A Study in Optimal Monetary Policy, Inflation and Capital Accumulation

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A thesis submitted for the degree of
Doctor of Philosophy at the University of Leicester.

September 2016

To my grandmother Anna and my mother Aimilia

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by

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Abstract

This thesis analyses the effect of optimal monetary policy in economies with imperfect labour and financial markets. It also studies how the interaction between structural characteristics and inflation can affect the economic growth.

Chapters 2 and 3 build on the literature on the New Keynesian model and assess quantitatively the optimal conduct of monetary policy. Chapter 2 analyses the optimal monetary policy in a currency union with involuntary unemployment and real wage rigidity by focusing on the role of labour market heterogeneity. Results show that in the presence of country-specific and aggregate productivity shocks, under the monetary policy regimes of optimal commitment or discretion, the welfare losses in the currency union increase monotonically with the degree of labour market heterogeneity.

Chapter 3 quantifies the welfare gains from the central bank using the discount window as a complementary instrument of monetary policy in an economy with a frictional financial market. Previous literature has characterised discount window as redundant. The novelty of chapter 3 is that it constructs a general-equilibrium model which is used to quantify the effect of discount window lending in households' welfare. In contrast with the previous literature, chapter 3 provides an argument in support of the view that discount window can be an effective instrument of monetary policy.

Chapter 4 studies the role of social status that is associated with investment projects in economic growth. By merging a growth model with a credit market framework, chapter 4 analyses the interaction between social status and inflation and the implications on the economy's long-term prospects. Results show that inflation is disruptive for capital accumulation. However, the latter is enhanced from the presence of social status concerns. Chapter 4 concludes that the role of structural characteristics can be important in determining an economy's long-term performance.

Acknowledgements

You only feel how difficult is to write this part of your thesis, when you start thinking all these people and memories. How can someone fit all these in a few pages?

I can only start by thanking my supervisors, Stephen G. Hall and Dimitris Varvarigos for being of great help and support all these years. Stephen, thank you for asking me challenging questions and suggesting great ideas that helped me to improve my work. Dimitri, you are the best supervisor that someone could ever have. Thank you for being always there for me, pushing me hard to improve my work and spending so much time to help me.

I also feel grateful for having other two advisors: Jesse Matheson, who I have been honoured to call a friend. Jesse thank you for the idea which ended up to be my favourite paper. Martin Jensen, thank you for giving me good advices and comments but mostly for being always available for me. You are a great person.

Going back in time I cannot forget the person that motivated me. This is my MSc supervisor in Edinburgh, Professor Tim Barmby. Seven years later, here I am Tim, in Leicester though and not in Aberdeen, but I really do hope that I have not disappointed you.

I am particularly thankful to Subir Bose. Subir you are always available for everyone. I was very lucky to have you as a PhD director. Special thanks to Chris Wallace. Chris, there are too many things that I could thank you for. Here, I thank you for your great help.

Special thanks to two friends that I really miss. Thank you Matthew Pollison for the time we spent discussing about life. Thank you Samuel Smithers for being the best office neighbour.

Thank you Samantha Hill, for your endless help. Special thanks to Andre, Fabrizio, Aris, Piercarlo and Curro for providing great advice and support.

I also want to thank my friends in Leicester for being a great company all these years. Thank you Vaggelis, Eleni, Alex, Stelios, Prodromos, Dimos, Stamatoula, Alessandro, Houra, Narges, Johan, Teddy, Ivy, Taha, Ioli, Sneha, and Seb.

Finally, I would like to spend a few lines thanking my family: Matteo, my bestman since September 2016, thank you so much for the great discussions we had.

Yes, I remember our promise and hope to see you in Victoria Park in 2023!

Aristi, the best partner in life and best partner in crime! Without you, this PhD would have finished a few months earlier! I will never forget your endless support and the great memories of us being in Leicester! You were always there next to me. I feel very lucky to have you.

Mano, Grandpa Gregory, Socrates my best-man, thank you for being a great family that was always next to me. I feel so lucky having you in my life.

Finally, my favourite persons in my life: my mother! Mama, this thesis is because of you and for you! My grandmother! Giagia, you left me early. This thesis is devoted to you.

Declaration

Chapters 2 and 3 of this thesis are single authored. Chapter 4 is co-authored with Dimitrios Varvarigos.

Chapter 2 has been awarded the Best-Paper Award at the International Network for Economic Research Workshop (INFER): The European Integration and its International Dimension. An earlier version of this chapter appears under the title: "Optimal Monetary Policy in a Currency Union with Labour Market Heterogeneity", INFER Working Paper Series, vol. 2015.04, Speyer, London.

I am grateful to the INFER for the monetary prize. I am also grateful to the Royal Economic Society for a RES Conference Grant as a contribution towards my attendance and presentation at the INFER Workshop. Chapter 2 has been also benefited from discussions in Dynare Summer School 2014 at CEPREMAP and Banque de France, as well as from comments of the internal seminar participants of the Department of Economics at the University of Leicester.

Chapter 3 has been benefited from comments of the internal seminar participants of the Department of Economics at the University of Leicester and discussions with the participants of the DSGE Modelling Advanced Course with Financial Frictions at the University of Surrey.

A revised version of chapter 4, entitled "Entrepreneurial Status, Social Norms, and Economic Growth" and co-authored with Dimitrios Varvarigos, appears as Discussion Paper no 17/05, School of Business (Economics Division), University of Leicester.

Finally, with regard to the writing of the entire book, I would like to acknowledge financial support from the University of Leicester, Department of Economics and the State Scholarships Foundation (IKY) of Greece.

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Chapter 1

Introduction

Modern macroeconomic theory should aim at studying what influences an economy's performance and evaluating the instruments and objectives of macroeconomic policy. The micro-founded models used for such analyses should be tractable. At the same time, they should incorporate elements that capture key features of the macroeconomic environment in order to provide useful lessons and more accurate predictions. The chapters 2, 3 and 4 of this thesis illustrate these views.

In particular, the aim of chapters 2 and 3 is to assess quantitatively the optimal conduct of monetary policy in economies with imperfect labour (chapter 2) and financial (chapter 3) markets. To do this, these chapters build on the literature on the New Keynesian (NK) model (e.g., Clarida, Gali, and Gertler (1999)), which in recent years has become the standard model for the analysis of monetary policy. Incorporating a frictional labour or financial market to the NK model allows us to extend the scope of the monetary policy objectives and include unemployment stabilisation and financial stability respectively. This is not new in the NK literature. However, chapters 2 and 3 provide novel elements that capture issues which have emerged in the aftermath of the recent financial crisis. Including these features to the NK model helps to address new research questions with regard to the effect of monetary policy in households' welfare. The aim of chapter 4 is different. Rather than evaluating macroeconomic policies, it focuses on how structural characteristics can affect an economy's growth performance. The particular characteristic considered in this chapter is the social status conferred to entrepreneurs who operate investment projects. By expanding a standard growth model (e.g., Acemoglu (2009)) with a credit market, chapter 4 also studies the role of inflation, the interaction between social status and inflation and the implications on the economy's long-term prospects. Thus, chapter 4 provides a lesson that highlights the importance of incorporating a credit market to popular macroeconomic models, such as an overlapping generations model.

What is the effect of optimal monetary policy in a currency union consisted of member states that have imperfect labour markets and different labour market characteristics? This is the main research question addressed in chapter 2. Empirical evidence suggests that since the financial crisis of 2008, the unemployment rates among the euro area member states have differed in variability and persistence. In contrary, inflation convergence has been maintained. Estrada, Gali, and Lopez-Salido (2013) suggest that different structural labour market characteristics between member states could explain unemployment divergence in the euro area. Taking some of these structural differences as given, chapter 2 examines whether these movements of unemployment and inflation could be characterised as an outcome of optimal monetary policy in a currency union. In addition, chapter 2 analyses the consequences of labour market heterogeneity on households' welfare under different optimal monetary policy regimes (optimal discretion or commitment).

The model developed in chapter 2 is a NK currency union (two-country) model (e.g., Benigno (2004), Wickens (2007), Gali and Monacelli (2008)) extended with a labour market framework. In each member state there is involuntary unemployment that arises from the presence of frictions in the search and matching process. Basing the labour market framework on the literature on Equilibrium Unemployment Theory (Diamond (1982), Mortensen and Pissarides (1994), Pissarides (2000), Shimer (2004), Hall (2005)) helps not only to keep the model tractable for monetary policy analysis, but also to derive a labour market heterogeneity index which is based on the degree of real wage rigidity (RWR) differential between the two member states. A linear-quadratic approach (e.g., Rotemberg and Woodford (1997), Woodford (2003)) for the normative analysis of monetary policy is followed and a novel feature emerges which is key for the model's results. The variance of the terms of trade of the two member states is derived as one of the central bank's stabilisation policy objectives.

The main results of chapter 2 can be summarised as follows: In the presence of a country-specific productivity shock, under any optimal monetary policy regime, commitment or discretion, the households' welfare losses in the currency union increase monotonically with the degree of RWR in the country hit by the shock. In the presence of a union-wide shock, the welfare losses increase monotonically with the degree of RWR differential, i.e., the value of the labour market heterogeneity index. The fluctuations of the terms of trade is crucial for these results, as they are positively linked with the degree of labour market heterogeneity. In addition, the terms of trade acts as a transmission mechanism of country-specific shocks from one member state to the other and intensifies the asymmetric effects of a union-wide shock. With regard to unemployment stabilisation, the presence of labour market heterogeneity generates a trade-off between optimal commitment and discretion. As the degree of heterogeneity increases, inflation becomes less responsive to unemployment changes. Under discretion, the central bank must react more strongly to fluctuations of unemployment and optimal discretion becomes more desirable than optimal commitment. Furthermore, the degree of labour market heterogeneity affects the frequency of stabilisation of all policy objectives. Nevertheless, it has more pronounced effects in the persistence of unemployment. Thus, chapter 2 provides an argument in support of the view that the unemployment divergence and the homogeneous inflation stabilisation observed in the euro area after the financial crisis could be an outcome of optimal monetary policy in a currency union consisted of member states with structurally different labour markets. However this outcome is sub-optimal.

The optimal monetary policy in a closed economy NK model with labour market frictions has been studied extensively (e.g., Thomas (2008), Blanchard and Gali (2010), Ravenna and Walsh (2011)). To the best of my knowledge, chapter 2 is the first that performs a normative analysis in a currency union framework. This adds to some important work in the literature, such as Benigno (2004), who studies a currency union model without involuntary unemployment and member states with heterogeneous nominal price rigidity. Chapter 2 also adds to Wickens (2007) who calls into question the sustainability of the euro area highlighting the role of individual exogenous characteristics of the member states in the persistence of inflation differential. In chapter 2, it is the different labour market characteristics that increase the fluctuations of the terms of trade and contribute to higher welfare losses in the currency union.

Assuming some degree of RWR is an essential characteristic for the Diamond, Mortensen and Pissarides model in order to generate realistic fluctuations of unemployment (Shimer (2004), Hall (2005)). In chapter 2, this is an assumption that is supported from empirical evidence (Babecky (2009), Messina (2010)). Combining these elements to currency union frameworks, such as in Benigno (2004) and Gali and Monacelli (2008), is considered as one of the novelties in chapter 2.

Chapter 3 analyses the effect of the central bank using the Discount Window (DW), as a complementary instrument of monetary policy in an economy with an imperfect financial market. In particular, the performance of DW lending is evaluated with reference to its consequences on households' welfare. The motivation of chapter 3 is twofold. First, it is driven from arguments in the literature that oppose to the view that DW is an effective instrument of monetary policy (e.g., Schwartz (1992)). These views, which are based on cost-benefit analyses of historical facts, have come to a conclusion that DW is redundant and should not be used by the central banks. The other motivation of this chapter has been the changes of monetary policy in practice after the financial crisis. The main change has been that the nominal interest rates have reached the zero lower bound, as a result of ongoing reductions made by the central banks. As a consequence, several central banks have adopted other, unconventional policies (e.g., large-scale asset purchase programmes) or have adjusted nominal interest rates below zero in order to combat the recession. Chapter 3 offers an alternative view. It provides a mainstream macroeconomic model that assesses the performance of the DW and suggests that it is effective. Thus, it concludes that the DW could be used more often by the central banks and should not be put on the shelf.

The core model used is a NK DSGE (e.g., Gali (2008)) with capital, which is merged with a model that analyses the behaviour of financial intermediaries in real business cycles (Gertler and Kiyotaki (2010), Gertler and Karadi (2011)). Financial market frictions generate borrowing constraints that create a spread between the intermediaries' real rate of lending and borrowing. In the presence of negative shocks that affect the financial and the goods market, the economy deviates from its desirable state. The goal of the central bank is to minimise households' welfare losses by conducting optimal monetary policy. It has two instruments at its disposal: a primary instrument, the nominal interest rate and a complementary instrument, the DW. The optimal monetary policy regimes are implemented by the central bank adjusting the nominal interest rate. DW is used as a secondary instrument to improve the financial market stability. In the model, DW lending requires the central bank to monitor the intermediaries' actions (Bordo (1989)), as there is a risk that DW loans could be misused in other than financial activities. Chapter 3 follows a linear-quadratic approach for the welfare analysis. This approach is useful, as the effect of a zero lower bound constraint on the nominal interest rate can be approximated. By limiting the variation of the nominal interest rate, the net effect of DW lending can be calculated.

The main result of chapter 3 is the following: when the ability of the central bank to monitor financial intermediaries' actions is low, DW lending can be welfare costly. This result confirms the doubts raised by the previous literature. However, when the ability of the central bank to monitor improves, DW lending is welfare enhancing. Thus, chapter 3 provides quantitative evidence that support the existence of welfare gains from DW lending, even if the central bank's ability to monitor is not perfect, or there are efficiency costs associated with DW lending. Furthermore, chapter 3 shows that the welfare effect from the joint use of DW and nominal interest rate is different when different policy regimes are followed, despite the presence of the zero lower bound constraint.

The main contribution of chapter 3 is that it provides a macroeconomic model that quantifies the effect of DW lending. In addition, it provides quantitative evidence in favour of the view that the DW is effective. The approximation of a zero lower bound constraint adds to the realism of the model and is a novel element. Another novel element is that DW is considered as a complementary instrument of monetary policy used by the central bank to improve efficiency in financial markets. This view is consistent with the formal position of major central banks, such as the Federal Reserve. This is not necessarily the same with the view that DW lending is a non-standard policy used only at exigent circumstances (financial crises), when the central bank acts as a lender of last resort. In other words, chapter 3 provides evidence which supports the view that DW lending could be effective during small recessions.

Chapter 4 analyses the effect of social status conferred to entrepreneurs who operate investment projects in economic growth. In addition, it studies the interaction between social status and inflation and its implications in the economy's performance. Chapter 4 adds to the literature (e.g., Weiss and Fershtman (1998), Varvarigos (2011)) by analysing status concerns that emerge from entrepreneurial decisions. The motivation for this analysis is the views in the literature that social status can be an important element of entrepreneurial decisions and performance (Van Praag (2011)).

The model developed in chapter 4 is an overlapping generations one extended with a framework for a credit market (e.g., Bose and Cothren (1996), Bose (2002)). In the model, individuals are workers and entrepreneurs. The latter borrow funds from workers to operate projects that produce capital. A key assumption in the model is that different projects vary with respect to their return as well as the status they confer to the entrepreneurs. Projects with high (low) monetary return require high (low) effort but confer relatively higher (lower) status.

The main results of chapter 4 can be summarised as follows. Capital accumulation is stimulated from the presence of social status concerns. In contrary, inflation has disruptive effect, because it decreases the real value of the funds available in the credit market. Furthermore, inflation reduces the lenders' real return inducing them to charge a higher loan rate. This decreases the number of entrepreneurs that undertake projects which produce relatively more capital. In addition, the effect of social status in the choice of investment projects is a source of transitional dynamics. The social status associated with the decision to devote effort and operate a high-return project is less pronounced in the cases where more entrepreneurs has taken the same decision in the past. In this case, the effect of the number of entrepreneurs who invest in the high-return project in capital is negative and dominates the positive effect.

Chapter 4 shows that structural characteristics, such as social status, can be responsible for the emergence of cycles and play an important role in determining the long-term economy's performance. This is a different approach to the examination of circumstances that generate a volatile economic environment. These circumstances can occur from structural characteristics (social status) rather than exogenous shocks. Chapter 4 concludes with some policy implications that involve policymakers' activities that give individuals the motivation to follow careers with high status rather than high wage.

Finally, Chapter 5 briefly concludes and provides a discussion for further research.

Chapter 2

Optimal Monetary Policy in a Currency Union with Labour Market Heterogeneity

Chapter Abstract

I evaluate the effect of optimal monetary policy in a New Keynesian two-country currency union model with rigid real wages and involuntary unemployment. I focus on the role of regional labour market heterogeneity which is quantified from the degree of real wage rigidity differential of the two countries. In the presence of a country-specific productivity shock, households' welfare losses increase monotonically with the degree of real wage rigidity in the country hit by the shock. In the presence of a union-wide shock, welfare losses increase monotonically with the degree of labour market heterogeneity. The positive relationship between the degree of labour market heterogeneity and the fluctuation of the terms of trade of the two countries is crucial for the results. With regard to unemployment stabilisation, the presence of labour market heterogeneity generates a trade-off between two optimal monetary policy regimes: optimal commitment and discretion. As the degree of heterogeneity increases, optimal discretion becomes more desirable than optimal commitment. Thus, this paper provides a quantitative argument in support of the view that labour market heterogeneity acts as an additional distortion in the currency union. It also attempts to explain the role of monetary policy in various facts observed in the euro area after the financial crisis, such as the homogeneous inflation stabilisation and the heterogeneous persistence of unemployment across member states.

2.1 Introduction

After the financial crisis of 2008—2009, unemployment in the euro area has not only increased but also diverged. Remarkable differences have been observed in the persistence of unemployment as well. Figure 2.1 illustrates these differences for some member states. In contrast, inflation in the euro area has been stabilised homogeneously and relatively fast.[1] This should not be surprising, as despite the social and political pressures caused by unemployment heterogeneity, price stability still is the primary policy objective of the European Central Bank (ECB).

Motivated from these facts, in this paper I study the optimal monetary policy

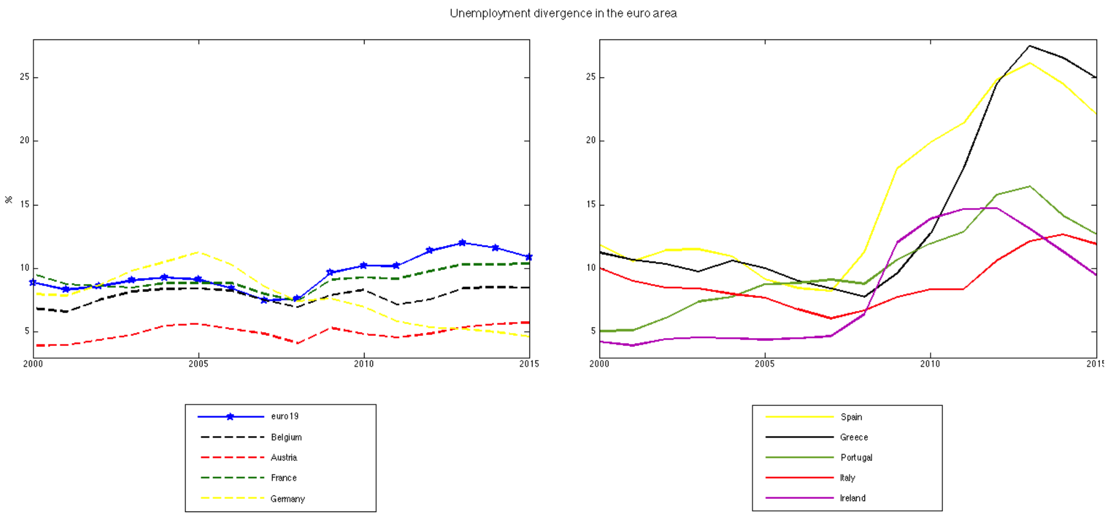

Figure 2.1: *Harmonised unemployment rates in euro area, % annually, OECD stats.*

in a currency union framework with involuntary unemployment. Estrada, Gali, and Lopez-Salido (2013) and Boeri and Jimeno (2015) highlight the presence of asymmetric shocks and transmission mechanisms as potential sources of unemployment divergence in the euro area. Following this context, the main objective of this paper is twofold: i) to evaluate the effect of optimal monetary policy by focusing on the role of structural differences between the labour markets of the member states and ii) to identify if this slow, heterogeneous recovery of unemployment and the relatively fast, homogeneous inflation stabilisation in the euro area can be an outcome of optimal monetary policy in a currency union.

I build on Benigno (2004) a New Keynesian (NK), two-country currency union

[1] See the EU LFS data for the HCPI inflation in the Appendix A.1.

model. In each country/member state, I introduce Diamond-Mortensen-Pissarides (DMP) labour market frictions in the search and matching process to generate involuntary unemployment.[2] I also allow for a form of real wage rigidity (RWR) in each member state. Following the critique for the DMP model in Shimer (2004, 2005) and Hall (2005), by assuming some degree of RWR the performance of DMP model is improved in terms of generating realistic fluctuations of unemployment. The assumption of RWR in the euro area is supported by empirical evidence. Examples include a firm-level survey by Babecky (2010) and a micro-study by Messina (2010).

I quantify the labour market heterogeneity in the currency union by constructing a simple index that is based on the degree of RWR differential among the two member states. In the presence of country-specific and union-wide productivity shocks, I evaluate the effect of optimal monetary policy for different values of the labour market heterogeneity index assuming that the central bank may follow one of two optimal regimes: the timeless perspective optimal commitment or optimal discretion. The criterion of policy evaluation is the households' welfare losses, which are generated from the presence of a shock. In particular, I follow a Linear-Quadratic (L-Q) approach introduced by Rotemberg and Woodford (1997) and Woodford (2003) and I derive a welfare criterion for the central bank. I find that the stabilisation objectives of the central bank are: fluctuations of inflation and unemployment in each member state and fluctuations in their terms of trade.

The main results are summarised as follows: Keeping the union average degree of RWR constant to avoid effects arisen from union-wide structural changes, in the presence of a country-specific shock, the welfare losses in both member states and thus, in the entire currency union, increase monotonically with the degree of RWR in the country hit by the shock. In addition, in the presence of a union-wide shock, the welfare losses increase monotonically with the degree of RWR differential, i.e., the value of the labour market heterogeneity index. I find that the positive relationship between the degree of labour market heterogeneity and the fluctuation of the terms of trade of the two countries has a key role for these results. That is because the terms of trade acts as a transmission mechanism of a country-specific shock from one member state to the other and intensifies the asymmetric effects of a union-

[2]Search and matching models of equilibrium unemployment have been merged with closed economy Real Business Cycle (RBC) models, as well as NK models. For RBC with search and matching frictions, see Merz (1995) and Andolfatto (1996). For NK models, see Cheron and Langot (2000), Walsh (2005), Trigari (2006), Thomas (2008), Blanchard and Gali (2010), Ravenna and Walsh (2011).

wide shock. Furthermore, with regard to unemployment stabilisation, I show that, in the case of a union-wide shock, the presence of labour market heterogeneity generates a trade-off between optimal commitment and discretion. This is because, as the degree of heterogeneity increases, inflation becomes less responsive to unemployment changes. Under discretion, the central bank must react more strongly to fluctuations of unemployment and optimal discretion becomes more desirable than optimal commitment.

In the quantitative analysis of the model, I find that under any monetary policy regime, unemployment exhibits strong persistence and is stabilised slowly. In contrast, inflation is stabilised relatively fast. The degree of labour market heterogeneity affects the frequency of stabilisation of all policy objectives, however it has more pronounced effects in the persistence of unemployment. Thus, this paper provides a quantitative argument in favour of the view that the facts observed in the euro area after the financial crisis could be an outcome of optimal monetary policy in a currency union consisted of member states with structurally different labour markets. However, these differences have a distortionary effect in the economy and are welfare costly for the entire union.

In recent years, the ECB has shown an increasing interest in the role of wages in unemployment persistence and divergence in the euro area. Among others, this is also demonstrated from the launch of the research group, Wage Dynamics Network, that studies various characteristics of the labour markets in the euro area. This direction taken by the ECB also justifies the need for a currency union framework with involuntary unemployment and rigid real wages that studies the optimal monetary policy. A normative analysis of monetary policy in this framework has been absent in the literature. Performing this analysis in this framework constitutes one of the novelties of this paper.

Existing work in currency union models that follows a L-Q approach in normative analysis of monetary policy includes Benigno (2004), Wickens (2007) and Gali and Monacelli (2008). Benigno (2004) focuses on the implications of nominal price rigidity for monetary policy. Wickens (2007) analyses the optimal monetary policy in the euro area under a discretionary regime highlighting that initial differences in the price level of the member states can have a permanent effect in inflation divergence in the euro area. Gali and Monacelli (2008) examine the implications of the coordination between the single monetary authority with fiscal policy-makers. This paper adds to this literature by deriving unemployment instead of output gap as policy objective of the central bank and by focusing in labour market differences. This paper also contributes to another literature of open economy NK models. There are a few papers which incorporate wage rigidity. The closest are by Andersen and Seneca (2010) and Fahr and Smets (2010). Andersen and Seneca (2010) study the role of heterogeneity in country size and nominal wage rigidity on the dynamic paths of inflation and output. Fahr and Smets (2010) incorporate nominal and real wage rigidity to study the effects of asymmetric productivity shocks on inflation. Both papers also find that the terms of trade acts as a transmission mechanism of the shocks and strengthens with the asymmetry of the degree of wage rigidity. The current paper adds to this literature by providing a normative analysis of monetary policy and by deriving unemployment as policy objective of the central bank. There are also other papers that incorporate labour market frictions and RWR in a NK currency union model but they do not focus on optimal monetary policy. Examples include Campolmi and Faia (2011) who study how the volatility of inflation in euro area countries is affected from the heterogeneous unemployment insurance and Abbritti and Mueller (2013) who focus on the implications of different degree of real wage rigidity on the inflation and unemployment differentials of the member states.

The rest of the paper is organised as follows. The currency union framework is described in Section 2.2. Section 2.3 describes the L-Q approach and the problem of optimal monetary policy in this framework. Section 2.4 describes the numerical solution of the model and presents the main results and implications. Section 2.5 concludes with a further discussion. An Appendix can be found in section A.

2.2 The model

The reference framework for the setup of the currency union is in Benigno (2004). The currency union model is formed of two countries or member states, A and B. There is a single monetary policy-maker, the central bank, and for simplicity fiscal authorities are absent. The currency union is occupied by a continuum of households on the interval $[0,1]$. Each household owns a firm that produces a homogeneous intermediate good and a firm that produces a differentiated final good. The firms and their owners are located in the same country. Consequently, in each country, the number of firms (per sector of production) is equal to the number of households. The population of households on the segment $[0,\zeta)$ live in country A and the one on the segment $[\zeta,1]$ live in country B. In each country, household members act as workers. However, some members may be unemployed involuntarily. For simplicity, I assume that there is not on-the-job search, so employment is immobile within and across countries. In addition, unemployed members cannot cross borders. Therefore, no migration can take place across countries at any point of time.

2.2.1 Preferences

Households which live within the same country have homogeneous preferences. Therefore, to make the analysis simpler, it is assumed the existence of a representative infinitely-lived household of country $j \in [A,B]$. At any given period of time, t, a fraction $u_t^j \in [0,1]$ of household members that live in country j are unemployed. Being unemployed entails the loss of labour income. However, assuming that the employed household members pool their income and distribute it equally across all members before making the optimal consumption-savings decision, guarantees perfect consumption insurance across all of the household members.[3] Given that within a country j all households are identical, u_t^j is considered as the unemployment rate of country j.

A representative household living in country $j \in [A,B]$ has preferences described by the additively separable, intertemporal utility function

$$U\left(C_t^j, N_t^j\right) = E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{\left(C_t^j\right)^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} - d_u \frac{\left(N_t^j\right)^{1+\varphi}}{1+\varphi} \right) \tag{2.1}$$

where C_t^j is an index of composite consumption of final private goods consumed in country $j \in [A,B]$ and $N_t^j \in [0,1]$ is the fraction of household members that are employed. The component $d_u > 0$ accounts for the disutility associated with labour and $\varphi > 0$ accounts for the inverse of the Frisch elasticity of labour supply.[4] House-

[3]The assumption of perfect consumption insurance has been presented by Merz (1995) and adopted by Thomas (2008) and Gertler and Trigari (2009), Blanchard and Gali (2010), amongst others.

[4]As in Blanchard and Gali (2010), this specification for the disutility of labour may occur from the sum of the disutility from working $d_u i^{\varphi}$ across household members $i \in [0,1]$. By summing across,

holds form rational expectations. The parameter $\beta \in (0,1)$ is the intertemporal discount factor and $\sigma > 0$ is a measure of household's risk aversion. The utility function is strictly increasing and strictly concave on C_t^j and strictly increasing and strictly convex on N_t^j.

The index C_t^j is defined as in Obstfeld and Rogoff (2000):

$$C_t^A \equiv \frac{\left(C_{At}^A\right)^{\zeta}\left(C_{Bt}^A\right)^{1-\zeta}}{\zeta^{\zeta}(1-\zeta)^{1-\zeta}} \tag{2.2}$$

and

$$C_t^B \equiv \frac{\left(C_{Bt}^B\right)^{\zeta}\left(C_{At}^B\right)^{1-\zeta}}{\zeta^{\zeta}(1-\zeta)^{1-\zeta}} \tag{2.3}$$

for countries A, B respectively. C_{At}^j and C_{Bt}^j are the Dixit-Stiglitz indices (baskets) of consumption across the continuum of differentiated final goods, consumed in country j and produced in countries A and B respectively. They are given by the CES functions:

$$C_{At}^j \equiv \left[\left(\frac{1}{\zeta}\right)^{\frac{1}{\gamma}} \int_0^{\zeta} c_t^j(a)^{\frac{\gamma-1}{\gamma}} da\right]^{\frac{\gamma}{\gamma-1}} \tag{2.4}$$

and

$$C_{Bt}^j \equiv \left[\left(\frac{1}{1-\zeta}\right)^{\frac{1}{\gamma}} \int_{\zeta}^{1} c_t^j(b)^{\frac{\gamma-1}{\gamma}} db\right]^{\frac{\gamma}{\gamma-1}} \tag{2.5}$$

for $a \in [0,\zeta)$, $b \in [\zeta,1]$ and $c_t^j(a)$, $c_t^j(b)$ denoting the variety of final goods produced in countries A and B respectively and $\gamma > 1$ being the within country elasticity of substitution of final goods, which is assumed to be the same for both countries.

From the definition of C_t^j above, it follows that ζ is the weight that households put on the domestically produced final goods (home bias in consumption). Thus, as it has been highlighted by Gali and Monacelli (2008), the term $(1-\zeta)$ reflects a natural index of openness.[5]

Given the current framework, the representative household takes an intertem-

we get: $\int_0^{N_t^j} d_u i^{\varphi} di = d_u \frac{\left(N_t^j\right)^{1+\varphi}}{1+\varphi}$

[5]Following Benigno (2004), I have assumed the across countries elasticity of substitution of final goods to be equal to one. See Benigno and Benigno (2003) for the case which the across country elasticity is higher than 1, and the implications for monetary policy.

poral consumption/savings decision and two intratemporal decisions: The optimal allocation of nominal spending between domestic and imported final goods, and the optimal allocation of the shares of nominal spending among the differentiated final goods produced in each country.

Households' optimal decisions

The optimal allocation of nominal spending between domestic and imported final goods requires the minimisation of total nominal spending $P^j_{ct} C^j_t$ given equations (2.2) and (2.3), where

$$P^A_{ct} \equiv P^{\zeta}_{At} P^{(1-\zeta)}_{Bt} \tag{2.6}$$

and

$$P^B_{ct} \equiv P^{\zeta}_{Bt} P^{(1-\zeta)}_{At} \tag{2.7}$$

is the Consumer Price Index (CPI) for countries A and B respectively, and P_{jt} is the Dixit-Stiglitz domestic price index for country j given by:

$$P_{At} \equiv \left[\frac{1}{\zeta} \int_0^{\zeta} p_t(a)^{1-\gamma} da \right]^{\frac{1}{1-\gamma}} \tag{2.8}$$

$$P_{Bt} \equiv \left[\frac{1}{1-\zeta} \int_{\zeta}^{1} p_t(b)^{1-\gamma} db \right]^{\frac{1}{1-\gamma}} \tag{2.9}$$

The solution of the problem yields the optimal shares for the representative household living in country A:

$$P_{At} C^A_{At} = \zeta P^A_{ct} C^A_t$$

and

$$P_{Bt} C^A_{Bt} = (1-\zeta) P^A_{ct} C^A_t$$

while similar conditions hold for country B.[6]

The optimal allocation of shares of nominal spending among the differentiated final goods requires the representative household to maximise the Dixit- Stiglitz indices given by eq. (2.4) and (2.5), for any given level of nominal spending. The so-

[6]For an analytical solution, see Appendix A.2.

lution of the problem yields the system of demand equations:

$$c_t^j(a) = \left(\frac{p_t(a)}{P_{At}}\right)^{-\gamma} S_t^{(1-\zeta)} C_t^j \tag{2.10}$$

for final differentiated goods $a \in [0, \zeta)$ produced in country A and

$$c_t^j(b) = \left(\frac{p_t(b)}{P_{Bt}}\right)^{-\gamma} S_t^{-\zeta} C_t^j \tag{2.11}$$

for final differentiated goods $b \in [\zeta, 1]$ produced in country B where I define

$$S_t \equiv \frac{P_{Bt}}{P_{At}} \tag{2.12}$$

as the terms of trade of country B.[7]

The intertemporal consumption/savings decision requires the representative household to choose the set of processes $\{C_t^j, B_t^j\}$ in order to maximize eq. (2.1), subject to a sequence of budget constraints. The budget constraint can take the form:

$$P_{ct}^j C_t^j + B_t^j \leq (1 + q_{t-1}) B_{t-1}^j + N_t^j W_t^j + \Pi_t^j \tag{2.13}$$

where the set of processes of $\{q_t\}$, $\{P_{ct}^j\}$, $\{W_t^j\}$, $\{\Pi_t^j\}$ are given. B_t^j is the one-period riskless bond, q_t is the nominal interest rate paid from bond holdings, W_t^j is the nominal wage in country j and Π_t^j are the nominal profits received from the ownership of firms. Given the solvency condition, $\lim_{T\to\infty} E_t B_T \geq 0$, for all t, the solution to the above problem yields:

$$\beta E_t \left\{ \left(\frac{C_{t+1}^j}{C_t^j}\right)^{-\frac{1}{\sigma}} \frac{P_{ct}^j}{P_{ct+1}^j} \right\} = \frac{1}{(1+q_t)} \tag{2.14}$$

which is a standard consumption Euler equation.[8]

2.2.2 International trade

The two countries are small open economies and are linked via trade. Following Gali and Monacelli (2008), in this section, I provide a definition of CPI of each country,

[7] For an analytical solution, see the Appendix A.3

[8] For the derivation of the Euler equation, see the Appendix A.4.

which is expressed in terms of their terms of trade. As in Benigno (2004), I assume that when the two economies trade, transaction costs are absent. Therefore, the price of a good produced in a country is the same for both countries, i.e. $P_{At} = P_{Bt}$, i.e., the law of one price holds across the currency union.[9] However CPI may be different due to different home biased preferences. Combining eq.(2.12) with eq.(2.6) and (2.7) the CPI of each country is given by:

$$P^A_{ct} = P_{At} S_t^{(1-\zeta)} \tag{2.15}$$

and

$$P^B_{ct} = P_{Bt} S_t^{(\zeta-1)} \tag{2.16}$$

In addition, in a currency union area there is a single nominal interest rate, q_t. Assuming symmetric households' preferences and initial conditions across countries implies that the Euler condition, eq.(2.14), is symmetric.[10] In Appendix A.5, I show that using the CPI definitions above and the definition of the terms of trade, the consumption indices of the two countries are linked:

$$E_t\left(\frac{C_t^A}{C_{t+1}^A}\right) = E_t\left\{\left(\frac{S_t}{S_{t+1}}\right)^{\sigma(2\zeta-1)}\left(\frac{C_t^B}{C_{t+1}^B}\right)\right\} \tag{2.17}$$

2.2.3 Technology

In each country there are two types of production. Production for intermediate and for final goods. I distinguish technology between intermediate and final goods production for tractability, as in Blanchard and Gali (2010). The representative intermediate good firm has a simple technology constituted by the input of labour N_t^j. In any country j $\in [A, B]$, the labour market is subject to frictions in the search and matching process, as for the representative intermediate good firm of each country posting a vacancy, v_t^j, is costly. In each country there are also final good firms which buy the homogeneous intermediate good in a perfectly competitive market and use it as the only input to produce a differentiated final good. The final good is sold in a monopolistically competitive market. Final good producers are subject to nominal

[9] This assumption also justifies the absence of a superscript j in $p_t(j)$ in the Dixit-Stiglitz domestic price indices.

[10] This is a remark from the assumption of complete securities markets, which is also assumed in other currency union models, such as in Gali and Monacelli (2008) and Abbritti and Mueller (2013).

price rigidity a la Calvo (1983).

The labour market

A job match is formed when a posted vacancy is filled by an unemployed worker. The number of new job matches in country $j \in [A, B]$ at any period t is given by the matching function, m_t^j, which takes the following form:

$$m_t^j = m^j(u_t^j, v_t^j) = \left(v_t^j\right)^{\kappa}\left(u_t^j\right)^{(1-\kappa)} \tag{2.18}$$

where v_t^j is the number of posted vacancies in country j. The specification of the matching function satisfies some standard properties: It is strictly increasing and strictly concave in both arguments, it exhibits constant returns to scale and it is homogeneous of degree 1.[11] In this case, $\kappa \in (0,1)$ is considered as the elasticity of matching function with respect to the number of vacancies. Given the properties of the matching function, I define labour market tightness of country j for a given period t, θ_t^j, as

$$\theta_t^j \equiv \frac{v_t^j}{u_t^j} \tag{2.19}$$

where the rate that job seekers find a job in country j at period t is $p(\theta_t^j) \equiv \frac{m(u_t^j, v_t^j)}{u_t^j}$, while the rate that firms fill a vacancy is $q(\theta_t^j) \equiv \frac{m(u_t^j, v_t^j)}{v_t^j}$. Combining with eq.(2.18), I can write the job-finding rate as:

$$p(\theta_t^j) = (\theta_t^j)^{\kappa} \tag{2.20}$$

and I can write the vacancy-matching rate as

$$q(\theta_t^j) = (\theta_t^j)^{\kappa-1} \tag{2.21}$$

From (2.20), $p(\theta_t^j)$ is increasing in θ_t^j, as for a job seeker there is higher probability to find a job as θ_t^j increases. Following the same intuition, by (2.21), $q(\theta_t^j)$ is decreasing in θ_t^j.

[11] See empirical evidence in Pissarides and Petrongolo (2001) that support the view that the matching function exhibits constant returns to scales. See in Pissarides (2000) ch. 1 for evidence that the matching function can be approximated by a log-linear Cobb-Douglas function.

A job match ends for exogenous reasons, at a constant separation rate δ. Given this and the definition of the matching function, I define the evolution of employment over time in each country as:

$$N_t^j = (1-\delta)N_{t-1}^j + m(u_t^j, v_t^j) \tag{2.22}$$

At every period the number of workers in country j is given by the number of those who continue to work from the previous period plus the number of new job matches (new workers). Unemployment evolves over time according to:

$$u_t^j = 1 - N_{t-1}^j + \delta N_{t-1}^j = 1-(1-\delta)N_{t-1}^j \tag{2.23}$$

which implies that unemployment is a predetermined variable at time t.[12] Equations (2.22)and (2.23) with (2.19) determine the Beveridge curve of the DMP model, i.e. the negative relationship between the number of vacancies and unemployment.

The intermediate good producers

The intermediate good, X_t^j, is sold in a perfectly competitive market in a real price $\phi_t^j \equiv \frac{(P_t^j)^I}{P_{ct}^j}$. Intermediate good producers deflate their income with CPI and not with the domestic price index, because in order to post vacancies they buy units of the tradable final good produced in both countries. [13] The technology is described by the production function:

$$X_t^j = Z_t^j N_t^j$$

where Z_t^j is the country-specific productivity. Letting $\log Z_t^j \equiv z_t^j$, productivity can be written in log deviation from the steady state terms. Assuming that it follows an AR (1) process, productivity is given by:

$$\hat{z}_t^j = \rho \hat{z}_{t-1}^j + \epsilon_t^j \tag{2.24}$$

[12]I use this law of motion of employment following Blanchard and Gali (2010) and Ravenna and Walsh (2011). Instead, $u_t^j = 1 - N_t^j$ could be used, but this measurement of unemployment considers those who do not end up with a job match at the end of period t.

[13]This is different with other currency union models with unemployment, such as in Abbritti and Mueller (2013) and Campolmi and Faia (2011) in which intermediate good firms deflate their nominal income with the domestic price index.

where $\rho \in (0,1)$ and $\epsilon_t \sim NID(0,\sigma^2)$.[14]

Intermediate good producers' decisions

Following a similar setup for a closed economy in Ravenna and Walsh (2011), I assume that intermediate good producers of country j buy $v_t^j(j)$ units of the differentiated final good in order to post vacancies, subject to the constraint

$$\left(\int_0^{\zeta} v_t^j(a)^{\frac{\gamma-1}{\gamma}} da\right)^{\frac{\gamma}{\gamma-1}} + \left(\int_{\zeta}^{1} v_t^j(b)^{\frac{\gamma-1}{\gamma}} db\right)^{\frac{\gamma}{\gamma-1}} = v_t^j(j)$$

The total nominal spending on posting vacancies in country j is given by:

$$\psi\left(\int_0^{\zeta} p_t(a) v_t^j(a) da + \int_{\zeta}^{1} p_t(b) v_t^j(b) db\right)$$

where ψ is the cost per posted vacancy (i.e., management or human resources costs).

The total number of vacancies expressed in terms of final goods, v_t^j is given by the Dixit-Stiglitz indices, which are equivalent with those given from (2.4) and (2.5):

$$v_{At}^j \equiv \left[\left(\frac{1}{\zeta}\right)^{\frac{1}{\gamma}} \int_0^{\zeta} v_t^j(a)^{\frac{\gamma-1}{\gamma}} da\right]^{\frac{\gamma}{\gamma-1}} \tag{2.25}$$

and

$$v_{Bt}^j \equiv \left[\left(\frac{1}{1-\zeta}\right)^{\frac{1}{\gamma}} \int_{\zeta}^{1} v_t^j(b)^{\frac{\gamma-1}{\gamma}} db\right]^{\frac{\gamma}{\gamma-1}} \tag{2.26}$$

Hence, intermediate good producers face similar intratemporal optimisation problems with consumers. These are the optimal choice of nominal spending and the optimal allocation of shares of income between the final goods produced domestically or abroad. Combining the solution of these problems, yields a system of de-

[14] Productivity is defined in non-linear form as $Z_t^j \equiv (Z^j)_{t-1}^{\rho} (Z^j)^{(1-\rho)} e^{\epsilon_t^j}$.

mand equations for final goods used for posting vacancies:

$$v_t^j(a) = \left(\frac{p_t(a)}{P_{At}}\right)^{-\gamma} S_t^{(1-\zeta)} \psi v_t^j \tag{2.27}$$

$$v_t^j(b) = \left(\frac{p_t(b)}{P_{Bt}}\right)^{-\gamma} S_t^{-\zeta} \psi v_t^j \tag{2.28}$$

Combining (2.10) with (2.27) and (2.11) with (2.28), I can write the total demand for the differentiated final goods

$$(y_t^j)^d(a) = \left(\frac{p_t(a)}{P_{At}}\right)^{-\gamma} S_t^{(1-\zeta)} \left\{C_t^j + \psi v_t^j\right\} \tag{2.29}$$

and

$$(y_t^j)^d(b) = \left(\frac{p_t(b)}{P_{Bt}}\right)^{-\gamma} S_t^{-\zeta} \left\{C_t^j + \psi v_t^j\right\} \tag{2.30}$$

produced in country A and B respectively.

The intertemporal problem of the intermediate good producers is, given the law of motion of employment, equation (2.22), to choose the number of vacancies, v_t^j, which maximise the expected present discounted sum of real profits. In Appendix A.6 I show that for country A the solution of the problem yields:

$$\frac{\psi}{q(\theta_t^A)} = \frac{(P_t^A)^I}{P_{ct}^A} Z_t^A - \frac{W_t^A}{P_{ct}^A} + (1-\delta) E_t \beta_{t,t+1} \left\{ \left(\frac{S_{t+1}}{S_t}\right)^{(1-\zeta)} \frac{\psi}{q(\theta_{t+1}^A)} \right\} \tag{2.31}$$

where I have used the definition for the stochastic discount factor in open economies,

$$\beta_{t,t+1} \equiv \beta \left(\frac{C_{t+1}^j}{C_t^j}\right)^{-\sigma^{-1}} \left(\frac{S_t}{S_{t+1}}\right)^{(1-\zeta)}$$

A similar condition holds for country B.[15]

Equation (2.31) implies that the optimal hiring decision by intermediate good firms requires the average cost per vacancy $\frac{\psi}{q(\theta_t^j)}$ to be equal to the value of the average job. That is given by the difference between the real marginal product and real marginal cost of labour, $\frac{(P_t^A)^I}{P_{ct}^A} Z_t^A - \frac{W_t^A}{P_{ct}^A}$, plus the expected continuation value of the job, $(1-\delta) E_t \beta_{t,t+1} \left\{ \left(\frac{S_{t+1}}{S_t}\right)^{(1-\zeta)} \frac{\psi}{q(\theta_{t+1}^A)} \right\}$. As intermediate good producers buy units of the final good, which is tradable, the expected continuation value of the job de-

[15] See appendix A.6 for details about country B.

pends on the terms of trade. Consequently, the optimal hiring decision requires firms to consider the changes of the terms of trade between two periods, i.e., the term $\left(\frac{S_{t+1}}{S_t}\right)^{(1-\zeta)}$.

2.2.4 Wage determination

Flexible real wage

The equilibrium in the labour market is concluded with the wage determination. As it is discussed in Pissarides (2000), the economic rent created from the costly search is typically shared through a Nash bargaining process of the real wage. Bargaining parties renegotiate at every time period taking into account the economic conditions, like the productivity changes. This implies a scheme of real wage flexibility.

The problem is formalised in terms of country A. I follow a procedure similar to Thomas (2008) in a closed economy NK model with unemployment. Let $\xi \in (0,1)$ be the bargaining power of a representative firm.[16] The firm's surplus, Λ^f_{at}, is given by the marginal value of an additional employment relationship. That is:

$$\Lambda^f_{at} = \phi^A_t Z^A_t - \frac{W^A_{at}}{P^A_{ct}} + (1-\delta)E_t\beta_{t,t+1}\left(\frac{S_{t+1}}{S_t}\right)^{(1-\zeta)}\Lambda^f_{at+1} \tag{2.32}$$

where recall that $\phi^A_t \equiv \frac{(P^A_t)^I}{P^A_{ct}}$ is the real price for intermediate good firms. The worker's surplus, Λ^w_{at}, is given by the marginal value added on the household's welfare criterion from an additional employment relationship. In order to express this value in terms of households' utility, I divide it by the marginal utility with respect to consumption. I obtain the following:

$$\begin{aligned}\Lambda^w_{at} = \frac{W^A_{at}}{P^A_{ct}} - d\left(N^A_t\right)^{\varphi}\left(C^A_t\right)^{\sigma^{-1}} - (1-\delta)E_t\beta_{t,t+1}\left(\frac{S_{t+1}}{S_t}\right)^{(1-\zeta)} p(\theta_{t+1})\Lambda^w_{at+1} \\ +(1-\delta)E_t\beta_{t,t+1}\left(\frac{S_{t+1}}{S_t}\right)^{(1-\zeta)}\Lambda^w_{at+1}\end{aligned} \tag{2.33}$$

[16]Symmetry implies that in equilibrium all firms behave in the same way hence I have dropped the subscript i.

In Appendix (A.7), I show that the solution of the Nash bargaining problem determines the real wage in country A:

$$\left(\frac{W_t^A}{P_{ct}^A}\right)^{Nash} = (1-\xi)\left(\phi_t^A Z_t^A + (1-\delta)\psi E_t \beta_{t,t+1}\left(\frac{S_{t+1}}{S_t}\right)^{(1-\zeta)}\theta_{t+1}^A\right) + \xi d_u \left(N_t^A\right)^{\varphi}\left(C_t^A\right)^{\sigma^{-1}} \tag{2.34}$$

A similar condition holds for country B. Notice that in the absence of search and matching frictions (i.e., $\psi = 0$) the current labour market would be perfect. Thus, the real wage would be equal to the workers' marginal rate of substitution between labour and consumption, $d_u \left(N_t^A\right)^{\varphi}\left(C_t^A\right)^{\sigma^{-1}}$ to be consistent with households' optimal intertemporal choices. In addition, in a perfect labour market, the real wage would be equal to the real marginal product of labour $\phi_t^A Z_t^A$, to be consistent with the profit maximisation behaviour of firms. However now, $\psi > 0$ creates a spread between the marginal rate of substitution and the marginal product of labour. From equation (2.34), the Nash bargaining real wage is determined by the weighted average of the higher wage that firms are willing to offer, and the lower wage that workers are willing to accept (reservation wage).[17] The weights are given by the bargaining power of both workers and firms. Firms use their bargaining power, ξ, to push the real wage down to the worker's reservation wage, $d_u \left(N_t^A\right)^{\varphi}\left(C_t^A\right)^{\sigma^{-1}}$. Workers use their bargaining power, $(1-\xi)$, to push the real wage up to the higher level of the wage that firms are keen to offer. That is the sum of firm's real marginal product of labour, $\phi_t^A Z_t^A$, and the savings from not posting a vacancy the next period, i.e. the continuation value of the job, $(1-\delta)\psi E_t \beta_{t,t+1}\left(\frac{S_{t+1}}{S_t}\right)^{(1-\zeta)}\theta_{t+1}^A$.

Real wage rigidity

The Nash bargaining real wage determination in the DMP model implies a real wage flexibility (Shimer (2004)). Firms and workers renegotiate the real wage every period, therefore the real wage adjusts to economic changes. This assumption has been criticised in Shimer (2004, 2005) and Hall (2005). It is highlighted that the adjustment of the Nash bargaining real wage to changes in the economic environment

[17]Notice that, as in the standard Diamond-Mortensen-Pissarides model, the job match will always be efficient. The real wage cannot be above the valued added to the firms' expected profits from an additional worker and cannot be below workers' reservation wage.

makes firms to not alter their optimal decision with regard to the number of vacancies posted. In this case, the response of unemployment in productivity shocks could be small and not reflect the unemployment fluctuations observed in reality. Instead, in this papers, it is supported that assuming a form of real wage rigidity (RWR) in the DMP model can solve the unemployment volatility puzzle. If wages are rigid, in the presence of shocks, the added value in firms' profits from an additional hiring changes affecting their optimal decision of posting vacancies.

In this paper, I build on this criticism and allow for some form of RWR. Assuming RWR in this paper serves three purposes. First, it is an assumption that is supported by empirical evidence for european countries (using euro or non-euro currency), thus, it adds more realism to the model. Babecky (2010) and Messina (2010) provide empirical evidence that support the assumption of downward RWR. In particular, Babecky (2010) analyses data obtained from firm-level surveys in 15 countries of the European Union. He finds that 17% of firms have applied a real wage indexation scheme. This scheme applies when firms link nominal wage with a wage deflator (i.e., CPI) making real wage more sluggish. In addition, Messina (2010) provides data for 13 sectors of three Eurozone countries (Spain, Portugal and Belgium) and Denmark. He finds evidence of downward RWR. However he highlights that the degree of RWR estimates varies across countries. Both authors also find that the degree of RWR depends on labour market corporate characteristics. In this paper, for simplicity it has been assumed that firms are homogenous within countries, hence, these characteristics are not taken into consideration.

The second reason of assuming a form of RWR in this paper is that it provides the convenience of a construction of a labour market heterogeneity index across countries. This index is just the degree of RWR differential between the two countries. Furthermore, assuming (real) wage rigidity in this NK model solves the divine coincidence problem and allows to observe stabilisation trade-offs between the optimal policy objectives. This remark has been highlighted first in closed economy NK models without unemployment (Erceg, Henderson, and Levin (2000), Blanchard and Gali (2007)), and then it has been delivered to closed economy NK models with involuntary unemployment (among others, Krause and Lubik (2007), Thomas (2008), Christoffel, Kuester, and Linzert (2009) and Blanchard and Gali (2010)).[18]

[18]Some empirical evidence has challenged the assumption of real wage rigidity. While also the aggregate data are supportive of this assumption, there is some ambiguity which lies on the compo-

I define the rigid real wage as a weighted average between the Nash real wage and a real wage norm similarly proposed by Hall (2005). This norm can be the last period wage or the steady-state real wage. The RWR scheme is then formalised as follows:

$$\left(\frac{W_t^j}{P_{ct}^j}\right)^{RWR} = (1-\mu^j)\left(\frac{W_t^j}{P_{ct}^j}\right)^{Nash} + \mu^j W^j \tag{2.35}$$

where W^j is the wage norm and can be $W^j \equiv W_{t-1}^j$ or $W^j \equiv \overline{W}^j$. The degree of RWR is μ^j, $j \in [A, B]$. Higher μ^j makes the real wage to be based more on the norm, thus, less responsive to current period shocks.

Equilibrium under RWR

Under RWR, the equilibrium in labour market is given by the Beveridge curve and equations (2.31), (2.35). These equations, combined with (2.34), give the optimal hiring decision under RWR, which is a key equation for the model. For country A, I obtain:

$$\begin{aligned}\frac{\psi}{q(\theta_t^A)} &= \phi_t^A Z_t^A - (1-\mu^A)(1-\xi^A)\left(\phi_t^A Z_t^A + (1-\delta)\psi E_t \beta_{t,t+1}\left(\frac{S_{t+1}}{S_t}\right)^{(1-\zeta)}\theta_{t+1}\right) - \\ &(1-\mu^A)\xi^A d_u \left(N_t^A\right)^{\varphi}\left(C_t^A\right)^{\sigma^{-1}} - \mu^A W^A + (1-\delta)E_t\beta_{t,t+1}\left(\frac{S_{t+1}}{S_t}\right)^{(1-\zeta)}\frac{\psi}{q(\theta_{t+1}^A)}\end{aligned} \tag{2.36}$$

while a similar condition holds for country B. In the presence of real wage rigidity firms' optimal number of posted vacancies requires a look backwards to the agreed real wage of the previous period.

2.2.5 Final good producers

In each country, the final good producers $a \in [0, \zeta)$, for $j = A$ and $b \in [\zeta, 1]$ for $j = B$ are monopolistic competitors and produce a differentiated final good which can sell in both countries without any transaction costs. In order to produce, they use the intermediate good which is bought from the domestic intermediate good producers

sition bias of real wages which time-series data cannot capture. This has been highlighted first by Solon, Barsky, and Parker (1994). In addition, Pissarides (2009) provides a survey of mixed evidence from panel data which support the real wage flexibility assumption for some euro area countries, the US and the UK. Also, Haefke, Sonntag, and van Rens (2013) recently provide evidence of real wage flexibility for the US. These results are in contrast with those of Babecky (2010) and Messina (2010).

at a price $\frac{(P_t^A)^I}{P_{At}} \equiv \phi^A_{t+s} S_{t+s}^{(1-\zeta)}$ in country A and $\frac{(P_t^B)^I}{P_{Bt}} \equiv \phi^B_{t+s} S_{t+s}^{\zeta-1}$ in country B. This price is the final good producers' real marginal cost in countries A and B respectivelly.[19] The production technology is given by:

$$y_t^j(\iota) = X_t^j(\iota)$$

for $j \in [A, B]$, and $\iota = a$ for $j = A$ and $\iota = b$ for $j = B$. The NK element of monopolistic competition creates a proper environment to invite the other element of nominal price rigidity which is crucial for monetary policy non-neutrality. For this reason, I assume that the price setting decision is subject to nominal price rigidity a la Calvo (1983). Every time period each producer faces a probability $(1-\omega)$ of resetting her own price, which is independent of the time since the last reset. Each producer chooses a price $p_t(j)$ for $j = a, b$ to maximise her expected discounted profits considering that her choice will be optimal at time $t+s$ with probability ω^s, subject to the demand equations (2.29) and (2.30) given by the consumers' problem. The optimal price setting for a final good firm in country A solves the problem:

$$\max_{p_t^*(a)} E_t \sum_{s=0}^{\infty} \omega^s \beta_{t,t+s} \left\{ (1+\tau) \frac{p_t(a)}{P_{At+s}} y^A_{t+s}(a) - \phi^A_{t+s} S_{t+s}^{(1-\zeta)} y^A_{t+s}(a) \right\}$$

for $s = 0, 1, 2, 3...$, subject to

$$(y_t^A)(a) = \left(\frac{p_t(a)}{P_{At}} \right)^{-\gamma} S_t^{(1-\zeta)} \left\{ C_t^j + \psi v_t^j \right\}$$

Following the standard NK literature, I assume that final output is subsidised by a constant rate τ which guarantees that the inefficiency caused from monopolistic competition will be eliminated.[20]

The first-order condition associated with the problem above, using the defini-

[19]The reason that the final good producers deflate their nominal marginal cost $\left(P_t^j\right)^I$ with the domestic price index is because in the setup of the model they buy the intermediate good from domestic producers only.

[20]This subsidy is financed through a lump-sum tax, thus, without causing further inefficiencies.

tion of the stochastic discount factor yields:

$$(1-\gamma)(1+\tau)E_t\sum_{s=0}^{\infty}\omega^s\beta_{t,t+s}y_{t+s}^A\frac{1}{P_{At+s}}+$$
$$\gamma E_t\sum_{s=0}^{\infty}\omega^s\beta_{t,t+s}\phi_{t+s}^A y_{t+s}^A\frac{1}{p_t^*(a)}=0$$

Rearranging, I can write the above equation as:

$$p_t^*(a)=\frac{\gamma}{(\gamma-1)(1+\tau)}\frac{E_t\sum_{s=0}^{\infty}\omega^s\beta_{t,t+s}\phi_{t+s}^A S_{t+s}^{(1-\zeta)}y_{t+s}^A}{E_t\sum_{s=0}^{\infty}\omega^s\beta_{t,t+s}y_{t+s}^A\frac{1}{P_{At+s}}} \tag{2.37}$$

A similar result holds for country B. Under the assumption of perfect price flexibility, $\omega=0$, this result is reduced to

$$\frac{p_t^*(a)}{P_{At}}=\frac{\gamma}{(\gamma-1)(1+\tau)}S_t^{(1-\zeta)}\phi_t \tag{2.38}$$

i.e. firms set the relative price equal to a markup over the real marginal cost. Notice that because the economies are open, the optimal price setting is affected from the terms of trade.

2.2.6 Market clearing in the goods market

The total demand for the final good at country level is given by the Dixit- Stiglitz aggregators:

$$Y_t^A\equiv\left[\left(\frac{1}{\zeta}\right)\int_0^{\zeta}y_t^j(a)^{\frac{\gamma-1}{\gamma}}da\right]^{\frac{\gamma}{\gamma-1}} \tag{2.39}$$

$$Y_t^B\equiv\left[\left(\frac{1}{1-\zeta}\right)\int_{\zeta}^{1}y_t^j(b)^{\frac{\gamma-1}{\gamma}}db\right]^{\frac{\gamma}{\gamma-1}} \tag{2.40}$$

The market clearing condition for the final good requires at every period the total quantity of final good to be consumed by households or be purchased by intermediate good producers. Combining equation (2.29) with (2.39), and (2.30) with (2.40), and using also the definition for the domestic price indices yields:

$$Y_t^A = S_t^{(1-\zeta)} \left\{ C_t^A + \psi v_t^A \right\} \tag{2.41}$$

$$Y_t^B = S_t^{-\zeta} \left\{ C_t^B + \psi v_t^B \right\} \tag{2.42}$$

The intermediate good is used only as input in the final good production and is sold domestically. Since a unit of the homogeneous intermediate good produces a unit of the final differentiated good, market clearing implies $y_t^A = X_t^A$ and $y_t^B = X_t^B$,

2.2.7 The social planner's problem

In this section, I analyse the social planner's problem to examine under which conditions can be replicated by the decentralised equilibrium and if this is attainable. The reason that I perform this exercise is because I use the efficient equilibrium as the reference state of the currency union. It is the aim of this paper to analyse optimal monetary policy, when in the presence of productivity shocks, the economy deviates from the efficient equilibrium. Ravenna and Walsh (2011) also use the efficient equilibrium as the reference state in a closed economy NK model with unemployment.

The social planner maximises households' welfare criterion in each country subject to the technology and the law of motions of employment and unemployment. The solution of the problem requires the choice of agents' control variables, C_t^j, v_t^j, and the choice of the economy's state variables. The problem is formalised for economy A:

$$\max_{C_t^A, v_t^A, N_t^A, u_t^A} E_0 \sum_{t=0}^{\infty} \beta^t \Bigg\{ \left(\frac{(C_t^A)^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} - d\frac{(N_t^A)^{1+\varphi}}{1+\varphi} \right) + \lambda_{1t}^A \Big(Z_t^A N_t^A - S_t^{(1-\zeta)} \big\{ C_t^A + \psi v_t^A \big\} \Big)$$
$$+ \lambda_{2t}^A \Big((1-\delta) N_{t-1}^A + (v_t^A)^{\kappa} (u_t^A)^{(1-\kappa)} - N_t^A \Big) + \lambda_{3t}^A \Big(u_t^A - 1 + (1-\delta) N_{t-1}^A \Big) \Bigg\}$$

In the Appendix A.8, I show that the combination of the first-order conditions yields the social planner's outcome:

$$\frac{\psi}{q(\theta_t^A)} = \kappa\left(S^{(\zeta-1)}Z_t^A - d_u\left(N_t^A\right)^{\varphi}\left(C_t^A\right)^{\sigma^{-1}}\right) + (1-\delta)\left(E_t\beta_{t,t+1}\left(\frac{S_{t+1}}{S_t}\right)^{(1-\zeta)}\frac{\psi}{q(\theta_{t+1}^A)}\right)$$
$$-(1-\delta)\left((1-\kappa)E_t\beta_{t,t+1}\left(\frac{S_{t+1}}{S_t}\right)^{(1-\zeta)}\psi\theta_{t+1}^j\right) \tag{2.43}$$

A similar condition holds for country B. Now, I compare the Social planner's outcome, equation (2.43), with the decentralised outcome, equation (2.36). From (2.43), efficiency requires $\phi_t^A = S_t^{(\zeta-1)}$ (and for country B $\phi_t^B = S_t^{(1-\zeta)}$). From (2.38), this requires prices to be flexible and particularly, the policy-maker of each country to eliminate the monopolistic mark-up by imposing a tax-financed subsidy to the final good sales equal to $\tau = \frac{1}{\gamma-1}$.[21] Moreover, comparing the two outcomes, efficiency requires $\kappa = \xi$. That is, the elasticity of the matching function with respect to vacancies must be equal to the firm's bargaining power. This is the Hosios (1990) condition which satisfies that job creation is efficient. Finally, comparing the two outcomes, we realise that any positive degree of RWR is undesirable, so efficiency requires $\mu = 0$.[22] In summary:

Proposition 1. *In a currency union NK model with labour market frictions and RWR, the decentralised outcome can replicate the efficient flexible-price equilibrium under three conditions for each country: i) The policy-maker imposes a tax-financed subsidy to the final good sales equal to $\tau = \frac{1}{\gamma-1}$. ii) The Hosios (1990) condition for efficient job creation which is, $\kappa = \xi$, holds and iii) The degree of RWR is equal to zero.*

In the Appendix A.8, I also show that the conditions for an efficient steady-state in the currency union can be summarised by:

$$\delta_1^j = -\beta\delta_2^j \tag{2.44}$$

[21] For assuming tax-financed subsidies see among others, Rotemberg and Woodford (1997). This is also used in Thomas (2008).

[22] Similar conditions for an efficient decentralised outcome in a currency union model with unemployment have also be proposed in Abbritti and Mueller (2013).

where:

$$\delta_1^A = \frac{1}{S^{(1-\zeta)}} - \frac{\psi V^A}{\delta\kappa N^A} - d_u (N^A)^{\varphi} (C^A)^{-\sigma} \tag{2.45}$$

$$\delta_1^B = \frac{1}{S^{-\zeta}} - \frac{\psi V^B}{\delta\kappa N^B} - d_u (N^B)^{\varphi} (C^A)^{-\sigma} \tag{2.46}$$

$$\delta_2^j = (1-\delta)\frac{\psi V^j}{\kappa}\left(\frac{1}{\delta N^j} - \frac{(1-\kappa)}{u^j}\right) \tag{2.47}$$

2.3 Optimal monetary policy. The L-Q Approach

To characterise the optimal monetary policy in the currency union, I follow a Linear-Quadratic (L-Q) approach introduced by Rotemberg and Woodford (1997) and Woodford (2003). The L-Q approach has been used in several closed economy NK models with involuntary unemployment like Thomas (2008), Blanchard and Gali (2010) and Ravenna and Walsh (2011). It has been also used in NK currency union models without involuntary unemployment like Benigno (2004) and Gali and Monacelli (2008). Following the L-Q approach, I first linearise the equilibrium conditions and resource constraints of the model and I then take a second order approximation of the households' utility function.

2.3.1 The log-linearised model

The linear representation of the model requires some extra notation. For any generic variable X_t, a small letter with a hat denotes the log deviation of variable X_t from its steady-state value X. That is: $\hat{x}_t = \log X_t - \log X$. The model is log-linearised by applying some standard techniques proposed by Uhlig (1997).

The final good market clearing condition for both countries, equations (2.41) and (2.42), are log-linearised according to:

$$\hat{y}_t^A = (1-\zeta)\hat{s}_t^A + \frac{S^{(1-\zeta)} C^A}{Y^A}\hat{c}_t^A + \psi \frac{S^{(1-\zeta)} v^A}{Y^A}\hat{v}_t^A \tag{2.48}$$

$$\hat{y}_t^B = -\zeta \hat{s}_t^B + \frac{S^{-\zeta} C^B}{Y^B}\hat{c}_t^B + \psi \frac{S^{-\zeta} v^B}{Y^B}\hat{v}_t^B \tag{2.49}$$

The intermediate good production function is given by $\hat{x}_t^j = \hat{z}_t^j + \hat{n}_t^j$ for $j = A, B$. Given the final good production function, $\hat{y}_t^j = \hat{x}_t^j$, I can write:

$$\hat{y}_t^j = \hat{z}_t^j + \hat{n}_t^j \tag{2.50}$$

while the productivity term is already given in log-linear form from equation (2.24). Here is repeated for convenience: $\hat{z}_t^j = \rho \hat{z}_{t-1}^j + \epsilon_t^j$. The Euler equation in log-linear form is:

$$\hat{c}_t^j = E_t \hat{c}_{t+1}^j + \sigma E_t \pi_{c\,t+1}^j - \sigma \hat{q}_t$$

where I have used that inflation is defined as $\pi_t = \log\left(\frac{P_t}{P_{t-1}}\right)$ Notice that $\hat{q}_t$ is the absolute deviation of the nominal interest rate from its steady-state, as it is already expressed in percentages. [23] The Fisher equation, which links the nominal with the real interest rate, in its linear form is given by:

$$\hat{r}_t^j = \hat{q}_t - E_t \pi_{c\,t+1}^j \tag{2.51}$$

By using the Fisher equation, I can rewrite the Euler equation:

$$\hat{c}_t^j = E_t \hat{c}_{t+1}^j - \sigma \hat{r}_t^j \tag{2.52}$$

The equations (2.15) and (2.16) which link CPI with the domestic price index, are log-linearised as follows:

$$\hat{p}_{c\,t}^A = \hat{p}_{A t} + (1-\zeta)\hat{s}_t \quad , \quad \hat{p}_{c\,t}^B = \hat{p}_{B t} + (\zeta - 1)\hat{s}_t$$

Subtracting their own one period lag, I get a link of CPI inflation with the domestic inflation and the terms of trade:

$$\pi_{c\,t}^A = \pi_{A t} + (1-\zeta)\Delta\hat{s}_t \tag{2.53}$$

$$\pi_{c\,t}^B = \pi_{B t} + (\zeta-1)\Delta\hat{s}_t \tag{2.54}$$

where inflation is expressed as a log-deviation from a zero steady-state and the log-

[23] For a derivation of the log-linear Euler equation, see the Appendix B.6.1. Also, see the Appendix of chapter 2 in Walsh (2010).

linear form of the terms of trade is given by $\hat{s}_t = \hat{p}_{Bt} - \hat{p}_{At}$. This also implies:

$$\Delta \hat{s}_t = \pi_{Bt} - \pi_{At} \tag{2.55}$$

The labour market tightness relationship:

$$\hat{\theta}_t^j = \hat{v}_t^j - \hat{u}_t^j \tag{2.56}$$

The law of motion of employment:

$$\hat{n}_t^j = (1-\delta)\hat{n}_{t-1}^j + \delta\left((\kappa^A - 1)\hat{\theta}_t^j + \hat{v}_t^j\right) \tag{2.57}$$

The unemployment relationship:

$$\hat{u}_t^j = -\alpha_0^j \hat{n}_{t-1}^j \tag{2.58}$$

where $\alpha_0^j = (1-\delta)\frac{N^j}{u^j}$
The intermediate good firm's optimal hiring decision in country A:

$$\begin{aligned}\psi(\theta^A)^{(1-\kappa^A)}(1-\kappa^A)\hat{\theta}_t^A = \gamma_1^A(\hat{\phi}_t^A + \hat{z}_t^A) + (1-\delta)\beta\psi\frac{\theta}{\sigma}\left(\gamma_0 - (\theta^A)^{-\kappa^A}\right)\left(\hat{c}_{t+1}^A - \hat{c}_t^A\right) \\ -(1-\delta)\beta\psi\theta^A\left(\gamma_0^A - (1-\kappa^A)(\theta^A)^{-\kappa}\right)\hat{\theta}_{t+1}^A \\ -(1-\mu)\xi^A\gamma_2^A\left(\varphi\hat{n}_t^A + \frac{1}{\sigma}\hat{c}_t^A\right)\end{aligned} \tag{2.59}$$

where I have already substituted the log-linearised version of the real wage with RWR and $\gamma_0^A = (1-\mu^A)(1-\xi^A)$, $\gamma_1^A = (1-\gamma_0^A)\phi^A$, $\gamma_2^A = d_u(N^A)^{\varphi}(C^A)^{\sigma^{-1}}$. A similar condition holds for country B.

From the assumption of nominal price rigidity and by using the domestic price index, equation (2.8), the average domestic price at time t in country $j \in [A, B]$ is given by:

$$P_{At}^{(1-\gamma)} = (1-\omega^A)p_t^{*(1-\gamma)}(a) + \omega^A P_{At-1}^{(1-\gamma)} \tag{2.60}$$

In the Appendix A.9, I show that by taking a first-order approximation of the optimal price setting decision, equation (2.37), and combining with equation (2.60), I derive the New Keynesian Phillips Curve (NKPC) in a currency union. For country A, this is given by:

$$\pi_{At} = \delta_p^A\left(\hat{\phi}_t^A + (1-\zeta)\hat{s}_t\right) + \beta E_t \pi_{At+1} \tag{2.61}$$

while for country B, it is given by:

$$\pi_{Bt} = \delta_p^B\left(\hat{\phi}_t^B + (\zeta-1)\hat{s}_t\right) + \beta E_t \pi_{Bt+1} \tag{2.62}$$

where $\delta_p^j = \frac{(1-\omega^j\beta)(1-\omega^j)}{\omega^j}$ is the elasticity of domestic inflation with respect to the real marginal cost of intermediate good firms, $\hat{\phi}_t^j$ and with respect to the terms of trade adjusted by an index for economic openness.

The NKPC implies that domestic inflation is a forward looking variable and has two driving forces, the real marginal cost and the terms of trade. The latter creates an extra cost channel on domestic inflation comparing to the NKPC in a standard closed economy NK model. Therefore there is an international spillover on domestic inflation through the terms of trade.

2.3.2 The reduced-form representation

The log-linearised model described in the previous section can be reduced to a system of difference equations in which the aggregate demand side of the member states is represented by a dynamic IS curve expressed in terms of domestic unemployment. The NKPC can be also expressed in terms of domestic unemployment.[24]

The dynamic IS for small open economies

Focusing on country A, by combining the production function, equation (2.50), and the market clearing condition, equation (2.48), I can solve for consumption:

$$\frac{S^{(1-\zeta)}C^A}{Y^A}\hat{c}_t^A = \hat{z}_t^A + \hat{n}_t^A - (1-\zeta)\hat{s}_t - \psi\frac{S^{(1-\zeta)}v^A}{Y^A}(\hat{\theta}_t^A + \hat{u}_t^A) \tag{2.63}$$

Taking the law of motion of unemployment, equation (2.58), one period forward, and then using the law of motion of employment, equation (2.57), to substitute for $\hat{n}_t^A$, yields:

$$\hat{u}_{t+1}^A = (1-\delta)\left(1 - \frac{\delta N^A}{u^A}\right)\hat{u}_t^A - \alpha_0^A\delta\kappa^A\hat{\theta}_t^A \tag{2.64}$$

[24] For this exercise, I have found the work by Ravenna and Walsh (2011) to be very useful.

Using (2.64) to solve for $\hat{\theta}_t^A$ and substitute in (2.63), I can also substitute for $\hat{n}_t^A$ from equation (2.57). Doing this, I get an expression for consumption in terms of unemployment, the terms of trade and the productivity shock. Taking this expression one period forward, I can use the Euler equation, (2.52), and substitute for $\hat{c}_t^A$ and $\hat{c}_{t+1}^A$ to get the dynamic IS expressed in terms of unemployment. For country A, this is:

$$\hat{u}_{t+1}^A = \frac{\eta_1^A}{\eta_1^A+\eta_2^A}\hat{u}_t^A + \frac{\eta_2^A}{\eta_1^A+\eta_2^A}E_t\hat{u}_{t+2}^A + \frac{\sigma^A}{\eta_1^A+\eta_2^A}\hat{r}_t^A + \frac{\alpha_1^A}{\eta_1^A+\eta_2^A}(1-\zeta)(E_t\hat{s}_{t+1}-\hat{s}_t) - \frac{\alpha_1^A}{\eta_1^A+\eta_2^A}\left(E_t\hat{z}_{t+1}^A-\hat{z}_t^A\right) \quad (2.65)$$

A similar condition as equation (2.64) holds for country B. Substituting as above, I get:

$$\hat{u}_{t+1}^B = \frac{\eta_1^B}{\eta_1^B+\eta_2^B}\hat{u}_t^B + \frac{\eta_2^B}{\eta_1^B+\eta_2^B}E_t\hat{u}_{t+2}^B + \frac{\sigma^B}{\eta_1^B+\eta_2^B}\hat{r}_t^B - \frac{\alpha_1^B}{\eta_1^B+\eta_2^B}\zeta(E_t\hat{s}_{t+1}-\hat{s}_t) - \frac{\alpha_1^B}{\eta_1^B+\eta_2^B}\left(E_t\hat{z}_{t+1}^B-\hat{z}_t^B\right) \quad (2.66)$$

where $\eta_1^j = -\alpha_2(\alpha_0\delta\kappa^j+\delta_u^j)$, $\eta_2^j = (\alpha_2^j-\alpha_3^j)$ and $\alpha_1^A = \frac{Y^A}{S^{(1-\zeta)}C^A}$, $\alpha_1^B = \frac{Y^B}{S^{-\zeta}C^B}$, $\alpha_2^j = \frac{\psi^j v^j}{\alpha_0^j\delta\kappa^j C^j}$, $\alpha_3^j = \frac{\alpha_1^j}{\alpha_0^j}$ and $\delta_u^j = ((1-\delta)-\alpha_0^j\delta)$

From the dynamic IS curves, equations (2.65) and (2.66) we understand that domestic unemployment has a forward looking and a backward looking component. It depends also on the relative change of the terms of trade.

The NKPC for small open economies

In order to get an NKPC with unemployment, I rearrange equation (2.59) and solve for the real marginal cost $\hat{\varphi}_t^j$. Then, I use the Euler equation to eliminate $E_t\hat{c}_{t+1}^j-\hat{c}_t^j$ and then I use the market clearing condition to express $\hat{c}_t^j$ in terms of employment. Using equation (2.64), I eliminate θ_t^j by expressing it in terms of unemployment. Finally, I use equation (2.57) to express n_t^j in terms of unemployment. Applying these changes, I get an expression of the NKPC:

$$\pi_{At} = \beta E_t \pi_{At+1} + \delta_p^A \rho_0^A \hat{u}_t^A + \delta_p^A \rho_1^A \hat{u}_{t+1}^A - \delta_p^A \rho_2^A E_t \hat{u}_{t+2}^A \\ + \delta_p^A \rho_3^A ((1-\zeta)\hat{s}_t - \hat{z}_t^A) + \delta_p^A \rho_4^A \hat{r}_t^A \tag{2.67}$$

$$\pi_{Bt} = \beta E_t \pi_{Bt+1} + \delta_p^B \rho_0^B \hat{u}_t^B + \delta_p^B \rho_1^B \hat{u}_{t+1}^B - \delta_p^B \rho_2^B E_t \hat{u}_{t+2}^B + \delta_p^B \rho_3^B \zeta \hat{s}_t \\ + \delta_p^B \Big(\rho_3^B + \frac{1-2\zeta}{\zeta}\Big)\hat{z}_t^B + \delta_p^B \rho_4^B \hat{r}_t^B \tag{2.68}$$

where the ρ^j coefficients are given in the Appendix A.11. Notice in equations (2.67) and (2.68) the presence of the domestic productivity term,$\hat{z}^j$, which acts as a cost-push shock on domestic inflation. In the Appendices A.10 and A.11 I show that the dynamic IS and the NKPC can be written in variables expressed as deviations from their efficient steady-state.

The equilibrium in the currency union is described by the linear system of difference equations (2.51) for $j \in [A, B]$, (2.53), (2.54), (2.55), (2.65), (2.66), (2.67), (2.68), the AR (1) process of the domestic productivity for $j \in [A, B]$ and a nominal interest-rate rule which satisfies the determinacy of equilibrium.

2.3.3 The quadratic welfare-criterion

In this section, I take a second order approximation of households' welfare criterion to derive an objective (loss) function for the central bank of the currency union. Drawing insights from this micro-founded currency union framework, I show that the monetary policy objectives are the domestic inflation, the domestic unemployment of the member states as well as their terms of trade. Thus, this paper extends the normative analysis in Benigno (2004) and Gali and Monacelli (2008) in which the policy objectives are the domestic inflation and the domestic output gap. Here, by introducing a specific framework for an imperfect labour market for each member state we can quantify the size of welfare losses arisen from unemployment fluctuations, as well as the relative strength of the monetary policy response.

The monetary policy-maker chooses the same efficient steady-state with the social planner. In the Appendix A.12, I show that the currency union's welfare loss function is given by the discounted weighted average of households' welfare criterion and a second-order approximation yields:

$$\begin{aligned}\Omega_{t+i} = U'(C)C\Bigg(\zeta\frac{\gamma}{2\delta_p^A}\sum_{i=0}^{\infty}\beta^i\pi_{At+i}^2 + (1-\zeta)\frac{\gamma}{2\delta_p^B}\sum_{i=0}^{\infty}\beta^i\pi_{Bt+i}^2\Bigg)\\ +U'(C)N\Bigg(\zeta\frac{\delta_3^A}{2(\alpha_0^A)^2}\sum_{i=0}^{\infty}\beta^i(\hat{u}_{t+1+i}^A)^2\\ +(1-\zeta)\frac{\delta_3^B}{2(\alpha_0^B)^2}\sum_{i=0}^{\infty}\beta^i(\hat{u}_{t+1+i}^B)^2\Bigg)\\ +U'(C)C\zeta(1-\zeta)\frac{1+\sigma}{2\sigma}\sum_{i=0}^{\infty}\beta^i\hat{s}_{t+i}^2\end{aligned} \tag{2.69}$$

where $\delta_3^j = \frac{N^j}{\sigma C^j}S + \frac{\varphi U'(N^j)}{U'(C^j)}$. Notice that the weights of domestic unemployment fluctuations do not depend on the degree of RWR. Also, notice that if one of the two member states becomes very small in size i, it can be neglected, i.e. $\zeta \to 0$ or $(1-\zeta) \to 0$, the terms of trade term is eliminated and the loss function takes a form similar to the closed economy case, like Blanchard and Gali (2010).

I evaluate two optimal monetary policy regimes: The timeless perspective optimal commitment and the optimal discretion. Under optimal commitment, the central bank makes credible announcements about future actions, thus, it can affect agents' expectations. Under optimal commitment the central bank chooses the sequence of the variables:

$$\left\{\pi_{jt+i}, \hat{u}_{t+1+i}^j, \hat{s}_{t+i}, \pi_{t+i}^W, \hat{u}_{t+i}^W\right\}_{i=0}^{\infty}$$

for $j \in [A, B]$ to minimise the welfare loss of the currency union:

$$E_0\sum_{i=0}^{\infty}\beta^t\left\{\Omega_{t+i}\right\}$$

Under optimal discretion, the central bank resets its actions every period. Therefore, current period actions are not restricted at the future and any announcements made by the central bank are not considered as credible by the agents. Thus their expectations are not affected.[25]. In this case the dynamic problem becomes a single-

[25] See Walsh (2010) chapter 8.4.3 for a comparison between optimal commitment and optimal discretion.

period problem and the central bank minimises the loss function by choosing

$$\pi_{jt}, \hat{u}^{j}_{t+1}, \hat{s}_t, \pi^W_t, \hat{u}^W_t$$

Notice that, as it has been highlighted in Svensson (1999), in the presence of endogenous persistence (i.e., lagged control variables affect current variables), the single-period problem of optimal discretion becomes dynamic. This is the case here, as unemployment depends on its lagged value, thus, it exhibits some endogenous persistence.

The constraints of the dynamic optimisation problem of the central bank are given by equations (2.51) for $j \in [A, B]$, (2.53), (2.54), (2.55), (2.65), (2.66), (2.67), (2.68), the AR (1) processes of domestic productivity and the union-wide constraints:

$$\pi^W_t = \zeta\pi_{At} + (1-\zeta)\pi_{Bt} \tag{2.70}$$

$$\hat{u}^W_t = \zeta\,\hat{u}^A_t + (1-\zeta)\hat{u}^B_t \tag{2.71}$$

2.4 Quantitative analysis

The dynamic model is stochastic and is solved numerically in DYNARE.[26] For the calibration part, I choose benchmark parameter values selected from estimations of the euro area and from standard parameter values used in multi-country NK models or NK models with unemployment. The calibrated values are summarized in table 2.1.

2.4.1 Calibration

Following Gali and Monacelli (2008) and Abbritti and Mueller (2013), I assume that both countries are symmetric except for the labour market heterogeneity that I introduce.

[26]For details about DYNARE visit http://www.dynare.org/. For details about the DYNARE code for optimal monetary policy under commitment and/or discretion please see Adjemian et al. (2011).

Description	Parameter	Value	Reference
Preferences			
Discount factor	β	.99	Quarterly time interval
Index of openness	ζ	.5	Member states of equal size
Relative risk aversion	σ	1	Log utility, Abbritti and Mueller (2013)
Labour supply elasticity	φ	0	Abbritti and Mueller (2013)
Labour market			
Prob. of filling a vacancy	$q(\theta^j)$	.97	Ravenna and Walsh (2011)
Exogenous job separation rate	δ^j	.10	Shimer (2005), Ravenna and Walsh (2011)
Elasticity of vacancies w.r.t matches	κ^j	.5	Pissarides and Petrongolo (2001), Campolmi and Faia (2011)
Bargaining power of firms	ξ^j	.5	Hosios (1990) condition
Posting vacancy cost	ψ^j	.097	$\frac{.01Y}{v^j}$, Walsh (2005), Thomas (2008), Blanchard and Gali (2010)
Union-average degree of RWR	$\overline{\mu}$	.5	Blanchard and Gali (2010), Abbritti and Mueller (2013)
domestic RWR	μ^j	.1 -.9	Abbritti and Mueller (2013)
Heterogeneity Index	$\Delta\mu$	$0-0.8$	
Technology			
Nominal price rigidity	ω^j	.75	Blanchard and Gali (2010)
Steady-state marginal cost	ϕ^j	.83	Inverse mark-up, Ravenna and Walsh (2011)
Initial steady-state values			
Terms of trade	S	1	Gali and Monacelli (2008)
Unemployment	u^j	.10	Euro average unemployment rate, Blanchard and Gali (2010)
Productivity shock			
Autocorrelation	ρ	.95	Euro area estimates, Abbritti and Mueller (2013)
Std. deviation	σ_z^j	.00624	Smets and Wouters (2003), Abbritti and Mueller (2013)
Correlation btw shocks	ρ_z	.258 - 1	Abbritti and Mueller (2013)

Table 2.1: Baseline Parameter Values

Preferences: I assume a quarterly frequency for the variables of the model. The value of the discount factor β for quarterly time interval is set equal to 0.99. I assume that the member states are of equal size, so I choose $\zeta = 0.5$. I assume a relative risk aversion coefficient, $\sigma = 1$. Following Abbritti and Mueller (2013), I assume the labour supply elasticity to be $\varphi = 0$, while other studies like Blanchard and Gali (2010) choose $\varphi = 1$.

Labour market: The probability for firms to fill a vacancy, $q(\theta^j)$, is set equal to 0.97, following Ravenna and Walsh (2011).[27] This value is relatively higher than $q(\theta) = 0.7$ which is used in other studies like Campolmi and Faia (2011) and Walsh (2005). However, as the robustness analysis highlights, $q(\theta)$ does not affect the results. The elasticity of matching function with respect to the number of vacancies, κ^j , is set equal to 0.5, following the estimations by Pissarides and Petrongolo (2001), while the same value is used by Thomas (2008) and Campolmi and Faia (2011). For the calibration of the bargaining power of firms, ξ^j, I assume that the Hosios (1990) efficient job-creation condition holds, so I set $\xi^j = \kappa^j$. The exogenous job separation rate, δ^j is set equal to 0.08 following Ravenna and Walsh (2011). This value is

[27]This is based on a calculation of a 5% daily probability times the average number of working days per month, times three, given that I treat time as quarters.

relatively higher to the value chosen by Blanchard and Gali (2010) (0.04). On the other hand,Campolmi and Faia (2011) choose 0.06 and Abbritti and Mueller (2013) choose 0.071.[28] The posting vacancy cost is calculated as a fraction 0.01 of the GDP and is set equal to 0.097. The same strategy is used by Walsh (2005), Thomas (2008), Blanchard and Gali (2010) and Abbritti and Mueller (2013). The value of the posting vacancy cost varies with the specification of the posting cost function.[29] For the calibration of the degree of real wage rigidity, the empirical evidence is mixed. In the benchmark calibration I choose an average $\overline{\mu} = 0.5$, following Blanchard and Gali (2010), Campolmi and Faia (2011) and Abbritti and Mueller (2013). Studying the labour market heterogeneity, I let the degree of RWR across countries to vary between $0.1-0.9$ like in Abbritti and Mueller (2013). Finally, I set a steady-state of unemployment, $u^j = 0.10$ following, Blanchard and Gali (2010) definition for the sclerotic European labour market. Abbritti and Mueller (2013) choose a value equal to 0.08.

Final good production: The degree of nominal price rigidity is calibrated following estimates that find that prices change every three to four quarters. Hence, I set $\omega^j = 0.75$, following Thomas (2008), Blanchard and Gali (2010) and Ravenna and Walsh (2011). I limit the analysis to the case where there is a symmetric degree of nominal price rigidity.[30]. Setting $\omega^j = 0.75$ implies an elasticity of inflation with respect to real marginal cost, $\delta_p = 0.086$. I set the elasticity of substitution among the differentiated final goods within country to $\gamma = 6$ like Ravenna and Walsh (2011). This implies a steady state real marginal cost of $\phi^j = 0.83$, where I have used that for symmetric countries $S = 1$ as in Gali and Monacelli (2008).

Productivity shock: I assume a persistent productivity shock by setting an autocorrelation coefficient, ρ, equal to 0.95 and a standard deviation productivity shock of σ_z^j equal to 0.00624 following the euro area estimates by Smets and Wouters (2003) and adopted by Abbritti and Mueller (2013) as well. Finally, the correlation of domestic productivity shocks varies. When it is set to 1, the productivity shock is union-

[28]These numbers are based on estimates and calculations for the European economy and they are very different than the more fluid US labour market in which the separation rate is set around $0.10-0.15$. For this reason, I calibrate the model by using a wide range for the value of the separation rate around $0.04-0.12$.

[29]For example, Thomas (2008) uses a convex posting vacancy cost following Gertler and Trigari (2009), while I use a simpler linear cost similar to Ravenna and Walsh (2011).

[30]See Benigno (2004) for a normative analysis of a currency union when there is an asymmetric degree of nominal price rigidity.

wide.

2.4.2 Weights of policy objectives

From the benchmark calibration, I calculate the weights of domestic inflation and domestic unemployment fluctuations, as well as of terms of trade fluctuations. These weights may reflect the strength of the response of the central bank on the monetary policy stabilisation objectives. The value of these weights is reported in table 2.2. The weight of domestic inflation fluctuations is relatively higher than the weight of domestic unemployment and the terms of trade fluctuations. This may reflect an incentive for the central bank to be more aggressive in stabilising the domestic inflation.

Table 2.2: Weights of central bank objectives

Policy Objective*	Description	Policy Weight**
π_{At}	Domestic inflation of country A	17.48
π_{Bt}	Domestic inflation of country B	17.48
$\hat{u}^A_{t+1}$	Domestic unemployment of country A	.0031
$\hat{u}^B_{t+1}$	Domestic unemployment of country B	.0031
$\hat{s}_t$	Terms of trade	.25

*Expressed as gap from the efficient steady-state.
**The weights are expressed in absolute value.

2.4.3 Optimal responses to country-specific productivity shocks

First, I analyse the optimal responses of the central bank to a country A-specific productivity shock. To focus on the role of labour market heterogeneity, I construct an index measured from the differential of the degree of RWR of the member states, $\Delta\mu = \mu^A - \mu^B$. Following Andersen and Seneca (2010), to avoid misinterpretation of the results arisen from union-wide level changes, I allow the degree of RWR to vary between $0.1-0.9$ across countries, but I keep the union-average degree of RWR constant and equal to $\overline{\mu} = \zeta\mu^A + (1-\zeta)\mu^B = 0.5$. Because the sizes of the two countries are calibrated to be equal, $\zeta = (1-\zeta) = 0.5$, then, $\mu^B = 1-\mu^A$. Therefore the heterogeneity index $\Delta\mu$ varies from -0.8 to 0.8.

The two figures below display the impulse responses of the stabilisation policy objectives and the real interest rates to a 0.624% standard deviation, positive, country A-specific shock, when the central bank follows the regime of optimal commitment (figure 2.2) or discretion (figure 2.3). In the cases illustrated here, μ^A varies between $0.5-0.9$ and μ^B varies between $0.1-0.5$. Therefore, the country that is hit by the domestic shock has greater or equal degree of RWR. Comparing the two figures, we observe immediately that while, the two optimal monetary policy regimes are different, the fluctuations of the policy objectives differ only in magnitude and not qualitatively.

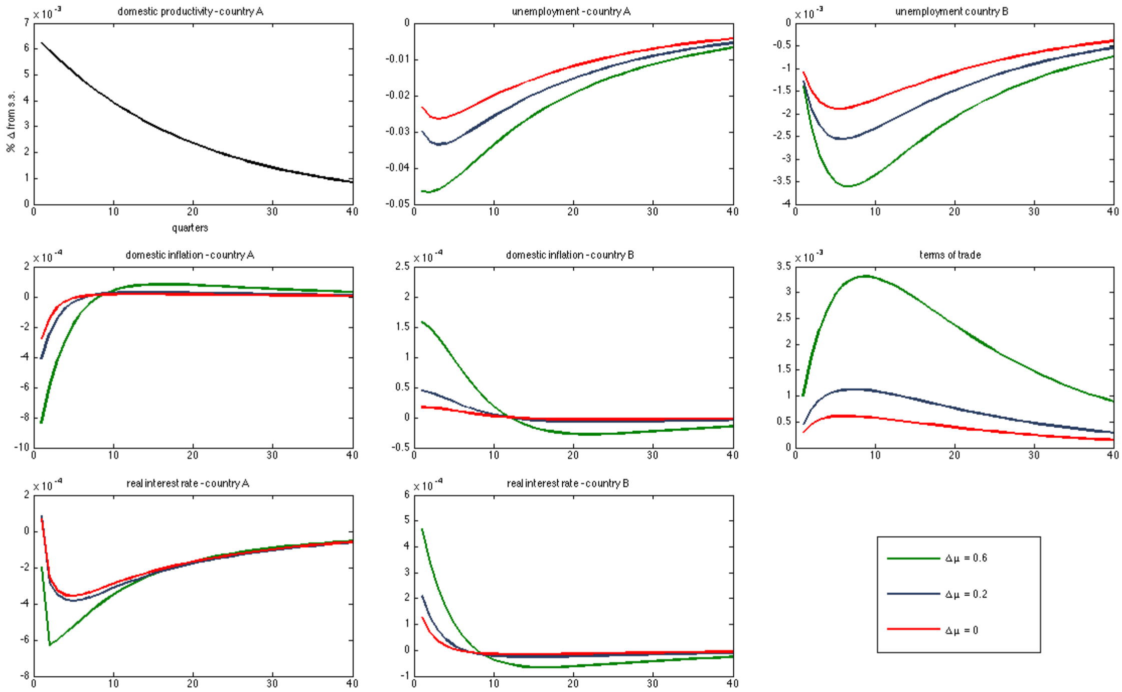

Figure 2.2: *Optimal commitment; country A-specific shock*

From figures 2.2 and 2.3, we realise that none of the five policy objectives are fully stabilised for positive values of $\Delta\mu$, regardless the type of optimal monetary policy regime. The positive productivity shock increases the marginal product of labour. However, the assumption of a high degree of RWR in country A ($\mu^A \geq 0.5$) implies that the real wage is not adjusted instantaneously to the new economic conditions. Therefore, the intermediate good producers in country A increase the optimal number of posting vacancies, thus, domestic unemployment decreases. Consequently, the production of final good increases, prices (set by the fraction of the producers not constrained to reset prices) fall and domestic inflation in country A declines.

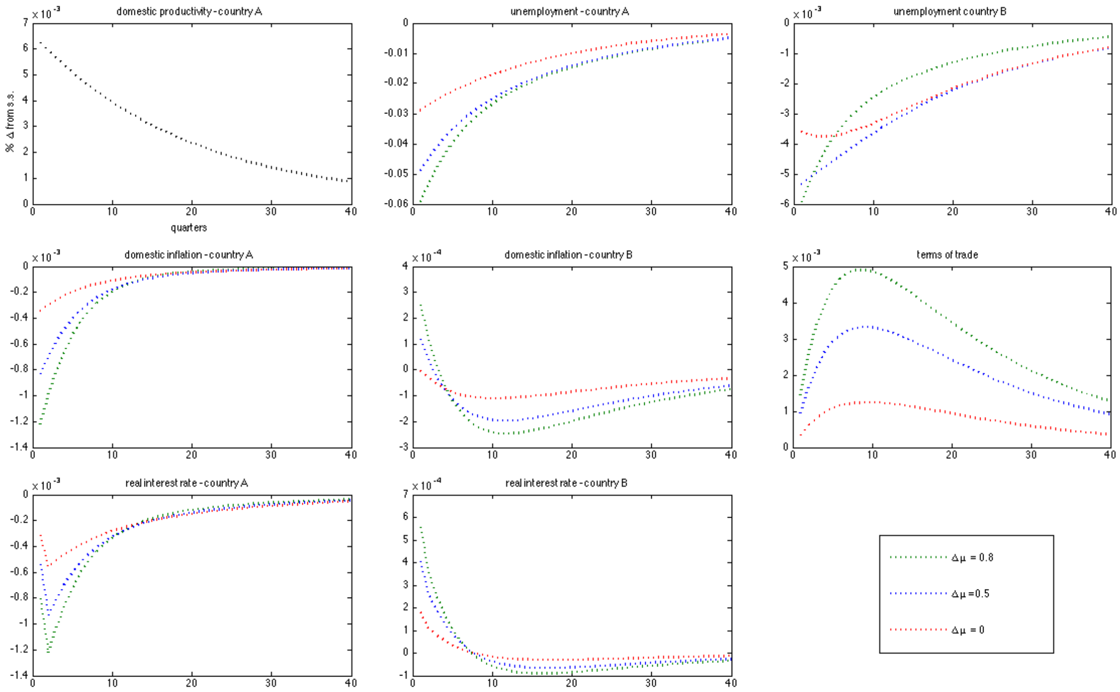

Figure 2.3: *Optimal discretion; country A-specific shock*

Because the weight of domestic inflation fluctuations is relatively higher than the weight of unemployment fluctuations, the central bank has an incentive to keep unemployment below the baseline, efficient level for several periods, in order to create inflationary pressure through expectations (under commitment). This policy stabilises domestic inflation in country A faster than domestic unemployment.

Country B is not hit by the country-A productivity shock directly. Nevertheless, the domestic inflation and unemployment in country B fluctuate for several periods. The reason lies on the terms of trade effect. Households buy baskets of final goods from both countries. The country-A specific shock affects the domestic inflation in country A. Thus, the relative competitiveness of the member states changes and this is translated into fluctuations of the terms of trade. Indeed, the terms of trade act as a transmission mechanism of the country A-specific shock and as a "cost-push" component on domestic inflation in country B. As it is shown in the two figures, the volatility of the terms of trade increases with the degree of RWR of country A (or with $\Delta\mu$) under any of the two optimal policy regimes. The higher is $\Delta\mu$, the more volatile is inflation in country B. Consequently the welfare losses in country B and the currency union increase regardless the fact that in country B, the degree of RWR decreases with $\Delta\mu$. This result cannot be captured by the closed economy NK models which highlight that welfare losses increase with the degree of RWR.

Analysing the role of labour market heterogeneity in the persistence of policy objectives, we can conclude the following: Under commitment higher degree of RWR in country A, increases the persistence of all policy objectives. However, it has a more pronounced effect in the persistence of unemployment in country B. Under discretion, labour market heterogeneity increases the persistence in country A.

Tables 2.3 and 2.4 display a moment analysis of the domestic productivity shock under optimal commitment and optimal discretion respectively. The standard deviations of the policy objectives are reported for different values of index $\Delta\mu \in [-0.8, 0.8]$. The reported welfare losses can be read as the average percentage reduction of households' steady state consumption.

From the last column, the volatility of the terms of trade increases monotonically with the degree of RWR of the country hit by the domestic shock. For example, when $\Delta\mu = -0.8$ ($\mu^A = 0.1$ and $\mu^B = 0.9$) the welfare losses are almost negligible, while when $\Delta\mu = 0.8$ ($\mu^A = 0.9$ and $\mu^B = 0.1$) the welfare losses are equal to a 2% reduction of households' steady-state consumption. From tables 2.3 and 2.4, the standard deviation of domestic inflation and domestic unemployment of country B increases with the standard deviation of the terms of trade. From the second column, we observe that the welfare losses of the currency union increase monotonically with the degree of RWR of the country hit by the shock (country A). It is noticeable that if we assume $\mu^A \geq \mu^B$, then the welfare losses increase monotonically with the value of the labour market heterogeneity index, $\Delta\mu$.

Table 2.3: Welfare losses under commitment - country A-specific shock

$\Delta\mu = \mu^A - \mu^B$	Welfare losses	$\sigma_{\pi_{At}}$	$\sigma_{\pi_{Bt}}$	$\sigma_{\hat{u}^A_{t+1}}$	$\sigma_{\hat{u}^B_{t+1}}$	$\sigma_{\hat{s}_t}$
-.8	.0001	.0000	.0000	.0196	.0005	.0000
-.6	.0003	.0000	.0000	.0342	.0020	.0002
-.4	.0007	.0001	.0000	.0516	.0031	.0005
-.2	.0016	.0002	.0000	.0724	.0052	.0013
0	.0030	.0003	.0000	.0968	.0077	.0026
.2	.0054	.0005	.0001	.1241	.0104	.0049
.4	.0093	.0008	.0002	.1503	.0131	.0088
.6	.0151	.0012	.0003	.1650	.0146	.0146
.8	.0223	.0017	.0005	.1511	.0131	.0220

Table 2.4: Welfare losses under discretion - country A-specific shock

$\Delta\mu = \mu^A - \mu^B$	Welfare losses	$\sigma_{\pi_{At}}$	$\sigma_{\pi_{Bt}}$	$\sigma_{\hat{u}^A_{t+1}}$	$\sigma_{\hat{u}^B_{t+1}}$	$\sigma_{\hat{s}_t}$
-.8	.0001	.0001	.0001	.0196	.0032	.0003
-.6	.0005	.0002	.0002	.0338	.0060	.0008
-.4	.0011	.0003	.0003	.0501	.0093	.0018
-.2	.0022	.0005	.0004	.0685	.0127	.0033
0	.0042	.0007	.0005	.0883	.0158	.0056
.2	.0074	.0010	.0006	.1084	.0179	.0087
.4	.0121	.0013	.0008	.1269	.0186	.0126
.6	.0184	.0017	.0010	.1417	.0174	.0170
.8	.0259	.0021	.0011	.1511	.0144	.0214

Given the analysis of moments and the impulse responses above, we can summarise with the following proposition:

Proposition 2. *In a currency union consisted of two member states that have imperfect, heterogeneous labour markets and are linked via trade of goods, in the presence of a country-specific productivity shock, welfare losses arise from fluctuations of domestic inflation, domestic unemployment and the terms of trade of the member states. The latter acts as a transmission mechanism of the shock from one member state to the other. The volatility of the terms of trade increases monotonically with the degree of RWR of the member state that is hit by the shock and intensifies the effect of the shock. Consequently, the welfare losses in the currency union increase monotonically with the degree of RWR of the country hit by the shock, for any optimal monetary policy regime.*

2.4.4 Optimal responses to union-wide shocks

In this section I examine the optimal responses of the central bank in the presence of an aggregate (union-wide) productivity shock. The case of a union-wide shock is this when the two country-specific shocks are perfectly correlated, i.e., $\rho_{a,b} = 1$. The impulse responses under optimal commitment and discretion are displayed in figures 2.4 and 2.5 respectively.

When there is an aggregate, positive productivity shock, the marginal product of labour increases in both countries. Because the real wage in countries A and B

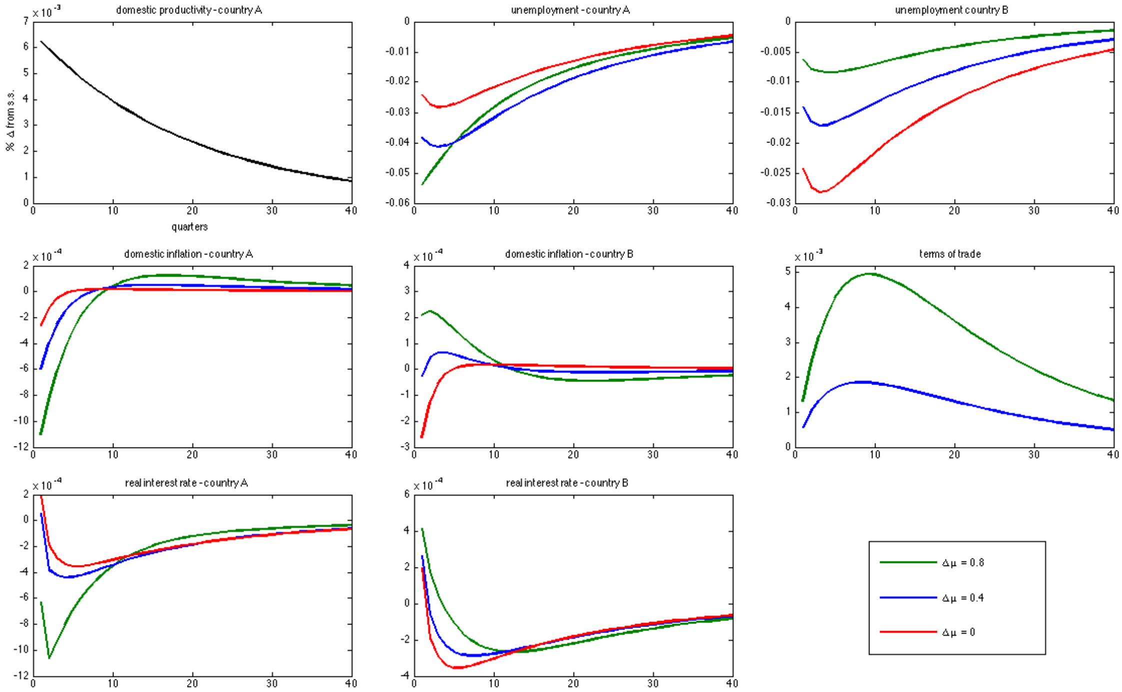

Figure 2.4: *Optimal commitment; aggregate productivity shock*

exhibits some degree of RWR, firms adjust their optimal hiring decision and unemployment in the currency union falls.

From figures 2.4 and 2.5, when $\mu^A = \mu^B = 0.5$, $\Delta\mu = 0$ the shock has symmetric effects. When there is perfect symmetry, the terms of trade effect is offset, the currency union is treated as a single economy by the central bank which responds to union-wide variables, i.e. the union-wide inflation and unemployment. This result could add to Benigno (2004) who shows that when two member states are identical the central bank reacts to the union-wide/CPI stabilisation.[31]

Not surprisingly, the asymmetric effects of the shock arise when $\mu^A \neq \mu^B$. In this scenario, in the country with the higher degree of RWR unemployment declines more. Because output in this country increases more, domestic inflation will decline more and the relative competitiveness of the two member states will change. The terms of trade fluctuate the higher is the degree of RWR differential between the two member states.

As in the case of a country-specific shock, the central bank finds optimal to keep

[31] In Benigno (2004) countries are identical when they have the same degree of nominal price rigidity.

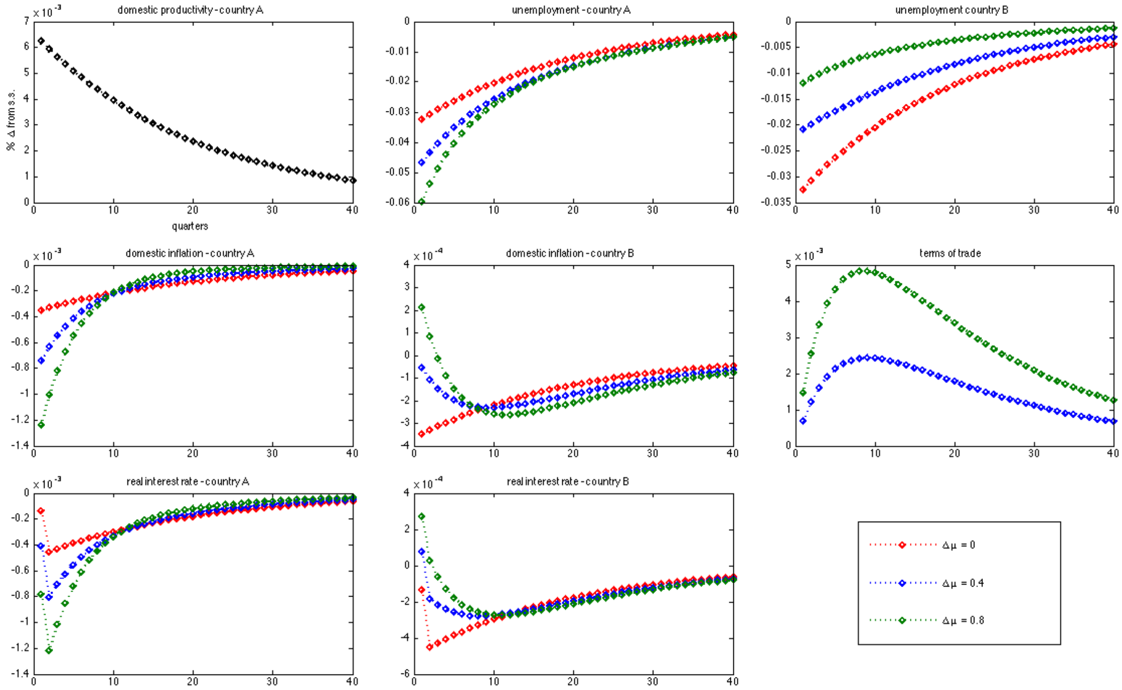

Figure 2.5: *Optimal discretion; aggregate productivity shock*

unemployment below the efficient steady-state for several periods in order to cause inflationary pressures. Under commitment, the central bank can affect expectations. Domestic inflation adjusts faster and the welfare losses are relatively smaller under commitment.

Analysing the effect of labour market heterogeneity in the persistence of the policy objectives, we can conclude that under any monetary policy regime, unemployment exhibits strong persistence and is stabilised slowly. In contrast, inflation is stabilised relatively fast. While the degree of labour market heterogeneity affects the frequency of stabilisation of all policy objectives, it has more pronounced effects in the persistence of unemployment. For example, when $\Delta\mu = 0.8$, unemployment in country B is stabilised relatively fast, while in country A unemployment remains further below the efficient steady-state for about 20 quarters.

The analysis of moments in tables 2.5 and 2.6 illustrates the same result. In the case of an aggregate productivity shock, the volatility of the terms of trade and consequently the welfare losses of the currency union increase monotonically with the value of the labour market heterogeneity index, $\Delta\mu$, regardless which country has greater degree of RWR. This result holds under commitment and discretion.

From these quantitative exercises, we notice that, in the presence of a union-wide shock, labour market heterogeneity has a distortionary effect on the currency union and it is inefficient. The single monetary authority of the currency union does not have a sufficient number of instruments to eliminate all distortions.This makes the stabilisation of policy objectives unfeasible. The following proposition summarises:

Table 2.5: Welfare losses under commitment - union-wide shock

$\Delta\mu = \mid \mu^A - \mu^B \mid$	Welfare losses	$\sigma_{\pi_{At}}$	$\sigma_{\pi_{Bt}}$	$\sigma_{\hat{u}^A_{t+1}}$	$\sigma_{\hat{u}^B_{t+1}}$	$\sigma_{\hat{s}_t}$
0	.0065	.0003	.0003	.1044	.1044	.0000
.2	.0074	.0005	.0002	.1293	.0827	.0036
.4	.0104	.0008	.0002	.1534	.0644	.0082
.6	.0157	.0012	.0003	.1665	.0484	.0144
.8	.0225	.0017	.0005	.1516	.0325	.0219

Table 2.6: Welfare losses under discretion - union-wide shock

$\Delta\mu = \mid \mu^A - \mu^B \mid$	Welfare losses	$\sigma_{\pi_{At}}$	$\sigma_{\pi_{Bt}}$	$\sigma_{\hat{u}^A_{t+1}}$	$\sigma_{\hat{u}^B_{t+1}}$	$\sigma_{\hat{s}_t}$
0	.0101	.0011	.0011	.1039	.1039	.0000
.2	.0113	.0013	.0010	.1209	.0863	.0054
.4	.0147	.0015	.0010	.1360	.0687	.0108
.6	.0201	.0018	.0011	.1475	.0511	.0162
.8	.0268	.0022	.0012	.1542	.0338	.0211

Proposition 3. *In a currency union consisted of two member states that have imperfect, heterogeneous labour markets and are linked via trade of goods, in the presence of a union-wide productivity shock, fluctuations of the terms of trade intensify the asymmetric effects of the shock. In particular, the volatility of the terms of trade increases monotonically with the value of the degree of RWR differential, i.e., the proposed labour market heterogeneity index. Consequently, the welfare losses in the currency union increase monotonically with the value of the labour market heterogeneity index, for any optimal monetary policy regime.*

2.4.5 Optimal commitment vs optimal discretion: Unemployment fluctuations

In this section I compare the two optimal monetary policy regimes. From the analysis of moments above, fluctuations of domestic inflation in both member states are lower under optimal commitment than discretion for any value of the heterogeneity index. Under commitment, the central bank has an additional instrument at its disposal in order to reduce the volatility of inflation, i.e., the public's expectations for future inflation. In this case, inflation is stabilised relatively faster. In contrast, under optimal discretion this instrument is not available, thus, fluctuations of domestic inflation are higher for both member states. This is the main reason that explains why optimal commitment generates lower welfare losses and is more desirable.

With regard to the role of labour market heterogeneity on inflation stabilisation, from tables 2.3 - 2.6, in the presence of a country-specific, or a union-wide shock, as $\Delta\mu$ increases, domestic inflation becomes gradually more volatile in both countries, under any regime. In addition, as $\Delta\mu$ increases, the difference between the volatilities of domestic inflation under commitment and discretion, i.e., $\sigma_{\pi_{jt}^{comm}} - \sigma_{\pi_{jt}^{discr}}$, remains almost unchanged in any member state. Hence, we can conclude that if a central bank focuses on domestic inflation stabilisation, optimal commitment is preferable for both member states and the degree of labour market heterogeneity does not alter the incentives of the central bank.

However, if a central bank focuses on domestic unemployment stabilisation, this does not necessarily hold. Comparing from the tables 2.3 and 2.4 the standard deviations of domestic unemployment between optimal commitment and discretion, $\sigma_{\hat{u}_{jt+1}^{comm}} - \sigma_{\hat{u}_{jt+1}^{discr}}$, in the presence of a country A-specific shock, fluctuations of domestic unemployment in the country hit by the shock are higher under commitment, while in the other country are higher under discretion. In addition, as the degree of RWR in the country hit by the shock increases, the volatility of domestic unemployment in this country increases relatively more under commitment than under discretion. In the other country the opposite holds. This result is visualised in the right graph of figure 2.6. Commitment and discretion generate similar domestic unemployment fluctuations, only for extreme degree of labour market heterogeneity, -0.8 or 0.8. In the other cases, the central bank faces a trade-off between the two policy regimes. This trade-off can be visualised from the distance between the blue (country A) and the red (country B) line. Summarising this result:

Proposition 4. *In a currency union consisted of two member states that have imperfect, heterogeneous labour markets, in the presence of a country-specific shock, fluctuations of domestic unemployment in the member state hit by the shock become larger under commitment than discretion, as the degree of RWR in this member state increases. In the member state that is not hit by the shock the opposite effect holds. Therefore, as the value of the labour market heterogeneity index increases, the central bank faces a trade-off between optimal commitment and optimal discretion with regard to domestic unemployment stabilisation. This-trade off is eliminated for extreme values of the index.*

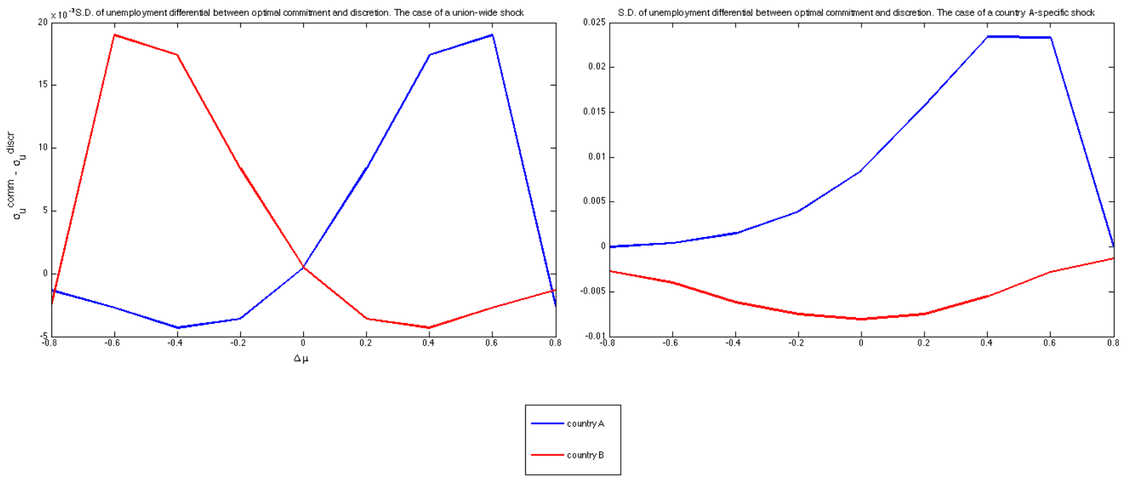

Figure 2.6: *Unemployment differentials - commitment vs discretion*

I perform the same exercise for the currency union in the presence of a union-wide shock. The standard deviation differentials of domestic unemployment are displayed in the left graph of figure 2.6. I find that the two regimes generate similar domestic unemployment fluctuations in a member state, when $\Delta\mu$ is −0.8, 0.8 or 0. For all values between 0−0.8 and -0.8−0, the central bank faces a trade-off between the two policy regimes. Optimal commitment is desirable for a member state with low degree of RWR. As the degree of RWR increases for this member state, optimal discretion becomes more desirable. The following proposition summarises:

Proposition 5. *In a currency union consisted of two member states that have imperfect, heterogeneous labour markets, when the degree of RWR in a member state increases, in the presence of a union-wide shock, fluctuations of domestic unemployment are larger under commitment than discretion in this member state. Consequently, as the value of the labour market heterogeneity index increases, the central bank faces a trade-off between optimal commitment and optimal discretion with regard to domestic unemployment stabilisation. This trade-off is eliminated for extreme values of the index or when there is perfect labour market homogeneity.*

The intuition behind propositions 4 and 5 is based on the sensitivity of inflation to unemployment changes, as it is calculated in the NKPC, and the role that expectations play under optimal commitment. Under optimal commitment, the central bank uses expectations for inflation to stabilise inflation. There is not strong motive for domestic unemployment to be used as an instrument for inflation stabilisation. Thus, domestic unemployment remains further below its efficient level and fluctuates relatively more. However, under optimal discretion, the central bank loses one instrument. In this case, there is a stronger incentive to stabilise unemployment relatively faster. Table 2.7 displays the elasticity of domestic inflation to changes of domestic unemployment (second column) for different degrees of RWR. The higher is the degree of RWR, the lower is the sensitivity of inflation with respect to unemployment changes. Thus, under discretion, when the RWR differential is relatively high, a stronger response to domestic unemployment is required. In this case, the difference in unemployment fluctuations between optimal discretion and commitment becomes relatively larger.

Table 2.7: RWR and sensitivity of domestic inflation

RWR	$\hat{u}^j_{t+1}$	$\hat{s}_t$	$\hat{s}_{t+1}$	$\hat{z}^j_t$
0.2	-.2321	.1665	.0418	-.4124
0.3	-.192	.1205	.1439	-.5144
0.4	-.1576	.0810	.2314	-.6018
0.5	-.1279	.0469	.3073	-.6776
0.6	-.1018	.0170	.3736	-.7439
0.7	-.0788	-.0094	.4322	-.8023
0.8	-.084	-.0329	.4843	-.8543

2.4.6 Further sensitivity analysis

In the quantitative analysis, I allow the member states to be heterogeneous only in the degree of RWR. This assumption has been motivated by the criticism in the literature of search and matching models with involuntary unemployment, which has addressed the importance of RWR in generating realistic unemployment fluctuations. Having this source of labour market heterogeneity unchanged, the results of the model are robust to different values of other labour market parameters, such as the exogenous job separation rate, δ^j, the elasticity of labour supply φ^j, the steady-state probability of firms to fill a vacancy, $q(\theta^j)$ and consequently, the steady-state labour market tightness, θ^j.

In particular, δ^j has been altered from 0.10 to 0.15, $q(\theta^j)$ from 0.7 to 0.97 and φ^j from 0 to 2, as these are other values found in the literature. Changing these parameters and worsening the conditions in the domestic labour markets, the results of the model are qualitatively the same. The only difference observed in most of the cases is a percentage increase of unemployment fluctuation in both countries, which results to an increase of total welfare losses by 0.002.

While the model can capture various currency unions of the world economy, the calibration strategy has been motivated by the euro area currency union. For this reason the union average steady-state unemployment has been assumed to be 0.10, as in Blanchard and Gali (2010). Making the labour market more sclerotic and increasing this value from 0.10 to 0.15, I do not obtain qualitative changes. However, the total welfare losses increase by 0.02 on average.

The parameters in the benchmark calibration are altered symmetrically across member states. If we allow asymmetric differences across member states along with different degree of RWR, given the robustness checks above, it is expected that the effect of the shocks will be more pronounced and the welfare losses will have higher magnitude.

The degree of nominal price rigidity, $\omega^j = 0.75$, has been kept unchanged in all quantitative exercises. Motivated from Benigno (2004), I expect asymmetry in the degree of nominal price rigidity to alter the incentives of the central bank and make monetary policy to react, even more strongly, to domestic inflation fluctuations of the member state with higher $\omega^j = 0.75$. Having asymmetric nominal price rigidity as well as RWR across the member states would make the analysis difficult to make robust inferences with regard to differences in the labour markets. However, it would be interesting to study cases in which countries have asymmetric degrees of rigidities.

Finally, making the member states asymmetric in size, but keeping the value of the labour market heterogeneity index unchanged, produces mixed results. For example, in the case of a country A-specific shock, fixing $\Delta\mu = 0.6$, for $\zeta = 0.33$, the welfare loss is a 1.32% reduction in households' steady-state consumption. For $\zeta = 0.66$, the welfare loss is 1.34%. However, as reported in the previous section, for $\zeta = 0.5$, the welfare loss is 1.51%. In contrary, for $\Delta\mu = 0.3$, the welfare loss increases monotonically with the size of the member state hit by the shock. In particular, for $\zeta = 0.33$, the welfare loss is 0.6%, for $\zeta = 0.5$ is 0.71% and for $\zeta = 0.66$ is 0.74%.

2.4.7 The role of fiscal policy

In the current model, fiscal policy is absent for simplicity. Introducing local governments in the model would add an extra variable in the resource constraint of each member states, i.e, government spending. Following the normative analysis in Gali and Monacelli (2008), this would add two extra policy objectives for stabilisation. Given that optimal monetary policy would require co-ordination with the independent fiscal authorities and a minimisation of the fiscal gap, this would make the analysis of this paper more complicated. On the other hand, while it is not the purpose of this model to study fiscal policies, ruling them out makes the model to lose some of its realism. An escape route in this case would be to assume that domestic government spending is constant over time. By taking a first-order approximation of the resource constraints would eliminate the constant component added in the right hand side.

2.5 Summary and further discussion

I have merged a frictional search and matching model of equilibrium unemployment into an otherwise standard NK currency union model in order to generate involuntary unemployment in the member states of a currency union. I have studied the optimal responses of central bank in country-specific and union-wide productivity shocks under two optimal regimes. commitment and discretion. I have focused on the role of labour market heterogeneity in the outcomes of optimal monetary policy. For this reason, I have constructed a simple index of labour market heterogeneity that is based on the differential of the degree of RWR of the member states. To the best of my knowledge, this paper is the first that provides an analysis of optimal monetary policy in a currency union with labour market heterogeneity.

The main results are derived by quantitative comparisons. In particular, I calculate the households' welfare losses under commitment and discretion for different values of the labour market heterogeneity index. I find that in the case of a country-specific shock, higher degree of RWR in the country hit by the shock implies higher welfare losses for each member state and consequently for the currency union. In the case of a union-wide shock, the welfare losses increase monotonically with the value of the labour market heterogeneity index. Finally, comparing the two regimes, I find that with regard to unemployment stabilisation, the central bank faces a trade-off between commitment and discretion. This trade-off is higher as the value of the index increases, excluding the cases where it takes extreme values. In the case of a union-wide shock, this monetary policy trade-off disappears when the labour markets are homogeneous. I find that the terms of trade plays a crucial role for the main results of the model. In particular, the terms of trade acts as a transmission mechanism of a country-specific shock from one member state to the other and intensifies the asymmetric effects of a union-wide shock. In any case, fluctuations of the terms of trade make even the country with low degree of RWR to suffer welfare losses, which in the absence of international trade would not be present. This result cannot be captured by closed economy NK models and adds to the novelty of this paper.

The comparison between the two optimal regimes could be useful for monetary policy in practice. The ECB has committed that the maintenance of price stability across the euro area is their primary goal. This is an objective that is publicly announced. This paper highlights a case, in which optimal commitment results to higher unemployment in member states with high degree of RWR. In this case, the paper adds an argument that could potentially explain why inflation is stabilised relatively fast and homogeneously, but unemployment persistence is heterogeneous across member states.

The results of the model also confirm the theory of optimum currency union areas introduced by Mundell (1961). In the presence of too many frictions, a currency union is far from being optimal. The imperfection of real wage rigidity and the no-migration assumption play an important role in the sub-optimality of the presented currency union. Mundell (1961) highlights the importance of wage flexibility and labour mobility for a currency union to be optimal. Assuming that unemployed workers can migrate would make the model far more complicated as it would require the construction of migration strategies, as in Ortega (2000). Allowing for migration in the presented framework and linking it with monetary policy is certainly the next step for further research. It is expected that labour mobility would absorb the welfare losses generated from the RWR differentials. However, adding extra frictions in migration could make things more complicated. Thus, such a paper could have further policy implications and trade-offs.

Finally, the construction of the current NK DSGE framework allows for other steps for further research. One of this is the estimation of the degree of RWR for the member states of the euro area using a Bayesian approach. Whether some euro area countries have rigid real wages is still an open research question. Thus, such an empirical study would add to the literature. Another extension of the model could incorporate a model with financial frictions. A macroeconomic framework which merges an imperfect financial and labour market would be useful for policy implications. Nevertheless, a normative analysis, as the one presented here, would be very complicated because of the potential presence of many state variables, such as capital and unemployment.

Chapter 3

The Welfare Gains from Discount Window Lending

Chapter Abstract

I study the role of the Discount Window (DW) as a complementary instrument of monetary policy by analysing the welfare consequences of the central bank providing DW loans to financial intermediaries. The analysis focuses on the role of the ability of the central bank to monitor the actions of financial intermediaries with regard to the effective use of the DW loans. By assessing several quantitative experiments in a New Keynesian DSGE model, I show that when the central bank's ability to monitor is low, DW lending can be welfare costly. However, when the ability of the central bank to monitor improves, DW lending is welfare enhancing, even if it is associated with efficiency costs. The performance of DW lending depends on the joint, primary monetary policy regime which is implemented by the adjustment of the nominal interest rate, even if this is constrained from a lower bound. Under a regime of optimal commitment, the welfare gains from DW lending could be negligible, but they are considerable when DW lending is used along with an optimal interest rate rule. The paper provides a formal argument in favour of the view that, as a complementary instrument of monetary policy, DW can be effective, in contrast with the views expressed in previous literature that DW lending should be redundant.

3.1 Introduction

Discount Window (DW) lending is a monetary policy operation where the central bank provides collateralised loans at a penalty rate to financial intermediaries that require funds to finance their activities but cannot obtain them from other sources, such as the interbank market. According to the Federal Reserve Discount Window Book, the role of the DW is complementary to the primary role of open market operations and it is used to help the central bank maintain financial stability.[1]

However, in practice, and especially before the financial crisis of $2007-2008$, this role has been vague. Central banks have been providing DW loans rarely and only at a small scale. The inactivity of the DW could be explained from a strand of the financial literature developed in the late $80's$, which has questioned the efficiency of DW lending. This literature has highlighted the difficulties for DW lending to be effective, mainly arguing that it is associated with a high risk that the loans will be misused by the financial intermediaries in non-financial activities. In some cases (Goodfriend and King (1988) and Schwartz (1992)) this literature has recommended that DW should be avoided by the central banks, unless there are exigent circumstances.

In this paper, I try to shed some light on these arguments. I study the role of the DW as a complementary instrument of monetary policy and I assess its performance by quantifying the welfare effect of the central bank providing DW loans to financial intermediaries. To do this, I construct a quantitative macroeconomic model with nominal frictions, which is expanded to incorporate an analysis of a financial market. In the model, the financial intermediaries face liquidity constraints that are disruptive for the financial market. In the presence of a disrupted financial market, the effect of a small financial shock is magnified and is transmitted to the real economy making households suffer some welfare losses. The main objective of the central bank is to minimise these losses having two policy instruments at its disposal: a primary instrument, the nominal interest rate and a complementary one, the DW. The central bank selects first an optimal policy regime to stabilise the real economy. This regime is implemented with the adjustment of the nominal interest rate. However, the deterioration of the financial market expands the scope of

[1] The electronic version of the Book can be found on the Federal Reserve Discount Window & Payment System Risk Website: *www.frbdiscountwindow.org*

monetary policy. The central bank uses DW as a secondary instrument to provide liquidity assistance and reduce the inefficiency in the financial market. Calomiris (1993) has highlighted that DW lending should be used only as a response to financial market disruptions. In this case, DW lending may stabilise investment and consequently output, supporting the role of the primary instrument. The contribution of DW lending to welfare is calculated from quantitative exercises, as the difference in welfare losses obtained by switching the DW off and on.

The objective of this paper is to make the analysis relevant to the criticism on the DW mentioned above. The friction that I assume in the financial market captures a main aspect of this criticism. In particular, I allow financial intermediaries to have the ability to misuse a fraction of the funds borrowed in non-financial activities, as in Gertler and Kiyotaki (2010). However, the central bank has some means of monitoring, unlike the other lenders, hence, it can limit the misuse of DW loans. I evaluate the performance of DW lending by focusing on the role of the central bank's ability to monitor the actions of financial institutions. The ability of the central bank to monitor the use of DW loans has been highlighted in Bordo (1989) as an important condition for the DW to be effective.

The main result of the paper is summarised as follows. When the central bank's ability to monitor is low, discount window lending is welfare costly. However, when the ability to monitor improves, DW lending is welfare enhancing. I also take the assumption that DW lending may be associated with some efficiency costs. Even if this is the case, there are cases that DW lending increases efficiency. Performing several quantitative experiments, I also compare the performance of DW lending with respect the primary monetary policy regime that is jointly conducted. I find that the performance of DW lending depends on the joint, primary regime even if the nominal interest rate is constrained from a lower bound. I find that under a regime of optimal commitment, the welfare gains from DW lending can be neglected. However, when DW lending is used along with an optimal interest rate rule, there are considerable welfare gains from DW lending.

The main contribution of this paper is that it provides a quantitative assessment, as a formal argument in favour of the view that, as a complementary instrument of monetary policy, the DW can be effective. While the financial literature that has studied the performance of DW is comprehensive, it has been based on a cost-benefit analysis rather than on the evaluation of a quantitative micro-founded model. The latter has several advantages, it can be used to derive the main objectives of the central bank and provide further insights with respect to the macroeconomic channels through which the DW operates.

The normative analysis follows the Linear-Quadratic (L-Q) approach introduced by Rotemberg and Woodford (1997) and Woodford (2003). This approach is accurate for small deviations from the steady-state. In the model, the benchmark calibration accounts for small deviations from the efficient steady-state. Therefore, the paper supports the view that, as long as the financial markets are disrupted, there are considerable welfare gains from DW, even when the role of DW is complementary. In contrast, Calomiris (1993) and Schwartz (1992) have justified the use of the DW at exigent circumstances only. Indeed, after the financial crisis of $2007-2008$, DW loans were provided on a large scale. In many of these occasions central banks acted as lenders of last resort and used DW to provide emergency liquidity assistance and prevent financial institutions from a possible bankruptcy.[2] It is the fact that the DW has been used substantially only during crises, that has made it to be closely linked with the role of the lender of last resort.

The literature has also tried to explain the inactivity of DW before $2007-2008$ from another view. It has been argued that there is reluctance of the financial institutions to borrow from the DW, as a result of the fear of revealing their poor financial condition and to be stigmatised.[3]

There is a long discussion whether the adjustment of the nominal interest rate is constrained from a Zero Lower Bound (ZLB). While there is no technical reason to argue why interest rates cannot be negative, in practice, some central banks are hesitant to reduce them below zero.[4] However, if the interest rates are close to zero, their

[2]A summary of these cases can be found on the policy report of the Bank for International Settlements: http://www.bis.org/publ/bppdf/bispap79.pdf.

[3]The "DW stigma" has been studied extensively in the literature and the current paper abstracts any feature related with it for simplicity. Clouse (1994) and Courtois and Ennis (2010) have highlighted the DW stigma as a reason to explain the reluctance of the depository institutions in the US to borrow from the Fed. It has been also mentioned in speeches by policymakers, such as by Bernanke (2009), as well as in the policy report by the Bank for International Settlements. In addition, Armantier et al. (2015) provide empirical evidence that supports the DW stigma effect, while Ennis and Weinberg (2013) provide a theoretical framework that incorporates the stigma effect. Both papers conclude that financial institutions are willing to borrow more expensively from other financial sources rather than borrowing from the DW of the central bank.

[4]Actually, since 2008, some major central banks have moved their short-term interest rates close to zero as a response to the financial crisis. In particular, on April 2016, the Bank of England (BoE) and the Federal Reserve (Fed) have moved their interest rate to 0.5%. In contrast, since 2014 a few central banks have moved their interest rates below zero. For example, the European Central Bank

adjustment below zero is somewhat limited revealing that a lower bound constraint may has some effect. In the model, I capture this effect. By utilising the L-Q approach, as in Levine, McAdam, and Pearlman (2008) and Cantore, Gabriel, Levine, Pearlman, and Yang (2013), I add an interest rate term to the welfare criterion of the central bank. This limits the variability of the nominal interest rate capturing a lower bound effect. In addition, limiting the effect of the interest rate, I can capture the net contribution of DW lending to the real economic activity. I also show that despite the limited adjustment of the interest rate, the choice of optimal policy regime plays an important role for the effectiveness of DW lending. In particular, I find that DW is more effective when it is jointly used under an optimal interest rate rule rather than an optimal commitment regime.

This paper adds to the value of the core New Keynesian (NK) models, such as Gertler, Gali, and Clarida (1999) and Gali (2008) by presenting a model that incorporates a frictional financial market and an additional policy instrument, taking into consideration the recent macroeconomic environment. It also expands further the analysis in Edge (2003) with regard to the optimal monetary policy in a quantitative macroeconomic model with investment. The paper also adds to the analysis in Gertler and Kiyotaki (2010), Gertler, Kiyotaki, and Queralto (2012) and Gertler and Karadi (2011). The core framework used in the latter is NK, so it is the one most close to this paper. The scope of the analysis of these papers is quite different. They focus on the effect of direct lending and unconventional monetary policy that is used to combat a financial crisis, rather than on the optimal monetary policy and the role of DW lending.

The rest of the paper is organised as follows. The core framework with financial intermediaries is described in Section 3.2. Section 3.3 analyses the role of monetary policy. Section 3.4 presents the quantitative results. Section 3.5 analyses DW lending in the U.S. Section 3.6 discusses some robustness checks and Section 3.7 concludes with a further discussion.

has set interest rate to −0.4%, while the Swedish and the Swiss interest rates have been set to −0.5% and −0.75% respectively.

3.2 The model

The model that I construct builds on the standard New Keynesian (NK) framework, e.g., Gertler, Gali, and Clarida (1999) and Gali (2008). The main elements of the NK model are monopolistic competition and nominal price rigidity. Monopolistic competition creates an environment for price setters that are subject to nominal price stickiness. The latter assumption is crucial, as it implies that adjustments of the nominal interest rate by the central bank affects the dynamic path of real variables.

In the standard NK model, capital is absent from the production process. In the current paper, capital is added to generate investments. A business cycle model with capital has been also analysed by Woodford (2003) and Edge (2003). A financial market framework is also missing from the standard NK model. Here, I add a financial market by incorporating a model that analyses the behaviour of financial institutions in business cycles, the Gertler and Kiyotaki (2010) model.

Financial institutions operate in the financial market. They act as financial intermediaries, because they have expertise and specialise in transferring funds from households and other intermediaries to non-financial firms. To do this, they raise funds from households' deposits and from other financial intermediaries in the interbank market. Then, they provide loans to non-financial, intermediate good producers. The latter use funds to acquire capital from investors. The capital (and labour) is used for the production of the intermediate good, which is then sold to the non-financial, final good producers. Households cannot provide loans to non-financial firms directly because they do not have the knowledge and the means of financial intermediaries.

The agents of the model are five. Households, financial intermediaries, intermediate good producers, final good producers and investors. Separating the type of agents makes the model tractable. The central bank is the monetary policymaker. As in Gertler and Kiyotaki (2010), I further assume that:

- In the economy, there is a continuum of locations of measure unity. Each location contains all types of agents.
- Capital is immobile, however labour is perfectly mobile across locations and firms.
- The probability of investors to invest and create new capital arrives randomly

only to a fraction of locations and is i.i.d across locations and time. Because capital is immobile, only the non-financial firms located in the investing locations can acquire new capital. The non-financial firms located in the non-investing locations can only use the capital produced in the existing investment projects.

- The financial intermediaries of locations with no new investment opportunities cannot provide funds to non-financial firms of the locations with new investment opportunities. However, they can provide funds to the intermediaries of these locations through the interbank market or they finance existing investment projects of their location.

The assumption of the i.i.d investment opportunity combined with the immobility of capital generates liquidity needs and creates an environment for an interbank market in which the financial intermediaries provide and acquire funds from each other. The optimising behaviour of each agent is described separately in the following subsections. The analysis of households, intermediate good producers, capital good producers and financial intermediaries is similar with the one in Gertler and Kiyotaki (2010).

3.2.1 Households

A continuum of identical, infinitely-lived households that lies on the interval $[0,1]$ lives in each location. A fraction of a representative household members are workers and the rest are owners of the financial intermediaries (from now on they will be called bankers). The workers are also the owners of the non-financial firms without this affecting their role as workers. It can be considered that they work in a different non-financial firm than the one they own. Workers supply labour and earn labour income, while the bankers provide ownership dividends. To avoid making bankers to have the incentive to accumulate dividends in favour of the financial intermediary that they own, when there are liquidity constraints, it is assumed that at every period workers and bankers can switch roles. In particular, there is an i.i.d probability that a banker will become a worker and the opposite. Thus the ratio of workers/bankers within households is fixed. The average time of survival for bankers is then $\frac{1}{1-\sigma}$. [5]

A representative household seeks to maximise the intertemporal utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\log C_t - \frac{{N_t}^{1+\varphi}}{1+\varphi} \right)$$

subject to a period budget constraint

$$\int_0^1 P_t(i) C_t(i) di + d_t \leq w_t N_t + (1+q_{t-1}) d_{t-1} + T_t$$

for $t = 0, 1, 2, ...,$ where C_t is the Dixit-Stiglitz index of consumption across the differentiated final goods, given by the constant elasticity of substitution (CES) function

$$C_t \equiv \left(\int_0^1 C_t(i)^{\frac{\gamma-1}{\gamma}} di \right)^{\frac{\gamma}{\gamma-1}}$$

$C_t(i)$ is the quantity of the differentiated final good i bought at a price $P_t(i)$ and consumed by the household in period t. Also, d_t is the quantity of deposits (in nominal terms) held to the financial intermediaries at period t and q_t is the nominal interest rate paid at the end of period t. Workers earn a nominal wage rate w_t and supply a measure of employment (e.g. hours of work) N_t. T_t are the dividends from the ownership of financial and non-financial firms. Notice that when there is a new banker, the household provides some initial funds, thus, T_t must be considered as the net dividends (profits from banks minus start up funds). In addition, $\gamma > 0$ is the elasticity of substitution between the differentiated goods and $\varphi > 0$ is the inverse of the Frisch elasticity of labor supply. Finally, the log utility of consumption implies a measure of household's risk aversion to consumption equal to 1.

Each representative household takes an intratemporal and an intertemporal decision. The intratemporal decision requires the optimal allocation of consumption expenditures among the differentiated final goods. This is given by the problem

$$\max_{C_t(i)} C_t \ s.t \int_0^1 P_t(i) C_t(i) di \equiv Z_t$$

[5]Given that the household members are infinitely-lived, the average survival time is a geometric series.

The solution to the problem, shown in the Appendix B.1, yields the system of demand equations for the final good i :

$$C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\gamma} C_t \tag{3.1}$$

$\forall i \in [0,1]$, where P_t is the Dixit-Stiglitz price index given by:

$$P_t \equiv \int_0^1 \left(P_t(i)^{1-\gamma} di\right)^{\frac{1}{1-\gamma}}$$

Combining the demand equations with the Dixit-Stiglitz price index, I can write:

$$\int_0^1 P_t(i) C_t(i) di = P_t C_t$$

Then, the budget constraint is rewritten as:

$$P_t C_t + d_t \leq w_t N_t + (1 + q_{t-1}) d_{t-1} + T_t \tag{3.2}$$

The intertemporal decision requires the optimal allocation of consumption/savings and the labour supply decision. It is formalised by the problem:

$$\max_{\{C_t, D_t, N_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left(\log C_t - \frac{{N_t}^{1+\varphi}}{1+\varphi}\right)$$

subject to equation (3.2) and a solvency condition $\lim_{T\to\infty} E_t D_T \geq 0$, $\forall t$. The first-order conditions are given in the Appendix B.2 and yield the optimal labour supply decision:

$$C_t {N_t}^{\varphi} = W_t \tag{3.3}$$

which simply equates the real wage, W_t, with the marginal rate of substitution between consumption and leisure. Combining the first-order conditions with the households' stochastic discount factor, $\beta_{t,t+s} \equiv \beta^s \frac{U'(c_{t+s})}{U'(c_t)}$, yields the optimal consumption/savings decision. This is given by the Euler equation, which equates the nominal interest rate with the marginal rate of substitution between present and future consumption (in real terms):

$$(1+q_t)E_t\beta_{t,t+1}\frac{P_t}{P_{t+1}}=1 \tag{3.4}$$

The nominal interest rate is also linked with the real interest rate, R_t, through the Fisher equation:

$$(1+q_t)=R_{t+1}\frac{E_tP_{t+1}}{P_t} \tag{3.5}$$

3.2.2 Final good producers

In each location there is a continuum of monopolistic competitors $i\in[0,1]$ that produces a differentiated final good $Y_t(i)$. The intermediate good, $X_t(i)$, is the only input for production and is bought from the intermediate good producers at a competitive price, ϕ_t.[6] The technology is given from:

$$Y_t(i)=X_t(i)$$

Final good producers take consumers' demand, equation (3.1), and the aggregate indices for consumption and prices as given. I assume that their optimal price setting decision is subject to nominal price rigidity a′ la Calvo (1983). As it is standard in the NK model, nominal price rigidity implies monetary policy non-neutrality. When nominal prices are sticky, the dynamic path of the real interest rate is determined by the monetary policymaker through the adjustments on the nominal interest rate.

Every period, a representative final good producer resets his/her own price with a probability $(1-\omega)$ to maximise profits. This probability is independent of the time of the last reset. Thus, the producer chooses a price that will remain until time $t+s$ with probability ω^s. This problem is formalised:

$$\max_{P_t^*(i)} E_t\sum_{s=0}^{\infty}\omega^s\beta_{t,t+s}\left\{\frac{P_t^*(i)}{P_{t+s}}Y_t(i)-\phi_{t+s}Y_t(i)\right\}$$

for $s=0,1,2,3...$, subject to

$$Y_t(i)=C_t(i)$$

[6]I distinguish between final and intermediate goods production to avoid the complexity arisen from the endogenous capital stock to the optimal price setting decision of the firms that are subject to nominal price rigidity. For further discussion see Woodford (2003) chapter 5.3.2.

i.e. any firm meets the demand for the good that it produces.

The solution of this problem can be found in the Appendix B.3 and yields the optimal price decision of the final good producers:

$$\frac{P_t^*(i)}{P_t} = \frac{\gamma}{(\gamma-1)} \frac{E_t \sum_{s=0}^{\infty} (\omega\beta)^s C_{t+s}^{-1} \phi_{t+s} \left(\frac{P_{t+s}}{P_t}\right)^{\gamma} Y_{t+s}}{E_t \sum_{s=0}^{\infty} (\omega\beta)^s C_{t+s}^{-1} \left(\frac{P_{t+s}}{P_t}\right)^{\gamma-1} Y_{t+s}} \tag{3.6}$$

Notice that in the absence of nominal price rigidity, $\omega = 0$, the optimal price setting decision is standard, i.e. the optimal relative price for a monopolistic competitor is equal to a markup over the real marginal cost.[7]

$$\frac{P_t^*(i)}{P_t} = \frac{\gamma}{\gamma-1} \phi_t \tag{3.7}$$

3.2.3 Intermediate good producers

In each location there are intermediate good producers that act under perfect competition. In particular, they buy new capital, I_t, from capital goods producers (investors) and hire labour units, N_t, offered by households. Then, they use the inputs along with existing depreciated capital and labour to produce a homogeneous intermediate good X_t, which is sold to the final good producers. The production technology is described by a Cobb-Douglas, constant returns to scale production function:

$$X_t = A_t K_t^{\alpha} N_t^{(1-\alpha)}$$

where A_t is the total factor productivity (TFP) and $0 \leq \alpha \leq 1$ is the capital share. Capital accumulates according to the law of motion

$$K_{t+1} = \psi_{t+1}\left(I_t + \tau^i(1-\delta)K_t + \tau^n(1-\delta)K_t\right) = \psi_{t+1}(I_t + (1-\delta)K_t) \tag{3.8}$$

[7]It is standard to assume that due to monopolistic competition, the production of output is less than the socially optimal. This inefficiency can be eliminated with an output subsidy at a constant rate. By subsidising output, the monopolistic distortion of the standard NK model is eliminated and the flexible price equilibrium is efficient. This subsidy is financed by the government with a lump-sum taxation system that avoids further distortionary effects.

where $\tau^i = 1 - \tau^n$ is the fraction of locations where new investment opportunities arise, I_t is the aggregate level of investment and $0 \le \delta \le 1$ is the fraction of capital depreciation during period t.[8] The model is stochastic, as it is assumed that the quality of capital, ψ_{t+1} , is exogenous. Assuming that the quality of capital is exogenous followsMerton (1973) and has been used by Gertler and Kiyotaki (2010), Gertler and Karadi (2011) and Gertler, Kiyotaki, and Queralto (2012). This may reflect some economic depreciation rather than a physical one. Because in the model securities are issued against capital, a shock in the quality of capital may affect asset (security) prices. In this sense, it can be used as a proxy for a financial shock. The (TFP) and the capital quality shock follow an $AR(1)$ process given by

$$A_t \equiv A_{t-1}^{\rho_a} A^{(1-\rho_a)} e^{\epsilon_{at}} \tag{3.9}$$

and

$$\psi_{t+1} \equiv \psi_t^{\rho_\psi} \psi^{(1-\rho_\psi)} e^{\epsilon_{\psi t+1}} \tag{3.10}$$

respectively, where ρ's are the autocorrelation coefficients and ϵ_t's are i.i.d exogenous disturbance terms. [9]

Intermediate goods producers choose optimally the amount of labour units to maximise profits. The first-order condition to this problem is

$$W_t = (1-\alpha)\phi_t \frac{X_t}{N_t} \tag{3.11}$$

and equates the real wage with the marginal product of labour. Intermediate good producers issue securities (equities) to obtain funds from the financial intermediaries. Then, those located in locations with new investment opportunities use these funds to acquire new capital at a perfectly competitive price, Q_t^i, that is determined endogenously by investors' profit maximisation decision. The net profit from the capital acquisition is zero, as perfect competition implies that any return from a unit of new capital acquired is equated to a unit of equity issued. The gross profits

[8] Notice that at each time t, the capital used for the production of output is of time t. This simply means that the new capital acquired at time t, I_t, is used one period after with no initial depreciation. Gertler and Karadi (2011) follow a similar logic.

[9] Capital is predetermined at time t. For the sake of terminology, I use ψ_{t+1} instead of ψ_t to denote that the realisation of the shock is known at the beginning of period $t+1$ after the investment opportunities are realised. Also, notice that new capital

from capital used in the production process are given by the difference of total revenues minus the labour cost, $\phi X_t - W_t N_t$. Using equation (3.11), the gross profits per unit of capital, Z_t, are given by:

$$Z_t = \alpha \phi_t \frac{X_t}{K_t} \tag{3.12}$$

3.2.4 Capital goods producers (Investors)

Investors act under perfect competition and those located in locations with new investment opportunities create new capital, I_t, by using $I_t(i)$ units of the final good. The new capital is sold at the competitive price Q_t^i. It is assumed that it is costly for the investors to increase or decrease their capital stock. Also the marginal change of capital stock is increasing in the size of the change. As it discussed in Woodford (2003), these assumptions can be captured by the convex adjustment cost function, $I(\cdot)$, given by:

$$I\left(\frac{I_t}{I_{t-1}}\right) \equiv \frac{\chi}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2$$

where χ is a constant capturing the sensitivity of investment costs. I have also assumed for simplicity a degree of convexity equal to 1. $I\left(\frac{I_t}{I_{t\,t-1}}\right)$ satisfies the properties $I(1) = I'(1) = 0$ and $I'\left(\frac{I_t}{I_{t\,t-1}}\right) > 0$, $I''\left(\frac{I_t}{I_{t\,t-1}}\right) > 0$.

Given that $I_t(i)$ is expressed in units of final good, I_t is given by the index:

$$I_t \equiv \left(\int_0^1 I_t(i)^{\frac{\gamma-1}{\gamma}} di\right)^{\frac{\gamma}{\gamma-1}}$$

and the total expenditure for investments are:

$$\int_0^1 P_t(i) I_t(i) di$$

It comes out that the intratemporal decision of the capital goods producers is equivalent to those of households. Solving a similar problem yields the system of demand equations for final good used in investment:

$$I_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\gamma} I_t$$

The intertemporal decision of capital goods producers is to maximise the expected discounted profits by choosing the amount of new capital to sell, subject to the adjustment costs. This is formalised as:

$$\max_{\{I_t\}_{t=0}^{\infty}} E_0 \sum_{s=0}^{\infty} \beta_{t,t+s} \left\{ Q^i_{t+s} I_{t+s} - \left(1 + I\left(\frac{I_{t+s}}{I_{t-1+s}}\right)\right) I_{t+s} \right\}$$

The solution to this problem is provided in the Appendix B.4 and yields the competitive price of capital:

$$Q^i_t = 1 + \chi\left(\frac{I_t}{I_{t-1}} - 1\right)\left(\frac{I_t}{I_{t-1}} + \frac{1}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)\right) - \chi E_t \beta_{t,t+1} \left(\frac{I_{t+1}}{I_t}\right)^2 \left(\frac{I_{t+1}}{I_t} - 1\right) \tag{3.13}$$

Notice that in the absence of adjustment costs, the asset price becomes $Q^i_t = 1$. Therefore, assuming adjustment costs is also used to capture the asset price variation as a result from changes of the aggregate level of investment.

3.2.5 Financial Intermediaries

In each location there is a continuum of financial intermediaries (investment and commercial banks) represented by $j \in [0,1]$ that provides loans to intermediate good producers. The latter can borrow funds only from financial intermediaries of the same location. Each financial intermediary may raise funds from households' deposits, D_t (in real terms), and from funds, B_{jt}, borrowed from other financial intermediaries in the interbank market. The interbank market is not location-specific, but operates in the economy as a whole. The real rate paid to households' deposits at the end of period t is R_{t+1}. From Fisher, equation (3.5), the real interest rate is determined by the nominal interest rate adjusted by the central bank, as well as the inflation rate. The interbank lending rate, $R_{b\,t+1}$, is determined endogenously by the representative intermediary's optimal behaviour, while $h = i, n$ denotes the type of the location that a financial intermediary operates, i.e. if at this specific location there are new investment opportunities or not.

A representative financial intermediary j can provide a total amount of loans equal to the total value of its assets, $Q_t^h S_{jt}^h$, where S_{jt}^h is the quantity of the intermediary's financial claims (assets) held by the borrowers, i.e. the intermediate good producers. Q_t^h is the price of this asset.

The representative financial intermediary j is balance sheet constrained:

$$Q_t^h S_{jt}^h = NW_{jt}^h + B_{jt}^h + D_{jt} + M_{jt}^h \tag{3.14}$$

At the end of period t, the total value of the intermediary j's assets must be equal to the total value of its liabilities. The latter is given from the sum of the intermediary's net worth, NW_{jt}^h, and the amount from borrowing. The last variable of the right hand side, M_{jt}^h, is the amount of funds borrowed from the DW, where its supply is controlled by the central bank. The net worth of the intermediary j, is a state variable and follows the following law of motion:

$$NW_{jt+1}^h = R_{kt+1}^h Q_t^h S_{jt}^h - R_{bt+1} B_{jt}^h - R_{t+1} D_{jt} - R_{mt+1} M_{jt}^h \tag{3.15}$$

where R_{kt+1}^h is the real lending rate (the return to the intermediary from lending non-financial firms) and R_{mt+1} is the discount rate determined by the central bank. The real rates are paid at the end of period t. From equation (3.15), it follows that the value of the intermediary j's net worth, at the beginning of period $t+1$, NW_{jt+1}^h, is predetermined at time t and is given from the difference between the revenue made from lending and the costs occurring from meeting the liabilities.

The intertemporal problem of the representative financial intermediary

The intertemporal problem of the representative financial intermediary j is to maximise the expected discounted value of its net worth

$$\Xi_{jt}^h = E_t \sum_{s=0}^{\infty} (1-\sigma)\sigma^{s-1} \beta_{t,t+s} NW_{jt+s}$$

where $\Xi_{jt}^h \equiv \Xi_{jt}(S_{jt}^h, B_{jt}^h, D_{jt}, M_{jt}^h)$. The term $(1-\sigma)\sigma^{s-1}$ reflects the fact that the intermediary's owner has a probability σ to survive but after s periods, with probability $(1-\sigma)\sigma^{s-1}$, he/she will become a worker.

To generate a liquidity constraint that is disruptive for the financial market, following Gertler and Kiyotaki (2010), Gertler and Karadi (2011) and Gertler, Kiyotaki, and Queralto (2012), I assume that the financial intermediaries' owners have the ability to cheat and misuse a fraction λ of the funds borrowed for their personal benefit and not for financial activities. For example, they divert a fraction of funds to large bonuses or dividends that are transferred to the owner's household. Goodfriend and King (1988) has stressed the importance of the central bank providing DW loans to solvent financial institutions only. In the model, the financial intermediaries are solvent, but if they are caught cheating, they return the remaining assets to the lenders and then default. The intermediaries' actions are perfectly observable to lenders. In this case, an endogenous borrowing constraint arises for the intermediaries, as lenders knowing the bank's actions may be willing to limit their funds.[10]

In the absence of DW lending ($M_{jt}^h = 0$), the liquidity constraint for the representative intermediary j is:

$$\Xi_{jt}^h \geq \lambda\left(Q_t^h S_{jt}^h - \omega_b B_{jt}^h\right)$$

where ω_b is the fraction of the funds obtained in the interbank market that cannot be diverted. I examine two cases: The case where the interbank market is frictional, i.e. $\omega_b = 0$ and the case where the interbank market is frictionless, $\omega_b = 1$.

Allowing for DW lending in the model, I assume that the owners of the financial intermediaries have the ability to misuse a fraction $\lambda(1-\omega_m)$ of the funds borrowed from the DW, as in Gertler and Kiyotaki (2010). The parameter ω_m is key for the results of the model. It reflects the ability of the central bank to monitor the financial intermediaries' actions with regard to the use of the funds obtained from the DW. This could be explained from the fact that a central bank may have access to means of monitoring (access to financial records, financial inspections etc), in contrast with the other borrowers. While, I do not construct explicitly a mechanism of monitoring, this is captured from ω_m. A higher ω_m implies a higher ability of the central bank to monitor.[11] Allowing for DW lending, the liquidity constraint is given

[10] Banks and lenders face a cost of their actions, which it is not expressed analytically in the model. In particular, bankers default after their decision to divert funds, while households face the cost of reclaiming their lost funds.

[11] As Gertler and Kiyotaki (2010) and Gertler and Karadi (2011) explain, the perfect interbank market case, $\omega_b = 1$, is isomorphic to the case where the interbank market is absent, since in aggregation B_{jt}^h is cancelled out. In this paper, it is more intuitive to consider that $\omega_b = 1$ describes an absent interbank market. In this case, I avoid unnecessary computations of assuming always $\omega_b < \omega_m$, as

by:

$$\Xi^h_{jt} \geq \lambda\left(Q^h_t S^h_{jt} - \omega_b B^h_{jt} - \omega_m M^h_{jt}\right)$$

The constraint implies that as long as the financial intermediary's expected present discounted value of net worth is not lower than the value from the funds misused, the lenders to this intermediary will not limit their funding.

The intertemporal problem of the representative financial intermediary j is constrained from the law of motion of the net worth and the liquidity constraint. Then, Ξ^h_{jt} must satisfy the Bellman equation given by:

$$\Xi_{jt}(S^h_{jt}, B^h_{jt}, D_{jt}, M^h_{jt}) = E_t \beta_{t,t+1} \sum_{h=i,n} \tau^h \Bigg\{ (1-\sigma) NW^h_{jt+1} + \sigma \max_{D_{jt+1}} \left[\max_{S^h_{jt+1}, B^h_{jt+1}, M^h_{jt+1}} \Xi_{jt+1}(S^h_{jt+1}, B^h_{jt+1}, D_{jt+1}, M^h_{jt+1}) \right] \Bigg\}$$

In the right hand side, the continuation value of the Bellman equation satisfies the assumption that the financial intermediary chooses optimally the level of households' deposits before the realisation of the idiosyncratic liquidity risk. In contrast, the optimal decision for lending and borrowing in the interbank market is made once the new investment opportunities have been realised.

Let $\chi^h_{mt} \equiv \frac{M^h_{jt}}{Q^h_t S^h_{jt}}$ be the fraction of the intermediary's j total assets that have been obtained from the DW. Then the borrowing constraint can be rewritten according to:

$$\Xi^h_{jt} \geq \lambda\Bigg((1-\omega_m \chi^h_{mt}) Q^h_t S^h_{jt} - \omega_b B^h_{jt}\Bigg) \tag{3.16}$$

I guess that $\Xi_{jt}(S^h_{jt}, B^h_{jt}, D_{jt}, M^h_{jt})$ is linear:

$$\Xi_{jt}(S^h_{jt}, B^h_{jt}, D_{jt}, M^h_{jt}) = \mathcal{V}^h_{sjt} S^h_{jt} - \mathcal{V}^h_{bjt} B^h_{jt} - \mathcal{V}_{djt} D_{jt} - \mathcal{V}^h_{mjt} M^h_{jt} \tag{3.17}$$

Using equation (3.14), I eliminate D_{jt} and I rewrite equation (3.17) according to:

$$\Xi_{jt}(S^h_{jt}, B^h_{jt}, M^h_{jt}, NW^h_{jt}) = \left(\frac{\mathcal{V}^h_{sjt}}{Q^h_t} - \mathcal{V}_{djt}\right) Q^h_t S^h_{jt} - \left(\mathcal{V}^h_{bjt} - \mathcal{V}_{djt}\right) B^h_{jt} - \left(\mathcal{V}^h_{mjt} - \mathcal{V}_{djt}\right) M^h_{jt} + \mathcal{V}_{djt} NW^h_{jt} \tag{3.18}$$

I argue that it is not realistic to assume that a central bank has less means of monitoring than the financial intermediaries, thus $\omega_b > \omega_m$.

where $\frac{\mathcal{V}^h_{sjt}}{Q^h_t}$ is the marginal value of lending and $\mathcal{V}^h_{bjt}, \mathcal{V}_{djt}, \mathcal{V}^h_{mjt}$ are the marginal costs of borrowing from other intermediaries, households and the central bank respectively. Letting:

$$\mu^h_{sjt} \equiv \frac{\mathcal{V}^h_{sjt}}{Q^h_t} - \mathcal{V}_{djt}$$

$$\mu^h_{bjt} \equiv \mathcal{V}^h_{bjt} - \mathcal{V}_{djt}$$

$$\mu^h_{mjt} \equiv \mathcal{V}^h_{mjt} - \mathcal{V}_{djt}$$

and using $\chi^h_{mt} \equiv \frac{M^h_{jt}}{Q^h_t S^h_{jt}}$, I rewrite equation (3.18) according to:

$$\Xi_{jt}(S^h_{jt}, B^h_{jt}, M^h_{jt}, NW^h_{jt}) = \left(\mu^h_{sjt} - \mu^h_{mjt}\chi^h_{mt}\right) Q^h_t S^h_{jt} - \mu^h_{bjt} B^h_{jt} + \mathcal{V}_{djt} NW^h_{jt} \qquad (3.19)$$

Then, the Bellman can be rewritten according to:

$$\begin{aligned}\Xi_{jt}(S^h_{jt}, B^h_{jt}, NW^h_t, M^h_{jt}) = & E_t\beta_{t,t+1} \sum_{h=i,n} \tau^h \Big\{ (1-\sigma) NW^h_{t+1} \\ & + \sigma \left[\max_{S^h_{jt+1}, B^h_{jt+1}, M^h_{jt+1}} \Xi_{jt+1}\left(S^h_{jt+1}, B^h_{jt+1}, NW^h_{jt+1}, M^h_{jt+1}\right)\right]\Big\}\end{aligned}$$

Therefore, the solution to the financial intermediary's problem is given from the solution to the problem:

$$\max_{S^h_{jt+1}, B^h_{jt+1}, M^h_{jt+1}} E_t\beta_{t,t+1} \sum_{h=i,n} \tau^h \; \Xi_{jt+1}\left(S^h_{jt+1}, B^h_{jt+1}, NW^h_{jt+1}, M^h_{jt+1}\right)$$

subject to the endogenous borrowing constraint, equation (3.16).

Frictionless interbank market

First, I consider the case where the interbank market is frictionless, thus $\omega_b = 1$. The associated Langrangian is given by:

$$\mathcal{L} = \sum_{h=i,n} \tau^h \left[\Xi^h_{jt} + l^h_t \left(\Xi^h_{jt} - \lambda\Big((1-\omega_m \chi^h_{mt}) Q^h_t S^h_{jt} - B^h_{jt}\Big)\right)\right] \qquad (3.20)$$

where l_t^h is the Langrangian multiplier. Using equation (3.19) and rearranging I obtain:

$$\mathscr{L} = \sum_{h=i,n} \tau^h (1+l_t^h)\Big[\Big(\mu^h_{sjt} - \mu^h_{mjt}\chi^h_{mt}\Big)Q_t^h S_{jt}^h - \mu^h_{bjt} B_{jt}^h + \mathscr{V}_{djt} NW_{jt}^h\Big] - \sum_{h=i,n} \tau^h l_t^h \lambda\Big((1-\omega_m \chi^h_{mt})Q_t^h S_{jt}^h - B_{jt}^h\Big)$$

The first-order and Kuhn-Tucker conditions for this problem are given by:[12]

$$S_{jt}^h : (1+l_t^h)\Big(\mu^h_{sjt} - \mu^h_{mjt}\chi^h_{mt}\Big) = l_t^h \lambda(1-\omega_m \chi^h_{mt})$$
$$M_{jt}^h : (1+l_t^h)\mu^h_{mjt} = l_t^h \lambda \omega_m$$
$$B_{jt}^h : (1+l_t^h)\mu^h_{bjt} = l_t^h \lambda$$
$$KT : l_t^h\Big[\Big(\mu^h_{sjt} - \mu^h_{mjt}\chi^h_{mt}\Big)Q_t^h S_{jt}^h - \mu^h_{bjt} B_{jt}^h + \mathscr{V}_{djt} NW_{jt}^h - \lambda\Big((1-\omega_m \chi^h_{mt})Q_t^h S_{jt}^h - B_{jt}^h\Big)\Big]$$
$$KT : l_t^h \geq 0$$

I assume that the constraint always binds, hence $l_t^h > 0$. From the first-order conditions, I get that when the interbank market is perfect:

$$\mu^h_{mjt} = \omega_m \mu^h_{bjt} = \omega_m \mu^h_{sjt} \tag{3.21}$$

From (3.21), $\mu^h_{bjt} = \mu^h_{sjt}$, which implies $\frac{\mathscr{V}^h_{sjt}}{Q_t^h} = \mathscr{V}^h_{bjt}$. In the absence of interbank frictions, it is also true that $\frac{\mathscr{V}^i_{sjt}}{Q_t^i} = \frac{\mathscr{V}^n_{sjt}}{Q_t^n}$ and $\mathscr{V}^i_{sjt} = \mathscr{V}^n_{sjt}$ therefore, $Q_t^i = Q_t^n = Q_t$. Hence, for the case of a frictionless interbank market, I can drop superscript h. Assuming that all banks behave in a symmetric way, I sum across banks and locations and I can drop subscript j. Using the definition of the leverage ratio, θ_t

$$\theta_t \equiv \frac{Q_t S_t}{NW_t} \tag{3.22}$$

[12] For the first order conditions and the guess and verify method, I have found very useful the handbook from Cantore and Levine (2015) and Gertler and Kiyotaki (2010).

and given that the constraint always binds, $l_t^h > 0$, from the Kuhn-Tucker condition, I get:

$$\theta_t = \frac{\mathcal{V}_{dt}}{\lambda(1-\omega_m \chi_{mt}) - (\mu_{st} - \mu_{mt}\chi_{mt})} \tag{3.23}$$

Verifying that $\Xi_{jt}(S_{jt}^h, B_{jt}^h, D_{jt}, M_{jt}^h)$ is linear, dropping the subscript j yields:

$$\Xi_t = E_t \beta_{t,t+1} \Omega_{t+1} (R_{kt+1} Q_t S_t - R_{bt+1} B_t - R_{t+1} D_t - R_{mt+1} M_t)$$

which implies:

$$V_{st} = E_t \beta_{t,t+1} \Omega_{t+1} Q_t R_{kt+1} \tag{3.24}$$

$$V_{dt} = E_t \beta_{t,t+1} \Omega_{t+1} R_{t+1} \tag{3.25}$$

$$V_{mt} = E_t \beta_{t,t+1} \Omega_{t+1} R_{mt+1} \tag{3.26}$$

where the real rate of lending, R_{kt+1}, is given by:

$$R_{kt+1} = \psi_{t+1} E_t \left(\frac{Z_{t+1} + (1-\delta) Q_{t+1}}{Q_t} \right)$$

Since the intermediate good producers earn zero profits from renting capital, they return to the intermediary the net earnings after the depreciation of capital, $Z_{t+1} + (1-\delta)Q_{t+1}^h$. The real lending rate is also affected from the exogenous quality of capital. In addition, $E_t\Omega_{t+1}$ is the marginal value of the bank's capital and is given by:

$$E_t \Omega_{t+1} = 1 - \sigma + \sigma E_t \left(\mathcal{V}_{dt+1} + (\mu_{st+1} - \mu_{mt+1}\chi_{mt+1})\theta_{t+1} \right) \tag{3.27}$$

I define the difference between the real rate of lending and the real rate of borrowing from households, as the financial spread, $Spread_{t+1} \equiv R_{kt+1} - R_{t+1}$. Now, combining equations (3.24 - 3.26) with the definitions of the marginal excess value from lending and marginal costs from borrowing, I summarise the key equations from the financial intermediaries' behaviour in the frictionless interbank market:

$$\mu_{st} = E_t \beta_{t,t+1} \Omega_{t+1} Spread_{t+1} \tag{3.28}$$

$$\mu_{mt} = E_t \beta_{t,t+1} \Omega_{t+1} \Big(R_{mt+1} - R_{t+1} \Big) \tag{3.29}$$

$$E_t \beta_{t,t+1} \Omega_{t+1} R_{mt+1} = \omega_m E_t \beta_{t,t+1} \Omega_{t+1} R_{kt+1} + (1-\omega_m) E_t \beta_{t,t+1} \Omega_{t+1} R_{t+1} \tag{3.30}$$

By definition, the central bank charges a penalty rate on the DW rate, R_{mt+1}, that is set above the real rate of borrowing from households. Notice that at the moment, nothing has been assumed with regard to the penalty rate. However, we have found that in equilibrium, $\mu_{mt} = \omega\mu_{st}$. That is, assuming that the central bank has some means of monitoring, but there is a risk that intermediaries may misuse the funds obtained from the DW, generates a penalty rate. This has been firstly highlighted by Gertler and Kiyotaki (2010). Equation (3.30) implies that the DW rate will be between the real rate of lending and the real rate of borrowing. Given that by definition, the penalty rate is equal to $PR_{t+1} \equiv R_{mt+1} - R_{t+1}$, combining with equation (3.30), then the penalty rate is given by:

$$E_t\beta_{t,t+1}\Omega_{t+1}PR_{t+1} = \omega_m E_t\beta_{t,t+1}\Omega_{t+1}Spread_{t+1} \tag{3.31}$$

From the equilibrium conditions, some further remarks are following:

- There is a spread between the real rate of lending and the real rate of borrowing from households, as long as the borrowing constraint is binding and therefore $\mu_{st} > 0$, i.e. the marginal value of lending is higher than the marginal cost of borrowing from households. From the first order conditions of the problem, this is true when $\lambda > 0$. Hence, given that the borrowing constraint binds always, the spread in the financial market is a result of the financial friction.

- The demand of financial intermediaries for borrowing from the DW is inelastic in the penalty rate, as long as the excess return from lending, i.e. the spread, is nonzero.

- From the first-order conditions, $E_t\beta_{t,t+1}\Omega_{t+1}R_{bt+1} \geq E_t\beta_{t,t+1}\Omega_{t+1}R_{mt+1}$. Therefore the intermediaries located at locations with no new investment opportunities may borrow from the DW and then lend the intermediaries located at locations with new investment opportunities.[13]

Frictional interbank market

In this section, I analyse the financial intermediaries' optimal behaviour when the interbank market is frictional. As Calomiris (1993) has argued, DW should be mostly

[13]Of course this holds as long as there is an interbank market. As it is mentioned in a comment above, the perfect interbank market case is isomorphic to the no interbank market case.

used in economies with disrupted interbank markets. Because for values $0 < \omega_b < 1$, the solution of the problem becomes complicated, I restrict the analysis for the case $\omega_b = 0$.[14] In the numerical exercises, I always allow for $\omega_b < \omega_m$, i.e. I assume that unlike the other borrowers, the central bank has some ability of monitoring. The fraction of the assets borrowed in the interbank market and from households that may be diverted is equal to λ.

Now, the associated Langrangian to the dynamic constrained maximisation problem is given by:

$$\mathscr{L} = \sum_{h=i,n} \tau^h (1 + l_t^h) \Big[\big(\mu_{sjt}^h - \mu_{mjt}^h \chi_{mt}^h \big) Q_t^h S_{jt}^h - \mu_{bjt}^h B_{jt}^h + \mathscr{V}_{djt} N W_{jt}^h \Big] \\ - \sum_{h=i,n} \tau^h l_t^h \lambda \Big((1 - \omega_m \chi_{mt}^h) Q_t^h S_{jt}^h \Big) \tag{3.32}$$

The first-order conditions with respect to the amount of loans and the amount of borrowing from DW are the same as before. The only first-order condition that is different now is the one with respect to B_{jt}^h. Together with the Kuhn-Tucker conditions, I now obtain:

$$B_{jt}^h : (1 + l_t^h) \mu_{bjt}^h = 0$$
$$KT : l_t^h \Big[\big(\mu_{sjt}^h - \mu_{mjt}^h \chi_{mt}^h \big) Q_t^h S_{jt}^h - \mu_{bjt}^h B_{jt}^h + \mathscr{V}_{djt} N W_{jt}^h - \lambda \Big((1 - \omega_m \chi_{mt}^h) Q_t^h S_{jt}^h \Big) \Big]$$
$$KT : l_t^h \geq 0$$

Again, I assume that the borrowing constraint binds always, hence, $l_t^h > 0$. From the first-order conditions, when the interbank market is imperfect, I get that $\mu_{bjt}^h = 0$, which implies that $\mathscr{V}_{bjt}^h = \mathscr{V}_{djt}$. Notice that, because the supply of assets is higher in the locations with new investment opportunities, the asset price will be lower. Since, the interbank market is imperfect the asset arbitrage between locations is imperfect, yielding $Q_t^i < Q_t^n$. This means $\mu_{sjt}^i > \mu_{sjt}^n \geq 0$. Summing across banks but separately for investing and non-investing locations, and also dropping the subscript j, I obtain $\mathscr{V}_{bt} = \mathscr{V}_{dt}$, hence, $R_{bt+1} = R_{t+1}$. This means that it is costly for the intermediaries located in the non-investing locations to borrow from the DW, as in this case lending these funds in the interbank market will yield a lower return. Only

[14] For a solution of the problem when $0 < \omega_b < 1$, see the Appendix in Gertler and Kiyotaki (2010).

the banks from the investing locations will borrow from the DW, as Gertler and Kiyotaki (2010) have highlighted. Given that the borrowing constraint binds always, $l_t^h > 0$, and combining the first-order conditions, I obtain:

$$\mu_{mt}^i = \mu_{mt} = \omega_m \mu_{st}^i \tag{3.33}$$

Using the definition of leverage:

$$\theta_t^i \equiv \frac{Q_t^i S_t^i}{NW_t^i} \quad and \quad \theta_t^n \equiv \frac{Q_t^n S_t^n}{NW_t^n} \tag{3.34}$$

I now obtain:

$$\theta_t^i = \frac{\mathscr{V}_{dt}}{\lambda(1-\omega_m \chi_{mt}) - (\mu_{st}^i - \mu_{mt}\chi_{mt})} \tag{3.35}$$

and

$$\theta_t^n = \frac{\mathscr{V}_{dt}}{\lambda - \mu_{st}^n} \tag{3.36}$$

Verifying again that $\Xi_{jt}(S_{jt}^h, B_{jt}^h, D_{jt}, M_{jt}^h)$ is linear yields:

$$\Xi_t^h = E_t \beta_{t,t+1} \Omega_{t+1}^h \Big(R_{kt+1}^h Q_t^h S_t^h - R_{bt+1}^h B_t^h - R_{t+1} D_t - R_{mt+1} M_t \Big)$$

which implies:

$$V_{st}^h = E_t \beta_{t,t+1} \Omega_{t+1}^h Q_t^h R_{kt+1}^h \tag{3.37}$$

$$V_{dt}^h = E_t \beta_{t,t+1} \Omega_{t+1}^h R_{t+1} \tag{3.38}$$

$$V_{mt} = E_t \beta_{t,t+1} \Omega_{t+1}^i R_{mt+1} \tag{3.39}$$

where now $E_t \Omega_{t+1}^h$ is given from:

$$E_t \Omega_{t+1}^h = 1 - \sigma + \sigma E_t \Big(\mathscr{V}_{dt+1} + (\mu_{st+1}^h - \mu_{mt+1}\chi_{mt+1})\theta_{t+1}^h \Big) \tag{3.40}$$

for $h = i$. When $h = n$ the same expression holds with setting $\mu_{mt+1} = 0$. Also, combining the equations (3.37 - 3.39) with the definitions for the marginal excess value from lending and the costs from borrowing, and using the simplifying assumption that $\mu_{st}^n = 0$ always, I get

$$E_t \beta_{t,t+1} \Omega_{t+1}^i R_{kt+1}^i > E_t \beta_{t,t+1} \Omega_{t+1}^n R_{kt+1}^n = E_t \beta_{t,t+1} \Omega_{t+1}^h R_{bt+1} = E_t \beta_{t,t+1} \Omega_{t+1}^h R_{t+1} \tag{3.41}$$

As $\mu^n_{st} = 0$, there is a financial spread only in the locations with new investment opportunities. That is $Spread^i_{t+1} \equiv R^i_{kt+1} - R_{t+1}$. Hence, I summarise that the key equations in the frictional interbank market are:

$$\mu^i_{st} = E_t \beta_{t,t+1} \Omega^i_{t+1} Spread^i_{t+1} \tag{3.42}$$

$$\mu_{mt} = E_t \beta_{t,t+1} \Omega^i_{t+1} \Big(R_{mt+1} - R_{t+1} \Big) \tag{3.43}$$

$$E_t \beta_{t,t+1} \Omega^i_{t+1} R^i_{mt+1} = \omega_m E_t \beta_{t,t+1} \Omega^i_{t+1} R^i_{kt+1} + (1-\omega_m) E_t \beta_{t,t+1} \Omega^i_{t+1} R_{t+1} \tag{3.44}$$

Now, the penalty rate charged to the DW rate is given by:

$$E_t \beta_{t,t+1} \Omega^i_{t+1} PR^i_{t+1} = \omega_m E_t \beta_{t,t+1} \Omega^i_{t+1} Spread^i_{t+1} \tag{3.45}$$

3.2.6 Aggregation and market clearing

In the model there are old and new bankers. Therefore, the total net worth of the financial intermediaries is given by:

$$NW^h_t = NW^h_{ot} + NW^h_{nt}$$

The net worth of the old bankers is given by:

$$NW^h_{ot} = \sigma \tau^h \left(R^h_{kt} Q^h_{t-1} S^h_{jt-1} - R_t D_{jt-1} - R_{mt} M^h_{jt-1} \right)$$

The new bankers begin with a start-up capital transferred from their household. For simplicity, this is assumed to be equal to a fraction $\frac{\xi}{1-\sigma}$ of the total value of the intermediaries' owners that exit:

$$NW^h_{nt} = \xi \tau^h R^h_{kt} Q^h_{t-1} S^h_{jt-1}$$

Summing across intermediaries, I can drop subscript j. Also, when the interbank market is frictionless, I can drop superscript h. Hence, the total net worth of financial intermediaries is given by:

$$NW_t = (\sigma + \xi) R_{kt} Q_{t-1} S_{t-1} - \sigma (R_t D_{t-1} + R_{mt} M_{t-1}) \tag{3.46}$$

When the interbank market is frictional, I can sum the net worth across intermediaries but separately for locations with new and no new investment opportunities. Recall that only intermediaries from locations with new investment opportunities will borrow from DW:

$$NW_t^i = \tau^i(\sigma+\xi)R_{kt}^i Q_{t-1}^i S_{t-1}^i - \tau^i\sigma\left(R_t D_{t-1} + R_{mt} M_{t-1}\right) \tag{3.47}$$

$$NW_t^n = \tau^n(\sigma+\xi)R_{kt}^n Q_{t-1}^n S_{t-1}^n - \tau^n\sigma R_t D_{t-1} \tag{3.48}$$

The securities S_t^h are issued against capital. In equilibrium:

$$S_t^i = I_t + (1-\delta)\tau^i K_t$$
$$S_t^n = (1-\delta)\tau^n K_t$$

Therefore, $S_t = S_t^i + S_t^n = I_t + (1-\delta)K_t$, which implies:

$$K_{t+1} = \psi_{t+1} S_t \tag{3.49}$$

The labour market clearing condition requires to equalise labour demand with labour supply, therefore:

$$(1-\alpha)\phi_t \frac{Y_t}{N_t} = C_t N_t^{\varphi} \tag{3.50}$$

where I have used that $Y_t = X_t$.

3.2.7 Government and aggregate resource constraint

The differentiated final goods are either consumed by households or used as input by investors. The market clearing condition in the final goods market requires that total demand is equal to the total supply. In the Appendix B.5, I show that by combining the Dixit-Stiglitz aggregator:

$$Y_t \equiv \left(\int_0^1 Y_t(i)^{\frac{\gamma-1}{\gamma}} di \right)^{\frac{\gamma}{\gamma-1}}$$

yields:

$$Y_t = C_t + I_t$$

Government spending, G_t, as in Gertler and Karadi (2011), is exogenously fixed at G_0, to avoid implications from the interaction of monetary and fiscal policy. In addition, lending through the DW the total amount of funds $M_t = \chi_{mt} Q_t S_t$ involves an efficiency per unit cost, c^{dw} for the central bank. In addition, the central bank finances DW loans by issuing riskless goverment bonds, B_{gt}, which are then sold to households with a return R_{t+1}. Therefore, the instantaneous net earnings of the central bank from DW lending are $(R_{mt+1} - R_{t+1}) B_{gt}$. Taking these into account, the aggregate resource constraint is given by:

$$Y_t = C_t + I_t + G_0 + c^{dw} \chi_{mt} Q_t S_t \tag{3.51}$$

and the government budget is given by:

$$G_0 + c^{dw} \chi_{mt} Q_t S_t = T_t + (R_{mt+1} - R_{t+1}) B_{gt}$$

where $B_{gt} = \chi_{mt} Q_t S_t$.

3.3 Monetary Policy

Nothing yet has been analysed with regard to the central bank's behaviour. I make an analysis of an optimal welfare-based monetary policy. In particular, I study the case, where the main objective of the central bank is to minimise the households' welfare losses that occur from a negative shock in the quality of capital.

I assume that the central bank follows a particular optimal monetary policy regime: A timeless perspective optimal commitment, or an optimal simple monetary rule. Any of these two regimes followed is then implemented with the adjustment of the nominal interest rate.

First, I solve the optimal monetary policy problem for both regimes assuming that the central bank has no access to a complementary instrument. Then, I switch the DW on and I solve the optimal monetary policy problem again. In this case, for any of the two regimes followed, the central bank is jointly adjusting the interest rate along with the DW. The difference in the welfare losses that occurs from switching

DW off and on is the contribution of DW lending on welfare.

The approach on the welfare analysis that I follow is the L-Q developed by Rotemberg and Woodford (1997) and Woodford (2003) and implemented by Levine, McAdam, and Pearlman (2008) and Benigno and Woodford (2012), among many other authors in the NK literature. The L-Q approach involves a second-order approximation of households' welfare criterion and a first-order approximation of the equilibrium conditions and resource constraints of the economy. The main reasons that I follow the L-Q approach are the following: First, I can derive an objective function for the central bank that involves variables relevant for monetary policy, such as inflation and output gap. The stabilisation of these variables is the main policy objective of central banks in reality. In addition, the micro-foundations of the model can provide intuition with regard to the sources of welfare loss and the relative strength put by the central bank. By decomposing the welfare loss and calculating the second moments of the central bank's targets under optimal policy, we can understand better the macroeconomic channels through which DW contributes on welfare. Last but not least, as in Woodford (2003) and Levine, McAdam, and Pearlman (2008), the effect of the ZLB constraint on the nominal interest rate can be approximated by adding an interest-rate term in the derived welfare loss function.

As Woodford (2003), Benigno and Woodford (2004) and Schmitt-Grohe and Uribe (2007) have highlighted, the L-Q approach is accurate as long as the deviation of the economy from steady-state is small and also the steady-state that the economy starts is close to the efficient outcome.[15] For this reason, first I derive and linearise the equilibrium conditions and resource constraints of the frictionless model, i.e. the model with fully flexible prices and a frictionless financial market. I name this state of the model as flexible-price equilibrium. This is close to the efficient outcome, or it is the efficient outcome, where all variables are at their natural level/rate, as long as there is also an output subsidy that offsets the monopolistic competition distortion and does not create an extra inefficiency.[16] Then, I derive and linearise

[15]For a further discussion and a review of the main assumptions that suggest that the L-Q approach is accurate, see Schmitt-Grohe and Uribe (2007).

[16]With regard to the inefficiently low output arising from the distortion of monopolistic competition, it is hard to assume that it is offset by an output subsidy, as normally, this is financed by a lump-sum tax imposed by the government, which is absent in the model for simplicity. This problem can be resolved by assuming that the subsidy is financed by the central bank. This would be equivalent with assuming a lump-sum tax, since both do not cause any further distortionary effect in the economy.

the equilibrium conditions and resource constraints of the frictional model and I write the model in log-deviations from the flexible price equilibrium.

The linear constraints of the central bank in conducting optimal monetary policy are given by the equilibrium conditions and the aggregate resource constraints of the model linearised following Uhlig (1997). These can be found in the Appendix B.6. Some extra notation is also needed. For any generic variable X_t, a small letter with a tilde, $\tilde{x}_t$, is defined as the log-deviation of X_t from its flexible-price equilibrium level. That is: $\tilde{x}_t = \hat{x}_t - \hat{x}_t^e$, where $\hat{x}_t$, denotes the log-deviation of X_t from its non-stochastic steady-state, X, that is $\hat{x}_t = \log X_t - \log X$, and $\hat{x}_t^e$ denotes the flexible price equilibrium log-deviation of X_t^e from its steady state, X^e, that is: $\hat{x}_t^e = \log X_t^e - \log X^e$. In that sense, the model captures how the economy deviates from equilibrium because of frictions in the real economy and the financial market.

3.3.1 The welfare criterion of the central bank

In the Appendix B.7, I show that a second order approximation of households' utility yields:

$$\sum_{t=0}^{\infty} \beta^t \left(\log C_t - \frac{{N_t}^{1+\varphi}}{1+\varphi} \right) \approx \sum_{t=0}^{\infty} \beta^t L_t + t.i.p. + O^3$$

where t.i.p. are the terms independent of policy, O^3 are the terms of third or higher order and L_t is the instantaneous welfare loss function given by:

$$\begin{aligned} L_t = \frac{1}{2} \left(\frac{\gamma \delta_p}{1-\delta_p} \pi_t^2 + \frac{(\alpha+\varphi) Y}{(1-\alpha) C} \tilde{y}_t^2 + \frac{\alpha^2 (1+\varphi) Y}{(1-\alpha) C} \tilde{k}_t^2 + \frac{I Y}{C^2} \tilde{i}_t^2 \right) \\ - \frac{Y}{C} \left(\frac{I}{C} \tilde{y}_t \tilde{i}_t + \frac{\alpha (1+\varphi)}{(1-\alpha)} \tilde{y}_t \tilde{k}_t \right) - G_t^e \end{aligned} \tag{3.52}$$

where:

$$G_t^e \equiv \frac{Y}{C} \left(\frac{K}{C} \tilde{k}_{t+1} \left(\Delta_{yi,t}^e - \Delta_{yi,t+1}^e \right) + \alpha \tilde{k}_t \Delta_{yi,t}^e \right)$$

and $\Delta_{yi,t}^e \equiv \hat{y}_t^e - \hat{i}_t^e$.

The loss function, 3.52, may have the following interpretation. The first part illustrates the average welfare loss per period given by the deviation of inflation from a zero steady-state as well as the deviation of output, investment and capital from their flexible-price equilibrium level. The second part illustrates the welfare gains from the covariance of output with capital and investments. These welfare gains arise from the fact that when capital is introduced in the model, the inefficient fluctuations in households' supply of labour units is reduced. The last part of the loss function, G_t^e, illustrates the effect from the covariance of capital with the flexible-price equilibrium output and investment.[17]

First, the central bank chooses to follow an optimal monetary policy regime. I assume that there are not strategic profiles on the selection of the optimal policy regime by the central bank, as in Levine, McAdam, and Pearlman (2008), but instead the central bank chooses to follow a timeless perspective optimal commitment, or an optimal simple monetary rule for exogenous reasons that are not explained.

When the central bank follows a regime of optimal commitment, this requires to choose the path for the current and future stabilisation policy objectives

$$\left\{\pi_t, \tilde{y}_t, \tilde{i}_t, \tilde{k}_{t+1}\right\}_{t=0}^{\infty}$$

to minimise the welfare loss function (3.52). That is, the optimal response by the central bank under commitment is given by the solution to the problem:

$$\min E_0 \sum_{t=0}^{\infty} \beta^t \left\{ L_t \right\}$$

subject to the constraints given from the linearised model. There is no reason to impose a constraint for the path of the nominal interest rate because, as Walsh (2010) shows, the optimal choice for the stabilisation policy objectives determines the interest rate path. Assuming a timeless perspective resolves the dynamic inconsistency problem that is associated with optimal commitment.[18]

When the central bank follows a regime of an optimal monetary rule, I assume that this takes the form of a Taylor-type rule proposed by Taylor (1993):[19]

$$q_t = \rho_q q_{t-1} + (1-\rho_q)(\kappa_\pi \pi_t + \kappa_y \tilde{y}_t + \kappa_k k_t + \kappa_i \tilde{i}_t) \tag{3.53}$$

[17] Notice that the loss function does not depend on purely quadratic terms only, neither is necessarily positive definite. However, this is not a problem as Levine, Pearlman, and Pierse (2006) show that positive definiteness is not necessary condition for optimality. For further discussion see also Levine, McAdam, and Pearlman (2008).

[18] See Walsh (2010), chapter 8, for a proof of dynamic inconsistency in a standard NK model.

[19] In models with a cashless economy, it is standard to assume that the central bank controls directly the nominal interest rate. Walsh (2010) shows that in a model with separable utility function, as this one, the quantity of money disappears from the equilibrium conditions, thus it has no effect.

where $0 \leq \rho_q \geq 1$ is the interest-rate smoothing parameter and $\kappa' s$ are the feedback parameters. Then, the central bank chooses $\kappa' s$ that minimise the expected discounted value of (3.52) subject to the constraints of the linearised model, which now includes the optimal simple rule, (3.53).[20]

When the complementary instrument of monetary policy is introduced, it is expressed as the fraction of the total assets, $\tilde{\chi}^h_{mt}$, that intermediaries finance through the DW. As in Cantore and Levine (2015) in a similar framework with outside equity, I assume that the complementary instrument follows a rule. In particular, the central bank uses DW to reduce the disruption in the financial market only. This is the case when there is a spread between the real rate of lending and the real rate of borrowing:

$$\tilde{\chi}^h_{mt} = \kappa_{\varrho}\left(\tilde{r}^h_{kt+1} - \tilde{r}_{t+1}\right) \tag{3.54}$$

The feedback parameter κ_{ϱ} captures how strongly the central bank responds to changes in the spread. The contribution of DW to welfare is calculated for different values of κ_{ϱ} considering both cases of a frictional and frictionless interbank market. In the frictionless interbank market scenario, the rule applies without the superscript h. In the frictional interbank market scenario, the rule applies only for $h = i$.

3.3.2 The Zero Lower Bound Constraint

After the 3^{rd} quarter of 2008, many major central banks cut their interest rates sharply as a response to the financial crisis. The graph in the Appendix B.10.2 illustrates the nominal interest rate adjustment for the period $2008-2015$. It is clear that during this period, the adjustment of the British and American interest rate has been constrained by a zero lower bound (ZLB). In addition, on April 2016 these interest rates have still been above zero (0.5 %). That was also true for the European interest rate until the end of 2014. Since then, the European Central Bank decided to decrease the nominal interest rate slightly below zero (-0.4 %). While there is no technical

[20] The minimum restriction that is imposed on the rule is that the determinacy of equilibrium is satisfied. This happens by applying the Taylor principle, $\kappa_\pi > 1$. In addition, because the term $\tilde{y}_t$ involves the natural level of output, which is unobserved by policymakers, issues of implementability of the rule may arise. In this case, the term $\hat{y}_t$ can be used instead. Finally, while it is unusual to include feedback parameters for capital and investment in the simple rule, here, I include them for robustness checks. The quantitative experiments suggest that setting $\kappa_k = \kappa_i = 0$ is more desirable.

reason to argue why the nominal interest rates cannot be below zero, the data show that at least some major central banks have been constrained by a ZLB constraint, or a lower bound constraint below zero. In any case, the adjustment of the interest rate becomes limited as long as it reaches the vicinity of zero.

I incorporate the effect of a lower bound constraint for two reasons: first, it is an assumption that captures the real, recent macroeconomic environment. In addition, limiting the adjustment of the nominal interest rate, I can quantify the net contribution of DW lending. Adding a lower bound constraint on the nominal interest rate, as Gertler and Karadi (2011) have highlighted, the effect of the complementary monetary policy instrument is not understated.

I approximate the effect of a lower bound constraint by utilising the L-Q framework, as in Woodford (2003) and Levine, McAdam, and Pearlman (2008). In this quantitative model, I want to limit the adjustment of the nominal interest rate and capture what happens in reality. While there is no actual reasoning to assume a ZLB constraint, to keep the analysis of the constrained maximisation problem of the central bank simpler, I assume that this lower bound is zero. In particular, in the non-linear model, I add the constraint that the expected discounted dynamic path of the nominal interest rate must be nonnegative:

$$E_0 \sum_{t=0}^{\infty} \beta^t q_t \geq 0 \tag{3.55}$$

According to Woodford (2003), chapter 6, this constraint can be approximated as:

$$E_0 \sum_{t=0}^{\infty} \beta^t q_t^2 \leq \frac{1}{1+\kappa_q^2} E_0 \left(\sum_{t=0}^{\infty} \beta^t q_t \right)^2 \tag{3.56}$$

That is, using expected discounted values, q_t^2 to be no greater than $\frac{1}{1+\kappa_q^2}$ times the square of the expected discounted value of q_t. Following the Appendix in Woodford (2003), page 700, in the Appendix B.8, I show that adding (3.55), (3.56) to the other constraints of the minimisation problem of the central bank and using the Kuhn-Tucker Theorem, yields an instantaneous Loss function L_t^{ZLB}:

$$L_t^{ZLB} = L_t + \omega_q (q_t - q^*)^2 \tag{3.57}$$

where ω_q is the Lagrangian multiplier of the constraint (3.56) and q^* is the new interest rate target. Notice that the latter is now higher than the targeted interest rate that is consistent with a zero steady-state of inflation (that is a targeted interest rate equal to zero).

The problem of the central bank now becomes to minimise (3.57) subject to the constraints of the model given in the previous sections. The difference now is that having the central bank an objective function such (3.57), the policy rule chosen, $E_0 \sum_{t=0}^{\infty} \beta^t q_t$, to minimise the welfare losses, will be subject to the additional constraints (3.55), (3.56).

Notice that the variability of the interest-rate term per se (the term $\omega_q(q_t - q^*)^2$) is not included in the calculation of households' welfare losses. Nevertheless, a positive ω_q implies that the constraint (3.56) binds and the central bank will reduce the adjustment of the nominal interest rate, allowing the policy objectives to be more volatile and increasing in that way the welfare losses. This approximates the ZLB effect as in reality. When the nominal interest rate is reaching the vicinity of zero, its volatility is reduced due to the lack of freedom of the central bank to impose freely further reductions. Therefore, the policy objectives of the central bank, such as inflation, output, investment and capital will be more volatile comparing to the no ZLB case.

The new distribution for q_t is now associated with a nonzero nominal interest rate with probability $1 - p_{ZLB}$ and a nonzero steady state of inflation. Following Levine, McAdam, and Pearlman (2008), the problem of the central bank is implemented by the numerical search for values of ω_q, such that $z_0(p_{ZLB})\sigma_q < q$, where $z_0(p_{ZLB})$ is the critical value of a standard normally distributed variable Z, $p_{ZLB} = Z \leq z_0$, σ_q is the standard deviation and q is the new steady state of the nominal interest rate which is now given by $q = R - 1 + \pi^*$. Also, the steady-state inflation, π^*, is given by:

$$\pi^* = \max\left[z_0(p_{ZLB})\sigma_q - (R-1), 0\right] \tag{3.58}$$

From the original, derived welfare loss function, we know that the steady-state inflation contributes to the welfare loss. The additional loss from the non-zero steady-state inflation, π^*, is given by $-\frac{1}{2}\frac{\gamma\delta_p}{1-\delta_p}\pi^{*2}$.

Taking into consideration the analysis above, I calculate the total households' welfare losses as:

$$L_t^{Total} = L_t^{ZLB} - \omega_q(q_t - q^*)^2 + \frac{1}{2}\frac{\gamma\delta_p}{1-\delta_p}\pi^{*2} \tag{3.59}$$

A higher value of ω_q makes the nominal interest rate less volatile, but increases the volatility of the policy objectives and consequently increases the welfare loss L_t^{ZLB}. However, from (3.58), lower σ_q reduces π^* and its contribution to the total welfare loss will be smaller. Therefore, the shift of the distribution of the nominal interest rate implies a trade-off for the central bank. In Levine, McAdam, and Pearlman (2008), the numerical exercises involve the search for ω_q from a range of positive parameter values and the selection of that ω_q that minimises L_t^{Total}. In this paper, i also consider the case of fixing ω_q to match a very low volatility for the nominal interest rate observed in the data after 2008.

3.4 Quantitative Analysis - Results

3.4.1 Calibration

Preferences: The value of the discount factor, β, is set equal to 0.99. The value of the labour supply elasticity is $\varphi = 2$, which varies from 0—2 in the literature. Gertler and Karadi (2011) choose 0.276 and Gertler, Kiyotaki, and Queralto (2012) choose 0.33.

Financial market: The calibration of the parameters in the financial market follows Gertler and Kiyotaki (2010) and Gertler and Karadi (2011). In the case of a frictionless interbank market, the fraction of the assets that financial intermediaries can divert is $\lambda = 0.383$. In the literature, this value is chosen to target a steady-state leverage approximately equal to 4. For the same reason, the survival rate of bankers is set equal to $\sigma = 0.972$, while the new bankers entry is set equal to $\xi = 0.003$.[21] In the scenario of a frictional interbank market, $\lambda = 0.129$ and $\xi = 0.002$. The fraction of locations with new investment opportunities is 0.25,so the probability to invest becomes 1 after one year. The steady-state spread is 0.0025, which implies a spread of 100 basis points per year for quarterly data.

Goods market: The calibration of the parameters in the real economy follows the

[21] The steady-state leverage ratio takes this value based on the average in different sectors of the financial market. For more details see Gertler and Kiyotaki (2010).

literature on the NK models. The nominal price rigidity parameter is $\omega = 0.75$. The steady-state real marginal cost is $\phi = 0.83$. It is calculated by setting the elasticity of substitution among the differentiated final goods equal to 6. The degree of convexity in the capital adjustment cost function is set equal to 2. The deprecation of capital and the capital share to the production are 0.025 and 0.33 respectively following the literature on the real business cycles.

Monetary policy: With regard to the regime of an optimal monetary rule, I set the benchmark values, $\kappa_\pi = 1.5$ and $\kappa_{\tilde{y}} = 0.125$ following Taylor (1993).[22] The interest rate smoothing parameter is set equal to $\rho_q = 0.2$, as in Gertler and Kiyotaki (2010). Furthermore, I choose a probability for the nominal interest rate to hit the ZLB equal to 2.5%. With regard to DW lending, I search for optimal values of κ_ϱ choosing values between $0-1000$.

Shocks: I allow for a 5% standard deviation decline in the quality of capital, as in Gertler and Kiyotaki (2010) and Gertler and Karadi (2011). The autoregressive coefficient is set equal to $\rho_\psi 0.75$, which captures a medium persistence of the shock. The productivity shock is switched off. I set an autocorrelation coefficient $\rho_a = 0.95$, following Smets and Wouters (2003), The parameter values are summarised in the Appendix, table B.1.

3.4.2 Impulse responses

The effect of the ZLB constraint on the nominal interest rate

To understand what is the effect of the lower bound constraint on the nominal interest rate to the optimal monetary policy regimes, first, I analyse a case where the DW is switched off. Just for illustration purposes, the interbank market is frictionless and the central bank follows the regime of the timeless perspective optimal commitment.[23]. Figure 4.1 displays the impulse response functions of the economy to a 5% standard deviation negative shock to the quality of capital. The dotted lines illustrate the case where there is no lower bound constraint on the nominal interest rate ($\omega_q = 0$), while the solid lines show the impulse responses when a lower bound constraint is imposed and the central bank has chosen a value $\omega_q = 47$, which min-

[22]The determinacy of equilibrium requires $\kappa_\pi > 1.01$ (Taylor principle). The search for the optimal weight $\kappa_{\tilde{y}}$ is also done by setting a zero value as a benchmark following Schmitt-Grohe and Uribe (2007).

[23]The case of a frictional interbank market is illustrated in B.10.1.

imises the total welfare losses, L_t^{Total}.

When there is a negative capital quality shock, capital loses its economic value and falls below its flexible price equilibrium value. Total securities decline because they are issued against capital. Consequently, the asset price falls and the total value of the assets of financial intermediaries falls as well. Hence, their total net worth declines. Tightening intermediaries' balance sheet causes a reduction in the total amount of loans to non-financial firms and consequently, investment falls. Because output depends on investment, it declines as well. From the reduction in the amount of loans, the intermediaries' marginal benefit from lending, $\tilde{r}_{kt+1}$ increases relatively to the marginal cost of borrowing, $\tilde{r}_{t+1}$. Therefore, the financial spread increases. From the NK Phillips curve (Appendix, equation B.10), inflation declines because depends on the real marginal cost of production, $\hat{\phi}_t$, which is affected from the capital quality shock.

Under optimal commitment, the central bank can affect agents' expectations. In particular, after an initial small rise in inflation, the central bank can reduce inflation expectations by keeping output, investment and capital below their flexible price equilibrium level. From figure 4.1, when there is no lower bound constraint, this is achieved by a relatively sharp cut of the nominal interest rate. The reduction in households' opportunity cost of spending gives a boost to the economy. The main policy objectives are stabilised relatively quickly, after $5-7$ quarters. Because the decline in investment is relatively small, the increase of the financial spread is relatively small as well. Under optimal commitment, the financial market is stabilised relatively quickly with the use of the primary monetary policy instrument only.

Nevertheless, when there is a small probability, $p_{ZLB} = 0.025$, that the nominal interest rate will reach the zero lower bound, the central bank shifts the distribution of the nominal interest rate to the right and chooses a new, positive steady-state inflation, which makes certain that the zero lower bound will be reached with $p_{ZLB} = 0.025$. In this case, under a negative capital quality shock, the reduction of the nominal interest rate is limited. As figure 4.1 illustrates, the effect of the shock is more pronounced and the return of the economy to the desired state is relatively slow. This results to an increase in the volatility of the policy objectives and, consequently, to the households' welfare losses. The central bank chooses a new, positive steady-state inflation that is equal to $\pi^* = 3.2\%$ annually. The positive steady-state inflation contributes to the total households' welfare losses, L_t^{ZLB}. Under commitment, the central bank will keep the rest policy objectives below the flexible price equilibrium level for several periods in order to stabilise inflation. The financial spread increases relatively more. The nominal interest rate cannot stabilise the financial market as before.

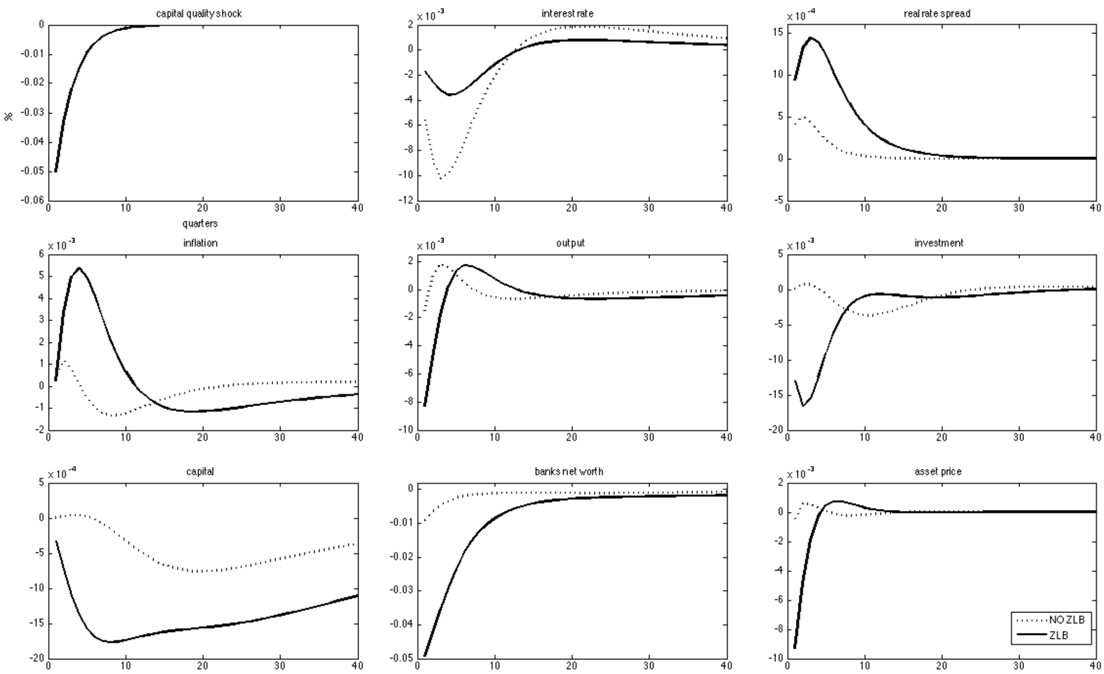

Figure 3.1: *ZLB constraint under optimal commitment; frictionless interbank market*

Discount Window lending. The case of a frictionless interbank market

Now, I switch DW. The first quantitative exercise is to understand what is the joint effect of the central bank providing loans through the DW along with following an optimal regime that is implemented with the adjustment of the nominal interest rate. I also allow for some probability that the interest rate will reach a lower bound. As the previous experiment showed, this is a case where the downturn of the economy is relatively large. A complementary monetary policy instrument, i.e., DW can be useful in this case. Figure 3.2 displays the impulse responses of the economy to a negative capital quality shock ($\sigma_\psi = 0.05$, $\rho_\psi = 0.75$), when the central bank uses DW along with following optimal commitment. In addition, under this scenario, the interbank market is frictionless and all financial intermediaries can borrow from the

DW. I fix the strength that the central bank reacts to the financial spread to be relatively moderate, so I calibrate $\kappa_{\varrho} = 100$. I also allow for a central bank that is characterised by a medium or a high ability to monitor the funds provided through DW, so I fix ω_m to be 0.5, or 1 respectively. Given these values, the central bank chooses optimally ω_q and the new positive steady-state inflation. I find that for an average ability to monitor, $\omega_m = 0.5$, the welfare losses are minimised with $\omega_q = 50$ and a new steady-state of annual inflation, $\pi^* = 2.4\%$. When monitoring is improved, i.e. $\omega_m = 1$, the optimal choice for the central bank is $\omega_q = 49$ and $\pi^* = 2\%$. Hence, a first remark could be that a more efficient use of DW loans reduces the welfare losses associated with the positive steady-state inflation.

As figure 3.2 illustrates, DW lending improves financial stability, as the financial

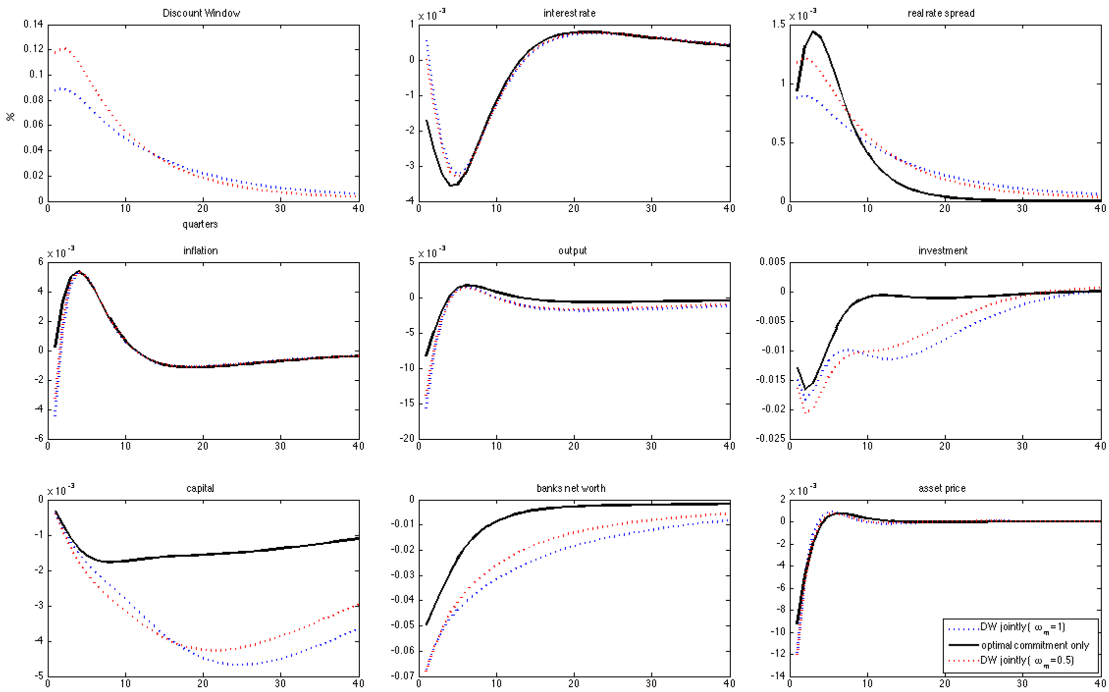

Figure 3.2: *DW lending along with optimal commitment; frictionless interbank market*

spread is reduced. However, the joint use of DW under optimal commitment makes the adjustment of the main policy objectives slower. In some cases (capital, investment) the volatility is excessive, even when the ability of the central bank to monitor the use of the funds provided through the DW is high. An explanation for this outcome may be the following: Under optimal commitment, the agents expect central bank to increase inflation. The central bank recognises this and tries to reduce expectations for future inflation. This requires to keep output, investment and capital below their desired level for several periods. In this scenario, DW lending that is supposed to improve investment, it just adds additional volatility to investment and output. In other words, DW lending worsens the stabilisation policy trade-off between inflation and output/ investment gap.

Now, I allow the central bank to use DW along with an optimal interest rate rule. Figure 3.3 displays the impulse responses of the economy in this case. For illustration purposes, the quantitative exercise is slightly different now. I fix the value of ω_q to match the low standard deviation of the nominal interest rate observed in the US data during the period that DW was active (2010−2013). For this reason, I calibrate ω_q to match $\sigma_q^2 = 0.1\%$. In this case, the choice of ω_q may be sub-optimal, nevertheless, we can draw some implications with regard to the use of DW in practice at a period where the US had been recovering from the financial crisis. I also allow for a slightly stronger response of the central bank ($\kappa_\varrho = 200$).

The impulse responses of the economy are different than the case of optimal

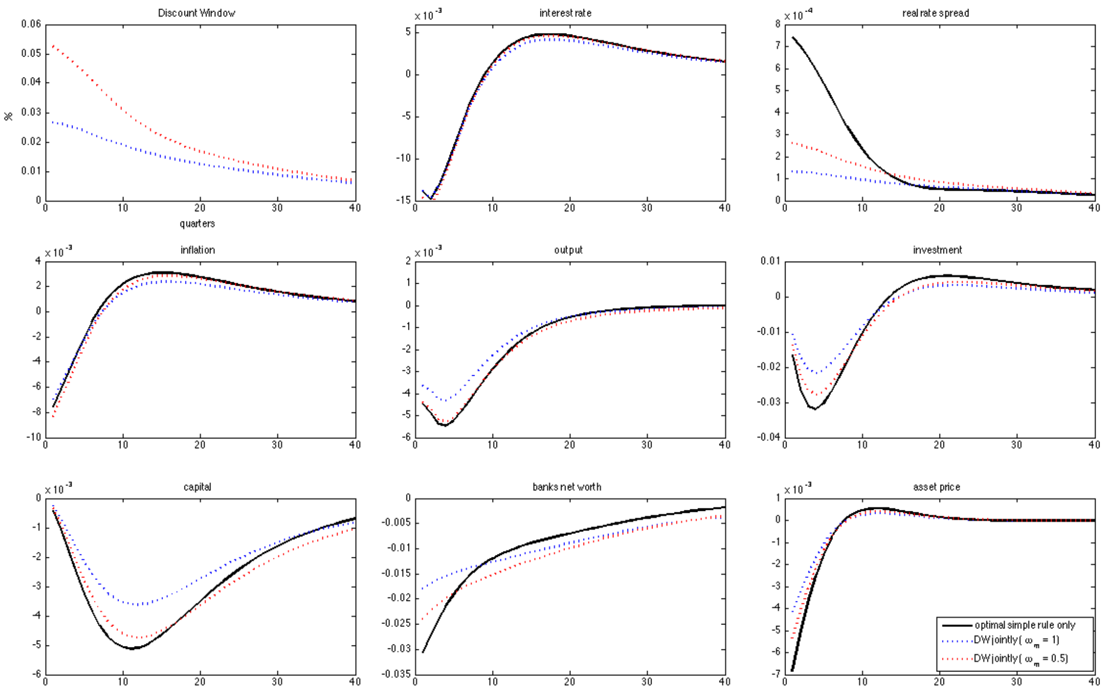

Figure 3.3: *DW lending along with an optimal interest rate rule; frictionless interbank market*

commitment. The main difference is observed in the dynamic path of inflation.

With an interest rate rule, the central bank allows inflation initially to fall. The central bank by relaxing commitment loses inflation expectations as an extra instrument in order to stabilise inflation. In this case, the central bank stabilises investment, output and capital relatively faster to influence expectations for future inflation. DW lending is now effective, as it gives a boost to investment and consequently, to output and capital. This makes the return of inflation to the steady-state smoother. This is true not only for a high ability of the central bank to monitor, but also for a medium one. The higher is the ability of the central bank to monitor (e.g. $\omega_m = 1$), the more effective is the use of loans obtained through DW in financing investment opportunities. Higher ability of the central bank to monitor, increases the efficient use of DW and helps the central bank to stabilise the economy relatively faster.

Discount Window lending. The case of a frictional interbank market

Assuming that the interbank market is frictional makes the negative shock to have more severe effects in the economy. Allowing financial intermediaries to have the ability to misuse the funds obtained in the interbank market tightens their balance sheet relatively more, causing a further decline to intermediaries' net worth and consequently to investment and the real economy. The effect from the use of the DW along with the regime of optimal commitment has similar effects with the case of a frictionless interbank market. As it is illustrated in figure 3.4, DW lending has a negligible effect. In addition, with $p_{ZLB} = 0.025$, DW lending does not contribute to the reduction of the steady-state inflation, which now is higher ($\pi^* = 4.1\%$).

Following the same calibration strategy with the case of the frictionless interbank market, in the case of a a frictional interbank market, DW is effective when it is jointly used with an optimal interest rate rule. Figure 3.5 illustrates the impulse responses.

3.4.3 Analysis of Moments and Welfare

In this section, I analyse the second moments of the variables that affect households' welfare, as they occur from the presence of the negative shock. These moments are used to calculate the contribution of DW lending to the reduction of the households' welfare losses. First, I switch DW off ($\kappa_{\varrho=0}$) and for $p_{ZLB} = 0.025$, I search

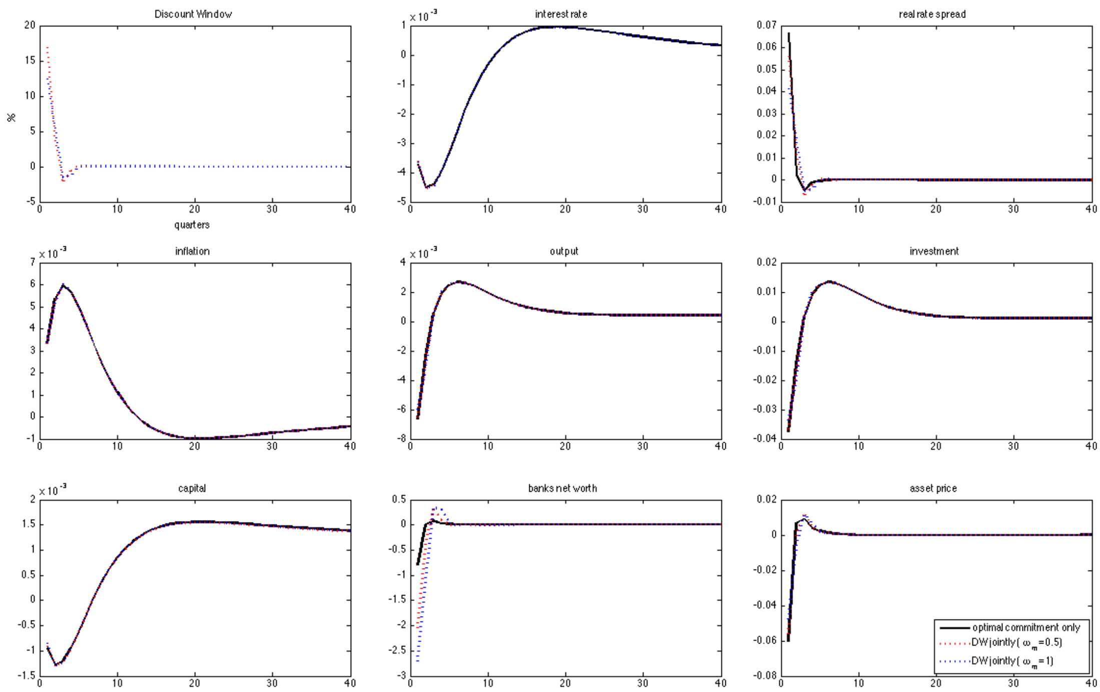

Figure 3.4: *DW lending along with optimal commitment; frictional interbank market*

for the parameter values of ω_q and π^* that minimise the households' welfare losses, $L_t^{Total}(\kappa_\varrho = 0)$. Then, I switch DW on ($\kappa_\varrho > 0$). As in Gertler and Kiyotaki (2010) and Gertler and Karadi (2011), I assume that the central bank does not have unlimited capacity to provide DW loans, so I fix the upper level of κ_ϱ. Then, by doing several experiments for all possible values, $\omega_m \in (0,1]$, that characterise the central bank's ability to monitor the misuse of DW loans, I search for the optimal values of ω_q, π^*, which are the new values that the central bank will choose by having DW at its disposal. I then calculate again the households' welfare losses, $L_t^{Total}(\kappa_\varrho > 0)$, and by subtracting them from those obtained when $\kappa_{\varrho=0}$, I obtain the contribution of DW lending to households' welfare.

The welfare gains/losses from DW lending are given in consumption equivalent terms, as in Levine, McAdam, and Pearlman (2008). That means they are interpreted as percentage change from the efficient steady-state consumption. The welfare losses L_t^{Total} are of the same order of variances. Therefore, expressed in percentage terms, the consumption equivalent welfare gains from DW lending are given from:

$$WG = \frac{L_t^{Total}(\kappa_\varrho = 0) - L_t^{Total}(\kappa_\varrho > 0)}{CU_c}$$

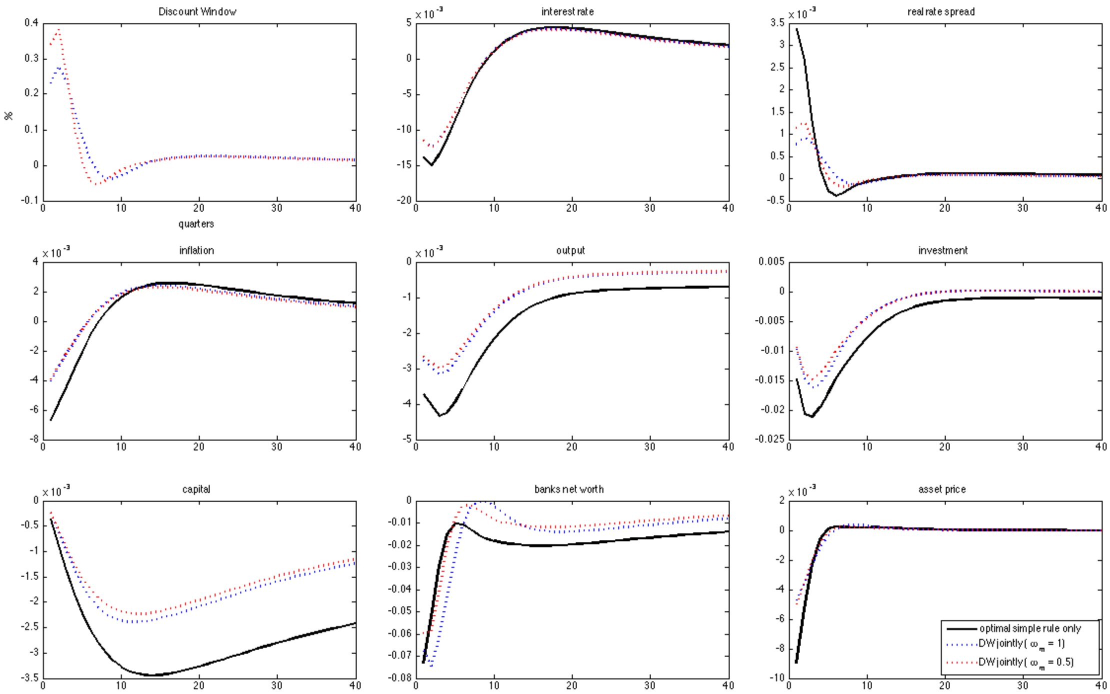

Figure 3.5: *DW lending along with an optimal simple rule; frictional interbank market*

where U_c is the marginal utility with respect to consumption. Table 3.1 displays the moments and the contribution of DW to welfare in the case where DW lending is jointly used under optimal commitment in a frictionless interbank market.

The first column reports the variances (for illustration are expressed as standard deviations) and covariances of the monetary policy objectives that contribute to households' welfare. The second column displays the second moments in the case which the ZLB constraint is absent ($\omega_q = 0$). The third column displays the second moments in the case which there is a probability 0.025 that the nominal interest rate will reach the ZLB, but DW is switched off. In this case, under optimal commitment, the central bank will choose an optimal value $\omega_q = 47$ and a new positive steady-state inflation 0.8% quarterly. From the last two rows, If the interest rate adjustment is not limited from a lower bound constraint, optimal commitment not only eliminates the welfare losses, but it is welfare enhancing. I find that the households' welfare will increase by a consumption equivalent 0.001.[24] However, when there is a lower bound constraint, the welfare losses are equal to 0.005.

[24]This result should not be surprising, given that in the objective function the terms related with supply of hours of work are welfare enhancing.

	Optimal Commitment		DW lending jointly					
ω_m	-	-	0.5			1		
κ_ϱ	0	0	100	200	300	100	200	300
ω_q	0	47	50	49	48	49	48	48
π^*	0	0.008	0.006	0.006	0.006	0.005	0.005	0.005
σ_q	0.024	0.009	0.008	0.008	0.008	0.008	0.008	0.008
σ_π	0.004	0.012	0.012	0.012	0.012	0.012	0.012	0.012
$\sigma_{\tilde{y}}$	0.004	0.011	0.018	0.020	0.021	0.021	0.022	0.022
$\sigma_{\tilde{i}}$	0.011	0.032	0.055	0.057	0.056	0.055	0.051	0.047
$\sigma_{\tilde{k}}$	0.004	0.010	0.025	0.028	0.029	0.028	0.028	0.026
$\rho_{\tilde{y},\tilde{k}}$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
$\rho_{\tilde{y},\tilde{i}}$	0.000	0.000	0.001	0.001	0.001	0.001	0.001	0.001
$\rho_{\tilde{k}_{+1},\hat{y}^e}$	0.000	0.001	0.003	0.003	0.003	0.003	0.003	0.002
$\rho_{\tilde{k}_{+1},\hat{i}^e}$	-0.002	-0.004	-0.010	-0.011	-0.011	-0.011	-0.010	-0.009
$\rho_{\tilde{k}_{+1},\hat{y}^e_{+1}}$	0.000	0.001	0.003	0.003	0.003	0.003	0.003	0.002
$\rho_{\tilde{k}_{+1},\hat{i}^e_{+1}}$	-0.002	-0.004	-0.010	-0.011	-0.011	-0.011	-0.010	-0.009
$\rho_{\tilde{k},\hat{y}^e}$	0.000	0.001	0.003	0.003	0.003	0.003	0.003	0.002
$\rho_{\tilde{k},\hat{i}^e}$	-0.002	-0.004	-0.010	-0.011	-0.011	-0.011	-0.010	-0.009
σ_{sprd}	0.001	0.003	0.003	0.003	0.002	0.003	0.002	0.002
L_t^{ZLB}	-0.001	0.005	0.001	0.000	-0.001	0.000	-0.001	-0.001
L_t^{Total}	-0.001	0.007	0.002	0.001	0.000	0.001	0.000	0.000
WG	-	-	0.005	0.007	0.007	0.007	0.007	0.007

Table 3.1: The welfare gains from DW lending along with optimal commitment; frictionless interbank market

The last six columns show the second moments in the case which DW is available. I search for κ_ϱ in the range 0—300. Under optimal commitment, an increase in the funds provided through DW worsens the volatility of the main policy objectives, π_t, $\tilde{y}_t$, $\tilde{i}_t$, $\tilde{k}_t$. However, from the last two rows, the welfare gains from DW lending are equal to a consumption equivalent increase of $0.005-0.007$. The reason that DW lending is welfare enhancing, even if the main policy objectives become more volatile, is the fact that it increases the covariances of capital with the desired output and investment. The explanation for this result could be given from the fact that DW lending reduces output and capital fluctuations through a reduction in the investment fluctuations. Therefore, their covariance increases. In contrast, this causes a relative reduction to the covariance between output and hours of labour. The latter contributes to the households' welfare negatively, thus, its reduction is welfare enhancing for households.

The contribution of DW lending is maximised when $\omega_q = 48$, $\kappa_\varrho = 300$. If the ability of the central bank to monitor the use of DW loans is medium, the new steady-state annual inflation, π^*, is reduced from 3.2% to 2.4%. If the ability of the central bank is high, then π^* is reduced to 2%. To conclude, if the central bank is limited from a lower bound constraint, under optimal commitment, the optimal choice of the parameters associated with DW lending can reduce the welfare cost associated with a positive steady-state inflation and can reduce the households' welfare losses. Figure 3.6 plots the welfare gains from DW lending with respect to ω_q and ω_m.

Table 3.2 displays the second moments of the policy objectives, when DW is

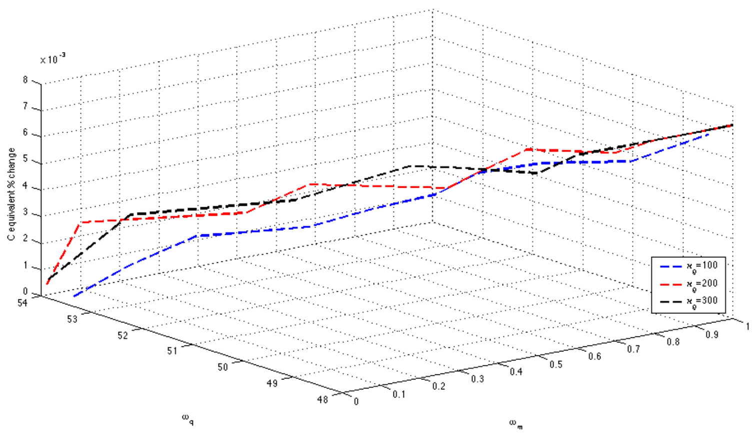

Figure 3.6: *The welfare gains from DW lending along with optimal commitment*

used along with an optimal interest rate rule in a frictionless interbank market. As in the standard NK model, in comparison with optimal commitment, the simple interest rule is sub-optimal and produces larger welfare losses, even if the adjustment of the nominal interest rate is relatively larger. I search for the optimal κ_ϱ between the range $0-1000$. I calibrate σ_q to hit a nominal interest rate variance of 0.001 when DW is off. I obtain that when the central bank reacts to the spread very strongly ($\kappa_\varrho = 1000$), financial stability is achieved ($\sigma_{sprd} \approx 0$), while the volatility of the main policy objectives, inflation, output, capital and investment is reduced almost by half. I find that the gains from DW lending can be a consumption equivalent increase of $0.04-0.07$.

By doing several quantitative experiments, I obtain that the ability of the cen-

	Optimal Simple Rule	**DW lending jointly**					
ω_m	-		0.5			1	
κ_ρ	0	100	500	1000	100	500	1000
ω_q	491			493			
π^*	0.059	0.061	0.056	0.050	0.060	0.048	0.042
σ_q	0.035	0.036	0.034	0.031	0.036	0.030	0.027
σ_π	0.017	0.018	0.014	0.010	0.017	0.009	0.005
$\sigma_{\tilde{y}}$	0.015	0.016	0.012	0.008	0.014	0.007	0.004
$\sigma_{\tilde{i}}$	0.080	0.077	0.053	0.035	0.067	0.032	0.016
$\sigma_{\tilde{k}}$	0.020	0.021	0.015	0.009	0.019	0.008	0.004
$\rho_{\tilde{y},\tilde{k}}$	0.000	0.000	0.000	0.000	0.000	0.000	0.000
$\rho_{\tilde{y},\tilde{i}}$	0.001	0.001	0.001	0.000	0.001	0.000	0.000
$\rho_{\tilde{k}_{+1},\hat{y}^e}$	0.003	0.003	0.002	0.001	0.003	0.001	0.001
$\rho_{\tilde{k}_{+1},\hat{i}^e}$	-0.008	-0.008	-0.006	-0.004	-0.008	-0.003	-0.001
$\rho_{\tilde{k}_{+1},\hat{y}^e_{+1}}$	0.003	0.003	0.002	0.001	0.003	0.001	0.001
$\rho_{\tilde{k}_{+1},\hat{i}^e_{+1}}$	-0.008	-0.009	-0.006	-0.004	-0.008	-0.003	-0.002
$\rho_{\tilde{k},\hat{y}^e}$	0.003	0.003	0.002	0.001	0.003	0.001	0.001
$\rho_{\tilde{k},\hat{i}^e}$	-0.008	-0.009	-0.006	-0.004	-0.008	-0.003	-0.002
σ_{sprd}	0.002	0.001	0.000	0.000	0.001	0.000	0.000
L_t^{ZLB}	0.012	0.013	0.008	0.004	0.011	0.004	0.001
L_t^{Total}	0.132	0.144	0.116	0.091	0.135	0.085	0.063
WG	-	–0.012	0.016	0.041	-0.003	0.047	0.069

Table 3.2: The welfare gains from DW lending along with simple monetary rule in a frictionless interbank market

tral bank to monitor the misuse of the DW loans matters for the effectiveness of DW lending in welfare. In particular, I calculate WG for $\omega_m = 0-1$ and $\kappa_\rho = 0-1000$ until the variance of the spread becomes negligible. Figure illustrates the welfare gains from DW lending. I obtain that when the ability of the central bank to monitor the misuse of DW loans is low, DW lending decreases households' welfare. In particular, for $\omega_m = 0.01-0.2$, welfare decreases for all values of κ_ρ. However, as the ability of the central bank to monitor improves, I find that DW lending is welfare enhancing. In particular, an average or high ability to monitor captured by $\omega_m = 0.3-1$, produces welfare gains equal to a consumption equivalent increase of $0.03-0.07$.

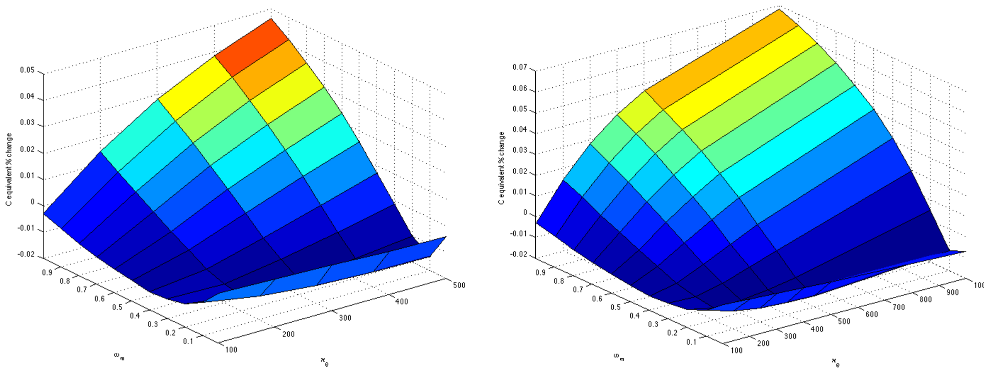

Figure 3.7: *The welfare gains from DW lending along with a simple monetary rule in a frictionless interbank market;* $\omega_q = 491 - 493$

I now perform the same analysis of moments, under the scenario where the interbank market is frictional. Table 3.3 reports the second moments that occur when DW is used along with optimal commitment. As the table shows, the welfare gains from DW lending are negligible. From the rows $6-9$ of the table, DW lending does not affect inflation, but it has some positive effect in the rest policy objectives. From the rows $10-11$, we can see that the covariances between capital and output as well as capital and investment, which are welfare enhancing, are reduced. A possible explanation for this result could be the fact that in a frictional interbank market, only financial intermediaries in locations with new investment opportunities will borrow from the DW. In this case, DW lending cannot reduce effectively the covariance between output and hours of labour, which is welfare decreasing. The reduction of the volatility of the main policy objectives is offset by the reduction of the welfare enhancing covariances resulting to negligible welfare gains from DW lending, for all possible values of ω_m and κ_ϱ.

As a last experiment, I also quantify the effect of DW lending when it is combined with an optimal interest rate rule in the case of a frictional interbank market. Again, DW lending is more effective when it is combined with an optimal interest rate rule. Fixing ω_q and searching for κ_ϱ from 0 to 400, I obtain that the welfare gains from DW lending can produce a consumption equivalent increase of steady state consumption equal to 0.045. Surprisingly, I find that higher ω_m does not improve welfare monotonically, as in the case of a frictionless interbank market. While DW lending reduces the variance of the main policy objectives, it also worsens the welfare enhancing covariances between output and capital as well as output and

	Optimal Commitment		**DW lending jointly**					
ω_m	-	-	0.5			1		
κ_ϱ	0	0	100	200	500	100	200	500
ω_q	0	66	66					
π^*	0	0.0102	0.0102	0.0102	0.0102	0.0102	0.0102	0.0102
σ_q	0.0282	0.0104	0.0104	0.0104	0.0104	0.0104	0.0104	0.0104
σ_π	0.0051	0.0137	0.0137	0.0137	0.0137	0.0137	0.0137	0.0138
$\sigma_{\tilde{y}}$	0.0043	0.0243	0.0237	0.0221	0.0190	0.0242	0.0224	0.0190
$\sigma_{\tilde{i}}$	0.0236	0.0803	0.0791	0.0755	0.0687	0.0800	0.0757	0.0680
$\sigma_{\tilde{k}}$	0.0023	0.0616	0.0600	0.0550	0.0454	0.0614	0.0560	0.0459
$\rho_{\tilde{y},\tilde{k}}$	0.0000	0.0014	0.0013	0.0011	0.0007	0.0014	0.0011	0.0008
$\rho_{\tilde{y},\tilde{i}}$	0.0001	0.0019	0.0018	0.0016	0.0012	0.0018	0.0016	0.0012
$\rho_{\tilde{k}_{+1},\hat{y}^e}$	-0.0001	-0.0008	-0.0008	-0.0008	-0.0008	-0.0008	-0.0008	-0.0008
$\rho_{\tilde{k}_{+1},\hat{i}^e}$	-0.0006	0.0046	0.0046	0.0046	0.0045	0.0046	0.0046	0.0046
$\rho_{\tilde{k}_{+1},\hat{y}^e_{+1}}$	-0.0001	-0.0008	-0.0007	-0.0007	-0.0007	-0.0007	-0.0007	-0.0008
$\rho_{\tilde{k}_{+1},\hat{i}^e_{+1}}$	-0.0005	0.0044	0.0044	0.0044	0.0044	0.0044	0.0044	0.0044
$\rho_{\tilde{k},\hat{y}^e}$	-0.0001	-0.0008	-0.0007	-0.0007	-0.0007	-0.0007	-0.0007	-0.0008
$\rho_{\tilde{k},\hat{i}^e}$	-0.0005	0.0044	0.0044	0.0044	0.0044	0.0044	0.0044	0.0044
σ_{sprd}	0.0220	0.0667	0.0636	0.0606	0.0537	0.0575	0.0506	0.0389
L_t^{ZLB}	-0.0010	0.0119	0.0119	0.0119	0.0120	0.0119	0.0119	0.0120
L_t^{Total}	-0.0010	0.0156	0.0156	0.0156	0.0156	0.0156	0.0156	0.0156
WG	-	-	0	0	0	0	0	0

Table 3.3: The welfare gains from DW lending along with optimal commitment in a frictional interbank market

investment. The results are summarised in the table 3.4 and also displayed in the figure 3.8.

3.4.4 DW lending with efficiency costs

So far in the welfare analysis, I have assumed that DW lending is not associated with efficiency costs, i.e., $c^{dw} = 0$. In this section, I allow for c^{dw} to be positive. In particular, I calibrate c^{dw} to take values between $0.05 - 0.25\%$. It is reasonable to assume that the efficiency cost per unit of DW lending will be a small fraction of DW lending. As the welfare gains from DW lending under optimal commitment are relatively small, the analysis focuses on the case where DW is used along with a monetary simple rule. Figure 3.9 displays the welfare gains of DW lending with respect to ω_m and

	Optimal Simple Rule	DW lending jointly					
ω_m	-		0.5			1	
κ_ϱ	0	100	200	400	100	200	400
ω_q				571			
π^*	0.0591	0.0565	0.0528	0.0465	0.0570	0.0533	0.0475
σ_q	0.0353	0.0340	0.0321	0.0289	0.0343	0.0324	0.0294
σ_π	0.0163	0.0151	0.0133	0.0109	0.0154	0.0138	0.0116
$\sigma_{\tilde{y}}$	0.0134	0.0118	0.0097	0.0068	0.0122	0.0101	0.0074
$\sigma_{\tilde{i}}$	0.0516	0.0464	0.0393	0.0289	0.0485	0.0419	0.0324
$\sigma_{\tilde{k}}$	0.0240	0.0203	0.0155	0.0093	0.0210	0.0164	0.0105
$\rho_{\tilde{y},\tilde{k}}$	0.0002	0.0002	0.0001	0.0000	0.0002	0.0001	0.0000
$\rho_{\tilde{y},\tilde{i}}$	0.0007	0.0005	0.0004	0.0002	0.0006	0.0004	0.0002
$\rho_{\tilde{k}_{+1},\hat{y}^e}$	0.0026	0.0023	0.0019	0.0013	0.0024	0.0020	0.0014
$\rho_{\tilde{k}_{+1},\hat{i}^e}$	-0.0083	-0.0073	-0.0059	-0.0037	-0.0076	-0.0062	-0.0041
$\rho_{\tilde{k}_{+1},\hat{y}^e_{+1}}$	0.0025	0.0022	0.0018	0.0012	0.0023	0.0019	0.0014
$\rho_{\tilde{k}_{+1},\hat{i}^e_{+1}}$	-0.0086	-0.0075	-0.0061	-0.0038	-0.0078	-0.0064	-0.0043
$\rho_{\tilde{k},\hat{y}^e}$	0.0025	0.0022	0.0018	0.0012	0.0023	0.0019	0.0014
$\rho_{\tilde{k},\hat{i}^e}$	-0.0086	-0.0075	-0.0061	-0.0038	-0.0078	-0.0064	-0.0043
σ_{sprd}	0.0046	0.0034	0.0025	0.0017	0.0029	0.0021	0.0013
L_t^{ZLB}	0.0068	0.0059	0.0049	0.0039	0.0063	0.0054	0.0044
L_t^{Total}	0.1288	0.1175	0.1022	0.0794	0.1200	0.1048	0.0832
WG	-	0.0113	0.0266	0.0494	0.0088	0.0240	0.0456

Table 3.4: The welfare gains from DW lending along with a simple monetary rule in a frictional interbank market

c^{dw}. The left graph presents the case of a frictionless interbank market, while the right one the case of a frictional one. To simplify the analysis, κ_ϱ is kept fixed and equal to 500 and 100 respectively.

Not surprisingly, in both cases, for any given level of ability of the central bank to monitor, the efficiency cost reduces welfare. In the case of a frictionless interbank market, DW lending can be welfare costly for low values of ω_m. However, for a medium value of ω_m, DW lending is welfare enhancing even if there are efficiency costs. The same is true for the case of a frictional interbank market. Efficiency costs reduce the effect of DW lending considerably, however the effect is positive even for low values of ω_m. The results are displayed analytically in tables 3.5 and 3.6.

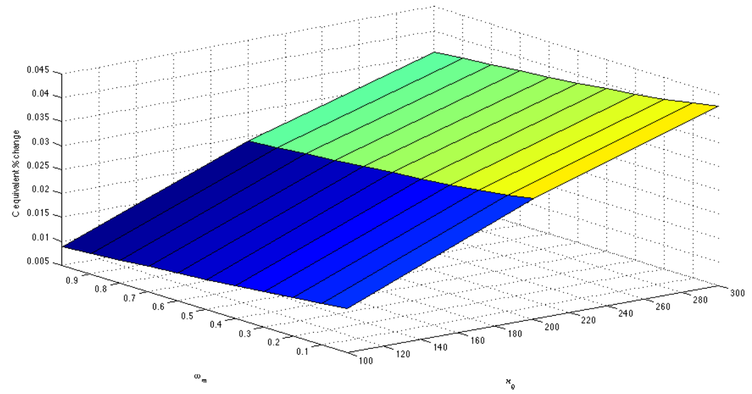

Figure 3.8: *The welfare gains from DW lending along with a simple monetary rule in a frictional interbank market;* $\omega_q = 571$

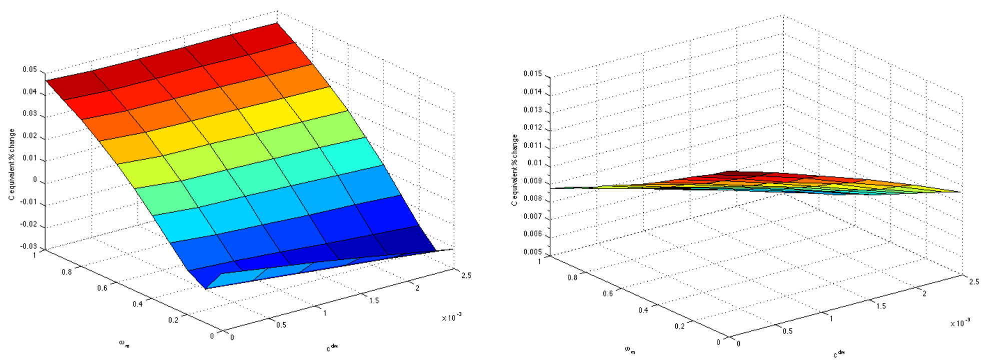

Figure 3.9: *The welfare gains from DW lending with efficiency costs*

ω_m	$c^{dw} = \mathbf{0}$	**0.05 %**	**0.1 %**	**0.15 %**	**0.2 %**	**0.25 %**
0.01	-0.0042	-0.0074	-0.0107	-0.0141	-0.0177	-0.0215
0.1	-0.0146	-0.0167	-0.0189	-0.0211	-0.0233	-0.0256
0.2	-0.0099	-0.0114	-0.0129	-0.0145	-0.0160	-0.0176
0.3	-0.0011	-0.0030	-0.0041	-0.0052	-0.0063	-0.0075
0.4	0.0074	0.0058	0.0050	0.0041	0.0033	0.0024
0.5	0.0161	0.0141	0.0135	0.0128	0.0122	0.0115
0.6	0.0235	0.0218	0.0213	0.0207	0.0202	0.0197
0.7	0.0302	0.0287	0.0283	0.0279	0.0275	0.0271
0.8	0.0363	0.0349	0.0346	0.0343	0.0339	0.0336
0.9	0.0417	0.0405	0.0402	0.0400	0.0397	0.0395
1	0.0467	0.0455	0.0453	0.0451	0.0450	0.0448

Table 3.5: Welfare gains from DW lending with efficiency costs in a frictionless interbank market

ω_m	$c^{dw} = \mathbf{0}$	**0.05 %**	**0.1 %**	**0.15 %**	**0.2 %**	**0.25 %**
0.01	0.0143	0.0135	0.0126	0.0117	0.0107	0.0096
0.1	0.0137	0.0129	0.0121	0.0111	0.0102	0.0092
0.2	0.0131	0.0123	0.0114	0.0106	0.0096	0.0087
0.3	0.0125	0.0117	0.0109	0.0100	0.0091	0.0082
0.4	0.0119	0.0111	0.0103	0.0095	0.0086	0.0077
0.5	0.0113	0.0106	0.0098	0.0090	0.0081	0.0072
0.6	0.0108	0.0100	0.0093	0.0085	0.0077	0.0068
0.7	0.0103	0.0095	0.0088	0.0080	0.0072	0.0063
0.8	0.0097	0.0090	0.0083	0.0075	0.0067	0.0059
0.9	0.0093	0.0086	0.0078	0.0071	0.0063	0.0055
1	0.0088	0.0081	0.0074	0.0066	0.0059	0.0051

Table 3.6: Welfare gains from DW lending with efficiency costs in a frictional interbank market

3.5 The Federal Reserve Discount Window

In the model, the fraction of the DW loans that can be misused, ω_m, is used to capture the ability of the central bank to monitor. Equilibrium in the financial market suggests a link between ω_m and the penalty rate charged on the DW rate. Ignoring the discount factors from both sides, Penalty Rate $= \omega_m$ Spread. Given that $\omega_m = 0-1$, we can conclude that the penalty rate that the central bank charges is proportional to the financial spread. It would be misleading to draw from the model any conclusion that links the ability of the central bank to monitor in practice and the size of the penalty rate. Nevertheless, we can use the equilibrium condition to extract some information with regard to the relative size of the penalty rate charged by the Fed.

To calculate the penalty rate on the DW rate, I use the Fed DW data that are available quarterly for the period $Q3/2010$ - $Q3/2013$.[25] The Fed DW credit is available on three different categories based on the financial condition of financial intermediaries. These are: i) primary credit, available for intermediaries with a healthy financial condition ii) secondary credit, for intermediaries that do not satisfy the criteria for a primary credit and iii) seasonal credit, available for smaller financial institutions.

Using the World Bank data for the bank lending rate and real rate of borrowing, I calculate the financial spread. Dividing the penalty rate by the spread, I obtain the values for ω_m. For seasonal credit, $\omega_m = 0.01-0.05$, for primary credit, $\omega_m = 0.18-0.21$ and for secondary credit, $\omega_m = 0.34-0.36$. In the Appendix B.10.3, figure B.3 summarises. We can conclude that the fraction between the penalty rate and the financial spread can be characterised as low.

3.6 Robustness analysis

The quantitative experiments are assessed for a wide range of key parameters, such as $\kappa_\varrho, \omega_q, \omega_m$ and for two different scenarios for the interbank market ($\omega_b = 0$ or $\omega_b = 1$). I also perform some other computations. When the optimal interest rate rule is computed, under the benchmark case, the optimal weights are $\kappa_\pi = 1.6, \kappa_{\tilde{y}} = 0.125$. These values are maintained for various configurations of the Taylor rule, such as i)$\kappa_\pi = 5, \kappa_{\tilde{y}} = 0$ ii) non-zero feedback parameters on investment and capital, $\kappa_{\tilde{i}}, \kappa_{\tilde{k}} \neq 0$.

There is a discussion whether a interest rate rule that uses $\tilde{y}_t$) instead of $\hat{y}_t$, is implementable, as the former requires central bank to know the desired level of output, which is unobservable. Using the “implementable” form of the Taylor rule, I do not obtain significant changes in the results. [26]

A different probability for the interest rate to hit the ZLB does not affect the welfare gains from DW lending, but it affects π^* and the standard deviation of the nominal interest rate. Allowing for a lower persistence of the capital quality shock,

[25] More recent data are not available, as according to the Fed, these are reported with a two-year lag. This information is available on: http://www.federalreserve.gov/newsevents/reform_discount_window.htm

[26] This discussion has been motivated from Cantore and Levine (2015).

$\rho_{\psi} = 0.66$, as in Gertler and Kiyotaki (2010) and Gertler and Karadi (2011) affects the volatility of the variables associated with the welfare criterion, thus, households' welfare losses. Qualitatively though, the effect of DW lending under commitment or an interest rate rule is similar.

3.7 Conclusion and Further Discussion

I have provided a formal model that can analyse the effectiveness of DW as a complementary monetary policy instrument. After the recent financial crisis, monetary policy in practice has changed and evaluating other monetary policy instruments has become of greater importance. The model that I have constructed is enriched with key elements of the recent macroeconomic environment, such as the limited adjustment of the nominal interest rate. The motivation for this paper has been the extensive literature developed the last three decades, which has been criticising the effectiveness of DW. In the paper, I provide quantitative evidence that is in favour of the view that DW lending is effective, which is in contrast with the views expressed in the previous literature.

The normative analysis suggests that there are welfare gains from the central bank using DW. The L-Q approach is accurate for small deviations from the steady-state, as those obtained in the model solution. This paper provides evidence that DW lending is effective in small recessions. Because the model is expressed in a form of deviations from the flexible-price equilibrium, a novelty of this paper is that obtains welfare costly fluctuations occurring from the nominal price rigidity. These implications have not been considered by the literature, even in models featuring the NK elements, such as in Gertler and Karadi (2011).

In this paper DW lending has been assessed by focusing on the ability of the central bank to monitor the actions of financial intermediaries with regard to the use of the funds obtained from the DW. Bordo (1989) has highlighted perfect monitoring as a prerequisite for DW lending to be effective. By combining the model of financial intermediation developed by Gertler and Kiyotaki (2010), I can capture monitoring and then provide quantitative arguments that support the view in Bordo (1989). In particular, I find that when the ability to monitor is low, DW lending is welfare costly. This result is obtained even without assuming any efficient cost of DW lending. Instead, the welfare losses occurs from the excessive volatility created when the central bank combines DW lending with other optimal policy regimes, or from the reduction of the associated welfare enhancing covariances between output and investment, as well as output and capital. When monitoring improves, DW lending is effective, as in this case it expands the balance sheet of financial intermediaries. I find that, when the primary and complementary monetary policies are combined optimally, the welfare gains from DW lending can reach a consumption equivalent increase of 0.04—0.07. Given that the fluctuations from the flexible price equilibrium are very small, this can be considered as a considerable contribution to welfare.

Another novelty of the paper is that while DW is the main policy instrument that is assessed, some remarks with regard to the conduct of the primary policy regimes, which are implemented by adjustments on the nominal interest rate. I find that there is a considerable difference in the effectiveness of DW lending, when it is combined under optimal commitment and when it is combined with an interest rate rule. I find that the welfare gains from DW lending, under optimal commitment, could be negligible. On the other hand, when DW is combined with an interest rate rule, the welfare gains increase.

In future work, the model could include the effect of the DW stigma (Ennis and Weinberg (2013)). Last but not least, the model implies a link between monitoring and the penalty rate charged by the central bank. A model that would explore this relation further could be used to evaluate DW lending focusing on the penalty rates. As Kaufman (1990) has highlighted, charging the optimal penalty rate is a challenging task for a central bank. A low DW rate is associated with high risk for the DW funds to never be used for investments. On the other hand, a high DW rate, DW lending will be very low. The current model could be used as a benchmark to explore further these issues.

Chapter 4

Social Status, Capital Accumulation, and Economic Growth

Chapter Abstract

We consider a monetary growth model in which entrepreneurs borrow funds to invest in projects that produce capital goods. In addition to their varying pecuniary returns, different projects also vary with respect to the status they confer to the entrepreneurs who operate them. We show that social status promotes capital accumulation, whereas inflation impedes it. We also show that, even when the status-induced increase of marginal utility is constant over time, the interaction between status and inflation is an additional source of transitional dynamics. When a social norm links this increase of marginal utility to past outcomes, however, the dynamics can generate endogenous cycles in the transition to the balanced growth path. Given these outcomes, we also derive implications for the relation between growth and volatility.

4.1 Introduction

The role of social status on decision making has long been recognised as an important determinant of economic outcomes (Weiss and Fershtman (1998); Heffetz and Frank (2011)). From a macroeconomic perspective, economists have investigated the effects of status on economic growth and social welfare by means of frameworks in which status concerns are associated with either a desire for high relative wealth per se[1], or conspicuous/positional consumption[2], or both (e.g. Zou (1994); Bakshi and Chen (1996); Corneo and Jeanne (1997, 2001); Rauscher (1997); Futagami and Shibata (1998); Tournemaine and Tsoukis (2008); Wendner (2010); Varvarigos (2011)). These effects have been shown to be ambiguous and to depend on various characteristics, such as the underlying source of social status. A similar ambiguity applies to the analysis of Fershtman et al. (1996). They employ a model with costly occupational choice where higher status is attached to the growth-enhancing occupation. Their results indicate that, in addition to its direct positive effect on growth, social status may also be a source of negative growth effects due to the fact that it attracts wealthy, but low-ability, individuals to the growth-enhancing occupation, hence reducing its average quality.

This paper is an attempt to take explicit account of status concerns that originate from entrepreneurial decisions, and present their implications for both transitional dynamics and long-term growth. The motivation behind our analysis is the view that entrepreneurship is yet another area of economic activity for which social status seems to be pertinent. Indeed, many analyses confirm the view that social status, and the characteristics that confer it, such as prestige, recognition, approval, and a sense of achievement, are important elements of entrepreneurial aspirations, decisions, and performance (e.g. Scheinberg and MacMillan (1988); Shane et al. (1991); Collins et al. (2004); Malach-Pines et al. (2005); Van Praag (2011)). In a similar vein, some researchers (e.g., Hollingshead (1975)) have argued that the scale of entrepreneurial activities - typically measured by the monetary value of firms - increases the status conferred to their proprietors.

In our model, entrepreneurs are individuals who borrow funds in order to operate projects that produce capital goods. In addition to their varying pecuniary

[1]This idea follows Weber (1904)'s notion of the 'spirit of capitalism'.

[2]See Veblen (1899) and Hirsch (1976).

returns, different projects also vary with respect to the status they confer to the entrepreneurs who operate them - an idea that is conceptually similar to the occupation-induced status of Fershtman et al. (1996). The model is a monetary one in the sense that there is a demand for money by lenders who face a liquidity constraint in their role as loan providers.[3] We show that, in addition to their effect on the economy's long-run growth rate, which is positive, status concerns have important implications for the shape of the economy's dynamics towards the balanced growth path. In fact, despite the presence of a production technology with permanently constant (social) returns to capital, the existence of status concerns generates transitional dynamics that do not allow an instantaneous adjustment to the balanced growth path. This happens even when the status-induced increase of marginal utility is constant over time: By increasing the growth rate and reducing the rate of inflation, the number of entrepreneurs who invested in the high-return/high-status project in the past has a positive effect on the incentive of the next generation's entrepreneurs to act similarly. The dynamics differ, however, under a social norm whereby status concerns are linked to past outcomes - specifically, when the status-induced increase of marginal utility is less pronounced in economies where the involvement with the high-return project was more common among entrepreneurs historically. Under this scenario, and in addition to sustaining a lower growth rate in the long-run, the economy's transitional dynamics can generate cycles endogenously, as it converges to its balanced growth path.[4]

Given that the characteristics of social status have implications for the shape of economic dynamics (monotonic or cyclical) and long-term economic performance (the growth rate), we also use the model's implications to provide a novel explanation for the relation between cyclical fluctuations and growth. This issue relates to empirical analyses that have shown a significant relation between the average growth rate and its volatility (e.g., Ramey and Ramey (1995); Martin and Ann Rogers (2000); Koren and Tenreyro (2007)). From a theoretical perspective, the more common approach in examining the underlying characteristics of this relation has been

[3]Other monetary models that include elements relevant to social status are those by Chang et al. (2000) and Gong and Zou (2001). These analyses show that, under the 'spirit of capitalism' assumption, inflation has real effects in circumstances where money would otherwise be neutral.

[4]Azariadis and Smith (1996) analyse a monetary growth model that generates damped fluctuations in the transition to the steady state. In their framework, the underlying cause of such cycles is the presence of credit market imperfections. In our model, we do not consider such imperfections; damped fluctuations are attributed solely to status concerns.

the construction of stochastic endogenous growth models in which the cycles generated by stochastic shocks impinge on the long-run growth rate (e.g., Femminis (2001); Canton (2002); Varvarigos (2010)). Our approach in inferring a relation between cyclical fluctuations and growth is rather different. Specifically, our argument is that the driving forces behind status considerations are (partially) responsible for the long-term prospects of the economy, and for the shape of its dynamics. In other words, the correlation between growth and cycles reflects the idea that cyclical growth converges to a lower value in the long-run, compared with a growth rate that is smoother (i.e., monotonic) during the transition.

The rest of the paper is organized as follows: In Section 2, we present the economic environment and derive the economy's equilibrium. In Section 3, we analyse the effects of social status, and its underlying characteristics, on the economy's dynamics and the (long-run) growth rate. Section 4 summarises and discusses some policy implications.

4.2 The Economy

Consider an economy populated by overlapping generations of individuals who live for three periods. The population mass of each age cohort is constant over time and equal to $2n$ ($n > 0$). Following their birth, nature divides individuals into two equal-sized groups of varying characteristics. Particularly, half of these individuals will spend their lifetimes as workers; the rest of them will spend their lifetimes as entrepreneurs. Irrespective of their type, all individuals are risk-neutral and enjoy utility from the consumption of goods during the last period of their lifetime.

Consider a worker born in period t. During the first period of her lifetime she is endowed with one unit of labour which she (inelastically) supplies to firms that produce the economy's final good. In exchange, she receives the competitive salary w_t . Subsequently, she explores opportunities for saving her income until the third period of her lifetime, during which she will receive the proceeds of her savings and use them to purchase consumption goods. One such opportunity is a storage technology that returns $1+q\ (q \geq 0)$ units of output in period $t+2$ for each unit of output stored in period t. Alternatively, she can agree to offer a loan to an entrepreneur, in a manner that will be described shortly.

Now let us consider an entrepreneur born in period t. She is largely inactive during the first period of her lifetime. In the second period, however, she is endowed with the ability to operate an investment project that generates $\varphi(j) > 0, \left(j = \{H, L\}\right)$ units of capital in period $t+2$ for each unit of output invested in period $t+1$. The entrepreneur will sell this capital to firms at a competitive price $r > 0$ per unit.[5] There are two such projects at her disposal, but she can choose to operate only one of them - a decision that, once made, is irreversible. The H project returns $\varphi(H) = \varphi$ units of capital for each unit of investment. In addition to its cost in terms of output, this project entails an effort cost for the entrepreneur. We assume that this effort cost is proportional to the scale of the project as it requires B units of effort per unit of output invested in it. We also assume that B is uniformly distributed across entrepreneurs, with support on $[0, n]$. The L project, on the other hand, does not entail such an effort cost. Nevertheless, it offers a lower return of $\varphi(L) = \left(1-\psi\right)\varphi$ units of capital $(0 < \psi < 1)$ for each unit of output invested in it. Note that, given the lack of own sources of income, entrepreneurs have no other option other than to borrow funds from workers in order to operate any of these two projects. Once an entrepreneur repays the loan in period $t+2$, she will use the residual income to purchase consumption goods.

The economy's final good can be used for both consumption and investment purposes. It is produced by a unit mass of perfectly competitive firms who combine labour from workers, denoted N_t, and capital purchased by entrepreneurs, denoted K_t, in order to produce Y_t units of output according to the following technology:

$$Y_t = AK_t^{\alpha}(\Gamma_t N_t)^{1-\alpha}, \quad \alpha \in (0,1), \tag{4.1}$$

Following Romer (1986), the variable Γ_t captures the productivity benefits that accrue as a result of an economy-wide, learning-by-doing externality that is related to the stock of capital per worker according to[6]

$$\Gamma_t = \frac{K_t}{n} \tag{4.2}$$

[5] Capital is assumed to depreciate completely during the production process.

[6] This externality is introduced as a means of allowing the emergence of an equilibrium with positive long-run growth.

4.2.1 Occupational Choice and Social Status

The differences between the two projects are not restricted to their varying (pecuniary) returns in terms of investment. On the contrary, we envisage a scenario where the choice of investment project generates direct utility effects that accrue to the entrepreneur who is involved in its operation. Such non-pecuniary differences are justified by alluding to the idea that an entrepreneur's occupational choice will have a direct impact on her utility due to social status concerns.

We formalise the aforementioned ideas by assuming that the marginal utility of an entrepreneur's consumption, denoted $X(j)$, is

$$X(j) = \begin{cases} x_{t+1} & if \quad j = H \\ 1 & if \quad j = L \end{cases}, x_{t+1} \geq 1. \tag{4.3}$$

The underlying idea is that the high-effort/high-return project confers a relatively higher social status to those entrepreneurs who undertake it. This may be because, given the H project's higher return, it is viewed as a more prestigious occupational choice for an entrepreneur, or because it is associated with a sense of accomplishment, as it reflects the entrepreneur's abilities and her willingness to strive for a more rewarding occupation. The recognition of these characteristics by a person's peers increases her status and, therefore, has a positive effect on her well-being. Note that the assumption through which social status impinges on the marginal utility of consumption is not an alien one. On the contrary, it is consistent with the existing literature on the economic implications of status (e.g., Fershtman et al. (1996); Becker et al. (2005); Hopkins (2011)).

We shall also consider two different scenarios regarding the driving forces behind such status considerations. The limiting scenario is one where the marginal utility of consumption is constant at $x_{t+1} = \bar{x} \geq 1 \ \forall t$. Nevertheless, it is also reasonable to consider a social norm whereby the status attached to an entrepreneur's occupational choice also depends on the society's perception on how much of an accomplishment the involvement with the high-return project actually is. Naturally, such perceptions will (among other factors) rely upon how common was the involvement with the H project historically. After all, it is reasonable to assume that the status (e.g., due to prestige; admiration etc.) enjoyed by entrepreneurs who operate the H project, albeit still higher compared with the status attached to the alternative low-return project, will not be as high in a society where the incidence of involvement with the high-return project was more common in the past. We capture this scenario by assuming that $x_{t+1} = x(\beta_t)\left(x^{'} < 0\right)$, where β_t is the number of the previous generation's entrepreneurs who invested in the H project. A general function that encompasses all the aforementioned scenarios is

$$x_{t+1} = \bar{x} - i\,(\bar{x} - 1)\,\frac{\beta_t}{n} \tag{4.4}$$

where $i = \{0, 1\}$ is a binary variable. Particularly, $i = 0$ captures the case where the utility benefit of social status is independent of outcomes that transpired in the past, whereas for $i = 1$ this benefit is mitigated by the fraction of the previous generation's entrepreneurs who devoted the effort necessary in order to operate the high-return project. Notice that the differences in social status that originate from an entrepreneur's choice of investment projects disappear when $\bar{x} = 1$.

4.2.2 The Market for Credit and Money

We follow others (Bencivenga and Smith (1993); Bose and Cothren (1996); Bose (2002)) in assuming that the credit market operates as follows. Loan contracts are agreed upon one period in advance of a capital-producing project's operation.[7] Therefore, in period t each worker announces a contract according to which she will offer loans in period $t+1$ at a rate R_{t+1} per unit, to be repaid during the next period (i.e., in $t+2$). Lenders will be approached by entrepreneurs, each of whom applies for a loan l_{t+1} . Furthermore, it is assumed that each entrepreneur can only submit one loan application.

The above imply that a worker willing to lend funds through the credit market, needs to have such funds available in period $t+1$. However, let us imagine that the storage technology is illiquid in the following sense: Despite the fact that it offers a (gross) return $1+q$ between t and $t+2$, if prematurely liquidated (i.e., in $t+1$) it entails a cost that is proportional to the amount of stored income. We normalise this proportional cost to 1, meaning that premature liquidation is prohibitively costly. Nevertheless, there is a liquid asset in the economy that allows the possibility of storage within one period. Henceforth, this asset will be called money.

[7] This assumption ensures there will be a positive demand for money by young workers who wish to offer loans to entrepreneurs.

Each unit of the good in period t is exchangeable for p_t units of money, where p_t is the price level. During the next period, each unit of money can be exchanged for $\frac{1}{p_{t+1}}$ units of goods. Subsequently, these are supplied to the credit market in the form of loanable funds that can be borrowed by entrepreneurs who undertake investments in capital projects. It follows that the overall return from lending to entrepreneurs is $\frac{1+R_{t+1}}{1+\pi_{t+1}}$, where $\pi_{t+1} = \frac{p_{t+1}-p_t}{p_t}$ is the inverse of the (net) period return on money holdings - i.e., the rate of inflation.

The stock of the liquid asset is controlled by a monetary authority that supplies a quantity of money m_t every period. Following other analyses of money in models of economic growth (e.g., Ireland (1994); Schreft and Smith (1997); Varvarigos (2010)) we assume that the monetary authority follows a rule whereby the supply of money evolves according to

$$m_{t+1} = \left(1+\mu\right) m_t, \quad \mu > 0. \tag{4.5}$$

4.2.3 Equilibrium

Let us begin with the reasonable assumption that the two-period return of the illiquid asset dominates the two-period return of holding money. Formally, $1+q = \frac{1}{(1+\pi_{t+1})(1+\pi_{t+2})}$. It follows that workers will be willing to offer their funds in the credit market as long as the overall return from doing so does not fall short of the overall return on storage. Given competition among workers in their role as loan providers, their net economic profit will be driven down to zero. Therefore, the equilibrium interest rate on loans is

$$R_{t+1} = (1+\pi_{t+1})\left(1+q\right) - 1 \tag{4.6}$$

Now, let us consider an entrepreneur who is contemplating which project to undertake after having secured a loan. Taking account of (4.3), the utility associated with operating the H project is[8]

$$u^H = x_{t+1}[r\varphi - (1+R_{t+1})]l_{t+1} - Bl_{t+1}, \tag{4.7}$$

[8]Each entrepreneur's consumption expenditures during maturity equal $r\varphi(j)l_{t+1} - (1+R_{t+1})\,l_{t+1}$. Given that all individuals are risk-neutral, the presence of x_{t+1} in Eq. (4.7) reflects the social status associated with operating the H project in the previous period.

whereas the utility associated with the L project is

$$u^L = [r(1-\psi)\varphi - (1+R_{t+1})]l_{t+1}, \tag{4.8}$$

where $r(1-\psi)\varphi \geq (1+R_{t+1})$ is imposed as a type of participation constraint, ensuring that all entrepreneurs will avoid bankruptcy.[9]

Entrepreneurs will choose which project to operate by comparing the corresponding utilities in (4.7) and (4.8), with the marginal entrepreneur being the one who is indifferent between the two. Setting $u^H = u^L$ defines a threshold

$$\beta_{t+1} = x_{t+1}[r\varphi - (1+R_{t+1})] - [r(1-\psi)\varphi - (1+R_{t+1})], \tag{4.9}$$

such that entrepreneurs with $0 \leq B \leq \beta_{t+1}$ ($\beta_{t+1} < B \leq n$) will operate the H (L) project. Naturally, β_{t+1} is also the number of entrepreneurs who invest in the high-return/high-status project in $t+1$. Note that the condition $r(1-\psi)\varphi > 1+R_{t+1}$ ensures that $\beta_{t+1} > 0$. Therefore, in order to guarantee that β_{t+1} is interior, we naturally assume that $x_{t+1}[r\varphi - (1+R_{t+1})] - [r(1-\psi)\varphi - (1+R_{t+1})] < n$ holds in equilibrium.[10] Furthermore, given the preceding analysis, it is straightforward to establish that $\frac{\partial u^H}{\partial l_{t+1}}, \frac{\partial u^L}{\partial l_{t+1}} > 0$. In other words, the amount of loan secured by each entrepreneur is bound by the amount of funds supplied by workers who offer loan contracts. Recalling that each entrepreneur can only make one loan application, and that the two groups of individuals are of equal size, it follows that

$$l_{t+1} = \frac{w_t}{1+\pi_{t+1}}, \tag{4.10}$$

i.e., the loan is equal to the amount of funds available to each worker in period $t+1$.[11]

Now, let us turn to the money market equilibrium. Given the earlier discussion, the demand for money during period t is $np_t w_t$. It follows that the equilibrium in

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[9]Note that $\frac{(1-a)(1+q)}{a(1-\psi)^2} < \left[\frac{(1-a)A\varphi}{1+\mu}\right]^2 < (1+\mu)^2(1+q)$ is a sufficient condition for both $1+q > \frac{1}{(1+\pi_{t+1})(1+\pi_{t+2})}$ and $r(1-\psi)\varphi > 1+R_{t+1}$ to hold simultaneously.

[10]A sufficient condition is $n > [\bar{x} - (1-\psi)]aA\varphi - (\bar{x}-1)\frac{(1+q)(1+\mu)^2}{(1-a)A\varphi}$.

[11]Note that the same outcomes associated with Eq. (4.6) and (4.10) would also apply if we dispel the idea behind a credit market altogether and assume, instead, that workers and entrepreneurs of the same age are randomly matched into pairs who agree on loan contracts. In that case, the loan rate would be the one that maximises the entrepreneur's utility subject to the lender's participation constraint.

the money market is characterised by $m_t = np_t w_t$. Substituting this condition in Eq. (4.5) yields

$$1+\pi_{t+1} = (1+\mu)\frac{w_t}{w_{t+1}}, \tag{4.11}$$

i.e., the familiar condition that links inflation to the relative growth rates of money and (real) income.

4.3 Capital Accumulation and the Dynamics of Growth

Using the production technology in (4.1), together with (4.2) and the labour market equilibrium condition $N_t = n$, we can solve the profit maximisation problem to derive the following results regarding the wage w_t and the return to capital r:

$$w_t = (1-a)A\frac{K_t}{n}, \tag{4.12}$$

$$r = aA. \tag{4.13}$$

Recall that the process of capital formation is driven by those entrepreneurs who operate the capital-producing projects H and L. Therefore, the aggregate stock of capital is given by

$$K_{t+2} = \int_0^{\beta_{t+1}} \varphi l_{t+1} dB + \int_{\beta_{t+1}}^{n} (1-\psi)\varphi l_{t+1} dB = [n(1-\psi)+\beta_{t+1}\psi]\varphi l_{t+1}. \tag{4.14}$$

Combining Eq. (4.9) and (4.14), a preliminary result comes in the form of

Proposition 1. *The presence of status concerns associated with the choice of entrepreneurial projects stimulates the process of capital accumulation.*

Proof. It is $\frac{\partial K_{t+2}}{\partial x_{t+1}} = \frac{\partial K_{t+2}}{\partial \beta_{t+1}}\frac{\partial \beta_{t+1}}{\partial x_{t+1}} = \varphi l_{t+1}\psi[r\varphi - (1+R_{t+1})] > 0$ by virtue of the condition $r(1-\psi)\varphi > 1+R_{t+1}$. □

This result is quite intuitive. As long as $x_{t+1} > 1$, the marginal utility of consumption associated with operating the H project is higher due to the social status attached to it. Consequently, it increases an entrepreneur's willingness to devote the effort required in order to operate the project that returns more units of capital for each unit of loan invested in it.

The expression in (4.14), when combined with our previous analysis, also allows us to derive the result that is formally presented in

Proposition 2. *The rate of inflation impedes the process of capital accumulation.*

Proof. It is $\frac{\partial K_{t+2}}{\partial \pi_{t+1}} = \frac{\partial K_{t+2}}{\partial l_{t+1}}\frac{\partial l_{t+1}}{\partial \pi_{t+1}} + \frac{\partial K_{t+2}}{\partial \beta_{t+1}}\frac{\partial \beta_{t+1}}{\partial \pi_{t+1}}$. Given $\frac{\partial K_{t+2}}{\partial l_{t+1}}\frac{\partial l_{t+1}}{\partial \pi_{t+1}} = -\frac{[n(1-\psi)+\beta_{t+1}\psi]\varphi w_t}{(1+\pi_{t+1})^2} < 0$ and $\frac{\partial K_{t+2}}{\partial \beta_{t+1}} = \psi\varphi l_{t+1} > 0$, the effect will be unambiguously negative as long as $\frac{\partial \beta_{t+1}}{\partial \pi_{t+1}} \leq 0$. Indeed, combining (4.6) and (4.9), it is straightforward to establish that $\frac{\partial \beta_{t+1}}{\partial \pi_{t+1}} = -(x_{t+1}-1)(1+q) \leq 0$. □

Inflation has two distinct, but both negative, effects on the process of capital formation. Firstly, it erodes the real value of the funds that are available in the credit market, i.e., the market where entrepreneurs seek to secure loans in order to operate their projects (see Eq. (4.10)). Furthermore, inflation reduces the workers' return from lending relative to the return of the storage technology – an outcome that induces them to charge a higher loan rate in order to compensate for this loss (see Eq. (4.6)). However, due to $x_{t+1} > 1$, the higher cost of borrowing has a more pronounced marginal effect on the utility of those who are attracted to the venture with the higher return. Consequently, the increased loan rate will induce fewer entrepreneurs to undertake the H project.

Our next step is to derive the economy's growth rate. To do this, we define

$$\frac{K_{t+2}}{K_{t+1}} = g_{t+2}. \tag{4.15}$$

Substituting (4.10), (4.11), (4.12) and (4.15) in (4.14) yields

$$g_{t+2} = \frac{(1-a)A\varphi}{n(1+\mu)}[n(1-\psi)+\beta_{t+1}\psi] = g(\beta_{t+1}). \tag{4.16}$$

As expected, given Proposition 1, the growth rate is increasing in the number of entrepreneurs who invest in the high-return project, i.e., $g' > 0$.

Now, let us substitute (4.4), (4.6), (4.11), (4.12), (4.13), and (4.16) in (4.9) to get

$$\beta_{t+1} = \left[\bar{x} - i(\bar{x}-1)\frac{\beta_t}{n}\right]\left[aA\varphi - \frac{(1+q)(1+\mu)}{g(\beta_t)}\right] - \left[aA(1-\psi)\varphi - \frac{(1+q)(1+\mu)}{g(\beta_t)}\right] = f(\beta_t). \tag{4.17}$$

Evidently, the entrepreneurial choice of investment projects is a source of dynamics that will permeate the economy's growth performance (see Eq. (4.16)). These transitional dynamics rest on the fact that, in the presence of status concerns, there are two distinct (and conflicting) effects that link intertemporally the number of entrepreneurs who opt for the H project. On the one hand, a higher β_t increases the growth rate of income and, therefore, reduces the rate of inflation (see Eq. (4.11)) – an outcome that increases the workers' return from lending relative to the return of the storage technology, hence inducing them to charge a lower loan rate in the competitive credit market (see Eq. (4.6)). Given $\bar{x} > 1$, the lower cost of borrowing has a more pronounced marginal effect on the utility of those who are attracted to the venture with the higher return. Consequently, the lower loan rate will attract more entrepreneurs towards the H project. On the other hand, however, a higher β_t may also have a direct effect on the status-induced utility increment of those entrepreneurs who invest in the high-return project, because of the social norm (see Eq. (4.3) and (4.4)). This effect mitigates the potential utility benefits that stem from an entrepreneur's choice to invest in the H project, hence reducing the fraction of entrepreneurs who ultimately decide to devote the effort required in order to operate it.

We shall begin our analysis of the economy's long-run equilibrium with the baseline scenario where there are no varying status considerations emanating from an entrepreneur's involvement with any of the two available investment projects. Of course, this is a case where $\bar{x} = 1$. The long-run equilibrium outcomes associated with this scenario are summarised in

Lemma 1. *Suppose that $\bar{x} = 1$. The number of entrepreneurs who invest in the high-return project does not vary over time. Therefore, irrespective of initial conditions, the economy adjusts instantaneously to a balanced growth path characterised by $\hat{g}$.*

Proof. Setting $\bar{x} = 1$ in Eq. (4.17) yields

$$\beta_{t+1} = aA\psi\varphi \equiv \hat{\beta} \ \forall t, \tag{4.18}$$

which can be substituted in Eq. (4.16) in order to get

$$g_{t+2} = \frac{(1-a)A\varphi}{n(1+\mu)}[n(1-\psi)+\hat{\beta}\psi] \equiv \hat{g} \ \forall t, \tag{4.19}$$

thus completing the proof. □

This result is not surprising given the discussion that followed Eq. (4.17) and the fact that the output production technology is (at the social level) linear to the stock of capital per person. The presence of status concerns is critical in generating the outcomes that ultimately shape the intertemporal profile of the variable β_t and, therefore, the growth rate of income. Consequently, as long as there are no forces that allow β_t to deviate from its steady state, the economy will not deviate from the balanced growth path characterised by Eq. (4.19).

Next, we turn our attention to the outcomes that transpire when status concerns play a role in an entrepreneur's occupational choice, i.e., when $\bar{x} > 1$. We summarise these in

Lemma 2. *Suppose that $\bar{x} > 1$.*

1. *If $i = 0$, the number of entrepreneurs who invest in the high-return project converges monotonically to a long-run equilibrium $\tilde{\beta}$. Therefore, the economy converges gradually and monotonically to a balanced growth path characterised by $\tilde{g}$.*

2. *If $i = 1$, the number of entrepreneurs who invest in the high-return project converges cyclically to a long-run equilibrium $\underset{\sim}{\beta} < \tilde{\beta}$. Therefore, the economy converges gradually and cyclically to a balanced growth path characterised by $\underset{\sim}{g} < \tilde{g}$.*

Proof. Combine (4.16) and (4.17) to calculate the derivative

$$f'(\beta_t) = -(\bar{x}-1)\frac{i(1-\psi)+\psi}{n(1-\psi)+\beta_t\psi}\left[\frac{in(1-\psi)+i\beta_t\psi}{in(1-\psi)+n\psi}aA\varphi - \frac{(1+q)(1+\mu)}{g(\beta_t)}\right]. \quad (4.20)$$

Furthermore, note that $f''(\beta_t) < 0$ and recall that $f(\beta_t) \in (0, n)$.

Firstly, consider $i = 0$. In this case, Eq. (4.20) becomes

$$f'(\beta_t) = (\bar{x}-1)\frac{\psi}{n(1-\psi)+\beta_t\psi}\frac{(1+q)(1+\mu)}{g(\beta_t)} > 0.$$

Thus, we conclude that there is a unique $\tilde{\beta}$, such that $\tilde{\beta} = f(\tilde{\beta})$ and $f'(\tilde{\beta}) < 1$. Moreover, for $\beta_t \neq \tilde{\beta}$, convergence is monotonic given $f'(\beta_t) > 0$. Substituting in Eq. (4.16) yields

$$\tilde{g} = \frac{(1-a)A\varphi}{n(1+\mu)}[n(1-\psi)+\tilde{\beta}\psi]. \quad (4.21)$$

Since $\beta_t \neq \tilde{\beta} \Rightarrow g(\beta_t) \neq \tilde{g}$, and given Eq. (4.16), we can infer that the growth rate will converge to its long-run equilibrium monotonically as well.

Secondly, consider $i = 1$. Now, Eq. (4.20) becomes

$$f'(\beta_t) = -\frac{(\bar{x}-1)}{n}\left[aA\varphi - \frac{n}{n(1-\psi)+\beta_t\psi}\frac{(1+q)(1+\mu)}{g(\beta_t)}\right] < 0,$$

given that $r(1-\psi)\varphi > 1 + R_{t+1}$ holds by assumption. Again, we conclude that there is a unique $\underset{\sim}{\beta}$, such that $\underset{\sim}{\beta} = f(\underset{\sim}{\beta})$. Since $f''(\beta_t) < 0$, then $f'(n) > -1$ is a sufficient condition to ensure that $f'(\underset{\sim}{\beta}) > -1$ holds as well – a condition necessary to establish the stability of the steady state equilibrium. Note that the expression $f'(n) > -1$ corresponds to

$$n - (\bar{x}-1)\left[aA\varphi - \frac{n}{n(1-\psi)+\beta_t\psi}\frac{(1+q)(1+\mu)}{g(\beta_t)}\right] > 0.$$

It is sufficient to show that this expression holds for the minimum possible n. Indeed, using the condition in Footnote 10, we can establish that

$$[\bar{x}-(1-\psi)]aA\varphi - (\bar{x}-1)\frac{(1+q)(1+\mu)^2}{(1-a)A\varphi} - (\bar{x}-1)\left[aA\varphi - \frac{n}{n(1-\psi)+\beta_t\psi}\frac{(1+q)(1+\mu)}{g(\beta_t)}\right] \Rightarrow$$

$$[\bar{x}-(1-\psi)]aA\varphi - (\bar{x}-1)\frac{(1+q)(1+\mu)}{g(n)} - (\bar{x}-1)aA\varphi + (\bar{x}-1)\frac{n}{n(1-\psi)+\beta_t\psi}\frac{(1+q)(1+\mu)}{g(\beta_t)} \Rightarrow$$

$$aA\psi\varphi - (\bar{x}-1)(1+q)(1+\mu)\left[\frac{1}{g(n)} - \frac{n}{n(1-\psi)+\beta_t\psi}\frac{1}{g(\beta_t)}\right]. \tag{4.22}$$

Given $\beta_t \leq n$ and $g' > 0$, the expression in Eq. (4.22) is unambiguously positive, thus establishing that $\underset{\sim}{\beta}$ is an asymptotically stable steady state. Furthermore, convergence towards the steady state is cyclical given $f'(\beta_t) < 0$. To obtain the long-run growth rate, we substitute in Eq. (4.16) to get

$$\underset{\sim}{g} = \frac{(1-a)A\varphi}{n(1+\mu)}[n(1-\psi) + \underset{\sim}{\beta}\psi]. \tag{4.23}$$

Since $\beta_t \neq \underset{\sim}{\beta} \Rightarrow g(\beta_t) \neq \underset{\sim}{g}$, and given Eq. (4.16), we can infer that the growth rate will converge to its long-run equilibrium through cycles (damped oscillations).

Finally, note that, by virtue of Eq. (4.17), we have $\frac{\partial f(\cdot)}{\partial i} < 0$. Consequently, $\underset{\sim}{\beta} < \tilde{\beta}$ – a result that can be used together with (4.21) and (4.23) to establish that $\underset{\sim}{g} < \tilde{g}$. □

In order to facilitate the exposition of the mechanisms underlying Lemma 2, recall the discussion that followed Eq. (4.17) and consider the effects of a higher

β_t on β_{t+1}. When the impact of social status on the marginal utility of consumption is positive, but independent of past outcomes (i.e., $\bar{x} > 1$ and $i = 0$), the effect is unambiguously positive due to the fact that an increase in β_t increases the growth rate, reduces inflation and the loan rate, thus attracting more entrepreneurs towards the high-return project because, with status concerns, the utility associated with this project is more responsive to these changes. Consequently, when $\beta_t < \tilde{\beta}$ ($\beta_t > \tilde{\beta}$), the number of entrepreneurs who invest in the H project will be increasing (decreasing) over time as it converges to its steady state (see Figure 4.1). Similarly, the growth rate will adjust gradually and monotonically to its long-run equilibrium since it is an increasing function of the fraction of entrepreneurs who operate the project that returns more capital goods per unit of investment.

Nevertheless, when the impact of social status on the marginal utility of consumption is positive but mitigated by outcomes that transpired in the past (i.e., $\bar{x} > 1$ and $i = 1$) there is an additional mechanism through which β_t impinges on β_{t+1}. By reducing the increment of the marginal utility of consumption – an effect that is attributed to the idea that the social status attached to the decision to devote effort and operate a more rewarding project is less pronounced in circumstances where more entrepreneurs took a similar decision in the past – this effect is a negative one. In fact, it dominates the positive effect to which we alluded earlier. As a result, when $\beta_t \neq \underset{\sim}{\beta}$, the number of entrepreneurs who invest in the H project converges to its steady state through cycles (see Figure 4.2). In terms of intuition, consider a relatively high (low) realisation of β_t. This will reduce (increase) the current utility benefits that stem from an entrepreneur's choice to operate the high-return project, hence reducing (increasing) the fraction of entrepreneurs who ultimately decide to invest in it. Given that the growth rate is an increasing function of the fraction of entrepreneurs who operate the project that returns more capital goods per unit of investment, the cyclical nature of β_t will be the underlying cause for the emergence of cycles in the economy's growth performance, as it gradually converges to the balanced growth path.

One implication from the preceding analysis is presented in

Proposition 3. *The impact of status concerns in the choice of entrepreneurial projects is an additional source of transitional dynamics, even when the status-induced increase of marginal utility is time-invariant.*

Proof. It follows from Lemmas 1 and 2. □

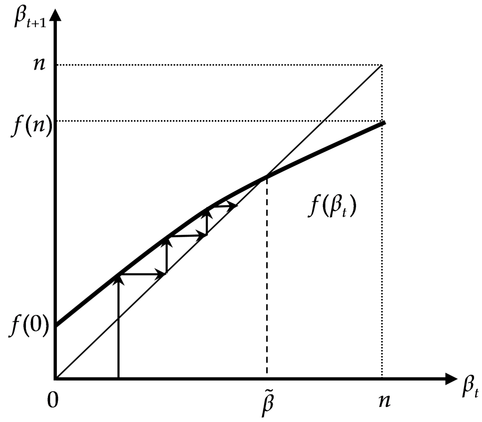

Figure 4.1: Phase diagram $(i = 0)$

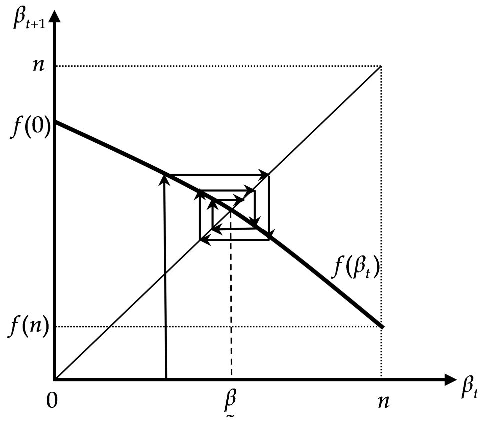

Figure 4.2: Phase diagram $(i = 1)$

It is well-known that in the presence of an AK-type technology, the economy adjusts instantaneously to a balanced growth path – i.e., a time-invariant growth rate – irrespective of initial conditions (e.g., Acemoglu (2009)). In our model, this outcome emerges only in the absence of any status considerations associated with an entrepreneur's occupational choice. Nevertheless, when status impinges on this choice, the adjustment to the balanced growth path is gradual, irrespective of whether the status-induced increase of marginal utility is fixed $(i = 0)$ or varies over time due to the social norm $(i = 1)$.

Despite the fact that transitional dynamics emerge regardless of the fundamental characteristics of the status-induced utility benefits, there are still important implications that emanate from the two different scenarios that capture these characteristics. Specifically, the shape of the economy's dynamics towards the long-run equilibrium, as well as the long-run equilibrium itself, differ in each case. The upshot from the comparison of these two cases is formally presented in

Proposition 4. *The underlying characteristics of social status generate a relation between cyclical volatility and growth in the sense that, when $i = 0$, the economy converges monotonically to a growth rate which is higher compared to the growth rate when $i = 1$, to which the economy converges cyclically.*

Proof. It follows from Lemmas 1 and 2. □

The majority of existing theories on the growth-volatility nexus have investigated this issue on the basis of stochastic growth models that allowed researchers to examine circumstances under which the (exogenous) volatility generated by stochastic terms impinges on the economy's long-term growth. Our paper offers a different approach which, contrary to these analyses, does not stem from the presence of exogenous shocks. In our framework, the structural characteristic (in this case, status) that is responsible for the emergence of cycles, is also an important characteristic in determining the long-term prospects of the economy. Put differently, here the correlation between growth and cycles reflects the idea that cyclical growth converges to a lower value in the long-run, compared with a growth rate that is smoother (i.e., monotonic) during the transition.

Note that the implications can be generalised to the case where, rather than treating i as a binary variable, we consider it as a parameter that takes values on $[0,1]$, thus measuring the magnitude of the social norm - i.e., the direct effect of past realisations of β_t on the current generation's perceptions regarding the status associated with the high-return project. Given the complexity of doing so, we are going to examine the implications for the function in Eq. (4.17) and, therefore, the economy's dynamics, by means of a numerical example.[12] In Figure 4.3, we employ a 3-dimensional plot of $f(\beta_t)$ against β_t and i. As we can see, these general results are consistent with the implications of Lemma 2 and Proposition 4. Specifically, we can see that the slope of $f(\beta_t)$ changes from positive to negative as we increase the value of i. Consequently, we can infer that the higher the strength of the social norm, the more likely it is that the steady state will lie on the downward-sloping part of $f(\beta_t)$, thus leading to damped oscillations (i.e., cycles) in the transition to the steady state.

4.4 Summary and Discussion

The model we developed in this paper represents yet another attempt to shed more light on the macroeconomic implications of social status concerns. Assuming that such concerns apply to the involvement with investment projects that produce capital goods, we have shown that the impact of status on the macroeconomic environment goes beyond its effect on the growth rate. In addition to its impact on long-

[12]The parameter values we use for this example are $a = 0.4$; $A = 2$; $\varphi = 2.5$; $q = 0.2$; $\mu = 0.75$; $n = 2$; and $\bar{x} = 3$.

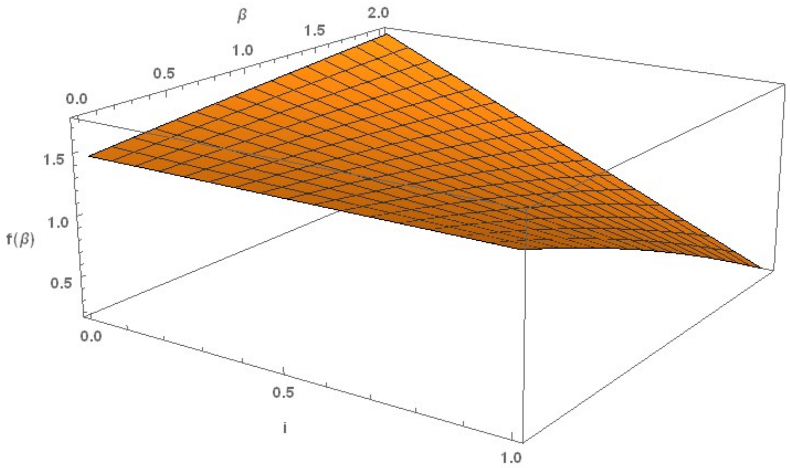

Figure 4.3: The slope of $f(\beta_t)$ when $i \in [0,1]$

term macroeconomic performance, the status-induced increase of the marginal utility of those entrepreneurs who devote effort and operate the high-return project, is also a source of transitional dynamics. The shape of these dynamics (monotonic or cyclical) depends on the underlying characteristics that drive status concerns. As a result, we have employed these characteristics as a means of inferring a relation between growth and cyclical volatility.

Our framework brings forth some interesting policy implications. Given the beneficial effect of status on macroeconomic performance, there is perhaps scope for supporting activities directed towards people's aspirations – for example, activities that will instil into successive generations of individuals the idea that the pursue of more rewarding/productive occupations results in benefits that are not solely restricted to high income. Instead, such occupations can offer additional rewards, such as recognition, admiration, prestige, and all other characteristics that confer status. Another policy implication relates to the negative relation between cyclical volatility and growth. Stochastic growth models that generate this relation suggest that conventional stabilisation policies – designed to eradicate the volatility stemming from exogenous shocks – may entail additional benefits in terms of improved growth performance. In our model, there is clearly no scope for such policies. This is because there is no underlying causal effect that underpins the cyclical volatility-growth nexus. On the contrary, both growth and cycles are endogenously determined by the relative strength of the social norm that governs the status accruing to entrepreneurs who invest in the more productive project. Hence, an appropriate policy should be one that can somehow impinge directly on people's perceptions and reduce the magnitude of this norm. Such a policy will improve the economy's growth prospects while, at the same time, alleviating the forces that are responsible for the emergence of cycles.

Chapter 5

Conclusion

This thesis has studied the effect of optimal monetary policy in economies with imperfect labour and financial markets. It has also analysed how structural characteristics, such as the social status associated in investment projects, can affect the economic growth.

Chapters 2 and 3 have contributed to the design of mainstream monetary policy. They have built on the New Keynesian model, which in recent years has become the standard model for the analysis of monetary policy. They have added key features of the macroeconomic environment in order to explore new issues that have attracted the interest of researchers as well as policymakers in the aftermath of the financial crisis. These aspects include the unemployment divergence in the euro area (chapter 2) and the presence of a lower bound constraint on the nominal interest rate (chapter 3).

Chapter 2 has been motivated from the divergence of unemployment and the convergence of inflation in the euro area after the financial crisis of $2008-2009$. It has examined whether these macroeconomic facts can be an outcome of optimal monetary policy in a currency union. Previous literature has focused on the structural differences of the labour markets to explain why there is unemployment divergence in the euro area. This literature has motivated chapter 2 to explore what are the implications of structurally different labour markets on monetary policy in a currency union. Based on this motivation, chapter 2 has analysed the optimal monetary policy in a currency union consisted of member states with heterogeneous labour markets. The main focus has been on how labour market heterogeneity affects the households' welfare under an optimal monetary policy regime.

The model that is constructed is a two-country currency union one with involuntary unemployment and real wage rigidity. To generate involuntary unemployment in the model, chapter 2 has assumed that in each member state there are Diamond-Mortensen-Pissarides (DMP) labour market frictions in the search and matching process. The assumption of real wage rigidity is supported from empirical evidence in the euro area. It is also an important assumption for the results of the model. Based on previous critiques of the DMP model, by assuming rigid real wages the unemployment volatility puzzle can be solved and realistic unemployment fluctuations are generated. In chapter 2, the real wage rigidity assumption has also served another purpose. It has been used to construct a labour market heterogeneity index and quantify the labour market differences between the member states of the currency union.

The model in chapter 2 is stochastic and has been solved numerically. The results have been obtained through calibration of the structural parameters and simulations. These can be summarised as follows: When a productivity shock hits a member state of the currency union and the central bank follows an optimal monetary policy regime (optimal commitment or optimal discretion), the welfare losses in the currency union increase monotonically with the degree of real wage rigidity in this member state. In addition, when an aggregate shock hits the economy, the welfare losses increase monotonically with the degree of the labour market heterogeneity index. The dynamic paths of unemployment and inflation diverge more, as heterogeneity between member states increases. Thus, labour market heterogeneity has a distortionary effect in the economy and is a source of sub-optimal monetary policy.

Chapter 2 has also shown that in the presence of a union-wide shock, labour market heterogeneity generates a trade off between optimal commitment and optimal discretion with regard to the stabilisation of unemployment. If the two member states are symmetric, optimal commitment and discretion generate the same unemployment fluctuations. However, in the presence of labour market heterogeneity discretion becomes more desirable. This is because the strength of the reaction of the central bank on fluctuations of unemployment increases with the degree of real wage rigidity relatively more under discretion than under commitment.

In conclusion, chapter 2 has provided an argument in support of the view that the unemployment divergence and the homogeneous inflation stabilisation observed in the euro area after the financial crisis can be an outcome of optimal monetary policy in a currency union consisted of member states with structurally different labour markets. In addition, chapter 2 is consistent with the well-known theory of Optimum Currency Areas developed by Mundell (1961). Wage flexibility is a prerequisite condition for an optimal currency area. Hence, a zero real wage rigidity differential that generates the lower welfare losses in the currency union can be considered as a special case of Mundell's theory. Chapter 2 also adds to the view expressed in Wickens (2007) that differences in the characteristics of the member states increase the inflation differential and constitute the main reason that the euro area could be sub-optimal. Chapter 2 provides similar results to this view by focusing on differences in labour market characteristics.

In recent years, the divergence of unemployment in the euro area has attracted the interest of the European Central Bank (ECB). In 2006, the ECB launched a research network, the Wage Dynamics Network (WDN), to research and analyse the changes in the labour markets of the member states. Chapter 2 can be considered that contributes to the research of the WDN. The main message of chapter 2 could be used as a policy advice: the presence of labour market heterogeneity among the member states of a currency union may require additional policies to reduce or eliminate the heterogeneity-generated welfare losses.

The model developed in chapter 2 could be expanded further to incorporate other recent key characteristics, such as migration. In this case, a next step would be to allow for labour mobility (workers can cross the borders) and then study the outcomes of optimal monetary policy. This work would be highly relevant due to the growing social phenomena of the persistent unemployment and migration. A future research work like this would provide a theoretical framework that links migration with monetary policy. This would contribute significantly to the novelty of the paper.

Chapter 3 has been motivated from the criticism on the effectiveness of discount window as an instrument of monetary policy. Previous literature has been based on historical facts and cost-benefit analyses to argue that the discount window is not effective because it is associated with the risk that the financial institutions will misuse the funds provided from the central bank. As a result of this criticism, the last few decades, the discount window facilities of the central banks have been inactive. However, in the aftermath of the financial crisis, the ability of the central banks to utilise the primary instrument of monetary policy, i.e., the nominal interest rate, has been limited due to the presence of the zero lower bound constraint. Chapter 3 has provided evidence which support the use of discount window as a complementary instrument of monetary policy. It has concluded that in the presence of the zero lower bound constraint, alternative, complementary instruments of monetary policy, such as the discount window, can be effective.

The model constructed in chapter 3 is a standard New Keynesian model extended with a framework that analyses the behaviour of financial intermediaries in business cycles. Two main frictions in the model have required the use of two instruments by the central bank. In the presence of a shock, the New Keynesian assumption of nominal price rigidity creates inefficient fluctuations of inflation and requires the central bank to adjust the nominal interest rate in order to stabilise the real economy. The assumption that financial intermediaries may be dishonest and not use the funds borrowed to financial activities requires the central bank to use a complementary monetary policy instrument, i.e., the discount window, in order to stabilise the financial market.

The main objective of chapter 3 has been to evaluate the effect of discount window lending. In particular, in the presence of a shock in the financial market, the central bank desires to minimise a loss function. This loss function has been derived in terms of the central bank's policy objectives, nevertheless, it captures a welfare criterion for the economy's households. The criterion for the evaluation of discount window lending is the difference in households' welfare losses, which has been occurred by switching discount window lending on and off.

It has been the intention of chapter 3 to make the analysis relevant to the criticism on the discount window. A strand of the literature (e.g., Bordo (1989)) has highlighted the ability of the central banks to monitor the intermediaries' actions as important for the effectiveness of the discount window. For this reason, the analysis has focused on the ability of the central bank to monitor the actions of the financial intermediaries with regard to the use of discount window loans. The main findings of chapter 3 can be summarised as follows: When the ability of the central bank to monitor the intermediaries' actions is low, the latter may misuse a fraction of the discount window loans to other than financial activities. In this case, discount window lending is not effective and does not contribute to the reduction of households' welfare losses. However, as the ability of the central bank to monitor improves, there are welfare gains from discount window lending, as the households' welfare losses are reduced. The welfare gains are maintained even if the discount window lending is associated with some efficiency costs.

By performing several quantitative experiments, chapter 3 has also compared two monetary policy regimes which have been followed along with the discount window and have been implemented with the adjustment of the nominal interest rate. These are the optimal commitment and an optimal interest rate rule. Chapter 3 has provided evidence that the welfare gains from discount window lending are different when it is operated along with these different regimes. It has been concluded that the welfare gains from discount window lending are larger when it is operated along with an optimal interest rate rule.

The main result of chapter 3 has given some reasoning to the doubts raised with respect to the effectiveness of discount window lending. Nevertheless, the evidence for welfare gains it has been in contrast with the views in previous literature (e.g., Schwartz (1992)), which have argued that the discount window should be redundant. Indeed, chapter 3 has been the first that constructs a macroeconomic framework that shows the positive effect of discount window lending in the economy.

In future work, the single economy described in chapter 3 could be expanded to an open economy. By doing this, several aspects could be added. One of these may include to explore the role of heterogeneous financial markets in a currency union. In the presence of country-specific and aggregate, asymmetric financial shocks, identifying a regime or policy tool that minimises the welfare losses would add to the novelty of this work.

Chapter 4 has analysed the macroeconomic consequences from the presence in the economy of structural characteristics, such as the social status conferred to entrepreneurs who operate projects that produce capital. In addition, it has examined the effect of the interaction between social status and key macroeconomic indicators, such as inflation.

The model that has been developed is an overlapping generations one which has been extended with a credit market. In the credit market, entrepreneurs borrow funds from workers in order to finance the production of the capital good. The key assumption in the model is that the investment projects differ in the social status they confer to the entrepreneurs who operate them. A project with a high (low) monetary return requires high (low) effort but confers a relatively higher (lower) status.

Chapter 4 has showed that the presence of social status concerns enhances the process of capital accumulation because higher status is linked positively with the effort required to operate a project that produces relatively more capital. However, capital accumulation is obstructed from the rate of inflation. Inflation deteriorates the real value of the funds that are available in the credit market. In addition, inflation reduces the lenders' real return and induces them to charge higher loan rate. This induces fewer entrepreneurs to undertake projects that produce relatively more capital.

Chapter 4 has also showed that the effect of social status in the choice of investment projects is a source of transitional dynamics. The social status associated with the decision to devote effort and operate a high-return project is less pronounced in the cases where more entrepreneurs has taken the same decision in the past. In this case, the effect of the number of entrepreneurs who invest in the high-return project in capital is negative and dominates the positive effect.

Chapter 4 has provided a different approach to the examination of circumstances under which volatility is generated in an economic environment suggesting structural characteristics (social status) as another candidate rather than stochastic terms (exogenous shocks). While chapter 4 has not analysed policy actions, it certainly has some interesting policy implications. These may involve policymakers' actions that support activities which give individuals the incentive to follow careers with higher status rather than higher salary only.

The monetary growth model in chapter 4 could be expanded with the incorporation of policy and welfare analysis. Adding monetary and fiscal policy in the model, would add to the novelty of the model. In this case, we could study how monetary and fiscal policy co-ordinate to maximise welfare in the presence of frictions that create inefficient fluctuations of inflation and capital.

This thesis has illustrated the view that modern macroeconomic theory should study what affects macroeconomic performance and how this can be enhanced from macroeconomic policy. The chapters 2, 3 and 4 of this thesis has built on standard models of the literature by adding key elements that capture features of the recent economic environment. The results of this thesis provide some new lessons for macroeconomic policy. The models that have been built also provide a solid benchmark for future work.

Appendices

Appendix A

Appendix to Chapter 2

A.1 Inflation stabilisation in the euro-area

I use annual European Union (EU) Labour Force Survey (LFS) data of CPI inflation from 1997 − 2014. Figures A.1, A.2 and A.3 display the CPI inflation in some member states of the euro area. After financial crisis, inflation has been stabilised to its steady-state, 2 − 3% for almost all member states, in contrast with unemployment stabilisation.

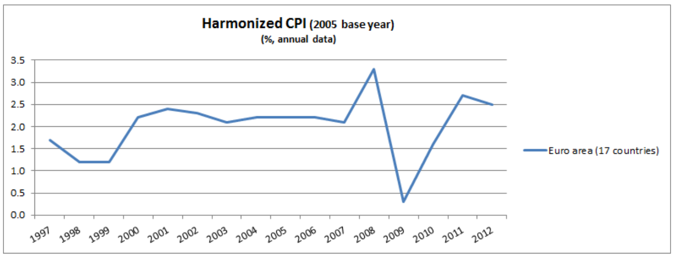

Figure A.1: *CPI inflation in the euro area*

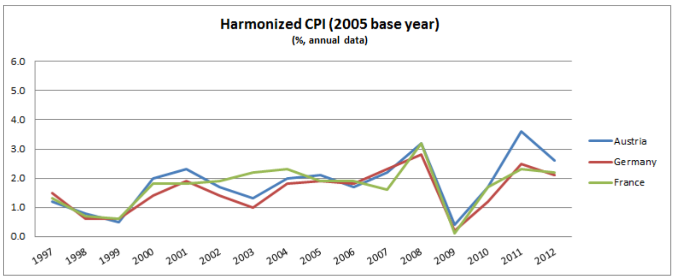

Figure A.2: *Inflation of euro area member states. i*

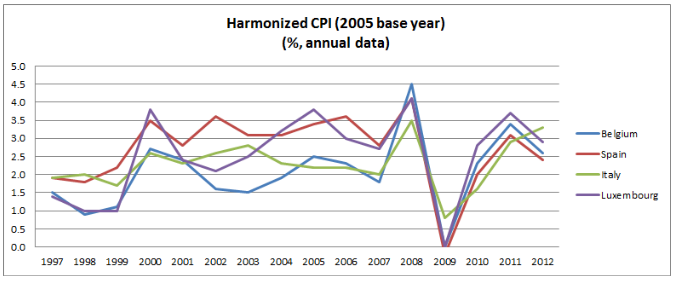

Figure A.3: *Inflation of euro area member states. ii*

A.2 Optimal allocation of nominal spending

The problem of the optimal allocation of nominal spending between domestic and imported final goods is formalised and solved like in Benigno (2004). For country A, the problem is formalised as follows:

$$\min_{P_{At}C^A_{At}, P_{Bt}C^A_{Bt}} \quad P^A_{ct}C^A_t \quad s.t \quad C^A_t \equiv \frac{(C^A_{At})^{\zeta}(C^A_{Bt})^{1-\zeta}}{\zeta^{\zeta}(1-\zeta)^{1-\zeta}}$$

By substituting the definition of CPI for country A, as well as the definition for the composite index:

$$P^A_{ct}C^A_t = P^{\zeta}_{At}P^{(1-\zeta)}_{Bt}\frac{\left(C^A_{At}\right)^{\zeta}\left(C^A_{Bt}\right)^{1-\zeta}}{\zeta^{\zeta}(1-\zeta)^{1-\zeta}} \tag{A.1}$$

The first order conditions with respect to $P_{At}C^A_{At}$ and $P_{Bt}C^A_{Bt}$ yield:

$$\left(\frac{\zeta}{(1-\zeta)}\frac{P_{Bt}C^A_{Bt}}{P_{At}C^A_{At}}\right)^{(1-\zeta)} = 0 \tag{A.2}$$

$$\left(\frac{(1-\zeta)}{\zeta}\frac{P_{At}C^A_{At}}{P_{Bt}C^A_{Bt}}\right)^{\zeta} = 0 \tag{A.3}$$

Combining the first order conditions yields:

$$C^A_{At} = \frac{\zeta}{(1-\zeta)}\frac{P_{Bt}}{P_{At}}C^A_{Bt} \tag{A.4}$$

or

$$C^A_{Bt} = \frac{(1-\zeta)}{\zeta}\frac{P_{At}}{P_{Bt}}C^A_{At} \tag{A.5}$$

By substituting (A.4),(A.5) to (A.1), I obtain:

$$P_{At}C_{At}^{A} = \zeta P_{ct}^{A} C_{t}^{A} \tag{A.6}$$

and

$$P_{Bt}C_{Bt}^{A} = (1-\zeta) P_{ct}^{A} C_{t}^{A} \tag{A.7}$$

Similar conditions holds for country B.

A.3 Optimal allocation of consumption expenditures

In this section, I solve the representative household's problem of the optimal allocation of any given consumption expenditures among the differentiated final goods produced in both countries. Here, the solution follows the basic steps provided in Gali (2008), Appendix of chapter 3, for the single economy problem. Solving first for goods produced in country A, the problem is formalised as follows:

$$\max_{c_t^j(a)} \; C_{At}^{j} \; s.t \int_{a=0}^{\zeta} p_t(a) c_t^j(a) da \equiv ZZ_t^A$$

Formalise the problem with the Langrangian:

$$\mathscr{L} = \left(\left(\frac{1}{\zeta}\right)^{\frac{1}{\gamma}} \int_{0}^{\zeta} c_t^j(a)^{\frac{\gamma-1}{\gamma}} da \right)^{\frac{\gamma}{\gamma-1}} - \lambda_0 \left(\int_{a=0}^{\zeta} p_t(a) c_t^j(a) da - ZZ_t^A \right)$$

The first-order condition with respect to $c_t^j(a)$ yields:

$$\left(\left(\frac{1}{\zeta}\right)^{\frac{1}{\gamma}} \int_{0}^{\zeta} c_t^j(a)^{\frac{\gamma-1}{\gamma}} da \right)^{\frac{1}{\gamma-1}} \left(\frac{1}{\zeta}\right)^{\frac{1}{\gamma}} c_t^j(a)^{-\frac{1}{\gamma}} = \lambda_0 p_t(a) \tag{A.8}$$

Notice that, from the derivation of the consumption index:

$$C_{At}^{j} \equiv \left(\left(\frac{1}{\zeta}\right)^{\frac{1}{\gamma}} \int_{0}^{\zeta} c_t^j(a)^{\frac{\gamma-1}{\gamma}} da \right)^{\frac{\gamma}{\gamma-1}}$$

$$\Rightarrow C_{At}^{j}{}^{\frac{1}{\gamma}} = \left(\left(\frac{1}{\zeta}\right)^{\frac{1}{\gamma}} \int_{0}^{\zeta} c_t^j(a)^{\frac{\gamma-1}{\gamma}} da\right)^{\frac{1}{\gamma-1}} \tag{A.9}$$

Combining equations (A.8) and (A.9) we can write:

$$\left(\tfrac{1}{\zeta}\right)^{\frac{1}{\gamma}} C_{At}^{j}{}^{\frac{1}{\gamma}} c_t^j(a)^{-\frac{1}{\gamma}} = \lambda_0 p_t(a), \quad \forall a \in [o, \zeta)$$

This expression holds for any two goods $\forall\ a, a' \in [o, \zeta)$. By combining each other and then rearranging, we can eliminate the lagrange multiplier and obtain:

$$c_t^j(a) = c_t^j(a')\left(\frac{p_t(a)}{p_t(a')}\right)^{-\gamma} \tag{A.10}$$

Substitute from equation (A.10) to the expression for the consumption expenditures ZZ_t^A:

$$ZZ_t^A = \int_{0}^{\zeta} p_t(a) c_t^j(a')\left(\frac{p_t(a)}{p_t(a')}\right)^{-\gamma} da$$

and then

$$ZZ_t^A = c_t^j(a') p_t(a')^{\gamma} \int_{0}^{\zeta} p_t(a)^{1-\gamma} da \tag{A.11}$$

The domestic price index:

$$P_{At} \equiv \left(\frac{1}{\zeta}\int_{0}^{\zeta} p_t(a)^{1-\gamma} da\right)^{\frac{1}{1-\gamma}}$$

can be re-written according to:

$$\zeta P_{At}^{(1-\gamma)} = \int_{0}^{\zeta} p_t(a)^{1-\gamma} da$$

By substituting to equation (A.11):

$$ZZ_t^A = c_t^j(a') p_t(a')^{\gamma} \zeta P_{At}^{1-\gamma}, \tag{A.12}$$

$\forall a' \in [0,\zeta)$, which implies:

$$ZZ_t^A = c_t^j(a) p_t(a)^\gamma \zeta P_{At}^{1-\gamma},$$

$\forall\ a \in [0,\zeta)$. After rearranging:

$$c_t^j(a) = \frac{1}{\zeta}\frac{ZZ_t^A}{P_{At}}\left(\frac{p_t(a)}{P_{At}}\right)^{-\gamma} \tag{A.13}$$

By substituting for $c_t^j(a)$ from equation (A.13) to the objective function yields:

$$C_{At}^j = \left\{\left(\frac{1}{\zeta}\right)^{\frac{1}{\gamma}} \int_0^\zeta \left[\frac{1}{\zeta}\frac{ZZ_t^A}{P_{At}}\left(\frac{p_t(a)}{P_{At}}\right)^{-\gamma}\right]^{\frac{\gamma-1}{\gamma}} da\right\}^{\frac{\gamma}{\gamma-1}}$$

Rearranging, using again the definition for the domestic price index and after some manipulations, we get:

$$C_{At}^j = ZZ_t^A (P_{At})^{-1}$$

or

$$ZZ_t^A = C_{At}^j P_{At} \tag{A.14}$$

which implies that:

$$\int_{a=0}^{\zeta} p_t(a) c_t^j(a) da = C_{At}^j P_{At}$$

Similarly:

$$\int_{b=\zeta}^{1} p_t(b) c_t^j(b) db = C_{Bt}^j P_{Bt}$$

Substituting for ZZ_t^A from equation (A.14) to equation (A.13) and rearranging in order to obtain the demand equation for final goods produced in country A:

$$c_t^j(a) = \frac{1}{\zeta}\left(\frac{p_t(a)}{P_{At}}\right)^{-\gamma} C_{At}^j \tag{A.15}$$

, $\forall\ a \in [0,\zeta)$

Following the same procedure for the differentiated final goods produced in country

B, the demand for final goods produced in B is given:

$$c_t^j(b) = \frac{1}{1-\zeta}\left(\frac{p_t(b)}{P_{Bt}}\right)^{-\gamma} C_{Bt}^j \tag{A.16}$$

, $\forall\ b \in [\zeta, 1]$

Combining equations (A.15),(A.16), the results of Appendix A.2, the definition of CPI, and the definition of the terms of trade, $S_t \equiv \frac{P_{Bt}}{P_{At}}$, yields the equations (2.10) and (2.11) of the text:

$$c_t^j(a) = \left(\frac{p_t(a)}{P_{At}}\right)^{-\gamma} S_t^{(1-\zeta)} C_t^j$$

for final goods $a \in [0, \zeta)$ produced in country A and

$$c_t^j(b) = \left(\frac{p_t(b)}{P_{Bt}}\right)^{-\gamma} S_t^{-\zeta} C_t^j$$

for final goods $b \in [\zeta, 1]$ produced in country B.

A.4 Optimal consumption/savings decision

For a representative household living in country A, the intertemporal optimisation problem is formalised as follows:

$$\max_{\{C_t^A, B_t^A\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{(C_t^A)^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} - d_u \frac{(N_t^A)^{1+\varphi}}{1+\varphi} \right)$$

subject to

$$\int_0^{\zeta} p_t(a) c_t(a) da + \int_{\zeta}^{1} p_t(b) c_t(b) db + B_t^A \leq (1+q_{t-1}) B_{t-1}^A + N_t^A W_t^A + \Pi_t^A$$

Combining the demand equations from Appendix A.3 with the Dixit-Stiglitz domestic price index, I can write:

$$\int_0^{\zeta} p_t(a)c_t(a)da = P_{At}C^A_{At}$$

and

$$\int_{\zeta}^{1} p_t(b)c_t(b)db = P_{Bt}C^A_{Bt}$$

Then, the total nominal spending of household living in country A can be written as:

$$P^A_{ct}C^A_t = P_{At}C^A_{At} + P_{Bt}C^A_{Bt}$$

Therefore the budget constraint for the representative household of country A becomes:

$$P^A_{ct}C^A_t + B^A_t \leq (1+q_{t-1})B^A_{t-1} + N^A_t W^A_t + \Pi^A_t$$

Given a solvency condition $\lim_{T\to\infty} E_t B_T \geq 0$, for all t, and the law of motion of employment

$$N^A_t = (1-\delta)N^A_{t-1} + m(u^A_t, v^A_t)$$

I solve the dynamic problem using the Lagrangian with period multipliers:

$$\mathscr{L} = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \left(\frac{(C^A_t)^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} - d_u \frac{(N^A_t)^{1+\varphi}}{1+\varphi} \right) - \sum_{t=0}^{\infty} \lambda_{1t} \left(C^A_t + \frac{B^A_t}{P^A_{ct}} - \frac{(1+q_{t-1})B^A_{t-1}}{P^A_{ct}} - \frac{\Pi^A_t}{P^A_{ct}} - \frac{N^A_t W^A_t}{P^A_{ct}} \right) \right\}$$

The first-order conditions are given as follows:

$$\begin{aligned}
C^A_t &: (C^A_t)^{-\frac{1}{\sigma}} = \lambda_{1t} \\
C^A_{t+1} &: (C^A_{t+1})^{-\frac{1}{\sigma}} = \lambda_{1t+1} \\
B^A_t &: \beta^{t+1} \frac{\lambda_{1t+1}(1+q_t)}{P^A_{ct+1}} = \beta^t \frac{\lambda_{1t}}{P^A_{ct}}
\end{aligned}$$

Substituting for λ_{1t}, λ_{1t+1} from the first two first-order conditions to the third one and then take expectations, I obtain the Euler condition, expressed in terms of country $j \in [A, B]$, equation (2.14) of the text:

$$\beta E_t \left\{ \left(\frac{C^j_{t+1}}{C^j_t} \right)^{-\frac{1}{\sigma}} \frac{P^j_{ct}}{P^j_{ct+1}} \right\} = \frac{1}{(1+q_t)}$$

A.5 International trade and risk sharing

In a currency union the real exchange rate among the member-states is defined according:

$$RER_{At} = \frac{P^B_{ct}}{P^A_{ct}} = S_t^{(2\zeta-1)}$$
$$RER_{Bt} = \frac{P^A_{ct}}{P^B_{ct}} = S_t^{(1-2\zeta)}$$

Assuming symmetric households' preferences and the same initial conditions across countries, the Euler condition is symmetric. This yields:

$$\left(\frac{C^A_{t+1}}{C^A_t} \right)^{-\frac{1}{\sigma}} = \left(\frac{C^B_{t+1}}{C^B_t} \right)^{-\frac{1}{\sigma}} \frac{RER_{At}}{RER_{At+1}} \tag{A.17}$$

Combining (2.14) of the text and (A.17) with the definitions of RER, CPI and the terms of trade, I can link the consumption indexes of both countries:

$$E_t \left(\frac{C^A_t}{C^A_{t+1}} \right) = E_t \left\{ \left(\frac{S_t}{S_{t+1}} \right)^{\sigma(2\zeta-1)} \left(\frac{C^B_t}{C^B_{t+1}} \right) \right\} \tag{A.18}$$

which is equation (2.17) of the text.

A.6 Optimal choice of number of vacancies

The representative intermediate good firm of country A chooses the number of vacancies v^A_t in order to maximize the expected sum of nominal profits subject to the law of motion of employment, given by equation (2.22) of the text. That is an intertermporal problem and it can be solved by the Lagrangian with period multipliers:

$$\begin{aligned} \mathscr{L} = {} & E_0 \sum_{s=0}^{\infty} \Xi_{t,t+s} \left\{ \left(P^A_{t+s}\right)^I Z^A_{t+s} N^A_{t+s} - W^A_{t+s} N^A_{t+s} - \psi P^A_{ct+s} v^A_{t+s} \right\} \\ & + E_0 \sum_{s=0}^{\infty} \Xi_{t,t+s} \left\{ \lambda_{2t+s} \left((1-\delta) N^A_{t+s-1} + q(\theta^A_{t+s}) v^A_{t+s} - N^A_{t+s} \right) \right\} \end{aligned}$$

where $\Xi_{t,t+s} = \beta^s \left(\frac{C^A_{t+s}}{C^A_t}^{-\frac{1}{\sigma}} \frac{P^A_{ct}}{P^A_{ct+s}} \right)$ is the discount factor for nominal payoffs. The first-order conditions with respect to v^A_t and N^A_t yield:

$$v^A_t : \psi P^A_{ct} = \lambda_{2t} q(\theta^A_t) \tag{A.19}$$

$$N^A_t : (P^A_t)^I Z^A_t - W^A_t - \lambda_2 t + (1-\delta) E_t \lambda_{2t+1} \Xi_{t,t+1} = 0 \tag{A.20}$$

Iterating equation (A.19) one period forward gives:

$$\psi E_t P^A_{ct+1} = E_t \lambda_{2t+1} q(\theta^A_{t+1})$$

Substituting this together with (A.19) to (A.20) in order to eliminate the Lagrangian multiplier. By using the definition of the the stochastic discount factor in open economies, $\beta_{t,t+1} \equiv \beta \left(\frac{C^j_{t+1}}{C^j_t} \right)^{-\sigma^{-1}} \left(\frac{S_t}{S_{t+1}} \right)^{(1-\zeta)}$, I obtain the firm's optimal hiring decision, equation (2.31) of the text:

$$\frac{\psi}{q(\theta^A_t)} = \frac{(P^A_t)^I}{P^A_{ct}} Z^A_t - \frac{W^A_t}{P^A_{ct}} + (1-\delta) E_t \beta_{t,t+1} \left\{ \left(\frac{S_{t+1}}{S_t} \right)^{(1-\zeta)} \frac{\psi}{q(\theta^A_{t+1})} \right\}$$

A similar condition holds for country B. In this case, the term $\frac{S_t}{S_{t+1}}$ is raised at power $-\zeta$, i.e. $\left(\frac{S_t}{S_{t+1}} \right)^{-\zeta}$

A.7 The Nash bargaining solution

As it is mentioned in the main text, the problem is formalised in terms of country A. For the solution, I follow the closed economy model with unemployment in Thomas (2008).

Assuming that all firms act symmetrically, we can drop the firm-specific index. Let $\xi \in (0,1)$ be the bargaining power of firms. Firms' surplus in country A, Λ^f_{At}, is given by the marginal value of an additional employment relationship, i.e. $\Lambda^f_{At} = \frac{dJ^A_t}{dN^A_t}$, where J^A_t is the sum of the expected discounted real profits:

$$J^A_t = \phi^A_t Z^A_t N^A_t - \frac{W^A_{at}}{P^A_{ct}} N^A_t - \psi v^A_t + E_t \beta_{t,t+1} \left(\frac{S_{t+1}}{S_t} \right)^{(1-\zeta)} J^A_{t+1}$$

Then,

$$\Lambda^f_{At} = \phi^A_t Z^A_t - \frac{W^A_{at}}{P^A_{ct}} + (1-\delta) E_t \beta_{t,t+1} \left(\frac{S_{t+1}}{S_t}\right)^{(1-\zeta)} \Lambda^f_{At+1} \tag{A.21}$$

Worker's surplus in country A, Λ^w_{At}, is given by the marginal value added from an additional employment relationship expressed in consumption terms, i.e. $\Lambda^w_{At} = \frac{dH^A_t}{dN^A_t}\frac{1}{U'(C_t)}$ where:

$$H^A_t = U(C_t, N_t) + \beta E_t H_{t+1}$$

Substituting for consumption from the budget constraint to H^A_t, that gives:

$$\begin{aligned}\Lambda^w_{At} = \frac{W^A_t}{P^A_{ct}} - d_u \left(N^A_t\right)^{\varphi} \left(C^A_t\right)^{\sigma^{-1}} - (1-\delta) E_t \beta_{t,t+1} \left(\frac{S_{t+1}}{S_t}\right)^{(1-\zeta)} p(\theta^A_{t+1}) \Lambda^w_{At+1} \\ + (1-\delta) E_t \beta_{t,t+1} \left(\frac{S_{t+1}}{S_t}\right)^{(1-\zeta)} \Lambda^w_{At+1}\end{aligned} \tag{A.22}$$

where it has been used that the law of motion of employment can be written as follows:

$$N^A_t = (1-\delta) N^A_{t-1} + p(\theta^A_t)\left(1-(1-\delta)N^A_{t-1}\right)$$

Given $\xi \in (0,1)$ and the total job match surplus $\Lambda^T_{At} = \Lambda^f_{At} + \Lambda^w_{At}$, Nash bargaining must satisfy:

$$\Lambda^f_{At} = \xi \Lambda^T_{At} \tag{A.23}$$

Combining (A.21), (A.22) and (A.23), I can solve for the real wage in country A:

$$\frac{W^A_t}{P^A_{ct}} = (1-\xi)\phi^A_t Z^A_t + \xi\left(d_u \left(N^A_t\right)^{\varphi} \left(C^A_t\right)^{\sigma^{-1}} + (1-\delta) E_t \beta_{t,t+1} \left(\frac{S_{t+1}}{S_t}\right)^{(1-\zeta)} \Lambda^w_{At+1}\right)$$

In order to eliminate the term Λ^w_{At+1}, I use (A.23) again, and I also use the condition for the firm's expected discounted marginal value from one hiring at period $t+1$. That is $\frac{\psi}{q(\theta^A_{t+1})} = \Lambda^f_{At+1}$. Finally, the Nash bargaining wage in country A is given by:

$$\begin{aligned}\left(\frac{W^A_t}{P^A_{ct}}\right)^{Nash} = (1-\xi)\left(\phi^A_t Z^A_t + (1-\delta)\psi E_t \beta_{t,t+1} \left(\frac{S_{t+1}}{S_t}\right)^{(1-\zeta)} \theta^A_{t+1}\right) \\ + \xi d_u \left(N^A_t\right)^{\varphi} \left(C^A_t\right)^{\sigma^{-1}}\end{aligned}$$

which is equation (2.34) of the text. A similar condition holds for country B, where in this case, the relative terms of trade between two consequent periods term is given by $\left(\frac{S_{t+1}}{S_t}\right)^{-\zeta}$.

A.8 The social planner's problem

The problem is formalised for country A. The dynamic constrained optimisation problem is solved with the method of Lagrangian with period multipliers. I follow a similar approach used in Ravenna and Walsh (2011) for a closed economy model with unemployment. The problem is formalised as follows:

$$\max_{C_t^A, v_t^A, N_t^A, u_t^A} E_0 \sum_{t=0}^{\infty} \beta^t \Bigg\{ \left(\frac{(C_t^A)^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} - d_u \frac{(N_t^A)^{1+\varphi}}{1+\varphi} \right) + \lambda_{1t}^A \Big(Z_t^A N_t^A - S_t^{(1-\zeta)} \big\{ C_t^A + \psi v_t^A \big\} \Big)$$
$$+ \lambda_{2t}^A \Big((1-\delta) N_{t-1}^A + (v_t^A)^{\kappa} (u_t^A)^{(1-\kappa)} - N_t^A \Big) + \lambda_{3t}^A \Big(u_t^A - 1 + (1-\delta) N_{t-1}^A \Big) \Bigg\}$$

where I have already substituted the production function of intermediate good producers, the matching function and the law of motion of employment and unemployment. Also λ_t's are the Lagrange multipliers. The first-order conditions of the problem yield:

$$C_t^A : (C_t^A)^{-\sigma^{-1}} - \lambda_{1t}^A S^{(1-\zeta)} = 0 \tag{A.24}$$

$$v_t^A : -\lambda_{1t}^A \psi S^{(1-\zeta)} + \lambda_{2t}^A \kappa q(\theta_t^A) = 0 \tag{A.25}$$

$$N_t^A : -d_u (N_t^A)^{\varphi} + \lambda_{1t}^A Z_t^A - \lambda_{2t}^A + (1-\delta)\beta E_t \lambda_{2t+1}^A + (1-\delta)\beta E_t \lambda_{3t+1}^A = 0 \tag{A.26}$$

$$u_t^A : \lambda_{2t}^A (1-\kappa) p(\theta_t^A) + \lambda_{3t}^A = 0 \tag{A.27}$$

Solving equation (A.24) with respect to λ_{1t}^A and then substituting to (A.25), I can eliminate λ_{1t}^A from (A.25). Then solving (A.25) w.r.t to λ_{2t}^A and substituting together with (A.24) to (A.26), I can eliminate λ_{2t}^A. Now, the only unknown in (A.26) is λ_{3t+1}^A. Now, by using (A.24), (A.25) again to substitute for λ_{1t}^A and λ_{2t}^A in (A.27), the only unknown in (A.27) is now λ^A_{3t}. Solving (A.27) with respect to λ^A_{3t} and iterate one period forward, I can get an expression for λ^A_{3t+1}. Substituting this expression to (A.26), I can eliminate λ^A_{3t+1} which is the only unknown of (A.26). This yields the social planner's outcome, equation (2.43) of the text:

$$\frac{\psi}{q(\theta^A_t)} = \kappa\left(S^{(\zeta-1)}Z^A_t - d_u\left(N^A_t\right)^{\varphi}\left(C^A_t\right)^{\sigma^{-1}}\right) + (1-\delta)\left(E_t\beta_{t,t+1}\left(\frac{S_{t+1}}{S_t}\right)^{(1-\zeta)}\frac{\psi}{q(\theta^A_{t+1})}\right)$$
$$-(1-\delta)\left((1-\kappa)E_t\beta_{t,t+1}\left(\frac{S_{t+1}}{S_t}\right)^{(1-\zeta)}\psi\theta^j_{t+1}\right)$$

In this case, the efficient steady-state in country A is given by:

$$\frac{1}{S^{(1-\zeta)}} - \frac{\psi V^A}{\delta\kappa N^A} - d_u(N^A)^{\varphi}(C^A)^{-\sigma} = -\beta\left((1-\delta)\frac{\psi v^A}{\kappa}\left(\frac{1}{\delta N^A} - \frac{(1-\kappa^A)}{u^A}\right)\right) \tag{A.28}$$

and a similar condition holds for country B. Following the closed economy model in Ravenna and Walsh (2011), I can find a condition for the efficient steady-state. Combining the law of motion of employment with the matching function, in the steady-state: $q(\theta^j) = \delta\frac{N^j}{v^j}$. Using this expression to the social planner's outcome above and defining:

$$\delta^A_1 = \frac{1}{S^{(1-\zeta)}} - \frac{\psi V^A}{\delta\kappa N^A} - d_u(N^A)^{\varphi}(C^A)^{-\sigma}$$
$$\delta^B_1 = \frac{1}{S^{-\zeta}} - \frac{\psi V^B}{\delta\kappa N^B} - d_u(N^B)^{\varphi}(C^A)^{-\sigma}$$
$$\delta^j_2 = (1-\delta)\frac{\psi V^j}{\kappa}\left(\frac{1}{\delta N^j} - \frac{(1-\kappa)}{u^j}\right)$$

I can summarise that the efficiency condition requires:

$$\delta^j_1 = -\beta\delta^j_2$$

A.9 The New Keynesian Phillips Curve in a currency union

I follow an approach found in any standard macroeconomics textbook, like in Walsh (2010) Appendix, chapter 8, from which I am borrowed the notation. The derivation in Walsh (2010) is for a closed economy. Starting from equation (2.37) of the text and noticing that it can be written as:

$$
\begin{aligned}
&E_t \sum_{s=0}^{\infty} (\omega\beta)^s {C^j_{t+s}}^{-\sigma^{-1}} (S_{t+s})^{(\zeta-1)} \left(\frac{P_{At+s}}{P_{At}}\right)^{\gamma-1} y^A_{t+s} R^A_t = \\
&\frac{\gamma}{(\gamma-1)(1+\tau)} E_t \sum_{s=0}^{\infty} (\omega\beta)^s {C^j_{t+s}}^{-\sigma^{-1}} (S_{t+s})^{(\zeta-1)} \phi^A_{t+s} S^{(1-\zeta)}_{t+s} \left(\frac{P_{At+s}}{P_{At}}\right)^{\gamma} y^A_{t+s}
\end{aligned}
\tag{A.29}
$$

where $R^A_t \equiv \frac{p^*_t(a)}{P_{At}}$ A first-order approximation of the L.H.S of (A.29) yields:

$$
\begin{aligned}
\frac{C^{-\frac{1}{\sigma}} S^{(\zeta-1)} Y}{1-\omega^A\beta} + \frac{C^{-\frac{1}{\sigma}} S^{(\zeta-1)} Y}{1-\omega^A\beta} \hat{R}^A_t + C^{-\frac{1}{\sigma}} S^{(\zeta-1)} Y \sum_{s=0}^{\infty} (\omega^A\beta)^s E_t \hat{y}^A_{t+s} \\
+(\gamma-1) C^{-\frac{1}{\sigma}} S^{(\zeta-1)} Y \sum_{s=0}^{\infty} (\omega^A\beta)^s E_t \hat{p}^A_{t+s} \\
+(\zeta-1) C^{-\frac{1}{\sigma}} S^{(\zeta-1)} Y \sum_{s=0}^{\infty} (\omega^A\beta)^s E_t \hat{s}_{t+s} \\
-\frac{1}{\sigma} C^{-\frac{1}{\sigma}} S^{(\zeta-1)} Y \sum_{s=0}^{\infty} (\omega^A\beta)^s E_t \hat{c}^A_{t+s} \\
-(\gamma-1) C^{-\frac{1}{\sigma}} S^{(\zeta-1)} Y \sum_{s=0}^{\infty} (\omega^A\beta)^s \hat{p}^A_t
\end{aligned}
$$

or by collecting terms:

$$
\begin{aligned}
&\frac{C^{-\frac{1}{\sigma}} S^{(\zeta-1)} Y}{1-\omega^A\beta} + \frac{C^{-\frac{1}{\sigma}} S^{(\zeta-1)} Y}{1-\omega^A\beta} \hat{R}^A_t \\
&+ C^{-\frac{1}{\sigma}} S^{(\zeta-1)} Y \sum_{s=0}^{\infty} (\omega^A\beta)^s \left\{ E_t \hat{y}^A_{t+s} + (\gamma-1)\Big(E_t \hat{p}^A_{t+s} - \hat{p}^A_t\Big) + (\zeta-1) E_t \hat{s}_{t+s} - \frac{1}{\sigma} E_t \hat{c}^A_{t+s} \right\}
\end{aligned}
\tag{A.30}
$$

where I have used that $\hat{R}_t^A = \frac{R_t^A - R^A}{R}$ and $R^A = 1$. Let $\Phi_{t+s}^A \equiv \phi_{t+s}^A S_{t+s}^{(1-\zeta)}$. Then, similarly, a first-order approximation of the R.H.S of (A.29) yields:

$$\frac{\gamma}{(\gamma-1)(1+\tau)}\left\{\frac{C^{-\frac{1}{\sigma}}S^{(\zeta-1)}Y}{1-\omega^A\beta}\Phi^A + C^{-\frac{1}{\sigma}}S^{(\zeta-1)}Y\Phi^A\sum_{s=0}^{\infty}(\omega^A\beta)^s\left[E_t\hat{y}_{t+s}^A + (\gamma-1)\left(E_t\hat{p}_{t+s}^A - \hat{p}_t^A\right)\right.\right.$$
$$\left.\left. + (\zeta-1)E_t\hat{s}_{t+s} + E_t\hat{\Phi}_{t+s}^A - \frac{1}{\sigma}E_t\hat{c}_{t+s}^A\right]\right\} \tag{A.31}$$

By letting (A.30) and (A.31) be equal, I have been left with:

$$\frac{1}{(1-\omega^A\beta)}\hat{R}_t^A = \sum_{s=0}^{\infty}(\omega^A\beta)^s\left(E_t\hat{\Phi}_{t+s}^A + E_t\hat{p}_{t+s}^A - \hat{p}_t^A\right)$$

where I can write it as:

$$\frac{1}{(1-\omega^A\beta)}\hat{R}_t^A = \sum_{s=0}^{\infty}(\omega^A\beta)^s\left(E_t\hat{\Phi}_{t+s}^A + E_t\hat{p}_{t+s}^A\right) - \frac{1}{(1-\omega^A\beta)}\hat{p}_t^A$$

or

$$\hat{R}_t^A + \hat{p}_t^A = (1-\omega^A\beta)\sum_{s=0}^{\infty}(\omega^A\beta)^s\left(E_t\hat{\Phi}_{t+s}^A + E_t\hat{p}_{t+s}^A\right)$$

This can be written as:

$$\hat{R}_t^A + \hat{p}_t^A = (1-\omega^A\beta)\left(\hat{\Phi}_t^A + \hat{p}_t^A\right) + (1-\omega^A\beta)\sum_{s=0}^{\infty}(\omega^A\beta)^s\left(E_t\hat{\Phi}_{t+s}^A + E_t\hat{p}_{t+s}^A\right) \tag{A.32}$$

Iterating (A.32) one period forward, taking expectations and then substituting back yields:

$$\hat{R}_t^A + \hat{p}_t^A = (1-\omega^A\beta)\left(\hat{\Phi}_t^A + \hat{p}_t^A\right) + \omega^A\beta\left(E_t\hat{R}_{t+1}^A + E_t\hat{p}_{t+1}^A\right) \tag{A.33}$$

Now, from the assumption of nominal price rigidity, I can use the domestic price index, equation (2.8) of the text, to derive a derivation of the average domestic price at time t. That is

$$P_{At}^{(1-\gamma)} = (1-\omega^A)p{*}_t^{(1-\gamma)} + \omega^A P_{At-1}^{(1-\gamma)} \tag{A.34}$$

which I can use to relate R_t^A with the domestic inflation of country A. Particularly, this yields $\hat{R}_t^A = \frac{\omega^A}{1-\omega^A}$

Therefore, I can substitute for $\hat{R}_t^A$ to equation (A.33). After some rearrangements, I obtain:

$$\pi_{At} = \frac{(1-\omega\beta)(1-\omega)}{\omega}\hat{\Phi}_t^A + \beta E_t \pi_{At+1} \tag{A.35}$$

Recall the definition of $\Phi^A_{t+s} \equiv \phi^A_{t+s} S^{(1-\zeta)}_{t+s}$. A first-order approximation implies: $\hat{\Phi}^A_t \approx \hat{\phi}^A_t + (1-\zeta)\hat{s}_t$ Substitute back to (A.35) in order to obtain the NKPC for an open economy:

$$\pi_{At} = \delta_p^A\left(\hat{\phi}_t^A + (1-\zeta)\hat{s}_t\right) + \beta E_t \pi_{At+1} \tag{A.36}$$

where $\delta_p^A = \frac{(1-\omega^A\beta)(1-\omega^A)}{\omega^A}$ is the elasticity of domestic inflation in country A with respect to the real marginal cost of intermediate good firms, $\hat{\phi}^A_t$ and with respect to the terms of trade adjusted by the economic openness,$(1-\zeta)\hat{s}_t$.
Similarly, for country B it holds:

$$\pi_{Bt} = \delta_p^B\left(\hat{\phi}_t^B + (\zeta-1)\hat{s}_t\right) + \beta E_t \pi_{Bt+1} \tag{A.37}$$

A.10 The efficient dynamic IS in a currency union

In A.8, I show that $\delta_1^j = -\beta\delta_2^j$. Following Ravenna and Walsh (2011), by using the definitions for δ_1^j and δ_2^j, I can write: $\eta_1^j = -\delta_2^j$ and $\eta_2^j = \delta_1^j + d_u(N^j)^{\varphi}(C^j)^{\sigma-1}$ which implies:

$$\frac{\eta_1^j}{\eta_1^j+\eta_2^j} = \frac{\delta_2^j}{(1+\beta)\delta_2^j - \gamma_2^j}$$

$$\frac{\eta_2^j}{\eta_1^j+\eta_2^j} = \frac{\beta\delta_2^j - \gamma_2^j}{(1+\beta)\delta_2^j - \gamma_2^j}$$

Recall that technology is an AR(1) process. Then $E_t\epsilon^j_{t+1} = 0$ and I can write $E_t\hat{z}^A_{t+1} - \hat{z}^A_t = (\rho-1)\hat{z}^A_t$. I can express the dynamic IS equations (2.65), (2.66) in terms of log gaps of the efficient steady state:

$$\begin{aligned}\tilde{u}^A_{t+1} = {} & \frac{\delta_2^A}{(1+\beta)\delta_2^A-\gamma_2^A}\tilde{u}_t^A + \frac{\beta\delta_2^A-\gamma_2^A}{(1+\beta)\delta_2^A-\gamma_2^A}E_t\tilde{u}^A_{t+2} + \frac{\sigma}{(1+\beta)\delta_2^A-\gamma_2^A}\tilde{r}_t^A \\ & + \frac{\alpha_1^A}{(1+\beta)\delta_2^j-\gamma_2^A}(1-\zeta)(E_t\tilde{s}_{t+1}-\tilde{s}_t) + (1-\rho^A)\frac{\alpha_1^A}{(1+\beta)\delta_2^A-\gamma_2^A}\tilde{z}_t^A\end{aligned} \tag{A.38}$$

$$\tilde{u}^B_{t+1} = \frac{\delta^B_2}{(1+\beta)\delta^B_2 - \gamma^B_2}\tilde{u}^B_t + \frac{\beta\delta^B_2 - \gamma^B_2}{(1+\beta)\delta^B_2 - \gamma^B_2}E_t\tilde{u}^B_{t+2} + \frac{\sigma}{(1+\beta)\delta^B_2 - \gamma^B_2}\tilde{r}^B_t$$
$$-\frac{\alpha^B_1}{(1+\beta)\delta^B_2 - \gamma^B_2}\zeta(E_t\tilde{s}_{t+1} - \tilde{s}_t) + (1-\rho^B)\frac{\alpha^B_1}{(1+\beta)\delta^B_2 - \gamma^B_2}\tilde{z}^B_t \qquad \text{(A.39)}$$

where the log deviation of $\hat{x}_t$ from the efficient steady state is expressed as $\tilde{x}_t = \hat{x}_t - x^e$, where x^e is the efficient steady state of variable X_t.

A.11 The efficient NKPC in a currency union

The NKPC can be expressed in terms of log-deviation of variables from their efficient steady state implies:

$$\pi_{At} = \beta E_t\pi_{At+1} + \delta_p\rho^A_0\tilde{u}^A_t + \delta_p\rho_1\tilde{u}^A_{t+1} - \delta_p\rho^A_2\tilde{u}^A_{t+2} + \delta_p\rho^A_3(1-\zeta)\tilde{s}_t$$
$$+\delta_p\rho^A_4\tilde{r}^A_t - \delta_p\rho^A_3\tilde{z}^A_t \qquad \text{(A.40)}$$

$$\pi_{Bt} = \beta E_t\pi_{Bt+1} + \delta_p\rho^B_0\tilde{u}^B_t + \delta_p\rho^B_1\tilde{u}^B_{t+1} - \delta_p\rho^B_2\tilde{u}^B_{t+2} + \delta_p\rho^B_3\zeta\tilde{s}_t$$
$$+\delta_p\rho^B_4\tilde{r}^B_t + \delta_p\Big(\rho^B_3 + \frac{1-2\zeta}{\zeta}\Big)\tilde{z}^B_t \qquad \text{(A.41)}$$

where the ρ^j coefficients are given by:

$$\rho^j_0 = \frac{\delta^j_u}{\alpha^j_0\delta\kappa\gamma^j_1}\left((1-\kappa)\psi\theta^{j(1-\kappa)} - \frac{(1-\mu^j)\psi\xi\upsilon\gamma^j_2}{\sigma C^j}\right) - \frac{(1-\mu^j)\psi\xi\upsilon\gamma^j_2}{\sigma C^j}$$

$$\rho^j_1 = \frac{1}{\alpha^j_0\delta\kappa\gamma^j_1}\Bigg\{\delta^j_u(1-\delta)\beta\theta^j\psi\Big(\gamma^j_0 - (1-\kappa)\Big) - (1-\kappa)\psi\theta^{j(1-\kappa)}$$
$$-(1-\mu^j)\xi\gamma^j_2\left(\frac{\alpha^j_1\delta\kappa}{\sigma} + \varphi\delta\kappa - \frac{\psi\upsilon}{\sigma C}\right)\Bigg\}$$

$$\rho_2^j = \frac{(1-\delta)\beta\theta^j\psi}{\alpha_0^j\delta\kappa\gamma_1^j}\left(\gamma_0^j - \frac{(1-\kappa)}{p(\theta^j)}\right)$$

$$\rho_3^j = 1 - \left((1-\mu^j)\xi\frac{\gamma_2^j\alpha_1^j}{\sigma\gamma_1^j}\right), \rho_3^B = (1-\mu^j)\xi\frac{\gamma_2^j\alpha_1^j}{\sigma\gamma_1^j} + \frac{(\zeta-1)}{\zeta}$$

$$\rho_4^j = \frac{(1-\delta)\beta\theta^j\psi}{\gamma_1^j}\left(\frac{1}{p(\theta^j)} - \gamma_0^j\right)$$

I can also eliminate $E_t\,\tilde{u}_{t+2}^j$, by solving for $E_t\,\tilde{u}_{t+2}^j$ and then substituting the IS to the NKPC. Then the NKPC will become:

$$\begin{aligned}\pi_{At} = \beta E_t\pi_{At+1} + \delta_p^A\left(\rho_0^A + \frac{\rho_2^A\phi_1^A}{\phi_2^A}\right)\tilde{u}_t^A + \delta_p^A\left(\rho_1^A - \frac{\rho_2^A}{\phi_2^A}\right)\tilde{u}_{t+1}^A + \delta_p^A\left(\rho_3^A - \frac{\rho_2^A\phi_4^A}{\phi_2^A}\right)(1-\zeta)\tilde{s}_t \\ +\delta_p^A\frac{\rho_2^A\phi_4^A}{\phi_2^A}(1-\zeta)E_t\,\tilde{s}_{t+1} + \delta_p^A\left(\rho_4^A + \frac{\rho_2^A\phi_3^A}{\phi_2^A}\right)\tilde{r}_t^A + \delta_p^A\left(\frac{(1-\rho^A)\rho_2^A\phi_4^A}{\phi_2^A} - \rho_3^A\right)\tilde{z}_t^A\end{aligned} \tag{A.42}$$

$$\begin{aligned}\pi_{Bt} = \beta E_t\pi_{Bt+1} + \delta_p^B\left(\rho_0^B + \frac{\rho_2^B\phi_1^B}{\phi_2^B}\right)\tilde{u}_t^B + \delta_p^B\left(\rho_1^B - \frac{\rho_2^B}{\phi_2^B}\right)\tilde{u}_{t+1}^B + \delta_p^B\left(\rho_3^B + \frac{\rho_2^B\phi_4^B}{\phi_2^B}\right)\zeta\,\tilde{s}_t \\ -\delta_p^B\frac{\rho_2^B\phi_4^B}{\phi_2^B}\zeta E_t\,\tilde{s}_{t+1} + \delta_p^B\left(\rho_4^B + \frac{\rho_2^B\phi_3^B}{\phi_2^B}\right)\tilde{r}_t^B + \delta_p^B\left(\frac{(1-\rho^B)\rho_2^B\phi_4^B}{\phi_2^B} + \rho_3^B + \frac{(1-2\zeta)}{\zeta}\right)\tilde{z}_t^B\end{aligned} \tag{A.43}$$

where:

$$\phi_1^j = \frac{\delta_2^j}{(1+\beta)\delta_2^j - \gamma_2^j},\ \phi_2^j = \frac{\beta\delta_2^j - \gamma_2^j}{(1+\beta)\delta_2^j - \gamma_2^j}$$

$$\phi_3^j = \frac{\sigma}{(1+\beta)\delta_2^j - \gamma_2^j},\ \phi_4^j = \frac{\alpha_1^j}{(1+\beta)\delta_2^j - \gamma_2^j}$$

This derivation of the NKPC has been motivated from Ravenna and Walsh (2011) as well.

A.12 The welfare loss function of the currency union

I derive the welfare-based currency union's loss function following the Linear-Quadratic approach suggested by Rotemberg and Woodford (1997) and Woodford (2003). In order to deriver the objective function of the central bank in a currency union with unemployment, I have also found very useful the work for a currency union by Benigno (2004) and the work for a closed economy model with unemployment by Blanchard and Gali (2010) and Ravenna and Walsh (2011).

The welfare criterion of the central bank in a currency union of two member states is the discounted weighted average of the households' utility function in the member states of the currency union:

$$\Omega = E_0 \sum_{i=0}^{\infty} \beta^i \Big(\zeta \omega^A_{t+i} + (1-\zeta)\omega^B_{t+i} \Big) \tag{A.44}$$

Households' welfare criterion

The households' welfare criterion in each member state is given by equation (2.1) of the text, which is repeated here for convenience:

$$\omega^j_t = \frac{\left(C^j_t\right)^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} - d_u \frac{\left(N^j_t\right)^{1+\varphi}}{1+\varphi} \tag{A.45}$$

for $j = A, B$

The households' welfare criterion is additively separable. I take a second-order approximation of each term separately. Let the first term denoted by $U(C^j_t)$ and the second term by $U(N^j_t)$. Then the second-order linear approximation of $U(C^j_t)$ around the steady state value C^j yields:

$$\frac{\left(C^j_t\right)^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} \approx \frac{\left(C^j\right)^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} + (C^j_t)^{1-\frac{1}{\sigma}} \Big(\frac{C^j_t - C^j}{C^j} \Big) - \frac{1}{2\sigma} (C^j_t)^{1-\frac{1}{\sigma}} \Big(\frac{C^j_t - C^j}{C^j} \Big)^2 + O^3 \tag{A.46}$$

I have used that up to a second order $\frac{C^j_t - C^j}{C^j} \approx \hat{c}^j_t + \frac{1}{2}(\hat{c}^j_t)^2$, therefore $\left(\frac{C^j_t - C^j}{C^j}\right)^2 \approx (\hat{c}^j_t)^2$, where a variable written with a small letter and a hat denotes the log-deviation from its steady state, i.e. for example for the variable X_t, $\hat{x}_t = \log X_t - \log X$. Also notice that the steady-state terms are t.i.p, i.e., terms independent of policy. Then, I obtain:

$$\frac{(C_t^j)^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} \approx (C^j)^{1-\frac{1}{\sigma}}\Big(\hat{c}_t^j + \frac{1-\sigma^{-1}}{2}(\hat{c}_t^j)^2\Big) + t.i.p + O^3 \tag{A.47}$$

or for later reference and using that $(C^j)^{1-\frac{1}{\sigma}} = U'(C)C$, which implies:

$$\frac{(C_t^j)^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} \approx U'(C)C\Big(\hat{c}_t^j + \frac{1}{2}(\hat{c}_t^j)^2 - \frac{1}{2\sigma}(\hat{c}_t^j)^2\Big) + t.i.p + O^3 \tag{A.48}$$

Similarly, the second-order linear approximation of $U(N_t^j)$ around the steady state value N^j yields:

$$d_u\frac{(N_t^j)^{1+\varphi}}{1+\varphi} \approx d\frac{(N^j)^{1+\varphi}}{1+\varphi} + d(N_t^j)^{1+\varphi}\Big(\frac{N_t^j - N^j}{N^j}\Big) + \frac{d\varphi}{2}(N_t^j)^{1+\varphi}\Big(\frac{N_t^j - N^j}{N^j}\Big)^2 + t.i.p + O^3$$

$$d\frac{(N_t^j)^{1+\varphi}}{1+\varphi} \approx d(N^j)^{1+\varphi}\Big(\hat{n}_t^j + \frac{1}{2}(1+\varphi)(\hat{n}_t^j)^2\Big) + t.i.p + O^3 \tag{A.49}$$

or for later reference and using that $d_u(N^j)^{1+\varphi} = U'(N)N$, then:

$$d\frac{(N_t^j)^{1+\varphi}}{1+\varphi} \approx U'(N)N\Big(\hat{n}_t^j + \frac{1}{2}(\hat{n}_t^j)^2 + \frac{\varphi}{2}(\hat{n}_t^j)^2\Big) + t.i.p + O^3 \tag{A.50}$$

Therefore, the households' welfare criterion is approximated as:

$$\omega_t^j = U'(C)C\Big(\hat{c}_t^j + \frac{1}{2}(\hat{c}_t^j)^2 - \frac{1}{2\sigma}(\hat{c}_t^j)^2\Big) - U'(N)N\Big(\hat{n}_t^j + \frac{1}{2}(\hat{n}_t^j)^2 + \frac{\varphi}{2}(\hat{n}_t^j)^2\Big) + t.i.p + O^3 \tag{A.51}$$

Market clearing condition

For this part, I have found very useful a similar procedure followed in Blanchard and Gali (2010) Appendix. In order to eliminate $\hat{c}_t^j$ from the welfare criterion, I use the final goods market clearing condition. These are equations (2.41), (2.42) of the text, repeated here for convenience:

$$Y_t^A = S_t^{(1-\zeta)} C_t^A D_t^A + \psi S_t^{(1-\zeta)} v_t^A \tag{A.52}$$

$$Y_t^B = S_t^{-\zeta} C_t^B D_t^B + \psi S_t^{-\zeta} v_t^B \tag{A.53}$$

where I have also used the fact that

$$\int c_t^j(j)dj = C_t^J \int \frac{c_t^j(j)}{C_t^J} da = C_t^J \Big(\frac{p_t(a)}{P_{At}}\Big)^{-\gamma} \equiv C_t^J D_t^J$$

where $D_t^A \equiv \frac{1}{\zeta}\int_0^{\zeta}\left(\frac{p_t(a)}{P_{At}}\right)^{-\gamma}$, $D_t^B \equiv \frac{1}{1-\zeta}\int_{\zeta}^{1}\left(\frac{p_t(b)}{P_{Bt}}\right)^{-\gamma}$ is a measure of price dispersion.

Showing work for country A first, notice that from $Y_t^A = X_t^A = Z_t^A N_t^A$, then also $Z_t^A N_t^A = S_t^{(1-\zeta)} C_t^A D_t^A + \psi S_t^{(1-\zeta)} v_t^A$. Therefore, I can rewrite the market clearing condition as:

$$\frac{S_t^{(1-\zeta)} C_t^A D_t^A}{S^{(1-\zeta)} C^A Z_t^A} = \frac{N_t^A}{S^{(1-\zeta)} C^A} - \frac{\psi S_t^{(1-\zeta)} v_t^A}{S^{(1-\zeta)} C^A Z_t^A} \tag{A.54}$$

Under the assumption that ψ is small enough, so the term $\psi \hat{x}_t$, for any generic variable X_t, is already of second order, a second-order approximation of the R.H.S implies:[1]

$$\frac{N_t^A}{S^{(1-\zeta)} C^A} - \frac{\psi S_t^{(1-\zeta)} v_t^A}{S^{(1-\zeta)} C^A Z_t^A} \approx 1 + \frac{N^A}{S^{(1-\zeta)} C^A}\Big(\hat{n}_t^A + \frac{1}{2}(\hat{n}_t^A)^2\Big) - \frac{\psi v^A}{C^A}\Big((1-\zeta)\hat{s}_t + \hat{v}_t^A - \hat{z}_t^A\Big) \tag{A.55}$$

Therefore:

$$\frac{S_t^{(1-\zeta)} C_t^A D_t^A}{S^{(1-\zeta)} C^A Z_t^A} \approx 1 + \frac{N^A}{S^{(1-\zeta)} C^A}\Big(\hat{n}_t^A + \frac{1}{2}(\hat{n}_t^A)^2\Big) - \frac{\psi v^A}{C^A}\Big((1-\zeta)\hat{s}_t + \hat{v}_t^A - \hat{z}_t^A\Big) \tag{A.56}$$

Taking logs in equation (A.56) and using that for any variable X_t, $\ln(1+\hat{x}_t) = \hat{x}_t - \frac{1}{2}\hat{x}_t^2$, I obtain, for the L.H.S of (A.56):

$$\ln\left(\frac{S_t^{(1-\zeta)} C_t^A D_t^A}{S^{(1-\zeta)} C^A Z_t^A}\right) = \hat{c}_t^A + (1-\zeta)\hat{s}_t + \hat{d}_t^A - \hat{z}_t^A \tag{A.57}$$

[1]This assumption is used for the cost of posting vacancy function in Blanchard and Gali (2010). They use a different specification than the one used here. Also, Ravenna and Walsh (2011) do not use this assumption for the main derivation of the welfare criterion. However, they admit its usefulness.

Also, the R.H.S of (A.56) will be:

$$\frac{N^A}{S^{(1-\zeta)}C^A}\Big(\hat{n}_t^A+\frac{1}{2}(\hat{n}_t^A)^2\Big)-\frac{\psi v^A}{C^A}\Big((1-\zeta)\hat{s}_t+\hat{v}_t^A-\hat{z}_t^A\Big)-\frac{1}{2}\bigg(\frac{N^A}{S^{(1-\zeta)}C^A}\bigg)^2(\hat{n}_t^A)^2 \tag{A.58}$$

Therefore, I can write (A.56) as:

$$\begin{aligned}\hat{c}_t^A=\hat{z}_t^A-(1-\zeta)\hat{s}_t-\hat{d}_t^A+\frac{N^A}{S^{(1-\zeta)}C^A}\Big(\hat{n}_t^A+\frac{1}{2}(\hat{n}_t^A)^2\Big)-\frac{\psi v^A}{C^A}\Big((1-\zeta)\hat{s}_t+\hat{v}_t^A-\hat{z}_t^A\Big)\\-\frac{1}{2}\bigg(\frac{N^A}{S^{(1-\zeta)}C^A}\bigg)^2(\hat{n}_t^A)^2\end{aligned} \tag{A.59}$$

Equation (A.59) also implies:

$$(\hat{c}_t^A)^2=(\hat{z}_t^A)^2+(1-\zeta)^2\hat{s}_t^2+\bigg(\frac{N^A}{S^{(1-\zeta)}C^A}\bigg)^2(\hat{n}_t^A)^2 \tag{A.60}$$

as the price dispersion term is already a second-order term.

Vacancies

Before substituting for $\hat{c}_t^A$ from equation (A.59) and evaluating the households' welfare criterion, I can eliminate $\hat{v}_t^A$ which is appeared in (A.59). For this section, I found very useful the work in Thomas (2008) and Ravenna and Walsh (2011). Notice that from the labour market tightness, equation (2.56) of the text, I get an expression for vacancies:

$$\hat{v}_t^A=\hat{u}_t^A+\hat{\theta}{j}_t^A \tag{A.61}$$

Taking a second-order approximation of the unemployment relationship, I get:

$$\hat{u}_t^A+\frac{1}{2}(\hat{u}_t^A)^2\approx-(1-\delta)\frac{N^A}{u^A}\Big(\hat{n}_{t-1}^A+\frac{1}{2}(\hat{n}_{t-1}^A)^2\Big)+t.i.p+O^3$$

which also implies:

$$(\hat{u}_t^A)^2=\alpha_0^A(\hat{n}_{t-1}^A)^2 \tag{A.62}$$

where $\alpha_0^A=(1-\delta^A)\frac{N^A}{u^A}$
Substituting back, I get:

$$\hat{u}_t^A\approx-\alpha_0^A\hat{n}_{t-1}^A-\frac{1}{2}\alpha_0^A(1+\alpha_0^A)(\hat{n}_{t-1}^A)^2 \tag{A.63}$$

Combining (A.63) with the law of motion of employment, equation (2.57) of the text, I obtain the following relationship for $\hat{\theta}{^j}_t^A$.

$$\hat{\theta}_t^A = \frac{1}{\delta\kappa}(\hat{n}_t^A - \delta_u^A \hat{n}_{t-1}^A) \tag{A.64}$$

where $\delta_u^A = ((1-\delta) - \alpha_3^A \delta)$ Substituting (A.63) and (A.64) in equation (A.61), I finally get an expression of vacancies in terms of employment:

$$\hat{v}_t^A = \frac{1}{\delta\kappa}(\hat{n}_t^A - \delta_u^A \hat{n}_{t-1}^A) - \alpha_0^A \hat{n}_{t-1}^A - \frac{1}{2}\alpha_0^A(1+\alpha_0^A)(\hat{n}_{t-1}^A)^2 \tag{A.65}$$

Therefore, I can substitute for vacancies from equation (A.65) to the expression for consumption above, equation (A.59). Notice that, the term $\psi\frac{1}{2}\alpha_0^A(1+\alpha_0^A)(\hat{n}_{t-1}^A)^2$ is a higher order term and will be eliminated. For country A, I obtain:

$$\begin{aligned}\hat{c}_t^A = \hat{z}_t^A - (1-\zeta)\hat{s}_t - \hat{d}_t^A + \frac{N^A}{S^{(1-\zeta)}C^A}\Big(\hat{n}_t^A + \frac{1}{2}(\hat{n}_t^A)^2\Big) - \frac{\psi v^A}{C^A}\Big\{(1-\zeta)\hat{s}_t + \frac{1}{\delta\kappa}(\hat{n}_t^A - \delta_u^A \hat{n}_{t-1}^A) \\ -\alpha_0^A \hat{n}_{t-1}^A - \hat{z}_t^A\Big\} - \frac{1}{2}\left(\frac{N^A}{S^{(1-\zeta)}C^A}\right)^2 (\hat{n}_t^A)^2\end{aligned} \tag{A.66}$$

Following a similar procedure for country B, I obtain:

$$\begin{aligned}\hat{c}_t^B = \hat{z}_t^B + \zeta\hat{s}_t - \hat{d}_t^B + \frac{N^B}{S^{-\zeta}C^B}\Big(\hat{n}_t^B + \frac{1}{2}(\hat{n}_t^B)^2\Big) - \frac{\psi v^B}{C^B}\Big\{-\zeta\hat{s}_t + \frac{1}{\delta\kappa}(\hat{n}_t^B - \delta_u^B \hat{n}_{t-1}^B) \\ -\alpha_0^B \hat{n}_{t-1}^B - \hat{z}_t^B\Big\} - \frac{1}{2}\left(\frac{N^B}{S^{-\zeta}C^B}\right)^2 (\hat{n}_t^B)^2\end{aligned} \tag{A.67}$$

Evaluating the households' welfare criterion

Now, I can substitute for consumption, from equations (A.66), (A.67) to (A.51). Starting from country A for convenience, after collecting terms and some rearrangements, I obtain:

$$
\begin{aligned}
\omega_t^A = & U'(C^A)C^A\Big((1+\frac{\psi v^A}{C^A})+\frac{1}{2}\Big(\frac{\sigma-1}{\sigma}\Big)\hat{z}_t^A\Big)\hat{z}_t^A \\
& + U'(C^A)N^A\Big(\delta_1^A\hat{n}_t^A+\delta_2^A\hat{n}_{t-1}^A\Big) \\
& - U'(C^A)C^A\hat{d}_t^A \\
& - U'(C^A)C^A(1+\frac{\psi v^A}{C^A})(1-\zeta)\hat{s}_t \\
& + \frac{1}{2}U'(C^A)C^A\Big(\frac{\sigma-1}{\sigma}\Big)(1-\zeta)^2\hat{s}_t^2 \\
& + \frac{1}{2}U'(C^A)N^A\Big(\delta_1^A+\frac{\psi v^A}{\delta\kappa N^A}\Big)(\hat{n}_t^A)^2 \\
& - \frac{1}{2}U'(C^A)N^A\Big(\frac{N^A}{\sigma C^A}S^{(2(1-\zeta))}+\frac{\varphi U'(N^A)}{U'(C^A)}\Big)(\hat{n}_t^A)^2
\end{aligned}
\tag{A.68}
$$

Following a similar procedure for country B, I obtain:

$$
\begin{aligned}
\omega_t^B = & U'(C^B)C^B\Big((1+\frac{\psi v^B}{C^B})+\frac{1}{2}\Big(\frac{\sigma-1}{\sigma}\Big)\hat{z}_t^B\Big)\hat{z}_t^B \\
& + U'(C^B)N^B\Big(\delta_1^B\hat{n}_t^B+\delta_2^B\hat{n}_{t-1}^B\Big) \\
& - U'(C^B)C^B\hat{d}_t^B \\
& + U'(C^B)C^B(1+\frac{\psi v^B}{C^B})\zeta\hat{s}_t \\
& + \frac{1}{2}U'(C^B)C^B\Big(\frac{\sigma-1}{\sigma}\Big)\zeta^2\hat{s}_t^2 \\
& + \frac{1}{2}U'(C^B)N^B\Big(\delta_1^B+\frac{\psi v^B}{\delta\kappa N^B}\Big)(\hat{n}_t^B)^2 \\
& - \frac{1}{2}U'(C^B)N^B\Big(\frac{N^B}{\sigma C^B}S^{(2\zeta)}+\frac{\varphi U'(N^B)}{U'(C^B)}\Big)(\hat{n}_t^B)^2
\end{aligned}
\tag{A.69}
$$

where:

$$\delta_1^A=\frac{1}{S^{(1-\zeta)}}-\frac{\psi V^A}{\delta\kappa N^A}-d(N^A)^{\varphi}(C^A)^{-\sigma},\delta_1^B=\frac{1}{S^{-\zeta}}-\frac{\psi V^B}{\delta\kappa N^B}-d(N^B)^{\varphi}(C^A)^{-\sigma}$$

and

$$\delta_2^j=(1-\delta)\frac{\psi V^j}{\kappa}\Big(\frac{1}{\delta N^j}-\frac{(1-\kappa)}{u^j}\Big)$$

The present discounted value of households' welfare criterion

Assuming that the welfare criterion of the currency union monetary policy maker is the discounted weighted average of households' utility, I need to obtain the weighted present discounted value of households utility. Focusing on country A, that is:

$$\begin{aligned}\zeta E_t \sum_{i=0}^{\infty} \beta^i \omega^A_{t+i} = {} & \zeta U'(C^A) C^A E_t \sum_{i=0}^{\infty} \beta^i \left((1+\frac{\psi v^A}{C^A}) + \frac{1}{2}\left(\frac{\sigma-1}{\sigma}\right)\hat{z}^A_{t+i} \right) \hat{z}^A_{t+i} \\ & + \zeta U'(C^A) N^A E_t \sum_{i=0}^{\infty} \beta^i \left(\delta^A_1 \hat{n}^A_{t+i} + \delta^A_2 \hat{n}^A_{t+i-1} \right) \\ & - \zeta U'(C^A) C^A E_t \sum_{i=0}^{\infty} \beta^i \hat{d}^A_{t+i} \\ & - \zeta(1-\zeta)\left(1+\frac{\psi v^A}{C^A}\right) U'(C^A) C^A E_t \sum_{i=0}^{\infty} \beta^i \hat{s}_{t+i} \\ & + \zeta(1-\zeta)^2 \frac{1}{2} U'(C^A) C^A \left(\frac{\sigma-1}{\sigma}\right) E_t \sum_{i=0}^{\infty} \beta^i \hat{s}^2_{t+i} \\ & + \zeta \frac{\delta^A_1}{2} U'(C^A) N^A E_t \sum_{i=0}^{\infty} \beta^i (\hat{n}^A_{t+i})^2 \\ & - \zeta \frac{1}{2} U'(C^A) N^A \left(\frac{N^A}{\sigma C^A} S^{(2(1-\zeta))} + \frac{\varphi U'(N^A)}{U'(C^A)} - \frac{\psi v^A}{\delta \kappa N^A} \right) E_t \sum_{i=0}^{\infty} \beta^i (\hat{n}^A_{t+i})^2 \end{aligned} \tag{A.70}$$

while a similar condition holds for country B.

The productivity shock term

Notice that the first term of the RHS, $\zeta U'(C^A) C^A E_t \sum_{i=0}^{\infty} \beta^i \left((1+\frac{\psi v^A}{C^A}) + \frac{1}{2}\left(\frac{\sigma-1}{\sigma}\right)\hat{z}^A_{t+i} \right) \hat{z}^A_{t+i}$, includes steady-state values and the productivity shock term only for any period $t+i$. Therefore is a t.i.p. and it can be eliminated. The same applies for country B.

First order term of employment

Here, I follow Ravenna and Walsh (2011). Given that the efficiency condition requires $\delta^A_1 = \beta \delta^A_2$, the term

$$\zeta U'(C^A) N^A E_t \sum_{i=0}^{\infty} \beta^i \left(\delta^A_1 \hat{n}^A_{t+i} + \delta^A_2 \hat{n}^A_{t+i-1} \right)$$

can be written as:

$$
\begin{aligned}
&-\zeta U'(C^A)N^A E_t \sum_{i=0}^{\infty} \beta^i \Big(\beta \delta_2^A \hat{n}_{t+i}^A + \delta_2^A \hat{n}_{t+i-1}^A\Big) = \\
&-\delta_2^A \zeta U'(C^A)N^A \Big(\beta \hat{n}_t^A - \hat{n}_{t-1}^A + \beta^2 \hat{n}_{t+1}^A - \beta \hat{n}_t^A + ...\Big) = \\
&-\delta_2^A \zeta U'(C^A)N^A \hat{n}_{t-1}^A
\end{aligned} \tag{A.71}
$$

Therefore this term as of time $t-1$ is independent of a policy taken at time t and it can be eliminated.

The price dispersion term

Here, I follow an approach similar to Thomas (2008). This derivation is also given in Woodford (2003) and it has been applied in several other studies like Blanchard and Gali (2010), Ravenna and Walsh (2011) etc. Starting from the definition of the price dispersion term and omitting the country-specific superscripts for simplicity:

$$
\begin{aligned}
D_t &= E_j \Big(\frac{p_t(j)}{P_t}\Big)^{-\gamma} \\
&= E_j \Big(e^{-\gamma \log(\frac{p_t(j)}{P_t})}\Big) \\
&= E_j \Big(e^{-\gamma \tilde{p}_t(j)}\Big)
\end{aligned} \tag{A.72}
$$

where $\tilde{p}_t(j) \equiv \log(\frac{p_t(j)}{P_t})$ A second-order approximation of (B.49) around a zero steady-state yields:

$$
\begin{aligned}
D_t &\approx E_j \Bigg(e^{-\gamma \tilde{p}(j)} - \gamma e^{-\gamma \tilde{p}(j)} \tilde{p}_t(j) + \frac{1}{2}\gamma^2 e^{-\gamma \tilde{p}(j)}\Bigg) \tilde{p}_t^2(j) + O^3 \\
&\approx D - \gamma D \tilde{p}_t(j) + \frac{\gamma^2}{2} D \tilde{p}_t^2(j) + O^3
\end{aligned}
$$

which implies:

$$
\begin{aligned}
\frac{D_t}{D} - 1 &\approx -\gamma \Big(E_j \tilde{p}_t(j) - \frac{\gamma}{2} E_j \tilde{p}_t^2(j)\Big) + O^3 \\
\hat{d}_t + \frac{1}{2}\hat{d}_t^2 &\approx -\gamma \Big(E_j \tilde{p}_t(j) - \frac{\gamma}{2} E_j \tilde{p}_t^2(j)\Big) + O^3
\end{aligned} \tag{A.73}
$$

Now, I use the Dixit-Stiglitz domestic price index of the text, for any country j:

$$1 = E_j\Big(\frac{p_t(j)}{P_t}\Big)^{(1-\gamma)}$$

By taking a second-order approximation of the R.H.S, following a similar procedure as before, I obtain:

$$1 \approx 1 + (1-\gamma)E_j\tilde{p}_t(j) + \frac{(1-\gamma)^2}{2}E_j\tilde{p}_t^2(j) + O^3$$

where I have used that $D = 1$ which implies:

$$E_j\tilde{p}_t(j) = \frac{\gamma-1}{2}E_j\tilde{p}_t^2(j) + O^3 \tag{A.74}$$

By combining equations (B.50) and (B.51) and noticing that $\hat{d}_t^2$ is a higher order term and it is eliminated, I obtain:

$$\hat{d}_t = \frac{\gamma}{2}E_j\tilde{p}_t^2(j) + O^3$$

and as $E_j\tilde{p}_t(j)$ is a second order term, the above implies:

$$\hat{d}_t = \frac{\gamma}{2}Var_j \log p_t(j) + O^3 \tag{A.75}$$

Based on the Proposition 6.3 in Woodford (2003), equation (B.52) implies an evolution of $Var_j \log p_t(j)$ over time:

$$Var_j \log p_t(j) = \omega Var_j \log p_{t-1}(j) + \frac{\omega}{1-\omega}\pi_{jt} + O^3 \tag{A.76}$$

where ω is the degree of nominal price rigidity in the main text and π_{jt} is the domestic price inflation of country $j = A, B$ at time t. Substituting for $\hat{d}_t$ from (B.52) to (B.53), implies:

$$\hat{d}_t^j = \omega\hat{d}_{t-1}^j + \frac{\gamma}{2}\frac{\omega}{1-\omega}\pi_{jt} + O^3 \tag{A.77}$$

Iterating forward equation (B.54), then taking discounted values in each term and noticing that the term $\hat{d}_{t-1}^j$ is t.i.p for a policy taken at time $t \geq 0$, I obtain the final result for the price dispersion term:

$$\sum_{i=0}^{\infty}\beta^i \hat{d}^j_{t+i} = \frac{\gamma}{2}\frac{1}{\delta^j_p}\sum_{i=0}^{\infty}\beta^i \pi^2_{jt+i} \tag{A.78}$$

where $\delta^j_p = \frac{(1-\omega\beta)(1-\omega)}{\omega}$ is the elasticity of domestic inflation with respect to the real marginal cost of intermediate good firms. This is standard in the NK literature that follows a L-Q approach for a normative analysis of monetary policy.

The terms of trade terms

Collecting all terms which contain the terms of trade, I can define:

$$\begin{aligned}\zeta E_t \sum_{i=0}^{\infty}\beta^i L^s_{At+i} \equiv &+\zeta(1-\zeta)\Big(1+\frac{\psi v^A}{C^A}\Big)U'(C^A)C^A E_t\sum_{i=0}^{\infty}\beta^i \hat{s}_{t+i}\\ &+\zeta(1-\zeta)^2\frac{1}{2}\Big(\frac{1-\sigma}{\sigma}\Big)U'(C^A)C^A E_t\sum_{i=0}^{\infty}\beta^i \hat{s}^2_{t+i}\end{aligned}$$

A similar term is used to define the terms of trade terms of country B.

$$\begin{aligned}(1-\zeta)E_t \sum_{i=0}^{\infty}\beta^i L^s_{Bt+i} \equiv &-\zeta(1-\zeta)\Big(1+\frac{\psi v^A}{C^A}\Big)U'(C^B)C^B E_t\sum_{i=0}^{\infty}\beta^i \hat{s}_{t+i}\\ &+(1-\zeta)\zeta^2\frac{1}{2}\Big(\frac{1-\sigma}{\sigma}\Big)U'(C^B)C^B E_t\sum_{i=0}^{\infty}\beta^i \hat{s}^2_{t+i}\end{aligned}$$

The term of employment squared

Notice that by using the efficiency condition, this implies: $\delta^A_1 = -\beta\delta^A_2$. However, a term in δ^A_2 depends on a product of ψ, therefore is of higher order and can be eliminated. Hence, for what remains I set $\delta^A_3 = \frac{N^A}{\sigma C^A}S^{(2(1-\zeta))} + \frac{\varphi U'(N^A)}{U'(C^A)}$. Then, I can summarize the $(\hat{n}^A_{t+i})^2$ terms according:

$$-\frac{\zeta}{2}\delta^A_3 U'(C^A)N^A\sum_{i=0}^{\infty}\beta^i(\hat{n}^A_{t+i})^2 \tag{A.79}$$

The final task is to eliminate employment and express the welfare criterion in terms of unemployment. Notice that from the second-order approximation of the unemployment relationship, I have obtained equation (A.62). Iterating this equation one period forward, I obtain:

$$(\hat{n}_t^j)^2 = \frac{1}{(\alpha_0^j)^2}(\hat{u}_{t+1}^j)^2 \tag{A.80}$$

Therefore, I can substitute above and obtain the term expressed in terms of unemployment:

$$-\frac{\zeta}{2}(\alpha_0^A)^2 \delta_3^A U'(C^A) N^A \sum_{i=0}^{\infty} \beta^i (\hat{u}_{t+1}^A)^2 \tag{A.81}$$

while the same term is also obtained for country B.

The welfare loss function of the currency union. Final result

Collecting all terms, I obtain for country A:

$$\begin{aligned}
\zeta \sum_{i=0}^{\infty} \beta^i \omega_{t+i}^A = & -\zeta \frac{\gamma}{2\delta_p^A} U'(C^A) C^A \sum_{i=0}^{\infty} \beta^i \pi_{At+i}^2 \\
& -\zeta \frac{\delta_3^A}{2(\alpha_0^A)^2} U'(C^A) N^A \sum_{i=0}^{\infty} \beta^i (\hat{u}_{t+1}^A)^2 \\
& -\zeta \sum_{i=0}^{\infty} \beta^i L_{At+i}^s
\end{aligned} \tag{A.82}$$

and for country B:

$$\begin{aligned}
(1-\zeta) \sum_{i=0}^{\infty} \beta^i \omega_{t+i}^B = & -(1-\zeta) \frac{\gamma}{2\delta_p^B} U'(C^B) C^B \sum_{i=0}^{\infty} \beta^i \pi_{Bt+i}^2 \\
& -(1-\zeta) \frac{\delta_3^B}{2(\alpha_0^B)^2} U'(C^B) N^B \sum_{i=0}^{\infty} \beta^i (\hat{u}_{t+1}^B)^2 \\
& -(1-\zeta) \sum_{i=0}^{\infty} \beta^i L_{Bt+i}^s
\end{aligned} \tag{A.83}$$

The welfare loss function in the currency union is the discounted weighted average of the average households' welfare criterion, i.e equation (A.44). Applying symmetry and then substituting the terms which contain the terms of trade,

$$\zeta \sum_{i=0}^{\infty} \beta^i L_{At+i}^s, \; (1-\zeta) \sum_{i=0}^{\infty} \beta^i L_{Bt+i}^s$$

with:

$$\zeta\sum_{i=0}^{\infty}\beta^{i}L^{s}_{At+i} \equiv U'(C)C\Bigg(\zeta(1-\zeta)\Big(1+\frac{\psi v^{A}}{C}\Big)E_t\sum_{i=0}^{\infty}\beta^{i}\hat{s}_{t+i} + \zeta(1-\zeta)^{2}\frac{1}{2}\Big(\frac{1-\sigma}{\sigma}\Big)E_t\sum_{i=0}^{\infty}\beta^{i}\hat{s}^{2}_{t+i}\Bigg)$$

and

$$(1-\zeta)\sum_{i=0}^{\infty}\beta^{i}L^{s}_{Bt+i} \equiv U'(C)C\Bigg(-\zeta(1-\zeta)\Big(1+\frac{\psi v^{A}}{C}\Big)E_t\sum_{i=0}^{\infty}\beta^{i}\hat{s}_{t+i} + (1-\zeta)\zeta^{2}\frac{1}{2}\Big(\frac{1-\sigma}{\sigma}\Big)E_t\sum_{i=0}^{\infty}\beta^{i}\hat{s}^{2}_{t+i}\Bigg)$$

I obtain the welfare loss of the currency union:

$$\begin{aligned}\Omega_{t+i} = -\Bigg\{ & U'(C)C\Bigg(\zeta\frac{\gamma}{2\delta^{A}_{p}}\sum_{i=0}^{\infty}\beta^{i}\pi^{2}_{At+i} + (1-\zeta)\frac{\gamma}{2\delta^{B}_{p}}\sum_{i=0}^{\infty}\beta^{i}\pi^{2}_{Bt+i}\Bigg) \\ & + U'(C)N\Bigg(\zeta\frac{\delta^{A}_{3}}{2(\alpha^{A}_{0})^{2}}\sum_{i=0}^{\infty}\beta^{i}(\hat{u}^{A}_{t+1+i})^{2} \\ & + (1-\zeta)\frac{\delta^{B}_{3}}{2(\alpha^{B}_{0})^{2}}\sum_{i=0}^{\infty}\beta^{i}(\hat{u}^{B}_{t+1+i})^{2}\Bigg) \\ & + U'(C)C\Bigg(\zeta(1-\zeta)\frac{1+\sigma}{2\sigma}\sum_{i=0}^{\infty}\beta^{i}\hat{s}^{2}_{t+i}\Bigg)\Bigg\}\end{aligned}$$

which is equation (2.69) of the text.

Notice that by using the efficiency conditions derived above and obtained from the social planner's problem solution, the policy objectives can be written in terms of deviations from the efficient steady-state. In this case, the log-deviations are expressed with a tilde instead of a hat and the constraints in the currency union (IS and NKPC) are given from their efficient form derived in the previous section of this Appendix.

Appendix B

Appendix to Chapter 3

B.1 Optimal allocation of nominal spending

The solution to the the intratemporal decision of the representative household follows from the basic steps from the Appendix, chapter 3, Gali (2008). The problem is given by:

$$\max_{C_t(i)} C_t \; s.t \int_0^1 P_t(i) C_t(i) di \equiv Z_t$$

Form the Langrangian:

$$\mathscr{L} = \left(\int_0^1 C_t(i)^{\frac{\gamma-1}{\gamma}} di \right)^{\frac{\gamma}{\gamma-1}} - \lambda_0 \left(\int_0^1 P_t(i) C_t(i) di - Z_t \right)$$

The first-order condition with respect to $C_t(i)$ yields:

$$\left(\int_0^1 C_t(i)^{\frac{\gamma-1}{\gamma}} di^{\frac{1}{\gamma-1}} C_t(i) \right)^{-\frac{1}{\gamma}} = \lambda_0 P_t(i) \tag{B.1}$$

Notice that, from the derivation of the consumption index:

$$C_t \equiv \left(\int_0^1 C_t(i)^{\frac{\gamma-1}{\gamma}} di \right)^{\frac{\gamma}{\gamma-1}}$$

$$\Rightarrow C_t^{\frac{1}{\gamma}} = \left(\int_0^1 C_t(i)^{\frac{\gamma-1}{\gamma}} di \right)^{\frac{1}{\gamma-1}} \tag{B.2}$$

Combining equations (B.1) and (B.2) we can write:

$$C_t^{\frac{1}{\gamma}} C_t(i)^{-\frac{1}{\gamma}} = \lambda_0 P_t(i), \; \forall i \in [0,1)$$

Hence, this holds for any two goods $\forall\ i, i' \in [0,1)$. Combing each other and then rearranging, we obtain:

$$C_t(i) = C_t(i')\left(\frac{P_t(i)}{P_t(i')}\right)^{-\gamma} \tag{B.3}$$

Substitute from equation (B.3) to the expression for the consumption expenditures Z_t:

$$Z_t = \int_0^1 P_t(i) C_t(i')\left(\frac{P_t(i)}{P_t(i')}\right)^{-\gamma} di$$

$$\Rightarrow Z_t = C_t(i') P_t(i')^{\gamma} \int_0^{\zeta} P_t(i)^{1-\gamma} di$$

From the price index:

$$P_t\left(\equiv \int_0^1 P_t(i)^{1-\gamma} di\right)^{\frac{1}{1-\gamma}}$$

$$\Rightarrow P_t^{(1-\gamma)} = \int_0^1 P_t(i)^{1-\gamma} di$$

Substitute to equation (B.3):

$$Z_t = C_t(i') P_t(i')^{\gamma} P_t^{1-\gamma},\ \forall\ i' \in [0,1]$$

which implies:

$$Z_t = C_t(i) P_t(i)^{\gamma} P_t^{1-\gamma},\ \forall\ i \in [0,1]$$

After rearranging:

$$\Rightarrow C_t(i) = \frac{Z_t}{P_t}\left(\frac{P_t(i)}{P_t}\right)^{-\gamma} \tag{B.4}$$

Substitute for $C_t(i)$ from equation (B.4) to the objective function:

$$C_t = \left(\int_0^1 \left(\frac{Z_t}{P_t}\left(\frac{P_t(i)}{P_t}\right)^{-\gamma}\right)^{\frac{\gamma-1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1}}$$

Rearranging and using again the definition for the price index yields:

$$C_t = Z_t (P_t)^{-1}$$

$$Z_t = C_t P_t$$

which also implies that:

$$\int_0^1 P_t(i)C_t(i)di = C_t P_t$$

Substituting for Z_t to equation (B.4) and rearranging, we obtain the demand equation:

$$C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\gamma} C_t \;\; \forall i \in [0,1]$$

B.2 Optimal consumption/savings decision

The problem is formalised according to:

$$\max_{\{C_t, D_t, N_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left(\log C_t - \frac{{N_t}^{1+\varphi}}{1+\varphi} \right)$$

subject to

$$P_t C_t + d_t \leq w_t N_t + (1+q_{t-1})d_t + T_t$$

and $\lim_{T\to\infty} E_t D_T \geq 0$. It is solved by using the Langrangian with period multipliers. First set up the Langrangian:

$$\mathscr{L} = E_0 \sum_{t=0}^{\infty} \beta^t \left(\log C_t - \frac{{N_t}^{1+\varphi}}{1+\varphi} + \sum_{t=0}^{\infty} \lambda_t \left[w_t N_t + (1+q_{t-1})d_t + T_t - P_t C_t - d_t \right] \right)$$

The first-order conditions are given by:

$$\begin{aligned}
&C_t : \beta^t \frac{1}{C_t} - \beta^t \lambda_t P_t = 0 \\
&N_t : -\beta^t N_t^{\varphi} + \beta^t \lambda_t w_t = 0 \\
&d_t : -\beta^t \lambda_t + E_t \beta^{t+1} \lambda_{t+1}(1+q_t) = 0
\end{aligned}$$

where here λ_t is the Langrangian multiplier at time t. Combining the first two yields the optimal labour supply decision:

$$C_t {N_t}^{\varphi} = W_t$$

Taking the first-order condition with respect to consumption one period forward, using the definition of the households' stochastic discount factor, $\beta_{t,t+s} \equiv \beta^s \frac{U'(c_{t+s})}{U'(c_t)}$ and combining with the first-order condition with respect the households' deposits yields the Euler equation:

$$(1+q_t)E_t\beta_{t,t+1}\frac{P_t}{P_{t+1}} = 1$$

B.3 Final good producer's intertemporal decision

The intertemporal problem of final good producers is formalised according to:

$$\max_{P_t^*(i)} E_t \sum_{s=0}^{\infty} \omega^s \beta_{t,t+s} \left\{ \frac{P_t^*(i)}{P_{t+s}} Y_t(i) - \phi_{t+s} Y_t(i) \right\}$$

for $s = 0, 1, 2, 3...$, subject to

$$Y_t(i) = C_t(i)$$

The first-order condition associated with the problem above, using the definition of the stochastic discount factor, yields:

$$E_t \sum_{s=0}^{\infty} (\omega\beta)^s \left\{ \left(\frac{C_t}{C_{t+s}} \right) \left((1-\gamma) \frac{P_t(i)^*}{P_{t+s}} + \gamma \phi_{t+s} \right) \left(\frac{P_t^*(i)}{P_{t+s}} \right)^{-\gamma} \frac{1}{P_t^*(i)} Y_{t+s} \right\} = 0$$

Rearranging we take the optimal price decision of the firms:

$$\frac{P_t^*(i)}{P_t} = \frac{\gamma}{(\gamma-1)} \frac{E_t \sum_{s=0}^{\infty} (\omega\beta)^s C_{t+s}^{-1} \phi_{t+s} \left(\frac{P_{t+s}}{P_t} \right)^{\gamma} Y_{t+s}}{E_t \sum_{s=0}^{\infty} (\omega\beta)^s C_{t+s}^{-1} \left(\frac{P_{t+s}}{P_t} \right)^{\gamma-1} Y_{t+s}}$$

B.4 Investors' optimal decision

The investors' problem is formalised as:

$$\max_{\{I_t\}_{t=0}^{\infty}} E_0 \sum_{s=0}^{\infty} \beta_{t,t+s} \left\{ Q_{t+s}^i I_{t+s} - \left(1 + I\left(\frac{I_{t+s}}{I_{t-1+s}} \right) \right) I_{t+s} \right\}$$

subject to the adjustment costs:

$$I\left(\frac{I_t}{I_{t-1}}\right) \equiv \frac{\chi}{2}\left(\frac{I_t}{I_{t-1}}-1\right)^2 I_t$$

The investors choose the quantity of new capital to produce. The first order condition of the problem is:

$$I_t : Q_t^i - \left(1+I\left(\frac{I_t}{I_{t-1}}\right)\right)' I_t - \left(1+I\left(\frac{I_t}{I_{t-1}}\right)\right) - E_t\beta_{t,t+1} I_{t+1}\left(1+I\left(\frac{I_{t+1}}{I_t}\right)\right)' = 0$$

$$Q_t^i - \frac{I_t}{I_{t-1}} I'\left(\frac{I_t}{I_{t-1}}\right) - \left(1+I\left(\frac{I_t}{I_{t-1}}\right)\right) + E_t\beta_{t,t+1}\left(\frac{I_{t+1}}{I_t}\right)^2 I'\left(\frac{I_{t+1}}{I_t}\right) = 0$$

after rearranging:

$$Q_t^i = 1 + I\left(\frac{I_t}{I_{t-1}}\right) + \frac{I_t}{I_{t-1}} I'\left(\frac{I_t}{I_{t-1}}\right) - E_t\beta_{t,t+1}\left(\frac{I_{t+1}}{I_t}\right)^2 I'\left(\frac{I_{t+1}}{I_t}\right)$$

Substituting the adjustment cost function to the expression above yields the competitive price of capital:

$$Q_t^i = 1 + \left(\chi \frac{I_t}{I_{t-1}}\left(\frac{I_t}{I_{t-1}}-1\right) + \frac{\chi}{2}\left(\frac{I_t}{I_{t-1}}-1\right)^2\right) - \chi E_t\beta_{t,t+1}\left(\frac{I_{t+1}}{I_t}\right)^2\left(\frac{I_{t+1}}{I_t}-1\right)$$

or:

$$Q_t^i = 1 + \chi\left(\frac{I_t}{I_{t-1}}-1\right)\left(\frac{I_t}{I_{t-1}} + \frac{1}{2}\left(\frac{I_t}{I_{t-1}}-1\right)\right) - \chi E_t\beta_{t,t+1}\left(\frac{I_{t+1}}{I_t}\right)^2\left(\frac{I_{t+1}}{I_t}-1\right)$$

B.5 Market clearing condition in the final good market

From the Dixit-Stiglitz aggregator

$$Y_t \equiv \left(\int_0^1 Y_t(i)^{\frac{\gamma-1}{\gamma}} di\right)^{\frac{\gamma}{\gamma-1}}$$

Each monopolistic competitor i will supply $Y_t(i)$. The final good i is either consumer or used by investors:

$$Y_t(i) = C_t(i) + I_t(i)$$

From the intratemporal decision problems of households and investors, we have obtained the system of demand equations. Recall:

$$C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\gamma} C_t$$

$$I_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\gamma} I_t$$

That is:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\gamma} (C_t + I_t)$$

Substitute to the Dixit-Stiglitz index:

$$\begin{aligned} Y_t &= \left(\int_0^1 \left(\left(\frac{P_t(i)}{P_t}\right)^{-\gamma} (C_t + I_t) \right)^{\frac{\gamma-1}{\gamma}} di \right)^{\frac{\gamma}{\gamma-1}} \\ &= \left[\frac{(C_t + I_t)^{\frac{\gamma-1}{\gamma}}}{P_t^{1-\gamma}} \int_0^1 P_t(i)^{1-\gamma} di \right]^{\frac{\gamma}{\gamma-1}} \end{aligned}$$

By using the Dixit-Stiglitz price index I get:

$$P_t^{1-\gamma} = \int_0^1 P_t(i)^{1-\gamma} di$$

Substituting yields the market clearing condition in the final goods market:

$$Y_t = C_t + I_t$$

B.6 The linear constraints

B.6.1 The real economy

To express the model in a form of deviations from the flexible price equilibrium, I first derive the flexible price equilibrium. The derivation of the flexible price equilibrium in the core NK model without capital is standard and can be found in textbooks like Gali (2008) or Walsh (2010). Here, the presence of capital in the production function makes things slightly more complicated. First consider the case where nominal price rigidity is absent. Assuming $\omega = 0$, all monopolistic competitors choose the same optimal price at every period, i.e. $P_t^*(i) = P_t$, then $\frac{1}{\mathcal{M}} = \phi_t$, where $\mathcal{M} \equiv \frac{\gamma}{(\gamma-1)}$. Notice that, if we assume that the government eliminates the monopolistic competition distortion with an output subsidy, $1+\tau$ then, $\mathcal{M} \equiv \frac{\gamma}{(\gamma-1)(1+\tau)}$, where the efficient mark-up is $\mathcal{M}^{eff} = 1$. This subsidy is also financed by a lump sum tax, so we avoid further distortionary effects. Under flexible prices, a constant mark-up implies $\hat{\phi}_t = 0$. Then, the market clearing condition in the labour market follows the first-order approximation:

$$\hat{y}_t = \hat{c}_t + (1+\varphi)\hat{n}_t$$

Now, I eliminate $\hat{n}_t$ in the expression above, by taking first a first-order approximation of the production function:

$$\hat{n}_t = \frac{1}{1-\alpha}\hat{y}_t - \frac{\alpha}{1-\alpha}\hat{k}_t - \frac{1}{1-\alpha}\hat{a}_t$$

Adding government spending in the model and solving the optimal monetary policy problem would be non-trivial, as G_t appears in the aggregate resource constraint and consequently, in the welfare loss function. This adds an extra policy goal: the stabilisation of the government spending gap, which in practice, is a goal of the government. I assume that government spending is constant, thus, $G_t = G_0$. The same assumption is also taken by Gertler and Karadi (2011). In addition, as it is highlighted by Cantore and Levine (2015), by assuming that government expenditures are financed by lump-sum taxes does not affect equilibrium in the real economy.

By taking a first-order approximation of the market-clearing condition in the market of final goods, given that $\hat{g}_t = 0$, yields:

$$\hat{c}_t = \frac{Y}{C}\hat{y}_t - \frac{I}{C}\hat{\imath}_t - c^{dw}\frac{M}{C}\hat{m}_t$$

and then substituting back, I can also eliminate consumption from the market clearing condition in the labour market. Notice that by taking a first-order approximation, the capital adjustment costs are disappeared, because of the assumption $I'(1) = 0$. Solving with respect to output, I get the flexible-price equilibrium condition between the natural level of output, investments and capital:

$$\left(\frac{\alpha+\varphi}{1-\alpha} + \frac{Y}{C}\right)\hat{y}_t^e = \frac{I}{C}\hat{\imath}_t^e + \frac{\alpha(1+\varphi)}{1-\alpha}\hat{k}_t^e + \frac{1+\varphi}{1-\alpha}\hat{a}_t \tag{B.5}$$

We observe that under flexible prices, in a NK model with capital, equilibrium output, $\hat{y}_t^e$, does not depend on the productivity shock only, $\hat{a}_t$, as in the standard model, but it depends on the existing capital, $\hat{k}_t^e$, and investment, $\hat{\imath}_t^e$, as well. Also, notice that this flexible price equilibrium, it is the efficient-frictionless one. In this case, $\hat{m}_t^e = 0$, as efficiency implies that the spread is zero.

The aggregate demand side of the economy is represented by a first-order difference equation, a dynamic IS relation. This describes a dynamic, negative relationship between the aggregate output and the real interest rate: First, I take a first-order approximation of the Euler equation. This yields:

$$\frac{1+q_t}{1+q}E_t\left(1 - \pi_{t+1} - \hat{c}_{t+1}\right) = 1 - \hat{c}_t$$

where I have used that $\frac{E_t P_{t+1}}{P_t} \equiv E_t\pi_{t+1}$. Then, from the Appendix in Walsh (2010), chapter 2, $\frac{1+q_t}{1+q} \approx q_t - q \equiv 1 + \hat{q}_t$ (in absolute deviation) and using from Uhlig (1997) that, in general, $\hat{y}_t\hat{x}_t \approx 0$, it yields the log-linear version of the Euler equation:

$$\hat{c}_t = E_t\hat{c}_{t+1} + E_t\pi_{t+1} - \hat{q}_t$$

By taking a first-order approximation of the Fisher equation:[1]

$$\hat{r}_{t+1} = \hat{q}_t - E_t\pi_{t+1}$$

[1] Notice that as the nominal and the real interest rate take small values, they are given in absolute values rather than as log deviation from their steady-state.

I can substitute the inflation term in the log-linear Euler equation, and thus, I can link consumption with the real interest rate:

$$\hat{c}_t = E_t \hat{c}_{t+1} - \hat{r}_{t+1}$$

Using also the market-clearing condition in the market of final goods:

$$\hat{c}_t = \frac{Y}{C}\hat{y}_t - \frac{I}{C}\hat{i}_t - c^{dw}\frac{M}{C}\hat{m}_t$$

I can eliminate consumption and then link output with the real interest rate. This is, the dynamic IS relation:

$$\hat{y}_t = E_t \hat{y}_{t+1} - \frac{I}{Y}(\hat{i}_{t+1} - \hat{i}_t) - c^{dw}\frac{M}{Y}(\hat{m}_{t+1} - \hat{m}_t) - \hat{r}_{t+1}$$

In order to express the terms of the dynamic IS as log-deviations from the flexible-price equilibrium, I add and subtract to the dynamic IS, $\hat{y}_t^e$. Rearranging, this yields the dynamic IS:

$$\tilde{y}_t = E_t \tilde{y}_{t+1} - \frac{I}{Y}(E_t \tilde{i}_{t+1} - \tilde{i}_t) - c^{dw}\frac{M}{Y}(\tilde{m}_{t+1} - \tilde{m}_t) - \tilde{r}_{t+1} \tag{B.6}$$

where

$$\tilde{r}_{t+1} \equiv (\hat{r}_{t+1} - r^e_{t+1}) \tag{B.7}$$

and r^e_{t+1} is the Wicksellian or natural real rate of interest, given by:

$$r^e_{t+1} \equiv \Delta \hat{y}^e_{t+1} - \frac{I}{Y}\Delta \hat{i}^e_{t+1} \tag{B.8}$$

A first-order approximation of the Fisher equation yields:

$$\hat{r}_{t+1} = q_t - E_t \pi_{t+1} \tag{B.9}$$

where $\pi_t \equiv \log \frac{P_t}{P_{t-1}}$. is the inflation rate. Notice that as the nominal and the real interest rate take small values, they are given in absolute value rather than log deviation from their steady-state.[2]

A first-order approximation of the optimal price setting decision of firms, equa-

[2]That follows the discussion in Walsh (2010), chapter 8.

tion (3.6) yields the aggregate supply side of the economy. That is described by a dynamic inflation equation, the New Keynesian Phillips Curve (NKPC):[3]

$$\pi_t = \delta_p \hat{\phi}_t + \beta E_t \pi_{t+1} \tag{B.10}$$

where δ_p is the elasticity of inflation with respect to the real marginal cost and is given by $\delta_p \equiv \frac{(1-\omega\beta)(1-\omega)}{\omega}$. The NKPC provides a link between inflation and output, investment and capital, through the marginal cost of final good producers, $\hat{\phi}_t$.

I can use the flexible price equilibrium to link the real marginal cost with a more convenient measure for output and capital gap. To obtain this, first, I take a first-order approximation of households' optimal labour supply decision. That is:

$$\hat{w}_t = \hat{c}_t + \varphi \hat{n}_t$$

Also, taking a first-order approximation of the optimal units of labour choice by the intermediate good producers yields:

$$\hat{w}_t = \hat{\phi}_t + \hat{y}_t - \hat{n}_t$$

This implies:

$$\hat{c}_t + \varphi \hat{n}_t = \hat{\phi}_t + \hat{y}_t - \hat{n}_t$$

From the final goods market clearing condition, I can also eliminate consumption. This yields:

$$(1+\varphi)\hat{n}_t = \hat{\phi}_t + \left(1 - \frac{Y}{C}\right)\hat{y}_t + \frac{I}{C}\hat{i}_t + c^{dw}\frac{M}{C}\hat{m}_t$$

A first-order approximation of the Cobb-Douglas production function of the intermediate good producers, using also that the production function of the final good producers is $Y_t = X_t$, yields:

$$\hat{n}_t = \frac{1}{1-\alpha}\left(\hat{y}_t - \alpha\hat{k}_t - \hat{\alpha}_t\right)$$

[3] The derivation of the NKPC is standard in textbooks, like Walsh (2010).

Substituting for n_t from the log-linear production function to the expression above and after re-arranging, I get an expression for the real marginal cost:

$$\hat{\phi}_t = \left(\frac{\alpha+\varphi}{1-\alpha} + \frac{Y}{C}\right)\hat{y}_t - \frac{I}{C}\hat{i}_t - \frac{\alpha(1+\varphi)}{1-\alpha}\hat{k}_t - c^{dw}\frac{M}{C}\hat{m}_t - \frac{1+\varphi}{1-\alpha}\hat{a}_t$$

To write the real marginal cost in terms expressed as deviations from the flexible-price equilibrium, I recall that:

$$\left(\frac{\alpha+\varphi}{1-\alpha} + \frac{Y}{C}\right)\hat{y}_t^e = \frac{I}{C}\hat{i}_t^e + \frac{\alpha(1+\varphi)}{1-\alpha}\hat{k}_t^e + \frac{1+\varphi}{1-\alpha}\hat{a}_t$$

Solving the flexible-price equilibrium for the productivity term and then substitute back to the expression of the real marginal cost, I get:

$$\hat{\phi}_t = \left(\frac{\alpha+\varphi}{1-\alpha} + \frac{Y}{C}\right)\tilde{y}_t - \frac{I}{C}\tilde{i}_t - \frac{\alpha(1+\varphi)}{1-\alpha}\tilde{k}_t - c^{dw}\frac{M}{C}\tilde{m}_t \tag{B.11}$$

That is, the real marginal cost is given by terms expressed as log-deviation from the flexible-price equilibrium.

Because the level of investment and capital, as well as of all the rest variables that depend on investment and capital, is different when prices are sticky and when flexible, they all can be written as log-deviations from the flexible-price equilibrium. According to this, a first-order approximation of the law of motion of capital yields:

$$\tilde{i}_t = \frac{1}{\delta}(\tilde{k}_{t+1} - (1-\delta)\tilde{k}_t) \tag{B.12}$$

Notice that, when the investment term is expressed as log-deviation from its flexible price equilibrium, the capital quality shock disappears. Similarly, a first-order approximation of the asset price yields:

$$\tilde{Q}_t = \chi\Delta\tilde{i}_t - \chi\beta\Delta\tilde{i}_{t+1} \tag{B.13}$$

the *gross profits per unit of capital* are expressed according to:

$$\tilde{z}_t = \left(\frac{1+\varphi}{1-\alpha} + \frac{Y}{C}\right)\tilde{y}_t - \frac{I}{C}\tilde{i}_t - \frac{1+\alpha\varphi}{1-\alpha}\tilde{k}_t \tag{B.14}$$

The rest flexible-price equilibrium conditions and equilibrium conditions under sticky prices that are given in terms expressed as log-deviations from the steady-state, are also part of the linear model. These are summarised as: A first-order approximation of the law of motion of capital yields:

$$\hat{i}_t = \frac{1}{\delta}(\hat{k}_{t+1} - (1-\delta)\hat{k}_t - \hat{\psi}_{t+1}) \tag{B.15}$$

This relation holds under flexible prices as well:

$$\hat{i}^e_t = \frac{1}{\delta}(\hat{k}^e_{t+1} - (1-\delta)\hat{k}^e_t - \hat{\psi}_{t+1}) \tag{B.16}$$

The asset price of capital is:

$$\hat{Q}_t = \chi\Delta\hat{i}_t - \chi\beta\Delta\hat{i}_{t+1} \tag{B.17}$$

and

$$\hat{Q}^e_t = \chi\Delta\hat{i}^e_t - \chi\beta\Delta\hat{i}^e_{t+1} \tag{B.18}$$

When prices are flexible, the real marginal cost is equal to a constant mark-up, therefore, the gross profits per unit of capital are given by:

$$\hat{z}^e_t = \hat{y}^e_t - \hat{k}^e_t \tag{B.19}$$

B.6.2 The financial market

With regard to the financial market, a first-order approximation of the equilibrium conditions is summarised below. For the case of a perfect interbank market, the log-linear equilibrium conditions hold without the superscript $h = i, n$, or the superscript i alone. For the case of an imperfect interbank market, the equilibrium conditions hold for $h = i, n$ separately, unless otherwise stated. *The fraction of assets financed through Discount Window*:

$$\tilde{\chi}^i_{mt} = \tilde{m}_t - \tilde{Q}^i_t - \tilde{s}^i_t \tag{B.20}$$

The definition of the leverage ratio:

$$\tilde{\theta}_t^h = \tilde{Q}_t^h - \tilde{s}_t^h - n\tilde{w}_t^h \tag{B.21}$$

Leverage and the marginal cost of borrowing from households:

$$\tilde{\theta}_t^i = \tilde{\mathcal{V}}_{dt} + \frac{\mu_s^i}{\lambda - \mu_s^i}\tilde{\mu}_{st}^i + \frac{\omega_m \chi_m}{1 - \omega_m \chi_m}\tilde{\chi}_{mt} \tag{B.22}$$

where this expression holds in the investing locations for the case of a frictional interbank market and for the case of a frictionless interbank market, but without the superscript i. Also, in the non-investing locations, for the case of a frictional interbank market it holds:

$$\tilde{\theta}_t^n = \tilde{\mathcal{V}}_{dt} + \frac{\mu_s^n}{\lambda - \mu_s^n}\tilde{\mu}_{st}^n \tag{B.23}$$

Banks' balance sheet constraint:

$$\tilde{d}_t = \frac{Q^i S^i}{D}\left(\tilde{Q}_t^i + \tilde{s}_t^i\right) + \frac{Q^n S^n}{D}\left(\tilde{Q}_t^n + \tilde{s}_t^n\right) - \frac{NW^h}{D}n\tilde{w}_t^h - \frac{NW^n}{D}n\tilde{w}_t^n \tag{B.24}$$

The banks' net worth:

$$\begin{aligned} E_t n\tilde{w}_{t+1}^i = \tau^i(\sigma + \xi)\frac{R_k^i Q^i S^i}{NW^i}\left(\tilde{r}_{kt+1}^i + \tilde{q}_t^i + \tilde{s}_t^i\right) - \tau^i \sigma \frac{RD}{NW^i}\left(\tilde{r}_{t+1} + \tilde{d}_t\right) \\ -\tau^i \sigma \frac{R_m M}{NW^i}\left(\tilde{r}_{mt+1} + \tilde{m}_t\right) \end{aligned} \tag{B.25}$$

where this expression holds in the investing locations for the case of a frictional interbank market and for the case of a frictionless interbank market, but without the superscript i. Also, in the non-investing locations, for the case of a frictional interbank market it holds:

$$E_t n\tilde{w}_{t+1}^n = \tau^n(\sigma + \xi)\frac{R_k^n Q^n S^n}{NW^n}\left(\tilde{r}_{kt+1}^n + \tilde{q}_t^n + \tilde{s}_t^n\right) - \tau^n \sigma \frac{RD}{NW^n}\left(\tilde{r}_{t+1} + \tilde{d}_t\right) \tag{B.26}$$

The real lending rate:

$$\tilde{r}_{kt+1}^h = \frac{\Psi Z}{R_k^h Q^h} E_t \tilde{z}_{t+1} + \Psi \frac{(1-\delta)}{R_k^h} E_t \tilde{Q}_{t+1}^h - \tilde{Q}_t^h \tag{B.27}$$

The excess marginal value of lending:

$$\tilde{\mu}^h_{st} = E_t \tilde{\Omega}^h_{t+1} + \frac{\beta \Omega^h R^h_k}{\mu^h_s} \left(\tilde{r}^h_{kt+1} - \tilde{r}_{t+1} \right) \tag{B.28}$$

The marginal value of the bank's capital:

$$\begin{aligned} E_t \tilde{\Omega}^i_{t+1} = \frac{\sigma \mathcal{V}_d}{\Omega^i} E_t \tilde{\mathcal{V}}^i_{dt+1} + \frac{\sigma \theta^i \mu^i_s}{\Omega^i} E_t \tilde{\mu}^i_{st+1} + \frac{\sigma \theta^i}{\Omega^i} \left(\mu^i_s - \mu_m \chi^i_m \right) E_t \tilde{\theta}^h_{t+1} \\ - \frac{\sigma \theta^i \mu_m \chi^i_m}{\Omega^i} E_t \left(\tilde{\mu}_{mt} + \tilde{\chi}^i_t \right) \end{aligned} \tag{B.29}$$

where this expression holds in the investing locations for the case of a frictional interbank market and for the case of a frictionless interbank market, but without the superscript i. Also, in the non-investing locations, for the case of a frictional interbank market it holds:

$$E_t \tilde{\Omega}^n_{t+1} = \frac{\sigma \mathcal{V}_d}{\Omega^n} E_t \tilde{\mathcal{V}}^n_{dt+1} + \frac{\sigma \theta^n \mu^n_s}{\Omega^n} E_t \tilde{\mu}^n_{st+1} + \frac{\sigma \theta^n}{\Omega^n} \mu^n_s E_t \tilde{\theta}^n_{t+1} \tag{B.30}$$

Marginal cost of borrowing and marginal value of the bank's capital:

$$\tilde{\mathcal{V}}^h_{dt} = E_t \tilde{\Omega}^h_{t+1} \tag{B.31}$$

The excess marginal cost of borrowing from the Discount Window:

$$\tilde{\mu}_{mt} = E_t \tilde{\Omega}_{t+1} + \frac{\beta R_m}{(\beta R_m - 1)} \left(\tilde{r}_{mt+1} - \tilde{r}_{t+1} \right) \tag{B.32}$$

The Discount Window rate:

$$\tilde{r}^i_{mt+1} = \frac{\omega_m R_k}{R_m} \tilde{r}^i_{kt+1} + \frac{(1 - \omega_m) R}{R_m} \tilde{r}_{t+1} \tag{B.33}$$

The securities market clearing condition:

$$\tilde{s}_t = \tilde{k}_{t+1} \tag{B.34}$$

when the interbank market is frictionless. If it is frictional, then it is given from:

$$\tilde{s}^i_t = \frac{I}{S^i} \delta \tilde{i}_t + \tau^i (1 - \delta) \frac{K}{S^i} \tilde{k}_t \tag{B.35}$$

and

$$\tilde{s}^n_t = \tilde{k}_t \qquad \text{(B.36)}$$

Finally, when the final good prices are fully flexible, the real rate of lending will be different, since it depends on the asset price. Thus, this yields:

$$\tilde{r}^{eh}_{kt+1} = \frac{\Psi Z}{R^h_k Q^h} E_t \tilde{z}^e_{t+1} + \Psi \frac{(1-\delta)}{R^h_k} E_t \tilde{Q}^{eh}_{t+1} - \tilde{Q}^{eh}_t + \hat{\psi}_{t+1} \qquad \text{(B.37)}$$

The linear model closes by assuming that together with fully flexible-prices, the financial market is also efficient.[4] That is $R^{eh}_{kt+1} = R^e_{t+1}$, which implies:

$$\hat{r}^{eh}_{kt+1} = \hat{r}^e_{t+1} \qquad \text{(B.38)}$$

In the case of a frictionless interbank market, under optimal commitment, the constraints are given from (3.54) of the main text and (B.5) - (B.22), (B.24) - (B.25), (B.27) - (B.29), (B.31) - (B.34), (B.37) - (B.38). Under an optimal simple interest rate rule, the constraint (3.53) of the main text is added. When the case of a frictional interbank market is considered, under commitment, the constraints are given from (3.54) of the main text and (B.5) - (B.33), (B.35) - (B.38). Under an optimal simple rule, the constraint (3.53) of the main text is added. The solution to the problem yields the dynamic path under an optimal joint monetary policy, for 31 variables in the frictionless interbank market scenario, and for 37 variables in the frictional interbank market case.

B.6.3 Exogenous variation

A first-order approximation of the productivity and the capital quality shock is given by:

$$\hat{a}_t = \rho_a \hat{a}_{t-1} + \epsilon_{at} \qquad \text{(B.39)}$$

$$\hat{\psi}_{t+1} = \rho_\psi \hat{\psi}_t + \epsilon_{\psi t+1} \qquad \text{(B.40)}$$

[4] Notice that the assumption of an efficient financial market under flexible prices is taken for simplicity. Otherwise, the central bank would also have to deal with another spread, $\hat{r}^{eh}_{kt+1} \geq \hat{r}^e_{t+1}$, which is mainly unobservable.

B.7 The welfare criterion of the central bank

B.7.1 The households' utility function

In order to derive the quadratic welfare criterion of the central bank I followw the L-Q approach pioneered by Rotemberg and Woodford (1997) and Woodford (2003). In addition, I have found very useful the work by Edge (2003). The instantaneous welfare criterion of households is given by:

$$U_t = \log C_t - \frac{{N_t}^{1+\varphi}}{1+\varphi}$$

The utility of households is additively separable. I take a second- order approximation of each argument around its steady-state separately. That is:

$$\log C_t \approx \hat{c}_t + t.i.p + O^3$$

where t.i.p are the terms independent of policy and O^3 are the terms higher than second-order. Similarly,

$$\frac{{N_t}^{1+\varphi}}{1+\varphi} \approx N^{1+\varphi}\Big(\frac{N_t - N}{N}\Big) + \frac{\varphi}{2}N^{1+\varphi}\Big(\frac{N_t - N}{N}\Big)^2 + t.i.p + O^3$$

which implies:

$$\frac{{N_t}^{1+\varphi}}{1+\varphi} \approx N^{1+\varphi}\Big(\hat{n}_t + \frac{1}{2}(1+\varphi){\hat{n}_t}^2\Big) + t.i.p + O^3$$

where I have used that for any generic variable X, $\frac{X_t - X}{X} \approx \hat{x}_t + \frac{1}{2}\hat{x}_t^2$ and therefore $\left(\frac{X_t - X}{X}\right)^2 \approx \hat{x}_t^2$. Then the households' utility is approximated by:

$$U_t = \hat{c}_t - N^{1+\varphi}\Big(\hat{n}_t + \frac{1}{2}(1+\varphi){\hat{n}_t}^2\Big) + t.i.p + O^3 \tag{B.41}$$

From the market clearing condition, I can eliminate consumption. This is done as follows: The market clearing condition can be written according to:

$$Y_t = C_t D_t + (1 + I(\cdot))I_t + c^{dw} M_t$$

where D_t is a price dispersion term. Solving for consumption and then taking a second-order approximation I obtain:

$$\frac{C_t D_t}{C} = 1 + \frac{Y}{C}\Big(\hat{y}_t + \frac{1}{2}\hat{y}_t^2\Big) - \frac{I}{C}\Big(\hat{i}_t + \frac{1}{2}\hat{i}_t^2\Big) + c^{dw}\frac{M}{C}\left(\hat{m}_t + \frac{1}{2}\hat{m}_t^2\right) + t.i.p + O^3 \quad \text{(B.42)}$$

Even if we ignore the fact that in the benchmark calibration $c^{dw} = 0$, but instead c^{dw} takes a small value, then $c^{dw}\frac{M}{C}\hat{m}_t$ is already of second order and $c^{dw}\frac{M}{C}\frac{1}{2}\hat{m}_t^2$ is eliminated as of higher order. Taking logs in equation (B.42) and by using that for any generic variable X_t, $\ln(1+\hat{x}_t) = \hat{x}_t - \frac{1}{2}\hat{x}_t^2$, I obtain, for the L.H.S of the equation (B.42):

$$ln\left(\frac{C_t D_t}{C}\right) \approx \hat{c}_t + \hat{d}_t + t.i.p + O^3 \quad \text{(B.43)}$$

where I have used the fact that price dispersion is already a second-order term. Then, the R.H.S of equation (B.42) is:

$$\frac{Y}{C}\Big(\hat{y}_t + \frac{1}{2}\hat{y}_t^2\Big) - \frac{I}{C}\Big(\hat{i}_t + \frac{1}{2}\hat{i}_t^2\Big) - c^{dw}\frac{M}{C}\hat{m}_t - \frac{1}{2}\left(\frac{Y}{C}\hat{y}_t - \frac{I}{C}\hat{i}_t\right)^2 + t.i.p + O^3$$

as the other terms are eliminated as of higher order. Therefore, the approximation for equation (B.42) is given by:

$$\hat{c}_t \approx -\hat{d}_t + \frac{Y}{C}\Big(\hat{y}_t + \frac{1}{2}\hat{y}_t^2\Big) - \frac{I}{C}\Big(\hat{i}_t + \frac{1}{2}\hat{i}_t^2\Big) - c^{dw}\frac{M}{C}\hat{m}_t - \frac{1}{2}\left(\frac{Y}{C}\hat{y}_t - \frac{I}{C}\hat{i}_t\right)^2 + t.i.p + O^3 \quad \text{(B.44)}$$

In addition, from a second order approximation on the Cobb Douglas production function, I can get an expression for the labour. That is:

$$\hat{n}_t = \frac{1}{1-\alpha}\left(\hat{y}_t - \alpha\hat{k}_t - \hat{\alpha}_t\right) \quad \text{(B.45)}$$

and

$$\hat{n}_t^2 = \frac{1}{(1-\alpha)^2}\left(\hat{y}_t - \alpha\hat{k}_t - \hat{\alpha}_t\right)^2 \quad \text{(B.46)}$$

Now I substitute the equations (B.44)(B.45) and (B.46) to the welfare criterion, equation (B.41), to eliminate consumption and labour. By doing some simplifications I get:

$$
\begin{aligned}
U_t = & -\hat{d}_t + \frac{\alpha Y}{C}\hat{k}_t - \frac{I}{C}\hat{i}_t - c^{dw}\frac{M}{C}\hat{m}_t + \frac{Y}{2C}\Big(1 - \frac{Y}{C} - \frac{1+\varphi}{1-\alpha}\Big)\hat{y}_t^2 - \frac{I}{2C}\Big(1+\frac{I}{C}\Big)\hat{i}_t^2 + \frac{IY}{C^2}\hat{y}_t\hat{i}_t \\
& - \frac{1+\varphi}{2(1-\alpha)}\frac{\alpha^2 Y}{C}\hat{k}_t^2 + \frac{\alpha(1+\varphi)Y}{(1-\alpha)C}\hat{y}_t\hat{k}_t + \left(\frac{(1+\varphi)Y}{(1-\alpha)C}\hat{y}_t - \frac{\alpha(1+\varphi)Y}{(1-\alpha)C}\hat{k}_t\right)\hat{\alpha}_t
\end{aligned}
\tag{B.47}
$$

where I have used that $I = \delta K$ and $\frac{N^{1+\varphi}}{1-\alpha} = \frac{Y}{C}$.

B.7.2 The productivity term

I can eliminate $\hat{\alpha}_t$ from the flexible-equilibrium condition, repeated for convenience:

$$
\left(\frac{\alpha+\varphi}{1-\alpha} + \frac{Y}{C}\right)\hat{y}_t^e = \frac{I}{C}\hat{i}_t^e + \frac{\alpha(1+\varphi)}{1-\alpha}\hat{k}_t^e + \frac{1+\varphi}{1-\alpha}\hat{a}_t
$$

and by solving for $\hat{\alpha}_t$ and then substituting back to equation (B.47), I get:

$$
\begin{aligned}
U_t = & -\hat{d}_t + \frac{\alpha Y}{C}\hat{k}_t - \frac{I}{C}\hat{i}_t - c^{dw}\frac{M}{C}\hat{m}_t + \frac{Y}{2C}\Big(1 - \frac{Y}{C} - \frac{1+\varphi}{1-\alpha}\Big)\hat{y}_t^2 - \frac{I}{2C}\Big(1+\frac{I}{C}\Big)\hat{i}_t^2 + \frac{IY}{C^2}\hat{y}_t\hat{i}_t \\
& - \frac{1+\varphi}{2(1-\alpha)}\frac{\alpha^2 Y}{C}\hat{k}_t^2 + \frac{\alpha(1+\varphi)Y}{(1-\alpha)C}\hat{y}_t\hat{k}_t - \frac{Y}{C}\left(1 - \frac{Y}{C} - \frac{1+\varphi}{1-\alpha}\right)\hat{y}\,\hat{y}_t^e - \frac{YI}{C^2}\hat{y}_t\hat{i}_t^e \\
& - \frac{\alpha(1+\varphi)Y}{(1-\alpha)C}\hat{y}_t\hat{k}_t^e + \frac{\alpha Y}{C}\left(1 - \frac{Y}{C} - \frac{1+\varphi}{1-\alpha}\right)\hat{k}_t\hat{y}_t^e + \frac{\alpha YI}{C^2}\hat{k}_t\hat{i}_t^e + - \frac{1+\varphi}{(1-\alpha)}\frac{\alpha^2 Y}{C}\hat{k}_t\hat{k}_t^e
\end{aligned}
\tag{B.48}
$$

B.7.3 The price dispersion term

The price dispersion term is linked with inflation. From the proposition 6.3 in Woodford (2003) and following a similar procedure with Thomas (2008):

$$
\begin{aligned}
D_t &= E_i\Big(\frac{P_t(i)}{P_t}\Big)^{-\gamma} \\
&= E_i\Big(e^{-\gamma \log(\frac{P_t(i)}{P_t})}\Big) \\
&= E_i\Big(e^{-\gamma \tilde{p}_t(j)}\Big)
\end{aligned}
\tag{B.49}
$$

where $\tilde{P}_t(i) \equiv \log(\frac{P_t(i)}{P_t})$ A second-order approximation of (B.49) around a zero steady-state yields:

$$\begin{aligned}\hat{d}_t &\approx E_i\left(e^{-\gamma\tilde{P}_t(i)} - \gamma e^{-\gamma\tilde{P}_t(i)}\tilde{P}_t(i) + \frac{1}{2}\gamma^2 e^{-\gamma\tilde{P}_t(i)}\right)\tilde{P}_t^2(i) + O^3 \\ &\approx d - \gamma d\tilde{P}_t(i) + \frac{\gamma^2}{2}D\tilde{P}_t^2(i) + O^3\end{aligned}$$

which implies:

$$\begin{aligned}\frac{\hat{d}_t}{d} - 1 &\approx -\gamma\Big(E_i\tilde{P}_t(i) - \frac{\gamma}{2}E_i\tilde{p}_t^2(j)\Big) + O^3 \\ \hat{d}_t + \frac{1}{2}\hat{d}_t^2 &\approx -\gamma\Big(E_i\tilde{P}_t(i) - \frac{\gamma}{2}E_i\tilde{P}_t^2(i)\Big) + O^3 \qquad \text{(B.50)}\end{aligned}$$

Now, I use the Dixit-Stiglitz domestic price index of the text, for any country j:

$$1 = E_i\Big(\frac{P_t(i)}{P_t}\Big)^{(1-\gamma)}$$

By taking a second-order approximation of the R.H.S, following a similar procedure as before, I obtain:

$$1 \approx 1 + (1-\gamma)E_i\tilde{P}_t(i) + \frac{(1-\gamma)^2}{2}E_i\tilde{P}_t^2(i) + O^3$$

where I have used that $D = 1$ which implies:

$$E_i\tilde{P}_t(i) = \frac{\gamma - 1}{2}E_i\tilde{P}_t^2(i) + O^3 \qquad \text{(B.51)}$$

By combining equations (B.50) and (B.51) and noticing that $\hat{d}_t^2$ is O^4 so it is eliminated, I obtain:

$$\hat{d}_t = \frac{\gamma}{2}E_j\tilde{P}_t^2(i) + O^3$$

and as $E_i\tilde{P}_t(i)$ is a second order term, the above implies:

$$\hat{d}_t = \frac{\gamma}{2}Var_i \log P_t(i) + O^3 \qquad \text{(B.52)}$$

The final part of the proof is based on the Proposition 6.3 on Woodford (2003). Equation (B.52) implies an evolution of $Var \log P_t(i)$ over time:

$$Var_i \log P_t(i) = \omega Var_i \log P_{t-1}(i) + \frac{\omega}{1-\omega}\pi_t + O^3 \tag{B.53}$$

where ω is the degree of nominal price rigidity in the text and π_t is the price inflation. Substituting for $\hat{d}_t$ from (B.52) to (B.53), implies:

$$\hat{d}_t = \omega \hat{d}_{t-1} + \frac{\gamma}{2}\frac{\omega}{1-\omega}\pi_t + O^3 \tag{B.54}$$

Integrating forward equation (B.54), taking then the discounted value of each term and noticing that the term $\hat{d}_{t-1}$ is t.i.p for a policy taken at time $t \geq 0$, I obtain the final result for the price dispersion term:

$$\sum_{i=0}^{\infty} \beta^i \hat{d}_{t+i} = \frac{\gamma}{2}\frac{1}{\delta_p}\sum_{i=0}^{\infty} \beta^i \pi_{t+i}^2 \tag{B.55}$$

where $\delta_p = \frac{(1-\omega\beta)(1-\omega)}{\omega}$ is the elasticity of domestic inflation with respect to the real marginal cost of intermediate good firms.

B.7.4 The welfare criterion

Assuming an efficient steady-state and using the law of motion of capital, the first-order terms are eliminated. Then, by taking the discounted sum of (B.48), using the result from equation (B.55) and using the notation for deviation from the flexible price equilibrium, i.e. $\tilde{x}_t = \hat{x}_t - \hat{x}_t^e$, after some rearrangements I get:

$$\sum_{t=0}^{\infty} \beta^t \left(\log C_t - \frac{N_t^{1+\varphi}}{1+\varphi} \right) \approx \sum_{t=0}^{\infty} \beta^t L_t + t.i.p. + O^3 \tag{B.56}$$

where L_t is the loss function given by

$$\begin{aligned} L_t =& -\frac{1}{2}\left(\frac{\gamma\delta_p}{1-\delta_p}\pi_t^2 + \frac{(\alpha+\varphi)Y}{(1-\alpha)C}\tilde{y}_t^2 + \frac{\alpha^2(1+\varphi)Y}{(1-\alpha)C}\tilde{k}_t^2 + \frac{IY}{C^2}\tilde{i}_t^2 \right) \\ &+ \frac{Y}{C}\left(\frac{I}{C}\tilde{y}_t \tilde{i}_t + \frac{\alpha(1+\varphi)}{(1-\alpha)}\tilde{y}_t \tilde{k}_t \right) + G_t^e \end{aligned} \tag{B.57}$$

where G_t^e is given by

$$G_t^e = \frac{Y}{C}\left(\frac{K}{C}\tilde{k}_{t+1}\left(\Delta_{yi,t}^e - \Delta_{yi,t+1}^e\right) + \alpha\tilde{k}_t\Delta_{yi,t}^n\right)$$

and $\Delta_{yi,t}^e \equiv \hat{y}_t^e - \hat{i}_t^e$.

B.8 The ZLB constraint in the L-Q framework

The following is based on Woodford (2003), chapter 6 and the proof of proposition 6.9, page 700.

The problem of the central bank is to minimise

$$\min E_0 \sum_{t=0}^{\infty} \beta^t \left\{ L_t \right\}$$

subject to the linear constraints given by the equilibrium conditions and resource constraints of the model. The ZLB effect is approximated by adding the non-linear constraints:

$$E_0 \sum_{t=0}^{\infty} \beta^t q_t \geq 0$$

$$E_0 \sum_{t=0}^{\infty} \beta^t q_t^2 \leq \frac{1}{1+\kappa_q^2} E_0 \left(\sum_{t=0}^{\infty} \beta^t q_t\right)^2$$

This can be written in an equivalent way:

$$E_0 \sum_{t=0}^{\infty} \beta^t q_t \geq m_1$$

$$E_0 \sum_{t=0}^{\infty} \beta^t q_t^2 \leq m_2$$

Now, notice that the first constraint, $E_0 \sum_{t=0}^{\infty} \beta^t q_t \geq 0$ never binds, as this violates the second one, for a non-zero policy. Therefore, $m_1 > 0$. Now, using the Kuhn-Tucker theorem, the policy, $E_0 \sum_{t=0}^{\infty} \beta^t q_t$ that minimises the loss function subject to the additional ZLB constraints, also minimises the following expression:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ L_t \right\} - \mu_1 E_0 \sum_{t=0}^{\infty} \beta^t q_t + \mu_2 E_0 \sum_{t=0}^{\infty} \beta^t q_t^2$$

or equivalently the policy m_1 minimises:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ L_t \right\} - \mu_1 m_1 + \mu_2 m_2$$

Notice that, when the second constraint

$$E_0 \sum_{t=0}^{\infty} \beta^t q_t^2 \leq \frac{1}{1+\kappa_q^2} E_0 \left(\sum_{t=0}^{\infty} \beta^t q_t \right)^2$$

binds, then it holds with equality. Equivalently, that is:

$$m_2 = \frac{1}{1+\kappa_q^2} {m_1}^2$$

Substitute for m_2 from the latter expression to the expression above, and then choose the policy m_1 to minimise the welfare losses (subject to the additional constraints), the first-order condition yields:

$$\mu_1 = 2 \frac{1}{1+\kappa_q^2} \mu_2 m_1$$

Substitute μ_1 from this expression to the expression in which we used the Kuhn-Tucker Theorem, and then re-arrange, yields (ignoring the discounted average terms):

$$L_t^{ZLB} = L_t + \mu_2 \left(q_t - \left(\frac{\mu_1}{2\mu_2} \right) \right)^2$$

Letting $\omega_q \equiv \mu_2$ and the new target of the nominal interest rate, associated with a non-zero inflation steady-state, to be $q^* \equiv \frac{\mu_1}{2\mu_2}$ yields the loss function with ZLB constraints of the text:

$$L_t^{ZLB} = L_t + \omega_q (q_t - q^*)^2$$

B.9 Benchmark Calibration

Description	Parameter	Value	Reference
Preferences			
discount factor	β	0.99	Quarterly time interval
labour supply elasticity	φ	2	Literature varies from $0-2$
Financial market - frictionless interbank			
fraction of divertable assets	λ	0.383	Gertler and Kiyotaki (2010)
survival rate of bankers	σ	0.972	"
new bankers entry	ξ	0.003	"
Financial market - frictional interbank			
	λ	0.129	"
	ξ	0.002	"
fraction of locations with new investment	τ^i	0.01 - 0.25	"
Goods market			
nominal price rigidity	ω	0.75	standard NK, quarterly data
steady-state marginal cost	ϕ	0.83	Inverse mark-up
depreciation of capital	δ	0.025	standard RBC
capital share	α	0.33	standard RBC
parameter on capital adjustment cost	χ	1	degree of convexity equal to 2
Primary instrument of monetary policy			
interest rate smoothing parameter	ρ_q	0.2 - 0.8	Gertler and Kiyotaki (2010)
benchmark inflation response	κ_π	1.5	Taylor (1993)
benchmark output response	$\kappa_{\tilde{y}}$	0.125	Schmitt-Grohe and Uribe (2007)
DW lending			
feedback parameter	κ_ϱ	$0-1000$	experiments
proxy for ability of CB to monitor	ω_m	0.01 - 1	experiments
efficiency cost per unit of lending	c^{dw}	0	Gertler and Karadi (2011)
Zero Lower Bound			
probability to reach ZLB	p_{ZLB}	0.025	Levine, McAdam, and Pearlman (2008)
Lagrangian multiplier on the ZLB constraint	ω_q	0-600	experiments
Capital quality shock			
std. deviation	σ_ψ	0.05	Gertler and Kiyotaki (2010)
quarterly autoregressive coefficient	ρ_ψ	0.75	
Productivity shock			
std. deviation	σ_a	-	
quarterly autoregressive coefficient	ρ_a	0.95	Smets and Wouters (2003)

Table B.1: Baseline Calibration

B.10 Further Impulse Responses and Graphs

B.10.1 The ZLB constraint in the frictional interbank market

Figure B.1 illustrates the effect of a lower bound constraint on the nominal interest rate in a scenario where the interbank market is frictional. The same logic as in figure 4.1 of the text applies. Comparing the cases of a frictionless and a frictional interbank market, the distortionary effect of the shock is more pronounced in the latter case.

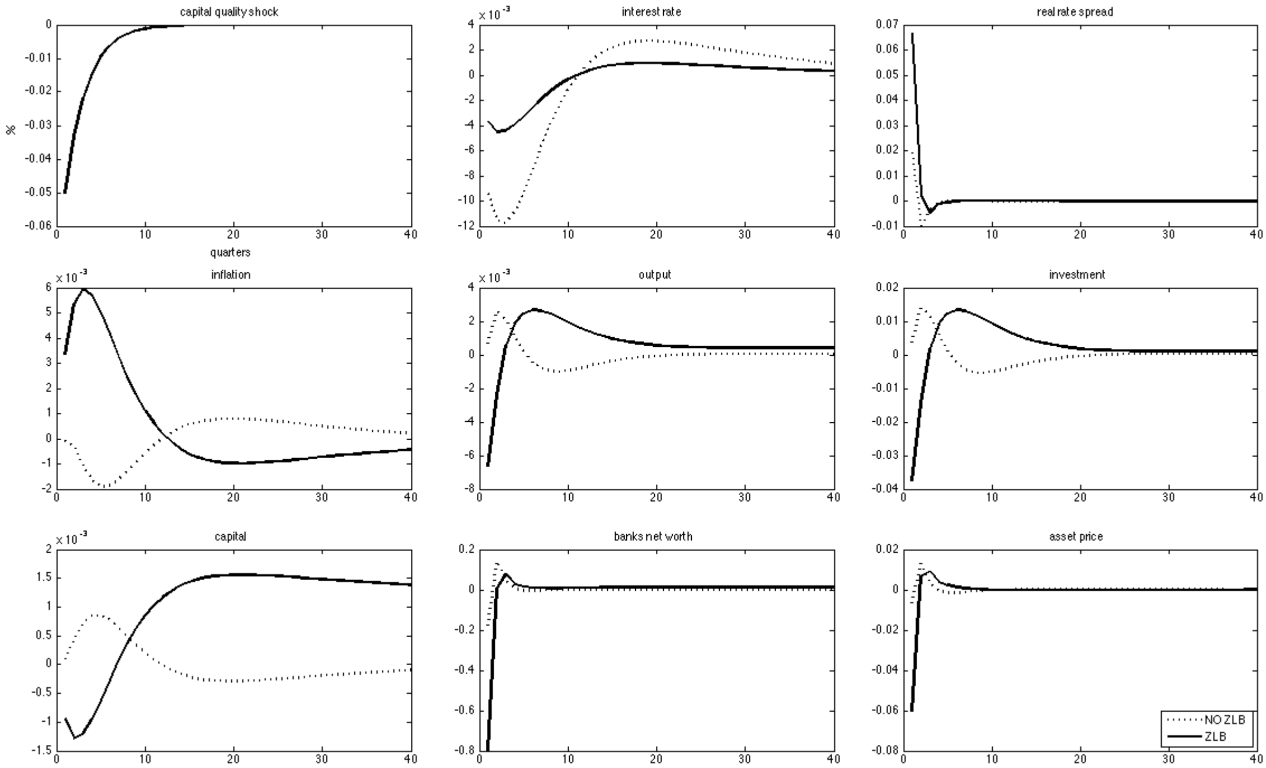

Figure B.1: *The effect of the ZLB constraint under optimal commitment. The case of a frictional interbank market,* $(\omega_q = 66)$.

B.10.2 The short-term interest rates

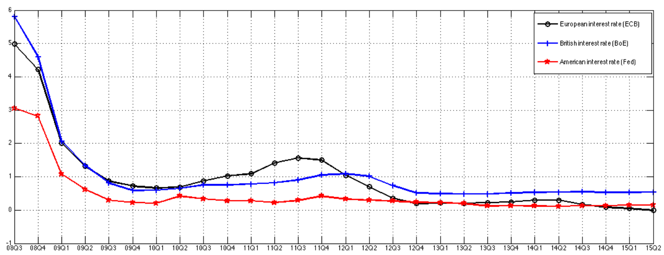

Figure B.2: *Short-term interest rates, % quarterly, Q3 2008 - Q2 2015, OECD Data.*

B.10.3 The discount window in the U.S.

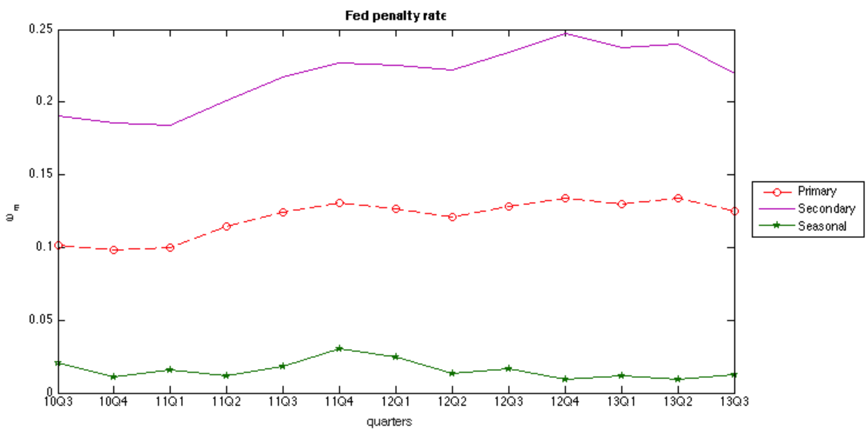

Figure B.3: *The discount window in the U.S.*

Appendix C

Dynare Codes

C.1 The code for the solution of the model in chapter 2.

```
var
u_a %unemployment in country A
u_b %unemployment in country B

pi_ad %domestic inflation country A
pi_bd %domestic inflation country B

s_ba %terms of trade

cpi_a %cpi inflation in country A
cpi_b %cpi inflation in country B

r_a %real interest rate in country A
r_b %real interest rate in country B

%q_ab %nominal interest rate

pi_wd %union wide inflation
u_w %union wide unemployment

z_a %productivity in country A
z_b; %productivity in country B

varexo
```

```
e_a %stochastic technology shock in country A
e_b; %stochastic technology shcok in country B

%parameters

parameters
%Preferences parameters
zeta %country size, openness
beta %discount factor
sigma %coefficient of relative risk aversion
gamma %elasticity of substitution among differentiated final goods
varphi %Frisch elasticity of labour
UCC %marginal utility of consumption times consumption.
UCN %marginal utility of consumption times labour.

%Final good firms parameters
omega_a %nominal price rigidity (probability at period t), country A
omega_b %nominal price rigidity (probability at period t), country B
delta_ap %elasticity of inflation with respect to the real marginal cost
delta_bp  % ''

%Labour market parameters (intermediate good firms)
q_theta %probability of firms to fill a vacancy
delta %exogenous separation rate
kappa %elasticity of matching function with respect to vacancies
xi_a %bargaining power of firms, country A
xi_b %bargaining power of firms, country B
mu_a %degree of real wage rigidity, country A
mu_b %degree of real wage rigidity, country B

%Labour market parameters (after linearization)
alpha_0 %parameter3 for optimal weight in unemployment
alpha_a1 alpha_b1
alpha_a2 alpha_b2
```

```
alpha_a3 alpha_b3
gamma_a0 gamma_b0
gamma_a1 gamma_b1
gamma_a2 gamma_b2
delta_a1 %efficient steady state parameter1
delta_2 %parameter1 for optimal weight in unemployment
delta_3 %parameter2 for optimal weight in unemployment
delta_u

%Steady-state levels. Because I linearise by hand
% I treat these as parameters

Z %steady state of productivity (level)
S %steady state of terms of trade (level)
phi %steady state of real marginal cost (level)
u %steady state of unemployment (level)
C_a %steady state value of consumption (level), country A
C_b %steady state value of consumption (level), country B
W %steady state of real wage
N %steady state value of employment (level)
Y %steady state value of output (level)
v %steady state value of vacancies (level)
theta %steady state of labour market tightness (level)
psi %cost of posting vacancies per vacancy
d_a %disutility of labour constant coefficient, country A
d_b %disutility of labour constant coefficient, country B

%Parameters of the constraints
%parameters in dynamic IS in country A
phi_a1 phi_a2 phi_a3 phi_a4

%parameters in dynamic IS in country B
phi_b1 phi_b2 phi_b3 phi_b4
```

```
%parameters in NKPC in country A
rho_a0 rho_a1 rho_a2 rho_a3 rho_a4

%parameters in NKPC in country B
rho_b0 rho_b1 rho_b2 rho_b3 rho_b4

%Autocorrelation parameters of AR(1) productivity
rho_a rho_b

%weights on the objective function
omega_pia omega_pib omega_ua omega_ub omega_sss;

% parameters values
beta=0.99;
sigma=1;
varphi=0;
kappa=0.5;
xi_a=0.5;
omega_a=0.75;
q_theta=0.97;
delta=0.1;
Z=1;
S=1;
phi=0.83;
u=0.1;


%%%%%%%%% Labour Market Heterogeneity Index %%%%%%%%%%
%%%%%%%%% Benchmark Scenario 0 %%%%%%%%%%%%%%%%%%%%%%%

mu_a=0.8;
mu_b = 1 - mu_a;
zeta=0.5;
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
/*
%%%%%%% Alternative Scenario 1 %%%%%%%%%%%%%%%%%%%%%%%
mu_a=0.7;
mu_b = 1.5 - 2*mu_a;
zeta=2/3;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
*/

/*
%%%%%%% Alternative Scenario 2 %%%%%%%%%%%%%%%%%%%%%%%
mu_a=0.9;
mu_b = (1.5 - mu_a)/2;
zeta=1/3;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
*/

rho_a=0.95;
rho_b=0.95;
gamma=6;
delta_p=0.0858;

%parameter values different for country B
xi_b=0.5;
omega_b=0.75;

% steady state values (levels)
N=(1-u)/(1-delta);
Y=Z*N;
v=(delta*N)/q_theta;
theta=v/u;
psi=(0.01*Y)/v;
C_a=Y*S^(zeta-1) - psi*v;
W=phi +(1-delta)*beta*(psi/q_theta) - psi/q_theta;
```

```
d_a=(W-(1-xi_a)*
*(phi + (1-delta)*beta*psi*theta))/( xi_a*(N^varphi)*C_a^(1/sigma));

% steady state values (levels) different for country B
C_b=Y*S^(zeta) - psi*v;
d_b=(W-(1-xi_b)*
*(phi + (1-delta)*beta*psi*theta))/( xi_b*(N^varphi)*C_b^(1/sigma));

%parameters after linearization, useful for the dynamic IS, country A
alpha_0=(1-delta)*(N/u);
alpha_a2=(psi*v)/(alpha_0*delta*kappa*C_a);
alpha_a1=Y/((S^(1-zeta))*C_a);
alpha_a3=alpha_a1/alpha_0;
delta_a1=1/(S^(1-zeta)) - (psi*v)/delta*kappa*N -
 d_a*(N^varphi)*(C_a^(1/sigma));
delta_2=(1-delta)*((psi*v)/kappa)*(1/(delta*N) - ((1-kappa)/u));
delta_3=(N/(sigma*C_a))*S^(2*(1-zeta));
delta_u=((1-delta)-alpha_0*delta);
gamma_a0=(1-mu_a)*(1-xi_a);
gamma_a1=(1-gamma_a0)*phi;
gamma_a2=(d_a)*(N^varphi)*C_a^(1/sigma);
delta_ap=((1-omega_a*beta)*(1-omega_a)/(omega_a));


%parameters after linearization, useful for the dynamic IS, country B
alpha_b1=Y/((S^(-zeta))*C_b);
alpha_b2=(psi*v)/(alpha_0*delta*kappa*C_b);
alpha_b3=alpha_b1/alpha_0;
delta_b1=1/(S^(-zeta)) -
(psi*v)/delta*kappa*N - d_b*(N^varphi)*(C_b^(1/sigma));
gamma_b0=(1-mu_b)*(1-xi_b);
gamma_b1=(1-gamma_b0)*phi;
gamma_b2=(d_b)*(N^varphi)*C_b^(1/sigma);
delta_bp=((1-omega_b*beta)*(1-omega_b)/(omega_b));
```

```
% rho's parameters for NKPC, country A
rho_a0=(delta_u/(alpha_0*delta*kappa*gamma_a1))*
*((1-kappa)*psi*theta^(1-kappa)-
(((1-mu_a)*xi_a*psi*v*gamma_a2)/(sigma*C_a)))
- ((1-mu_a)*xi_a*psi*v*gamma_a2)/(sigma*C_a);

rho_a1=(1/(alpha_0*delta*kappa*gamma_a1))*
*(delta_u*(1-delta)*beta*theta*psi*(gamma_a0-
(1-kappa))-(1-kappa)*psi*theta^(1-kappa)-
(1-mu_a)*xi_a*gamma_a2*((((alpha_a1)*
*delta*kappa)/sigma)+varphi*delta*kappa
- ((psi*v)/(sigma*C_a))));

rho_a2=(((1-delta)*beta*theta*psi)/(alpha_0*delta*kappa*gamma_a1))*
*(gamma_a0-(1-kappa)*theta^(-kappa));

rho_a3=1 - ((1-mu_a)*xi_a*
*((gamma_a2*alpha_a1)/(sigma*gamma_a1)));

rho_a4=((((1-delta)*beta*theta)*psi)/gamma_a1)*
*(theta^(-kappa)-gamma_a0);

% rho's parameters for NKPC country B
rho_b0=(delta_u/(alpha_0*delta*kappa*gamma_b1))*
*((1-kappa)*psi*theta^(1-kappa)-(((1-mu_b)
*xi_b*psi*v*gamma_b2)/(sigma*C_b)))
- ((1-mu_b)*xi_b*psi*v*gamma_b2)/(sigma*C_b);

rho_b1=(1/(alpha_0*delta*kappa*gamma_b1))*
(delta_u*(1-delta)*beta*theta*psi*(gamma_b0-(1-kappa))-
(1-kappa)*psi*theta^(1-kappa)-
(1-mu_b)*xi_b*gamma_b2*
```

```
*((((alpha_b1)*delta*kappa)/sigma)+
varphi*delta*kappa - ((psi*v)/(sigma*C_b))));

rho_b2=(((1-delta)*beta*theta*psi)/(alpha_0*
*delta*kappa*gamma_b1))*(gamma_b0-(1-kappa)*theta^(-kappa));
rho_b3=(((1-mu_b)*xi_b**
((gamma_b2*alpha_b1)/(sigma*gamma_b1)))
+ ((zeta-1)/zeta));
rho_b4=((((1-delta)*beta*theta)*psi)/gamma_b1)*(theta^(-kappa)-gamma_b0);

%eta parameters, useful for IS
eta_a1=((psi*v)/C_a)*
(((1-delta)/(alpha_0*delta*kappa))*(1-((delta*N)/u))+1);

eta_a2=(1/alpha_0*C_a)*
(psi*v/(delta*kappa) - Y/(S^(1-zeta)));

eta_b1=((psi*v)/C_b)*(((1-delta)/(alpha_0*delta*kappa))*
(1-((delta*N)/u))+1);

eta_b2=(1/alpha_0*C_b)*
(psi*v/(delta*kappa) - Y/(S^(-zeta)));


% phi's parameters, IS, country A
phi_a1=eta_a1/(eta_a1 + eta_a2);
phi_a2=eta_a2/(eta_a1 + eta_a2);
phi_a3=sigma/(eta_a1 + eta_a2);
phi_a4=alpha_a1/(eta_a1 + eta_a2);

% phi's parameters, IS, country B
phi_b1=eta_b1/(eta_b1 + eta_b2);
phi_b2=eta_b2/(eta_b1 + eta_b2);
phi_b3=sigma/(eta_b1 + eta_b2);
```

```
phi_b4=alpha_b1/(eta_b1 + eta_b2);

UCC = C_a*(C_a)^(-1/sigma);
UCN = N*(C_a)^(-1/sigma);

% weights on the policy objectives
omega_pia=UCC*(zeta*gamma/(2*delta_ap));
omega_piab=UCC*((1-zeta)*gamma/(2*delta_bp));
omega_ua=UCN*((zeta*delta_3)/(2*(alpha_0)^2));
omega_ub=UCN*(((1-zeta)*delta_3)/(2*(alpha_0)^2));
omega_sss=UCC*zeta*(1-zeta)*((1+sigma)/(2*sigma));




model (linear);
%dynamic IS expressed in terms of unemployment
u_a = phi_a1*u_a(-1) + phi_a2*u_a(+1) - phi_a3*r_a -
phi_a4*(1-zeta)*s_ba(+1) + phi_a4*(1-zeta)*s_ba  - phi_a4*(1-rho_a)*z_a;

u_b = phi_b1*u_b(-1) + phi_b2*u_b(+1) - phi_b3*r_b +
phi_b4*zeta*s_ba(+1) - phi_b4*zeta*s_ba  - phi_b4*(1-rho_b)*z_b;

%NKPC expressed in terms of unemployment
pi_ad = beta*pi_ad(+1) + delta_ap*rho_a0*u_a(-1) + delta_ap*rho_a1*u_a
- delta_ap*rho_a2*u_a(+1) + delta_ap*rho_a3*(1-zeta)*s_ba
+ delta_ap*rho_a4*r_a - delta_ap*rho_a3*z_a;

pi_bd = beta*pi_bd(+1) + delta_bp*rho_b0*u_b(-1) + delta_bp*rho_b1*u_b
- delta_bp*rho_b2*u_b(+1) + delta_bp*zeta*rho_b3*s_ba
+ delta_bp*rho_b4*r_b + delta_bp*rho_b3*z_b;

%Link terms of trade with domestic inflation
s_ba = s_ba(-1) + pi_bd - pi_ad;
```

```
%union wide inflation
pi_wd = zeta*pi_ad + (1-zeta)*pi_bd;

%union wide unemployment
u_w = zeta*u_a + (1-zeta)*u_b;

/* %For Optimal Policy these are commented out.
%Fisher equations for country A and B
r_a = q_ab - cpi_a(+1);
r_b = q_ab - cpi_b(+1);
*/


%link of cpi with domestic inflation and terms of trade
cpi_a = pi_ad + (1-zeta)*(s_ba - s_ba(-1));
cpi_b = pi_bd + (zeta-1)*(s_ba - s_ba(-1));

%AR(1) technology in countries A and B
z_a = rho_a*z_a(-1) + e_a;
z_b = rho_b*z_b(-1) + e_b;
end;


shocks;
var e_a;
stderr 0.00624;

var e_b;
stderr 0;

var e_a, e_b= 0*0.00624*0.00624;
```

```
end;

planner_objective(UCC*((zeta*gamma/(2*delta_ap))*(pi_ad)^2 +
 ((1-zeta)*gamma/(2*delta_bp))*(pi_bd)^2 +
 UCN*(((zeta*delta_3)/(2*(alpha_0)^2))*(u_a)^2 +
 (((1-zeta)*delta_3)/(2*(alpha_0)^2))*(u_b)^2)) +
 zeta*(1-zeta)*((1+sigma)/(2*sigma))*(s_ba)^2);




%%%%%%%%% Optimal Commitment %%%%%%%%%
ramsey_policy(planner_discount=0.99)pi_ad pi_bd u_a u_b s_ba r_a r_b z_a;



/*
%%%%%%%%% Optimal Discretion %%%%%%%%%
discretionary_policy(planner_discount=0.99, instruments=(r_a r_b))
pi_ad pi_bd u_a u_b s_ba r_a r_b z_a;
*/



stoch_simul (irf=0);

%%%% Calculate the welfare loss from the planner_objective%%%%%%%%%
oo_.planner_objective_value

%%%%%Calculate standard deviations%%%%%%%%%

stdr_ua =((oo_.var(1,1))^(1/2))

stdr_ub =((oo_.var(2,2))^(1/2))
```

```
stdr_piad =((oo_.var(3,3))^(1/2))

stdr_pibd =((oo_.var(4,4))^(1/2))

stdr_sba =((oo_.var(5,5))^(1/2))


write_latex_dynamic_model;
```

C.2 The code for the solution of the model in chapter 3.

```
%  The current .mod file solves the Optimal Monetary Policy problem
%of the paper: '' Welfare Gains from Discount Window Lending''
%for the case of a frictionless interbank market
%with no efficiency costs.
%Due to lack of space the code for the case of a
%frictional interbank market as well as the
%case of DW lending with efficiency costs
%are available upon request

%  (c) Nikolas Kontogiannis, Department of Economics,
%University of Leicester

%%% Some Notation:

% For any generic variable X_t:
% X_t denotes the level of variable X at time t.
% X_flex denotes the level of variable X at time t,
% when prices are fully flexible
```

```
% X_ss denotes the steady state.
% X_hat = logX_t - logX_ss,
% X_flex = logX_flex - logX_ss,
% X_tilde = X_hat - X_flex, is the log-deviation from the flex price
% equilibrium
%%%%%%%%%

% Declaration of Variables %

var

R_hat % real interest rate
QNOM_hat % nominal interest rate
Pi % inflation, given in absolute deviaton from ss
Phi_hat % real marginal cost
A_hat % productivity
Psi_hat % quality of capital
Spread_hat % the spread or
Y_hat
RK_hat

Y_tilde % output
I_tilde % investment (new capital)
K_tilde % capital stock
R_tilde % real interest deviation from the Wicksellian
Z_tilde % gross profit per unit of capital use
Q_tilde % capital (asset) price
D_tilde % households' deposits
S_tilde % total securities issued
NW_tilde % banks' net worth (capital)
RK_tilde % real interest of banks' lending
Theta_tilde % leverage ratio, private intermediation
VD_tilde % banks' marginal value from lending
Mu_tilde % banks' excess value of lending
```

```
Omega_tilde % stochastic marginal value of net worth
MuM_tilde
Spread_tilde % the spread o
KAUXLAG_tilde % auxiliary v

chiM_tilde % unconventional policy (discount window instrument)
M_tilde % discount window (aggregate)
Rm_tilde % real interest- discount window

Y_flex % flexible price equilibrium level of output.
I_flex % natural level of investment
K_flex % natural level of capital
R_flex % Wicksellian real interest rate
Z_flex % gross profits per unit of capital use
Q_flex % capital (asset) price
RK_flex % real interest of banks' lending

%QNOM_hatZLB

YAUXLEAD_flex
IAUXLEAD_flex;


% Declaration of the Shocks
varexo

E_A % productivity shock
E_Psi; % capital quality shock


% Declaration of the Parameters

parameters
```

```
% Steady-State of levels, expressed as parameters %

C_ss % consumption
R_ss % real interest rate
N_ss % labour or measure of participation to work
QNOM_ss % nominal interest rate
Phi_ss % real marginal cost
Y_ss % output of final goods
A_ss % productivity
K_ss % capital stock
I_ss % investment (new capital)
Psi_ss % quality of capital
Z_ss % gross profits per unit of capital use
Q_ss % capital (asset) price
D_ss % households' deposits
S_ss % total securities issued
NW_ss % banks' net worth (capital)
RK_ss % real interest of banks' lending
Theta_ss % leverage ratio, private intermediation
VD_ss % banks' marginal value from lending
Mu_ss % banks' excess value of lending
Omega_ss % stochastic marginal value of net worth
Spread_ss % the spread
NWK_ss % the steady state of the ratio NW/K
DK_ss % the steady state of the ratio D/K
YK_ss % the steady state of the ratio Y/K
CK_ss % the steady state of the ratio C/K
%Pi_star %inflation steady state when the ZLB binds
chiM_ss
MK_ss
Rm_ss
MuM_ss
M_ss
Z_null
```

```
% Structural Parameters %%

beta % static discount factor
alpha % capital share
varphi % Frisch inverse elasticity of labour
delta % depreciation of capital
gamma % elasticity of substitution among the differ
delta_p % elasticity of inflation wrt the real marginal cost
M % markup
chi % parameter in convex adjustment cost function
omega % degree of nominal price rigidity,
rho_a % degree of autocorrelation, productivity
rho_psi %d egree of autocorrelation, capital quality
sigma %survival rate of bankers
xi % transfer to entering bankers, perfect interbank
lambda % fraction of divertable assets
omegaG % advantage of CB on monitoring the banks' asset
delta_q % weight to interest rate in the loss function

kappa_y % elasticity of nominal interest rate w.r.t. output
kappa_pi % elasticity of nominal interest rate w.r.t. inflation
rho_q

rho_chiM % outside equity smoothing
kappa_varrhochiM %unconv policy parameter

delta_1
delta_2
delta_3
delta_4
delta_5
delta_6
delta_7
delta_8;
```

```
%Calibration

beta=0.99; % static discount factor
alpha=0.33; %capital share
varphi=2; %Frisch inverse elasticity of labour
delta=0.025; %depreciation of capital
gamma=6; %elasticity of substitution (to match the markup)
M=gamma/(gamma-1); %markup
omega=0.75; %degree of nominal price rigidity, to match quarterly data
delta_p=((1-omega*beta)*(1-omega)/(omega)); %elasticity of inflation wrt
%the real marginal cost

chi=1; % parameter in convex adj

R_ss=1/beta; % the SS from euler equation. Useful to find the R_ss

sigma=0.97; %survival rate of bankers
xi=0.003; %transfer to entering bankers, perfect interbank l
lambda=0.383; %fraction of diver
%kappa_zlb = 0; %from Woodford ch.6
chiM_ss = 0.000000001;

%chiM_ss = 0.15;
omegaG = 0.0000000001; % small advantage of CB on
%omegaG = 1;
kappa_varrhochiM =0;
delta_q = 47; %
Z_null = 1.96; % p for ZLB is 2.5%
%Z_null = 1.04; % p for ZLB is 15%
%Z_null = 0.09; % p(ZLB) = 0.4641%

QNOM_ss=R_ss-1; % Fisher equation
```

```
Psi_ss=1; % steady-state of quality of capital shock
A_ss=1; % steady-state of productivity shock
Q_ss=1; % price of new-capital good (tobin's q) steady-state,
Phi_ss=1/M; % price flexibility value of real marginal

Spread_ss = 0.0025;
RK_ss = Spread_ss + R_ss;
Z_ss = RK_ss - (1-delta);
Rm_ss = omegaG*RK_ss + (1-omegaG)*R_ss;
MK_ss = chiM_ss*Q_ss;
DK_ss = (1- (sigma+xi)*RK_ss)*(Q_ss/(1 - (sigma*R_ss))) -
(((1- sigma*Rm_ss))/(1 - (sigma*R_ss)))*MK_ss;

NWK_ss = (sigma+xi)*RK_ss*Q_ss - sigma*R_ss*DK_ss -
sigma*Rm_ss*MK_ss;

Theta_ss = (NWK_ss)^(-1);

Omega_ss = (1 - sigma)/(1 - sigma -
(sigma*Theta_ss*beta*(1-omegaG*chiM_ss)*Spread_ss));

Mu_ss=beta*Omega_ss*Spread_ss;
MuM_ss=omegaG*Mu_ss;
VD_ss=Omega_ss;

YK_ss=(1/(alpha*Phi_ss))*Z_ss;
CK_ss=YK_ss - delta;
N_ss=(Phi_ss*(1-alpha)*YK_ss*(CK_ss)^(-1))^(1/(1+varphi));
K_ss=(N_ss)*(YK_ss)^((-1/(1-alpha)));
I_ss=delta*K_ss;
C_ss=CK_ss*K_ss;
Y_ss=YK_ss*K_ss;
S_ss=K_ss;
NW_ss=NWK_ss*K_ss;
```

```
D_ss=DK_ss*K_ss;
M_ss=MK_ss*K_ss;

kappa_pi=1.5;
kappa_y=0.5/4;
rho_q=0.2; %low interest smoothing

rho_chiM = 0; % discount window feedback parameter smoothing

% autocorrelation of the shocks
rho_a=0.95;
rho_psi=0.75;

delta_1 = (gamma/2)/delta_p;
delta_2 = (1/2)*(Y_ss/C_ss)*(((alpha+varphi)/(1-alpha)));
delta_3 = (1/2)*(I_ss*Y_ss)/(C_ss^2);
delta_4 = ((alpha^2)*(1+varphi)*Y_ss)/(2*(1-alpha)*C_ss);
delta_5 = (alpha*Y_ss/C_ss)*((1+varphi)/(1-alpha));
delta_6 = 2*delta_3;
delta_7 = (K_ss*Y_ss)/(C_ss^2);
delta_8 = (alpha*Y_ss)/C_ss;


%STEADY STATE


initval;
% choose all variables as zeros
end;

model (linear);

%% New Keynesian model with capital
```

```
Y_tilde = Y_tilde(+1) - (I_ss/Y_ss)*(I_tilde(+1) - I_tilde) - R_tilde;
% Dynamic IS

R_tilde = R_hat - R_flex;
% Definition of the R_tilde

R_hat = QNOM_hat - Pi(+1);
% Fisher equation.

R_flex = Y_flex(+1) - Y_flex - (I_ss/Y_ss)*(I_flex(+1) - I_flex);
% Definition of the Wicksellian (natural) interest rate

(((alpha+varphi)/(1-alpha)) + (Y_ss/C_ss))*Y_flex =
(I_ss/C_ss)*I_flex + ((alpha*(1+varphi))/(1-alpha))*K_flex(-1)
+ ((1+varphi)/(1-alpha))*A_hat;
% Flexible price equilibrium

Pi = beta*Pi(+1) + delta_p*Phi_hat;
% New Keynesian Phillips Curve

Phi_hat = (((alpha+varphi)/(1-alpha)) + (Y_ss/C_ss))*Y_tilde -
(I_ss/C_ss)*I_tilde -
((alpha*(1+varphi))/(1-alpha))*K_tilde(-1);
% Marginal cost (linked with variables expressed in tildes).

K_tilde = delta*I_tilde + (1 - delta)*K_tilde(-1);
% Law of motion of capital.
%Notice that the capital quality shock disappears
%when we express the variables in tildes.

Z_tilde = Phi_hat + Y_tilde - K_tilde(-1);
% Gross profits per capital use

Q_tilde = chi*(I_tilde - I_tilde(-1))
```

```
- chi*beta*(I_tilde(+1) - I_tilde);
% Asset price.

Y_hat = Y_tilde + Y_flex;

%Taylor rule should be removed when
%calculate ramsey/discretion policy%%

%QNOM_hat = rho_q*QNOM_hat(-1) +
%(1-rho_q)*(kappa_pi*Pi + kappa_y*Y_tilde);

A_hat = rho_a*A_hat(-1) - E_A; % AR(1) productivity.

Psi_hat = rho_psi*Psi_hat(-1) - E_Psi; % AR(1) capital quality.

KAUXLAG_tilde = K_tilde(-1);
YAUXLEAD_flex = Y_flex(+1);
IAUXLEAD_flex = I_flex(+1);                 "                              "


% Financial Intermediation %
%%% The capital quality shock disappears
%from the RK when we use tildes %%%

S_tilde = K_tilde;

chiM_tilde = M_tilde - Q_tilde - S_tilde;

S_tilde = Theta_tilde + NW_tilde - Q_tilde;

Theta_tilde = VD_tilde + (Mu_ss/(lambda - Mu_ss))*Mu_tilde +
((omegaG*chiM_ss)/(1-omegaG*chiM_ss))*chiM_tilde;

D_tilde = (Q_ss*S_ss/D_ss)*Q_tilde +
```

```
(Q_ss*S_ss/D_ss)*S_tilde - (NW_ss/D_ss)*NW_tilde - (M_ss/D_ss)*M_tilde;

NW_tilde = ((sigma+xi)*(RK_ss*Q_ss*S_ss)/NW_ss)*(RK_tilde + Q_tilde(-1) +
S_tilde(-1)) - sigma*(R_ss*D_ss/NW_ss)*(R_tilde +
D_tilde(-1)) - sigma*(Rm_ss*M_ss/NW_ss)*(Rm_tilde + M_tilde(-1));

RK_tilde = - Q_tilde(-1) + (Psi_ss*Z_ss/RK_ss*Q_ss)*Z_tilde +
((1-delta)*Psi_ss/RK_ss)*Q_tilde; % The shock disappears when we use tildes

Rm_tilde = (omegaG*RK_ss/Rm_ss)*RK_tilde +
(((1-omegaG)*R_ss)/Rm_ss)*R_tilde;

Mu_tilde = ((beta*Omega_ss*RK_ss)/Mu_ss)*(RK_tilde(+1) -
R_tilde(+1)) + Omega_tilde(+1);

MuM_tilde = ((beta*Rm_ss)/((beta*Rm_ss) - 1))*(Rm_tilde(+1) -
R_tilde(+1)) + Omega_tilde(+1);

Omega_tilde = ((sigma*VD_ss)/Omega_ss)*VD_tilde +
((sigma*Theta_ss*Mu_ss)/Omega_ss)*Mu_tilde -
(sigma*MuM_ss*chiM_ss*Theta_ss/Omega_ss)*(MuM_tilde + chiM_tilde) +
 (sigma*Theta_ss/Omega_ss)*(Mu_ss - MuM_ss*chiM_ss)*Theta_tilde;

VD_tilde =  Omega_tilde (+1);

Spread_tilde = RK_tilde(+1) - R_tilde(+1);

Spread_hat = RK_hat(+1) - R_hat(+1);

RK_hat = RK_tilde + RK_flex;


%% Discount Window %
```

```
chiM_tilde = rho_chiM*chiM_tilde(-1) +
(1-rho_chiM)*kappa_varrhochiM*Spread_tilde;

 %% FLEXI PART (Notice that R_flex is given above)

Q_flex = chi*(I_flex - I_flex(-1)) - chi*beta*(I_flex(+1) - I_flex);
% Asset price (flexi).

Z_flex = Y_flex - K_flex(-1);
% Gross profits per capital use (flexi)

K_flex = Psi_hat(+1) + delta*I_flex + (1 - delta)*K_flex(-1);
% Law of motion of capital ( flexi )

RK_flex = Psi_hat - Q_flex(-1) + (Psi_ss*Z_ss/RK_ss*Q_ss)*Z_flex +
((1-delta)*Psi_ss/RK_ss)*Q_flex;

RK_flex(+1) = R_flex(+1);



end;

% SHOCKS %

shocks;

var E_Psi;
stderr 0.05;


/*
var E_A;
stderr 0.00624;
```

```
var E_Psi, E_A = 0.3*0.05*0.00624;

*/

end;


%% SOLUTION %%

% Optimal Commitment

planner_objective (delta_q*(QNOM_hat)^2 +
 delta_1*(Pi^2) + delta_2*(Y_tilde^2) + delta_3*(I_tilde^2) +
 delta_4*(KAUXLAG_tilde^2) - delta_5*Y_tilde*KAUXLAG_tilde -
 delta_6*Y_tilde*I_tilde - delta_7*K_tilde*Y_flex +
 delta_7*K_tilde*I_flex +
 delta_7*K_tilde*YAUXLEAD_flex - delta_7*K_tilde*IAUXLEAD_flex -
 delta_8*KAUXLAG_tilde*Y_flex + delta_8*KAUXLAG_tilde*I_flex);

ramsey_policy(planner_discount=0.99)

oo_.planner_objective_value



/*

% Optimal simple rule %

check;
optim_weights;
QNOM_hat delta_q;
Pi delta_1;
Y_tilde delta_2;
```

```
I_tilde delta_3;
KAUXLAG_tilde delta_4;
Y_tilde, KAUXLAG_tilde -delta_5;
Y_tilde, I_tilde -delta_6;
K_tilde, Y_flex -delta_7;
K_tilde ,I_flex delta_7;
K_tilde, YAUXLEAD_flex delta_7;
K_tilde, IAUXLEAD_flex -delta_7;
KAUXLAG_tilde, Y_flex -delta_8;
KAUXLAG_tilde, I_flex delta_8;

end;

%oo_.osr.objective_function
%oo_.var


*/

Pi_star= max(Z_null*((oo_.var(10,10))^(1/2)) - ((1/beta) - 1),0)

asymptotic_loss= delta_1*oo_.var(1,1) +
delta_2*oo_.var(2,2) + delta_3*oo_.var(3,3) +
delta_4*oo_.var(4,4) - delta_5*oo_.var(2,4) -
delta_6*oo_.var(2,3) - delta_7*oo_.var(9,5) +
delta_7*oo_.var(9,6) + delta_7*oo_.var(9,7) -
delta_7*oo_.var(9,8) - delta_8*oo_.var(4,5) +
delta_8*oo_.var(4,6)

asymptotic_lossTotal = asymptotic_loss + delta_1*Pi_star^2


stdr_interest =((oo_.var(10,10))^(1/2))
```

```
stdr_spread =((oo_.var(11,11))^(1/2))

stdr_inflation =((oo_.var(1,1))^(1/2))

stdr_output =((oo_.var(2,2))^(1/2))

stdr_invest =((oo_.var(3,3))^(1/2))

stdr_capital =((oo_.var(4,4))^(1/2))

covar_yk = (oo_.var(2,4))^1

covar_yinv = (oo_.var(2,3))^1

covar_kleadyf = (oo_.var(9,5))^1

covar_kleadinvf = (oo_.var(9,6))^1

covar_kleadyflead = (oo_.var(9,7))^1

covar_kleadinvflead = (oo_.var(9,8))^1

covar_kyf = (oo_.var(4,5))^1

covar_kinvf = (oo_.var(4,6))^1

stdr_assets =((oo_.var(12,12))^(1/2))

stdr_networth =((oo_.var(13,13))^(1/2))

stdr_leverage =((oo_.var(14,14))^(1/2))

stdr_costDW =((oo_.var(15,15))^(1/2))
```

```
stdr_margcostnetw =((oo_.var(16,16))^(1/2))
```

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