Three Essays on Non-linear Effects in Dynamic Macroeconomic Models

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by

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I dedicate this thesis to my husband Afshin and my son Aryaz.

Abstract

This thesis has aimed to analyse non-linearity in dynamic models. Attention has focused on the class of dynamic models that accommodate the possibility of distributional modification in the models. In chapter 1, I have studied the non-linear effects of policy shocks in the classical DSGE model. The analysis of such model is subject to two types of shocks, technology and monetary policy. I have extended the analysis of classical model by allowing for the distributional modification of monetary policy shock using WSN distribution. This study reveals the extent to which the distribution of macroeconomic variables may response to policy actions and outcomes involved. Moreover, in classical monetary model the long run behaviour of the level of inflation with respect to the inflation uncertainty has investigated.

I have also analysed the dynamic model of AR-GARCH time series. I have investigated the possible non-linear and asymmetric effects of distributional assumptions on the behaviour of the QMLE of the parameters in AR(1)-GARCH(1,1) model. A Monte Carlo experiment is set up to evaluate the distributional misspecification in aforementioned model by applying both symmetric and asymmetric WSN distribution across a range of mean and volatility persistence. The other contribution in chapter 2 is computing the quantiles under distributional misspecification in AR-GARCH model. In terms of the accuracy of the estimated quantiles, I have implemented the bootstrap technique.

In addition, in chapter 3 the attention has concentrated on the procedures with suitable technique for the analysis of unit root tests. The usefulness of bootstrap technique is investigated in the context of unit root test applying in stock indices and exchange rate series. I evaluate the popular unit root tests including Augmented Dickey Fuller(ADF) and Phillips Perron(PP) as well as DF-GLS. Furthermore, this chapter attempts to answer the question of how the difference in frequency of empirical data say, monthly, weekly, and daily might affect the unit root results.

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Introduction

In recent years, there has been considerable interest in non-linear models and their effects in economics. The growth in the power of computing more sophisticated models in recent years has made it possible to analyse and develop a very wide variety of non-linear models.

This thesis has aimed to analyse non-linearity in dynamic models. Attention has focused on the class of dynamic models that accommodate the possibility of modification (including distributional assumption) in the models. I have considered their application in the study of non-linear effects of policy shocks in the classical DSGE model. Regarding the classical monetary model, money balance in utility function has been considered in the general equilibrium model of a monetary economy (see, Woodford (2003)). Similarly, Walsh (2003), analyzes the equilibrium properties of a real business cycle model with money in the utility function. He concludes that real variables are independent of monetary policy even under a utility which is non-separable in real balances. However, Gali (2007) notes that in contrast with real variables, monetary policy can have an important implication for the behaviour of nominal variables and in particular, of inflation. Therefore, it is important to consider the effect of policy shocks on the nominal variables in the economy. Hence, the first chapter focuses on the analysis of classical monetary model. The analysis of such model which is found in Gali (2007) is subject to two types of shocks, technology and monetary policy. Gali has investigated the effect of these shocks on macroeconomic variables in which the shocks are defined as AR(1) processes with normally distributed error terms. As a consequence of this stochastic assumption, in equilibrium the conditional distribution of macroeconomic variables are normal distribution which can not explain the empirical behaviour and asymmetries of such variables, specially during period of financial distress. Therefore, in this chapter I have extended the analysis of classical model by allowing for the distributional modification of monetary policy shock, motivated by the policy actions involved.

The analysis of classical model is conducted by changing the stochastic assumption of model which is normally distributed policy shock and using WSN distribution instead. This distribution was recently developed by Charemza, Diaz and Makarova (2013) where the parameters of this distribution can be interpreted economically related to the monetary policy actions and outcomes. In chapter 1, first, the non-linearity effect of policy actions on the volatility of policy shock is analysed using stochastic simulation. Then, the extent to which the distribution of macroeconomic variables may response to policy actions and outcomes is investigated. The dynamics of classical model has been analysed by deriving impulse response analysis. Finally, in order to explain the response of inflation's mean to policy shock, the relationship between inflation's level and uncertainty are examined.

Chapter 2 analyses the dynamic model of AR-GARCH time series. The GARCH model was first introduced by Bollerslev (1986) as an extension of the ARCH model developed by Engle (1982) in time series modelling. As the GARCH process has received considerable attention from applied and theoretical point of view, many researchers have tried to expand and use these type of models in several applications. In the extension model of AR-GARCH the conditional mean is given as an AR process where the error term of this process follows a GARCH process. I have also investigated the possible non-linear and asymmetric effects of distributional assumptions on the behaviour of the quasi maximum likelihood estimation of an interest parameter in AR(1)-GARCH(1,1) model. The contribution of this chapter is to evaluate distributional misspecification in aforementioned model by applying both symmetric and asymmetric WSN distribution. The properties of quasi maximum likelihood estimator of model parameters are investigated under both correct and incorrect model specification respectively normal and WSN distribution. A Monte Carlo experiment is set up in which a range of WSN distribution is employed in comparison with normal distribution considering a range of mean and volatility persistence. The Monte Carlo experiment is used to explore the issue of how bias, RMSE, skewness and kurtosis of parameters' estimators vary with sample size under the distributional misspecification.

The other contribution also in chapter 2 is computing the quantiles under distributional misspecification in AR-GARCH model. These quantiles are used in hypothesis testing and practical problems as critical values. In terms of the accuracy of the estimated quantiles, the attention has focused on the implementation of bootstrap technique. This technique can be used for estimating the standard errors when the theoretical calculation is difficult or mathematically intractable.

In addition, in chapter 3 attention has concentrated on the procedures with suitable technique for the analysis of unit root tests. The problem of determining whether a time series contains a unit root has received a great attention among the both statisticians and econometricians. The bootstrap technique in recent years also has become increasingly popular and has been applied to a wide range of topics including nonstationary time series. The bootstrap method introduced by Efron (1979) is a resampling procedure which is designed to approximate the sampling distribution of a statistic of interest. Bootstrap tests constitute an attractive approaches rather than tests based on asymptotic distributions. Therefore, the usefulness of bootstrap technique is investigated in the context of unit root test obtained from Park (2003) applying in stock market and exchange rate data. Chapter 3 evaluates the traditional and popular unit root tests including Augmented Dickey Fuller(ADF) and Phillips Perron(PP) as well as DF-GLS. Furthermore, this chapter attempts to answer the question of how the difference in frequency of empirical data say, monthly, weekly, and daily might affect the unit root results.

All of the analysis in this thesis has been done using MATLAB programming where the codes are available in Appendixes.

Chapter 1

Using the WSN distribution to examine the monetary shocks in the classical DSGE model

1.1 Introduction

Monetary policy shock is one of the main factors which economists have attempted to quantify the effect of that in the economy. In this chapter the effect of monetary policy shocks is investigated not empirically but through a particular real business cycle model. In early equilibrium real business cycle models which were influenced by the monetarist tradition notably Milton Friedman, unanticipated changes in the money supply played a key role in generating fluctuations in aggregate real variables and explaining the correlation between real and nominal variables(Lucas (1972)). In the early 1980s, researchers have focused on the models in which fluctuations associated with the business cycle, are subject to exogenous technology shocks. These models, which were originally developed by Kydland and Prescott (1982) and Long and Plosser (1983), have been criticized for not taking money into account. It has been argued that money causes changes in real variables in economy(see e.g. Bollerslev (1986), Eichenbaum and Singleton (1986)). Lucas(1987) also argued that money has some role over and above technology shocks. After that, a monetary section in real business cycle model is introduced by Cooley and Hansen (1989). They studied the role of money in a real business cycle model through the anticipated inflation operating. They explained that people substitute away from the activities that require cash, such as consumption for activities without any need for cash, such as leisure. Therefore, in this structure there is not any role for unanticipated money. This real business cycle model with monetary section is referred to as the classical monetary model.

By "classical", Gali (2007) defines that the economy's equilibrium is described by the assumption of perfect competition in all markets and fully flexible prices and wages and no other frictions (other than those associated with the existence of money). The classical monetary model focuses on the behaviour of the economy by analysing the interaction of many microeconomic decisions. Regarding the money, its effect is considered by assuming the non-separability of real balance in the model. However, under the seperable utility function equilibrium values for nominal variables like inflation, output and nominal interest rate can be determined without any reference to monetary policy and the description of monetary policy includes only the quantity of money in circulation that the central bank needs to supply in order to support the nominal interest rate in equilibrium. In summary, monetary policy is neutral with respect to those variables. Therefore, introducing real balances in the non-separable utility function, where the elasticity of substitution is different from the relative weight of real balances in utility, can solve the problem of neutrality of money and monetary policy.

At the same time in this model, the economy is affected by another random shock such as technological changes. Gali (1999) identifies the assumption that the level of labour productivity can be affected only by technology shocks in the long run. According to his estimates with positive technology shocks, there is a negative correlation between output and the hours worked. However, Christiano et al. (2003) applying the same identifying assumption as Gali (1999), Gali et al. (2003), and Francis and Ramey (2005) but with different specification of hours worked. They argue that hours of worked is increasing with the same technology shock. Gali and Rabanal (2005) estimated the empirical effects of technology shocks on macroeconomic variables and evaluated quantitatively the contribution of those shocks and also policy shocks to business-cycle fluctuations. They found that technology shocks can explain 22% of the variability of output growth while the effect of monetary policy shocks on the volatility of inflation is 27%.

From other point of view, Walsh (2003) discusses the classical monetary models with cash-in-advance constraints and their implications for the role of monetary policy. Furthermore, Woodford (2003) introduces the monetary assets in the model and assumes that the transaction costs can be reduced by a household's holding of monetary assets. He analyses the implications of this assumption on the specification of utility and concludes that the real variables are not affected by monetary policy in the economy. Therefore, the specification of monetary policy in the classical monetary model can play a role only for the determination of nominal variables.

However, according to the fact that monetary policy is neutral with respect to real variables and this is at odds with the effectiveness of policy on the output and employment in the economy, the classical monetary model is adopted to basic New Keynesian model by assuming the product of differentiated goods, monopolistic competition, and staggered price setting (see Gali and Gertler (2007)).

The existing literature has not found a generally accepted framework that provides a unified explanation of monetary policy asymmetries in the classical model and the movements of economic aggregate. However, in the vast majority of literature, regime switching models have been used in business cycle analysis since they were introduced by Hsieh (1989). Recently, Foerster (2016) examines the effect of two different monetary policy switching assumptions on the long-run behaviour of macroeconomic variables. He studies the extent to which the distribution of macroeconomic variables may response to policy switches of changing inflation targets and varying inflation responses. He finds that the level of the economy is affected by switching inflation targets, whereas the variance is affected by switching inflation responses. Moreover, in order to examine the impact of monetary policy shocks on key macroeconomic variables Allen and Robinson (2015) utilize a threshold vector autoregression (TVAR) model. They investigate the influence of the policy under multiple regime classified as tight, neutral, and loose. They find that there is an asymmetric response to policy action as the response to the tight regime is in the opposite direction of the neutral monetary policy regime. They prove that the response to the exchange rate and the inflation has longer and more persistent effects under neutral monetary policy regime. In addition, in some other applications the regime switching models characterize recessions (expansions) as switches by negative (positive) shocks and have been highly productive paths for empirical macroeconomic analysis. Recently work by Tenreyro and Thwaites (2015) has addressed the analysis of monetary policy in expansions and recessions using regime switching in US data. They utilize impulse response analysis and investigate how a range of real and nominal variables may response to the estimate of monetary policy shocks introduced by Romer and Romer (2004). They employ local projection model and conclude that there is a strong evidence of the powerful impact of monetary policy on real and nominal variables in expansions than in recessions. In contrast, in classical model in order to address the different conditions in the economy such as recessions and expansions, I apply an alternative approach where monetary policy shocks are generated by WSN distribution with different set of parameters. This distribution could shed some light on the analysis of classical model by considering monetary policy actions and outcomes.

Although a majority of empirical research has focused on studying the impact of monetary policy shocks on the economy, changes in the volatility of monetary policy is still one of the important concerns for policy makers. According to Primiceri (2005) the estimated volatility of the U.S. monetary policy shock raises by more than 100% during the early 1980s. Moreover, Mumtaz and Zanetti (2013) study the time varying variance of monetary policy shocks using a structural vector autoregression model. The results of their analysis explain that an increase in the volatility of the monetary policy shock causes a decline in the nominal interest rate, inflation and output growth (further studies concerning the estimation of VAR with heteroscedastic shocks see e.g. Bernanke and Mihov (1998); Cogley and Sargent (2005); Sims and Zha (2006)). Overall, the literature has missed the relevance of monetary policy volatilities in the classical monetary model. Hence, I intend to assess how monetary policy actions and outcomes might change the volatility of policy shock in classical monetary model by applying WSN distribution.

According to the classical monetary model and using a stochastic framework, Gali (2007) assumes AR(1) process with normally distributed error term specifying monetary policy shocks in the model. As a consequence of this assumption, the conditional distribution of macroeconomic variables in equilibrium are normally distributed. Also, skewness and excess kurtosis of empirical data can not be specified by normal distribution which is the drawback of this distribution. Although, in order to address the significant distributional asymmetries the Gaussian Mixture distribution has been used widely specially in the field of finance, but an alternative approach is to use the WSN distribution with a different selection of parameters to be able to address the asymmetry and skewness of the monetary policy distribution. Moreover, the empirical behaviour of economic variables presents asymmetries in the tails of the distribution of variables particularly if there is a financial disturbance. However, as heavy tail characteristic in macroeconomic data is not as extreme as financial data, the study on heavy tail with macroeconomic data is limited. Significant positive skewness of US inflation data was reported by Corrado and Holly (2003). Charemza et al. (2005) claim that the distribution of inflation is often leptokurtic, differing significantly in shape from the Normal distribution. Recently, heavy tail characteristic of macroeconomic data was confirmed by Hurlimann (2012) who also found in the autoregressive(AR) process of inflation that the observed sample residual errors indicate more skewness and also higher kurtosis than is allowed by a normal distribution. Furthermore, some authors test for skewness in macroeconomic time series. For example, Bai and Ng (2005) reject asymmetry for output, industrial production and unemployment in U.S. but find the evidence of skewness in inflation. Grabek et al. (2011) reports skewness coefficients for five macro aggregates in Australia, Canada, Newzealand and United Kingdom. They find that in all of the countries inflation is positively skewed. Real output growth rate, in turn, tends to be more often below the average which has negative skewness. Nominal interest rate and exchange rate also reveal moderate and positive skewness, whereas absolute changes of terms of trade do not exhibit a consistent pattern of skewness. Hence, the study of various time series, consist of inflation data, has discovered a clear evidence of dependence and asymmetry in the distributional characteristic of the data. Given these considerations and as distributional problems are missed from the classical monetary models, this chapter builds on the classical monetary model in Gali (2007) and explores some methodological modifications to the distribution of monetary policy shock, motivated by the policy actions involved. It is also interesting to investigate the extent to which the distribution of macroeconomic variables may response to policy actions and outcomes.

On the other hand, identifying the effects of monetary policy actions on macroeconomic variables specially inflation require confronting a challenges of inflation uncertainty. The relationship between inflation and inflation uncertainty is still arguable since the publication of Friedman (1977) Nobel lecture. Friedman (1977) was the first who studied the causal influence of inflation on inflation uncertainty. Then, Ball (1992) formalized Friedman's result by considering an asymmetric information about the type of policy maker which leads to the uncertainty. Therefore, the basic idea regarding positive causality from inflation to inflation uncertainty as proposed by Friedman (1977) and thereafter its formalization by Ball (1992) in a game theoretic framework is known as Friedman-Ball hypothesis (for empirical studies see e.g. Fountas (2001)). In contrast to the Friedman-Ball view, Cukierman and Meltzer (1986) show that average inflation is rising by higher inflation uncertainty. They claimed that in the presence of more inflation uncertainty, inflation surprises can be created by policy makers in order to stimulate output growth. In support of Cukierman-Meltzer hypothesis Grier and Perry (1998) found the evidence for Japan and France among the G7 countries. Moreover, Fountas and Karanasos (2007) performed the analysis of inflation for UK, Germany, Canada and Japan using GARCH model. Their analysis revealed the strong evidence of Cukierman-Meltzer hypothesis in the UK. Therefore, this chapter also has a fresh look in relationship between inflation and its uncertainty in the classical model in which the variance of policy shock is used as a measure of uncertainty.

In summary, the application of WSN distribution as the engine of analysis for the monetary policy in classical model raises a number of issues. First, how does the volatility of monetary policy change by different policy actions? Second, what are the dynamics of classical model in response to technology shock, and different monetary policy shocks defined by WSN distribution? Third, what is the relationship between average inflation and inflation uncertainty in the classical model? Hence, the analysis of these issues, which is focusing on the behaviour of macroeconomic dynamics under monetary policy shocks, forms the core of the present chapter.

Therefore, the contents of this chapter is as follows. First it defines the distribution of policy shock to WSN distribution which was recently developed by Charemza, Diaz and Makarova (2013). The advantage of WSN is that parameters of this distribution can be interpreted in relation to the monetary policy outcomes and actions. It allows me to assess not only how the variation of monetary policy outcomes and actions might change the volatility of policy shock, but also how these actions and outcomes represented by WSN parameters can affect the behaviour of the first two moments of inflation, nominal interest rate and output in the economy through classical model. This is conducted by simulating the macroeconomic dynamics in classical monetary model as explained in Gali (2007). In the stochastic simulation I replace the assumption of normally distributed policy shocks by WSN distribution. The results of the analysis show the extent to which the responses of the macroeconomic variables change due to the different monetary policy strength and high forecast accuracy. This chapter's major contribution to the existing literature is the finding that in the long run response of the mean of inflation, the greater is the uncertainty the higher would be the level of inflation. Therefore, the classical monetary model represented by Gali can confirm the Cukierman-Meltzer hypothesis.

The remainder of this chapter is organized as follows. Section 2 provides an overview of the classical monetary model. Section 3 presents the definition of WSN distribution and its parameters' interpretation related to the monetary policy actions and outcomes. Section 4 briefly describes the simulation design for analysing the dynamics of economy. Section 5 describes the simulation results. Section 6 concludes all of the findings.

1.2 Model overview

This section summarizes a classical monetary model to investigate how changes in the monetary policy actions and outcomes translate into movements in inflation, nominal interest rate and the output distributional characteristic. To implement the analysis, a classical monetary model which is documented by Gali (2007) is modified by changing the distributional assumption of policy shock. According to Gali (2007) it is assumed that firms maximize profits. It is also assumed that the households tend to maximize a non-separable utility function over consumption, labour effort and real balance.

1.2.1 Households

According to Gali (2007) the representative household would maximize the objective function

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, \frac{M_t}{P_t}, N_t) \tag{1.1}$$

where $\beta \in (0, 1)$ is the discount factor and C_t is the consumption quantity of the single good. In this function, M_t stands for holding of money in period t and N_t denotes hours of work. A key feature of the discounted utility is that an individual's preferences over the factors in utility function in any period are independent of that factors in any other period.

The household's utility function is assumed to have the nonseperable functional form

$$U(C_t, \frac{M_t}{P_t}, N_t) = \frac{X_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}$$

where $\frac{1}{\varphi}$ represents the elasticity of labour supply and measures how labour supply increases when the real wage increases. $\frac{1}{\sigma}$ denotes the elasticity of intertemporal substitution in X_t. Composite index of consumption and real balances is represented by X_t which $U(C_t)$ depends on $\frac{M_t}{P_t}$ in any period. Thus, X_t [1] can be defined as:

$$X_t \equiv \left[(1-\vartheta)C_t^{1-\nu} + \vartheta\left(\frac{M_t}{P_t}\right)^{1-\nu}\right]^{\frac{1}{1-\nu}} \quad for \quad \nu \neq 1 \tag{1.2}$$

$$X_t \equiv C_t^{1-\vartheta}\left(\frac{M_t}{P_t}\right)^{\vartheta} \quad for \quad \nu = 1 \tag{1.3}$$

with $\frac{1}{\nu}$ representing the elasticity of substitution between consumption and real balances, and ϑ is the relative weight of real balances in utility. As Gali (2007) explained maximization of (1.1) is subject to a sequence of flow budget constraints, taking the form

$$P_tC_t + Q_tB_t + M_t \leq B_{t-1} + M_{t-1} + W_tN_t - T_t \tag{1.4}$$

where P_t is the price of consumption good and W_t expresses the nominal wage. B_t represents the quantity of one-period discount bonds purchased in period t and maturing in period $t+1$ and Q_t [2] is the price of each bond at maturity. T_t represents lump-sum additions or subtractions to period income. By assuming $F_t \equiv B_{t-1}+M_{t-1}$ refer to total financial assets at the beginning of the period t, the budget constraint (1.4) can be rewritten as

$$P_tC_t + Q_tF_{t+1} + (1-Q_t)M_t \leq F_t + M_{t-1} + W_tN_t - T_t \tag{1.5}$$

Now, the representation of the previous budget constraint can be corresponding to that an economy in which all financial assets yield a gross nominal return $Q_t^{-1} = exp\{i_t\}$, and agents can purchase the utility yielding services of money balances at a unit price $(1-Q_t) = 1 - exp\{-i_t\} \simeq i_t$. Hence, the implicit price for money services

[1] It is standard to have real money balance in the utility of household as a shortcut to getting money valued in equilibrium. Therefore, it is assumed that household gain utility from holding money. This utility approximates benefits from using money in transactions.

[2] The yield on the one period bond is defined by $Q_t \equiv (1+yield)^{-1}$. Notice that i_t corresponds to the log of the gross yield on the one-period bond and referred to as the nominal interest rate, where $i_t \equiv -logQ_t = log(1+yield_t) \simeq yield_t$

approximately refers to the nominal interest rate.

According to Gali (2007) the optimality conditions of household's problem, implied by the maximization of (1.1) subject to (1.5) plus solvency constraint, for $t = 0, 1, 2, ...$ are given by

$$-\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t} \quad \Rightarrow \quad \frac{W_t}{P_t} = N_t^{\varphi} X_t^{\sigma-\nu} C_t^{\nu} (1-\vartheta)^{-1} \tag{1.6}$$

$$Q_t = \beta E_t \{ \frac{U_{c,t+1}}{U_{c,t}} \frac{P_t}{P_{t+1}} \} \quad \Rightarrow \quad Q_t = \beta E_t \left\{ \left(\frac{C_t + 1}{C_t} \right)^{-\nu} \left(\frac{X_t + 1}{X_t} \right)^{\nu-\sigma} \frac{P_t}{P_t + 1} \right\} \tag{1.7}$$

$$\frac{U_{m,t}}{U_{c,t}} = 1 - exp\{-i_t\} \quad \Rightarrow \quad \frac{M_t}{P_t} = C_t (1 - exp\{-i_t\})^{-\frac{1}{\nu}} \left(\frac{\vartheta}{1-\vartheta} \right)^{\frac{1}{\nu}} \tag{1.8}$$

Equation (1.6) implies labour supply equation and its log linear form can be obtained by

$$w_t - p_t = \sigma c_t + \varphi n_t + (\nu - \sigma)(c_t - x_t) \tag{1.9}$$

and determining the quantity of labour supplied as a function of real wage given the marginal utility of consumption which is influenced by the level of real balances through the dependence of the X_t.

Equation (1.7) reflects the household's preferences for consumption smoothing. The level of real balances can effect the preferences of households over the dependence of the index X_t. Also, the level of real balances, in turn, is influenced by the nominal interest rate as in (1.8) which is defined as money demand equation. These characteristics indicate that monetary policy is not neutral with nonseparable utility function.

The log-linear form of equation (1.8) can be obtained by applying the first-order Taylor approximation[3] as

$$m_t - p_t = c_t - \eta i_t$$

[3]First order Taylor approximation: $log(1 - exp\{-i_t\}) \simeq const + \frac{i_t}{exp\{i\}-1}$

where $\eta \equiv \frac{1}{\nu(exp\{i\}-1)} \simeq \frac{1}{\nu i}$[4] is the implied interest semi-elasticity of money demand. Therefore, according to Gali (2007) after some simple manipulation and combining the resulting expression with (1.8) log-linearization of (1.6) around a zero inflation steady state can be defined as

$$w_t - p_t = \sigma c_t + \varphi n_t + \chi\eta(\nu - \sigma)i_t \tag{1.10}$$

where $\chi \equiv \frac{\vartheta^{\frac{1}{\nu}}(1-\beta)^{1-\frac{1}{\nu}}}{(1-\vartheta)^{\frac{1}{\nu}}+\vartheta^{\frac{1}{\nu}}(1-\beta)^{1-\frac{1}{\nu}}} \in [0,1)$.

Optimality condition (1.10) can be rewritten in terms of the steady state ratio $k_m \equiv \frac{\frac{M}{P}}{c}$ which is the inverse consumption velocity. Using the money demand equation, $k_m = \left(\frac{\vartheta}{(1-\beta)(1-\vartheta)}\right)^{\frac{1}{\nu}}$ and $\chi = \frac{k_m(1-\beta)}{1+k_m(1-\beta)}$ and using the definition of η evaluated at the zero inflation steady state, the optimality condition (1.10) can be rewritten as

$$w_t - p_t = \sigma c_t + \varphi n_t + \omega i_t \tag{1.11}$$

where $\omega \equiv \frac{k_m\beta(1-\frac{\sigma}{\nu})}{1+k_m(1-\beta)}$ and represents the quantity of labour supply at any given real wage. According to Gali (2007), the sign of $\nu - \sigma$ determines the sign of the effect of the nominal interest rate on labour supply. When $\nu > \sigma$ which is implying $\omega > 0$, a rise in the nominal rate, declines the real balances. Then a fall in the real balances reduces the marginal utility of consumption which is followed by decreasing the quantity of labour supplied at any given real wage. The opposite effect can be obtained when $\nu < \sigma$.

1.2.2 Firms

According to Gali (2007) a representative firm seeks to maximize profits

$$P_t Y_t - W_t N_t \tag{1.12}$$

[4] i coresponds to a constant interest rate for all t which is consistent with a steady state with zero inflation.

subject to the production function

$$Y_t = A_t N_t^{1-\alpha} \tag{1.13}$$

where α is the partial elasticity of output with respect to labour. A_t represents the level of technology and $a_t \equiv logA_t$ indicate exogenously some stochastic process. The firm's optimality condition by maximization of (1.12) subject to (1.13) becomes

$$\frac{W_t}{P_t} = (1-\alpha)A_t N_t^{-\alpha} \tag{1.14}$$

which means that the firm hires labour up to the point that marginal product equals to the real wage. The log-linear form given by

$$w_t - p_t = a_t - \alpha n_t + log(1-\alpha) \tag{1.15}$$

can be interpreted as labour demand equation, determining the quantity of labour demanded as a function of the real wage, given the level of technology.

1.2.3 Equilibrium Dynamics

In order to determine the equilibrium path of output, inflation and nominal interest rate, on the one hand, combining households' optimality condition (1.11) with the labour demand equation (1.15) generates the labour market clearing condition (Gali (2007))

$$\sigma c_t + \varphi n_t + \omega i_t = y_t - n_t + log(1-\alpha)$$

This condition can be rewritten, using the goods market clearing condition ($y_t = c_t$) and the log-linear production relationship ($y_t = a_t + (1-\alpha)n_t$), as

$$y_t = \psi_{ya} a_t - \psi_{yi} i_t \tag{1.16}$$

where $\psi_{ya} \equiv \frac{1+\varphi}{\sigma(1-\alpha)+\varphi+\alpha}$, $\psi_{yi} \equiv \frac{\omega(1-\alpha)}{\sigma(1-\alpha)+\varphi+\alpha}$.

Alternatively, log-linear approximation to equation (1.7), combining with goods market clearing condition ($y_t = c_t$) is determined as

$$y_t = E_t\{y_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - \omega E_t\{\Delta i_{t+1}\} - \rho) \tag{1.17}$$

which is relating the nominal interest rate to the expected path of output and expected inflation. Moreover, the central bank adjusts the nominal interest rate according to the simple inflation-based rule as follows

$$i_t = \rho + \phi_\pi \pi_t + \upsilon_t \tag{1.18}$$

where υ_t represents an exogenous policy disturbance, assumed to follow the stationary AR(1) process ($0 < \rho_\upsilon < 1$) with i.i.d (independently and identically distributed) error term which is distributed by normal with zero mean and variance equals to 1 as

$$\upsilon_t = \rho_\upsilon \upsilon_{t-1} + \varepsilon_t^\upsilon$$

It is worth to mention that monetary policy shocks are formed when monetary authorities receive signals from forecasters related to a possible inflationary disturbance and the authorities therefore act if they think this disturbance can be a nuisance for the policy conducted. Also, assuming that the technology parameter a_t follows the stationary AR(1) process ($0 < \rho_a < 1$) with i.i.d error term, which is distributed by normal distribution with mean and variance equal to 0 and 1 respectively.

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a$$

Therefore, according to Gali (2007) the equilibrium level of inflation, nominal interest rate, and output are determined by using (1.18) to eliminate the nominal rate in (1.16) and (1.17) as follows

$$\pi_t = -\frac{\sigma(1-\rho_a)\psi_{ya}}{\phi_\pi(1+\omega\psi)(1-\Theta\rho_a)} a_t - \frac{1+(1-\rho_\upsilon)\omega\psi}{\phi_\pi(1+\omega\psi)(1-\Theta\rho_\upsilon)} \upsilon_t \tag{1.19}$$

$$i_t = -\frac{\sigma(1-\rho_a)\psi_{ya}}{(1+\omega\psi)(1-\Theta\rho_a)}a_t - \frac{\rho_\upsilon}{\phi_\pi(1+\omega\psi)(1-\Theta\rho_\upsilon)}\upsilon_t \tag{1.20}$$

$$y_t = \psi_{ya}\left(1+\frac{\sigma(1-\rho_a)\psi_{yi}}{(1+\omega\psi)(1-\Theta\rho_a)}\right)a_t + \frac{\rho_\upsilon\psi_{yi}}{\phi_\pi(1+\omega\psi)(1-\Theta\rho_\upsilon)}\upsilon_t \tag{1.21}$$

where $\Theta \equiv \frac{1+\omega\psi\phi_\pi}{(1+\omega\psi)\phi_\pi}$ and $\psi \equiv \frac{\alpha+\varphi}{\sigma(1-\alpha)+\varphi+\alpha}$.

It can be seen that the equilibrium dynamics of inflation, nominal interest rate and output are affected by changes in the level of technology and monetary policy disturbance. However, as Gali(2007) stated in classical monetary model, rate of change of inflation with respect to the rate of change of interest rate is defined as

$$\frac{d\pi_t}{di_t} = \frac{d\pi_t/d\upsilon_t}{di_t/d\upsilon_t} = (1+(1-\rho_\upsilon)\omega\psi)\rho_\upsilon^{-1} > 0$$

The above equation expresses a relation between inflation and the interest rate, considering the effect of monetary policy. In response to a monetary policy shock that increases the nominal interest rate and decreases output, inflation tends to rise. This is in contrast with the economy's response to the contractionary monetary policy shock. With regard to the definition of contractionary monetary policy, the money supply needs to be reduced and this is expected to lower inflation in the economy. Hence, the conflict between the theoretical model and evidence is the drawback of the classical monetary models, where the response of inflation to monetary policy shock is at odds with the evidence. In this regard, Castelnuovo and Surico (2005) study empirically the effect of contractionary monetary policy shocks on inflation in the period of pre and post-inflation targeting using UK data. They argue that the effectiveness is significantly different whereas according to the impulse response analysis, inflation responses positively and largely in the pre inflation targeting period, but the response is negative and small after inflation targeting regime. However, in order to address the effect of contractionary monetary policy on the inflation in the classical model, this chapter focuses on the analysis of the relationship between inflation and its uncertainty.

1.2.4 Baseline Calibration

In order to analyse the dynamic effects of a monetary policy shock in the economy, I need to define the numerical values for the model's parameters. I have chosen the values from the baseline calibration of the model as they are consistent with the empirical evidence. In the baseline calibration of the model according to Gali (2007), it is assumed that discount factor $\beta = 0.99$, which explains a steady state real return on financial assets of about 4 percent. It is also assumed that $\sigma = 1$(inverse elasticity of intertemporal substitution among goods), $\phi = 1$(inverse elasticity of labour supply) and $\alpha = \frac{1}{3}$(partial elasticity of output with respect to labour). Coefficient in interest rate rule equals 1.5 ($\phi_\pi = 1.5$) which is consistent with observed variation of U.S. Federal Funds rate over the Greenspan era. Also, interest semi elasticity of money demand η which is proportional to the elasticity of substitution between real balances and consumption ν^{-1} is equal to 4. According to the definition of ω, and noting that $\nu \simeq \frac{1}{i\eta}$ is likely to be larger than σ for any reasonable values of η and σ and also by using $M2$ as the definition of money, $k_m \simeq 3$, and so $\omega = 3$. Moreover, autoregressive coefficients equal 0.5 ($\rho_a = \rho_v = 0.5$) which implies a moderate persistent shock.

1.3 Weighted Skew Normal distribution

The classical monetary model in Gali (2007) is characterized by two shocks, technology and monetary policy. According to the classical model, it is assumed that both shocks are AR(1) process with normal error term. However, I change the stochastic assumption of normality only in monetary policy shock's process and apply WSN distribution instead.

The Skew Normal distribution is a continuous probability distribution that extends the normal distribution, allowing for the presence of skewness which is proposed by Azzalini (1985). The distribution of a random variable Z defined by skew normal distribution with parameter λ denoted by $Z \sim SN(\lambda)$, if its density is given by $f(z, \lambda) = 2\Phi(\lambda z)\phi(z)$ where Φ and ϕ respectively are the standard normal cumulative distribution function and probability density function, and z and λ are real numbers.

$SN(\lambda)$ is limited by the fact that, moderate values of (λ) causes almost all the mass accumulates, either on the positive or negative numbers, as determined by the sign of (λ). To relax this limitation, Weighted Skew Normal(WSN) distribution exhibits a better behaviour, specially at the side with smaller mass. WSN distribution includes the Skew Normal distribution as a special case.

WSN distribution is recently introduced by Charemza, Diaz and Makarova (2013). WSN distribution consists of a family of distribution possibly skewed with all moments finite. As macroeconomic distributions frequently display moderate skewness, WSN distribution allows us to consider the nature of the asymmetry in the analysis and macroeconomic modelling.

According to Charemza, Diaz and Makarova (2013) WSN distribution with parameters $\alpha^*, \beta^*, m, k, r$ is defined by

$$Z = X + \alpha^*.Y.I_{Y>m} + \beta^*.Y.I_{Y<k} \tag{1.22}$$

where $(X,Y) \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma^2 & r\sigma^2 \\ r\sigma^2 & \sigma^2 \end{bmatrix}\right)$ with $0 \leq r < 1$

$$I_{Y>m} = \begin{cases} 1 & \text{if } Y > m; \\ 0 & \text{otherwise.} \end{cases}$$

and

$$I_{Y<k} = \begin{cases} 1 & \text{if } Y < k; \\ 0 & \text{otherwise.} \end{cases}$$

and $\alpha^* \in R$, $\beta^* \in R$, $m \in R$, $k \in R$.

Probability density function of WSN distribution is given by

$$f_{WSN}(t) = \frac{1}{\sqrt{A_{\alpha^*}}}\phi\left(\frac{t}{\sqrt{A_{\alpha^*}}}\right)\Phi\left(\frac{B_{\alpha^*}t - mA_{\alpha^*}}{\sqrt{A_{\alpha^*}(1-r^2)}}\right)$$
$$+\frac{1}{\sqrt{A_{\beta^*}}}\phi\left(\frac{t}{\sqrt{A_{\beta^*}}}\right)\Phi\left(\frac{B_{\beta^*}t + kA_{\beta^*}}{\sqrt{A_{\beta^*}(1-r^2)}}\right)+\phi(t).\left[\Phi\left(\frac{m-rt}{\sqrt{1-r^2}}\right) - \Phi\left(\frac{k-rt}{\sqrt{1-r^2}}\right)\right] \tag{1.23}$$

where ϕ, Φ denote respectively, the density and cumulative distribution functions of the standard normal distribution and

$A_{\alpha^*} = 1 + 2\alpha^* r + \alpha^{*2}$ $\qquad B_{\alpha^*} = \alpha^* + r$

$A_{\beta^*} = 1 + 2\beta^* r + \beta^{*2}$ $\qquad B_{\beta^*} = \beta^* + r$

For $\alpha^* = -2r$ and $\beta^* = m = 0$ in 1.23, pdf of WSN distribution corresponds to Azzalini skew normal $SN(\lambda)$ distribution with pdf function of $f_{SN}(t;\lambda) = 2\phi(t)\Phi(\lambda t)$ where $\lambda = \frac{-r}{\sqrt{1-r^2}}$. If $\alpha^* = \beta^* = 0$ the WSN distribution coincides with normal distribution.

Figure 1.1 depicts Normal, symmetric and asymmetric WSN densities. For the sake of comparison, the symmetric and asymmetric densities' parameters include as:

Symmetric WSN parameters: $\alpha* = -0.9, \beta^* = -0.9, m = -k = 1, r = 0.9$

Asymmetric WSN parameters: $\alpha* = -0.9, \beta^* = -0.1, m = -k = 1, r = 0.9$

According to Charemza, Diaz and Makarova (2013) WSN distribution has a

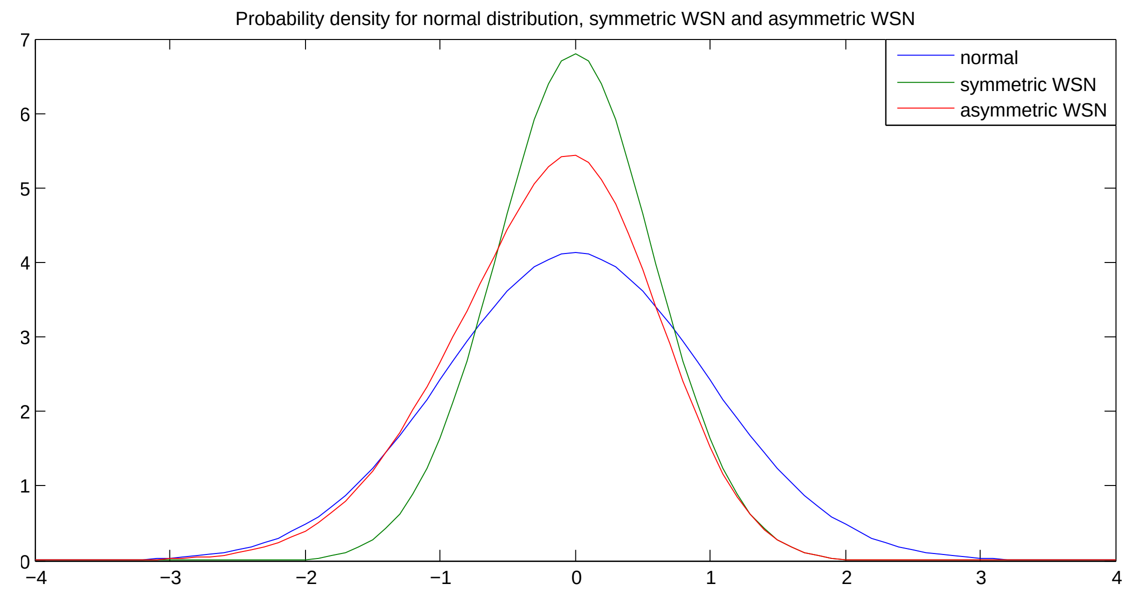

Figure 1.1: Probability density for normal, symmetric WSN and asymmetric WSN distributions

moment generating function given by

$$R_{WSN}(u) = e^{\frac{u^2}{2}A_{\alpha^*}}\Phi(B_{\alpha^*}u - m) + e^{\frac{u^2}{2}A_{\beta^*}}\Phi(k - B_{\beta^*}u) + e^{\frac{u^2}{2}}[\Phi(m - ru) - \Phi(k - ru)] \tag{1.24}$$

The expected value, variance and skewness of Z can be derived using the moment generating function 1.24 as follows:

$$E(Z) = \alpha^*.\phi(m) - \beta^*.\phi(k)$$

$$E(Z^2) = A_{\alpha^*} + [1 - A_{\alpha^*}]\Phi(m) + [B_{\alpha^*}^2 - r^2]m\phi(m) + [A_{\beta^*} - 1]\Phi(k) + [B_{\beta^*}^2 - r^2]k\phi(k)$$

$$\begin{aligned} E(Z^3) = \phi(m).\left\{B_{\alpha^*}.\left[3A_{\alpha^*} + B_{\alpha^*}^2(m^2-1)\right] - r.\left[3 + r^2(m^2-1)\right]\right\} \\ + \phi(k).\left\{-B_{\beta^*}.\left[3A_{\beta^*} + B_{\beta^*}^2(k^2-1)\right] - r.\left[3 + r^2(k^2-1)\right]\right\} \end{aligned}$$

$$Var(Z) = E(Z^2) - [E(Z)]^2$$

$$Sk(Z) = \frac{E(Z^3) - 3.E(Z^2).E(Z) + 2.\left[E(Z)\right]^3}{\left[Var(Z)\right]^{\frac{3}{2}}}$$

1.3.1 Interpretation of WSN parameters

It is assumed that monetary policy is undertaken in association with the level of inflation. Monetary authorities are undertaking an anti-inflationary and pro-inflationary decisions if they receive a signal from forecasters about high and low level of future inflation respectively. According to Charemza et al. (2014), it is assumed that inflation in time $t+h$, π_{t+h}, is a random variable consists of two parts: predictable and non predictable from the past. Within the context of Gali model the predictable part is the non stochastic part of the model which is indexed by $t-1$, and non predictable part from the past corresponds to the stochastic part which is given at time t. Monetary authorities are making decisions in time t regarding the inflation's forecast of time $t+h$, where h is the forecast horizon.

Let us consider the distribution of monetary policy shock as WSN in which

$\varepsilon_t^v \sim WSN(\alpha^*, \beta^*, m, k, r)$ and interpret the parameters of distribution related to monetary policy as in Charemza et al. (2014).

1. Random variable X consists of two components; the first part is related to the fact that future inflation is the random and the second part is related to the incomplete knowledge of the inflation's forecaster.

2. Random variable Y relates to the disagreement of expert forecasters where each forecaster has their own source of information. It is necessary to mention that monetary policy is based on experts' forecasts Y and monetary authorities react only to information passed to them through Y.

3. Parameter r is the correlation coefficient between X and Y and expressed the accuracy of experts' forecasts. $r = 0$ states that X is totally unpredictable or the experts are ignorant. The higher is the value of r, the experts become more knowledgeable (e.g. the knowledge becomes perfect if $r = 1$). It is also assumed that the variances of X and Y are identical which is denoted as σ^2. This assumption shows that in the absence of knowledge of forecasters in X, disagreement between the experts has the same variability as the random behaviour of the future inflation in X. Furthermore, as the experts' forecasts can not be negatively related to the X, therefore $0 \leq r < 1$.

4. The parameters m and k represent respectively high and low thresholds for imperfect knowledge Y. If m is reached from below, it is a signal to the policy makers the necessity of undertaking an anti-inflationary policy, or pro-inflationary policy if k is reached from above.

5. The parameters α^* and β^* denote respectively the effectiveness of anti-inflationary and pro-inflationary policies. They express the strength of these policies and to what extent the anti-inflationary and pro-inflationary policies can effect the inflation and other variables in the economy. The greater the absolute value of α^* and β^*, the higher will be the effects of the monetary policy on macroeconomic variables.

Regarding the rational behaviour of the policy makers and forecasters, the parameters space can be defined as: $\alpha^* \leq 0$, $\beta^* \leq 0$, $m \geq 0$, $k \leq 0$ and $0 \leq r \leq 1$.

1.3.2 Volatility and skewness of policy shock with respect to the policy actions

In order to assess the distributional characteristic of policy shocks it is interesting to evaluate the dependence of the variance which is defined as volatility and also the skewness of policy shocks to the strength of policies which is described by α^* and β^* and to the accuracy of inflation forecast measured by r. I simulate volatility and skewness of policy shocks with respect to the different policy strengths.

In Figure 1.2 left panel represents variance of WSN distribution with respect to the parameter α^* and parameter $r \in \{0.1, 0.3, 0.5, 0.7, 0.9\}$ in symmetric case where $\alpha^* = \beta^*$ and $m = -k = 1$. Therefore, in this case the strength of both policies including anti-inflationary and pro-inflationary are identical and monetary policy shocks' volatility is plotted when strength of policy is varying from 0 to -0.99 as weakest to strongest respectively. It is obvious from Figure 1.2 that with the most accurate forecast when $r = 0.9$, the higher is the strength of policy, the smaller would be the volatility of policy shock. However, with the worst forecast accuracy when $r = 0.1$ even the strength of monetary policy actions can not decrease the policy shocks' volatility(volatility is increasing from 1 to 1.6 for weakest and strongest policy respectively). Moreover, right panel displays the non-linear characteristic between the parameters and volatility of monetary policy shock. The minimum volatility is about 0.36 for the most accurate forecast and the strong monetary policy. The variance also reaches maximum of about 1.78 for the strongest policy and the worst forecast.

Figure 1.3 indicates the case of an asymmetric policy where only anti-inflationary policy is effective and $m = 1$, $\beta^* = 0$. In this case, in the same way as symmetric case the nonlinearity between the parameters of policy shock distribution and its volatility is obvious. However in compare to the symmetric case with strong monetary policy, volatility is about 0.62 and 1.34 for the best and worst forecast respectively.

Figures 1.2 and 1.3 provide interesting insight into the differences between the effects of forecast accuracy on the volatility of policy shock with the strongest policy in symmetric and asymmetric cases. In symmetric case with the 22% increase in forecast accuracy, the volatility of policy shocks is falling 45% . However, with the same increasing rate of forecast accuracy in asymmetric case the volatility declines by the rate of 19%. Hence, the rate of change of volatility in symmetric case decreases twofold in comparison with asymmetric case. This is suggestive of a stronger effect of forecast accuracy when both anti and pro-inflationary policies are of identical strength.

Figure 1.4 describes the skewness of monetary policy shock distribution with respect to the policy's strength and forecast accuracy in symmetric and asymmetric cases. In the case of symmetric (right panel) the skewness of distribution of policy shock is zero. However, in asymmetric case (left panel) there is a nonlinear characteristic between the parameters of policy shock and its skewness. With the best forecast and strongest policy the degree of skewness is -0.42. Moreover, as obvious from Figures 1.2-1.4 with no policy the skewness and variance of policy shock is 0 and 1 respectively. Hence, if there is no effective policy, then policy shock distribution would be standard normal distribution. Therefore, comparing the WSN and the normal distributions the advantage of the WSN is that with changing the range of parameters of the distribution, it is possible to consider the monetary policies with different strength and asymmetries. However, the normal distribution is the special case of WSN distribution where the effectiveness of policies is equal to zero.

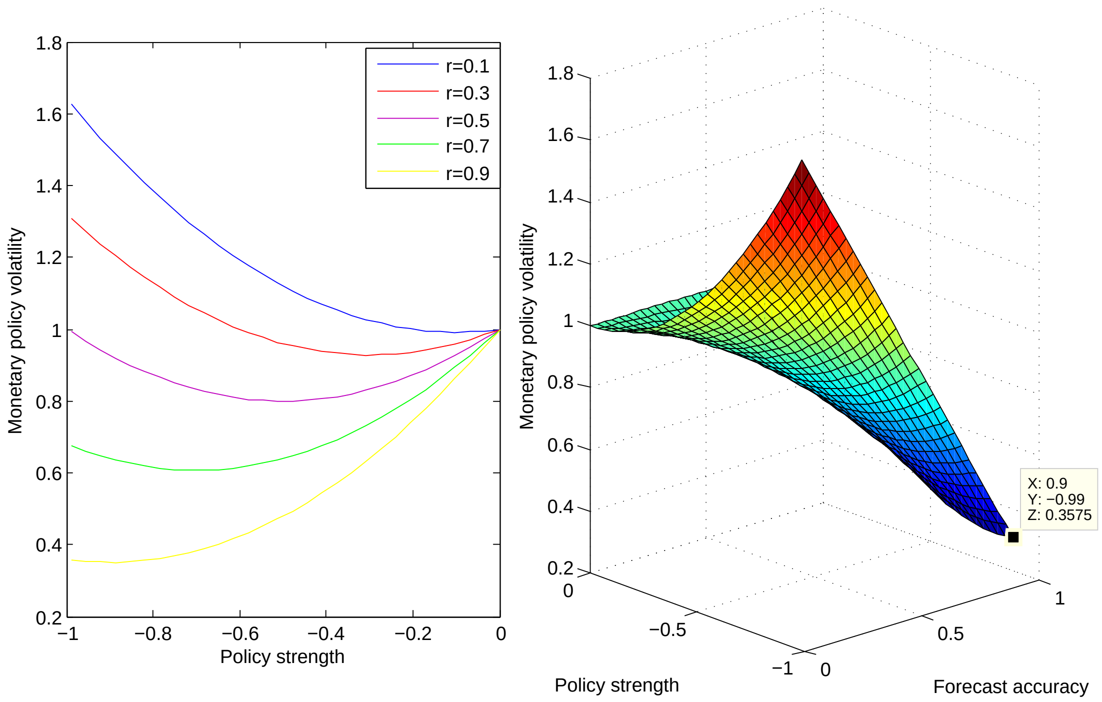

Figure 1.2: Volatility of monetary policy with respect to the policy strength and forecast accuracy in symmetric case ($\alpha^* = \beta^*, m = -k = 1$)

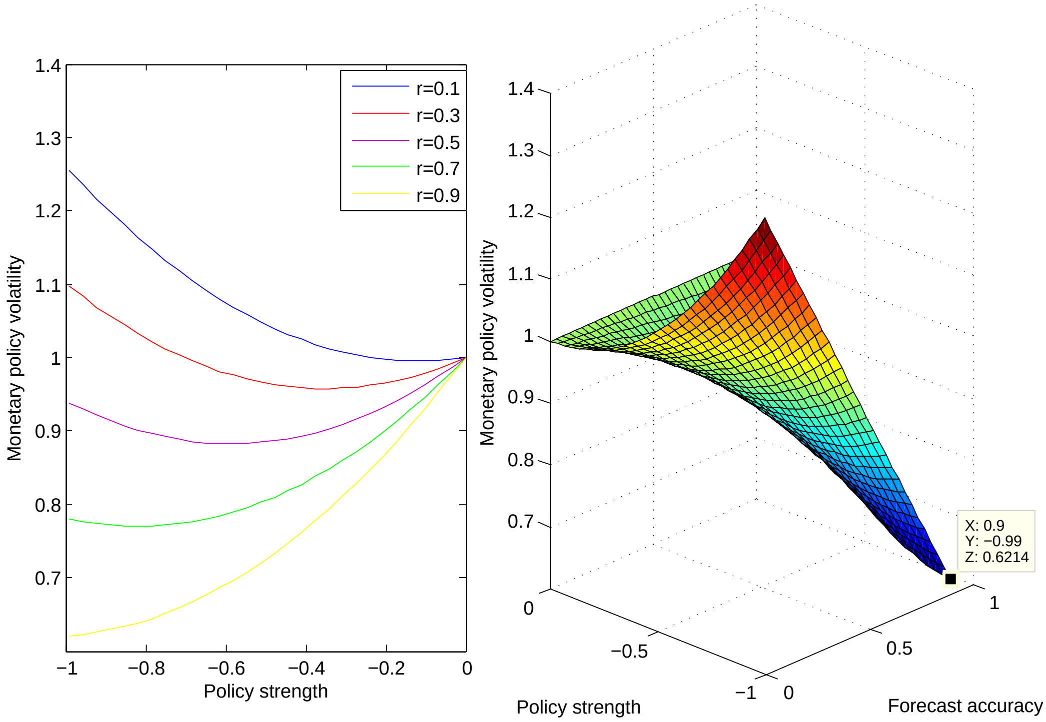

Figure 1.3: Volatility of monetary policy with respect to the policy strength and forecast accuracy in asymmetric case ($\beta^* = 0, m = -k = 1$)

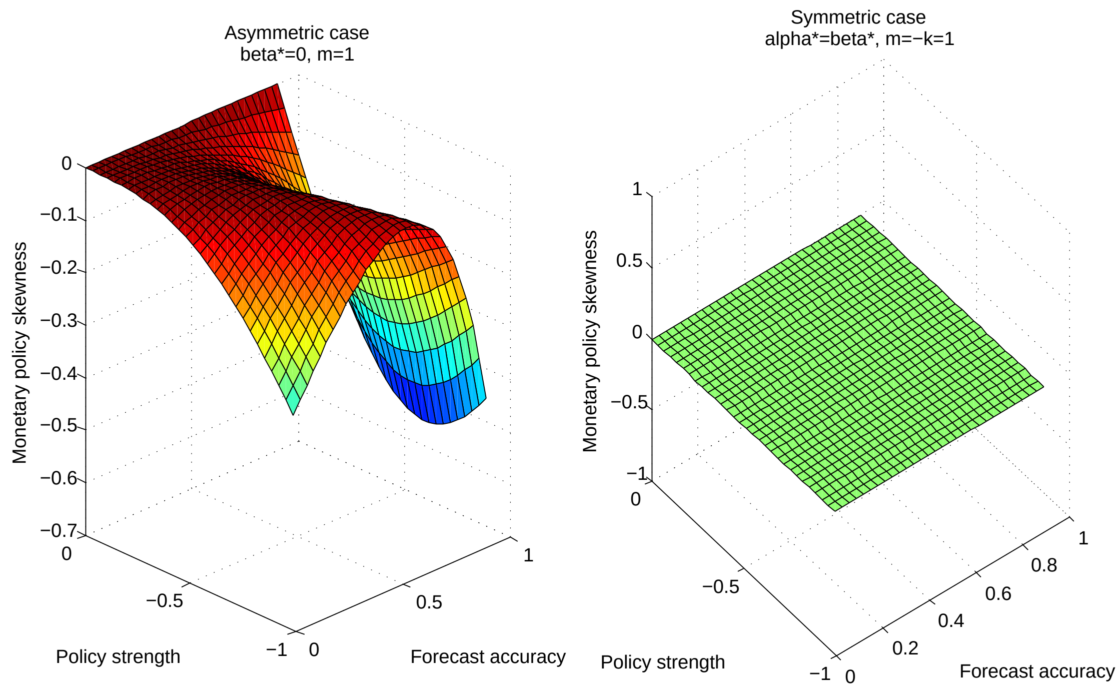

Figure 1.4: Skewness of monetary policy with respect to the policy strength and forecast accuracy in symmetric(right) and asymmetric(left) cases

1.4 Stochastic Simulation

Now, in order to answer to the question that how monetary policy actions affect the distribution of macroeconomic variables, I use a stochastic scheme to simulate the variables and shocks using random number generators. Also, in order to randomly assign the policy shocks, I need a different random numbers for each simulated variables.

According to Kleijnen et al. (2003), simulation based study can be more productive if statistical theories have been applied in the design of the experiment. Montgomery (2008) argues that the experiment can be defined as a test or as a series of tests. He explains that the proposed changes are applied on the input variables of a process or system to observe how the output variable will change. Proper design of the experiment will improve the performance of the output variable, instead of a trial-and-error method.

Simulation may not always produce accurate results as it works on logical manipulation, whereas analytical results are truthful as these are found by proven mathematical manipulation. However, simulation is most useful when there is a lack of analytic tractability or when we have a highly multidimensional problem.

The term "Stochastic Simulation" refers to the simulation method in which the variables can change randomly with certain probabilities. Its necessary technique is to use random number generators. A random number generator produces a sequence of numbers that are draws from a specific independently and identically distributed random variable. Therefore, this is a mathematical algorithm that creates a series of so-called pseudo random numbers. Also, in simulation experiments apart from random number generator, one should specify the statistical model and Data Generating Process(DGP). This implies that the assumption of the deterministic parts of the model as well as the exact parameters of the distribution of the stochastic term is necessary.

1.4.1 Random Number Generation

According to Charemza, Diaz and Makarova (2013), the algorithm for generating random numbers from $WSN(\alpha^*, \beta^*, m, k, r)$ is:

Step(1) generate a pair of random numbers (x, y) from a bivariate normal distribution with zero means, unitary variance and covariance equal to r.

Step(2)

(a) if $y \leq m$ and $y \geq k$, return $z = x$;

(b) if $y > m$, return $z = x + \alpha^* y$;

(c) if $y < k$, return $z = x + \beta^* y$.

1.4.2 Data Generating Process

I generate the data from the equations 1.19-1.21 as shown in the below with the parameter values as in the baseline calibration in 1.2.4 and with the following AR(1) processes,

$$\pi_t = -\frac{\sigma(1-\rho_a)\psi_{ya}}{\phi_\pi(1+\omega\psi)(1-\Theta\rho_a)}a_t - \frac{1+(1-\rho_\upsilon)\omega\psi}{\phi_\pi(1+\omega\psi)(1-\Theta\rho_\upsilon)}\upsilon_t$$

$$i_t = -\frac{\sigma(1-\rho_a)\psi_{ya}}{(1+\omega\psi)(1-\Theta\rho_a)}a_t - \frac{\rho_\upsilon}{\phi_\pi(1+\omega\psi)(1-\Theta\rho_\upsilon)}\upsilon_t$$

$$y_t = \psi_{ya}\left(1 + \frac{\sigma(1-\rho_a)\psi_{yi}}{(1+\omega\psi)(1-\Theta\rho_a)}\right)a_t + \frac{\rho_\upsilon\psi_{yi}}{\phi_\pi(1+\omega\psi)(1-\Theta\rho_\upsilon)}\upsilon_t$$

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a \qquad \varepsilon_1^a \sim N(0,1) \qquad t = 1, 2, ..., 10 \tag{1.25}$$

$$\upsilon_t = \rho_\upsilon \upsilon_{t-1} + \varepsilon_t^\upsilon \qquad \varepsilon_1^\upsilon \sim WSN(\alpha^*, \beta^*, m, k, r) \qquad t = 1, 2, ..., 10 \tag{1.26}$$

where $\rho_a = \rho_\upsilon = 0.5$ are fixed coefficients. The initial value in (1.25) and (1.26) is set to 0 ($a_0 = \upsilon_0 = 0$). Regarding WSN parameters, a set of parameters is used to obtain symmetric and asymmetric policy actions based on the high forecast accuracy. In symmetric case where anti-inflationary and pro-inflationary policy has the identical strength, parameter $\alpha^* = \beta^*$ and $m = -k = 1$ and $r = 0.9$. In asymmetric case where only anti-inflationary policy is effective, parameter $\beta^* = 0$, $m = -k = 1$ and $r = 0.9$. In order to analyse the effect of policy from weakest to strongest in both symmetric and asymmetric cases parameter $\alpha^* \in \{0, -0.25, -0.50, -0.75, -0.99\}$.

1.4.3 Impulse Response Analysis

In this study impulse response describes the reaction of two first moments of endogenous macroeconomic variables with respect to the volatility of monetary policy shocks. The variables include inflation, nominal interest rate and output, obtained at the occurrence time of the monetary policy shock and over subsequent points in time. In order to design the simulation, single technology and monetary policy shocks are generated in $t = 1$ as in (1.25) and (1.26) which are not related to each other. Then according to the equations (1.19), (1.20) and (1.21) I simulate endogenous variables and set the parameters' values as in baseline calibration (1.2.4) setting. I simulate 10000 realisations of inflation, nominal interest rate and output $\pi_t^{(R)}$, $i_t^{(R)}$ and $y_t^{(R)}$, $R = 1, 2, ..., 10000$, $t = 1$. Then I compute the mean and variance of macroeconomic variables in $t = 1, ..., 10$.

1.5 Simulation Results

In this section, I analyse the mean and variance of three macroeconomic variables inflation, interest rate and output by impulse response analysis considering the normal distribution for technology shock and the WSN distribution for monetary policy shock in three cases:

Case1. Normal policy shock

Case2. Symmetric policy shock

Case3. Asymmetric policy shock

1.5.1 Case1

The impulse response of macroeconomic variables are simulated when the technology and policy shocks are both normally distributed. According to Figure 1.5, inflation's and output's mean decline by -0.85×10^{-3} and -0.008 respectively whereas interest rate's mean increases in response to these shocks reaching about 1.3×10^{-4}. Moreover, volatility of macroeconomic variables raises by approximately about 0.7, 0.05 and 1.2 respectively for inflation, interest rate and output.

1.5.2 Case2

The effect of fully symmetric monetary policy shock is investigated for the different forecast accuracy, high, medium and low when the policy strength is varying. The parameters of the policy shocks' distribution are:

$\alpha^* = \beta^* \in \{0, -0.25, -0.50, -0.75, -0.99\}$, $m = -k = 1$ and $r = 0.9, 0.5, 0$ for high, moderate and low accuracy of forecast respectively.

Figure 1.6, 1.7 and 1.8 plot the impulse response of inflation, interest rate and output to the technology and symmetric monetary policy shocks in three different scenarios with low, moderate and high forecast accuracy respectively. It can be seen that the inflation's mean decreases in response to these shocks in the cases of low and moderate forecast accuracy where the stronger is the policy, the smaller would be the inflation's mean. However, with the most accurate forecast, inflation's mean increases reaching about 0.25×10^{-3} and 1.5×10^{-3} respectively for weakest and strongest policy. Furthermore, volatility of inflation rises as policy strength is increasing in the both cases of low and moderate forecast accuracy. But with high forecast accuracy, volatility declines from 0.68 to 0.27 as policy changes from weak to strong. Interest rate's mean follows an inverse pattern of inflation's mean in the case of most accurate forecast, rising by 4.2×10^{-4} and 7.2×10^{-4} for the weak and strong policy respectively. However, volatility of interest rate decreases from 0.048 to 0.022 as the policy strength is increasing. It is also interesting to note that in the case of high forecast accuracy with the 63% declining in the volatility of policy shock (according to Figure 1.2), volatility of inflation and interest rate fall 60% and 54% respectively. However, the changes in the volatility of the aforementioned variables is very slight when the policy is varying from moderate to strongest. In terms of output, it is obvious that its mean and variance are affected only by the technology shock and variation in both, strength of symmetric monetary policy and forecast accuracy has no effect on the output.

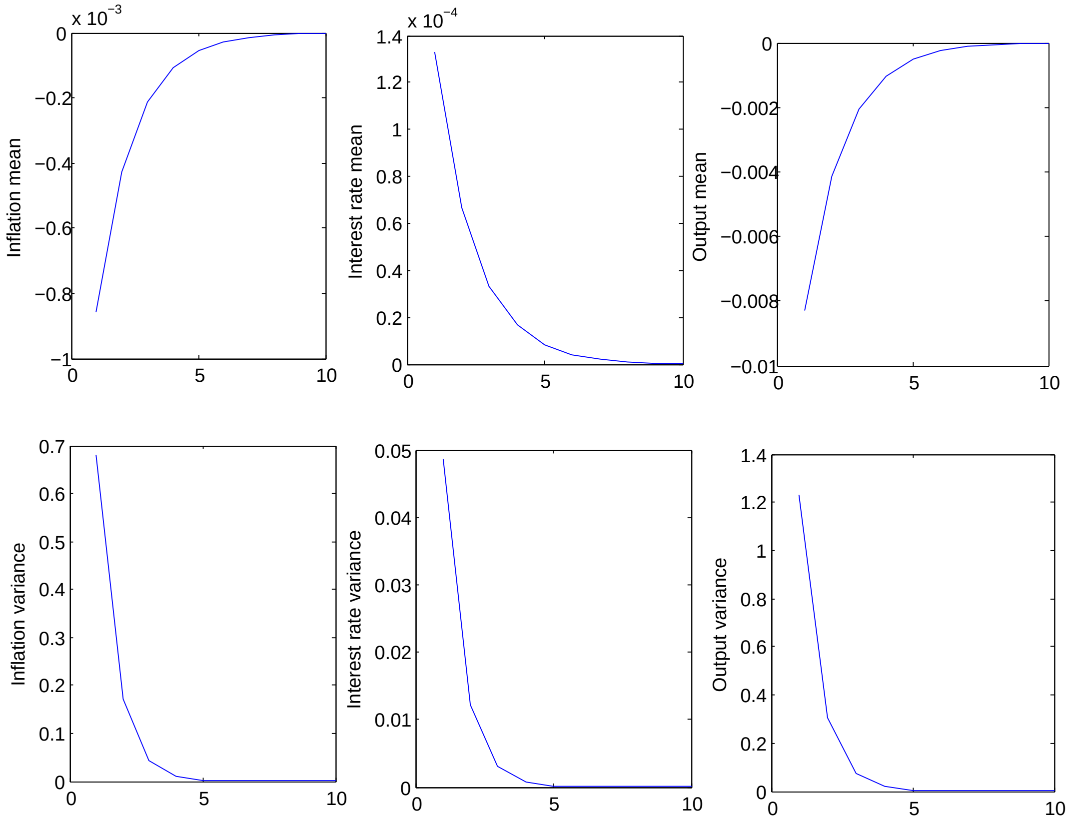

Figure 1.5: Effects of the Technology and Normal Monetary Policy Shocks on macroeconomic variables

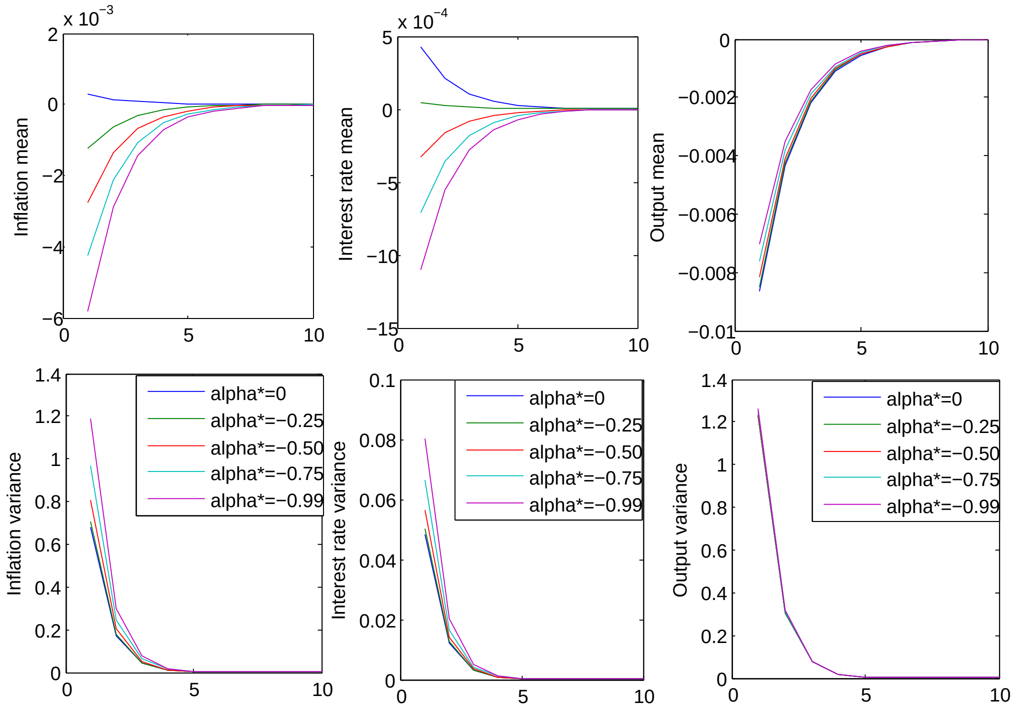

Figure 1.6: Effects of the Technology and Symmetric Monetary Policy Shocks on macroeconomic variables with low forecast accuracy

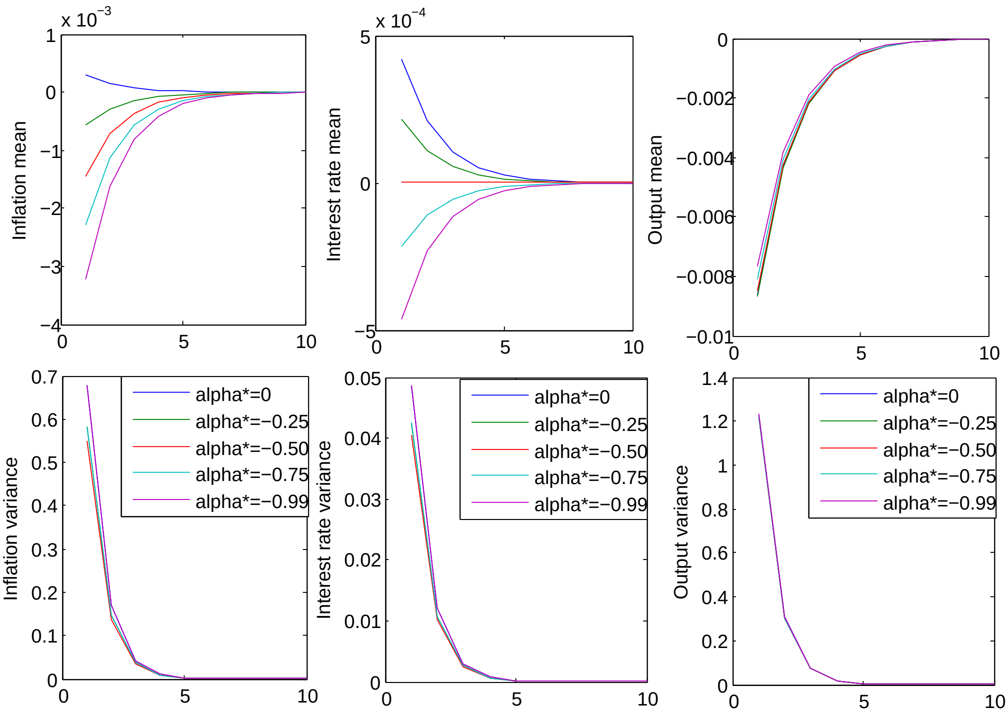

Figure 1.7: Effects of the Technology and Symmetric Monetary Policy Shocks on macroeconomic variables with moderate forecast accuracy

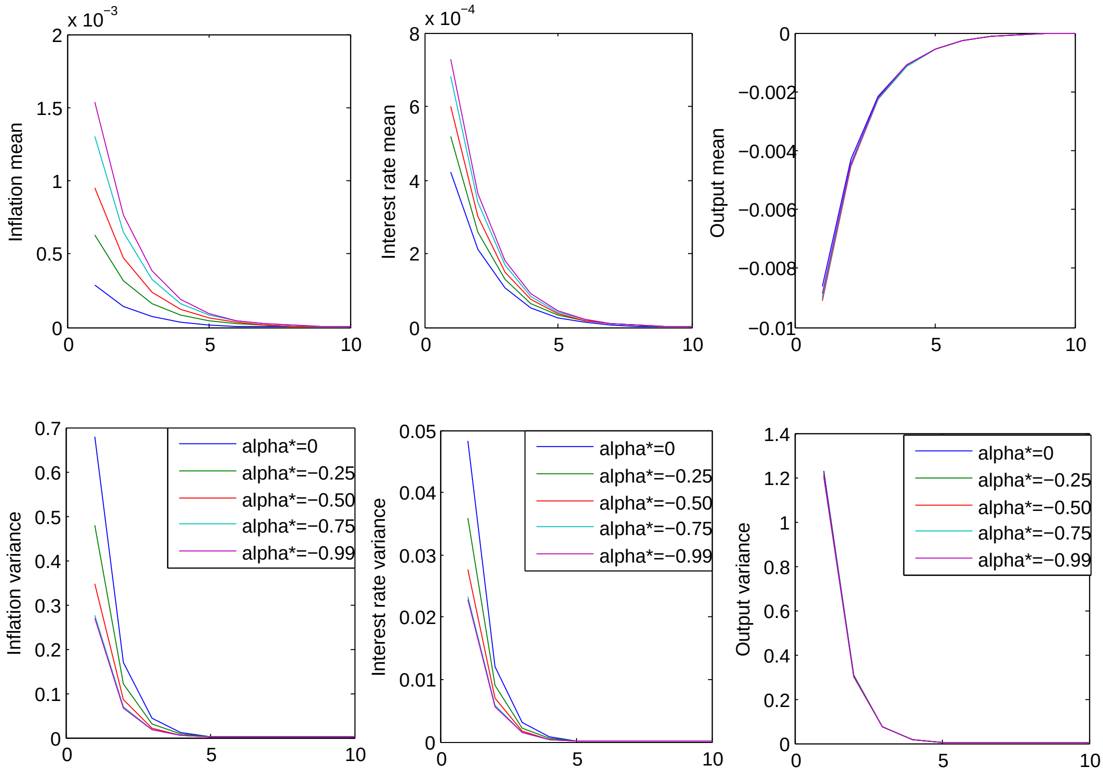

Figure 1.8: Effects of the Technology and Symmetric Monetary Policy Shocks on macroeconomic variables with high forecast accuracy

1.5.3 Case3

The effect of the technology shock and the asymmetric monetary policy shock is investigated in the model with the high, moderate and low forecast accuracy when the policy strength is varying. The variation of policy strength is expressed by parameter α^* of the WSN distribution as: $\alpha^* \in \{0, -0.25, -0.50, -0.75, -0.99\}$ and also because of asymmetric policy shock $\beta^* = 0$, $m = -k = 1$ and due to high, moderate and low forecast accuracy $r = 0.9, 0.5, 0$.

Figures 1.9, 1.10 and 1.11 display the mean and variance of macroeconomic variables with low, moderate and high forecast accuracy respectively when only anti-inflationary policy is undertaking. In this case the direction of changes in the mean and variance of variables is in the same way as symmetric case. However, with the strongest anti-inflationary policy, inflation's and interest rate's mean increases about 0.2 and 0.05 respectively which is higher than symmetric case. Regarding the variance of variables, inflation's volatility declines from 0.67 to 0.44 whereas this reduction is from 0.048 to 0.034 for interest rate. Thus, the amount of decreasing in the volatility of inflation and interest rate correspond to the strength of monetary policy, in asymmetric case is lower than symmetric case. It can be also seen that 38% reduction in the variance of policy shock according to the weakest to strongest policy can explain 34% and 29% of decline in the inflation's and interest rate's volatility respectively. Moreover, as shown in Figure 1.11 the mean of output is falling by around -0.053 for the strongest policy where its variance is not affected by the change of anti-inflationary policy strength.

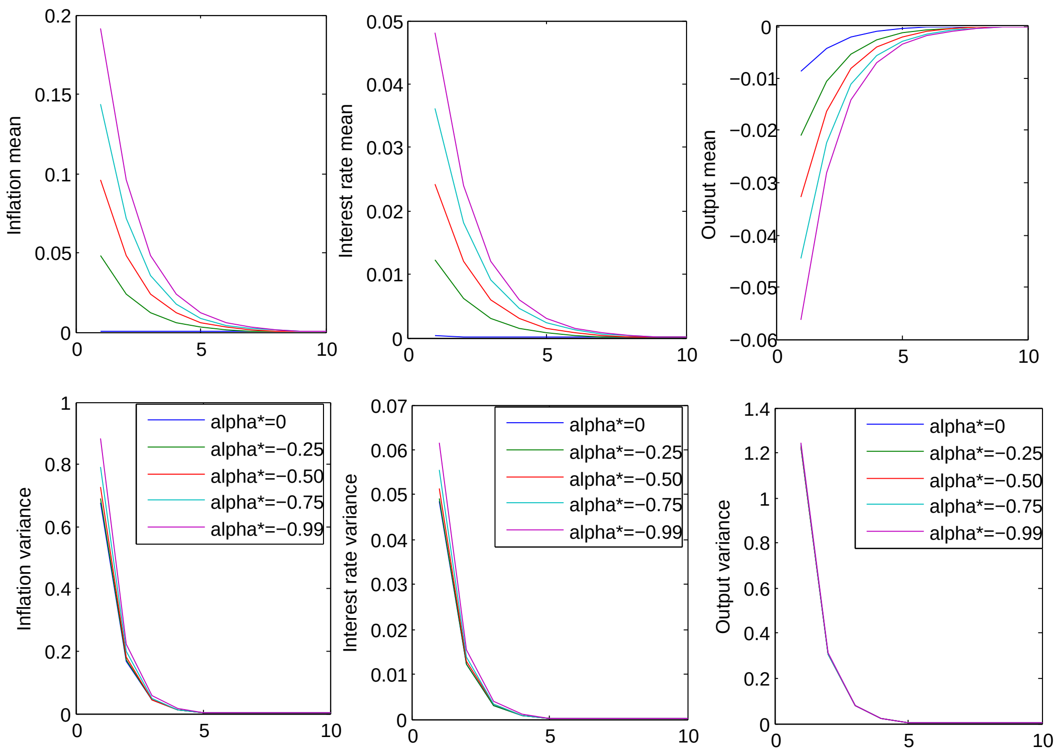

Figure 1.9: Effects of the Technology and Asymmetric Monetary Policy Shocks on macroeconomic variables with low forecast accuracy

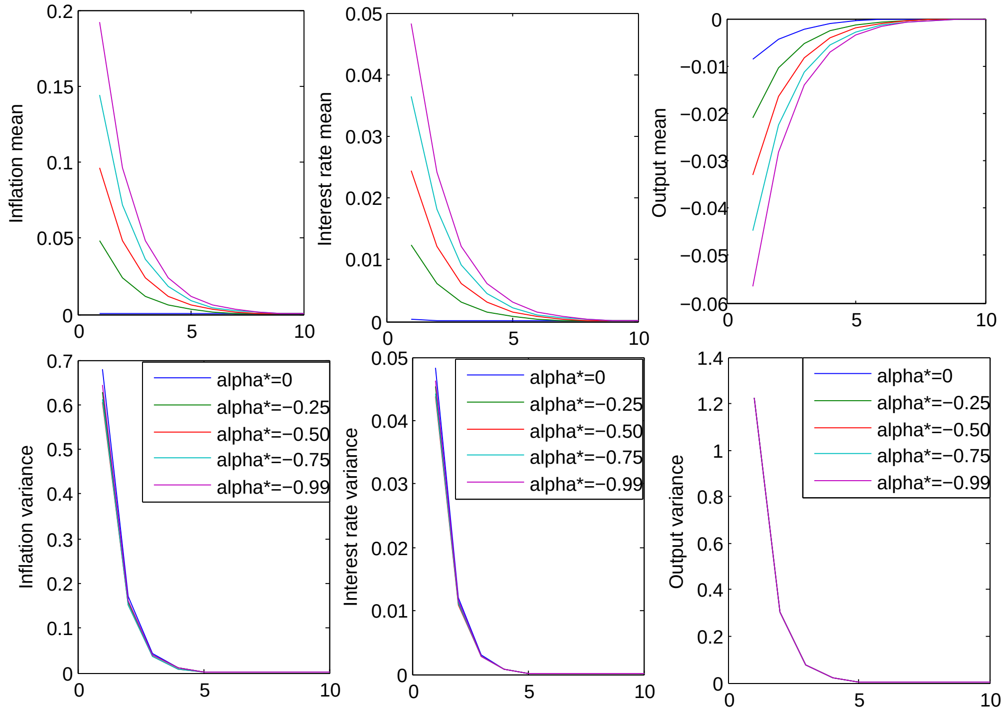

Figure 1.10: Effects of the Technology and Asymmetric Monetary Policy Shocks on macroeconomic variables with moderate forecast accuracy

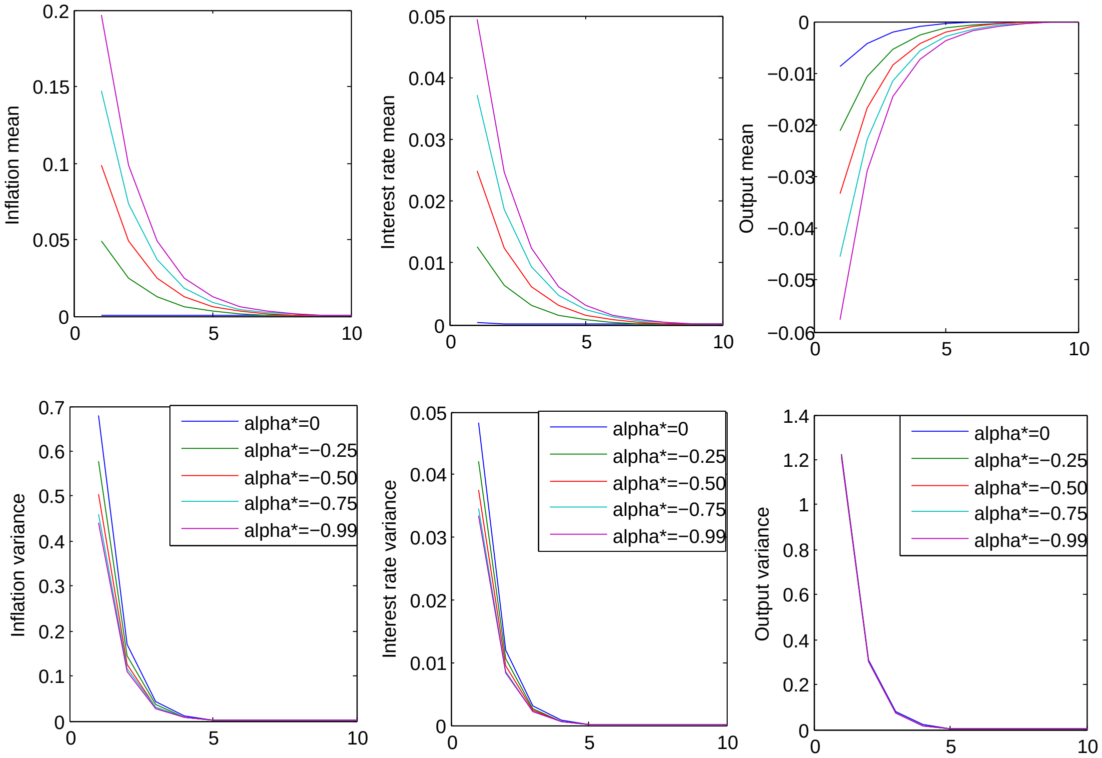

Figure 1.11: Effects of the Technology and Asymmetric Monetary Policy Shocks on macroeconomic variables with high forecast accuracy

1.5.4 Discussion

Using stochastic simulation and comparing the impulse responses of macroeconomic variables with respect to the normal, symmetric WSN and asymmetric WSN distributions, I found that the mean of inflation is increasing in Gali model when anti-inflationary policy (asymmetric WSN case) is undertaking. This is the drawback of classical monetary models that the response of inflation to contractionary monetary policy is at odds with the evidence. I am going to explain this phenomena in the light of Cukierman-Meltzer hypothesis. Cukierman and Meltzer (1986) analysed the relationship between inflation uncertainty and the level of inflation. They showed that increses in inflation uncertainty raise the optimal average inflation rate. Hence, there is a positive causality from inflation uncertainty to inflation level.

In order to explain the Cukierman-Meltzer hypothesis in Gali model I analyse the long-run response of the mean of inflation with respect to the increases of uncertainty. The overall uncertainty is defined by the variance of policy shock. In Figure 1.12 I have plotted two scenarios with the same variances. The first one is the case with no monetary policy in which $\alpha^* = \beta^* = 0$ and it is shown with blue solid line in the Figure 1.12. In the second case I introduce monetary policy with non zero α^* and β^* and is plotted with the red solid line. It can be seen that in the long-run the greater is the uncertainty, the higher would be the mean of inflation. Therefore, using the analysis of long run mean of inflation, I have provided strong evidence in favour of the hypothesis that high inflation uncertainty in Gali model are associated with high inflation's level. This analysis can shed light on the explaining the drawback of Gali model.

The analysis of inflation uncertainty in Gali model carry noteworthy implications for policy making and macroeconomic modelling. According to the validation of Cukierman-Meltzer hypothesis in Gali model it can be suggested that central banks and policy makers might adjust their rate of money growth differently to inflation uncertainty depending on their relative preference toward inflation and output stabilization.

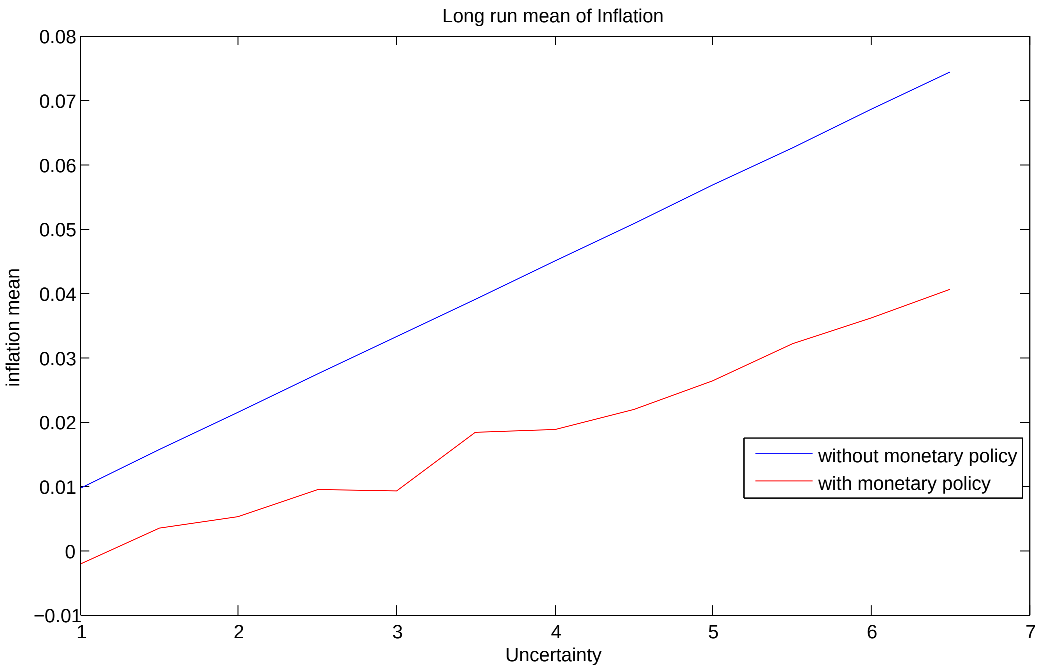

Figure 1.12: Long run response of mean of inflation

1.6 Conclusion

This chapter relaxes the stochastic assumption of normality for monetary policy shock in the classical monetary model and applies WSN distribution instead. This is an important insight as research often depends on the distributional assumptions of the shocks. This modification allows for analysing the macroeconomic variables in the model considering the monetary policy actions and outcomes which are represented by parameters of WSN distribution. WSN parameters have potential implication related to monetary policy actions using the assumption that monetary authorities are making decisions on the basis of forecasts of future's inflation. The contribution of this chapter is twofold. First, it investigates the volatility of monetary policy in two cases, symmetric and asymmetric.

The analysis of monetary policy's volatility in both symmetric and asymmetric cases shows that there is a nonlinear characteristic between forecast accuracy, policy action and volatility of monetary policy shock. It also displays that the movements in the volatility of monetary policy is highly affected by accuracy of inflation's forecast where the lowest volatility is related to the high forecast accuracy.

Second, it provides the distributional analysis of macroeconomic variables with respect to the different monetary policy shocks distribution, normal, symmetric WSN and asymmetric WSN using impulse response analysis.

The policy shocks with normal distribution implies that the inflation's and output's mean decline whereas interest rate's mean increases. In addition, variance of all three macroeconomic variables raises where the output's variance has the largest change among the others.

Regarding the distributional effect of symmetric and asymmetric policy shocks on macroeconomic variables, this study reveals the extent to which the responses of the endogenous variables change through time, due to the different monetary policy strength with different forecast accuracy. The results show that with high forecast accuracy, the percentage change in the volatility of inflation and interest rate becomes small as strength of policy is growing. However, monetary policy strength in both symmetric and asymmetric cases has no effect on output's volatility. Also comparing the normal case, symmetric and asymmetric WSN cases it can be said that output's volatility is affected only by technology shock. Moreover, monetary policy shock affects the level of output only in the case that anti-inflationary policy is effective. In terms of inflation and interest rate, the results show that the greater is the strength of monetary policy, the bigger would be the impact of the asymmetric shocks on the variable's distribution. However, the responses of mean of inflation which is increasing with anti-inflationary policy is at odds with the evidence. The inflation is increasing when the anti-inflationary policy is undertaking in Gali model. This phenomena explained by Cukierman-Meltzer hypothesis and conclude that in classical monetary model represented by Gali there is a positive causality from inflation uncertainty to inflation level. Therefore, the increases of inflation level might be caused by inflation uncertainty.

Chapter 2

Performance of the QMLE under misspecification

2.1 Introduction

Time series models of conditional heteroskedasticity have a long history in statistics and econometrics. Engle (1982) proposed a popular model of conditional heteroskedasticity. His concept of autoregressive conditional heteroskedasticity (ARCH) literally revolutionized empirical work for example in financial economics, where primarily stock returns, foreign exchange rates, and interest rates have been modelled with this type of time series. ARCH specifies the conditional variance as a linear function of the squares of the previous innovations which can be estimated by maximum likelihood method. Recent contributions have extended the ARCH model to a wider class of specifications, the most important of which is the generalized ARCH (GARCH) model of Bollerslev (1986).

In earlier literature from GARCH models maximum likelihood estimation is based on the conditional Gaussian assumption on the innovation distribution. Although, unconditional distribution of ARCH/GARCH residuals might not be Gaussian, but there is more evidence that financial returns are not well approximated by Gaussian distribution. In particular, it is often found that market returns distribution has negative skewness and excess kurtosis. This extreme realization of returns can have adverse effect on the performance of estimation. But this type of realizations is particularly true for ARCH and GARCH models whose estimation of variances are sensitive to large innovations. Some of empirical evidence has addressed heavy tailed and asymmetric distribution of innovation. Bollerslev (1987) estimated a GARCH (1,1) model with a conditional Student-t distribution for daily observations of the U.S. dollar/British pound and the U.S. dollar/deutsche mark from 1980 to 1985. Hsieh (1988) had the longest daily time series so far for five currencies and for nine years of data, from 1974 to 1983, but the model he estimated was a restricted ARCH model. He concluded that generally, the ARCH and GARCH models are successful in accounting for most of the heteroscedasticity of exchange rate data, but in none of these works the type of heteroscedasticity is identified properly in the data generation process. Hence, Hsieh (1989) uses a wider class of non-normal error densities to improve the fit of the model. He found that residuals from ARCH/GARCH models using standard normal density are highly leptokurtic but non-normal distributions fit the data quite well. However, Hall and Yao (2003) presented that for heavy tailed errors, whose squares have regularly varying tail with index $\alpha \in [1, 2)$, the asymptotic distributions of QMLE of parameters in ARCH and GARCH models are nonnormal, and are specially not easy to estimate directly using standard parametric methods. Chen et al. (2012) consider a modification of quasi-maximum likelihood estimator(QMLE) for GARCH(1,1) process with the errors of α-stable distribution, whose squares have regularly varying tail with index α, $\alpha > 0$. They show that the estimator is unbiased and the asymptotic distribution of that is normality regardless of the non-normal error distribution. In addition, regarding the impact of an incorrect error distribution of GARCH model, the performance of QMLE has been investigated by several researchers. For instance, Engle and Gonzalez-Rivera (1991) confirm a loss of efficiency of the QMLE of the model parameters under non-normal innovations. Gonzlez-Rivera and Drost (1999) find that the efficiency of the QMLE is affected by the skewness as well as the kurtosis of the conditional error distribution. Furthermore, Bellini and Bottolo (2008) identify the impact of misspecification on the volatilities through Monte Carlo simulation study and fitting the GARCH(1,1) model. Their results display a systematic overestimate of volatilities in the case that the tails of the underlying innovations are heavier than the fitting innovations.

In this chapter I investigate QMLE properties in a GARCH model with dynamics introduced into the mean equation, including AR(1)-GARCH(1,1) when a normal log-likelihood is maximized but distributional misspecification in conditional error term is assumed and normality assumption is violated. The quasi-maximum likelihood estimation is very popular amongst various GARCH type models. This approach has the advantage that it does not rely on the distribution information of the process. The procedure starts by imposing a postulated distribution on the i.i.d. innovation process, whose actual distribution is unknown. In practice, the most common substitute is the Gaussian distribution that the obtained estimator is known as the Gaussian QMLE. In contrast, there is a limited number of studies focusing on non-Gaussian QMLE which is based on non-Gaussian likelihoods. Recently, Fan et al. (2014) consider non-Gaussian QMLE of GARCH model with heavy tailed likelihoods, and investigate the consistency of the estimator. They identify the inconsistency of the estimator because of the density misspecification, and propose a novel approach by introducing a scale adjustment parameter and a three step quasi maximum likelihood procedure with non-Gaussian likelihood function. However, a majority of literature prefer applying Gaussian QMLE as this is robust against misspecification of error distribution, while using non-Gaussian QMLE is not. Bollerslev and Woodridge (1992) provide Monte Carlo evidence for the AR(1)-GARCH(1,1) model to investigate finite sample performance of estimators in both the normally distributed case and under the assumption of student t distribution. They find that for the sample sizes of 100, 200 and 400 the biases in the QMLE are relatively minor. However, they study the efficiency of the QMLE under non-normality and conclude that the QMLE loses little efficiency with symmetrically t-distributed errors, but the efficiency loss can be marked under asymmetric error distributions. In contrast, Yaya et al. (2014) study AR-GARCH process under misspecified probability distribution. They sampled the GARCH process using one of the distributions of normal, student t, and generalized error distribution (GED). Then they analysed the performance of AR(1)-GARCH(1,1) model based on the parameter estimation, volatility, excess kurtosis, and forecast evaluation criteria. The Monte Carlo simulation results reveal that the AR-GARCH model when data generating process assumed GED, performed better on the three assumed distribution. Moreover, Iglesias and Phillips (2008) provide simulation results concerning the finite sample properties of QMLE in AR(1)-ARCH(1) model. They find that mean square error of the estimator of the AR(1) parameter is significantly reduced as conditional heteroskedasticity increases. In another example Lumsdaine (1995) investigates the finite sample properties of the maximum likelihood estimator in GARCH(1,1) and

IGARCH(1,1) models via a Monte Carlo simulation. He concludes that the estimators of the parameters particularly those of the ARCH parameters are skewed in small samples.

From other point of view regarding the impact of a range of mean and volatility persistence on the performance of QMLE for AR(1)-GARCH(1,1) model there is not enough related literature. However, Zhou (2000) studies a Monte Carlo experiment on QMLE, Maximum Likelihood Estimation (MLE), Efficient Method of Moments (EMM), and Generalised Method of Moments (GMM) for a continuous time square root model under two scenarios; mean persistence and volatility cluster. He finds that in both scenarios MLE get the highest efficiency and QMLE stands out in the second place. He also concludes that QMLE is straight forward to implement and it can be reliable in the case that the specification information is included in the conditional mean and variance.

To examine the effects of distributional misspecification in the AR(1)-GARCH(1,1) models in this chapter, I consider the light tail WSN distribution in both symmetric and asymmetric cases. I investigate the properties of the QMLE of model parameters under both correct and incorrect model specifications. I study the bias, root mean square error(RMSE), skewness, and kurtosis of the QMLE of model's parameters derived under the assumption of Normal distribution (i.e., under correct model specification), as well as WSN distribution (i.e., under model misspecification). Here, I numerically evaluate how quasi maximum likelihood estimate of parameters vary according to the misspecification in distribution of innovations and whether the mean and volatility persistence has an impact on the behaviour of QMLE. I focus on maximum likelihood estimators because they are used widely in practice. I also investigate maximum likelihood estimators of quantiles under a distributional misspecification in the models using simulation.

In practical problems and hypothesis testing, quantiles are proper distributional summaries as facilitate interpretation of attribute values represented in graphics such as histograms or density plots. According to Breidt (2004) quantiles of a known distribution are often difficult to obtain analytically or numerically. Therefore, practically it is common to use the order statistics for estimating the quantiles of the distribution.

Now the question is that how accurate is the estimation of quantiles? The bootstrap method is a general methodology for answering this question. It is a computer-based method which is proposed by Efron (1979) to study the properties of parameter estimates or test statistics. One approach of bootstrap technique is in hypothesis testing. Although testing a hypothesis is a central concern of econometrics, the distributions of the most frequently used test statistics are identified only asymptotically. Thus inference on the basis of asymptotic distribution can be risky. Therefore, the bootstrap technique is an approach developed to solve this issue. In the last two decades, using the bootstrap technique to present hypothesis testing in econometrics has become common; see e.g. Horowitz (1994), Horowitz (1997), Nankervis and Savin (1996) and Davidson and MacKinnon (1999). Moreover, for estimating the standard errors the bootstrap technique can be used as an alternative method while mathematically the theoretical calculation is intractable. This method can be implemented by constructing a number of re-samples with replacement from original data. According to Efron (1979) the non-parametric bootstrap might be applied as a valid appropriate tool for statistical inference. However, Bickel and Freedman (1981) show that the non-parametric bootstrap can fail in a situation like the consistent estimate of the distribution function of the quantiles with the maximum of a sample size n(referred to as n out of n bootstrap). They rectify the method by re-sampling smaller bootstrap sample m instead of re-sampling bootstrap samples of size n, where $m \to \infty$ and $\frac{m}{n} \to 0$. This adaptation of the bootstrap sample size is called the m out of n bootstrap. In a similar way Janssen et al. (2001) show that in the estimate of the distribution function of the U-quantiles by using n out of n re-sampling scheme, the rate of consistency is slower than the rate obtained by using m out of n bootstrap (further studies concerning the m out of n bootstrap, see e.g. Swanepoel (1986), Bickel et al. (2012), Bickel and Sakov

(2008) and Cheung and Lee (2005)). However, computation of sample quantiles in the tailes (e.g. quantile is close to 0 or 1) is inefficient when sample size is small. In the Monte Carlo simulation, one solution is to use importance sampling for variance reduction. It can apply by generating more samples around a neighbourhood of the interesting quantile. Hult and Svensson (2009) show that design of importance sampling algorithms can significantly improve the inefficiency of the extreme quantiles. In this chapter I apply n out of n bootstrap technique in order to compute the standard errors of quantiles.

Since reviewing the literature on Monte Carlo investigation into GARCH type models which nothing has been done with WSN distribution, it is interesting to examine the behaviour of QMLE with symmetric and asymmetric WSN distributions. The purpose of this chapter is twofold. First, it investigates the finite sample properties of QMLE in AR-GARCH model with symmetric and asymmetric WSN which is defined as misspecified error distribution using a range of mean and volatility persistence as well as correctly specified error terms. Second, this chapter computes quantiles of the simulated finite sample distributions of test statistics and also computes bootstrapped standard errors of the estimated quantiles.

The structure of this chapter is as follows. Section 2 provides an overview of the time series models. Section 3 presents the definition of bootstrap technique, its application in hypothesis testing and computing standard errors. Section 4 briefly describes the simulation design and data generating processes for analysing the finite sample properties and computing the quantiles. Section 7 concludes all of the findings.

2.2 Time series model

In earlier literature, ARCH/GARCH models are employed to analyse the volatilities of financial and economic data. However, according to Li et al. (2001) although the volatilities of the data might be key interest of researchers, the specification of the conditional mean and the estimation of it are still significant. Therefore, many researchers have tried to expand and use these type of models in several applications. One of these extensions is AR-GARCH model.

2.2.1 AR(1)-GARCH(1,1) model

In AR-GARCH model the conditional mean is given as an AR model and the error term in AR process follows a GARCH process. This is the type of model discussed by Li and Li (1996), Ling (1999), Li et al. (2001), and Meitz and Saikkonen (2011). Specifically, the AR(1)-GARCH(1,1) model has the form

$$y_t = \rho y_{t-1} + \varepsilon_t \tag{2.1}$$

$$\varepsilon_t = \sqrt{h_t}\eta_t \qquad h_t = \omega + \alpha\varepsilon_{t-1}^2 + \beta h_{t-1} \tag{2.2}$$

where η_t is a sequence of independently and identically distributed (i.i.d.) random variables with zero mean and unit variance. The parameters in (2.1) and (2.2) consist of two sets: one set includes the parameter of the conditional mean, denoted by θ, and another set includes the parameters of the conditional variance h_t, denoted by λ. Hence, $\theta = (\rho)$, $\lambda = (\omega, \alpha, \beta)'$ are the parameter vectors, with restriction specified by

$$|\rho| < 1 \quad , \quad \omega > 0 \quad , \quad \alpha > 0 \quad , \quad \beta > 0 \tag{2.3}$$

Therefore, $\Theta \subset]-1, 1[\times]0, +\infty[\times]0, +\infty[\times]0, +\infty[$ denotes parameter space. The above condition 2.3 is quite standard in the AR and GARCH literature. Bollerslev (1986) specifies the conditions as $\alpha \geq 0$ and $\beta \geq 0$. However, noticing that when

$\beta = 0$ the model reduces to an ARCH case and when $\alpha = 0$ it becomes rather trivial, I want to exclude these scenarios by using the strict inequality as in 2.3.

2.2.2 Estimation of AR(1)-GARCH(1,1)

Corresponding to the model, let $\{y_t\}$ be a random sample of length T generated by stationary and ergodic process defined by equations 2.1 and 2.2 and let $\vartheta_0 = (\theta_0, \lambda_0) = (\rho_0, \omega_0, \alpha_0, \beta_0)$ denotes the true parameter values which is assumed to be known. The parameter of interest is autoregressive parameter ρ which is estimated by maximum likelihood estimation method. Pantula (1989) shows that the MLE is more efficient than the LSE for the AR model with ARCH(1) errors. Since the conditional error η_t is not assumed to be normal, the resulting estimator is called the quasi-maximum likelihood estimator (QMLE). According to Ling and Li (1997c), the Gaussian quasi-likelihood function of this model(ignoring constants) is given by

$$L_T(\theta) = \frac{1}{T}\sum_{t=1}^{T} l_t(\theta) \quad , \quad l_t(\theta) = -\frac{1}{2}\ln h_t - \frac{\varepsilon_t^2}{2h_t}$$

where h_t is treated as a function of y_t and θ, and is calculated through the following recursion:

$$h_t = \omega + \alpha\varepsilon_{t-1}^2 + \beta h_{t-1}$$

and h_0=a positive constant.

The QMLE is then defined as

$$\hat{\theta} = argminL_T(\theta)$$

Here (θ) is just the argument of the quasi-likelihood function that needed to be minimized to obtain the QMLE.

Weiss (1986) shows that the QMLE is consistent and asymptotically normal under the existence of the finite fourth moment($E(\eta_t^4) < \infty$). From Ling and Li

(1997c), also for the ARMA-GARCH model there exists a locally consistent and asymptotically normal QMLE if it has finite fourth moment. However, Ling and McAleer (2002b) show the global consistency of QMLE for the ARMA-GARCH model under a finite second moment condition($E(\eta_t^2) < \infty$). For the GARCH(1,1) model also Lumsdaine (1996) proves that the quasi-maximum likelihood estimators of the parameters are consistent and asymptotically normal even in the absence of a finite fourth moment.

In order to have the consistent and asymptotically normal QMLE, I follow Francq and Zakoian (2004) and make the following assumptions about the true parameter vector, $\vartheta_0 = (\rho_0, \omega_0, \alpha_0, \beta_0)$, and the distribution of η_t.

1. η_t is a sequence of i.i.d. random variables such that $E[\eta_t] = 0$;

2. η_t^2 has a non-degenerate distribution;

3. $E(\eta_t^4) = \kappa < \infty$ for all t;

4. $\theta_0 \in \Theta$ and Θ is compact;

5. $\alpha_0 + \beta_0 < 1$, the roots of the characteristic polynomial evaluated at the true parameters are outside the unit circle;

6. $Elog\left\{\alpha_0\eta_t^2 + \beta_0\right\} < 0$.

Assumption 2 is necessary to ensure that h_t is not almost surely (a.s.) a constant. When assumption 3 holds the strong consistency and asymptotic normality of its global QMLE were proved by Francq and Zakoian (2004), while the consistency and asymptotic normality of its local QMLE were given by Ling and Li (1997c). Assumption 4 is necessary for the asymptotic normality of QMLE and implies the stationarity, invertibility and identifiability of 2.1. Assumption 5 is sufficient condition to ensure that GARCH process 2.2 has a finite variance. Assumption 6 implies that ε_t in 2.2 is strictly stationary and ergodic and $\varepsilon_t = \sqrt{h_t(\lambda_0)}\eta_t$ is the unique strictly stationary and ergodic solution of the GARCH(1,1) model specified by equations 2.2. Taking a closer look at assumption 6 by Jensen's inequality we know that

$$E \log\left\{\alpha_0\eta_t^2 + \beta_0\right\} \leq \log E\left\{\alpha_0\eta_t^2 + \beta_0\right\} = \log\{\alpha_0 + \beta_0\}$$

Therefore, when $\alpha_0 + \beta_0 < 1$ we have $E \log \{\alpha_0 \eta_t^2 + \beta_0\} < \log 1 = 0$ in which case the GARCH process ε_t is strictly stationary and ergodic.

2.3 Bootstrap Technique

The bootstrap method is a resampling procedure which is designed to approximate the sampling distribution of a statistic of interest. It is becoming more popular method because of its wide application and the high capability of computers. The term "bootstrap" is a English phrase which refers to the process that can proceed from an existing sample without using external samples.

As mentioned in the introduction, Efron (1979) introduced a general re-sampling method that attempts to estimate or approximate the sampling distributions of statistics. In time series literature, when asymptotic theory delivers poor approximations to the finite sample distributions bootstrap appear to be quite useful method to improve small sample properties. Efron and Tibshirani (1985) summarize that one of the important benefits of the bootstrap methodology is that it can answer questions which are too complicated to analyse by traditional statistical methods. MacKinnon (2006) explains that bootstrap method involves estimating a model many times using simulated data. Then quantities computed from the simulated data are used to make inferences from the actual data.

2.3.1 Bootstrap in AR-GARCH model

In regression and time series models I re-sample the residuals which bootstrap procedure is as follows:

First, consider the following AR(1)-GARCH(1,1) model for $t = 1, ..., T$:

$$y_t = \rho y_{t-1} + \varepsilon_t \tag{2.4}$$

$$\varepsilon_t = \sqrt{h_t}\eta_t \qquad \eta_t \sim iid(0,1) \qquad h_t = \omega + \alpha\varepsilon_{t-1}^2 + \beta h_{t-1} \tag{2.5}$$

Given a time series sample $\varepsilon_1, ..., \varepsilon_T$, I estimate the AR(1)-GARCH(1,1) model by means of QMLE, yielding the vector of estimated model parameters $\hat{\vartheta} = (\hat{\rho}, \hat{\omega}, \hat{\alpha}, \hat{\beta})$. Then the residual sample bootstrap consists of the following steps. A random sample of length T is drawn with replacement from the estimated AR-GARCH model's returns $\hat{\eta}_1, ..., \hat{\eta}_T$, yielding $\hat{\eta}_1^*, ..., \hat{\eta}_T^*$. Next $\hat{\vartheta}$ and Equations 2.4-2.5 are used to recursively generate values $\varepsilon_1^*, ..., \varepsilon_T^*$ and then $y_1^*, ..., y_T^*$.

2.3.2 Bootstrap Standard Errors

The bootstrap was primarily suggested as a method for computing standard errors; see Efron (1979) and Engle (1982). It can be valuable for this purpose when other methods are computationally difficult, are unreliable or are not available at all. Therefore, bootstrap technique can be used to avoid the calculation of the estimated asymptotic variance, or in other words, to obtain standard error without programming out the asymptotic variance formula. This can be perhaps the most common reason to use bootstrap in applied work in economics because many economic models are so complicated that computing the standard error analytically using asymptotic distribution is very difficult.

If $\hat{\theta}$ is a parameter estimate, $\hat{\theta}_j^*$ is the corresponding estimate for the j^{th} bootstrap replication, and the $\bar{\theta}^*$ is the mean of the $\hat{\theta}_j^*$, then the bootstrap standard error is

$$s^*(\hat{\theta}) = \left(\frac{1}{B-1} \sum_{j=1}^{B} (\hat{\theta}_j^* - \bar{\theta}^*)^2 \right)^{\frac{1}{2}}$$

This is simply the sample standard deviation of the $\hat{\theta}_j^*$. We can use $s^*(\hat{\theta})$ in the same way as we would use any other asymptotically valid standard error to construct asymptotic confidence intervals or perform asymptotic tests.

2.4 Simulation Studies

In this section, the results from two simulation studies are presented to illustrate the performance of the QMLE under misspecification. The simulations' results are demonstrated across the range of mean and volatility persistence.

2.4.1 Data Generating Process

In the simulations, I generate the data from following process as:

DGP: I consider AR(1) model that residuals of this model follow generalised autoregressive conditional heteroskedasticity process of order (1,1). Thus, GARCH process is presented in residuals and data is generated from AR(1)-GARCH(1,1) model given by

$$y_t = \rho y_{t-1} + \varepsilon_t$$

$$\varepsilon_t = \sqrt{h_t}\eta_t$$

$$h_t = \omega + \alpha\varepsilon_{t-1}^2 + \beta h_{t-1}$$

where η_t is a sequence of independent and identically distributed (i.i.d.) random variable such that $E(\eta_t^2) = \sigma_\eta^2$. h_t is the conditional variance and can be defined as an autoregressive moving average[ARMA(1,1)] process in the innovation ε_t^2. Also, true value of parameters in AR(1)-GARCH(1,1) process is set as explained in the next subsection. In data generating process, three different choices of error distributions are considered:

(i) i.i.d. Normal;

(ii) i.i.d. symmetric WSN-Weighted Skew Normal distribution;

(iii) i.i.d. asymmetric WSN

The symmetric and asymmetric WSN distributions are generated by following set of WSN parameters, P1 and P2 respectively.

P1: $a = -0.6145$, $b = -0.6145$, $m = 1$, $k = -1$, $r = 0.6$

P2: $a = -0.9217$, $b = 0$, $m = 1$, $k = -1$, $r = 0.784$

The first set of WSN parameters (P1) corresponds to symmetric WSN by setting $a = b$ and $m = -k$ where the second set of parameters explain asymmetric WSN. In second case, asymmetry is demonstrated by $b = 0$ and $m = -k$.

It is noteworthy that in DGP, mean and variance of innovations with normal distribution as well as WSN distributions are 0 and 0.7117 respectively. The reason of choosing 0.7117 for the variance is that variance of WSN distribution is equal to 1 only for $a = b = 0$. Therefore, it is impossible to get variance 1 for symmetric distribution with $a = b \neq 0$ as well as asymmetric case. The variance is plotted for aforementioned three cases as shown in Figure 2.1 which the minimum variance is 0.7117. For the sake of comparison I also consider the normally distributed innovations in both DGPs with variance equals to 0.7117. I have plotted the variance of WSN distribution respect to the variation of parameter a which is varying from -1 to 0. I have also assumed the value of parameter r equals to 0.6 and 0.784 in symmetric and asymmetric cases respectively by try and error just in order to find the equal minimum variance (here is 0.7117) in both cases. Then I have found in both cases the optimal value of parameter a corresponds to the minimum variance.

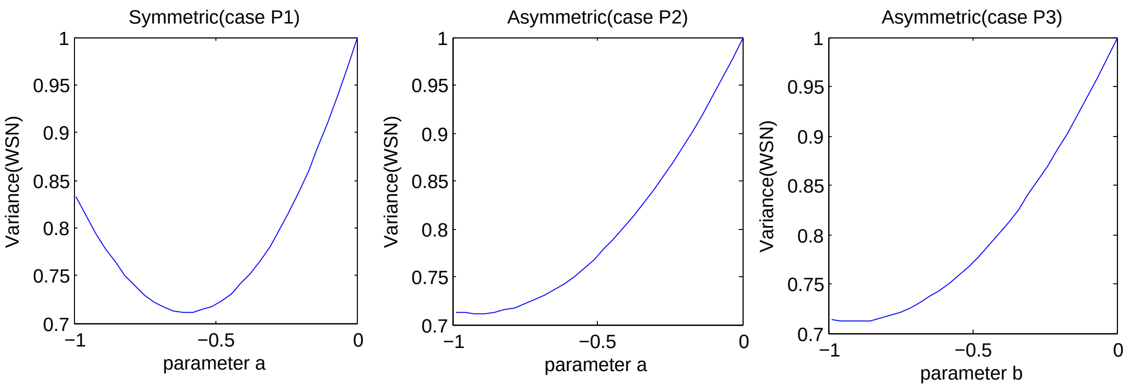

Figure 2.1: Variance of WSN distribution respect to the variation of parameter a

2.4.2 Different set of mean and volatility persistence

I analyse the performance of QMLE under misspecification in where the error distribution is misspecified by symmetric and asymmetric WSN distribution. The analysis is expressed in different scenarios according to the wide range of mean and volatility persistence as follows:

A) $\rho_0 = 0.5$, $\omega_0 = 0.01$, $\alpha_0 = 0.05$, $\beta_0 = 0.8$

B) $\rho_0 = 0.5$, $\omega_0 = 0.01$, $\alpha_0 = 0.05$, $\beta_0 = 0.85$

C) $\rho_0 = 0.5$, $\omega_0 = 0.01$, $\alpha_0 = 0.05$, $\beta_0 = 0.9$

D) $\rho_0 = 0.5$, $\omega_0 = 0.01$, $\alpha_0 = 0.1$, $\beta_0 = 0.8$

E) $\rho_0 = 0.5$, $\omega_0 = 0.01$, $\alpha_0 = 0.1$, $\beta_0 = 0.85$

where in scenarios A, B, C the variance parameter β_0 is increased by 0.05. Regarding the scenarios A and D with the fixed variance parameter $\beta_0 = 0.8$, the mean parameter α_0 is increased from 0.05 to 0.1. Also, comparing two scenarios of B and E with the fixed $\beta_0 = 0.85$ mean parameter is raised from 0.05 to 0.1. Furthermore, two scenarios of A and E describe the change in the mean and variance persistence simultaneously that mean parameter α_0 is increased from 0.05 to 0.1, and variance parameter β_0 is increased from 0.8 to 0.85.

I run the simulation using ALICE High Performance Computing Facility at University of Leicester. I have developed the MATLAB code written by Junhui Qian [1] from maximum likelihood estimation of GARCH(1,1) process, to AR(1)-GARCH(1,1). For maximization of likelihood function I used the constrained optimization algorithm and defined all the parameter restrictions in the QMLE. Therefore, optimization needed to compute the QMLE was performed using MATLAB's *fmincon*[2] function in optimization toolbox with medium-scale "SQP" algorithm[3]. The starting values for the optimization were taken from the true parameter values. The number of bootstrap replication used to calculate the standard errors of criti-

[1] available at www.jhqian.org/ts/index.htm

[2] *fmincon* finds a minimum of a constrained nonlinear multivariable function.

[3] SQP (Sequential Quadratic Programming) algorithm allows to mimic Newton's method for constrained optimization. At each iteration, an approximation is made of the Hessian of the Lagrangian function using a quasi-Newton updating method. An overview of SQP is found in Fletcher (1987) and Gill et al. (1981)

cal values in simulation study 2 is $n = 100, 500, 1000$ respectively for sample sizes $T = 100, 500, 1000$.

2.4.3 Simulation Study 1: Finite Sample Properties of parameter estimators in AR(1)-GARCH(1,1) model

In this study I examine the finite sample properties of the estimator using 5000 Monte Carlo replication. I consider the impact of varying the sample size T among $T = 100, 500, 1000$ in the performance of the estimator of the parameters across the range of mean and volatility persistence. As T increases, we may investigate the convergence of QMLE as well as its distributional properties. I consider the skewness and kurtosis of estimator with making distributional assumption in the model. The following steps are carried out to obtain the bias and RMSE of parameter estimation using Monte Carlo simulation method.

1- Specify the DGP and generate a sample of T

2- Estimate the interest parameter in DGP using maximum likelihood estimation method

3- Repeat steps 1 and 2, M times to obtain $\hat{\vartheta}_1(T), \hat{\vartheta}_2(T), ..., \hat{\vartheta}_M(T)$

4- Compute the bias and RMSE as

$$\bar{\vartheta} = \frac{1}{M}\sum_{i=1}^{M}\hat{\vartheta}_i$$

$$bias = \bar{\vartheta} - \vartheta$$

$$RMSE = \sqrt{\frac{1}{M}\sum_{i=1}^{M}(\hat{\vartheta}_i - \vartheta)^2}$$

5- Compute skewness and kurtosis of estimated parameter in step 3

Tables 2.1-2.5 report the bias and RMSE as well as the skewness and kurtosis of the estimators based on the three different distributional assumptions, and five different set of mean and volatility persistence. Negative bias shows that model parameters are underestimated. The overall convergence trend also is quite apparent from the tables. A pattern of convergence to zero of the bias and RMSE is obvious from the simulations when T increases. It is clear that RMSE decreases steadily as the sample size increases. Therefore, a larger sample size enables us to obtain more accurate estimates, and the estimates will eventually converge to the true value as sample size increases. But, comparing the tables with symmetric and asymmetric WSN distributions, it can be seen that the asymmetric is the innovations, the larger would be the bias and RMSE of the estimators. Moreover, the coefficients of skewness and kurtosis suggest that the empirical distribution of the QML estimator of parameters particularly ARCH and GARCH parameters are different from the symmetrical distribution; the distribution of $\hat{\rho}$ and $\hat{\alpha}$ is always negatively skewed while the skewness coefficient of $\hat{\beta}$ is positive. However, it is convincing from simulations that the distribution of estimators will converge slowly to a Gaussian distribution which skewness and kurtosis are 0 and 3 respectively. In addition, the results of simulation suggest that QMLE is consistent even if the true distribution of the process misspecified with WSN distribution. The basic statistics of estimator with symmetric WSN are very close to normal case. Moreover, under the asymmetric WSN innovations, the rate of convergence is slower in general. This fact indicates that although the asymmetric WSN may not have an impact on the consistency of estimator, it may have an influence on other aspects such as efficiency of estimates, rate of convergence, etc. Also, the results confirm that in the models that residuals follow GARCH process, skewness and kurtosis are higher than the simple AR model. Comparing the simulation results with those of Lange et al. (2011) and Jardet et al. (2009) I found that the performance of QMLE is consistent with their estimators. Lange et al. (2011) examine finite sample properties of modified QMLE (MQMLE) in autoregressive model with autoregressive conditional heteroscedastic errors (AR-ARCH) and normally distributed innovations by selecting the censoring constant to avoid the need for moment restrictions. The performance of their estimator of the autoregressive parameter confirms negative bias and negative skewness in the finite sample distribution of estimator. Jardet et al. (2009) also investigate finite sample and asymptotic properties of OLS estimator in AR(1) model with normally distributed error term. They found the left asymmetry in the distribution of OLS estimator of autoregressive parameter.

2.4.3.1 Comparing a range of volatility persistence

In order to provide clear comparison of how low and high volatility persistence might have an impact on the estimation of parameters in AR(1)-GARCH(1,1) model with misspecified WSN distribution, I compared the results of Tables 2.1-2.3.

According to the results of bias of QMLE of AR parameter ρ, I found that the performance of estimator is quite similar in both normal and symmetric WSN cases using different volatility persistence. However, using asymmetric WSN distribution it is obvious that the higher is the volatility persistence, the smaller would be the bias of AR parameter estimator. Moreover, the results of RMSE show that volatility persistence affects the estimator performance only in the case of asymmetric WSN distribution with large sample size.

According to the results of bias and RMSE in QMLE of ARCH parameter it can be said that in the cases of symmetric and asymmetric WSN distribution with large sample size, as the volatility persistence is getting higher, the bias and RMSE of estimator is getting bigger. However, with small sample size the amount of bias and RMSE is getting smaller. In the case of normal distribution, higher is the volatility persistence, smaller would be the bias and RMSE of QMLE.

According to the results of bias and RMSE in QMLE of GARCH parameter, it is obvious that the same as QMLE of ARCH parameter with large sample size the bias and RMSE of estimator are decreasing as the volatility persistence is increasing. Also, the bias and RMSE are increasing corresponding to the change of volatility persistence from low to high.

Comparing the results of skewness and kurtosis, the empirical distribution of estimated parameters exhibit third and forth moment higher than the normal value of 0 and 3 respectively, which represents the fat tail. Moreover, it can be said that volatility persistence does not have any impact on the skewness and kurtosis of estimator as the amount of these two statistics does not change with volatility persistence movements. However, in the case of estimated ARCH and GARCH parameters with three scenarios of normal, symmetric and asymmetric WSN, the higher is the volatility persistence, the fatter would be the tail of empirical distribution.

2.4.3.2 Comparing a range of mean persistence

I compared the simulation results of Tables 2.1, 2.4 and 2.2, 2.5 to demonstrate how the results change across the range of mean persistence.

According to the results of bias and RMSE, mean persistence does not have any impact on the performance of QMLE of AR parameter in normal and symmetric WSN cases. However, in asymmetric WSN case the amounts of bias and RMSE are decreasing in order to respond to high mean persistence.

According to the results of skewness and kurtosis, mean persistence does not have noticeable impact on the empirical distribution of QMLE of AR parameter using three distributional assumption of normal, symmetric and asymmetric WSN.

According to the results of bias and RMSE in the case of ARCH parameter estimator using normal distribution, I found that the higher is the mean persistence, the smaller would be the bias and RMSE. However, using symmetric WSN distribution the changes of bias and RMSE are in the opposite direction of normal case whereas high mean persistence increases the amount of bias and RMSE. Moreover, in the case of asymmetric WSN within the small sample size, higher mean persistence has a negative effect whereas within the larger sample size the amount of bias and RMSE are increasing, and higher mean persistence has a positive impact on the performance of QMLE.

According to the results of skewness and kurtosis it can be said that the higher is the mean persistence, the smaller would be the skewness and kurtosis of empirical distribution of QMLE of ARCH parameter in all three distributional cases of normal, symmetric and asymmetric WSN.

According to the results of bias and RMSE of GARCH parameter estimator, I noticed that the mean persistence has a negative impact on the performance of QMLE as in all three distributional cases of normal, symmetric and asymmetric WSN, the amount of bias and RMSE are decreasing.

According to the results of skewness and kurtosis, it can be said that the higher is the mean persistence in the model, the greater would be the skewness and kurtosis of the empirical distribution. This fact indicates that the fatter tail of empirical distribution of QMLE of GARCH parameter in all three distributional assumptions of normal, symmetric and asymmetric WSN corresponds to the higher mean persistence.

2.4.3.3 Comparing a range of mean and volatility persistence simultaneously

Simulation results of Tables 2.1 and 2.5 demonstrate the case that mean and volatility persistence change simultaneously.

According to the results of bias and RMSE of AR parameter estimator, it is clear that changing both mean and volatility persistence do not seem to affect the performance of QMLE of AR parameter in the cases of normal and symmetric WSN distribution. However, in the case of asymmetric WSN distribution with small sample sizes, the higher is the mean and volatility persistence, the lower is the bias of AR parameter estimator while the bias is getting larger in large sample size.

According to the results of skewness and kurtosis, it can be observed that empirical distribution of QMLE of AR parameter is not affected by changing in mean and volatility persistence using three distributional assumption of normal, symmetric and asymmetric WSN.

According to the results of bias and RMSE of ARCH parameter estimator, in the case of normal distribution, it is clear that how the bias and RMSE decreases while the mean and volatility persistence increases. However, in the cases of symmetric

WSN, and asymmetric WSN with large sample size I can observe how the higher mean and volatility persistence value, the greater value the bias and RMSE have.

According to the results of skewness and kurtosis of ARCH parameter estimator, it is noted that the value of third and forth moments of estimator's distribution decrease corresponding to the changes of the mean and volatility persistence from low to high.

According to the simulation results of bias and RMSE of GARCH parameter estimator, in all three distributional assumption of normal, symmetric and asymmetric WSN distributions with small sample sizes, the value of bias is not affected by the movement of both mean and volatility persistence, while with large sample size the higher are the mean and volatility persistence, the lower would be the bias and RMSE of the estimator.

According to the simulation results of skewness and kurtosis of GARCH parameter estimator, it can be said that the higher mean and volatility persistence increase the value of third and forth moment of empirical distribution of the estimator.

Table 2.1: Finite sample properties of parameter estimators in AR(1)-GARCH(1,1) model- case A ($\rho_0 = 0.5$, $\omega_0 = 0.01$, $\alpha_0 = 0.05$, $\beta_0 = 0.8$)

			AR	ARCH	GARCH
normal	T=100	Bias	-0.0300	0.0208	-0.1751
		RMSE	2.1218	1.4706	12.3839
		Skewness	-0.3057	1.6389	-0.7580
		Kurtosis	3.1184	6.3537	2.3150
	T=500	Bias	-0.0052	0.0073	-0.1268
		RMSE	0.3659	0.5177	8.9678
		Skewness	-0.1532	0.9209	-1.2875
		Kurtosis	3.0179	4.0440	3.8405
	T=1000	Bias	-0.0028	-0.0040	-0.0805
		RMSE	0.1989	0.2859	5.6920
		Skewness	-0.0755	0.7057	-1.7675
		Kurtosis	3.0490	3.7869	6.1330
symmetric WSN	T=100	Bias	-0.0297	0.0110	-0.1711
		RMSE	2.1010	0.7813	12.965
		Skewness	-0.3345	1.9608	-0.7380
		Kurtosis	3.0292	7.6015	2.2660
	T=500	Bias	-0.0056	-0.0055	-0.1469
		RMSE	0.3981	0.3872	10.3870
		Skewness	-0.1950	1.1581	-1.0684
		Kurtosis	3.0425	4.8242	3.1045
	T=1000	Bias	-0.0026	-0.0090	-0.1131
		RMSE	0.1829	0.6378	8.0001
		Skewness	-0.1559	0.8295	-1.3837
		Kurtosis	3.0050	3.9076	4.1776
asymmetric WSN	T=100	Bias	-0.0291	0.0167	-0.1775
		RMSE	2.0597	1.1818	12.5532
		Skewness	-0.2922	1.9320	-0.7242
		Kurtosis	3.0948	8.6011	2.2416
	T=500	Bias	-0.0037	-0.00054	-0.1414
		RMSE	0.2650	0.0388	9.9955
		Skewness	-0.1688	1.0535	-1.1320
		Kurtosis	3.0011	4.4053	3.3242
	T=1000	Bias	-0.00077	-0.0028	-0.0953
		RMSE	0.0547	0.1952	6.7357
		Skewness	-0.1656	0.7306	-1.5913
		Kurtosis	2.9015	3.8206	5.2322

Table 2.2: Finite sample properties of parameter estimators in AR(1)-GARCH(1,1) model- case B ($\rho_0 = 0.5$, $\omega_0 = 0.01$, $\alpha_0 = 0.05$, $\beta_0 = 0.85$)

			AR	ARCH	GARCH
normal	T=100	Bias	-0.0300	0.0198	-0.2052
		RMSE	2.1247	1.3973	14.5115
		Skewness	-0.3018	1.6756	-0.8726
		Kurtosis	3.1115	6.7174	2.5548
	T=500	Bias	-0.0052	0.0076	-0.1212
		RMSE	0.3650	0.5387	8.5704
		Skewness	-0.1493	0.9765	-1.7332
		Kurtosis	3.0098	4.4431	5.4485
	T=1000	Bias	-0.0028	0.0037	-0.0634
		RMSE	0.1985	0.2646	4.4830
		Skewness	-0.0768	0.7413	-2.5252
		Kurtosis	3.0461	4.1060	10.6759
symmetric WSN	T=100	Bias	-0.0297	0.0104	-0.2117
		RMSE	2.1016	0.7353	14.9678
		Skewness	-0.3303	1.9686	-0.7927
		Kurtosis	3.0278	7.7171	2.3867
	T=500	Bias	-0.0056	-0.0054	-0.1566
		RMSE	0.3967	0.3785	11.0725
		Skewness	-0.1955	1.2012	-1.3161
		Kurtosis	3.0437	5.1831	3.7954
	T=1000	Bias	-0.0026	-0.0090	-0.1081
		RMSE	0.1817	0.6348	7.6447
		Skewness	-0.1567	0.8958	-1.8072
		Kurtosis	3.0034	4.2776	5.8258
asymmetric WSN	T=100	Bias	-0.0290	0.0159	-0.2153
		RMSE	2.0513	1.1278	15.2253
		Skewness	-0.2899	1.9498	-0.7994
		Kurtosis	3.0823	8.6333	2.3887
	T=500	Bias	-0.0036	0.00039	-0.1404
		RMSE	0.2560	0.0277	9.9283
		Skewness	-0.1695	1.1218	-1.4822
		Kurtosis	3.0076	4.8551	4.4344
	T=1000	Bias	-0.00067	-0.0029	-0.0821
		RMSE	0.0475	0.2056	5.8073
		Skewness	-0.1636	0.8257	-2.2221
		Kurtosis	2.9016	4.2961	8.4021

Table 2.3: Finite sample properties of parameter estimators in AR(1)-GARCH(1,1) model- case C ($\rho_0 = 0.5$, $\omega_0 = 0.01$, $\alpha_0 = 0.05$, $\beta_0 = 0.9$)

			AR	ARCH	GARCH
normal	T=100	Bias	-0.0302	0.0166	-0.2268
		RMSE	2.1358	1.1706	16.0351
		Skewness	-0.3056	1.7619	-1.0266
		Kurtosis	3.1102	7.4001	2.9046
	T=500	Bias	-0.0051	0.0060	-0.0887
		RMSE	0.3615	0.4214	6.2733
		Skewness	-0.1538	1.0160	-2.6473
		Kurtosis	3.0099	4.8403	10.3308
	T=1000	Bias	-0.0028	0.0024	-0.0345
		RMSE	0.1967	0.1674	2.4416
		Skewness	-0.0768	0.7340	-4.4561
		Kurtosis	3.0539	4.2722	30.6368
symmetric WSN	T=100	Bias	-0.0296	0.0079	-0.2442
		RMSE	2.0929	0.5573	17.2664
		Skewness	-0.3322	2.0277	-0.8964
		Kurtosis	3.0318	8.1243	2.6194
	T=500	Bias	-0.0056	-0.0061	-0.1429
		RMSE	0.3961	0.4291	10.1041
		Skewness	-0.2003	1.2945	-1.7935
		Kurtosis	3.0516	5.8930	5.4838
	T=1000	Bias	-0.0025	-0.0097	-0.0816
		RMSE	0.1796	0.6856	5.7665
		Skewness	-0.1587	0.9805	-2.6903
		Kurtosis	3.0083	4.7557	10.7038
asymmetric WSN	T=100	Bias	-0.0290	0.0126	-0.2351
		RMSE	2.0491	0.8929	16.6224
		Skewness	-0.2884	1.9797	-0.9588
		Kurtosis	3.0853	8.7899	2.7424
	T=500	Bias	-0.0034	-0.00064	-0.1114
		RMSE	0.2434	0.0459	7.8774
		Skewness	-0.1653	1.2120	-2.1933
		Kurtosis	3.0083	5.6024	7.5403
	T=1000	Bias	-0.00052	-0.0039	-0.0517
		RMSE	0.0369	0.2755	3.6563
		Skewness	-0.1587	1.0143	-3.5967
		Kurtosis	2.8982	5.5841	18.9410

Table 2.4: Finite sample properties of parameter estimators in AR(1)-GARCH(1,1) model- case D ($\rho_0 = 0.5$, $\omega_0 = 0.01$, $\alpha_0 = 0.1$, $\beta_0 = 0.8$)

			AR	ARCH	GARCH
normal	T=100	Bias	-0.0299	0.0100	-0.1666
		RMSE	2.1125	0.7081	11.7811
		Skewness	-0.3002	1.2835	-0.9116
		Kurtosis	3.1230	5.3692	2.7817
	T=500	Bias	-0.0050	0.0037	-0.0549
		RMSE	0.3534	0.2595	3.8812
		Skewness	-0.1408	0.5556	-2.3490
		Kurtosis	2.9977	3.5842	10.5640
	T=1000	Bias	-0.0027	0.00084	-0.0205
		RMSE	0.1914	0.0598	1.4474
		Skewness	-0.0752	0.3204	-2.2162
		Kurtosis	3.0606	3.2391	15.5631
symmetric WSN	T=100	Bias	-0.0297	-0.0138	-0.1741
		RMSE	2.0995	0.9738	12.3130
		Skewness	-0.3313	1.5662	-0.8027
		Kurtosis	3.0504	6.0721	2.4983
	T=500	Bias	-0.0056	-0.0239	-0.0844
		RMSE	0.3963	1.6865	5.9690
		Skewness	-0.1942	0.8128	-1.7519
		Kurtosis	3.0350	4.1966	6.0617
	T=1000	Bias	-0.0025	-0.0265	-0.0427
		RMSE	0.1745	1.8728	3.0161
		Skewness	-0.1601	0.4943	-2.3022
		Kurtosis	2.9895	3.4897	10.8801
asymmetric WSN	T=100	Bias	-0.0274	-0.00092	-0.1765
		RMSE	1.9359	0.0651	12.4818
		Skewness	-0.2938	1.4490	-0.8362
		Kurtosis	3.0995	6.2068	2.5704
	T=500	Bias	-0.0020	-0.0112	-0.0655
		RMSE	0.1391	0.7912	4.6340
		Skewness	-0.1656	0.7426	-2.0639
		Kurtosis	3.0010	4.0771	8.2083
	T=1000	Bias	0.00093	-0.0130	-0.0299
		RMSE	0.0662	0.9184	2.1137
		Skewness	-0.1611	0.4749	-2.4179
		Kurtosis	2.8956	3.6981	13.7964

Table 2.5: Finite sample properties of parameter estimators in AR(1)-GARCH(1,1) model- case E ($\rho_0 = 0.5$, $\omega_0 = 0.01$, $\alpha_0 = 0.1$, $\beta_0 = 0.85$)

			AR	ARCH	GARCH
normal	T=100	Bias	-0.0300	0.0062	-0.1766
		RMSE	2.1181	0.4365	12.4861
		Skewness	-0.3059	1.2810	-1.1685
		Kurtosis	3.1359	5.4738	3.4227
	T=500	Bias	-0.0050	0.0018	-0.0315
		RMSE	0.3533	0.1277	2.2274
		Skewness	-0.1511	0.5257	-3.1657
		Kurtosis	3.0047	3.6932	21.4439
	T=1000	Bias	-0.0027	0.00007	-0.0115
		RMSE	0.1895	0.0055	0.8134
		Skewness	-0.0783	0.2669	-1.4626
		Kurtosis	3.0764	3.0847	9.8310
symmetric WSN	T=100	Bias	-0.0296	0.156	-0.1948
		RMSE	2.0904	1.1029	13.7725
		Skewness	-0.3322	1.6283	-0.9833
		Kurtosis	3.0384	6.5456	2.8688
	T=500	Bias	-0.0056	-0.0245	-0.0649
		RMSE	0.3925	1.7320	4.5875
		Skewness	-0.1930	0.9085	-2.6005
		Kurtosis	3.0387	4.8840	11.1045
	T=1000	Bias	-0.0025	-0.0271	-0.0276
		RMSE	0.1733	1.9184	1.9519
		Skewness	-0.1559	0.8295	-3.5154
		Kurtosis	2.9982	3.8392	24.4687
asymmetric WSN	T=100	Bias	-0.0273	-0.0041	-0.1885
		RMSE	2.1181	0.4365	13.3304
		Skewness	-0.3059	1.2810	-1.0658
		Kurtosis	3.1359	5.4738	3.0936
	T=500	Bias	-0.0016	-0.0122	-0.0462
		RMSE	0.1157	0.8622	3.2683
		Skewness	-0.1624	0.7854	-3.1194
		Kurtosis	3.0048	4.7310	16.6558
	T=1000	Bias	0.0013	-0.0136	-0.0175
		RMSE	0.0891	0.9647	1.2389
		Skewness	-0.1548	0.4301	-2.9176
		Kurtosis	2.8966	3.5099	23.1638

2.4.4 Simulation Study 2: Quantiles of Estimated AR parameter in AR(1)-GARCH(1,1) model

I compute quantiles of the simulated finite sample distribution of test statistic under the null ($\rho = 0$) which might be used as critical values. The data is generated by DGP in 2.4.1 . Simulation design is in the same way as simulation study 1 in 2.4.3, but instead of steps 4 and 5, I compute the quantiles. However the standard error for each quantile is calculated using the bootstrap method. The n out of n bootstrap technique is carried out by simulating T bootstrap samples from the sample of residuals. Then after estimating the parameter in each bootstrap sample and obtaining the t-statistics, quantiles are computed in each Monte Carlo replication. Hence, in order to compute the standard errors of quantiles, I compute the sample standard deviation of each quantile. I report simulation of the quantile estimates, and of the standard errors in parentheses in Tables 2.6-2.10.

The bootstrapped standard errors in the above tables consistent with the fact that the smallest standard error corresponds to the estimation of median. Thus, the standard errors increase in the side of estimated quantiles in which standard errors of estimated quantiles in the tails are bigger than the standard error of estimated median. This result is aligned with the argument concluded by Cuddington and Navidi (2011). They found that there is greater uncertainty associated with the critical values for extreme quantiles. However, aforementioned standard errors are decreasing in right tail quantiles as sample size and consequently bootstrap replication increases. Therefore, the simulation study illustrates the estimation of quantiles with small bootstrapped standard errors and determines that when the model is misspecified by error distribution, the quantiles' estimator can allow us to conduct valid hypothesis test.

Also, I have compared the results of Tables 2.6-2.10 in order to find out the extent to which the impact of range of mean and volatility persistence on quantile estimation of AR parameter $\hat{\rho}$ in the AR-GARCH model. According to the Tables 2.6, 2.7 and 2.8 which correspond to the range of volatility persistence, and Tables

2.6 and 2.9 which correspond to the mean persistence I found that by considering distributional assumptions of normal, symmetric and asymmetric WSN, mean and volatility persistence do not have any noticeable impact on the quantile estimation of AR parameter $\hat{\rho}$.

Table 2.6: Estimated quantiles of $\hat{\rho}$ - Case A ($\rho_0 = 0.5$, $\omega_0 = 0.01$, $\alpha_0 = 0.05$, $\beta_0 = 0.8$)

		0.50	0.90	0.95	0.975	0.99
	T=100	-0.1922	1.0809	1.3646	1.6309	1.8602
		(0.0899)	(0.0967)	(0.1019)	(0.1215)	(0.1295)
Normal	T=500	-0.0587	1.0838	1.5153	1.8947	2.4405
		(0.0801)	(0.0949)	(0.1163)	(0.1227)	(0.1356)
	T=1000	-0.1172	1.2945	1.6890	2.0551	2.5890
		(0.0878)	(0.0958)	(0.1087)	(0.1201)	(0.1326)
	T=100	-0.2285	0.9232	1.1605	1.4132	1.5968
		(0.0806)	(0.0901)	(0.1040)	(0.1122)	(0.1310)
Symmetric WSN	T=500	-0.0875	1.0688	1.3511	1.5968	1.9838
		(0.0809)	(0.0915)	(0.1098)	(0.1236)	(0.1332)
	T=1000	-0.1566	1.1228	1.5297	1.9191	2.3449
		(0.0857)	(0.0932)	(0.1038)	(0.1291)	(0.1378)
	T=100	-0.2603	0.8889	1.2798	1.5381	1.8157
		(0.0845)	(0.0921)	(0.0968)	(0.1174)	(0.1365)
Asymmetric WSN	T=500	-0.0123	1.1372	1.4910	1.8736	1.9676
		(0.0888)	(0.0909)	(0.1097)	(0.1201)	(0.1334)
	T=1000	0.0500	1.3190	1.5826	1.8910	2.1701
		(0.0813)	(0.0996)	(0.1047)	(0.1143)	(0.1393)

Table 2.7: Estimated quantiles of $\hat{\rho}$ - Case B ($\rho_0 = 0.5$, $\omega_0 = 0.01$, $\alpha_0 = 0.05$, $\beta_0 = 0.85$)

		0.50	0.90	0.95	0.975	0.99
Normal	T=100	-0.1911	1.0798	1.3533	1.6446	1.8697
		(0.0856)	(0.0965)	(0.1064)	(0.1237)	(0.1295)
	T=500	-0.0531	1.0685	1.5285	1.8356	2.4273
		(0.0896)	(0.0974)	(0.1104)	(0.1272)	(0.1319)
	T=1000	-0.1107	1.2826	1.6761	2.0616	2.6013
		(0.0864)	(0.0943)	(0.1190)	(0.1241)	(0.1296)
Symmetric WSN	T=100	-0.2387	0.9282	1.1495	1.4279	1.6104
		(0.0881)	(0.0994)	(0.1027)	(0.1150)	(0.1311)
	T=500	-0.0782	1.0555	1.3483	1.5959	1.9724
		(0.0873)	(0.0988)	(0.1083)	(0.1162)	(0.1302)
	T=1000	-0.1491	1.1119	1.5466	1.9192	2.3470
		(0.0831)	(0.0905)	(0.1009)	(0.1173)	(0.1264)
Asymmetric WSN	T=100	-0.2566	0.9085	1.2734	1.5556	1.8511
		(0.0889)	(0.0917)	(0.1089)	(0.1143)	(0.1366)
	T=500	-0.0241	1.1645	1.5382	1.8114	1.9937
		(0.0828)	(0.0952)	(0.1056)	(0.1129)	(0.1298)
	T=1000	0.0523	1.3344	1.5650	1.8727	2.1563
		(0.0876)	(0.0956)	(0.1034)	(0.1197)	(0.1343)

Table 2.8: Estimated quantiles of $\hat{\rho}$ - Case C ($\rho_0 = 0.5$, $\omega_0 = 0.01$, $\alpha_0 = 0.05$, $\beta_0 = 0.9$)

		0.50	0.90	0.95	0.975	0.99
Normal	T=100	-0.1958	1.0936	1.3755	1.6446	1.8749
		(0.0835)	(0.0909)	(0.1079)	(0.1138)	(0.1346)
	T=500	-0.0545	1.0715	1.4724	1.7463	2.3840
		(0.0861)	(0.0910)	(0.0968)	(0.1118)	(0.1261)
	T=1000	-0.1200	1.3039	1.6737	2.0543	2.6057
		(0.0750)	(0.0825)	(0.0856)	(0.1081)	(0.1104)
Symmetric WSN	T=100	-0.2306	0.9414	1.1692	1.4208	1.6498
		(0.0812)	(0.0894)	(0.0937)	(0.1147)	(0.1204)
	T=500	-0.0814	1.0552	1.3512	1.6254	1.9508
		(0.0959)	(0.0998)	(0.1088)	(0.1133)	(0.1211)
	T=1000	-0.1460	1.1329	1.5806	1.9011	2.3397
		(0.0816)	(0.0993)	(0.1048)	(0.1059)	(0.1163)
Asymmetric WSN	T=100	-0.2605	0.9186	1.2399	1.5680	1.7895
		(0.08468)	(0.0923)	(0.0966)	(0.1120)	(0.1396)
	T=500	-0.0237	1.1878	1.5890	1.7805	1.9873
		(0.0771)	(0.0885)	(0.0923)	(0.1157)	(0.1320)
	T=1000	0.0644	1.3597	1.5670	1.8416	2.1514
		(0.0816)	(0.0860)	(0.0950)	(0.1040)	(0.1255)

Table 2.9: Estimated quantiles of $\hat{\rho}$ - Case D ($\rho_0 = 0.5$, $\omega_0 = 0.01$, $\alpha_0 = 0.1$, $\beta_0 = 0.8$)

		0.50	0.90	0.95	0.975	0.99
Normal	T=100	-0.2021	1.0851	1.4109	1.6734	1.8803
		(0.0885)	(0.0967)	(0.1010)	(0.1236)	(0.1358)
	T=500	-0.0444	1.0847	1.5252	1.8053	2.4355
		(0.0868)	(0.0953)	(0.1031)	(0.1179)	(0.1310)
	T=1000	-0.1196	1.2693	1.6205	2.0521	2.5937
		(0.0895)	(0.0964)	(0.1013)	(0.1250)	(0.1342)
Symmetric WSN	T=100	-0.2370	0.9469	1.1754	1.4896	1.6439
		(0.0820)	(0.0991)	(0.0998)	(0.1002)	(0.1279)
	T=500	-0.0650	1.0757	1.3842	1.6516	1.9800
		(0.0905)	(0.0974)	(0.1026)	(0.1158)	(0.1205)
	T=1000	-0.1614	1.1464	1.5538	1.9585	2.3632
		(0.0885)	(0.0958)	(0.1072)	(0.1262)	(0.1363)
Asymmetric WSN	T=100	-0.2366	0.9665	1.3120	1.6496	1.7615
		(0.0860)	(0.0941)	(0.0989)	(0.1105)	(0.1244)
	T=500	0.0169	1.2161	1.6126	1.8966	2.0836
		(0.0912)	(0.0989)	(0.1051)	(0.1165)	(0.1237)
	T=1000	0.0819	1.3796	1.6334	1.9209	2.2948
		(0.0818)	(0.0949)	(0.1048)	(0.1162)	(0.1202)

Table 2.10: Estimated quantiles of $\hat{\rho}$ - Case E($\rho_0 = 0.5$, $\omega_0 = 0.01$, $\alpha_0 = 0.1$, $\beta_0 = 0.85$)

		0.50	0.90	0.95	0.975	0.99
Normal	T=100	-0.1897	1.0850	1.3914	1.6827	1.8929
		(0.0866)	(0.0916)	(0.1160)	(0.1233)	(0.1340)
	T=500	-0.0431	1.0961	1.5142	1.7879	2.3948
		(0.0863)	(0.0958)	(0.1017)	(0.1289)	(0.1323)
	T=1000	-0.1361	1.2476	1.6176	2.0391	2.5695
		(0.0850)	(0.0915)	(0.1057)	(0.1210)	(0.1315)
Symmetric WSN	T=100	-0.2384	0.9487	1.2443	1.4696	1.6394
		(0.0826)	(0.0914)	(0.1118)	(0.1248)	(0.1384)
	T=500	-0.0695	1.0459	1.3852	1.6586	1.9430
		(0.0897)	(0.1013)	(0.1075)	(0.1218)	(0.1327)
	T=1000	-0.1622	1.1788	1.5942	1.9617	2.3804
		(0.0831)	(0.0914)	(0.1092)	(0.1125)	(0.1387)
Asymmetric WSN	T=100	-0.2377	0.9220	1.2542	1.6403	1.8082
		(0.0865)	(0.0947)	(0.1093)	(0.1123)	(0.1387)
	T=500	0.0647	1.2117	1.5473	1.8873	2.0875
		(0.0929)	(0.1068)	(0.1141)	(0.1224)	(0.1376)
	T=1000	0.1075	1.3890	1.6348	1.9317	2.2754
		(0.0809)	(0.1037)	(0.1242)	(0.1261)	(0.1391)

2.5 Conclusion

This chapter has examined the AR(1)-GARCH(1,1) model from the quasi maximum likelihood viewpoint, and presented numerical simulation results regarding the finite sample properties of QMLE for this model considering a range of mean and volatility persistence. Finite sample properties of estimator is examined using Monte Carlo simulation. The impact of different sample sizes as well as various mean and volatility persistence are investigated in the performance of the estimator of the model parameters. The bias, RMSE and distributional properties of estimators are analysed in three cases when the distribution of the error term in the model is normal distribution and when it is misspecified with symmetric and asymmetric WSN distributions. Also, quantiles of the simulated finite sample distribution of test statistic is computed in the case of AR parameter in the model.

Regarding the effect of sample size, in all three cases of different distributional assumption, this study reveals a pattern of convergence to zero of a bias and RMSE when sample size increases. Moreover, the empirical distribution of estimated parameters exhibits skewness and kurtosis higher than the normal value of 0 and 3 respectively, and therefore, represents the fatter tail. However, the results of skewness and kurtosis of empirical distributions change across the range of mean and volatility persistence. I found that with all three distributional assumptions, mean and volatility persistence have no effect on the distributional properties of QMLE of AR parameter in the model. Moreover, in the case of QMLE of ARCH parameter, the higher is the mean persistence, the smaller would be the skewness and kurtosis of empirical distribution of estimator. In contrast, the effect of volatility persistence is in the other direction as higher volatility persistence corresponds to the larger skewness and kurtosis of empirical distribution of QMLE of ARCH parameter. Furthermore, using a range of mean and volatility persistence, the results of skewness and kurtosis of QMLE of GARCH parameter indicate that in all three distributional assumptions of normal, symmetric and asymmetric WSN the higher is the mean and volatility persistence, the fatter would be the tail of empirical distribution of estimator.

Regarding the effect of mean and volatility persistence on the performance of AR parameter estimator, I found that there is no noticeable impact in the cases of normal and symmetric WSN as the amounts of bias and RMSE are not changing according to the range of mean and volatility persistence. However, in asymmetric WSN case the higher is the volatility persistence, the greater would be the bias and RMSE and therefore, less accurate would be the estimate of the parameter. Also, regarding the mean persistence in the case of asymmetric WSN, the higher is the mean persistence, the more accurate would be the estimate of AR parameter.

Regarding the QMLE of ARCH and GARCH parameters, I found that in the cases of symmetric and asymmetric WSN distribution with small sample size, higher volatility persistence corresponds to the smaller bias and RMSE of both parameter estimators. However, in the cases of symmetric WSN as well as asymmetric WSN with large sample size, the higher is the mean persistence, the greater would be the amount of bias and RMSE of ARCH parameter estimator. But, in the case of QMLE of GARCH parameter, with all three distributional assumption of normal, symmetric and asymmetric WSN, I noticed that higher mean persistence results in a smaller bias and RMSE and consequently more accurate estimate of parameter.

In addition, from the simulations I observed that the changes in bias, RMSE, skewness, and kurtosis of QMLE of parameters corresponding to simultaneous mean and volatility persistence are greater than the cases of either mean or volatility persistence.

This chapter also has computed the quantiles of estimated AR parameter in AR(1)-GARCH(1,1) model across a range of mean and volatility persistence by considering distributional assumption of normal, symmetric and asymmetric WSN distributions. Then, the standard error of each quantile is calculated using bootstrap technique. I found that mean and volatility persistence can not significantly affect on the quantiles of estimated AR parameter in the model.

Chapter 3

Empirically comparing the p-values of unit root tests obtained from the Park bootstrap and asymptotics

3.1 Introduction

In recent years the application of bootstrap to non-stationary series has become increasingly popular due to the fact that this technique has a good performance in finite samples for stationary processes. This chapter will consider one specific application of the bootstrap, namely unit root testing. The problem of determining whether a time series contains a unit root has received a great deal of attention in both the statistics and econometrics literature (see e.g., Dickey and Fuller (1979); Said and Dickey (1984); Phillips and Perron (1988); Phillips and Xiao (1998) for a survey).

Bootstrap theories have previously been investigated in unit root models by, among others, Basawa et al. (1991a), Basawa et al. (1991b), Datta (1996), Park (2002), Chang and Park (2003), Park (2003), Paparoditis and Politis (2003) and Smeekes (2006). Most studies of bootstrap methods for unit root tests have been concerned with type I errors and, in particular, the question of whether the sizes of the tests correspond to the nominal levels. Ferretti and Romo (1996) consider a bootstrap unit root test in AR(l) model. Through Monte Carlo experiment they compare the power of the bootstrap test with the previously existing methods like the Ljung-Box statistic, the Dickey-Fuller statistic and the Phillips-Perron statistic. They find that bootstrap test is more powerful for small samples.

Bootstrap tests constitute an attractive approaches rather than the tests based on the asymptotic distributions. Bootstrap procedures are able to take into account the impact of factors such as sample size, initial conditions and error distributions which typically do not affect asymptotic distributions. In the context of hypothesis testing, this implies that the empirical size of bootstrapped tests is in overall closer to the nominal size than that based on asymptotic critical values. Therefore, the bootstrap unit root tests relying on the bootstrap critical values, appear particularly attractive in this respect. Chang et al. (2016) consider parametric bootstrap method to the Covariates Augmented Dickey Fuller (CADF) unit root test which is suggested in Hansen (1995). They compared the finite sample performances of CADF and bootstrap CADF tests with popular univariate unit root tests. They argue that the asymptotic and the finite sample size performances of the CADF test is improved significantly by using bootstrap method. Also, by applying bootstrap CADF to the empirical data they find that compare to the univariate unit root tests, bootstrap CADF can reject the null hypothesis of a unit root for more series. For most of the commonly used unit root tests, the size distortions are known to be large and often too large for the tests to be any reliable. It is now well perceived that the bootstrap, if applied appropriately, helps to compute the empirical distribution of the estimated parameter or test statistics and also critical values more accurately in finite samples. Beran (1988) showed through simulation experiment that the test using bootstrap-based critical value can provide better control over the rejection probability than the test using asymptotic-theory-based critical value(see also Hall (1992) and Horowitz (1994)).

The two popular bootstrap technique for the test of a unit root are the sieve bootstrap and the block bootstrap. The sieve bootstrap introduced by Buhlmann (1997) attempts to approximate the model using a parametric model. However, the block bootstrap methods which is proposed by Kunsch (1989), attempts to use blocks of consecutive observations instead of individual observations in resampling. Palm et al. (2008) study the behaviour of a set of bootstrap unit root tests based on the use of block bootstrap that blocks of residuals are resampled, and sieve bootstrap that fits an AR model to the residuals and then resamples the residuals obtained from this model. They find that sieve tests perform better than block tests in terms of size. They also prefer sieve bootstrap practically as the selection of lag length based on an information criterion is quite easy. But there are no satisfactory methods to choose the block length. Moreover, the sieve bootstrap has been applied to Dickey-Fuller unit root test by Park (2003) and Chang and Park (2003). This method seems to work quite well and they argue that the bootstrap can provide some improvements over the asymptotic.

It has indeed been observed by various authors including Ferretti and Romo

(1996) and Nankervis and Savin (1996) that the bootstrap tests have actual rejection probabilities that are much closer to their nominal values based on the assumption that the innovations are iid in an AR model with one lag. Furthermore, Gospodinov and Tao (2011) propose a bootstrap unit root test in models with GARCH(1,1) errors and study empirically this test as well as bootstrap DF test in U.S. interest rates data. The data used in the analysis include the Federal Funds rate, 3-month Treasury bill rate, 1-, 5- and 10-year Treasury bond yields and the default premium rate. They find that the bootstrap p-values of the DF test provide no evidence against the null of a unit root. However, the bootstrap p-values of their proposed test can reject the null of a unit root at 5% significance level for all interest rates except for the 10-year yield. Moreover, Park (2003) investigates through simulation the effectiveness of the bootstrap method in rejection probabilities. He considers unit root models driven by Gaussian and non-Gaussian innovations. His results are consistent with the results obtained by Ferretti and Romo (1996) and Nankervis and Savin (1996). He suggests that the distributional characteristics of innovations, the sample size and the presence of deterministic trends can affect the magnitude of improvements of rejection probabilities. He finds that bootstrap provides better approximations for the models including time trends and for the samples of small sizes.

Since reviewing the literature on empirically studies of unit root tests obtained from the bootstrap and not finding any studies that consider how different frequencies of empirical data can affect the unit root results, it is interesting to investigate through empirical studies how much evidence against unit root tests in stock market data and exchange rate series can be found by using bootstrap technique proposed in Park (2003). The analysis includes monthly, weekly and daily frequencies of these series and compare the bootstrap method in unit root models including Augmented Dickey-Fuller (ADF), Phillips-Perron (PP), and DF-GLS which was developed by Elliott, Rothenberg and Stock (1996) as a modification of ADF test. I compute the p-values under the null hypothesis of unit root in exchange rate of 30 countries as well as 30 stock market indices.

The rest of the chapter is organized as follows. The model and simulation design of bootstrapped unit root tests are introduced briefly in Section 2. Section 3 describes the empirical results of bootstrap unit root tests. Section 4 concludes.

3.2 Bootstrap unit root tests

In this section three unit root tests including ADF, PP and DF-GLS tests are described in terms of the model and bootstrap simulation design.

3.2.1 Augmented Dickey-Fuller(ADF) test

3.2.1.1 The model

For a time series process y_t without and with deterministic trend, the ADF test is carried out by estimating the following equation for $t = 1, ..., T$ when $H_0 : \rho = 1$,

$$y_t = \rho y_{t-1} + \sum_{i=1}^{p} \rho_i \triangle y_{t-i} + \varepsilon_t \tag{3.1}$$

$$y_t = D_t + \rho y_{t-1} + \sum_{i=1}^{p} \rho_i \triangle y_{t-i} + \varepsilon_t \tag{3.2}$$

where 3.1 defines as the ADF equation without deterministic trend and D_t in 3.2 is the deterministic trend specified as μ or $\mu + \beta t$, $\triangle$ is the difference operator and the augmented terms, $\triangle y_{t-i}$, are included to ensure a lack of serial correlation in the disturbances, ε_t.

Practically, selection of lag length p in ADF test is a major concern. With the selection of too small p, the test will be biased with the remaining serial correlation in the errors. Also, with too large p, the test will lost the power. Said and Dickey (1984) suggested that the order of $T^{\frac{1}{3}}$ is sufficient for the select of lag length. Ng and Perron (1995) proposed to set the upper bound p_{max} for p and estimate the

ADF test regression with $p = p_{max}$. For determination of p_{max}, Ng and Perron (1995) used the rule suggested by Schwert (1989) as $p_{max} = \left[12.\left(\frac{T}{100}\right)^{\frac{1}{4}} \right]$. Other suggestions in the literature include the determination of lag length by using the Akaike Information Criterion (AIC) or the Schwartz Bayesian Criterion(SBC) to insure that residuals in 3.2 is white noise. These order selection criterions set the AR order increasing at a logarithmic rate.

A unit root test is a hypothesis test for testing if y_t is a unit root process based on the observations $y_1, ..., y_T$ and our interest is in the null hypothesis $H_0 : \rho = 1$ which implies the y_t is integrated of order one.

The unit root hypothesis is tested using the t-statistic on ρ in regression 3.1. The parameter ρ is estimated by the ordinary least square (OLS) estimator and according to Park (2003) the test statistic based on $\hat{\rho}$ is the ADF test statistic as

$$F_n = \frac{\hat{\rho} - 1}{\sigma_n \left(\sum_{t=1}^{n} {}_p y_{t-1}^2\right)^{-\frac{1}{2}}}$$

where σ_n^2 is the variance estimator of the regression errors and ${}_p y_{t-1}$ defined as

$${}_p y_{t-1} = y_{t-1} - \left(\sum_{t=1}^{n} y_{t-1} x_{t-1}'\right) \left(\sum_{t=1}^{n} x_{t-1} x_{t-1}'\right)^{-1} x_{t-1}$$

$$x_{t-1} = (\triangle y_{t-1}, ..., \triangle y_{t-p})'$$

The sample distribution of the statistic F_n is unknown and it has the Dickey Fuller type of asymptotic distribution (see, e.g.,Stock (1994) and Park (2003)),

$$F_n \to_d F = \frac{\int_0^1 W_t dW_t}{\left(\int_0^1 W_t^2 dt\right)^{\frac{1}{2}}}$$

where W is standard Brownian motion. When the innovation sequence $\{\varepsilon_t\}$ is independent and identically distributed, the limiting distribution of the statistics are independent of nuisance parameters. Since F does not depend on any nuisance parameter, the asymptotic distribution of F_n applies for ADF as well.

For ADF model with deterministic trends as explained in Chang and Park (2002) the asymptotic distribution is given with demeaned (W_t^{μ}) and detrended (W_t^{β}) Brownian motions in place of standard Brownian motion W as

$$W_t^{\mu} = W_t - \int_0^1 W_s d_s$$

$$W_t^{\beta} = W_t + (6t-4)\int_0^1 W_s d_s - (12t-6)\int_0^1 sW_s d_s$$

3.2.1.2 Design of Bootstrapped ADF test

For implementation of the bootstrap method in the unit root model, I follow the method which is proposed in Park (2003).

1. Fit the regression model $\triangle y_t = D_t + \sum_{i=1}^{p} \rho_i \triangle y_{t-i} + \varepsilon_t$ into the original data and obtain the coefficient estimate ($\hat{\rho}_i$), test statistic F_n and fitted residuals ($\hat{\varepsilon}_t$);

2. Obtain the centred residuals ($\hat{\varepsilon}_t - \frac{\sum_{i=1}^{T} \hat{\varepsilon}_i}{T}$) to get the zero mean of bootstrap samples;

3. Draw the bootstrap samples for the innovations (ε_t^*) from centred residuals;

4. Construct the values for (u_t^*) recursively from (ε_t^*) as

$$u_t^* = \sum_{i=1}^{p} \hat{\rho}_i u_{t-i}^* + \varepsilon_t^*$$

starting from $(u_0, ..., u_{1-p})$ equal zero;

5. Obtain bootstrap samples y_t^* for y_t as

$$y_t^* = y_{t-1}^* + u_t^* = y_0^* + \sum_{i=1}^{t} u_i^*$$

starting from $y_0^* = y_0 = 0$;

6. Calculate the bootstrap test statistic F^* from the bootstrap sample in the same way as F_n is constructed from original data;

7. Set the number of bootstrap replications B and repeat step 3 to step 6 B times

to obtain B bootstrap statistics $\{F_1^*, ..., F_B^*\}$;

8. Calculate the P-value of the bootstrap test as

$$p = \frac{1}{B-1} \sum_{b=1}^{B} I\{F_b^* < F_n\}$$

It is worth to mention that with respect to obtaining the bootstrap residuals in step 2, two different approaches can be distinguished, the difference-based approach and the residual-based approach. In the difference-based approach, the bootstrap errors are obtained with the restriction $\rho = 1$, i.e. $\hat{\varepsilon_t} = y_t - y_{t-1}$. This approach is used in, for example, Psaradakis (2001) and Park (2003). In the residual-based approach, on the other hand, the bootstrap errors are obtained by estimating ρ without restriction, i.e.$\hat{\varepsilon_t} = y_t - \hat{\rho} y_{t-1}$. This approach is used in Paparoditis and Politis (2003). In general, both approaches do not affect the consistency of the bootstrap. However, as argued in Paparoditis and Politis (2005), the residual-based approach will have a better power property, especially when the true DGP is from the alternative hypothesis.

3.2.2 Phillips-Perron(PP) Test

3.2.2.1 The model

Phillips and Perron (1988) developed a generalization of the ADF test procedure to test for the existence of a unit root. PP test is a nonparametric modification to the standard Dickey-Fuller test statistic to account for the autocorrelation of the error terms. While the ADF methodology addresses this issue by adding lagged differenced terms in the regression equation, the PP test makes a correction to the t-statistic of the autoregressive coefficient from the regression model. The unit root is tested with hypothesis $H_0 : \rho = 1$ and $H_1 : \rho < 1$ based on the following AR(1)regression with deterministic trends as:

$$y_t = \rho y_{t-1} + \varepsilon_t \qquad t = 1, ..., T \tag{3.3}$$

$$y_t = D_t + \rho y_{t-1} + \varepsilon_t \qquad t = 1, ..., T \tag{3.4}$$

where D_t is the deterministic trend defined as μ or $\mu + \beta(t - \frac{1}{2}T)$ and $\varepsilon_t \sim (0, \sigma^2_\varepsilon)$. According to Phillips and Perron (1988) consistent estimate of variance of errors (σ^2_ε) is provided by $s^2_\varepsilon = T^{-1}\sum_1^T (y_t - y_{t-1})^2$ under the null hypothesis $\rho = 1$. Also, consistent estimate of the long-run variance or the variance of the sum of errors (σ^2) are provided in the context of weighted variance estimates by Newey and West (1987) as

$$s^2_{Tl} = T^{-1}\sum_{t=1}^{T}\varepsilon^2_t + 2T^{-1}\sum_{s=1}^{l} w_{sl}\sum_{t=s+1}^{T}\varepsilon_t\varepsilon_{t-s}$$

where $w_{sl} = 1 - \dfrac{s}{l+1}$ is nonnegative. They showed that s^2_{Tl} is a consistent estimator of σ^2 if $l \to \infty$ as $T \to \infty$ such that $l = o\left(T^{\frac{1}{4}}\right)$. Therefore, Phillips and Perron (1988) defined the transformation of the t statistic of parameter ρ from regression 3.4 while $D_t = \mu$ as

$$Z(t_{\hat{\rho}}) = (\frac{s_\varepsilon}{s_{Tl}})G_n - \lambda' \frac{s_{Tl}}{\bar{m}^{\frac{1}{2}}_{yy}}$$

and also it is defined with $D_t = \mu + \beta(t - \frac{1}{2}T)$ as

$$Z(t_{\hat{\rho}}) = (\frac{s_\varepsilon}{s_{Tl}})G_n - \lambda' \frac{s_{Tl}}{M^{\frac{1}{2}}}$$

where G_n is the usual regression test statistic for testing the null hypothesis in 3.4 and

$$\lambda' = \frac{(s^2_{Tl} - s^2_\varepsilon)}{2s^2_{Tl}}$$

$$\bar{m}_{yy} = T^{-2}\sum(y_t - \bar{y})^2$$

$$M = (1 - T^{-2})m_{yy} - 12m^2_{ty} + 12(1 + T^{-1})m_{ty}m_y - (4 + 6T^{-1} + 2T^{-2})m^2_y$$

$$m_{yy} = T^{-2}\sum y^2_t$$

$$m_{ty} = T^{\frac{-5}{2}}\sum ty_t$$

$$m_y = T^{\frac{-3}{2}}\sum y_t$$

3.2.2.2 Design of Bootstrapped PP test

With the observations $y_1, ..., y_T$, a bootstrap unit root test can be conducted as follows:

1. Fit the regression model $\triangle\, y_t = D_t + \varepsilon_t$ into the original data and obtain the coefficient estimate $(\hat{\rho})$, test statistic (G_n) and fitted residuals $(\hat{\varepsilon}_t)$;

2. Obtain the centred residuals $(\hat{\varepsilon}_t - \frac{\sum_{i=1}^{T} \hat{\varepsilon}_i}{T})$;

3. Obtain the bootstrap errors, $\varepsilon_1^*, ..., \varepsilon_T^*$ from centred residuals;

4. Construct the bootstrap samples $y_1^*, ..., y_T^*$ with $y_t^* = y_{t-1}^* + \varepsilon_t^*$, starting from $y_0^* = y_0 = 0$;

5. Calculate the bootstrap test statistic G^* from the bootstrap sample in the same way as step 1;

6. Repeat step 3 to step 5 B times to obtain B bootstrap statistics $\{G_1^*, ..., G_B^*\}$;

7. Calculate the p-value of the bootstrap test with $p = \frac{1}{B-1} \sum_{b=1}^{B} I\{G_b^* < G_n\}$

3.2.3 DF-GLS Unit Root Test

In order to improve the Dickey-Fuller type tests, Elliott, Rothenberg and Stock (1996) advocate a local-to-unity GLS detrending procedure. This is the so-called DF-GLS unit root test, that is a DF test (see Dickey and Fuller (1979)) applied to the regression residual, which results from the GLS estimators employed in the original regressions (Vougas (2007)). The reason for considering the DF-GLS test, originally proposed by Elliott, Rothenberg and Stock (1996) is that this test has much smaller size distortions than traditional unit root tests when the errors have strong negative serial correlation.

DF-GLS proceeds by first detrending the series as

$$y_t^d = y_t - \hat{\beta} x_t$$

where $\hat{\beta}$ is obtained by regressing $\bar{y}$ on $\bar{x}$ which is defined as follows,

$$\bar{y} = [y_1, (1 - \rho L)y_2, ..., (1 - \rho L)y_T]$$

$$\bar{x} = [x_1, (1 - \rho L)x_2, ..., (1 - \rho L)x_T]$$

where y_t is the original time series and $\rho = 1 - \frac{c}{T}$. Elliott, Rothenberg and Stock (1996) determine the value of $c = -7$ when $x_t = 1$ in which the local power of the test is close to that of the optimal test and show that DF-GLS test has the same asymptotic distribution as ADF t-test. Also, they determine $c = -13.5$ when $x_t = [1, t]$ which in this case the asymptotic distribution of DF-GLS test is different from the ADF t-test. However, they demonstrate that asymptotic power of DF-GLS test is the same as point optimal test and also is higher than DF t-test against local alternatives.

After detrending the series, the DF-GLS proceeds on the similar way as the traditional ADF test with null hypothesis of a unit root ($H_0 : \rho = 0$) which can be tested in the following regression:

$$\triangle y_t^d = \rho y_{t-1}^d + \sum_{j=1}^{k} \beta_j \triangle y_{t-j}^d + u_t$$

According to Stock (1994) and Park (2003) the asymptotic distribution of local to unity model can be defined as

$$F_n \rightarrow_d F(c) = -c \left(\int_0^1 W_c(t)^2 dt \right)^{\frac{1}{2}} + \frac{\int_0^1 W_c(t) dW(t)}{\left(\int_0^1 W(t)^2 dt \right)^{\frac{1}{2}}}$$

where $W_c(t) = W_t - c \int_0^t e^{-c(t-s)} W(s) ds$ is the Ornstein-Uhlenbeck process which is determined as the solution to the stochastic differential equation $dW_c(t) = -cW_c(t)dt + dW(t)$.

After detrending the series, a bootstrap unit root test can be designed in the similar way as ADF test in 3.2.1.2.

3.3 Empirical results

This chapter investigates the hypothesis of a unit root in stock market indices and exchange rate series. The data is collected from Datastream and consists of monthly, weekly and daily data of 30 major stock indices as well as 30 exchange rate series. The list of indices are AEX, BEL20, CAC 40, DAX30 Performance, Dow Jones Industrials, Euro Stoxx, Euro Stoxx50, FTSE 100, FTSE 250, Hang Seng, IBEX35, MDAX Frankfurt, MSCI Europe, MSCI World, Nasdaq Composite, Nasdaq100, EuroNext100, Next150, Nikkei225 Stock Average, Nyse Composite, OMX Stockholm30, Russell2000, SBF120, S&P 200, S&P 500 Composite, S&P 500 Growth, Stoxx Europe600E, Stoxx Europe50, Swiss Market, and Topix. For the analysis of exchange rate series, the list of countries including: Argentina, Australia, Brazil, Bulgaria, Canada, Chile, China, Colombia, Croatia, Czech Republic, Denmark, Germany, Greece, Hong Kong, Hungary, Iceland, Indonesia, Israel, Japan, Malaysia, Mexico, Norway, Russia, Singapore, South Africa, South Korea, Sweden, Switzerland, Taiwan, and Thailand. The monthly, weekly and daily analysis covers the period of 2009.01.01 to 2016.12.01, 2009.01.01 to 2016.12.15, and 2009.01.01 to 2016.12.16 respectively with the sample size of 96, 416, and 2077.

In this chapter, I focus on the time series properties of the stock indices and exchange rate series, which are drawing more attention among researchers in economics. The purpose of this chapter is to investigate how much evidence against unit roots in these series with different frequencies of monthly, weekly, and daily is found using the bootstrap method. The empirical analysis covers the result of three unit root tests including ADF, PP and DF-GLS. I analyse and compare the results of the both bootstrap and asymptotic p-values based on the proportions statistical test. The results of p-values using three aforementioned unit root tests with drift and trend reported in Tables 3.1 to 3.12. I have selected the lag order of the ADF test using SBC and also for the bandwidth parameter in the PP test I have chosen 12.

In the case of ADF test with drift, according to the Table 3.1 the results of the asymptotic p-values show that the unit root null can be rejected in 9 stock indices (1 at 1%, 3 at 5% , and 5 at 10% significance level) using weekly data. However, the bootstrap p-values represent that the null hypothesis of the unit root can be rejected for 1, 4, and 4 respectively at 1%, 5% and 10% level of significance. In the case of daily stock market data, both asymptotic and bootstrap p-values provide evidence of rejection for 3 out of 30 stock indices. Also, according to Table 3.7 both asymptotic and bootstrap p-values show that the ADF test can reject the null hypothesis for 2 and 3 countries respectively using monthly and weekly exchange rate data. However, using daily exchange rate data, the bootstrap p-values provide evidence of rejecting one more countries relative to the asymptotic p-values.

In the case of ADF test with trend as shown in the Tables 3.2 and 3.8, the bootstrap can provide the significant improvement over the asymptotic in the case of monthly data of stock indices and exchange rate, in which the bootstrap can reject the unit root null of 20 stock indices at 1% significance level and 15 exchange rate series where the asymptotic can not reject the null hypothesis neither in stock market nor in exchange rate series. Furthermore, using weekly stock market and exchange rate data and based on the bootstrap p-values, ADF test provides enough evidence against the unit root hypothesis(13 out of 30), whereas the asymptotic can reject the null of the unit root for 10 and 6 out of 30 series respectively using market indices and exchange rate series. However, the results with daily data show that the bootstrap can provide enough evidence of improvement over the asymptotic only in the case of exchange rate series.

In the case of PP test with drift and using monthly stock indices, the results of Table 3.3 show that according to the asymptotic p-values the unit root null can be rejected in 2 indices, 1 at 10% and 1 at 5% significance level, where using the bootstrap technique only 1 index at 5% significance level is stationary. However, using the monthly stock indices according to the results of Table 3.9 it is possible to reject the null for one more index using the bootstrap relative to the asymptotic.

In the case of PP test with trend the results of Table 3.4 indicate that only in monthly stock data, the bootstrap can make an improvement over the asymptotic as it can reject the null of unit root test in 3 indices whereas based on the asymptotic p-values only in one index the unit root null can be rejected. Furthermore, according to Table 3.10 there is no evidence of improvement using the bootstrap method over the asymptotic in exchange rate data with different frequencies of monthly, weekly, and daily.

In the case of DF-GLS test with drift, the results of the asymptotic and the bootstrap p-values in Table 3.5 demonstrate that using weekly and daily stock indices, there is enough evidence that the bootstrap can reject the null of the unit root in 3 and 2 more indices respectively respect to the asymptotic. However, according to Table 3.11 the number of stationary series using both the asymptotic and the bootstrap p-values is equal(1 and 3 respectively for weekly and daily exchange rate data). Moreover, there is no evidence against the unit root hypothesis in monthly exchange rate data. But, the bootstrap can reject the null of unit root in one country at 10% significance level using monthly data.

In the case of DF-GLS test with trend, the results of Tables 3.6 and 3.12 show that the number of stationary series with different frequencies is the same as the case of DF-GLS test with drift. Also, it is obvious that in the case of weekly stock indices, the power of rejection of the unit root null can be improved significantly as the stationary series at 10% significance level based on the asymptotic p-values is corresponding to the 5% significance level using the bootstrap p-values.

Now, in order to compare the results of the bootstrap and asymptotic p-values I conduct a hypothesis test of two population proportions to determine whether the difference between two proportions of stationary series using the bootstrap and asymptotic p-values is significant. The appropriate test statistic is a z-value defined by the following equation as:

$$H_0 : P_{boot} = P_{asymp}$$

$$H_1 : P_{boot} \neq P_{asymp}$$

$$z = \frac{(\hat{P}_{boot} - \hat{P}_{asymp})}{\sqrt{\frac{\hat{P}_0(1-\hat{P}_0)}{n_1} + \frac{\hat{P}_0(1-\hat{P}_0)}{n_2}}}$$

$$\hat{p}_0 = \frac{n_1\hat{p}_{boot} + n_2\hat{p}_{asymp}}{n_1 + n_2}$$

where $\hat{P}_{boot}$ and $\hat{P}_{asymp}$ are the proportions of stationary series using bootstrap and asymptotic p-values respectively. $\hat{p}_0$ defines the estimate for the common overall proportion. n_1 is the size of sample of the bootstrap p-values and n_2 is the sample size of the asymptotic p-values. In this case $n_1 = n_2 = n = 60$ (30 stock indices and 30 exchange rate series). I calculate the number of stationary series based on the asymptotic and bootstrap p-values of ADF, PP and DF-GLS tests with different data frequencies which are presented in Tables 3.1 to 3.12. I also determine the proportions of stationary series in relation to the bootstrap and the asymptotic p-values. Then, I calculate the test statistic of a z-value as well as a p-value associated with the test statistic using the standard normal table. I can decide whether to accept the null or the alternative hypothesis by examining the p-values. The null hypothesis is rejected at 5% significance level if $p - value \leq 0.05$. It is failed to reject if $p - value > 0.05$.

Table 3.13 shows the results of sample proportions of bootstrap and asymptotic as well as the test statistics and p-values corresponding to ADF, PP and DF-GLS using data frequencies of monthly, weekly and daily. The results reveal that in the case of ADF test with trend using monthly and weekly data, the null hypothesis is rejected at 5% and 10% significance level respectively. Therefore, there is a significant evidence of a difference between the bootstrap and the asymptotic in proportions of stationary series. Hence, the bootstrap technique can provide enough evidence of an improvement over the asymptotic in the case of ADF unit root test with trend using monthly and weekly frequencies of data. In contrast, in the cases of PP and DF-GLS tests the null hypothesis can not be rejected and so the implementation of the bootstrap does not seem to improve the number of stationary series compared to the asymptotic.

However, according to the Table 3.4 and using monthly stock indices data, the empirical results show that the null hypothesis of a unit root is rejected for two more series by the PP test with trend. Moreover, using exchange rate series the results of Table 3.9 show that there is one more stationary series by using the bootstrap technique in the PP with drift unit root test. In the case of DF-GLS test according to Tables 3.5, 3.6, 3.11 and 3.12 the null hypothesis of a unit root is rejected for more series using weekly and daily stock indices as well as monthly exchange rate series. So, there is an improvement of the bootstrap in comparison with the asymptotic in the cases of the PP and DF-GLS unit root tests. Therefore, in order to explain the reason for no improvement of the bootstrap based on the proportions statistical test, I can expect that because of the small number of stationary series corresponding to the PP and DF-GLS tests, the conditions of hypothesis testing for the proportions which are defined by $n\hat{p}_0 \geq 10$ and $n(1-\hat{p}_0) \geq 10$, are not satisfied.

Table 3.1: Stock Market Data-ADF test with drift

	Monthly		Weekly		Daily	
	Asymp p-value	boot p-value	Asymp p-value	boot p-value	Asymp p-value	boot p-value
AEX	0.8548	0.7917	0.1633	0.1418	0.4148	0.4054
Bel20	0.8845	0.7813	0.3689	0.3341	0.4969	0.5046
CAC40	0.8164	0.7917	0.3284	0.2885	0.3353	0.3365
DAX30	0.7106	0.7500	0.3763	0.3413	0.4566	0.4714
DJI	0.4086	0.6250	0.2453	0.2260	0.5859	0.6033
Euro Stoxx	0.7987	0.7500	0.3010	0.2692	0.3536	0.3515
Euro Stoxx50	0.6485	0.5625	0.2395	0.2212	0.2079	0.2051
FTSE100	0.4208	0.5729	0.0151**	0.0072**	0.1400	0.1435
FTSE250	0.6301	0.6771	0.0511***	0.0481**	0.1313	0.1314
Hang Seng	0.2689	0.3646	0.0010*	0.0000*	0.0038*	0.0024*
IBEX35	0.1588	0.1042	0.2961	0.2548	0.2148	0.2104
MDAX	0.5575	0.6979	0.0932***	0.1082***	0.4357	0.4429
MSCI Europe	0.5704	0.5833	0.0177**	0.0120**	0.0586**	0.0510**
MSCI World	0.6580	0.7500	0.0932***	0.0962***	0.2872	0.2624
Nasdaq Composite	0.6930	0.7396	0.2850	0.2500	0.4520	0.4665
Nasdaq100	0.5704	0.6875	0.1740	0.1683	0.3552	0.3587
EuroNext100	0.8467	0.8333	0.3084	0.2861	0.4420	0.4429
Next150	0.8210	0.7708	0.2197	0.1995	0.3354	0.3332
Nikkei225	0.8847	0.8021	0.8207	0.7933	0.8110	0.8122
NYSE Composite	0.6232	0.6875	0.0970***	0.0986***	0.3248	0.3149
OMX Stockholm30	0.5978	0.6250	0.0817***	0.0841***	0.0826***	0.0838***
Russell2000	0.5086	0.5208	0.2337	0.2139	0.4687	0.4699
SBF120	0.8398	0.8333	0.3436	0.3125	0.4013	0.4011
S&P 200	0.7556	0.7396	0.1161	0.1034	0.1537	0.1507
S&P 500	0.6565	0.7396	0.2963	0.2524	0.5671	0.5883
S&P 500 Growth	0.5879	0.7083	0.2733	0.2428	0.5365	0.5561
Stoxx Europe600E	0.7175	0.7500	0.1041	0.1106	0.3251	0.3110
Stoxx Europe50	0.6799	0.7396	0.0553**	0.0673**	0.1863	0.1844
Swiss Market	0.7471	0.6563	0.2618	0.2308	0.4773	0.4752
Topix	0.8373	0.7396	0.8176	0.7909	0.4773	0.4752

* reject at 1% level of significance

** reject at 5% level of significance

*** reject at 10% level of significance

Table 3.2: Stock Market Data-ADF test with trend

	Monthly		Weekly		Daily	
	Asymp p-value	boot p-value	Asymp p-value	boot p-value	Asymp p-value	boot p-value
AEX	0.4910	0.0000*	0.1369	0.1322	0.1631	0.1517
Bel20	0.5650	0.0000*	0.5080	0.4976	0.4985	0.4834
CAC40	0.5602	0.0000*	0.4013	0.4207	0.2471	0.2195
DAX30	0.6059	0.0000*	0.1420	0.1298	0.0871***	0.0896***
DJI	0.8591	0.8750	0.0940***	0.0962***	0.0844***	0.0934***
Euro Stoxx	0.5531	0.0000*	0.4037	0.4303	0.2878	0.2571
Euro Stoxx50	0.5003	0.0000*	0.4651	0.4874	0.2841	0.2513
FTSE100	0.6087	0.6875	0.0379**	0.0361**	0.1264	0.1247
FTSE250	0.4973	0.0104*	0.0524***	0.0553***	0.1263	0.1234
Hang Seng	0.4200	0.0000*	0.0034*	0.0000*	0.0334**	0.0000*
IBEX35	0.4088	0.0000*	0.5644	0.5769	0.4904	0.4762
MDAX	0.5628	0.0000*	0.0450**	0.0433**	0.1714	0.1675
MSCI Europe	0.9112	0.8854	0.1735	0.1635	0.2852	0.2653
MSCI World	0.8323	0.8021	0.1000***	0.0865***	0.1620	0.8469
Nasdaq Composite	0.8213	0.7604	0.0920***	0.0721***	0.0593***	0.0631***
Nasdaq100	0.8512	0.7917	0.0605***	0.0457**	0.0416**	0.0409**
EuroNext100	0.5839	0.0000*	0.3528	0.3606	0.2709	0.2393
Next150	0.5593	0.0000*	0.1360	0.0000*	0.2809	0.0000*
Nikkei225	0.3459	0.0000*	0.6349	0.0024*	0.6188	0.6216
NYSE Composite	0.8077	0.7188	0.0977***	0.0841***	0.1756	0.1825
OMX Stockholm30	0.3627	0.0000*	0.0876***	0.0913***	0.0840***	0.0838***
Russell2000	0.6541	0.5833	0.1079	0.1082	0.1172	0.1156
SBF120	0.5710	0.0000*	0.3539	0.3654	0.2361	0.2099
S&P 200	0.3678	0.0000*	0.1853	0.1971	0.1577	0.1632
S&P 500	0.8812	0.8333	0.1520	0.1442	0.1084	0.1016
S&P 500 Growth	0.9540	0.9375	0.1792	0.1707	0.0841***	0.0838***
Stoxx Europe600E	0.6444	0.0000*	0.1738	0.1587	0.2280	0.2138
Stoxx Europe50	0.6854	0.0000*	0.1712	0.1755	0.1938	0.1873
Swiss Market	0.7578	0.0000*	0.7334	0.7308	0.5573	0.5527
Topix	0.3955	0.0000*	0.5987	0.0000*	0.6582	0.6533

* reject at 1% level of significance
** reject at 5% level of significance
*** reject at 10% level of significance

Table 3.3: Stock Market Data-PP test with drift

	Monthly		Weekly		Daily	
	Asymp p-value	boot p-value	Asymp p-value	boot p-value	Asymp p-value	boot p-value
AEX	0.4295	0.5104	0.4534	0.5000	0.3845	0.3727
Bel20	0.5139	0.5938	0.5397	0.6106	0.5112	0.5238
CAC40	0.4556	0.4792	0.4156	0.4688	0.3343	0.3178
DAX30	0.6092	0.7083	0.6439	0.7115	0.6046	0.6105
DJI	0.7291	0.7396	0.7507	0.7981	0.7109	0.7039
Euro Stoxx	0.4671	0.5208	0.4492	0.5144	0.3913	0.3784
Euro Stoxx50	0.3058	0.3438	0.2772	0.2837	0.2057	0.2022
FTSE100	0.3399	0.4063	0.3321	0.3582	0.2538	0.2378
FTSE250	0.1003***	0.1979	0.1552	0.1707	0.1931	0.1940
Hang Seng	0.0280**	0.0417**	0.0171**	0.0216**	0.0154**	0.0164**
IBEX35	0.1883	0.2292	0.1658	0.1707	0.1527	0.1478
MDAX	0.5228	0.6042	0.6250	0.6947	0.6300	0.6394
MSCI Europe	0.1839	0.1771	0.1317	0.1659	0.1147	0.1141
MSCI World	0.4379	0.4271	0.4532	0.4688	0.4287	0.4160
Nasdaq Composite	0.3824	0.3646	0.5182	0.5625	0.5333	0.5359
Nasdaq100	0.1475	0.1667	0.3422	0.3293	0.4177	0.4131
EuroNext100	0.5300	0.6042	0.5272	0.6106	0.4623	0.4540
Next150	0.3137	0.3438	0.3582	0.3846	0.3641	0.3736
Nikkei225	0.8314	0.8646	0.8428	0.8606	0.8254	0.8190
NYSE Composite	0.4788	0.4896	0.4838	0.5000	0.4518	0.4357
OMX Stockholm30	0.1275	0.1458	0.1822	0.1923	0.1877	0.1834
Russell2000	0.6156	0.5938	0.6234	0.6683	0.5872	0.5912
SBF120	0.5269	0.5833	0.4999	0.5721	0.4298	0.4194
S&P 200	0.2971	0.2813	0.2840	0.3197	0.2094	0.2277
S&P 500	0.6705	0.7083	0.7142	0.7740	0.6849	0.6885
S&P 500 Growth	0.4890	0.5000	0.6148	0.6707	0.6187	0.6351
Stoxx Europe600E	0.3907	0.4375	0.4211	0.4543	0.3848	0.3693
Stoxx Europe50	0.3296	0.3750	0.2934	0.2861	0.2243	0.2162
Swiss Market	0.5227	0.5521	0.5369	0.5841	0.4853	0.4689
Topix	0.8300	0.8542	0.8404	0.8702	0.8337	0.8334

* reject at 1% level of significance
** reject at 5% level of significance
*** reject at 10% level of significance

Table 3.4: Stock Market Data-PP test with trend

	Monthly		Weekly		Daily	
	Asymp p-value	boot p-value	Asymp p-value	boot p-value	Asymp p-value	boot p-value
AEX	02675	0.3229	0.1415	0.1346	0.0940***	0.1011***
Bel20	0.4502	0.5521	0.4110	0.4014	0.3870	0.3601
CAC40	0.2624	0.3125	0.1588	0.1587	0.1367	0.1444
DAX30	0.2330	0.3021	0.0930***	0.0793***	0.0844***	0.0867***
DJI	0.0951***	0.0729***	0.1242	0.1346	0.0866***	0.0871***
Euro Stoxx	0.3448	0.4167	0.2224	0.1947	0.1979	0.1979
Euro Stoxx50	0.3064	0.3958	0.2166	0.1947	0.1810	0.1709
FTSE100	0.4207	0.5208	0.2042	0.2284	0.1263	0.1295
FTSE250	0.3351	0.4167	0.1734	0.2091	0.1373	0.1565
Hang Seng	0.1729	0.2188	0.0950***	0.1298	0.0875***	0.0958***
IBEX35	0.4713	0.5521	0.4319	0.4183	0.4089	0.3722
MDAX	0.5231	0.6042	0.2075	0.2115	0.1933	0.2027
MSCI Europe	0.5184	0.6146	0.3403	0.3822	0.3141	0.3028
MSCI World	0.3336	0.3333	0.2347	0.2813	0.1866	0.1950
Nasdaq Composite	0.1748	0.2083	0.1211	0.1250	0.0641***	0.0727***
Nasdaq100	0.1672	0.1979	0.0827***	0.0721***	0.0369**	0.0414**
EuroNext100	0.3198	0.4063	0.1844	0.1827	0.1514	0.1550
Next150	0.3493	0.4696	0.2486	0.2572	0.2775	0.2730
Nikkei225	0.4942	0.6250	0.5172	0.5457	0.4969	0.5002
NYSE Composite	0.2739	0.2500	0.2262	0.2620	0.1687	0.1748
OMX Stockholm30	0.2995	0.3229	0.1358	0.1298	0.1152	0.1353
Russell2000	0.1985	0.1979	0.1997	0.2139	0.1500	0.1623
SBF120	0.2827	0.3438	0.1576	0.1466	0.1388	0.1497
S&P 200	0.2419	0.2604	0.2616	0.2716	0.1642	0.1738
S&P 500	0.1172	0.0938***	0.1210	0.1394	0.0780***	0.0780***
S&P 500 Growth	0.1180	0.0938***	0.1017	0.1178	0.0460**	0.0462**
Stoxx Europe600E	0.4518	0.5833	0.1957	0.1683	0.1558	0.1598
Stoxx Europe50	0.3993	0.4792	0.1711	0.1683	0.1304	0.1271
Swiss Market	0.6561	0.7500	0.5172	0.5361	0.4344	0.4068
Topix	0.5364	0.6250	0.5329	0.5457	0.5411	0.5455

* reject at 1% level of significance
** reject at 5% level of significance
*** reject at 10% level of significance

Table 3.5: Stock Market Data-DF GLS test with drift

	Monthly		Weekly		Daily	
	Asymp p-value	boot p-value	Asymp p-value	boot p-value	Asymp p-value	boot p-value
AEX	0.8269	0.6250	0.4290	0.3005	0.3425	0.2248
Bel20	0.7287	0.5417	0.3852	0.2452	0.3655	0.2403
CAC40	0.5494	0.3542	0.0591***	0.0409**	0.0425**	0.0250**
DAX30	0.7826	0.5833	0.3521	0.2236	0.2445	0.1502
DJI	0.9017	0.7813	0.3772	0.2644	0.3398	0.2061
Euro Stoxx	0.5562	0.3750	0.1231	0.0721***	0.0782***	0.0496**
Euro Stoxx50	0.3452	0.1979	0.0503**	0.0361**	0.0254**	0.0159**
FTSE100	0.7796	0.5729	0.3810	0.2332	0.2994	0.1767
FTSE250	0.8760	0.7500	0.8426	0.7139	0.8053	0.6827
Hang Seng	0.6381	0.3958	0.4696	0.3005	0.5111	0.3558
IBEX35	0.0192**	0.0208**	0.0804***	0.0505**	0.0354**	0.0226**
MDAX	0.8173	0.6250	0.5375	0.3846	0.5484	0.4073
MSCI Europe	0.5576	0.3333	0.4696	0.3317	0.4092	0.2715
MSCI World	0.8755	0.7083	0.5246	0.3966	0.4555	0.3202
Nasdaq Composite	0.9315	0.8333	0.7821	0.6538	0.6403	0.5176
Nasdaq100	0.9362	0.8333	0.8899	0.7909	0.7903	0.6765
EuroNext100	0.7256	0.5208	0.2633	0.1587	0.1962	0.1213
Next150	0.8545	0.6875	0.6287	0.4976	0.6112	0.4906
Nikkei225	0.4560	0.3646	0.2308	0.1298	0.1708	0.0876***
NYSE Composite	0.8697	0.6979	0.4680	0.3486	0.4252	0.2759
OMX Stockholm30	0.8379	0.6875	0.7718	0.6250	0.6481	0.5123
Russell2000	0.8673	0.7292	0.3340	0.1947	0.3482	0.2287
SBF120	0.6695	0.4792	0.1387	0.0769***	0.0931***	0.0587***
S&P 200	0.6933	0.4583	0.3919	0.2332	0.2940	0.1776
S&P 500	0.8883	0.7396	0.5028	0.3798	0.4400	0.2990
S&P 500 Growth	0.8987	0.8021	0.7083	0.5913	0.6016	0.4766
Stoxx Europe600E	0.8062	0.6458	0.5380	0.4255	0.4479	0.3303
Stoxx Europe50	0.6649	0.4792	0.4142	0.2740	0.2760	0.1738
Swiss Market	0.6223	0.4271	0.4926	0.3389	0.3657	0.2422
Topix	0.2962	0.2292	0.1858	0.0986***	0.1868	0.0992***

* reject at 1% level of significance
** reject at 5% level of significance
*** reject at 10% level of significance

Table 3.6: Stock Market Data-DF GLS test with trend

	Monthly		Weekly		Daily	
	Asymp p-value	boot p-value	Asymp p-value	boot p-value	Asymp p-value	boot p-value
AEX	0.8435	0.6771	0.4520	0.3221	0.3462	0.2277
Bel20	0.7655	0.5521	0.4063	0.2692	0.3690	0.2436
CAC40	0.5817	0.3958	0.0632***	0.0457**	0.0428**	0.0250**
DAX30	0.8218	0.6354	0.3754	0.2524	0.2484	0.1521
DJI	0.9231	0.8021	0.4094	0.2837	0.3444	0.2109
Euro Stoxx	0.5866	0.3958	0.1334	0.0817***	0.0791***	0.0496**
Euro Stoxx50	0.3659	0.1979	0.0531***	0.0361**	0.0255**	0.059**
FTSE100	0.8075	0.5938	0.4028	0.2572	0.3030	0.1796
FTSE250	0.8887	0.7500	0.8530	0.7260	0.8077	0.6875
Hang Seng	0.6481	0.4167	0.4774	0.3077	0.5126	0.3572
IBEX35	0.0198**	0.0208**	0.0808***	0.0505**	0.0354**	0.0226**
MDAX	0.8518	0.6667	0.5687	0.4135	0.5539	0.4121
MSCI Europe	0.5662	0.3333	0.4763	0.3365	0.4105	0.2725
MSCI World	0.8908	0.7188	0.5449	0.4135	0.4590	0.3216
Nasdaq Composite	0.9470	0.8437	0.8066	0.6827	0.6457	0.5234
Nasdaq100	0.9505	0.8750	0.9024	0.80209	0.7960	0.6798
EuroNext100	0.7604	0.5521	0.2831	0.1731	0.1987	0.1233
Next150	0.8705	0.6875	0.6475	0.5144	0.6153	0.4940
Nikkei225	0.4906	0.3958	0.2383	0.1394	0.1718	0.0886***
NYSE Composite	0.8850	0.7083	0.4902	0.3678	0.4289	0.2831
OMX Stockholm30	0.8497	0.7083	0.7868	0.6418	0.6519	0.5185
Russell2000	0.8865	0.7604	0.3625	0.2188	0.3525	0.2306
SBF120	0.7051	0.5313	0.1504	0.0841***	0.0942***	0.0597***
S&P 200	0.7099	0.4688	0.4078	0.2380	0.2970	0.1791
S&P 500	0.9133	0.8125	0.5331	0.4014	0.4449	0.3014
S&P 500 Growth	0.9228	0.8229	0.7376	0.6130	0.6068	0.4800
Stoxx Europe600E	0.8212	0.6458	0.5555	0.4423	0.4510	0.3312
Stoxx Europe50	0.6789	0.4792	0.4273	0.2933	0.2785	0.1762
Swiss Market	0.6424	0.4479	0.5048	0.3534	0.3677	0.2455
Topix	0.3076	0.2396	0.1871	0.0986***	0.1870	0.0992***

* reject at 1% level of significance
** reject at 5% level of significance
*** reject at 10% level of significance

Table 3.7: Exchange rate Data-ADF test with drift

	Monthly		Weekly		Daily	
	Asymp p-value	boot p-value	Asymp p-value	boot p-value	Asymp p-value	boot p-value
Argentina	0.8080	0.6771	0.9519	0.9495	0.9350	0.9355
Australia	0.8691	0.8021	0.5041	0.5048	0.4584	0.4646
Brazil	0.9454	0.9167	0.9299	0.8942	0.9524	0.9514
Bulgaria	0.9077	0.8958	0.8239	0.7644	0.8261	0.8170
Canada	0.8074	0.6563	0.8584	0.8221	0.8652	0.8642
Chile	0.8969	0.8434	0.8691	0.8558	0.8000	0.7949
China	0.4200	0.1875	0.8563	0.8245	0.9453	0.9129
Colombia	0.6938	0.5000	0.9056	0.9135	0.9326	0.9268
Croatia	0.8840	0.8333	0.8273	0.7740	0.8177	0.8031
Czech republic	0.9484	0.9479	0.8659	0.8221	0.7839	0.7800
Denmark	0.8922	0.8646	0.8168	0.7764	0.8121	0.7944
Germany	0.9025	0.9063	0.8270	0.7813	0.8182	0.8084
Greece	0.9068	0.8958	0.8250	0.7957	0.8249	0.8161
Hong Kong	0.3315	0.3438	0.1762	0.1442	0.0154**	0.0135**
Hungary	0.9374	0.9375	0.8331	0.8221	0.7738	0.7891
Iceland	0.0069*	0.0000*	0.2279	0.2476	0.0817**	0.0852**
Indonesia	0.8473	0.7604	0.9282	0.9231	0.9488	0.9446
Israel	0.0906***	0.0313**	0.0277**	0.0264**	0.2373	0.2523
Japan	0.6305	0.5521	0.8345	0.8077	0.8916	0.8931
Malaysia	0.9589	0.9063	0.9508	0.9495	0.9770	0.9803
Mexico	0.9968	0.9792	0.9987	0.9976	0.9914	0.9918
Norway	0.9504	0.9271	0.9465	0.9231	0.9319	0.9283
Russia	0.7921	0.7188	0.8902	0.8582	0.9192	0.9201
Singapore	0.4867	0.4792	0.2966	0.2552	0.3977	0.4174
South Africa	0.8784	0.7188	0.9491	0.9351	0.9554	0.9547
South Korea	0.3213	0.3229	0.0045*	0.0048*	0.0115**	0.025**
Sweden	0.9568	0.9271	0.8475	0.8149	0.8240	0.8291
Switzerland	0.3817	0.3646	0.0943***	0.0841***	0.1081	0.0944***
Taiwan	0.4604	0.3958	0.2236	0.1995	0.2700	0.2465
Thailand	0.7159	0.7396	0.6406	0.6514	0.7180	0.7323

* reject at 1% level of significance
** reject at 5% level of significance
*** reject at 10% level of significance

Table 3.8: Exchange rate Data-ADF test with trend

	Monthly		Weekly		Daily	
	Asymp p-value	boot p-value	Asymp p-value	boot p-value	Asymp p-value	boot p-value
Argentina	0.5216	0.0000*	0.6535	0.6538	0.5970	0.6004
Australia	0.4937	0.5625	0.0956***	0.0962***	0.2600	0.2340
Brazil	0.4494	0.0000*	0.1249	0.0000*	0.1539	0.8479
Bulgaria	0.7599	0.0000*	0.4073	0.0000*	0.3908	0.3818
Canada	0.6139	0.5938	0.0990***	0.0865***	0.2408	0.2340
Chile	0.6711	0.5833	0.4289	0.3966	0.2872	0.8830
China	0.9990	0.9792	0.9990	0.9952	0.9990	0.9494
Colombia	0.6946	0.0000*	0.4283	0.4255	0.7301	0.8960
Croatia	0.5931	0.0000*	0.1647	0.5000	0.2137	0.8811
Czech republic	0.6488	0.6458	0.1721	0.1923	0.1117	0.8050
Denmark	0.7102	0.0000*	0.3837	0.0000*	0.3474	0.0000*
Germany	0.7523	0.0000*	0.4087	0.0000*	0.3662	0.2662
Greece	0.7659	0.0104*	0.4008	0.0000*	0.3766	0.0019*
Hong Kong	0.1590	0.0104*	0.3146	0.2500	0.0444**	0.0371**
Hungary	0.5385	0.0313**	0.0284**	0.0000*	0.1477	0.1526
Iceland	0.0248	0.0000*	0.5659	0.5841	0.2686	0.8946
Indonesia	0.4239	0.0000*	0.0117**	0.0120**	0.2159	0.8782
Israel	0.2551	0.0000*	0.1050	0.0000*	0.5308	0.5460
Japan	0.1848	0.0000*	0.3520	0.7909	0.8041	0.0000*
Malaysia	0.8242	0.7396	0.7718	0.0000*	0.8989	0.8657
Mexico	0.9875	0.9792	0.9277	0.9303	0.8944	0.8994
Norway	0.8727	0.8333	0.6064	0.6322	0.4672	0.4665
Russia	0.5897	0.0000*	0.5644	0.9327	0.5873	0.9008
Singapore	0.7145	0.6667	0.7146	0.7043	0.8505	0.8117
South Africa	0.2097	0.2813	0.0097*	0.0024*	0.0293**	0.0270**
South Korea	0.7403	0.8229	0.0835***	0.0986***	0.0988***	0.8440
Sweden	0.9017	0.8646	0.6648	0.6514	0.7401	0.7415
Switzerland	0.8431	0.7292	0.4910	0.4519	0.4614	0.4588
Taiwan	0.6362	0.5833	0.4778	0.8462	0.5832	0.8926
Thailand	0.3519	0.3229	0.4558	0.6274	0.6695	0.9547

* reject at 1% level of significance
** reject at 5% level of significance
*** reject at 10% level of significance

Table 3.9: Exchange rate Data-PP test with drift

	Monthly		Weekly		Daily	
	Asymp p-value	boot p-value	Asymp p-value	boot p-value	Asymp p-value	boot p-value
Argentina	0.9373	0.8958	0.9316	0.8990	0.9391	0.9336
Australia	0.4278	0.3750	0.4847	0.5144	0.5139	0.5388
Brazil	0.9057	0.8750	0.9401	0.9231	0.9555	0.9523
Bulgaria	0.7054	0.7708	0.9519	0.7716	0.7608	0.7679
Canada	0.8026	0.7708	0.8638	0.8534	0.8531	0.8532
Chile	0.7223	0.7500	0.7677	0.7596	0.7686	0.7761
China	0.6826	0.6875	0.8650	0.8630	0.9293	0.9292
Colombia	0.9190	0.8750	0.9080	0.8966	0.9312	0.9384
Croatia	0.7199	0.7708	0.7424	0.7716	0.7718	0.7694
Czech republic	0.8090	0.8542	0.8044	0.7885	0.7930	0.7785
Denmark	0.6989	0.8500	0.7560	0.7716	0.7725	0.7718
Germany	0.7312	0.7708	0.7921	0.8029	0.8005	0.7944
Greece	0.7328	0.7708	0.7950	0.7957	0.8032	0.7930
Hong Kong	0.0025*	0.0000*	0.0040*	0.0024*	0.0030*	0.0043*
Hungary	0.5941	0.7188	0.5740	0.5962	0.5924	0.6206
Iceland	0.0963***	0.1250	0.0846***	0.0865***	0.0785***	0.0862***
Indonesia	0.9090	0.9063	0.9302	0.9183	0.9557	0.9605
Israel	0.1236	0.0938***	0.1426	0.1466	0.1932	0.2003
Japan	0.7853	0.7708	0.8554	0.8870	0.8886	0.8864
Malaysia	0.9733	0.9688	0.9722	0.9736	0.9808	0.9812
Mexico	0.9972	0.9896	0.9951	0.9928	0.9865	0.9884
Norway	0.8669	0.8333	0.9340	0.9279	0.9222	0.9119
Russia	0.9387	0.9479	0.8629	0.8750	0.9266	0.9206
Singapore	0.4854	0.4792	0.5538	0.5913	0.5803	0.6134
South Africa	0.9385	0.9167	0.9493	0.9519	0.9413	0.9302
South Korea	0.1624	0.0521***	0.1154	0.1250	0.0933***	0.1021
Sweden	0.7944	0.7708	0.8482	0.8389	0.8307	0.8315
Switzerland	0.2481	0.2500	0.1981	0.2067	0.1660	0.1714
Taiwan	0.3855	0.3854	0.3866	0.4207	0.4397	0.4506
Thailand	0.5904	0.6250	0.6548	0.6923	0.7505	0.7684

* reject at 1% level of significance
** reject at 5% level of significance
*** reject at 10% level of significance

Table 3.10: Exchange rate Data-PP test with trend

	Monthly		Weekly		Daily	
	Asymp p-value	boot p-value	Asymp p-value	boot p-value	Asymp p-value	boot p-value
Argentina	0.5473	0.5521	0.5484	0.6058	0.5652	0.5806
Australia	0.4024	0.3750	0.3787	0.4351	0.3846	0.3813
Brazil	0.2050	0.2396	0.1834	0.2188	0.1811	0.1873
Bulgaria	0.3971	0.4167	0.4435	0.4952	0.4863	0.5017
Canada	0.4567	0.4792	0.4259	0.4784	0.4103	0.4237
Chile	0.2366	0.1771	0.2156	0.2019	0.2225	0.2186
China	0.9990	0.9896	0.9990	0.9976	0.9990	0.9995
Colombia	0.8424	0.8229	0.7046	0.7284	0.7198	0.7439
Croatia	0.2776	0.2396	0.3032	0.3341	0.3723	0.3654
Czech republic	0.4203	0.4896	0.3316	0.3582	0.3335	0.3182
Denmark	0.3913	0.4167	0.4474	0.5168	0.5015	0.5094
Germany	0.4095	0.4271	0.4607	0.5240	0.5024	0.5094
Greece	0.4093	0.4375	0.4636	0.5240	0.5056	0.5099
Hong Kong	0.0086*	0.0104*	0.00928	0.0168**	0.0072*	0.0116**
Hungary	0.1507	0.1354	0.1010***	0.1466	0.1504	0.1608
Iceland	0.3308	0.3125	0.2966	0.3510	0.2748	0.2677
Indonesia	0.3970	0.3646	0.3664	0.3606	0.3606	0.3399
Israel	0.3804	0.3646	0.4032	0.4255	0.4788	0.4762
Japan	0.6987	0.7813	0.7389	0.7572	0.8011	0.7978
Malaysia	0.9740	0.9583	0.9359	0.9231	0.9397	0.9345
Mexico	0.9780	0.9583	0.9302	0.9447	0.8704	0.8690
Norway	0.4732	0.4688	0.4729	0.5216	0.4539	0.4405
Russia	0.8145	0.7813	0.6519	0.7043	0.8037	0.7915
Singapore	0.9359	0.9063	0.9324	0.9519	0.9307	0.9297
South Africa	0.1859	0.1458	0.1881	0.2239	0.1824	0.1926
South Korea	0.4421	0.4271	0.4037	0.4111	0.3485	0.3481
Sweden	0.8841	0.9167	0.8588	0.9063	0.8167	0.8223
Switzerland	0.6174	0.6354	0.5541	0.5962	0.5018	0.4935
Taiwan	0.7983	0.7708	0.7292	0.7452	0.7769	0.7848
Thailand	0.7452	0.7917	0.6650	0.7067	0.7046	0.7222

* reject at 1% level of significance
** reject at 5% level of significance
*** reject at 10% level of significance

Table 3.11: Exchange rate Data-DF GLS test with drift

	Monthly		Weekly		Daily	
	Asymp p-value	boot p-value	Asymp p-value	boot p-value	Asymp p-value	boot p-value
Argentina	0.3146	0.3021	0.5265	0.4207	0.4208	0.3317
Australia	0.3819	0.3438	0.5300	0.5433	0.5486	0.5431
Brazil	0.1751	0.1875	0.3343	0.3149	0.3677	0.3062
Bulgaria	0.2222	0.2396	0.2073	0.2115	0.2416	0.2369
Canada	0.1636	0.1250	0.4569	0.4784	0.4740	0.4627
Chile	0.2544	0.1563	0.4096	0.2764	0.4428	0.2975
China	0.2415	0.3229	0.5021	0.4111	0.5119	0.3606
Colombia	0.2183	0.1563	0.4070	0.2236	0.3632	0.2119
Croatia	0.2256	0.1667	0.2328	0.1875	0.2234	0.3191
Czech republic	0.2294	0.1875	0.2857	0.1827	0.3016	0.1955
Denmark	0.2126	0.1771	0.2019	0.1370	0.2373	0.1324
Germany	0.1815	0.2188	0.2035	0.2212	0.2414	0.2167
Greece	0.1851	0.1771	0.2034	0.1490	0.2442	0.1358
Hong Kong	0.3633	0.3021	0.1777	0.1178	0.0302**	0.0221**
Hungary	0.5132	0.4167	0.3120	0.2115	0.2109	0.1358
Iceland	0.1414	0.0833***	0.3018	0.2091	0.0849***	0.0448**
Indonesia	0.1156	0.1354	0.4217	0.2885	0.3989	0.2504
Israel	0.1309	0.1250	0.0161**	0.0096*	0.0999***	0.0655***
Japan	0.1761	0.1875	0.3222	0.2284	0.2880	0.1550
Malaysia	0.3362	0.3021	0.3893	0.3510	0.3724	0.2600
Mexico	0.2766	0.1771	0.3155	0.2452	0.3028	0.1907
Norway	0.2024	0.1667	0.3287	0.2716	0.3861	0.2667
Russia	0.1732	0.2083	0.2701	0.1563	0.3335	0.2460
Singapore	0.4412	0.4063	0.5095	0.5000	0.5165	0.4275
South Africa	0.1130	0.1146	0.3561	0.2716	0.3919	0.2687
South Korea	0.5100	0.3333	0.3976	0.2644	0.3045	0.1892
Sweden	0.3690	0.2500	0.4157	0.3173	0.4045	0.2754
Switzerland	0.4469	0.4479	0.3514	0.3558	0.3346	0.3385
Taiwan	0.4659	0.3438	0.4475	0.3149	0.4386	0.3062
Thailand	0.4223	0.3229	0.4856	0.3101	0.4680	03115

* reject at 1% level of significance
** reject at 5% level of significance
*** reject at 10% level of significance

Table 3.12: Exchange rate Data-DF GLS test with trend

	Monthly		Weekly		Daily	
	Asymp p-value	boot p-value	Asymp p-value	boot p-value	Asymp p-value	boot p-value
Argentina	0.3536	0.3229	0.5492	0.4327	0.4253	0.3341
Australia	0.3836	0.3438	0.5315	0.5433	0.5489	0.5441
Brazil	0.1747	0.1875	0.3343	0.3149	0.3677	0.3062
Bulgaria	0.2483	0.2604	0.2081	0.2115	0.2416	0.2369
Canada	0.1645	0.1250	0.4572	0.4784	0.4741	0.4627
Chile	0.2548	0.1563	0.4099	0.2764	0.4429	0.2975
China	0.2473	0.3229	0.5025	0.411	0.5119	0.3603
Colombia	0.2191	0.1563	0.4070	0.2236	0.3632	0.2191
Croatia	0.2701	0.1875	0.2347	0.1875	0.2235	0.1391
Czech republic	0.2341	0.1875	0.2857	0.1827	0.3015	0.1955
Denmark	0.2371	0.1875	0.2027	0.1370	0.2372	0.1324
Germany	0.1947	0.2188	0.2039	0.2212	0.2413	0.2167
Greece	0.1989	0.1771	0.2037	0.1490	0.2441	0.1358
Hong Kong	0.3771	0.3125	0.1810	0.1178	0.0304**	0.0221**
Hungary	0.6024	0.5208	0.3164	0.2188	0.2114	0.1367
Iceland	0.1434	0.0833***	0.3022	0.2091	0.0850***	0.0448**
Indonesia	0.1155	0.1250	0.4216	0.2885	0.3989	0.2504
Israel	0.1381	0.1458	0.0164**	0.0096*	0.1000***	0.0655***
Japan	0.1755	0.1875	0.3221	0.2260	0.2881	0.1550
Malaysia	0.3360	0.3021	0.3893	0.3510	0.3723	0.2600
Mexico	0.2768	0.1771	0.3156	0.2452	0.3028	0.1907
Norway	0.2016	0.1667	0.3285	0.2692	0.3861	0.2667
Russia	0.1724	0.2083	0.2714	0.1587	0.3339	0.2475
Singapore	0.4477	0.4167	0.5113	0.5000	0.5168	0.4275
South Africa	0.1132	0.1146	0.3562	0.2716	0.3920	0.2687
South Korea	0.5214	0.3542	0.4036	0.2740	0.3055	0.1892
Sweden	0.3686	0.2500	0.4159	0.3173	0.4046	0.2756
Switzerland	0.4587	0.4479	0.3553	0.3606	0.3353	0.3394
Taiwan	0.4751	0.3542	0.4507	0.3149	0.4392	0.3072
Thailand	0.4254	0.3229	0.4865	0.3125	0.4682	0.3115

* reject at 1% level of significance
** reject at 5% level of significance
*** reject at 10% level of significance

Table 3.13: Comparison of the results of the bootstrap and asymptotic p-values based on proportions statistical test

		$\hat{P}_{boot}$	$\hat{P}_{asymp}$	z-value	p-value
	monthly	0.033	0.033	0.0000	1.0000
ADF test-drift	weekly	0.200	0.200	0.0000	1.0000
	daily	0.117	0.100	0.2937	0.7718
	monthly	0.583	0.000	7.0294	0.0000*
ADF test-trend	weekly	0.433	0.267	1.9139	0.0561**
	daily	0.2171	0.167	0.6958	0.4839
	monthly	0.067	0.067	0.0000	1.0000
PP test-drift	weekly	0.050	0.050	0.0000	1.0000
	daily	0.050	0.067	-0.3895	0.6965
	monthly	0.067	0.033	0.8377	0.4009
PP test-trend	weekly	0.050	0.067	-0.3895	0.6965
	daily	0.150	0.150	0.0000	1.0000
	monthly	0.033	0.017	0.5847	0.5619
DF GLS test-drift	weekly	0.117	0.067	0.9491	0.3421
	daily	0.167	0.133	0.5113	0.6100
	monthly	0.033	0.017	0.5847	0.5619
DF GLS test-trend	weekly	0.117	0.067	0.9491	0.3421
	daily	0.167	0.133	0.5113	0.6100

* reject at 5% level of significance

** reject at 10% level of significance

3.4 Conclusion

This chapter attempts to answer the two following questions: Does the use of bootstrap technique in 3 unit root tests including ADF, PP and DF-GLS result in significant support for stationarity of exchange rate and stock market data? How does the difference in data frequency affect the unit root results?

I tested the stationarity of 30 stock indices and 30 exchange rate series, both with the frequencies of monthly, weekly and daily. I used aforementioned three unit root tests with drift and trend. I applied the bootstrap technique proposed by Park (2003) and conducted a hypothesis test based on the proportions statistical test. According to the results of z-value and p-value, I found that in ADF test with trend and data including monthly stock and exchange rate, bootstrap technique can provide enough evidence of improvement over the asymptotic whereas the bootstrap can reject the unit root null of 20 stock indices at 1% significance level and 15 exchange rate series (1 at 5% and 14 at 1% significance level) where the asymptotic can not reject the null hypothesis neither in stock indices nor in exchange rate series. Therefore, the bootstrap technique increases significantly the number of stationary series with monthly and weekly data using ADF unit root test with trend.

Chapter 4

Concluding Remarks

The thesis has made significant contributions in six main areas.

First, it has contributed to the development of the analysis of dynamic models including classical monetary models, focusing on the monetary policy shocks' distribution. It modifies the stochastic assumption of normality of monetary policy shocks' distribution and employs WSN distribution, both symmetric and asymmetric, instead. This is an important insight as research often depends on the distributional assumptions of the shocks. The monetary policy's volatility is analysed considering the monetary policy actions and outcomes. The results revealed the non linearity between policy actions and the volatility of policy shock in both symmetric and asymmetric cases. It also showed that the volatility of policy shock is highly affected by accuracy of inflation's forecast. The higher is the forecast accuracy, the lower would be the volatility of policy shock.

Second, in order to investigate the dynamics of macroeconomic variables in the economy appropriate technique has been derived. Specifically, the impulse response analysis has been investigated using normal, symmetric and asymmetric WSN distributions correspond to the monetary policy shock. The impulse response analysis can be useful in characterising the extent to which shocks can effect the first and second moments of macroeconomic variables.

Regarding the distributional effect of both symmetric and asymmetric policy shocks on macroeconomic variables, I investigated the extent to which the responses of the endogenous variables correspond to the various monetary policy strength and forecast accuracy. The results explained that in both symmetric and asymmetric cases with high forecast accuracy, the percentage change in the volatility of inflation and interest rate becomes small while strength of policy is growing. However, output's volatility is not affected by monetary policy shock, instead the technology shock has an impact on it. In addition, monetary policy shock affects the level of output only in the case that anti-inflationary policy is effective. Also, in the cases of inflation's and interest rate's level, the greater is the strength of monetary policy, the bigger would be the impact of the asymmetric shocks on the variable's mean.

Third, Using stochastic simulation and comparing the impulse responses of macroeconomic variables with respect to the normal, symmetric WSN and asymmetric WSN distributions, I found that the responses of mean of inflation which is increasing with anti-inflationary policy is at odds with the evidence. According to this fact I provided a clear illustration of the role of Cukirman-Meltzer hypothesis in classical monetary model. Therefore, in classical monetary model represented by Gali there is a positive causality from inflation uncertainty to inflation level. Hence, the increases of inflation level when anti-inflationary policy is undertaking might be caused by inflation uncertainty.

Fourth, significant progress has been made in the evaluation of the AR(1)-GARCH(1,1) with distributional misspecification in the model. The finite sample properties of QMLE of model's parameters has been examined using Monte Carlo simulation and considering a range of mean and volatility persistence. The bias, RMSE and distributional properties of estimator including skewness and kurtosis have been analysed when the error term in the model is misspecified with symmetric and asymmetric WSN distribution. The simulation results revealed a pattern of convergence to zero of a bias and RMSE when sample size increases. Also, skewness and kurtosis of parameters' estimator represent the fatter tail of empirical distribution.

Regarding the effect of the range of mean and volatility persistence on the performance of QMLE of parameters, I found that neither distributional properties nor bias and RMSE of QMLE of AR parameter are affected significantly by mean and volatility persistence in the cases of normal and symmetric WSN distributions. However, in asymmetric WSN case the higher is the volatility persistence, the greater would be the bias and RMSE and therefore, less accurate would be the estimate of the AR parameter. Moreover, in the cases of symmetric and asymmetric WSN with small sample size, higher volatility persistence corresponds to the smaller bias and RMSE of QMLE of ARCH and GARCH parameters.

In addition, from the simulations I observed that the changes in bias, RMSE, skewness, and kurtosis of QMLE of parameters corresponding to simultaneous mean and volatility persistence are greater than the cases of either mean or volatility persistence.

Fifth the quantiles of the simulated finite sample distribution of test statistic was computed under the null hypothesis of autoregressive parameter equals to zero to obtain the critical values. These critical values can be used in hypothesis testing which might be addressed in future work. A bootstrap technique offer a practical alternative way to calculate the standard error when the theoretical calculation is difficult. The results of bootstrapped standard errors illustrate the possibility of conducting valid hypothesis test when the model is misspecified by WSN distribution.

Sixth, the bootstrap technique has been applied to the problem of unit root test in order to analyse stock market and exchange rate data. Three unit root tests including ADF, PP, and DF-GLS all with drift and trend have been applied to the empirical data with the frequencies of monthly, weekly, and daily. Comparing the p-values of the unit root tests obtained from the Park bootstrap as well as the asymptotic and applying the proportions statistical test, it can be concluded that bootstrap technique can affect the results of the ADF test with trend using monthly and weekly data. Therefore, the bootstrap can provide enough evidence of an improvement over the asymptotic in the case of ADF test with trend. In contrast, in the cases of PP and DF-GLS tests the null hypothesis of the unit root can not be rejected at 5% significance level and so the implementation of the bootstrap does not seem to improve the number of stationary series compared to the asymptotic.

Appendix A

MATLAB codes-chapter1

```
%% ------------------WSN Distribution----------------%%

clear all
clc
close all

alpha=-0.9;
beta=-0.9;
m=1;
k=-1;
rho=0.9;
i=-4:0.1:4;

f2=zeros(numel(i),1);
X1=zeros(numel(i),1);
X2=zeros(numel(i),1);
X3=zeros(numel(i),1);
X4=zeros(numel(i),1);
X5=zeros(numel(i),1);
X6=zeros(numel(i),1);

A_alpha=1+2*alpha*rho+alpha^2;
A_beta=1+2*beta*rho+beta^2;
B_alpha=alpha+rho;
B_beta=beta+rho;
i_=0;
for t=-4:0.1:4;

    for i_=i_+1;
    X1=t/sqrt(A_alpha);
    X2=(B_alpha*t-m*A_alpha)/(sqrt(A_alpha*(1-rho^2)));
    X3=t/sqrt(A_beta);
    X4=(-B_beta*t+k*A_beta)/(sqrt(A_beta*(1-rho^2)));
    X5=(m-rho*t)/sqrt(1-rho^2);
    X6=(k-rho*t)/sqrt(1-rho^2);
```

```
        X7=t;
        f=(1/sqrt(A_alpha)).*normpdf(X1).*normcdf(X2)+(1/sqrt(
            A_beta)).*normpdf(X3).*normcdf(X4)+normpdf(X7).*(
            normcdf(X5)-normcdf(X6));
        f1=log(f);

        f2(i_,1)=f;

        end
end
plot(i,f2);
```

****

```
%% -------3D Graph of WSN Variance and Skewness in Symmetric
    case-------%%


close all
clear all
clc

n=20;
m=1;
k=-1;
Na=30;
Nr=30;
MZ=zeros(Na,Nr);
VZ=zeros(Na,Nr);
[a_,r_]=meshgrid(linspace(-0.99,0,Na),linspace(0,0.9,Nr));
i_=0;
for a=linspace(-0.99,0,Na),

        b=a;
        home
        i_=i_+1;
        ij=0;

        for r=linspace(0,0.9,Nr),
            ij=ij+1;

        x=randn(n,2);
        c=horzcat(1,r);
        d=horzcat(r,1);
        covar=vertcat(c,d);
        ch=chol(covar);
        x=(ch'*x')';
        z=x(:,1)+a*x(:,2).*gt(x(:,2),m)+b*x(:,2).*le(x(:,2),k);
        %% WSN-moments
        aa=1+2*a*r+a^2;
```

```
        ab=1+2*b*r+b^2;
        ba=a+r;
        bb=b+r;
        ez=a*normpdf(m,0,1)-b*normpdf(k,0,1);
        ez2=aa+(1-aa)*normcdf(m,0,1)+(ba^2-r^2)*m*normpdf(m,0,1)
            +(ab-1)*normcdf(k,0,1)-(bb^2-r^2)*k*normpdf(k,0,1);
        varz=ez2-ez^2;
        ez3=normpdf(m,0,1)*(ba*(3*aa+ba^2*(m^2-1))-r*(3+r^2*(m
            ^2-1)))+normpdf(k,0,1)*(-bb*(3*ab+bb^2*(k^2-1))+r*(3+
            r^2*(k^2-1)));
        skz=(ez3-3*ez2*ez+2*ez^3)/varz^(3/2);


        MZ(ij,i_)=ez;
        VZ(ij,i_)=varz;
        SK(ij,i_)=skz;
        end
end

figure(1)
surf(r_,a_,SK)
xlabel('r')
ylabel('a')
zlabel('Skewness')


figure(2)
surf(r_,a_,VZ)
xlabel('r')
ylabel('alpha*')
zlabel('Variance(WSN)')
```

****

```

%% Plotting macroeconomic variables' mean and variance in
   Normal cases for varying alpha* and constant Phi_pi


clear all
clc
close all


n=10;
Na=5;
NMC=10000;
t=1:n;
sigma1=1;

```

```
vt_ALL=zeros(NMC,n);
at_ALL=zeros(NMC,n);
pit_ALL=zeros(NMC,n);
it_ALL=zeros(NMC,n);
yt_ALL=zeros(NMC,n);

af1=zeros(n+1,1);
pa=zeros(n+1,1);

 for nMC=1:NMC,

   sa=normrnd(0,sigma1,n+1,1);      %normal random
      number
   z1=normrnd(0,sigma1,n+1,1);      %normal random
      number

   af1(2,1)=z1(2,1);
   pa(2,1)=sa(2,1);
   eps_vt=(af1)';
   eps_at=(pa)';

   rho_a=0.5;        % AR coeff
   rho_v=0.5;        % AR coeff
   at(1)=0;          % initial value
   vt(1)=0;          % initial value

   %% Constant Parameters(Baseline Calibration)
   phi=1;
   sigma=1;
   alpha=1/3;
   omega=3;
   phi_pi=1.5;


   %% Parameters in Gali model

   Psi_ya=(phi+1)/(sigma+phi+alpha*(1-sigma));
   Psi_yi=omega*(1-alpha)/(sigma+phi+alpha*(1-sigma));
   Psi=(alpha+phi)/(sigma*(1-alpha)+alpha+phi);
   Theta=(1+omega*Psi*phi_pi)/((1+omega*Psi)*phi_pi);
   A=-(sigma*Psi_ya*(1-rho_a))/(phi_pi*(1+omega*Psi)
      *(1-Theta*rho_a));
   B=-(1+(1-rho_v)*omega*Psi)/(phi_pi*(1+omega*Psi)*(1-
      Theta*rho_v));
   C=-(sigma*Psi_ya*(1-rho_a))/(1+omega*Psi)*(1-Theta*
      rho_a);
   D=-rho_v/(phi_pi*(1+omega*Psi)*(1-Theta*rho_v));
```

```
        E=Psi_ya*(1+(sigma*Psi_yi*(1-rho_a))/(1+omega*Psi)
            *(1-Theta*rho_a));
        F=(rho_v*Psi_yi)/(phi_pi*(1+omega*Psi)*(1-Theta*
            rho_v));

        %% To speed up loop calculations!
        at=zeros(1,numel(t));
        vt=zeros(1,numel(t));
        pit=zeros(1,numel(t));
        it=zeros(1,numel(t));
        yt=zeros(1,numel(t));


        for  iter=2:n+1;
         at(iter)=rho_a*at(iter-1)+eps_at(iter);    %
            technology shock
         vt(iter)=rho_v*vt(iter-1)+eps_vt(iter);    %monetary
             policy shock

         pit(iter)=A*at(iter)+B*vt(iter);        %inflation in
             Gali model
         it(iter)=C*at(iter)+D*vt(iter);         %interest
            rate in Gali model
         yt(iter)=E*at(iter)+F*vt(iter);         %output in
            Gali model


        end

            at=at(2:n+1);
            vt=vt(2:n+1);
            pit=pit(2:n+1);
            it=it(2:n+1);
            yt=yt(2:n+1);

            at_ALL(nMC,:)=at;
            vt_ALL(nMC,:)=vt;
            pit_ALL(nMC,:)=pit;
            it_ALL(nMC,:)=it;
            yt_ALL(nMC,:)=yt;

      end
            M=mean(pit_ALL);        %mean of inflation(change
               it for interest rate and output)
            V=var(pit_ALL);         %variance of inflation(
               change it for interest rate and output)


```

```
subplot(211), plot(M'), ylabel('Inflation mean')
subplot(212), plot(V'), ylabel('Inflation variance')
```

****

```
%% Plotting macroeconomic variables' mean and variance in
   symmetric and asymmetric cases for varying alpha* and
   constant Phi_pi


clear all
clc
close all


n=10;
m=1;
k=-1;
r=0.9;
b=0;                    %asymmetric case
Na=5;
NMC=10000;
t=1:n;
sigma1=1;

for i_n=1:Na
i_=0;
    for a=linspace(0,-0.99,Na)
  %  b=a                           %symmetric case%
    home
    i_=i_+1;

     vt_ALL=zeros(NMC,n);
     at_ALL=zeros(NMC,n);
     pit_ALL=zeros(NMC,n);
     it_ALL=zeros(NMC,n);
     yt_ALL=zeros(NMC,n);

     af1=zeros(n+1,1);
     pa=zeros(n+1,1);

       for nMC=1:NMC,
    %% WSN random number
         randn('seed',nMC+i_);
         x=randn(n,2);
         c=horzcat(1,r);
         d=horzcat(r,1);
         covar=vertcat(c,d);
         ch=chol(covar);
```

```
x=(ch'*x')';
z=x(:,1)+a*x(:,2).*gt(x(:,2),m)+b*x(:,2).*le(x(:,2),k);

sa=normrnd(0,sigma1,n+1,1);       %normal random number

af1(2,1)=z(2,1);
pa(2,1)=sa(2,1);
eps_vt=(af1)';
eps_at=(pa)';

rho_a=0.5;          %  AR coeff
rho_v=0.5;          %  AR coeff
at(1)=0;            % initial value
vt(1)=0;            % initial value

%% Constant Parameters(Baseline Calibration)
phi=1;
sigma=1;
alpha=1/3;
omega=3;
phi_pi=1.5;


%% Parameters in Gali model

Psi_ya=(phi+1)/(sigma+phi+alpha*(1-sigma));
Psi_yi=omega*(1-alpha)/(sigma+phi+alpha*(1-sigma));
Psi=(alpha+phi)/(sigma*(1-alpha)+alpha+phi);
Theta=(1+omega*Psi*phi_pi)/((1+omega*Psi)*phi_pi);
A=-(sigma*Psi_ya*(1-rho_a))/(phi_pi*(1+omega*Psi)*(1-Theta*rho_a));
B=-(1+(1-rho_v)*omega*Psi)/(phi_pi*(1+omega*Psi)*(1-Theta*rho_v));
C=-(sigma*Psi_ya*(1-rho_a))/(1+omega*Psi)*(1-Theta*rho_a);
D=-rho_v/(phi_pi*(1+omega*Psi)*(1-Theta*rho_v));
E=Psi_ya*(1+(sigma*Psi_yi*(1-rho_a))/(1+omega*Psi)*(1-Theta*rho_a));
F=(rho_v*Psi_yi)/(phi_pi*(1+omega*Psi)*(1-Theta*rho_v));

%% To speed up loop calculations!
at=zeros(1,numel(t));
vt=zeros(1,numel(t));
pit=zeros(1,numel(t));
it=zeros(1,numel(t));
```

```
            yt=zeros(1,numel(t));


            for   iter=2:n+1;
             at(iter)=rho_a*at(iter-1)+eps_at(iter);    %
                technology shock
             vt(iter)=rho_v*vt(iter-1)+eps_vt(iter);    %monetary
                 policy shock

             pit(iter)=A*at(iter)+B*vt(iter);       %inflation in
                 Gali model
             it(iter)=C*at(iter)+D*vt(iter);        %interest
                rate in Gali model
             yt(iter)=E*at(iter)+F*vt(iter);        %output in
                Gali model


            end

                at=at(2:n+1);
                vt=vt(2:n+1);
                pit=pit(2:n+1);
                it=it(2:n+1);
                yt=yt(2:n+1);

                at_ALL(nMC,:)=at;
                vt_ALL(nMC,:)=vt;
                pit_ALL(nMC,:)=pit;
                it_ALL(nMC,:)=it;
                yt_ALL(nMC,:)=yt;

          end
                M=mean(yt_ALL);       %mean of inflation(change
                   it for interest rate and output)
                V=var(yt_ALL);        %variance of inflation(
                   change it for interest rate and output)

                 M1(i_,:)=M;
                 V1(i_,:)=V;

    end



end

subplot(211), plot(M1'), ylabel('Output mean')
subplot(212), plot(V1'), ylabel('Output variance')
```

****

```
%% Long run response of mean of inflation

clear all
clc
close all

n=12;
m=1;
k=-1;
r=0.9;
Na=5;
NMC=10000;
t=1:n;
sigma1=1;
sigma2=1:0.5:6.5;

for iii=1:n;

for i_n=1:Na
i_=0;
for a=linspace(0,-0.99,Na)
      b=a;
      home
      i_=i_+1;

 vt_ALL=zeros(NMC,n);
 at_ALL=zeros(NMC,n);
 pit_ALL=zeros(NMC,n);
 it_ALL=zeros(NMC,n);
 yt_ALL=zeros(NMC,n);

 af1=zeros(n+1,1);
 pa=zeros(n+1,1);

for nMC=1:NMC,

randn('seed',nMC+i_);
x=normrnd(0,sigma2(iii),n,2);
c=horzcat(1,r);
d=horzcat(r,1);
covar=vertcat(c,d);
ch=chol(covar);
x=(ch'*x')';
z=x(:,1)+a*x(:,2).*gt(x(:,2),m)+b*x(:,2).*le(x(:,2),k);

sa=normrnd(0,sigma1,n+1,1);
af1(2,1)=z(2,1);
```

```
pa(2,1)=sa(2,1);

eps_vt=(af1)';
eps_at=(pa)';

rho_a=0.5;    % AR coeff
rho_v=0.5;    % AR coeff
at(1)=0;      % initial value
vt(1)=0;      % initial value

%% Constant Parameters (Baseline Calibration)
phi=1;
sigma=1;
alpha=1/3;
omega=3;
phi_pi=1.5;

%% fixed parameters in Gali model

Psi_ya=(phi+1)/(sigma+phi+alpha*(1-sigma));
Psi_yi=omega*(1-alpha)/(sigma+phi+alpha*(1-sigma));
Psi=(alpha+phi)/(sigma*(1-alpha)+alpha+phi);
Theta=(1+omega*Psi*phi_pi)/((1+omega*Psi)*phi_pi);
A=-(sigma*Psi_ya*(1-rho_a))/(phi_pi*(1+omega*Psi)*(1-Theta*rho_a));
B=-(1+(1-rho_v)*omega*Psi)/(phi_pi*(1+omega*Psi)*(1-Theta*rho_v));
C=-(sigma*Psi_ya*(1-rho_a))/(1+omega*Psi)*(1-Theta*rho_a);
D=-rho_v/(phi_pi*(1+omega*Psi)*(1-Theta*rho_v));
E=Psi_ya*(1+(sigma*Psi_yi*(1-rho_a))/(1+omega*Psi)*(1-Theta*rho_a));
F=(rho_v*Psi_yi)/(phi_pi*(1+omega*Psi)*(1-Theta*rho_v));


%% To speed up loop calculations!
at=zeros(1,numel(t));
vt=zeros(1,numel(t));
pit=zeros(1,numel(t));
it=zeros(1,numel(t));
yt=zeros(1,numel(t));


for  iter=2:n+1;
     at(iter)=rho_a*at(iter-1)+eps_at(iter);      %technology shock
     vt(iter)=rho_v*vt(iter-1)+eps_vt(iter);      %monetary policy shock

```

```
        pit(iter)=A*at(iter)+B*vt(iter);                     %inflation in Gali model
        it(iter)=C*at(iter)+D*vt(iter);                      %interest rate in Gali model
        yt(iter)=E*at(iter)+F*vt(iter);                      %output in Gali model


end

at=at(2:n+1);
vt=vt(2:n+1);
pit=pit(2:n+1);
it=it(2:n+1);
yt=yt(2:n+1);

at_ALL(nMC,:)=at;
vt_ALL(nMC,:)=vt;
pit_ALL(nMC,:)=pit;
it_ALL(nMC,:)=it;
yt_ALL(nMC,:)=yt;

end
      M=mean(pit_ALL);
      M1(i_,:)=M;

end

end
M11=sum(M1');
p_bar(:,iii)=M11';
end

plot(sigma2,p_bar')
title('Long run mean of Inflation')
xlabel('Uncertainty')
ylabel('inflation mean')
```

****

Appendix B

MATLAB codes-chapter2

```
%% plotting WSN variance with different parameter 'a' values in symmetric case when a=b
%% finding optimal 'a' corresponding to minimum variance

clear all
clc
close all

%% WSN parameters
n=100;
m=1;
k=-1;
r=0.6;
Na=30;

i_=0;
for a=linspace(-0.99,0,Na),
    b=a
    home
    i_=i_+1


%%WSN random number
x=randn(n,2);
c=horzcat(1,r);
d=horzcat(r,1);
covar=vertcat(c,d);
ch=chol(covar);
x=(ch'*x')';
z=x(:,1)+a*x(:,2).*gt(x(:,2),m)+b*x(:,2).*le(x(:,2),k);

%%WSN variance
aa=1+2*a*r+a^2;
ab=1+2*b*r+b^2;
ba=a+r;
```

```
bb=b+r;
ez=a*normpdf(m,0,1)-b*normpdf(k,0,1);
ez2=aa+(1-aa)*normcdf(m,0,1)+(ba^2-r^2)*m*normpdf(m,0,1)+(ab-1)*normcdf(k,0,1)-(bb^2-r^2)*k*normpdf(k,0,1);
varz=ez2-ez^2;

VZ(i_,:)=[a varz];

end

MIN=find(min(VZ(:,2))==VZ(:,2));    %minimum variance%
a_best=VZ(MIN,1);                   %parameter a corresponding to minimum variance%
plot(VZ(:,1),VZ(:,2))
xlabel('parameter a')
ylabel('Variance(WSN)')
```

****

```
%% plotting WSN variance with different parameter 'a' values in asymmetric case when b=0
%% finding optimal 'a' corresponding to minimum variance

clear all
clc
close all

%% WSN parameters
n=100;
m=1;
k=-1;
r=0.784;
b=0;
Na=30;

i_=0;
for a=linspace(-0.99,0,Na),
    home
    i_=i_+1


%%WSN random number
x=randn(n,2);
c=horzcat(1,r);
d=horzcat(r,1);
covar=vertcat(c,d);
ch=chol(covar);
x=(ch'*x')';
z=x(:,1)+a*x(:,2).*gt(x(:,2),m)+b*x(:,2).*le(x(:,2),k);

```

```
%%WSN variance
aa=1+2*a*r+a^2;
ab=1+2*b*r+b^2;
ba=a+r;
bb=b+r;
ez=a*normpdf(m,0,1)-b*normpdf(k,0,1);
ez2=aa+(1-aa)*normcdf(m,0,1)+(ba^2-r^2)*m*normpdf(m,0,1)+(ab
    -1)*normcdf(k,0,1)-(bb^2-r^2)*k*normpdf(k,0,1);
varz=ez2-ez^2;

VZ(i_,:)=[a varz];

end

MIN=find(min(VZ(:,2))==VZ(:,2));   %minimum variance%
a_best=VZ(MIN,1);                   %parameter a corresponding
    to minimum variance%
plot(VZ(:,1),VZ(:,2))
xlabel('parameter a')
ylabel('Variance(WSN)')
```

****

```
%%Simulation study 1

clear all
clc
close all

 a=0.5;     %AR coefficient
 n=100;     %Sample size
 m=n+1;
 m1=1;
 k=-1;
 r=0.6;
 a1=-0.6145;
 b1=-0.6145;
 %b1=0;
 nmc=5000;    %number of Monte Carlo replication
 X_all=zeros(n+1,nmc);
 e_all=zeros(n,nmc);
 w_all=zeros(n,nmc);
 a_hat_all=zeros(1,nmc);
 alpha_hat_all=zeros(1,nmc);
 beta_hat_all=zeros(1,nmc);


 for NMC=1:nmc;
 %% WSN random numbers for case 'skewed'
 randn('seed',NMC);
```

```
x1=randn(n+1,2);
c1=horzcat(1,r);
d=horzcat(r,1);
covar=vertcat(c1,d);
ch=chol(covar);
x1=(ch'*x1')';
z=x1(:,1)+a1.*x1(:,2).*gt(x1(:,2),m1)+b1*x1(:,2).*le(x1
    (:,2),k);


%% GARCH induced by normal and wsn
n1 = 2;
cc = 0.01; aa = 0.05; bb = 0.8;
w = zeros(m-1,1);
eps = zeros(m,1);

SELECT_MOD='garch_wsn';

            switch SELECT_MOD
                case 'garch_normal'
                    eps=randn(m,1);

                case 'garch_wsn'
                    eps=z;

            end



sigma2 = zeros(m,1);   %
sigma2(1) = cc/(1-aa-bb);

for ii=2:m       %
    sigma2(ii) = cc + aa*w(ii-1)^2 + bb * sigma2(ii-1);
    w(ii) = sqrt(sigma2(ii)) * eps(ii);
end

w=w(2:m);
x=w;
e=w;

%% AR simulation

X = zeros(n+1,1);
X(1) = 0;

for i = 2:n
```

```
    X(i) = e(i) + a'*X(i-1);

end
%% Estimation of AR(1)-GARCH(1,1) model
Mdl=garchset('R',1,'P',1,'Q',1,'Display','off');
[Coeff,Errors]=garchfit(Mdl,X);

a_hat=Coeff.AR;           %estimated AR parameter%
alpha_hat=Coeff.ARCH;     %estimated ARCH parameter%
beta_hat=Coeff.GARCH;     %estimated GARCH parameter%

X_all(:,NMC)=X;
e_all(:,NMC)=e;
a_hat_all(:,NMC)=a_hat;
alpha_hat_all(:,NMC)=alpha_hat;
beta_hat_all(:,NMC)=beta_hat;


 end

a_bar=sum(a_hat_all)/nmc;
bias1=a_bar-a;
RMSE1=sqrt((sum(a_hat_all-a)^2)/nmc);
S1=skewness(a_hat_all);
K1=kurtosis(a_hat_all);

alpha_bar=sum(alpha_hat_all)/nmc;
bias2=alpha_bar-aa;
RMSE2=sqrt((sum(alpha_hat_all-aa)^2)/nmc);
S2=skewness(alpha_hat_all);
K2=kurtosis(alpha_hat_all);

beta_bar=sum(beta_hat_all)/nmc;
bias3=beta_bar-bb;
RMSE3=sqrt((sum(beta_hat_all-bb)^2)/nmc);
S3=skewness(beta_hat_all);
K3=kurtosis(beta_hat_all);

disp('bias_a:')
disp(bias1)

disp('RMSE_a:')
disp(RMSE1)

disp('skewness_a:')
disp(S1)

disp('kurtosis_a:')
```

```
disp(K1)


disp('bias_alpha:')
disp(bias2)

disp('RMSE_alpha:')
disp(RMSE2)

disp('skewness_alpha:')
disp(S2)

disp('kurtosis_alpha:')
disp(K2)

disp('bias_beta:')
disp(bias3)

disp('RMSE_beta:')
disp(RMSE3)

disp('skewness_beta:')
disp(S3)

disp('kurtosis_beta:')
disp(K3)
```

****

```
%%Simulation Study 2

clear all
clc
close all

 a=0.5;
 n=100;
 m=n+1;
 N=100;
 m1=1;
 k=-1;
 r=0.6;
 a1=-0.6145;
 b1=-0.6145;
 %b1=0;
 nmc=500;
 X_all=zeros(n+1,nmc);
 e_all=zeros(n,nmc);
 w_all=zeros(n,nmc);
 a_hat_all=zeros(1,nmc);
```

```
  alpha_hat_all=zeros(1,nmc);
  beta_hat_all=zeros(1,nmc);

XX=zeros(n,1);
se_all=zeros(1,nmc);
t_stat_all=zeros(1,nmc);
prcboot=zeros(nmc,5);


  for NMC=1:nmc;
  %% WSN random number
  randn('seed',NMC);

  x1=randn(n+1,2);
  c1=horzcat(1,r);
  d=horzcat(r,1);
  covar=vertcat(c1,d);
  ch=chol(covar);
  x1=(ch'*x1')';
  z=x1(:,1)+a1.*x1(:,2).*gt(x1(:,2),m1)+b1*x1(:,2).*le(x1
      (:,2),k);



%% GARCH induced by normal and wsn
n1 = 2;
cc = 0.01; aa = 0.05; bb = 0.8;
w = zeros(m-1,1);
eps = zeros(m,1);

SELECT_MOD='garch_wsn';

                switch SELECT_MOD
                    case 'garch_normal'
                        eps=randn(m,1);

                    case 'garch_wsn'
                        eps=z;

                end



sigma2 = zeros(m,1);    %
sigma2(1) = cc/(1-aa-bb);

for ii=2:m          %
    sigma2(ii) = cc + aa*w(ii-1)^2 + bb * sigma2(ii-1);
```

```
    w(ii) = sqrt(sigma2(ii)) * eps(ii);
end

w=w(2:m);
x=w;
e=w;

%% AR simulation

X = zeros(n+1,1);
X(1) = 0;

for i = 2:n
    X(i) = e(i) + a'*X(i-1);

end
%% Estimation of AR(1)-GARCH(1,1) model
Mdl=garchset('R',1,'P',1,'Q',1,'Display','off');
[Coeff,Errors]=garchfit(Mdl,X);

a_hat=Coeff.AR;
alpha_hat=Coeff.ARCH;
beta_hat=Coeff.GARCH;
cc_hat=Coeff.K;

X_all(:,NMC)=X;
e_all(:,NMC)=e;
a_hat_all(:,NMC)=a_hat;
alpha_hat_all(:,NMC)=alpha_hat;
beta_hat_all(:,NMC)=beta_hat;

se=Errors.AR;
se_all(1,NMC)=se;
t_stat_all=(a_hat_all-a)./se_all;

 for j=2:n
        e_hat(j)=X(j)-a_hat*X(j-1);
        sigma2_hat(j)=cc_hat+alpha_hat*e_hat(j-1).^2+beta_hat*
            sigma2(j-1);
        eps_hat(j)=e_hat(j)/sqrt(sigma2_hat(j));
 end


 rng default;
 nN=N*N;
 Xboot=randsample(eps_hat,nN,true);
 Xboot=reshape(Xboot,N,N);

```

```
  prcboot_t_all=zeros(N,5);
  a_hatboot_all=zeros(1,N);
  seboot_a_all=zeros(1,N);
  tboot_stat_all=zeros(1,N);

  for B=1:N
      eps_hat_boot=Xboot(:,B);

      for jj=2:N
          e_hat_boot(jj)=sqrt(sigma2_hat(jj))*eps_hat_boot(jj
              );
          XX(jj)=e_hat_boot(jj)+a_hat*XX(jj-1);
      end

 Mdl_B=garchset('R',1,'P',1,'Q',1,'Display','off');
 [CoeffB,ErrorsB]=garchfit(Mdl_B,XX);


  a_hatboot=CoeffB.AR;
  seboot=ErrorsB.AR;

  seboot_a_all=seboot(1,:);
  a_hatboot_all(:,B)=a_hatboot;

  tboot_stat=(a_hatboot-a)./seboot;
  tboot_stat_all(:,B)=tboot_stat;

  end
  prcboot_t=prctile(tboot_stat_all,[50,90,95,97.5,99]);
  prcboot(NMC,:)=prcboot_t;


 end
 prc_t=prctile(t_stat_all,[50,90,95,97.5,99]);
 SE=std(prcboot);

 disp('Quantiles:')
 disp(prc_t)

 disp('SE-Quantiles:')
 disp(SE)
```

****

Appendix C

MATLAB codes-chapter3

Bootstrapped ADF test

```

%%Bootstrap ADF unit root test

clear all
clc
close all

T = 3;
n=92;                    % sample size
N=n+T;
XX=zeros(n+T,1);
Y_star=zeros(n+T,1);
Y=zeros(n+T,1);

A=load('Data');
Y=log(A.Data);

[h,pValue,stat,cValue,reg]=adftest(Y,'model','TS','lags',12);
ADF=stat;                %t-statistic of original data%
pval=pValue;             %p-value of original data%
se1=reg.se;
e=reg.res;               %residuals%
e_hat=e-mean(e);         %centered residuals%
%% Bootstrapping
rng default;
nN=(N)*(N);
Xboot=randsample(e_hat,nN,true);     %resample residuals with replacement by n*n bootstrap technique%
Xboot=reshape(Xboot,N,N);
stat_all=zeros(1,N);
coeff_all=zeros(1,N);
se_all=zeros(1,N);
t_all=zeros(1,N);
```

```
u_star(1)=0;
u_star(2)=0;
u_star(3)=0;
u_star(4)=0;
u_star(5)=0;
u_star(6)=0;
u_star(7)=0;
u_star(8)=0;
u_star(9)=0;
u_star(10)=0;
u_star(11)=0;
u_star(12)=0;
y_star(1)=Y(1);
y_star(2)=Y(2);
y_star(3)=Y(3);
y_star(4)=Y(4);
y_star(5)=Y(5);
y_star(6)=Y(6);
y_star(7)=Y(7);
y_star(8)=Y(8);
y_star(9)=Y(9);
y_star(10)=Y(10);
y_star(11)=Y(11);
y_star(12)=Y(12);
y_star_all=zeros(N,n+T-2);
rho1=reg.coeff(3);
rho2=reg.coeff(4);
rho3=reg.coeff(5);
rho4=reg.coeff(6);
rho5=reg.coeff(7);
rho6=reg.coeff(8);
rho7=reg.coeff(9);
rho8=reg.coeff(10);
rho9=reg.coeff(11);
rho10=reg.coeff(12);
rho11=reg.coeff(13);
rho12=reg.coeff(14);


for B=1:N
e_star=Xboot(:,B);
    for jj=13:N
    u_star(jj)=rho1*u_star(jj-1)+rho2*u_star(jj-2)+rho3*
        u_star(jj-3)+rho4*u_star(jj-4)+rho5*u_star(jj-5)+rho6
        *u_star(jj-6)+rho7*u_star(jj-7)+rho8*u_star(jj-8)+
        rho9*u_star(jj-9)+rho10*u_star(jj-10)+rho11*u_star(jj
        -11)+rho12*u_star(jj-12)+e_star(jj);

```

```
    y_star(jj)=y_star(jj-1)+u_star(jj);

    end

    y_star_all(:,B)=y_star;
    [h,pValue,stat,cValue,reg]=adftest(y_star','model','TS',
        'lags',12);
    stat_all(1,B)=stat;

end

greater_t=find(stat_all<=ADF);
pvalue_boot=numel(greater_t)/(B+1);

disp('p-value:')
disp(pval)

disp('bootstrap p-value:')
disp(pvalue_boot)
```

```
*****************************************************************************

%%Bootstrap PP unit root test

clear all
clc
close all

T = 3;
n=92;                    % sample size
N=n+T;
XX=zeros(n+T,1);
 Y_star=zeros(n+T,1);
Y= zeros(n+T,1);

A=load('Data');
Y=log(A.Data);

[h,pValue,stat,cValue,reg]=pptest(Y,'model','TS','lags',12);
rho=reg.coeff(2);
pp1=stat;
se1=reg.se(2);
e=reg.res;
pval=pValue;
e_hat=e-mean(e);
%% Bootstrapping
rng default;
nN=(N)*(N);
```

```
Xboot=randsample(e_hat,nN,true);      %resample residuals
    with replacement by n*n bootstrap technique
Xboot=reshape(Xboot,N,N);
stat_all=zeros(1,N);
coeff_all=zeros(1,N);
se_all=zeros(1,N);
t_all=zeros(1,N);
u_star(1)=0;
y_star(1)=Y(1);
y_star_all=zeros(N,n+T-2);
for B=1:N
e_star=Xboot(:,B);
     for jj=2:N
     y_star(jj)=y_star(jj-1)+e_star(jj);
     end

     y_star_all(:,B)=y_star;
     [h,pValue,stat,cValue,reg]=pptest(y_star,'model','TS','
        lags',12);
     stat_all(1,B)=stat;
     coeff_all(1,B)=reg.coeff(2);
     se_all(1,B)=reg.se(2);
     t_all(1,B)=(coeff_all(1,B)-1)./se_all(1,B);
end


greater_t=find(stat_all<=pp1);
pvalue_boot=numel(greater_t)/(B+1);

disp('p-value:')
disp(pval)

disp('bootstrap p_value:')
disp(pvalue_boot)
```
****

```

%%Bootstrap DF-GLS unit root test

clear all
clc
close all

T = 3;
n=92;
N=n+T;
XX=zeros(n+T,1);
Y_star=zeros(n+T,1);
Y=zeros(n+T,1);
```

```
Y2=zeros(n+T+1,1);
c_bar=-13.5;

alpha_bar=1+(c_bar/(n+T+1));
x=ones(1,n+T+1);

A=load('Data');
Y=log(A.Data);

y_bar=zeros(1,n+T+1);
x_bar=zeros(1,n+T+1);
y_bar(1)=Y(1);
x_bar(1)=x(1);

for k=2:n+T+1
    y_bar(k)=Y(k)-alpha_bar*Y(n+T);
    x_bar(k)=x(k)-alpha_bar*x(n+T);
end
A1=mean(x_bar);
A2=mean(y_bar);
B1=x_bar-A1;
B2=y_bar-A2;
B3=B1.*B2;
B4=B1.*B1;
beta_hat=sum(B3)/sum(B4);

Y_mad=Y'-(beta_hat*x);


for t=2:n+T+1
    A1(t)=Y(t-1)*Y(t);
    B1(t)=Y(t-1)*Y(t-1);
end
rho_hat=sum(A1)/sum(B1);


[h,pValue,stat,cValue,reg]=adftest(Y_mad,'lags',12);

ADF=stat;
se1=reg.se;
e=reg.res;
pval=pValue;
e_hat=e-mean(e);

%% Bootstrapping
rng default;
nN=(N)*(N);
```

```
Xboot=randsample(e_hat,nN,true);      %resample residuals
   with replacement by n*n bootstrap technique
Xboot=reshape(Xboot,N,N);
stat_all=zeros(1,N);
coeff_all=zeros(1,N);
se_all=zeros(1,N);
t_all=zeros(1,N);
u_star(1)=0;
u_star(2)=0;
u_star(3)=0;
u_star(4)=0;
u_star(5)=0;
u_star(6)=0;
u_star(7)=0;
u_star(8)=0;
u_star(9)=0;
u_star(10)=0;
u_star(11)=0;
u_star(12)=0;
y_star(1)=Y(1);
y_star(2)=Y(2);
y_star(3)=Y(3);
y_star(4)=Y(4);
y_star(5)=Y(5);
y_star(6)=Y(6);
y_star(7)=Y(7);
y_star(8)=Y(8);
y_star(9)=Y(9);
y_star(10)=Y(10);
y_star(11)=Y(11);
y_star(12)=Y(12);
y_star_all=zeros(N,n+T-2);
rho1=reg.coeff(2);
rho2=reg.coeff(3);
rho3=reg.coeff(4);
rho4=reg.coeff(5);
rho5=reg.coeff(6);
rho6=reg.coeff(7);
rho7=reg.coeff(8);
rho8=reg.coeff(9);
rho9=reg.coeff(10);
rho10=reg.coeff(11);
rho11=reg.coeff(12);
rho12=reg.coeff(13);

for B=1:N
e_star=Xboot(:,B);
      for jj=13:N
```

```
    u_star(jj)=rho1*u_star(jj-1)+rho2*u_star(jj-2)+rho3*
        u_star(jj-3)+rho4*u_star(jj-4)+rho5*u_star(jj-5)+rho6
        *u_star(jj-6)+rho7*u_star(jj-7)+rho8*u_star(jj-8)+
        rho9*u_star(jj-9)+rho10*u_star(jj-10)+rho11*u_star(jj
        -11)+rho12*u_star(jj-12)+e_star(jj);

    y_star(jj)=y_star(jj-1)+u_star(jj);

    end

    y_star_all(:,B)=y_star;
    [h,pValue,stat,cValue,reg]=adftest(y_star','lags',12);
    stat_all(1,B)=stat;

end


greater_t=find(stat_all<=ADF);
pvalue_boot=numel(greater_t)/(B+1);

disp('p-value:')
disp(pval)

disp('bootstrap p_value:')
disp(pvalue_boot)
```

Bibliography

Allen, N., &. Robinson, J. (2015). Monetary Policy Effects in a Regime Switching Model.

Azzalini, A. (1985). A class of distributions which includes the normal ones. *Scandinavian journal of statistics*, 171-178.

Bai, J., &. Ng, S. (2005). Tests for skewness, kurtosis, and normality for time series data. *Journal of Business &. Economic Statistics,* 23(1), 49-60.

Ball, L. (1992). Why does high inflation raise inflation uncertainty?. *Journal of Monetary Economics*, 29(3), 371-388.

Basawa, I. V., Mallik, A. K., McCornick, W. P., Reeves, J. H., &. Taylor, R. L. (1991a). Bootstrap test of significance and sequential bootstrap estimation for unstable first order autoregressive processes. *Communications in Statistics-Theory and Methods*, 20(3), 1015-1026.

Basawa, I. V., Mallik, A. K., McCormick, W. P., Reeves, J. H., &. Taylor, R. L. (1991b). Bootstrapping unstable first-order autoregressive processes. *The Annals of Statistics*, 1098-1101.

Bellini, F., &. Bottolo, L. (2008). Misspecification and domain issues in fitting GARCH (1, 1) models: a Monte Carlo investigation. *Communications in Statistics-Simulation and Computation*, 38(1), 31-45.

Beran, R. (1988). Prepivoting test statistics: a bootstrap view of asymptotic refinements. *Journal of the American Statistical Association*, 83(403), 687-697.

Bernanke, B. S., &. Mihov, I. (1998). Measuring Monetary Policy. *The Quarterly Journal of Economics*, 113(3), 869-902.

Bickel, P. J., &. Freedman, D. A. (1981). Some asymptotic theory for the bootstrap. *The Annals of Statistics*, 1196-1217.

Bickel, P. J., Gtze, F., &. van Zwet, W. R. (2012). Resampling fewer than n observations: gains, losses, and remedies for losses (pp. 267-297). *Springer New York*.

Bickel, P. J., &. Sakov, A. (2008). On the choice of m in the m out of n bootstrap and confidence bounds for extrema. *Statistica Sinica*, 18(3), 967-985.

Bollerslev, T. (1986). Generalized autoregressive conditional heteroskedasticity. *Journal of econometrics*, 31(3), 307-327.

Bollerslev, T. (1987). A conditionally heteroskedastic time series model for speculative prices and rates of return. *The review of economics and statistics*, 542-547.

Bollerslev, T., &. Wooldridge, J. M. (1992). Quasi-maximum likelihood estimation and inference in dynamic models with time-varying covariances. *Econometric reviews*, 11(2), 143-172.

Breidt, F. J. (2004). Simulation estimation of quantiles from a distribution with known mean. *Journal of Computational and Graphical Statistics*, 13(2).

Buhlmann, P. (1997). Sieve bootstrap for time series. *Bernoulli*, 3(2), 123-148.

Castelnuovo, E., &. Surico, P. (2005). The price puzzle and indeterminacy. Bank of England Working Paper no 288.

Chang, Y., &. Park, J. Y. (2002). On the asymptotics of ADF tests for unit roots. *Econometric Reviews*, 21(4), 431-447.

Chang, Y., &. Park, J. Y. (2003). A sieve bootstrap for the test of a unit root. *Journal of Time Series Analysis*, 24(4), 379-400.

Chang, Y., Sickles, R. C., &. Song, W. (2016). Bootstrapping unit root tests with covariates. *Econometric Reviews*, 1-20.

Charemza, W., Hristova, D., &. Burridge, P. (2005). Is inflation stationary?. *Applied Economics*, 37(8), 901-903.

Charemza, W., Diaz, C., &. Makarova, S. (2013). Inflation Fan Charts, Monetary Policy and Skew Normal Distributions, *University of Leicester* Working Paper 13/06.

Charemza, W., Diaz, C., &. Makarova, S. (2014). Term structure of inflation forecast uncertainties and skew normal distributions, *University of Leicester* Working Paper 14/01.

Chen, H., You, C., &. Chen, D. (2012). Parameters estimator of GARCH (1,1) process with heavy tailed errors. *7th International Forum on Strategic Technology (IFOST).*

Cheung, K. Y., &. Lee, S. M. (2005). Variance estimation for sample quantiles using the m out of n bootstrap. *Annals of the Institute of Statistical Mathematics*, 57(2), 279-290.

Christiano, L. J., Eichenbaum, M., &. Vigfusson, R. (2003). What happens after a technology shock? *National Bureau of Economic Research.*(No. w9819).

Cogley, T., &. Sargent, T. J. (2005). Drifts and volatilities: monetary policies and outcomes in the post WWII US. *Review of Economic dynamics*, 8(2), 262-302.

Cooley, T. F., &. Hansen, G. D. (1989). The inflation tax in a real business cycle model. *The American Economic Review*, 733-748.

Corrado, L., &. Holly, S. (2003). Nonlinear Phillips curves, mixing feedback rules and the distribution of inflation and output. *Journal of Economic Dynamics and Control*, 28(3), 467-492.

Cuddington, J. T., &. Navidi, W. (2011). A critical assessment of simulated critical values. *Communications in StatisticsSimulation and Computation*, 40(5), 719-727.

Cukierman, A., &. Meltzer, A. H. (1986). A theory of ambiguity, credibility, and inflation under discretion and asymmetric information. *Econometrica: Journal of the Econometric Society*, 1099-1128.

Datta, S. (1996). On asymptotic properties of bootstrap for AR (1) processes. *Journal of Statistical Planning and Inference*, 53(3), 361-374.

Davidson, R., &. MacKinnon, J. G. (1999). The size distortion of bootstrap tests. *Econometric theory*, 15(03), 361-376.

Dickey, D. A., &. Fuller, W. A. (1979). Distribution of the estimators for autoregressive time series with a unit root. *Journal of the American statistical association*, 74(366a), 427-431.

Efron, B. (1979). Bootstrap methods: another look at the jackknife. *The annals of Statistics*, 1-26.

Efron, B., &. Tibshirani, R. (1985). The bootstrap method for assessing statistical accuracy. *Behaviormetrika*, 12(17), 1-35.

Eichenbaum, M., &. Singleton, K. I. (1986). Do equilibrium real business cycle theories explain postwar US business cycles?. *In NBER Macroeconomics Annual 1986*, Volume 1 (pp. 91-146). MIT Press.

Elliott, G., Rothenberg, T. J., &. Stock, J. H. (1996). Efficient tests for an autoregressive unit root. *Econometrica*, 64(4), 813-836.

Engle, R. F. (1982). Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation. Econometrica: *Journal of the Econometric Society*, 987-1007.

Engle, R. F., &. Gonzalez-Rivera, G. (1991). Semiparametric ARCH models. *Journal of Business &. Economic Statistics*, 9(4), 345-359.

Fan, J., Qi, L., &. Xiu, D. (2014). Quasi-maximum likelihood estimation of GARCH models with heavy-tailed likelihoods. *Journal of Business &. Economic Statistics*, 32(2), 178-191.

Ferretti, N., &. Romo, J. (1996). Unit root bootstrap tests for AR(1) models. *Biometrika*, 83(4), 849-860.

Fletcher, R. (1987). Practical methods of optimization. *John Wiley &. Sons.*

Foerster, A. T. (2016). Monetary policy regime switches and macroeconomic dynamics. *International Economic Review*, 57(1), 211-230.

Fountas, S. (2001). The relationship between inflation and inflation uncertainty in the UK: 18851998. *Economics Letters*, 74(1), 77-83.

Fountas, S., &. Karanasos, M. (2007). Inflation, output growth, and nominal and real uncertainty: empirical evidence for the G7. *Journal of International Money and finance*, 26(2), 229-250.

Francis, N., &. Ramey, V. A. (2005). Is the technology-driven real business cycle hypothesis dead? Shocks and aggregate fluctuations revisited. *Journal of Monetary Economics*, 52(8), 1379-1399.

Francq, C., &. Zakoian, J. M. (2004). Maximum likelihood estimation of pure GARCH and ARMA-GARCH processes. *Bernoulli*, 10(4), 605-637.

Friedman, M. (1977). Nobel lecture: inflation and unemployment. *The Journal of Political Economy*, 451-472.

Gali, J. (1999). Technology, employment, and the business cycle: Do technology shocks explain aggregate fluctuations?. *The American Economic Review*, 89(1), 249.

Gali, J. (2007). *Monetary Policy, Inflation, and the Business Cycle*. Princeton UP.

Gali, J., &. Gertler, M. (2007). Macroeconomic modeling for monetary policy evaluation. *National Bureau of Economic Research.*(No. w13542).

Gali, J., &. Rabanal, P. (2005). Technology Shocks and Aggregate Fluctuations: How Well Does the Real Business Cycle Model Fit Postwar US Data?. *NBER Macroeconomics Annual 2004*, MIT Press, Cambridge, MA. Volume 19 (pp. 225-318).

Gali, J., Lpez-Salido, J. D., &. Valls, J. (2003). Technology shocks and monetary policy: assessing the Fed's performance. *Journal of Monetary Economics*, 50(4), 723-743.

Gill, P. E., Murray, W., &. Wright, M. H. (1981). Practical optimization. *London, Academic Press.*

Gonzlez-Rivera, G., &. Drost, F. C. (1999). Efficiency comparisons of maximum-likelihood-based estimators in GARCH models. *Journal of Econometrics*, 93(1), 93-111.

Gospodinov, N., &. Tao, Y. (2011). Bootstrap unit root tests in models with GARCH (1,1) errors. *Econometric Reviews*, 30(4), 379-405.

Grabek, G., Klos, B., &. Koloch, G. (2011). Skew-normal shocks in the linear state space form DSGE model. *National Bank of Poland* Working Paper, (101).

Grier, K. B., &. Perry, M. J. (1998). On inflation and inflation uncertainty in the G7 countries. *Journal of International Money and Finance*, 17(4), 671-689.

Hall, P. (1992). The bootstrap and Edgeworth expansion. *New York: Springer-Verlag.*

Hall, P., &. Yao, Q. (2003). Inference in ARCH and GARCH models with heavy-tailed errors. *Econometrica*, 285-317.

Hansen, B. E. (1995). Rethinking the univariate approach to unit root testing. *Econometric Theory*, 11(5), 1148-71.

Horowitz, J. L. (1994). Bootstrap-based critical values for the information matrix test. *Journal of Econometrics*, 61(2), 395-411.

Horowitz, J. L. (1997). Bootstrap methods in econometrics: theory and numerical performance. *Econometric Society Monographs*, 28, 188-222.

Hsieh, D. A. (1988). The statistical properties of daily foreign exchange rates: 19741983. *Journal of international economics*, 24(1), 129-145.

Hsieh, D. A. (1989). Modeling heteroscedasticity in daily foreign-exchange rates. *Journal of Business &. Economic Statistics*, 7(3), 307-317.

Hult, H., &. Svensson, J. (2009). Efficient calculation of risk measures by importance sampling–the heavy tailed case. *arXiv preprint arXiv*:0909.3335.

Hurlimann, W. (2012). On non-Gaussian AR (1) inflation modeling. *Journal of Statistical and Econometric Methods*, 1(1), 93-101.

Iglesias, E. M., &. Phillips, G. D. (2008). Finite sample theory of QMLE in ARCH models with dynamics in the mean equation. *Journal of Time Series Analysis*, 29(4), 719-737.

Janssen, P., Swanepoel, J., &. Veraverbeke, N. (2001). Modified bootstrap consistency rates for U-quantiles. *Statistics &. probability letters*, 54(3), 261-268.

Jardet, C., Monfort, A., &. Pegoraro, F. (2009). Persistence, bias, prediction and averaging estimators. *Banque de France Working Paper*.

Kleijnen, J. P., Sanchez, S. M., Lucas, T. W., &. Cioppa, T. M. (2003). A user's guide to the brave new world of designing simulation experiments. Tilburg University.

Kunsch, H. R. (1989). The jackknife and the bootstrap for general stationary observations. *The Annals of Statistics*, 1217-1241.

Kydland, F. E., &. Prescott, E. C. (1982). Time to build and aggregate fluctuations. Econometrica: *Journal of the Econometric Society*, 1345-1370.

Lange, T., Rahbek, A., &. Jensen, S. T. (2011). Estimation and asymptotic inference in the AR-ARCH model. *Econometric Reviews*, 30(2), 129-153.

Li, C. W., &. Li, W. K. (1996). On a doublethreshold autoregressive heteroscedastic time series model. *Journal of applied econometrics*, 11(3), 253-274.

Li, W. K., Ling, S., &. MacAleer, M. (2001). A survey of recent theoretical results for time series models with GARCH errors (No. 545). *ISER Discussion Paper, Institute of Social and Economic Research, Osaka University.*

Ling, S. (1999). On the stationarity and the existence of moments of conditional heteroskedastic ARMA models. *Statistica Sinica*, 9(4), 1119-1170.

Ling, S., &. Li, W. K. (1997c). On fractionally integrated autoregressive moving-average time series models with conditional heteroscedasticity. *Journal of the American Statistical Association*, 92(439), 1184-1194.

Ling, S., &. McAleer, M. (2002b). Asymptotic theory for a vector ARMA-GARCH model. *Econometric Theory*, 19(2), 280-310.

Long Jr, J. B., &. Plosser, C. I. (1983). Real business cycles. *The Journal of Political Economy*, 39-69.

Lucas, R. E. (1972). Expectations and the Neutrality of Money. *Journal of economic theory*, 4(2), 103-124.

Lumsdaine, R. L. (1995). Finite-sample properties of the maximum likelihood estimator in GARCH (1,1) and IGARCH (1,1) models: A Monte Carlo investigation. *Journal of Business &. Economic Statistics*, 13(1), 1-10.

Lumsdaine, R. L. (1996). Consistency and asymptotic normality of the quasi-maximum likelihood estimator in IGARCH (1,1) and covariance stationary GARCH (1,1) models. *Econometrica: Journal of the Econometric Society*, 575-596.

MacKinnon, J. G. (2006). Bootstrap methods in econometrics*. *Economic Record*, 82(s1), S2-S18.

Meitz, M., &. Saikkonen, P. (2011). Parameter estimation in nonlinear ARGARCH models. *Econometric Theory*, 27(06), 1236-1278.

Montgomery, D. C. (2008). Design and analysis of experiments. *John Wiley &. Sons.*

Mumtaz, H., &. Zanetti, F. (2013). The impact of the volatility of monetary policy shocks. *Journal of Money, Credit and Banking*, 45(4), 535-558.

Nankervis, J. C., &. Savin, N. E. (1996). The level and power of the bootstrap t test in the AR (1) model with trend. *Journal of Business &. Economic Statistics*, 14(2), 161-168.

Newey, W. K., &. West, K. D. (1987). Hypothesis testing with efficient method of moments estimation. *International Economic Review*, 777-787.

Ng, S., &. Perron, P. (1995). Unit root tests in ARMA models with data-dependent methods for the selection of the truncation lag. *Journal of the American Statistical Association*, 90(429), 268-281.

Palm, F. C., Smeekes, S., &. Urbain, J. P. (2008). Bootstrap UnitRoot Tests: Comparison and Extensions. *Journal of Time Series Analysis*, 29(2), 371-401.

Pantula, S. G. (1989). Estimation of autoregressive models with ARCH errors. Sankhy: *The Indian Journal of Statistics*, Series B, 119-138.

Paparoditis, E., &. Politis, D. N. (2003). ResidualBased Block Bootstrap for Unit Root Testing. *Econometrica*, 71(3), 813-855.

Paparoditis, E., &. Politis, D. N. (2005). Bootstrapping unit root tests for autoregressive time series. *Journal of the American Statistical Association*, 100(470), 545-553.

Park, J. Y. (2002). An invariance principle for sieve bootstrap in time series. *Econometric Theory*, 18(02), 469-490.

Park, J. Y. (2003). Bootstrap unit root tests. *Econometrica*, 71, 1845-1895.

Perron, P. (1988). Trends and random walks in macroeconomic time series: Further evidence from a new approach. *Journal of economic dynamics and control*, 12(2), 297-332.

Phillips, P. C., &. Perron, P. (1988). Testing for a unit root in time series regression. *Biometrika*, 75(2), 335-346.

Phillips, P. C., &. Xiao, Z. (1998). A primer on unit root testing. *Journal of Economic Surveys*, 12(5), 423-470.

Primiceri, G. E. (2005). Time varying structural vector autoregressions and monetary policy. *The Review of Economic Studies*, 72(3), 821-852.

Psaradakis, Z. (2001). Bootstrap tests for an autoregressive unit root in the presence of weakly dependent errors. *Journal of Time Series Analysis*, 22(5), 577-594.

Romer, C. D., &. Romer, D. H. (2004). A new measure of monetary shocks: Derivation and implications. *The American Economic Review*, 94(4), 1055-1084.

Said, S. E., &. Dickey, D. A. (1984). Testing for unit roots in autoregressive-moving average models of unknown order. *Biometrika*, 71(3), 599-607.

Schwert, G. W. (1989). Tests for Unit Roots: A Monte Carlo Investigation. Journal of Business &. *Economic Statistics*, 147-159.

Sims, C. A., &. Zha, T. (2006). Were there regime switches in US monetary policy?. *The American Economic Review*, 54-81.

Smeekes, S. (2006). Bootstrapping unit root tests. *Medium Econometrische Toepassingen*, 14(4), 24-28.

Stock, J. H. (1994). Unit roots, structural breaks and trends. *Handbook of econometrics*, 4, 2739-2841.

Swanepoel, J. W. (1986). A note on proving that the (modified) bootstrap works. *Communications in Statistics-Theory and Methods*, 15(11), 3193-3203.

Tenreyro, S., &. Thwaites, G. (2015). Pushing on a string: US monetary policy is less powerful in recessions.

Vougas, D. V. (2007). GLS detrending and unit root testing. *Economics Letters*, 97(3), 222-229.

Walsh, C. E. (2003). *Monetary theory and policy.* Second Edition, MIT press, Cambridge, MA.

Weiss, A. A. (1986). Asymptotic theory for ARCH models: estimation and testing. *Econometric theory*, 2(01), 107-131.

Woodford, M. (2003). *Interest and Prices: Foundations of a Theory of Monetary Policy.* Princeton University Press.

Yaya, O. S., Olubusoye, O. E., &. Ojo, O. O. (2014). Estimates and Forecasts of GARCH Model under Misspecified Probability Distributions: A Monte Carlo Simulation Approach. Journal of Modern Applied Statistical Methods, 13(2), 28.

Zhou, H. (2000). A study of the finite sample properties of EMM, GMM, QMLE, and MLE for a square-root interest rate diffusion model. *Board of Governors of the Federal Reserve System (US)*,(No. 2000-45).