers. There are two different timing of the game: one where the government can credibly commit to the level of environmental policy; and the second timing where the government cannot credibly commit, and adjust optimally the policy after the firms innovate. This work shows that firms invest more in research when the government can credibly commit to the chosen level of policy. Chapter 4 investigates the role of pollution as a source of income inequality. Blackburn and Chivers (2015), in an overlapping generation model without credit market imperfections but in presence of loss aversion and uncertain return of investment, model agents that inherited from their parent and leave as a bequest to their offspring a positive amount of human capital. If the human capital is below a certain threshold, the loss aversion strongly influence agents, thus avoiding the investment. This reduces their possibility of realising profits and agents may end up in a low-income growth equilibrium with persistent income inequality. We extend their model introducing the pollution flow, which reduces the productivity of human capital and an abatement policy, which mitigates the negative effect of pollution. This work

shows that in the presence of pollution, income inequality may increase and that the government can mitigate it through pollution abatement.

Acknowledgements

I wish to thank the University of Leicester and the Department of Economics for the great opportunity they offered me, accepting my candidature as a Ph.D. student in economics. They gave me the possibility to teach in University, allowing me to realize one of my greatest ambitions. Even if just as a Teaching Assistant, I am very grateful for the unmissable opportunity. My Ph.D. adventure would not have been possible without the constant and perspicacious guide of my Supervisor, Professor Vincenzo Denicolò. He gave me a continuous support, especially during the tough moment of the research life. I am extremely grateful to my second supervisor, Dr. Dimitrios Varvarigos. His knowledge and his patience with my attempts has offered to me not only a precious guide for my research activities, but also an outstanding example of dedication to the professional aspect of this job, an example that I will conserve throughout my career. I wish to thank also the academic staff for the continuous support, especially Dr. Subir Bose and Dr. Piercarlo Zanchettin, for the support and the help they gave me throughout these years. A special thank to Samantha Hill, for the invaluable administrative support she gave me during the time I spent in the University of Leicester. With the submission of this thesis, I ideally completed a journey that has started several years ago, in the school I attended during my academic life. I wish to thank all the professors I had, for having shared with me the passion for studying and the curiosity for new research. I wish to thank especially Professor Sanfile, for believing in me.

Such a tough journey would have not been possible without the marvellous people I met during these years. The friends that I know since I was born, Francesco and Mattia, like brothers to me. A friend that has kept looking for me, Emanuele. A special thanks to my beloved Daria, for sharing with me all her thoughts and not leaving me alone. I want to thank also Irene and her wonderful family, for sharing part of this journey with me. Thank also to the people I shared the office with during these years: Alex, Pasqua, Taha, Ioli, Zara, Sebastian, Yamin, Rui, Eleni, Deniz. Life would not have been the same without you. I have been blessed also with the companion of magnificent people. Vaggelis, for giving me strength and support. Giacomo, for our wonderful conversations. Matteo, for the precious suggestions and our chess matches. Sonia, for being so patient with me and for the closeness, even when it is very hard. Sneha, because we started together and I would not have been able to write this thesis without your help. I am sure that, with all of you, our friendship will last longer than this period we have spent together. I wish to thank also Roberto, Alberto and Alessio, for being so close even if they are far away. I wish to thank my second father, Maurizio Cerrato. He taught me to be strong and to honour with hard work all my activities. Finally, I want to thank the people without whom nothing of this would have been possible. My family, my aunt Silvana, my grandmother Lucia and my grandfather Rolando. My sister Giulia, to whom I do not show all the love that I feel for her. My father Sergio, the guide of my life.

And my mother Lucia, who made this journey with me, in every single moment.

Declaration

A version of Chapter 2 was presented under the title "Pollution and growth in a private and public health expenditure framework" at the following seminar:

- Internal PhD Conference - June 2013 - University of Leicester - Leicester

A version of Chapter 3 was presented under the title "Emission tax versus grandfathered permits in a Cournot competition with research spillovers" at the following seminar:

- Internal PhD Conference - October 2012 - University of Leicester - Leicester

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Chapter 1

Introduction

Environmental pollution is one of the worst and most diffused threats to human welfare and prosperity nowadays. Global warming has been widely accepted and recognised as a source of dramatic change to the ecosystem, whose aftermaths are difficult to measure. Despite the importance that this pressing problem has finally obtained, there are still many questions related to pollution and how it affects economic activities that remain unanswered. This thesis is an attempt to offer an answer to some of these questions.

Chapter 2 investigates the long-term growth in an overlapping generation model with capital accumulation and learning-by-doing externality *à la* Romer (1986), where pollution affects agents' health. Pollution is among the primary causes of premature death worldwide.[1] Air, water and soil pollution have been indicated among the primary sources of premature death, sever illness and morbidity. In our model, agents may invest privately in health care, to reduce the negative impact of pollution. Investing in private health reduces aggregate savings, necessary to guarantee a positive rate of capital accumulation. The government can intervene with public health expenditure. This generates a complementarity between savings and public health investment, since the latter acts as a subsidy for private spending for healthcare. Thus, the government intervention affects positively the capital accumulation dynamic. The interaction between the detrimental effects of pollution on agent's health and the positive effect of public health spending generates non-monotonic dynamics. The pollution elasticity with respect to the production function plays a crucial role. The elasticity can be considered as the "dirtiness" level of the adopted technology. When the pollution elasticity is lower than 1, the pollution flow grows less than proportionally with re-

[1]See the WHO report, 2012, on exposure to air pollution.

spect to the output production. When the elasticity is greater than 1, the pollution grows more than proportionally with respect to the output production. The Chapter shows the different possible long-run equilibria as a function of the initial capital endowment of the economy and the interactions between pollution elasticity with respect to the public health spending elasticity. The difference between the two elasticities can be considered as the "net dirtiness" of the adopted technology, which reveals crucial aspects of our model. When the pollution elasticity is greater than the public health spending elasticity, pollution is affecting agents' health. Without private health expenditure, the economy would be condemned to a low growth scenario. Since agents can spend for their private health, if the "net dirtiness" is not very large (the difference is within the unit), and the initial level of capital is above a certain threshold, the economy can reach non-monotonically and sustained long-term growth. This is possible thanks to the investment in public health undertaken by the government and by the learning-by-doing externality, that generates positive spillovers, increasing the productivity of capital. If the "net dirtiness" increases (the difference is above one), the detrimental effect of pollution on agents' health increases so heavily that, in the steady-state, all resources are devoted to maintain an acceptable health status. Thus, the economy reaches an asymptotically stable steady state. The greater the difference between the pollution elasticity and the public health elasticity, the greater the oscillations of the capital accumulation process in approaching the steady-state. If the difference is greater than 1 but limited, capital dynamics reaches the steady state non-monotonically but stably. If the difference increases, the accumulation process shows oscillations towards the steady-state, or a periodic cycle around it. Finally, when the elasticity of pollution is lower than the elasticity of the public health spending, the economy can reach a sustained long-term growth, if the initial level of capital is above a certain threshold. In this case the public health care is so efficient that agents can reduce the investment in private health and saving. The increase in aggregate saving and the positive effect of the learning-by-doing spillovers let the capital accumulation grows period after period.

The scope of this thesis is also to address the role of the government in setting optimal environmental policy. Through environmental policy, firms internalize pollution costs and receive the right incentives for investing in abatement research. In undergraduate courses, pollution is always considered as the typical example of market failure, caused by firms that do not internalize the social cost that their activity is provoking. Government intervention is not only desirable, but also necessary. Pollution is a wide-phenomena and the transaction costs are too high for the Coase theorem to be successfully applied.[2]

Chapter 3 considers a partial equilibrium model, with a large number of firms competing *à la* Cournot with research spillovers and non tournament innovation. The government intervenes in the economy introducing an environmental policy, to let firms internalize the pollution externality. A second scope of the government is to provide firms with the incentive to invest in emissions abatement research. It has been largely recognized that the environmental policies should be evaluated on the ground of the incentive they provide to firms to invest in clean technologies and emission reduction R&D. The technological change towards more sustainable methods of production is still considered as the best option we have to solve the trade-off between environmental quality and output growth. In the model presented in this chapter, the government can introduce either an emission tax or a free-permit (non auctioned). In the case of permit, the government chooses the optimal number of permits to be issued to firms for free. Permit allocation represents a cap to the emissions. If firms want to pollute more than the allocated number of permits, they have to buy the permits they need on the market. Firms that pollute below their permit allowances can sell on the market the excess number of permits, obtaining an extra profit.

Firms can invest in abatement R&D, but the incentives are reduced by the limited appropriability of research. In this model firms invest in R&D to reduce their marginal emissions, but there is no patent race. Firms do not invest to innovate and become a monopolist. The Chapter ranks the different policies according to the incentive they provide in terms of investment in

[2] Coase, 1960.

research and in terms of total welfare. The presence of the spillover parameter forces agent to under-invest in R&D, to avoid that other firms free-ride on their investment. There exists a negative relationship between the number of firms operating in the market and the aggregate investment in research. Environmental permits tend to perform better than the environmental tax for low number of firms in terms of incentive to invest in R&D and welfare. The situation is entirely in favour of the emission tax for higher number of firms. A possible intuition is represented by the possibility of free riding that permits offer with respect to the environmental tax. This effect is well known in the environmental policy literature (see Requate and Unold, 2003, or Denicolò, 1999, among others). If the number of firms investing in research is high enough, the demand for permits will decrease, as the firms' emissions rate drops. Thus, as the permit price decreases, also the incentives of polluting firms to buy permits decreases. This effect, absent under the environmental tax, explains why the environmental tax performs better than permits for larger number of firms. The model presented allows also for the comparison of different market structures (*à la* Kamien et al., 1992). Firms can decide to cooperate fully or partially on the R&D side. Following Kamiet et al. (1992), we can separate two different cases. If firms maximize the joint profit function but do not share information fully, then there is a Research Cartelization.[3] If firms share also information, then we have a Research Joint Venture. In our model we find that when cooperation takes place, aggregate research increases under both environmental policies.

Pollution can be seen also as a source of income inequality. Several authors shows evidence that poorest members of the society are the ones mostly exposed to environmental pollution.[4] Chapter 4 considers pollution as a possible source of permanent income inequality. Previous literature has focused mostly on income inequality in relation to growth (Piketty, 1997; Aghion and Bolton, 1997, among others).

The role of pollution as the cause, and not only the effect of income inequality,

[3]Poyago-Theotoky (2007) uses the same definition for the Environmental Research Cartelization (ERC), that we adopted in our model.

[4]See, among others, Bell et al. (2006).

has found little attention so far. This Chapter tries to fill this gap addressing this important issue in an overlapping generation model presented by Blackburn and Chivers (2015), where agents inherited from previous generations and leave as a bequest to future ones a certain amount of human capital. Once the agents receive the inherited human capital, they decide whether to undertake the investment or not. Given that the investment's outcome is uncertain, the agent may suffer a strong loss aversion and avoid the investment. The greater the agents' aspiration level, the higher the inherited human capital necessary to undertake the investment. If agents fail to invest in human capital, they miss a profitable opportunity, thus incurring in a lower growth path that may causes persistent income inequality. The Chapter shows that, by introducing pollution, income inequality may significantly increase. Furthermore, it provides a theoretical framework that explains the substantial evidence found in the literature, i.e. that pollution has a detriment effect on human capital accumulation. By reducing the final output, pollution reduces the profitability of investing in human capital. Thus, a higher inherited level of human capital is necessary to avoid loss aversion and undertake investments. This intuition is captured in our model through a change in the human capital threshold with respect to Blackburn and Chivers (2015). If the treshold is higher than the original model, more agents will be discouraged from investing, ending in a low growth path. Moreover, their bequests to the next generation will be low too, allowing for persistent high income inequality. If the threshold is lower, the opposite is true and the persistence of income inequality is lowered. This outcome is possible through the intervention of the government, by mean of abatement policies. The abatement policy reduces the detrimental impact of pollution on the production function. This increases the profitability of investing in human capital, and it reduces the loss aversion towards the possible investment failure. More individuals will borrow funds from the credit market to invest in human capital, ending in a greater growth path. Since these agents will leave richer bequests to their offspring, the income inequality is mitigated through generations. In the model, this corresponds to a reduction of the human capital threshold.

Finally, Chapter 5 concludes and summarises the results presented.

Chapter 2

Endogenous growth in a private and public health expenditure framework

Chapter Abstract

This paper introduces agents' health condition in a standard Overlapping Generation Model with capital. Young agents decide whether to allocate their wage towards savings or private health expenditure to benefit a better health condition once old. The government collects tax revenue to finance public health expenditure schemes, which reduce private investments and increase agents' savings. The pollution effect on agents' health and the intervention of the public sector generate non-monotonic dynamics of the capital accumulation function, which leads to complex (non chaotic) dynamics.

2.1 Introduction

There is an increasing consensus and an overwhelming literature that points out how the pollution generated by human activities represents a serious threat to humans' health. Pollution affects air, water and soil. Air pollution is considered among the major environmental risks to health. In 2012 the World Health Organizations' estimates revealed that around 3.7 million people died as a result of ambient air pollution exposure. A great majority of deaths occurred in low-and-middle income countries (LMI), where 88% of diseases are caused by ambient air pollution.[1] According to the WHO report, ambient air pollution is the major cause of ischaemic heart diseases (40% of total deaths due to air pollution), stroke (40%), chronic obstructive pulmonary disease (COPD) (11%), lung cancer (6%), and acute lower respiratory infections in children (3%), and it is also a recognized source of diseases as cancer of the urinary tract/bladder, and both chronic and acute respiratory diseases, including asthma.[2] According to Brook (2008), "...studies from across the world have consistently shown that both short- and long-term exposures to PM (particulate matter) are associated with a host of cardiovascular diseases, including myocardial ischaemia and infarctions, heart failure, arrhythmias, strokes and increased cardiovascular mortality".[3] Air pollution is ranked as the 13^{th} cause of mortality; moreover, long term exposure increases the chronic genesis of atherosclerosis (Brook, 2008). There is an increasing evidence that reducing pollution augments life expectancy. Pope III, Ezzati and Dockery (2009) showed that in the United States, reducing air pollution increases life expectancy up to 15%. Soil and water pollution both represent serious threats to humans' health, too. Evidence of an increasing soil pollution of polycyclic aromatic hydrocarbons (PAHs) and heavy metals (HMs) has been founded by Kahn et al., (Kahn et al., 2008), which are a direct cause for different types of cancer (Boström et al., 2002), if present in the atmosphere or in the food or water (Ramesh et

[1] Burden of disease from Ambient Air Pollution for 2012, WHO 2012.

[2] For a substantial critical review of the health effect of particulate matter (PM), especially on cardiopulmonary morbidity and mortality, refer to Pope III and Dockery (2006)

[3] Cardiovascular effects of air pollution, R. D. Brook, 2008, p. 175.

al., 2004). A deterioration of health not only reduces life expectancy, but it reduces propensity to consume too. Finkelstein et al., (2013) found that a one percent point increase of the chronic diseases corresponds to a 10 - 25% decline in the marginal utility of consumption, relative to the marginal utility of consumption of agents with good health. If the data clearly indicate that a negative relationship between water, air and soil pollution and health has been clearly established, less clear are the effects of pollution on long-term growth.

Our model tries to explain the consequences of health deterioration due to pollution on growth. The channel between growth and pollution is human health. The model we propose considers the morbidity caused by pollution that decreases agents' utility (as found by Finkelstein et al., 2013 and modelled by Pautrel, 2012a.). We introduce an OLG model with capital accumulation and learning-by-doing externality *à la* Romer (1986), where agents consume only when old. Agents work when young and decide whether to spend their wages for private health care to mitigate the detrimental effect of pollution in the second period of their life, or saving to consume in the next period. The older agents' utility is augmented by the health status, which increases with private and public health spending and decreases with pollution. Public health investments are financed with a linear tax on wages. Pollution is presented as a flow function.[4] Our main result are the following. If agents' health is not affected by pollution, the economy experiences a sustained long-run growth. The agents do not spend for private health; they only save to consume in the next period. The capital accumulation shows an increasing and monotonic dynamic, due to the positive spillover guaranteed by the learning-by-doing externality presented in the production function. When pollution affects agents' health, higher level of pollution reduces the utility of consumption of older agents through the health channel. To reduce the burden of pollution, young agents reduce their saving and increase the spending in private health. This reduces the capital accumulation and slows the economy growth. In this scenario, the difference of the elasticity of pollution with respect to the elasticity of public health spending plays

[4]Following Gradus and Smulder (1993) and Pautrel (2008).

a crucial role in determining the long-run equilibrium of the model. The elasticity of pollution measures the "dirtiness" of the technology adopted. The greater the elasticity, the stronger the detrimental effect of pollution on agents' health. The elasticity of public health, instead, measures the effectiveness of the public investment in improving agents' condition. The difference of the two elasticities can be considered as the "net dirtiness" of the technology adopted, i.e. the effective detrimental effect of pollution on the agents' health. If this difference is limited (i.e. the difference is within the unity), the economy may experience a long-run growth, given that the initial capital is above a certain threshold. In this case, agents are negatively affected by pollution, but the efficiency of the public health spending in reducing the detrimental effect is such that allows agents to do not spend all their resources for private spending but to save part of them to consume when old. This allows a non-monotonic, sustained long-term growth. If the initial level of economy is below the threshold, agents do not have enough resources to reach a minimum amount of saving able to guarantee a positive capital accumulation. The low level of growth reduces pollutants emissions, but they are not reduced to zero; thus, agents have to spend to mitigate the negative effects of pollution on their health. The agents do not have the necessary resource for a spending optimally in private health and to generate a sufficient level of saving to improve the capital accumulation, nor the government can raise the necessary tax revenues to improve agents' health through public health spending. The economy ends in a poverty trap. When the difference between the elasticity of pollution and the elasticity of public spending is greater than the unity, the economy experiences a long-run "no-growth" equilibrium. There is only one steady state and no poverty-trap is present here. The economy shows a non-monotonic capital accumulation dynamic, that ends either in an asymptotically stable long-run steady state or in an unstable steady-state (a limit cycle). The economy grows up to the point where pollutant emissions affect agents' health so heavily that all the extra resources produced have to be invested in private health care. The elasticity of public health spending is too low to guarantee a better health condition to agents. The level of saving is thus insufficient to guarantee a sustained growth. The interesting result of this scenario is about how the economy approaches the steady-state. The capital accumulation may experience non-oscillatory dynamics towards the steady-state, oscillatory but stable dynamics or a periodic cycle, according to the magnitude of the difference between pollution elasticity and public health elasticity. As the economy approaches the steady-state, the higher level of growth causes also a higher level of pollution. Agents are forced to reduce their savings to mitigate the negative impact of pollution on their health status. This slows down the economy; emissions are reduced too. Agents can now invest in savings, as the threat on their health is reduced. The economy grows again, as does pollution. This process can conducts the economy either to the steady-state (the oscillations tend to disappear period after period, up to the steady-state equilibrium, where the economy is asymptotically stable), or they can show a periodic cycle around the steady-state, where the aforementioned process continues indefinitely. Finally, when public health elasticity is greater than pollution elasticity, the economy may either experience a poverty-trap equilibrium, if the initial level of capital of the economy is below a certain threshold, or a sustained long-run growth, if the initial level of capital is above the threshold. In this case, the public health spending is so efficient with respect to the "dirtiness" of the technology adopted (or the technology adopted is very "clean"), that agents can limit their investment in private health care and save more, increasing capital accumulation and reaching a sustained long-run growth. In the long-run, the greater level of pollution is not detrimental to agents' health, given the public health spending efficiency, leading to a sustained long-term growth.

The closest literature to our paper is the new growth theory, where the capital accumulation process is affected by negative externalities due to pollution. The endogenous growth literature, indeed, created a new interest in issues relative to the long-run growth.[5] The different endogenous growth models offered a new theoretical framework able to describe the problematic inherent to a sustainable growth. The environmental regulation necessary to reduce pollution (tax, permits, etc), has a detrimental effect on growth.

[5]After the seminal contribution of Romer (1986, 1990) and Lucas (1988), the endogenous growth literature flourished. For a comprehensive review, see Aghion and Howitt, 1998.

In the endogenous growth model, the trade off between growth and a better environment can be partially mitigated. The stringency of the environmental regulation poses an extra cost to firms, but it also represents an incentive for them to invest in polluting-abatement research. The investment in research is the core of growth of endogenous models; thus, a more stringent regulation can be a strong incentive for firms to invest in research, which can leads to more growth and overcomes the detrimental effect of the regulation on firms' production.[6] Among others, first attempts to model an environmental externalities in endogenous growth model are Stockey (1998), John and Pecchenino (1994), Jones and Manuelli (1995), and Gradus and Smulders (1993). Stockey (1998) tried to answer the question whether the environmental issue would have represented a major obstacle to long-run growth or not, since increasing environmental cost tends to reduce the benefit of growth. She found an inverted U-share relationship between per-capita income growth and environmental quality. This relationship, known also as the Environmetnal Kuznets Curve (EKC), is a very challenged topic.[7] The question of whether a sustainable growth was possible in an overlapping generation model (OLG) with environmental pollution affecting multiple generations has been addressed by John and Pecchenino (1994), where young agents are taxed for environmental maintenance that can lead to multiple inefficient Pareto equilibria and to an overinvestment in environmental maintenance; and in Jones and Manuelli (1995), where a tax charged on the young agent reduces the capital production and so the environmental pollution, to have a greater environmental quality once old. Finally, Gradus and Smulder (1993) were among the first to use an endogenous growth model with human capital accumulataion *à la* Lucas (1988), with learning abilities affected by pollution.

A more recent literature have tried to unveil the connection between pollution and the economy dynamics using agents' health as the channel between long-term growth and pollution. This new approach has been possible following the seminal contribution of Blackburn and Cipriani (1998), the first

[6] For an exhaustive review of the sustainability in endogenous growth model see the Chapter 5 of Aghion and Howitt, 1998 and Smulders, 1999. For a more general review of the literature on the growth-environment nexus, see Brock and Taylor, 2004.

[7] For a comprehensive review of the rise and fall of the EKC in the economic literature see Stern, 2004.

to introduce mortality in an overlapping generation model. Using an OLG model were agents can spend time to raise their offspring and invest in private health to increase their life expectancy, they reveal the negative correlation between fertility and mortality with respect to income per capita. They allow for public health spending, which acts as a partial substitute for private health investment. In their paper, they do not model explicitly for pollution; the spending on health is essential to reduce the detrimental effect on child welfare of a broad series of causes, ranging from the "household production activity" (accident at home, hereditary disorder, etc), and negative externalities coming from production activities, including pollution among others. Following the seminal contribution of Blackburn and Cipriani (1998), several authors explicitly modelled a link between pollution as a cause of a reduction in life expectancy; among others, Jouvet et al., (2010), Gutierrez (2008), Mariani et al., (2010), Varvarigos (2010, 2013).

Varvarigos (2013), investigates how pollution affects volatility in an OLG model with pollution stock, abatement capital and endogenous life expectancy. There is a positive correlation between environmental quality, life expectancy and capital accumulation. The author finds that if the pollution rate of the economy is above a certain threshold, the economy shows oscillatory (i.e. growth volatility) dynamics towards the steady state, ending in a lower equilibrium than the equilibrium reached monotonically when the pollution rate is below the aforementioned threshold. Varvarigos (2010), shows multiple equilibria arising (and poverty traps) in an OLG model with endogenous life expectancy. Agents can reduce detrimental effects of pollution on life expectancy through investment in health care. The role of the government is to collect taxes to fund public health investment. As in our model, the author focuses on the "environment-related parameters" such as the elasticity of the environmental damage due to pollution and the elasticity of environmental improvements due to abatement policies, that can lead to multiple equilibria (including a poverty trap). Gutierrez (2008) considers an OLG model with pollution externalities, formed by a pollution stock function that affects agents' health when old. The pollution-growth nexus is given by the incentive of young agents to save to have the resources necessary to spend more once old. Higher savings mean a greater capital accumulation and so more growth, which increases the amount of pollution. She focuses on an optimal tax able to reduce the dynamic inefficiencies caused by the presence of pollution. Jouvet et al., (2010), analyse in an OLG model the ambiguous effect of public health spending to reduce the impact of pollution on agents health. The increase in agents' welfare given the increased life expectancy is reduced by a congestion effect due to a longer life span. They find that in the decentralized equilibrium it is optimal to tax capital income and to subsidy rather than tax health spending. Mariani et al., (2010) find that an OLG model with life expectancy and private spending for environmental maintenance leads to multiple long-run growth equilibrium and poverty trap. Government is not spending for public health, and there is human capital accumulation. Parents care for their offsprings' education, so human capital and environmental quality are substitutes. It is worth to notice that pollution does not affect learning abilities. A greater human capital accumulation increases growth, which increases pollution.

A second line of research introduces the human capital accumulation as the engine of long-term growth in OLG model with endogenous mortality. Mortality increases (or agents' health is reduced) by the increasing of pollution.[8] The authors that contributed to this line of research are Aloi and Tournemaine (2011), Pautrel (2008, 2012a, 2012b), among others.

Pautrel (2008) in a continuous time OLG model a la' Blanchard (1985) shows that an endogenous mortality caused by pollution diminishes the negative

[8] They move from the seminal papers of Blackburn and Cipriani (2002) and Chakraborty (2004), who treated endogenous life expectancy with growth based on human capital accumulation, which is negatively affected by lower life expectancy. They did not model explicitly for pollution, but considered instead more generic threats to humans' health. Blackburn and Cipriani (2002) modelled an OLG with three periods, with endogenous life expectancy and human capital investment. Longer life expectancy increases the opportunity cost of rising children instead of investing in human capital. This explains the lower fertility rate and the higher life expectancy in higher income per capita level country. Chakraborty (2004) in an OLG model with endogenous mortality finds that high mortality societies do not grow fast due to the detrimental effect of shorter life expectancy on the saving rate and on the investment in human capital, as the productivity of investment is reduced. Poor economic conditions affect positively the mortality rate, which is reduced by public investment in health. Economies may end up in poverty trap if the initial level of human capital is below a certain threshold.

consequences on growth due to the stringency of environmental pollution, even if pollution does not affect directly human capital accumulation. A higher level of pollution increases mortality, reducing human capital accumulation at an aggregate level. A minimum level of environmental policy is required to have a positive growth. Pautrel (2012a) instead of focusing on mortality, focuses on morbidity and illness caused by pollution. Agents can spend for private health and government undertakes abatement activities funded with an environmental tax. Finally, in Pautrel (2012b) the author takes into account the "generational turnover effects".[9] Considering that new generations have lower financial asset than the dying ones, they reduce the consumption and so the growth rate of the economy. The author shows that this "generational turnover effects" itself is able to reduce pollution. Aloi and Tournemaine (2011) tried to unveil the link between health and growth through an endogenous growth model with investments in R&D. The focus on their paper is to unveil the connection between health status and growth, with the presence of an environmental regulation. A stricter environmental policy results in a lower pollution level and so in higher productivity due to the better health status of the agent. This helps the economy to reduce the cost of the environmental regulation. Of course a tighter environmental regulation increases incentives to invest more in R&D.

Most recent contributions are Mathieu-Bolh and Pautrel (2016), Pautrel (2015) and Wang, Zhao and Bhatthacharya (2015). Mathieu-Bolh and Pautrel (2016) introduce in a three period OLG model agents' choice about their retirement; they make an explicit link between productivity of labour and health over the life cycle. Labour between young and old is supposed to be substitute or complement. Agents invest time in health when young, to collect benefit once old in terms of better health.[10] The introduction of a retirement choice allows the authors to mitigate the negative effect of the environmental tax on investment (the crowding-out effect). Pautrel (2015), in an OLG *à la* Blanchard (1985) shows the effect of different technologies used in the abatement sector. The level of human capital required in the

[9]See also Pautrel, (2009).

[10]The time considered by the authors is the time spent by agents in health improving activities, as sports discipline, training, sleeping, etc.

abatement technology is crucial to determine the effect of the environmental policy, with respect the final output. Finally, Wang, Zhao and Bhatthacharya (2015) introduce in an OLG model two different private health spendings: a full-private health insurance and a pay-as-you-go publicly funded health insurance. Agents invest in their health when young, to have a good health status once old. Older agents can spend for their health too. So, if expectations of high pollution level increase, young will save more to face the future higher health expenditure. Thus higher saving forms higher capital, creating an environment growth nexus. Pollution increases the likelihood of death, while in other papers the connection between pollution and increase in mortality is certain.

Finally, interactions between private and public health expenditure in a growth model with agents facing a limited survival probability have been investigated by Bhattacharya and Qiao (2007) and Varvarigos and Zakaria (2013),[11] although in their analysis the pollution threat to agents' health is absent. Bhattacharya and Qiao (2007) present an OLG model with capital accumulation with agents facing a limited survival probability. In their framework agents can spend in private health to increase their life expectancy. They also introduce a public health investment, complementary to private spending. The greater the public investment, the higher the productivity of private spending for health care.[12] A longer life expectancy increases the saving rate of young agents, which increases investment that, combined with a healthier workforce, has a positive effect on the growth rate. The complementarity between private and public health spending shows a trade-off between saving and private health expenditure, which generates non monotonic capital dynamics, that can result in chaotic equilibria. Varvarigos and Zakaria (2013), model an OLG framework with fertility and endogenous mortality, where agents can consume in both periods (not only when old as in Bhattacharya and Qiao, 2007), and the investment in private health occurs only in the second period of agents' life. They show that a higher public spending, by increasing the life expectancy, augments also savings and pri-

[11] Moving from the seminal contributions of Blackburn and Cipriani (2002)

[12] The authors consider the public spending in health care as spending for infrastructure, medical R&D, better preparation of medical staff, etc.

vate health spendings, due to the increased productivity in both investment. With respect to the aforementioned authors, in our model there exists a trade-off between private and public health spending. Public health investments crowd out private health expenditures. This is a novelty with respect to both the authors mentioned above. Moreover, we find a complementarity between public health spending and savings. With higher level of public health spending, agents can increase their saving, as the government replaces their efforts for mitigating the negative effects of pollution.

The closest to our paper is Palivos and Varvarigos (2016). In a OLG model with learning-by-doing externalities *à la* Romer (1986) with public expenditure for health care and pollution abatement, they show the existence of multiple equilibria and even limit cycles given that the limitless environmental degradation and the decrease in longevity introduces non-monotonicities in the dynamic of capital accumulation. They consider public health spending (without private health spending), and pollution abatement, which can be either a process-integrated or an end-of-pipe technology abatement. Our model differs from Palivos and Varvarigos (2016) as we do not include endogenous mortality, but the morbidity caused by pollution as the element that reduces consumption utility of old agents. Moreover, in our model agents consume only in the second period. In their framework technology abatement is necessary to reduce the dynamics volatility of capital accumulation and to have sustained long-term growth. In their end-of-pipe abatement technologies scenario, they also model an interaction between elasticities similar to ours. They consider the elasticity of public investment in health and the elasticity of the environmental degradation, which is the result of pollution minus the application of the end-of-pipe abatement technologies. Their model shows similar results to our case when pollution abatement activities are absent. Our model is different since we are able to guarantee a positive long term growth when the elasticity of pollution is greater than the elasticity of public health spending (when the difference is within the unit), thanks to the role of the private health spending in mitigating the negative effects of pollution on agents health, which is absent in their model. Our framework allows for a long-term growth even when pollution has a stronger detrimental effect on agents' health than the public health spending, as long as the initial level of capital is above a certain threshold. In Palivos and Varvarigos (2016), with end-of-pipe abatement capital and the elasticity of pollution greater than the elasticity of public health spending, the economy can experience a positive long run growth only if the initial level of capital is above a threshold and abatement policies are in place; while our model guarantees a positive growth even without abatement policies.

The rest of the paper is organized as follows. Section 2 introduces the model and the solution of the agent's participation problem. Section 3 shows the temporary equilibrium, while section 4 shows the main dynamics of the economy. Proofs are provided in the Appendix.

2.2 The Economic Framework

In the subsequent analysis we develop an overlapping generation model where agents consume only in the second period what they have saved in the first one. We introduce a health status index h_{t+1}. The health function augments the utility that old agents derive from consumption.[13] In this framework there are only two generations, young and old ones. People work when young, earning a competitive wage that can be spent in private health care x_t or in saving S_t.

The agent's utility function is equal to:

$$U(h_{t+1}, C_{t+1}) \tag{2.1}$$

where $U'_{h_{t+1}}(h_{t+1}, C_{t+1}) > 0$, $U'_{C_{t+1}}(h_{t+1}, C_{t+1}) > 0$, $U''_{h_{t+1}}(h_{t+1}, C_{t+1}) < 0$, $U''_{h_{t+1}}(h_{t+1}, C_{t+1}) < 0$, $U(0) = 0$, $U(0, C_{t+1}) = 0$, $U(h_{t+1}, 0) = 0$ and C is

[13] Similar to Varvarigos and Zakaria (2013). In Bhattacharya and Qiao, (2007), the utility function is augmented by life expectancy, while the health index does not enter directly the utility function. Moreover, it follows the evidence provided by Finkelstein et al., (2013), about the existence of a correlation of deterioration of health and decrease in utility of consumption.

consumption. The agent's health index is represented by h_{t+1}, equal to:

$$h_{t+1} = (\gamma x_t g_t^{\theta} - e v_t)^{\epsilon} \tag{2.2}$$

with $\epsilon \in (0, 1)$, $\gamma > 0$, and $\theta > 0$; x_t and g_t are respectively private and public health spending, $e \in [0, 1]$ is a parameter that measures the magnitude of the impact of pollution v_t on the agents' health h_{t+1}. A minimum level of public spending is necessary to provide the initial health infrastructure and it increases the efficiency of the private spending. The net wage is $\hat{\omega}_t = \omega_t(1 - \tau)$, where τ is the income tax used by government to collect revenues that are entirely spent for public health spending (ω_t is the gross wage). The public health spending per capita can be represented as:

$$g_t = \frac{\Upsilon_t}{L_t} \tag{2.3}$$

where $\Upsilon_t = \omega_t \tau L_t$ is the gross capital spending. Second period consumption is given by:

$$C_{t+1} = (1 + r_{t+1}) S_t \tag{2.4}$$

where $S_t = \hat{\omega}_t - x_t$. For simplicity we can rewrite the utility function (2.1) as $U(h_{t+1}C_{t+1}) = h_{t+1}C_{t+1}$. By substituting (2.2) and (2.4) into (2.1) we obtain:

$$h_{t+1}C_{t+1} = (\gamma x_t g_t^{\theta} - e v_t)^{\epsilon}(1 + r_{t+1})(\hat{\omega}_t - x_t) \tag{2.5}$$

The utility function U_t is concave with respect to x_t.[14] The pollution function

[14]The first derivative $\frac{\partial U()}{\partial x_t} = 0$ is:

$$\epsilon(\gamma x_t g_t^{\theta} - e\rho_t)^{\epsilon-1} \left(\gamma g_t^{\theta}\right)^2 (1 + r_{t+1})(\hat{\omega}_t - x_t) + (\gamma x_t g_t^{\theta} - e\rho_t)^{\epsilon} \left(-(1 + r_{t+1})\right)$$

and the second order derivative $\frac{\partial^2 U(.)}{\partial x_t^2}$ is equal to $\epsilon\gamma g_t^{\theta}(\hat{\omega}_t - x_t) - \gamma g_t^{\theta} + e\rho_t$, which, rearranged, is equal to:

$$-\epsilon\gamma g_t^{\theta} - \gamma g_t^{\theta} < 0$$

v_t is equal to:[15]

$$v_t = \mu Y_t^{\phi} \tag{2.6}$$

with $\mu > 0$, which represents the damage coefficient given the production Y_t, (the sensibility of the agents to pollution). It is worth to notice that:

$$\frac{\partial v_t}{\partial Y_t}\frac{Y_t}{v_t} = \phi$$

i.e., $\phi > 0$ is the elasticity of the pollution function. We can consider ϕ as the "dirtiness" of the technology used in the industry.[16]

When $\phi \in (0, 1)$ the elasticity of pollution with respect to production is less than proportional. The pollution function v_t is monotonically increasing, i.e. the incidence of pollution on the morbidity caused by the environmental externalities increases, but less than proportionally to the production Y_t. In this case, the curve resembles the typical Environmental Kuznets Curve, concave with respect to the level of output of the economy. For values of $\phi > 1$, the pollution function is convex and the incidence of pollution on agents' illness is more than proportional with respect to Y_t. The parameter ϕ becomes then crucial to unveil the implications for the stability of the model in the long-term.

The production function Y_t is a typical Cobb-Douglas with a learning-by-doing externality:

$$Y_t = AK_t^{\alpha}(Q_t L_t)^{1-\alpha} \tag{2.7}$$

where $Q = \left(\frac{\bar{K_t}}{L_t}\right) = \bar{k_t}$ (Frankel, 1962; Romer, 1986), i.e. the productivity of labour is augmented by the stock of capital per worker. In equilibrium $L_t = 1\ \forall t$, since each worker supplies one unit of labour inelastically and we have a constant population normalised to one. In the intensive form (7) becomes $y_t = Ak^{\alpha}Q_t^{1-\alpha}$, where $y_t = \frac{Y_t}{L_t}$ and $k_t = \frac{K_t}{L_t}$.

[15] I treat the pollution as a flow variable, following, among others, Pautrel (2008) and Jones and Manuelli (2001).

[16] Bhattacharya and Qiao (2007) consider the elasticity of longevity with respect to private investment in health, which is influenced by public health investment. We use a similar definition of "dirtiness" as Varvarigos (2010); in his work, pollution function abatement technologies are present too, which are absent in ours.

2.3 Optimal health spending

Given the concavity of $U(\cdot)$ with respect to x_t, we can maximize the utility function (2.5) to obtain the optimal level of private health spending x_t^*. The first derivative of $U()$ with respect to x_t gives us the optimal private health spending x_t^*, equal to:

$$x_t^* = \frac{\epsilon\gamma\hat{\omega}_t + \frac{ev_t}{g_t^\theta}}{\gamma(1+\epsilon)} \tag{2.8}$$

The optimal level of private health spending x_t^* increases with respect to gross wage $\hat{\omega}_t$, with respect to the pollution function v_t and it is positively related to the elasticity ϵ; it decreases with respect to the public health spending g_t; the higher public spending crowds out the private health spending, contrary to Bhattacharya and Qiao (2007) and Varvarigos and Zakaria (2013). While in their model an increase of public health investment augments the life expectancy and so the productivity of private health, in our model private and public health can be considered substitutes.[17] We can substitute x_t^* into (2.4) to obtain the optimal saving function S_t^*, equal to:

$$S_t^* = \hat{\omega}_t - x_t^*$$

which, with appropriate substitutions, becomes:

$$S_t^* = \frac{\gamma\hat{\omega}_t - \frac{ev_t}{g_t^\theta}}{\gamma(1+\epsilon)} \tag{2.9}$$

The saving function S_t^* increases for higher level of wage $\hat{\omega}_t$ (both second period consumption, saving and private health spending are normal goods), and it decreases for higher level of pollution v_t. The intuition is straightforward: higher level of pollution forces young agents to spend more in private health care x_t, thus decreasing the level of saving. Public health spending g_t increases the level of saving; by increasing the productivity of the optimal private health spending x_t^*, it is possible to obtain the same level of health

[17] Our result is close to Brown and Finkelstein (2008), who showed that even an incomplete public funded health insurance (as Medicair) can have a crowding out effect for private health insurance.

care with less private resources.[18] Given the hypothesis of perfect competitive markets and the assumption that $L_t = 1$ in equilibrium (so $L_t = 1$, $\frac{K_t}{L_t} = \bar{k}_t = k_t$), the equilibrium gross wage ω_t is equal to:

$$\omega_t = (1 - \alpha)Ak_t \tag{2.10}$$

By substituting $\hat{\omega_t} = \omega_t(1 - \tau)$, we have the equilibrium net wage:

$$\hat{\omega}_t = (1 - \tau)\omega_t = (1 - \tau)(1 - \alpha)Ak_t \tag{2.11}$$

By appropriate substitution we obtain:

$$g_t = \tau\omega_t = \tau(1 - \alpha)Ak_t \tag{2.12}$$

Given $g_t = \frac{\Upsilon_t}{L_t}$, the ratio of the pollution function v_t with respect to the public health spending g_t is equal to:

$$\frac{v_t}{g_t^{\theta}} = \frac{\mu A^{\phi-\theta} k^{\phi-\theta}}{[\tau(1 - \alpha)]^{\theta}} \tag{2.13}$$

The capital accumulation function $K_{t+1} = S_t L_t$ in equilibrium becomes

$$k_{t+1} = \frac{K_{t+1}}{L_t} = S_t \tag{2.14}$$

By substituting (2.3), (2.6), (2.9), (2.11) and (2.12) into (2.14) we obtain:

$$k_{t+1} = \frac{(1 - \tau)(1 - \alpha)A}{(1 + \epsilon)} k_t - \frac{e\mu A^{\phi-\theta}}{\gamma(1 + \epsilon)\left[\tau(1 - \alpha)\right]^{\theta}} k_t^{\phi-\theta} = Q(k_t) \tag{2.15}$$

[18]This result is in line with Varvarigos and Zakaria (2013), while in Bhattacharya and Qiao (2007) savings decreases for higher level of public health investment.

We can use composite parameter terms to simplify the analysis:

$$\Gamma = \frac{(1-\tau)(1-\alpha)A}{(1+\epsilon)}$$
$$\Lambda = \frac{e\mu A^{\phi-\theta}}{\gamma(1+\epsilon)\left[\tau(1-\alpha)\right]^{\theta}}$$

We can rewrite (2.15), which becomes:

$$k_{t+1} = \Gamma k_t - \Lambda k_t^{\phi-\theta} = Q(k_t) \tag{2.16}$$

where:

- $Q^{'}(k_t) = \Gamma - (\phi-\theta)\Lambda k_{t+1}^{\phi-\theta-1}$

- $Q^{''}(k_t) = -(\phi-\theta-1)(\phi-\theta)\Lambda k_{t+1}^{\phi-\theta-2}$

2.4 Dynamic Equilibrium

The dynamic of the capital accumulation of the economy is given by:

$$K_{t+1} = S_t L_t$$

In the subsequent analysis, we distinguish whether pollution affects agents' health or not. When environmental pollution does not affect agents' health, i.e. $e = 0$, agents do not have to spend for private health, as they do not need to face any detrimental effect from pollution. When pollution affects agents' health, i.e. $e > 0$, agents need private health investment, to reduce the detrimental effect on their health. We can distinguish among different cases according to the possible values of the difference of pollution elasticity and public health spending elasticity $(\phi - \theta)$. The difference $(\phi - \theta)$ can be considered as the relative "net dirtiness" of the adopted technology, i.e. how polluting is the technology with respect to production Y_t (and so how heavily it affects agents' health), decreased by the health improving effect of public spending, represented by the elasticity θ. This is the major novelty of this paper.[19] The incidence of the adopted technology "dirtiness" on agents' health, represented by ϕ, is the cause of the reduction in capital accumulation. Agents are forced to reduce their savings and to increase the private health investment to reduce the negative impact on pollution when they will be older. The "dirtiness" effect can be mitigated by public health expenditure. The greater the elasticity of public intervention, i.e. the more efficient the investment made by the government on agents' health, the lower the detrimental impact of pollution on health. Agents divert their resources from the private health spending and increase their savings to consume more once old, as the utility derived from consumption is augmented by the health index. The "net dirtiness" $(\phi - \theta)$ allows us to capture this crucial effect into our model, and it is at the base of the different scenarios that we analyse. It may takes different values. It can be (i) $(\phi - \theta) \in (0, 1)$, (ii) $(\phi - \theta) > 1$ or (iii) $(\phi - \theta) < 0$. In the first case (i), the elasticity of pollution is greater than the elasticity of the public health spending, but the difference is limited. In this case the agents are facing a detrimental effect of pollution on their health. The public health expenditure is partially effective in mitigating the negative effect of pollution. In the second case (ii), either the "dirtiness" of the adopted technology is very high, or the efficiency of public health expenditure is very low. In this case, the effect of pollution on agents' health can be extremely harmful, leading to significant reduction on agents' condition. Finally, in the last case (iii), the technology adopted can be considered "environmental-friendly", or the efficiency of public health investment is at its highest with respect to the harmful effect of pollution. The "net-dirtiness" reveals that public health spending efficiency is more than sufficient to overcome the negative effect of pollution on agents' health. This let agents invest their resources in savings and increase the capital accumulation.

[19] Bhattacharya and Qiao (2007) consider only the elasticity of public health spending. Palivos and Varvarigos (2016) use a similar framework with the elasticity of public health spending and the elasticity of environmental degradation (net pollution) in presence of end-of-pipe technology abatement, but in our model the "net dirtiness" offers a richer results.

For the subsequent analysis we need the following Assumption:

Assumption 1: $\Gamma > 1$

Assumption 1 implies that:

$$A > \frac{1+\epsilon}{(1-\tau)(1-\alpha)}$$

The level of total factor productivity A should be greater than a certain level to guarantee a positive growth in the baseline scenario, when pollution is not affecting agent's health ($e = 0$), in order to rule out the non-interesting case of no growth. It is possible to consider it as the minimum amount of input factor productivity necessary to assure to the economy the possibility of growing.

2.4.1 Baseline scenario: $e = 0$

Suppose $e = 0$. In this scenario, emissions resulting from production activities do not affect agents' health status. In this case we can set $\Lambda = 0$, then $Q(k_t) = \Gamma k_t$. There is only one single steady state, $\hat{k} = 0$, which is unstable given that $Q'(\hat{k}) = \Gamma > 1$.

There is a balanced and positive growth. If we consider the growth process equal to $\frac{k_{t+1}-k_t}{k_t}$ it becomes $\Gamma - 1 > 0$, which is always positive given Assumption 1. We can summarise this result in the following Proposition:[20]

Proposition 1: *When emissions do not affect the agents' health status, the economy shows a positive and balanced growth as long as Assumption 1 holds and $k_0 > 0$. (This result resembles the typical OLG model with endogenous growth and without externalities).*

When agents' health status is not affected by emissions level ($e = 0$), if Assumption 1 holds the economy is productive enough to guarantee a positive balanced growth period after period. Agents do not need to spend for

[20]The proof of this and the following propositions are in the Appendix.

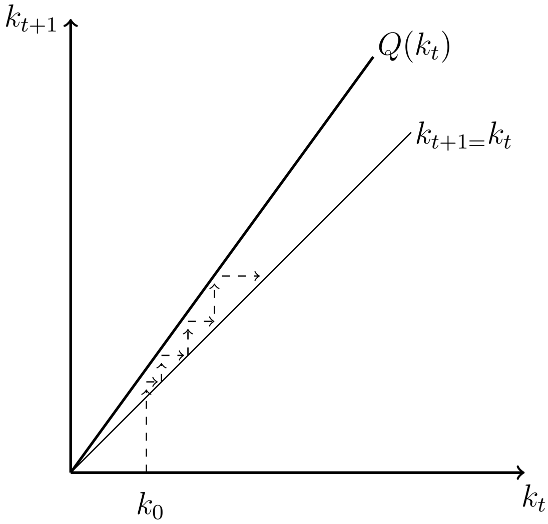


Figure 2.1: Baseline scenario with e=0

private health, thus wages are saved and invested in capital accumulation. The presence of learning-by-doing externalities in the production function guarantees that the social marginal returns to capital are high enough to obtain long-term growth. The result is the typical outcome of the classic OLG model without negative externalities, based on the work of Allais (1947), Samuelson (1958) and Diamond (1965). In our model we enrich their seminal contributions with the introduction of learning-by-doing externalities, which guarantee the existence of positive growth in the long-run. The outcome is shown in Figure 2.1.

2.4.2 Agents' health is affected by emissions: $e > 0$ and $(\phi - \theta) > 0$

Now we assume $e > 0$; agents' health is affected negatively by emissions. In this scenario, agents need to spend in private health, reducing their savings. We have two steady state: the trivial $k = 0$, which is unstable and an interior steady-state $\hat{k} \neq 0$, which is equal to:

$$\hat{k} = \left[\frac{\Gamma - 1}{\Lambda}\right]^{\frac{1}{\phi-\theta-1}} \tag{2.17}$$

Given the steady-state $\hat{k}$, we can distinguish between three different scenarios, according to the different values that $(\phi - \theta)$ may take

i) $(\phi - \theta) \in (0, 1)$

ii) $(\phi - \theta) > 1$

iii) $(\phi - \theta) < 0$

2.4.3 1^{st} Scenario: $(\phi - \theta) \in (0, 1)$

In the first scenario we have $(\phi - \theta) \in (0, 1)$. The difference between the elasticity of pollution ϕ and the elasticity of public health θ is positive but lower than one. Accordingly, the steady state (4.23) can be rearranged as:

$$\hat{k} = \left[\frac{\Lambda}{\Gamma - 1}\right]^{\frac{1}{1-(\phi-\theta)}} \tag{2.18}$$

By setting $Q^{'}(k_t) = 0$ we can define the minimum of $Q(k_t)$ as $\tilde{k}$, such that $\Gamma - (\phi - \theta)\Lambda\tilde{k}^{\phi-\theta-1} = 0$; [21] we can rearrange it as:

$$\tilde{k} = \left[\frac{(\phi - \theta)\Lambda}{\Gamma}\right]^{\frac{1}{1-(\phi-\theta)}}$$

We can now define the shape of $Q(k_t)$ when $(\phi - \theta) \in (0, 1)$:

$$Q^{'}(k_t) = \begin{cases} < 0 \text{ for } k_t < \tilde{k} \\ > 0 \text{ for } k_t > \tilde{k} \end{cases}$$

It is important to notice that there are two levels of capital k for which $Q(k_t) = 0$. These two are $k = 0$ and $k^* = \left[\frac{\Lambda}{\Gamma}\right]^{\frac{1}{1-(\phi-\theta)}}$. From the above, it is clear that $\tilde{k} < k^* < \hat{k}$. We can rule out negative values of $Q(k_t)$, hence:

[21] When $(\phi - \theta) \in (0, 1)$, $Q^{''}(k_t) > 0$.

$$Q^{'}(k_t) = \begin{cases} 0 \text{ for } k_t < k^* \\ \Gamma k_t - \Lambda k^{\phi-\theta_t} \text{ for } k_t > k^* \end{cases}$$

At the steady state, the first derivative of $Q^{'}(\hat{k}) > 1$. The steady state is unstable

We can present these results formally.

Proposition 2: *Given* $k_0 > 0$,

i) if $k_0 < \hat{k}$ *the economy converges to the poverty trap* $k_t = 0$

ii) if $k_0 > \hat{k}$ *the economy converges to a"positive growth" equilibrium, where* $k_{t+1} > k_t$.

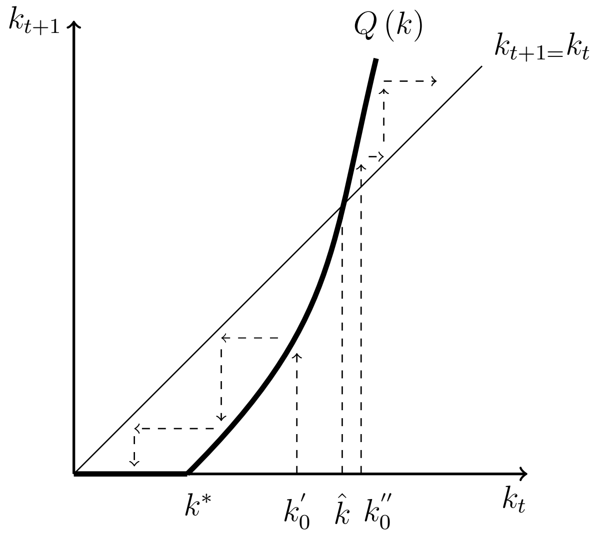


Figure 2.2: Long-term growth when $e > 0$ and $(\phi - \theta) \in (0, 1)$

In Figure 2.2 it is possible to see that $\hat{k}$ represents a capital threshold. An increase in k_t has two effects on k_{t+1} through savings: a positive linear case through Γ and a negative one through health damages. The latter is not constant, it is more pronounced at lower k_t; this is why positive capital accumulation can only be achieved at relatively high values of k_t. Indeed, when $k_0^{'} < \hat{k}$, the economy converges toward a poverty trap equilibrium up to $k_{t+1} = k_t = 0$. When the initial capital is lower than the threshold level, although total pollution is relatively low, the economy cannot produce enough income to overcome the more pronounced negative effects of the morbidity due to pollution. Agents are forced to spend the low income in private health; savings are not enough to guarantee a sufficient level of capital and the economy slowly backs up to the "poverty-trap" steady state. The public health spending is not enough to increase agents' health either, due to the economy low income. The economy is trapped in the underdeveloped equilibrium. If $k_0^{''} > \hat{k}$, then the economy has enough capital accumulation to escape the poverty trap and to maintain a sustainable and positive growth overtime. The positive linear effect of Γ overcomes the negative one given by the health damage. When the initial endowment of capital is above the threshold, the economy can generate enough revenues to allow agents to spend in private health care and to save up to the creation of a continuum flow of savings able to sustain the capital accumulation required to have a positive growth. Pollution is affecting population's health but, given the low pollution intensity of the production sector (or the high elasticity of the public health spending), the morbidity caused by pollution is not sufficient to force agents to diverts the majority of their resources towards private health spending. The final result is a sustained long-term growth equilibrium, which guarantees $k_{t+1} > k_t$ period after period. This result is absent from the case with end-of-pipe pollution abatement analysed in Palivos and Varvarigos (2016). In their model, when pollution abatement is absent and the elasticity of environmental deterioration is greater than the elasticity of public health spending, no positive growth is possible. To have a sustained growth, the presence of pollution abatement is necessary. In our model we show how the possibility of agents to spend for private health guarantees a positive level of long-term growth, as the detrimental effect of pollution is mitigated both by public and by private health investment.

2.4.4 2^{nd} Scenario: $(\phi - \theta) > 1$

When the difference between pollution elasticity ϕ and public health spending elasticity θ is greater than 1, different scenarios may occur. We can present these results in the following Proposition:

Proposition 3: *Consider $k_0 > 0$. Then:*

(i) If $(\phi - \theta) \in \left(1, \frac{\Gamma}{\Gamma - 1}\right)$ the economy converges to the asymptotically steady state.

(ii) If $(\phi - \theta) \in \left(\frac{\Gamma}{\Gamma - 1}, \frac{\Gamma + 1}{\Gamma - 1}\right)$ the economy shows an oscillatory but convergent trend toward the steady-state .

(iii) If $(\phi - \theta) > \frac{\Gamma + 1}{\Gamma - 1}$ we have limited cycles around the steady state.

Results are shown in Figure 2.3 - 2.5.

The steady state $k = 0$ is unstable, given Assumption 1, which guarantees that the productivity of the economy is sufficient to have a positive growth even with low level of capitals, thanks also to the learning-by-doing spillovers, which guarantee a minimum return of social capital high enough for a positive capital accumulation. The intuition for Proposition 3 is the following. When $(\phi - \theta) > 1$, the negative effect of k_t on k_{t+1} due to the health damaged is more pronounced compared to the constant Γ at relatively high levels of k_t. This is what generates the non-monotonic effects that are responsible for cycles. As the difference $(\phi - \theta)$ increases, the capital accumulation dynamic shows increasing oscillation in approaching the steady-state, up to the limit case of periodic cycle. The harmful effect of pollution increases period after period. Thus, the increase in capital accumulation increases pollutant emissions, which affects heavier humans' condition. This reduces savings, as agents need to spend in private health. The reduction in capital accumulation reduces pollution, which ameliorates agents' conditions. They reduce their private health spending and increase the saving investment, which will lead to a greater capital accumulation and, consequently, more pollution. This process increases as the difference $(\phi - \theta)$ rises. Specifically, we can analyse each case separately.

(i) When $(\phi - \theta) \in \left(1, \frac{\Gamma}{\Gamma - 1}\right)$, the economy is stable and converges toward a "no-growth" equilibrium. The steady-state $\hat{k}$ is lower than the maximum of $Q(k_t)$. In this scenario, the economy increases up to $\hat{k}$, then it remains stable at the steady-state and $k_{t+1} = k_t$. The equilibrium is asymptotically stable. The phase diagram is shown in Figure (2.3)

(ii) In this scenario, the economy shows an oscillatory but stable pattern toward the steady state. The intuition is straightforward. The greater productions increases agents' income but also the pollution level. The morbidity caused by pollution increases, thus agents are forced to reduce savings and increase private health spending x_t. Lower savings reduce production and, consequently, the pollution level. Agents can now reduce the private health spending and increase savings again, which increases capital and so production. This cyclical process decreases period after period and converges toward the steady-state. Once the equilibrium is reached, the economy remains in the "no-growth" equilibrium also in the long run. The phase diagram is shown in Figure (2.4)

(iii) In this scenario the cycle is permanent and stable. The economy shows a continuous movement around the steady-state $\hat{k}$. There is now a stable equilibrium because the increase in production leads to a significant increase in private health spending, which reduces savings and future capital accumulation. The reduction in pollution given by the lower capital is greater than the previous scenario, so both the increase in agents' health condition and the reduction in private health spending in favour of savings are more significant. Consequently, capital increases and so does production and emissions. This cycle, due to the magnitude of the change in capital and private health spending, is perpetual. An example with a 2-period cycle is shown in Figure 2.5. When $k = k_2^*$, the capital level is greater than the stead state. The pollution production increases and, given the high harmful effect of pollution relative to the elasticity of public health spending, agents increase their spending for private health and reduce savings. Economy decreases, as the positive linear effect of Γ is lower than the health damage effect. The economy shifts back to k_1^*. Capital accumulation is low, so the economy produces less pollution. Agents can decrease the amount they invest in private health and increase savings, capital accumulation increases and so pollution. This process continues indefinitely.

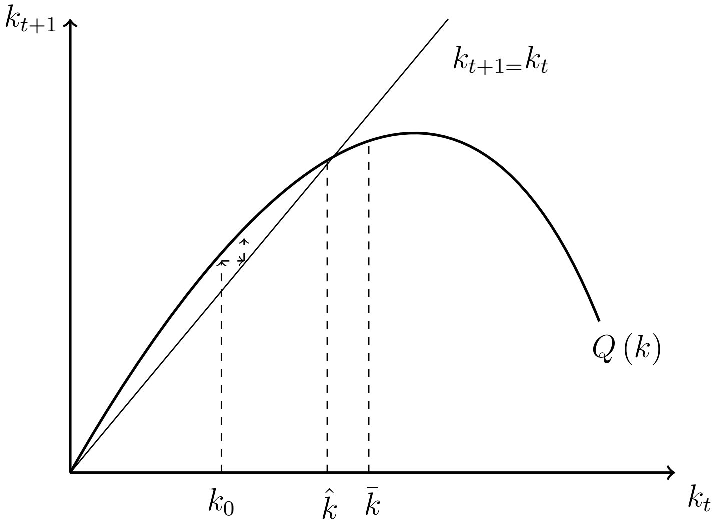


Figure 2.3: $e > 0$ and $(\phi - \theta) \in \left(1, \frac{\Gamma}{\Gamma-1}\right)$

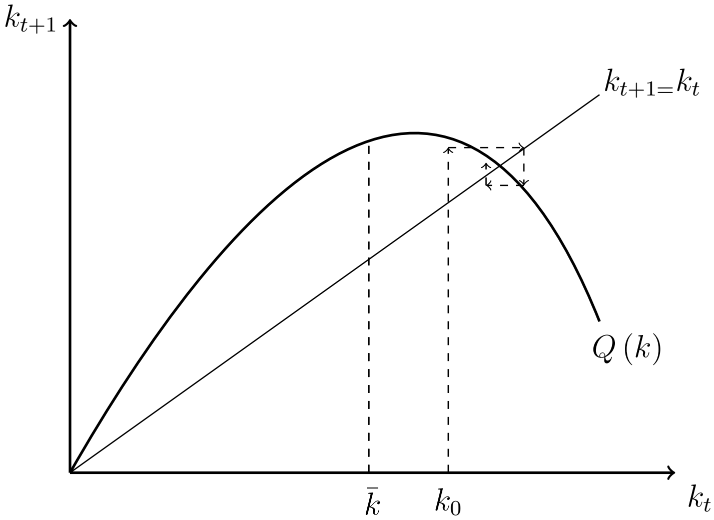


Figure 2.4: $e > 0$ and $(\phi - \theta) \in \left(\frac{\Gamma}{\Gamma-1}, \frac{\Gamma+1}{\Gamma-1}\right)$

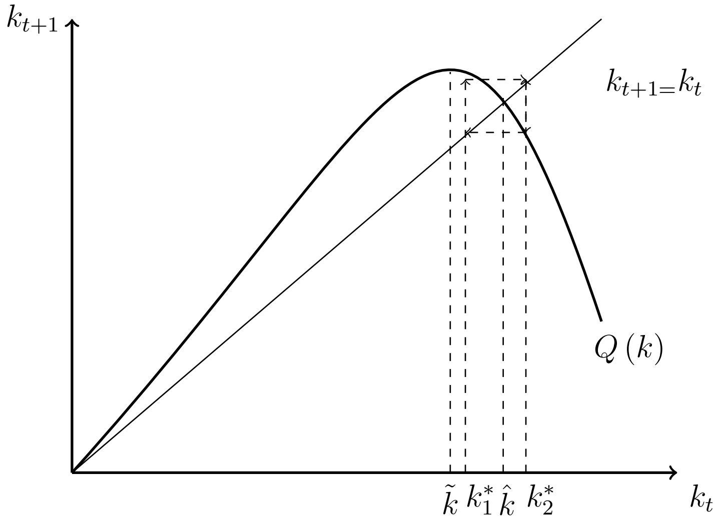


Figure 2.5: $e > 0$ and $(\phi - \theta) > \frac{\Gamma+1}{\Gamma-1}$. An example of a period 2-cycle, (k_1^*, k_2^*)

2.4.5 3^{rd} Scenario: $e > 0$ and $(\phi - \theta) < 0$

In this scenario, public health spending elasticity θ is greater than pollution elasticity ϕ. The technology "dirtiness" and its detrimental effects on agents' health can be reduced by public health investment.[22]

The steady state is $\hat{k} = \left[\frac{\Gamma}{\Lambda-1}\right]^{\frac{1}{1+\theta-\phi}}$.

The function (k_t) is everywhere concave.[23] Since $Q(0) = -\infty$, there is a $\check{k}_t$ such that $Q(\check{k}_t) = 0$. This is given by:

$$Q(\check{k}) = \Gamma\check{k} - \frac{\Lambda}{\check{k}^{\theta-\phi}} = 0 \longrightarrow \left(\check{k}\right)^{1+\theta-\phi} = \frac{\Lambda}{\Gamma} \longrightarrow \check{k} = \left[\frac{\Lambda}{\Gamma}\right]^{\frac{1}{1+\theta-\phi}} \quad (2.19)$$

For values of $k_t < Q(\check{k}_t)$, $Q(k_t) = 0$. So:

[22] This section is qualitatively similar to Palivos and Varvarigos (2016), in the case of end-of-pipe technology abatement and elasticity of public health spending greater than the elasticity of environmental degradation, and to Chakraborty (2004).

[23] $Q^{'}(k_t) > 0 \ \forall k_t$ and $Q^{''}(k_t) < 0 \ \forall k_t$.

$$Q(k_t) = \begin{cases} 0 \text{ for } k_t < \check{k}_t \\ \Gamma k_t - \Lambda k^{\phi-\theta_t} \text{ for } k_t > \check{k}_t \end{cases}$$

Proposition 4:*If $(\phi - \theta) < 0$ and $k_0 < \hat{k}$, the economy converges toward a "poverty trap" equilibrium. If the initial level of capital $k_0 > \hat{k}$, the economy experiences a sustained and continuous "long-run" growth.*

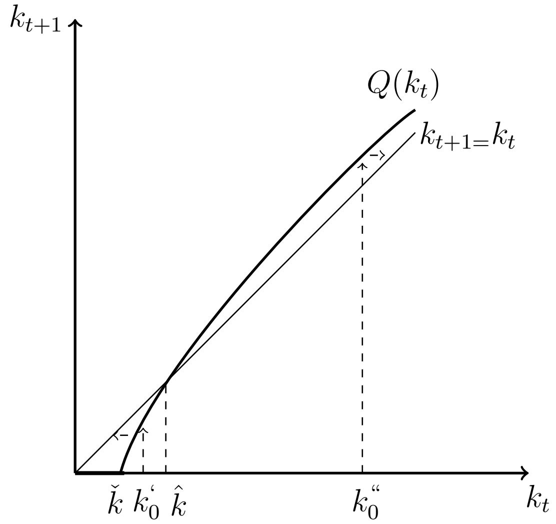


Figure 2.6: $e > 0$ and $\phi - \theta < 0$

In this scenario, if $k_0^{'} < \hat{k}$ the economy ends up in a "poverty trap". Despite the public health spending elasticity θ is higher than ϕ, the initial level of capital is so low that even a scarce level of private health spending reduces saving and the capital accumulation irremediably (the intuition explained for the previous result shown hold). If $k_0^{''} > \hat{k}$ the economy reaches a "long-run" growth outcome. Here the effect of k_t on health is positive, as the public health elasticity exceeds the pollution elasticity. Effectively, health and capital accumulation are complementary to each other. Only if capital starts at a sufficiently high level, the rate of capital accumulation is high enough to guarantee an increase of capital stock over time. Given that th epublic health spending elasticity is greater than the pollution elasticity ϕ, even low levels of public health expenditure are sufficient to reduce the detrimental effect of pollution on agents' health. Individuals may thus reduce their spending for private health and increase their savings, leading the economy to a long-term growth. The increase in pollution due to the higher growth, even if affects agents' health, is not as strong as it should be to force agents to stop saving and invest more in private health. The phase diagram is shown in Figure (2.6).

2.5 Conclusion

We construct a two period OLG model to investigate the connection between pollution and long-term growth. Pollution affects the capital accumulation process through the agents' health channel, which is negatively affected by environmental externalities. In our model agents work in the first period of their life and decide whether to allocate their wage towards savings or private health care, to mitigate the detrimental effect of pollution on health in their second period. Health status augments the utility of older agents, and it is an increasing function with respect to the private and to the public health spending. In the baseline scenario, when pollution does not affect agents' health, the model shows a long-term growth similar to the one of the classic OLG model with endogenous growth, thanks to the positive capital accumulation due to the learning-by-doing externality presents in the production function. When pollution affects agents' health, the technology "dirtiness" (i.e. the pollution elasticity with respect to the total production), and the public health spending elasticity are crucial to determine the long-run equilibrium. If the initial level of capital is below a certain threshold and the pollution "dirtiness" is high, the economy ends up in a "poverty trap": pollution forces agents to spend in private health to mitigate pollution morbidity, reducing savings and capital accumulation. If the capital initial level is above the threshold, interactions between pollution elasticity and public health spending elasticity can generate different results: either a non-monotonic convergence towards the long-run asymptotically stable steady state, a limited oscillatory equilibrium or a persistent cycle (non chaotic) around the steady-state. If the technology "dirtiness" is lower than the public health expenditure elasticity, the economy may ends up either in a poverty-trap or in sustained long-term growth. Further researches could investigate the role of government in the model, to find the optimal allocation of public resources that maximizes agents' health, and to assess the possible effects on the steady-state and the capital threshold.[24]

[24] In the working paper of Palivos and Varvarigos (2016), the author endogenize the government' choice for an optimal tax, showing the effects of the optimal allocation of resources between pollution abatement activities and health expenditure.

Chapter 3

Emission tax versus grandfathered permits in a Cournot competition with research spillovers

Chapter Abstract

We present a Cournot competition model with n-firms where the government introduces an environmental policy, either an emission tax or free permits (i.e. non auctioned). The government may or may not be able to credibly commit to the environmental policy. Firms invest in abatement research and can partially imitate other firms' research, due to the presence of research spillovers *à la* d'Aspremont C., Jacquemin A., (1988). When the research is non cooperative and the government's commitment is credible, the two policies are equivalent. When the government cannot credibly commit, for a low number of firms and low social damages caused by pollution, the environmental permits allow the economy to reach a greater level of social welfare and provide stronger incentives to firms to invest in R&D; when the number of firms increases, the environmental tax guarantees stronger incentives and a greater welfare. When the research is cooperative, firms can form an environmental research cartel (ERC), *à la* Poyago-Theotoky (2007). Firms invest more in research in the ERC case than in the non-cooperative research, but the results in terms of welfare are the opposite.

3.1 Introduction

To address the pressing issue of global warming and its severe aftermaths, caused by human pollution, different environmental policies have been adopted worldwide in the past two decades. But how can we evaluate the effectiveness of these policies? Since environmental threats are caused mostly by humans' production activities, the best option to reduce pollution without affecting output production too severely would be to invest in technological change, in order to develop (or adopt) less pollutant technologies. The famous quote from Kneese and Schulze (1975) captures optimally this idea: "Over the long haul, perhaps the most important single criterion on which to judge environmental policies is the extent to which they spur new technology toward the efficient conservation of the environment".[1] Several authors have tried to establish a rank among different environmental policies, according to incentives they provide in terms of R&D investment towards the invention, or the adoption, of less pollutant technology. Literature has investigated environmental policies under two different innovation processes: tournament (patent race or product innovation), and non-tournament innovation (or process innovation).[2] In the tournament type, several firms invest in innovation but only one can get the prize, i.e. the patent and the right of exploiting the possibility of producing the innovation. This line of research follows the Schumpeter's approach (Schumpeter, 1943), which indicates the pursue of a monopolistic position to be the strongest incentive to invest in R&D to produce an innovation, starting from an ideal situation of perfect competition.

In the non-tournament type (or process innovation), firms invest in research to diminish the marginal cost or, in presence of an environmental policy, to reduce the emissions rate in order to reduce the policy burden. In this case, firms do not license a new innovation, but invest in emissions reduction. Innovation is protected by secrets, or it can be thought as "good strategies" to reduce emissions, where there is not a specific innovation but a series of

[1]Kneese and Schulze (1975, p. 38).

[2]For a comprehensive review of the problems related to the patent race and the technology diffusion, see Tirole (1988), Chapter 10.

methods and good practices. In both innovation processes, innovation can be copied or imitated by other firms.

The game timing plays a crucial role to determine the environmental policy effectiveness. There are two possible timing structures. In the first case, the government moves first by setting the policy and credibly committing to it. In the second stage, firms invest in innovation, maximizing their profit function with respect to research. In the second case, first firms choose the investment in research, then the government sets the environmental policy, in order to optimally adjust to the new level of emissions. In this case, the government is said to be non-credible (or, time consistent, as specified by Petrakis and Xepapadeas, 2003). A similar analysis has been conducted by Lutz, Lyon and Maxwell (2000), who found that the innovation leader firm may influence the government's decision about which standard to adopt, by setting its standard first. The final outcome is a lower welfare than it would have been possible if the government would have set the standard first.

Among others,[3] the authors that investigate the effect of environmental policies in presence of tournament innovation are Milliman and Prince (1989), Downing and White (1986), Jung et al., (1996), Fischer et al., (2003), Requate and Unold (2003), Denicolo' (1999). The scope of the literature is to establish a clear rank among the different policies in terms of incentives provided to invest in R&D and in social welfare gained.

Milliman and Prince (1989) examined the effects of five different policy instruments (direct controls, emission subsidies, emission taxes, free marketable permits, and auctioned marketable permits) in promoting three different phases, each crucial in the process of technological change: innovation, diffusion, and optimal agency response. In a tournament innovation framework, they find that auctioned permits and tax are the best instruments to provide incentives for firms to innovate, and that market based instruments offer greater incentive than command-and-control policies. Downing and White (1986) consider three models: a model where a firm produces an innovation

[3] The literature on technological change and the environment prospered in the last 30 years. For a comprehensive review of the existing literature, see Jaffe et al., (2002), Jaffe et al., (2003), Popp et al., (2010).

that does not significantly impact the marginal cost of pollution, an innovation that does impact pollution cost but where the government cannot optimally adjust to exploit the extra profit realized by the innovative firm, and a final case where the government adjusts optimally. They find that with a low impact innovation, market based instruments are better than command and control, while with high impact innovation, a clear ranking is not possible. Jung et al., (1996), introduce firm heterogeneity in marginal abatement costs and firm size. The rank that emerges is the following: auctioned permits, emission taxes and subsidies, free permits and differentiated performance standards. A more recent analysis is provided by Requate and Unold (2003), who challenge the previous analyses that typically consider a wide industry adoption of the innovation and the resulting cost saving at an aggregate level. Requate and Unold (2003) consider the hypothesis that a single firm can act as a free rider in the case of auctioned permits; if the majority of firms adopt the innovation, demand for permits will decrease and permit price decreases accordingly. Thus, incentives for firms to adopt the new technology are lower. The authors find that under an environmental tax, all firms adopt the new technology, while there could be partial adoption under an environmental permits policy. Moreover, they find that when the government commits to the chosen environmental policy, standards may offer stronger incentives than permits.[4] Denicolò (1999), in a tournament innovation model, compares auctioned permits and taxes. He finds that when government is not credible (i.e. government adjusts environmental policy after the innovation occurs), the two policies are fully equivalent, while if government commits credibly, the two policies lead to different results. If post innovation profits are the same, incentives to innovate increase as taxation increases; when tax is too high and it has a too strong detrimental effect on output, permits offer stronger incentive to innovate. Denicolò (1999) analyses also which policy offers better results in term of welfare, and he finds that when social damages due to pollution are low (high), tax performs better (worse) than permits in

[4] In a similar framework, Requate and Unold (2001) introduce firm heterogeneity (as Jung et al., 1996). They find that environmental tax offers more incentives than permits and that environmental tax (permit) leads to an overinvestment (underinvestment) in R&D.

terms of welfare.[5] Fischer et al., (2003), introduce the possibility of other firms to imitate around the innovation licensed by the innovator, which has a limited appropriability of profits generated by the innovation produced. The authors find that auctioned permits are preferred to tax and to free permits, but the rank may change if there are significant differences on innovation cost, innovator's appropriability and on the possibility of imitating around innovations.

The non tournament innovation started from the seminal papers of d'Aspremont and Jacquemin (1988), Kamien et al., (1992) and Suzumura (1992), followed by Katsoulacos and Xepapadeas (1996), Chiou and Hu (2001), Poyago-Theotoky (2007), Lambertini et al., (2015) among others. In this literature, market structure is crucial to determine policy efficiency in providing incentives to firms to invest in R&D. Moreover, given the absence of a patent protecting innovations, firms can imitate (partially or completely) around other firms' investment in R&D, through research knowledge spillover. This represents a strong disincentive for firms to invest in research.

The seminal contributions are d'Aspremount and Jacquemin (1988), Kamien et al., (1992) and Suzumura (1992). d'Aspremount and Jacquemin (1988), in a Cournot-duopolistic model, introduce research effort limited appropriability, i.e. firms can imitate around other firms R&D investment. In the non tournament game, knowledge is a public good. This idea is captured by a spillover parameter. They show different effects of market structure (i.e. full competition, competition in output sector but collusion in research sector and collusion in both sectors) on incentives to innovate. Kamien et al., (1992) extended their framework to a Cournot-model with n-firms and with different spillovers level, to include research joint venture (RJV), where firms maximize their joint profit function with respect to research and spillover effect is at its maximum (i.e. there is full information sharing about their research); R&D cartelization, where firms do not share information completely (the spillover parameter is lower than its maximum level), but maximize their joint profit; finally the R&D competition, which is close to d'Aspremount and Jacquemin (1988) case, where firms maximize their own research effort in-

[5] Similar results are obtained in Scotchmer (2011).

dependently and the spillover rate is exogenously given. Finally, Suzumura (1992), in a non tournament innovation analysed the effects of spillovers on research investment and on optimal social welfare function from an R& D investment perspective. The author finds that with large spillover, both non cooperative and cooperative research are insufficient and any investment improvement policy is needed to further increase social optimum, while when there are no spillover, the cooperative (non cooperative) equilibrium R&D is less (more) than optimal and a specific policy is needed to increase (reduce) it.

Moving from their seminal contribution, several authors developed their model to unveil the relationship between incentives to invest in "green" R&D and market structure, in presence of pollution and environmental policy. Katsoulacos and Xepapadeas (1996) were among the first to address this question. In a Cournot-duopolistic model with polluting firms, research spillovers and a government that commits credibly to the chosen policy, they introduce an environmental tax and a research subsidy to address the two externalities present in their framework, i.e. pollution and underprovision of R&D investment. They find that investment in R&D is far from optimal as firms do not take into account consumer surplus; tax is set less than optimally, as output is already far from social optimal level due to oligopolistic condition,[6] and firms invest strategically in R&D to gain a greater market share. The final result can be an over or an underinvestment in R&D. A more recent extension of d'Aspremount and Jacquemin (1988) and Kamien et al. (1992) is done by Chiou and Hu (2001) and by Poyago-Theotoky (2007). Chiou and Hu (2001), extended the work done by Kamien et al., (1992) to compare and list the impact that environmental R&D cartelization and research joint venture (RJV competition and RJV cartelization) have on final output and abatement investment. They found that, under an environmental tax and with high R&D spillover, RJV cartelization perform better than RJV competition, despite its negative impact on final output. Poyago-Theotoky (2007), introduced, in a Cournot-duopolistic model, a non credible government where firms can decide whether to engage in R&D competition

[6] A result seen in Barnett (1980).

or to form an environmental R&D cartel (ERC) *à la* Kamien et al., (1992). She found that the ERC performs better in terms of incentive to invest in R&D and in social welfare than the non-cooperative R&D, when damages caused by pollution is low; the same results applies when social damage is high, but R&D is inefficient, i.e. there is strategic over - or underinvestment in research.[7] Finally, Lambertini et al., (2015) extend the model of Poyago-Theotoky (2007) to n-firms, to investigate the effect of competition on investment in "green" R&D. They find that an inverted-U relationship exists between competition and investment in R&D when the regulator can pre-commits.[8]

Our model follows the non-tournament innovation research with knowledge spillover, in presence of a government that can either commit or not commit to the environmental policy. The model is similar to Lambertini et al., (2015) and to Poyago-Theotoky (2007), where firms invest in R&D to reduce their emissions and they can partially imitate on abatement investment made by others firms. We extend their model by introducing free permits and comparing the two policies to rank them from an R&D incentive and welfare perspective, both in the commitment and in the non commitment scenario.

The main purpose of our research is to examine the effects that environmental tax and permits have on incentive to innovate and on welfare under a credible/non credible government. To the best of our knowledge, this framework has not been used to make a welfare comparison between taxes and permits under different commitment scenarios, while it has been used mostly to show the impact on research level of different level of competition in oligopolistic competitive markets. This analysis can answer questions such as which environmental policy is performing better in terms of research investment and emission reduction, and which one is better with respect to social welfare.

About permit market, we follow Requate (1993). In a Cournot-duopolistic

[7] In a similar framework, Poyago-Theotoky (1999) analyses the effects of an endogenous spillover rate on emission reduction R&D.

[8] Instead of focusing on the effect of market structure on R&D investment, Gil-Molto and Dijkstra (2011), focused on the effect of a stricter tax on emission-to-output ratio; they found that a stricter environmental tax does not necessarily correspond to a stronger incentive to invest in R&D, as the emission-to-output ratio has a U-shaped relation with respect to tax.

model, he compares environmental tax and free permits. For the permit market, he considers a stage where government allocates permits to firms for free. Firms then trade permits on market, according to their need. We differ from his paper in assuming that firms have constant zero marginal cost and by introducing the possibility of adopting abatement technologies. On the opposite, Requate does not allow for an abatement reduction and differentiates the two firms of the duopolistic model according to their efficiency and level of pollution, performing then comparative statics according to the stringency of the environmental regulation.

We assume a competitive permit market, similar to Requate (1993). Firms trade in the permit market the allocated permits; market clears (as Sartzetakis, 1997) and the result of the condition of equilibrium is the permit price. In our model we do not consider firms' possible strategic behaviour in the permit market, as investigated by the seminal contribution of Hahn (1984) and von der Fehr (1993), where firms act strategically to manipulate rivals firms' costs.[9]

The model presents two different timings of the games, following the work of Denicolò (1999), Petrakis and Xepapadeas (2003) and Dijkstra and Gil Molto (2011). The first is the so-called *commitment* case; the Government moves first by imposing an environmental tax or the permits number. In this case, the Government can commit to keep the tax, or the number of the permit, fixed. There are three separate stages. In the first stage, the Government sets the environmental policy (either a tax or a pollution cap with tradable permits), and commits itself to the chosen level of policy. In the second stage, the firms choose their amount of research to reduce their emissions. Finally, in the last stage, the firms compete in output market with a typical Cournot-Nash equilibrium.

In the second scenario, called *non commitment*, the firms move first by deciding the optimal amount of research.[10] The Government is tempted to "steal" the extra-rent that firms have obtained in investing in research after

[9]This literature is close to the "raising rival costs" literature; see Hintermann (2011) and the literature within.

[10]This is the time-consistent case of Petrakis and Xepapadeas, 2003.

the introduction of the environmental regulation. The timing of this scenario is the following: the firms move first by choosing the optimal level of research, then the Government sets the environmental regulation and finally the firms decide the amount of output. Both models are solved by backward induction, starting from the last stage. Results obtained for each scenario are compared to establish which policy offers more incentive to invest in R&D and a greater social welfare.

To compare the results, we follow Poyago-Theotoky (2007), by evaluating the difference of the results obtained in each case. Due to the complexity of the equation, an explicit solution was impossible to find. Thus, we evaluate our results through numerical simulations. Following Kamien et al., (1992) and Poyago-Theotoky (2007), together with the non-cooperative R&D case, where firms do not cooperate in research investment, we also analyse the environmental research cartel (ERC) case, where firms maximize their joint research effort, but they do not share their research information completely, and the research joint venture (RJV) case, where firms share information completely. We consider this case only under a credible government (i.e. the commitment case).

Comparing environmental tax and permit policy in the non-cooperative research case, we find that for a low number of firms (the duopolistic case) and low social environmental damage, total aggregate research is greater under environmental permits than under environmental tax. For a larger number of firms, the environmental tax provides stronger incentive to invest in R&D, for any level of research spillover or social environmental damage.

Regarding social welfare, for low (high) levels of damages due to pollution, the welfare gained under the environmental tax (environmental permit) regime is greater than the welfare obtained under the environmental permit (environmental tax). As the number of firms increases, welfare gained under the environmental tax is greater for all parameters values. In the ERC case, for low values of social environmental damage due to pollution, the environmental tax guarantees a greater production of research than the environmental permits. Despite this, the total welfare gained under the environmental permits regulation is greater than the environmental tax under any parameter values.

The results show that, for low social environmental damage caused by pollution and low number of firms, environmental permits are better than the environmental tax in providing incentives to invest in R&D and in social welfare gains. For a stronger market competition, the environmental tax turns out to be the best policy both under research incentive and social welfare. Surprisingly, there is no difference between the two policies under the commitment regime. When the government is credible, both policies reach the same results in terms of incentives to research and of social welfare . When the government is not credible, results differ under the two policies. Moreover, both policies provide better abatement research incentives and welfare gains under commitment than non commitment scenario (a similar result found by Lutz, Lyon and Maxwell, 2000).

If the firms can rely on government's choice, they invest optimally in research. This reduces emissions and so the detrimental effect on social welfare. If, instead, firms cannot rely on government, they reduce investment in research. Finally, the ERC provides stronger incentive to firms to invest in R&D, but the social welfare gains are greater under the non-cooperative scenario than under research cartelization.

The paper is organized as follows: in section 2 we introduce the general framework with environmental tax and permits. In section 3 we compare the main results obtained under environmental tax and environmental permits. Finally, in section 4, we analyse the ERC case and compare the results obtained with the non-cooperative R&D case.

3.2 The Model

In the following section, we analyse first the environmental taxation case and then the free permits case. In both situations we consider a government that can or cannot credibly commit. For both policies, preliminary results are showed.

3.2.1 Environmental Tax - Commitment case

The environmental tax scenario follows the work of Lambertini et al., (2015) for the Commitment case, while it follows the work of Poyago-Theotoky (2007) extended to n-firms for the Non Commitment scenario. The comparison between the two different cases relies on our own work.

The Commitment timing is the following:

1) Firms set the profit maximizing level of output q_t;

2) The firms maximize their research; effort x_t,

3) Finally, the Government sets the optimal tax t.

The price P is equal to:

$$P = a - Q \tag{3.1}$$

where $Q_i = \sum_{i=1}^{n} q_i$.

Following Poyago-Theotoky (2007), firms' i emissions can be formulated as:

$$e_i = \theta_i q_i - x_i - \beta \sum_{i=1}^{n-1} x_{-i} \tag{3.2}$$

where $\theta_i \in [0, 1]$ is the emission rate per unit of production specific for each firm, x_i is the amount of research aimed to reduce the pollution rate or, more generally, to reduce activities that causes pollution. Following d'Aspremont and Jacquemin (1988) and Katsoulacos and Xepapadeas (1996), we introduce the spillover parameter $\beta \in [0, 1]$; when $\beta > 0$, firms can partially imitate from others' research activities. Firms research cost function is quadratic,[11] and equal to: $\frac{\gamma x^2}{2}$; $\gamma > 0$ is the productivity of research. Cost function is

[11] The quadratic cost function has been adopted by d'Aspremont and Jacquemin (1988) and then it has become a standard in the environmental policy models; it has been used by Katsoulacos and Xepapadeas (1996), Petrakis and Poyago-Theotoky (2002), Gil Molto and Djikstra (2011 and 2013) among others.

equal to $c(q_i) = cq_i$. To simplify the analysis, marginal costs c is set equal to zero. The Government in the first stage sets the effluent tax on firms' emissions t. Total cost of emissions is $t\left[\theta q_i - x_i - \beta\sum_{i=1}^{n-1} x_{-i}\right]$.

In the third stage firms maximize profits function Π_i with respect to their own quantity q_i:

$$\Pi_i = [a - Q_{-i} - q_i]\, q_i - t\left[\theta_i q_i - x_i - \beta\sum_{i=1}^{n-1} x_{-i}\right] - \frac{\gamma x_i^2}{2} \tag{3.3}$$

where $Q_{-i} = \sum_{j\neq i=1}^{n-1} q_j$.

In order to simplify our analysis, we assume that the emission rate θ_i is the same for all $i = 1...n$ firms, so $\theta_i = \theta$. Given that, the first order condition with respect to q_i is equal to:[12]

$$q_i = \frac{a - t\theta}{n+1} \tag{3.4}$$

We assume that $a > t\theta$ in order to assure positive production. The result is the typical symmetric Nash-Cournot equilibrium with n-firms, so $q_i = q_{-i} = q$. Production q is decreasing with respect to the tax t, to the emission ratio θ and to the number of firms n.

Total production $Q_i = Q_{-i} = \bar{Q}$ is equal to $Q = \left[n\frac{a-t\theta}{n+1}\right]$.

Using (3.4), total profits for firm (i) is:

$$\Pi_i = \left[\frac{a - t\theta}{n+1}\right]^2 + t\left[x_i + \beta\sum_{i=1}^{n-1} x_{-i}\right] - \frac{\gamma x_i^2}{2} \tag{3.5}$$

Firms maximize their profit with respect the amount of research x_i to find

[12]The second order derivative $\frac{\partial^2\Pi_i}{\partial q_i^2} = -2 < 0$.

the best response research function:[13]

$$x_i = \frac{t}{\gamma} \tag{3.6}$$

Research function is symmetric, so $x_i = x_{-i} = x$. Given the best response function of quantity and research, the Government can maximize the Social Welfare function in order to find the optimal effluent tax t. The social welfare function is equal to:

$$W(t) = \int_0^Q (a - S)dS - \frac{\gamma \sum_{i=1}^n x_i^2}{2} - \Omega \left[\sum_{i=1}^n e_i \right]^2 \tag{3.7}$$

where the first term represents the consumer surplus, the second term the firms' R&D costs and the final term the pollution social cost; $\Omega \in [0, 1]$ is the environmental social damage caused by pollution, assumed to be quadratic. By solving the integral and substituting (3.6) into (3.7), the total welfare function becomes:

$$W(t) = a \left[\frac{n(a - \theta t)}{1 + n} \right] - \frac{1}{2} \left[\frac{n(a - \theta t)}{1 + n} \right]^2 - \frac{nt^2}{2\gamma} - \Omega \left[\theta n \frac{(a - \theta t)}{1 + n} - nt \frac{[1 + \beta(n - 1)]}{\gamma} \right]^2 \tag{3.8}$$

The result of the first order condition of $W(t)$ with respect to t is the optimal tax t, which is a linear function of $t(a, \beta, n, \gamma, \theta, \Omega)$.[14] With this, we solve the

[13]The second order derivative $\frac{\partial^2 \Pi_i}{\partial x_i^2} = -\gamma < 0$.

[14]The second derivative is equal to:

$$-\frac{\theta^2 n^2}{(n + 1)^2} - 2\Omega \left[-\frac{n(-\beta + \beta n + 1)}{\gamma} - \frac{\theta^2 n}{n + 1} \right]^2 - \frac{n}{\gamma}$$

which is always negative.

first stage of the game:

$$t_C = \frac{a\gamma\theta\left[2n\Omega\left(\beta(n^2-1)+n+1\right)+\gamma\left(2\theta^2 n\Omega-1\right)\right]}{\gamma(n+1)\left(4\beta\theta^2 n^2\Omega+n\left(1-4(\beta-1)\theta^2\Omega\right)+1\right)+ 2n\Omega\left(\beta(n^2-1)+n+1\right)^2+\gamma^2\theta^2 n\left(2\theta^2\Omega+1\right)} \tag{3.9}$$

A sufficient condition for a positive tax is $n \geq 1$ and $\Omega = \bar{\Omega} > \frac{1}{2\theta^2 n}$. For values of Ω lower than $\bar{\Omega}$, the tax turns to be a subsidy. The environmental damage is so low that it is optimal, for the Government, to subsidize the firms to have more research.

By substituting t in (3.6) we obtain the optimal level of research x_C^t:[15]

$$x_C^t = \left[\frac{a\theta\left[2n\Omega\left(\beta(n^2-1)+n+1\right)+\gamma\left(2\theta^2 n\Omega-1\right)\right]}{\gamma(n+1)\left(4\beta\theta^2 n^2\Omega+n\left(1-4(\beta-1)\theta^2\Omega\right)+1\right)+ 2n\Omega\left(\beta(n^2-1)+n+1\right)^2+\gamma^2\theta^2 n\left(2\theta^2\Omega+1\right)}\right] \tag{3.10}$$

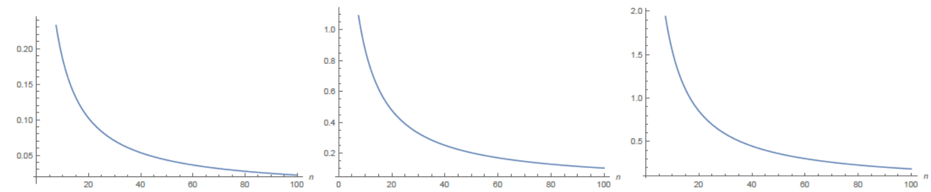
The research is positive for $n \geq 1$, (which is given by assumption) and for $\Omega > \frac{1}{2n\theta^2}$. A minimum level of environmental damage is required for the firms to have an incentive to invest in research. The aggregate research X_C^t is equal to nx, i.e.:

$$X_C^t = \left[\frac{an\theta\left[2n\Omega\left(\beta(n^2-1)+n+1\right)+\gamma\left(2\theta^2 n\Omega-1\right)\right]}{\gamma(n+1)\left(4\beta\theta^2 n^2\Omega+n\left(1-4(\beta-1)\theta^2\Omega\right)+1\right)+ 2n\Omega\left(\beta(n^2-1)+n+1\right)^2+\gamma^2\theta^2 n\left(2\theta^2\Omega+1\right)}\right] \tag{3.11}$$

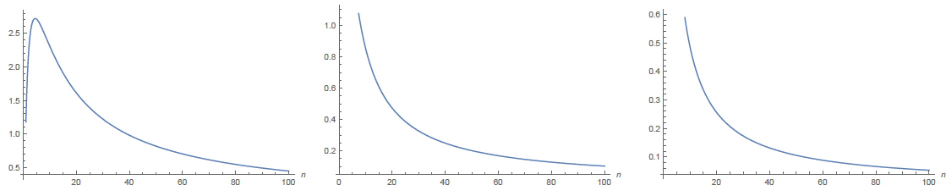
In Figure 3.1 we can show how the aggregate research function reacts to change in parameters. The aggregate research increases as the intensity of the emissions per firms θ increases. Given that firms pollute more, they have to face a greater tax burden; this represents a stronger incentive for firms to invest more in research (Figure 3.1 (a), (b) and (c)). Not surprisingly, the aggregate research is decreasing as the spillover rate β increases (Figure 3.1 (d), (e) and (f)). The function has a convex shape when $\beta > 0$, while it

[15]Where the superscript t stands for the environmental tax and the subscript C for the Commitment scenario.

becomes concave for $\beta = 0$. Following Lambertini et al. (2015), we can say that, when the spillovers are absent, firms can fully appropriate their own research effort. So, even with a greater number of firms, aggregate research is increasing. But, when the research appropriability decreases, a greater firms competition represents a strong impediment for firms to invest in research, thus the aggregate research decreases. That eliminates the negative competitive effect given by the reduced market power. The result is shown in Figure 3.2.

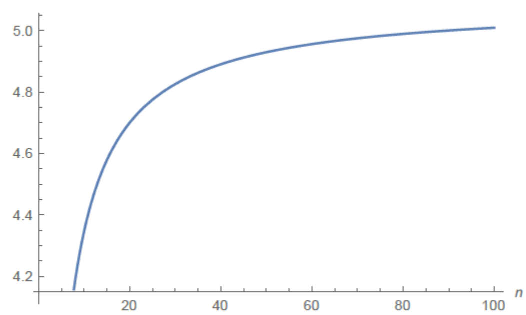


(a) $a = 5$, $\underline{\theta = 0.1}$, $\gamma = 0.5$, $\Omega = 0.5$, $\beta = 0.5$ (b) $a = 5$, $\underline{\theta = 0.5}$, $\gamma = 0.5$, $\Omega = 0.5$, $\beta = 0.5$ (c) $a = 5$, $\underline{\theta = 0.9}$, $\gamma = 0.5$, $\Omega = 0.5$, $\beta = 0.5$



(d) $a = 5$, $\theta = 0.5$, $\gamma = 0.5$, $\Omega = 0.5$, $\underline{\beta = 0.1}$ (e) $a = 5$, $\theta = 0.5$, $\gamma = 0.5$, $\Omega = 0.5$, $\underline{\beta = 0.5}$ (f) $a = 5$, $\theta = 0.5$, $\gamma = 0.5$, $\Omega = 0.5$, $\underline{\beta = 0.9}$

Figure 3.1: Total research X_C^t with respect to the number of firms n



(a) $a = 5$, $\theta = 0.5$, $\gamma = 0.5$, $\Omega = 0.5$, $\beta = 0$

Figure 3.2: Total research X_C^t for $\beta = 0$

Our result doubles the finding of Lambertini and al., (2015). Following their analysis, it is possible to show that the aggregate research function is concave with respect the number of firms. We can formulate our first Lemma:[16]

Lemma 1: The aggregate research obtained in the Commitment scenario has an inverted-U shape with respect to the number of firms.

The result, even under different circumstances, resembles the finding of Aghion et al., (2005) which found, empirically and theoretically, an inverted U-relationship between market dimension and innovation.

Substituting (3.9) into (3.8), we can obtain the total welfare, which is equal to:

$$W_C^t = \frac{a^2 n \left[2n\Omega \left[\gamma\theta^2(2\beta(n-1)+1) + (n+2)(\beta(n-1)+1)^2\right] + \gamma\left(\gamma\theta^2 + n + 2\right)\right]}{4n\Omega \left[\gamma\theta^2 + \beta(n^2-1) + n + 1\right]^2 + 2\gamma\left[\gamma\theta^2 n + (n+1)^2\right]} \tag{3.12}$$

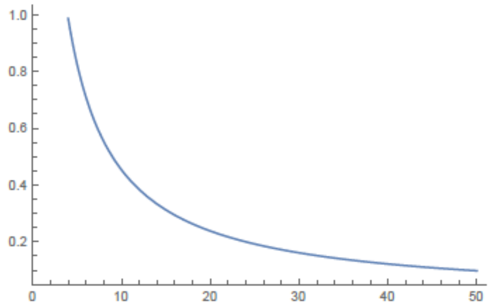
It is possible to verify the effectiveness of the environmental policy in reducing total emissions. Total emissions are equal to:

$$E_C^t = \frac{a\gamma\theta n \left[\gamma\theta^2 + \beta(n-1) + n + 2\right]}{2n\Omega \left[\gamma\theta^2 + \beta(n^2-1) + n + 1\right]^2 + \gamma\left[\gamma\theta^2 n + (n+1)^2\right]} \tag{3.13}$$

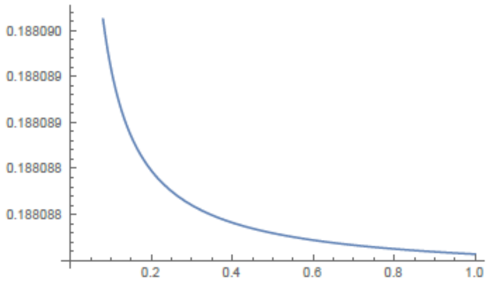
By substituting the optimal tax in (3.4), it is possible to find the quantity produced by a single firm:

$$q_C^t = \frac{a\left[2n\Omega(\beta(n-1)+1)\left(\gamma\theta^2 + \beta(n^2-1) + n + 1\right) + \gamma\left(\gamma\theta^2 + n + 1\right)\right]}{2n\Omega \left[\gamma\theta^2 + \beta(n^2-1) + n + 1\right]^2 + \gamma\left[\gamma\theta^2 n + (n+1)^2\right]} \tag{3.14}$$

[16]This and all the subsequent proofs are shown in the Appendix.



(a) $a = 5$, $\theta = 0.6$, $\gamma = 0.5$, $\beta = 0.5$, $\Omega = 0.7$



(b) $a = 5$, $\theta = 0.6$, $\gamma = 0.5$, $\beta = 0.5$, $n = 25$

Figure 3.3: Single firm quantity q_C^t with respect to the number of firms n (a) and environmental damage Ω (b)

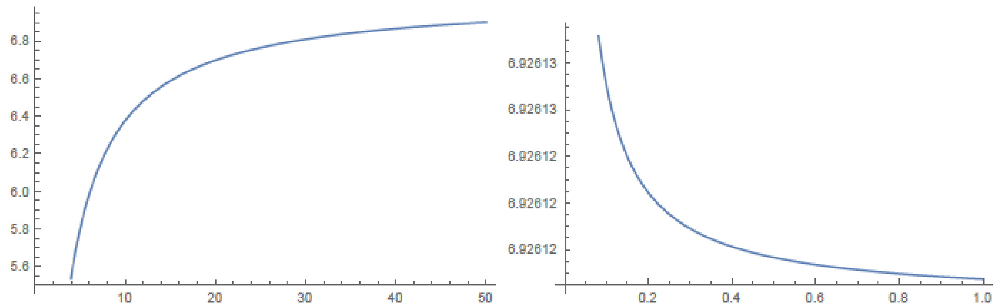
As shown in Figure 3.3, q_C^t is a convex function with respect to the number of firms (Figure 3.3 (a)) and with respect to the environmental damage Ω (Figure 3.3 (b)). The convexity of q_C^t with respect to the number of firms reveal the intuitive condition that increasing competition reduces the quantity produced by each firm. Moreover, as the damage caused by pollution increases, the stringency of the environmental regulation increases too, forcing firms to reduce their production.

Total quantity is equal to:

$$Q_C^t = \frac{an\left[2n\Omega(\beta(n-1)+1)\left(\gamma\theta^2+\beta(n^2-1)+n+1\right)+\gamma\left(\gamma\theta^2+n+1\right)\right]}{2n\Omega\left[\gamma\theta^2+\beta(n^2-1)+n+1\right]^2+\gamma\left[\gamma\theta^2 n+(n+1)^2\right]} \tag{3.15}$$

positive for $n \geq 1$.

As it is shown in Figure 3.4, with respect to the quantity produced by a single firm, the total quantity Q_C^t is a concave function with respect to the number of firm and it is convex with respect to the social environmental damage. The limit of the quantity for n that goes to infinity is equal to the market dimension, a.



(a) $a = 5$, $\theta = 0.6$, $\gamma = 0.5$, $\beta = 0.5$, $\Omega = 0.7$

(b) $a = 5$, $\theta = 0.6$, $\gamma = 0.5$, $\beta = 0.5$, $n = 50$

Figure 3.4: Total Quantity Q_C^t with respect number of firms n (a) and environmental damage Ω (b)

3.2.2 Environmental Tax - Non Commitment case

In the Non Commitment scenario, the timing is the following:

1) The firm sets the profit maximizing output q_{NC};

2) The Government sets the optimal tax t_{NC};

3) The firm sets the research level x_{NC};

First stage is identical to the Commitment case. Firms maximize (3.3) with respect to q_i; that leads to the best quantity response function (3.4). In the second stage, the Government sets the optimal environmental tax. The welfare function is:

$$W(t) = \int_0^Q (a - S)dS - \Omega \left[\sum_{i=1}^{n} e_i \right]^2 \tag{3.16}$$

where the cost of research $\frac{\gamma\left[\sum_i^n x_i\right]^2}{2}$ are non counted as they have been already included in the total welfare when the government set the tax in the first instance.

Given that the government maximizes the welfare function considering the aggregate research, (3.16) can be rewritten as:

$$W(t) = a\left[\frac{n(a-\theta t)}{1+n}\right] - \frac{1}{2}\left[\frac{n(a-\theta)t}{2(n+1)}\right]^2 - \Omega\left[\theta\left[n\frac{(a-\theta t)}{n+1}\right] - \sum_{i=1}^{n} x_i\left[1+\beta(n-1)\right]\right]^2 \quad (3.17)$$

The first derivative of (3.17) with respect to t gives the tax t_{NC}, which is equal to:[17]

$$t_{NC} = \frac{a\left[2\theta^2 n\Omega - 1\right] - 2\theta\Omega\left[\beta(n^2-1)+n+1\right]\sum_{i=1}^{n} x_i}{\theta n\left(2\theta^2\Omega+1\right)} \quad (3.18)$$

With (3.18) we solve the second stage. We can substitute it into the profit function (3.5) and, by considering research symmetry $x_i = x_{-i} = x$, we can obtain the research function x^t_{NC}, which solve the last and final stage of the game, equal to:

$$x^t_{NC} = \left[\frac{a\left[4\theta^4 n^2\Omega^2 + 2\theta^2\Omega(n(2\beta+n-1)-2\beta+2)-n\right]}{\theta n(4\theta^2\Omega^2(-(\beta-1)^2+\beta(\beta+1)n^3+((2-3\beta)\beta+1)n^2+n(\beta(3\beta-5)+\gamma\theta^2+2)) + 2\Omega((n+1)(\beta(n-1)+1)(\beta(n-1)+n+1)+2\gamma\theta^2 n)+\gamma n)}\right] \quad (3.19)$$

[17]The second order derivative is equal to:

$$-\frac{2\theta^4 n^2\Omega}{(n+1)^2} - \frac{\theta^2 n^2}{(n+1)^2}$$

always negative.

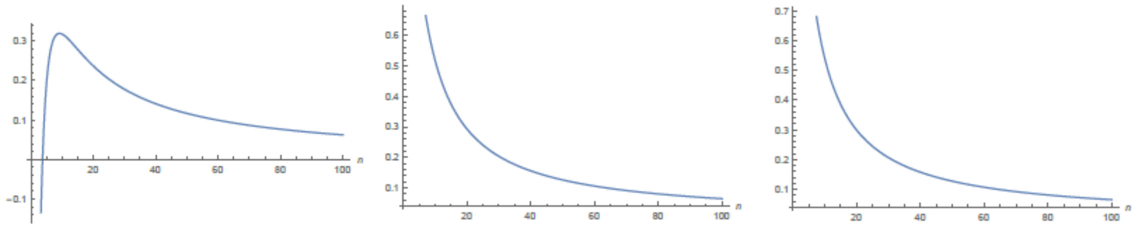
The aggregate research is:

$$X^t_{NC} = \left[\frac{a\,[4\theta^4 n^2\Omega^2 + 2\theta^2\Omega(n(2\beta + n - 1) - 2\beta + 2) - n]}{\theta(4\theta^2\Omega^2(-(\beta-1)^2 + \beta(\beta+1)n^3 + ((2-3\beta)\beta+1)n^2 + n(\beta(3\beta-5) + \gamma\theta^2 + 2)) + 2\Omega((n+1)(\beta(n-1)+1)(\beta(n-1)+n+1) + 2\gamma\theta^2 n) + \gamma n)} \right] \tag{3.20}$$

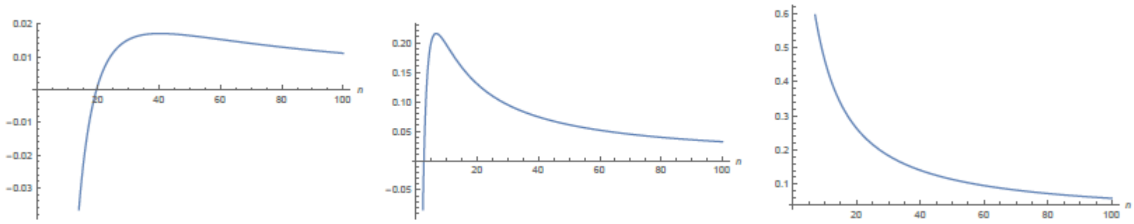
The aggregate research X^t_{NC} has the same characteristics of the aggregate research obtained under the Commitment scenario. We can thus formulate Lemma 2:

Lemma 2: The aggregate research is concave with respect to the number of firms n and single-peaked.

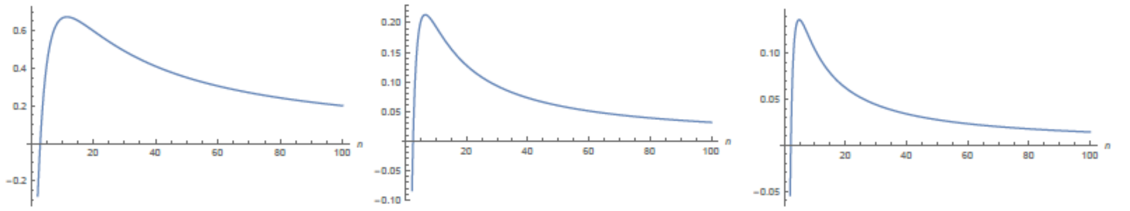
Given Lemma 2, it is possible to show how the total research reacts to a parameter change. Again, our results double the ones found by Lambertini et al., (2015). The total research shows an inverted-U relationship with respect to the number of firms. Increase in competition forces firms to reduce their investment in research, due to limited appropriability of their own research effort, represented by the spillover β. Some preliminary conclusion can be drawn from Figure 3.5 (a), (b) and (c). Total research is increasing with respect to the social environmental damage Ω. The government is forced to increase the stringency of environmental regulation, so firms react increasing their research effort. The same happens as the emission rate θ increases: firms that causes more emissions, to avoid a higher tax burden invest in reduction emissions research (Figure 3.5, (d), (e) and (f)). For low emission rate θ, the research is even negative for $n < 20$ (Figure 3.5 (d)). Firms have no incentive at all to invest in research due to the lack of stringency of the environmental regulation. It is straightforward to see (Figure 3.5 (g), (h), (i) that as β increases, the total effort in research decreases. When the spillover are absent ($\beta = 0$), total research does not show an inverted-U shape relationship with respect to number of firms, but it becomes a concave function. Given that firms can appropriate fully their effort in research, the increased competitive effect represented by the increasing number of firms does not affect the research investment (Figure 3.6).



(a) $a = 5$, $\theta = 1$, $\gamma = 0.5$, $\underline{\Omega = 0.1}$, $\beta = 0.5$
(b) $a = 5$, $\theta = 1$, $\gamma = 0.5$, $\underline{\Omega = 0.5}$, $\beta = 0.5$
(c) $a = 5$, $\theta = 1$, $\gamma = 0.5$, $\underline{\Omega = 0.9}$, $\beta = 0.5$

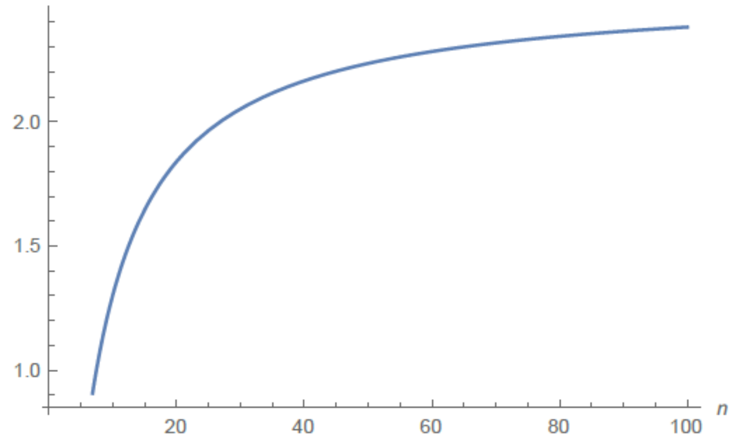


(d) $a = 5$, $\underline{\theta = 0.2}$, $\gamma = 0.5$, $\Omega = 0.5$, $\beta = 0.5$
(e) $a = 5$, $\underline{\theta = 0.5}$, $\gamma = 0.5$, $\Omega = 0.5$, $\beta = 0.5$
(f) $a = 5$, $\underline{\theta = 0.9}$, $\gamma = 0.5$, $\Omega = 0.5$, $\beta = 0.5$



(g) $a = 5$, $\theta = 0.5$, $\gamma = 0.5$, $\Omega = 0.5$, $\underline{\beta = 0.1}$
(h) $a = 5$, $\theta = 0.2$, $\gamma = 0.5$, $\Omega = 0.5$, $\underline{\beta = 0.5}$
(i) $a = 5$, $\theta = 0.5$, $\gamma = 0.5$, $\Omega = 0.5$, $\underline{\beta = 0.9}$

Figure 3.5: Total research X^t_{NC} with no spillover effect



(a) $a = 5$, $\theta = 0.5$, $\gamma = 0.5$, $\Omega = 0.5$, $\beta = 0$

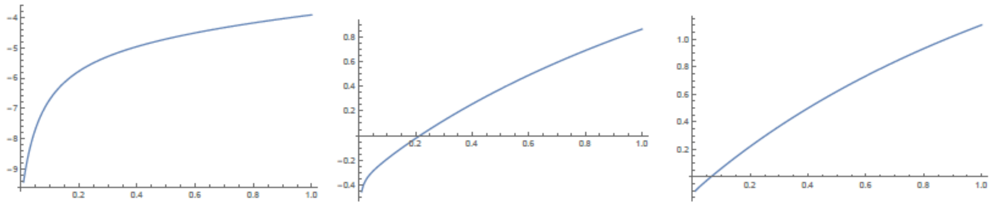
Figure 3.6: Total research X^t_{NC} with no spillover effect

Given (3.21), the optimal tax t_{NC} is equal to:

$$t_{NC} = \Bigg[\frac{a\,(2\theta^2 n\Omega - 1)}{\theta n\,(2\theta^2\Omega + 1)} + \qquad (3.21)$$

$$-\frac{2a\Omega(\beta(n^2-1)+n+1)(4\theta^4 n^2\Omega^2 + 2\theta^2\Omega(-2\beta + n(2\beta + n - 1) + 2) - n)}{\theta n(2\theta^2\Omega+1)(4\theta^2\Omega^2(-(\beta-1)^2 + \beta(\beta+1)n^3 + ((2-3\beta)\beta+1)n^2 + n(\beta(3\beta-5) + \gamma\theta^2 + 2)) + 2\Omega((n+1)(\beta(n-1)+1)(\beta(n-1)+n+1) + 2\gamma\theta^2 n) + \gamma n)}\Bigg]$$

The optimal tax is an increasing function of the number of firms n, of the social environmental damage Ω and of the firms' emission rate θ. In Figure 3.7 it is possible to see that for low values of Ω and θ and few firms, the research produced is so low that the tax turns out to be a subsidy. As the number of firms increases, the tax becomes positive.



(a) $a = 5$, $\theta = 0.5$, $\gamma = 0.5$, $\Omega = 0.5$, $\beta = 0.5$, $n = 1$

(b) $a = 5$, $\theta = 0.5$, $\gamma = 0.5$, $\Omega = 0.5$, $\beta = 0.5$, $n = 10$

(c) $a = 5$, $\theta = 0.5$, $\gamma = 0.5$, $\Omega = 0.5$, $\beta = 0.5$, $n = 30$

Figure 3.7: Optimal Tax t_{NC}

By substituting the optimal tax t_{NC} and the optimal level of research x_{NC} into (3.17), we obtain the final welfare level as a function of $(a, n, \theta, \Omega, \gamma)$.[18]

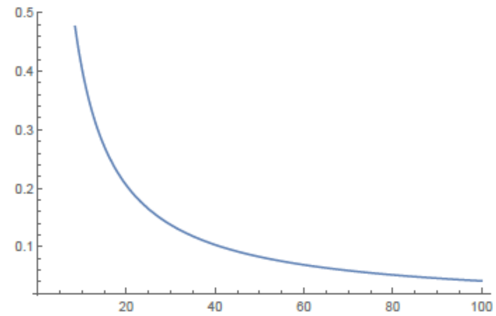
$$W(t) = a\left[\frac{n(a - t_{NC}\theta)}{1+n}\right] - \frac{1}{2}\left[\frac{n(a - t_{NC}\theta)}{1+n}\right]^2 - \Omega\left[\theta\left[\frac{n(a - t_{NC}\theta)}{1+n}\right] - \sum_{i=1}^{n} x_i(1 + \beta(n-1)\right]^2 \tag{3.22}$$

Optimal quantity for a single firm is equal to:

$$q_{NC}^{t} = \frac{a\left[2\gamma\theta^2\Omega + \gamma + 2\Omega(\beta(n-1)+1)^2\left(2n\left(\theta^2\Omega+1\right)+1\right)\right]}{n\left[\gamma\left(2\theta^2\Omega+1\right)^2 + 4\Omega(\beta(n-1)+1)^2\left(2\theta^2 n\Omega + n + 1\right)\right]} \tag{3.23}$$

As for the Commitment case, also in the Non Commitment case the quantity of the single firm q_{NC}^{t} is a convex function with respect to the environmental social damage. It decreases as Ω increases, and it is convex with respect to the number of firms n. This result is shown in Figure 3.8.

[18] The explicit form of the welfare function is shown in the appendix (see function B.5).



(a) $a = 5$, $\theta = 0.6$, $\gamma = 0.5$, $\beta = 0.5$, $\Omega = 0.7$

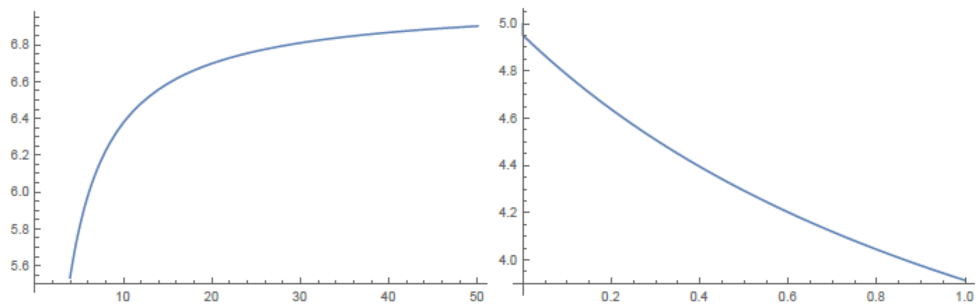
(b) $a = 5$, $\theta = 0.6$, $\gamma = 0.5$, $\beta = 0.5$, $n = 25$

Figure 3.8: Single firm quantity q^t_{NC} with respect to the number of firms n (a) and environmental damage Ω (b)

The total quantity is equal to:

$$Q_{NC} = \frac{a\left[2\gamma\theta^2\Omega + \gamma + 2\Omega(\beta(n-1)+1)^2\left(2n\left(\theta^2\Omega+1\right)+1\right)\right]}{\gamma\left[2\theta^2\Omega+1\right]^2 + 4\Omega\left[\beta(n-1)+1\right]^2\left[2\theta^2 n\Omega + n + 1\right]} \tag{3.24}$$

The total quantity Q_{NC} is concave with respect to the total number of firms n, and with respect to the social environmental damage Ω (Figure 3.9).



(a) $a = 5$, $\theta = 0.6$, $\gamma = 0.5$, $\beta = 0.5$, $\Omega = 0.7$

(b) $a = 5$, $\theta = 0.6$, $\gamma = 0.5$, $\beta = 0.5$, $n = 50$

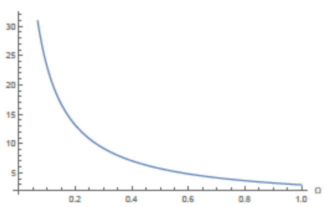
Figure 3.9: Total Quantity Q^t_{NC} with respect to the number of firms n (a) and environmental damage Ω (b)

3.2.3 Preliminary Result

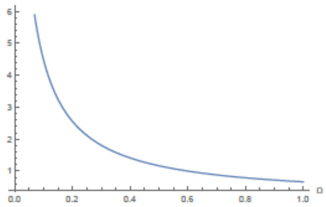
The results obtained so far allow us to make a comparison between the different levels of aggregate research and Welfare gained under the Commitment (X_C^t) and the Non Commitment (X_{NC}^t) scenario. Due to the equation complexity, providing an explicit solution is infeasible, but through numerical solutions and graphs it is possible to reach some general results. We follow the analysis done by Poyago-Theotoky (2007), providing a simple difference of the research level obtained under the Commitment (X_C^t) and the Non Commitment (X_{NC}^t), and the total Welfare, (W_C^t and W_{NC}^t respectively), showing the difference in graphs with different parameters values. We start first with the difference between the research level X_C^t and X_{NC}^t in the duopolistic case. In Figure 3.10 it is possible to see that the research produced is greater under the Commitment scenario than under the Non Commitment for any values of θ and γ. In Figure 3.10 (a), (b) and (c), with θ increasing, the difference is decreasing in favour of the Non Commitment research. The same result is obtained when the firms' emission rate γ is increasing (Figure 3.10 (c), (d) and (f)). In both cases, as the social environmental damage Ω increases, the difference is slightly reduced in favour of the research reached under the Non Commitment scenario. As the effectiveness of the pollution damages and the emission rate per firms both increase, the firms have more incentive to invest in research under the Non Commitment than the Commitment case, but the difference remains entirely positive. As the competition increases, for $\beta > 0$ and any level of emission rate θ and research productivity γ, research under Non Commitment is always inferior, as shown in Figure 3.11. A possible intuition is that firms are aware that their research effort will not be protected under a non credible Government. Thus, they do not have the proper incentives to invest in research as much as under a credible government. We can summarize the results found so far in Proposition 1:

Proposition 1: In the duopolistic case the research is always greater under the Commitment scenario than under the Non Commitment. This difference is entirely in favour of Commitment research for increasing competition (n increasing).

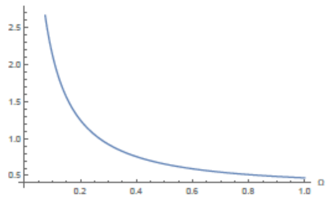
The idea that credible government can obtain higher social welfare has been largely investigated in the literature. The result obtained is analogous to the result derived by Kydland and Prescott (1977). In their seminal paper, they showed that discretionary policy are time inconsistent and they lead to suboptimal policy result. Thus, in dynamic models, better results are reached with committed policy-makers. Their analysis holds here, showing that consistent policy lead to better outcomes. The result obtained resembles the one found by Lutz et al. (2000), too. They found that quality choice made by firms are higher when the government moves first, by setting a quality standard before firms can choose by their own. These results hold for the following propositions, too.



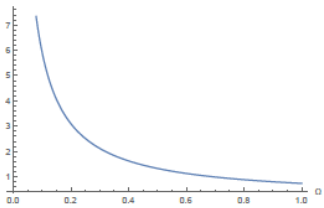
(a) $a = 5$, $\theta = 0.1$, $\gamma = 0.5$, $\beta = 0.5$, $n = 2$



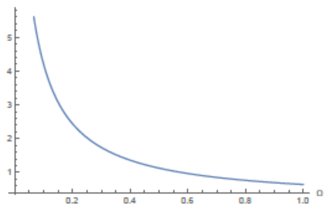
(b) $a = 5$, $\theta = 0.5$, $\gamma = 0.5$, $\beta = 0.5$, $n = 2$



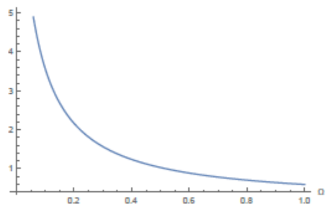
(c) $a = 5$, $\theta = 0.9$, $\gamma = 0.5$, $\beta = 0.5$, $n = 2$



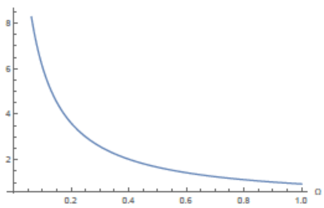
(d) $a = 5$, $\theta = 0.5$, $\gamma = 0.1$, $\beta = 0.5$, $n = 2$



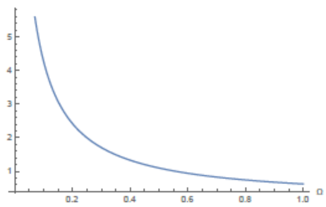
(e) $a = 5$, $\theta = 0.5$, $\gamma = 0.6$, $\beta = 0.5$, $n = 2$



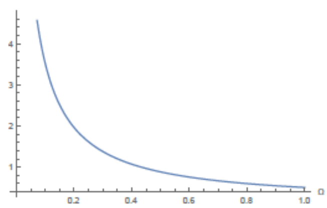
(f) $a = 5$, $\theta = 0.5$, $\gamma = 0.9$, $\beta = 0.5$, $n = 2$



(g) $a = 5$, $\theta = 0.5$, $\gamma = 0.5$, $\beta = 0.1$, $n = 2$

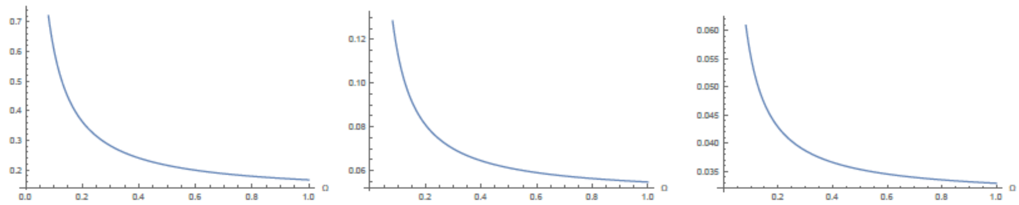


(h) $a = 5$, $\theta = 0.5$, $\gamma = 0.5$, $\beta = 0.6$, $n = 2$



(i) $a = 5$, $\theta = 0.5$, $\gamma = 0.5$, $\beta = 0.9$, $n = 2$

Figure 3.10: $X_C^t - X_{NC}^t$. Difference in Aggregate Research under the Commitment and the Non Commitment scenario with respect to Ω when $n = 2$



(a) $a = 5$, $\theta = 0.5$, $\gamma = 0.5$, $\beta = 0.5$, $\underline{n = 10}$ (b) $a = 5$, $\theta = 0.5$, $\gamma = 0.5$, $\beta = 0.5$, $\underline{n = 30}$ (c) $a = 5$, $\theta = 0.5$, $\gamma = 0.5$, $\beta = 0.5$, $\underline{n = 50}$

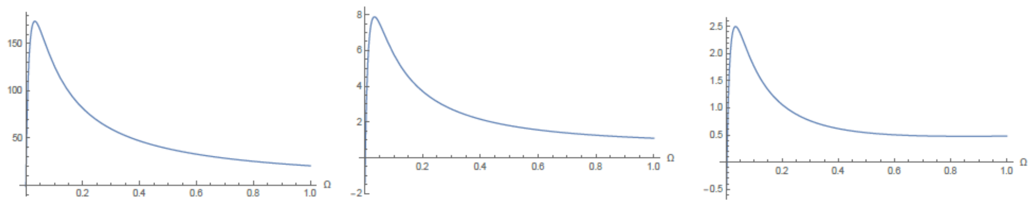
Figure 3.11: $X_C^t - X_{NC}^t$. Difference in Aggregate Research under the Commitment and the Non Commitment scenario with respect to Ω

The results are less straightforward for the total welfare difference. According to the level of emission rate θ, the research spillover β, the research productivity γ and to the number of firms n, different results may be obtained. First we analyse the duopolistic case. When $n = 2$, for high values of γ and θ (above 0.5), the difference is entirely positive, meaning that the welfare gained under the environmental tax with a credible government is greater than the welfare gained under the non commitment case (Figure 3.12, (a) - (c)). As β increases, the difference slightly increases in favour of the Commitment case (Figure 3.12, (d), (e) and (f)), still remaining entirely positive. An interesting case is represented when $\beta < 0.5$ and firms' emission rate θ and productivity of research γ are close to 1. For low level of β ($\beta < 0.1$) and high values of θ and γ (respectively $\theta > 0.8$ and $\gamma > 0.9$) the difference becomes negative for $\Omega > 0.6$ (Figure 3.12 (g), (h) and (i)). This is the only case where, for great values of the social environmental damage Ω, the welfare difference turns out to be in favour of the Non Commitment scenario. For larger number of firms, the difference is entirely in favour of the welfare gained under the Commitment scenario, for any parameter values. Thus, we can conclude that higher spillover rate let the difference increase in favour of the Welfare gained under the Commitment case. For low spillover rate and high emission rate θ and research productivity γ, the social welfare is greater under the Non Commitment scenario than the Commitment one. As the number of firms increases, the difference turns out to be in favour of the Commitment case, due the greater effort in research provided under the Commitment case than under the Non Commitment. For larger number of firms, the result is entirely in favour of the Commitment case.

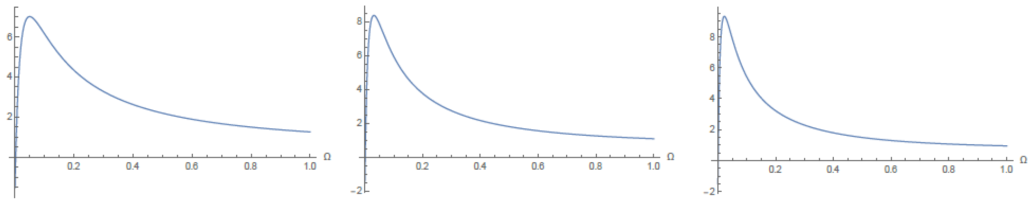
Results can be summarised by Proposition 2:

Proposition 2: In the duopolistic case, for high values of β ($\beta > 0.5$), the welfare gained under the Commitment scenario is greater than the welfare obtained under the Non Commitment. We obtain the opposite result for $\beta < 0.5$ and high firms emission rate θ and research productivity γ. In this case, for values of $\Omega > 0.5$, the welfare is greater under the Non Commitment case. As the market competition increases, the difference is entirely in favour of the Commitment case.

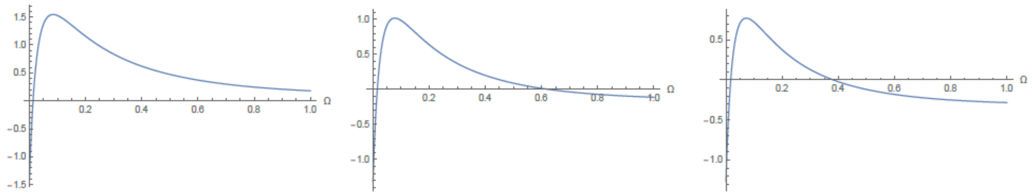
The emergence of a critical level of β as a turning point has been found by d'Aspremont and Jacquemin in their seminal paper. Authors discriminate between social profitability of different R&D coordination schemes according to different spillover level. In their paper, they discriminate between level of $\beta > 0.5$, for which closer cooperation in R&D allows for greater research effort, while when spillover rate is low ($\beta < 0.4$), non-cooperative R&D guarantees greater level of research. Here and elsewhere in the rest of the paper, different level of β discriminates between the various level of research obtained when government commits or not to the chosen environmental policy.



(a) $a = 5$, $\underline{\theta = 0.1}$, $\gamma = 0.5$, $\beta = 0.5$, $n = 2$ (b) $a = 5$, $\underline{\theta = 0.5}$, $\gamma = 0.5$, $\beta = 0.5$, $n = 2$ (c) $a = 5$, $\underline{\theta = 0.9}$, $\gamma = 0.5$, $\beta = 0.5$, $n = 2$

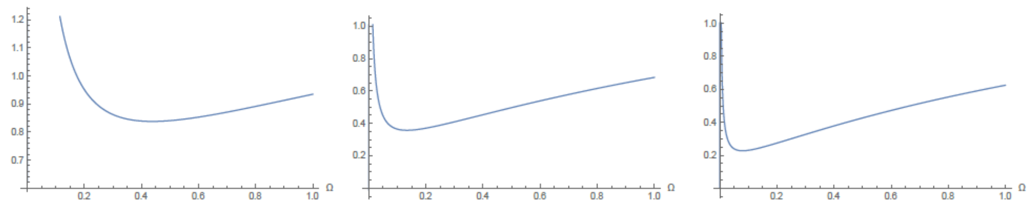


(d) $a = 5$, $\theta = 0.5$, $\gamma = 0.5$, $\underline{\beta = 0.1}$, $n = 2$ (e) $a = 5$, $\theta = 0.5$, $\gamma = 0.5$, $\underline{\beta = 0.5}$, $n = 2$ (f) $a = 5$, $\theta = 0.5$, $\gamma = 0.5$, $\underline{\beta = 0.9}$, $n = 2$



(g) $a = 5$, $\beta = 0.25$, $\gamma = 0.5$, $\underline{\theta = 0.75}$, $n = 2$ (h) $a = 5$, $\beta = 0.25$, $\gamma = 0.5$, $\underline{\theta = 0.9}$, $n = 2$ (i) $a = 5$, $\beta = 0.25$, $\gamma = 0.5$, $\underline{\theta = 1}$, $n = 2$

Figure 3.12: $W_C^t - W_{NC}^t$. Difference in Welfare under the Commitment and the Non Commitment scenario under environmental tax with respect to Ω and $n > 2$



(a) $a = 5$, $\theta = 0.5$, $\gamma = 0.5$, $\underline{n = 10}$, $\beta = 0.5$ (b) $a = 5$, $\theta = 0.5$, $\gamma = 0.5$, $\underline{n = 10}$, $\beta = 0.5$ (c) $a = 5$, $\theta = 0.5$, $\gamma = 0.5$, $\underline{n = 10}$, $\beta = 0.5$

Figure 3.13: $W_C^t - W_{NC}^t$. Difference in Welfare under the Commitment and the Non Commitment scenario under environmental tax with respect to Ω and $n > 2$

3.2.4 Environmental Permits

3.2.5 Permit Price

Under the environmental permit regulation, the government sets the optimal number of permits and allocates them to firms for free. The number of permits allocated correspond to the maximum pollution emission available. If firms pollute more than the allocated limit, they will need to buy the extra allowance needed; obviously, this represents a cost. If they pollute less, they can sell the permits surplus on the market. Following Sartzetakis (1997), permit price Υ is found by setting the permit net demand equal to zero, i.e. $\sum_{i=1}^{n} ND = 0$, where $ND = \sum_{i=1}^{n} [e_i - \phi]$, $e_i = \theta q_i - x_i - \beta \sum_{i=1}^{n-1} x_{-1}$ and $\phi = \sum_{i=1}^{n} \phi_i$ is the total sum of permits granted by the Government to the firms for free (i.e., the so-called *grandfathering*).

The permit price is equal to:

$$\Upsilon = \frac{a\theta n - [\beta (n^2 - 1) + n + 1] \sum_{i=1}^{n} x_i - n(n+1)\phi}{\theta^2 n} \tag{3.25}$$

which is positive for any number of permits ϕ such that:

$$\phi < \frac{a\theta n - [\beta(n^2 - 1) + n + 1] \sum_{i=1}^{n} x_i}{n(n+1)} \tag{3.26}$$

The intuition is straightforward. If the number of permits exceed ϕ, the oversupply forces the permit price to zero.

3.2.6 Environmental Permits - Commitment case

The timing is the following:

1) Firms set their profit maximizing level of output q_i;

2) Firms maximize their own level of research x_i;

3) The Government sets the optimal number of permits ϕ and issues them

to the firms. That corresponds to the maximum emission capacity allocated to each firms.

Price charged by firms is:

$$P = a - Q_i \tag{3.27}$$

where $Q_i = \sum_{i=1}^{n} q_i$. The firm's i profit function is:

$$\Pi_i = [a - Q_i]\, q_i - \Upsilon \left[\theta_i q_i - x_i - \beta \sum_{i=1}^{n-1} x_{-i} - \phi_i \right] - \frac{\gamma x_i^2}{2} \tag{3.28}$$

By symmetry, $\theta_i = \theta_{-i} = \theta$. Consequently, given that each firm produces the same amount of emissions, the Government provides each firm with the same number of permits; so $\phi_i = \phi_{-i} = \phi$ and the total number of permits $\phi = \sum_{i=1}^{n} \phi$, becomes equal to $\phi = n\phi$. Firms maximizes (3.28) with respect to q_i to obtain the best response function q_i:

$$q_i = \frac{a - \Upsilon\theta}{1 + n} \tag{3.29}$$

Optimal quantity is symmetric, so $q_i = q_{-i} = q$. It is straightforward to see that the optimal quantity q_i increases with respect to the market dimension a, and it decreases with respect to the permit price Υ and the emission rate θ.

Given (3.29), the profit function is equal to:

$$\Pi_i = \left[\frac{a - \Upsilon\theta}{n + 1} \right]^2 + \Upsilon \left[x_i + \beta \sum_{i=1}^{n-1} x_{-i} + \phi \right] - \frac{\gamma x_i^2}{2} \tag{3.30}$$

Firm's i maximizes (3.30) with respect to their own level of research x_i to obtain the non-cooperative optimal research:

$$x_i = \frac{\Upsilon}{\gamma} \tag{3.31}$$

Research is symmetric, so $x_i = x_{-i} = \bar{x}_C^P$, where the superscript P stands for permit and the subscript C for Commitment. We can substitute $\bar{x}_C^P$ into the permit price (3.25), to obtain:

$$\Upsilon = \frac{\gamma\left[a\theta - \phi(n-1)\right]}{\gamma\theta^2 + \beta(n^2-1) + n + 1} \tag{3.32}$$

The permit price is positive for a number of permit $\phi < \frac{a\theta}{n-1}$ and for $n \geq 1$.

The Social Welfare function is equal to (Lambertini et al, 2015):

$$\sum_{i=1}^{n} \pi_i + \frac{Q^2}{2} - \Omega\left[\sum_{i=1}^{n} e_i\right]^2 \tag{3.33}$$

where $\frac{Q^2}{2}$ is the consumer surplus. By substituting (3.28), (3.29), (3.31) and (3.32) into (3.33), the total welfare can be reduced to:

$$W(\Upsilon) = aQ - \frac{1}{2}Q^2 - \frac{\gamma\sum_{i=1}^{n} x_i^2}{2} - \Upsilon\theta Q + \Upsilon\sum_{i=1}^{n}\left[x_i + \beta\sum_{i=1}^{n-1} x_{-i}\right] + \Upsilon n\phi - \Omega\left[\sum_{i=1}^{n} e_i\right] \tag{3.34}$$

It is worth mentioning that permits issued to firms for free represents a net wealth transfer from government to firms, so they increase total welfare. With respect to welfare under the environmental tax, government does not collect any tax revenues, given that permits are issued for free.

Maximizing (3.34) with respect to the total number of permit ϕ, we can find the optimal number of permits.[19]

$$\phi = \frac{a\gamma\theta\left[\gamma\theta^2 + \beta(n-1) + n + 2\right)}{2n\Omega\left[\gamma\theta^2 + \beta(n^2-1) + n + 1\right]^2 + \gamma\left[\gamma\theta^2 n + (n+1)^2\right]} \tag{3.35}$$

We can substitute the optimal number of permits into (3.32) to find the permit price:

[19]It satisfies the second order conditions.

$$\Upsilon = \frac{a\gamma\theta\left[2n\Omega\left[\gamma\theta^2 + \beta(n^2-1) + n + 1\right] - \gamma\right]}{2n\Omega\left[\gamma\theta^2 + \beta(n^2-1) + n + 1\right]^2 + \gamma\left[\gamma\theta^2 n + (n+1)^2\right]} \tag{3.36}$$

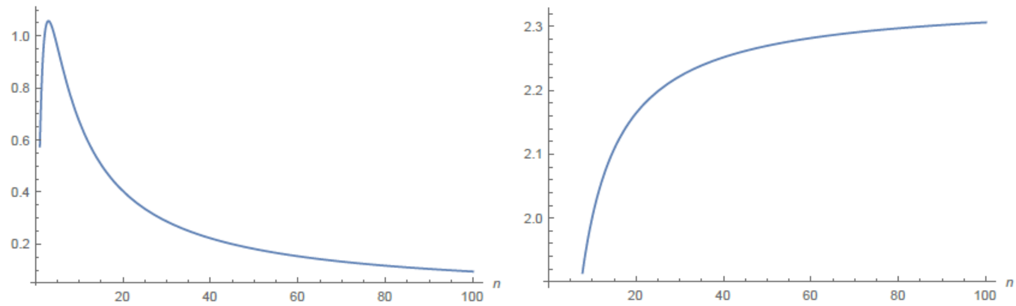
which is positive for $n \geq 1$. Given (3.36) and substituting it into (3.31), we can find the optimal research x_C^P, equal to:

$$x_C^P = \frac{a\theta\left(2n\Omega\left(\beta(n^2-1) + n + 1\right) + \gamma\left(2\theta^2 n\Omega - 1\right)\right)}{\begin{array}{l}\gamma(n+1)(4\beta\theta^2 n^2\Omega + n(1-4(\beta-1)\theta^2\Omega) \\ \quad + 1) + 2n\Omega(\beta(n^2-1)+n+1)^2 + \gamma^2\theta^2 n(2\theta^2\Omega+1)\end{array}} \tag{3.37}$$

research is positive for $n \geq 1$ and for $\Omega > \frac{1}{2\theta^2 n}$. The aggregate research $\bar{X}_C^P$ is:

$$X_C^P = \frac{an\theta(2n\Omega(\beta(n^2-1)+n+1) + \gamma(2\theta^2 n\Omega - 1))}{\begin{array}{l}\gamma(n+1)(4\beta\theta^2 n^2\Omega + n(1-4(\beta-1)\theta^2\Omega) + 1) \\ \quad + 2n\Omega(\beta(n^2-1)+n+1)^2 + \gamma^2\theta^2 n(2\theta^2\Omega+1)\end{array}} \tag{3.38}$$

As for the environmental tax case, the aggregate research is convex when spillover are positive, showing a decreasing amount of investment in research when number of firms are increasing. When the spillover are absent and firms can fully appropriate their research effort, the function becomes concave (Figure 3.14 (a) and (b)).



(a) $a = 5$, $\theta = 0.5$, $\gamma = 0.5$, $\beta = 0.5$, $\Omega = 0.5$

(b) $a = 5$, $\theta = 0.5$, $\gamma = 0.5$, $\beta = 0$, $\Omega = 0.5$

Figure 3.14: Aggregate research X_C^P with respect n

We can summarize the result in the following Lemma:

Lemma 3: The aggregate research is concave with respect to the number of firms n and single-peaked.

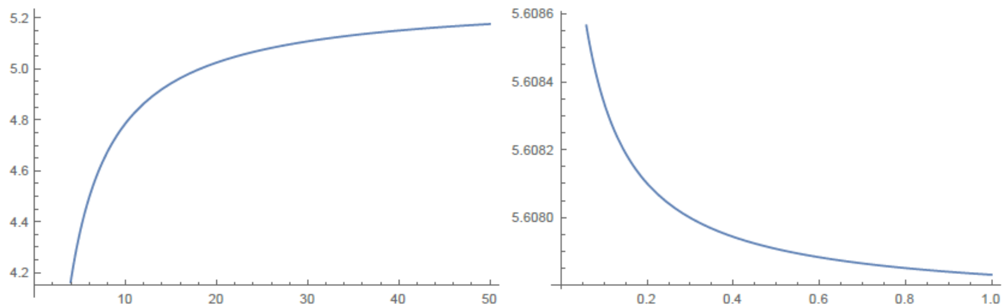
By substituting (3.35), (3.36) and (3.37) into (3.34), we obtain the total welfare W_C^P as a function of $(a, n, \gamma, \beta, \theta, \Omega)$.

$$W_C^P = \frac{a^2 n \left[2n\Omega \left[\gamma\theta^2(2\beta(n-1)+1) + (n+2)(\beta(n-1)+1)^2\right] + \gamma\left(\gamma\theta^2 + n + 2\right)\right]}{4n\Omega \left[\gamma\theta^2 + \beta(n^2-1) + n + 1\right]^2 + 2\gamma\left[\gamma\theta^2 n + (n+1)^2\right]} \tag{3.39}$$

Finally, the total quantity Q_C^P is equal to:

$$Q_C^P = \left[\frac{n}{n+1}\right]\left[a - \frac{a\gamma\theta^2\left(2n\Omega\left(\gamma\theta^2+\beta(n^2-1)+n+1\right)-\gamma\right)}{2n\Omega\left(\gamma\theta^2+\beta(n^2-1)+n+1\right)^2 + \gamma n\left(\gamma\theta^2+(n+1)^2\right)}\right] \tag{3.40}$$

As it is possible to see in Figure 3.15, the function is concave with respect to the total number of firms n (Fig. 3.15 (a)), and convex with respect to the environmental damage (Fig. 3.15 (b)).



(a) $a = 5$, $\theta = 0.6$, $\gamma = 0.5$, $\beta = 0.5$, $\Omega = 0.7$

(b) $a = 5$, $\theta = 0.6$, $\gamma = 0.5$, $\beta = 0.5$, $n = 20$

Figure 3.15: Total quantity Q_C^P with respect to the number of firms n (a) and to the social environmental damage Ω (b)

3.2.7 Environmental Permits - Non Commitment case

In the Non Commitment scenario, timing is the following:

1) Firms set the profit maximizing quantity q_i;

2) The Government adjusts the optimal number of permits ϕ;

3) Firms set the research level x_i.

As usual we solve the model by backward induction, starting from the last stage, which is equal to the Commitment case. The optimal quantity is given by (3.29).[20]

The welfare in the Non Commitment case is equal to:

$$W_{NC}^{P} = \sum_{i=1}^{n} \pi_i + \frac{Q^2}{2} + \frac{\gamma \sum_{i=1}^{n} x_i^2}{2} - \Omega \left[\sum_{i=1}^{n} e_i \right]^2 \tag{3.41}$$

The welfare function can be rearranged as:

$$\begin{aligned} W_{NC}^{P} = aQ - \frac{1}{2}Q^2 - \Upsilon\theta Q + \Upsilon \left[\sum_{i=1}^{n} x_i(1+\beta(n-1)) \right] + n\Upsilon\phi \\ -\Omega \left[\theta Q - \sum_{i=1}^{n} x_i(1+\beta(n-1)) \right]^2 \end{aligned} \tag{3.42}$$

By maximizing (3.42) with respect to the number of permit ϕ, we find the optimal number of permits.[21]

$$\phi = \frac{a\theta - (1+\beta(n-1)) \sum_{1}^{n} x_i}{2\theta^2 n\Omega + n} \tag{3.43}$$

[20]Due to space constraints, the main findings of this section are shown in the Appendix.
[21]The second order conditions are satisfied.

Given (3.43), we can obtain the permit price Υ:

$$\Upsilon = \frac{a\left(2\theta^2 n\Omega - 1\right) - 2\theta\Omega\left(\beta\left(n^2-1\right)+n+1\right)\sum_1^n x_i}{\theta n\left(2\theta^2\Omega+1\right)} \tag{3.44}$$

Finally we can substitute (3.44) and (3.43) into (3.30) and maximize it with respect to x, to solve the final stage:

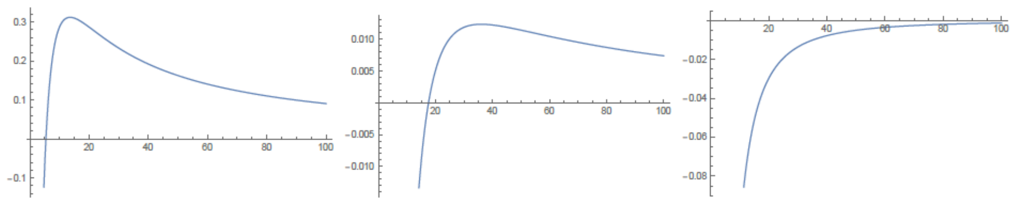
$$x_{NC}^P = \left[\frac{a(4\theta^4 n^2\Omega^2 + 2\theta^2\Omega(-\beta + n(3\beta - 2\beta n + n - 3) + 1) + (\beta-1)(n-1))}{\begin{array}{r}\theta n(-2\Omega((\beta-1)(n^2-1)(\beta(n-1)+1) - 2\gamma\theta^2 n) + 4\theta^2\Omega^2(-(\beta-1)^2 + \\ \beta(\beta+1)n^3 + ((2-3\beta)\beta+1)n^2 + n(\beta(3\beta-5)+\gamma\theta^2+2)) + \gamma n)\end{array}}\right] \tag{3.45}$$

the aggregate research X_{NC}^P is equal to:

$$X_{NC}^P = \left[\frac{a(4\theta^4 n^2\Omega^2 + 2\theta^2\Omega(-\beta + n(3\beta - 2\beta n + n - 3) + 1) + (\beta-1)(n-1))}{\begin{array}{r}\theta(-2\Omega((\beta-1)(n^2-1)(\beta(n-1)+1) - 2\gamma\theta^2 n) + 4\theta^2\Omega^2(-(\beta-1)^2 + \\ \beta(\beta+1)n^3 + ((2-3\beta)\beta+1)n^2 + n(\beta(3\beta-5)+\gamma\theta^2+2)) + \gamma n)\end{array}}\right] \tag{3.46}$$

The aggregate research obtained under the environmental permits with a non credible government shows a slight different behaviour with respect to the aggregate research results obtained so far. The spillover rate has a deeper impact on the aggregate research. For high spillover rate, the total research turns out to be negative (Figure 3.16). Given a high number of firms, incentive to invest in research given the environmental regulation are not enough and, as the incidence of spillover increases, firms prefer to do not invest in research at all. Finally, the aggregate research shows the characteristics U-shaped behaviour for $\beta > 0$ and increasing number of firms, as summarised by the following Lemma:

Lemma 4: The total research is concave with respect the number of firms n and single-peaked.



(a) $a = 5$, $\theta = 0.5$, $\gamma = 0.5$, $\beta = 0.2$, $\Omega = 0.7$ (b) $a = 5$, $\theta = 0.6$, $\gamma = 0.5$, $\beta = 0.2$, $n = 20$ (c) $a = 5$, $\theta = 0.6$, $\gamma = 0.5$, $\beta = 0.2$, $n = 20$

Figure 3.16: Aggregate Research X^P_{NC} with respect to the number of firms n

Due to space constraints, in the appendix are shown the optimal number of permits issued by the government (B.9), the total permit price (B.10) and the aggregate quantity (B.11). By substituting (3.46), (B.9) and (B.10) into (3.42), we can find the total welfare W^P_{NC} as a function of $(a, \gamma, \beta, n, \Omega, \theta)$.

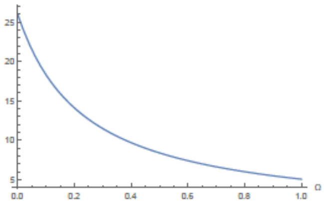
3.2.8 Preliminary Results

As for the Environmental Tax section, we can draw some preliminary results. We can analyse the difference between the aggregate research level obtained under the Commitment scenario minus the Non Commitment (X^P_C and X^P_{NC} respectively), for the duopolistic case. When $n = 2$, the research difference is decreasing when θ and γ increase (Figure (3.17 (a) - (f)); the difference is decreasing for β increasing (Figure 3.17 (g), (h), (i)). When $\gamma = \theta = 1$, for $\beta < 0.64$ the difference is entirely positive for any values of the social environmental damage Ω. When $\beta > 0.64$, to have a positive difference we need to have $\Omega > 0.0002$.[22] (Figure 3.17 (j), (k) and (l)). For values of γ and θ strictly lower than 1, but increasing, the difference between the research done under the Commitment scenario is always greater than the research obtained under the Non Commitment. The difference is negative when the productivity of research and the emission rate per firms are at their highest level and the spillover rate is above $\beta > 0.64$, but only for very low level of social environmental damage Ω (Figure 3.17 (j), (k) and (l)). These results

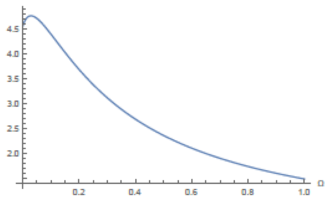
[22] When $\beta = 1$, $\Omega > 0.02$.

confirm what we obtained in the environmental tax scenario. Firms invest more in research under a credible government than under a non credible one. As the level of competition increases, the difference decreases in favour of the Non Commitment case (Figure 3.18 (a), (b) and (c)). In Figure 3.18 (e), (f) and (g) it is shown that as β increases, the difference decreases in favour of the Non Commitment, but still it is entirely positive. For higher market competition, the difference is entirely in favour of the research produced under the Commitment scenario. The results can be summarised in the following Proposition:

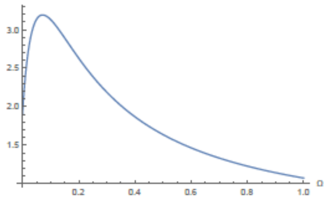
Proposition 3: In the duopolistic case, for $\beta > 0.5$, the total investment in aggregate research under the Commitment regime is greater than the total investment under Non Commitment. For $\beta < 0.5$ and θ and γ close to 1, the difference is in favour of the Non Commitment case for values of $\Omega > 0.5$. For a larger number of firms, the difference is entirely in favour of the Commitment case.



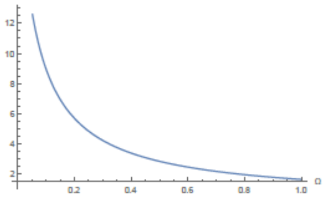
(a) $a = 5$, $\underline{\theta = 0.1}$, $\gamma = 0.5$, $\beta = 0.5$, $n = 2$



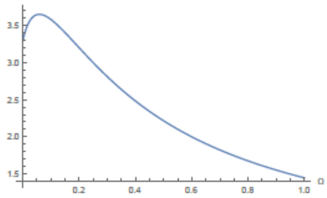
(b) $a = 5$, $\underline{\theta = 0.5}$, $\gamma = 0.5$, $\beta = 0.5$, $n = 2$



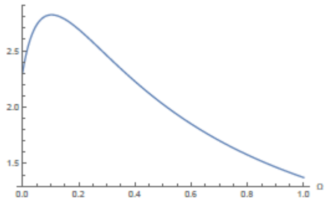
(c) $a = 5$, $\underline{\theta = 0.9}$, $\gamma = 0.5$, $\beta = 0.5$, $n = 2$



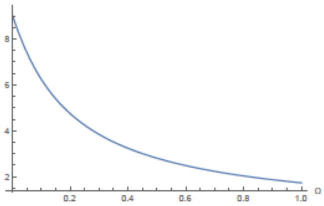
(d) $a = 5$, $\theta = 0.5$, $\underline{\gamma = 0.1}$, $\beta = 0.5$, $n = 2$



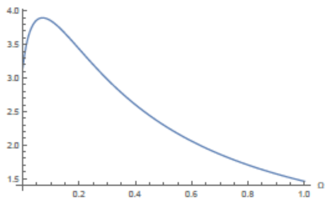
(e) $a = 5$, $\theta = 0.5$, $\underline{\gamma = 0.6}$, $\beta = 0.5$, $n = 2$



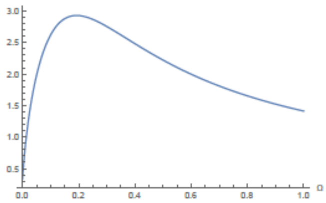
(f) $a = 5$, $\theta = 0.5$, $\underline{\gamma = 0.9}$, $\beta = 0.5$, $n = 2$



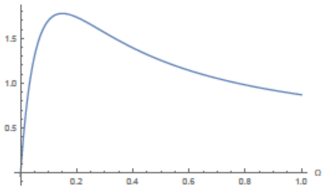
(g) $a = 5$, $\theta = 0.5$, $\gamma = 0.5$, $\underline{\beta = 0.1}$, $n = 2$



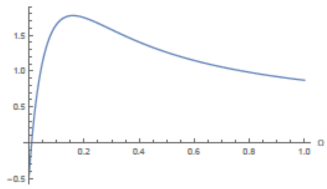
(h) $a = 5$, $\theta = 0.5$, $\gamma = 0.5$, $\underline{\beta = 0.6}$, $n = 2$



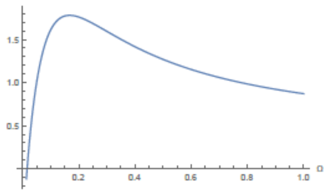
(i) $a = 5$, $\theta = 0.5$, $\gamma = 0.5$, $\underline{\beta = 0.9}$, $n = 2$



(j) $a = 5$, $\theta = 1$, $\gamma = 1$, $\underline{\beta = 0.65}$, $n = 2$

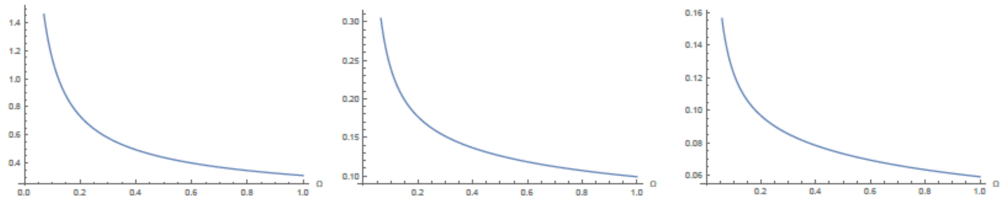


(k) $a = 5$, $\theta = 1$, $\gamma = 1$, $\underline{\beta = 0.8}$, $n = 2$

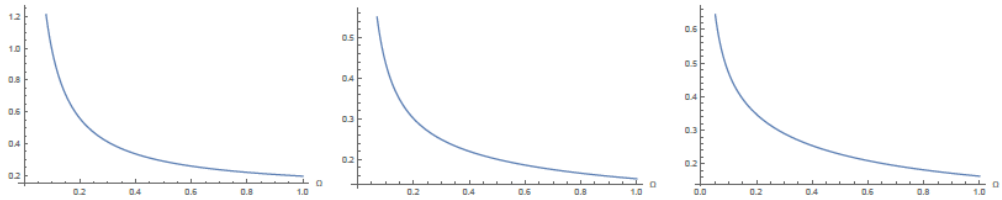


(l) $a = 5$, $\theta = 1$, $\gamma = 1$, $\underline{\beta = 0.95}$, $n = 2$

Figure 3.17: $X_C^P - X_{NC}^P$. Difference in Aggregate Research under the Commitment and the Non Commitment scenario when $n = 2$



(a) $a = 5$, $\theta = 0.5$, $\gamma = 0.5$, $\beta = 0.5$, $\underline{n = 10}$ (b) $a = 5$, $\theta = 0.5$, $\gamma = 0.5$, $\beta = 0.5$, $\underline{n = 30}$ (c) $a = 5$, $\theta = 0.5$, $\gamma = 0.5$, $\beta = 0.5$, $\underline{n = 50}$



(d) $a = 5$, $\theta = 0.5$, $\gamma = 0.5$, $\underline{\beta = 0.1}$, $n = 20$ (e) $a = 5$, $\theta = 0.7$, $\gamma = 0.5$, $\underline{\beta = 0.5}$, $n = 20$ (f) $a = 5$, $\theta = 0.7$, $\gamma = 0.5$, $\underline{\beta = 0.9}$, $n = 20$

Figure 3.18: $X_C^P - X_{NC}^P$. Difference in Aggregate Research under the Commitment and the Non Commitment scenario under environmental permits for $n > 2$

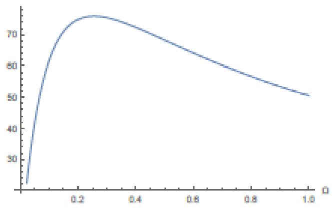
The same analysis is conducted for the welfare levels gained under the two regimes. The difference is evaluated between the welfare obtained under the Commitment scenario minus the Non Commitment (W_C^P and W_{NC}^P respectively). The difference in welfare for the duopolistic case decreases in γ and θ,[23] as shown in Figure 3.19 (a)-(f), while increases in β (Figure 3.19 (g), (h) and (i)).[24] For an increasing number of firms, the difference turns out to be in favour of the Commitment Scenario (as it is possible to see in Figure 3.19 (j), (k), (l)). The results show a similar pattern to the results we obtained for the difference in the aggregate research. The research is always positive for $\Omega > 0.07$ and for any values of the others parameters. The welfare gained under the Commitment scenario is greater than the welfare gained under the Non Commitment. The intuition for this counterintuitive results can be explained by the fact that the firms invest more in research under the Commitment scenario than the Non Commitment, in order to reduce the

[23]For an increasing θ, the minimum level of social environmental damage Ω required to have a positive difference tends to zero for $\theta \to 0$ and we must have $\Omega > 0.022$ for $\theta = 1$. For an increasing θ, the minimum level required is $\Omega > 0.04$ when $\gamma = 1$, and $\Omega > 0$ when $\gamma \to 0$.

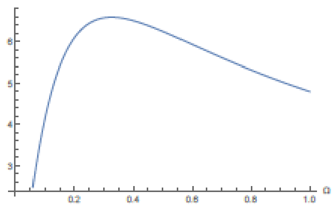
[24]The difference is positive for $\Omega > 0.07 >$ when $\gamma = \theta = 0.5$ and $\beta = 1$, while it is sufficient to have $\Omega > 0$ when $\beta = 0$

burden of the environmental regulation. Due to the lower research investment, under the Non Commitment scenario firms produce more emissions (Figure 3.20 (a), (b) and (c)). That represents a cost for the society as a whole and it has a detrimental effect on the total welfare. The results, which state again the supremacy of a credible government against a non credible one, can be summarised in the following Proposition:

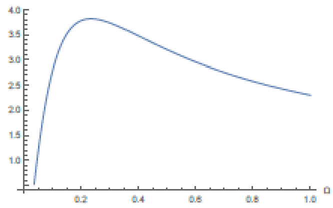
Proposition 4: For $\Omega > 0.07$, the difference in total welfare is always in favour of the Commitment case.



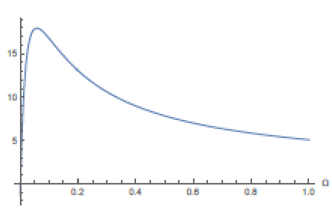
(a) $a = 5$, $\underline{\theta = 0.1}$, $\gamma = 0.5$, $\beta = 0.5$, $n = 2$



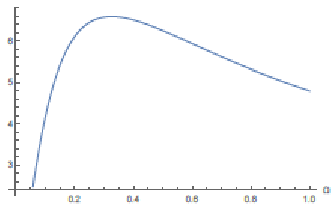
(b) $a = 5$, $\underline{\theta = 0.5}$, $\gamma = 0.5$, $\beta = 0.5$, $n = 2$



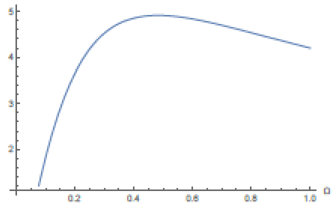
(c) $a = 5$, $\underline{\theta = 0.9}$, $\gamma = 0.5$, $\beta = 0.5$, $n = 2$



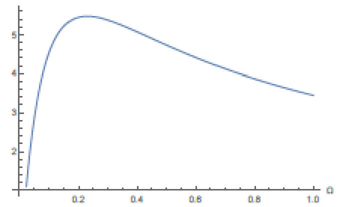
(d) $a = 5$, $\theta = 0.5$, $\underline{\gamma = 0.1}$, $\beta = 0.5$, $n = 2$



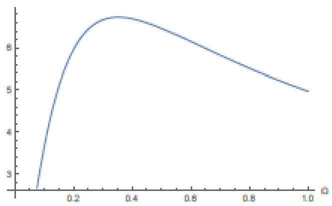
(e) $a = 5$, $\theta = 0.5$, $\underline{\gamma = 0.5}$, $\beta = 0.5$, $n = 2$



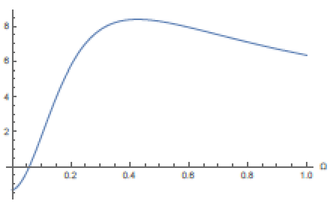
(f) $a = 5$, $\theta = 0.5$, $\underline{\gamma = 0.9}$, $\beta = 0.5$, $n = 2$



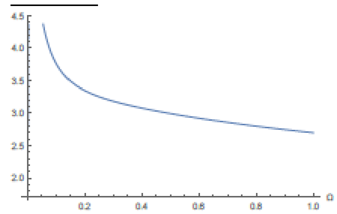
(g) $a = 5$, $\theta = 0.5$, $\gamma = 0.5$, $\underline{\beta = 0.1}$, $n = 2$



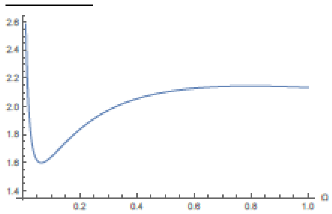
(h) $a = 5$, $\theta = 0.5$, $\gamma = 0.5$, $\underline{\beta = 0.6}$, $n = 2$



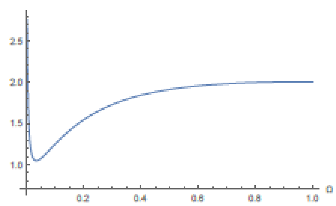
(i) $a = 5$, $\theta = 0.5$, $\gamma = 0.5$, $\underline{\beta = 0.9}$, $n = 2$



(j) $a = 5$, $\theta = 0.5$, $\gamma = 0.5$, $\beta = 0.5$, $\underline{n = 10}$

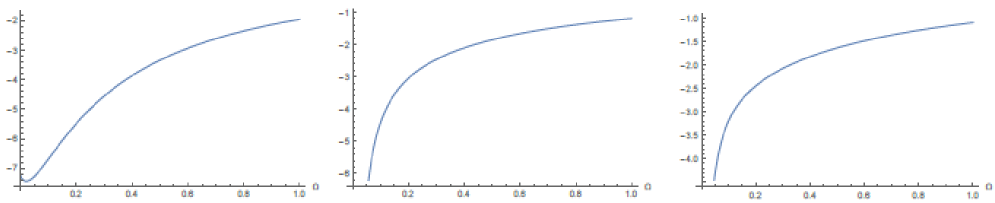


(k) $a = 5$, $\theta = 0.5$, $\gamma = 0.5$, $\beta = 0.5$, $\underline{n = 30}$



(l) $a = 5$, $\theta = 0.5$, $\gamma = 0.5$, $\beta = 0.5$, $\underline{n = 50}$

Figure 3.19: $W_C^P - W_{NC}^P$. Difference in Welfare under the Commitment and the Non Commitment scenario ($W_C^P - W_{NC}^P$)



(a) $a = 5$, $\theta = 0.5$, $\gamma = 0.5$, $\beta = 0.5$, $n = 2$ (b) $a = 5$, $\theta = 0.5$, $\gamma = 0.5$, $\beta = 0.5$, $n = 20$ (c) $a = 5$, $\theta = 0.5$, $\gamma = 0.5$, $\beta = 0.5$, $n = 50$

Figure 3.20: $W_C^P - W_{NC}^P$. Difference in Aggregate Emissions under the Commitment and the Non Commitment scenario ($E_C^P - E_{NC}^P$)

3.3 Results

We can now summarize the results we have found so far and do a comparative static analysis of the aggregate research and of the total welfare obtained under the two different regimes. In both scenarios, commitment leads to better results and, surprisingly, total research and total welfare under both regimes are the same when the government is credible. Firms maximize their own effort in research, reaching the same optimal investment, which gives exactly the same total welfare. Results are different for the Non Commitment case. We have already seen that a credible government obtains a greater level of welfare and firms tend to invest more in research, under both scenarios. Under a non credible government, results are less straightforward and we can run interesting comparative analysis. The different level of aggregate research obtained under the two regimes ($X_{NC}^t - X_{NC}^P$) shows that, in the duopolistic case, the difference is increasing in favour of the research produced under the tax scenario when θ increases (Figure 3.21 (a), (b) and (c)).[25] It is interesting to notice that, for low values of θ and high values of β, the difference turns out to be entirely negative. (Figure 3.21 (d), (e)

[25] To have a positive difference when $\gamma = 0.5$ and θ are increasing, $\Omega > \left[\frac{1}{4}\left(\frac{3\beta^2-4\theta^2-3}{\theta^2(\beta(5\beta+16)+5\theta^2+11)} + \sqrt{\frac{9(\beta^2-1)^2-4(\beta-17)(\beta+1)\theta^2+36\theta^4}{\theta^4(\beta(5\beta+16)+5\theta^2+11)^2}}\right)\right]$, which is a convex and increasing function with respect to θ. For $\beta \leq 0.5$, $\Omega > -\frac{31}{9(20\theta^2+81)} - \frac{1}{36\theta^2} + \frac{3}{4}\sqrt{\frac{64\theta^4+176\theta^2+9}{\theta^4(20\theta^2+81)^2}}$, while for $\beta > 0.5$, $\Omega > \frac{1}{2}\sqrt{\frac{9\theta^2+32}{\theta^2(5\theta^2+32)^2}} + \frac{1}{-5\theta^2-32}$. For $\beta \to 1$ and low values of θ, the difference turns out to be entirely negative.

and (f)). The difference decreases for γ increasing.[26](Figure 3.21 (g), (h) and (i)). Finally, the difference is slightly increasing for greater values of β (Figure 3.21 (j), (k) and (l)).[27] With a larger number of firms, the difference is decreasing, but the minimum level of Ω required to have a positive difference is diminishing too (Figure 3.22 (a), (b) and (c)). The effect of competition on the difference $X_{NC}^t - X_{NC}^P$ shows that the greater the number of firms, the greater the research performed under the environmental tax with respect to the environmental permit. Thus, as the number of firms increases, lower values of θ or greater values of γ are required to have a larger research under permits than under tax. The graphs show that, for low values of social environmental damage, the environmental permit regulation guarantees greater incentive for firms to invest in research than the environmental tax. The result is entirely in favour of the research produced under the permit scenario for low firms emission rate ($\theta < 0.1$) and high spillover rate ($\beta > 0.6$). As the firms' rate emission θ and the social environmental damage Ω both increase, the environmental tax regulation turns out to be more incisive in assuring a positive amount of investment to reduce the emission rate. When the number of firms increases, the environmental tax turns out to be the best policy to induce firms in investing in aggregate research. As the firms market power decreases, the environmental tax results more incisive. For low values of social damage, the permit regulation is more stringent and provide stronger incentives to firm to invest in research. For higher values of Ω, the opposite is true. We can summarize these results in the following Proposition:

Proposition 5: For the duopolistic case, for low level of Ω, total aggregate research is greater under the environmental permits than the environmental tax. The level of Ω required to have a positive difference increases for high values of β and low values of θ. For larger number of firms, the difference is entirely in favour of the environmental tax.

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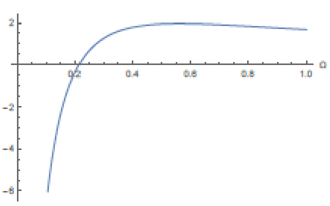
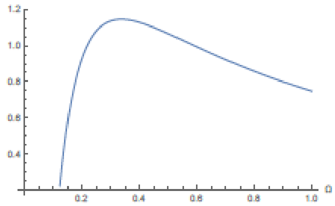
[26] To have a positive difference, for γ increasing and $\theta = 0.5$, $\Omega > 2\left(\sqrt{\frac{9(\beta^2-1)^2-2(\beta-17)(\beta+1)\gamma+9\gamma^2}{(2\beta(5\beta+16)+5\gamma+22)^2}} + \frac{3\beta^2-2\gamma-3}{2\beta(5\beta+16)+5\gamma+22}\right)$, which is a convex and increasing function with respect to γ. When $\gamma = \beta = 1$, $\Omega > 0.19$ to have a positive difference.

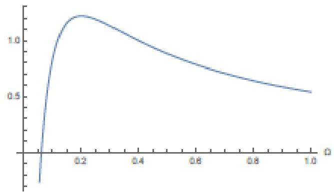
[27] For $\gamma = \theta = 0.5$, to have a positive difference $\Omega > 2\sqrt{\frac{4\beta(9\beta^3-19\beta+16)+113}{(4\beta(5\beta+16)+49)^2}} + \frac{4\left(3\beta^2-4\right)}{4\beta(5\beta+16)+49}$, which is a positive and convex function with respect to β, with a range equal to $[0.107, 0.145]$.



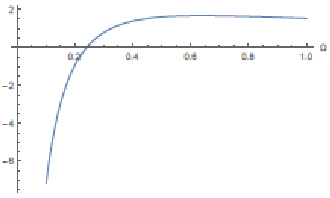
(a) $a = 5$, $\underline{\theta = 0.1}$, $\gamma = 0.5$, $\beta = 0.5$, $n = 2$



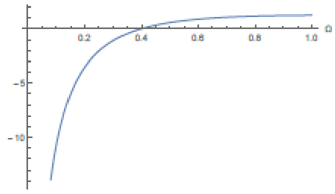
(b) $a = 5$, $\underline{\theta = 0.5}$, $\gamma = 0.5$, $\beta = 0.5$, $n = 2$



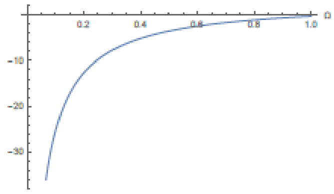
(c) $a = 5$, $\underline{\theta = 0.9}$, $\gamma = 0.5$, $\beta = 0.5$, $n = 2$



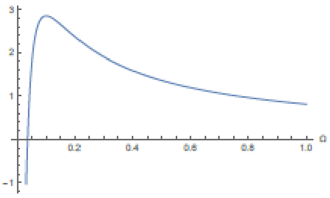
(d) $a = 5$, $\theta = 0.1$, $\gamma = 0.5$, $\underline{\beta = 0.6}$, $n = 2$



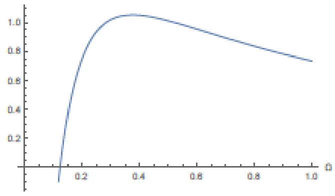
(e) $a = 5$, $\theta = 0.1$, $\gamma = 0.5$, $\underline{\beta = 0.8}$, $n = 2$



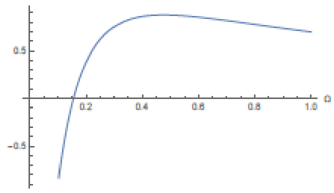
(f) $a = 5$, $\theta = 0.05$, $\gamma = 0.5$, $\underline{\beta = 0.95}$, $n = 2$



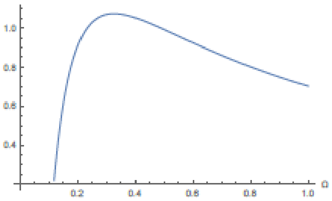
(g) $a = 5$, $\theta = 0.5$, $\underline{\gamma = 0.1}$, $\beta = 0.5$, $n = 2$



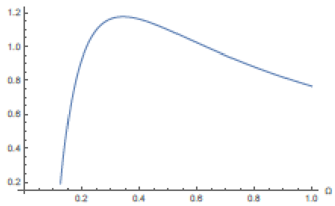
(h) $a = 5$, $\theta = 0.5$, $\underline{\gamma = 0.6}$, $\beta = 0.5$, $n = 2$



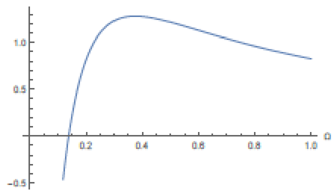
(i) $a = 5$, $\theta = 0.5$, $\underline{\gamma = 0.9}$, $\beta = 0.5$, $n = 2$



(j) $a = 5$, $\theta = 0.5$, $\gamma = 0.5$, $\underline{\beta = 0.1}$, $n = 2$

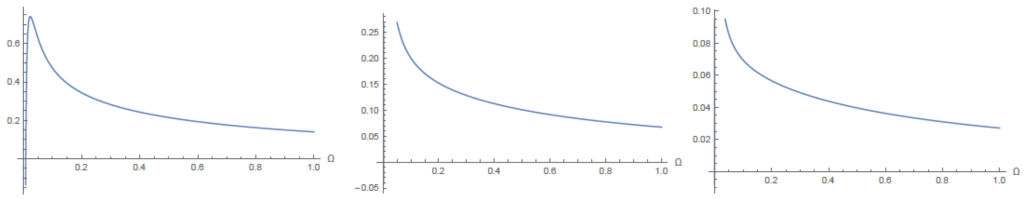


(k) $a = 5$, $\theta = 0.5$, $\gamma = 0.5$, $\underline{\beta = 0.6}$, $n = 2$



(l) $a = 5$, $\theta = 0.5$, $\gamma = 0.5$, $\underline{\beta = 0.9}$, $n = 2$

Figure 3.21: Difference of $\bar{X}^t_{NC}$ minus $\bar{X}^P_{NC}$ under the Non Commitment scenario in the duopolistic case



(a) $a = 5$, $\theta = 0.5$, $\gamma = 0.5$, $\beta = 0.5$, $\underline{n = 10}$ (b) $a = 5$, $\theta = 0.5$, $\gamma = 0.5$, $\beta = 0.5$, $\underline{n = 20}$ (c) $a = 5$, $\theta = 0.5$, $\gamma = 0.5$, $\beta = 0.5$, $\underline{n = 50}$

Figure 3.22: Difference of X^t_{NC} minus X^P_{NC} under the Non Commitment scenario for $n > 2$.

The same analysis can be conducted for the difference between the welfare obtained under the environmental tax and the environmental permits, when the government is not credible ($W^t_{NC} - W^P_{NC}$). The difference in total welfare is decreasing with respect to θ.[28] For values of θ sufficiently close to zero and high values of β, the difference turns out to be entirely negative (for example, when $\theta \leq 0.0857$ and $\beta \geq 0.9$), as shown in Figure 3.23 (a)-(f). This result is confirmed by the difference of the aggregate researches with the same parameter values. For low firms emissions rate θ and high spillover rate β, the higher level of research produced under the environmental permits guarantees a greater level of welfare too, due to the reduced detrimental effect of the emission. For γ increasing, the difference is decreasing in favour of the permit regulation (Figure 3.23 (g), (h) and (i)).[29] Finally, as β increases, the difference is increasing in favour of the welfare gained under the environmental tax (Figure 3.23, (j), (k) and (l)).[30] As the number of firms increases, the environmental tax guarantees a greater welfare than the permit one (Figure 3.24 (a), (b) and (c)). These results show that for low values of social environmental damage, the welfare gained under the environmental permit scenario ($\Omega < 0.2$) is greater than the welfare gained under the environmen-

[28]The value of Ω for which the difference is positive, for $\gamma = 0.5$, is shown in the Appendix (function (B.13)).

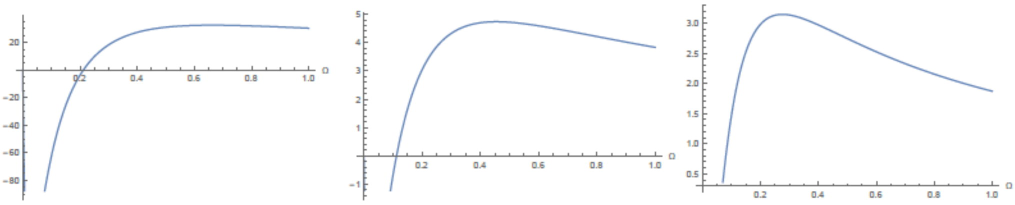
[29]The difference, for $\theta = 0.5$, is positive when $\Omega > 2\left(\sqrt{\frac{9(\beta^2-1)^2-2(\beta-17)(\beta+1)\gamma+9\gamma^2}{(2\beta(5\beta+16)+5\gamma+22)^2}} + \frac{3\beta^2-2\gamma-3}{2\beta(5\beta+16)+5\gamma+22}\right)$, which is a convex increasing function with respect to *gamma* and *beta* and that for $\gamma = \beta = 1$ is equal to 0.19.

[30]The difference, when $\gamma = \beta = 0.5$, is positive for $\Omega > 2\sqrt{\frac{4\beta(9\beta^3-19\beta+16)+113}{(4\beta(5\beta+16)+49)^2}} + \frac{4(3\beta^2-4)}{4\beta(5\beta+16)+49}$, which, for $\beta \in [0,1]$ takes values of $[0.107, 0.146]$.

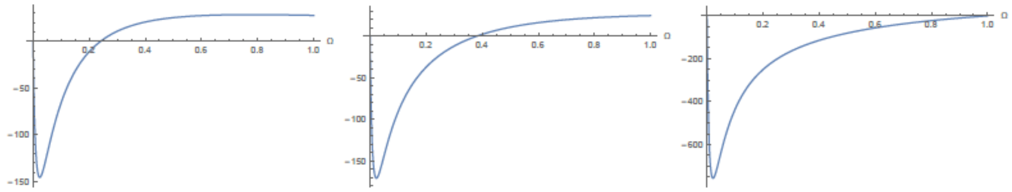
tal tax, for any parameters value. The intuition is that the environmental permit regulation is more stringent than the environmental tax when Ω is low, inducing firms to invest more in research and so to reduce emissions. For greater values of Ω, the situation is the opposite. The environmental tax becomes more stringent than the permit, inducing firms to invest more. This result does not hold for low emission rate per firms θ. When firms do not pollute much, they have more incentives to invest in research under the permit regulation. For greater market competition, the environmental tax is outperforming the environmental permit both in incentive to firms in investing in research and in the total welfare. The Proposition follows:

Proposition 6: In the duopolistic case, for low (high) levels of Ω, the welfare gained under the environmental tax (environmental permit) regime is greater than the welfare obtained under the environmental permit (environmental tax). For low firms emissions rate θ and high spillover β, the difference is entirely in favour of the welfare gained under the environmental permit regime. For higher market competition, the welfare gained under the environmental tax is greater for any parameters values.

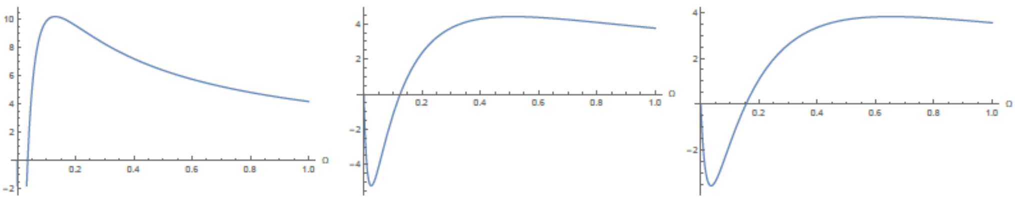
Environmental permits tend to perform better than the environmental tax for low number of firms in terms of incentive to invest in R&D and welfare. The situation is entirely in favour of the emission tax for higher number of firms. A possible intuition is represented by the possibility of free riding that the permits offer with respect to the environmental tax for a large number of firms. This effect is well known in the environmental policy literature (see Requate and Unold (2003), or Denicolò (1999), among others). If the number of firms investing in research is high enough, the demand for permits will decreases, as the firms' emissions rate drops. Thus, as the permit price decreases, also the incentives of polluting firms to buy the permit decreases. This effect, absent under the environmental tax, explains why the environmental tax performs better than permits for larger number of firms.



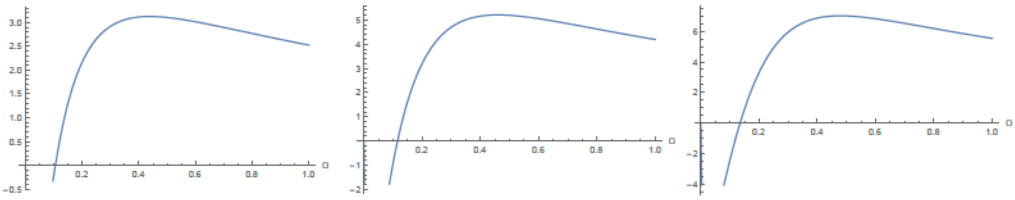
(a) $a = 5$, $\underline{\theta = 0.1}$, $\gamma = 0.5$, $\beta = 0.5$, $n = 2$ (b) $a = 5$, $\underline{\theta = 0.5}$, $\gamma = 0.5$, $\beta = 0.5$, $n = 2$ (c) $a = 5$, $\underline{\theta = 0.9}$, $\gamma = 0.5$, $\beta = 0.5$, $n = 2$



(d) $a = 5$, $\theta = 0.1$, $\gamma = 0.6$, $\underline{\beta = 0.5}$, $n = 2$ (e) $a = 5$, $\theta = 0.1$, $\gamma = 0.5$, $\underline{\beta = 0.8}$, $n = 2$ (f) $a = 5$, $\theta = 0.05$, $\gamma = 0.5$, $\underline{\beta = 0.95}$, $n = 2$

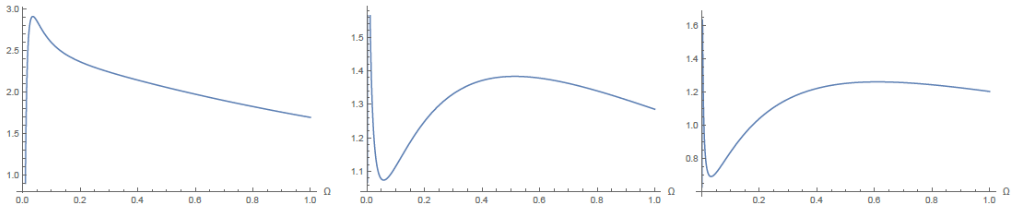


(g) $a = 5$, $\theta = 0.5$, $\underline{\gamma = 0.1}$, $\beta = 0.5$, $n = 2$ (h) $a = 5$, $\theta = 0.5$, $\underline{\gamma = 0.6}$, $\beta = 0.5$, $n = 2$ (i) $a = 5$, $\theta = 0.5$, $\underline{\gamma = 0.9}$, $\beta = 0.5$, $n = 2$



(j) $a = 5$, $\theta = 0.5$, $\gamma = 0.5$, $\underline{\beta = 0.1}$, $n = 2$ (k) $a = 5$, $\theta = 0.5$, $\gamma = 0.5$, $\underline{\beta = 0.6}$, $n = 2$ (l) $a = 5$, $\theta = 0.5$, $\gamma = 0.5$, $\underline{\beta = 0.9}$, $n = 2$

Figure 3.23: Difference of W^t_{NC} minus W^P_{NC} under the Non Commitment scenario in the duopolistic case



(a) $a = 5$, $\theta = 0.5$, $\gamma = 0.5$, $\beta = 0.5$, $\underline{n = 10}$ (b) $a = 5$, $\theta = 0.5$, $\gamma = 0.5$, $\beta = 0.5$, $\underline{n = 30}$ (c) $a = 5$, $\theta = 0.5$, $\gamma = 0.5$, $\beta = 0.5$, $\underline{n = 50}$

Figure 3.24: Difference of W^t_{NC} minus W^P_{NC} under the Non Commitment scenario for $n > 2$

3.4 Research Cartelization

Following d'Aspremont and Jacquemin (1988), Kamien et al., (1992) and Poyago-Theotoky (2007), we can extend our model to investigate R&D cooperation among firms. Under a credible government, in the second stage of the game firms maximize their joint profit with respect to their own research level. Third and last stage remain non-cooperative, given that cartelization at output level is strictly banned. According to the different spillover value, following the specification of Kamien et al., (1992), if $\beta < 1$ we have research cooperation (or research cartelization); firms coordinate their research activities, but do not share their information completely. If $\beta = 1$, we have a Research Joint Venture (RJV), where research effort and research results are spread equally among firms. We limit our analysis to the case where the government commits credibly to the environmental regulation chosen in the first stage.

3.4.1 Research cartelization under environmental tax

Price is equal to:

$$P = a - Q \tag{3.47}$$

The third and final stage is the same as the previous cases under the environmental tax. Cournot-Nash quantity is equal to (3.4) and the total profit is equal to (3.5). At the second stage, firms maximize their joint profits, so $\sum_{i=1}^{n} \Pi_i$, which is equal to:

$$\sum_{i=1}^{n} \Pi_i = n\left[\frac{a-t\theta}{1+n}\right]^2 + \sum_{i=1}^{n} t\left[x_i + \beta \sum_{i=1}^{n-1} x_{-i}\right] - \frac{\gamma \sum_{i=1}^{n} x_i^2}{2} \tag{3.48}$$

We consider the symmetric solution $x_i = x_{-i} = x$; by maximizing (3.48) with respect to x, we obtain the optimal research x_{RC}[31], where RC stands for *Research Cartelization*:

$$x_{RC} = \frac{t\left[1+\beta(n-1)\right]}{\gamma} \tag{3.49}$$

Research function is increasing with respect to the environmental tax t, the number of firms n and the spillover rate β. To obtain the optimal tax t, the government maximizes the total welfare with respect to t; that solves the last stage of the game:

$$t_{RC} = \frac{a\gamma\theta\left(2n(n+1)\Omega(\beta(n-1)+1)^2 + \gamma\left(2\theta^2 n\Omega - 1\right)\right)}{2n\Omega\left(\gamma\theta^2 + (n+1)(\beta(n-1)+1)^2\right)^2 + \gamma\left(\gamma\theta^2 n + (n+1)^2\right)} \tag{3.50}$$

Substituting (3.51) into (3.49), gives us the single firm research x_{RC}^t:

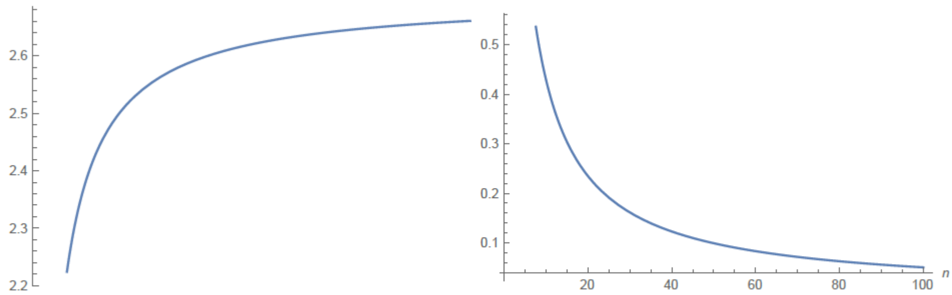
$$x_{RC}^t = \frac{a\theta(\beta(n-1)+1)\left(2n(n+1)\Omega(\beta(n-1)+1)^2 + \gamma\left(2\theta^2 n\Omega - 1\right)\right)}{2n\Omega\left(\gamma\theta^2 + (n+1)(\beta(n-1)+1)^2\right)^2 + \gamma\left(\gamma\theta^2 n + (n+1)^2\right)} \tag{3.51}$$

[31] The second order condition is satisfied.

The aggregate research is equal to:

$$X_{RC}^{t} = \frac{a\theta n(\beta(n-1)+1)\left(2n(n+1)\Omega(\beta(n-1)+1)^2+\gamma\left(2\theta^2 n\Omega-1\right)\right)}{2n\Omega\left(\gamma\theta^2+(n+1)(\beta(n-1)+1)^2\right)^2+\gamma\left(\gamma\theta^2 n+(n+1)^2\right)} \tag{3.52}$$

The aggregate research X_{RC}^{t} is convex for $\beta = 0$ and concave for $\beta > 1$, confirming the trend that the aggregate research have shown so far (Figure 3.25) .



(a) $a = 5$, $\theta = 0.5$, $\gamma = 0.5$, $\beta = 0$, $\Omega = 0.5$

(b) $a = 5$, $\theta = 0.5$, $\gamma = 0.5$, $\beta = 0.5$, $\Omega = 0.5$

Figure 3.25: Aggregate research X_{RC}^{t} with respect to the number of firms n

Finally, total welfare is equal to:

$$W^{t} = \left[\frac{a^2 n(2n\Omega(\gamma\theta^2 (2\beta(n-1)(\beta(n-1)+2)+1)+(n+2)(\beta(n-1)+1)^4)+\gamma(\gamma\theta^2+n+2))}{2(2n\Omega(\gamma\theta^2+(n+1)(\beta(n-1)+1)^2)^2+\gamma(\gamma\theta^2 n+(n+1)^2))}\right] \tag{3.53}$$

3.4.2 Research cartelization under environmental permits

The third and last stage in this scenario is the same of the environmental permits under a credible government. Thus, firm's optimal quantity is given by (3.29) and the total profit is given by (3.30).

In the second stage, firms maximize the joint profit $\sum_{i=1}^{n} \pi_i$ with respect to x, to obtain the research function x:

$$x = \frac{\Upsilon\left[1+\beta(n-1)\right]}{\gamma} \tag{3.54}$$

The permit price is equal to:

$$\Upsilon = \frac{a\gamma\theta - \gamma(n+1)\phi}{\gamma\theta^2 + (n+1)(\beta(n-1)+1)^2} \tag{3.55}$$

By maximizing the welfare with respect to the number of permits ϕ, we can obtain the optimal number of permits[32]:

$$\phi = \frac{a\gamma\theta\left[\gamma\theta^2 + (\beta(n-1)+1)^2\right]}{n\left[\gamma\theta^2\left(2\gamma\theta^2\Omega + \gamma + 4(n+1)\Omega(\beta(n-1)+1)^2\right) + 2(n+1)^2\Omega(\beta(n-1)+1)^4\right]} \tag{3.56}$$

The optimal price is equal to:

$$\Upsilon = \frac{a\gamma\theta\left[2n(n+1)\Omega(\beta(n-1)+1)^2 + \gamma\left(2\theta^2 n\Omega - 1\right)\right]}{n\left[\gamma\theta^2\left(2\gamma\theta^2\Omega + \gamma + 4(n+1)\Omega(\beta(n-1)+1)^2\right) + 2(n+1)^2\Omega(\beta(n-1)+1)^4\right]} \tag{3.57}$$

By substituting (3.57) into (3.54), we find the optimal research function, equal to:

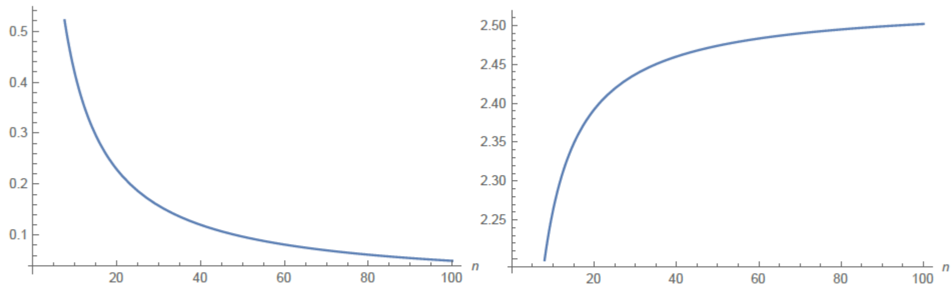
$$x_{RC}^{P} = \frac{a\theta\left[\beta(n-1)+1\right]\left[2n(n+1)\Omega(\beta(n-1)+1)^2 + \gamma\left(2\theta^2 n\Omega - 1\right)\right]}{n\left[\gamma\theta^2\left(2\gamma\theta^2\Omega + \gamma + 4(n+1)\Omega(\beta(n-1)+1)^2\right) + 2(n+1)^2\Omega(\beta(n-1)+1)^4\right]} \tag{3.58}$$

[32]The second order condition is satisfied.

and the aggregate research is:

$$X_{RC}^{P} = \frac{a\theta\left[\beta(n-1)+1\right]\left[2n(n+1)\Omega(\beta(n-1)+1)^2+\gamma\left(2\theta^2 n\Omega-1\right)\right]}{\gamma\theta^2\left(2\gamma\theta^2\Omega+\gamma+4(n+1)\Omega(\beta(n-1)+1)^2\right)+2(n+1)^2\Omega(\beta(n-1)+1)^4} \tag{3.59}$$

The aggregate research under the environmental permit is concave for $\beta = 0$, while it becomes convex and decreasing with respect to the number of firms for n increasing, as shown in Figure 3.26.



(a) $a = 5$, $\theta = 0.5$, $\gamma = 0.5$, $\beta = 0.5$, $\Omega = 0.5$

(b) $a = 5$, $\theta = 0.5$, $\gamma = 0.5$, $\beta = 0$, $\Omega = 0.5$

Figure 3.26: Aggregate research X_{RC}^{t} with respect to the number of firms n

Total welfare is equal to:

$$W_{RC}^{P} = \frac{a^2(\gamma\theta^2(\gamma+4n\Omega(\beta(n-1)+1)^2)+2n(n+2)\Omega(\beta(n-1)+1)^4)}{2(\gamma\theta^2(2\gamma\theta^2\Omega+\gamma+4(n+1)\Omega(\beta(n-1)+1)^2)+2(n+1)^2\Omega(\beta(n-1)+1)^4)} \tag{3.60}$$

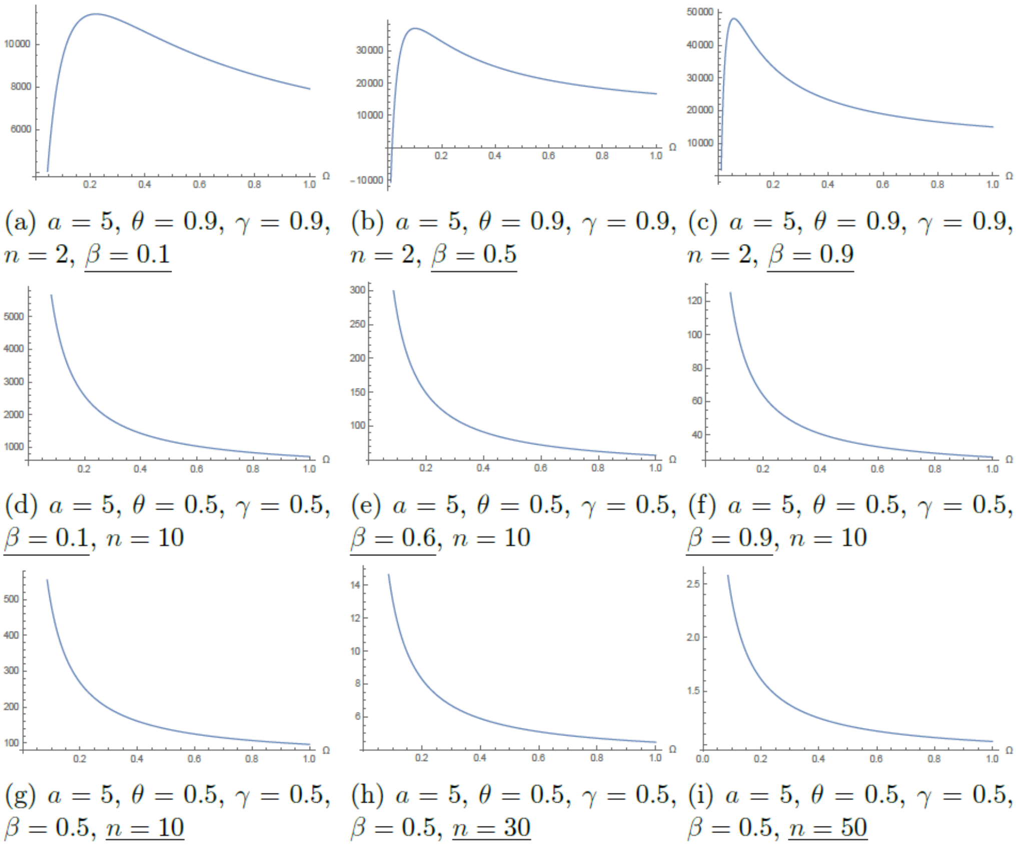
3.4.3 Results

The first analysis can be conducted between the aggregate research evaluated under the competitive commitment scenario (X_C^t, given in (11)) and the aggregate research obtained under the research cartelization case (X_{RC}^t, given in (52)), with $\beta < 1$.

In the duopolistic case, for high γ and θ, the difference is negative for low values of Ω.[33] As the rate of spillover increases, the difference tends to be entirely positive. The difference of the research obtained under the cartelization minus the level of research under the competitive scenario $(X^t_{RC} - X^t_C)$ for $n > 2$, shows that the research performed under the cartelization scenario is greater than the research done under the competitive case (Figure 3.27). The difference is increasing for higher values of θ and γ and decreasing for β and the number of firms n increasing.

The most interesting result is given by the reduction in the aggregate difference obtained under the cartelization with respect to the competitive scenario for increasing level of spillover β (Figure 3.27, (a) - (f)). The greater the ability of firms to imitate around other firms' research, the lower the advantage represented by the cartelization scenario of coordinating the research activity to avoid duplication of effort. Finally, for larger number of firms, the difference is decreasing (Figure 3.27, (g), (h) and (i)).

[33] For $\gamma = \theta = 0.9$ and $\beta = 0.1$, $\Omega > 0.025$. For $\beta = 0.5$, $\Omega > 0.015$ and, finally, for $\beta = 0.9$, $\Omega > 0.009$. Clearly, as β increases the minimum level of Ω needed to have a positive difference tends to zero.



(a) $a = 5$, $\theta = 0.9$, $\gamma = 0.9$, $n = 2$, $\beta = 0.1$
(b) $a = 5$, $\theta = 0.9$, $\gamma = 0.9$, $n = 2$, $\beta = 0.5$
(c) $a = 5$, $\theta = 0.9$, $\gamma = 0.9$, $n = 2$, $\beta = 0.9$
(d) $a = 5$, $\theta = 0.5$, $\gamma = 0.5$, $\beta = 0.1$, $n = 10$
(e) $a = 5$, $\theta = 0.5$, $\gamma = 0.5$, $\beta = 0.6$, $n = 10$
(f) $a = 5$, $\theta = 0.5$, $\gamma = 0.5$, $\beta = 0.9$, $n = 10$
(g) $a = 5$, $\theta = 0.5$, $\gamma = 0.5$, $\beta = 0.5$, $n = 10$
(h) $a = 5$, $\theta = 0.5$, $\gamma = 0.5$, $\beta = 0.5$, $n = 30$
(i) $a = 5$, $\theta = 0.5$, $\gamma = 0.5$, $\beta = 0.5$, $n = 50$

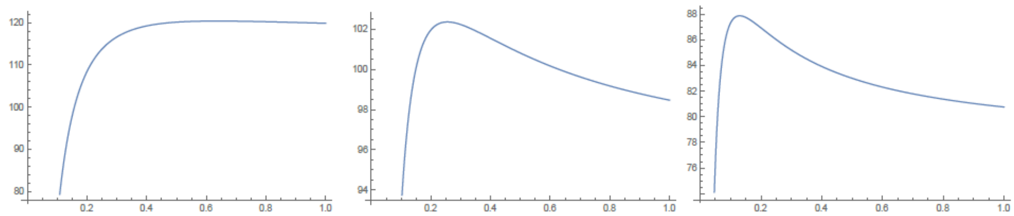
Figure 3.27: Difference of Research Cartelization (X^t_{RC}) minus Research Non Cartelization (X^t_C)

The difference between the aggregate research obtained under the cartelization scenario (X^P_{RC}) and the aggregate research of the competitive scenario (X^P_C) for the duopolistic case shows a similar trend with the difference of the aggregate researches analysed for the environmental tax case. The level of Ω needed to have a positive difference is greater under the environmental permits than the environmental tax,[34](Figure 3.28, (a), (b) and (c)). For larger number of firms, the difference is entirely positive, as summarised in Figure 3.28 ((d), (e) and (f)), decreasing for greater spillovers level of β (Figure 3.28 (g), (h) and (i)).

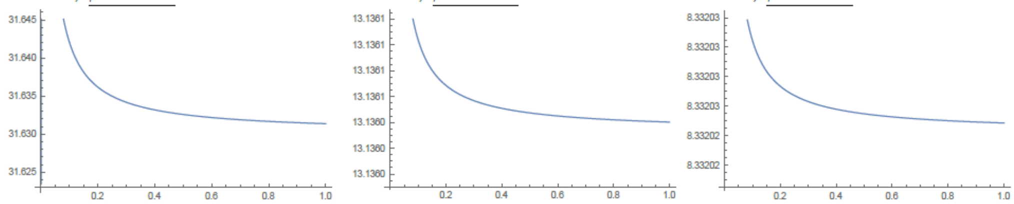
The results show that, for low number of firms ($n = 2$) and low values of social environmental damage Ω, the difference is negative in both cases, as

[34]For $\gamma = \theta = 0.9$ and $\beta = 0.1$, to have a positive difference $\Omega > 0.05$; for $\beta = 05$, $\Omega > 0.028$ and for $\beta = 0.9$, $\Omega > 0.017$.

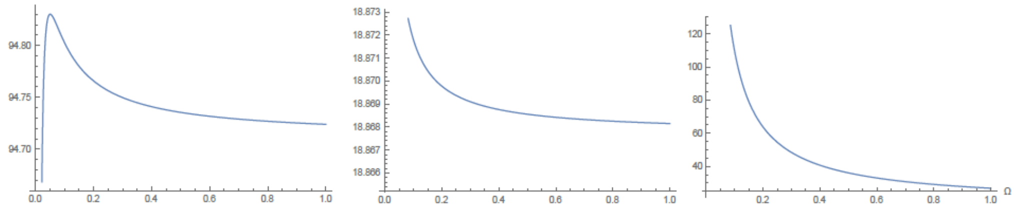
the research produced under the competitive cases in both the environmental regulations is greater than the research obtained under the cartelization case. The difference is decreasing for higher spillover rate β, but the difference turns out to be positive in the environmental tax case for lower values of Ω than the environmental permits case. For larger number of firms, both differences are entirely positive.



(a) $a = 5$, $\theta = 0.9$, $\gamma = 0.9$, $n = 2$, $\underline{\beta = 0.1}$ (b) $a = 5$, $\theta = 0.9$, $\gamma = 0.9$, $n = 2$, $\underline{\beta = 0.5}$ (c) $a = 5$, $\theta = 0.9$, $\gamma = 0.9$, $n = 2$, $\underline{\beta = 0.9}$



(d) $a = 5$, $\theta = 0.5$, $\gamma = 0.5$, $\beta = 0.5$, $\underline{n = 10}$ (e) $a = 5$, $\theta = 0.5$, $\gamma = 0.5$, $\beta = 0.5$, $\underline{n = 30}$ (f) $a = 5$, $\theta = 0.5$, $\gamma = 0.5$, $\beta = 0.5$, $\underline{n = 50}$



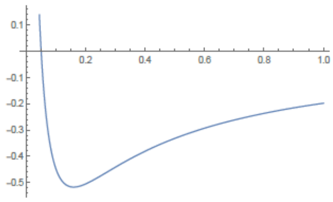
(g) $a = 5$, $\theta = 0.5$, $\gamma = 0.5$, $\underline{\beta = 0.1}$, $n = 10$ (h) $a = 5$, $\theta = 0.5$, $\gamma = 0.5$, $\underline{\beta = 0.6}$, $n = 10$ (i) $a = 5$, $\theta = 0.5$, $\gamma = 0.5$, $\underline{\beta = 0.9}$, $n = 10$

Figure 3.28: Difference of Research Cartelization minus Research Non Cartelization under Permit

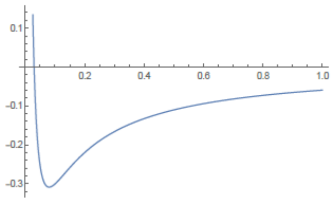
We can finally compare the aggregate researches produced under the environmental tax and the environmental permits with the research cartelization regime. The aggregate research produced under the environmental tax (X^t_{RC}) is greater than the research obtained under the environmental permit (X^P_{RC}) for high values of γ and θ and low values of Ω [35](Figure 3.29, (a), (b) and

[35]For $\gamma = \theta = 0.9$ and $\beta = 0.1$, to have a positive difference $\Omega < 0.052$. For $\beta = 0.5$, $\Omega < 0.03$ and for $\beta = 0.9$, $\Omega < 0.019$.

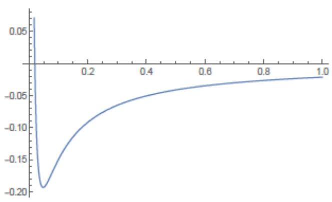
(c)). For a larger number of firms, the research produced under the environmental permits is always greater than the research obtained under the environmental tax, for any parameters values (Figure 3.29, (d), (e) and (f)). For low values of social environmental damage Ω, the environmental tax induces firms to produce a higher research level than the environmental level. This result contradicts the same scenario seen under the competitive case with a non credible government. When the social damage level Ω increases, the environmental permits regime induces firms to invest more in research. For higher number of firms, under research cartelization firms invest more in research under the environmental permits than the environmental tax. Thus, the environmental permits regulations is more incisive for higher social environmental damage level than the environmental tax, which is more stringent for low level of Ω.



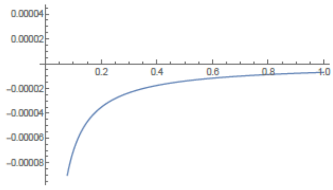
(a) $a = 5$, $\theta = 0.9$, $\gamma = 0.9$, $\underline{\beta = 0.1}$, $n = 2$



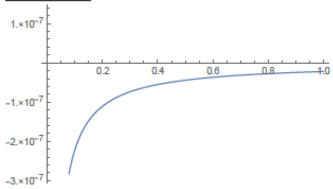
(b) $a = 5$, $\theta = 0.9$, $\gamma = 0.9$, $\underline{\beta = 0.5}$, $n = 2$



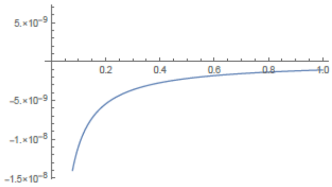
(c) $a = 5$, $\theta = 0.9$, $\gamma = 0.9$, $\underline{\beta = 0.9}$, $n = 2$



(d) $a = 5$, $\beta = 0.5$, $\underline{n = 10}$, $\theta = 0.5$, $\gamma = 0.5$



(e) $a = 5$, $\beta = 0.5$, $\underline{n = 30}$, $\theta = 0.5$, $\gamma = 0.5$



(f) $a = 5$, $\beta = 0.5$, $\underline{n = 50}$, $\theta = 0.5$, $\gamma = 0.5$

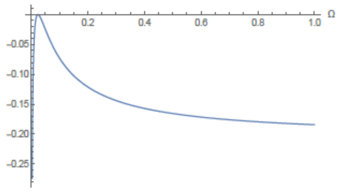
Figure 3.29: Difference of aggregate research under environmental tax and Research Cartelization minus the aggregate research under the environmental permit $(X^t_{RC} - X^P_{RC})$

The difference of the welfare gained under the environmental tax minus the environmental permit under the Research Cartelization $(W^t_{RC} - W^P_{RC})$ is entirely in favour of the welfare obtained under the environmental permit, both in the duopolistic case (Figure 3.30, (a), (b) and (c)), and for larger number of firms (Figure 3.30, (d), (e) and (f)). Despite the results shown for the difference in research, the welfare difference indicates that for any values of social environmental damage Ω, the total welfare is greater under the environmental permits than under tax. As the number of firms increases, the difference is increasing further.[36] We can summarise these results in the following Proposition:

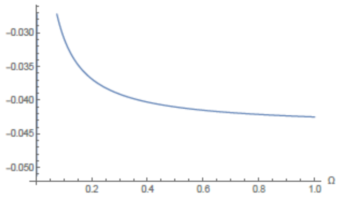
Proposition 7: Under the Research Cartelization scenario and with a credible government, for low values of Social Environmental damage Ω, the environmental tax offers greater incentives in research than the environmental permits. Despite this, the total welfare gained under the environmental permits regulation is greater than the environmental tax under any parameter values.

A crucial aspect of the whole analysis, stated in Proposition 7 and in the previous Propositions, is that the government has to face the following trade-off: it has to maximise the social welfare and, at the same time, to obtain the least possible polluting technology. There are two negative externalities here, notably environmental pollution and limited appropriability of the research effort (a typical side effect of R&D investment), and a single instrument (environmental tax or permits) to address both externalities. Reaching optimally two targets with a single instrument is not possible, so the government is forced to reach a compromise between the two objectives. The results found in our paper are the outcomes of this compromise. In their seminal paper, Katsoulacos and Xepapadeas (1996) use two different instruments, an environmental tax and a research subsidy, to address both targets. In our model we use a single instrument, to enlight the effect of the government commitment on final results.

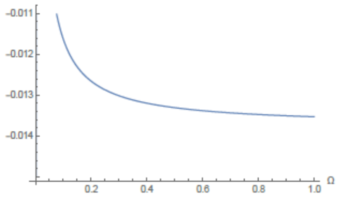
[36] Even if the difference is entirely in favour of the permits, this difference is very little.



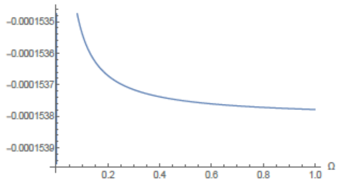
(a) $a = 5$, $\theta = 0.5$, $\gamma = 0.5$, $\underline{\beta = 0.1}$, $n = 2$



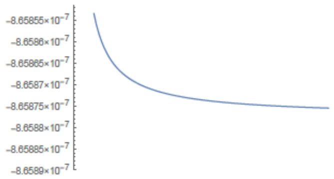
(b) $a = 5$, $\theta = 0.5$, $\gamma = 0.5$, $\underline{\beta = 0.5}$, $n = 2$



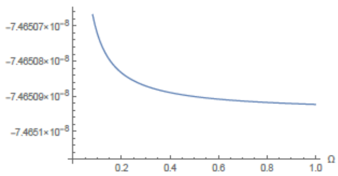
(c) $a = 5$, $\theta = 0.5$, $\gamma = 0.5$, $\underline{\beta = 0.9}$, $n = 2$



(d) $a = 5$, $\theta = 0.5$, $\gamma = 0.5$, $\underline{n = 10}$, $\beta = 0.5$



(e) $a = 5$, $\theta = 0.5$, $\gamma = 0.5$, $\underline{n = 30}$, $\beta = 0.5$



(f) $a = 5$, $\theta = 0.5$, $\gamma = 0.5$, $\underline{n = 50}$, $\beta = 0.5$

Figure 3.30: Difference of Welfare Tax minus Research Permit under Research Cartelization

3.5 Conclusion

We presented a Cournot competition model with pollution externalities, research spillovers and a government that can or cannot credibly commits to the chosen level of environmental policy. The policies analysed are environmental tax and free (i.e. no auctioned) permits. In our paper we make a comparison between these two policies, to establish which one offers greater incentives in investing in R&D and leads to a greater social welfare. The evaluation has been undertaken under different scenario, i.e. a government that can credibly commit to the policy level, and a scenario where the government does not credibly commit. We also evaluated the two policies when firms compete on R&D and when they cooperate, forming an environmental research cartel. The model shows that firms invest more under a credible government than under a non credible one. In the non commitment case, for low number of firms and low social environmental damage due to pollution, the free permits guarantee greater incentive in investing in R&D and in social welfare with respect to the optimal tax. The situation is reversed for a large number of firms, showing that when market competition increases, the environmental tax performs better. This result is possible due to the less stringency of the environmental permit when firms invest in innovation. As firms reduce their emission rate, the demand for the permit decreases. The price is reduced, and firms that have not yet invested in R&D, do not have the same incentive as before, due to the lower permit price (Requate and Unold, 2003; Denicolò, 1999). Finally, the Environmental Research Cartelization case showed that firms have stronger incentive to firms to invest in R&D, but the social welfare gains are greater under the non-cooperative scenario than under research cartelization.

Chapter 4

The nexus between pollution and income inequality. A theoretical approach

Chapter Abstract

Despite increasing evidence of the detrimental role of pollution on low income agents' health and their productive capacity, few theoretical models have tried to explain the connection between environmental degradation and inequality. We try to obviate this gap by extending Blackburn and Chivers' (2015) model. They tried to explain the persistence of income inequality using an overlapping generation model without credit market imperfections and with uncertainty about human capital investment. Agents that inherited low level of human capital are so concerned about loss aversion that they restrain from investing. We extend their framework by introducing pollution and abatement policies. The model shows that pollution increases income inequality, by raising the initial level of human capital required to invest.

4.1 Introduction

There is an increasing consensus about the link between inequality and environmental degradation. Empirical evidence shows that low income population is more affected by pollution, rather than wealthier population. For example, Villeneuve et al., (2012), found that the presence of green areas close to urban environments reduces significantly mortality of people leaving close to green spots. In urban areas, poorer neighbourhoods have a limited access to green areas, as established, among others, by the OECD (OECD, 2004); this has a detrimental impact on their health. Moreover, as showed by Dadvand et al., (2012), exposition to green areas increased birth weight on lowest socio-economic group, showing that if poorer social groups are able to enjoy better environmental quality, their health status increases significantly. Not only exposure to green areas benefits the population poorest groups, but so do stricter environmental policies. As poors are the most exposed to the negative effects of environmental pollution, they are also the agents that benefit the most from a reduction of negative externalities. Bell et al., (2006), showed that the more exposed groups to pollution (i.e. the poorest sector of the population) are the ones that face the highest burden in terms of morbidity and premature death. Cifuentes et al., (2002), showed the positive results in terms of lower morbidity and premature death by introducing stricter environmental policies to reduce emissions. In their optimal survey, O'Neill et al., (2003) list all the possible effects that a prolonged exposure to atmospheric pollutants may have on agents' health. They find that agents' socio economic position is a factor that increases damages caused by pollution and that prevents them from successfully mitigating its detrimental effects, reducing their productivity and future earnings. Since lowest sectors of the economy are the most exposed, it is clear how a reduction of pollution through a stricter environmental policy may result in reducing inequality.

Despite numerous evidences shown in the aforementioned literature, few theoretical works have been produced where environmental degradation is treated as a cause of income inequality. A notable exception is Aloi and Tournemaine (2013). They developed a model where human capital accumulation *à la* Lucas (1988) is the engine of long-term growth. Pollution affects negatively the process of human capital accumulation, thus reducing the possibility of long-term growth. They introduce heterogeneous skilled agents, and two different locations where agents can choose to live in. The first location is close to the productive area. It is more polluted, but the inhabitants can save on transportation costs. The second area is farther from the productive district. Inhabitants do not suffer from pollution exposure, but invest more in commuting. The authors assume that unskilled agents live closer to the polluted area, while skilled labourers prefer to commute and to live away from the production district. The introduction of an environmental policy undoubtedly increases the welfare conditions of poorer agents, mitigating inequality, as low income individuals benefit higher return from human capital investment when the detrimental effect of pollution is reduced.

The nexus between environmental quality and income has been debated for a long time. After the seminal contribution of Kuznets (1955, 1963), who found a negative relationship between income inequality and income level, a similar approach has been applied to the relationship between environmental quality and income level, through the so called Environmental Kuznets Curve (EKC). The inverted-U shape of the EKC curve is the result of three different effects, notably the scale, the composition and the technology effect. The scale effect is characterised by the environmental worsening that follows an income increase, as more output causes more pollution; the composition effect and the technology effect, instead, try to counterbalance the negative scale effect. The composition effect hypothesizes that, as income in a country increases, there is a shift from more capital intensive (and polluting) sector towards more human capital intensive sectors (e.g. the service sectors), while the technology effects shows that with higher income, a greater amount of resources can be spent towards abatement reducing technologies.[1] Although the environmental Kuznets curve (EKC) has been a very debated topic,[2]

[1] See Torras and Boyce, 1998.

[2] For a theoretical analysis of the EKC, see, among others, Stern et al., (1996), Dinda (2005). For an overview of the existing literature, see the survey of Dinda (2004) and Stern (2004).

it has been the first attempt to establish a clear link between income and environmental quality.

More recent literature has tried to establish a nexus between income inequalities as a source of environmental degradation. One of the first attempts to model this nexus is Boyce (1994). Using a public choice approach framework, the author shows that more power and income inequality lead inevitably to a worsening of environmental quality. This is due to the unbalanced distribution of power and income towards who benefits from the polluting activities, which leads to a deterioration of the environment that cannot be mitigated by the less powerful and poorer agents.[3] Heerink et al., (2001), partially confirm Boyce (1994) results. They indicate the relationship between environmental damage and household income as a "micro EKC", finding that the redistribution of income may lead to an environmental degradation when the household pressure on the environment is positivly related with income. Also Scruggs (1998) challenges Boyce (1994)'s results, showing that if Boyce's assumption of constant marginal degradation with respect to income is relaxed, then greater inequality is not necessarily positively related to greater environmental degradation. Magnani (2000) finds that, considering heterogeneous individuals, environmental consciousness increases with income per capita (so justifying partially the EKC curve), but that the greater the income inequality, the lower the agents' willingness to pay for environmental maintenance, with a detrimental effect on environmental quality.[4] A more recent paper by Baek and Gweisah (2013), using country-specific time series data for the United States, found that lower income inequality has a positive effect on environmental quality both in the short and in the long run. Finally, Berthe and Elie, (2015) developed a comprehensive analysis of the existing literature of the effects of inequality on environmental quality.

A close field of research has been the connection between environmental pressure, made by lobbies or citizens, and environmental quality. Among others, Fredriksson et al., (2005), showed that there is a positive relationship between lobbying activities and political competition, favoured by citizens' participa-

[3]His hypothesis is confirmed empirically by Torras and Boyce (1998).

[4]As specified by the author, these results hold only for high-income countries.

tion, and environmental quality. A similar result is reached by Farzin and Bond (2006), which shows that countries with stronger democratic institutions deal better with pollution and a stronger income inequality exacerbates environmental degradation.

The contribution of Aloi and Tournemaine (2013) is one of the few that directly addresses pollution as a source of inequality. In the existing literature, negative externalities generated by polluting activities has been introduced mostly in growth theory literature, to investigate pollution effects on long term growth. Specifically, the effects of pollution on human capital accumulation have been largely investigated. Since the seminal contribution of Gradus and Smulder (1993), who shown in a growth model *à la* Lucas (1988) that when pollution affects agents' ability to learn, profitability of investing in human capital accumulation is reduced and so growth falls. Following Gradus and Smulder (1993), several authors modelled the impact of pollution on agents' ability to learn and on long-term income growth.[5] Main focus of these papers is to establish the long-term growth effect of pollution through human capital accumulation. A stronger pollution effect reduces agents' ability to learn, thus the productivity of investing in human capital, or it reduces life expectancy, leading to similar results. The focus is thus on inter-generational aspects, leaving intra-generational aspects, such as income inequality.

The intra-generational aspects of inequality have been extensively investigated within the literature; still, they have not considered pollution as a possible source of inequality. Galor and Zeira (1993) were among the first to model persistent inequality in an endogenous growth model, following the new growth literature started with Romer (1986) and Lucas (1988). In an overlapping generation model, with human capital accumulation function as the engine of long-term growth, Galor and Zeira (1993) introduce heterogeneous agents, where the difference is given by the dynasty they belong,

[5]Among others, Pautrel (2015), shows that an environmental tax may increase the long-term growth if the abatement sector is more human capital intense than physical capital. Mariani et al., (2010), in an OLG model with human capital accumulation and endogenous life expectancy, shows that higher pollution forces parents to invest in their offsprings' health, reducing investment in human capital.

which could be a rich or a poor one. Rich dynasties inherited a conspicuous legacy, which let them invest in human capital accumulation. They become skilled workers, earn a high wage and are able to leave large bequests to their offspring. Poor dynasties, on the other hand, do not have enough resources to invest in education. Without credit market imperfections, anyway, both agents can invest in human capital accumulation. The opportunity cost of capital is entirely covered by the future returns of investment in education. In presence of credit market imperfections, access to financial resources for poor dynasty agents is limited. Thus, inequality between rich and poor dynasties tends to be perpetual. Galor and Moav (2004) extend the model of Galor and Zeira (1993) to show the transition from an economy based on physical capital accumulation to an economy based on a human capital accumulation. The role of bequests and imperfect credit market is crucial to explain the persistence of income inequality among generations.

Income inequality has been largely investigated in relation to growth, too. Aghion et al., (1999), review the existing literature on inequality and growth in light of the new growth theory. They find that lower income inequality enhances growth, but there is debate whether it can cause a virtuous circle, by reducing inequality period after period, or a vicious one, that increases inequality. The role of credit market imperfection is central in Piketty (1997), which shows how, in a Solow growth model, imperfect credit market can lead to multiple equilibria, affecting agents' saving behaviour. In Aghion and Bolton (1997), agents should protect themselves through insurance, given the unpredictable returns from investment projects. Due to credit market imperfections, agents cannot insure themselves optimally; this conducts the economy to a persistent income inequality. In their model, Aghion and Bolton (1997) endogenize the interest rate, which is the result of the interaction between supply and demand of funds in credit market. The "trickle-down" effect of a stronger capital accumulation mitigates income inequality, but it is not sufficient to eliminate it completely. A political redistribution is necessary to increase growth, as the reduction in income inequality provided poorest agents with more resources; thus, they need less access to the imperfect credit market, reducing the distortionary effect.

To address our question about how pollution can affect income inequality, we move from a recent paper of Blackburn and Chivers (2015). In an overlapping generation model without credit market imperfections, where returns on human capital investment are uncertain, they try to explain income inequality persistence across generations through the uncertainty that individuals face when they invest in a project with an unknown outcome. By using recent development in decision theory, notably the “aspiration-induced loss aversion” (Blackburn and Chivers,2015, p. 346) the authors develop a model where agents have concerns about successfully obtaining a final wealth level greater than the aspiration level, while any level below that is considered as a failure. Agents want to buffer themselves against bad scenarios. Thus, low income agents may be less willing to invest in risky project than high income agents, as the possibility of obtaining an insurance (for example, a minimum level of savings) are lower. The authors explain that the persistence of income inequality is due to the fact that low income agents, facing the possibility of failing (i.e. obtaining a final level of wealth below the aspiration level), decide to desist from investing in potentially advantageous, but risky, projects. The authors are among the first to introduce uncertainty and loss-aversion to explain income inequalities.[6] They refer to aspiration levels to include the proved attitude of agents to evaluate both chances of winning and losing when undertaking risky investments. The aspiration level is a threshold, above which agents may undertake risky behaviour, investing in projects aimed to maintain their actual wealth level; below the aspiration level, agents are too concerned about the possible loss, thus they prefer to avoid any risky behaviour and refuse to invest in any project.

The model we introduce follows the same steps as Blackburn and Chivers (2015). We use their framework to introduce a flow pollution function (following Aloi and Tournmenaine, 2013), and an abatement function funded by the government through an output tax. By introducing the pollution function, we are able to unveil the link between pollution and income inequality. We are aware of the possible existence of causal connection and endogene-

[6] For the literature on loss-aversion, see Blackburn and Chivers (2015) and the literature therein.

ity between damages caused by pollution and inequality, i.e. the possibility that pollution is more present and pervasive due to inequality, e.g. because lower income segment of population have no possibilities to invest in any abatement (or less polluting) technologies, while richer agents may do. In our model we analyse the other possible effect, i.e. pollution as a factor that increases income inequality, leaving the other nexus to further research.

Pollution affects negatively the production process; this reduces the incentives for agents to invest in human capital accumulation, as the investments returns are lower. Due to pollution, agents need to have greater expected income from human capital investment. This will discourage low income agents to invest, because for precautionary motives they do not want to borrow capital on financial markets. Main intuition for this result is that introduction of pollution function increases the threshold level of inherited human capital necessary to invest. The detrimental effect of pollution is mitigated by the introduction of abatement policies, which increase the productivity of human capital investment and reduce the human capital threshold. Thus, in presence of pollution abatement, income inequality is reduced as more agents will invest in human capital. Abatement policies are funded through a linear tax on production that, obviously, has a detrimental effect on production. We show that for low values of tax, the positive effect of abatement is stronger than the detrimental one, while for higher tax, the opposite is true. Finally, we compare our results with the findings of Blackburn and Chivers (2015) and we show that our model can either lead to lower income inequality, for low level of pollution damage relative to the abatement policies, or to higher level of income inequality, when damages caused by pollution are greater with respect to their work. Our paper differs from Aloi and Tournemaine (2013), as in their framework they consider pollution affecting directly human capital accumulation, and agents are affected by pollution heterogeneously, according to their location choice. The results we obtained are similar to Galor and Zeira (1993), since the role of inheritance received from previous generations is crucial to determine whether agents belong to a low or to a high growth path. The novelty with respect to Galor and Zeira (1993), and the literature close to their seminal paper, is that in this framework it is possible to obtain persistent income inequalities without credit market imperfections. Moreover, we introduce pollution and abatement policies, that were absent in Galor and Zeira (1993), or in the previous literature on income inequality and growth.

The paper is organised as follows. In section 2 we develop the model. In section 3 we provide the equilibrium. In section 4 we provide the main results and section 5 concludes.

4.2 The Model

Our model follows closely the one by Blackburn and Chivers (2015). We consider an overlapping generation model with agents living for two periods. As in Galor and Zeira (1993), they belong to dynastic families. Every agent has one parent and one child; she inherited a certain amount of human capital from his parents, and leaves a bequest to her offspring. In the first period of her life, she decides whether to invest in a risky project or not (we can consider the project as an investment in human capital), while in the second period she decides whether to be an output producer or consumer.

Agents have the same utility $u_t = u(x_{t+1})$, where x_{t+1} is the consumption at time $t + 1$. Following Blackburn and Chivers (2015), we depart from the expected utility theory to develop an approach based on aspiration level theory. The utility function $u()$ is considered to be linear, such that $u(x_{t+1}) = x_{t+1} - x^*$ (Blackburn and Chivers, 2015). The agent attaches a positive value to the possibility of succeeding (P^s) or failing (P^f). By representing the aspiration level as x^*, the probability of failing to acquire the desired aspiration level, P^f, can be rewritten as $P^f = P(x_{t+1} < x^*)$ and the probability of succeess, P^s, as $P^s = P(x_{t+1} > x^*)$ (Blackburn and Chivers, 2015). We can consider the agent maximizing the value V_t as:

$$V_t = E(u_t) + \mu P^s + \lambda P^f \tag{4.1}$$

where $\mu > 0$ is the weight attached to the probability of success and $\lambda > 0$

the weight attached to the probability of failing. Since the agent is more concerned about the idea of failing than the possibility of having success, we can set $\mu = 0$ (Blackburn and Chivers, 2015).

The expected payoff is equal to:

$$V_t = E(x_{t+1} - x^*) - \lambda P(x_{t+1} < x^*) \tag{4.2}$$

The agent decides the amount of the investment i_t in the risky project in the first period of her life. The investment cost is fixed and equal to $k > 0$. Agents can decide either not to invest, i.e. $i_t = 0$, or to invest, $i_t = k$ (Blackburn and Chivers, 2015). The financial constraint reveals that agents are endowed with zero resources in their first stage. They need to access financial markets to borrow the capital required to invest in the project. Given the assumption that financial markets are perfectly competitive and there are no constraints, agents do not have to behave strategically to access financial markets.[7]

The human accumulation function H_t is equal to:

$$H_{t+1} = \begin{cases} \beta H_t + b(1 + \gamma_{t+1}) & \text{if } i_t = 0 \\ \beta H_t + B(1 + \gamma_{t+1}) & \text{if } i_t = k \end{cases} \tag{4.3}$$

where $\beta \in (0, 1)$ and $B > b > 0$.

Following Blackburn and Chivers (2015), we define the term γ_{t+1} as an uniform distributed variable over the interval $(-c, c)$, whit probability distribution function $f(\gamma_{t+1}) = \frac{1}{2c}$, where $c < 1$.[8] The role of γ_{t+1} is crucial to introduce uncertainty in the model. It can be considered as innate human abilities (as Aloi and Tournemaine, 2013), or the responsiveness of the agents to morbidity caused by pollution: for low values of γ_{t+1} the pollution affects severely the agents' ability to learn or to be productive, and for high values

[7]Blackburn and Chivers (2015) do not consider any financial constraint that limits the access of the agents to the financial markets.

[8]The expected value is equal to 0 and the variance is $\frac{c^2}{3}$.

of γ_{t+1} the opposite is true.

The agent produces output in the second period of his life, according to the production function Y_{t+1}, equal to:

$$Y_{t+1} = \tilde{A}h_{t+1} \tag{4.4}$$

where $\tilde{A}$ represent our major contribution to the model of Blackburn and Chivers (2015). The total factor productivity $\tilde{A}$ is a decreasing function of pollution P_t and an increasing function of abatement activities D_t, such that:

$$\tilde{A} = Af\left[\frac{D_t}{P_t}\right] \tag{4.5}$$

For simplicity, we can use a functional form of $f\left[\frac{D_t}{P_t}\right]$ equal to $\left[\frac{D_t}{P_t}\right]$. The pollution function P_t represents the flow of pollution, and is equal to:

$$P_{t+1} = \rho Y_{t+1} \tag{4.6}$$

The pollution function is similar to Aloi and Tournemaine (2013), and the parameter $\rho \in (0, 1)$ represents the effective damage caused by pollution. The abatement function D_t is equal to:

$$D_{t+1} = \tau Y_{t+1} \tag{4.7}$$

where $\tau \in (0, 1)$ is the linear tax introduced by the government to raise the revenues necessary to invest in abatement technologies.[9] The abatement technologies reduces the detrimental effect of pollution on human capital accumulation. By substituting (4.6) and (4.7) into the functional form of (4.5), we obtain:

$$\tilde{A} = A\left[\frac{\tau}{\rho}\right] \tag{4.8}$$

[9]In our model, we consider generic abatement technologies. For an analysis of the effects of different abatement technologies in an overlapping generation model with endogenous life expectancy, see Palivos and Varvarigos, (2016).

The total factor productivity increases in the linear tax τ and decreases with respect to the damage rate of pollution ρ. To finance the abatement activities, the government charges a tax on the total output Y_{t+1}, such that the disposable income Y^d_{t+1} is equal to $Y^d_{t+1} = (1-\tau)Y_{t+1}$. By substituting (4.8), (4.6) and (4.7) in Y^d_{t+1}, we obtain the disposable income, equal to:

$$Y^d_{t+1} = (1-\tau)A\left[\frac{\tau}{\rho}\right]H_{t+1} \tag{4.9}$$

It is interesting to appreciate the effect of the tax on the disposable income. As long as $\tau < \frac{1}{2}$, the final output is increasing as the tax is increasing. For $\tau > \frac{1}{2}$, the effect is the opposite. It is straightforward to see that the final output decreases for ρ increasing.

From now on, our analysis follows the model of Blackburn and Chivers (2015). They introduce agents' consumption as the direct result of the action taken by the agent when young, i.e. if the agent invested in human capital or not (if $i_t = 0$ or $i_t = k$). We can show the alternative choices as:

$$x_{t+1} = \begin{cases} \tilde{A}\left[\beta H_t + b(1+\gamma_{t+1})\right] & \text{if } i_t = 0 \\ \tilde{A}\left[\beta H_t + b(1+\gamma_{t+1})\right] - (1+r)k & \text{if } i_t = 0 \end{cases} \tag{4.10}$$

where $(1+r)k$ is the size of an agents' loan repayment (Blackburn and Chivers, 2015). The rate of interest r is the world rate of interest, which results from the transactions on the competitive financial intermediaries, where the demand for funds matches with a perfectly elastic supply of loanable funds. Agents consume all of their realised output if they did not invest in human capital when young. If they invested, agents can consume what is remaining after having paid back the loan.

When $i_t = 0$, (4.2) is equal to:

$$V_t \mid_{i_t=0} = \tilde{A}\left[\beta h_t + b\right] - x^* \tag{4.11}$$

Agents invest in the first period, according whether the aspiration level x^*

may or may not be reached. In term of the expected payoff, agents invest when (4.2) is realised, $\tilde{A}\left[\beta H_t + b(1+\gamma_{t+1}\right] - (1+r)k \geq x^*$, while they do not invest when $\tilde{A}\left[\beta H_t + b(1+\gamma_{t+1}\right] - (1+r)k < x^*$. Blackburn and Chivers (2015) find that there exists a critical level of γ_{t+1}, that they call $\hat{\gamma}_{t+1}$, such that if $\gamma_{t+1} \geq \hat{\gamma}_{t+1}$, the agent invests in the project (i.e., in human capital), while if $\gamma_{t+1} < \hat{\gamma}_{t+1}$, she refuses to invest, as it is too risky. Following Blackburn and Chivers (2015), we can now evaluate the agent's expected payoff:

$$V_t \mid_{i_t=k} = \int_{-c}^{c} \left\{ (1-\tau)\left[\frac{\tau}{\rho}\right] A\left[\beta H_t + B(1+\gamma_{t+1})\right] - (1+r)k \right\} f(\gamma_{t+1}) d\gamma_{t+1} \tag{4.12}$$

$$-\lambda \int_{-c}^{\hat{\gamma}_{t+1}} f(\gamma_{t+1}) d\gamma_{t+1} - x^*$$

where the second term of the right-hand side represents the expected utility loss given the probability of failing, i.e. when $P(x_{t+1} < x^*) = P(\gamma_{t+1} < \hat{\gamma}_{t+1}) = -\lambda \int_{-c}^{\hat{\gamma}_{t+1}} f(\gamma_{t+1}) d\gamma_{t+1} = \frac{\hat{\gamma}_{t+1}+c}{2c}$ (Blackburn and Chivers, 2015). The agent will invest in human capital if the expected payoff is at least equal to the payoff obtained in the case of no investment, i.e. if $V_t \mid_{i_t=k} \geq V_t \mid_{i_t=0}$(Blackburn and Chivers, 2015).

4.3 Equilibrium outcomes

Given $\hat{\gamma}_{t+1}$ and (4.10), we can define the equilibrium condition for agents that want to invest in the risky project as:

$$(1-\tau)\left[\frac{\tau}{\rho}\right] A\left[\beta H_t + b(1+\hat{\gamma}_{t+1}\right] = (1+r)k + x^* \tag{4.13}$$

We can rearrange (4.13) to obtain $\hat{\gamma}_{t+1}$, which is equal to:

$$\hat{\gamma}_{t+1} = \frac{[(1+r)k + x^*] \frac{\rho}{\tau(1-\tau)} - A[\beta H_t + B]}{AB} \equiv \gamma(H_t) \tag{4.14}$$

Given (4.10), (4.14) is negative. In Blackburn and Chivers (2015), the probability of failing (i.e. $\gamma_{t+1} < \hat{\gamma}_{t+1}$) is greater the lower the inherited level of human capital h_{t+1}. In our model this result is enriched by the introduction of the linear tax τ and the damaging equation ρ. For an increasing tax (up to $\tau = \frac{1}{2}$), $\hat{\gamma}_{t+1}$ is decreasing. A stronger abatement policies increases the productivity of the human capital, and so the incentives for the agent to invest in human capital. When $\tau > \frac{1}{2}$, the detrimental effect of the tax on final output outweighs the beneficial effect of human capital accumulation given by abatement activities. Thus, the agent has lower incentive to invest in human capital due to lower return, and the value of $\hat{\gamma}_{t+1}$ increases accordingly. Agents need to inherit a higher value of human capital to invest in the risky activity. The value of $\hat{\gamma}_{t+1}$ is increasing for a greater damage caused by pollution, measured by ρ. The higher the detrimental effect of pollution on human capital accumulation, the lower the incentive for the agent to invest in human capital.

We can compare the result we obtained with the results obtained by Blackburn and Chivers (2015), without abatement policies and pollution function. By setting $\tau(1-\tau) = \rho$, we can rewrite (4.14) as:

$$\hat{\gamma}'_{t+1} = \frac{[(1+r)k + x^*] - A[\beta H_t + B]}{AB} \tag{4.15}$$

We can compare the two critical levels of γ_{t+1}, $\hat{\gamma}_{t+1}$ and $\hat{\gamma}'_{t+1}$, to investigate when the two frameworks offer different results. We can formally define in Lemma 1:

Lemma 1: Given the role of γ in our model, when $\rho < \tau(1-\tau)$, the required investment in human capital is lower than the level required with no abatement policies, i.e. $\hat{\gamma}_{t+1} < \hat{\gamma}'_{t+1}$. When $\rho > \tau(1-\tau)$, the opposite is true ($\hat{\gamma}_{t+1} > \hat{\gamma}'_{t+1}$). For $\rho = \tau(1-\tau)$, the two variables are equivalent.

Proof: The proof is straightforward. The difference $\hat{\gamma}_{t+1} - \hat{\gamma}'_{t+1} = 0$ is equal to $\frac{\rho}{\tau(1-\tau)} - 1 = 0$. So, $\hat{\gamma}_{t+1} < (>) \hat{\gamma}'_{t+1}$ for $\rho < (>) \tau(1-\tau)$.

□

The intuition for this result is the following. For pollution damage ρ lower than $\tau(1-\tau)$, the abatement investment undertaken by the government increases the investment productivity in human capital accumulation. Thus, the presence of the abatement policy represents an incentive for the agent to invest more and to undertake the risky project. When $\rho > \tau(1-\tau)$, the opposite is true. The pollution is so detrimental for human capital accumulation that even in presence of abatement technology, return of investment in human capital are low and agents require a higher $\hat{\gamma}_{t+1}$ to undertake risky activities. Of course in this case $\hat{\gamma}'_{t+1}$ is lower than $\hat{\gamma}_{t+1}$, since Blackburn and Chivers (2015) do not consider the pollution problem. In our case, the presence of pollution may either reduce the critical value $\hat{\gamma}_{t+1}$, when $\rho < \tau(1-\tau)$, or increase it, when $\rho > \tau(1-\tau)$.

We can solve (4.15) using (4.14), to obtain:

$$V_t \mid_{i_t=k} = (1-\tau)\left[\frac{\tau}{\rho}\right] A\left[\beta H_t + B\right] - (1+r)k - \lambda\left[\frac{\gamma(H_t)+c}{2c}\right] \tag{4.16}$$

The agent will invest in human capital as long as $V_t \mid_{i_t=k} \geq V_t \mid_{i_t=0}$. Using (4.11), we can rewrite (4.16) as:[10]

$$(1-\tau)\left[\frac{\tau}{\rho}\right] A\left[B - b\right] - (1+r)k \geq \lambda\left[\frac{\gamma(H_t)+c}{2c}\right] \tag{4.17}$$

where the left hand side represents the expected difference in wage from investing and not investing in human capital. In order to have investment, the left hand side must be greater than the expected disutility form not

[10] As Blackburn and Chivers, (2015), we also assume that the left hand side is positive, to rule out non-interesting cases in which there are no investments.

reaching the aspirational level, which is represented by the right hand side of (4.17), and it is equal to:

$$\lambda \left[\frac{\frac{[(1+r)k+x^*]\frac{\rho}{\tau(1-\tau)} - A[\beta H_t + B]}{AB} + c}{2c} \right] \tag{4.18}$$

The expected disutility from not reaching the aspirational level is increasing in c (since $\gamma(H_t)$ is negative), decreasing in H_t and in the tax τ,[11] and increasing in ρ and λ. The intuition is the following. The greater the weight given to the probability of not reaching the aspiration level λ, the greater the expected income for agents to invest; when H_t increases, the expected income may decrease, as the agent has inherited a greater human capital and needs a lower expected income to decide whether to invest or not. A higher rate of pollution ρ induces a greater aspirational level; the detrimental effect of pollution reduces the potential output, so the agent needs a greater expected wage in order to undertake the risky activity. For any parameter that increases the aspirational level, the agent's willingness to borrow from the market and to invest is reduced, as the risk of not attaining the aspirational level is greater. We can draw some preliminary results. With greater damage caused by pollution, the expected income necessary to invest in the risky project must be higher. Thus, the agent will be less willing to invest in human capital, limiting her possibility of improving her wage and so increasing income inequality.

4.4 Human capital distribution and income inequality

Following Blackburn and Chivers (2015), we study how the aspiration-induced loss aversion determines the income inequality and its persistence. We also compare our results with Blackburn and Chivers (2015)'s benchmark model.

[11] For values of $\tau < \frac{1}{2}$.

Agents invest in human capital accumulation as long as:

$$(1-\tau)A\frac{\tau}{\rho}\left[B-b\right]-(1+r)k=\lambda\left[\frac{\gamma(\hat{H})+c}{2c}\right] \tag{4.19}$$

where $\hat{H}$ is the critical inheritance of human capital that acts as a threshold. If the initial level of human capital inherited by agents is below $\hat{H}$, agents won't find optimal to borrow from financial markets to invest in the risky project; if $H_t > \hat{H}$, then agents will borrow and invest. We can solve (4.19) for $\hat{H}$, to find the threshold value (Blackburn and Chivers, 2015):

$$\hat{H}=\left[\left[\left(\frac{(1+r)k+x^{*}}{AB}\right)\frac{\rho}{\tau(1-\tau)}\right]-\frac{2c}{\lambda}\left[(1-\tau)A\left(\frac{\tau}{\rho}\right)(B-b)-(1+r)k\right]+c-\frac{1}{A}\right]\frac{B}{\beta} \tag{4.20}$$

The critical level of $\hat{H}$ increases for higher level of damage caused by pollution ρ. As the damage level increases, it increases the value of the critical value $\hat{\gamma}$, i.e. the probability of failing to achieve aspirations is higher. Thus, the agent needs a higher human capital inheritance to borrow funds on the market to invest in the risky project. This shows that pollution increases inequality, as only agents that belong to richer dynasties will be able to borrow the necessary funds to invest in human accumulation. The same result is obtained for greater values of λ; the greater value attached to the probability of not attaining the aspirational level P^f induces the agent to necessitate a stronger human capital inheritance. The threshold is reduced for higher values of tax τ, as long as $\tau < \frac{1}{2}$. For values of the tax higher than $\frac{1}{2}$, the detrimental effect on the output production overcomes the benefits of the abatement activities, resulting in lower incentives for agents to invest in human capital; accordingly, the required threshold increases. We can compare this result with the benchmark case shown in Blackburn and Chivers (2015). If we set $\rho = \tau(1-\tau)$, the human capital threshold $\hat{H}$ becomes:

$$\bar{H}=\left[\left[\left(\frac{(1+r)k+x^{*}}{AB}\right)\right]-\frac{2c}{\lambda}\left[A(B-b)-(1+r)k\right]+c-\frac{1}{A}\right]\frac{B}{\beta} \tag{4.21}$$

By comparing the two thresholds, we can summarize the following result:[12]

Proposition 1: For $\rho < \tau(1-\tau)$, the human capital treshold required to invest is lower than the one needed in the benchmark model ($\hat{H} < \bar{H}$); for $\rho > \tau(1-\tau)$, the opposite is true ($\hat{H} > \bar{H}$). Finally, for $\rho = \tau(1-\tau)$, $\hat{H} = \bar{H}$.

The main intuition for this result is the following. When damages caused by pollution are lower than the beneficial effect of pollution abatement, $\rho < \tau(1-\tau)$, the presence of the policy increases the productivity of investments in human research. The increased productivity of human investment reduces the critical value $\hat{\gamma}$, so more agents are willing to borrow funds in the financial market to invest in risky projects. The required capital threshold is reduced with respect to the benchmark model, i.e. $\hat{H} < \bar{H}$, as a lower inherited human capital level is required for agents to invest. In the opposite case, when $\rho > \tau(1-\tau)$, the damages caused by pollution overcome the abatement policy benefits and the threshold increases with respect to the benchmark model ($\hat{H} > \bar{H}$). The investment in human capital is less productive and the probability of failing in achieving aspirations increases. Thus, agents invest only if they receive a higher inherited human capital. When the rate of pollution ρ is equal to $\tau(1-\tau)$, the two cases are equivalent.

Now, followign Blackburn and Chivers (2015), we can define the intergenerational dynamics of the human capital accumulation. Given (4.19) and (4.10), the dynamics for an agent are equal to:

$$H_{t+1} = \begin{cases} \beta H_t + b(1+\gamma_{t+1}) & \text{if } H_t < \hat{H} \\ \beta H_t + B(1+\gamma_{t+1}) & \text{if } H_t \geq \hat{H} \end{cases} \tag{4.22}$$

In Figure (1) we can see the dynamics of the human capital accumulation in the benchmark case (the case shown in Blackburn and Chivers, 2015). The transition equation are bounded according to the value of γ_{t+1}, where $\gamma_{t+1} \in (-c, c)$. The steady states are (Blackburn and Chivers, 2015):

[12] Proof in the appendix.

$$H^* = \frac{b(1 \pm c)}{1-\beta}; \qquad H^{**} = \frac{B(1 \pm c)}{1-\beta} \tag{4.23}$$

According to Proposition 1, when $\rho < \tau(1-\tau)$, the threshold level $\hat{H}$ moves to the left. When it happens, the required level of initial human capital H_0 necessary to switch toward the higher capital accumulation is lower. Given that it is necessary to have $H_0 > \hat{H}$ in order to have a high long-term growth, when the human capital threshold is reduced by the abatement pollution policies, more agents will become investors, increasing their own income and reducing inequality. The opposite happens when $\rho > \tau(1-\tau)$. In this case, the threshold moves to the right. More initial human capital is needed to obtain a higher long-term growth. Fewer agents become investors as the loss aversions caused by aspirations increases due to pollution. The agent receives a lower inheritance, which reduces her willingness to risk. She prefers not to invest in human capital, ending in the low long-run growth. As a result, she will leave a lower bequest to her offspring, causing inequality to persist generation after generation. In our framework, the presence of the abatement policy may be a good instrument to reduce the persistence of income inequality through generations. The government can increase the abatement policies (keeping the level of tax $\tau < \frac{1}{2}$), increasing the productivity of the investment in human capital and reducing the loss aversion due to the aspiration level. This reduces the human capital threshold necessary to invest, leading more agents towards the high long-term growth equilibrium.

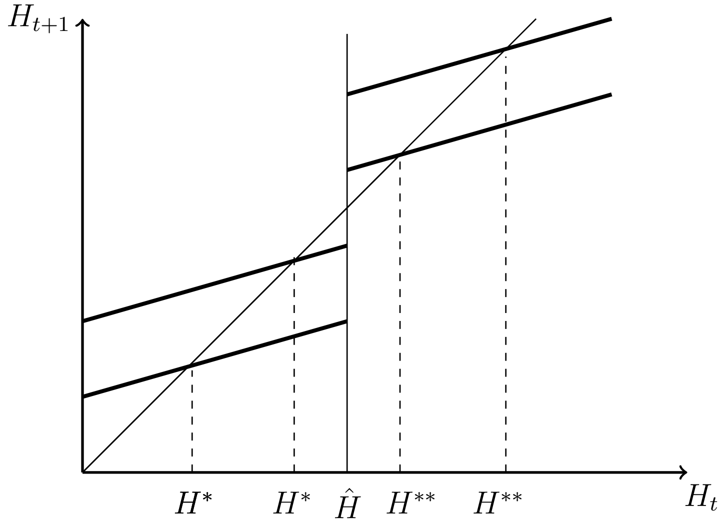


Figure 4.1: Human capital distribution when $\rho = \tau(1-\tau)$

4.5 Conclusion

We developed the model of Blackburn and Chivers (2015) to investigate the role of pollution as a source and a direct cause of income inequality. Few theoretical papers tried to unveil this nexus, despite the growing empirical evidence of how pollution affects mostly low income segments of the population. The greater exposure to pollution forces agents to spend part of their poor resources in private health care, as it reduces their labour productivity or their possibility of investing in education.

We tried to address this question using the model of Blackburn and Chivers (2015). They explain the persistence of income inequality through generations without using credit market imperfection, as widely adopted in the previous literature (see, among others, Galor and Zeira, 1993, or Aghion and Bolton, 1997). They introduce loss aversion caused by the aspiration induced to reach a specific target. Agents are concerned about the probability of failing their aspiration level. This induces agents to adopt precautionary behaviours, if the level of human capital inherited from previous generation is not sufficient. With an insufficient initial human capital, agents prefer not to invest in risky activities, loosing the opportunity to realize a profit that may help in reducing income inequality. Given that the agent earns a lower income, she will leave a lower bequest to her offspring, reinforcing the persistence of income inequality. In this model, we introduce pollution and abatement policies. Pollution has a detrimental effect on the final output. It reduces the returns of the investment in human capital and increases agents loss aversion.

Pollution, by increasing the probability of failing to achieve aspirations, increases the human capital threshold necessary to invest in profitable projects, as it increases expected disutility from not achieving the aspiration. Thus, the stronger the damage caused by pollution, the higher the income inequality, due to less agents investing. The government can mitigate the negative effect of pollution with abatement policies. They increase the productivity of investment in human health and reduce the loss aversion of failing to achieve aspiration. The role of the government is ambiguous, as the positive effect of the abatement policies are mitigated by the opportunity cost, represented by the linear tax on output. If the tax is too high, it has a detrimental effect on output and on the willingness of agent to invest.

Finally, we compare the result we obtained with the original model of Blackburn and Chivers (2015), showing that our framework offers a richer interpretation. Specifically, due to the presence of pollution and abatement policies, the minimum level of human capital inherited necessary to invest may differ from the original model. For low (high) level of damage caused by pollution with respect to the tax, the threshold is lower (greater) than the original model.

Chapter 5

Conclusion

This thesis studied the effect of pollution on economic activity. Chapter 2 presents an overlapping generation model with capital accumulation and pollution affecting agents' health. Individuals live only for two periods, working in the first period of their life and consuming once old. The wage earned can either be saved or spent in private health care, to reduce the detrimental effect of pollution. The government can invest in public health care; the public investment in health has a complementary effect on savings and crowds out private investment. If pollution does not affect agents' health, the economy shows a positive growth, thanks to the presence of the learning-by-doing spillover in the production function, resembling a typical OLG model with endogenous growth. When the pollution affects agents' health, the possible combinations of the initial level of capital of the economy and the "net dirtiness", i.e. the difference between the elasticity of pollution with respect to production minus the elasticity of the public health investment generates different long-run growth equilibria. When the difference between the elasticity of pollution and the elasticity of public health spending is positive, but within the unity, the economy may experience either a long-run positive growth or a low-income growth (a poverty trap), according to whether the initial allocation of capital is below or above a certain threshold.

If the economy is below the threshold, resources are not sufficient to guarantee a positive capital accumulation, and the economy ends in the poverty-trap. If the initial level of capital is above the threshold, the economy shows a positive long-run growth. When the "net difference" is greater than the unit, the capital accumulation process ends in a "no-growth" equilibrium, where the effect of pollution on agents' health is so intense that individuals are forced to spend their resources in private health. If the difference between the elasticity of pollution and the elasticity of the public health further increases, the capital accumulation experiences oscillations toward the stable steady state or a periodic cycle around the steady state. Finally, when the elasticity of public health expenditure is greater than the elasticity of pollution, if the initial level of capital is below a certain threshold the economy ends in a poverty trap, while if it is above the threshold, the economy experiences positive long-run growth.

In Chapter 3 firms compete in a Cournot-competition model, with non-tournament innovation and research spillover. The government introduces an environmental policy, either an environmental tax or free (non-auctioned) permits, to induce firms to internalize the costs of the pollutant emissions and to provide incentives to invest in emission abatement research. The government can credibly commit to the environmental policy level, or it can be time consistent and adjusts optimally the policy after firms innovate. There is evidence of an U-inverse relationship between aggregate research and the level of competition, both under the environmental tax and under the permits (Aghion et al., 2005; Lambertini et al. 2015). Due to the greater market competition, firms prefer to decrease the investment in research. Chapter 3 presents a comparison between the two policies with respect to R&D investment and social welfare level. Both policies perform better in terms of R&D incentive and welfare when the government commits credibly. When the government does not commit, ambiguous results appear. For low number of firms and low social environmental damage, environmental permits perform better in terms of research and welfare. When the number of firms increases, the environmental tax offers more incentive to firms for investing in R&D and reaches higher level of welfare. This effect is due to the lower stringency of the environmental permits when a large number of firms invest in abatement research. As the emission rate decreases, the demand for permits decreases too. So firms have lower incentives in investing in abatement R&D (Requate and Unold, 2003; Denicolò, 1999). When firms can cooperate in the research sector, forming an Environmental Research Cartel (ERC), with a credible government, for low values of social environmental damage, the environmental tax guarantees greater incentives to firms to invest in research than the environmental permits. Despite this, the total welfare gained under the environmental permits regulation is greater than the environmental permit under any parameter values.

Finally, Chapter 4 extends the model of Blackburn and Chivers (2015) to provide a theoretical framework to the nexus between pollution and income inequality. They explain the persistence of income inequality through generation without using credit market imperfection, but using loss aversion caused by aspiration level. This induces agents to adopt precautionary behaviours, if the level of human capital inherited from previous generation is not sufficient. With an insufficient initial human capital, agents prefer not to invest in risky activities. Given that agents earn a lower income, they will leave a lower bequest to their offspring, reinforcing income inequality persistence. Pollution has a detrimental effect on final output. It reduces the returns of the investment in human capital and increases agents loss aversion. Pollution, by increasing the probability of failing to achieve aspirations, increases the threshold of human capital necessary to invest in profitable projects. Thus, the stronger the damage caused by pollution, the higher the income inequality, due to less agents investing. The government can mitigate the negative effect of pollution with abatement policies, which increase the productivity of investment in human health and reduce the loss aversion of failing to achieve aspiration. Finally, we compare the result we obtained with the results from the original model of Blackburn and Chivers (2015), showing that our framework offers a richer interpretation. Specifically, due to the presence of pollution and abatement policies, the minimum level of human capital inherited necessary to invest may differ from the original model. For low (high) level of damage caused by pollution with respect to the tax, the threshold is lower (greater) than the original model.

Appendices

Appendix A

Appendix to Chapter 2

A.1 Omitted Proofs

A.1.1 Proof of Proposition 1

We have balanced growth if $\frac{\partial k_{t+1}}{\partial k_t} \mid_{k_t=0} > 1$.

The first derivative of (16) with respect to k_t is equal to:

$$\frac{(1-\alpha)(1-\tau)A)}{1+\epsilon} > 1 \tag{A.1}$$

which is always true as long as Assumption 1 holds.

□

A.1.2 Proof of Proposition 2

Substituting $\hat{k}$ into $Q'(k_t)$, we have $Q'(\hat{k}) = \Gamma - (\phi - \theta)\Lambda \left[\left[\frac{\Lambda}{\Gamma - 1} \right]^{\frac{1}{1-(\phi-\theta)}} \right]^{\phi - \theta - 1}$ which is equal to:

$$\Gamma - (\phi - \theta)(\Gamma - 1) \longrightarrow \Gamma \left[1 - (\phi - \theta)\right] + \phi - \theta > 1$$

The steady state $\hat{k}$ is unstable.

□

A.1.3 Proof of Proposition 3

Given that $(\phi-\theta) > 1$, the steady-state $\hat{k}$ can be rewritten as $\hat{k} = \left[\frac{\Gamma-1}{\Lambda}\right]^{\frac{1}{\phi-\theta-1}}$, while the turning point $\tilde{k} = \left[\frac{\Gamma}{(\phi-\theta)\Lambda}\right]^{\frac{1}{\phi-\theta-1}}$, now is a maximum, since $Q(k_t)$ is concave when $(\phi-\theta) > 1$. The second derivative is equal to:

$$Q^{''}(k_t) = -(\phi-\theta-1)(\phi-\theta)k_t^{\phi-\theta-2}$$

which is lower than zero for $\phi-\theta > 1$. In addition to $\hat{k} > 0$, there is also a steady state at $k_t = 0$. However, it is unstable because $Q^{'}(0) = \Gamma > 1$ when $\phi-\theta > 1$.

Therefore:

$$Q^{'}(k_t) = \begin{cases} > 0 \text{ for } k_t < \tilde{k} \\ < 0 \text{ for } k_t > \tilde{k} \end{cases}$$

As for $\hat{k}$, it is $Q^{'}(\hat{k}) = \Gamma\left[1-(\phi-\theta)\right] + \phi-\theta$, but with $(\phi-\theta) > 1$, there are three possible different scenarios:

(i) Given $Q^{'}(\hat{k}) > 0$ we have $\Gamma\left[1-(\phi-\theta)\right] + \phi-\theta > 0$ which rearranged gives us:

$$\phi-\theta < \frac{\Gamma}{\Gamma-1}$$

Note that in this case, it is also true that $Q(\hat{k}) < 1$. Thus, $Q^{'}(\hat{k}) \in (0,1)$ and $\hat{k}$ is an asymptotically stable steady state. Furthermore, for $1 < \phi-\theta < \frac{\Lambda}{\Lambda-1}$, we have that $\tilde{k} > \hat{k}$. The phase diagram is shown in Figure 3, where $\phi-\theta \in \left[1, \frac{\Gamma}{\Gamma-1}\right]$ and it corresponds to $0 < Q^{'}(\hat{k}) < 1$.

(ii) Given the previous results, when $(\phi-\theta) > \frac{\Gamma}{\Gamma-1}$, then $Q^{'}(\hat{k}) < 0$. But $Q^{'}(\hat{k}) > -1$ when $\Gamma\left[1-(\phi-\theta)\right] + \phi-\theta$, which rearranged gives us $(\phi-\theta) < \frac{\Lambda+1}{\Lambda-1}$. Thus, $-1 < Q^{'}(\hat{k}) < 0$ so $\hat{k}$ is a oscillatory but convergent toward the stable steady state. Furthermore, note that for $(\phi-\theta) > \frac{\Gamma}{\Gamma-1} \longrightarrow \tilde{k} < \hat{k}$. The phase diagram is shown in Figure 4.

(iii) Given the previous results, when $(\phi-\theta) > \frac{\Lambda-1}{\Lambda+1}$, then $Q^{'}(\hat{k}) < -1$, so $\hat{k}$ is unstable. In this case, we have limit cycles. In Figure 5 is shown an example of a period 2-cycle.

□

A.1.4 Proof of Proposition 4

Given that $Q^{'}(\hat{k}) = \Gamma\left[1-(\phi-\theta)\right]+(\phi-\theta)$, which can be rearranged as $\Gamma+(\Gamma-1)(\theta-\phi)>1$, given $\Gamma>1$ and $(\theta>\phi)$.

So $\hat{k}$ is unstable and $\hat{k}>\check{k}$. Therefore, given that $Q(k_t)>0$, only for $k_t>\check{k}$.

□

Appendix B

Appendix to Chapter 1

B.1 Omitted Proofs

B.1.1 Proof of Lemma 1

To prove that the aggregate research has an inverted-U shape function with respect to the number of firms, we can show that for a very low number of firms (i.e., $n = 1$), the slope of the aggregate research function is positive and that for the number of firms that tends to infinite, the function goes to zero.

Accordingly, the first derivative of the aggregate research for $n \to 1$ is:

$$\left.\frac{\partial \bar{X}_C^t}{\partial n}\right|_{n=1} = \frac{2a\theta\Omega\left[\gamma\left(3(2\beta+3)\gamma\theta^2+16(\beta+1)+\gamma^2\theta^4\right)+2\Omega\left(\gamma\theta^2+2\right)^2\left(-2\beta+\gamma\theta^2+1\right)\right]}{\left[\gamma\left(\gamma\theta^2+4\right)\left(2\theta^2\Omega+1\right)+8\Omega\right]^2} \tag{B.1}$$

A sufficient condition for (57) to be positive is that $\gamma > \bar{\gamma} = \frac{2\beta-1}{\theta^2}$.

The limit of $\bar{X}$ that tends to infinite is zero.

$$\lim_{n\to\infty} \bar{X}_C^t = 0$$

Following Lambertini et al. (2015), it is possible to apply the Mean Value Theorem to show that the aggregate research function $\bar{X}$ is concave.

□

B.1.2 Proof of Lemma 2

The firs derivative with respect to n, when n=1, is equal to:

$$\left.\frac{\partial \bar{X}^t_{NC}}{\partial n}\right|_{n=1} = \frac{\begin{array}{r} a[\beta(4\theta^2\Omega(\theta^2\Omega+1)-1)[\gamma(2\theta^2\Omega+1)^2-8\Omega(\theta^2\Omega+1)] \\ +2\Omega(2\theta^2\Omega+1)(\gamma(2\theta^3\Omega+\theta)^2+2)] \end{array}}{\theta[\gamma(2\theta^2\Omega+1)^2+8\Omega(\theta^2\Omega+1)]^2} \tag{B.2}$$

The first derivative is positive for:

$$\gamma > \tilde{\gamma} = \frac{4\Omega\left(2\beta\left(\theta^2\Omega+1\right)\left(4\theta^2\Omega\left(\theta^2\Omega+1\right)-1\right)-2\theta^2\Omega-1\right)}{\left(2\theta^2\Omega+1\right)^2\left(4(\beta+1)\theta^4\Omega^2+2(2\beta+1)\theta^2\Omega-\beta\right)} > 0 \tag{B.3}$$

We can simplify by assuming $\theta = 1$, so (B.3) is satisfied for $\beta < 1$ and $\Omega \in \left[\frac{1}{2}\left(\sqrt{2}-1\right), \frac{1-2\beta}{2\beta-2}\right]$.[1]

$$\lim_{n\to\infty} \bar{X}^t_{NC} = 0$$

□

B.1.3 Environmental Tax - Commitment case - Welfare Function

$$W^t_{NC} = \left[\frac{a^2}{2\left(\gamma\left(2\theta^2\Omega+1\right)^2+4\Omega(\beta(n-1)+1)^2\left(2\theta^2 n\Omega+n+1\right)\right)^2}\right]$$

$$\Big[\gamma^2\left(2\theta^2\Omega+1\right)^3+4\gamma\Omega\left(2\theta^2\Omega+1\right)^2(\beta(n-1)+1)^2\left(2n\left(\theta^2\Omega+1\right)+1\right)+$$

$$+\frac{2\Omega(\beta(n-1)+1)^4\left(2\theta^2\Omega\left(2n\left(2\theta^2\Omega+1\right)\left(n\left(3\theta^2\Omega+2\right)+3\right)+3\right)-1\right)}{\theta^2}\Big] \tag{B.4}$$

[1]The denominator is positive for $\Omega > \frac{-1-2\beta\sqrt{8\beta^2+8\beta+1}}{4(\beta+1)}$.

B.1.4 Environmental Tax - Non commitment case - Welfare Function

$$\begin{gathered}\big[a^2(2n\Omega(2\gamma\theta^2(\beta(n-1)+1)(-\beta+(\beta+1)n^2+n+1)\\ -n(\beta(n-1)+1)^2+3\gamma^2\theta^4 n)+\gamma^2\theta^2 n^2+16\theta^6 n^2\Omega^4\\ (\beta(n-1)+1)(-2(\beta-1)^2+\beta(2\beta+1)n^3+((5-6\beta)\beta+1)n^2\\ +2n(\beta(3\beta-5)+\gamma\theta^2+2))+8\theta^4\Omega^3(2\gamma\theta^2 n\\ (\beta(n-1)+1)(-\beta+(\beta+3)n^2+n+1)+\gamma^2\theta^4 n^2+\\ (\beta(n-1)+1)^2(2n^4+8n^3+2\beta(n-1)(n(2n^2+n+4)-3)+\\ \beta^2(n-1)^3(n+3)+6n-3))+4\theta^2\Omega^2(3\gamma^2\theta^4 n^2+(\beta(n-1)+1)^2\\ ((\beta-1)^2+(\beta+1)^2 n^4+2(\beta+2)n^3+\\ (1-2(\beta-2)\beta)n^2-6(\beta-1)n)+2\gamma\theta^2 n(\beta(n-1)+1)\\ (-2\beta+n((2\beta+3)n+2)+2)))\big]\end{gathered} \tag{B.5}$$

$$\left[\frac{1}{\begin{gathered}2(4\theta^3\Omega^2(-(\beta-1)^2+\beta(\beta+1)n^3+((2-3\beta)\beta+1)n^2+n(\beta(3\beta-5)+\gamma\theta^2+2))+\\ +2\theta\Omega((n+1)(\beta(n-1)+1)(\beta(n-1)+n+1)+2\gamma\theta^2 n)+\gamma\theta n)^2\end{gathered}}\right]$$

B.1.5 Proof of Lemma 3

It is possible to show that the aggregate research function is concave. The limit of the aggregate research that goes to infinity is equal to zero.

$$\lim_{n\to\infty}\bar{X}_C^P=0$$

The first derivative when $n \to 1$ is equal to:

$$\left.\frac{\partial\bar{X}_C^P}{\partial n}\right|_{n=1}=$$

$$\frac{2a\theta\Omega\left[\gamma\left(3(2\beta+3)\gamma\theta^2+16(\beta+1)+\gamma^2\theta^4\right)+2\Omega\left(\gamma\theta^2+2\right)^2\left(-2\beta+\gamma\theta^2+1\right)\right]}{\left[\gamma\left(\gamma\theta^2+4\right)\left(2\theta^2\Omega+1\right)+8\Omega\right]^2} \tag{B.6}$$

Which is positive for values of $\gamma > \bar{\gamma} = \frac{2\beta-1}{\theta^2}$. Given the Mean Value Theorem, since the value of the research at infinite is equal to zero and the first derivative positive when $n \to 1$, we can say that the aggregate research is a concave function.

□

B.1.6 Environmental permit - Commitment case - Emission function

Total emissions are equal to:

$$E_C^P = \frac{a\gamma\theta(-\beta + \gamma\theta^2 + n^2 + \beta n + n + 1}{\gamma^2(2\theta^4\Omega + \theta^2) + 2\Omega(-\beta + \beta n^2 + n + 1)^2 + \gamma(n+1)(-4(\beta - 1)\theta^2 \Omega + 4\beta\theta^2 n\Omega + n + 1)} \tag{B.7}$$

B.1.7 Proof of Lemma 4

The limit of the research for n that goes to infinity is equal to zero.

$$\lim_{n\to\infty} \bar{X}_{NC}^P = 0$$

and the first derivative with respect to n, with $n \to 1$ is equal to:

$$\left.\frac{\partial X_{NC}^P}{\partial n}\right|_{n=1} = \frac{a\left(2\theta^2\Omega - 1\right)\left[-\gamma\left(2\theta^2\Omega + 1\right)^2\left(\beta - 2\theta^2\Omega - 1\right) - 16\beta\theta^4\Omega^3\right]}{\theta\left[\gamma\left(2\theta^2\Omega + 1\right)^2 + 8\theta^2\Omega^2\right]^2} \tag{B.8}$$

which, for $a = 5$ and $\beta = \gamma = 0.5$, is positive for $\theta > \frac{1}{\sqrt{2}}$ and $\Omega > \frac{1}{2\theta^2}$

□

B.1.8 Environmental Permit - Non Commitment case - Results

The optimal number of permit is equal to:

$$\phi = \left[\frac{a(-(\beta-1)^2(2\theta^2\Omega+1)+2\beta^2\theta^2n^3\Omega-(\beta-1)\beta n^2(6\theta^2\Omega+1)+ n(2\beta^2+2\theta^2\Omega(\beta(3\beta-5)+\gamma\theta^2+2)-3\beta+\gamma\theta^2+1))}{\theta n(-2\Omega((\beta-1)(n^2-1)(\beta(n-1)+1)-2\gamma\theta^2 n)+4\theta^2\Omega^2(-(\beta-1)^2+ \beta(\beta+1)n^3+((2-3\beta)\beta+1)n^2+n(\beta(3\beta-5)+\gamma\theta^2+2))+\gamma n)} \right] \tag{B.9}$$

The permit price with research and number of permits is equal to:

$$\Upsilon =$$

$$\left[\frac{a(4\theta^2(n-1)\Omega^2(\beta(n-1)+1)^2+\gamma(2\theta^2\Omega+1)(2\theta^2 n\Omega-1))}{\theta(-2(\beta-1)^2\Omega(2\theta^2\Omega+1)+2\beta n^3\Omega(2(\beta+1)\theta^2\Omega-\beta+1)-2(\beta-1)n^2\Omega(2(3\beta+1) \theta^2\Omega-\beta+1)+n(2(\beta-1)\Omega(2(3\beta-2)\theta^2\Omega+\beta)+\gamma(2\theta^2\Omega+1)^2))} \right] \tag{B.10}$$

Total Quantity is equal to:

$$Q^P_{NC} = \frac{a\left[2\gamma\theta^2\Omega+\gamma+4\theta^2 n\Omega^2(\beta(n-1)+1)^2\right]}{\gamma\left[2\theta^2\Omega+1\right]^2+8\theta^2 n\Omega^2\left[\beta(n-1)+1\right]^2} \tag{B.11}$$

Aggregate quantity is concave with respect the number of firms and convex with respect the environmental social damage. By substituting (45), (47) and (48) into (42), we obtain the welfare Non Commitment function W^P_{NC} as a function of $(a, n, \beta, \gamma, \theta, \Omega)$.

$$W^P_{NC} =$$

$$\begin{gathered}
\big[a^2(8\theta^4\Omega^3(-2\gamma\theta^2n^2(-\beta+(\beta-3)n+1)(\beta(n-1)+1)+ \\
+\gamma^2\theta^4n^2-(\beta(n-1)+1)^2(-2\beta+\beta^2(3n-1)(n-1)^3+ \\
\beta n(n(7n-12)+6)-2n(n(n+3)-3)(n-1)+1))+4\theta^2\Omega^2 \\
2(-2\gamma\theta^2n^2(\beta(n-1)+1)(2\beta(n-1)-3n+2)+(\beta-1)(n-1) \\
(\beta(n-1)+1)^2(2(\beta-1)-n(n^2+n-6)+ \\
\beta n(n(n+3)-6))+3\gamma^2\theta^4n^2)+\gamma^2\theta^2n^2+16\theta^6n^2\Omega^4(\beta(n-1)+1) \\
(-2(\beta-1)^2+\beta(2\beta+1)n^3+((5-6\beta)\beta+1)n^2+ \\
2n(\beta(3\beta-5)+\gamma\theta^2+2))-2\Omega((\beta-2)\beta((\beta-2)\beta+2)+(\beta-1) \\
\beta n^4((\beta-1)\beta+2\gamma\theta^2)-2(\beta-1)(2\beta-1)n^3((\beta-1)\beta+\gamma\theta^2)+ \\
n^2(2(\beta-1)^2\gamma\theta^2+(\beta-1)^2(6(\beta-1)\beta+1)-3\gamma^2\theta^4) \\
-2(\beta-1)^3(2\beta-1)n)-2\Omega)\big]
\end{gathered} \tag{B.12}$$

$$\left[\frac{1}{\begin{array}{l}2(2\theta\Omega((\beta-1)(1-n^2)(\beta(n-1)+1)+2\gamma\theta^2n)+4\theta^3\Omega^2(-(\beta-1)^2+ \\ \quad\beta(\beta+1)n^3+((2-3\beta)\beta+1)n^2+n(\beta(3\beta-5)+\gamma\theta^{\ 2}+2))+\gamma\theta n)^2\end{array}}\right]$$

B.1.9 Difference of welfare obtained under tax and permit - Non Commitment case

Here are shown the values of Ω for which the difference is positive, given that $\gamma = 0.5$

$$\begin{gathered}
0<\theta<0.0857 \quad \wedge \\
0<\beta<\frac{\sqrt{\frac{-200\theta^6+52\theta^4+106\theta^2+15}{(10\theta^2-3)^2}}}{\sqrt{2}}-\frac{16\theta^2}{10\theta^2-3} \quad \wedge \\
\frac{3\beta^2-4\theta^2-3}{4\theta^2(5\beta^2+16\beta+5\theta^2+11)}+\frac{1}{4}\sqrt{\frac{9\beta^4-4\beta^2\theta^2-18\beta^2+64\beta\theta^2+36\theta^4+68\theta^2+9}{\theta^4(5\beta^2+16\beta+5\theta^2+11)^2}} \\
<\Omega<1
\end{gathered} \tag{B.13}$$

$$0.0857 \leq \theta < 1 \quad \wedge$$

$$0 < \beta < 1 \quad \wedge$$

$$\frac{3\beta^2 - 4\theta^2 - 3}{4\theta^2\left(5\beta^2 + 16\beta + 5\theta^2 + 11\right)} + \frac{1}{4}\sqrt{\frac{9\beta^4 - 4\beta^2\theta^2 - 18\beta^2 + 64\beta\theta^2 + 36\theta^4 + 68\theta^2 + 9}{\theta^4\left(5\beta^2 + 16\beta + 5\theta^2 + 11\right)^2}}$$

$$< \Omega < 1$$

Appendix C

Appendix to Chapter 3

C.1 Proof of Proposition 1

We can solve only for the case when $\hat{H} < \bar{H}$ to prove the proposition. By rearranging the two equations and simplifying, we obtain:

$$-\frac{2c}{\lambda}\left[(1-\tau)\frac{\tau}{\rho}-1\right][A(B-b)]+\left[\frac{\rho}{\tau(1-\tau)}-1\right]\left[\frac{(1+r)k+x^*}{AB}\right] \quad \text{(C.1)}$$

A sufficient condition for the inequality to hold is that $\rho < \tau(1-\tau)$. The other conditions are solved accordingly.

□

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