Essays on Dependence Modelling with Vine Copulas and its Applications

Thesis submitted for the degree of

Doctor of Philosophy

at the University of Leicester

by

Xinghua Xia

School of Business

University of Leicester

March 2018

Abstract

This thesis contains three essays on dependence modelling with high dimension vine copulas and its applications in credit portfolio risk, asset allocation and international financial contagion.

In the first essay, we demonstrate the superiority of vine copulas over multivariate Gaussian copula when modelling the dependence structure of a credit portfolio risk factors. We introduce the vine copulas to modelling the dependence structure of multi risk factors log returns in the combined framework of both threshold model and mixture model credit risk modelling.

The second essay studies asset allocation decisions in the presence of regime switching on asset allocation with alternative investments. We find evidence that two regimes, characterized as bear and bull states, are required to capture the joint distribution of stock, bond and alternative investments returns. Optimal asset allocation varies considerably across these states and changes over time. Therefore, in order to capture observed asymmetric dependence and tail dependence in financial asset returns, we introduce high dimensional vine copula and construct a multivariate vine copula regime-switching model, which account for asymmetric dependence and tail dependence in high dimensional data.

The third essay explores the cross-market dependence between six popular equity indices (S&P 500, NASDAQ 100, FTSE 100, DAX 30, Euro Stoxx 50 and Nikkei 225), and their corresponding volatility indices (VIX, VXN, VFTSE, VDAX, VSTOXX and VXJ). In particular, we propose a novel dynamic method that combine the Generalised Autoregressive Score (GAS) Method with high dimension R-vine copula approach which is able to capture the time-varying tail dependence coefficient (TDC) of index returns.

Acknowledgement

First and foremost, I would like to express my deep gratitude to my supervisors Dr Alexandra Dias and Professor Emmanuel Haven for their continued support throughout my studies, providing me with the patient guidance and helpful suggestions. I am most grateful for their advice, constructive comments and willingness to share their views and knowledge. I have gained invaluable experience. I am greatly indebted to my beloved parents. Without their continuous support and encouragement, I would not complete this thesis. Finally, I would like to thank my friends. Their warm friendship support me a lot.

Contents

1 Introduction **11**

1.1 The Challenges of Gaussian Copula Modelling 11

1.2 The Superiority of Vine Copula Modelling 12

1.3 Motivation and Objectives 13

2 Credit Portfolio Risk Modelling with Vine Copulas **15**

2.1 Introduction 17

2.2 Multi-factor Credit Portfolio Model Setup 21

2.2.1 Threshold Model Setup 21

2.2.2 Threshold Model represented as Mixture Model 25

2.3 Review of Vine Copula 29

2.3.1 Construction of Vine Copula 29

2.3.2 Inference of Vine Copula 30

2.4 Modelling Marginal Model 32

2.5 Simulation Study 33

2.6 Data 39

2.7 Application of Vine Copula to Credit Portfolio 55

2.7.1 Selection and Estimation of Vine Copula Models 55

2.7.2 Homogeneous Credit Portfolio 61

2.7.3 Heterogeneous Credit Portfolio 76

2.8 Systematic Risk Factor Contributions 81

2.8.1 Risk Contribution Method 82

2.8.2 Homogeneous Credit Portfolio Risk Contribution 84

2.8.3 Heterogeneous Credit Portfolio Risk Contribution 87

2.9 Loss Function-based Backtesting 98

2.9.1 The Model Confidence Set Procedure 101
2.9.2 Loss function-based backtesting results 106
2.10 Conclusion 107

3 Asset Allocation Benefits of Alternative Investments: Markov Regime Switching Regular Vine Copula Method 125
3.1 Introduction 127
3.2 The Markov Regime Switching Regular Vine Copula Model 131
3.2.1 Regular Vine Distribution 132
3.2.2 Markov Regime Switching Vine Copula Model 133
3.3 Marginal Model 135
3.4 Parameter Estimate of Markov Regime Switching Vine Copula Model . . 136
3.4.1 EM Algorithm for Markov Regime Switching Copula Model . . . 137
3.5 Optimization of the Investor's Utility Function 139
3.6 Data 142
3.7 Empirical Results 144
3.7.1 Marginal Model Estimate 149
3.7.2 Efficient Frontiers with and without Alternative Investments . . . 149
3.7.3 Vine Copula based Regime Switching Model Estimate 151
3.7.4 Out-of-sample Risk-adjusted Portfolio Performance of Different Asset Allocation Strategies 156
3.7.5 Economic Performance 181
3.8 Value-at-Risk (VaR) Backtesting 189
3.9 Conclusion 192

4 Modelling International Financial Contagion: Generalised Autoregressive Score Regular Vine Copula Method 197
4.1 Introduction 199
4.2 Literature Review 201
4.3 Review of Vine Copula 205
4.3.1 Construction of Vine Copula 206
4.3.2 Inference of Vine Copula 207

4.4 Generalized Autoregressive Score Regular Vine Dynamic Copula Model Setting 209
4.5 Modelling Marginal Model 212
4.6 Estimation of GAS Vine Copula Parameters 214
4.7 Data 215
4.7.1 Descriptive statistics 219
4.8 Selection of Vine Copula Structure 219
4.9 GAS Regular Vine Copula Parameters Estimate of Stock Indices and Volatility Indices 225
4.10 Assessing Contagion 231
4.11 Backtesting 239
4.12 Conclusion 243

5 Conclusion **248**

List of Tables

2.1 **Given Vine Copula** 35

2.2 **Estimated Vine Copula** 35

2.3 **Impact of copula dimension and number of observation on estimation performance** 38

2.4 **Estimation Results for Marginal Model** 42

2.5 **Estimation Results for Marginal Model** 43

2.6 **Estimation Results for Marginal Model** 44

2.7 **Estimation Results for Marginal Model** 45

2.8 **Estimation Results for Marginal Model** 46

2.9 **Estimation Results for Marginal Model** 47

2.10 **Estimation Results for Marginal Model** 48

2.11 **Estimation Results for Marginal Model** 49

2.12 **Estimation Results for Marginal Model** 50

2.13 **Estimation Results for Marginal Models** 51

2.14 **Estimation Results for Marginal Model** 52

2.15 **Estimation Results for Marginal Model** 53

2.16 **Estimation Results for Marginal Model** 54

2.17 **Vuong test results** 57

2.18 **Clarke test results** 57

2.19 **Tree 1 Parameters Estimation of R-vine mixed Copula Model** 62

2.20 **Tree 2 Parameters Estimation of R-vine mixed Copula Model** 64

2.21 **Tree 3 Parameters Estimation of R-vine mixed Copula Model** 66

2.22 **Tree 1 Parameters Estimation of C-vine mixed Copula Model** 68

2.23 **Tree 2 Parameters Estimation of C-vine mixed Copula Model** 70

2.24 **Tree 3 Parameters Estimation of C-vine mixed Copula Model** 72

2.25 **Homogeneous Small Portfolio Comparison of VaR and CVaR for different copula settings** 77
2.26 **Homogeneous Large Portfolio Comparison of VaR and CVaR for different copula settings** 78
2.27 **Heterogeneous Small Portfolio Comparison of VaR and CVaR for different copula settings** 80
2.28 **Heterogeneous Large Portfolio Comparison of VaR and CVaR for different copula settings** 81
2.29 **Small Homogeneous Portfolio VaR and CVaR Risk Contribution Ranking for C-vine Mixed Copula setting** $r = 0.2$ 88
2.30 **Small Homogeneous Portfolio VaR and CVaR Risk Contribution Ranking for C-vine Mixed Copula setting** $r = 0.3$ 89
2.31 **Small Homogeneous Portfolio VaR and CVaR Risk Contribution Ranking for C-vine Mixed Copula setting** $r = 0.4$ 90
2.32 **Small Homogeneous Portfolio VaR and CVaR Risk Contribution Ranking for C-vine Mixed Copula setting** $r = 0.5$ 91
2.33 **Small Homogeneous Portfolio VaR and CVaR Risk Contribution Ranking for C-vine Mixed Copula setting** $r = 0.6$ 92
2.34 **Small Homogeneous Portfolio VaR and CVaR Risk Contribution Ranking for R-vine Mixed Copula setting** $r = 0.2$ 93
2.35 **Small Homogeneous Portfolio VaR and CVaR Risk Contribution Ranking for R-vine Mixed Copula setting** $r = 0.3$ 94
2.36 **Small Homogeneous Portfolio VaR and CVaR Risk Contribution Ranking for R-vine Mixed Copula settings** $r = 0.4$ 95
2.37 **Small Homogeneous Portfolio VaR and CVaR Risk Contribution Ranking for R-vine Mixed Copula setting** $r = 0.5$ 96
2.38 **Small Homogeneous Portfolio VaR and CVaR Risk Contribution Ranking for R-vine Mixed Copula setting** $r = 0.6$ 97
2.39 **Small Heterogeneous Portfolio VaR and CVaR Risk Contribution Ranking for C-vine Mixed Copula setting** 99
2.40 **Small Heterogeneous Portfolio VaR and CVaR Risk Contribution Ranking for R-vine Mixed Copula setting** 100

2.41 **Value at Risk Backtesting under R-vine Gaussian Copula Benchmark Assumption** . . . 108
2.42 **Value at Risk Backtesting under R-vine t Copula Benchmark Assumption** . . . 109
2.43 **Value at Risk Backtesting under R-vine mixed Copula Benchmark Assumption** . . . 110
2.44 **Value at Risk Backtesting under C-vine t Copula Benchmark Assumption** . . . 111
2.45 **Value at Risk Backtesting under C-vine mixed Copula Benchmark Assumption** . . . 112
46 Descriptive Statistics of Risk Factors . . . 114
47 Descriptive Statistics of Risk Factors . . . 115
48 Descriptive Statistics of Risk Factors . . . 116
49 Descriptive Statistics of Risk Factors . . . 117
50 Descriptive Statistics of Risk Factors . . . 118
51 Descriptive Statistics of Risk Factors . . . 119
52 Descriptive Statistics of Risk Factors . . . 120
53 Descriptive Statistics of Risk Factors . . . 121
54 Descriptive Statistics of Risk Factors . . . 122
55 **Risk Factors Equity Index** . . . 123
56 **Bivariate Copula Family Employed as Building Blocks** . . . 124

3.1 **Proxy Indices** . . . 145
3.2 **Descriptive statistics from the weekly return distribution of all asset classes** . . . 146
3.3 **Pearson Correlation matrix** . . . 147
3.4 **Kendall's τ Correlation matrix** . . . 148
3.5 **Estimation Results for Marginal Model** . . . 150
3.6 **Efficient frontiers with and without alternative investments** . . . 152
3.7 **Four Assets Regime Switching Estimation Results** . . . 157
3.8 **Four Assets Regime Switching Estimation Results Continued** . . . 158
3.9 **Four Assets Regime Switching Estimation Results Continued** . . . 159
3.10 **Eight Assets Regime Switching Estimation Results** . . . 162

3.11 **Eight Assets Regime Switching Estimation Results Continued** 163
3.12 **Eight Assets Regime Switching Estimation Results Continued** 164
3.13 **Four Traditional Assets' asset allocation strategies Out-of-sample risk-adjusted performance ignoring regime switching (short sales constrained)**173
3.14 **Four Traditional Assets' asset allocation strategies Out-of-sample risk-adjusted performance ignoring regime switching (short sales unconstrained)** 174
3.15 **Four Traditional Assets' asset allocation strategies Out-of-sample risk-adjusted performance considering regime switching (short sales constrained)** 175
3.16 **Four Traditional Assets' asset allocation strategies Out-of-sample risk-adjusted performance considering regime switching (short sales unconstrained)** 176
3.17 **Eight Assets' asset allocation strategies Out-of-sample risk-adjusted performance ignoring regime switching (short sales constrained)** . . . 177
3.18 **Eight Assets' asset allocation strategies Out-of-sample risk-adjusted performance ignoring regime switching (short sales unconstrained)** . 178
3.19 **Eight Assets' asset allocation strategies Out-of-sample risk-adjusted performance considering regime switching (short sales constrained)** . 179
3.20 **Eight Assets' asset allocation strategies Out-of-sample risk-adjusted performance considering regime switching (short sales unconstrained)** 180
3.21 **Economic measures of asset allocation strategies out-of-sample performance ignoring regime switching (short sales constrained)** 184
3.22 **Economic measures of asset allocation strategies out-of-sample performance ignoring regime switching (short sales unconstrained)** . . . 185
3.23 **Economic measures of asset allocation strategies out-of-sample performance considering regime switching (short sales constrained)** . . . 186
3.24 **Economic measures of asset allocation strategies out-of-sample performance considering regime switching (short sales unconstrained)** . 187
3.25 **Value-at-Risk (VaR) backtesting of asset allocation strategies ignoring regime switching** 193

3.26 **Value-at-Risk (VaR) backtesting of asset allocation strategies considering regime switching** . . . 194
27 **An integer defining the bivariate copula family** . . . 196

4.1 **Considered Indices separated by Regions** . . . 216
4.2 **Summary statistics of daily stock index returns and volatility index returns** . . . 220
4.3 **Estimated marginal model for Stock index returns** . . . 221
4.4 **Estimated marginal model for Volatility index returns** . . . 222
4.5 **Best Fitting Vine Copula for Stock Indices and Volatility Indices** . . . 226
4.6 **Goodness-of-fit Test of Vine Copula for Stock Indices and Volatility Indices** . . . 227
4.7 **Tree Structure of Vine Copula for Stock Indices and Volatility Indices** 228
4.8 **GAS R-vine Copula Parameters Estimates of Stock Indices and Volatility Indices** . . . 232
4.9 **Hypothesis tests for contagion and for increasing tail dependence** . . . 240
4.10 **Hypothesis tests for contagion and for increasing tail dependence** . . . 241
4.11 **Value at risk evaluation of Stock Indices and Volatility Indices** . . . 244
12 **Bivariate copula family employed in Vine copula construction** . . . 247

List of Figures

2.1 **Comparison of given vine structure with the estimated one** 36

2.2 **Comparison of given bivariate copula parameters with the estimated one** 36

2.3 **Comparison of given vine copula with the estimated one on an aggregate level** 37

3.1 Four assets regime switching smooth probability 160

3.2 Eight assets regime switching smooth probability 165

3.3 **Regime Switching Copula Model Cumulative Returns (short sales constrained)** 188

3.4 **Regime Switching Copula Model Cumulative Returns (short sales unconstrained)** 188

4.1 Time series of Stock index returns and their volatility index returns 217

4.2 Time series of Stock index returns and their volatility index returns 218

4.3 Evolution of upper tail dependence of stock indices 233

4.4 Evolution of upper tail dependence of volatility indices 234

4.5 Evolution of lower tail dependence of stock indices 235

4.6 Evolution of lower tail dependence of volatility indices 236

4.7 Evolution of Kendall's τ of stock indices 237

4.8 Evolution of Kendall's τ of volatility indices 238

Chapter 1

Introduction

1.1 The Challenges of Gaussian Copula Modelling

The multivariate Gaussian distribution is the most extensively used model for statistical dependence model in the literature. However, the asymmetric dependence and fat tail characteristics of financial returns trigger the growing demand for non-Gaussian models (Cherubini et al. (2004)). Though the Student t copula allows for symmetric tail dependence as measured by the tail dependence coefficient or tail dependence function (Joe et al. (2010)), it just has a single parameter to control tail dependence for all pairs of variables. The two class of elliptical copula (Fang et al. (2002); Frahm et al. (2003)) and Archimedean copula (Nelsen (2005)) received lots of attention. Elliptical copula normally consists symmetric Gaussian and Student t copula (Demarta and McNeil (2005)), while the class of Archimedean copula contains the tail asymmetric copula, such as Clayton and Gumbel copula. Standard Archimedean multivariate copula allows for tail asymmetry, however, it is still governed by a single parameter. Traditional bivariate Gaussian and Archimedean copulas particularly exhibit two drawbacks. Only simple, symmetric and therefore unrealistic dependence structures can be modelled by Gaussian copula. When dimension higher than two, the applicable bivariate copula families is restricted to either the elliptical family or the Archimedean family, secondly, though Archimedean copula family, such as Clayton and Gumbel copula can capture asymmetric dependence, in high dimension case, where if there are different dependence between different pairs of variables, it is unable to be all captured by single Archimedean copula structure. Some researchers try to extend the class of Archimedean copula (Joe (1997), Savu and Trede

(2010), and Hofert (2011)), however, these extended models generate additional parameter restrictions. Puzanova (2011) introduce the novel hierarchical Archimedean copula (HAC) to modelling tail dependent asset returns, while the building blocks of hierarchical Archimedean copula can only be chosen from the Archimedean copula family, which still exhibit limitations. Modelling the dependence structure of financial returns mainly based on the Gaussian copula, which has even received much criticism in a non-academic context, see Salmon (2012).

1.2 The Superiority of Vine Copula Modelling

Against the above background of these criticism, it is necessary to introduce a flexible and effective model to capture the asymmetric tail dependence of the different types of financial returns. The Sklar (1959) theorem allows to construct general multivariate distributions separately from copula and marginal distributions. The specification of copula can be done independently from the margins. Very recently, the vine copula become increasingly popular in modelling large sets of financial returns dependence structure, the vine structure split the dependence of large sets of returns into pair of returns, and easily employ abundant of bivariate copulas as the building blocks to capture the dependence structure between these pairs of variables. Aas et al. (2009) start to construct a class of multivariate copula employing only bivariate copula specifications as dependency models for the distribution of certain pairs of variables conditional on a specified set of variables. These independent building blocks are called pair-copula. This approach can trace back to Joe (1997) and was investigated and organized systematically by Bedford and Cooke (2001) and Bedford and Cooke (2002). Aas et al. (2009) also proposed two subclass of regular vines, canonical vines (C-vines) and drawable vines (D-vines). C-vines possess star structure in their tree sequence, while D-vine have path structures. Kurowicka and Cooke (2006) focused on vine distributions with Gaussian pair-copula, but Aas et al. (2009) allowed for several different pair-copula families, such as the bivariate Student t copula, bivariate Gumbel and bivariate Clayton copula. D-vine has been employed in many applications (Fischer et al. (2009); Min and Czado (2010); Chollete et al. (2009); Hofmann and Czado (2010); Mendes and De Melo (2010); Salinas-Gutiérrez et al. (2010); Erdorf et al. (2011); Mercier and Frison (2009); Smith et al. (2012)), C-vines are less commonly used (Heinen and Valdesogo (2008); Czado (2010); Nikoloulopoulos et al. (2012)) consider both classes. The much general class of R-vine distributions has very few applications. One reason for this is the enormous number of possible R-vine tree sequences to choose from. The importance of a good selection choice has also been noted by Garcia and Tsafack (2011). The vine structure makes the Student t copula and Gaussian copula as a special case. By decomposing a multivariate density into a cascade of conditional bivariate copulas, vine copulas, or say pair-copula construction, circumvent these problems. There are numerous bivariate copula families with different properties can be served as building blocks for vine copulas, see Joe (1997) and Nelsen (2007). This variety of bivariate copulas are exploited to form a rich and powerful multivariate distribution in large dimension data case, which can model asymmetric and complex dependence structures.

1.3 Motivation and Objectives

In principle, modelling the dependence structure of financial variables is a non-trivial task due to the complex dynamics of individual variables on the one hand, and the time-varying dependence structure between the variables on the other hand. In order to solve these problems, this PhD thesis attempt to develop multivariate modelling procedures that address the complex dependence modelling challenges found in financial data and improve methods for credit portfolio risk management, asset allocation and financial contagion. The thesis is divided into three main chapters focusing on the aforementioned risk management and asset allocation topics. In the chapter of Credit Portfolio Risk Modelling with Vine Copulas, we introduce the vine copulas to modelling the dependence structure of multi-risk factors log returns in the combined framework of both threshold model and mixture model credit risk modelling. Since the dependence structure among the multi-factor is complex and various, traditional multivariate Gaussian Copula is not applicable to modelling the dependence between each pair of factors, and also is unable to capture the different dependence between different pair variables with only one single parameter, in this sense, it is necessary to introduce vine copula into modelling their complex dependence. Empirical results demonstrate that two regimes, characterized as bear and bull states, are required to capture the joint distribution of stock, bond and alternative investments returns. In order to take into consider both regime switching and multi assets asymmetric dependence, we apply vine copula into asset allocation topic in the chapter of Asset Allocation Benefits of Alternative Investments: Markov Regime Switching Regular Vine Copula Method. In Chapter of Modelling International Financial Contagion: Generalised Autoregressive Score Regular Vine Copula Method, as we know, the dependence structure of financial returns is not always static, since then we set time-varying dependence structure in each pair bivariate copula in this chapter, in this sense, our vine copula modelling not only capture the complex and asymmetric dependence among stock indices returns and volatility indices returns, and also take into consider the time-varying dependence structure. Our empirical results and backtesting results strongly support that vine copulas are superior to conventional Gaussian copula in dependence modelling.

Chapter 2

Credit Portfolio Risk Modelling with Vine Copulas

Abstract

In this paper, we demonstrate the superiority of vine copulas over multivariate Gaussian copula when modelling the dependence structure of a credit portfolio risk factors. We introduce the vine copulas to modelling the dependence structure of multi risk factors log returns in the combined framework of both threshold model and mixture model credit risk modelling. In previous literature, Gaussian copula is always adopted to modelling the dependence structure of multi risk factors, while Gaussian copula can only capture the symmetric dependence and cannot capture the fat tails. In such case, we introduce the high dimension vine copulas in order to capture the asymmetric and fat tails characteristics of multi risk factors log returns. In our study, we compare the R-vine mixed copula, R-vine *t* copula, C-vine mixed copula, C-vine *t* copula with the traditional multivariate Gaussian copula. We find that the vine copulas largely improve the ability of threshold model and mixture model credit risk model, the conventional multivariate Gaussian copula is deficient in modelling the dependence structure of a credit portfolio. In depth, we also calculate the out-of-sample risk measure VaR, CVaR and their industry sector risk contribution for credit portfolio separately based on various vine copulas and multivariate Gaussian copula setting, we find VaR and CVaR are seriously underestimated based on multivariate Gaussian copula. In backtesting test, we introduce the Loss function based backtesting method-Model Confidence Set method to select and rank the best copula modelling settings for multi risk factors, the R-vine mixed copula setting outperform other settings.

Keywords: Portfolio Credit Risk, Vine Copulas, C-vine copula, R-vine copula, Tail Dependence, Credit Portfolio, Dependence Structure

2.1 Introduction

Interdependent default events, when defaults of different counterparties tend to occur simultaneously, pose major challenges for an adequate assessment of credit risk in banks' lending and corporate bond portfolios. A prerequisite for accurate estimation of the associated extreme losses requires, therefore, a credit portfolio model is capable of capturing dependence between rare events. Under the structural approach for credit risk modelling, a firms failure results from the asset value of the counterparty falling below the value of its outstanding debt. Due to this direct link between the default and asset value of an institution, interdependent default events is able to be modelled based on the joint distribution of asset values or, equivalently, asset returns. As a consequence, the tail dependence properties of the joint distribution of asset returns play a vital role and would determine frequency of low probability events, such as the simultaneous defaults of several obligors, can actually occur. This would eventually affect the amount of portfolio unexpected loss and the capital buffer required as protection against losses.

The above chain of reasoning demonstrates that correctly using portfolio credit risk models with incorporated tail dependence determine a single bank's ability to remain solvent as well as the sustainability of the entire banking sector. From the perspective of methodology, the Gaussian copula does not allow for heavy tails, the employment of Gaussian copula of Li (2000) in credit portfolio risk is commonly blamed as the contributor of financial crisis of 2007-2009 (Salmon (2009)). In particular, valuation of credit debt obligations (CDOs) is, to a large extent, depend on measuring the underlying pool of loan portfolio credit risk. An overview of CDO-related write-downs at major financial institutions can be found in Barnett-Hart (2009). An analysis of the financial crisis in a wider scope can be found in Crouhy et al. (2008) or Hull (2008).

In spite of this, Gaussian dependence structures have been extensively employed by practitioners and regulators. The symmetric assumption of Gaussian copula or Student t copula and their lack of lower tail dependence coefficient are considered over simplistic to capture the asymmetric tail dependence and fat tail characteristics of the risk factor log returns, leading to a systematic underestimation of portfolio credit risk and capital requirements and, in turn, endangering banks solvency. Therefore, it is crucial for the portfolio credit risk models to incorporate tail dependence for bank's solvency and sustainability consideration. Modelling the dependence structure of a credit portfolio mainly based on the Gaussian copula, which even has received much criticism in a non-academic context, see Salmon (2012). Against the above background of these criticism, we introduce vine copulas (also referred to as pair-copula constructions) to the credit portfolio risk modelling and try to prove that vine copulas are superior to conventional Gaussian copula in credit portfolio risk modelling.

The multivariate Gaussian distribution is the most extensively used model for statistical dependence model in the literature. However, the asymmetric dependence and fat tail characteristics of financial returns trigger the growing demand for non-Gaussian models (Cherubini et al. (2004)). In this case, there emerges a growing need for more flexible copula. Though the Student t copula allows for symmetric tail dependence as measured by the tail dependence coefficient or tail dependence function (Joe et al. (2010)), it just has a single parameter to control tail dependence for all pairs of variables. The two class of elliptical copula (Fang et al. (2002); Frahm et al. (2003)) and Archimedean copula (Nelsen (2005)) received lots of attention. Elliptical copula normally consists symmetric Gaussian and Student t copula (Demarta and McNeil (2005)), while the class of Archimedean copula contains the tail asymmetric copula, such as Clayton and Gumbel copula. Standard Archimedean multivariate copula allows for tail asymmetry, however, it is still governed by a single parameter. Traditional bivariate Gaussian and Archimedean copulas particularly exhibit two drawbacks. Only simple, symmetric and therefore unrealistic dependence structures can be modelled by Gaussian copula. When dimension higher than two, the applicable bivariate copula families is restricted to either the elliptical family or the Archimedean family, secondly, though Archimedean copula family, such as Clayton and Gumbel copula can capture asymmetric dependence, in high dimension case, where if there are different dependence between different pairs of variables, it is unable to be all captured by single Archimedean copula structure. Some researchers try to extend the class of Archimedean copula (Joe (1997), Savu and Trede (2010), and Hofert (2011)), however, these extended models generate additional parameter restrictions. Puzanova (2011) introduce the novel hierarchical Archimedean copula (HAC) to modelling tail dependent asset returns which can be helpful for measuring portfolio credit risk within the structural framework. While the building blocks of hierarchical Archimedean copula can only be chosen from the Archimedean copula family, which still exhibit limitations.

In this context, it is necessary to introduce a flexible and effective model to capture the asymmetric tail dependence of the different types of risk factor log returns. The Sklar (1959) theorem allows to construct general multivariate distributions separately from copula and marginal distributions. The specification of copula can be done independently from the margins. Very recently, the vine copula become increasingly popular in modelling large sets of financial returns dependence structure, the vine structure split the dependence of large sets of returns into pair of returns, and easily employ abundant of bivariate copulas as the building blocks to capture the dependence structure between these pairs of variables. Aas et al. (2009) start to construct a class of multivariate copula employing only bivariate copula specifications as dependency models for the distribution of certain pairs of variables conditional on a specified set of variables. These independent building blocks are called pair-copula. This approach can trace back to Joe (1997) and was investigated and organized systematically by Bedford and Cooke (2001) and Bedford and Cooke (2002). Aas et al. (2009) also proposed two subclass of regular vines, canonical vines (C-vines) and drawable vines (D-vines). C-vines possess star structure in their tree sequence, while D-vine have path structures. Kurowicka and Cooke (2006) focused on vine distributions with Gaussian pair-copula, but Aas et al. (2009) allowed for several different pair-copula families, such as the bivariate Student t copula, bivariate Gumbel and bivariate Clayton copula. D-vine has been employed in many applications (Fischer et al. (2009); Min and Czado (2010); Chollete et al. (2009); Hofmann and Czado (2010); Mendes and De Melo (2010); Salinas-Gutiérrez et al. (2010); Erdorf et al. (2011); Mercier and Frison (2009); Smith et al. (2012)), C-vines are less commonly used (Heinen and Valdesogo (2008); Czado (2010); Nikoloulopoulos et al. (2012)) consider both classes. The much general class of R-vine distributions has very few applications. One reason for this is the enormous number of possible R-vine tree sequences to choose from. The importance of a good selection choice has also been noted by Garcia and Tsafack (2011). The vine structure makes the Student t copula and Gaussian copula as a special case. By decomposing a multivariate density into a cascade of conditional bivariate copulas, vine copulas, or say pair-copula construction, circumvent these problems. There are numerous bivariate copula families with different properties can be served as building blocks for vine copulas, see Joe (1997) and Nelsen (2007). This variety of bivariate copulas are exploited to form a rich and powerful multivariate distribution in large dimension data case, which can model asymmetric and complex dependence structures.

However, the employment of this flexible vine copulas comes with the first question that there are several different vine tree structures to choose from and it is a priori not clear which structure to choose. Standard procedures to tackle this problem have been studied and evolved over the past years (see Dissmann et al. (2013), for instance). In our analysis, we will show how these approaches can be applied in the credit portfolio risk context. Comparing to the work of Changqing et al. (2015), we are not restricted to D-vines and allow for higher dimensions. For a comprehensive collection of research on vine copulas, we refer to Kurowicka and Joe (2011) or Aas et al. (2009).

Credit portfolio risk is mainly determined and driven by two components, namely obligor-specific default risk and cross-obligor dependencies. To model the former, we apply latent variable model, or say, threshold model, structural model, which originates in Merton (1974) and in which default happens when the asset value of a company falls below its liabilities (Alternative obligor-specific default risk models are reduced form (or intensity) models, see Jarrow and Turnbull (1995) and Duffie and Singleton (1999)). For a comparison of both model classes, see Jarrow and Protter (2004) or Arora et al. (2005).

Most of the criticism of structural models concerns the accuracy of credit spread predictions, whereas the focus of this paper is on the dependence structure among obligors. Therefore, we set aside these criticism. In our analysis, the dependence structure of the credit portfolio is modelled by vine copula functions. An alternative approach to model the dependence structure are factor models, see Gordy (2003) or Dorfleitner et al. (2012). Mixture model, which possess several superior characteristics to latent variable model, is another type of credit risk model extensively used in financial industry. The mixture model seems employ different default rules comparing to latent variable model, nevertheless, Gordy (2003) proves that latent variable model and mixture model can be mapped to each other. In this sense, in our study, we construct a common framework following Gordy (2003) to test and prove the superiority of vine copula dependence modelling based on both of latent variable and mixture credit risk models.

Inferring credit or default correlations from the equity market may seem problematic at a first glance. The classical threshold model-Merton model propose the use of asset values to calibrate credit correlations, while this approach has some limitations pointed out by Frye (2008). Furthermore, owing to various difficulties concerning the accessibility of asset values, equity prices are widely adopted as a substitute. Many banks rely on equity data for the calibration of the dependence structure of their internal credit portfolio models. For instance, the dependence structure of CreditMetrics model is estimated via equity data (see Morgan (1997)) and Hull and White (2004) propose that default correlation between two companies is usually assumed to be the same as the correlation between their equity returns. In addition, equity returns are the most important variable in predicting defaults, since they provide an empirical link between equity data and credit risk, see Fabozzi et al. (2010). Thus, from a practical point of view, though the use of equity returns is justified, we still adopt equity returns as the substitute of asset returns.

Based on the analysis above, in this paper, we fit both C-vine and R-vine and traditional multivariate Gaussian copula separately to monthly equity returns of 92 multiple industry sector equity indices log returns. As bivariate building blocks, we employ the Gaussian (no tail dependence), the Student t (with symmetric tail dependence) and the Clayton copula (lower, but not upper tail dependence). For the Clayton family, we also include the rotated versions (90, 180 and 270 degrees) as well as survival version.

The remainder of the paper is structured as follows: Section 2 lists related literature of vine copula application and portfolio credit risk study. Section 3 outlines the credit portfolio model setting used in the paper. Section 4 describes and analyses the data we adopt in this paper. Section 5 talks about the selection of vine copula. In section 6, we apply monte carlo simulation method to obtain portfolio loss distribution and measure the credit risk of our test portfolio. Then we investigate the risk factor VaR and CVaR contribution of various industry sectors in Section 7. Finally, we summarise the main results and draw conclusions in section 8.

2.2 Multi-factor Credit Portfolio Model Setup

2.2.1 Threshold Model Setup

In the analysis of mechanisms for dependent credit events, existing credit risk models are normally divided into two classes: latent variable models, or say threshold model, structural model, such as KMV (Kealhofer and Bohn (2001), Crosbie and Bohn (2003)) or *CreditMetrics* (Morgan (1997)) which essentially descend from the firm-value model of Merton (1974); in latent variable models default occurs if a random variable X (termed a latent variable even if in some models X may be observable) falls below some threshold.

Dependence between defaults is caused by dependence between the corresponding latent variables.

Another class, Bernoulli mixture models such as *CreditRisk*$^+$ developed by Credit Suisse Financial Products (Suisse (1997)) and more generally the reduced form models from the credit derivatives literature such as Lando (1998) or Duffie and Singleton (1999), where default events have a conditional independence structure conditional on common economic factors. In the mixture models the default probability of a company is assumed to depend on a set of economic factors, given these factors, defaults of the individual obligors are conditionally independent. This division reflects the way these models are conventionally presented rather than any fundamental structural difference and the recognition that *CreditMetrics* (usually presented as a latent variable model) and *CreditRisk*$^+$ (a mixture model) can be mapped into each other dates back to Gordy (2000) and also Lagrado and Osher (1997).

Nevertheless, latent variable model still takes several drawbacks, therefore, in our study, in order to avoid the bad effects of shortcomings of threshold model and investigate whether our vine copula model applicable to in both of the two classes credit risk model. As mentioned above, due to the two model classes can be mapped into each other, thus we work one step further and reduced to a common framework. The useful mapping direction is to rewrite latent variable models as Bernoulli mixture models, which we will discuss in details in subsequent section.

The Vasicek single factor threshold model (Vasicek (1987); Vasicek (1991); Vasicek (2002)), KMV and *CreditMetrics* can be considered to descend from the firm-value model of Merton (1974), where default is modelled as occurring when the asset value of a company falls below its liabilities. In statistical literatures, such as Joe (1997), such models are under the general heading of latent variable models. Based on above credit models, we first construct our multi-factor latent variable model, then rewrite it as mixture model.

Following the CAPM framework, risk is divided into systematic risk and idiosyncratic risk. According to modern portfolio theory, the idiosyncratic risk, which is the firm-specific risk, can be diversified, while the systematic risk is impossible to be diversified. It is assumed that the systematic risk of counterparty is adequately described by a set of risk factors. In both KMV and *CreditMetrics*, they consider a random vector $\mathbf{X}$ =

$(X_1, ..., X_m)'$, in which dependence is described by a multivariate Gaussian copula, where X_i is an underlying latent variable for counterparty i at time T. In order to improve this threshold model, we employ the high dimensional vine copula instead of multivariate Gaussian copula to describe the dependence structure of multiple systematic risk factors, and then we evaluate the risk measure of VaR and CVaR for portfolio loss distribution and compare to the case that the risk factors following the multivariate Gaussian distribution and various different vine copula settings.

In our case, we just take into account the default events without consideration of the credit immigration. Let $H_i = \mathbf{1}_{\{Y_i \leq d_i\}}$ be the indicator function for counterparty k at time horizon T. Therefore, the H_i is assumed to take the value of 0 or 1, when the counterparty default, which means the counterparty falls below the threshold, the H_i takes the value 1, and when there's non-default, it takes the value 0. Let Y_i be a random variable with continuous distribution function $F_i(x) = P[Y_i \leq x]$, and let $d_i \in R$ such that H_i=1 if and only if $Y_i \leq d_i$. The parameter d_i is called the default threshold and (Y_i, d_i) is the latent variable model for H_i, as described in Frey et al. (2001). As in KMV model (Crosbie and Bohn (2002)), Y_i represents the asset value monthly log return of counterparty k. The model can be formulated mathematically in the following way,

$$Y_i = r_i X_i + \sqrt{1 - {r_i}^2}\epsilon_i, \quad X_i, \quad \epsilon_i \sim N(0, 1) \tag{2.1}$$

where r_i denotes the systematic risk factor loading and X_i denotes the composite risk factor which is defined as

$$X_i = \sum_{k=1}^{K} \alpha_{ik} Z_k, \quad Z_k \sim N(0, 1) \tag{2.2}$$

It must hold that $\sum_{k=1}^{K} \alpha_{ik}^2 = 1$ in order for X_i to satisfy unit variance and the asset return correlation in-between obligors i and j is thereby fully determined through the set of systematic risk factors

$$\rho = corr(Y_i, Y_j) = r_i r_j \sum_{k=1}^{K} \alpha_{ik} \alpha_{jk} \tag{2.3}$$

When $K = 1$, the model turns to be a single-factor model, while $K > 1$ corresponds to a multi-factor model. Where the X_i represents the composite systematic risk factors, which contain a set of industry sector factors representing the systematic risk of industry sector. The parameter r_i is the coefficient of determination for systematic risk (how much of the variance can be explained by the risk factors). Let π be the (unconditional) probability of default for counterparty i, i.e. π=$F(d)$. π is assumed to be given from some internal or external rating system or other procedures. The dependence structure of these risk factors are captured by vine copulas in our paper. The α_{ik} are the composite risk factor loading for the ith instrument ($\sum_{k=1}^{K} \alpha_{ik}^2 < 1$), represent the sensitivity of the ith obligor to the kth systematic risk factor. The default probabilities are prescribed exogenously, for example, from a bank's internal credit rating model, with the probability of default over the time horizon for the ith name denoted by d_i and the $\epsilon_i \sim$ N(0,1) are standard normal variables representing the idiosyncratic risk factor independent of Z, E[Z]=0.

Conditional on the systematic risk factors $Z = (Z_1, ..., Z_K)^T$, obligor defaults are independent and conditional default probabilities for each name are given by

$$Q(Z) = P[Y_i \leq d_i|Z] = \Phi\left(\frac{F_i^{-1}(d_i) - r_i X_i}{\sqrt{1 - r_i^2}}\right) \tag{2.4}$$

where Φ denotes the standard normal distribution function. As we know, in classical industrial models such as KMV and CreditMetrics, the Y_i is assumed to follow standard normal distribution, whereas in our case, the distribution of Y_i is unknown. Therefore, we work with the estimated conditional probability of default $Q(Z)$ obtained by replacing $F_i^{-1}(d_i)$ by the empirical quantile estimate $\hat{F}_i^{-1}(d_i)$.

Now we consider a credit portfolio consists I obligors, $i = 1, 2, 3, ..., I$, can be characterized by three parameters: the exposure at default denoted by EAD_i, loss given default denoted by LGD_i and the probability of default PD_i. Therefore, the credit portfolio loss incurred due to default of obligor i is given by

$$L_i = EAD_i \cdot LGD_i \cdot H_i = w_i \cdot H_i, \tag{2.5}$$

where $w_i = EAD_i \cdot LGD_i$ is the effective exposure of obligor i. Then the portfolio loss is defined as

$$L = \sum_{i=1}^{I} L_i. \tag{2.6}$$

Regarding the distribution of the portfolio loss variable PL, the Value at Risk at a prespecified confidence level q (VaR_q) and for the Conditional Value at Risk ($CVaR_q$). VaR is commonly used in risk management and controlling as a measure of portfolio credit risk, although it is incoherent (not sub-additive in general). It quantifies the minimum portfolio loss in the worst $(1 - q) \times 100$ percent of cases. $VaR_q(PL)$ equals the value of the quantile function of the random variable PL

$$VaR_q(PL) := F_{PL}^{-1}(q) \tag{2.7}$$

CVaR is a coherent risk measure which quantifies the expected portfolio loss in the worst $(1 - q) \times 100$ percent of cases. $CVaR_q(PL)$ equals the conditional tail expectation beyond the q-quantile of the portfolio loss distribution

$$CVaR_q(PL) := E[PL|PL \geq VaR_q(PL)] \tag{2.8}$$

2.2.2 Threshold Model represented as Mixture Model

Another class of main credit risk model, the mixture model, such as Bernoulli mixture format, which has a number of advantages over the threshold format. Bernoulli mixture models exhibit more applicable to Monte Carlo simulation risk studies. Mixture models are considered to be more convenient for statistical fitting purposes. Especially for large size portfolio, whose behaviour modeled by Bernoulli mixtures model can be understood and analysed in terms of the behaviour of the distribution of the common economic factors. Due to these advantages, a question raised, can we adopt mixture model as a substitute of threshold model? Despite the format of mixture models seem to have different structure from the threshold models at first glance, it is important to realize that the majority of useful threshold models can be represented as Bernoulli mixture models mathematically.

To motivate the subsequent analysis we begin by computing the mixture model representation of the multi-factor threshold model as we set up in previous section. It is convenient to substitute the factor Z_k in the threshold representation with the variable negative $\mathbf{\Psi}$ in the mixture representation; this yields conditional default probabilities that are increasing in $\mathbf{\Psi}$ and obtains the formula that are in line with the Basel IRB formula. With Z_k=-$\mathbf{\Psi}$, the multi-factor model takes the form

$$Y_i = -r_i \sum_{k=1}^{K} \alpha_{ik}\Psi + \sqrt{1 - r_i{}^2}\epsilon_i \tag{2.9}$$

Under threshold model default definition, company i defaults if and only if $Y_i \leq d_i$ and hence if and only if $\sqrt{1 - r_i{}^2}\epsilon_i \leq d_i + r_i \sum_{k=1}^{K} \alpha_{ik}\Psi$. Since the variables $\epsilon_1, ..., \epsilon_m$ and Ψ are independent, default events are independent conditional on Ψ and we can compute

$$\begin{aligned} p_i(\psi) = P(Y_i = 1|\Psi = \psi) &= P(\sqrt{1 - r_i{}^2}\epsilon_i \leq d_i + r_i \sum_{k=1}^{K} \alpha_{ik}\Psi|\Psi = \psi) \\ &= \Phi\left(\frac{d_i + r_i \sum_{k=1}^{K} \alpha_{ik}\psi}{\sqrt{1 - r_i{}^2}}\right) \end{aligned} \tag{2.10}$$

where we have used the fact that ϵ_i is standard normally distributed. The threshold is typically set so that the default probability matches an exogenously chosen value p_i, so that $d_i = F^{-1}(p_i)$. In this case we obtain

$$p_i(\psi) = \Phi\left(\frac{F^{-1}(p_i) + r_i \sum_{k=1}^{K} \alpha_{ik}\psi}{\sqrt{1 - r_i{}^2}}\right) \tag{2.11}$$

In the following part, we want to extend this representation to more general threshold models with a factor structure, which can be matched to our multi-factor model setting, and different copula setting for the comparison study of different competing vine copula models. Therefore, we first give a condition that ensures that a threshold model can be written as a Bernoulli mixture model.

Definition. (McNeil et al. (2015)) A random vector $\mathbf{X}$ has a p-dimensional conditional independence structure with conditioning variable Ψ if there is some $p < m$ and a p-dimensional random vector $\Psi = (\Psi_1, ..., \Psi_p)'$ such that, conditional on Ψ, the random variables $X_1, ..., X_m$ are independent.

In our case, the conditioning variable is taken to be $\Psi = -Z_k$. The next lemma generalizes the computations to any threshold model with a conditional independence structure.

Lemma. (McNeil et al. (2015)) Let $(\mathbf{X}, \mathbf{d})$ be a threshold model for an m-dimensional random vector $\mathbf{X}$. If $\mathbf{X}$ has a p-dimensional conditional independence structure with conditioning variable Ψ, then the default indicators $Y_i = I_{\{X_i \leq d_i\}}$ follow a Bernoulli mixture model with factor Ψ, where the conditional default probabilities are given by $p_i(\psi) = P(X_i \leq d_i|\Psi = \psi)$.

Proof. For $\mathbf{y} \in \{0, 1\}^m$ define the set $B := 1 \leq i \leq m : y_i = 1$ and let $B^c = 1, ..., m \setminus B$. We have

$$P(\mathbf{Y} = \mathbf{y}|\Psi = \psi) \tag{2.12}$$

$$= P(\bigcap_{i \in B} X_i \leq d_i \bigcap_{i \in B^c} X_i > d_i|\Psi = \psi)$$

$$= \prod_{i \in B} P(X_i \leq d_i|\Psi = \psi) \prod_{i \in B^c} (1 - P(X_i \leq d_i|\Psi = \psi))$$

Hence, conditional on $\Psi=\psi$, the Y_i are independent Bernoulli variables with success probability $p_i(\psi) := P(X_i \leq d_i|\Psi = \psi)$.

Poisson Mixture Models and *CreditRisk*$^+$

Since defaults is typically a rare event, it is possible to approximate Bernoulli indicator random variables for default with Poisson random variables and to approximate Bernoulli mixture models with Poisson mixture models. By choosing independent gamma distributions for risk factors Ψ and using the Poisson approximation, we obtain a particularly tractable model for portfolio losses, known as *CreditRisk*$^+$.

Poisson approximation and Poisson mixture models. (McNeil et al. (2015)) To be more precise, assume that, given the factors Ψ, the default indicator variables $Y_1, ..., Y_m$ for a particular time horizon are conditionally independent Bernoulli variables satisfying $P(Y_i = 1|\Psi = \psi) = p_i(\psi)$. Moreover, assume that the distribution of Ψ is such that the conditional default probabilities $p_i(\psi)$ tend to be very small. In this case, the Y_i variables can be approximated by conditionally independent Poisson variables $\tilde{Y}_i$ satisfying $\tilde{Y}_i|\Psi = \psi \sim Poi(p_i(\psi))$, since

$$P(\tilde{Y}_i = 0|\Psi = \psi) = e^{-p_i(\psi)} \approx 1 - p_i(\psi), \tag{2.13}$$

$$P(\tilde{Y}_i = 1|\Psi = \psi) = p_i(\psi)e^{-p_i(\psi)} \approx p_i(\psi). \tag{2.14}$$

Moreover, the portfolio loss $L = \sum_{i=1}^{m} e_i\delta_i Y_i$ can be approximated by $\tilde{L} = \sum_{i=1}^{m} e_i\delta_i\tilde{Y}_i$. Of course, it is possible for a company to "default more than once" in the approximating Poisson model with a very low probability.

We now give a formal definition of a Poisson mixture model for counting variables that parallels the definition of a Bernoulli mixture model.

Definition (Poisson mixture model). (McNeil et al. (2015)) Give some $p < m$ and a

p-dimensional random vector $\Psi = (\psi_1, ..., \psi_p)'$, the random vector $\tilde{\mathbf{Y}}=(\tilde{Y}_1, ..., \tilde{Y}_m)'$ follows a Poisson mixture model with factors Ψ if there are functions $\lambda_i : \mathbb{R}^p \rightarrow (0, \infty)$, $1 \leq i \leq m$, such that, conditional on $\Psi=\psi$, the random vector $\mathbf{Y}$ is a vector of independent Poisson distributed random variables with rate parameter $\lambda_i(\psi)$.

If $\tilde{\mathbf{Y}}$ follows a Poisson mixture model and if we define the indicators $Y_i = I_{\{\tilde{Y}_i \geq 1\}}$, then $\mathbf{Y}$ follows a Bernoulli mixture model and the mixing variables are related by $p_i(\cdot) = 1 - e^{-\lambda_i(\cdot)}$.

CreditRisk$^+$ **model.** The *CreditRisk*$^+$ model for credit risk was proposed by Credit Suisse Financial Products in 1997 (see Credit Suisse Financial Products 1997). It has the structure of the Poisson mixture model, where the factor vector Ψ consists of p independent gamma-distributed random variables. The distributional assumptions and functional forms imposed in *CreditRisk*$^+$ make it possible to compute the distribution of the number of defaults and the aggregate portfolio loss fairly explicitly using techniques for compound distributions and mixture distributions.

The stochastic parameter $\lambda_i(\Psi)$ of the conditional Poisson distribution for firm i is assumed to take the form

$$\lambda_i(\Psi) = k_i \mathbf{w}_i' \Psi \tag{2.15}$$

for a constant $k_i > 0$, for non-negative factor weights $\mathbf{w}_i = (w_{i1}, ..., w_{ip})'$ satisfying $\sum_j w_{ij} = 1$, and for p independent Gamma(α_j,β_j) distributed factors Ψ_i, $i = 1, ...p$, with parameters set to be $\alpha_j = \beta_j = \sigma_j^{-2}$ for $\sigma_j > 0$ and $j = 1, ...p$. This parametrization of the gamma variables ensures that we have $E(\Psi_j) = 1$ and $var(\Psi_j) = \sigma_j^2$.

It is easy to verify that

$$E(\tilde{Y}_i) = E(E(\tilde{Y}_i|\Psi)) = E(\lambda_i(\Psi)) = k_i E(\mathbf{w}_i' \Psi) = k_i, \tag{2.16}$$

so that k_i is the expected number of defaults for obligor i over the time period. Setting $Y_i = I_{\{\tilde{Y}_i \geq 1\}}$ we also observe that

$$P(Y_i = 1) = E(P(\tilde{Y}_i > 0|\Psi)) = E(1 - exp(k_i \mathbf{w}_i' \Psi)) \approx k_i E(\mathbf{w}_i' \Psi) = k_i \tag{2.17}$$

for k_i small, so that k_i is approximately equal to the default probability.

2.3 Review of Vine Copula

In order to accurately describe the dependence structure of multiple systematic risk factors, in our study, we employ the high dimensional vine copula to capture the asymmetric dependence of systematic risk factors. Vine copula is a type of high dimensional copula which can individually choose their building blocks from a wide range of bivariate copula families, so that it can easily to capture the asymmetric dependence characteristics between pairs of variables. In this section, following Nikoloulopoulos et al. (2012), we briefly review the vine copula construction and inference.

2.3.1 Construction of Vine Copula

A d-variate copula $C(u_1, ..., u_d)$ is a cumulative distribution function (cdf) with uniform marginals on the unit interval, see examples in Joe (1997) and Nelsen (2007). Regarding the theorem of Sklar (1959) for multivariate case, if $F_j(y_j)$ is the cdf of a univariate continuous random variable Y_j, then $C(F_1(y_1), ..., F_d(y_d))$ is a d-variate distribution for $\mathbf{Y} = (Y_1, ..., Y_d)$ with marginal distributions $F_j, j = 1, ..., d$. Conversely, if H is a continuous d-variate cdf with univariate marginal cdfs $F_1, ..., F_d$, then there exists a unique d-variate copula C satisfy that

$$F(\mathbf{y}) = C(F_1(y_1), ..., F_d(y_d)), \quad \forall \mathbf{y} = (y_1, ..., y_d). \tag{2.18}$$

The corresponding density is

$$f(\mathbf{y}) = \frac{\partial^d F(\mathbf{y})}{\partial y_1 \ldots \partial y_d} = c(F_1(y_1), ..., F_d(y_d)) \prod_{j=1}^{d} f_j(y_j), \tag{2.19}$$

where $c(u_1, ..., u_d)$ is the d-variate copula density and $f_j, j = 1, ..., d$, are the corresponding marginal densities. As we know, a copula C has reflection symmetry if $(U_1, ..., U_d) \sim$ C implies that $(1 - U_1, ..., 1 - U_d)$ has the same distribution C. When we require the copula models have the characteristics of reflection asymmetry and flexible lower or upper tail dependence, then vine copulas (see Bedford and Cooke (2001); Bedford and Cooke (2002); Kurowicka and Cooke (2006) and Joe (1997)) become the best choice.

A d-dimensional vine copulas are constructed through sequential mixing of $d(d-1)/2$ linked bivariate copulas by trees and their cdfs involve lower dimensional integrals. Since the densities of multivariate vine copulas can be factorized in terms of linked bivariate copulas and lower dimension marginals, they show the advantage of computationally tractable.

According to the different types of tree structures, various vine copulas can be constructed. Two special cases are D-vines and C-vines while R-vines is their more general format.

With respect to the d-dimensional C-vine copula, the pairs at tree 1 are 1, i, for $i = 2, ..., d$, and for tree $l(2 \leq l < d)$, the (conditional) pairs are $l, i|1, ..., l-1$ for $i = l+1, ..., d$, the conditional copulas are specified for variables l and i given those indexed as 1 to $l-1$. For C-vines density is given by (Aas et al. (2009)),

$$f(y) = \prod_{k=1}^{d} f_k(y_k) \prod_{j=1}^{d-1} \prod_{i=1}^{d-j} c_{i,i+j|i+1,...,i+j-1}(F_{i|i+1...i+j-1}(y_i|y_{i+1:i+j-1}), F_{i+j|i+1,...,i+j-1}(y_{i+j}|y_{i+1:i+j-1})), \tag{2.20}$$

where $y_{k_1:k_2} = (y_{k_1}, ..., y_{k_2})$, index j denotes the tree, while i runs over the edges in each tree.

Regarding the d-dimensional D-vine copula, the pairs at tree 1 are $i, i+1$, for $i = 1, ..., d-1$, and for tree $l(2 \leq l < d)$, the (conditional) pairs are $i, i+l|i+1, ..., i+l-1$ for $i = 1, ..., d-l$, the conditional copulas are specified for variables i and $i+l$ given the variables indexed in between.

$$f(y) = \prod_{k=1}^{d} f_k(y_k) \prod_{j=1}^{d-1} \prod_{i=1}^{d-j} c_{j,j+i|1,...,j-1}(F_{j|1...j-1}(y_j|y_1, ..., y_{j-1}), F_{j+i|1...j-1}(y_{j+i}|y_1, ..., y_{j-1})), \tag{2.21}$$

where $y_{k_1:k_2} = (y_{k_1}, ..., y_{k_2})$, index j denotes the tree, while i runs over the edges in each tree.

For more general d-dimension regular vines, there are $d-1$ pairs at tree 1, $d-2$ pairs in tree 2 where each pair has one element in common, and for $l = 2, ..., d-1$, there are $d-l$ pairs in level l where each pair has $l-1$ elements in common. Other conditions for regular vines can be found in Bedford and Cooke (2001) and Bedford and Cooke (2002).

2.3.2 Inference of Vine Copula

In this part we discuss the parameter estimate of the C-vine (canonical vine copula) density given by (20). We omit the discussion of estimate of D-vine (drawable vine copula) density because we don't employ D-vine in modelling the dependence structure of risk factors in our analysis. Inference for the general regular vine is also feasible though not straightforward, details of R-vine inference can be found in Dissmann et al. (2013).

Here we follow the inference method of Aas et al. (2009). Assume that we observe n variables at time T time. Let $\mathbf{x}_i = (x_{i,1}, ..., x_{i,T}); i = 1, ..., n$, denote the data set. For simplicity, we assume that the T observations of each variable are independent over time. Independence assumption is not a limiting condition, in our empirical analysis, we will adopt univariate time series model fit to the margins and analyze the obtained residuals.

Since the margins are unknown, the parameter estimation must rely on the normalised ranks of the data. The approximate uniform and independence means what is being maximised is a pseudo-likelihood maximization. We extend the method of maximum pseudo-likelihood originally proposed for copula by Oakes (1994), and proved to be asymptotically normal and consistent both by Genest et al. (1995) and Shih and Louis (1995). Moreover, by adopting simulation method, Kim et al. (2007) indicate that the maximum pseudo-likelihood method outperform the maximum likelihood method when the marginal distributions are unknown.

For the canonical vine, the log-likelihood is given by

$$\sum_{j=1}^{n-1}\sum_{i=1}^{n-j}\sum_{t=1}^{T} log[c_{j,j+i|1,...,j-1}\{F(x_{j,t}|x_{1,t}, ..., x_{j-1,t}), F(x_{j+i,t}|x_{1,t}, ..., x_{j-1,t})\}]. \tag{2.22}$$

For each bivariate copula there is at least one parameter to be estimated which depends on which kind of bivariate copula is chosen. The log-likelihood must be numerically maximised over all parameters.

The marginal conditional distribution in vine copula construction is given by Joe (1997), for each j,

$$F(x|\mathbf{v}) = \frac{\partial C_{x,v_j|\mathbf{v}_{-j}}\{F(x|\mathbf{v}_{-j}), F(v_j|\mathbf{v}_{-j})\}}{\partial F(v_j|\mathbf{v}_{-j})} \tag{2.23}$$

where $C_{i,j|k}$ is a bivariate copula distribution function. For the special case where v is univariate, we have

$$F(x|v) = \frac{\partial C_{x,v}\{F(x), F(v)\}}{\partial F(v)} \tag{2.24}$$

Then we introduce h function (Aas et al. (2009)), $h(x, v, \Theta)$ denotes this conditional distribution function when x and v are uniform, i.e., $f(x) = f(v) = 1$, $F(x) = x$ and $F(v) = v$.

That is,

$$h(x, v, \Theta) = F(x|v) = \frac{\partial C_{x,v}(x, v, \Theta)}{\partial v}, \tag{2.25}$$

where the parameter v denotes the conditioning variable and Θ represents the set of parameters for the copula of the joint distribution function of x and v. Let $h^{-1}(x, v, \Theta)$ be the inverse of the h-function with respect to the first variable u, or say the inverse of the conditional distribution function. $\Theta_{j,i}$ is the set of parameters of the corresponding copula density $c_{j,j+i|1,...,j-1}(\cdot, \cdot)$, $h(\cdot)$ is given by (23), and element t of $\mathbf{v}_{j,i}$ is $v_{j,i,t} = F(x_{i+j,t}|x_{1,t}, ..., x_{j,t})$. Further, $L(\mathbf{x}, \mathbf{v}, \Theta)$ is the log-likelihood of the chosen bivariate copula with parameters Θ given the data vectors $\mathbf{x}$ and $\mathbf{v}$. Which is,

$$L(\mathbf{x}, \mathbf{v}, \Theta) = \sum_{t=1}^{T} logc(x_t, v_t, \Theta). \tag{2.26}$$

where $c(u, v, \Theta)$ is the density of the bivariate copula with parameters Θ. According to the setting above, we can first estimate the parameters of the copula of tree 1 with the original data, then compute conditional distribution functions for tree 2 using the copula parameters from tree 1 and the h-function, repeat the process, estimate the parameters of the copula of tree 2 using the observations in last step, and then continue to repeat last step process until obtain all parameters. Finally, we can obtain the starting value of the parameters for numerical maximisation.

2.4 Modelling Marginal Model

Since Sklar (1959) theorem demonstrates that we can model the marginals and dependence structure separately, we therefore discuss the marginal modelling in this section.

Let the random process r_t denote the financial asset returns which can be characterized by an autoregressive moving-average (ARMA) model as follows

$$r_t = a_0 + \sum_{i=1}^{p} a_i r_{t-i} + \sum_{j=1}^{q} b_j \epsilon_{t-j} + \epsilon_t \tag{2.27}$$

where a_0 is a constant; p and q are the order of autoregressive and moving average processes respectively for the conditional mean. The error term ϵ_t can be split into a stochastic part x_t and a time-dependent standard deviation σ_t so that $\epsilon_t = \sigma_t x_t$. The conditional variance σ_t^2 is characterized by an asymmetric GARCH model, namely GJR-GARCH(1,1) (see Glosten et al. (1993)).

$$\sigma_t^2 = \omega_i + \alpha_i \epsilon_{i,t-1}^2 + \beta_i \sigma_{i,t-1}^2 + \gamma_i \epsilon_{i,t-1}^2 I_{i,t-1} \tag{2.28}$$

where $I_{i,t-1} = 1$ if $\epsilon_{i,t-1} < 0$, and $I_{i,t-1} = 0$ if $\epsilon_{i,t-1} \geq 0$.

The filtered returns $x_t = \epsilon_t/\sigma_t$, $t = 1, ..., T$; follow a strong white noise process with a zero mean and unit variance. In our empirical work, we adopt Hansen (1994)'s skewed Student t distribution $x_t \sim skT(0, 1; \nu, \zeta)$, with $\nu > 2$ and ζ denoting the degrees of freedom (dof) and asymmetry parameters, respectively. Its PDF is give by, [1]

$$f(x; \nu, \zeta) = \begin{cases} bc\left(1 + \dfrac{1}{\nu - 2}\left(\dfrac{bz + a}{1 - \zeta}\right)^2\right)^{-\frac{\nu+1}{2}}, & if \;\; z < -\dfrac{a}{b} \\ bc\left(1 + \dfrac{1}{\nu - 2}\left(\dfrac{bz + a}{1 + \zeta}\right)^2\right)^{-\frac{\nu+1}{2}}, & if \;\; z \geq -\dfrac{a}{b} \end{cases}$$

where $a = 4\zeta c\frac{\nu-2}{\nu-1}$, $b^2 = 1 + 3\zeta^2 - a^2$, $c = \frac{\Gamma(\frac{\nu+1}{2})}{\sqrt{\pi(\nu-2)}\Gamma(\frac{\nu}{2})}$. The skewed Student t distribution is quite general as it nests the Student t distribution and the Gaussian density. Previous studies advocate this parametrization for the margins as able to capture the autocorrelation, volatility clustering, skewness and heavy tails exhibited typically by financial asset returns; see e.g. Jondeau and Rockinger (2006) and Kuester et al. (2006). In our empirical work, we adopt GJR-GARCH(1,1) and select the best ARMA p and q among 1, 2,..., 10 by minimizing the Akaike Information Criterion (AIC). The model parameters are estimated by quasi-maximum likelihood (QML). Uniform (0, 1) margins denoted $u_n = F_n(x_n)$, n = 1, 2, can be obtained from each filtered return series via the probability integral transform. Once the vector $\mathbf{u} = (u1, u2)'$ is formed, the copula parameter vector can be estimated by maximum pseudo-likelihood method discussed above.

2.5 Simulation Study

In order to investigate whether the vine copula calibration is feasible, we conduct a simulation study in which we sample from a known vine copula, apply the estimation approach

[1] There are other Student t distribution that the skewness is introduced in different ways, see Fernández and Steel (1998) and Aas and Haff (2006).

to the sampled values and compare the known with the fitted vine copula. As in practise, we neither know which type of vine tree structure is the best fitting one to our data nor which kind of bivariate copula families we should choose as building blocks nor bivariate copula parameters (bivariate copula families are a little exception in our case since we restrict ourselves to the Gauss, Student *t* and Clayton family and their rotated and survival version). Given abundant possible vine structures can be sampled from, we attempt to carry out our simulation study to be more realistic. Therefore, we randomly choose five UK equity indices, fit an R-vine to their equity time series and use the resulting R-vine as the known copula in the simulation study (R-vines are the general form of C-vines and D-vines, which is why we employ R-vines in our simulation study). Consequently, our vine tree structure, bivariate copula and their parameters can be considered as realistic. It turns out that all three bivariate copula families, Gaussian, Student t and Clayton copula, are all included in the known vine copula setting. Then we generate 200 random samples from the known vine copula, as this has the same time series length with the data in subsequent empirical analysis.

The left side of Figure 1 shows the given vine structure from which we draw 200 samples, while the right side shows the vine structure which results from fitting a known R-vine to the 200 observations. The results display that estimated tree structure is identical to the given one, which is actually quite remarkable given that there are 480 different R-vines on five variables. In addition, the selected building blocks of bivariate copula families are also pretty close to the known ones, as eight out of ten bivariate copula families are correctly estimated. Next, we compare the parameters of the bivariate copulas from the given copulas with the estimated ones (see Figure 2). The Table 1 and Table 2 indicate that, especially in the ground level trees, the parameter match is especially good, while the match deviates a little when check the higher level trees. The largest parameter deviation occurs in the Tree 2 and Tree 3, in which the real Clayton parameter is separately underestimated (0.59 vs. 0.47) and overestimated quite a bit (0.31 vs. 0.37). We highlighted parameter deviation of less than 15% in green.

What we concerned is whether the observed deviation in bivariate copula parameters will impact on model overall level, therefore, we adopt QQ plot to test it. As we focus on credit portfolio risk modelling, the aggregate portfolio behavior make more sense to us rather than the behavior of a single creditor given the portfolio been well diversified.

Table 2.1: **Given Vine Copula**

tree	edge	No.	family	par	par2	τ	UTD	LTD
1	3,5	1	N	0.71	0.00	0.50	-	-
	3,2	2	t	0.62	4.78	0.42	0.29	0.29
	4,1	1	N	0.66	0.00	0.46	-	-
	4,3	2	t	0.61	30.00	0.42	0.01	0.01
2	2,5;3	1	N	0.28	0.00	0.18	-	-
	4,2;3	3	C	0.59	0.00	0.23	-	0.31
	3,1;4	3	C	0.48	0.00	0.19	-	0.24
3	4,5;2,3	13	SC	0.10	0.00	0.05	0.00	-
	1,2;4,3	13	SC	0.31	0.00	0.14	0.11	-
4	1,5;4,2,3	3	C	0.15	0.00	0.07	-	0.01
type: R-vine	logLik: 257.05	AIC: -490.1	BIC: -450.52					

Note: This table lists estimated first four trees parameters of given R-vine mixed copula model fitted to five UK equity indices as risk factors. Selected copula families are explained in Appendix Table 56.

Table 2.2: **Estimated Vine Copula**

tree	edge	No.	family	par	par2	τ	UTD	LTD
1	3,5	1	N	0.69	0.00	0.48	-	-
	3,2	2	t	0.64	3.49	0.44	0.37	0.37
	4,1	1	N	0.62	0.00	0.42	-	-
	4,3	1	N	0.60	0.00	0.41	-	-
2	2,5;3	1	N	0.30	0.00	0.19	-	-
	4,2;3	3	C	0.47	0.00	0.19	-	0.23
	3,1;4	3	C	0.51	0.00	0.20	-	0.26
3	4,5;2,3	1	N	0.06	0.00	0.04	-	-
	1,2;4,3	13	SC	0.37	0.00	0.16	0.16	-
4	1,5;4,2,3	3	C	0.13	0.00	0.06	-	0.00
type: R-vine	logLik: 255.73	AIC: -489.45	BIC: -453.17					

Note: This table lists estimated first four trees parameters of estimated R-vine mixed copula model fitted to five UK equity indices as risk factors. Selected copula families are explained in Appendix Table 56.

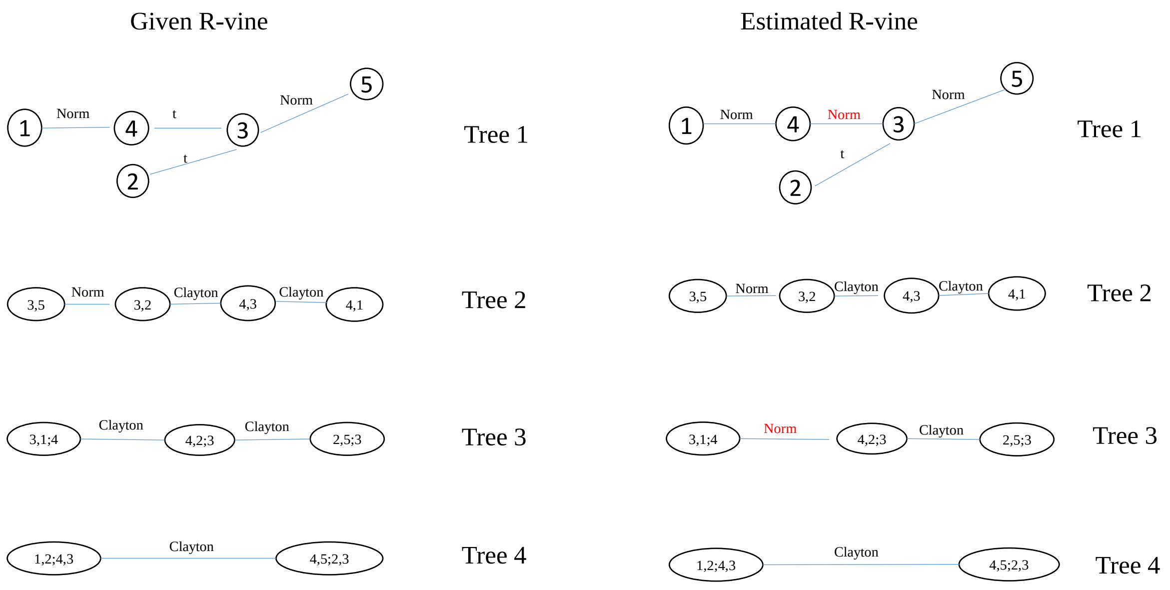


Figure 2.1: **Comparison of given vine structure with the estimated one**

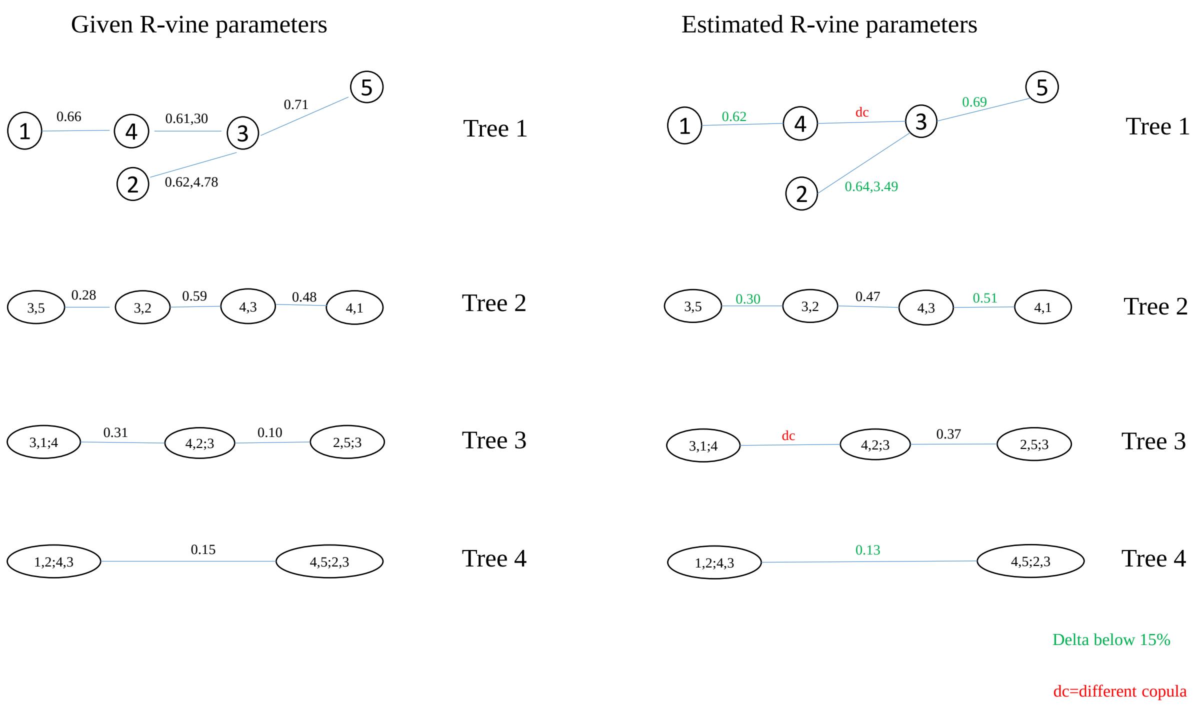


Figure 2.2: **Comparison of given bivariate copula parameters with the estimated one**

For all the 200 given vine samples, we sum up the entries of each five-dimensional vine sample to a single value describing the aggregate portfolio. Accordingly, we generate 200 vine samples from the fitted vine copula and also calculate the aggregate value of the samples. Then we plot the quantiles of the aggregate given vine sample against the quantiles of the aggregate sample from the fitted vine copula in Figure 3. On the aggregate portfolio level, we can see the overall fit perform rather good, which demonstrates that we can bear some deviations in bivariate copula parameters if the vine tree structure and bivariate copula families are well selected.

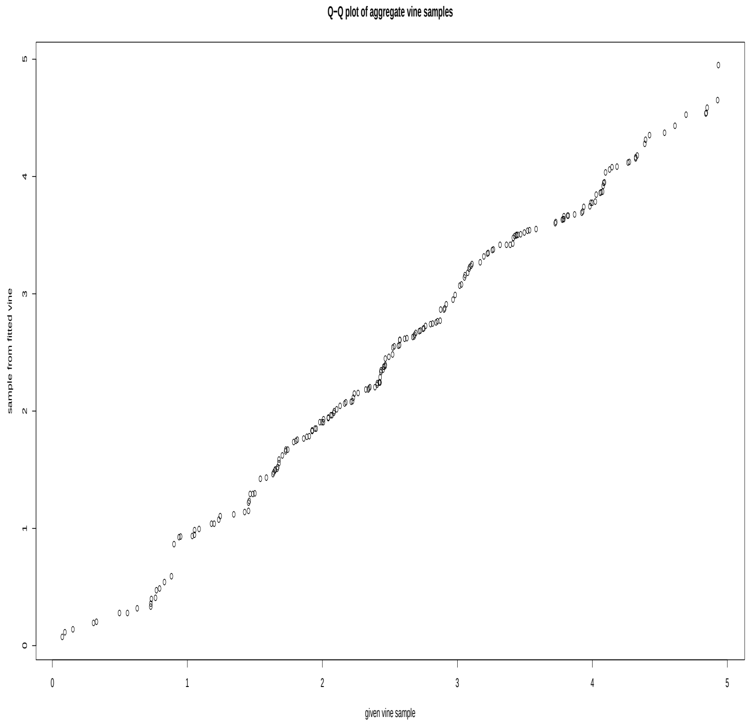


Figure 2.3: **Comparison of given vine copula with the estimated one on an aggregate level**

Conclusion cannot be simply drawn by a single simulation experiment, so that we repeat the simulation above many times. And the comparison of known vine to fitted vine copula on tree structure, bivariate copula families and copula parameters doesn't work on more than two or three simulation experiments, therefore, we need to set a single metric to check if the fitted copula close to the known copula. Hence we adopt the AIC ratio which defined as:

$$AIC \ \ ratio = \frac{AIC(fitted \ \ vine, \ \ 200 \ \ observations)}{AIC(known \ \ vine, \ \ 200 \ \ observations)} \tag{2.29}$$

The simulation experiment that we analyzed in detail has an AIC ratio of 99.87%, which serves as a reference point of interpreting AIC ratios. For investigating the impact of the number of observations N, we keep the copula dimension fixed at five, and repeat our simulation experiment 50 times for N = 100, N = 200 and N = 500 each. Table 3, in which AIC ratios averaged over the 50 iterations are displayed, indicating the estimation results become better when the number of observations increase. AIC ratio drops from 128.17% to 98.82% when N is increased from 100 to 500 (see left panel of Table 3). As a consequence, we can draw the conclusion that the rather good fit from the simulation experiment can be generalized, because the average AIC ratio of 50 iterations is very close to the AIC ratio of the single experiment (99.86% vs. 99.87%) for N = 200. We can generalize our conclusion from the single simulation experiment that in dimension five, 200 observations are sufficient to get a fairly good estimation result.

Table 2.3: **Impact of copula dimension and number of observation on estimation performance**

	Dimension = 5			N = 200		
	N = 100	N = 200	N = 500	dim = 5	dim = 10	dim = 16
Average AIC ratio	128.17%	99.86%	98.82%	99.86%	104.66%	108.35%

Note: This table shows AIC ratios averaged over 50 iterations for a different copula dimensions and numbers of observations.

Since dimension of five we analyzed above is clearly very low dimension from credit portfolio management perspective(Normally, it is impossible that there just five underlying assets in the credit portfolio in real world), we have to analyze how the estimation results behave when facing the high dimension case by vine copula. So that we fix the number of observations at N = 200, and increase the dimension to dim = 10 and dim = 16 (because we select 16 UK equity indices in subsequent empirical study). Note that N = 200 must not be mistaken for the absolute number of observations, but for the number of dim-dimensional observation vectors (e.g., in dim = 10, we have a 10×200 observations matrix and 2000 observations in absolute terms).

The results display the copula dimension hardly has large effect on estimation performance which is quite comforting for using vine copulas in high dimension of 92. When the copula dimension is increased from five to 16, the average AIC ratio changes a little (from 99.86% to 104.66%). We can easily explain the estimation's robustness towards the dimension by looking at the ground level trees of the vine. No matter what the dimension is, at each bivariate estimation step, there is a 200×2 observation matrix available to estimate the bivariate copula from. Obviously, the estimation results of the ground level trees do not deteriorate if we repeat the bivariate copula estimation with increasing dimension. The same argument holds for the higher level trees. Therefore, we can conclude that estimation performance increases with number of observations and that the copula dimensions have hardly any effect.

2.6 Data

Inferring credit or default correlations from the equity market data may seem problematic at a first glance. The classical Merton model advocates the use of asset values to calibrate credit correlations, nevertheless, even this method has its limitations. Moreover, owing to various problems concerning the accessibility of asset values, equity prices are widely adopted as a substitute. Many banks rely on equity data for the calibration of the dependence structure of their internal credit portfolio models. For instance, the correlation structure of CreditMetrics model is estimated via equity data and Hull and White indicate that default correlation between two companies is often assumed to be the same as the correlation between their equity returns. In addition, equity indices returns are also the important variable in predicting defaults, which provides an empirical link between equity data and credit risk. Therefore, from a practical point of view, despite the employment of equity returns as the substitute of asset returns is justified, it still a reasonable and feasible substitute.

Hence we consider an internationally diversified credit portfolio with K counterparties. It is assumed that the systematic risk of each counterparty is adequately described by a set of risk factors. We mimic Daul et al. (2003) who followed the risk factors selection of CreditMetrics handbook by Morgan (1997) in which they choose 92 country/industry equity indices from UK, France and Germany in Europe Area, US, Canada in North America, Japan in Asia and Australia. All these countries cover main areas around the world and they have comparatively complete industrial sectors. Since copula based models have been widely applied in the area of multivariate modelling of financial returns. Here we introduce high dimension vine copula to modelling dependence of these risk factors log returns.

In order to compare our vine copula model with other multivariate Gaussian copula based industry model, we also select 92 country/industry equity indices monthly returns as the risk factors similar with Daul et al. (2003) from January 2002 to December 2016. We then fit vine copulas and conventional Gaussian copulas separately to these monthly equity indices returns. The dependency among these log returns is then modeled using vine copula after a transformation to marginally uniform data using either an empirical or probability integral transformation. Since there has been empirical evidence that different asymmetric and tail dependencies are presented in different pairs of variables, which cannot be captured either by a multivariate Gaussian nor Student t copula with a common degree of freedom (Longin and Solnik (1995); Longin and Solnik (2001) and Ang and Bekaert (2002a)). In this context, D-vines have been employed modelling of financial returns successfully (Aas et al. (2009); Min and Czado (2010) and Mendes and De Melo (2010)), and also the C-vines have been applied (Czado (2010)). While in our question of portfolio credit risk modelling, it is hard to pre-determine the order of equity index risk factors, which is important for checking the dependence structure between each factors. Regarding the path structure of D-vine copula, one factor can only has no more than two connection with other two factors. Therefore, D-vine copula is not suitable for multi factor credit risk modelling because these risk factors are interconnected between each other. The C-vine copula is especially appropriate to be selected when there is a pivotal element among all the variables, otherwise, the C-vine tree structure will be somewhat restrictive. However, in our multi-factor analysis, one of our purpose is to check if there is a pivotal factor among the 92 risk factors, and compare the C-vine structure to other vine structure, so that we employ C-vine copula as a candidate tree structure. The R-vine tree structure, as the general form of C-vine and D-vine, have much more flexibility which is not restrict to pivotal element selection and no number of connection restriction, therefore, R-vine is naturally included in our candidate tree structure. These 92 equity indices are listed in Appendix Table 55.

We fit the different vine copulas to end-of-month equity log-returns of the time period from January 2002 to December 2016, which has 200 observations in total spanning the period of the global financial crisis of 2007-2009 and European sovereign debt crisis of 2010-2011. Equity returns are all obtained from Datastream. Descriptive statistics of returns are presented in Appendix Table 46-54. The skewness of the returns are non-zero while most of the kurtosis of the returns are significantly higher than 3 indicating that the empirical distributions of returns display heavy tails characteristics comparing to Gaussian distribution. The basic statistics and the p-values of JarqueBera test show solid evidence against the assumption of normality. Using the Ljung-Box Q-test, the null hypothesis of no autocorrelation is rejected at lag 10 for all the series. The ARCH test of Engle (1982) indicates the significance of ARCH effects in all the series. Overall, the descriptive statistics show the non-normality, asymmetry, autocorrelation and heteroscedasticity of equity returns.

According to Sklar theorem, before modelling the joint distribution of returns, the first step is to select a suitable model for the marginal return distribution, because misspecification of the univariate model probably result in biased copula parameter estimates. To allow for autocorrelation, heteroscedasticity and asymmetry, firstly, we adopt the Akaike Information Criterion (AIC) to select the optimal order of the AR model for the conditional mean up to order 10. Second, to allow for the heteroskedasticity of each series, we consider a group of GARCH models as candidates and find that the asymmetric GJR-GARCH model is preferred to the others based on their likelihood values. We then consider the GJR-GARCH class of up to order (2, 2, 2) and select the optimal order by using BIC. The model parameters are estimated by using maximum likelihood estimation (MLE) and the results of AR and GJR-GARCH estimations are presented in Table 4-16. We find that, for each series, the variance persistence implied by the model is close to 1. For all the series, the leverage effect parameters γ are significantly positive implying that a negative return on the series increases volatility more than a positive return with the same magnitude. The obvious skewness and high kurtosis of returns leads us to consider the skewed Student t distribution of Hansen (1994) for residual modelling. We report the estimation results also in Table 4-16. To evaluate the goodness-of-fit for the skewed Student t distribution, the Kolmogorov-Smirnov (KS) test are implemented and the p-values are reported in Table 4-16. Our results suggest that the skewed Student t distribution is suitable for residual modelling. Thus, in general, the diagnostics provide evidences that our marginal distribution models are well-specified and therefore, we can reliably use the combination of AR, GJR-GARCH and skewed Student t distribution, allied to vine copulas to model the dependence structure.

Table 2.4: **Estimation Results for Marginal Model**

	Australia						
	BANKS	MEDIA	ENERGY	INSURANCE	TRANS.	MATERIALS	PHARM.& BIOTECH.
AR							
a	0.45670	-0.1084059	0.877529	0.059300	0.50903	1.129681	1.57119**
	(0.51007)	(0.0312038)	(0.053113)	(0.107613)	(0.07420)	(0.023029)	(0.54839)
ar1	-0.03077	-0.0495858	-0.050380	0.013542	0.07585	-0.052436	-0.05539
	(0.07219)	(0.0026123)	(0.006392)	(0.012216)	(0.01086)	(0.002535)	(0.07348)
GARCH							
ω	3.61208	0.0257322	0.548927	0.258500	0.15470	0.458599	22.99136
	(2.61987)	(0.0043680)	(0.224877)	(0.100022)	(0.06834)	(0.361310)	(14.57553)
$\alpha1$	0.02944	0.0319285	0.168828*	0.074973**	0.08975*	0.126632	0.07512
	(0.07454)	(0.063267)	(0.068256)	(0.027669)	(0.04126)	(0.065752)	(0.12157)
$\gamma1$	1.00000	1.0000000	0.546847	1.000000	0.97565	0.849547 **	0.99625
	(2.19856)	(0.0001417)	(0.363171)	(0.003152)	(0.06667)	(0.258617)	(0.50950)
$\beta1$	0.87152	0.9629889	0.666309	0.865139	0.86449	0.578922*	0.41736
	(0.07824)	(0.0051405)	(0.097408)	(0.045177)	(0.05326)	(0.262107)	(0.41061)
δ	2.00000	0.1303805	0.474420	0.500987	0.34967	0.176808	2.00000
	(1.58150)	(0.317612)	(0.312591)	(0.323842)	(0.30335)	(0.439045)	(2.61766)
Skew t							
skew	0.72121	0.8001046	0.769795	0.693323	0.72326	0.857033	0.87276
	(0.08965)	(0.1033713)	(0.080517)	(0.076624)	(0.07057)	(0.119100)	(0.08721)
shape	5.1333 *	9.68800**	9.018690	9.54321	8.53631	9.768306*	9.27141
	(2.35937)	(3.6089666)	(5.002614)	(5.487742)	(5.14261)	(4.444043)	(4.79199)
K-S test	0.7112	0.7112	0.4375	0.6272	0.5441	0.06177	0.3275

This table reports parameter estimation from AR and GJR-GARCH models for conditional mean and conditional variance of risk factor log returns, their p-values list in the parenthesis. We estimate all parameters using the sample from January 2002 to December 2016, which correspond to a sample of 200 observations for 92 risk factors. We use * and ** to indicate the significance of estimate at the 5% and 10% significance level respectively. We also report the p-values of Kolmogorov-Smirnov (KS) test for the skewed Student t distribution.

Table 2.5: **Estimation Results for Marginal Model**

	Australia	Canada					
	RETALING	METALS& MINING	BANKS	TRANSPT	AUTO&COMPO	BCAST	CHEMICALS
AR							
a	0.13780	1.098039	0.59417	1.007213	0.46160	0.16737	0.965669
	(0.57937)	(0.042033)	(0.37182)	(0.130889)	(0.72782)	(0.63871)	(0.571614)
ar1	0.12661	-0.039205	0.02679	-0.044459*	0.14594	0.10743	0.008762
	(0.08928)	(0.005047)	(0.07030)	(0.020945)	(0.07733)	(0.078461)	(0.079572)
GARCH							
ω	10.80645	0.499368	0.37678*	0.468925*	5.23165	0.57607**	0.517838
	(6.22323)	(0.326960)	(0.16746)	(0.210322)	(6.49458)	(0.18826)	(0.462943)
$\alpha 1$	0.15396	0.106083**	0.09274**	0.128326**	0.04582	0.08211**	0.130330*
	(0.08884)	(0.034973)	(0.03103)	(0.047660)	(0.13897)	(0.02706)	(0.056646)
$\gamma 1$	0.23790	0.833255	1.00000	1.000000	1.00000	1.00000	0.201777
	(0.40496)	(0.231977)	(0.02849)	(0.003805)	(1.54532)	(0.02044)	(0.244522)
$\beta 1$	0.68762**	0.647009	0.85666	0.752294	0.84628	0.84719	0.860848
	(0.21480)	(0.196622)	(0.04806)	(0.096297)	(0.07707)	(0.06333)	(0.071032)
δ	2.00000	0.280927	0.91154	0.507873	2.00000	0.87602	1.160622
	(2.42424)	(0.57033)	(0.50054)	(0.313331)	(3.98874)	(0.50377)	(1.040065)
Skew t							
skew	0.64511	0.888214	0.75153	0.730786	0.78148	0.81818	0.853771
	(0.08488)	(0.100934)	(0.07491)	(0.089355)	(0.11168)	(0.03148)	(0.114896)
shape	9.58700*	9.015490*	9.54321	9.868407*	9.758306*	5.66047 *	7.010423
	(4.43553)	(4.453740)	(6.25409)	(4.163756)	(5.00085)	(2.34556)	(4.070995)
K-S test	0.9874	0.1638	0.27	0.9639	0.1122	0.2202	0.09959

This table reports parameter estimation from AR and GJR-GARCH models for conditional mean and conditional variance of risk factor log returns, their p-values list in the parenthesis. We estimate all parameters using the sample from January 2002 to December 2016, which correspond to a sample of 200 observations for 92 risk factors. We use * and ** to indicate the significance of estimate at the 5% and 10% significance level respectively. We also report the p-values of Kolmogorov-Smirnov (KS) test for the skewed Student t distribution.

Table 2.6: **Estimation Results for Marginal Model**

	Canada						
	INSURANCE	PHARMA.	FD/BEV/TOB	ELEC. COMP.& EQU	HT/REST/LEIS	ENERGY	MET & MIN
AR							
a	0.74442	0.42746	9.849e-01 *	-0.09105	1.0930350	0.876055	0.51325
	(0.44305)	(0.89611)	(3.970e-01)	(0.23860)	(0.0319512)	(0.509945)	(0.84943)
ar1	-0.04937	-0.02889	6.000e-02	0.01402	-0.0959986	0.005968	-0.07574
	(0.07077)	(0.06845)	(6.842e-02)	(0.07296)	(0.0045259)	(0.077219)	(0.09960)
GARCH							
ω	3.21086	23.07992	2.565e+01	1.47159	0.2026372**	6.047271*	10.44261
	(2.53094)	(29.88659)	(27.77458)	(1.69646)	(0.0672562)	(2.898743)	(20.08294)
$\alpha 1$	0.09721	0.02190	1.202e-01	0.29184*	0.0318580	0.155979*	0.08328
	(0.08365)	(0.15205)	(0.14106)	(0.12937)	(0.0203937)	(0.077127)	(0.06906)
$\gamma 1$	0.16640	1.00000	5.691e-01	0.17511	1.0000000	0.190918	0.20786
	(0.41984)	(7.49005)	(1.45375)	(0.30394)	(0.0005195)	(0.289849)	(1.34264)
$\beta 1$	0.82588	0.81978	1.000e-08	0.66033	0.8275492	0.689975	0.81819*
	(0.08664)	(0.20456)	(0.0403150)	(0.11626)	(0.0514370)	(0.118808)	(0.35649)
δ	2.00000	2.00000	2.000e+00	2.00000	0.1383896	1.791741	2.00000
	(1.15761)	(2.17478)	(1.672e+00)	(1.33050)	(0.1208172)	(1.107650)	(5.15409)
Skew t							
skew	0.82086	0.76196	9.671e-01	0.83928	0.7421260	0.787095	0.79199
	(0.08413)	(0.07820)	(9.286e-02)	(0.08080)	(0.0832353)	(0.096630)	(0.11927***)
shape	3.99637 **	4.21798**	7.857e+00	3.98874**	6.3961010 **	8.76453*	6.29978
	(1.22413)	(1.42912)	(4.315e+00)	(1.26793)	(1.9879910)	(4.688143)	(3.26414)
K-S test	0.3927	0.7112	0.3275	0.27	0.3927	0.142	0.1932

This table reports parameter estimation from AR and GJR-GARCH models for conditional mean and conditional variance of risk factor log returns, their p-values list in the parenthesis. We estimate all parameters using the sample from January 2002 to December 2016, which correspond to a sample of 200 observations for 92 risk factors. We use * and ** to indicate the significance of estimate at the 5% and 10% significance level respectively. We also report the p-values of Kolmogorov-Smirnov (KS) test for the skewed Student t distribution.

Table 2.7: **Estimation Results for Marginal Model**

	Canada		France				
	MEDIA	UTILITIES	FRANCE AUTO & PARTS	FRANCE BANKS	FRANCE CHEMICALS	FRANCE CON & MAT	FRANCE FD & DRUG RTL
AR							
a	-0.0182116	0.53078	0.373388	0.01683	0.75153	0.318271 **	-0.46324
	(0.0339581)	(0.39186)	(0.038532)	(0.54556)	(0.42293)	(0.099404)	(0.28594)
ar1	0.0641673	0.00198	-0.060644	0.05984	-0.23087 **	0.022382	-0.05132
	(0.0054671)	(0.08395)	(0.003569)	(0.07050)	(0.07471)	(0.013131)	(0.04689)
GARCH							
ω	0.1985082 **	3.77503	0.183961 *	0.65667 *	6.79739	0.447200 *	0.70151
	(0.0682603) **	(7.69393)	(0.072873)*	(0.30653)*	(4.27053)	(0.228107)*	(0.60448)
$\alpha 1$	0.0486755	0.17310	0.098619	0.13656 **	0.12994	0.121707 **	0.07573
	(0.0591951)	(0.16679)	(0.009097)	(0.04383)	(0.11746)	(0.043643)**	(0.04793)
$\gamma 1$	1.0000000	0.15816	0.974792	1.00000	0.28656	1.000000	1.00000
	(0.0005296)	(0.27923)	(0.079928)	(0.01742)	(0.47730)	(0.005656)	(0.01508)
$\beta 1$	0.8312335	0.62758	0.836579	0.83498	0.67728 **	0.788360	0.79176
	(0.0653003)	(0.69014)	(0.057774)	(0.04977)	(0.21551)	(0.085850)	(0.16928)
δ	0.1894172	1.76694	0.281086	1.05849*	2.00000	0.572161	0.71788
	(0.3259272)	(3.87831)	(0.2369383)	(0.48712)	(1.26231)	(0.363491)	(0.89431)
Skew t							
skew	0.6982688	0.86500	0.754807	0.74183	0.78854	0.692746	0.82662
	(0.0730302)	(0.10673)	(0.083636)	(0.08077)	(0.07917)	(0.080487)	(0.08672)
shape	8.2724507*	9.3834608*	6.784687**	5.47711 *	9.688081	9.589092	9.6987092
	(4.1888371)	(4.82469)	(2.389309)	(2.35535)	(6.14425)	(5.690091)	(5.40922)
K-S test	0.792	0.05224	0.3275	0.08787	0.2202	0.142	0.8643

This table reports parameter estimation from AR and GJR-GARCH models for conditional mean and conditional variance of risk factor log returns, their p-values list in the parenthesis. We estimate all parameters using the sample from January 2002 to December 2016, which correspond to a sample of 200 observations for 92 risk factors. We use * and ** to indicate the significance of estimate at the 5% and 10% significance level respectively. We also report the p-values of Kolmogorov-Smirnov (KS) test for the skewed Student t distribution.

Table 2.8: **Estimation Results for Marginal Model**

	Germany						
	AUTOMOBILE	CHEMICALS	CONSTRUCTION	INSURANCE	TRANSPORT & LOGIS.	UTILITIES	FINANCIAL SERVICES
AR							
a	0.833086	0.76023	0.77291	0.04863	0.03567	0.197087	0.64663
	(0.068425)	(0.57734)	(0.63104)	(0.11712)	(0.56083)	(0.149894)	(0.55051)
ar1	-0.011690	0.01699	0.06587	-0.02663*	0.05349	0.009367	0.09135
	(0.006896)	(0.07578)	(0.08026)	(0.01144)	(0.07510)	(0.023527)	(0.08328)
GARCH							
ω	0.909302	1.47533	4.35304 *	0.37767 **	4.03761	0.574589 **	7.73886
	(0.590477)	(0.86371)	(1.92453)*	(0.13832)**	(2.95067)	(0.215151)**	(4.66615)
$\alpha 1$	0.125999	0.08577	0.09923	0.14289 **	0.03904	0.114165*	0.08135
	(0.069210)	(0.06140)	(0.05576)	(0.05030)**	(0.08086)	(0.048242)*	(0.13997)
$\gamma 1$	0.999327	1.00000	1.00000	0.99856	1.00000	1.000000	1.00000
	(0.127704)	(0.11801)	(0.12010)	(0.756615)	(0.84424)	(0.007872)	(0.56014)
$\beta 1$	0.507004	0.80874	0.71908	0.79419	0.85919	0.763070	0.71749
	(0.286843)	(0.07677)	(0.11733)	(0.05534)	(0.06742)	(0.091468)	(0.16879)
δ	0.356898	1.31066	1.50088	0.51745	2.00000	0.580989	1.98279
	(0.317478)	(0.95942)	(0.82705)	(0.32746)	(2.04509)	(0.414403)	(2.49769)
Skew t							
skew	0.812307	0.76135	0.66979	0.71997	0.68746	0.762995	0.98211
	(0.081599)	(0.07839)	(0.10128)	(0.07391)	(0.08207)	(0.088471)	(0.10793)
shape	9.865011	8.61640	9.7693342	9.886538	5.99584	9.564411*	7.88564
	(7.651163)	(5.63776)	(5.51498)	(5.30552)	(3.59985)	(4.616435)*	(4.54418)
K-S test	0.3275	0.6272	0.9228	0.4653	0.7112	0.05084	0.5441

This table reports parameter estimation from AR and GJR-GARCH models for conditional mean and conditional variance of risk factor log returns, their p-values list in the parenthesis. We estimate all parameters using the sample from January 2002 to December 2016, which correspond to a sample of 200 observations for 92 risk factors. We use * and ** to indicate the significance of estimate at the 5% and 10% significance level respectively. We also report the p-values of Kolmogorov-Smirnov (KS) test for the skewed Student t distribution.

Table 2.9: **Estimation Results for Marginal Model**

	Germany			Japan			
	FOOD & BEVERAGES	TECHNOLOGY	MEDIA	BANKS	CONSTRUCTION	INFO & COMMUNICATION	INSURANCE
AR							
a	0.38041 *	-0.140424	-0.19232	-0.4190312	0.669667	0.05301	0.36191
	(0.15924) *	(0.293990)	(0.61927)	(0.0374839)***	(0.419059)	(0.36297)	(0.50519)
ar1	0.03295	0.132207 *	0.19283 **	-0.0235927	-0.004003	-0.08058	-0.09310
	(0.03785)	(0.067094)*	(0.07004)	(0.0036746)	(0.088479)	(0.07397)	(0.07584)
GARCH							
ω	0.09920	0.198318 *	15.36220	0.0266711	4.472979	1.92848	12.25627
	(0.07695)	(0.091361) *	(9.50445)	(0.016584)	(3.922202)	(1.811200)	(8.43232)
$\alpha 1$	0.06570 *	0.104289	0.12377	0.0147308 **	0.248054	0.08582	0.15669
	(0.02900)	(0.023287)	(0.15347)	(0.0037004) **	(0.128324)	(0.13216)	(0.08578)
$\gamma 1$	0.95462	1.000000	1.00000	1.0000000	0.157349	0.28440	0.20123
	(0.85407)	(0.005612)	(1.10663)	(0.0002161)	(0.197929)	(0.03584)	(0.25862)
$\beta 1$	0.93668	0.895230	0.64100 **	0.9697624	0.612637 **	0.84196	0.64328
	(0.03289)	(0.028105)	(0.19490) **	(0.0037943)	(0.220929)**	(0.07588)	(0.19145)
δ	0.68065	0.628398	2.00000 *	0.0985919	1.817309 *	2.00000	2.00000
	(2.66726)	(0.326207)	(0.98906)	(0.001)	(0.923404)	(0.001)	(2.68877)
Skew t							
skew	0.77932	0.718401	0.86085	1.0186868	1.179874	0.90095	0.85284
	(0.09382)	(0.083067)	(0.08847)	(0.0788871)	(0.140477)	(0.08998)	(0.10011)
shape	9.64235	9.78954 *	4.02756 **	4.2954199 **	7.227208 *	8.95406	9.77443*
	(6.59523)	(4.680673)	(1.24153)	(1.4823691)**	(3.557318) *	(5.30260)	(4.24875)
K-S test	0.7112	0.5441	0.3968	0.2984	0.05224	0.06809	0.9639

This table reports parameter estimation from AR and GJR-GARCH models for conditional mean and conditional variance of risk factor log returns, their p-values list in the parenthesis. We estimate all parameters using the sample from January 2002 to December 2016, which correspond to a sample of 200 observations for 92 risk factors. We use * and ** to indicate the significance of estimate at the 5% and 10% significance level respectively. We also report the p-values of Kolmogorov-Smirnov (KS) test for the skewed Student t distribution.

Table 2.10: **Estimation Results for Marginal Model**

	Japan						
	MACHINERY	MINING	PHARMACEUTICAL	PULP & PAPER	ELEC.POWER & GAS	OIL & COAL PRDS.	CHEMICAL
AR							
a	0.52387	0.180229	0.106540	-0.32299	-0.106734	0.24452	0.22635
	(0.46481)	(0.608701)	(0.024248)	(0.48797)	(0.057727)	(0.52734)	(0.34797)
ar1	0.15617 *	0.007176	-0.119072	-0.09958	-0.080812	-0.04442	0.02806
	(0.07669)	(0.080195)	(0.004752)	(0.07310)	(0.008231)	(0.07457)	(0.07622)
GARCH							
ω	7.31841	18.182273	0.305024	18.13569	0.689604	3.69913	0.14252
	(4.63531)	(13.304480)	(0.230924)	(26.22149)	(0.001)	(3.99875)	(0.10676)
$\alpha 1$	0.14277	0.113266	0.085916	0.22066	0.201973	0.05330	0.03598 *
	(0.10857)	(0.087305)	(0.069392)	(0.12029)	(0.054635)	(0.05144)	(0.01599)
$\gamma 1$	0.18113	0.131464	0.866561 *	0.12628	0.120666	0.22455	1.00000
	(0.32353)	(0.350804)	(0.369174)	(0.32308)	(0.001)	(0.44540)	(0.01366)
$\beta 1$	0.67002	0.654702 **	0.724652	0.44459	0.560206	0.87823	0.93339
	(0.17685)	(0.203296)	(0.184141)	(0.69192)	(0.111991)	(0.09117)	(0.03791)
δ	2.00000	2.000000	0.234523	2.00000	0.508813	2.00000	0.75867
	(1.17204)	(1.233688)	(0.292068)	(2.44450)	(0.001)	(1.70075)	(0.68803)
Skew t							
skew	0.88677	0.934403	0.869571	1.17185	0.819515	0.89543	0.90152
	(0.10898)	(0.117444)	(0.097619)	(0.11361)	(0.075776)	(0.10477)	(0.08925)
shape	9.847200 *	9.56345 *	6.947608 **	5.20348 *	5.279515 **	9.568703 *	8.99654
	(4.39964)	(4.479625)	(2.435153)	(2.39648)	(1.719174)	(4.24371)	(5.58833)
K-S test	0.792	0.1638	0.3927	0.1777	0.3927	0.27	0.08787

This table reports parameter estimation from AR and GJR-GARCH models for conditional mean and conditional variance of risk factor log returns, their p-values list in the parenthesis. We estimate all parameters using the sample from January 2002 to December 2016, which correspond to a sample of 200 observations for 92 risk factors. We use * and ** to indicate the significance of estimate at the 5% and 10% significance level respectively. We also report the p-values of Kolmogorov-Smirnov (KS) test for the skewed Student t distribution.

Table 2.11: **Estimation Results for Marginal Model**

	Japan				UK		
	ELECTRIC APPLIANCES	FOODS	TEXTILES AND APPARELS	TRANSPORT EQU.	BANKS	AUTO & PARTS	CHEMICALS
AR							
a	-0.520250	0.34582	2.805e-01	0.17036	-0.475848	0.2785194	0.792663
	(0.063896)	(0.31324)	(3.962e-01)	(0.39751)	(0.067276)	(0.0367505)	(0.057917)
ar1	0.060755 **	-0.01316	4.945e-02	0.09544	0.052257 **	-0.0082446	-0.012689
	(0.019567)	(0.08596)	(9.225e-02)	(0.07049)	(0.008141)	(0.0046182)	(0.007329)
GARCH							
ω	0.263889 *	5.42804 *	1.528e+01	0.84813	0.120684	0.0835012 **	0.124717
	(0.123215)	(2.50094)	(1.231e+01)	(0.73240)	(0.064682)	(0.0308615)	(0.023104)
$\alpha 1$	0.083280 *	0.27412	2.960e-01	0.02402	0.084156	0.0539341	0.026647
	(0.036714)	(0.14772)	(1.512e-01)	(0.03840)	(0.108073)	(0.001)	(0.006073)
$\gamma 1$	1.000000	0.19333	7.117e-03	1.00000 **	0.999860	0.9999997	1.000000
	(0.004021)	(0.20581)	(3.197e-01)	(0.30811)	(0.001564)	(0.0003524)	(0.000167)
$\beta 1$	0.851205	0.48804 *	1.000e-08	0.91516	0.883185	0.9196917	0.859719
	(0.059508)	(0.19328)*	(6.037e-01)	(0.06640)	(0.038598)	(0.0258122)	(0.022364)
δ	0.497126	2.00000 *	1.751e+00	1.58159	0.289822	0.2009111	0.025032 **
	(0.356351)	(0.88043)*	(1.786e+00)	(2.08383)	(0.639316)	(0.001)	(0.007964)
Skew t							
skew	0.741734	0.90115	1.060e+00	0.84464	0.772102	0.8502853	0.859696
	(0.081707)	(0.09785)	(1.088e-01)	(0.08815)	(0.106154)	(0.0880822)	(0.099872)
shape	9.67221 *	9.77754	7.542e+00	9.883245	9.91654 *	6.5708108 *	6.681560
	(4.425696)	(5.38607)	(4.128e+00)	(5.71605)	(4.185186)	(3.2132231)	(3.838833)
K-S test	0.2222	0.142	0.8643	0.9639	0.06177	0.7112	0.27

This table reports parameter estimation from AR and GJR-GARCH models for conditional mean and conditional variance of risk factor log returns, their p-values list in the parenthesis. We estimate all parameters using the sample from January 2002 to December 2016, which correspond to a sample of 200 observations for 92 risk factors. We use * and ** to indicate the significance of estimate at the 5% and 10% significance level respectively. We also report the p-values of Kolmogorov-Smirnov (KS) test for the skewed Student t distribution.

Table 2.12: **Estimation Results for Marginal Model**

	UK						
	CON & MAT	ELTRO/ELEC EQ	FD PRODUCERS	FORESTRY & PAP	H/C EQ & SVS	INDS TRANSPT	MEDIA
AR							
a	0.74255	-0.25604	0.4541632	0.17114	0.450303	0.05271	-0.019333
	(0.46409)	(0.63677)	(0.001)	(0.49734)	(0.444080)	(0.45531)	(0.415090)
ar1	0.01971	-0.03708	-0.0634430	0.01146	-0.001988	0.13902 *	-0.044424
	(0.07270)	(0.05912)	(0.001)	(0.05955)	(0.065804)	(0.07067)	(0.071694)
GARCH							
ω	11.24237	0.31469 0	0.1078420	0.23469	5.991478	2.92377	0.107015 *
	(6.57037)	(0.55519)	(0.0202800)	(0.17607)	(6.421191)	(1.92092)	(0.052740)
$\alpha 1$	0.04121	0.02461	0.0229328	0.12819 *	0.027292	0.02814	0.059652
	(0.001)	(0.01897)	(0.001)	(0.05555)	(0.057037)	(0.04956)	(0.017808)
$\gamma 1$	1.00000	0.99957	1.0000000	0.97150	0.992487	1.00000	1.000000
	(0.93717)	(0.52968)	(0.0004257)	(0.13438)	(0.856234)	(1.02652)	(0.004301)
$\beta 1$	0.66170	0.94764	0.9020182	0.85362	0.790899	0.87528	0.927668
	(0.09639)	(0.01593)	(0.0218763)	(0.08159)	(0.210848)	(0.05771)	(0.022962)
δ	2.00000	1.79305 **	0.1334707	0.52002	2.000000	2.00000	0.584564
	(0.001)	(0.60985)	(0.001)	(0.30960)	(2.209671)	(1.52441)	(0.344682)
Skew t							
skew	0.77062	0.61877	0.8862561	0.87916	0.821365	0.78443	0.711611
	(0.08489)	(0.06861)	(0.0770774)	(0.08729)	(0.081928)	(0.09776)	(0.076399)
shape	9.55634	5.00847 **	9.80347	4.44724 **	5.290602 *	6.05035 *	9.667312
	(5.94712)	(1.79677)	(1.7188505)	(1.43118)	(2.199799)	(3.01716)	(6.775597)
K-S test	0.2984	0.1777	0.9228	0.1777	0.9228	0.5441	0.2222

This table reports parameter estimation from AR and GJR-GARCH models for conditional mean and conditional variance of risk factor log returns, their p-values list in the parenthesis. We estimate all parameters using the sample from January 2002 to December 2016, which correspond to a sample of 200 observations for 92 risk factors. We use * and ** to indicate the significance of estimate at the 5% and 10% significance level respectively. We also report the p-values of Kolmogorov-Smirnov (KS) test for the skewed Student t distribution.

Table 2.13: **Estimation Results for Marginal Models**

	UK						US
	MINING	OIL & GAS PROD	PHARM & BIO	S/W & COMP SVS	TRAVEL & LEIS	LIFE INSURANCE	AUTOMOBILES
AR							
a	0.706358	0.01178	-0.05040	0.1803808	4.922e-01	1.721e-01	-0.733219
	(0.025513)	(0.45652)	(0.33638)	(0.1018025)	(3.859e-02)	(6.918e-03)	(0.586435)
ar1	-0.110424	-0.22839 **	-0.15557 *	-0.0219238	7.208e-02	8.040e-02	0.001812
	(0.002663)	(0.08298)	(0.07708)	(0.0120232)	(6.089e-03)	(1.159e-03)	(0.077408)
GARCH							
ω	0.214017 **	1.26736	3.37547	0.1481400	7.961e-02	1.126e-01	3.069714
	(0.080344)	(0.87115)	(3.08144)	(0.1437837)	(8.030e-03)	(7.710e-03)	(2.084633)
$\alpha 1$	0.122103 *	0.07529	0.03783	0.0808972	2.277e-02	2.290e-02	0.053681
	(0.052785)	(0.04860)	(0.06123)	(0.1836149)	(0.001)	(0.001)	(0.062065)
$\gamma 1$	0.912096	1.00000	1.00000	1.0000000	1.000e+00	1.000e+00	1.000000
	(0.132340)	(0.04516)	(0.74734)	(0.0007788)	(2.988e-04)	(1.462e-04)	(0.575680)
$\beta 1$	0.755159	0.78458	0.78586	0.8770812	9.250e-01	8.730e-01	0.863325
	(0.075473)	(0.15474)	(0.18279)	(0.0600394)	(1.271e-02)	(1.050e-02)	(0.077379)
δ	0.163990	1.14896	2.00000	0.3011731	1.329e-01	2.041e-02	2.000000
	(0.147762)	(1.70722)	(1.69682)	(1.0949732)	(0.001)	(3.400e-04)	(1.406654)
Skew t							
skew	0.840190	0.83349	0.89726	0.8288519	6.946e-01	6.579e-01	0.904518
	(0.115690)	(0.08941)	(0.08291)	(0.0867455)	(9.005e-02)	(0.001)	(0.101606)
shape	7.183876 **	8.87661	9.16873	8.2942704	1.000e+01 *	1.000e+01	9.88745
	(2.317458)	(5.79534)	(7.32774)	(5.3847538)	(5.050e+00)	(0.001)	(6.754818)
K-S test	0.3968	0.3275	0.9228	0.08787	0.9228	0.9228	0.142

This table reports parameter estimation from AR and GJR-GARCH models for conditional mean and conditional variance of risk factor log returns, their p-values list in the parenthesis. We estimate all parameters using the sample from January 2002 to December 2016, which correspond to a sample of 200 observations for 92 risk factors. We use * and ** to indicate the significance of estimate at the 5% and 10% significance level respectively. We also report the p-values of Kolmogorov-Smirnov (KS) test for the skewed Student t distribution.

Table 2.14: **Estimation Results for Marginal Model**

	US						
	BANKS	BCAST	CHEMICALS	INSURANCE	MACHINERY	TRANSPORTATION	CONSTRUCTION MATERIALS
AR							
a	0.12186	0.11574	0.62875 *	0.10199	0.78090	0.593497 **	0.65748
	(0.35883)	(0.51362)	(0.31824)	(0.37940)	(0.46078)	(0.184226)	(0.57690)
ar1	-0.13843 *	-0.05400	-0.15351 **	-0.05829	-0.13286	-0.009009	-0.04355
	(0.06389)	(0.08633)	(0.05557)	(0.07349)	(0.07717)	(0.027211)	(0.07198)
GARCH							
ω	0.28061	12.13941 *	1.05447 **	7.25712 *	2.07550	0.764012	5.54530
	(0.14561)	(5.92015)	(0.40625)	(3.27406)	(1.28008)	(0.526501)	(3.20575)
$\alpha 1$	0.07946 *	0.18420	0.18580	0.08956	0.06578	0.126774 *	0.04177
	(0.03187)*	(0.16414)	(0.05519)	(0.08757)	(0.04672)	(0.063674)	(0.05832)
$\gamma 1$	1.00000	0.66724	1.00000	0.91035	1.00000 *	1.000000	1.00000
	(0.05185)	(0.72487)	(0.01938)	(0.73060)	(0.40041)	(0.008644)	(1.03098)
$\beta 1$	0.89879	0.58615 *	0.66822	0.61252 **	0.82662	0.659779 **	0.84119
	(0.02640)	(0.23228)*	(0.10134)	(0.19141)	(0.08052)	(0.235526)	(0.06652)
δ	1.23892 *	2.00000	1.03050	2.00000	1.68187 *	0.599932	2.00000
	(0.48944)	(1.53311)	(0.60262)	(1.15035)	(0.77656)	(0.564505)	(1.05661)
Skew t							
skew	0.60704	0.79190	0.80219	0.68635	0.61679	0.759219	0.85887
	(0.06120)	(0.08534)	(0.08370)	(0.06461)	(0.09172)	(0.087166)	(0.09956)
shape	5.02345 **	6.21557 *	8.65789	4.81430 *	8.76223 *	8.22765	9.85278
	(1.77922)	(2.75822)	(5.62063)	(2.23116)	(4.35905)	(6.283526)	(7.22331)
K-S test	0.05224	0.2222	0.8643	0.1195	0.4653	0.6272	0.4653

This table reports parameter estimation from AR and GJR-GARCH models for conditional mean and conditional variance of risk factor log returns, their p-values list in the parenthesis. We estimate all parameters using the sample from January 2002 to December 2016, which correspond to a sample of 200 observations for 92 risk factors. We use * and ** to indicate the significance of estimate at the 5% and 10% significance level respectively. We also report the p-values of Kolmogorov-Smirnov (KS) test for the skewed Student t distribution.

Table 2.15: **Estimation Results for Marginal Model**

	US						
	FOOD PRODUCTS	METALS & MINING	ELECTRICAL COMP & EQUIP	TEXTILES & APPAREL	UTILITIES	PUBLISHING &PRINTING	ENERGY
AR							
a	0.76701 **	0.098672	0.66911	0.2698976	0.6322169	-0.03870	0.63392
	(0.26218)	(0.111137)	(0.44043)	(0.0170099)	(0.0568292)	(0.37626)	(0.41141)
ar1	-0.10304	-0.082286	-0.14221	-0.0720872	-0.0265335 **	0.07999	-0.11365
	(0.06821)	(0.009722)	(0.09728)	(0.0034332)	(0.0086379)	(0.08062)	(0.07147)
GARCH							
ω	2.21980	0.343181	13.03586	0.0271950	0.1287594	0.40521	0.97576 *
	(1.58245)	(0.393690)	(3.92017)	(0.0064676)	(0.0666699)	(0.50540)	(0.43518)
$\alpha 1$	0.08626	0.111245	0.31322	0.0539448	0.0302446	0.16970 **	0.08345 *
	(0.06006)	(0.151421)	(0.16274)	(0.001)	(0.0579780)	(0.06109)	(0.03467)
$\gamma 1$	0.09825	0.827306	0.30085	0.9999677	0.9999998	0.42629	1.00000
	(0.27413)	(0.464818)	(0.32063)	(0.0003686)	(0.0008088)	(0.32416)	(0.01139)
$\beta 1$	0.76065	0.762361 **	0.47215 *	0.9594537	0.8893876	0.84078	0.67993
	(0.14295)	(0.272257)	(0.19451)	(0.0083607)	(0.0333414)	(0.06774)	(0.13891)
δ	2.00000	0.355359	1.99470	0.2787755	0.2042096	1.55938	0.71341
	(1.37711)	(1.069653)	(1.81240)	(0.001)	(0.6637106)	(1.67168)	(0.72911)
Skew t							
skew	0.84655	0.790940	0.50608	0.5763635	0.7197873	0.90661	0.69714
	(0.08346)	(0.122918)	(0.11502)	(0.0793941)	(0.0679317)	(0.08555)	(0.07648)
shape	4.86250 **	8.67713	8.12765 **	7.0122789 **	7.5694118	8.59425	8.48098
	(1.79985)	(5.122434)	(3.72318)	(2.6746387)	(4.2771394)	(6.08556)	(6.16261)
K-S test	0.27	0.1777	0.6272	0.9228	0.9228	0.05224	0.7112

This table reports parameter estimation from AR and GJR-GARCH models for conditional mean and conditional variance of risk factor log returns, their p-values list in the parenthesis. We estimate all parameters using the sample from January 2002 to December 2016, which correspond to a sample of 200 observations for 92 risk factors. We use * and ** to indicate the significance of estimate at the 5% and 10% significance level respectively. We also report the p-values of Kolmogorov-Smirnov (KS) test for the skewed Student t distribution.

Table 2.16: **Estimation Results for Marginal Model**

	US							
	HOTELS REST & LEISURE IN	HEALTH CARE EQUIP & SERV	OIL & GAS REFING & MARK	SOFTWARE & SERVICES	TELECOM SERV	AIRLINES	MOVIES & ENTERTAINMENT	PAPER PACKAGING
AR								
a	5.582e-01	0.80805 **	1.039854	0.36876	-0.1537190	-0.10020	0.06950	0.363132
	(8.202e-02)	(0.31186)	(0.625288)	(0.32064)	(0.001)	(0.58235)	(0.43041)	(0.043107)
ar1	2.477e-02	-0.02629	-0.009416	-0.08437	0.0105817	0.12652	0.01608	-0.105499
	(1.586e-02)	(0.08094)	(0.070823)	(0.06149)	(0.001)	(0.07654)	(0.07558)	(0.006592)
GARCH								
ω	4.352e-02	6.38900 *	6.098403	0.92462 *	0.1234391	2.17067	7.25163 *	0.246322
	(1.090e-02)	(2.96227)	(5.727967)	(0.41134)	(0.0151067)	(1.27509)	(3.11176)	(0.056698)
$\alpha1$	4.606e-02	0.06570	0.111256	0.16266	0.0365286	0.12403*	0.12157	0.077757
	(NA)	(0.08622)	(0.068657)	(0.04645)	(0.001)	(0.04896)	(0.20466)	(0.001)
$\gamma1$	1.000e+00	1.00000	0.080159	1.00000	1.0000000	1.00000	1.00000	0.999994
	(3.334e-04)	(0.66954)	(0.255513)	(0.04651)	(0.0004141)	(0.001)	(1.65141)	(0.001681)
$\beta1$	9.510e-01	0.50895 *	0.816788	0.72443	0.8880068	0.69889	0.63010	0.830097
	(1.238e-02)	(0.25274)	(0.087584)	(0.09775)	(0.0242214)	(0.13976)	(0.14785)	(0.060110)
δ	2.631e-01	2.00000	2.000000	1.03096	0.1688294	1.06772	2.00000	0.336041
	(NA)	(1.24746)	(1.335886)	(0.60482)	(0.001)	(0.58947)	(1.05792)	(0.001)
Skew t								
skew	6.211e-01	0.68003	0.773079	0.61449	0.8640050	0.83093	0.74691	0.805329
	(7.786e-02)	(0.07633)	(0.077747)	(0.07438)	(0.0925968)	(0.10347)	(0.08020)	(0.102194)
shape	1.000e+01	9.11674	8.62453	9.04532 *	9.10465 *	8.67120 *	9.85702	8.133729 **
	(5.410e+00)	(7.66931)	(6.269791)	(4.38116)	(4.2069342)	(4.09763)	(6.47483)	(3.109171)
K-S test	0.4653	0.142	0.9972	0.2222	0.3927	0.1122	0.125	0.05224

This table reports parameter estimation from AR and GJR-GARCH models for conditional mean and conditional variance of risk factor log returns, their p-values list in the parenthesis. We estimate all parameters using the sample from January 2002 to December 2016, which correspond to a sample of 200 observations for 92 risk factors. We use * and ** to indicate the significance of estimate at the 5% and 10% significance level respectively. We also report the p-values of Kolmogorov-Smirnov (KS) test for the skewed Student t distribution.

2.7 Application of Vine Copula to Credit Portfolio

2.7.1 Selection and Estimation of Vine Copula Models

To investigate the practical consequences of using vine copulas compared to conventional multivariate Gaussian copula, we consider a credit portfolio of 92 risk factors represented by industry sector equity indices downloaded from Datastream database. We fit the different vine copulas to end-of-month equity log-returns of the time period from January 2002 to December 2016, which has 200 observations in total.

For model selection we want to demonstrate the superior fit of vine copulas with individually chosen pair-copula families and assess the gain over vine copula with only bivariate Student *t*, with only Gaussian pair-copula as well as over C-vine mixed copula and R-vine mixed copula model. We need to select a bivariate copula for every pair of variables. In this study, we take into consideration of the following copula: Gaussian/Normal (tail symmetric, no tail dependence), Student *t* (tail symmetric, symmetric tail dependence), and Clayton copula (tail asymmetric, low tail dependence) and their corresponding Survival and Rotated version. (See Appendix Table 56.) Given these bivariate copula options we still have to decide which copula fits ”best”. In this case, we adopt the AIC (Akaike (1974)) criteria which corrects the log likelihood of a copula for the number of parameters. Bivariate copula selection using the AIC has previously investigated by Manner (2007) and Brechmann (2010) who find that it is quite reliable criterion, particularly in comparison to alternative criteria such as copula goodness-of-fit tests. Selection proceeds by computing the AICs for each possible family and then choosing the copula with smallest AIC.

In order to investigate which type of vine copula model is preferred to describe the dependence of risk factors, we employ two likelihood ratio based goodness-of-fit test-Vuong test and Clarke test, to compare multivariate Gaussian copula with other vine copula models. Therefore, we set

Null hypothesis: M1 = Multivariate Gaussian copula

Alternatives: M2 = R-vine *t* copula, R-vine mixed copula, C-vine *t* copula, C-vine mixed copula.

multivariate Gaussian copula ($R-vine\ \ Gaussian\ \ copula$): R-vine with each pair-copula terms chosen as bivariate Gaussian copula, i.e., this corresponds to a multivariate Gaussian copula, where unconditional correlations can be obtained from conditional ones by inverting a generalized version.

$R-vine\ \ t\ \ copula$: R-vine with each pair-copula terms chosen as bivariate Student t copula. If the degrees of freedom parameter of a pair is estimated to be larger than 30, we set the copula to be the Gaussian.

$R-vine\ \ mixed\ \ copula$: R-vine with pair-copula terms chosen individually from six bivariate copula types (Gauss, Student t, Clayton, survival Clayton, rotated Clayton (90° and 270°)).

$C-vine\ \ t\ \ copula$: C-vine with each pair-copula terms chosen as bivariate Student t copula. If the degrees of freedom parameter of a pair is estimated to be larger than 30, we set the copula to be the Gaussian.

$C-vine\ \ mixed\ \ copula$: C-vine with pair-copula terms chosen individually from six bivariate copula types (see above).

The likelihood-ratio based test proposed by Vuong (1989) can be used for comparing non-nested models. For this let c_1 and c_2 be two competing vine copulas in terms of their densities and with estimated parameter sets θ_1 and θ_2. We then compute the standardized sum, ν, of the log differences of their pointwise likelihoods $m_i := log[\frac{c_1(u_i|\hat{\theta}_1)}{c_2(u_i|\hat{\theta}_2)}]$ for observations $u_i \in [0,1]$, $i = 1, ..., N$, i.e.,

$$statistic := \nu = \frac{\frac{1}{n}\sum_{i=1}^{N} m_i}{\sqrt{\sum_{i=1}^{N}(m_i - \overline{m})^2}} \tag{2.30}$$

Vuong (1989) shows that ν is asymptotically standard normal. According to the null-hypothesis

$$H_0 : E[m_i] = 0 \ \ \forall i = 1, ..., N, \tag{2.31}$$

we hence prefer vine model 1 to vine model 2 at level α if

$$\nu > \Phi^{-1}(1 - \frac{\alpha}{2}), \tag{2.32}$$

where Φ^{-1} denotes the inverse of the standard normal distribution function. If $\nu < -\Phi^{-1}(1 - \frac{\alpha}{2})$, we choose model 2. If, however, $|\nu| < \Phi^{-1}(1 - \frac{\alpha}{2})$, no decision among the models is possible.

Like AIC and BIC, the Vuong test statistic may be corrected for the number of parameters used in the models. There are two possible corrections, the Akaike and the Schwarz corrections, which correspond to the penalty terms in the AIC and the BIC, respectively.

Table 2.17: **Vuong test results**

	Multivariate Gaussian	R-vine t	R-vine mixed	C-vine t	C-vine mixed
Log likelihood	10808.04	10215.88	12254.29	11503.89	12306.09
Vuong tests					
no correction		4.051351	-9.135201	-6.430959	-9.168386
		(5.092265e-05)	(6.528561e-20)	(1.268011e-10)	(4.801543e-20)
Akaike corr.		32.69053	-7.347637	32.25543	-7.460845
		(0.00)	(2.017413e-13)	(0.00)	(8.596902e-14)
Schwarz corr.		79.92107	-4.399661	96.05543	-4.64484
		(0.00)	(1.0842e-05)	(0.00)	(3.403395e-06)

Note: Log likelihoods for all models as well as results of the Vuong tests (test statistics and p-values in parentheses) comparing the multivariate Gaussian copula model to all other vine copula models. The negative values of Vuong test statistics indicate that the test favors the R-vine and C-vine mixed copula model over other alternative models.

Table 2.18: **Clarke test results**

	Multivariate Gaussian	R-vine t	R-vine mixed	C-vine t	C-vine mixed
Log likelihood	10808.04	10215.88	12254.29	11503.89	12306.09
Clarke tests					
no correction		110	15	52	12
		(0.178964)	(1.979423e-38)	(7.261224e-12)	(8.113776e-42)
Akaike corr.		200	21	199	17
		(0.00)	(1.946966e-32)	(0.00)	(2.508977e-36)
Schwarz corr.		200	40	200	40
		(0.00)	(3.384016e-18)	(0.00)	(3.384016e-18)

Note: Log likelihoods for all models as well as results of the Clarke tests (test statistics and p-values in parentheses) comparing the multivariate Gaussian copula model to all other vine copula models. The smaller values of Clarke test statistics indicate that the test favors the R-vine and C-vine mixed copula model over other alternative models.

The test proposed by Clarke (2007) also allows to compare non-nested models. For this model, let c_1 and c_2 be two competing vine copulas in terms of their densities and with estimated parameter sets $\hat{\theta}_1$ and $\hat{\theta}_2$. The null hypothesis of statistical indistinguishability of the two models is

$$H_0 : P(m_i > 0) = 0.5 \ \ \forall i = 1, ..., N, \tag{2.33}$$

where $m_i := log[\frac{c_1(u_i|\hat{\theta}_1)}{c_2(u_i|\hat{\theta}_2)}]$ for observations u_i; $i = 1, ..., N$.

Since under statistical equivalence of the two models, the log likelihood ratios of the single observations are uniformly distributed around zero and in expectation 50% of the log likelihood ratios greater than zero, the test statistic

$$statistic := B = \sum_{i=1}^{N} \mathbf{1}_{(0,\infty)}(m_i), \tag{2.34}$$

where **1** is the indicator function, which follows Binomial distribution with parameters N and p=0.5, and critical values can easily be obtained. Model 1 is interpreted as statistically equivalent to model 2 if B is not significantly different from the expected value $N_p = N/2$.

Like AIC and BIC, the Clarke test statistic also may be corrected for the number of parameters used in the models. There are two possible corrections, the Akaike and the Schwarz corrections, which correspond to the penalty terms in the AIC and the BIC, respectively.

Vuong test copula selection results for all models are summarized in Table 17. The first row gives the log likelihood after joint optimization of the chosen regular vine tree specification and copula types. From the results of log likelihood, the value of C-vine mixed copula log likelihood is larger than other copula models, which means the C-vine mixed copula is superior to other model. And the second row gives the test statistics together with the p-values in parentheses of a Vuong test with and without Akaike and Schwarz corrections, respectively, testing the multivariate Gaussian model against the alternative vine copula setting indicated by the respective column. From the Vuong tests results we see that only the C-vine mixed copula and the R-vine mixed copula have all negative values of Vuong test statistics, according to Vuong test criterion, hence the C-vine mixed copula and the R-vine mixed copula are to be preferred over other vine copula setting and multivariate Gaussian copula. Overall Vuong test demonstrates the usefulness of vine copula with individually chosen copula types for each pair copula term.

Clarke test copula selection results for all models are summarized in Table 18. We also list in the first row the log likelihood value after joint optimization of the chosen regular vine tree specification and copula types. And the second row gives the test statistics together with the p-values in parentheses of a Clarke test with and without Akaike and Schwarz corrections, respectively, also testing the multivariate Gaussian model against the alternative vine copula setting indicated by the respective column. From the Clarke tests we see that the C-vine mixed copula and the R-vine mixed copula have the smallest values of Clarke test statistics, according to Clarke test criterion, hence the C-vine mixed copula and the R-vine mixed copula are to be preferred over other vine copula setting and multivariate Gaussian copula. Overall Clarke test again demonstrates the usefulness of vine copula with individually chosen copula types for each pair copula term.

As mentioned above, five different vine copula models including R-vine Gaussian, R-vine t, R-vine mixed, C-vine t and C-vine mixed are estimated for our credit portfolio. The selection of copula families for each pair-copula in the vine structure specification is mainly based on the Akaike Information Criterion (AIC). There are two main reasons behind this choice. Firstly, it is practically impossible, in high dimension case, for one to investigate every single unconditional and conditional pair-copula in the vine structure and define accordingly an appropriate copula family for each of these pairs. As a result, we adopt the AIC, which is the most frequently used criterion in copula selection literature. The range of all possible copula families chosen from by the criterion is defined in Appendix Table 56. The second main reason that drives our copula selection strategy is related to the theoretical and empirical results of the studies by Joe et al. (2010) and Nikoloulopoulos et al. (2012).

Based on our Vuong test and Clarke test results, we present the maximum likelihood estimation results of C-vine mixed copula model and R-vine mixed copula model, their Kendall's τ, and upper and lower tail dependence parameters of first three level trees in Table 19-24.

Joe et al. (2010) indicate that vine copulas can have a different upper and lower tail dependence for each bivariate margin when asymmetric bivariate copulas with upper/lower tail dependence are used in tree 1 of the vine. In other words, in order for a vine copula to have tail dependence for all bivariate margins, it is necessary for the bivariate copulas in tree 1 to have tail dependence but it is not necessary for the conditional bivariate copulas in trees $2, ..., d-1$ to have tail dependence. At trees 2 or higher, Gaussian copulas might be adequate to model the dependency structure. Moreover, Nikoloulopoulos et al. (2012) show that vine copulas with bivariate Student t linking copulas tend to be preferred in likelihood-based selection methods because they provide a better fit in the middle for the first level of the vine. They suggest that for inference involving the tails, the ”best-fitting” copula should not be entirely likelihood-based but also depend on matching the non-parametric tail dependence measures and extreme quantiles. Taking these results into account, based on above Vuong test and Clarke test results, we both consider C-vine mixed copula and R-vine mixed copula model selected by goodness-of-fit test and AIC, and try to compare and verify if our results are in line with Joe et al. (2010) and Nikoloulopoulos et al. (2012). We expect to get more accurate risk measure estimates from these vine copula models that allow asymmetries.

After filtering the original return series with the appropriate ARMA-GJR-GARCH models, the resulting standardised residual series are transformed to uniform pseudo observations. From the results of Table 19-24, we find the majority of the selected copula families correspond to the Student t copula in Tree 1 of R-vine mixed copula setting. In particular, 5 out of 8 selected copula families in Tree 1 belong to the Student t copula. The empirical results of our R-vine mixed likelihood-based copula selection procedure seem to agree with the empirical findings of Nikoloulopoulos et al. (2012). While not in line with Nikoloulopoulos et al. (2012), Clayton copula takes up largest percentage of the selected copula families in Tree 1 of C-vine mixed copula setting. The reason probably is that, with respect to C-vine copula structure definition, when fitting C-vine copula, a pivotal factor should be selected in first step. If this pivotal factor has an asymmetric dependence with remaining factors, the asymmetric dependence bivariate copula, such as Clayton copula, would naturally be selected as bivariate margin. Due to the characteristics of low tail dependence, most frequent bivariate margin Clayton copula in C-vine mixed copula Tree 1, is able to more precisely capture the dependence between number 27 risk factor, which is the pivotal factor, and other factors, therefore, C-vine mixed copula setting outperform the R-vine mixed setting, the better performance of C-vine mixed copula can also find support from their likelihood values results. In R-vine structure, a pivotal factor is not required and a more general vine structure can be fitted to data, therefore, the dependence structure of factors in Tree 1 is described by various and more diversified bivariate copulas, just as our empirical results demonstrated in Tree 1 R-vine copula parameters estimation. For levels $2, ..., d-1$, the selection of the appropriate copula family is based on the AIC. Regarding Tree 2 and 3 we list, more asymmetric bivariate copulas are selected as bivariate margin both in C-vine mixed copula and R-vine mixed copula model setting. Though previous research demonstrate that in Tree 2 till Tree $d-1$, the asymmetric bivariate copulas are not necessary, but the supply of asymmetric bivariate copulas in our model make our Tree $2, ..., d-1$ dependence structure be more precisely described.

In sum, from our goodness-of-fit results, which is actually surprising that given the more flexibility in the tree structures, the R-vine tree structure underperform the C-vine tree structure. From the statistical fit point of view, we believe C-vine mixed copula model can better fit to our data. Because a pivotal factor is required to be selected among our data sets which would affects each remaining factor. Nevertheless, when we investigate this problem in depth, we find the pivotal element selected is the number 27 factor, which is France Construction and Materials sector. Economically speaking, it is hard to believe this factor can determinate and make extensive effects on all other industry sectors. Therefore, whether the C-vine mixed copula is superior to R-vine mixed copula in fitting to our data sets is still justified and need further exploitation, such as comparison of the accuracy of the estimate of risk measure in subsequent section. The Student *t* copula is the most frequent chosen copula among all bivariate copula families of Tree 1 in both C-vine mixed copula and R-vine mixed copula setting. Therefore, the advantage of vine copulas does not come solely from the flexible tree structure, but the flexibility of mixing different bivariate copula families as their building blocks is used to beat the classical Gaussian and Student *t* copula.

2.7.2 Homogeneous Credit Portfolio

Though the goodness-of-fit test in previous section indicate that C-vine mixed copula and R-vine mixed copula model are the "best fitting" model for our risk factors log returns data, in further step, with respect to credit risk management, what we would like to know is whether these two vine copula setting can help us to improve the computation of risk measure, such as VaR and CVaR, in comparison to traditional multivariate Gaussian copula setting. In addition, whether the performance of the C-vine mixed copula and R-vine mixed copula in estimating VaR and CVaR are in line with the goodness-of-fit test results and which of the two model performs better. In order to answer these questions, in this section, we employ C-vine mixed copula and R-vine mixed copula including other vine structure model and multivariate Gaussian copula model to separately estimate risk measure of VaR and CVaR for our credit portfolios. We both consider homogeneous credit portfolio and heterogeneous credit portfolio, and small portfolio and large portfolio separately.

As described in above sections, we consider the underlying portfolio of 92 equity indices, using data from January 2002 to December 2016. One year default probabilities are implied from credit default swap spreads. And assume a multi factor model with a set of sector factors Z_S representing the systematic risk of industry sectors.

In homogeneous credit portfolio setting, now we mimic the numerical examples of

Table 2.19: **Tree 1 Parameters Estimation of R-vine mixed Copula Model**

tree	edge	No.	family	par	par2	tau	UTD	LTD
1	51,47	2	t	0.51	5.14	0.34	0.21	0.21
	51,45	1	N	0.75	0.00	0.54	-	-
	91,89	1	N	0.56	0.00	0.38	-	-
	72,83	1	N	0.62	0.00	0.43	-	-
	91,72	2	t	0.77	21.63	0.56	0.10	0.10
	91,88	2	t	0.70	5.24	0.49	0.33	0.33
	22,13	2	t	0.80	2.94	0.60	0.55	0.55
	10,15	2	t	0.74	7.64	0.53	0.28	0.28
	2,91	1	N	0.66	0.00	0.46	-	-
	4,7	2	t	0.63	30.00	0.43	0.01	0.01
	41,51	1	N	0.66	0.00	0.46	-	-
	49,41	1	N	0.62	0.00	0.43	-	-
	49,46	2	t	0.55	4.88	0.37	0.24	0.24
	39,42	1	N	0.78	0.00	0.57	-	-
	40,39	2	t	0.68	7.24	0.48	0.24	0.24
	52,40	1	N	0.68	0.00	0.48	-	-
	49,52	2	t	0.82	3.37	0.61	0.54	0.54
	43,49	1	N	0.87	0.00	0.68	-	-
	50,53	1	N	0.72	0.00	0.51	-	-
	48,44	1	N	0.73	0.00	0.52	-	-
	43,48	1	N	0.61	0.00	0.42	-	-
	50,43	1	N	0.83	0.00	0.63	-	-
	37,50	3	C	1.16	0.00	0.37	-	0.55
	27,28	3	C	1.23	0.00	0.38	-	0.57
	4,1	2	t	0.73	4.86	0.52	0.38	0.38
	5,4	2	t	0.70	4.87	0.49	0.35	0.35
	23,17	3	C	1.08	0.00	0.35	-	0.53
	84,65	2	t	0.82	7.30	0.62	0.40	0.40
	23,82	1	N	0.51	0.00	0.34	-	-
	84,87	1	N	0.65	0.00	0.45	-	-
	20,84	1	N	0.85	0.00	0.65	-	-
	20,23	2	t	0.63	8.00	0.43	0.19	0.19
	20,18	2	t	0.66	9.71	0.46	0.17	0.17
	3,20	1	N	0.76	0.00	0.55	-	-
	6,3	2	t	0.81	8.93	0.60	0.33	0.33
	8,5	2	t	0.74	5.58	0.53	0.35	0.35
	2,8	2	t	0.69	5.33	0.49	0.32	0.32
	9,6	1	N	0.99	0.00	0.93	-	-
	63,2	1	N	0.67	0.00	0.46	-	-
	63,61	1	N	0.53	0.00	0.36	-	-
	22,10	3	C	1.49	0.00	0.43	-	0.63
	63,22	3	C	1.38	0.00	0.41	-	0.60
	29,55	1	N	0.68	0.00	0.47	-	-
	24,29	1	N	0.80	0.00	0.59	-	-
	63,60	2	t	0.66	12.42	0.46	0.12	0.12
	58,16	2	t	0.41	4.71	0.27	0.18	0.18

63,38	1	N	0.66	0.00	0.46	-	-
27,31	2	t	0.77	6.66	0.56	0.35	0.35
24,37	2	t	0.70	4.94	0.50	0.35	0.35
35,36	1	N	0.45	0.00	0.30	-	-
63,35	1	N	0.72	0.00	0.51	-	-
27,24	2	t	0.75	10.09	0.54	0.23	0.23
86,78	3	C	0.93	0.00	0.32	-	0.48
74,71	1	N	0.68	0.00	0.48	-	-
74,86	1	N	0.63	0.00	0.43	-	-
27,33	2	t	0.75	4.52	0.54	0.42	0.42
63,56	1	N	0.72	0.00	0.51	-	-
63,58	1	N	0.72	0.00	0.52	-	-
68,62	2	t	0.70	5.25	0.50	0.34	0.34
63,67	1	N	0.73	0.00	0.52	-	-
32,74	1	N	0.64	0.00	0.44	-	-
68,59	3	C	1.21	0.00	0.38	-	0.56
27,57	1	N	0.71	0.00	0.50	-	-
63,27	1	N	0.77	0.00	0.56	-	-
63,68	1	N	0.82	0.00	0.61	-	-
69,63	2	t	0.80	6.94	0.59	0.38	0.38
54,25	1	N	0.74	0.00	0.53	-	-
69,54	3	C	2.05	0.00	0.51	-	0.71
32,69	2	t	0.78	8.84	0.57	0.30	0.30
30,32	2	t	0.76	10.80	0.55	0.23	0.23
64,9	1	N	0.91	0.00	0.73	-	-
30,66	2	t	0.46	5.14	0.31	0.18	0.18
30,34	3	C	1.25	0.00	0.38	-	0.57
30,26	2	t	0.77	5.01	0.56	0.41	0.41
75,80	1	N	0.85	0.00	0.64	-	-
76,90	2	t	0.66	7.87	0.46	0.21	0.21
76,11	2	t	0.73	5.13	0.52	0.36	0.36
85,19	3	C	1.24	0.00	0.38	-	0.57
12,70	3	C	1.34	0.00	0.40	-	0.60
79,64	1	N	0.84	0.00	0.64	-	-
73,14	1	N	0.62	0.00	0.43	-	-
73,30	2	t	0.71	30.00	0.50	0.03	0.03
75,12	1	N	0.66	0.00	0.46	-	-
85,81	2	t	0.70	30.00	0.49	0.02	0.02
76,85	3	C	1.44	0.00	0.42	-	0.62
75,76	2	t	0.73	6.38	0.52	0.31	0.31
79,21	2	t	0.78	3.60	0.57	0.49	0.49
75,79	1	N	0.71	0.00	0.50	-	-
75,73	1	N	0.79	0.00	0.58	-	-
75,77	1	N	0.56	0.00	0.38	-	-
92,75	2	t	0.67	5.19	0.46	0.31	0.31

Note: This table lists estimated Tree 1 parameters of R-vine mixed copula model fitted to 92 risk factors. Selected copula families are explained in Appendix Table 56.

Table 2.20: **Tree 2 Parameters Estimation of R-vine mixed Copula Model**

tree	edge	No.	family	par	par2	tau	UTD	LTD
2	45,47;51	3	C	0.11	0.00	0.05	-	0.00
	41,45;51	1	N	0.21	0.00	0.13	-	-
	88,89;91	3	C	0.36	0.00	0.15	-	0.15
	91,83;72	1	N	0.23	0.00	0.15	-	-
	2,72;91	3	C	0.39	0.00	0.16	-	0.17
	2,88;91	1	N	0.33	0.00	0.21	-	-
	63,13;22	33	C270	-0.08	0.00	-0.04	-	-
	22,15;10	3	C	0.38	0.00	0.16	-	0.16
	63,91;2	3	C	0.64	0.00	0.24	-	0.34
	1,7;4	2	t	0.15	7.69	0.10	0.03	0.03
	49,51;41	1	N	0.36	0.00	0.23	-	-
	46,41;49	1	N	0.28	0.00	0.18	-	-
	52,46;49	13	SC	0.27	0.00	0.12	0.08	-
	40,42;39	1	N	0.32	0.00	0.21	-	-
	52,39;40	2	t	0.31	7.50	0.20	0.07	0.07
	49,40;52	13	SC	0.43	0.00	0.18	0.20	-
	43,52;49	3	C	0.38	0.00	0.16	-	0.16
	50,49;43	1	N	0.37	0.00	0.24	-	-
	43,53;50	2	t	0.26	6.09	0.17	0.08	0.08
	43,44;48	13	SC	0.36	0.00	0.15	0.15	-
	50,48;43	23	C90	-0.19	0.00	-0.09	-	-
	37,43;50	3	C	0.10	0.00	0.05	-	0.00
	24,50;37	3	C	0.14	0.00	0.07	-	0.01
	24,28;27	1	N	0.16	0.00	0.10	-	-
	5,1;4	1	N	0.39	0.00	0.25	-	-
	8,4;5	3	C	0.48	0.00	0.19	-	0.23
	20,17;23	3	C	0.26	0.00	0.12	-	0.07
	20,65;84	1	N	0.27	0.00	0.17	-	-
	20,82;23	3	C	0.23	0.00	0.10	-	0.05
	20,87;84	2	t	0.02	4.24	0.01	0.07	0.07
	23,84;20	1	N	-0.15	0.00	-0.10	-	-
	18,23;20	2	t	0.35	8.50	0.23	0.06	0.06
	3,18;20	2	t	0.16	4.66	0.10	0.09	0.09
	6,20;3	1	N	0.21	0.00	0.13	-	-
	9,3;6	2	t	-0.17	7.31	-0.11	0.01	0.01
	2,5;8	3	C	0.50	0.00	0.20	-	0.25
	63,8;2	3	C	0.38	0.00	0.16	-	0.16
	64,6;9	1	N	-0.13	0.00	-0.08	-	-
	22,2;63	1	N	0.32	0.00	0.21	-	-
	35,61;63	1	N	0.22	0.00	0.14	-	-
	63,10;22	2	t	0.15	8.55	0.10	0.02	0.02
	35,22;63	2	t	0.30	7.10	0.19	0.07	0.07
	24,55;29	1	N	0.24	0.00	0.15	-	-
	27,29;24	1	N	0.26	0.00	0.17	-	-
	56,60;63	2	t	0.20	11.16	0.13	0.01	0.01

63,16;58	1	N	0.11	0.00	0.07	-	-
35,38;63	3	C	0.37	0.00	0.16	-	0.16
33,31;27	3	C	0.41	0.00	0.17	-	0.18
27,37;24	3	C	0.50	0.00	0.20	-	0.25
63,36;35	1	N	0.14	0.00	0.09	-	-
27,35;63	1	N	0.42	0.00	0.27	-	-
33,24;27	2	t	0.34	6.91	0.22	0.08	0.08
74,78;86	2	t	0.21	3.57	0.13	0.15	0.15
86,71;74	13	SC	0.12	0.00	0.06	0.00	-
32,86;74	13	SC	0.15	0.00	0.07	0.01	-
63,33;27	2	t	0.38	5.99	0.25	0.12	0.12
58,56;63	1	N	0.38	0.00	0.25	-	-
67,58;63	1	N	0.37	0.00	0.24	-	-
63,62;68	3	C	0.21	0.00	0.10	-	0.04
68,67;63	1	N	0.31	0.00	0.20	-	-
69,74;32	2	t	0.27	6.66	0.17	0.07	0.07
63,59;68	3	C	0.14	0.00	0.07	-	0.01
63,57;27	1	N	0.28	0.00	0.18	-	-
69,27;63	2	t	0.31	9.75	0.20	0.04	0.04
69,68;63	1	N	0.34	0.00	0.22	-	-
54,63;69	1	N	0.17	0.00	0.11	-	-
69,25;54	1	N	0.30	0.00	0.19	-	-
32,54;69	1	N	0.15	0.00	0.09	-	-
30,69;32	3	C	0.27	0.00	0.12	-	0.08
26,32;30	3	C	0.44	0.00	0.18	-	0.21
79,9;64	1	N	0.29	0.00	0.19	-	-
34,66;30	3	C	0.20	0.00	0.09	-	0.03
26,34;30	13	SC	0.31	0.00	0.13	0.11	-
73,26;30	1	N	0.21	0.00	0.13	-	-
76,80;75	3	C	0.43	0.00	0.18	-	0.20
11,90;76	23	C90	-0.29	0.00	-0.13	-	-
75,11;76	13	SC	0.31	0.00	0.13	0.11	-
76,19;85	3	C	0.32	0.00	0.14	-	0.11
75,70;12	1	N	0.32	0.00	0.21	-	-
21,64;79	1	N	0.12	0.00	0.08	-	-
30,14;73	1	N	0.19	0.00	0.12	-	-
75,30;73	3	C	0.49	0.00	0.20	-	0.24
76,12;75	1	N	0.19	0.00	0.12	-	-
76,81;85	3	C	0.26	0.00	0.12	-	0.07
75,85;76	3	C	0.39	0.00	0.16	-	0.17
73,76;75	2	t	0.29	5.69	0.19	0.10	0.10
75,21;79	33	C270	-0.40	0.00	-0.17	-	-
73,79;75	1	N	0.26	0.00	0.17	-	-
92,73;75	13	SC	0.38	0.00	0.16	0.16	-
92,77;75	3	C	0.42	0.00	0.17	-	0.19

Note: This table lists estimated Tree 2 parameters of R-vine mixed copula model fitted to 92 risk factors. Selected copula families are explained in Appendix Table 56.

Table 2.21: **Tree 3 Parameters Estimation of R-vine mixed Copula Model**

tree	edge	No.	family	par	par2	tau	UTD	LTD
3	41,47;45,51	13	SC	0.11	0.00	0.05	0.00	-
	49,45;41,51	13	SC	0.13	0.00	0.06	0.00	-
	2,89;88,91	3	C	0.23	0.00	0.10	-	0.05
	2,83;91,72	2	t	0.23	7.34	0.15	0.05	0.05
	63,72;2,91	13	SC	0.12	0.00	0.06	0.00	-
	63,88;2,91	3	C	0.24	0.00	0.11	-	0.05
	35,13;63,22	1	N	0.05	0.00	0.03	-	-
	63,15;22,10	1	N	0.24	0.00	0.15	-	-
	22,91;63,2	3	C	0.29	0.00	0.13	- 0.09	
	5,7;1,4	1	N	0.11	0.00	0.07	-	-
	46,51;49,41	1	N	0.17	0.00	0.11	-	-
	52,41;46,49	13	SC	0.07	0.00	0.03	0.00	-
	40,46;52,49	1	N	0.18	0.00	0.12	-	-
	52,42;40,39	2	t	0.11	6.03	0.07	0.05	0.05
	49,39;52,40	13	SC	0.20	0.00	0.09	0.03	-
	43,40;49,52	3	C	0.12	0.00	0.06	-	0.00
	50,52;43,49	33	C270	-0.17	0.00	-0.08	-	-
	48,49;50,43	1	N	0.12	0.00	0.08	-	-
	48,53;43,50	23	C90	-0.14	0.00	-0.07	-	-
	50,44;43,48	33	C270	-0.12	0.00	-0.06	-	-
	37,48;50,43	1	N	-0.07	0.00	-0.04	-	-
	24,43;37,50	1	N	0.08	0.00	0.05	-	-
	27,50;24,37	1	N	0.12	0.00	0.07	-	-
	29,28;24,27	2	t	0.11	5.40	0.07	0.06	0.06
	8,1;5,4	2	t	0.18	21.68	0.11	0.00	0.00
	2,4;8,5	13	SC	0.33	0.00	0.14	0.12	-
	84,17;20,23	3	C	0.09	0.00	0.04	-	0.00
	23,65;20,84	3	C	0.15	0.00	0.07	-	0.01
	84,82;20,23	3	C	0.40	0.00	0.17	-	0.18
	23,87;20,84	23	C90	-0.17	0.00	-0.08	-	-
	18,84;23,20	3	C	0.24	0.00	0.11	-	0.05
	3,23;18,20	33	C270	-0.04	0.00	-0.02	-	-
	6,18;3,20	2	t	0.22	5.57	0.14	0.08	0.08
	9,20;6,3	3	C	0.17	0.00	0.08	-	0.02
	64,3;9,6	23	C90	-0.12	0.00	-0.06	-	-
	63,5;2,8	1	N	0.24	0.00	0.15	-	-
	22,8;63,2	13	SC	0.20	0.00	0.09	0.03	-
	79,6;64,9	13	SC	0.10	0.00	0.05	0.00	-
	10,2;22,63	3	C	0.18	0.00	0.08	-	0.02
	22,61;35,63	23	C90	-0.10	0.00	-0.05	-	-
	35,10;63,22	13	SC	0.17	0.00	0.08	0.02	-
	36,22;35,63	3	C	0.09	0.00	0.04	-	0.00
	27,55;24,29	3	C	0.36	0.00	0.15	-	0.14
	37,29;27,24	3	C	0.20	0.00	0.09	-	0.03

58,60;56,63	33	C270	-0.10	0.00	-0.05	-	-
56,16;63,58	33	C270	-0.11	0.00	-0.05	-	-
27,38;35,63	3	C	0.16	0.00	0.07	-	0.01
24,31;33,27	3	C	0.32	0.00	0.14	-	0.11
33,37;27,24	1	N	0.19	0.00	0.12	-	-
27,36;63,35	1	N	0.02	0.00	0.01	-	-
33,35;27,63	1	N	0.22	0.00	0.14	-	-
63,24;33,27	1	N	0.09	0.00	0.06	-	-
32,78;74,86	3	C	0.25	0.00	0.11	-	0.06
32,71;86,74	1	N	-0.04	0.00	-0.03	-	-
69,86;32,74	1	N	0.15	0.00	0.10	-	-
69,33;63,27	3	C	0.44	0.00	0.18	-	0.21
67,56;58,63	3	C	0.15	0.00	0.07	-	0.01
68,58;67,63	3	C	0.05	0.00	0.02	-	0.00
67,62;63,68	1	N	0.15	0.00	0.10	-	-
69,67;68,63	1	N	0.12	0.00	0.07	-	-
54,74;69,32	3	C	0.35	0.00	0.15	-	0.14
69,59;63,68	13	SC	0.28	0.00	0.12	0.08	-
69,57;63,27	1	N	0.23	0.00	0.15	-	-
68,27;69,63	13	SC	0.21	0.00	0.09	0.04	-
54,68;69,63	3	C	0.23	0.00	0.10	-	0.05
25,63;54,69	3	C	0.10	0.00	0.05	-	0.00
32,25;69,54	2	t	0.30	4.99	0.20	0.12	0.12
30,54;32,69	3	C	0.13	0.00	0.06	-	0.01
26,69;30,32	2	t	0.11	3.97	0.07	0.10	0.10
73,32;26,30	13	SC	0.25	0.00	0.11	0.06	-
21,9;79,64	3	C	0.20	0.00	0.09	-	0.03
26,66;34,30	3	C	0.20	0.00	0.09	-	0.03
73,34;26,30	33	C270	-0.05	0.00	-0.02	-	-
75,26;73,30	1	N	-0.08	0.00	-0.05	-	-
11,80;76,75	2	t	0.16	5.94	0.10	0.06	0.06
75,90;11,76	33	C270	-0.10	0.00	-0.05	-	-
73,11;75,76	3	C	0.17	0.00	0.08	-	0.02
75,19;76,85	3	C	0.26	0.00	0.12	-	0.07
76,70;75,12	1	N	0.10	0.00	0.06	-	-
75,64;21,79	1	N	0.12	0.00	0.08	-	-
75,14;30,73	2	t	0.10	4.30	0.07	0.09	0.09
79,30;75,73	1	N	0.16	0.00	0.10	-	-
85,12;76,75	3	C	0.17	0.00	0.08	-	0.02
75,81;76,85	1	N	0.03	0.00	0.02	-	-
73,85;75,76	13	SC	0.19	0.00	0.09	0.03	-
92,76;73,75	1	N	0.18	0.00	0.11	-	-
73,21;75,79	1	N	-0.13	0.00	-0.08	-	-
92,79;73,75	3	C	0.16	0.00	0.07	-	0.01
77,73;92,75	3	C	0.28	0.00	0.12	-	0.08

Note: This table lists estimated Tree 3 parameters of R-vine mixed copula model fitted to 92 risk factors. Selected copula families are explained in Appendix Table 56.

Table 2.22: **Tree 1 Parameters Estimation of C-vine mixed Copula Model**

tree	edge	No.	family	par	par2	tau	UTD	LTD
1	27,83	1	N	0.52	0.00	0.35	-	-
	27,30	2	t	0.74	30.00	0.53	0.04	0.04
	27,3	1	N	0.55	0.00	0.37	-	-
	27,35	1	N	0.74	0.00	0.53	-	-
	27,62	3	C	1.15	0.00	0.37	-	0.55
	27,11	1	N	0.51	0.00	0.34	-	-
	27,17	3	C	0.69	0.00	0.26	-	0.37
	27,87	3	C	0.77	0.00	0.28	-	0.41
	27,25	1	N	0.72	0.00	0.51	-	-
	27,53	1	N	0.44	0.00	0.29	-	-
	27,18	1	N	0.46	0.00	0.30	-	-
	27,37	3	C	1.34	0.00	0.40	-	0.60
	27,61	1	N	0.47	0.00	0.31	-	-
	27,14	2	t	0.46	6.72	0.30	0.13	0.13
	27,79	2	t	0.61	10.56	0.42	0.12	0.12
	27,68	1	N	0.73	0.00	0.52	-	-
	27,42	3	C	0.60	0.00	0.23	-	0.31
	27,70	1	N	0.53	0.00	0.35	-	-
	27,16	2	t	0.29	4.19	0.19	0.15	0.15
	27,88	2	t	0.58	30.00	0.39	0.01	0.01
	27,45	3	C	0.18	0.00	0.08	-	0.02
	27,81	1	N	0.45	0.00	0.30	-	-
	27,41	1	N	0.30	0.00	0.19	-	-
	27,90	3	C	0.33	0.00	0.14	-	0.12
	27,65	3	C	0.97	0.00	0.33	-	0.49
	27,49	3	C	0.76	0.00	0.28	-	0.40
	27,59	3	C	0.97	0.00	0.33	-	0.49
	27,82	3	C	0.51	0.00	0.20	-	0.25
	27,28	3	C	1.23	0.00	0.38	-	0.57
	27,58	1	N	0.62	0.00	0.42	-	-
	27,77	3	C	0.97	0.00	0.33	-	0.49
	27,36	2	t	0.39	4.66	0.25	0.17	0.17
	27,5	1	N	0.60	0.00	0.41	-	-
	27,31	2	t	0.77	6.66	0.56	0.35	0.35
	27,7	3	C	0.69	0.00	0.26	-	0.37
	27,10	1	N	0.52	0.00	0.35	-	-
	27,80	3	C	1.24	0.00	0.38	-	0.57
	27,46	3	C	0.44	0.00	0.18	-	0.21
	27,91	1	N	0.59	0.00	0.40	-	-
	27,24	2	t	0.75	10.09	0.54	0.23	0.23
	27,43	3	C	0.80	0.00	0.28	-	0.42
	27,34	2	t	0.61	8.56	0.42	0.16	0.16
	27,55	1	N	0.65	0.00	0.45	-	-
	27,89	1	N	0.43	0.00	0.29	-	-
	27,48	3	C	0.47	0.00	0.19	-	0.23
	27,20	2	t	0.54	30.00	0.36	0.00	0.00

27,9	1	N	0.58	0.00	0.39	-	-
27,54	1	N	0.66	0.00	0.46	-	-
27,33	2	t	0.75	4.52	0.54	0.42	0.42
27,19	1	N	0.48	0.00	0.32	-	-
27,47	13	SC	0.13	0.00	0.06	0.00	-
27,86	3	C	0.67	0.00	0.25	-	0.36
27,4	2	t	0.61	30.00	0.42	0.01	0.01
27,12	1	N	0.50	0.00	0.33	-	-
27,38	1	N	0.61	0.00	0.42	-	-
27,57	1	N	0.71	0.00	0.50	-	-
27,78	3	C	0.79	0.00	0.28	-	0.42
27,13	1	N	0.42	0.00	0.28	-	-
27,8	3	C	1.10	0.00	0.35	-	0.53
27,32	3	C	1.43	0.00	0.42	-	0.62
27,56	2	t	0.69	30.00	0.48	0.02	0.02
27,75	3	C	1.43	0.00	0.42	-	0.62
27,64	1	N	0.58	0.00	0.40	-	-
27,60	1	N	0.56	0.00	0.38	-	-
27,71	1	N	0.39	0.00	0.26	-	-
27,44	3	C	0.43	0.00	0.18	-	0.20
27,66	1	N	0.40	0.00	0.26	-	-
27,52	3	C	0.58	0.00	0.22	-	0.30
27,72	1	N	0.53	0.00	0.35	-	-
27,67	2	t	0.63	30.00	0.43	0.01	0.01
27,85	3	C	1.00	0.00	0.33	-	0.50
27,21	1	N	0.39	0.00	0.26	-	-
27,26	1	N	0.70	0.00	0.49	-	-
27,15	3	C	0.97	0.00	0.33	-	0.49
27,39	1	N	0.27	0.00	0.17	-	-
27,29	1	N	0.70	0.00	0.49	-	-
27,1	3	C	0.87	0.00	0.30	-	0.45
27,23	2	t	0.42	8.20	0.27	0.08	0.08
27,74	2	t	0.55	5.20	0.37	0.23	0.23
27,63	1	N	0.77	0.00	0.56	-	-
27,40	2	t	0.30	8.56	0.19	0.05	0.05
27,2	2	t	0.61	26.51	0.41	0.01	0.01
27,84	3	C	0.93	0.00	0.32	-	0.47
27,76	3	C	0.91	0.00	0.31	-	0.47
27,69	2	t	0.73	12.52	0.52	0.17	0.17
27,50	3	C	0.88	0.00	0.30	-	0.45
27,51	3	C	0.40	0.00	0.17	-	0.17
27,6	1	N	0.60	0.00	0.41	-	-
27,73	2	t	0.66	17.07	0.46	0.07	0.07
27,22	1	N	0.60	0.00	0.41	-	-
92,27	3	C	1.19	0.00	0.37	-	0.56

Note: This table lists estimated Tree 1 parameters of C-vine mixed copula model fitted to 92 risk factors. Selected copula families are explained in Appendix Table 56.

Table 2.23: **Tree 2 Parameters Estimation of C-vine mixed Copula Model**

tree	edge	No.	family	par	par2	tau	UTD	LTD
2	22,83;27	1	N	0.36	0.00	0.24	-	-
	22,30;27	1	N	0.26	0.00	0.17	-	-
	22,3;27	1	N	0.33	0.00	0.22	-	-
	22,35;27	2	t	0.35	9.50	0.23	0.05	0.05
	22,62;27	1	N	0.28	0.00	0.18	-	-
	22,11;27	2	t	0.45	5.93	0.29	0.15	0.15
	22,17;27	1	N	0.43	0.00	0.29	-	-
	22,87;27	1	N	0.18	0.00	0.11	-	-
	22,25;27	13	SC	0.21	0.00	0.10	0.04	-
	22,53;27	1	N	0.22	0.00	0.14	-	-
	22,18;27	2	t	0.55	6.80	0.37	0.17	0.17
	22,37;27	1	N	0.30	0.00	0.19	-	-
	22,61;27	13	SC	0.21	0.00	0.10	0.04	-
	22,14;27	2	t	0.28	30.00	0.18	0.00	0.00
	22,79;27	1	N	0.32	0.00	0.21	-	-
	22,68;27	1	N	0.34	0.00	0.22	-	-
	22,42;27	3	C	0.09	0.00	0.05	-	0.00
	22,70;27	3	C	0.42	0.00	0.18	-	0.20
	22,16;27	1	N	0.15	0.00	0.10	-	-
	22,88;27	1	N	0.43	0.00	0.29	-	-
	22,45;27	3	C	0.18	0.00	0.08	-	0.02
	22,81;27	3	C	0.25	0.00	0.11	-	0.06
	22,41;27	3	C	0.13	0.00	0.06	-	0.01
	22,90;27	1	N	0.20	0.00	0.13	-	-
	22,65;27	1	N	0.28	0.00	0.18	-	-
	22,49;27	3	C	0.23	0.00	0.10	-	0.05
	22,59;27	3	C	0.25	0.00	0.11	-	0.07
	22,82;27	3	C	0.32	0.00	0.14	-	0.11
	22,28;27	3	C	0.12	0.00	0.05	-	0.00
	22,58;27	3	C	0.34	0.00	0.14	-	0.13
	22,77;27	2	t	0.12	4.73	0.08	0.08	0.08
	22,36;27	1	N	0.22	0.00	0.14	-	-
	22,5;27	3	C	0.48	0.00	0.19	-	0.23
	22,31;27	1	N	0.16	0.00	0.10	-	-
	22,7;27	2	t	0.30	8.17	0.20	0.05	0.05
	22,10;27	3	C	0.96	0.00	0.32	-	0.48
	22,80;27	2	t	0.38	9.24	0.25	0.06	0.06
	22,46;27	23	C90	-0.04	0.00	-0.02	-	-
	22,91;27	3	C	0.74	0.00	0.27	-	0.39
	22,24;27	1	N	0.18	0.00	0.12	-	-
	22,43;27	3	C	0.25	0.00	0.11	-	0.06
	22,34;27	3	C	0.17	0.00	0.08	-	0.02
	22,55;27	3	C	0.29	0.00	0.13	-	0.09
	22,89;27	1	N	0.35	0.00	0.23	-	-
	22,48;27	3	C	0.15	0.00	0.07	-	0.01

22,20;27	3	C	0.51	0.00	0.20	-	0.26
22,9;27	1	N	0.31	0.00	0.20	-	-
22,54;27	3	C	0.43	0.00	0.18	-	0.20
22,33;27	2	t	0.30	7.44	0.19	0.06	0.06
22,19;27	1	N	0.36	0.00	0.23	-	-
22,47;27	33	C270	-0.06	0.00	-0.03	-	-
22,86;27	3	C	0.40	0.00	0.17	-	0.18
22,4;27	1	N	0.39	0.00	0.26	-	-
22,12;27	3	C	0.51	0.00	0.20	-	0.25
22,38;27	1	N	0.21	0.00	0.14	-	-
22,57;27	3	C	0.16	0.00	0.07	-	0.01
22,78;27	3	C	0.17	0.00	0.08	-	0.02
22,13;27	2	t	0.75	4.94	0.54	0.40	0.40
22,8;27	1	N	0.37	0.00	0.24	-	-
22,32;27	1	N	0.25	0.00	0.16	-	-
22,56;27	3	C	0.35	0.00	0.15	-	0.14
22,75;27	2	t	0.37	6.00	0.24	0.12	0.12
22,64;27	1	N	0.24	0.00	0.16	-	-
22,60;27	3	C	0.32	0.00	0.14	-	0.11
22,71;27	2	t	0.30	8.77	0.20	0.05	0.05
22,44;27	1	N	0.14	0.00	0.09	-	-
22,66;27	1	N	0.24	0.00	0.15	-	-
22,52;27	3	C	0.12	0.00	0.06	-	0.00
22,72;27	3	C	0.64	0.00	0.24	-	0.34
22,67;27	3	C	0.56	0.00	0.22	-	0.29
22,85;27	1	N	0.33	0.00	0.21	-	-
22,21;27	1	N	0.21	0.00	0.14	-	-
22,26;27	2	t	0.15	4.73	0.09	0.09	0.09
22,15;27	1	N	0.49	0.00	0.32	-	-
22,39;27	3	C	0.10	0.00	0.05	-	0.00
22,29;27	1	N	0.26	0.00	0.16	-	-
22,1;27	1	N	0.36	0.00	0.24	-	-
22,23;27	1	N	0.42	0.00	0.28	-	-
22,74;27	3	C	0.52	0.00	0.21	-	0.27
22,63;27	3	C	0.57	0.00	0.22	-	0.30
22,40;27	1	N	0.06	0.00	0.04	-	-
22,2;27	1	N	0.45	0.00	0.29	-	-
22,84;27	1	N	0.30	0.00	0.20	-	-
22,76;27	1	N	0.27	0.00	0.18	-	-
22,69;27	3	C	0.45	0.00	0.18	-	0.21
22,50;27	1	N	0.22	0.00	0.14	-	-
22,51;27	3	C	0.11	0.00	0.05	-	0.00
22,6;27	1	N	0.33	0.00	0.21	-	-
22,73;27	1	N	0.21	0.00	0.13	-	-
92,22;27	3	C	0.21	0.00	0.10	-	0.04

Note: This table lists estimated Tree 2 parameters of C-vine mixed copula model fitted to 92 risk factors. Selected copula families are explained in Appendix Table 56.

Table 2.24: **Tree 3 Parameters Estimation of C-vine mixed Copula Model**

tree	edge	No.	family	par	par2	tau	UTD	LTD
3	73,83;22,27	1	N	0.29	0.00	0.19	-	-
	73,30;22,27	1	N	0.41	0.00	0.27	-	-
	73,3;22,27	1	N	0.17	0.00	0.11	-	-
	73,35;22,27	13	SC	0.15	0.00	0.07	0.01	-
	73,62;22,27	3	C	0.18	0.00	0.08	-	0.02
	73,11;22,27	2	t	0.45	11.30	0.30	0.05	0.05
	73,17;22,27	3	C	0.23	0.00	0.10	-	0.05
	73,87;22,27	2	t	0.17	6.71	0.11	0.05	0.05
	73,25;22,27	3	C	0.18	0.00	0.08	-	0.02
	73,53;22,27	3	C	0.22	0.00	0.10	-	0.04
	73,18;22,27	1	N	0.10	0.00	0.06	-	-
	73,37;22,27	1	N	0.28	0.00	0.18	-	-
	73,61;22,27	1	N	0.19	0.00	0.12	-	-
	73,14;22,27	1	N	0.47	0.00	0.31	-	-
	73,79;22,27	2	t	0.41	30.00	0.27	0.00	0.00
	73,68;22,27	2	t	0.20	10.93	0.13	0.02	0.02
	73,42;22,27	1	N	0.16	0.00	0.10	-	-
	73,70;22,27	1	N	0.30	0.00	0.19	-	-
	73,16;22,27	1	N	0.06	0.00	0.04	-	-
	73,88;22,27	2	t	0.35	4.70	0.23	0.15	0.15
	73,45;22,27	1	N	-0.03	0.00	-0.02	-	-
	73,81;22,27	1	N	0.30	0.00	0.19	-	-
	73,41;22,27	23	C90	-0.08	0.00	-0.04	-	-
	73,90;22,27	1	N	0.20	0.00	0.13	-	-
	73,65;22,27	1	N	0.21	0.00	0.14	-	-
	73,49;22,27	3	C	0.15	0.00	0.07	-	0.01
	73,59;22,27	3	C	0.17	0.00	0.08	-	0.02
	73,82;22,27	1	N	-0.05	0.00	-0.03	-	-
	73,28;22,27	3	C	0.03	0.00	0.02	-	0.00
	73,58;22,27	3	C	0.27	0.00	0.12	-	0.08
	73,77;22,27	3	C	0.37	0.00	0.16	-	0.15
	73,36;22,27	1	N	0.03	0.00	0.02	-	-
	73,5;22,27	13	SC	0.11	0.00	0.05	0.00	-
	73,31;22,27	1	N	0.17	0.00	0.11	-	-
	73,7;22,27	3	C	0.23	0.00	0.10	-	0.05
	73,10;22,27	1	N	0.23	0.00	0.15	-	-
	73,80;22,27	1	N	0.44	0.00	0.29	-	-
	73,46;22,27	13	SC	0.05	0.00	0.03	0.00	-
	73,91;22,27	2	t	0.34	8.18	0.22	0.06	0.06
	73,24;22,27	1	N	0.30	0.00	0.20	-	-
	73,43;22,27	3	C	0.21	0.00	0.09	-	0.04
	73,34;22,27	1	N	0.10	0.00	0.06	-	-
	73,55;22,27	3	C	0.38	0.00	0.16	-	0.16
	73,89;22,27	1	N	0.10	0.00	0.07	-	-

73,48;22,27	2	t	0.05	8.93	0.03	0.01	0.01
73,20;22,27	1	N	0.22	0.00	0.14	-	-
73,9;22,27	1	N	0.32	0.00	0.20	-	-
73,54;22,27	2	t	0.16	6.51	0.10	0.05	0.05
73,33;22,27	1	N	0.20	0.00	0.13	-	-
73,19;22,27	3	C	0.41	0.00	0.17	-	0.19
73,47;22,27	33	C270	-0.08	0.00	-0.04	-	-
73,86;22,27	1	N	0.27	0.00	0.17	-	-
73,4;22,27	3	C	0.30	0.00	0.13	-	0.10
73,12;22,27	1	N	0.33	0.00	0.21	-	-
73,38;22,27	2	t	0.13	6.29	0.08	0.05	0.05
73,57;22,27	1	N	0.22	0.00	0.14	-	-
73,78;22,27	3	C	0.23	0.00	0.10	-	0.05
73,13;22,27	3	C	0.17	0.00	0.08	-	0.02
73,8;22,27	1	N	0.11	0.00	0.07	-	-
73,32;22,27	1	N	0.29	0.00	0.19	-	-
73,56;22,27	1	N	0.35	0.00	0.23	-	-
73,75;22,27	1	N	0.58	0.00	0.39	-	-
73,64;22,27	1	N	0.41	0.00	0.27	-	-
73,60;22,27	2	t	0.21	5.71	0.14	0.08	0.08
73,71;22,27	1	N	0.30	0.00	0.19	-	-
73,44;22,27	3	C	0.09	0.00	0.05	-	0.00
73,66;22,27	3	C	0.18	0.00	0.08	-	0.02
73,52;22,27	3	C	0.06	0.00	0.03	-	0.00
73,72;22,27	1	N	0.29	0.00	0.18	-	-
73,67;22,27	3	C	0.18	0.00	0.08	-	0.02
73,85;22,27	2	t	0.36	7.35	0.24 0.08	0.08	
73,21;22,27	1	N	0.20	0.00	0.13	-	-
73,26;22,27	1	N	0.31	0.00	0.20	-	-
73,15;22,27	2	t	0.26	5.18	0.17	0.10	0.10
73,39;22,27	2	t	0.11	8.69	0.07	0.02	0.02
73,29;22,27	3	C	0.42	0.00	0.17	-	0.19
73,1;22,27	1	N	0.09	0.00	0.06	-	-
73,23;22,27	2	t	0.12	4.83	0.08	0.08	0.08
73,74;22,27	1	N	0.38	0.00	0.24	-	-
73,63;22,27	1	N	0.24	0.00	0.15	-	-
73,40;22,27	13	SC	0.03	0.00	0.01	0.00	-
73,2;22,27	13	SC	0.13	0.00	0.06	0.00	-
73,84;22,27	1	N	0.34	0.00	0.22	-	-
73,76;22,27	2	t	0.50	7.78	0.33	0.12	0.12
73,69;22,27	2	t	0.28	7.75	0.18	0.05	0.05
73,50;22,27	3	C	0.25	0.00	0.11	-	0.06
73,51;22,27	33	C270	-0.18	0.00	-0.08	-	-
73,6;22,27	1	N	0.31	0.00	0.20	-	-
92,73;22,27	2	t	0.41	6.84	0.27	0.11	0.11

Note: This table lists estimated Tree 3 parameters of C-vine mixed copula model fitted to 92 risk factors. Selected copula families are explained in Table 56.

Glasserman (2004) and Pykhtin (2004), assume our 92 risk factors have identical default probabilities, $p_i = 0.03$, $i = 1, ..., k$, $k = 92$, (loss given default adjusted) exposures set as 1000000, loss given default equals 0.45, composite risk factor loadings is equally set, and systematic risk factor loadings are set as $\beta_1 = \beta_2 = ... = \beta_k = 0.2; 0.3; 0.4; 0.5; 0.6$ separately, which are common in credit modelling, see Pykhtin (2004), Daul et al. (2003). We list all credit VaR and CVaR money value results estimated under five different copula model setting at $\alpha = 0.9, 0.95, 0.99, 0.995, 0.999$ confidence level and various systematic risk ratios level separately in Table 25-26.

Homogeneous small portfolio, M = 100. In the case of small homogeneous credit portfolio incorporating 100 obligors, we set the ratio of systematic risk to total risk from 20% to 60%. Plenty of empirical results demonstrate that in various different industries, the ratio of systematic risk in total risk ranges from 20% to 60%, since our industry factors cover various industries in several countries, we set the ratio from 20% to 60% in order to approach the real world case. From the Table 25, firstly, we find that with the increase of the proportion of systematic risk, the values of VaR and CVaR under various copula setting and various confidence levels are all increasing, indicating that the greater the proportion of the systematic risk, the greater the risk of the entire credit portfolio. These results matches the asset pricing theory. As we know, the financial risk can be divided as systematic risk and nonsystematic risk, nonsystematic risk can be diversified by portfolio management. So as the increase of the proportion of systematic risk, the value of VaR and CVaR also increase. One exception is that when systematic risk ratio being 30%, under 95%, 99%, and 99.5% confidence level, the values of the VaR and CVaR are both less than 20% case, while the other cases are all consistent with the above description.

Then we investigate VaR and CVaR results under different copula settings in detail. As expected, VaR and CVaR values under the setting of R-vine mixed and C-vine mixed copula are always large than R-vine Gaussian, R-vine t, C-vine t copula setting at each systematic risk ratios under various confidence levels. For example, at 95% confidence level, 20% systematic risk ratios setting, the VaR and CVaR value under the C-vine mixed copula setting are largest among all models, which are 569000 and 743166.3 separately. The second largest money value of VaR and CVaR are the results under R-vine mixed copula setting, its VaR equals 575000 and CVaR equals 746396.1. And VaR of C-vine t equals 567000, CVaR is 735432.9, R-vine t VaR equals 566000, CVaR is 736783.0.

The lowest VaR and CVaR money value are obtained under multivariate Gaussian copula setting, in which VaR equals 563000, CVaR equals 728608.8. At 99.9% confidence level, 60% systematic risk ratios setting, the VaR of C-vine mixed copula is 148500, the CVaR equals 1656745.1, the VaR set by R-vine mixed copula equals 148200, the CVaR is 1631843.1, and the VaR of the C-vine t is 145800, CVaR equals 1602176.5, R-vine t is 146600, CVaR equals 1619269.2, R-vine Gaussian is the lowest, in which VaR equals 1446000, CVaR is 1605764.7.

Homogeneous large portfolio, M = 1000. Now we take a look at large homogeneous credit portfolio case which includes 1000 obligors, we set the ratio of systematic risk to total risk also from 20% to 60%, which is the same as M=100 case. From the Table 26, we find similar results with small portfolio case, firstly, with the increase of the proportion of systematic risk, the values of VaR and CVaR under various copula setting and various confidence levels are all increasing, indicating that the greater the proportion of the systematic risk, the greater the risk of the entire credit portfolio. One exception is that when systematic risk ratio being 30%, under 95%, 99%, and 99.5% confidence level, the values of the VaR and CVaR are both less than 20% case, while the other cases are all consistent with the above description.

Then we investigate VaR and CVaR results under different copula settings in detail. As expected, VaR and CVaR values under the setting of R-vine mixed and C-vine mixed copula are always large than R-vine Gaussian, R-vine t, C-vine t copula setting at each systematic risk ratios under various confidence levels. For example, at 95% confidence level, 20% systematic risk ratios setting, the VaR and CVaR value under the C-vine mixed copula setting are largest among all models, which are 758000 and 959044.7 separately. The second largest money value of VaR and CVaR are the results under R-vine mixed copula setting, its VaR equals 754000 and CVaR equals 948288.2. And VaR of C-vine t equals 75700, CVaR equals 949092.6, R-vine t VaR equals 761000, CVaR is 955590.6. The lowest VaR and CVaR money value are obtained under multivariate Gaussian copula setting, in which VaR equals 756000, CVaR equals 958503. The VaR of C-vine mixed copula is 2723000, the CVaR equals 2961392, the VaR obtained by R-vine mixed copula setting is 2657000, the CVaR equals 2905745, and the VaR of the C-vine t is 2657000, CVaR equals 2905745, R-vine t is 2651000, CVaR equals 2945333, R-vine Gaussian exhibits the lowest value, in which VaR is 2643000, CVaR equals 2882275.

First of all, the above homogeneous portfolio results show that the rich bivariate copula families allow us to be more flexible and more appropriate to choose copula to describe the dependency of different pairs of stock indices, secondly, as we discussed above, the superiority of vine structure not only come from plenty of bivariate copula families as building blocks of vines, but also because they provide a more refined and rational structure to connect each equity indices. An example is that R-vine mixed copula outperform C-vine t copula setting in estimating VaR and CVaR in our results. Therefore, we can draw a conclusion that vine copula structures help us to more precisely estimate credit portfolio VaR and CVaR, so as to more accurately measure the portfolio credit risk, while the traditional multivariate Gaussian copula setting, and the inappropriate R-vine t, C-vine t copula underestimate the risk of our credit portfolio.

2.7.3 Heterogeneous Credit Portfolio

In practical point of view, homogeneous portfolio is a simplified version of real credit portfolio, while heterogenous portfolio setting can considered to be more realistic one. Hence, in this section, we would like to in further investigate whether R-vine mixed copula and C-vine mixed copula setting outperform other copula model setting in estimating VaR and CVaR in a more realistic heterogenous credit portfolio. In heterogeneous credit portfolio setting, we also mimic the setting of numerical examples in Glasserman (2004) and Pykhtin (2004).

Heterogeneous small portfolio, M = 100. In the case of small heterogeneous credit portfolio including 100 obligors, following Glasserman (2004) and Pykhtin (2004), we set the each industry sector's proportion of systematic risk in total risk to randomly selected from 20% to 60%. As mentioned in the homogeneous case, plenty of empirical results demonstrate that, in various different industries, the ratio of systematic risk in total risk ranges from 20% to 60%, since our industry factors cover various industries in several countries, we set the ratio randomly selected from 20% to 60% in heterogeneous case in order to approach the real world case. Composite risk factor loading is randomly chosen from 0 to 1, but the sum of each composite risk factor loadings should be equal to 1. In credit risk management, normally, probabilities of default of investment bond and speculative bond ranges from 0 to 0.1. Default probabilities randomly set from 0 to 0.1, which represents we distinguish different credit rating of obligors. (loss given default adjusted)

Table 2.25: **Homogeneous Small Portfolio Comparison of VaR and CVaR for different copula settings**

	VaR				CVaR			
	95%	99%	99.5%	99.9%	95%	99%	99.5%	99.9%
$r = 0.2$								
R vine Gaussian	563000	823000	927000	1160000	728608.8	974140.6	1083726.2	1341019.6
R vine t	566000	836000	947000	1204000	736783.0	997960.2	1111789.7	1347980.4
R vine mixed	575000	844000	970000	1210000	746396.1	1006700.6	1117976.1	1356862.7
C vine t	567000	833000	934000	1175000	735432.9	980251.5	1084075.1	1320846.2
C vine mixed	569000	840000	954000	1224000	743166.3	1007974.2	1127000.0	1400882.4
$r = 0.3$								
R vine Gaussian	444000	749000	832000	1040000	626789.2	871944.1	957158.7	1168607.8
R vine t	444000	750000	839000	1079000	626354.8	883950.4	978442.2	1186588.2
R vine mixed	448000	750000	848000	1086000	637882.0	895424.9	997091.6	1227647.1
C vine t	444000	737000	827000	1080000	621675.2	880345.9	983705.2	1218098.0
C vine mixed	447000	751000	844000	1108000	641193.4	894530.9	993191.4	1207803.9
$r = 0.4$								
R vine Gaussian	566000	848000	957000	1200000	746497	1002873	1115988	1363608
R vine t	568000	845000	973000	1207000	750388.8	1013473.1	1130354.6	1372862.7
R vine mixed	572000	854000	976000	1215000	752640.4	1017874.8	1133111.6	1386235.3
C vine t	569000	848000	958000	1205000	747801.3	1002834.7	1111374.5	1345666.7
C vine mixed	573000	857000	992000	1244000	759010.4	1036528.9	1157525.9	1406288.5
$r = 0.5$								
R vine Gaussian	706000	995000	1129000	1385000	879943	1171214	1284243	1517745
R vine t	706000	1020000	1139000	1414000	885201.7	1191662.7	1304747.0	1590352.9
R vine mixed	708000	1022000	1143000	1430000	887340.4	1196736.1	1313047.6	1568942.3
C vine t	704000	1012000	1140000	1405000	880306.7	1183820.4	1295608.7	1557882.4
C vine mixed	713000	1025000	1158000	1460000	898014.8	1212441.4	1335948.4	1605372.5
$r = 0.6$								
R vine Gaussian	719000	1047000	1166000	1446000	916430.1	1218428.3	1336888.9	1605764.7
R vine t	728000	1050000	1175000	1466000	923479.4	1227215.1	1345329.4	1619269.2
R vine mixed	729000	1063000	1195000	1482000	931669.2	1246748.0	1369523.6	1631843.1
C vine t	725000	1052000	1182000	1458000	922221.1	1229960.2	1348718.3	1602176.5
C vine mixed	728000	1053000	1196000	1485000	926897.1	1245304.2	1376478.1	1656745.1

Note: This table reports homogeneous small credit portfolio VaR and CVaR money value at different confidence level q estimated by different copula models: R-vine Gaussian, R-vine t, R-vine mixed, C-vine t, C-vine mixed. Results are given for test portfolio containing 100 credit exposures.

Table 2.26: **Homogeneous Large Portfolio Comparison of VaR and CVaR for different copula settings**

	VaR				CVaR			
	95%	99%	99.5%	99.9%	95%	99%	99.5%	99.9%
$r = 0.2$								
R vine Gaussian	756000	1098000	1221000	1493000	958503	1274515	1395223	1673863
R vine t	761000	1088000	1211000	1491000	955590.6	1269862.5	1393654.8	1677921.6
R vine mixed	754000	1075000	1189000	1496000	948288.2	1251757.5	1379063.7	1666490.2
C vine t	757000	1077000	1198000	1498000	949092.6	1256371.0	1380832.7	1665711.5
C vine mixed	758000	1101000	1229000	1499000	959044.7	1283884.2	1405434.3	1683000.0
$r = 0.3$								
R vine Gaussian	840000	1176000	1315000	1640000	1051563	1372622	1508661	1784308
R vine t	838000	1194000	1347000	1611000	1053763	1388653	1517319	1780529
R vine mixed	834000	1202000	1347000	1669000	1056593	1401821	1537837	1824176
C vine t	837000	1185000	1326000	1622000	1050399	1379078	1512202	1789392
C vine mixed	838000	1191000	1336000	1619000	1053334	1384250	1514937	1788824
$r = 0.4$								
R vine Gaussian	1161000	1605000	1793000	2125000	1433341	1845771	2004450	2342137
R vine t	1162000	1586000	1756000	2163000	1427975	1826790	1993671	2358216
R vine mixed	1161000	1609000	1795000	2130000	1438217	1856226	2019056	2337078
C vine t	1171000	1607000	1778000	2124000	1435619	1838727	1996287	2321275
C vine mixed	1170000	1622000	1793000	2165000	1441855	1851061	2007861	2325608
$r = 0.5$								
R vine Gaussian	1276000	1717000	1875000	2228000	1544893	1943203	2099538	2420039
R vine t	1272000	1707000	1872000	2225000	1543354	1942505	2100189	2447059
R vine mixed	1274000	1737000	1914000	2249000	1553257	1967171	2118239	2454902
C vine t	1271000	1717000	1899000	2224000	1543884	1959116	2116162	2448647
C vine mixed	1275000	1722000	1908000	2297000	1553129	1982082	2152190	2553942
$r = 0.6$								
R vine Gaussian	1547000	2056000	2236000	2643000	1856063	2316808	2493311	2882275
R vine t	1543000	2050000	2232000	2651000	1852713	2323267	2510992	2945333
R vine mixed	1559000	2075000	2267000	2679000	1868878	2341281	2521924	2896471
C vine t	1556000	2061000	2266000	2657000	1862892	2337217	2526363	2905745
C vine mixed	1547000	2057000	2265000	2723000	1860287	2338747	2533179	2961392

Note: This table reports homogeneous large credit portfolio VaR and CVaR money value at different confidence level q estimated by different copula models: R-vine Gaussian, R-vine t, R-vine mixed, C-vine t, C-vine mixed. Results are given for test portfolio containing 1000 credit exposures.

exposures randomly set from 1000 to 1000000. These settings are common in financial risk which is easily applicable to monte carlo simulation (random selection) and to check if our model perform well in different level of default probabilities (different credit rating) and different composite risk factor loadings. In addition, all these setting is close to real world case. The estimated money value of VaR and CVaR based on various copula model setting list in Table 27-28. In general, we find that, under different copula settings, different confidence levels, the maximum value of both VaR and CVaR always come from the setting by R-vine mixed and C-vine mixed copula, which exceed the corresponding value of multivariate Gaussian copula, R-vine t, C-vine t copula setting. These results primarily demonstrate that multivariate Gaussian copula, R-vine t, C-vine t copula setting underestimate the VaR and CVaR of credit portfolio. For example, in the case of 95% confidence level, the VaR of the C-vine mixed copula equals 946000, the CVaR equals 1237909, the VaR value under the R-vine mixed copula setting is 942000, the CVaR equals 1235755, VaR of the C-vine t is 945000, the CVaR is 1236162, VaR under R-vine t setting equals 934000, CVaR is 1229494. In line with homogeneous case, values under multivariate Gaussian copula setting are the lowest, VaR is 935000, CVaR is 1222331.

Heterogeneous large portfolio, M = 1000. Then we further check the case of large heterogeneous credit portfolio which consists 1000 obligors. Similarly with M=100 case setting, we also set the each sector's proportion of systematic risk in total risk to randomly select from 0.2 to 0.6, which is the same with small heterogeneous portfolio case. We find that the highest values of VaR and CVaR still always originate from the setting of R-vine mixed and C-vine mixed copula at various confidence levels, which are higher than those of R-vine Gaussian, R-vine t, C-vine t copula, for example, in the case of 95% low confidence level, the VaR of the C-vine mixed copula is 1827000, the CVaR is 2248750, R-vine mixed copula setting's VaR equals 1818000, the CVaR is 2245065, the C-vine t is 1806000, the CVaR is 2228142, the R-vine t is 1817000, the CVaR is 2225689, the multivariate Gaussian copula setting values are also the lowest, the VaR is 1811000, the CVaR is 2228888. In the case of 99.9% high confidence level, the VaR of C-vine mixed copula is 3311000, the CVaR is 3632353, the VaR set by R-vine mixed copula is 3368000, the CVaR is 3717902, and VaR of the C-vine t is 3262000, CVaR equals 3583647, VaR set by R-vine t is 3290000, CVaR is 3563431, R-vine Gaussian setting exhibits the lowest VaR which is 3293000, and CVaR equals 3613865. All these results are consist with heterogeneous small portfolio case.

According to the similar results of heterogeneous portfolio with homogeneous portfolio, also the small and large portfolio cases, we can draw a conclusion that, first, the VaR and CVaR results under the traditional multivariate Gaussian copula setting are all lower than other four vine copula setting, which demonstrate that flexible vine structure is able to more precisely capture the dependence of equity indices, so as to estimate portfolio credit risk VaR and CVaR more precisely. Without the vine structure, the traditional multivariate Gaussian copula is inferior to vine copula setting which leads to the underestimation of VaR and CVaR. Secondly, why we say R-vine t copula and C-vine t copula setting which have the vine structure still underestimate the VaR and CVaR compared with the R-vine mixed copula and C-vine mixed copula. R-vine mixed copula and C-vine mixed copula allow the user to choose building blocks from various bivariate copulas, including both symmetric and asymmetric copulas, to capture tail dependence and asymmetric dependence, however, despite R-vine t copula and C-vine t copula possess flexible vine structure, they restrict to Student t copula as their building blocks. It becomes the restriction of R-vine t and C-vine t that the Student t copula can only capture tail dependence but can not capture asymmetric dependence, which results in lower ability and actuality of capturing dependency. Therefore, R-vine mixed copula and C-vine mixed copula which taking both above advantages are able to be more precisely estimate VaR and CVaR, while the traditional multivariate Gaussian copula, and R-vine t, C-vine t copula underestimate the risk of the credit portfolio.

Table 2.27: **Heterogeneous Small Portfolio Comparison of VaR and CVaR for different copula settings**

	VaR				CVaR			
	95%	99%	99.5%	99.9%	95%	99%	99.5%	99.9%
R vine Gaussian	935000	1403000	1562000	1942000	1222331	1637701	1796243	2146549
R vine t	934000	1396000	1586000	2013000	1229494	1663330	1851207	2246843
R vine mixed	942000	1422000	1607000	2079000	1235755	1684307	1867230	2267588
C vine t	945000	1412000	1599000	2057000	1236162	1686226	1878746	2313784
C vine mixed	946000	1427000	1608000	2026000	1237909	1681067	1856161	2228549

Note: This table reports heterogeneous small credit portfolio VaR and CVaR money value at different confidence level q estimated by different copula models: R-vine Gaussian, R-vine t, R-vine mixed, C-vine t, C-vine mixed. Results are given for test portfolio containing 100 credit exposures.

Table 2.28: **Heterogeneous Large Portfolio Comparison of VaR and CVaR for different copula settings**

	VaR				CVaR			
	95%	99%	99.5%	99.9%	95%	99%	99.5%	99.9%
R vine Gaussian	1811000	2483000	2756000	3293000	2228888	2858986	3109869	3613865
R vine t	1817000	2474000	2722000	3290000	2225689	2823528	3062230	3563431
R vine mixed	1818000	2515000	2772000	3368000	2245065	2886840	3150805	3717902
C vine t	1806000	2497000	2744000	3262000	2228142	2847221	3085375	3583647
C vine mixed	1827000	2520000	2792000	3311000	2248750	2879094	3110687	3632353

Note: This table reports heterogeneous large credit portfolio VaR and CVaR money value at different confidence level q estimated by different copula models: R-vine Gaussian, R-vine t, R-vine mixed, C-vine t, C-vine mixed. Results are given for test portfolio containing 1000 credit exposures.

2.8 Systematic Risk Factor Contributions

Decomposing portfolio risk into its different sources is a fundamental problem in financial risk management. Once risk measure has been selected, VaR and CVaR in our case, and the risk of a portfolio has been calculated, a question naturally be raised is: where does these risk come from? Hence, we develop an extension of the Euler allocation that applies to nonlinear functions of a set of risk factors in our vine copula setting framework. The technique is based on the Hoeffding decomposition, originally developed for statistical applications (see, for example, Van der Vaart (2000); Sobol (1993)). The thoughts of this method simply is that though we cannot write the portfolio loss as a sum of functions of individual risk factors, the application of the Hoeffding decomposition allows us to express it as a sum of functions of all subsets of risk factors. The standard Euler allocation machinery can then be applied to the new loss decomposition. The price paid for this methodology is that we have to consider contributions not only from single risk factors, but also from the interaction of every possible collection of risk factors.

We firstly briefly review the theory of risk contributions, with particular emphasis on marginal contributions (also known as the Euler allocation rule). For a more complete discussion of the theory of capital allocation, focusing in particular on credit risk management, see Mausser and Rosen (2007). For a survey of results on the Euler allocation rule, see Tasche (2007), or McNeil et al. (2015).

We consider the total portfolio loss as a sum of the losses of individual positions (instruments or sub-portfolios):

$$L = \sum_{n=1}^{N} w_n L_n$$

where L_n is the random variable giving the loss per dollar of exposure in instrument n, and w_n is the amount of money invested in position n. The total risk of the portfolio is $\rho(L)$, where ρ is a risk measure mapping random variables to real numbers.

We are interested in defining a measure C_n of the contribution of the nth position to the total portfolio risk. Different methods of calculating risk contributions have been studied for different purposes. We present a brief list of the alternatives that are popular in practice.

2.8.1 Risk Contribution Method

Stand-alone contributions

$$C_n = \rho(w_n L_n)$$

The stand-alone contribution of a position is simply its risk if it were held as a portfolio in isolation. It ignores the distributions of all other positions, and therefore does not take into account any diversifying or hedging effects resulting from its inclusion in the institution's portfolio. It is considered to be useful in measuring the reduction of risk due to diversification, and in measuring diversification factors for portfolios (see Cespedes et al. (2006); Tasche (2006)).

It can also be considered as an upper bound on the contribution to the risk for any reasonable allocation rule. That is, for any allocation rule, we would expect to have $C_n \leq \rho(w_n L_n)$. This condition features in axiomatizations of capital contributions, e.g. Kalkbrener (2005), as well as the interpretation of the Euler allocation rule in terms of the theory of cooperative games, e.g. Denault (2001) or Koyluoglu and Stoker (2002). If the risk measure is subadditive, then the sum of the stand-alone contributions provides an upper bound for the total portfolio risk:

$$\rho \leq \sum_{n=1}^{N} \rho(w_n L_n)$$

Coherent risk measures such as expected shortfall, or say CVaR, are subadditive. It is well known that Value-at-Risk and Economic Capital are not subadditive risk measures, and for them the above inequality can be violated.

Incremental contributions

The incremental risk contribution of a position is the change in total risk arising from including the position in the portfolio.

$$C_n = \rho(L) - \rho(\sum_{m \neq n} w_m L_m)$$

This is a useful measure for one considering adding the position L_n to their portfolio. When w_n is small, it may also be regarded as a finite difference approximation to the marginal risk contribution discussed below. It is typically not the case that incremental contributions of positions add up to the total portfolio risk, and it should also be noted that this definition of risk contribution is motivated by applications where additivity is not necessarily desirable.

Marginal contributions (Euler allocation)

We consider a risk measure that is positive homogeneous (i.e. $\rho(\lambda \cdot L) = \lambda \rho(L)$ for $\lambda \geq 0$) which normally includes measures such as standard deviation (δ_L), Value-at-Risk ($VaR(L)$), Economic Capital ($EC(L)$) and Conditional Value-at-Risk ($CVaR(L)$), also known as expected shortfall, Haezendonck risk measures (see, e.g. Bellini and Gianin (2008)), spectral risk measures (see, e.g. Adam et al. (2008)), and any risk measure satisfying the coherence axioms of Artzner et al. (1999). Under technical differentiability assumptions on ρ, Euler's theorem for positive homogeneous functions can immediately implies,

$$\rho(L) = \sum_{n=1}^{N} C_n$$

where

$$C_n = w_n \frac{d\rho}{d\epsilon}(L + \epsilon L_n)|_{\epsilon=0} = w_n \frac{\partial \rho(L)}{\partial w_n}(w).$$

The nth term in the sum, C_n is then interpreted as the contribution of the nth position's loss (L_n) to the overall portfolio risk $\rho(L)$.

Explicit formulas for marginal risk contributions are available for some of the most important risk measures. For standard deviation,

$$C_n^{\sigma} = w_n \frac{cov(L_n, L)}{\sigma_L},$$

where σ_L is the standard deviation of L. For Value-at-Risk at the confidence level α, subject to technical conditions, Gourieroux et al. (2000) and Tasche (1999) showed that,

$$C_n^{VaR} = w_n \mathbb{E}[L_n | L = VaR_\alpha(L)].$$

Finally, for CVaR, and again subject to technical conditions, Tasche (1999) showed that,

$$C_n^{CVaR} = w_n \mathbb{E}[L_n | L \geq VaR_\alpha(L)].$$

2.8.2 Homogeneous Credit Portfolio Risk Contribution

From above section, we find that R-vine mixed copula and C-vine mixed copula settings are able to more accurately estimate VaR and CVaR, while other copula settings under-estimate VaR and CVaR. In this section, we therefore calculate different industry sector's VaR and CVaR risk contribution based on R-vine mixed copula and C-vine mixed copula settings. Risk contributions for both VaR and CVaR as a function of β for all confidence levels. Risk contributions are calculated based on a Monte-Carlo simulation using ten million scenarios. VaR contributions are calculated using a kernel estimator for the conditional expectation with equal weights. Observe that even with a large number of scenarios, VaR contributions are subject to significant estimation error, which could likely be reduced using importance sampling (e.g. Glasserman and Li (2005); Merino and Nyfeler (2004)). We consider risk contributions to both VaR and CVaR, at the confidence levels $\alpha = 0.9, 0.95, 0.99, 0.995, 0.999$ separately.

For the C-vine copula setting case, we focus on the small homogeneous credit portfolio. In the same way with VaR and CVaR estimation, we also set the ratio of systematic risk in the total risk from 20% to 60%, in order to examine the change of each sector's risk contribution of VaR and CVaR with the increase of percentage of systematic risk to total risk. We list the top ten sector risk contribution rankings from high to low in Table 29-33, where systematic risk ratio is set 20%, 30%, 40%, 50% and 60% separately. In the case of systematic risk ratio equals 20%, we find that both of the VaR and CVaR risk contributions of the banking and financial services sector rank high at all 95%, 99%, 99.5% and 99.9% levels among the top ten risk contributors, followed by the mining industry and information industry which also provides a great risk contribution. Regarding countries of risk sources, in the case of systematic risk ratio equals 20%, the Japan occupies the most positions in the top ten of the risk contributors, such as, under the high 99.9% levels, the Japan occupies the first three, seventh and ninth places of top ten VaR risk contributors, the second, fourth and seventh places of CVaR risk contributors. At 99.5% levels, Japan accounts for third, fourth, seventh, eighth and tenth, five places in total of VaR risk contribution among the top ten. Among CVaR risk contribution of the top ten, Japan occupies also five positions, second, fourth, fifth, sixth and seventh.

Next question is whether there will be some changes and what kind of changes of the largest risky contributors of sector and country when we change the proportion of systematic risk to total risk. Therefore, we increase the proportion of systematic risk in the total risk from 20% to 30%, 40%, 50% and 60%. Then we find that the industry departments that provide the largest risk contribution move from mainly financial industry, such as banking and financial services industry to the manufacturing sectors, such as, automotive industry, auto parts industry, transportation industry and petrochemical industry, which demonstrates the risk come from financial sector would be diversified. When the systematic risk increase to the high proportion of 60%, at each confidence level, VaR and CVaR risk contribution of the top ten moves to Chemicals, Materials, primarily the pharmaceutical industry. As we know, these industries are least affected by macro economy. From the point of view of the country of the risk sources, the country provide most risk changes from Japan to UK and US when the proportion of systematic risk move from 20% to 30%, 40%, 50% and 60%, however, what interesting is, the countries which make the largest risk contribution in 30%, 40%, 50% and 60% cases are much more decentralized comparing to the 20% case. Particularly in 20% case, Japan always take up around half of the places of the top ten risk contribution countries, however, at 30%, 40%, 50% and 60% cases, the US plus UK take up around half of top ten places.

Then we move to R-vine mixed copula setting case, in our case, we just consider the sector risk factors without considering macro economic variable as the pivotal element of vine copula modelling, therefore, as expected, the R-vine mixed copula without pivotal variable requirement would be much effective for capturing and modelling the dependence of various sector risk factors here. In depth, the R-vine structure, which is the general form of the C-vine with star structure and D-vine with path structure, possess more flexible structure to capture the asymmetric tail dependence of risk factors and the fat tails characteristics. However, from the section of goodness-of-fit, the test results demonstrate that C-vine mixed copula setting outperforms R-vine mixed copula setting. The reason probably would be though there is no general macro economic variable which can affect all risk factors, due to the C-vine structure, when we fit the C-vine copula to data, a factor should be selected as the pivotal factor from statistical perspective, and this selected sector factor has a great effect on and strong correlation with all other factors. While R-vine loses some accuracy of modelling dependence in our case results from the ignorance of above consideration.

In R-vine mixed copula setting, we also take a look at the case of small homogeneous credit portfolios. We first similarly set the percentage of systematic risk in the total risk from 20% to 30%, 40%, 50% and 60% to examine the changes in the risk contribution of VaR and CVaR with the increase of systematic risk proportion. We list the top ten risk contributors of sector and their country of origin from high to low. In the case of ratio of 20%, we find that at 95%, 99%, 99.5%, 99.9% levels, similar to the C-vine mixed copula setting, the VaR and CVaR risk contributions of the banking, insurance and financial services sectors are at the forefront and are the most important sources of risk. While the slight difference is that under the R-vine mixed copula setting, not the mining industry and the information industry provide the second greatest risk contribution following financial industry, but the power and utility industry provide the second largest risk contribution following the banking and finance industry. When systematic risk proportion increase to 60%, the main risk contributor industry change to power, energy, and pharmaceutical industry.

From the point of view of the risk country of origin, in the case of 20%, Japan occupies the largest number of places among the top ten risk contributors, such as at 99% level, Japan occupies the second, sixth, seventh and eighth positions of VaR risk contributors of the top ten. Among CVaR risk contribution to the top ten risk contributor, Japan accounts for the third, fifth, sixth and tenth places. At 99.5% confidence level, regarding VaR risk contribution to the top ten, Japan accounts for the third, eighth, ninth and tenth positions, and accounts for the second, fourth, fifth, sixth, seventh and eighth positions of CVaR risk contribution to the top ten. When we adjust systematic risk percentage to 30%, both of the most risky sector and their country of origin are basically the same with the situation of 20%, however, as we increase the systematic risk proportion of total risk to 40%, 50%, we find that the sector which provide greatest risk change from the banking and financial services and information industry into the mining, transportation, chemical industry, and this is basically the same with C-vine mixed copula setting case, because C-vine copula is a special form of R-vine copula, and the risk of the banking and financial industry would be diversified. From the point of view of the origin country of risk, in the case of 20%, 30%, Japan accounts for most places among the top ten risk contributors, while in 40% case, Canada's risk contribution increases, and in 50% case, three countries, including UK, US and Japan, are the most risky countries, while when the proportion of the systematic risk increased to 60%, similarly with C-vine mixed copula setting, the countries of risk sources are more decentralized.

2.8.3 Heterogeneous Credit Portfolio Risk Contribution

Next, we examine the heterogeneous credit portfolio risk contribution, where we randomly set the systematic risk weights for each industry department among the range of 20% to 60%. First of all, let us take a look at the C-vine mixed copula setting case, which we find that vine copula setting is effective at all 95%, 99 %, 99.5 % and 99.9 % confidence level. The financial industry, such as banks, insurance industry account for half of the risk contribution of the top ten most risky industrial sectors. For example, at the low 95% confidence level, VaR contribution of the bank insurance industry take the first, second, fourth, seventh and eighth, five places in total, while the first, fifth, sixth and tenth of VaR contribution are taken place by insurance industry, regarding measure of CVaR, the bank insurance industry is the first, sixth and tenth risk contributor, which indicating that the banking insurance industry undertakes more risk compared with other manufacturing industry, like construction and mining.

Investigating the risk sources from the perspective of country, at all the 95 %, 99 %, 99.5 % and 99.9 % levels, the UK and Germany provide the most risk among the top ten risk source countries. For example, at the 95% confidence level of the VaR, the first, fourth, fifth, sixth and ninth places of risk all source from UK and Germany. Under the high confidence level 99.9%, the second, fourth, fifth and seventh place of VaR contribution, the first, sixth and tenth places of CVaR are occupied by UK and Germany.

We now transfer to investigate the risk contribution of R-vine mixed copula setting. Very similarly, at all the 95 %, 99 %, 99.5 % and 99.9 % levels, the financial industry,

Table 2.29: **Small Homogeneous Portfolio VaR and CVaR Risk Contribution Ranking for C-vine Mixed Copula setting** $r = 0.2$

VaR											
		95%			99%			99.5%			99.9%
Sector	Country		Sector	Country		Sector	Country		Sector	Country	
S39 Banks	Japan	11824.83513	S85 Hotel&Leisure	US	53245.58452	S35 Financial Service	Germany	26473.33333	S60 Forestry&Pap	Japan	49808.2192
S89 Telecom Service	US	11074.47322	S39 Banks	Japan	17195.12195	S9 Metal&Mining	Australia	24050.00000	S40 Construction	Japan	47232.8767
S35 Financial Service	Germany	10892.07013	S35 Financial Service	Germany	16171.57275	S41 Info&Commu.	Japan	23391.66667	S41 Info&Commu.	Japan	46691.7808
S76 Transportation	US	10860.54367	S41 Info&Commu.	Japan	15513.87721	S60 Forestry&Pap	Japan	22893.33333	S2 Media	Australia	46027.3973
S56 Chemicals	UK	10332.31462	S9 Metal&Mining	Australia	15416.31623	S13 Broadcast	Canada	22400.00000	S35 Financial Service	Germany	45808.2192
S2 Media	Australia	10304.64854	S2 Media	Australia	15071.48865	S2 Media	Australia	22400.00000	S13 Broadcast	Canada	42082.1918
S85 Hotel&Leisure	US	10183.20733	S56 Chemicals	UK	14950.37847	S40 Construction	Japan	22268.33333	S40 Construction	Japan	39219.1781
S9 Metal&Mining	Australia	10162.45778	S13 Broadcast	Canada	14210.26072	S39 Banks	Japan	21750.00000	S9 Metal&Mining	Australia	37397.2603
S24 Auto&Parts	France	10064.01802	S60 Forestry&Pap	Japan	13931.03448	S89 Telecom Service	US	20541.66667	S39 Banks	Japan	32773.9726
S40 Construction	Japan	9999.67830	S63 Media	UK	13458.36838	S40 Construction	Japan	19768.33333	S76 Transportation	US	31794.5205
CVaR											
		95%			99%			99.5%			99.9%
S39 Banks	Japan	28285.87918	S35 Financial Service	Germany	28310.75697	S35 Financial Service	Germany	43298.8048	S2 Media	Australia	70274.5098
S89 Telecom Service	US	26490.95806	S40 Construction	Japan	26615.53785	S40 Construction	Japan	39494.0239	S40 Construction	Japan	59156.8627
S35 Financial Service	Germany	26054.63640	S2 Media	Australia	25880.47809	S2 Media	Australia	37482.0717	S35 Financial Service	Germany	49176.4706
S76 Transportation	US	25979.22278	S9 Metal&Mining	Australia	25637.45020	S41 Info&Commu.	Japan	35147.4104	S39 Banks	Japan	34117.6471
S56 Chemicals	UK	24715.65987	S89 Telecom Service	US	24551.79283	S60 Forestry&Pap	Japan	33800.7968	S33 Transport&Logis.	Germany	33235.2941
S2 Media	Australia	24649.48057	S39 Banks	Japan	24262.94821	S39 Banks	Japan	32928.2869	S76 Transportation	US	33098.0392
S85 Hotel&Leisure	US	24358.98422	S60 Forestry&Pap	Japan	24143.42629	S40 Construction	Japan	30960.1594	S60 Forestry&Pap	Japan	31686.2745
S40 Construction	Japan	23919.96922	S41 Info&Commu.	Japan	23964.14343	S13 Broadcast	Canada	29067.7291	S9 Metal&Mining	Australia	30588.2353
S64 Mining	UK	23878.79954	S13 Broadcast	Canada	23713.14741	S89 Telecom Service	US	27091.6335	S13 Broadcast	Canada	30117.6471
S41 Info&Commu.	Japan	23760.67718	S40 Construction	Japan	22812.74900	S76 Transportation	US	25219.1235	S35 Financial Service	Germany	30098.0392

Note: VaR and CVaR risk contribution at different confidence levels q estimated by C-vine mixed copula model.

Table 2.30: **Small Homogeneous Portfolio VaR and CVaR Risk Contribution Ranking for C-vine Mixed Copula setting** $r = 0.3$

VaR											
		95%			99%			99.5%			99.9%
Sector	Country		Sector	Country		Sector	Country		Sector	Country	
S73 Chemical	US	11795.88944	S60 Forestry&Pap	UK	24380.80495	S66 Pharm&Bio	UK	34249.63716	S36 Food&Beverages	Germany	49840.90909
S47 Elec. Power&Gas	Japan	11766.47768	S66 Pharm&Bio	UK	24014.70588	S36 Food&Beverages	Germany	33139.33237	S21 Metal&Mining	Canada	47840.90909
S60 Forestry&Pap	UK	11757.61871	S36 Food&Beverages	Germany	23962.84830	S73 Chemical	US	31149.49202	S66 Pharm&Bio	UK	39727.27273
S61 Health Equip.&Ser.	UK	11728.56130	S14 Chemical	Canada	21834.36533	S47 Elec. Power&Gas	Japan	29673.43977	S4 Insurance	Australia	37909.09091
S36 Food&Beverages	Germany	11382.35294	S73 Chemical	US	20340.55728	S60 Forestry&Pap	UK	28650.21771	S49 Chemical	Japan	37439.39394
S21 Metal&Mining	Canada	11338.05811	S83 Publishing&Printing	US	20274.76780	S21 Metal&Mining	Canada	28107.40203	S47 Elec. Power&Gas	Japan	36250.00000
S72 Broadcast	US	10660.05197	S47 Elec. Power&Gas	Japan	20201.23839	S4 Insurance	Australia	23603.77358	S60 Forestry&Pap	UK	35000.00000
S79 Metal&Mining	US	10585.99102	S72 Broadcast	US	19570.43344	S83 Publishing&Printing	US	23396.22642	S73 Chemical	US	33181.81818
S80 Elec. Comp&Equip.	US	10564.37515	S21 Metal&Mining	Canada	18573.52941	S72 Broadcast	US	22583.45428	S84 Energy	US	31000.00000
S16 Pharmaceutical	Canada	10545.71226	S61 Health Equip.&Ser.	UK	17113.00310	S14 Chemical	Canada	22226.41509	S80 Elec. Comp&Equip.	US	29848.48485
CVaR											
		95%			99%			99.5%			99.9%
S36 Food&Beverages	Germany	15770.67519	S66 Pharm&Bio	UK	39884.92063	S21 Metal&Mining	Canada	51789.68254	S4 Insurance	Australia	65411.7647
S14 Chemical	Canada	15456.65202	S21 Metal&Mining	Canada	34248.01587	S36 Food&Beverages	Germany	47607.14286	S47 Elec. Power&Gas	Japan	59705.8824
S60 Forestry&Pap	UK	15437.47503	S73 Chemical	US	33023.80952	S47 Elec. Power&Gas	Japan	41428.57143	S21 Metal&Mining	Canada	57784.3137
S66 Pharm&Bio	UK	15014.78226	S60 Forestry&Pap	UK	32500.00000	S73 Chemical	US	39976.19048	S36 Food&Beverages	Germany	53117.6471
S47 Elec. Power&Gas	Japan	14598.48182	S47 Elec. Power&Gas	Japan	31934.52381	S66 Pharm&Bio	UK	39884.92063	S84 Energy	US	51058.8235
S83 Publishing&Printing	US	14007.59089	S36 Food&Beverages	Germany	30714.28571	S60 Forestry&Pap	UK	30000.00000	S49 Chemical	Japan	48450.9804
S72 Broadcast	US	13987.21534	S4 Insurance	Australia	28130.95238	S4 Insurance	Australia	28130.95238	S66 Pharm&Bio	UK	42843.1373
S73 Chemical	US	13824.21095	S83 Publishing&Printing	US	26386.90476	S84 Energy	US	23619.04762	S73 Chemical	US	34352.9412
S61 Health Equip.&Ser.	UK	13651.61806	S46 Paper&Pulp	Japan	24488.09524	S80 Elec. Comp&Equip.	US	21888.88889	S39 Bank	Japan	32549.0196
S28 Food&Drug	France	13308.82940	S84 Energy	US	24357.14286	S14 Chemical	Canada	20789.68254	S72 Broadcast	US	30509.8039

Note: VaR and CVaR risk contribution at different confidence levels q estimated by C-vine mixed copula model.

Table 2.31: **Small Homogeneous Portfolio VaR and CVaR Risk Contribution Ranking for C-vine Mixed Copula setting** $r = 0.4$

VaR											
		95%			99%			99.5%			99.9%
Sector	Country		Sector	Country		Sector	Country		Sector	Country	
S24 Auto&Parts	France	20822.99297	S24 Auto&Parts	France	29607.45176	S24 Auto&Parts	France	49992.08861	S24 Auto&Parts	France	72588.65248
S36 Food&Beverages	Germany	20822.84347	S24 Auto&Parts	France	27273.45309	S89 Telecom Services	US	37509.49367	S51 Food	Japan	64638.29787
S70 Automobiles	US	20739.12393	S51 Food	Japan	26854.29142	S51 Food	Japan	34335.44304	S65 Oil&Gas Prod.	UK	45957.44681
S50 Electric appliances	Japan	20434.44461	S36 Food&Beverages	Germany	26227.54491	S36 Food&Beverages	Germany	31879.74684	S10 Bank	UK	44255.31915
S24 Auto&Parts	France	19661.53386	S70 Automobiles	US	25995.34265	S10 Bank	UK	31822.78481	S89 Telecom Services	US	42659.57447
S51 Food	Japan	19594.55823	S41 Info&Commu.	Japan	25829.67399	S65 Oil&Gas Prod.	UK	31443.03797	S21 Metal&Mining	Canada	41808.51064
S51 Food	Japan	18893.85558	S65 Oil&Gas Prod.	UK	24143.71257	S10 Bank	UK	30278.48101	S36 Food&Beverages	Germany	40382.97872
S65 Oil&Gas Prod.	UK	18535.50605	S21 Metal&Mining	Canada	24055.88822	S24 Auto&Parts	France	29052.21519	S2 Hotel&Leisure	US	37063.82979
S10 Bank	Canada	18346.53909	S21 Metal&Mining	Canada	23639.38789	S10 Bank	Canada	28509.49367	S10 Bank	UK	35659.57447
S89 Telecom Services	US	18164.59859	S50 Electric appliances	Japan	23405.18962	S89 Telecom Services	US	27917.72152	S70 Automobiles	US	34248.22695
CVaR											
		95%			99%			99.5%			99.9%
S24 Auto&Parts	France	23514.10409	S24 Auto&Parts	France	53293.41317	S24 Auto&Parts	France	58043.47826	S24 Auto&Parts	France	85576.9231
S36 Food&Beverages	Germany	21230.03576	S70 Automobiles	US	36802.39521	S51 Food	Japan	51462.45059	S51 Food	Japan	75115.3846
S70 Automobiles	US	20406.43623	S51 Food	Japan	35516.96607	S65 Oil&Gas Prod.	UK	44395.25692	S41 Info&Commu.	Japan	63538.4615
S24 Auto&Parts	France	20357.56853	S65 Oil&Gas Prod.	UK	31904.19162	S70 Automobiles	US	36438.73518	S9 Metal&Mining	Australia	54923.0769
S51 Food	Japan	20174.01669	S10 Bank	Canada	31780.43912	S36 Food&Beverages	Germany	36355.73123	S65 Oil&Gas Prod.	UK	49846.1538
S50 Electric appliances	Japan	19928.48629	S36 Food&Beverages	Germany	29724.55090	S9 Metal&Mining	Australia	33727.27273	S2 Hotel&Leisure	US	46384.6154
S89 Telecom Services	US	19277.31426	S50 Electric appliances	Japan	27532.93413	S40 Construction	Japan	33118.57708	S40 Construction	Japan	46038.4615
S41 Info&Commu.	Japan	19033.77036	S10 Bank	Canada	26570.85828	S21 Metal&Mining	Canada	32620.55336	S78 Food Prod.	US	45769.2308
S21 Metal&Mining	Canada	18580.45292	S24 Auto&Parts	France	26421.15768	S89 Telecom Services	US	31699.60474	S21 Metal&Mining	Canada	45346.1538
S65 Oil&Gas Prod.	UK	18193.08701	S21 Metal&Mining	Canada	25886.22754	S10 Bank	UK	31466.40316	S9 Metal&Mining	Australia	42807.6923

Note: VaR and CVaR risk contribution at different confidence levels q estimated by C-vine mixed copula model.

Table 2.32: **Small Homogeneous Portfolio VaR and CVaR Risk Contribution Ranking for C-vine Mixed Copula setting** $r = 0.5$

VaR											
		95%			99%			99.5%			99.9%
Sector	Country		Sector	Country		Sector	Country		Sector	Country	
S53 Transport.Equip.	Japan	16745.59526	S70 Automobiles	US	36457.86963	S70 Automobiles	US	37646.34146	S70 Automobiles	US	69631.57895
S4 Insurance	Australia	15960.64548	S53 Transport.Equip.	France	36324.32432	S53 Transport.Equip.	France	35512.19512	S53 Transport.Equip.	France	51087.71930
S18 Elec comp.&Equip.	Canada	14981.72238	S36 Food&Beverages	Germany	33453.89507	S36 Food&Beverages	Germany	29713.41463	S18 Elec comp.&Equip.	Canada	50105.26316
S70 Automobiles	US	14523.29985	S4 Insurance	Australia	32245.62798	S4 Insurance	Australia	29291.15854	S54 Banks	UK	45614.03509
S36 Food&Beverages	Germany	14443.27351	S24 Auto&Parts	France	29360.09539	S24 Auto&Parts	France	28490.85366	S73 Chemical	US	42122.80702
S24 Auto&Parts	France	13922.27894	S18 Elec comp.&Equip.	Canada	24648.64865	S53 Transport.Equip.	Japan	28371.95122	S78 Food products	US	38789.47368
S54 Banks	UK	13765.84884	S53 Transport.Equip.	Japan	23294.11765	S18 Elec comp.&Equip.	Canada	27987.80488	S4 Insurance	Australia	37456.14035
S53 Transport.Equip.	Japan	13693.39700	S68 Trvel&Leis	UK	23007.94913	S78 Food products	US	27576.21951	S45 Pharmaceutical	Japan	37052.63158
S87 Oil&Gas refining	US	13468.13766	S38 Media	Germany	22863.27504	S54 Banks	UK	26829.26829	S44 Mining	Japan	36377.19298
S91 Movies&Entertainment	US	13267.41314	S78 Food products	US	22368.83943	S82 Utilities	US	25289.63415	S68 Trvel&Leis	UK	35263.15789
CVaR											
		95%			99%			99.5%			99.9%
S53 Transport.Equip.	France	28424.58101	S70 Automobiles	US	42251.49701	S70 Automobiles	US	56223.10757	S70 Automobiles	US	95117.64706
S4 Insurance	Australia	25388.26816	S53 Transport.Equip.	Japan	40239.52096	S53 Transport.Equip.	Japan	48191.23506	S45 Pharmaceutical	Japan	75294.11765
S70 Automobiles	US	24636.87151	S36 Food&Beverages	Germany	34485.02994	S36 Food&Beverages	Germany	42358.56574	S54 Banks	UK	62745.09804
S18 Elec comp.&Equip.	Canada	24095.77015	S18 Elec comp.&Equip.	Canada	31760.47904	S18 Elec comp.&Equip.	Canada	42262.94821	S73 Chemical	US	60529.41176
S24 Auto&Parts	France	23617.31844	S4 Insurance	Australia	31534.93014	S68 Trvel&Leis	UK	38438.24701	S18 Elec comp.&Equip.	Canada	56000.00000
S36 Food&Beverages	Germany	22980.84597	S54 Banks	UK	31137.72455	S53 Transport.Equip.	Japan	37864.54183	S68 Trvel&Leis	UK	55176.47059
S54 Banks	UK	22027.13488	S24 Auto&Parts	France	30199.60080	S4 Insurance	Australia	37426.29482	S38 Media	Germany	54078.43137
S91 Movies&Entertainment	US	21869.91221	S78 Food products	US	29688.62275	S54 Banks	UK	35059.76096	S53 Transport.Equip.	Japan	52705.88235
S53 Transport.Equip.	Japan	21174.78053	S44 Mining	Japan	27842.31537	S24 Auto&Parts	France	33685.25896	S5 Transportation	Australia	44941.17647
S38 Media	Germany	20438.94653	S53 Transport.Equip.	Japan	27664.67066	S22 Media	Canada	30438.24701	S24 Auto&Parts	France	43627.45098

Note: VaR and CVaR risk contribution at different confidence levels q estimated by C-vine mixed copula model.

Table 2.33: **Small Homogeneous Portfolio VaR and CVaR Risk Contribution Ranking for C-vine Mixed Copula setting** $r = 0.6$

VaR											
		95%			99%			99.5%			99.9%
Sector	Country		Sector	Country		Sector	Country		Sector	Country	
S74 Insurance	US	19138.65070	S30 Chemical	Germany	38641.10429	S74 Insurance	US	39564.54121	S74 Insurance	US	66947.36842
S67 Software&Comp Svs.	UK	19111.47064	S74 Insurance	US	37067.48466	S45 Pharmaceutical	Japan	39284.60342	S1 Banks	Australia	64000.00000
S1 Banks	Australia	18554.92639	S67 Software&Comp Svs.	UK	36432.51534	S80 Elec. comp&Equip	US	36679.62675	S30 Chemical	Germany	63142.85714
S30 Chemical	Germany	17519.81880	S80 Elec. comp&Equip	US	35149.53988	S30 Chemical	Germany	36432.34837	S80 Elec. comp&Equip	US	60225.56391
S27 Con&Materials	France	17463.19366	S1 Banks	Australia	34699.38650	S1 Banks	Australia	35533.43701	S69 Life Insurance	UK	56857.14286
S45 Pharmaceutical	Japan	17436.66073	S45 Pharmaceutical	Japan	34545.24540	S67 Software&Comp Svs.	UK	34612.75272	S80 Elec. comp&Equip	US	52142.85714
S25 Banks	France	16968.12813	S25 Banks	France	31501.53374	S32 Insurance	Germany	33629.86003	S67 Software&Comp Svs.	UK	48270.67669
S80 Elec. comp&Equip	US	16486.81443	S32 Insurance	Germany	27533.74233	S25 Banks	France	30583.20373	S45 Pharmaceutical	Japan	47481.20301
S48 Oil&Gas Prds.	Japan	16089.30594	S27 Con&Materials	France	27055.21472	S80 Elec. comp&Equip	US	29517.88491	S41 Info&Commun.	Japan	41578.94737
S79 Metal&Minig	US	15286.52322	S41 Info&Commun.	Japan	25141.87117	S27 Con&Materials	France	29393.46812	S54 Banks	UK	34601.50376
CVaR											
		95%			99%			99.5%			99.9%
S74 Insurance	US	28990.00400	S74 Insurance	US	43161.67665	S45 Pharmaceutical	Japan	55350.59761	S80 Elec. comp&Equip	US	78529.41176
S45 Pharmaceutical	Japan	28616.55338	S1 Banks	Australia	41133.73253	S80 Elec. comp&Equip	US	54960.15936	S74 Insurance	US	74823.52941
S67 Software&Comp Svs.	UK	28407.83687	S30 Chemical	Germany	39700.59880	S30 Chemical	Germany	54589.64143	S1 Banks	Australia	70274.50980
S30 Chemical	Germany	27746.50140	S45 Pharmaceutical	Japan	39495.00998	S74 Insurance	US	48988.04781	S30 Chemical	Germany	69333.33333
S1 Banks	Australia	27048.38065	S67 Software&Comp Svs.	UK	39297.40519	S1 Banks	Australia	46406.37450	S88 Software&Svs.	US	60862.74510
S80 Elec. comp&Equip	US	25799.68013	S80 Elec. comp&Equip	US	39081.83633	S67 Software&Comp Svs.	UK	42629.48207	S67 Software&Comp Svs.	UK	58745.09804
S25 Banks	France	24811.67533	S25 Banks	France	34017.96407	S41 Info&Commun.	Japan	42490.03984	S45 Pharmaceutical	Japan	57784.31373
S27 Con&Materials	France	24686.12555	S32 Insurance	Germany	33389.22156	S69 Life Insurance	UK	36470.11952	S32 Insurance	Germany	56000.00000
S48 Oil&Gas Prds.	Japan	23569.37225	S80 Elec. comp&Equip	US	32055.88822	S80 Elec. comp&Equip	US	36354.58167	S48 Oil&Gas Prds.	Japan	55039.21569
S83 Publishing&Printing	Germany	23413.43463	S90 Airlines	US	30427.14571	S32 Insurance	Germany	35760.95618	S71 Banks	US	42941.17647

Note: VaR and CVaR risk contribution at different confidence levels q estimated by C-vine mixed copula model.

Table 2.34: **Small Homogeneous Portfolio VaR and CVaR Risk Contribution Ranking for R-vine Mixed Copula setting** $r = 0.2$

VaR											
		95%			99%			99.5%			99.9%
Sector	Country		Sector	Country		Sector	Country		Sector	Country	
S47 Elec power&Gas	Japan	12221.61589	S1 Banks	Australia	18230.96664	S15 Insurance	Canada	25948.05195	S47 Elec power&Gas	Japan	54993.1034
S1 Banks	Australia	10835.83447	S47 Elec power&Gas	Japan	17810.94953	S1 Banks	Australia	25948.05195	S1 Banks	Australia	48993.1034
S23 Utilities	Canada	10429.58921	S34 Utilities	Germany	17772.45509	S47 Elec power&Gas	Japan	23012.98701	S15 Insurance	Canada	48993.1034
S34 Utilities	Germany	10412.00546	S15 Insurance	Canada	17471.34303	S58 Eltro/Elec Eq.	UK	22805.19481	S71 Banks	US	39358.6207
S15 Insurance	Canada	10230.10459	S58 Eltro/Elec Eq.	UK	16147.98973	S66 Pharm&Bio	France	22243.50649	S24 Auto&Parts	Japan	37924.1379
S47 Elec power&Gas	Japan	9869.63771	S24 Auto&Parts	Japan	15559.45252	S71 Banks	US	22092.53247	S66 Pharm&Bio	France	33531.0345
S51 Food	Japan	9847.35486	S51 Food	Japan	15307.10009	S34 Utilities	Germany	22025.97403	S34 Utilities	Germany	32165.5172
S60 Forestry&Pap	UK	9728.05821	S42 Insurance	Japan	15189.05047	S48 Oil&Coal Prds.	Japan	21642.85714	S72 Broadcast	US	31482.7586
S24 Auto&Parts	Japan	9425.64802	S66 Pharm&Bio	France	15124.03764	S42 Insurance	Japan	21305.19481	S48 Oil&Coal Prds.	Japan	30648.2759
S42 Insurance	Japan	9420.34258	S23 Utilities	Canada	14802.39521	S24 Auto&Parts	Japan	21287.33766	S58 Eltro/Elec Eq.	UK	30275.8621
CVaR											
		95%			99%			99.5%			99.9%
S47 Elec power&Gas	Japan	31867.98419	S15 Insurance	Canada	28359.28144	S15 Insurance	Canada	38608.6957	S47 Elec power&Gas	Japan	60803.9216
S1 Banks	Australia	28254.54545	S1 Banks	Australia	28359.28144	S47 Elec power&Gas	Japan	36770.7510	S1 Banks	Australia	52235.2941
S34 Utilities	Germany	27149.40711	S47 Elec power&Gas	Japan	25642.71457	S1 Banks	Australia	35098.8142	S47 Elec power&Gas	Japan	45058.8235
S15 Insurance	Canada	26675.09881	S71 Banks	US	24534.93014	S42 Insurance	Japan	32039.5257	S71 Banks	US	43039.2157
S24 Auto&Parts	Japan	24577.47036	S24 Auto&Parts	Japan	24485.02994	S48 Oil&Coal Prds.	Japan	30339.9209	S58 Eltro/Elec Eq.	UK	36823.5294
S66 Pharm&Bio	France	24283.79447	S48 Oil&Coal Prds.	Japan	23385.22954	S47 Elec power&Gas	Japan	30276.6798	S15 Insurance	Canada	34823.5294
S71 Banks	US	23598.41897	S23 Utilities	Canada	23025.94810	S24 Auto&Parts	Japan	30094.8617	S27 Con&Materials	France	33823.5294
S76 Transportation	US	22580.63241	S34 Utilities	Germany	22850.29940	S51 Food	Japan	29213.4387	S24 Auto&Parts	Japan	33176.4706
S58 Eltro/Elec Eq.	UK	22557.31225	S76 Transportation	US	22473.05389	S11 Transport	Canada	28537.5494	S46 Paper Pulp	Japan	32078.4314
S92 Paper Packaging	US	21653.75494	S42 Insurance	Japan	22343.31337	S76 Transportation	US	26371.5415	S48 Oil&Coal Prds.	Japan	31686.2745

Note: VaR and CVaR risk contribution at different confidence levels q estimated by R-vine mixed copula model.

Table 2.35: **Small Homogeneous Portfolio VaR and CVaR Risk Contribution Ranking for R-vine Mixed Copula setting** $r = 0.3$

VaR											
		95%			99%			99.5%			99.9%
Sector	Country		Sector	Country		Sector	Country		Sector	Country	
S3 Energy	Australia	14864.17083	S42 Insurance	Japan	30989.18919	S42 Insurance	Japan	38684.21053	S79 Metal&Mining	US	64852.94118
S42 Insurance	Japan	14006.62252	S41 Info&Commun.	Japan	24123.55212	S41 Info&Commun.	Japan	33684.21053	S42 Insurance	Japan	45397.05882
S32 Insurance	Germany	13527.77402	S88 Software&Svs.	US	23630.11583	S79 Metal&Mining	US	32354.06699	S28 Food&Drug Retail	France	38235.29412
S79 Metal&Mining	US	13410.59603	S79 Metal&Mining	US	23497.29730	S3 Energy	Australia	31033.49282	S78 Food Prod.	US	35911.76471
S21 Metal&Mining	Canada	13321.93540	S28 Food&Drug Retail	France	22548.26255	S32 Insurance	Germany	30106.85805	S41 Info&Commun.	Japan	35588.23529
S13 Broadcast	Canada	12854.16948	S32 Insurance	Germany	22373.74517	S78 Food Prod.	US	29859.64912	S3 Energy	Australia	34213.23529
S3 Energy	Australia	12726.58467	S78 Food Prod.	US	22314.28571	S88 Software&Svs.	US	26808.61244	S21 Metal&Mining	Canada	33080.88235
S41 Info&Commun.	Japan	12666.57656	S3 Energy	Australia	21884.94208	S19 HT/REST/LEIS	Japan	26258.37321	S32 Insurance	Germany	32279.41176
S88 Software&Svs.	US	12290.98527	S3 Energy	Australia	20091.89189	S28 Food&Drug Retail	France	25518.34131	S19 HT/REST/LEIS	Japan	31705.88235
S57 Con&Materials	UK	12134.61279	S21 Metal&Mining	Canada	19265.63707	S57 Con&Materials	UK	25306.22010	S2 Media	Australia	29154.41176
CVaR											
		95%			99%			99.5%			99.9%
S42 Insurance	Japan	40090.90909	S42 Insurance	Japan	39531.87251	S42 Insurance	Japan	49195.21912	S21 Metal&Mining	Canada	70788.46154
S32 Insurance	Germany	38720.30948	S41 Info&Commun.	Japan	35059.76096	S79 Metal&Mining	US	49195.21912	S3 Energy	Australia	65076.92308
S79 Metal&Mining	US	38384.91296	S3 Energy	Australia	34547.80876	S41 Info&Commun.	Japan	45577.68924	S85 Hotel&Leisure	US	52230.76923
S41 Info&Commun.	Japan	36255.31915	S79 Metal&Mining	US	32503.98406	S3 Energy	Australia	42131.47410	S88 Software&Svs.	US	49730.76923
S88 Software&Svs.	US	15005.80271	S78 Food Prod.	US	32430.27888	S88 Software&Svs.	US	39494.02390	S28 Food&Drug Retail	France	46153.84615
S78 Food Prod.	US	14485.10638	S32 Insurance	Germany	32356.57371	S32 Insurance	Germany	38478.08765	S42 Insurance	Japan	42403.84615
S3 Energy	Australia	14072.72727	S19 HT/REST/LEIS	Japan	30454.18327	S28 Food&Drug Retail	France	38247.01195	S41 Info&Commun.	Japan	42307.69231
S19 HT/REST/LEIS	Japan	13951.25725	S88 Software&Svs.	US	29191.23506	S19 HT/REST/LEIS	Japan	35920.31873	S3 Energy	Australia	39711.53846
S28 Food&Drug Retail	France	13771.76015	S28 Food&Drug Retail	France	28685.25896	S78 Food Prod.	US	34051.79283	S78 Food Prod.	US	39134.61538
S3 Energy	Australia	13740.03868	S85 Hotel&Leisure	US	27051.79283	S3 Energy	Australia	29617.52988	S50 Electric appliances	Japan	38461.53846

Note: VaR and CVaR risk contribution at different confidence levels q estimated by R-vine mixed copula model.

Table 2.36: **Small Homogeneous Portfolio VaR and CVaR Risk Contribution Ranking for R-vine Mixed Copula settings** $r = 0.4$

VaR											
		95%			99%			99.5%			99.9%
Sector	Country		Sector	Country		Sector	Country		Sector	Country	
S31 Construction	Germany	22553.40891	S31 Construction	Germany	27109.57960	S31 Construction	Germany	33958.13953	S64 Mining	UK	55870.96774
S66 Pharm&Bio	UK	20935.65384	S91 Movies& Entertainment	US	26629.91041	S11 Transpt	Canada	33565.89147	S26 Chemical	France	51361.29032
S20 Energy	Canada	20910.42631	S64 Mining	UK	25365.26533	S64 Mining	UK	32894.57364	S31 Construction	Germany	49025.80645
S26 Chemical	France	20873.06403	S11 Transpt	Canada	25365.26533	S91 Movies& Entertainment	US	32558.13953	S91 Movies& Entertainment	US	48774.19355
S90 Airlines	US	20809.99521	S49 Chemical	Japan	24198.48380	S20 Energy	Canada	29200.00000	S66 Pharm&Bio	UK	42580.64516
S2 Media	Australia	20803.12949	S66 Pharm&Bio	UK	23046.17505	S49 Chemical	Japan	29162.79070	S20 Energy	Canada	39561.29032
S16 Pharmaceuticals	Canada	20404.75810	S16 Pharmaceuticals	Canada	22812.54307	S53 Transport Equ.	Japan	29026.35659	S11 Transpt	Canada	36316.12903
S64 Mining	UK	19911.22465	S20 Energy	Canada	22337.69814	S16 Pharmaceuticals	Canada	28582.94574	S23 Utilities	Canada	35812.90323
S11 Transpt	Canada	19842.08846	S2 Media	Australia	22226.05100	S2 Media	Australia	28000.00000	S16 Pharmaceuticals	Canada	35141.93548
S91 Movies& Entertainment	US	19782.85167	S11 Transpt	Canada	21663.68022	S66 Pharm&Bio	UK	27968.99225	S90 Airlines	US	33522.58065
CVaR											
		95%			99%			99.5%			99.9%
S31 Construction	Germany	20318.18182	S31 Construction	Germany	36507.96813	S64 Mining	UK	46577.68924	S64 Mining	UK	81698.1132
S91 Movies& Entertainment	US	19760.76555	S64 Mining	UK	31914.34263	S91 Movies& Entertainment	US	40159.36255	S91 Movies& Entertainment	US	63396.2264
S64 Mining	UK	19336.52313	S20 Energy	Canada	30537.84861	S66 Pharm&Bio	UK	38565.73705	S31 Construction	Germany	59037.7358
S16 Pharmaceuticals	Canada	18878.38915	S11 Transpt	Canada	29326.69323	S26 Chemical	France	35055.77689	S90 Airlines	US	57188.6792
S49 Chemical	Japan	18166.66667	S91 Movies& Entertainment	US	29282.86853	S11 Transpt	Canada	32776.89243	S23 Utilities	Canada	56396.2264
S26 Chemical	France	18043.06220	S2 Media	Australia	29123.50598	S23 Utilities	Canada	32322.70916	S42 Insurance	Japan	49584.9057
S66 Pharm&Bio	UK	17894.73684	S66 Pharm&Bio	UK	28047.80876	S20 Energy	Canada	31410.35857	S26 Chemical	France	47433.9623
S20 Energy	Canada	17813.39713	S26 Chemical	France	26709.16335	S2 Media	Australia	30836.65339	S66 Pharm&Bio	UK	41509.4340
S11 Transpt	Canada	17782.69537	S23 Utilities	Canada	26368.52590	S31 Construction	Germany	30274.90040	S11 Transpt	Canada	40849.0566
S21 Metal& Mining	Canada	17699.36204	S16 Pharmaceuticals	Canada	25039.84064	S84 Energy	US	28398.40637	S19 HT/REST/LEIS	Japan	34716.9811

Note: VaR and CVaR risk contribution at different confidence levels q estimated by R-vine mixed copula model.

Table 2.37: **Small Homogeneous Portfolio VaR and CVaR Risk Contribution Ranking for R-vine Mixed Copula setting** $r = 0.5$

VaR											
		95%			99%			99.5%			99.9%
Sector	Country		Sector	Country		Sector	Country		Sector	Country	
S73 Chemical	US	26024.18861	S44 Mining	Japan	34644.03292	S44 Mining	Japan	41313.63636	S5 Transportation	Australia	67384.6154
S66 Pharm&Bio	UK	25547.30557	S64 Mining	UK	32647.46228	S64 Mining	UK	41212.12121	S64 Mining	UK	56468.5315
S44 Mining	Japan	25456.82792	S73 Chemical	US	30521.26200	S5 Transportation	Australia	41145.45455	S44 Mining	Japan	50013.9860
S60 Forestry&Pap	UK	24677.58726	S74 Insurance	US	29787.37997	S74 Insurance	US	36484.84848	S49 Chemical	Japan	48965.0350
S5 Transportation	Australia	23938.45683	S60 Forestry&Pap	UK	29561.04252	S60 Forestry&Pap	UK	31998.48485	S39 Banks	Japan	48671.3287
S39 Banks	Japan	23241.73301	S66 Pharm&Bio	UK	28995.19890	S73 Chemical	US	31689.39394	S66 Pharm&Bio	UK	46678.3217
S74 Insurance	US	23040.41641	S5 Transportation	Australia	28539.09465	S39 Banks	Japan	30318.18182	S60 Forestry&Pap	UK	45209.7902
S90 Airlines	US	14012.55358	S39 Banks	Japan	27746.91358	S12 Auto&Compo	US	28993.93939	S74 Insurance	US	45104.8951
S64 Mining	UK	13468.30986	S12 Auto&Compo	US	27676.26886	S88 Software&Svs.	US	28342.42424	S12 Auto&Compo	US	34909.0909
S12 Auto&Compo	US	13437.84446	S35 Financial Svs.	Germany	27193.41564	S49 Chemical	Japan	28290.90909	S73 Chemical	US	34230.7692
CVaR											
		95%			99%			99.5%			99.9%
S44 Mining	Japan	25140.40686	S5 Transportation	Australia	44586.826347	S64 Mining	UK	57341.26984	S44 Mining	Japan	87647.0588
S73 Chemical	US	24495.41284	S64 Mining	UK	43263.473054	S5 Transportation	Australia	50404.76190	S49 Chemical	Japan	83901.9608
S66 Pharm&Bio	UK	23962.90387	S44 Mining	Japan	40149.700599	S44 Mining	Japan	47892.85714	S5 Transportation	Australia	77294.1176
S64 Mining	UK	23903.07140	S74 Insurance	US	35189.620758	S39 Banks	Japan	43154.76190	S74 Insurance	US	59019.6078
S60 Forestry&Pap	UK	23209.01476	S39 Banks	Japan	33862.275449	S49 Chemical	Japan	41678.57143	S64 Mining	UK	58333.3333
S74 Insurance	US	22640.60630	S60 Forestry&Pap	UK	32690.618762	S74 Insurance	US	39246.03175	S88 Software&Svs.	US	54627.4510
S5 Transportation	Australia	22013.56203	S73 Chemical	US	31976.047904	S60 Forestry&Pap	UK	37626.98413	S66 Pharm&Bio	UK	52352.9412
S88 Software&Svs.	US	21431.99043	S12 Auto&Compo	US	30722.554890	S73 Chemical	US	33551.58730	S39 Banks	Japan	51176.4706
S39 Banks	Japan	21342.24172	S49 Chemical	Japan	30281.437126	S12 Auto&Compo	US	33015.87302	S12 Auto&Compo	US	48941.1765
S49 Chemical	Japan	20947.34743	S88 Software&Svs.	US	29393.213573	S14 Chemical	Canada	32250.00000	S14 Chemical	Canada	45529.4118

Note: VaR and CVaR risk contribution at different confidence levels q estimated by R-vine mixed copula model.

Table 2.38: **Small Homogeneous Portfolio VaR and CVaR Risk Contribution Ranking for R-vine Mixed Copula setting** $r = 0.6$

VaR											
		95%			99%			99.5%			99.9%
Sector	Country		Sector	Country		Sector	Country		Sector	Country	
S47 Elec. Power&Gas	Japan	19322.34432	S15 Insurance	Canada	30953.35516	S47 Elec. Power&Gas	Japan	35955.15695	S61 H/C EQ&SVS	UK	49902.09790
S15 Insurance	Canada	19264.06926	S3 Energy	Australia	30257.77414	S3 Energy	Australia	35762.33184	S3 Energy	Australia	48671.32867
S45 Pharmaceutical	Japan	19017.48252	S47 Elec. Power&Gas	Japan	28317.51227	S58 Eltro/Elec EQ	UK	32852.01794	S67 Software&Comp Svs.	UK	45944.05594
S67 Software&Comp Svs.	UK	18815.18482	S15 Insurance	Canada	28186.57938	S15 Insurance	Canada	32593.42302	S63 Media	UK	44531.46853
S3 Energy	Australia	18613.88611	S62 Inds Transpt	UK	27851.06383	S30 Chemical	Germany	32399.10314	S44 Mining	Japan	43951.04895
S15 Insurance	Canada	18584.08258	S45 Pharmaceutical	Japan	27788.05237	S83 Publishing&Printing	US	32085.20179	S15 Insurance	Canada	43566.43357
S46 Pulp&Paper	Japan	18251.74825	S36 Food&Beverages	Germany	27773.32242	S38 Media	Germany	31713.00448	S90 Airlines	US	43076.92308
S62 Inds Transpt	UK	17662.33766	S30 Chemical	Germany	27127.65957	S62 Inds Transpt	UK	31713.00448	S30 Chemical	Germany	41608.39161
S89 Telecom Svs.	US	17454.54545	S67 Software&Comp Svs.	UK	26523.73159	S21 Metal&Mining	US	30780.26906	S47 Elec. Power&Gas	Japan	41314.68531
S38 Media	Germany	17367.96537	S58 Eltro/Elec EQ	UK	26311.78396	S46 Pulp&Paper	Japan	30560.53812	S46 Pulp&Paper	Japan	39545.45455
CVaR											
		95%			99%			99.5%			99.9%
S15 Insurance	Canada	29962.15139	S3 Energy	Australia	40808.38323	S44 Mining	Japan	46555.55556	S3 Energy	Australia	76764.70588
S3 Energy	Australia	28075.69721	S47 Elec. Power&Gas	Japan	37904.19162	S38 Media	Germany	45603.17460	S90 Airlines	US	60392.15686
S45 Pharmaceutical	Japan	27935.85657	S30 Chemical	Germany	36477.04591	S89 Telecom Svs.	US	44444.44444	S67 Software&Comp Svs.	UK	60117.64706
S27 Con&Mat	France	27334.26295	S46 Pulp&Paper	Japan	34730.53892	S62 Inds Transpt	UK	43849.20635	S45 Pharmaceutical	Japan	57647.05882
S47 Elec. Power&Gas	Japan	27236.65339	S38 Media	Germany	34407.18563	S3 Energy	Australia	41428.57143	S58 Eltro/Elec EQ	UK	55862.74510
S15 Insurance	Canada	26750.59761	S44 Mining	Japan	34289.42116	S30 Chemical	Germany	40476.19048	S63 Media	UK	54627.45098
S44 Mining	Japan	25707.56972	S58 Eltro/Elec EQ	UK	34119.76048	S45 Pharmaceutical	Japan	40250.00000	S38 Media	Germany	52000.00000
S67 Software&Comp Svs.	UK	25477.29084	S89 Telecom Svs.	US	33980.03992	S47 Elec. Power&Gas	Japan	40190.47619	S39 Banks	Japan	52000.00000
S62 Inds Transpt	UK	24829.48207	S62 Inds Transpt	UK	33524.95010	S27 Con&Mat	France	39884.92063	S62 Inds Transpt	UK	50000.00000
S61 H/C EQ&SVS	UK	24698.80478	S21 Metal&Mining	US	33197.60479	S61 H/C EQ&SVS	UK	38936.50794	S47 Elec. Power&Gas	Japan	49647.05882

Note: VaR and CVaR risk contribution at different confidence levels q estimated by R-vine mixed copula model.

such as banks, insurance industry occupies around half of places of risk contribution of top ten industries. For instance, at 95% confidence level, the financial industry, such as banks, insurance industry account for half of the risk contribution of the top ten most risky industrial sectors. At the 95% confidence level, VaR contribution of the bank insurance industry takes the first, second, fourth, seventh and eighth places, while the first, fifth, sixth and tenth of VaR contribution are taken up by insurance industry, regarding measure of CVaR, the bank insurance industry is the first, sixth and tenth risk contributor, which indicating that the banking insurance industry undertake higher degree of risk than other manufacturing industry, like construction and mining sector.

From the perspective of risk country of origin, at all the 95%, 99%, 99.5% and 99.9% levels, the UK and Germany provide the most risk among the top ten risk source countries. For example, at the 95% confidence level of the VaR, the first, fourth, fifth, sixth and ninth place of risk source come from UK and Germany. Under the high confidence level 99.9%, the second, fourth, fifth and seventh place of VaR contribution, the first, sixth and tenth places of CVaR are occupied by UK and Germany.

2.9 Loss Function-based Backtesting

Classic market risk backtesting, see Kupiec (1995) unconditional coverage test, the conditional coverage test proposed by Christoffersen (1998) and the duration-based Weibull test of independence by Christoffersen and Pelletier (2004), have a common spirit that compare a given risk metric, which is derived from a risk model and a forecast about the future trend, with ex-post observations. For this mechanism, the time frames of data is split into two non-overlapping parts. The first time frame, normally called estimation period, is used to calibrate the model. At the end of the estimation period, the risk metric as a prediction of the up-coming future is derived from the calibrated model. Then, what we want to know is the accuracy of the model's prediction about the future. Therefore, a test period is defined, usually begin from the end of the estimation period, and the model user checks the performance of risk metric in test period, and whether it is consistent with the model's prediction. If the risk metric in the test period largely deviates from the prediction of the model, it suggests there exists model misspecification.

While in this paper, due to the adoption of hypothesized credit portfolio, we can-

Table 2.39: **Small Heterogeneous Portfolio VaR and CVaR Risk Contribution Ranking for C-vine Mixed Copula setting**

VaR											
		95%			99%			99.5%			99.9%
Sector	Country		Sector	Country		Sector	Country		Sector	Country	
S56 Chemicals	UK	98930.48	S1 Banks	Australia	167920.7	S1 Banks	Australia	176272.5	S1 Banks	Australia	243239.43662
S1 Banks	Australia	90415.47	S56 Chemicals	UK	140583.4	S56 Chemicals	UK	170598.8	S56 Chemicals	UK	250140.84507
S16 Pharmaceutical	Canada	53791.44	S16 Pharmaceutical	Canada	114549.7	S16 Pharmaceutical	Canada	147125.7	S16 Pharmaceutical	Canada	213401.40845
S54 Banks	UK	53771.42	S31 Construction	Germany	80780.85	S31 Construction	Germany	98940.12	S31 Construction	Germany	141901.40845
S31 Construction	Germany	42659.54	S54 Banks	UK	80591.62	S54 Banks	UK	84492.51	S54 Banks	UK	103239.43662
S37 Technology	Germany	32942.55	S37 Technology	Germany	55646.97	S37 Technology	Germany	66130.24	S39 Banks	Japan	68478.87324
S74 Insurance	US	29969.01	S74 Insurance	US	46937.92	S74 Insurance	US	56691.62	S37 Technology	Germany	65492.95775
S39 Banks	Japan	29696.97	S39 Banks	Japan	45290.95	S64 Mining	UK	49230.54	S75 Machinery	US	63690.14085
S64 Mining	UK	28892.64	S75 Machinery	US	43434.55	S39 Banks	Japan	47640.72	S46 Pulp&Paper	Canada	61478.87324
S46 Pulp&Paper	Canada	28130.40	S63 Media	UK	40188.48	S75 Machinery	US	47029.94	S74 Insurance	US	60957.74648
CVaR											
		95%			99%			99.5%			99.9%
S1 Banks	Australia	129578.3	S1 Banks	Australia	203286.9	S1 Banks	Australia	252321.4	S1 Banks	Australia	261666.6667
S56 Chemicals	UK	125107.9	S56 Chemicals	UK	188685.3	S56 Chemicals	UK	202619.0	S56 Chemicals	UK	246666.6667
S16 Pharmaceutical	Canada	85762.59	S16 Pharmaceutical	Canada	151727.1	S16 Pharmaceutical	Canada	188500.0	S16 Pharmaceutical	Canada	224823.5294
S54 Banks	UK	70897.68	S31 Construction	Germany	102757.0	S31 Construction	Germany	121539.7	S31 Construction	Germany	205450.9804
S31 Construction	Germany	59918.47	S54 Banks	UK	93450.20	S37 Technology	Germany	86111.11	S75 Machinery	US	130666.6667
S37 Technology	Germany	45533.57	S37 Technology	Germany	69472.11	S54 Banks	UK	81444.44	S54 Banks	UK	129352.9412
S74 Insurance	US	43677.86	S39 Banks	Japan	53709.16	S46 Pulp&Paper	Canada	63511.90	S46 Pulp&Paper	Canada	85588.2353
S39 Banks	Japan	41691.45	S14 Chemicals	Canada	47729.08	S39 Banks	Japan	61388.89	S37 Technology	Germany	75980.3922
S46 Pulp&Paper	Canada	39932.05	S75 Machinery	US	47410.36	S75 Machinery	US	56666.67	S88 Software&Services	US	57882.3529
S75 Machinery	US	37288.57	S46 Pulp&Paper	Canada	47340.64	S74 Insurance	US	51523.81	S39 Banks	Japan	52000.0000

Note: VaR and CVaR risk contribution at different confidence levels q estimated by C-vine mixed copula model.

Table 2.40: **Small Heterogeneous Portfolio VaR and CVaR Risk Contribution Ranking for R-vine Mixed Copula setting**

VaR											
		95%			99%			99.5%			99.9%
Sector	Country		Sector	Country		Sector	Country		Sector	Country	
S56 Chemicals	UK	98466.44	S1 Banks	Australia	151346.58665	S1 Banks	Australia	168253.8	S16 Pharmaceutical	Canada	231320.61069
S1 Banks	Australia	93758.52	S56 Chemicals	UK	139339.83496	S56 Chemicals	UK	156525.7	S1 Banks	Australia	203740.45802
S16 Pharmaceutical	Canada	61374.55	S16 Pharmaceutical	Canada	127795.94899	S16 Pharmaceutical	Canada	147552.9	S56 Chemicals	UK	192061.06870
S54 Banks	UK	57904.77	S31 Construction	Germany	90093.02326	S31 Construction	Germany	110725.1	S31 Construction	Germany	166122.13740
S31 Construction	Germany	49768.95	S54 Banks	UK	88531.88297	S54 Banks	UK	109577.0	S54 Banks	UK	123099.23664
S64 Mining	UK	33367.44	S46 Pulp&Paper	Canada	52029.25731	S37 Technology	Germany	66194.86	S37 Technology	Germany	94656.48855
S37 Technology	Germany	30555.15	S74 Insurance	US	49919.72993	S74 Insurance	US	58534.74	S74 Insurance	US	78465.64885
S46 Pulp&Paper	Canada	30439.92	S37 Technology	Germany	47674.41860	S64 Mining	UK	55679.76	S63 Media	UK	77099.23664
S74 Insurance	US	30190.61	S39 Banks	Japan	45426.85671	S39 Banks	Japan	46146.53	S46 Pulp&Paper	Canada	62938.93130
S39 Banks	Japan	27677.05	S63 Media	UK	38793.69842	S75 Machinery	US	41397.28	S14 Chemicals	Canada	54870.22901
CVaR											
		95%			99%			99.5%			99.9%
S1 Banks	Australia	123666.53339	S1 Banks	Australia	182232.1	S16 Pharmaceutical	Canada	217750.00000	S16 Pharmaceutical	Canada	256941.1765
S56 Chemicals	UK	115097.96082	S56 Chemicals	UK	161507.9	S56 Chemicals	UK	190873.01587	S31 Construction	Germany	221254.9020
S16 Pharmaceutical	Canada	93656.13754	S16 Pharmaceutical	Canada	159250.0	S1 Banks	Australia	190019.84127	S56 Chemicals	UK	217647.0588
S54 Banks	UK	72098.36066	S54 Banks	UK	122166.7	S54 Banks	UK	133801.58730	S1 Banks	Australia	200098.0392
S31 Construction	Germany	68643.74250	S31 Construction	Germany	105547.6	S31 Construction	Germany	127936.50794	S54 Banks	UK	158098.0392
S46 Pulp&Paper	Canada	45377.84886	S37 Technology	Germany	691964.3	S37 Technology	Germany	79960.31746	S37 Technology	Germany	136764.7059
S74 Insurance	US	43911.63535	S74 Insurance	US	611845.2	S74 Insurance	US	68698.41270	S63 Media	UK	102980.3922
S37 Technology	Germany	40593.76250	S46 Pulp&Paper	Canada	567757.9	S46 Pulp&Paper	Canada	67361.11111	S74 Insurance	US	95470.5882
S39 Banks	Japan	38350.25990	S64 Mining	UK	495416.7	S63 Media	UK	59317.46032	S82 Utilities	US	75137.2549
S64 Mining	UK	35307.87685	S39 Banks	Japan	491111.1	S14 Chemicals	Canada	47539.68254	S46 Pulp&Paper	Canada	66568.6275

Note: VaR and CVaR risk contribution at different confidence levels q estimated by R-vine mixed copula model.

not obtain the credit portfolio average return as the benchmark for traditional market risk backtesting framework, in this sense, we introduce loss function based backtesting method for our credit VaR forecasting. The idea of employing loss functions to assess risk measure performance was firstly introduced by Lopez (1998) and Lopez (1999). The loss function evaluation method is not based on a hypothesis-testing framework, but rather on assigning to risk measure estimates a numerical score that reflects the evaluator's specific concerns. As such, it provides a measure of relative performance that can be utilised to assess the performance of risk measure estimates. Under this approach, a model which minimises the loss is preferred to other models (Lopez (1998)). We forecast the one-day-ahead VaR of equally weighted credit portfolios. The five competing copula models are specified the same as above sections. The in-sample forecasting period for the credit portfolio corresponds to the period from January 2002 to December 2010. We evaluate all risk metrics separately at 95%, 99%, 99.5% and 99.9% confidence levels, since they constitute the levels most commonly used for model evaluation both in literature and in financial markets.

2.9.1 The Model Confidence Set Procedure

The comparison and selection of a number of competing models raises the question of requiring a statistical method or procedure that delivers the best model with respect to a given criterion. Moreover, model selection issue is regard to be necessary to reduce the uncertainty when the usual comparison procedures do not provide a unique result. For example, when a series of models are compared in terms of their predictive ability, models that exhibit better forecast accuracy are preferred. However, in practice, when evaluating the performances of different forecasting models, it is not always possible to establish which model clearly outperforms the remaining available competing ones. This issue occurs especially when the set of competing alternatives similarly to each other, such as our copula models case. As discussed by Hansen and Lunde (2005) and Hansen et al. (2011), from the practical perspective view, it probably unrealistic to obtain a single model which dominates all the other competitors, which may either owing to the different model specifications exhibit statistically equivalence or because there lack of sufficient information obtained from the data to discriminate the candidate models. Nevertheless, though it is hard to deliver the unique discriminant model, the ranking of the competing models still make great sense for model users to select the best performance model.

Recently, a number of alternative procedures have been developed to deliver the ”best fitting” model, such as the Reality Check (RC) of White (2000), the Stepwise Multiple Testing procedure of Romano and Wolf (2005), the Superior Predictive Ability (SPA) test of Hansen (2005) and the Conditional Predictive Ability (CPA) test of Giacomini and White (2006). Among these multiple testing procedures, the Model Confidence Set procedure (MCS) proposed by Hansen et al. (2003) and Hansen et al. (2011) consists of a sequence of statistic tests which allows to construct the so called Superior Set of Models (SSM), where the null hypothesis of equal predictive ability (EPA) is not rejected at certain confidence level α. The EPA statistic test can be evaluated for an arbitrary loss function, which essentially means that it is possible to test models on preferred aspects depending on the chosen loss function. The possibility to specify model user supplied loss functions enhances the flexibility of the procedure that can be used to test different aspects. In this paper, we compare the different vine copula and traditional multivariate Gaussian copula models setting for our credit portfolio by investigating their VaR forecasts ability and actuality similar to Caporin and McAleer (2014) and Chen and Gerlach (2013). Since the object of interest is the conditional quantile of the portfolio loss distribution, we choose the asymmetric linear loss function proposed in González-Rivera et al. (2004) and Giacomini and White (2006). The asymmetric loss function compares the performances of two or more forecasting models, by evaluating the forecasts with a pre-specified loss function. The best performance forecast model is the model that produces the smallest expected loss.

As the procedure mentioned above, the MCS procedure starts from an initial set of m competing models, denoted by M^0, and results in a smaller set of superior models, the SSM, denoted by $\hat{M}^*_{1-\alpha}$. Of course, the best scenario is when the final set consists of a single discriminating model. At each iteration, the MCS procedure tests the null hypothesis of EPA among the competing models and ends with the creation of the SSM only if the null hypothesis is accepted, otherwise the MCS is iterated again and the EPA is tested on a smaller set of models obtained by eliminating the worst one at the previous step. The availability of several alternative model specifications being able to adequately describe the unobserved data generating process (DGP) raises the question of selecting the ”best fitting model” according to a given optimality criterion.

Formally, let y_t denotes the observation at time t and let $\hat{l}_{i,t}$ be the output of model i at time t, the loss function $l_{i,t}$ associated to the ith model is defined as

$$l_{i,t} = l(y_t, \hat{y}_{i,t}) \tag{2.35}$$

and measures the difference between the output $\hat{y}_{i,t}$ and the "posteriori" realisation y_t. In this paper, we develop a method named "Rotation-Substitution" that we adopt VaR value generated by one vine copula setting as the benchmark realisation y_t, comparing the realisation with output generated by other four vine copula settings. For example, when we assume the VaR generated by multivariate Gaussian copula setting as benchmark realisation, then we compare the other four vine copula setting VaR to this benchmark. We then repeat this process five times then we get five series of results. As a consequence, this process can provide a cross verification of the best model.

González-Rivera et al. (2004) use a loss function to compare the forecasting ability of different GARCH specifications. By employing their method, Bernardi and Catania (2015) specify the asymmetric VaR loss function in order to predict extreme loss within the framework of high-frequency financial data setting. Here, we similarly adopt their asymmetric VaR loss function to investigate the VaR forecasting ability of our vine copula models. The VaR loss function of González-Rivera et al. (2004) is defined as

$$l(y_t, VaR_t^\tau) = (\tau - d_t^\tau)(y_t - VaR_t^\tau) \tag{2.36}$$

where VaR_t^τ denotes the τ-level predicted VaR at time t, given information up to time $t-1$, F_{t-1}, and $d_t^\tau = 1(y_t < VaR_t^\tau)$ is the τ-level quantile loss function. The asymmetric VaR loss function represents the natural candidate to backtest quantile based risk measures since it penalises more heavily observations below the τth quantile level, i.e. $y_t < VaR_t^\tau$. Details about the loss function specifications can be find in Hansen and Lunde (2005). We can also consider the following alternative loss function,

$$l(r_t, VaR_t^\tau) = (\tau - m_\delta(r_t, VaR_t^\tau))(r_t - VaR_t^\tau) \tag{2.37}$$

where $m_\delta(a, b) = [1 + exp\{\delta(a - b)\}]^{-1}$. Note that the δ parameter controls the function smoothness.

We now briefly describe how the MCS procedure is implemented. The procedure starts from an initial set of models M^0 of dimension m, encompassing all the alternative copula model specifications, and delivers, for a given confidence level α, a smaller set, the superior set of models (SSM), $\hat{M}^*_{1-\alpha}$, of dimension $m^* \leq m$. The SSM, $\hat{M}^*_{1-\alpha}$, contains all the models having superior predictive ability according to the selected loss function. Of course, the best scenario is when the final set consists of a single model, i.e., $m^* = 1$. Formally, let $d_{ij,t}$ denote the loss differential between models i and j at time t:

$$d_{ij,t} = l_{i,t} - l_{j,t}, \quad i, j = 1, ..., m, \quad t = 1, ..., n, \tag{2.38}$$

and let

$$d_{i\cdot,t} = (m-1)^{-1} \sum_{j \in M \setminus \{i\}} d_{ij,t}, \quad i = 1, ..., m, \tag{2.39}$$

be the average loss of model i relative to any other model j at time t. The EPA hypothesis for a given set of models M can be formulated in two alternative ways,

$$H_{0,M} : c_{ij} = 0, \quad for \;\; all \;\; i, j = 1, 2, ..., m \tag{2.40}$$

$$H_{A,M} : c_{ij} \neq 0, \quad for \;\; some \;\; i, j = 1, ..., m, \tag{2.41}$$

or

$$H_{0,M} : c_{i\cdot} = 0, \quad for \;\; all \;\; i = 1, 2, ..., m \tag{2.42}$$

$$H_{A,M} : c_{i\cdot} \neq 0, \quad for \;\; some \;\; i = 1, 2, ..., m, \tag{2.43}$$

where $c_{ij} = \mathbb{E}(d_{ij})$ and $c_{i\cdot} = \mathbb{E}(d_{i\cdot})$ are assumed to be finite and time independent. According to Hansen et al. (2011), the two hypothesis defined in equations (39)-(42) can be tested by constructing the following two statistics

$$t_{ij} = \frac{\bar{d}_{ij}}{\sqrt{\widehat{var}(\bar{d}_{ij})}} \tag{2.44}$$

$$t_{i,\cdot} = \frac{\bar{d}_{i,\cdot}}{\sqrt{\widehat{var}(\bar{d}_{i,\cdot})}}, \tag{2.45}$$

for $i, j \in M$, where $\bar{d}_{i,\cdot} = (m-1)^{-1} \sum_{j \in M \setminus \{i\}} \bar{d}_{ij}$ is the average loss of the ith model relative to the average losses across the models belonging to the set M, and $\bar{d}_{ij} = n^{-1}\sum_{t=1}^{n} d_{ij,t}$ measures the relative average loss between models i and j. The variances $\widehat{var}(\bar{d}_{i,\cdot})$ and $\widehat{var}(\bar{d}_{ij})$ are bootstrapped estimates of $var(\bar{d}_{i,\cdot})$ and $var(\bar{d}_{ij})$ respectively. The bootstrapped variances $\widehat{var}(\bar{d}_{i,\cdot})$ and $\widehat{var}(\bar{d}_{ij,t})$ are calculated by performing a block-bootstrap procedure where the block length p is set as the maximum number of significants parameters obtained by fitting an AR(p) process on the d_{ij} terms. The statistic t_{ij} is used in the well know test for comparing two forecasts; see e.g., Diebold and Mariano (2002) and West (1996), while the second one is used in Hansen et al. (2003) and Hansen et al. (2011). As discussed in Hansen et al. (2011), the two EPA null hypothesis presented in equations (39)-(42) map naturally into the two test statistics

$$T_{R,M} = \max_{i,j \in M} |t_{ij}| \;\; and \;\; T_{max,M} = \max_{i \in M} t_{i\cdot}, \tag{2.46}$$

where t_{ij} and $t_{i\cdot}$ are defined in equations (43)(44). Since the asymptotic distributions of the two test statistics is nonstandard, the relevant distributions under the null hypothesis is estimated using a bootstrap procedure similar to that used to estimate $var(\bar{d}_{i,\cdot})$ and $var(\bar{d}_{ij})$. For further details about the bootstrap procedure, see e.g., White (2000), Hansen et al. (2003), Hansen (2005), Kilian (1999) and Clark and McCracken (2001).

The MCS procedure consists of a sequential testing procedure, which eliminates at each step the worst model, until the hypothesis of equal predictive ability (EPA) is accepted for all the models belonging to the SSM. At each step, the choice of the worst model to be eliminated has been made using an elimination rule that is coherent with the statistic test defined in equations (46)-(47) which are

$$e_{R,M} = \underset{i}{argmax}\Big\{ \sup_{j \in M} \frac{\bar{d}_{ij}}{\sqrt{\widehat{var}(\bar{d}_{ij})}} \Big\}, \tag{2.47}$$

$$e_{max,M} = \underset{i \in M}{argmax} \frac{\bar{d}_{i,\cdot}}{\sqrt{\widehat{var}(\bar{d}_{i,\cdot})}}, \tag{2.48}$$

respectively. Therefore, the MCS procedure to obtain the SSM can summarise that, firstly, we set M = M^0, then test for EPA-hypothesis. If EPA is accepted, then terminate the algorithm and set $\hat{M}^*_{1-\alpha} = M$, otherwise use the elimination rules defined to determine and remove the worst model, and repeat the previous step.

2.9.2 Loss function-based backtesting results

We obtain the loss functions VaR forecast performance based on various different copula setting as benchmark. Tables 41-45 report the numerical scores of average out-of-sample VaR estimates of random systematic risk factor loading heterogenous credit portfolio.

In general, our empirical results suggest that, during periods of financial instability, such as the recent Global Financial Crisis (GFC) of 2007-2008 and the recent European Sovereign debt crisis, R-vine mixed copula and C-vine mixed copula specification VaR deliver better forecasts. The numerical scores of the VaR-based loss functions are highly supportive of the R-vine mixed copula model at 95%, 99%, 99.5% and 99.9% confidence level. In particular, almost all numerical scores for every possible credit portfolio combination tend to favour the R-vine mixed copula model over the rest of the models at various confidence levels. Let us investigate these results in details. When we adopt the multivariate Gaussian copula as the benchmark realisation, at each confidence level, R-vine mixed copula setting dominates other four competing copula settings. C-vine mixed copula setting also performs well, it ranks second three times. In the case of R-vine t copula as benchmark, R-vine mixed copula setting ranks first at 99%, 99.5% confidence level, and it ranks second when C-vine mixed copula ranks first at 95%, 99.9% level. Similarly, when C-vine t copula setting being the benchmark, R-vine mixed copula and C-vine mixed copula setting separately ranks first and second twice. As expected, when R-vine mixed copula becomes the realisation benchmark, the C-vine mixed copula setting dominates all other competing copula setting at each confidence level, similarly, when C-vine mixed copula setting becomes the realisation benchmark, the R-vine mixed copula setting also dominates all other competing copula setting at each confidence level. These numerical scores ranking results strongly support the vine structure is superior to capture the dependency of various risk factors. Moreover, the availability and flexibility of the abundant bivariate copula families becomes the key step of precisely modelling dependence when vine structure has been selected.

In sum, the portfolio loss of R-vine mixed copula model setting are minimal 12 out of 20 cases at 95%, 99%, 99.5% and 99.9% confidence level in all these five ”Rotation-Substitution” test when VaR-based loss functions are employed. The C-vine mixed copula model setting also produces satisfactory results under each confidence level, 8 out of 20 cases portfolio loss are minimal by C-vine mixed copula model.

Therefore, the findings of loss function based backtesting in this section strongly support our vine copula approach for credit portfolio risk factors dependence modelling. It suggests that R-vine mixed and C-vine mixed copula model can most successfully and precisely describe the dependence structure of systematic risk factors and provide better fit in the tails. Moreover, these findings support the theoretical and empirical findings of Joe et al. (2010) and Nikoloulopoulos et al. (2012). Based on the loss function results at 95%, 99%, 99.5% and 99.9% confidence level, it demonstrates that the VaR forecasts produced by the R-vine mixed and C-vine mixed copula model outperform other vine copula and multivariate Gaussian copula model setting.

There is not the case preference towards a multivariate Gaussian copula model, among the 20 test cases, multivariate Gaussian copula model is eliminated in 9 out of 20 cases, additionally, in rest 11 cases, its VaR-based loss function numerical scores ranks last twice. Therefore, multivariate Gaussian copula model is regarded as the least preferred models according to the VaR-based loss function numerical scores at each 95%, 99%, 99.5% and 99.9% confidence level. These results support that the multivariate Gaussian copula which lack of tail dependence and asymmetric dependence characteristics is hard to capture the dependency structure of various systematic risk factors and provide good fit in the tails.

2.10 Conclusion

Especially in the context of high dimension, Archimedean and elliptical copulas suffer from their inability to model asymmetric and complex dependence structures. The financial crisis of 2007-2008 established the need for an improved approach to model the dependence structure of credit portfolio. Vine copulas, therefore, are an intuitive and convenient alternative to conventional copulas, which circumvent their shortcomings. In this paper, we present how vine copulas can be used to derive a more accurate and more reliable estimate of the VaR and CVaR of a multi factor credit portfolio in credit risk management. We employ a common framework of latent variable and mixture credit risk model to construct a multi factors credit portfolio risk model, then we fit conventional multivariate Gaussian copula and various vine copulas separately to monthly equity returns of 92 sectors systematic risk factors. After having fitted C-vine, R-vine and tradi-

Table 2.41: **Value at Risk Backtesting under R-vine Gaussian Copula Benchmark Assumption**

95%						
Superior Set of Models						
C-vine t copula eliminated						
	Rank(M)	v(M)	MCS(M)	Rank(R)	v(R)	MCS(R)
R-vine t copula	3	0.9146	1	3	1.317	0.051
R-vine mixed copula	1	-1.3689	1	1	-1.059	1.000
C-vine mixed copula	2	0.4597	1	2	1.059	0.872
Number of eliminated models:1 Statistic:Tmax						
99%						
Superior Set of Models						
No Model eliminated						
	Rank(M)	v(M)	MCS(M)	Rank(R)	v(R)	MCS(R)
R-vine t copula	4	0.7053	1	4	1.468	0.0000
R-vine mixed copula	1	-1.6926	1	1	-1.298	1.0000
C-vine t copula	2	0.4172	1	3	1.297	0.6726
C-vine mixed copula	3	0.5957	1	2	1.405	0.0002
Number of eliminated models:0 Statistic:Tmax						
99.5%						
Superior Set of Models						
No Model eliminated						
	Rank(M)	v(M)	MCS(M)	Rank(R)	v(R)	MCS(R)
R-vine t copula	4	0.7426	1	4	1.482	0.0252
R-vine mixed copula	1	-1.6968	1	1	-1.319	1.0000
C-vine t copula	3	0.5168	1	3	1.360	0.3350
C-vine mixed copula	2	0.4488	1	2	1.319	0.5430
Number of eliminated models:0 Statistic:Tmax						
99.9%						
Superior Set of Models						
No Model eliminated						
	Rank(M)	v(M)	MCS(M)	Rank(R)	v(R)	MCS(R)
R-vine t copula	3	0.3496	1	3	1.0939	1
R-vine mixed copula	1	-1.4335	1	1	-0.7527	1
C-vine t copula	4	1.3027	0.319	4	1.6674	0
C-vine mixed copula	2	-0.2164	1	2	0.7528	1
Number of eliminated models:0 Statistic:Tmax						

Table 2.42: **Value at Risk Backtesting under R-vine t Copula Benchmark Assumption**

95%						
Superior Set of Models						
C-vine t copula eliminated R-vine Gaussian copula eliminated						
	Rank(M)	v(M)	MCS(M)	Rank(R)	v(R)	MCS(R)
R-vine mixed copula	2	0.4019	0.7106	2	0.4019	0.7074
C-vine mixed copula	1	-0.4098	1.0000	1	-0.4098	1.0000
Number of eliminated models:2 Statistic:Tmax						
99%						
Superior Set of Models						
C-vine t copula eliminated C-vine mixed copula eliminated R-vine Gaussian copula eliminated						
	Rank(M)	v(M)	MCS(M)	Rank(R)	v(R)	MCS(R)
R-vine mixed copula	1	-0.995	1	1	-0.995	1
Number of eliminated models:3 Statistic:Tmax						
99.5%						
Superior Set of Models						
No Model eliminated						
	Rank(M)	v(M)	MCS(M)	Rank(R)	v(R)	MCS(R)
R-vine t copula	3	0.23406598	1	3	1.087085	1
R-vine mixed copula	1	-1.53285871	1	1	-1.001936	1
C-vine t copula	4	1.21983325	1	4	1.677009	0
C-vine mixed copula	2	0.09019975	1	2	1.002329	1
Number of eliminated models:0 Statistic:Tmax						
99.9%						
Superior Set of Models						
R-vine Gaussian copula eliminated						
	Rank(M)	v(M)	MCS(M)	Rank(R)	v(R)	MCS(R)
R-vine mixed copula	2	0.4143	1.0000	2	1.030	0.9830
C-vine t copula	3	0.9576	0.9998	3	1.343	0.3848
C-vine mixed copula	1	-1.3709	1.0000	1	-1.031	1.0000
Number of eliminated models:1 Statistic:Tmax						

Table 2.43: **Value at Risk Backtesting under R-vine mixed Copula Benchmark Assumption**

95%						
Superior Set of Models						
No Model eliminated						
	Rank(M)	v(M)	MCS(M)	Rank(R)	v(R)	MCS(R)
R-vine Gaussian copula	2	-0.2466	1	2	0.7578	1.0000
R-vine t copula	4	1.1781	1	4	1.6148	0.0012
C-vine t copula	3	0.5449	1	3	1.2359	1.0000
C-vine mixed copula	1	-1.4739	1	1	-0.7588	1.0000
99%						
Superior Set of Models						
C-vine t copula eliminated R-vine Gaussian copula eliminated						
	Rank(M)	v(M)	MCS(M)	Rank(R)	v(R)	MCS(R)
R-vine t copula	2	0.8511	0.4498	2	0.8511	0.4456
C-vine mixed copula	1	-0.8582	1.0000	1	-0.8582	1.0000
Number of eliminated models:2 Statistic:Tmax						
99.5%						
Superior Set of Models						
R-vine Gaussian copula eliminated C-vine t copula eliminated						
	Rank(M)	v(M)	MCS(M)	Rank(R)	v(R)	MCS(R)
R-vine t copula	2	0.8888	0.2972	2	0.8888	0.29
C-vine mixed copula	1	-0.8977	1.0000	1	-0.8977	1.00
Number of eliminated models:2 Statistic:Tmax						
99.9%						
Superior Set of Models						
R-vine t copula eliminated R-vine Gaussian copula eliminated						
	Rank(M)	v(M)	MCS(M)	Rank(R)	v(R)	MCS(R)
C-vine t copula	2	0.9857	0.291	2	0.9857	0.2784
C-vine mixed copula	1	-0.9910	1.000	1	-0.9910	1.0000
Number of eliminated models:2 Statistic:Tmax						

Table 2.44: **Value at Risk Backtesting under C-vine t Copula Benchmark Assumption**

95%						
Superior Set of Models						
No Model eliminated						
	Rank(M)	v(M)	MCS(M)	Rank(R)	v(R)	MCS(R)
R-vine Gaussian copula	1	-1.5769267	1	1	-0.9446589	1.0000
R-vine t copula	4	0.9460699	1	4	1.5399211	0.0780
C-vine t copula	3	0.6982672	1	3	1.3907887	0.4782
C-vine mixed copula	2	-0.0524004	1	2	0.9438430	1.0000
99%						
Superior Set of Models						
C-vine mixed copula eliminated						
	Rank(M)	v(M)	MCS(M)	Rank(R)	v(R)	MCS(R)
R-vine Gaussian copula	3	1.1164134	0.9696	3	1.3905417	0
R-vine mixed copula	1	-1.2947261	1.0000	1	-0.8553522	1
R-vine t copula	2	0.1825572	1.0000	2	0.8555191	1
Number of eliminated models:1 Statistic:Tmax						
99.5%						
Superior Set of Models						
R-vine Gaussian copula eliminated R-vine t copula eliminated						
	Rank(M)	v(M)	MCS(M)	Rank(R)	v(R)	MCS(R)
R-vine mixed copula	2	0.5297738	0.3214	2	0.5297738	0.3188
C-vine mixed copula	1	-0.5327003	1.0000	1	-0.5327003	1.0000
Number of eliminated models:2 Statistic:Tmax						
99.9%						
Superior Set of Models						
R-vine t copula eliminated R-vine Gaussian copula eliminated						
	Rank(M)	v(M)	MCS(M)	Rank(R)	v(R)	MCS(R)
C-vine t copula	2	0.7633700	0.7194	2	0.7633700	0.7194
C-vine mixed copula	1	-0.7630053	1.0000	1	-0.7630053	1.0000
Number of eliminated models:2 Statistic:Tmax						

Table 2.45: **Value at Risk Backtesting under C-vine mixed Copula Benchmark Assumption**

95%						
Superior Set of Models						
C-vine t copula eliminated						
	Rank(M)	v(M)	MCS(M)	Rank(R)	v(R)	MCS(R)
R-vine Gaussian copula	3	0.7104	1	3	1.218	0
R-vine t copula	2	0.6987	1	2	1.211	0
R-vine mixed copula	1	-1.3983	1	1	-1.211	1
Number of eliminated models:1 Statistic:Tmax						
99%						
Superior Set of Models						
C-vine t copula eliminated						
	Rank(M)	v(M)	MCS(M)	Rank(R)	v(R)	MCS(R)
R-vine Gaussian copula	2	-0.07671984	1.0000	2	0.6263107	1.0000
R-vine mixed copula	1	-1.14027652	1.0000	1	-0.6256664	1.0000
R-vine t copula	3	1.22770328	0.0242	3	1.3579953	0.0526
Number of eliminated models:1 Statistic:Tmax						
99.5%						
Superior Set of Models						
R-vine Gaussian copula eliminated						
	Rank(M)	v(M)	MCS(M)	Rank(R)	v(R)	MCS(R)
R-vine mixed copula	1	-1.20555308	1.0000	1	-0.6870983	1.0000
R-vine t copula	3	1.22756358	0.0118	3	1.4008996	0.0096
C-vine t copula	2	-0.02112572	1.0000	2	0.6864368	1.0000
Number of eliminated models:1 Statistic:Tmax						
99.9%						
Superior Set of Models						
R-vine Gaussian copula eliminated R-vine t copula eliminated						
	Rank(M)	v(M)	MCS(M)	Rank(R)	v(R)	MCS(R)
R-vine mixed copula	1	-0.9928	1.000	1	-0.9928	1.0000
C-vine t copula	2	0.9908	0.069	2	0.9908	0.0648
Number of eliminated models:2 Statistic:Tmax						

tional mutilative Gaussian copula to our data, we find that the vine copulas for modelling the dependence structure of risk factors returns largely improve the risk estimate ability of both threshold and mixture credit risk models, the conventional multivariate Gaussian copula is deficient in modelling the dependence structure of the risk factors for credit portfolio. In depth, we also calculate the out-of-sample risk measure VaR, CVaR and also adopt Euler allocation to calculate and discuss the VaR and CVaR risk contribution of various industry sectors at different systematic risk ratios for the corresponding homogeneous and heterogeneous credit portfolio separately based on various vine copulas and multivariate Gaussian copula settings. We find VaR and CVaR are seriously underestimated by multivariate Gaussian copula model. In backtesting test, we introduce the Loss function based backtesting method - Model Confidence Set method - to select and rank the best copula modelling settings for multi risk factors, the R-vine mixed copula setting outperform other copula settings. Classical multivariate Gaussian copula offers the worst statistical fit (as measured by goodness-of-fit test, such as Voung test, Clarke test and Akaike's information criterion, briefly AIC) to the data, the Gaussian copula underestimates risk measure VaR and ES, while the vine structure provide a better statistical fit to the data than the classic Gaussian copula.

In our study, we compare various copula setting approaches both from a statistical and economic perspective. Vine copulas enable us to model a more flexible and less restricted dependence structure compared to classical Gaussian copula, as replacing the latter by the former leads to an increased AIC. The better statistical fit to the data suggests that the modeled dependence structure is a more realistic model of the actual dependence structure and, consequently, vine copula should be preferred to conventional Gaussian copula. When classic Gaussian copula is replaced by vine copula structures, the VaR and CVaR are all increased. C-vine mixed copula and R-vine mixed copula in turn lead to a higher risk measure than multivariate Gaussian copula. Flexible building blocks chosen from bivariate copula families in a vine structure results in more accurate and reliable estimate for VaR and CVaR. Therefore, we obtain statistically well-founded arguments that support the criticism of the role of the Gaussian copula in the financial crisis. We present the convenient and applicable alternative model-vine copula mixed model-which are supposed to be adopted by risk managers in order to improve the methodology of credit portfolio risk modelling.

Table 46: Descriptive Statistics of Risk Factors

	Australia									Canada
	BANKS	MEDIA	ENERGY	INSURANCE	TRANS.	MATERIALS	PHARM & BIO.	RETAILING	METALS & MINING	BANKS
min	-29.7196555	-32.26579898	-35.31251406	-29.21448958	-32.14837678	-34.3580689	-19.2158207	-39.75084542	-34.70207532	-23.31428636
max	27.22949242	23.61146553	21.47321418	19.99405165	24.60583643	20.98235781	32.62394338	26.76508749	22.6090864	21.54114845
median	1.075718926	-0.010199282	1.415121338	1.048626841	1.544453408	1.05053797	2.051837589	1.491423023	0.874698372	1.327271867
mean	0.581362628	-0.340114852	0.624026693	0.160788429	0.615810415	0.646423087	1.225774116	0.331498132	0.621705528	0.730892595
var	59.66720822	82.91255229	70.80246877	58.11238331	56.72846803	73.55601854	55.85355426	77.38366651	84.55927975	37.92239064
std.dev	7.724455206	9.105632998	8.414420287	7.623147861	7.531830324	8.576480545	7.47352355	8.796798651	9.195611983	6.158115835
skewness	-0.709689846	-0.445245662	-0.660745336	-0.696751667	-0.846666359	-0.66164362	0.041623348	-0.746903014	-0.56148573	-0.454986167
kurtosis	3.320580812	2.759202672	2.415644771	2.734323276	4.845232842	2.238469098	2.557787984	3.33021313	2.540367085	3.519225379
JB test	0.000	0.004	0.000	0.000	0.000	0.000	0.0155	0.000	0.025	0.000
Ljung-Box(10)	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
ARCH(10)	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Note: This table reports descriptive statistics for log returns of 92 industry sectors equity index over the period from January, 2002 to December, 2016, which correspond to a sample of 200 observations for each market. We report the p-value of JB test for the normality of series, p-value of LB test lag 10, the Ljung-Box Q-test for autocorrelation at lags 10. In addition, we report the p-value of Engles Lagrange Multiplier test for the ARCH effect on the residual series.

Table 47: Descriptive Statistics of Risk Factors

	Canada									
	TRANSPT	AUTO&COMPO	BCAST	CHEMICALS	INSURANCE	PHARMACEUTICALS	FD&BEV&TOB	ELEC&COMP&EQU	HT&REST&LEIS	ENERGY
min	-31.73478801	-38.70033555	-33.10830974	-42.35895304	-39.86854211	-80.29994382	-20.19363415	-66.94000479	-32.26710814	-31.99291069
max	18.69807052	26.70478903	23.26535688	26.9931354	25.70685445	30.96748054	19.49078491	37.65352238	37.31767227	23.4330464
median	1.453318564	1.133060549	0.775216578	1.06777294	1.176444634	2.017146167	1.047226668	0.096038594	0.897538669	1.050524905
mean	1.191103315	0.636821457	-0.021946584	0.946613093	0.527331921	-0.128530308	0.996148453	-1.140938911	0.798104787	0.656264186
var	45.44358816	84.87132198	62.66472442	89.6418376	61.64524702	186.338433	31.03437588	80.26570728	56.06746586	61.45845064
std.dev	6.741185961	9.212563269	7.916105382	9.467937346	7.851448721	13.65058362	5.570850553	8.959113085	7.487821169	7.839544033
skewness	-0.69139831	-0.577735969	-0.658499532	-0.986462113	-1.229662862	-1.669536905	-0.163977722	-2.302442879	-0.0767963	-0.634148249
kurtosis	2.254096472	2.659738506	2.12142789	3.762484776	6.823903899	6.713271886	2.360916927	18.97327036	3.683374714	2.671490043
JB test	0.000	0.001	0.000	0.000	0.000	0.000	0.0025	0.000	0.000	0.001
Ljung-Box(10)	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
ARCH(10)	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Note: This table reports descriptive statistics for log returns of 92 industry sectors equity index over the period from January, 2002 to December, 2016, which correspond to a sample of 200 observations for each market. We report the p-value of JB test for the normality of series, p-value of LB test lag 10, the Ljung-Box Q-test for autocorrelation at lags 10. In addition, we report the p-value of Engles Lagrange Multiplier test for the ARCH effect on the residual series.

Table 48: Descriptive Statistics of Risk Factors

	Canada			France					Germany	
	MET&MIN	MEDIA	UTILITIES	AUTO&PARTS	BANKS	CHEMICALS	CON...MAT	FD...DRUG.RTL	AUTOMOBILE	CHEMICALS
min	-53.13520868	-19.02669961	-18.58642098	-43.52023418	-34.03559181	-23.5998384	-26.37103122	-30.44070978	-26.88877411	-28.49967031
max	32.48456833	14.72287369	12.81857203	30.85691073	33.44723881	15.87129207	22.93399587	21.12385797	22.41709298	24.95158369
median	1.410752692	0.746637986	0.803621317	0.728370117	1.318014376	1.337243151	0.876963983	0.239682173	1.378791097	1.839302282
mean	0.419244392	-0.018954317	0.57813253	0.280487822	0.102004913	0.592589703	0.126548639	-0.432363614	0.663522022	0.809264106
var	108.3044983	29.02428267	24.45569537	101.9632825	105.6228551	41.04859496	67.48196024	60.70892236	77.6727917	62.21231394
std.dev	10.40694471	5.387418924	4.94527	10.09768699	10.27729804	6.406917742	8.214740424	7.791593056	8.813216876	7.887478301
skewness	-0.609321334	-0.655496739	-0.344622272	-0.588286481	-0.490313231	-0.51067723	-0.521391139	-0.386859311	-0.523494075	-0.461107684
kurtosis	3.302667209	2.761468131	2.6165143	2.427631526	2.876385084	3.025259567	2.251187691	2.753402823	2.802368599	2.345994779
JB test	0.000	0.000	0.0305	0.000	0.000	0.0025	0.002	0.0135	0.005	0.0015
Ljung-Box(10)	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
ARCH(10)	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Note: This table reports descriptive statistics for log returns of 92 industry sectors equity index over the period from January, 2002 to December, 2016, which correspond to a sample of 200 observations for each market. We report the p-value of JB test for the normality of series, p-value of LB test lag 10, the Ljung-Box Q-test for autocorrelation at lags 10. In addition, we report the p-value of Engles Lagrange Multiplier test for the ARCH effect on the residual series.

Table 49: Descriptive Statistics of Risk Factors

	Germany							Japan		
	CONSTRUCTION	INSURANCE	TRANSPORT&LOGIS	UTILITIES	FINANCIAL&SERVICES	FOOD&BEVERAGES	TECHNOLOGY	MEDIA	BANKS	CONSTRUCTION
min	-39.78039312	-39.91703674	-43.3449393	-28.71359635	-32.81134823	-27.07975278	-32.82897643	-65.66909645	-32.60915206	-21.01011209
max	25.92688487	44.58794504	22.97720603	25.27668151	30.84744174	25.51384182	46.75999207	33.74565963	26.52811358	22.00618848
median	0.864444342	0.074958564	0.241759746	-0.049196293	0.177839303	0.449365089	0.132903143	-0.62350043	-0.486099182	0.328785153
mean	0.666934969	0.682761951	0.607296566	0.605290661	0.61628711	0.606921184	0.820471167	0.754801897	0.577694158	0.424802392
var	9.431884784	9.655712107	8.588470397	8.560102626	8.715615897	8.583161704	11.60321452	10.67451079	8.169829128	6.007613044
std.dev	10.91092199	128.8139947	35.52481567	-173.9989366	49.00837866	19.10064201	87.30579494	-17.12029419	-16.80691808	18.27215429
skewness	-2.057113095	-1.458856329	-2.943463036	-1.385641888	-1.303841387	-1.093319061	-0.315431655	-4.124424085	-0.478799466	0.120173299
kurtosis	2.482125414	6.90095006	5.173422701	2.318410723	2.748044393	2.420464104	2.581351049	10.56860577	2.135657735	2.810567043
JB test	0.000	0.000	0.000	0.004	0.002	0.004	0.001	0.000	0.0035	0.0055
Ljung-Box(10)	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
ARCH(10)	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Note: This table reports descriptive statistics for log returns of 92 industry sectors equity index over the period from January, 2002 to December, 2016, which correspond to a sample of 200 observations for each market. We report the p-value of JB test for the normality of series, p-value of LB test lag 10, the Ljung-Box Q-test for autocorrelation at lags 10. In addition, we report the p-value of Engles Lagrange Multiplier test for the ARCH effect on the residual series.

Table 50: Descriptive Statistics of Risk Factors

	Japan									
	INFO&COMMUNICATION	INSURANCE	MACHINERY	MINING	PHARMACEUTICAL	PULP&PAPER	ELEC&POWER&GAS	OIL&COAL&PRDS	CHEMICAL	ELECTRIC& APPLIANCES
min	-30.17452819	-24.88727973	-27.17072918	-38.41423386	-18.58587005	-25.08343515	-38.91638067	-22.2586293	-15.17826939	-23.54963193
max	18.73286425	16.74262238	15.66984519	21.956573	13.75267138	22.97711565	16.29209934	19.07641753	15.78704609	14.91128858
median	0.130363247	0.627784076	1.015113477	0	0.291160503	-0.882417039	0.676838975	0.334395886	0.318716116	0.068513478
mean	-0.383395523	0.281620585	0.29008313	0.080171325	0.089451533	-0.332595552	-0.125506225	0.182712964	0.18329166	-0.326113455
var	44.96785977	55.55585785	41.96535202	79.9215317	24.31557353	53.57580362	38.42642988	55.50757968	25.4234962	42.74148051
std.dev	6.705807913	7.453580204	6.478066997	8.939884323	4.931082389	7.319549414	6.198905539	7.45034091	5.042171775	6.537696881
skewness	-0.711295294	-0.304550504	-0.510877845	-0.476883985	-0.370631111	0.141712772	-1.567519532	-0.187175724	-0.1496768	-0.473484492
kurtosis	3.857841589	3.178961528	3.285527303	2.881979962	3.170262078	2.220482328	11.56999184	3.088202966	2.616605576	2.287568574
JB test	0.000	0.01555	0.0025	0.001	0.0095	0.001	0.000	0.0492	0.0232	0.022
Ljung-Box(10)	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
ARCH(10)	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Note: This table reports descriptive statistics for log returns of 92 industry sectors equity index over the period from January, 2002 to December, 2016, which correspond to a sample of 200 observations for each market. We report the p-value of JB test for the normality of series, p-value of LB test lag 10, the Ljung-Box Q-test for autocorrelation at lags 10. In addition, we report the p-value of Engles Lagrange Multiplier test for the ARCH effect on the residual series.

Table 51: Descriptive Statistics of Risk Factors

	Japan			UK						
	FOODS	TEXTILES&AND&APPARELS	TRANS.&EQU	BANKS	AUTO&PARTS	CHEMICALS	MAT	ELTRO&ELEC&EQ	FOOD PRODUCERS	FORESTRY&PAP
min	-20.09440689	-18.73340986	-18.09336315	-46.13264308	-63.05304122	-38.12108827	-30.15529747	-74.46892047	-16.61303697	-50.74143308
max	12.32326404	19.39965884	16.53520532	28.91958401	55.59733572	26.92322832	17.41659708	36.39912546	18.00500568	37.00137534
median	0.576866758	-0.097751742	0.536365048	-0.269078196	0.587753178	1.285914222	1.526578952	0.150398003	0.735776761	0.672280556
mean	0.360110237	0.140428054	0.202438523	-0.462525236	0.223473502	0.706399666	0.58834372	-0.390360207	0.393748545	0.422650619
var	21.57648247	31.37972578	32.64181703	70.49764522	124.4913866	60.02690959	48.35153856	156.5547696	30.64532548	119.051098
std.dev	4.645049243	5.601760954	5.713301762	8.396287586	11.15757082	7.747703504	6.953527059	12.51218484	5.535822024	10.91105394
skewness	-0.582323354	0.096757803	-0.424343701	-0.649378999	-0.687777531	-0.705363691	-0.862815183	-1.592481639	-0.164151991	-0.605856824
kurtosis	2.389508078	2.675168235	3.030117463	8.594756396	11.65747528	5.613512403	3.068468224	11.22024282	2.73936716	5.275702591
JB test	0.000	0.008	0.0095	0.000	0.000	0.000	0.000	0.000	0.0148	0.000
Ljung-Box(10)	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
ARCH(10)	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Note: This table reports descriptive statistics for log returns of 92 industry sectors equity index over the period from January, 2002 to December, 2016, which correspond to a sample of 200 observations for each market. We report the p-value of JB test for the normality of series, p-value of LB test lag 10, the Ljung-Box Q-test for autocorrelation at lags 10. In addition, we report the p-value of Engles Lagrange Multiplier test for the ARCH effect on the residual series.

Table 52: Descriptive Statistics of Risk Factors

	UK									US
	H.C.EQ&SVS	INDS&TRANSPT	MEDIA	MINING	OIL&GAS&PROD	PHARM&BIO	S&W&COMP&SVS	TRAVEL&LEIS	LIFE&INSURANCE	AUTOMOBILES
min	-24.67356707	-32.67548665	-28.62529563	-41.20292409	-19.45018805	-19.08002944	-33.42237328	-32.39717835	-39.48541135	-59.43551809
max	20.65177132	21.29000694	16.13801886	33.31258536	20.03057769	14.57891065	26.06080421	14.89825475	22.5452911	59.52685586
median	1.012461831	0.767044557	0.958957502	0.976165144	0.159429123	0.328169018	0.232013047	1.10152791	0.352969332	-0.459767232
mean	0.465850522	-0.017778989	-0.176721888	0.480246986	0.001177969	-0.039972367	-0.490064921	0.279835238	-0.090785063	-0.429962889
var	38.74627643	52.54383048	46.3220551	111.184432	43.64029128	25.94608989	89.25312684	38.15076945	70.99221873	116.8925127
std.dev	6.224650707	7.248712332	6.806030789	10.54440288	6.606079872	5.093730449	9.447387302	6.176630914	8.425688027	10.81168408
skewness	-0.495392691	-0.909362608	-0.689496509	-0.522878089	-0.067418894	-0.367584849	-0.667403449	-1.10047525	-0.65190942	0.236787831
kurtosis	2.346341065	5.084458218	2.401155364	2.215732312	2.482969603	2.989295399	2.284581983	5.234289454	3.625646932	12.15008774
JB test	0.001	0.000	0.000	0.0015	0.524	0.003	0.000	0.000	0.000	0.000
Ljung-Box(10)	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
ARCH(10)	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Note: This table reports descriptive statistics for log returns of 92 industry sectors equity index over the period from January, 2002 to December, 2016, which correspond to a sample of 200 observations for each market. We report the p-value of JB test for the normality of series, p-value of LB test lag 10, the Ljung-Box Q-test for autocorrelation at lags 10. In addition, we report the p-value of Engles Lagrange Multiplier test for the ARCH effect on the residual series.

Table 53: Descriptive Statistics of Risk Factors

	US										
	BANKS	BCAST	CHEMICALS	INSURANCE	MACHINERY	TRANS.	CONS. MATERIALS	FOOD PRODUCTS	METALS&MINING	ELEC. COMP&EQUIP	TEXTILES&APPAREL
min	-47.39867195	-49.66278278	-24.23898041	-30.40471655	-27.65622609	-20.26609843	-36.58563162	-16.14822007	-42.00847068	-23.39089256	-22.7957962
max	22.20345858	61.67460985	21.82599173	23.80241456	22.19926412	17.42644943	20.44897214	15.32263668	29.25179254	21.65629151	21.17651383
median	1.271548862	0.66529297	0.70550813	0.95577056	1.223569001	1.101662822	0.399193369	0.960907314	0.225202079	0.694969749	1.004876334
mean	0.171641359	0.152520387	0.498192128	0.105745472	0.742110483	0.64959712	0.455321399	0.667242663	0.059669486	0.411388644	0.819272275
var	67.14702811	118.2013909	35.77392498	48.02113496	53.41514682	35.8306384	84.9201714	14.94712478	98.85880576	50.9684589	42.5076405
std.dev	8.194329021	10.87204631	5.981130744	6.929728346	7.308566674	5.985869895	9.215214126	3.866151158	9.942776562	7.139219768	6.519788378
skewness	-1.72773022	-0.167364158	-0.216129786	-1.178234929	-0.571795651	-0.572892092	-0.690078763	-0.573238662	-0.57974313	-0.359835425	-0.430674992
kurtosis	8.200936579	8.520293969	2.33335971	5.044507883	2.006153293	2.282478984	2.591602654	2.66990305	2.443999864	2.918851367	2.58165966
JB test	0.000	0.000	0.000	0.000	0.000	0.000	0.002	0.000	0.000	0.01	0.0015
Ljung-Box(10)	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
ARCH(10)	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Note: This table reports descriptive statistics for log returns of 92 industry sectors equity index over the period from January, 2002 to December, 2016, which correspond to a sample of 200 observations for each market. We report the p-value of JB test for the normality of series, p-value of LB test lag 10, the Ljung-Box Q-test for autocorrelation at lags 10. In addition, we report the p-value of Engles Lagrange Multiplier test for the ARCH effect on the residual series.

Table 54: Descriptive Statistics of Risk Factors

	US										
	UTILITIES	PUBLISHING&PRINTING	ENERGY	HOTELS&REST&LEISURE	HEALTH CARE&EQUIP&SERV	OIL&GAS&REFING&MARK	SOFTWARE&SERVICES	TELECOM&SERV	AIRLINES	MOVIES&ENTERTAIN.	PAPER&PACKAGING
min	-18.19418649	-25.08661385	-20.32458108	-20.5327678	-22.89822474	-33.80768889	-20.80354317	-15.45020502	-39.73582128	-24.62683559	-25.69962617
max	11.60488926	25.96806188	15.837492	15.97851601	13.54292865	30.15832393	22.31350935	26.70571647	20.65195655	27.33192283	26.67000034
median	0.775104511	0.308935517	0.842738478	1.433309145	1.423065866	2.44799394	0.91121611	0.136327318	0.095831366	0.877768903	0.417444599
mean	0.216371439	0.003869539	0.451551664	0.790945621	0.729265509	0.944106224	0.133153443	-0.337739057	0.057993187	0.036396204	0.4171176
var	24.07899002	50.87035151	39.88960386	26.94881781	23.91880113	92.44854459	47.5321828	33.44984676	86.72170907	61.60302456	52.10156364
std.dev	4.907034749	7.132345443	6.315821709	5.191225078	4.890685139	9.615016619	6.894358767	5.783584248	9.312449144	7.848759428	7.218141287
skewness	-0.864943732	-0.227174256	-0.679782409	-0.708533864	-1.434539166	-0.562494135	-0.339303369	0.130634924	-0.645390029	-0.326661096	-0.04356556
kurtosis	2.779597813	3.413383013	2.505928175	2.483308241	6.858371255	2.217533798	2.948791094	2.806996532	2.861669822	2.436836282	2.74649278
JB test	0.000	0.000	0.0015	0.000	0.000	0.000	0.003	0.000	0.000	0.0015	0.000
Ljung-Box(10)	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
ARCH(10)	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Note: This table reports descriptive statistics for log returns of 92 industry sectors equity index over the period from January, 2002 to December, 2016, which correspond to a sample of 200 observations for each market. We report the p-value of JB test for the normality of series, p-value of LB test lag 10, the Ljung-Box Q-test for autocorrelation at lags 10. In addition, we report the p-value of Engles Lagrange Multiplier test for the ARCH effect on the residual series.

Table 55: **Risk Factors Equity Index**

	Asset Category	Index
Australia	BANKS	1S&P/ASX 300 BANKS
	MEDIA	2S&P/ASX 300 MEDIA
	ENERGY	3S&P/ASX 300 ENERGY
	INSURANCE	4S&P/ASX 300 INSURANCE
	TRANSPORTATION	5S&P/ASX 300 TRANSPORTATION
	MATERIALS	6S&P/ASX 300 MATERIALS
	PHARM.&BIOTECHNOLOGY	7S&P/ASX 300 PHARM.& BIOTECHNOLOGY
	RETALING	8S&P/ASX 300 RETALING
	METALS&MINING	9S&P/ASX 300 METALS& MINING
Canada	BANKS	10S&P/TSX COMP BANKS
	TRANSPT	11S&P/TSX COMP TRANSPT
	AUTO&COMPO	12S&P/TSX COMP AUTO&COMPO
	BCAST	13S&P/TSX COMP BCAST
	CHEMICALS	14S&P/TSX COMP CHEMICALS
	INSURANCE	15S&P/TSX COMP INSURANCE
	PHARMACEUTICALS	16S&P/TSX COMP PHARMACEUTICALS
	FD/BEV/TOB	17S&P/TSX COMP FD/BEV/TOB
	ELEC. COMP.& EQU	18S&P/TSX COMP ELEC. COMP.& EQU
	HT/REST/LEIS	19S&P/TSX COMP HT/REST/LEIS
	ENERGY	20S&P/TSX COMP ENERGY
	MET & MIN	21S&P/TSX COMP MET & MIN
	MEDIA	22S&P/TSX COMP MEDIA
	UTILITIES	23S&P/TSX COMP UTILITIES
France	AUTO & PARTS	24FTSE FRANCE AUTO & PARTS
	BANKS	25FTSE FRANCE BANKS
	CHEMICALS	26FTSE FRANCE CHEMICALS
	CON & MAT	27FTSE FRANCE CON & MAT
	FD & DRUG RTL	28FTSE FRANCE FD & DRUG RTL
Germany	AUTOMOBILE	29DAX AUTOMOBILE (XETRA)
	CHEMICALS	30DAX CHEMICALS (XETRA)
	CONSTRUCTION	31DAX CONSTRUCTION (XETRA)
	INSURANCE	32DAX INSURANCE (XETRA)
	TRANSPORT & LOGIS.	33DAX TRANSPORT & LOGIS. (XETRA)
	UTILITIES	34DAX UTILITIES (XETRA)
	FINANCIAL SERVICES	35DAX FINANCIAL SERVICES (XETRA)
	FOOD & BEVERAGES	36DAX FOOD & BEVERAGES XETRA
	TECHNOLOGY	37DAX TECHNOLOGY (XETRA)
	MEDIA	38DAX MEDIA (XETRA)
Japan	BANKS	39TOPIX BANKS
	CONSTRUCTION	40TOPIX CONSTRUCTION
	INFO & COMMUNICATION	41TOPIX INFO & COMMUNICATION
	INSURANCE	42TOPIX INSURANCE
	MACHINERY	43TOPIX MACHINERY
	MINING	44TOPIX MINING
	PHARMACEUTICAL	45TOPIX PHARMACEUTICAL
	PULP & PAPER	46TOPIX PULP & PAPER
	ELEC.POWER & GAS	47TOPIX ELEC.POWER & GAS
	OIL & COAL PRDS.	48TOPIX OIL & COAL PRDS.
	CHEMICAL	49TOPIX CHEMICAL
	ELECTRIC APPLIANCES	50TOPIX ELECTRIC APPLIANCES
	FOODS	51TOPIX FOODS
	TEXTILES AND APPARELS	52TOPIX TEXTILES AND APPARELS
	TRANSPORT EQU.	53TOPIX TRANSPORT EQU.
UK	BANKS	54FTSE 350 BANKS
	AUTO & PARTS	55FTSE 350 AUTO & PARTS
	CHEMICALS	56FTSE 350 CHEMICALS
	CON & MAT	57FTSE 350 CON & MAT
	ELTRO/ELEC EQ	58FTSE 350 ELTRO/ELEC EQ
	FD PRODUCERS	59FTSE 350 FD PRODUCERS
	FORESTRY & PAP	60FTSE 350 FORESTRY & PAP
	H/C EQ & SVS	61FTSE 350 H/C EQ & SVS
	INDS TRANSPT	62FTSE 350 INDS TRANSPT
	MEDIA	63FTSE 350 MEDIA
	MINING	64FTSE 350 MINING
	OIL & GAS PROD	65FTSE 350 OIL & GAS PROD
	PHARM & BIO	66FTSE 350 PHARM & BIO
	S/W & COMP SVS	67FTSE 350 S/W & COMP SVS
	TRAVEL & LEIS	68FTSE 350 TRAVEL & LEIS
	LIFE INSURANCE	69FTSE 350 LIFE INSURANCE
US	AUTOMOBILES	70S&P500 AUTOMOBILES
	BANKS	71S&P500 BANKS
	BCAST	72S&P500 BCAST
	CHEMICALS	73S&P500 CHEMICALS
	INSURANCE	74S&P500 INSURANCE
	MACHINERY	75S&P500 MACHINERY
	TRANSPORTATION	76S&P500 TRANSPORTATION
	CONSTRUCTION MATERIALS	77S&P500 CONSTRUCTION MATERIALS
	FOOD PRODUCTS	78S&P500 FOOD PRODUCTS
	METALS & MINING	79S&P500 METALS & MINING
	ELECTRICAL COMP & EQUIP	80S&P500 ELECTRICAL COMP & EQUIP
	TEXTILES & APPAREL	81S&P500 TEXTILES & APPAREL
	UTILITIES	82S&P500 UTILITIES IG
	PUBLISHING & PRINTING	83S&P500 PUBLISHING & PRINTING
	ENERGY	84S&P500 ENERGY IG
	HOTELS REST & LEISURE	85S&P500 HOTELS REST & LEISURE IN
	HEALTH CARE EQUIP & SERV	86S&P500 HEALTH CARE EQUIP & SERV
	OIL & GAS REFING & MARK	87S&P500 OIL & GAS REFING & MARK
	SOFTWARE & SERVICES	88S&P500 SOFTWARE & SERVICES
	TELECOM SERV	89S&P500 TELECOM SERV
	AIRLINES	90S&P500 AIRLINES
	MOVIES & ENTERTAINMENT	91S&P500 MOVIES & ENTERTAINMENT
	PAPER PACKAGING	92S&P500 PAPER PACKAGING

Note: This table lists 92 equity indices we employ as sector risk factors, which contain 9 indices from Australia, 14 from Canada, 5 from France, 10 from Germany, 15 from Japan, 16 from UK, 23 from US.

Table 56: **Bivariate Copula Family Employed as Building Blocks**

1 = Gaussian copula
2 = Student t copula (t-copula)
3 = Clayton copula
13 = rotated Clayton copula (180 degrees; survival Clayton)
23 = rotated Clayton copula (90 degrees)
33 = rotated Clayton copula (270 degrees)

Note: This table lists all bivariate copula families we employ as vine copula building blocks.

Chapter 3

Asset Allocation Benefits of Alternative Investments: Markov Regime Switching Regular Vine Copula Method

Abstract

This paper studies asset allocation decisions in the presence of regime switching on asset allocation with alternative investments. We find evidence that two regimes, characterized as bear and bull states, are required to capture the joint distribution of stock, bond and alternative investments returns. Optimal asset allocation varies considerably across these states and changes over time. Therefore, in order to capture observed asymmetric dependence and tail dependence in financial asset returns, we introduce high dimensional vine copula and construct a multivariate vine copula regime-switching model, which account for asymmetric dependence and tail dependence in high dimensional data. We model dependence with one Gaussian copula and various kinds of vine copulas separately for regime switching modelling. We discover that R-vine model with individually chosen various bivariate copulas as building blocks, which provides a very flexible way of characterizing dependence in multivariate settings, generally dominates alternative dependence structures. Second, the choice of vine copula model setting is vital for asset allocation, since it modifies the Value-at-Risk (VaR) of strategic asset allocation and produces a better out-of-sample VaR performance. And we also show that ignoring asymmetric dependence and regime-switching in asset allocation strategy leads to significant costs for investor.

Keywords: Copula, R-vine, C-vine, financial returns, pair-copula construction, Markov regime switching, asset allocation

3.1 Introduction

Traditional financial asset returns have been extensively investigated that they exhibit asymmetric dependence. This asymmetry means that in times of crisis, asset returns tend to exhibit increasing dependence than in quiet times. This phenomenon has an important implication to portfolio construction and asset allocation strategy selection. In particular, it implies that, due to increased dependence in crisis period, investors might lose the portfolio diversification benefits because of the underestimation of risk. The presence of such asymmetric dependence adds a cost to portfolio diversification, which requires to consider the portfolio strategy in different market regime. In another aspect, alternative investments asset class, such as hedge funds, commodities, REIT (Real Estate Investment Trust), PE (Private Equity) and VC (Venture Capital), given the increasing importance to investors, also exhibit similar time-varying asymmetric dependence with traditional financial assets. In this sense, another question naturally be proposed is whether alternative investments asset class will improve the risk-return characteristics of an existing portfolio, and whether it will benefit from including alternative investments into portfolio.

Therefore, in this paper, we provide further evidence on asymmetric dependence by introducing and estimating an innovative Markov regime switching regular vine copula model for the dependence of multi-asset portfolio including both traditional and alternative investments asset class. Our contribution comes from several aspects. The previous Gaussian distribution model proposed by Pelletier (2006), is just able to capture symmetric dependence and assume dependence between all returns follow the same Gaussian distribution. In order to overcome the shortcomings of Gaussian model, we employ a regime switching regular vine copula model. The tree structure of regime switching regular vine copula provides a flexible and realistic way to model the dependence of different types of asset returns. Plenty of bivariate copula families can be chosen as the building blocks of our vine structure provides more accurate tail dependence modelling between different assets. The use of copulas makes it possible to separate the dependence model from the marginal distributions, which makes us be able to modelling the time-varying dependence conveniently. As a high dimension multivariate model, our regime switching regular vine copula method exactly applies to our multi-asset portfolio including traditional asset and alternative investments assets. The new type of the high dimension regular vine copula was introduced by Aas et al. (2009) in finance, which allows for very extensive types of dependence. In the bivariate case, it is easy to model dependence with bivariate copula, however, it becomes much more difficult in high dimension case, given that the choice of copulas always restrict to the symmetric Gaussian or the Student t copula. Both of these copulas are only able to capture linear dependence and symmetric dependence. In particular, Gaussian copula suffers from the shortcoming that it lacks of tail dependence, and the multivariate Student t copula is too restrictive in the sense that, though the tail dependence is a function of the correlation and the degrees of freedom, it restricts the symmetric dependence that the upper tail dependence should be equal to lower tail dependence. While the assumption of tail independence is acceptable for positive returns, it is clearly not for negative returns. Regular vine copulas provide the way to overcome these limitations.

Our paper is related to extant research in three aspects, mainly including asset allocation considering asymmetric dependence, asset allocation with regime switching consideration and portfolio diversification benefits of alternative investments.

Regarding asymmetric dependence, adopting the constant conditional correlation (CCC) model proposed by Bollerslev (1990), Longin and Solnik (1995) analyze the correlation between stock market during a period of 30 years. They find that correlation between stock markets are not constant while increase over the sample period. Additionally, correlations are much higher when market are more volatile and depend on some economic variables, such as dividend yields and interest rates. Longin and Solnik (2001) continues their study and employ extreme value theory combined with the method proposed by Ledford and Tawn (1996) to put up the concept of exceedance correlation, which defined as the correlation above a certain threshold which exists between returns. Based on comparing empirical and model-based conditional correlations, a test for asymmetric correlations, Ang and Bekaert (2002a) specify a Gaussian Markov regime switching model for international returns, and they identify two market regime, a bear regime exhibits negative returns, high volatilities and high correlations, a bull regime displays positive mean, low volatilities and low correlations. Patton (2004) find significant asymmetry exist in both marginal distribution and dependence structure of financial returns, considering asymmetric dependence will lead to significant gains for investor with no short sales constraints. Patton (2006a) and Patton (2006b) model foreign exchange series by using conditional copulas and time varying models of bivariate dependence coefficients.

Regime switching model was firstly introduced in econometrics by Hamilton (1989) and since then it has been widely applied in finance. Ang and Bekaert (2002b), Guidolin and Timmermann (2006a) and Guidolin and Timmermann (2006b) apply regime switching models to interest rates modelling. Ang and Bekaert (2002a) and Guidolin and Timmermann (2008) employ a regime switching model for international financial returns. Pelletier (2006) use regime switching model in correlation when the marginals are modeled by GARCH model. Despite his model lies between the CCC model proposed by Bollerslev (1995) and the dynamic conditional correlation (DCC) model of Engle (2002), it still stays in the Gaussian framework. Our model extends the Pelletier (2006) model to the non-Gaussian case. As it is well known that financial returns exhibit non Gaussian distribution, therefore, we discard the Gaussian assumption, while still remain the appealing properties of regime switching structure for dependence. Thus we introduce and employ the flexible high dimensional vine copula to substitute the Gaussian copula in regime switching model. In another aspect, by using vine copula, we can separate asymmetry in marginals from asymmetry in dependence that Gaussian regime switching is unable to work. We therefore can model the marginal distribution by adopting skewed Student t GARCH model of Hansen (1994) instead of Gaussian setting.

With regard to asset allocation benefits of alternative investments, previous literatures normally focus on exploring the effects of adding one alternative investment class into a traditional mixed-asset portfolio. Adding hedge funds makes a positive effects on portfolio performance (see e.g., Amin and Kat (2002); Lhabitant and Learned (2002); Amin et al. (2003); Gueyie and Amvella (2006); Kooli (2007). In addition, incorporation of private equity also improves the portfolio performance (see, e.g., Chen et al. (2002); Schmidt (2003); Ennis and Sebastian (2005)), and also real estate investment trusts (REITs) can increase portfolio diversification benefits (see, e.g., Chandrashekaran (1999); Hudson-Wilson et al. (2003); Stephen and Simon (2005); Chiang and National (2007)). For the case of commodities, there is no consensus on whether or not incorporating them will add portfolio value. Gorton and Rouwenhorst (2006) and Conover et al. (2010) find positive effects from their addition. In contrast, Erb and Harvey (2006) and Daskalaki and Skiadopoulos (2011) find no such effects. Huang and Zhong (2013) takes into account several alternative investments asset classes, including commodities, REITs, and treasury inflation-protected securities (TIPS), and they find adding these alternative investments provides positive diversification benefits to investment portfolio.

In literatures, researchers have started to combine copulas and regime switching models in bivariate data case. Rodriguez (2007) and Okimoto (2008) estimate regime switching copulas for pairs of international stock indices. Okimoto (2008) work on the US-U.K. pairs of stock indices, while Rodriguez (2007) focus on pairs of Latin American and Asian countries. They specify a structure following Ramchand and Susmel (1998) that variances, means, and correlations all switch together. For multivariate regime switching modelling, Garcia and Tsafack (2011) estimate a regime switching model for four variables of domestic and foreign stocks and bonds by developing a mixture of bivariate copula to model the dependence between all possible pairs of these four variables. Nevertheless, the mixture copula model can only capture limited dependence and it lacks of generalization applies to higher dimensions modelling.

To best of our knowledge, we are the first that adopt multivariate high dimension vine copula to modelling a variety of alternative investments (e.g., private equity, buyout, hedge funds, and real estate investment trusts) combined with traditional investments (stocks, government bonds and risk free asset) with regime switching consideration. Previously, Rodriguez (2007) spilt the multi-asset returns series into sub-samples according to four different bivariate copula and multivariate student t copula density from regime switching model. While we are the first to spilt the multi-asset returns series into sub-samples according to six different vine copula density from regime switching model, and calculate the portfolio Sharpe ratio, Sortino ratio, Omega ratio of each regime switching model, and compared with the benchmark conventional asset allocation strategy. To summarize our approach, we estimate regime switching models with one symmetric R-vine Gaussian (multivariate Gaussian) copula representing normal market regime and a R-vine t, a C-vine t, a C-vine (canonical vine) mixed independence copula, a R-vine (regular vine) mixed independence copula, a C-vine (canonical vine) mixed copula, a R-vine (regular vine) mixed copula representing market crisis regime separately. We find that regime switching C-vine mixed model performs best in terms of the likelihood. We then show with an out-of-sample portfolio performance exercise that our regime switching C-vine mixed model dominates alternative models. We investigate economic performance of each competing model and conduct the backtesting by the Value-at-Risk (VaR) and compare them to the conventional model. All results support that our regime switching vine copula applies to multi-asset case and bring portfolio diversification benefits.

The remainder of the paper is organized in the following manner. In Section 2 we present the model. We discuss the regular vine copula, then we present the Markov regime switching regular vine model for dependence, as well as the marginal models. Section 3 describes the inference method of the model, the EM algorithm. In Section 4 we evaluate the out-of-sample portfolio performance of the various models and Section 5 conduct the backtesting with VaR. Section 6 concludes.

3.2 The Markov Regime Switching Regular Vine Copula Model

Abundant empirical evidence has been reported in the finance literature supporting regime-switching behaviour for international stock markets. Specifically, a bearish stock market tends to be associated with higher correlations with other stock markets. This results in reduced portfolio diversification benefits of investors. If alternative investments exhibit the same type of regime-switching behaviour with stocks, the diversification benefits reported earlier might be increased by adding alternative investments into portfolio. To explore the diversification benefits of multi-asset portfolio with regime switching consideration, we set up a Markov regime switching regular vine copula model.

The Markov switching model has been established in statistics and econometrics by Hamilton (1989), which focusing on the multivariate dependence modelling. Markov regime switching model constitutes a special class of regime switching models, in which the regime switching process has a Markov structure. In the financial application case, a hidden underlying process is assumed to be the ”state” of the world or the economy, which has an impact on the development of return time series.

In order to model the multivariate dependence of our multivariate data, we employ a Markov regime switching regular vine copula method. In financial literature, two to six regimes all had been considered before, in our case, we follow Pelletier (2006) and Garcia and Tsafack (2011) that allow for two regimes, characterized by different shapes or levels of dependence. Our Markov regime switching regular vine copula model can be considered as a multivariate extension of Rodriguez (2007) model or as an extension to more realistic dependence of the Pelletier (2006) model. For the reason that we take into account the nonlinearity and employ copula model, our model are more closer to Pelletier (2006) in the spirit of modelling the marginal distributions separately from the dependence structure and not allow the marginal distributions depend on regime.

This specification is also in line with modelling approach underlying the DCC model of Engle (2002) and Engle and Sheppard (2001). To best of our knowledge, Chollete et al. (2009) and Garcia and Tsafack (2011) are the only study that apply regime switching copula for modelling multivariate time series. In the remainder of this section, we present the Markov regime switching vine copula model that allows different dependence structures over different subsamples.

3.2.1 Regular Vine Distribution

Employing notation and methods from graph theory, R-vines has been firstly introduced by Bedford and Cooke (2002) for the construction of multivariate distributions. An R-vine v on d variables, which consists of a sequence of connected trees $T_1, ..., T_{d-1}$, with nodes N_i and edges E_i, $1 \leq i \leq d-1$. In order to satisfy the needs for statistical application, the nodes and edges are required to satisfy the following properties (Bedford and Cooke (2001)):
T_1 is a tree with nodes $N_1 = 1, \ldots,$ d and corresponding set of edges E_1;
For i ≥ 2, T_i is a tree with nodes $N_i = E_{i-1}$ and edges E_i;
If two nodes in T_{i+1} are joined by an edge, the corresponding edges in T_i must share a common node. (*proximity condition*)

To build up a statistical model based on this graph theoretic structure, we associate each edge $e = j(e), k(e)|D(e)$ in the vine with a bivariate copula. This bivariate copula will be the copula corresponding to the bivariate conditional marginal distribution of $X_{j(e)}$ and $X_{k(e)}$ given $\mathbf{X}_{D(e)}$. R-vines normally has two subclasses. If in each tree T_i, there is one node which has edges with all $d-i$ other nodes, this kind of R-vine is called *Canonical vine* (C-vine). We call R-vine *Drawable vine* (D-vine) if each node has at most two edges. Examples of regular vine tree structures can be found in Czado (2010). The notation we employ in our paper follows Czado (2010).

Let $\mathbf{X} = (X_1, ..., X_d)$ be a random vector with marginal densities $f_1, ..., f_d$, respectively. To build up a statistical model using the R-vine, we associate to each edge $j(e)$, $k(e)|D(e)$ in E_i, for $1 \leq i \leq d-1$, a bivariate copula density $c_{j(e),k(e)|D(e)}$. We call $j(e)$ and $k(e)$ the *conditioned set* while $D(e)$ is the *conditioning set*. Let $\mathbf{X}_{D(e)}$ denote the subvector of $\mathbf{X}$ determined by the set of indices $D(e)$. For the definition of the R-vine distribution, we associate the bivariate copula densities $c_{j(e),k(e)|D(e)}$ with the conditional densities of $X_{j(e)}$ and $X_{k(e)}$ given $\mathbf{X}_{D(e)}$ represented as $c_{j(e),k(e)|D(e)}(F_{j(e)|D(e)}(x_{j(e)}|\mathbf{X}_{D(e)}), F_{k(e)|D(e)}(x_{k(e)}|\mathbf{X}_{D(e)}))f_{j(e)|D(e)}f_{k(e)|D(e)}$. In general, $c_{j(e),k(e)|D(e)}$ can depend on the values of variables which are conditioned on. In order to keep the number of parameters tractable, we always assume the conditional copula is constant, i.e. $c_{j(e),k(e)|D(e)}(.,.|\mathbf{X}_{D(e)}) = c_{j(e),k(e)|D(e)}(.,.)$ (see discussion in Stöber and Czado (2012), Hofmann and Czado (2010) and Acar et al. (2012)). The joint density of $\mathbf{X}$ is then uniquely determined by

$$f_{1,...,d}(x_1, ..., x_d) = \prod_{i=1}^{d} f_i(x_i) \cdot \prod_{i=1}^{d-1} \prod_{e \in E_i} c_{j(e),k(e)|D(e)}(F_{j(e)|D(e)}(x_{j(e)}|\mathbf{X}_{D(e)}), F_{k(e)|D(e)}(x_{k(e)}|\mathbf{X}_{D(e)})) \tag{3.1}$$

as given by Czado (2010). If the marginal densities are uniform on [0,1], we call the distribution in (1) an R-vine copula. Given an R-vine ν, a set of corresponding parametric bivariate copulas $\mathbf{B}$ and their parameter vector θ, we denote the R-vine copula density by $c(.|\nu, \mathbf{B}, \theta)$.

Due to other iterative decompositions of a multivariate density into bivariate copulas and marginal densities are also possible, R-vine distributions have the particularly appealing feature that the values for $F(x_{j(e)}|\mathbf{X}_{D(e)})$ and $F(x_{k(e)}|\mathbf{X}_{D(e)})$ appearing in (1) can be derived recursively without high dimensional integrations (see details in Dissmann et al. (2013)).

3.2.2 Markov Regime Switching Vine Copula Model

In our study, we aim to develop a model for a multivariate financial time series $\{\mathbf{X}_t = (X_{1t}, ..., X_{dt}), t = 1, ..., T\}$ by using R-vine copulas combined with the general Markov switching approach introduced by Hamilton (1989). Many financial time series, like stock returns or exchange rates are influenced by external factors like the state of the economy or monetary policies which are not directly observable and which are therefore included in the hidden state variable. In this context, the dependency among $\mathbf{X}_t$ depends on a hidden latent state variable S_t, which takes on only finitely many values $k = 1, ..., p$. These are called regimes and represent the different "states" of the world or economy mentioned above. As it is usual in the Markov switching approach, we assume that S_t, $t = 1, ..., T$

is a homogeneous Markov chain (MC) in discrete time. For simplicity, we restrict the model to a first order Markov chain, which is characterized by its transition matrix P with elements $P_{k,k'} := P(S_t = k^{'}|S_{t-1} = k)$, and $k, k' \in 1, 2$. Where the $p_{i,j}$ represents the probability of moving from state i at time t to state j at time $t+1$. If we are currently in State 1, the probability of remaining in the same state is given by P_{11} and the probability of transitioning to State 2 is therefore given by $1 - P_{11}$. On the other hand, if we are currently in State 2, P_{22} denotes the probability of staying in State 2. Note that high estimated values of P_{11} and P_{22} imply regime persistency. The Markov switching model allows data to be drawn from two possible distributions (regimes). At a given point in time, there is a non-zero probability that the process given will stay in the same state or switch to the other state in the next period.

We adopt a copula based approach to model the dependency of $\mathbf{X}_t$ in regime $k(S_t = k)$. Thus we assume that we know or can estimate the marginal distributions of X_{it} for $i = 1, ..., d$. In particular we can have (pseudo) copula data $\mathbf{u}_t = (u_{1t}, ..., u_{dt}) \in [0, 1]^d$ for $t = 1, ..., T$ through parametric or semi-parametric transform method. Therefore, the Markov switching R-vine (MS-RV) copula for $\mathbf{u}_t$ is now fully characterized by specifying conditional densities as follows

$$c(\mathbf{u}_t|(\nu, \mathbf{B}, \theta)_{1,...,p}, S_t) = \sum_{k=1}^{p} \mathbf{1}_{\{s_t=k\}} \cdot c(\mathbf{u}_t|(\nu, \mathbf{B}, \theta)_k). \tag{3.2}$$

In our model the regime only affects the dependence structure. Therefore, we switch between two density functions to describe the data. The complete MS-RV copula model is thus specified in terms of p R-vine copula specifications and the transition matrix P which contains the parameters of the underlying Markov chain. For inference, we will always assume the R-vine structures ν_k and corresponding sets of copulas $\mathbf{B}_k$, $k = 1, ..., p$, to be given and thus suppress them in the following notation. Thus MS-RV copula is then able to completely described by its parameters

$$\eta' = (\theta'_{cop}, \theta'_{MC}) = ((\theta'_1, ..., \theta'_p), \theta'_{MC}), \tag{3.3}$$

where the subscript "cop" represents copula parameters and "MC" stands for parameters needed for the transition matrix P. In particular, θ_k are the copula parameters corresponding to regime k. While this model does not include switching margins, the switching copula regimes induce serial dependence. The individual marginal time series $(u_{i,t})_{t=1,2,...}$, however, are i.i.d. uniform for $i = 1, ..., d$.

Our Markov regime switching vine copula model is able to capture cyclical behaviors. Moreover, the regimes can be efficiently and endogenously determined by the asset returns data alone without reference to other economic information. Finally, the MS-RV model can be exploited ex ante to enhance the return of the portfolio in different regimes as demonstrated by Ang and Bekaert (2002a).

3.3 Marginal Model

In our Markov regime switching R-vine copula model, we separate the marginal distribution from the dependence structure, and just allow for regime switching of the dependence structure. Thus, we firstly model the marginal distribution of our data.

Let the random process r_t denote the financial asset returns which can be characterized by an autoregressive moving-average (ARMA) model as follows

$$r_t = a_0 + \sum_{i=1}^{p} a_i r_{t-i} + \sum_{j=1}^{q} b_j \epsilon_{t-j} + \epsilon_t \tag{3.4}$$

where a_0 is a constant; p and q are the order of autoregressive and moving average processes respectively for the conditional mean. The error term ϵ_t can be split into a stochastic part x_t and a time-dependent standard deviation σ_t so that $\epsilon_t = \sigma_t x_t$. The conditional variance σ_t^2 is characterized by an asymmetric GARCH model, namely GJR-GARCH(1,1) (see Glosten et al. (1993)). A negative λ corresponds to left skewed density, which means that there is more probability of observing large negative than large positive returns. This is what we expect, since it captures the large negative returns associated to market crashes that are the cause of the skewness.

$$\sigma_t^2 = \omega_i + \alpha_i \epsilon_{i,t-1}^2 + \beta_i \sigma_{i,t-1}^2 + \gamma_i \epsilon_{i,t-1}^2 I_{i,t-1} \tag{3.5}$$

where $I_{i,t-1} = 1$ if $\epsilon_{i,t-1} < 0$, and $I_{i,t-1} = 0$ if $\epsilon_{i,t-1} \geq 0$.

The filtered returns $x_t = \epsilon_t / \sigma_t$, $t = 1, ..., T$; follow a strong white noise process with a zero mean and unit variance. In our empirical work, we adopt Hansen (1994)'s skewed Student t distribution $x_t \sim skT(0, 1; \nu, \zeta)$, with $\nu > 2$ and ζ denoting the degrees of freedom (dof) and asymmetry parameters, respectively. Its PDF is give by, [1]

$$f(x;\nu,\zeta)=\begin{cases} bc\left(1+\dfrac{1}{\nu-2}\left(\dfrac{bz+a}{1-\zeta}\right)^2\right)^{-\frac{\nu+1}{2}}, & if\ \ z<-\dfrac{a}{b}\\ bc\left(1+\dfrac{1}{\nu-2}\left(\dfrac{bz+a}{1+\zeta}\right)^2\right)^{-\frac{\nu+1}{2}}, & if\ \ z\geq-\dfrac{a}{b}\end{cases}$$

where $a=4\zeta c\frac{\nu-2}{\nu-1}$, $b^2=1+3\zeta^2-a^2$, $c=\frac{\Gamma(\frac{\nu+1}{2})}{\sqrt{\pi(\nu-2)}\Gamma(\frac{\nu}{2})}$. The skewed Student t distribution is quite general as it nests the Student t distribution and the Gaussian density. Previous studies motivate this parametrization for the margins as able to capture the autocorrelation, volatility clustering, skewness and heavy tails exhibited typically by financial asset returns (see Jondeau and Rockinger (2006) and Kuester et al. (2006)). In our empirical work, we adopt GJR-GARCH(1,1) and select the best ARMA p and q among 1, 2,..., 10 by minimizing the Akaike Information Criterion (AIC). The model parameters are estimated by quasi-maximum likelihood (QML). Uniform (0, 1) margins denoted $u_n=F_n(x_n)$, n = 1, 2, can be obtained from each filtered return series via the probability integral transform. Once the vector $\mathbf{u}=(u1,u2)'$ is formed, the copula parameter vector can be estimated mentioned in above section.

3.4 Parameter Estimate of Markov Regime Switching Vine Copula Model

For the parameter estimation problems, we assume the specification of ν_k and $\mathbf{B}_k$, for all $1<k<n$, to be given such that only the sets of parameters θ are subject to estimation.

The first difficulty to overcome in developing inference methods for Markov regime switching copula model is that we face unobserved latent variables. In order to derive an expression for the full likelihood of $\tilde{\mathbf{u}}_T=(\mathbf{u}_1,...,\mathbf{u}_T)$, we first decompose the joint density $f(\tilde{\mathbf{u}}_T|\eta)$ into conditional densities:

$$f(\tilde{\mathbf{u}}_T|\eta)=f(\mathbf{u}_1|\eta)\cdot\prod_{t=2}^{T}f(\mathbf{u}_t|\tilde{\mathbf{u}}_{t-1},\eta)$$

[1] There are other Student t distribution that the skewness is introduced in different ways, see Fernández and Steel (1998) and Aas and Haff (2006).

$$= \Big[\sum_{k=1}^{p} f(\mathbf{u}_1|S_1 = k, \theta_k)P(S_1 = k|\theta_{MC})\Big] \cdot \prod_{t=2}^{T} \Big[\sum_{k=1}^{p} f(\mathbf{u}_t|S_t = k, \theta_k) \cdot P(S_t = k|\tilde{\mathbf{u}}_{t-1}, \theta_{MC})\Big], \tag{3.6}$$

where $\tilde{\mathbf{u}}_t := (\mathbf{u}_1, ..., \mathbf{u}_t)$ and $f(\mathbf{u}_t|S_t = k, \theta_k)$ is known from (2) for $t = 1, ..., T$. The unconditional probabilities $P(S_1 = k)$ in this expression are known from the stationary distribution of the Markov chain, which we assume to be given. To obtain the state prediction probabilities $\Omega_{t|t-1} \in \Delta^p \subset \mathbb{R}^{p\times 1}$ on the p-simplex with elements

$$(\Omega_{t|t-1}(\eta))_k := P(S_t = k|\tilde{u}_{t-1}, \eta) \ \ for \ \ k = 1, ..., p \tag{3.7}$$

we can apply the filter of Hamilton (1989). Assuming $\Omega_{t-1|t-1}$ to be given,

$$\Omega_{t|t-1}(\eta) = P^{'} \cdot \Omega_{t-1|t-1}(\eta) \ \ and$$

$$\Omega_{t|t}(\eta) = \frac{\Omega_{t|t-1}(\eta) \odot (f(\mathbf{u}_t|S_t = k, \tilde{\mathbf{u}}_{t-1}, \theta_k))_{k=1,...,p}}{\sum_{k=1}^{p}(\Omega_{t|t-1}(\eta))_k \odot f(\mathbf{u}_t|S_t = k, \tilde{\mathbf{u}}_{t-1}, \theta_k)},$$

and we obtain all probabilities which are required to evaluate the density (6) recursively. The operator $\odot$ denotes componentwise multiplication of two vectors. Similarly, the probability $(\Omega_{t|T}(\eta))_{s_t} := P(S_t = s_t|\tilde{\mathbf{u}}_T, \eta)$, to which we will refer as the ”smoothed” probability of being in state s_t at time t, can be determined by applying the following backward iterations.

$$(\Omega_{t|T}(\eta))_{s_t} = \left(\left(P \cdot \frac{\Omega_{t+1|T}(\eta)}{\Omega_{t+1|t}(\eta)}\right) \odot \Omega_{t|t}(\eta)\right)_{s_t}, \tag{3.8}$$

where also the division is to be understood componentwise.

3.4.1 EM Algorithm for Markov Regime Switching Copula Model

Hamilton (1989) proposed to solve the problem of maximum likelihood estimation for an Markov switching model having unobserved state variable by adopting an EM type (Dempster et al. (1977)) algorithm, constitutes an iterative procedure consisting of two steps, form the conditional expectation of unobserved variables and maximize the likelihood, replacing the latent state variables with their conditional expectation. This algorithm iteratively determines parameter estimates η^l, $l = 1, 2, ...$, which converge to the ML estimate for $l \to \infty$. Let us consider the expected pseudo log likelihood function $Q(\eta^{l+1}; \tilde{u}_T, \eta^l)$ for η^{l+1}, given observations $\tilde{u}_T$ and the current parameter estimate η^l,

$$Q(\eta^{l+1};\tilde{\mathbf{u}}_T,\eta^l) := \int_{\tilde{\mathbf{S}}_T} log(f(\tilde{\mathbf{u}}_T,\tilde{\mathbf{S}}_T|\eta^{l+1}))P(\tilde{\mathbf{S}}_T|\tilde{\mathbf{u}}_T,\eta^l)$$

$$\propto \sum_{t=1}^{T}\int_{\tilde{\mathbf{S}}_T} log(f(\mathbf{u}_t|\mathbf{S}_t,\theta_{cop}^{l+1}))P(\tilde{\mathbf{S}}_T|\tilde{\mathbf{u}}_T,\eta^l)$$

$$+\int_{\tilde{\mathbf{S}}_T}\Big[\sum_{t=2}^{T} log(P(S_t|S_{t-1},\theta_{MC}^{l+1})) + log(P(S_1)^{l+1})\Big]\cdot P(\tilde{\mathbf{S}}_T|\tilde{\mathbf{u}}_T,\eta^l) \tag{3.9}$$

where we write $\tilde{\mathbf{S}}_t := (S_1,...,S_t)$ and $\int_{\tilde{\mathbf{S}}_T} g(\tilde{\mathbf{S}}_T) := \sum_{s_1=1}^{n}...\sum_{s_t=1}^{n} g(S_1 = s_1,...,S_T = s_T)$ for an arbitrary function g of $\tilde{\mathbf{S}}_T$. The algorithm iterates the following steps:
Expectation step: Obtain the smoothed probabilities $\Omega_{t|T}(\eta^l)$ of the latent states $\tilde{\mathbf{S}}_t := (S_1,...,S_t)$ given the current parameter vector η^l.
Maximization step: Maximize $Q(\eta^{l+1};\tilde{\mathbf{u}}_T,\eta^l)$ with respect to η^{l+1}.

Using the Markov property of $\tilde{\mathbf{S}}_T$, Kim et al. (1999) show that the maximum of the pseudo likelihood is attained at

$$P_{i,j}^{l+1} = \frac{\sum_{t=1}^{T} P(S_t = j, S_{t-1} = i|\tilde{\mathbf{u}}_T,\eta^l)}{\sum_{t=1}^{T} P(S_{t-1} = i|\tilde{\mathbf{u}}_T,\eta^l)}$$

similarly $P(S_1 = k)^{l+1} = P(S_1 = k|\tilde{\mathbf{u}}_T,\eta^l), k = 1,...,p$.

Compared with the model originally considered by Hamilton where all maximization steps could be performed analytically, it is not possible for the maximization with respect to the copula parameters θ_{cop}^{l+1} in our case. This means though the transition probabilities can be obtained directly, the second part of the maximization step has to be performed using numerical optimization methods. Since a d-dimensional R-vine copula specification, in which each pair copula has k parameters, contains $d(d-1)/2\cdot k$ parameters, this is computationally still very challenging. To circumvent this issue, we can exchange the joint maximization with respect to θ_{cop}^{l+1} with the stepwise maximization procedure of Aas et al. (2009) which is modified to weight each observation by $P(S_t = s_t|\tilde{\mathbf{u}}_T,\eta^l)$.

This is called the *stepwise EM algorithm.* Since tree-wise estimation of copula parameters is asymptotically consistent (Haff et al. (2013)), this constitutes a close approximation (Haff (2012)) to the "proper" EM algorithm. While there are theoretical results on the convergence of the EM algorithm (Wu (1983)), we loosen these properties with our approximation. All limit theorems however do rely on proper maximization at each step of the algorithm. It is almost impossible to guarantee in our case where we are faced with high dimensional optimization problems and have to rely on numerical techniques. While all existing models for time-varying dependence structures in high dimensions suffer from the computational burden for numerical estimation, we do only need to maximize the likelihoods of bivariate copulas in this tree-wise procedure. This reduces computation time and avoids the curse of dimensionality. The obtained estimate is given by

$$(\hat{\eta}^{EM})' = \left((\hat{\theta}^{EM}_{cop})' = \left((\hat{\theta}^{EM}_{1})', ..., (\hat{\theta}^{EM}_{p})'\right), (\hat{\theta}^{EM}_{MC})'\right). \tag{3.10}$$

3.5 Optimization of the Investor's Utility Function

We consider US investor as the representative investor holding US equities, Emerging markets equities, US government bonds and risk free treasury bills. We would like to examine the effects on the risk-return tradeoff of adding alternative investments to their existing portfolios. Specifically, we estimate the Sharpe ratio, Sortino ratio and Omega ratio of the portfolio with the four existing assets. We then re-estimate the Sharpe ratio, Sortino ratio and Omega ratio when alternative investments are added to each representative investor's portfolio. Any statistically significant improvement in the Sharpe ratio, Sortino ratio and Omega ratio will prove that alternative investments indeed add diversification benefits to the investor's portfolio.

One of the most important elements of Markowitz (1952)'s Modern Portfolio Theory is the notion of efficient frontier in the mean-variance space. A classical mean-variance (MV) portfolio strategy consists of minimizing the portfolio risk, proxied by the variance of the joint distribution, subject to a target portfolio return.

The classical mean variance (MV) portfolio strategy consists minimizing the portfolio risk, where the risk is proxied by the variance of the joint distribution, subject to the target portfolio return.

$$\min_{\mathbf{w}} \; Variance = \min_{\mathbf{w}} w' \sum w \tag{3.11}$$

subject to

$$w'\mu = g$$

$$w'\mathbf{1} = 1$$

where $\sum$ represents the estimated covariance matrix of asset returns, μ denotes the estimated vector of expected asset returns, **1** represents a vector of ones, g denotes the a *priori* chosen portfolio target return, and w represents the resulting optimal vector of weights. The efficient frontier is then constructed by solving the problem for different values of g.

Employing variance as the portfolio risk measure implicitly assume the symmetry or say equal probabilities of losses and profits, which probably underestimates the occurrence of rare adverse events. Due to these reasons, though the computation of variance is simple, it probably leads to the underestimation of risk. Since the variance measure has been widely adopted by banks, the Basel Committee for banking supervision began to draw up some of the risk management requirements in terms of percentiles, in particular, the Value-at-Risk (VaR) of loss distributions.

Current regulations impose capital requirements on banks and financial institutions proportional to the VaR of a portfolio. VaR has established itself as the most popular risk metric for determining the largest size of losses in trading books at a given confidence level. Thus, for instance, 95% VaR is an estimate of the maximum portfolio loss which is exceeded with 5% probability. Empiricals find financial returns always display non-normal distributions, however, VaR is not coherent and it fails to satisfy the subadditivity property in mathematics under non-normal distribution. Due to these shortcomings, VaR is still inappropriate for portfolio optimization. Fortunately, an alternative coherent risk metric proposed by Rockafellar and Uryasev (2000) which called Conditional Value at Risk (CVaR) or Expected Shortfall. CVaR is defined as the expected loss exceeding VaR and thus it represents an upper bound for VaR. Given the focus on lower tail dependence, it makes sense for us to select an portfolio optimization strategy that has a meaningful downside risk emphasis. It is suitable for an investor who focus on minimizing downside risk and is indifferent (or might even prefer) upside variance. Furthermore, it generates an efficient frontier that incorporates non-normality. Thus, if asset returns exhibit lower tail dependence, more emphasis will be placed on reducing this risk in comparison to mean-variance portfolios that assume quadratic utility and ignore all higher moments of the returns distribution. Formally,

$$CVaR^{\alpha} \equiv \mathbb{E}(-r > VaR^{\alpha}), \tag{3.12}$$

where VaR^{α} denotes the maximum loss at confidence level $\alpha \in (0,1)$ typically chosen at 0.99 or 0.95 and r denotes the portfolio loss. It follows from $CVaR^{\alpha} \geqslant VaR^{\alpha}$ that, if the risk manager can control CVaR then he can control VaR but not the other way round. Accordingly, we choose to minimize CVaR in preference to Value-at-Risk (VaR).

A shortcoming of CVaR is the difficulty of computation. Let $r(w,\mu)$ be a portfolio return function where w and μ are vectors of weights and expected asset returns, respectively. We can rewrite (15) as follows

$$CVaR^{\alpha}(\mathbf{w}) = \frac{1}{\alpha}\int_{-f(\mathbf{w},\mathbf{r})>VaR^{\alpha}(\mathbf{w},r)} f(\mathbf{w},\mathbf{r})p(\mathbf{r})d\mathbf{r} \tag{3.13}$$

where $f(r)$ denotes the multivariate pdf of asset returns. Rockafellar and Uryasev (2000) proposed an alternative simpler function

$$F^{\alpha}(\mathbf{w},d) = d + \frac{1}{\alpha}\int_{-f(\mathbf{w},\mathbf{r})>d} (-f(\mathbf{w},\mathbf{r}) - d)f(\mathbf{r})d\mathbf{r} \tag{3.14}$$

and demonstrate that $F^{\alpha}(\mathbf{w},d)$ is a convex function with respect to d, and that VaR is a minimum point of this function with respect to d. So in the mean-CVaR framework, where variance is replaced by CVaR as the relevant risk metric, the optimization problem becomes $min_{\mathbf{w},d}\, F^{\alpha}(\mathbf{w},d) = min_{\mathbf{w}}\, CVaR^{\alpha}(\mathbf{w})$. Rockafellar and Uryasev (2000) and Andersson et al. (2001) suggest to approximate (17) by Monte Carlo simulation as follows

$$F^{\alpha}(\mathbf{w},d) = d + \frac{1}{\alpha q}\sum_{i=1}^{q}(-f(\mathbf{w},\mathbf{r}_i - d)^{+}), \tag{3.15}$$

where q denotes the number of samples generated by Monte Carlo simulation, and $z^{+} = max(0,z)$. d represents VaR, 1 is a vector of ones, and α represents the threshold value. As we consider the investor who is averse to extreme downside losses, we set 1-α to 0.99 analogous to an investor who wishes to minimize losses at the 1% level of CVaR, similar to di Basilea per la vigilanza bancaria (2004) requirements. r_k is the kth vector of simulated returns. The vector of portfolio weights, w, is extracted from the optimization procedure to generate the portfolio that minimizes CVaR for a given R. This optimization can be approached as a linear programming problem

$$\min_{\mathbf{w},z,d} d + \frac{1}{\alpha q}\sum_{i=1}^{q} z_i \tag{3.16}$$

$$subject\ \ to\ \ z_i \geqslant -f(\mathbf{w}, \mathbf{r}_i) - d;$$

$$z_i \geqslant 0;$$

$$w'\mathbf{1} = 1;$$

$$w'\mathbf{r}_i = g.$$

where **w** is the Mean-CVaR optimal vector of weights.

3.6 Data

In our study, we choose not only traditional financial assets, such as stocks, bonds and risk free asset, but also consider adding alternative investments into our portfolio. Since publication of seminal paper on portfolio theory by Markowitz (1952), the literature acknowledges us that diversification can increase expected portfolio returns while reducing volatility. However, investors should not blindly add another asset class into their portfolios without carefully investigating its properties in the context of their portfolios. A naively chosen allocation to the newly added asset class may not improve the risk return profile, while might even worsen it. This raises the questions of whether alternative investments really improve the risk-adjusted performance of a mixed multi-asset portfolio and whether they should be included in the strategic asset allocation.

In order to investigate whether the alternative investments are able to improve the portfolio performance, we take into account following indices as proxies for both traditional and alternative investments asset class. Regarding the traditional asset classes, we choose S&P 500 Total Return Index and MSCI Emerging Markets Total Return Index representing stock asset, and JP Morgan US Government Bonds Total Return Index representing government bonds, also the US Treasury bills is considered. And four alternative investment assets include private equity, which subdivided into buyout and venture capital, separately employing US Buyout index and US Venture Capital index, Hedge Fund Research, Inc. is selected as the proxy of hedge funds, and FTSE EPRA/NAREIT Total Return Index chosen as the REITs asset. To obtain excess returns we subtract the 90-day T-bill rate from these returns. All time series in our investigation are based on a weekly data with a July 1998 inception date, and the end date of the time series is June 2017.

Data from April 2004 to June 2017 are not used for model selection or parameter estimation in order to maintain a genuine out-of-sample period data. All data are obtained from Datastream.

Table 2 provides the descriptive statistics of each asset class returns considered. These descriptive statistics presented in Table 2 show that venture capital and buyout have the highest mean return (0.28, 0.27) and also high standard deviation (3.91,3.59), followed by emerging markets and hedge fund, with a mean return of both 0.11, in which emerging markets has a high standard deviation of 3.14. Though the REIT has the low mean return of 0.08, it has high standard deviation of 3.32.

The higher moments (skewness and kurtosis) are additional potential sources of risk. All series show very clear signs of non-normality with negative skewness except for Japan and Argentina, which have small positive skewness. Further evidence of non-normality is given by the fact that all series have a kurtosis that is well above 3. In particular, emerging markets exhibit the lowest skewness, -0.99 (kurtosis 6.53), whereas hedge fund shows the highest kurtosis, 29.98 (skewness -0.61), among all asset classes. Therefore, emerging market and hedge funds show the most unfavorable higher-moment properties, because negative skewness and positive excess kurtosis indicate that the outliers are on the left side of the return distribution and occur more often than expected under the normal distribution (known as tail risk). The kurtosis for all asset classes exceed 3 which means all asset return exhibit high kurtosis.

Analysing the higher moments of the return distribution for the asset classes shows that some return distributions do not follow a normal distribution. The Jarque and Bera (1980) test rejects the null hypothesis of a normally distributed return distribution for all asset class returns at the 5% level. Both the Ljung-Box Q test and Engle's ARCH LM test reject the null of no autocorrelation for lags in returns and squared returns confirming, respectively, serial dependence and heteroskedasticity. Thus, relying on a simple mean-variance framework and ignoring the higher moments does not adequately capture the risk-return profile. Failure to consider higher moments increases the probability of maintaining biased and suboptimal weight estimations, as well as underestimating tail losses. These results support the employment of mean-CVaR framework as our portfolio optimization strategy.

Table 3 provides insight into the diversification potential of each asset class. Hedge funds have a high diversification potential because the correlation to all other asset classes is statistically significant not different from zero. Similar diversification potential applies to government bonds, which also have a correlation to all other asset classes statistically significant not different from zero. It is worth noting that there is no significantly negative correlation between asset classes. After reviewing the descriptive statistics of the return distributions, we cannot determine a priori that one asset class is a substitute for another. Therefore, we consider all the asset classes for the portfolio construction. To create optimal investor portfolios, our model considers the characteristics of the asset classes.

The results indicate that we have a high and a low dependence regime. The copula correlation coefficient in the more dependent regime is higher for all pairs of indices, which means that the whole assets together is more dependent when the economy is in that regime. This regime is characterized by some very large correlations. For instance, $S\&P$ 500 and Buyout have a Pearson correlation coefficient of 0.71 in Table 3, that translates into a Kendall's τ of 0.68 in Table 4, which is very high dependence. More generally, the highest correlations are between the $S\&P$ 500 and Buyout index and $S\&P$ 500 and VC index.

3.7 Empirical Results

We first estimate Markov regime switching vine copula models parameters using the two-step estimation and EM algorithm mentioned above, then investigate the applicability of the different Markov regime switching vine copula models in the context of investors who wish to minimize the event of extreme losses within their portfolio. First, we perform an in-sample study to observe the efficient frontiers produced from historical data of indices excess returns for portfolios with and without alternative investments. We perform this analysis to observe if the alternative investments have diversification benefits for asset allocation. Second, we perform a multi-period, long-term, out-of-sample study which uses the Markov regime switching vine copula models and optimize our portfolios to minimize CVaR. We employ a wide range of statistical and economic metrics, including Sharpe ratio, Sortino ratio, Omega ratio and VaR backtesting, to assess the superiority of each asset allocation strategies in an out-of-sample portfolio management context, and we also discuss the economic performance of out-of-sample portfolio.

Table 3.1: **Proxy Indices**

Asset class	Proxy Index	Frequency	Inception date	End date	Source information
US Stocks	S&P 500 Composite-Total Return Index	Weekly	July 1998	June 2017	Datastream
Emerging Markets Stocks	MSCI Emerging Markets-Total Return Index	Weekly	July 1998	June 2017	Datastream
US Treasure Bonds	JP Morgan US Govt. Bond-Total Return Index	Weekly	July 1998	June 2017	Datastream
US Treasure Bill	US 90 Day T-bill	Weekly	July 1998	June 2017	Datastream
Real Estate Investment Trusts	FTSE EPRA NAREIT-Total Return Index	Weekly	July 1998	June 2017	Datastream
Hedge Funds	HFRI Fund of Hedge-fund Composite Index	Weekly	July 1998	June 2017	hedgefundresearch.com
Buyout	Thomson Reuters VentureXpert	Weekly	July 1998	June 2017	Thomsonreuters
Venture Capital	Thomson Reuters VentureXpert	Weekly	July 1998	June 2017	Thomsonreuters

Note: This table reports the proxy indices for each asset class. The frequencies, inception dates, end dates, and additional information sources are given for the proxy time series.

Table 3.2: **Descriptive statistics from the weekly return distribution of all asset classes**

	S&P 500	Emerging Markets	Gov. Bond	REIT	Hedge Fund	Venture Capital	Buyout
Mean	0.07	0.11	0.03	0.08	0.11	0.28	0.27
Standard Deviation	2.39	3.14	1.88	3.32	0.86	3.91	3.59
Kurtosis	4.52	6.53	3.18	9.45	29.98	5.75	6.22
Skewness	-0.61	-0.99	-0.06	-0.75	-0.61	-0.80	0.34
Minimum	-16.45	-26.06	-7.41	-24.48	-8.06	-31.53	-15.14
Maximum	10.18	16.76	7.77	21.79	8.03	14.63	25.87
Median	0.22	0.44	0.08	0.21	0.02	0.56	0.25
25th Percentile	-0.0108	-0.154	-0.0109	-0.01296	-0.0173	-0.0187	-0.0098
75th Percentile	0.0135	0.0206	0.1119	0.0166	0.0195	0.0243	0.0190
MaxDD	0.2663	0.3399	0.1518	0.3657	0.1488	0.4616	0.3851
JarqueBera	46.822	107.51	47.296	1798.6	291.72	731.09	41.889
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)

This table reports the mean, standard deviation, skewness, kurtosis, minimum, maximum, median, 25th percentile, 75th percentile, and the Maximum Drawdown (MaxDD) of the weekly return distributions of the S&P 500, MSCI Emerging Markets, JPM US Government Bonds, FTSE EPRA/NA REIT, HFRI Fund of Funds, US Buyout, and US Venture Capital asset returns from July 1998 to June 2017. We also report the Jarque-Bera test results and its p-value for normality test.

Table 3.3: **Pearson Correlation matrix**

	S&P 500	MSCI.EM	Gov. bond	REIT	Hedge fund	Buyout	Venture Capital
S&P 500	1.00000000	0.67290439	-0.28479011	0.68401943	0.15466295	0.71122931	0.75141859
MSCI.EM	0.67290439	1.00000000	-0.25744608	0.50155944	0.16933536	0.49631010	0.49209987
Gov.bond	-0.28479011	-0.25744608	1.00000000	-0.10023496	-0.05901482	-0.22921772	-0.23429135
REIT	0.68401943	0.50155944	-0.10023496	1.00000000	0.11312018	0.40317285	0.38120185
Hedge.fund	0.15466295	0.16933536	-0.05901482	0.11312018	1.00000000	0.14909670	0.14532352
Buyout	0.71122931	0.49631010	-0.22921772	0.40317285	0.14909670	1.00000000	0.57234174
Venture Capital	0.75141859	0.49209987	-0.23429135	0.38120185	0.14532352	0.57234174	1.00000000

Note: This table reports the Pearson correlation matrix of the proxy indices for each asset class returns.

Table 3.4: **Kendall's τ Correlation matrix**

	S&P 500	MSCI.EM	Gov. bond	REIT	Hedge fund	Buyout	Venture Capital
S&P 500	1.00000000	0.47512257	-0.20052187	0.44118480	0.05992722	0.68512165	0.59237583
MSCI.EM	0.47512257	1.00000000	-0.17561895	0.29936703	0.09560183	0.39417994	0.36332447
Gov.bond	-0.20052187	-0.17561895	1.00000000	-0.05736681	-0.02788939	-0.17257336	-0.16083132
REIT	0.44118480	0.29936703	-0.05736681	1.00000000	0.05456992	0.34735855	0.28291594
Hedge fund	0.05992722	0.09560183	-0.02788939	0.05456992	1.00000000	0.04163329	0.04650949
Buyout	0.68512165	0.39417994	-0.17257336	0.34735855	0.04163329	1.00000000	0.48172747
Venture Capital	0.59237583	0.36332447	-0.16083132	0.28291594	0.04650949	0.48172747	1.00000000

Note: This table reports the Kendall's τ correlation matrix of the proxy indices for each asset class, which is more robust and have been recommended if the data do not necessarily come from a bivariate normal distribution.

3.7.1 Marginal Model Estimate

In this section we first fit the skewed t AR-GJR-GARCH model to our asset returns, and present the results of the marginal model estimation. Table 3 presents the estimate results of each of the univariate skewed Student t AR-GJR-GARCH model. Seen from the table, the degrees-of-freedom parameters of most series is around 8, which corresponds to tails of the conditional distribution that are somewhat fatter than those of the normal distribution. A well-specified model for the marginals is crucial, because misspecification can result in biased copula parameter estimates(Fermanian and Scaillet (2005)). Therefore, we apply the KolmogorovSmirnov test. We also present the p-values of the LjungBox test of autocorrelation in the squared residuals of the skewed Student t innovations of the GARCH models. Using the Ljung-Box Q-test, the null hypothesis of no autocorrelation is rejected at lag 1,2,5 for all the returns. The ARCH test indicates the significance of ARCH effects in all the series. The table shows that each one of the marginal models is well specified.

3.7.2 Efficient Frontiers with and without Alternative Investments

In this section, we adopt a simple way to primarily investigate diversification benefits of alternative investments. We estimate and compare the Sharpe ratios of the tangency portfolios with and without alternative investments using full sample of excess return data from January 1998 to June 2017. This ex post approach is the most common practice and is also used here. The short sales constrained and short sales unconstrained case results for the US investor are reported separately in Panel A and Panel B of Table 5. Column 1 lists the portfolio weights of each asset class the case with alternative investments and the case without alternative investments respectively when mean-CVaR portfolio optimization strategy applied. Column 2 to 4 of each panel lists the mean, CVaR of each portfolio. The Sharpe ratios corresponding to the cases with and without alternative investments are reported in Column 5. In all cases, the addition of alternative investments leads to an improvement in the Sharpe ratio. This confirms the well-known fact stated earlier that the portfolio performance of the optimized portfolios with extra asset class can be considered improved. Nevertheless, any conclusion based on this observed improvement in performance is meaningless without statistical testing. The results of the statistical tests based

Table 3.5: **Estimation Results for Marginal Model**

	S&P500	MSCI.EM	Gov.bond	REIT	Hedge fund	Buyout	Venture Capital
AR-GJR-GARCH							
μ	0.073946	0.142157	0.046800	0.158806	0.025589	0.13409	0.275145
	(0.050936)	(0.088916)	(0.052661)	(0.070728)	(0.000398)	(0.057634)	(0.094633)
ar1	-0.111235	0.056886	-0.032239	-0.045629	0.001305	-0.11110	-0.074620
	(0.031469)	(0.034407)	(0.031949)	(0.033791)	(0.000883)	(0.032988)	(0.031045)
ω	0.168501	0.352305	0.050428	0.412670	0.000858	0.37108	0.290499
	(0.058014)	(0.118290)	(0.022996)	(0.120553)	(0.000006)	(0.120898)	(0.145801)
$\alpha 1$	0.000000	0.042036	0.077793	0.042642	0.016472	0.14752	0.083148
	(0.024877)	(0.026338)	(0.015992)	(0.032000)	(0.000067)	(0.039571)	(0.029945)
$\beta 1$	0.841640	0.865730	0.951720	0.810668	0.923533	0.69635	0.886576
	(0.035674)	(0.030391)	(0.014142)	(0.039110)	(0.002150)	(0.037729)	(0.034258)
$\gamma 1$	0.250246	0.087993	-0.085258	0.177531	-0.082622	0.34259	0.022891
	(0.054354)	(0.032804)	(0.018349)	(0.058192)	(0.000440)	(0.083158)	(0.036889)
Skew t							
skew	0.724667	0.747526	0.992974	0.847135	1.071546	0.78403	0.799771
shape	8.422077	19.699745	20.376726	6.496324	2.010000	5.22510	7.424783
K-S test	0.3275	0.6272	0.9228	0.4653	0.7112	0.5084	0.5441
Ljung-Box Test							
Lag[1]	0.3291	0.2654	0.7007	0.5308	0.4975	0.7728	0.695
Lag[2]	0.7289	0.52045	0.9994	0.9596	0.9086	0.9999	0.9977
Lag[5]	0.9134	0.04046	0.8204	0.6071	0.9199	0.9975	0.7112
ARCH LM Tests							
Lag[3]	0.9841	0.6625	0.1582	0.1776	0.354458	0.8131	0.47959
Lag[5]	0.8420	0.6577	0.3922	0.3662	0.1609	0.7857	0.06131
Lag[7]	0.7051	0.7758	0.5402	0.569	0.1756	0.7477	0.11517

This table reports parameter estimates from AR and GJR-GARCH models for conditional mean and conditional variance of risk factor log returns, their standard errors of the parameters list in the parenthesis. We estimate all parameters using the sample from July 1998 to June 2017, which correspond to a sample of 200 observations for 92 risk factors. We report the p-value of Ljung-Box Test and ARCH LM Tests for serial dependence and heteroskedasticity. We also report the p-values of Kolmogorov-Smirnov (KS) test for the skewed Student t distribution.

on the simulation method will be discussed in the section of Backtesting.

3.7.3 Vine Copula based Regime Switching Model Estimate

To examine if the diversification benefits behave differently across the two regimes of the assets market, the first task is to spilt the return data into two regimes. The estimated smooth probabilities will be a useful tool. As a byproduct of the maximum likelihood estimation, the endogenously determined probability of realizing a particular state or regime can also be extracted at any point in time. For example, the *filter probability*, $p(S_{t=1}|r_{t-1}, r_{t-2}, r_{t-3}, ..., r_0)$, represents the conditional probability that assets market is in State 1 at t given the observed time-series of returns up to the beginning of that period. Alternatively, we can compute the *smooth probability*, $p(S_{t=1}|r_T, r_{T-1}, r_{T-2}, ..., r_0)$, in which the inference about the state is now based on all return data up to the end of the sample period (i.e. at time T). The evolution of the smooth probability over time is governed by both the magnitudes of the transition probabilities (i.e. P_{11} and P_{22}) defined and the prevailing asset returns. For example, the higher the values of P_{11} and P_{22}, the more persistent is the smooth probability given the lower chance of switching between the two states. A large jump in the index asset returns will however disrupt the persistency of the smooth probability. The realization of a high (low) return at time t will lead us to assign a lower (higher) probability of State 1 being realized at time t (i.e. the value of the smooth probability at time t). The smooth probability therefore captures our assessment of the relative chances of the two states being realized in a particular time period given all the observations of the index asset returns. The higher probability of regime 1 (the low return state) can be thought of as a bearish assets market, whereas the lower probability of regime 2 can be considered as a bullish assets market. A smooth probability of 0.5 indicates equal probabilities of realizing regimes 1 and 2. The return series of various asset classes can then be split into two sub-samples corresponding to the low and high return regimes based on whether the smooth probability of asset returns is higher or lower than 0.5. [2] [3] So that we divide return data into two sub-samples according to our six kinds of

[2]As described earlier, the smooth probability is estimated using the entire sample period. Using the entire sample enables us to more accurately capture the switching between the two regimes.

[3]A smooth probability of 0.5 corresponds to the condition of equal chance of realizing the two regimes at a particular time. Based on our regime-switching model, it results in a smaller subsample for the high-return state (State 2) than the low-return state (State 1). As a robustness test, we repeat the analysis by partitioning the data series based on the median value of the estimated smooth probabilities, thus resulting

Table 3.6: **Efficient frontiers with and without alternative investments**

Panel A: Short sales constrained																	
Portfolios with alternative investments											Portfolios without alternative investments						
Portfolio weights								mean	CVaR	Sharpe ratio	Portfolio weights				mean	CVaR	Sharpe ratio
S&P US	MSCIEAFE	Gov. bond	Real Est	Hedge	Buyout	VC	T-bills				S&P US	MSCIEAFE	Gov. bond	T-bills			
0.0600	0.0408	0.1935	0.0360	0.3320	0.0380	0.0120	0.2807	0.0649	1.209	0.0607	0.060	0.052	0.146	0.738	0.0127	0.8227	0.0364
Panel B: Short sales unconstrained																	
Portfolios with alternative investments											Portfolios without alternative investments						
Portfolio weights								mean	CVaR	Sharpe ratio	Portfolio weights				mean	CVaR	Sharpe ratio
S&P US	MSCIEAFE	Gov. bond	Real Est	Hedge	Buyout	VC	T-bills				S&P US	MSCIEAFE	Gov. bond	T-bills			
0.480	-0.340	0.496	-0.318	-0.126	0.194	0.142	0.470	0.0646	3.378	0.0757	-0.4720	0.4780	0.4947	0.4940	0.0292	3.22	0.0451

Diversification benefits of alternative investments: portfolio weights and Sharpe ratios of tangency portfolios with and without alternative investments are reported. Optimal portfolios are obtained with short sales unconstrained and also short sales constrained and based on the full sample period from July 1998 to June 2017.

different Markov regime switching vine copula model, Model 1 to Model 6.

To test which kind of Markov regime switching vine copula model is the best fitting model for our data, and investigate whether eight assets regime switching outperform the four assets case, we estimate six competing Markov regime switching vine copula models for both the four traditional assets case and eight assets indices data including stock, bond and alternative investments indices. The four assets regime switching results are presented in Table 6 to Table 8, and eight assets results list in Table 9 to Table 11. We specify in Model 1 that market state switch from an R-vine Gaussian copula state, the normal market state, to an R-vine *t* regime (first column to fourth column), Model 2 that market state switch from an R-vine Gaussian copula state to a C-vine *t* copula regime (fifth column to eighth column), Model 3 that market state switch from an R-vine Gaussian copula state to an R-vine independence mixed copula regime (first column to fourth column), and Model 4 that market state switch from an R-vine Gaussian copula state to an C-vine independence mixed copula regime (fifth column to eighth column), Model 5 that market state switch from an R-vine Gaussian copula state to an R-vine mixed copula regime (first column to fourth column), and Model 6 that market state switch from an R-vine Gaussian copula state to an C-vine mixed copula regime (fifth column to eighth column). The details of employed vine copula model are presented as follows.

For model selection we want to demonstrate the superior fit of vine copula with individually chosen pair-copula families and assess the gain over R-vines or C-vines with only bivariate t or with only Gaussian pair-copula. In particular, we apply the selection algorithm to select among seven different R-vine classes given by,

$R-vine\ mixed\ copula$: R-vine with pair-copula terms chosen individually from bivariate copula families.

$R-vine\ independence\ mixed\ copula$: R-vine with pair-copula terms chosen individually from bivariate copula families with independence copula.

$C-vine\ mixed\ copula$: C-vine with pair-copula terms chosen individually from bivariate copula families (see above).

$C-vine\ independence\ mixed\ copula$: C-vine with pair-copula terms chosen individually from bivariate copula families with independence copula (see above).

$R-vine\ t\ copula$: R-vine with each pair-copula term chosen as bivariate Student t copula.

in equal length of subsamples in States 1 and 2. Our conclusions remain essentially the same.

If the degrees of freedom parameter of a pair is estimated to be larger than 30, we set the copula to the Gaussian.

$C-vine\ t\ copula$: C-vine with each pair-copula term chosen as bivariate Student t copula. If the degrees of freedom parameter of a pair is estimated to be larger than 30, we set the copula to the Gaussian.

multivariate Gaussian copula ($R-vine\ Gaussian\ copula$): R-vine with each pair-copula term chosen as bivariate Gaussian copula, i.e., this corresponds to a R-vine Gaussian copula, where unconditional correlations can be obtained from conditional ones by inverting a generalized version.

Our filtering technique is similar to that of Gray (1996) and Hamilton (1989) conducting the maximum likelihood estimation of the parameters. This approach is also employed by Ramchand and Susmel (1998) and Ang and Chen (2002). We first estimate parameters of the best fitting bivariate copula, which as the building blocks of vine copula, in each regime and the regime switching transition probabilities, the results are reported in Table 6 to Table 11. From Table 6 to Table 11, State 1 can be characterized by both its low expected return and low return volatility. Whereas State 2 is the state when both expected return and volatility are high. The persistency of regime as indicated by the high values of P_{11} and P_{22} is statistically significant in both states.

In our regime switching vine copula specification, the R-vine t copula corresponds to the lower dependence regime. The difference between the models is that, unlike the R-vine Gaussian copula, the R-vine t copula with all Student t copula as building blocks which is capable to capturing tail dependence, but it implies equal upper and lower tail dependence. We also show the results of a switching model with an R-vine Gaussian and a C-Vine mixed copula regime which employ the C-vine structure and choose bivariate copula building blocks from an abundant of bivariate copula families (sixth column to tenth column). The class of possible canonical vines is evidently extremely large. We follow Aas et al. (2009) for the specification of the copula. First, we order the variables by decreasing correlations, choosing the variable with the largest correlation as the first one to condition on. This leads us to place $S\&P$ 500 index as the pivotal element of the C-vine tree structure, followed by MSCI Emerging Markets index, JPM US Government Bonds index, US T-bill, FTSE EPRA/NAREIT index, HFRI Fund of Funds index, US Private equity index, and US Buyout index. By so doing, we intend that most of the dependence structure in the copula will be captured in the lower stages of the canonical vine, leaving only very little dependence to be modelled as we move to copulas that are conditional on more indices. The difference in Model 3, 5 is that we employ more flexible R-vine copula structure comparing to Model 4, 6, regarding the R-vine structure, we therefore do not need to choose a pivotal index comparing to C-Vine structure, and it is more flexible for us to construct the correlation among different asset indices. In the Model 3, we adopt R-Vine independence mixed copula as the low dependence state, while in the Model 4, the R-Vine mixed copula structure is employed to describe the structure of the low dependence regime. In Model 3, 6, the independence copula as the bivariate copula building blocks candidate is considered.

By using the likelihood as a criterion for selecting models, in general, the eight asset cases are all taking larger likelihood value than four asset case. In eight asset context, we note that the likelihood of C-vine and R-vine model increases comparing to the R-vine *t* model in general. Adopting R-vine structure in Model 3 to 6, abundant of bivariate copulas are selected as the building blocks for C-vine and R-vine model. As displayed in the Tables, among all mixed vine copula models, we can see that the likelihood value of C-vine mixed copula model is highest compared with the all other competing models due to the flexible structure of C-vine copula which capturing dependence of different types of assets indices.

Regarding the regime persistence, when we investigate the results of transition probability, all models for the eight assets case are characterized by very high persistence in both regimes comparing to four asset case results. The six different vine copula based regime switching models' filter probabilities are plotted in Figure 1, 2 for four asset and eight asset case separately. When we examine the plot of the smooth probabilities of the six model being in the high-dependence regime of Figure 1, 2, the high-dependence regime is the dominant one from 1998 onward, except some crisis period, such as 2000 Internet Bubble, 2001 ”911” event, 2007-2009 Global Financial Crisis and 2011 Euro Debt Crisis. A probability close to unity (zero) suggests it is very likely the market state is in State 1 (State 2). Figures nicely confirm what is generally accepted as the several major bearish market environments in the crisis period mentioned above. One factor explaining this might be the increased integration of global financial markets. More generally, the returns from the all eight assets indices have all become much more highly dependent.

We found the filtered probabilities differ a little from one model to another and the dependence within each regime, as measured by the unconditional Kendall's τ, seems to change a little from one model to another, which demonstrate the dependence increase in period of crisis is a general fact no matter which model is employed.

The results also indicate that we truly have a high and a low dependence regime. We can notice that the bivariate copula correlation coefficient in the more dependent regime is higher than all pairs of asset returns in low dependence regime, which means that the market state together is more dependent when the economy is in that regime. This regime is characterized by larger correlations. For instance, in R-vine mixed copula model, asset 1 and 6 have a correlation coefficient of 0.93, that translates into a Kendall's τ of 0.75, which is very high dependence. More specifically, the highest correlations are between the $S\&P500$ index and Buyout index.

When we check the high dependence regime in our most preferred eight asset Model 6 - "R-vine Gaussian regime transfer to R-vine mixed copula regime", the bivariate copula chosen to capture the pair asset return indices contains Gaussian copula, Student t copula, Survival Gumbel copula, Survival BB1 copula, Survival Joe copula. Some of these copulas can capture both asymmetric dependence and tail dependence with upper tail dependence and lower tail dependence, such as Survival BB1 copula, some with only one tail dependence, such as Survival Gumbel copula and Survival Joe copula, and Student t only have symmetric tail dependence. These results imply that tail dependence between all pairs of variables indeed exist and support the flexibility and superiority of vine copula for capturing tail dependence between different asset types.

Strictly speaking, we can not use the likelihood as a criterion for selecting models that are not nested, we nonetheless note that the C-vine model increases the likelihood compared with the other competing models, with the same number of parameters. Of course we can by no means claim that we have chosen the best possible copula, since more theoretical work is needed about model selection of vine copulas in general.

3.7.4 Out-of-sample Risk-adjusted Portfolio Performance of Different Asset Allocation Strategies

In this section, we investigate whether there exist diversification benefits provided by alternative investments in an out-of-sample setting. To this end, we calculate optimal port-

Table 3.7: **Four Assets Regime Switching Estimation Results**

	Model 1				Model 2		
Regime 1							
	R-vine Gaussian				R-vine Gaussian		
		Coef	τ			Coef	τ
1,2	Gaussian	0.64	0.44	1,2	Gaussian	0.64	0.44
3,1	Gaussian	0.29	0.19	3,1	Gaussian	0.29	0.19
4,3	Gaussian	0.11	0.07	4,3	Gaussian	0.11	0.07
Regime 2							
	R-vine t				C-vine t		
		Coef	τ			Coef	τ
1,2	Student t	0.64,9.69	0.45	2,1	Student t	0.64,9.69	0.45
3,1	Student t	0.29,6.36	0.19	2,3	Student t	0.26,14.33	0.17
4,3	Student t	0.11,27.5	0.07	4,2	Student t	0.04,8.73	0.02
	Transition probabilities				Transition probabilities		
	Coef	t-stat			Coef	t-stat	
P_{11}	0.5009	0.0019			0.4957	0.0073	
P_{22}	0.7371	5.8987			0.7322	5.7173	
LogL	325.6538				325.3881		

This table presents parameter estimates of the regime switching dependence structure from our Markov regime switching regular vine copula model for the four asset case. Model 1 (first column to fourth column): Regime 1 (low dependence) described by a R-vine Gaussian copula switch to Regime 2 (high dependence) described by a R-vine t copula. Model 2 (fifth column to eighth column): Regime 1 (low dependence) described by a R-vine Gaussian copula switch to Regime 2 (high dependence) described by a C-vine t copula. In column one and five, 1 represents $S\&P$ 500 index, 2 represents MSCI Emerging Markets index, 3 represents JPM US Government Bonds index, 4 represents FTSE EPRA/NAREIT index, 5 represents HFRI Fund of Funds index, 6 represents US Buyout index, 7 represents US Venture Capital index, and 8 represents US Treasury bill. The table reports bivariate copula selected as building blocks for vine copula including parameters and Kendall's τ, also the diagonal elements of the transition probability P_{11} and P_{22} and t statistics. The log likelihood is reported in the last row.

Table 3.8: **Four Assets Regime Switching Estimation Results Continued**

	Model 3				Model 4		
Regime 1							
	R-vine Gaussian				R-vine Gaussian		
		Coef	τ			Coef	τ
1,2	Gaussian	0.64	0.44	1,2	Gaussian	0.64	0.44
3,1	Gaussian	0.29	0.19	3,1	Gaussian	0.29	0.19
4,3	Gaussian	0.11	0.07	4,3	Gaussian	0.11	0.07
Regime 2							
	R-vine ind. mixed				C-vine ind. mixed		
		Coef	τ			Coef	τ
1,2	Survival Gumbel	1.77	0.44	2,1	Survival Gumbel	1.77	0.44
3,1	Student t	0.29,6.36	0.19	2,3	Rotated BB8 90 degrees	2.04,0.73	0.17
4,3	Frank	0.69	0.08	4,2	Independence		
	Transition probabilities				Transition probabilities		
	Coef	t-stat			Coef	t-stat	
P_{11}	0.9660	4.080			0.9597	3.356	
P_{22}	0.9917	4.493			0.9885	3.743	
LogL	340.0641				336.1411		

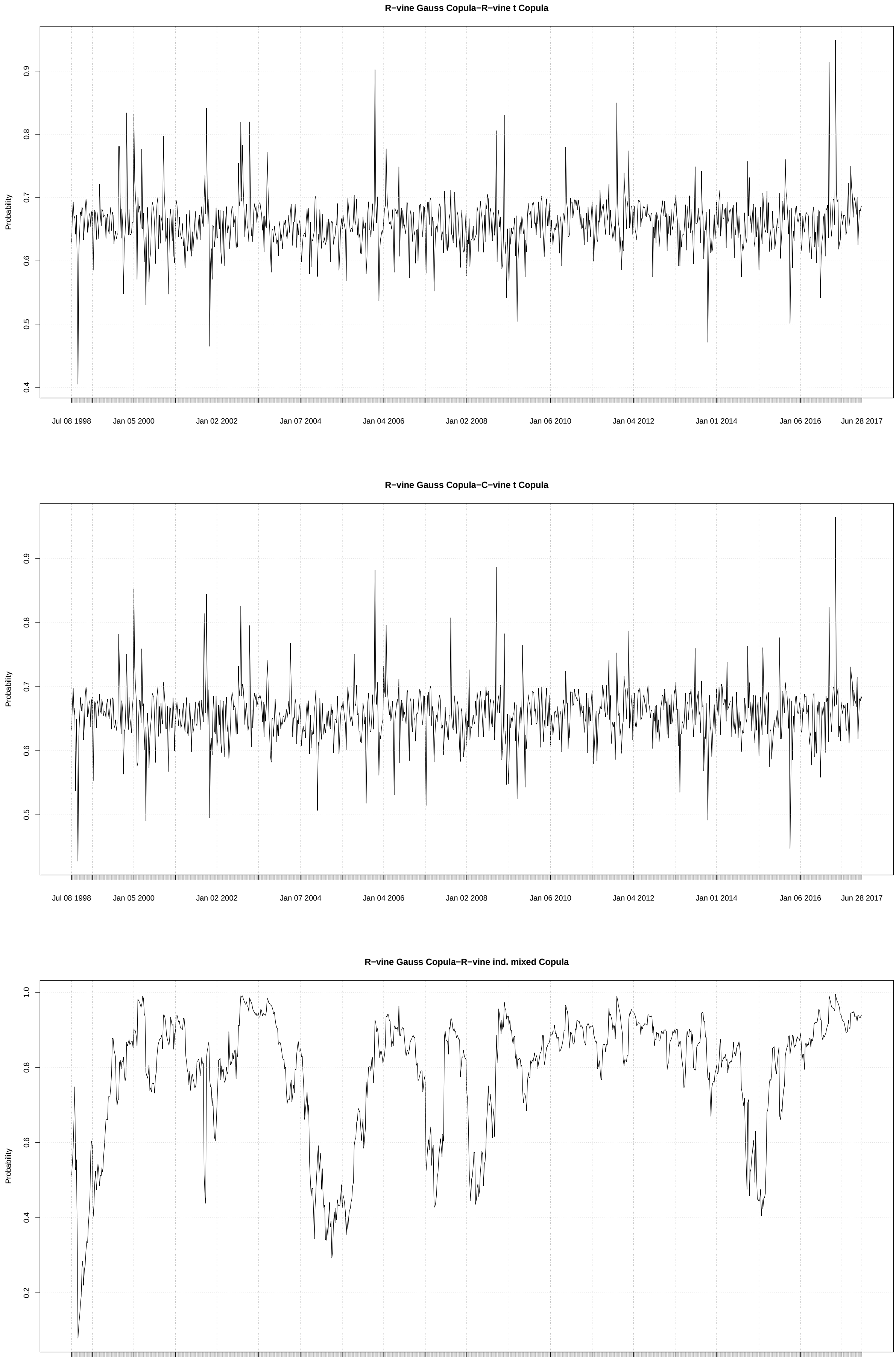
This table presents parameter estimates of the regime switching dependence structure from our Markov regime switching regular vine copula model for the four asset case. Model 3 (first column to fourth column): Regime 1 (low dependence) described by a R-vine Gaussian copula switch to Regime 2 (high dependence) described by a R-vine independence mixed copula. Model 4 (fifth column to eighth column): Regime 1 (low dependence) described by a R-vine Gaussian copula switch to Regime 2 (high dependence) described by a C-vine independence mixed copula. In column one and five, 1 represents $S\&P$ 500 index, 2 represents MSCI Emerging Markets index, 3 represents JPM US Government Bonds index, 4 represents FTSE EPRA/NAREIT index, 5 represents HFRI Fund of Funds index, 6 represents US Buyout index, 7 represents US Venture Capital index, and 8 represents US Treasury bill. The table reports bivariate copula selected as building blocks for vine copula including parameters and Kendall's τ, also the diagonal elements of the transition probability P_{11} and P_{22} and t statistics. The log likelihood is reported in the last row.

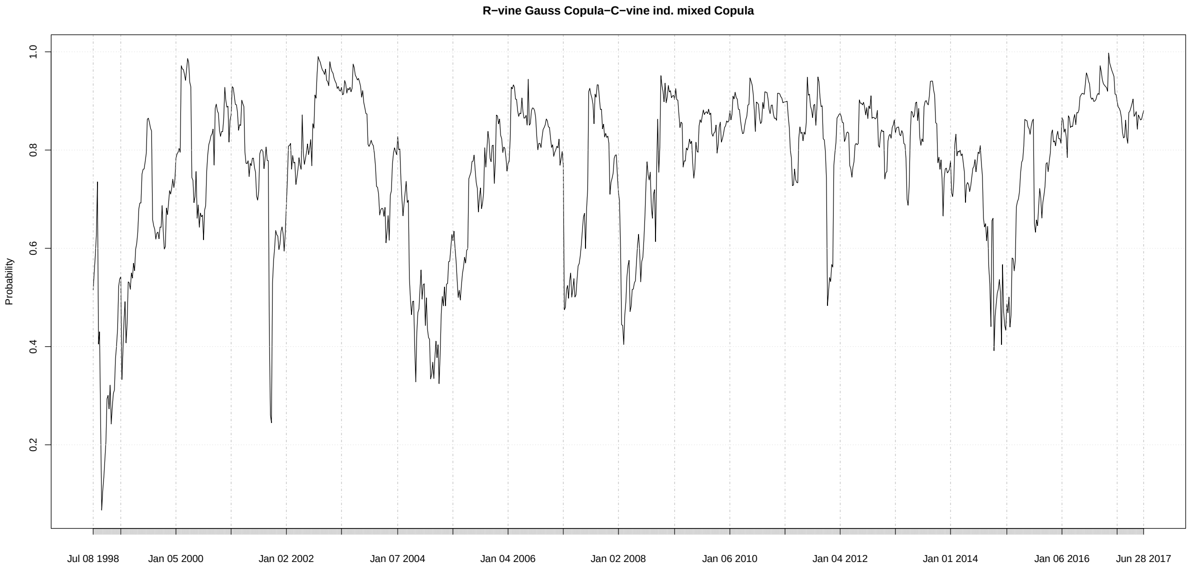
Table 3.9: **Four Assets Regime Switching Estimation Results Continued**

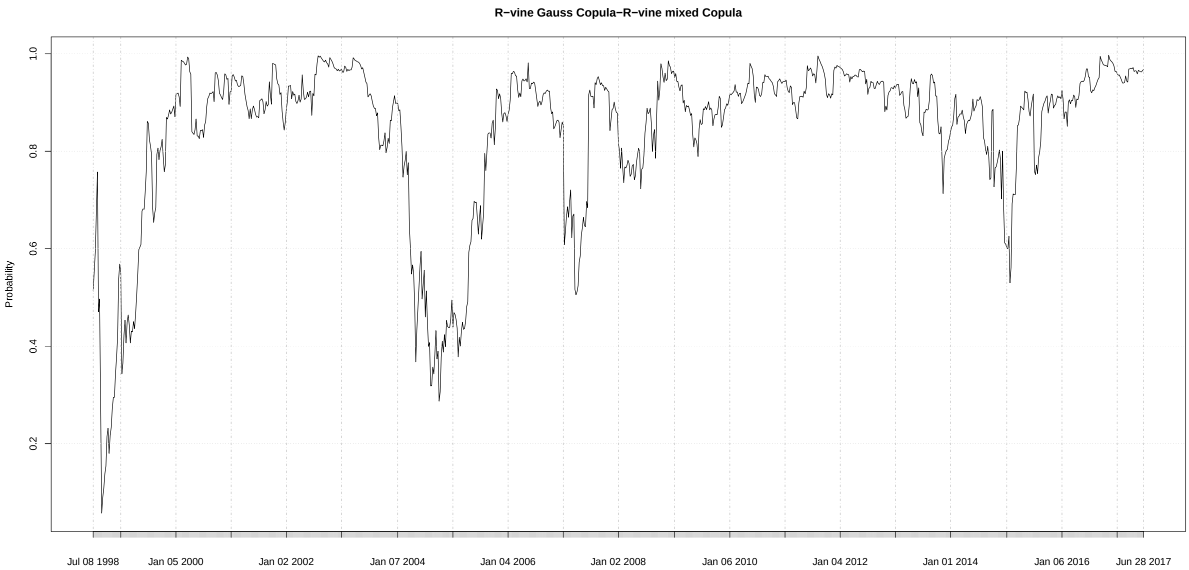
	Model 5				Model 6		
Regime 1							
	R-vine Gaussian				R-vine Gaussian		
		Coef	τ			Coef	τ
1,2	Gaussian	0.64	0.44	1,2	Gaussian	0.64	0.44
3,1	Gaussian	0.29	0.19	3,1	Gaussian	0.29	0.19
4,3	Gaussian	0.11	0.07	4,3	Gaussian	0.11	0.07
Regime 2							
	R-vine mixed				C-vine mixed		
		Coef	τ			Coef	τ
1,2	Survival Gumbel	1.77	0.44	1,2	Survival Gumbel	1.77	0.44
3,1	Student t	0.29,6.36	0.19	3,1	Student t	0.29,6.36	0.19
4,3	Frank	0.69	0.08	4,3	Frank	0.69	0.08
	Transition probabilities				Transition probabilities		
	Coef	t-stat			Coef	t-stat	
P_{11}	0.9712	4.114			0.9732	4.165	
P_{22}	0.9956	4.274			0.9963	4.294	
LogL	342.3656				343.3494		

This table presents parameter estimates of the regime switching dependence structure from our Markov regime switching regular vine copula model for the four asset case. Model 5 (first column to fourth column): Regime 1 (low dependence) described by a R-vine Gaussian copula switch to Regime 2 (high dependence) described by a R-vine mixed copula. Model 6 (fifth column to eighth column): Regime 1 (low dependence) described by a R-vine Gaussian copula switch to Regime 2 (high dependence) described by a C-vine mixed copula. In column one and five, 1 represents $S\&P$ 500 index, 2 represents MSCI Emerging Markets index, 3 represents JPM US Government Bonds index, 4 represents FTSE EPRA/NAREIT index, 5 represents HFRI Fund of Funds index, 6 represents US Buyout index, 7 represents US Venture Capital index, and 8 represents US Treasury bill. The table reports bivariate copula selected as building blocks for vine copula including parameters and Kendall's τ, also the diagonal elements of the transition probability P_{11} and P_{22} and t statistics. The log likelihood is reported in the last row.

Figure 3.1: Four assets regime switching smooth probability




R−vine Gauss Copula−C−vine ind. mixed Copula
Probability
0.2
0.4
0.6
0.8
1.0
Jul 08 1998
Jan 05 2000
Jan 02 2002
Jan 07 2004
Jan 04 2006
Jan 02 2008
Jan 06 2010
Jan 04 2012
Jan 01 2014
Jan 06 2016
Jun 28 2017


R−vine Gauss Copula−R−vine mixed Copula
Probability
0.2
0.4
0.6
0.8
1.0
Jul 08 1998
Jan 05 2000
Jan 02 2002
Jan 07 2004
Jan 04 2006
Jan 02 2008
Jan 06 2010
Jan 04 2012
Jan 01 2014
Jan 06 2016
Jun 28 2017

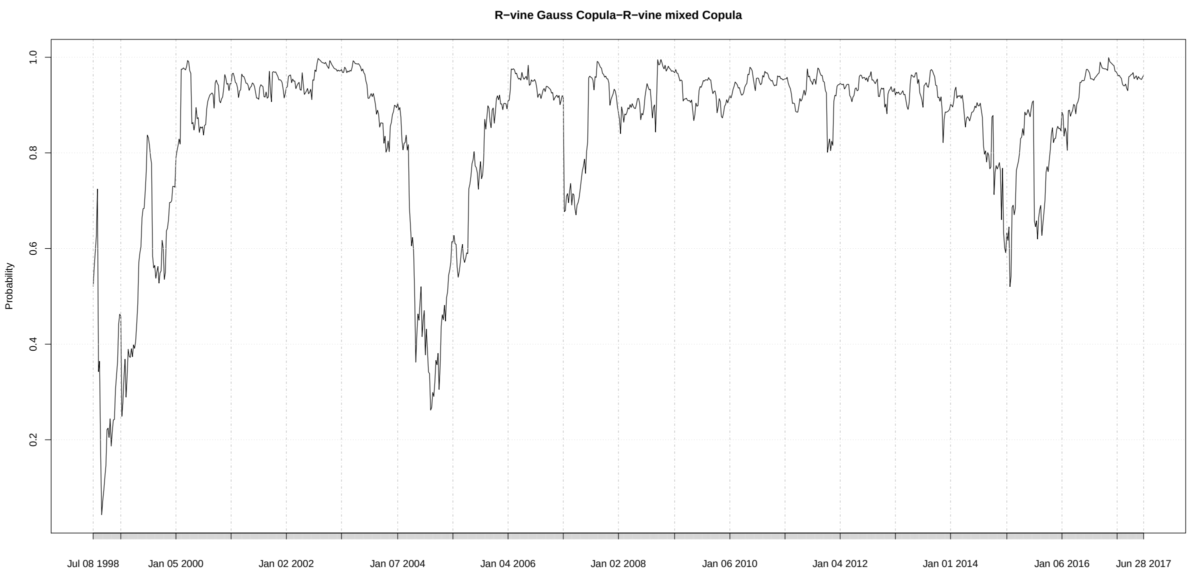

R−vine Gauss Copula−R−vine mixed Copula
Probability
0.2
0.4
0.6
0.8
1.0
Jul 08 1998
Jan 05 2000
Jan 02 2002
Jan 07 2004
Jan 04 2006
Jan 02 2008
Jan 06 2010
Jan 04 2012
Jan 01 2014
Jan 06 2016
Jun 28 2017

Table 3.10: **Eight Assets Regime Switching Estimation Results**

	Model 1				Model 2		
Regime 1							
	R-vine Gaussian				R-vine Gaussian		
		Coef	τ			Coef	τ
2,5	Gaussian	0.14	0.09	2,5	Gaussian	0.14	0.09
3,8	Gaussian	0.11	0.07	3,8	Gaussian	0.11	0.07
1,2	Gaussian	0.64	0.44	1,2	Gaussian	0.64	0.44
1,3	Gaussian	0.29	0.19	1,3	Gaussian	0.29	0.19
1,4	Gaussian	0.62	0.42	1,4	Gaussian	0.62	0.42
1,6	Gaussian	0.85	0.65	1,6	Gaussian	0.85	0.65
7,1	Gaussian	0.79	0.58	7,1	Gaussian	0.79	0.58
Regime 2							
	R-vine t				C-vine t		
		Coef	τ			Coef	τ
2,5	Student t	0.14,30	0.09	1,5	Student t	0.12,30	0.07
3,8	Student t	0.11,27.5	0.07	1,6	Student t	0.93,2	0.75
1,2	Student t	0.64,9.69	0.45	1,4	Student t	0.63,4.31	0.43
1,3	Student t	0.29,6.36	0.19	1,2	Student t	0.64,9.69	0.45
1,4	Student t	0.63,4.31	0.43	1,7	Student t	0.8,6.43	0.59
1,6	Student t	0.93,2	0.75	1,3	Student t	0.29,6.36	0.19
7,1	Student t	0.8,6.43	0.59	8,1	Student t	0.01,18.05	0.01
	Transition probabilities				Transition probabilities		
	Coef	t-stat			Coef	t-stat	
P_{11}	0.8729	3.007			0.8728	2.978	
P_{22}	0.9885	6.415			0.9885	6.378	
LogL	2055.621				2056.248		

This table presents parameter estimates of the regime switching dependence structure from our Markov regime switching regular vine copula model for the eight asset case. Model 1 (first column to fourth column): Regime 1 (low dependence) described by a R-vine Gaussian copula switch to Regime 2 (high dependence) described by a R-vine t copula. Model 2 (fifth column to eighth column): Regime 1 (low dependence) described by a R-vine Gaussian copula switch to Regime 2 (high dependence) described by a C-vine t copula. In column one and five, 1 represents $S\&P$ 500 index, 2 represents MSCI Emerging Markets index, 3 represents JPM US Government Bonds index, 4 represents FTSE EPRA/NAREIT index, 5 represents HFRI Fund of Funds index, 6 represents US Buyout index, 7 represents US Venture Capital index, and 8 represents US Treasury bill. The table reports bivariate copula selected as building blocks for vine copula including parameters and Kendall's τ, also the diagonal elements of the transition probability P_{11} and P_{22} and t statistics. The log likelihood is reported in the last row.

Table 3.11: **Eight Assets Regime Switching Estimation Results Continued**

	Model 3				Model 4		
Regime 1							
	R-vine Gaussian				R-vine Gaussian		
		Coef	τ			Coef	τ
2,5	Gaussian	0.14	0.09	2,5	Gaussian	0.14	0.09
3,8	Gaussian	0.11	0.07	3,8	Gaussian	0.11	0.07
1,2	Gaussian	0.64	0.44	1,2	Gaussian	0.64	0.44
1,3	Gaussian	0.29	0.19	1,3	Gaussian	0.29	0.19
1,4	Gaussian	0.62	0.42	1,4	Gaussian	0.62	0.42
1,6	Gaussian	0.85	0.65	1,6	Gaussian	0.85	0.65
7,1	Gaussian	0.79	0.58	7,1	Gaussian	0.79	0.58
Regime 2							
	R-vine ind. mixed ind.				C-vine ind. mixed		
		Coef	τ			Coef	τ
2,5	BB8	1.23,0.95	0.09	1,5	Gaussian	0.12	0.08
3,8	Frank	0.69	0.08	1,6	Student *t*	0.93,2	0.75
1,2	Survival Gumbel	1.77	0.45	1,4	Student *t*	0.63,4.31	0.43
1,3	Student *t*	0.29,6.36	0.19	1,2	Survival Gumbel	1.77	0.45
1,4	Student *t*	0.63,4.31	0.43	1,7	Survival BB1	0.11,2.27	0.58
1,6	Student *t*	0.93,2	0.75	1,3	Student *t*	0.29,6.36	0.19
7,1	Survival BB1	0.11,2.27	0.59	8,1	Independence		
	Transition probabilities				Transition probabilities		
	Coef	t-stat			Coef	t-stat	
P_{11}	0.8247	2.571			0.8419	2.850	
P_{22}	0.9835	6.824			0.9852	6.913	
LogL	2076.3				2076.332		

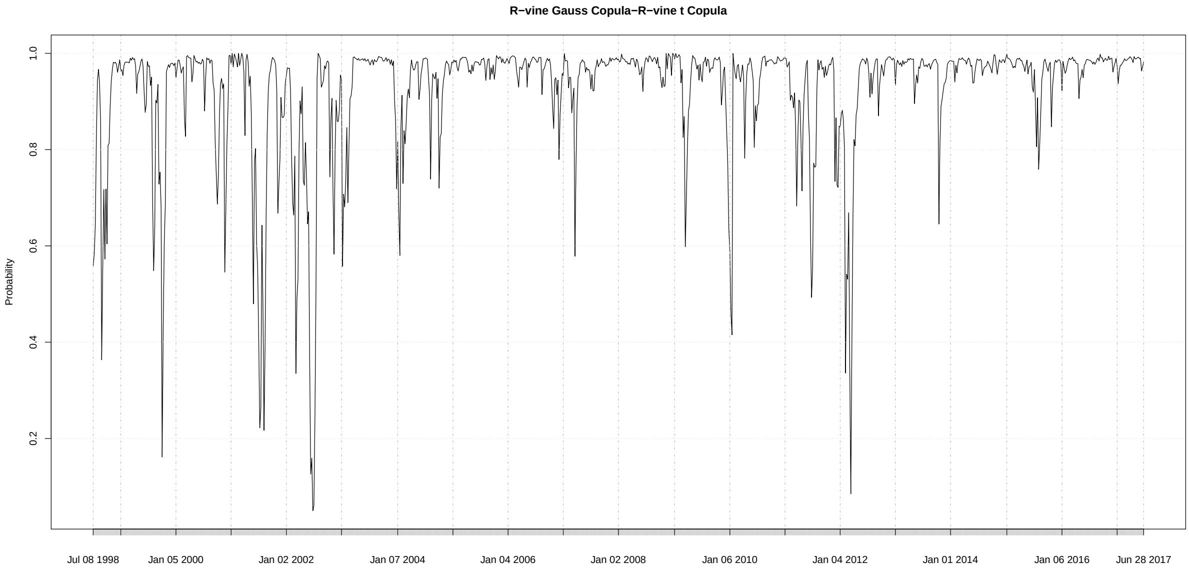
This table presents parameter estimates of the regime switching dependence structure from our Markov regime switching regular vine copula model for the eight asset case. Model 3 (first column to fourth column): Regime 1 (low dependence) described by a R-vine Gaussian copula switch to Regime 2 (high dependence) described by a R-vine independence mixed copula. Model 4 (fifth column to eighth column): Regime 1 (low dependence) described by a R-vine Gaussian copula switch to Regime 2 (high dependence) described by a C-vine independence mixed copula. In column one and five, 1 represents $S\&P$ 500 index, 2 represents MSCI Emerging Markets index, 3 represents JPM US Government Bonds index, 4 represents FTSE EPRA/NAREIT index, 5 represents HFRI Fund of Funds index, 6 represents US Buyout index, 7 represents US Venture Capital index, and 8 represents US Treasury bill. The table reports bivariate copula selected as building blocks for vine copula including parameters and Kendall's τ, also the diagonal elements of the transition probability P_{11} and P_{22} and t statistics. The log likelihood is reported in the last row.

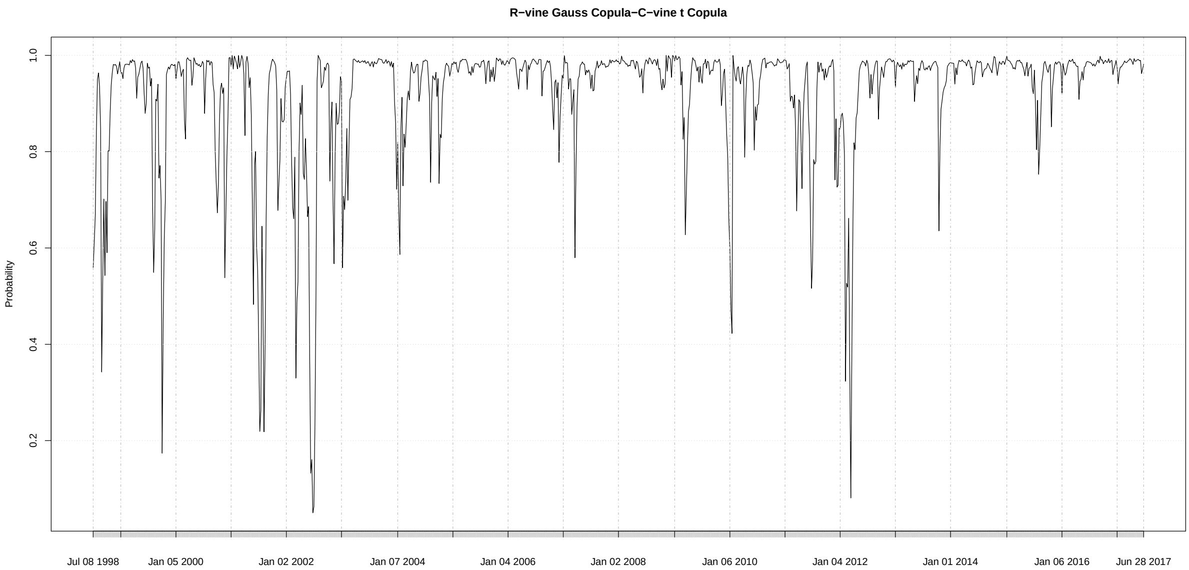
Table 3.12: **Eight Assets Regime Switching Estimation Results Continued**

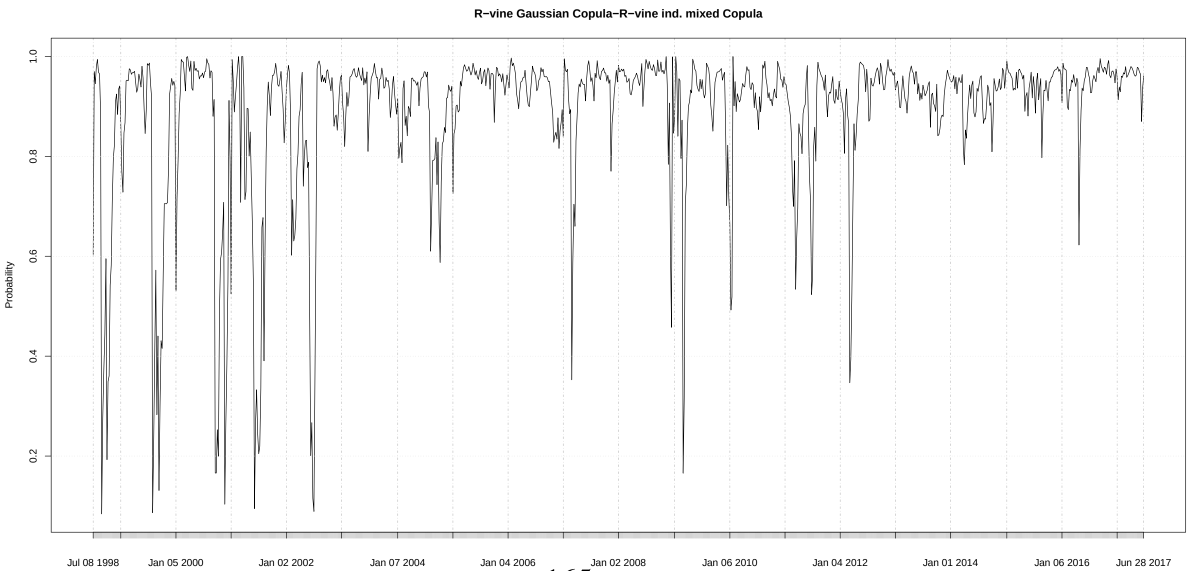
	Model 5				Model 6		
Regime 1							
	R-vine Gaussian				R-vine Gaussian		
		Coef	τ			Coef	τ
2,5	Gaussian	0.14	0.09	2,5	Gaussian	0.14	0.09
3,8	Gaussian	0.11	0.07	3,8	Gaussian	-0.11	0.07
1,2	Gaussian	0.64	0.44	1,2	Gaussian	0.64	0.44
1,3	Gaussian	0.29	0.19	1,3	Gaussian	-0.29	0.19
1,4	Gaussian	0.62	0.42	1,4	Gaussian	0.62	0.42
1,6	Gaussian	0.85	0.65	1,6	Gaussian	0.85	0.65
7,1	Gaussian	0.79	0.58	7,1	Gaussian	0.79	0.58
Regime 2							
	R-vine mixed				C-vine mixed		
		Coef	τ			Coef	τ
2,5	BB8	1.23,0.95	0.10	1,5	Gaussian	0.12	0.08
3,8	Frank	0.69	0.08	1,6	Student *t*	0.93,2	0.75
1,2	Survival Gumbel	1.77	0.45	1,4	Student *t*	0.63,4.31	0.44
1,3	Student *t*	0.29,6.36	0.19	1,2	Survival Gumbel	1.77	0.45
1,4	Student *t*	0.63,4.31	0.43	1,7	Survival BB1	0.11,2.27	0.59
1,6	Student *t*	0.93,2	0.75	1,3	Student *t*	-0.29,6.36	0.19
7,1	Survival BB1	0.11,2.27	0.59	8,1	Survival Joe	1.04	0.02
	Transition probabilities				Transition probabilities		
	Coef	t-stat			Coef	t-stat	
P_{11}	0.7898	1.936			0.8169	2.204	
P_{22}	0.9859	5.851			0.9866	6.129	
LogL	2100.604				2114.291		

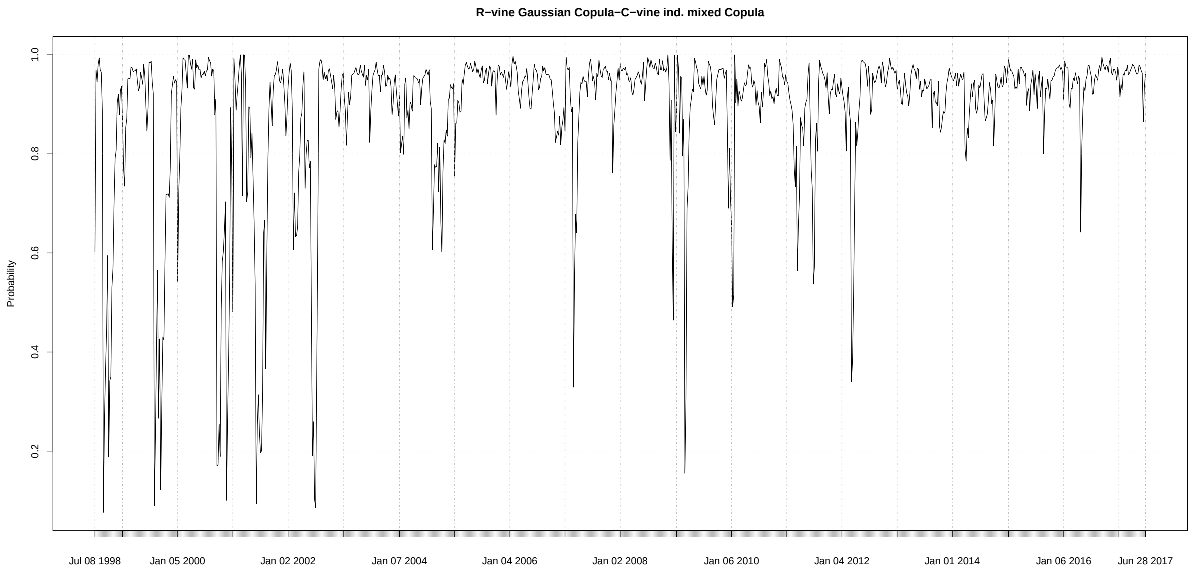
This table presents parameter estimates of the regime switching dependence structure from our Markov regime switching regular vine copula model for the eight asset case. Model 5 (first column to fourth column): Regime 1 (low dependence) described by a R-vine Gaussian copula switch to Regime 2 (high dependence) described by a R-vine mixed copula. Model 6 (fifth column to eighth column): Regime 1 (low dependence) described by a R-vine Gaussian copula switch to Regime 2 (high dependence) described by a C-vine mixed copula. In column one and five, 1 represents $S\&P$ 500 index, 2 represents MSCI Emerging Markets index, 3 represents JPM US Government Bonds index, 4 represents FTSE EPRA/NAREIT index, 5 represents HFRI Fund of Funds index, 6 represents US Buyout index, 7 represents US Venture Capital index, and 8 represents US Treasury bill. The table reports bivariate copula selected as building blocks for vine copula including parameters and Kendall's τ, also the diagonal elements of the transition probability P_{11} and P_{22} and t statistics. The log likelihood is reported in the last row.

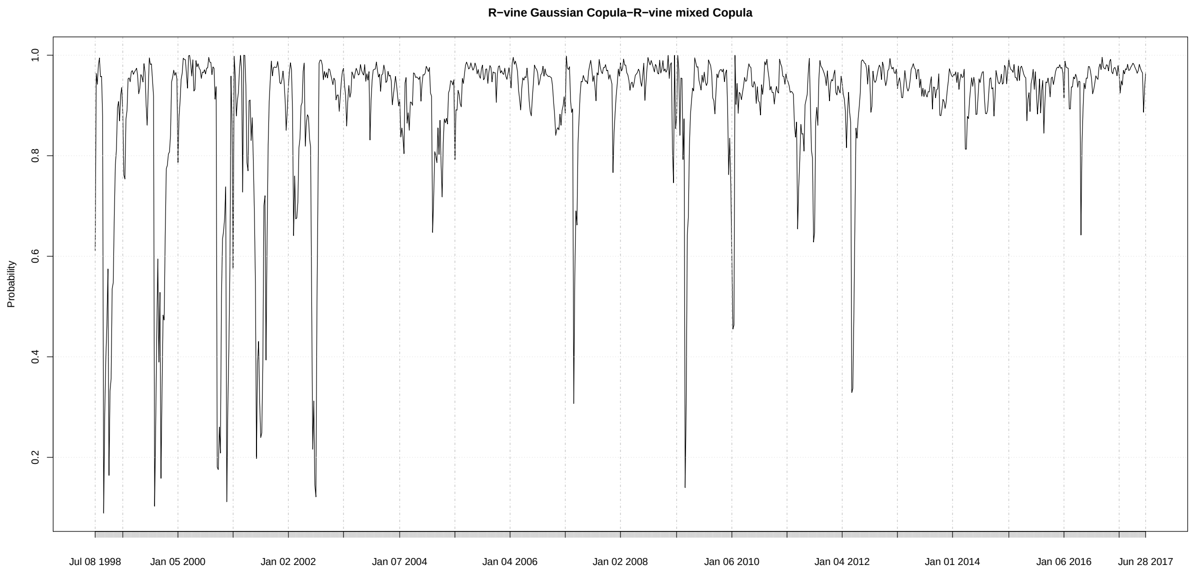
Figure 3.2: Eight assets regime switching smooth probability

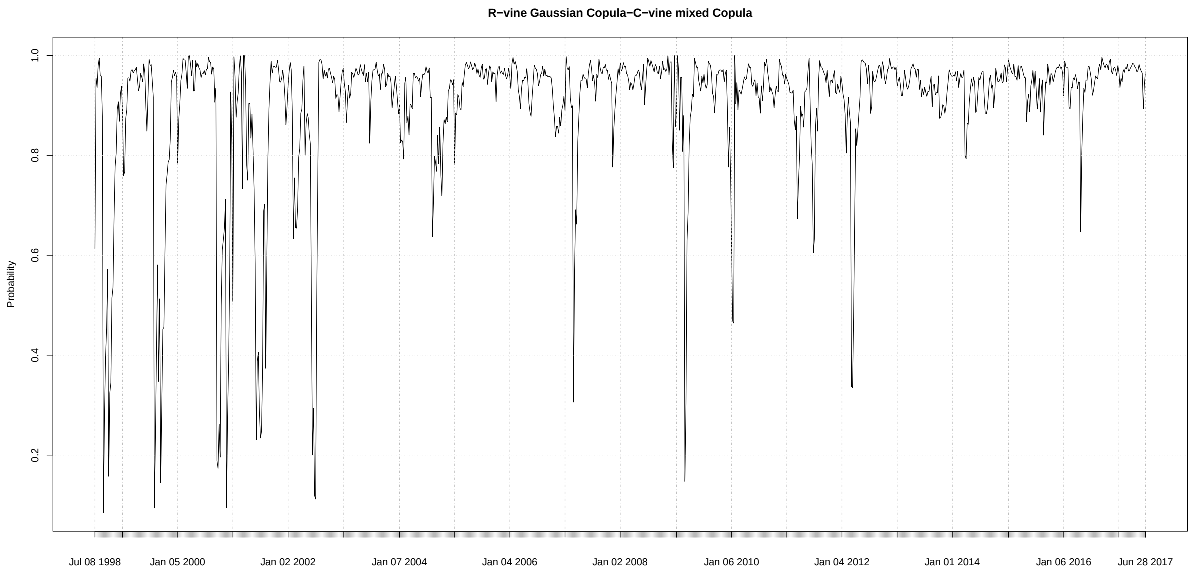







R−vine Gaussian Copula−C−vine ind. mixed Copula
Probability
0.2
0.4
0.6
0.8
1.0
Jul 08 1998
Jan 05 2000
Jan 02 2002
Jan 07 2004
Jan 04 2006
Jan 02 2008
Jan 06 2010
Jan 04 2012
Jan 01 2014
Jan 06 2016
Jun 28 2017


R−vine Gaussian Copula−R−vine mixed Copula
Probability
0.2
0.4
0.6
0.8
1.0
Jul 08 1998
Jan 05 2000
Jan 02 2002
Jan 07 2004
Jan 04 2006
Jan 02 2008
Jan 06 2010
Jan 04 2012
Jan 01 2014
Jan 06 2016
Jun 28 2017


R−vine Gaussian Copula−C−vine mixed Copula
Probability
0.2
0.4
0.6
0.8
1.0
Jul 08 1998
Jan 05 2000
Jan 02 2002
Jan 07 2004
Jan 04 2006
Jan 02 2008
Jan 06 2010
Jan 04 2012
Jan 01 2014
Jan 06 2016
Jun 28 2017

folios separately for an asset universe that only includes traditional asset classes (stock, bond, risk-free asset) and an augmented one that also includes alternative investments. Among them, we calculate optimal portfolios for both cases of considering regime switching and ignorance of regime respectively. Then we evaluate the relative performance of all these asset allocation strategies in an out-of-sample setting which is the ultimate test given that at any given point in time, the investor decides on the portfolio weights and the portfolio returns to be realized.

We study the use of Markov regime switching vine copula model incorporating asymmetries within the dependence structure in portfolio asset returns, therefore we adopt the portfolio optimization strategy of minimizing CVaR. Our out-of-sample analysis is performed in a long-run, multi-period investment horizon from January 2010 to June 2017. As mention above section, we consider the six Markov regime switching vine copula model which describe Regime 1 using R-vine Gaussian copula, and describe Regime 2 separately using R-vine *t* copula, C-vine independence mixed copula, R-vine independence mixed copula, R-vine mixed copula and C-vine mixed copula. We explore out-of-sample portfolio performance based on these copula strategy in relation to each other and against the benchmark of the conventional 1/N asset allocation strategy. We conduct out-of-sample analysis aiming to examine whether the vine copula regime switching asset allocation strategy outperforms the conventional asset allocation strategy. All portfolio strategies comparison are discussed both short-sales constrained and unconstrained.

To examine if the diversification benefits behave differently across the two regimes of the asset market, we first need to partition the asset return data into two regimes. As mentioned in previous section, the estimated smooth probabilities, as a useful tool, reported in Figure 1, 2, in which Figure 1 reports the case of four assets and Figure 2 for eight assets case. The higher probability of regime 1 (the low return state) can be thought of as a bearish asset market, whereas the low probability of regime 2 can be considered as a bullish asset market. A smooth probability of 0.5 indicates equal probabilities of realizing regimes 1 and 2. Then the return series of various asset classes can be partitioned into two sub-samples corresponding to the low and high return regimes based on whether the smooth probability of the regime model is higher or lower than 0.5. With these two sub-samples of return data corresponding to the two regimes, the analyses on the diversification benefits of adding alternative investments can be conducted for each of the two regimes.

Under this methodology, the regime switching copula models and conventional asset allocation models are estimated using information available only up to time t. The process is repeated every month until June 2017, the end of the sample period. Assume that the investor rebalances their portfolio once a month, our first analysis uses historical data from January 1998 to January 2010 to construct various asset allocation models as of January 2010. Together, we evaluate the portfolio performance by adopting Sharpe ratio, Sortino ratio and Omega ratio realized by the various portfolio strategies-six kinds of vine copula with and without regime switching consideration for both four assets and eights assets cases and conventional mean-variance and equal weights 1/N asset allocation strategy.

The results for the US investor are reported in Table 13 to Table 20. In the portfolio strategy with regime switching consideration, the State 1 (low asset returns) and State 2 (high asset returns) results are listed separately in the tables.

The Effect of Vine Structure and Dimension

In the comparison of our various competing portfolio strategy performance, we separately fit R-vine Gauss copula, R-vine t copula, R-vine independence mixed copula, C-vine independence mixed copula, R-vine mixed copula, C-vine mixed copula, where ignoring the regime switching, to our multi asset returns, and also fit R-vine Gauss copula representing market state 1, transfer to market state 2 characterized by R-vine independence mixed copula, C-vine independence mixed copula, R-vine mixed copula, C-vine mixed copula respectively to our returns when considering the regime switching. For both cases, according to the investment ratios results which we will discuss in details in subsequent section, R-vine t copula model performs poorly across all vine copula models for portfolios. This finding suggests that the R-vine t copula models' building blocks of bivariate t copula lack of the characteristics to capture the asymmetric dependence, therefore, it is unable to meaningfully capture asymmetric dependence of different kinds of multi-assets. Furthermore, it demonstrates that though the R-vine t copula has the vine structure, their bivariate building blocks lack of capabilities of capturing asymmetric dependence still make it not perform well.

However, when we turn to investigate the effect of dimensions, we find in the four assets case that the C-vine copula model underperforms the R-vine t copula model. We therefore are able to conclude, in low dimension case, it is not suitable to choose a pivotal asset to construct C-vine structure. This contrast is potentially due to the fact that there is little or no benefit to be gained by using a complex model of the dependence structure for simpler, smaller, low dimension portfolios. In such cases, using advanced models induces estimation error which swamps any benefits from the modelling, resulting in poor portfolio decisions.

Investment Ratios and Portfolio Performance

We now turn to analyse the investment ratios and relative performance measures of the optimal portfolio returns resulting from the different Markov regime switching vine copula portfolio strategies, vine copula strategy without regime switching consideration, conventional mean-variance strategy and equal weights strategy. Results are reported in Table 13 to Table 20 for various specifications of the preferences. We compute the Sharpe, Sortino and Omega investment ratios, given, respectively, by

$$Sharpe\ ratio = \frac{u_P - r_f}{\delta_P},$$

$$Sortino\ ratio = \frac{u_P - r_f}{\sqrt{q_2^l(r_f)}},$$

$$Omega\ ratio = \frac{q_1^u(r_f)}{q_1^l(r_f)},$$

where u_P is the average realized portfolio return, δ_P is the realized portfolio volatility, r_f is the risk-free rate, and $q_m^l(r_f)$ and $q_m^u(r_f)$ are the lower and upper partial moments of order m with target value equal to the risk-free rate. The Sortino ratio modifies the Sharpe ratio by dividing the excess return of the portfolio by the downside standard deviation or square root of semi-variance. The Omega ratio can be interpreted as the probability weighted ratio of gains to losses, relative to the risk-free rate and it measures the combined effect of all returns moments, rather than the individual effects of any of them. The Sharpe ratio penalizes the entire standard deviation of portfolio returns, whereas the Sortino ratio penalizes only downside standard deviation. The Omega ratio is a practical measure that makes no assumptions regarding investor risk preferences or utility functions except that investors prefer more to less. The higher the values of these ratios, the better the portfolio performance.

Seen from our results of various competing asset allocation strategies from Table 13 to Table 20, we firstly can draw a general conclusion that the C-vine mixed copula regime switching model outperforms all other vine copula regime switching model, vine copula model ignoring the regime switching and also conventional asset allocation model on the risk-adjusted return basis.

In particular, we first take a look at short sales constrained results. Table 13, 15, 17, 19 provide the outcomes of the portfolio optimization with and without alternative investments for our representative US investor in the short sales constrained condition. In regime switching case in Table 15, 19, the low asset return state and the high asset return state results are listed separately. Across the Sharpe, Sortino and Omega Ratios, it is observed that as the portfolio increases from the case without alternative investments to alternative investments included case, so does produce the higher ranked outcome. As the numbers indicate, for example, in Table 19, the eight asset case considering the regime switching, the Sharpe ratio of R-vine t strategy equals 0.0596, C-vine t strategy equals 0.0643, R-vine independence mixed strategy equals 0.0732, C-vine independence mixed strategy equals 0.0688, R-vine mixed strategy equals 0.0762, C-vine mixed strategy equals 0.0827, which are all much better than the conventional non-regime dependent strategy where Sharpe ratio equals 0.0376 in Table 13. Similarly, the Sortino ratio and Omega ratio for the regime switching model are also higher than the conventional non-regime dependent strategy. When we compare the eight assets and four assets case, no matter the case considering regime switching or not, the eight assets risk adjust measure results are better than the four assets case. For instance, in Table 15, the Sharpe ratio of R-vine t strategy equals 0.0336, C-vine t strategy equals 0.0288, R-vine independence mixed strategy equals 0.0382, C-vine independence mixed strategy equals 0.0398, R-vine mixed strategy equals 0.0458, C-vine mixed strategy equals 0.0477, which are all lower than the eight assets' mentioned in Table 19. Among the six Markov regime switching vine copula asset allocation strategies, the C-vine mixed copula regime switching model outperforms other vine copula regime switching model, vine copula model without regime consideration and also conventional asset allocation according to the risk-adjusted return indicators, which confirm that the flexible structure of the vine model makes it capture the asymmetric dependence and lower tail dependence accurately and this result is also in line with the log likelihood value result in previous section.

Then we turn to the short sales unconstrained case. Table 14, 16, 18, 20 reports the results of the portfolio optimization with and without alternative investments for our representative US investor in the short sales unconstrained condition. Similar to the short sales constrained results reported previously, in regime switching case in Table 16, 20, the low asset return state and the high asset return state results are listed separately. Across the Sharpe, Sortino and Omega Ratios, it is observed that as the portfolio increases from the case without alternative investments to alternative investments included case, so does produce the higher ranked outcome. As the numbers indicate, for example, in Table 20, the eight asset case considering the regime switching, the Sharpe ratio of R-vine *t* strategy equals 0.0691, C-vine *t* strategy equals 0.0749, R-vine independence mixed strategy equals 0.0772, C-vine independence mixed strategy equals 0.0780, R-vine mixed strategy equals 0.0849, C-vine mixed strategy equals 0.0877, which are all much better than the conventional non-regime dependent strategy where Sharpe ratio equals 0.0370 in Table 14. Similarly, the Sortino ratio and Omega ratio for the regime switching model are also higher than the conventional non-regime dependent strategy. When we compare the eight assets and four assets case, no matter the case considering regime switching or not, the eight assets risk adjust measure results are better than the four assets case. For instance, in Table 16, the Sharpe ratio of R-vine *t* strategy equals 0091, C-vine *t* strategy equals 0205, R-vine independence mixed strategy equals 0.0372, C-vine independence mixed strategy equals 0.0376, R-vine mixed strategy equals 0.0529, C-vine mixed strategy equals 0.0538, which are all lower than the eight assets' mentioned in Table 20. Among the six Markov regime switching vine copula asset allocation strategies, the C-vine mixed copula regime switching model outperforms other vine copula regime switching model, vine copula model without regime consideration and also conventional asset allocation according to the risk-adjusted return indicators, which confirm that the flexible structure of the vine model makes it capture the asymmetric dependence and lower tail dependence accurately and this result is also in line with the log likelihood value result in previous section.

Across the Sharpe, Sortino and Omega Ratios, it is observed that as the portfolio increases from four assets to eight assets case, so does the level of model complexity required in the model to produce good performance of portfolio. For both the Markov regime switching vine copula strategy and non-dependent vine copula strategy, eight assets case outperforms its counterpart model in four assets case across the Sharpe, Sortino and Omega ratios. This finding suggests that the Markov regime switching vine copula model perform well in multi-asset case, the small portfolio is not suitable to be modelling with this complex model. This contrast is potentially due to the fact that there is little or no benefit to be gained by using a complex model of the dependence structure and marginal for simpler, smaller portfolios. In such cases, using advanced models induces estimation error which swamps any benefits from the modelling, resulting in poor portfolio decisions. In another aspect, these results confirm that including alternative investments substantially add value into the portfolio. Generally speaking, these results provide the evidence that increases in model complexity and parameterization for small portfolios have little or even negative benefits due to noise-prone estimation. At eight assets case, C-vine mixed regime switching copula asset allocation strategy consistently achieves the highest rank across all portfolio metrics. As the number of assets within the portfolio increases, the greater degree of parameterization in the modelling process of C-vine mixed regime switching copula strategy produces various out-of-sample benefits including improved risk-adjusted returns and performance benefits.

When we investigate the Mean/CVaR metrics, Markov regime switching vine copula strategy consistently produces the highest ranked outcomes for eight asset case, which indicate that regime switching vine copula strategy method is able to produce a higher portfolio return without a substantial increase in downside exposure.

In summary, the out-of-sample tests reasonably demonstrate consistent outperformance of our Markov regime switching vine copula asset allocation strategy comparing to the non-regime dependent vine copula strategy and conventional asset allocation strategy. The regime switching vine copula allocation strategy helps investor establish a defensive portfolio in the bear market regime (i.e. regime 1) that hedges against higher correlations and low returns in international assets markets. Additionally, since the vine copula regime switching allocation strategy relies less on the historical moments, it is likely that the resulting optimal portfolio could even be more internationally diversified (Ang and Bekaert (2002a)). As a consequence, it is equally possible to add value to the portfolios as the presence of a bear market (and characterized by high correlation) regime whereas not necessarily erode the benefits of full portfolio diversification. The out-of-sample results

Table 3.13: **Four Traditional Assets' asset allocation strategies Out-of-sample risk-adjusted performance ignoring regime switching (short sales constrained)**

Short sales constrained										
		Asset Allocation Strategies								
		Equal weights	Conventional	R-vine Gauss Copula	R-vine t Copula	C-vine t Copula	R-vine ind. mixed Copula	C-vine ind. mixed Copula	R-vine mixed Copula	C-vine mixed Copula
Portfolio weights	S&P US	0.25	0.060	0.336	0.022	0.008	0.028	0.246	0.2442	0.046
	MSCI.EM	0.25	0.038	0.220	0.034	0.014	0.026	0.188	0.2482	0.030
	Gov. bond	0.25	0.132	0.356	0.046	0.014	0.040	0.378	0.4272	0.068
	T-bills	0.25	0.772	0.094	0.890	0.956	0.916	0.198	0.0711	0.854
	Measure									
	Mean	0.2276	0.264	0.0551	0.0539	0.345	0.2902	0.2188	0.1985	0.2389
	Standard deviation	1.485	1.574	0.7254	0.7407	0.6141	0.8597	0.9563	0.6908	0.8245
	CVaR	4.817	4.912	4.283	4.275	4.119	4.932	3.048	3.803	5.852
	Sharpe ratio	0.0376	0.0110	0.0339	0.0351	0.0462	0.0518	0.0548	0.0507	0.0624
	Sortino ratio	0.0514	0.0154	1.559	0.0441	0.6179	0.0607	0.0873	0.0744	0.0922
	Omega ratio	0.1176	0.0372	3.2718	0.1222	1.3837	0.2204	0.2634	0.1571	0.1887

This table reports a range of risk-adjusted measures for the out-of-sample asset allocation strategies. Portfolio weights and Sharpe ratios, Sortino ratio, and Omega ratio are reported. Optimal portfolios are obtained based on the out-of-sample period from January 2010 to June 2017. Here vine copula strategies are non-regime dependent.

Table 3.14: **Four Traditional Assets' asset allocation strategies Out-of-sample risk-adjusted performance ignoring regime switching (short sales unconstrained)**

Short sales constrained										
		Asset Allocation Strategies								
		Equal weights	Conventional	R-vine Gauss Copula	R-vine t Copula	C-vine t Copula	R-vine ind. mixed Copula	C-vine ind. mixed Copula	R-vine mixed Copula	C-vine mixed Copula
Portfolio weights	S&P US	0.25	0.218	0.238	0.320	0.386	0.458	0.3162	0.318	0.334
	MSCI.EM	0.25	0.192	0.330	0.260	0.266	0.260	0.2816	0.280	0.318
	Gov. bond	0.25	0.438	0.492	0.434	0.500	0.426	0.4320	0.486	0.484
	T-bills	0.25	0.152	-0.056	-0.018	-0.160	-0.154	-0.0339	-0.090	-0.134
	Measure									
	Mean	0.492	0.0481	0.1254	0.0554	0.0446	0.1432	0.1099	0.1189	0.1990
	Standard deviation	0.6378	0.6414	0.6828	0.6339	0.7284	0.6073	0.1081	0.0923	0.1354
	CVaR	2.359	2.454	2.872	2.684	3.525	2.847	1.275	1.209	2.138
	Sharpe ratio	0.0376	0.0370	0.0329	0.0349	0.0490	0.0520	0.0699	0.0724	0.0768
	Sortino ratio	0.0500	0.0492	0.0523	0.0510	0.0655	0.0711	0.0944	0.1006	0.1098
	Omega ratio	0.1076	0.1059	0.1286	0.1037	0.1470	0.1507	0.2643	0.2362	0.2389

This table reports a range of risk-adjusted measures for the out-of-sample asset allocation strategies. Portfolio weights and Sharpe ratios, Sortino ratio, and Omega ratio are reported. Optimal portfolios are obtained based on the out-of-sample period from January 2010 to June 2017. Here vine copula strategies are non-regime dependent.

Table 3.15: **Four Traditional Assets' asset allocation strategies Out-of-sample risk-adjusted performance considering regime switching (short sales constrained)**

Short sales constrained							
		Asset Allocation Strategies					
		R-vine t Copula	C-vine t Copula	R-vine ind. mixed Copula	C-vine ind. mixed Copula	R-vine mixed Copula	C-vine mixed Copula
Regime 1							
Portfolio weights	S&P US	0.048	0.078	0.0971	0.218	0.2186	0.052
	MSCI.EM	0.116	0.082	0.1623	0.114	0.1496	0.056
	Gov. bond	0.818	0.410	0.1409	0.426	0.5380	0.162
	T-bills	0.008	0.0140	0.0485	0.248	0.0940	0.740
	Measure						
	Mean	1.76	0.8276	0.0098	0.0888	0.1646	-0.0148
	Standard deviation	0.2911	0.2177	0.1678	0.0683	0.0271	0.167
	CVaR	0.7464	1.188	0.9065	1.67	2.034	0.7588
Regime 2							
Portfolio weights	S&P US	0.0776	0.0180	0.0520	0.026	0.0709	0.058
	MSCI.EM	0.0268	0.0192	0.0440	0.034	0.0631	0.038
	Gov. bond	0.1405	0.1167	0.1980	0.162	0.1737	0.136
	T-bills	0.7521	0.4420	0.3700	0.286	0.6928	0.766
	Measure						
	Mean	0.0104	0.0093	0.0111	0.0086	0.0119	0.0124
	Standard deviation	0.3054	0.3059	0.3135	0.3175	0.3144	0.327
	CVaR	0.7653	0.7283	0.7307	0.7501	0.9512	0.7456
	Sharpe ratio	0.0336	0.0288	0.0382	0.0398	0.0458	0.0477
	Sortino ratio	0.0438	0.0381	0.0507	0.0533	0.0962	0.1578
	Omega ratio	0.0998	0.0876	0.1104	0.1149	0.2093	0.3415

This table reports a range of risk-adjusted measures for the out-of-sample asset allocation strategies. Portfolio weights and Sharpe ratios, Sortino ratio, and Omega ratio are reported. Optimal portfolios are obtained based on the out-of-sample period from January 2010 to June 2017. Here vine copula strategies are regime dependent, such as "C-vine mixed Copula" represents regime switches from R-vine Gaussian to C-vine mixed copula.

Table 3.16: **Four Traditional Assets' asset allocation strategies Out-of-sample risk-adjusted performance considering regime switching (short sales unconstrained)**

Short sales constrained							
		Asset Allocation Strategies					
		R-vine t Copula	C-vine t Copula	R-vine ind. mixed Copula	C-vine ind. mixed Copula	R-vine mixed Copula	C-vine mixed Copula
Regime 1							
Portfolio weights	S&P US	0.062	0.170	0.1960	0.486	0.230	0.288
	MSCI.EM	0.036	0.132	0.1160	-0.348	0.126	0.468
	Gov. bond	0.470	0.262	0.4660	0.396	0.500	-0.260
	T-bills	0.436	0.428	0.2246	0.468	0.140	0.494
	Measure						
	Mean	0.5383	0.0232	0.0894	0.1146	-0.0302	-1.919
	Standard deviation	1.218	0.6328	0.5468	0.5198	0.5977	1.153
	CVaR	0.6627	1.716	1.671	1.426	2.241	8.288
Regime 2							
Portfolio weights	S&P US	0.0180	0.0520	0.112	0.198	-0.462	0.060
	MSCI.EM	0.0192	0.0440	0.114	0.158	0.500	0.028
	Gov. bond	0.1167	0.1980	0.276	0.392	0.458	0.158
	T-bills	0.4420	0.3700	0.498	0.252	0.496	0.384
	Measure						
	Mean	0.0280	0.0342	0.0241	0.0334	0.0495	0.0278
	Standard deviation	0.6575	0.6391	0.6441	0.6649	0.6513	0.6436
	CVaR	1.499	1.738	1.479	2.166	3.118	1.531
	Sharpe ratio	0.0091	0.0205	0.0372	0.0376	0.0529	0.0538
	Sortino ratio	0.0112	0.0255	0.0494	0.0501	0.0904	0.1009
	Omega ratio	0.0272	0.0627	0.1081	0.1077	0.1946	0.2169

This table reports a range of risk-adjusted measures for the out-of-sample asset allocation strategies. Portfolio weights and Sharpe ratios, Sortino ratio, and Omega ratio are reported. Optimal portfolios are obtained based on the out-of-sample period from January 2010 to June 2017. Here vine copula strategies are regime dependent, such as "C-vine mixed Copula" represents regime switches from R-vine Gaussian to C-vine mixed copula.

Table 3.17: **Eight Assets' asset allocation strategies Out-of-sample risk-adjusted performance ignoring regime switching (short sales constrained)**

Short sales constrained										
		Asset Allocation Strategies								
		Equal weights	Conventional	R-vine Gauss Copula	R-Vine t Copula	C-Vine t Copula	R-Vine ind. mixed Copula	C-Vine ind. mixed Copula	R-Vine mixed Copula	C-Vine mixed Copula
	S&P US	0.125	0.0745	0.028	0.062	0.1233	0.0704	0.0684	0.1516	0.2053
	MSCI.EM	0.125	0.0827	0.088	0.094	0.0553	0.1180	0.0977	0.0182	0.1443
	Gov. bond	0.125	0.1572	0.210	0.160	0.1653	0.2800	0.1255	0.1263	0.1880
Portfolio weights	Real Est	0.125	0.0004	0.160	0.092	0.1225	0.0260	0.1210	0.1253	0.1262
	Hedge	0.125	0.5025	0.446	0.398	0.1415	0.2980	0.3620	0.3642	0.1160
	Buyout	0.125	0.0476	0.034	0.096	0.0774	0.0600	0.0970	0.0617	0.1480
	VC	0.125	0.0868	0.042	0.098	0.0779	0.0600	0.0896	0.0648	0.0586
	T-bills	0.125	0.0460	0.002	0.008	0.2434	0.0840	0.0309	0.0927	0.0151
	Measure									
	Mean	0.0120	0.0107	0.1249	0.0027	0.0057	0.0162	0.1058	0.0769	0.0154
	Standard deviation	0.312	0.306	0.0410	0.0728	0.0699	0.0464	0.0549	0.0343	0.0617
	CVaR	0.7473	0.7485	2.266	0.3256	0.1389	0.2591	2.141	2.41	0.3512
	Sharpe ratio	0.0376	0.0349	0.0568	0.0430	0.0604	0.0682	0.0695	0.0757	0.0777
	Sortino ratio	0.0500	0.0462	0.0689	0.0580	0.0916	0.0977	0.1684	0.1077	0.1083
	Omega ratio	0.1076	0.0999	0.2014	0.1199	0.1836	0.1979	0.3812	0.2227	0.2254

This table reports a range of risk-adjusted measures for the out-of-sample asset allocation strategies. Portfolio weights and Sharpe ratios, Sortino ratio, and Omega ratio are reported. Optimal portfolios are obtained based on the out-of-sample period from January 2010 to June 2017. Here vine copula strategies are non-regime dependent.

Table 3.18: **Eight Assets' asset allocation strategies Out-of-sample risk-adjusted performance ignoring regime switching (short sales unconstrained)**

Short sales unconstrained										
		Asset Allocation Strategies								
		Equal weights	Conventional	R-vine Gauss Copula	C-vine t Copula	R-vine t Copula	R-Vine ind. mixed Copula	C-vine ind. mixed Copula	R-vine mixed Copula	C-vine mixed Copula
	S&P US	0.125	0.3900	-0.4863	0.248	0.1380	0.316	0.354	0.4520	-0.484
	MSCI.EM	0.125	0.3520	0.3791	0.430	0.3685	0.334	0.234	0.2960	0.494
	Gov. bond	0.125	0.4940	0.4811	0.478	0.4980	-0.426	-0.408	-0.4135	0.460
Portfolio weights	Real Est	0.125	0.4360	-0.2332	-0.272	0.3760	-0.372	0.204	0.2660	0.470
	Hedge	0.125	-0.4800	0.3039	0.476	-0.4480	0.394	-0.004	-0.4513	-0.434
	Buyout	0.125	-0.2720	0.3938	0.228	0.1889	0.308	-0.104	0.1951	0.338
	VC	0.125	0.3241	-0.3013	-0.206	0.2719	0.286	0.246	0.1960	0.380
	T-bills	0.125	-0.2420	0.4703	-0.374	-0.3976	0.160	0.488	0.4640	-0.218
	Measure									
	Mean	0.0988	0.0929	0.0987	0.1105	0.1375	0.0917	0.1100	0.1317	0.1627
	Standard deviation	0.0931	0.1644	0.0913	0.1069	0.0907	0.303	0.7275	0.7001	0.6338
	CVaR	1.468	1.492	1.472	1.548	1.317	1.972	3.0440	2.8910	2.9780
	Sharpe ratio	0.0376	0.0292	0.0286	0.0559	0.0502	0.0601	0.0746	0.0786	0.0789
	Sortino ratio	0.0514	0.0428	0.0401	0.0749	0.0702	0.0822	0.1115	0.1092	0.1109
	Omega ratio	0.1176	0.1368	0.1487	0.1703	0.1651	0.1798	0.2262	0.2266	0.2245

This table reports a range of risk-adjusted measures for the out-of-sample asset allocation strategies. Portfolio weights and Sharpe ratios, Sortino ratio, and Omega ratio are reported. Optimal portfolios are obtained based on the out-of-sample period from January 2010 to June 2017. Here vine copula strategies are non-regime dependent.

Table 3.19: **Eight Assets' asset allocation strategies Out-of-sample risk-adjusted performance considering regime switching (short sales constrained)**

Short sales constrained							
		Asset Allocation Strategies					
		R-vine t Copula	C-vine t Copula	R-vine ind. mixed Copula	C-vine ind. mixed Copula	R-vine mixed Copula	C-vine mixed Copula
Regime 1							
	S&P US	0.0971	0.1280	0.078	0.1220	0.1280	0.030
	MSCI.EM	0.1623	0.2045	0.082	0.1658	0.1540	0.072
	Gov. bond	0.1409	0.0200	0.410	0.0120	0.0200	0.370
Portfolio weights	Real Est	0.1014	0.0805	0.116	0.0980	0.1120	0.076
	Hedge	0.3044	0.3250	0.2065	0.2188	0.2376	0.280
	Buyout	0.1057	0.0868	0.0760	0.0401	0.1040	0.024
	VC	0.0403	0.0731	0.0120	0.0296	0.0400	0.040
	T-bills	0.0485	0.0880	0.0140	0.3075	0.1980	0.102
	Measure						
	Mean	-0.3164	-0.0313	-0.2634	0.2651	0.2075	0.0946
	Standard deviation	0.6364	1.574	0.7616	0.5659	0.643	0.637
	CVaR	3.145	3.104	2.936	8.416	8.821	3.407
Regime 2							
	S&P US	0.0520	0.056	0.0180	0.026	0.032	0.060
	MSCI.EM	0.0440	0.050	0.0192	0.034	0.022	0.028
	Gov. bond	0.1980	0.312	0.1167	0.162	0.162	0.158
Portfolio weights	Real Est	0.0255	0.080	0.0100	0.044	0.034	0.010
	Hedge	0.2220	0.236	0.3400	0.394	0.412	0.286
	Buyout	0.0360	0.066	0.0240	0.014	0.050	0.036
	VC	0.0533	0.100	0.0260	0.038	0.044	0.028
	T-bills	0.3700	0.102	0.4420	0.286	0.244	0.384
	Measure						
	Mean	0.03176	0.1708	0.1411	0.3359	0.3354	0.2907
	Standard deviation	0.6729	0.4891	0.5997	0.6908	1.426	0.4518
	CVaR	4.272	6.714	4.872	4.200	3.796	3.096
	Sharpe ratio	0.0596	0.0643	0.0732	0.0688	0.0762	0.0827
	Sortino ratio	0.0830	0.0882	0.1030	0.0972	0.1095	0.1191
	Omega ratio	0.1939	0.1943	0.2281	0.2113	0.2336	0.2516

This table reports a range of risk-adjusted measures for the out-of-sample asset allocation strategies. Portfolio weights and Sharpe ratios, Sortino ratio, and Omega ratio are reported. Optimal portfolios are obtained based on the out-of-sample period from January 2010 to June 2017. Here vine copula strategies are regime dependent, such as "C-vine mixed Copula" represents regime switches from R-vine Gaussian to C-vine mixed copula.

Table 3.20: **Eight Assets' asset allocation strategies Out-of-sample risk-adjusted performance considering regime switching (short sales unconstrained)**

Short sales unconstrained							
		Asset Allocation Strategies					
		R-vine t Copula	C-vine t Copula	R-vine ind. mixed Copula	C-vine ind. mixed Copula	R-vine mixed Copula	C-vine mixed Copula
Regime 1							
Portfolio weights	S&P US	-0.498	-0.4868	0.488	0.484	-0.282	-0.460
	MSCI.EM	0.324	0.4873	-0.416	-0.120	0.500	0.498
	Gov. bond	0.454	0.4620	0.446	0.286	0.286	0.312
	Real Est	-0.488	-0.2361	0.422	0.440	0.390	-0.308
	Hedge	0.400	0.1343	0.230	0.478	0.500	0.264
	Buyout	0.382	0.4960	0.438	-0.152	-0.152	0.474
	VC	0.236	-0.1748	-0.474	-0.348	-0.174	-0.134
	T-bills	0.184	0.3181	-0.142	-0.076	-0.064	0.354
	Measure						
	Mean	-0.3263	-0.9001	-0.259	-0.5775	-0.3184	-0.2778
	Standard deviation	0.1581	0.1081	0.1839	0.1644	0.0990	0.2391
	CVaR	4.522	6.087	2.263	3.688	4.66	1.892
Regime 2							
Portfolio weights	S&P US	0.486	0.2765	0.2440	0.1733	0.134	0.4980
	MSCI.EM	-0.438	0.2879	0.2559	0.2297	0.176	-0.4160
	Gov. bond	0.330	-0.4800	-0.4648	-0.4855	-0.262	0.4243
	Real Est	0.224	0.2296	0.2900	0.2320	0.128	-0.2900
	Hedge	0.468	-0.3916	0.4607	0.2198	0.088	0.0240
	Buyout	0.420	0.2920	0.2900	0.2220	0.154	0.2340
	VC	-0.474	0.2997	0.2163	0.1140	0.160	0.1340
	T-bills	-0.024	0.4932	-0.2822	0.3044	0.422	0.3880
	Measure						
	Mean	0.0706	0.1046	0.06044	0.0732	0.089	0.06534
	Standard deviation	0.3059	0.3013	0.3019	0.3201	0.3301	0.3133
	CVaR	1.25	2.082	0.7063	1.045	1.138	1.017
	Sharpe ratio	0.0691	0.0749	0.0772	0.0780	0.0849	0.0877
	Sortino ratio	0.1025	0.1074	0.1106	0.1132	0.1260	0.1319
	Omega ratio	0.2272	0.2259	0.2392	0.2376	0.2536	0.2787

This table reports a range of risk-adjusted measures for the out-of-sample asset allocation strategies. Portfolio weights and Sharpe ratios, Sortino ratio, and Omega ratio are reported. Optimal portfolios are obtained based on the out-of-sample period from January 2010 to June 2017. Here vine copula strategies are regime dependent, such as "C-vine mixed Copula" represents regime switches from R-vine Gaussian to C-vine mixed copula.

also indicate constructing portfolio by both traditional assets and alternative investments by our Markov regime switching vine copula allocation strategy outperform the case just taking into account traditional assets, which document the necessity of adding alternative investments asset class into asset allocation such that obtain more benefits of portfolio diversification in vine copula regime switching allocation strategy framework.

3.7.5 Economic Performance

Table 21 to Table 24 reports three alternative economic metrics across the all considered asset allocation strategies with and without regime switching consideration in short sales constrained and unconstrained case. Specifically, we model portfolio terminal wealth by hypothetically investing \$100 at the start of the out-of-sample periods for each asset allocation strategy. To gauge the amount of trading required to implement each portfolio strategy we also calculate the average turnover requirement and the effect of transaction costs on each portfolio strategy. The average turnover is defined as the average sum of the absolute value of the trades across the N available assets following DeMiguel et al. (2009):

$$Average\ turnover = \frac{1}{T-M}\sum_{t=1}^{T-M}\sum_{j=1}^{N}(|w_{k,j,t+1} - w_{k,j,t^{+}}|)$$

where N is the total number of assets in the portfolio, here we set N=4 and 8 separately. T is total length of the time series, M is the sample period used to parameterize the forecast models, $w_{k,j,t+1}$ is the desired target portfolio weight for asset j at time $t+1$ using strategy k, which we have obtained by various asset strategies in previous section, and $w_{k,j,t^{+}}$ is the counterpart portfolio weight before re-balancing. Similar to DeMiguel et al. (2009), we apply proportional transaction costs of 1 basis point per transaction (as assumed in Balduzzi and Lynch (1999) based on studies of transaction costs by Fleming et al. (1995) for trades on futures contracts on the $S\&P$ 500 index). The turnover quantity defined above can be interpreted as the average percentage of wealth traded in each period. For the benchmark of the equal weights portfolio strategy, we report its absolute turnover, and for all the other strategies, we report their turnover relative to that of the benchmark strategy. This shows that the higher degree of parameterization of vine copula regime switching model leads to performance benefits above the traditional model for larger portfolios.

In particular, we first take a look at the results of short sales constrained case listed in

Table 21 and Table 22. Table 21 reports the four assets and eight assets results when ignoring regime switching, Table 22 lists the results when considering regime switching. For eight assets including alternative investments, C-vine mixed copula strategy produces the largest terminal wealth regardless of whether transaction costs are included or not. R-vine mixed copula strategy is the second best performing strategy irrespective of transaction costs but exhibits much higher turnover requirements compared to C-vine mixed copula strategy. All eight assets terminal wealth of each strategy higher than its corresponding strategy in four asset case.

Then we turn to the case considering regime switching, for eight assets including alternative investments, C-vine mixed copula strategy still displays the largest terminal wealth among all eight asset strategies regardless of whether transaction costs are included or not. R-vine mixed copula is the second best performing strategy irrespective of transaction costs and exhibits much lower turnover requirements comparing to C-vine mixed copula strategy. And it is observed that all eight asset strategy terminal wealth higher than the four assets' when considering the regime switching. In another aspect, when compared with non-regime case, the eight asset vine copula regime switching strategy terminal wealth all suppress its corresponding strategy in non-regime case, which confirm the benefits of taking into account of regime switching. For the four asset case, each strategy produces larger terminal wealth when ignoring regime switching over considering regime switching, which support that employing complex model for simpler portfolio will not bring more portfolio diversification benefits. Regarding average turnover, for both four asset and eight asset case, the high return regime 2 exhibits higher average turnover than the low return regime 1. Though eight asset strategy when considering regime switching displays larger average turnover, as mentioned above, they still produce higher terminal wealth.

Now we turn to investigate the short sales unconstrained case in Table 23 and Table 24. Table 23 reports the four assets and eight assets results when ignoring regime switching, Table 24 lists the results when considering regime switching. Similar to short sale constrained case, for eight assets including alternative investments, C-vine mixed copula strategy produces the largest terminal wealth regardless of whether transaction costs are included or not with and without regime switching consideration. R-vine mixed copula strategy is the second best performing strategy irrespective of transaction costs but exhibits much higher turnover requirements comparing to C-vine mixed copula strategy. All eight assets terminal wealth of each strategy are higher than its corresponding strategy in four asset case.

Now we turn to the case considering regime switching, for eight assets including alternative investments, C-vine mixed copula strategy still displays the largest terminal wealth among all eight asset strategies regardless of whether transaction costs are included or not. R-vine mixed copula is the second best performing strategy irrespective of transaction costs and exhibits much lower turnover requirements compared to C-vine mixed copula strategy. And it is observed that all eight asset strategy terminal wealth higher than the four assets' when considering the regime switching. In another aspect, when compared with non-regime case, the eight asset vine copula regime switching strategy terminal wealth all suppress its corresponding strategy in non-regime case, which confirm the benefits of taking into account of regime switching. For the four asset case, each strategy produces larger terminal wealth when ignoring regime switching over considering regime switching, which support that employing complex model for simpler portfolio will not bring more portfolio diversification benefits. With respect to average turnover, a little different from short sales constrained case, for both four asset and eight asset case, the high return regime 2 not consistently exhibits higher average turnover than the low return regime 1. What attracts our most notification is that, vine copula regime switching strategy exhibits lower average turnover compared to non-regime dependent strategy, whereas produce higher terminal wealth, which substantially demonstrate that, among all cases, our Markov regime switching C-vine mixed copula asset allocation strategy perform best in all aspects in short sale unconstrained case when considering regime switching.

Figure 3 and Figure 4 separately shows end of the sample period (June 2017) cumulative returns obtained from various regime switching vine copula asset allocation strategy in short sales constrained and unconstrained case. Observed from the figures, whether in short sales constrained and unconstrained case, C-vine mixed copula and R-vine mixed copula regime switching strategy perform similarly with other portfolio strategy from January 2010 till October 2008, these two regime switching vine copula strategies do not substantially exceed other strategies. However, beyond October 2008, around global financial crisis period, regime switching C-vine mixed copula strategy start to outperform other competing strategies. For the end of the sample period (June 2017) cumulative

Table 3.21: **Economic measures of asset allocation strategies out-of-sample performance ignoring regime switching (short sales constrained)**

Economic metric	Method	Four Traditional Assets without alternative investments	Eight Assets including alternative investmentss
Terminal wealth exc. transaction cost	Equal weights	122.9080	135.5517
	Conventional	64.3284	115.4229
	R-Vine Gauss Copula	109.3096	291.5669
	R-Vine t Copula	115.7464	203.764
	C-Vine t Copula	221.8739	298.9115
	R-Vine ind. mixed Copula	240.1205	304.8535
	C-Vine ind. mixed Copula	272.5492	318.0968
	R-Vine mixed Copula	231.2009	341.1287
	C-Vine mixed Copula	300.3711	360.6844
Terminal wealth inc. transaction cost	Equal weights	111.9043	114.7011
	Conventional	59.7651	99.9987
	R-Vine Gauss Copula	98.0203	281.4413
	R-Vine t Copula	102.8899	187.0076
	C-Vine t Copula	200.3465	290.1000
	R-Vine ind. mixed Copula	221.2305	292.3747
	C-Vine ind. mixed Copula	264.6756	303.5631
	R-Vine mixed Copula	209.0340	328.6612
	C-Vine mixed Copula	283.0872	343.3451
Average turnover	Equal weights		
	Conventional	1.042	0.6596
	R-Vine Gauss Copula	0.378	0.872
	R-Vine t Copula	1.42	0.608
	C-Vine t Copula	0.3637	0.3438
	R-Vine ind. mixed Copula	1.322	0.53
	C-Vine ind. mixed Copula	1.21	0.3721
	R-Vine mixed Copula	1.288	0.8217
	C-Vine mixed Copula	0.246	0.4829

This table shows the hypothetical terminal wealth generated by each assets allocation strategy. Terminal wealth is modeled as the final portfolio value (either excluding or including transaction costs) assuming an initial investment \$100 at the start of the out-of-sample period for each strategy. The turnover required to implement each strategy is also reported and can be interpreted as the average percentage of portfolio wealth traded in each period. The final portfolio value including transaction costs assumes transaction costs of 1 bps per transaction. Equal weights strategy set as benchmark case. Here vine copula strategies are non-regime dependent.

Table 3.22: **Economic measures of asset allocation strategies out-of-sample performance ignoring regime switching (short sales unconstrained)**

Economic metric	Method	Four Traditional Assets without alternative investments	Eight Assets including alternative investments
Terminal wealth exc. transaction cost	Equal weights	125.0641	143.7958
	Conventional	116.2608	73.3539
	R-Vine Gauss Copula	99.6333	69.0059
	R-Vine t Copula	114.4627	286.1966
	C-Vine t Copula	225.5948	227.2484
	R-Vine ind. mixed Copula	243.1151	295.8179
	C-Vine ind. mixed Copula	320.1745	330.9229
	R-Vine mixed Copula	325.7187	387.6978
	C-Vine mixed Copula	344.7119	395.1645
Terminal wealth inc. transaction cost	Equal weights	107.8899	129.4383
	Conventional	102.7832	68.7602
	R-Vine Gauss Copula	91.9984	61.4567
	R-Vine t Copula	103.2278	269.6785
	C-Vine t Copula	213.0968	216.3325
	R-Vine ind. mixed Copula	234.5183	278.4989
	C-Vine ind. mixed Copula	304.2716	314.8116
	R-Vine mixed Copula	308.9765	365.8476
	C-Vine mixed Copula	321.8645	383.2234
Average turnover	Equal weights		
	Conventional	0.376	2.799
	R-Vine Gauss Copula	0.64	2.462
	R-Vine t Copula	0.812	2.1869
	C-Vine t Copula	0.77	1.792
	R-Vine ind. mixed Copula	0.5637	2.2339
	C-Vine ind. mixed Copula	0.798	3.028
	R-Vine mixed Copula	0.532	2.7401
	C-Vine mixed Copula	0.674	2.096

This table shows the hypothetical terminal wealth generated by each assets allocation strategy. Terminal wealth is modeled as the final portfolio value (either excluding or including transaction costs) assuming an initial investment $100 at the start of the out-of-sample period for each strategy. The turnover required to implement each strategy is also reported and can be interpreted as the average percentage of portfolio wealth traded in each period. The final portfolio value including transaction costs assumes transaction costs of 1 bps per transaction. Equal weights strategy set as benchmark case. Here vine copula strategies are non-regime dependent.

Table 3.23: **Economic measures of asset allocation strategies out-of-sample performance considering regime switching (short sales constrained)**

Economic metric	Method	Four Traditional Assets without alternative investments	Eight Assets including alternative investmentss
Terminal wealth exc. transaction cost	R-Vine t Copula	108.2812	291.7141
	C-Vine t Copula	69.6705	303.7409
	R-Vine ind. mixed Copula	186.2246	327.7284
	C-Vine ind. mixed Copula	198.5752	306.3545
	R-Vine mixed Copula	207.6511	343.9701
	C-Vine mixed Copula	225.5948	416.8074
Terminal wealth inc. transaction cost	R-Vine t Copula	97.6633	279.3322
	C-Vine t Copula	64.1313	291.8832
	R-Vine ind. mixed Copula	173.8878	315.0743
	C-Vine ind. mixed Copula	184.8438	292.0001
	R-Vine mixed Copula	192.7649	332.7893
	C-Vine mixed Copula	215.0043	400.9989
Average turnover			
Regime 1	R-Vine t Copula	1.146	0.806
	C-Vine t Copula	0.736	0.7385
	R-Vine ind. mixed Copula	0.5512	0.4646
	C-Vine ind. mixed Copula	0.97	0.4416
	R-Vine mixed Copula	0.346	0.6404
	C-Vine mixed Copula	0.5758	0.5591
Regime 2	R-Vine t Copula	1.0072	0.916
	C-Vine t Copula	0.7881	1.0681
	R-Vine ind. mixed Copula	0.576	0.8292
	C-Vine ind. mixed Copula	1.034	0.886
	R-Vine mixed Copula	0.564	0.936
	C-Vine mixed Copula	0.8851	0.594

This table shows the hypothetical terminal wealth generated by each assets allocation strategy. Terminal wealth is modeled as the final portfolio value (either excluding or including transaction costs) assuming an initial investment \$100 at the start of the out-of-sample period for each strategy. The turnover required to implement each strategy is also reported and can be interpreted as the average percentage of portfolio wealth traded in each period. The final portfolio value including transaction costs assumes transaction costs of 1 bps per transaction. R-vine Gaussian copula strategy set as benchmark case. Here vine copula strategies are regime dependent, such as ”C-vine mixed Copula” represents regime switches from R-vine Gaussian to C-vine mixed copula.

Table 3.24: **Economic measures of asset allocation strategies out-of-sample performance considering regime switching (short sales unconstrained)**

Economic metric	Method	Four Traditional Assets without alternative investments	Eight Assets including alternative investmentss
Terminal wealth exc. transaction cost	R-Vine t Copula	20.4222	314.7465
	C-Vine t Copula	67.3861	340.5760
	R-Vine ind. mixed Copula	116.2608	351.0663
	C-Vine ind. mixed Copula	116.7101	386.4857
	R-Vine mixed Copula	261.1401	508.3595
	C-Vine mixed Copula	263.1456	560.8195
Terminal wealth inc. transaction cost	R-Vine t Copula	16.2708	302.8874
	C-Vine t Copula	55.8848	323.1187
	R-Vine ind. mixed Copula	103.7788	338.3390
	C-Vine ind. mixed Copula	104.5532	373.8736
	R-Vine mixed Copula	247.0954	492.3340
	C-Vine mixed Copula	251.7072	543.8866
Average turnover			
Regime 1	R-Vine t Copula	1.01	2.466
	C-Vine t Copula	0.808	2.554
	R-Vine ind. mixed Copula	0.388	2.806
	C-Vine ind. mixed Copula	0.504	2.348
	R-Vine mixed Copula	0.4294	2.384
	C-Vine mixed Copula	1.198	2.5454
Regime 2	R-Vine t Copula	0.638	2.614
	C-Vine t Copula	0.7881	2.1103
	R-Vine ind. mixed Copula	0.576	2.0039
	C-Vine ind. mixed Copula	1.416	0.848
	R-Vine mixed Copula	0.548	1.2527
	C-Vine mixed Copula	0.288	2.2505

This table shows the hypothetical terminal wealth generated by each assets allocation strategy. Terminal wealth is modeled as the final portfolio value (either excluding or including transaction costs) assuming an initial investment $100 at the start of the out-of-sample period for each strategy. The turnover required to implement each strategy is also reported and can be interpreted as the average percentage of portfolio wealth traded in each period. The final portfolio value including transaction costs assumes transaction costs of 1 bps per transaction. R-vine Gaussian copula strategy set as benchmark case. Here vine copula strategies are regime dependent, such as ”C-vine mixed Copula” represents regime switches from R-vine Gaussian to C-vine mixed copula.

returns, it is observed that regime switching C-vine mixed copula strategy substantially exceed other vine copula strategy, conventional strategy and equal weights strategy.

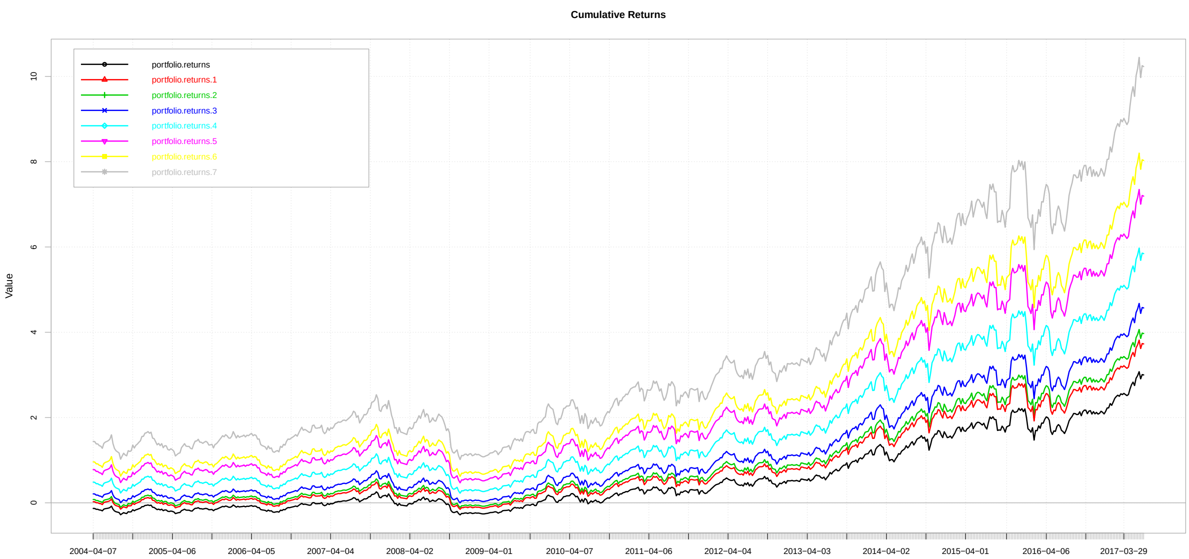


Figure 3.3: **Regime Switching Copula Model Cumulative Returns (short sales constrained)**

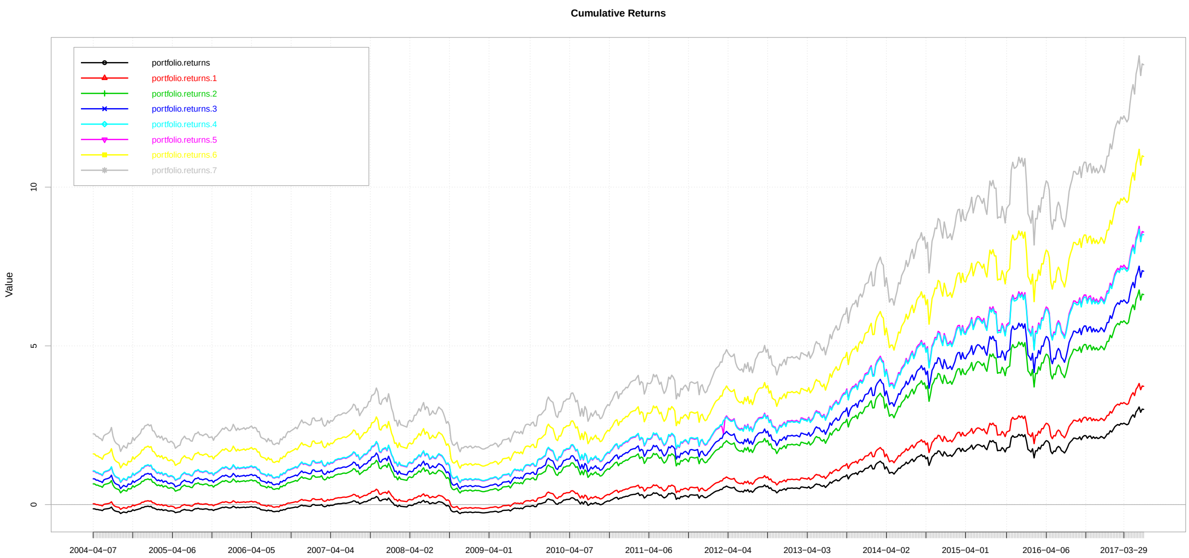


Figure 3.4: **Regime Switching Copula Model Cumulative Returns (short sales unconstrained)**

[3]portfolio.returns is obtained by C-Vine mixed Copula Strategy, portfolio.returns.1 is obtained by R-Vine mixed Copula Strategy, portfolio.returns.2 is obtained by C-Vine ind. mixed Copula Strategy, portfolio.returns.3 is obtained by R-Vine ind. mixed Copula Strategy, portfolio.returns.4 is obtained by R-Vine t Copula Strategy, portfolio.returns.5 is obtained by C-Vine t Copula Strategy, portfolio.returns.6 is obtained by R-Vine Gaussian Copula Strategy, portfolio.returns.7 is obtained by Conventional Strategy.

3.8 Value-at-Risk (VaR) Backtesting

Our backtesting aims to examine whether the VaR estimates coming from competing portfolio strategy model satisfy the appropriate theoretical statistical properties. We forecast the one-day-ahead VaR of portfolios constructed by different portfolio strategy models. The forecasting period for the portfolio corresponds to the period from January 2010 to June 2017. The 1-day-ahead VaR is an α-quantile prediction of the future portfolio profit and loss (P/L) distribution. It provides a measure of the maximum future losses over a time span [t, t+1], which can be formalized as

$$P[R_{t+1} \leq VaR^{\alpha}_{t+1} | I_t] = \alpha$$

where R_{t+1} denotes the portfolio return on day $t+1$; and I_t is the information set available on day t. The nominal coverage $0 < \alpha < 1$ is typically set at 0.01 or 0.05 for long trading positions (i.e., left tail) meaning that the risk manager seeks a high degree of statistical confidence, 99% and 95%, respectively, that the portfolio loss on trading day $t+1$ will not exceed the VaR extracted from information up to day t.

A well-specified VaR model should produce statistically meaningful VaR forecasts. In this sense, the exceedance proportion should approximately equal the VaR confidence level (unconditional coverage) whereas the exceedances should be independent instead of occurring in clusters. Specifically, a well-specified model should produce low VaR forecasts in period of low volatility and high VaR forecasts when volatility is high, which means exceedances are spread over the entire sample period instead of in clusters. A model failing to capture the volatility dynamics of the underlying return distribution will lead to a clustering of failures, though it can provide correct unconditional coverage. Therefore, conditional coverage, which takes both of above properties, is necessary to be included in backtesing. Therefore, we employ the Kupiec (1995) unconditional coverage test, the conditional coverage test by Christoffersen (1998), which are very popular in the literature for testing the above two properties.

Kupiec (1995) test for unconditional coverage (LR_{uc})

The popular failure rates test, unconditional coverage test, proposed by Kupiec (1995) is a kind of likelihood-ratio test, which measures whether the number of exceedances is in line with the confidence level. Since the null hypothesis is set as the model is well-specified, the number of exceedances should follow binomial distribution. Thus the spirit of the unconditional coverage test aim to examine whether the observed failure rate $\hat{\pi}$ is significantly different from α, the failure rate implied by the confidence level. The likelihood-ratio test statistic is given by

$$LR_{uc} = -2log[\frac{\alpha^{n_1}(1-\alpha)^{n_0}}{\hat{\pi}^{n_1}(1-\hat{\pi})^{n_0}}] \sim \chi^2_{(1)},$$

where n_1 is the number of exceedances, n_0 is the number of non-exceedances, α is the confidence level at which VaR measures are estimated and $\hat{\pi} = n_1/(n_0 + n_1)$ is the MLE estimate of α. Under the null hypothesis, LR_{uc} is asymptotically χ^2 distributed with one degree of freedom.

Unconditional coverage test exhibits several drawbacks. This test only take into account the frequency of exceedances whereas ignore the time exceedances occur, which will lead to the failure of rejecting a model which suffers from exceedances cluster. Additionally, unconditional coverage test is statistically real with sample sizes in line with the current regulatory framework (one year). Therefore, backtesting entirely rely on unconditional coverage will lead to inaccurate conclusion, clustered exceedances should also be considered.

Christoffersen (1998) test for conditional coverage (LR_{cc})

Christoffersen (1998) proposed conditional coverage test to overcome the drawbacks of unconditional coverage test, which jointly examines whether the total number of exceedances is consistent with the expected one, and whether the VaR failures are independently distributed. The test is carried out, given the realisation of return series r_t and the ex-ante VaR for a $\alpha\%$ coverage rate by first defining an indicator function $I_t + 1$ that gets the value of 1 if a VaR violation occurs and 0 otherwise.

Under the null hypothesis that model is well specified, an exception today should not depend on whether or not an exception occurred on previous day and the total number of exceedances should be consistent with the confidence level. Combine this test statistic of independence (LR_{ind}) with Kupiec's unconditional coverage test statistic (LR_{uc}), we obtain a conditional coverage test $LR_{cc} = LR_{uc} + LR_{ind}$ that jointly test the correct VaR failures and the independence of the exceedances. The LR_{cc} test statistic for the correct conditional coverage is given by

$$LR_{cc} = -2log[\frac{(1-\alpha)^{n_0}\alpha^{n_1}}{(1-\hat{\pi}_{01})^{n_{00}}\hat{\pi}_{01}^{n_{01}}(1-\hat{\pi}_{11})^{n_{10}}\hat{\pi}_{11}^{n_{11}}}] \sim \chi^2_{(2)},$$

where n_{ij} is the number of i values followed by a j value in the I_{t+1} series $(i, j = 0, 1)$, $\pi_{ij} = Pr\{I_{t+1} = i|I_t = j\}, (i, j = 0, 1)$, $\hat{\pi}_{01} = n_{01}/(n_{00} + n_{01})$, $\hat{\pi}_{11} = n_{11}/(n_{00} + n_{01})$. LR_{cc} follows χ^2 distributed with two degrees of freedom. The above tests are employed for detecting misspecified risk models when the temporal dependence in the sequence of VaR violations is a simple first-order Markov structure.

Table 25 and Table 26 separately show the performance of our various asset allocation strategies with and without regime switching consideration using a range of VaR backtesting at the 1% level, similar to Basel (2004) requirements. During each out-of-sample period, a VaR violation is recorded when the portfolio strategy return is less than the 1% VaR value of the total forecast return series for all constituent assets within the portfolio (Christoffersen (2012)). Lower unconditional coverage test and conditional coverage test p-values are indicative that the proposed portfolio management strategy systematically understates or overstates the portfolio's underlying level of risk. Therefore, a superior strategy results in a higher p-value of test statistic. Following Christoffersen (2012), we report the p-values for these test statistics where the null hypothesis is that the portfolio management model is correct on average.

We first take a look at the case ignoring regime switching, non-regime dependent C-vine mixed copula and R-vine mixed copula strategy are the best performing models in eight assets case. At eight asset case, C-vine mixed copula strategy exhibits a substantial performance improvement for unconditional coverage test compared to conventional strategy and equal weights strategy. For four assets case, p-value of unconditional coverage test statistic indicates similar performance among vine copula strategy and conventional and equal weights strategy. However, when we take into account independence property of VaR backtesting using conditional coverage test, C-vine mixed copula and R-vine mixed copula exhibits superior performance compared to conventional strategy and equal weights strategy. Therefore, incorporation of return asymmetry in forecasting improves the independence property as the likelihood of having a sequence of VaR violations is reduced. We also observe that when accounting for independence property, the performance of conventional strategy and equal weights strategy deteriorates from four assets to eight assets case, which again support that, in low dimension portfolio, it is not necessary to adopt complex dependence model.

Then we turn to the case taking into account regime switching, regime switching R-vine mixed copula and regime switching C-vine mixed copula strategy are the best performing models in eight assets case. At eight asset case, regime switching C-vine mixed copula strategy exhibits a substantial performance improvement for unconditional coverage test compared to conventional strategy and equal weights strategy. For four assets case, p-value of unconditional coverage test statistic still lower than their corresponding strategy of eight assets, which indicates underperformance of four assets regime switching vine copula strategy comparing to eight assets strategy in unconditional coverage test. When we account for independence property of VaR backtesting using conditional coverage test, regime switching C-vine mixed copula and regime switching R-vine mixed copula exhibits superior performance compared to C-vine mixed copula and R-vine mixed copula in non-regime dependent case, conventional strategy and equal weights strategy.

Therefore, incorporation of both return asymmetry and regime switching in forecasting improves the independence property as the likelihood of having a sequence of VaR violations is reduced. Similar to non-regime dependent case, we again observe that when taking into account independence property, the performance of conventional strategy and equal weights strategy deteriorates from four assets to eight assets case, which support that in low dimension portfolio, it is not necessary to adopt complex dependence and regime switching model.

In general, the VaR backtesting results are consistent with our regime switching vine copula out-of-sample portfolio performance results, and support our findings and conclusions in previous section that regime switching C-vine mixed copula portfolio strategy improve portfolio performance as there is reduced frequency of exceedances and increased independence of VaR violations in eight assets case which including alternative investment asset classes.

3.9 Conclusion

Given the importance of alternative investments as an investment vehicle for investors to gain portfolio diversification benefits, and as traditional mean-variance portfolio strategy

Table 3.25: **Value-at-Risk (VaR) backtesting of asset allocation strategies ignoring regime switching**

VaR backtest metric	Method	Four Assets (short sales constrained)	Four Assets(short sales unconstrained)	Eight Assets (short sales constrained)	Eight Assets(short sales unconstrained)
Unconditional coverage test	Equal weights	0.353	0.353	0.353	0.353
	Conventional	0.353	0.353	0.353	0.353
	R-Vine Gauss	0.652	0.652	0.446	0.274
	R-Vine t	0.785	0.155	0.553	0.801
	C-Vine t	0.652	0.938	0.320	0.652
	R-Vine ind mixed	0.801	0.039	0.208	0.672
	C-Vine ind mixed	0.785	0.923	0.274	0.446
	R-Vine mixed	0.057	0.652	0.553	0.652
	C-Vine mixed	0.652	0.801	0.155	0.633
Conditional coverage test	Equal weights	0.166	0.166	0.585	0.585
	Conventional	0.166	0.166	0.585	0.585
	R-Vine Gauss	0.461	0.461	0.740	0.524
	R-Vine t	0.544	0.356	0.569	0.965
	C-Vine t	0.461	0.800	0.596	0.826
	R-Vine ind mixed	0.703	0.104	0.434	0.659
	C-Vine ind mixed	0.544	0.963	0.524	0.475
	R-Vine mixed	0.057	0.820	0.569	0.826
	C-Vine mixed	0.820	0.965	0.327	0.633
Actual %	Equal weights	5.8%	5.8%	5.8%	5.8%
	Conventional	5.8%	5.8%	5.8%	5.8%
	R-Vine Gauss	4.6%	4.6%	5.6%	5.9%
	R-Vine t	4.8%	6.2%	5.2%	5.2%
	C-Vine t	4.6%	5.1%	4.6%	4.6%
	R-Vine ind mixed	5.2%	6.8%	6.1%	5.4%
	C-Vine ind mixed	4.8%	4.9%	5.6%	5.6%
	R-Vine mixed	6.7%	4.6%	6.1%	6.1%
	C-Vine mixed	4.6%	5.2%	6.2%	6.2%

This table reports VaR backtesting performed at the 1% level. The Unconditional coverage test (Kupiec (1995)) measures only unconditional coverage. The conditional coverage test (Christoffersen (2012)) is a simultaneous test of both the unconditional coverage and independence properties of VaR violations. The actuality percentage are also reported. Here vine copula strategies are non-regime dependent.

Table 3.26: **Value-at-Risk (VaR) backtesting of asset allocation strategies considering regime switching**

VaR backtest metric	Method	Four Assets (short sales constrained)	Four Assets(short sales unconstrained)	Eight Assets (short sales constrained)	Eight Assets(short sales unconstrained)
Unconditional coverage test	R-Vine t	0.353	0.785	0.446	0.553
	C-Vine t	0.672	0.353	0.785	0.553
	R-Vine ind mixed	0.353	0.353	0.353	0.353
	C-Vine ind mixed	0.274	0.274	0.353	0.353
	R-Vine mixed	0.353	0.353	0.274	0.446
	C-Vine mixed	0.274	0.274	0.353	0.353
Conditional coverage test	R-Vine t	0.166	0.062	0.646	0.689
	C-Vine t	0.139	0.166	0.544	0.689
	R-Vine ind mixed	0.166	0.166	0.585	0.585
	C-Vine ind mixed	0.164	0.164	0.585	0.585
	R-Vine mixed	0.166	0.166	0.512	0.646
	C-Vine mixed	0.164	0.061	0.585	0.585
Actual %	R-Vine t	5.8%	4.8%	5.6%	5.5%
	C-Vine t	5.4%	5.8%	4.8%	5.5%
	R-Vine ind mixed	5.8%	5.8%	5.8%	5.8%
	C-Vine ind mixed	5.9%	5.9%	5.8%	5.8%
	R-Vine mixed	5.8%	5.8%	5.9%	5.6%
	C-Vine mixed	5.9%	5.9%	5.8%	5.8%

This table reports VaR backtesting performed at the 1% level. The Unconditional coverage test (Kupiec (1995)) measures only unconditional coverage. The conditional coverage test (Christoffersen (2012)) is a simultaneous test of both the unconditional coverage and independence properties of VaR violations. The actuality percentage are also reported. Here vine copula strategies are regime dependent, such as ”C-vine mixed Copula” represents regime switches from R-vine Gaussian to C-vine mixed copula.

does not account for asymmetry in returns distributions, it is quite plausible that there is a need for more advanced portfolio management strategies that incorporate asymmetries especially when market regime changes over time. Therefore, our paper introduces a Markov regime switching regular vine copula asset allocation model in international assets markets and focuses on investigating, as the presence of regimes, whether the regime switching vine copula model is able to produce superior investment performance in the multi-asset case which including alternative investments compared to traditional models.

We find evidence of regimes detected in international asset markets through the selection the best-fitting regime switching vine copula model. The asymmetric dependence between the various classes of asset returns are higher in the bear market regime than in the bull market regime. The risk-return characteristics for the optimal portfolio in the bear market regime is different from those of the portfolio in the bull market regime, which requires different portfolio construction strategy dependent on market regime.

We first explore the efficient frontiers produced by four traditional asset and eight assets including alternative investment, the higher Sharpe ratio of eight assets exhibits better portfolio diversification when incorporate alternative investment. Then we primarily compare several competing Markov regime switching regular vine copula model. Flexible vine tree structure and asymmetric dependence bivariate copula as building blocks make regime switching C-vine mixed and R-vine mixed copula to be the best fitting model. Subsequent out-of-sample portfolio performance from each model in a long-run multi-period investment support the superiority of regime switching vine copula model in improving portfolio diversification benefits across a broad range of metrics. Despite the regime switching C-vine mixed copula strategy having high turnover requirements, even when transaction costs are incorporated, it still exhibits greater economic benefits relative to the other competing strategies. Regime switching C-vine mixed copula strategy also exhibits the best performance when a series of Value-at-Risk (VaR) backtests are applied to four asset and eight asset portfolios. The p-value of VaR of C-vine mixed copula are substantially higher than for the R-vine t, R-vine Gauss copula and conventional models, which implies that using the latter models can lead to underestimating the risk of a portfolio and affect the portfolio performance.

Accordingly, we can draw the conclusion that regime switching C-vine mixed copula strategy bring more portfolio diversification benefits when managing multi-asset high dimension portfolio due to their ability to better capture asymmetries within the dependence structure as the presence of regime than other traditional multivariate Gaussian models.

Table 27: **An integer defining the bivariate copula family**

0 = independence copula
1 = Gaussian copula
2 = Student t copula (t-copula)
3 = Clayton copula
4 = Gumbel copula
5 = Frank copula
6 = Joe copula
7 = BB1 copula
8 = BB6 copula
9 = BB7 copula
10 = BB8 copula
13 = rotated Clayton copula (180 degrees; survival Clayton)
14 = rotated Gumbel copula (180 degrees; survival Gumbel)
16 = rotated Joe copula (180 degrees; survival Joe)
17 = rotated BB1 copula (180 degrees; survival BB1)
18 = rotated BB6 copula (180 degrees; survival BB6)
19 = rotated BB7 copula (180 degrees; survival BB7)
20 = rotated BB8 copula (180 degrees; survival BB8)
23 = rotated Clayton copula (90 degrees)
24 = rotated Gumbel copula (90 degrees)
26 = rotated Joe copula (90 degrees)
27 = rotated BB1 copula (90 degrees)
28 = rotated BB6 copula (90 degrees)
29 = rotated BB7 copula (90 degrees)
30 = rotated BB8 copula (90 degrees)
33 = rotated Clayton copula (270 degrees)
34 = rotated Gumbel copula (270 degrees)
36 = rotated Joe copula (270 degrees)
37 = rotated BB1 copula (270 degrees)
38 = rotated BB6 copula (270 degrees)
39 = rotated BB7 copula (270 degrees)
40 = rotated BB8 copula (270 degrees)
104 = Tawn type 1 copula
114 = rotated Tawn type 1 copula (180 degrees)
124 = rotated Tawn type 1 copula (90 degrees)
134 = rotated Tawn type 1 copula (270 degrees)
204 = Tawn type 2 copula
214 = rotated Tawn type 2 copula (180 degrees)
224 = rotated Tawn type 2 copula (90 degrees)
234 = rotated Tawn type 2 copula (270 degrees)

Note: This table lists all bivariate copula families we employ as Vine copula building blocks.

Chapter 4

Modelling International Financial Contagion: Generalised Autoregressive Score Regular Vine Copula Method

Abstract

This paper explores the cross-market dependence between six popular equity indices (S&P 500, NASDAQ 100, FTSE 100, DAX 30, Euro Stoxx 50 and Nikkei 225), and their corresponding volatility indices (VIX, VXN, VFTSE, VDAX, VSTOXX and VXJ). In particular, we propose a novel dynamic method that combine the Generalised Autoregressive Score (GAS) Method with high dimension R-vine copula approach which is able to capture the time-varying tail dependence coefficient (TDC) of index returns. Our empirical findings demonstrate the existence of international financial contagion and significant asymmetric tail dependence in some major international equity markets. Although in some cases contagion cannot be clearly detected by stock index movements, it can be captured by dependence of volatility indices. The results imply that contagion is not only reflected in the first moment of index returns, but also the second moment, the volatility indices. Results also indicate that dependence of volatility indices tend to be influenced by financial shocks and reflects the instantaneous information faster than the stock market indices. At last, our backtesting test prove that the forecast ability of our dynamic GAS R-vine method outperform Gaussian-DCC, t-DCC and static R-vine copula method.

Keywords: Financial contagion, Asymmetric dependence, Financial crisis, Vine copula, Volatility index, Generalised Autoregressive Score

4.1 Introduction

In the last 20 years, global financial market have experienced a series of financial crises, such as the Tequila effect in Mexico in 1994, the Asian Flu in 1997, the Russian default in 1998, the Brazilian Sneeze in 1999, the Nasdaq fall in 2000, the Argentine crisis in 2001, the subprime crisis, which began in 2007 and developed into the global financial crisis after the collapse of Lehman Brothers bank in September of 2008, and the Euro crisis in 2011. Normally, the phenomenon that a financial crisis occurs in one country will then spreads to other countries is known as financial contagion, which has been one of the most studied issues in international finance.

In the literatures, there are several different definitions and measure methods of contagion (see Forbes and Rigobon (2002) and Pelletier (2006)). A well-known definition proposed by Ciccarelli and Rebucci (2007) considers that, following a shock (crises) in one or more markets, contagion occurs when there is a change or shift in the cross-market linkages. According to this definition, we should pay attention to several aspects, which including the presence of a crisis, the movements or changes of the dependence linkages and the measure method of linkages. A simple method to measure a linkage is through correlation. Forbes and Rigobon (2002) defined contagion as a scenario that occurs between two or more markets when the correlation between them increases after a crisis event. In this sense, increase in the correlation can only be regarded as contagion if a crisis occurs; otherwise, this increase only demonstrate the deepen of financial integration between underlying markets. They also found that the increase in the dependence during turmoil periods could results from the increase in the volatility of the markets instead of a shock. Therefore, the evidence for contagion is unreliable when the model ignoring heteroscedasticity. They did not find any evidence of contagion in the major countries that were analysed. However, stock markets and volatility indices markets tend to show comprehensive linkages and interdependence which might be better described by nonlinear dependence measure rather than linear ones. In this context, Rodriguez (2007), Chen and Poon (2007), Arakelian and Dellaportas (2012) addressed the time varying dependence between stock markets by adopting the copula method. Contrary to Forbes and Rigobon (2002), their study demonstrate evidence of contagion.

Copulas has been considered and used for modelling multivariate financial time series since twenty years ago. Copulas have been extensively applied to financial contagion and also financial risk management (see Giacomini et al. (2009)), portfolio management and option pricing (see Cherubini et al. (2004)). In the copula framework, according to Sklar (1959) theorem, the density of a multivariate time series of financial returns is expressed as the product of its marginal (univariate) densities and a copula function, which is able to capture all the dependence between the financial returns. One superiority of copula method is that several dependence measures can be derived from copula function. Two most widely employed dependence measures in finance are tail dependence coefficients, normally including lower tail dependence and upper tail dependence, which describe the comovements of extreme losses and extreme gains respectively; and correlation of Kendall's τ.

High dimension vine copula is able to capture the extreme comovements (tail dependence) that a simple linear correlation and traditional bivariate copula fails to model. Recently, Patton (2006b) Patton (2006a) propose a dynamic copula approach combined with other evaluation models to measure market dependence. Xu and Li (2009) adopt three kinds of Archimedean copulas to estimate tail dependence across three Asian future markets; other researchers employed other copula approaches to explore relationships between financial markets, while they mainly concentrates on equity indices (e.g., Hu (2006) Hu (2010); Nikoloulopoulos et al. (2012); Rodriguez (2007)). Ammann and Süss (2009) proposed to study the dependence between equity indices and their corresponding volatility indices. However, to the best of our knowledge, there is so far no literature on measuring cross-market volatility indices with vine copula models. Individually chosen bivariate copula as building blocks from a plenty of candidates, our vine copula is able to provide more flexibility in modelling asymmetric tail dependence compared with the bivariate copula method suggested by Patton (2006b) and Ammann and Süss (2009).

In this paper, we model the stock market and corresponding volatility indices market dependence and assess financial contagion. Rather than only focusing on stock market, we also investigate the volatility indices market. The development of volatility indices motivates the research on investigating the relationships between volatility indices across different index markets, such as volatility spillover and market integration (Nikkinen and Sahlström (2004); Äijö (2008)). However, the dependence between different volatility indices has not been previously discussed in the literature. Our study aim to investigate the dependence of volatility indices in the US, European and Japanese index markets. It is well known that the rate of change of market volatility far exceeds the rate of change of market return. As a consequence, cross-market volatilities are probably to reflect the dynamics of market interdependence much effective than stock market returns. In addition, considering the volatility index as a proxy for the second moment of returns owing to implied volatility better reflects the investors' expectation on future market volatility, and it reflects more market information comparing to realised volatility and model based volatility (e.g., Fleming et al. (1995); Blair et al. (2010); Giot (2005)). Comparing to the stock indices return, volatility index return exhibits the characteristics, such as non-Gaussian, much higher volatility and significant asymmetry (Low (2004)). Taking into account above characteristics, we employ a novel vine copula method which can flexibly capture asymmetric dependence and tail dependence between variables. In another aspect, a well-known stylized fact of multivariate financial time series is the time varying distributions, which means the dependence structure between the time series also naturally evolves with time. Therefore, we combine the dynamic generalised autoregressive score model and regular vine copula method (Aas et al. (2009)) to estimate the bivariate time-varying dependence among the stock markets and corresponding volatility indices market. The pair copula decomposition has received much attention because of its flexibility in defining higher-dimension copula models; see, for example, Joe and Kurowicka (2011) for developments on this subject. Financial contagion, in our study, is defined as an increase in dependence following a crisis, and two measures of dependence are used, which are time varying correlations and time varying tail dependence coefficients introduced by Patton (2006b) together with conditional copula.

The remainder of the paper is structured as follows, section 2 provides a literature review on financial contagion and cross-market dependence. Section 3 introduces the vine copula methodology, section 4 discusses the data and the empirical results, and section 5 concludes.

4.2 Literature Review

Research on measuring the cross-market dependence, and the dependence between stock market and financial contagion began since 30 years ago. A number of different conclusions are drawn based on various methodologies and data. The markets investigated cover not only international stock market, also across exchange market (Bollerslev (1990)), bond market (Loretan and English (2000)), spot-future market(Fung et al. (2005)) as well as output growth rates.

Financial contagion is a widely defined as the shock or spillover effect between countries, especially in financial crisis period. It means the dependence probably will change significantly in financial turmoil, which leads to dependence crash. Claessens and Forbes (2001) conducted an estimation of the correlation of international markets when financial contagion occurs. Whether the financial contagion exists or not is controversial since the beginning of this research. Despite there formed some common outlook (Koch and Koch (1991)), the existence of financial contagion was again overturned by the end of 1990s (Forbes and Rigobon (2002)). Nevertheless, some evidence supporting the financial contagion were found subsequently in certain markets during some periods since the beginning of 2000s until now (Rigobon (2003)). Despite all this, consensus are still not agreed in academy that whether indeed the correlation crash in the period of financial crisis.

There have been some study focusing on the non-linearity of cross-market correlation during the financial turmoil (see Boyer et al. (1997)). The application of different statistical models in this area, such as multi-variable generalized autoregressive conditional heteroskedasticity (MGARCH) model or Markov switching method are discussed (Bollerslev (1990); Tse (2002); Pelletier (2006)).

Moreover, international financial markets also exhibit asymmetric dependence structures. Their correlation is higher in a bear market and lower in a bull market. Ignoring the stylised fact of asymmetry in financial markets will results in an underestimation of the lower tail risk, and then leads to suboptimal international diversification benefits. Therefore, the specification of an appropriate model to capture asymmetric cross-market dependence is crucial to risk management of international portfolio. So far, the asymmetric dependence in many different financial areas has been investigated for both developed and emerging markets. These financial market areas cover stock and stock indices (Longin and Solnik (2001); Hu (2010); Hu (2006); Nikoloulopoulos et al. (2012); Rodriguez (2007), ADRs and their underlying stocks (Alaganar and Bhar (2002)), large and small companies portfolio (Kroner and Ng (1998)), future markets (Xu and Li (2009)), exchange rates (Patton (2006b)) and interest rates (Chowdhury and Sarno (2004)).

In addition, recent empirical research on market dependence not only focuses on market returns, but also focuses on volatility returns (see Ammann and Süss (2009)). Their results demonstrate that market volatility move together with correlation, the higher market volatility corresponds to the higher cross market correlation. Soriano and Climent (2005) provided a review of the relationship between financial markets based on volatility transmission models. Moreover, Longin and Solnik (2001) employed an extreme theory model for multivariate distributions to test whether the correlation increase in international stock market during high volatility periods (see Bekiros and Georgoutsos (2008)). Their null hypothesis of normality is only rejected on the lower tail but cannot be rejected on the upper tail, which means that the correlation increase only appears in the bear market but not in the bull market. While Poon et al. (2003) criticize some extreme theory models simply assume asymptotic dependence between the estimated variables is incorrect in most cases and may lead to overestimation of financial market risk. Their estimates suggest that the asymptotic dependence between European countries (UK, Germany, and France) is true and truly increase over time, but asymptotic independence is observed between Europe, United States and Japan.

Moreover, the relationship between different market volatility indices has also been studied. Skiadopoulos (2004) adopt the regression model to investigate the relationship between the constructed Greek volatility index (GVIX) and the volatility indices (VIX and VXN) of the US market. The result displays the contemporaneous spillover effect between their changes, but the US volatility index has no lead effect on GVIX. Wagner and Szimayer (2004) provide an analysis of cross-market relationships, including volatility indices, and study the shock spillover effects between the VXO and the old VDAX with a stochastic volatility jump model. However, they did not explicitly analyze their variation of correlation. In addition, VXO and the old VDAX have different maturity, which may affect the accuracy of the results. Nikkinen and Sahlström (2004) analyse the implied volatilities for the US, the UK, Germany and Finland for market integration of uncertainty. The results show that the US, UK and German markets are closely related. Uncertainty changes in the US market are transferred to other markets under survey ; and changes in uncertainty in German market are delivered to other European markets investigated. Äijö (2008) estimates the implied volatility term structure of the new VDAX, VSMI and VSTOXX volatility sub-indices (see Krylova et al. (2009)). The results show that the implied volatility term structure of DAX, SMI and STOXX is highly correlated and the random behavior of VSMI and VSTOXX can be explained by the DAX model.

However, these studies of cross-market dependence between different volatility indices do not propose an effective approach to capture tail dependence, because the correlation coefficients employed in previous studies restrict to measuring linearity and symmetry. With respect to asymmetric cross-market dependence and financial contagion analysis, it is necessary to extend the analysis to the extreme nonlinear co-movements of volatility indices with copula models for investigating financial contagion and asymmetric dependence at higher moments.

Multivariate distributions modeling is crucial to risk management and asset allocation. Due to the difficulty of modelling the conditional mean of financial assets, many studies only focus on modeling the conditional volatility and dependency. The multivariate GARCH (Bauwens et al. (2006)) and stochastic volatility models (Harvey et al. (1994); Yu and Meyer (2006)) provide some ways to extend the univariate volatility model to multivariate case. Nevertheless, in general, the resulting multivariate model still assumes (conditional) multivariate normality. The copula based multivariate model provides an effective alternative method because non-elliptical multivariate distributions can be constructed in an tractable and flexible way. The advantage of using copulas to construct a multivariate volatility model is that the marginal model, the univariate volatility model, can be selected from a variety of bivariate copula family and is possible to capture the asymmetric dependence and tail dependence. In particular, the measure of financial risk takes into account the lower tail dependence.

However, most studies only focus on bivariate copula which is considered as a restriction of copula method for practical problem. Another limitation is that previous copula based model always assume the dependence parameters are time-constant, which is contradict with the empirically observed time-varying correlations. The emergence of vine copula method is able to solve above issues. High dimension vine copulas other than Gaussian or Student t copulas have become available through the introduction of hierarchical Archimedean copulas by Savu and Trede (2010) and Okhrin et al. (2013), factor copula models by Oh and Patton (2017a), and the class of pair copula construction proposed by Aas et al. (2009). Pair copulas construction, or call vine copula construction, are widely used recently due to its flexibility and the possibility of estimating a large number of parameters sequentially. Examples of financial applications of vine copula models can be found in Chollete et al. (2009) and Dissmann et al. (2013). Brechmann and Czado (2015) proposed a vine copula-based model for both cross-sectional and serial dependence.

Patton (2006b) introduced copulas with time varying parameters to model changing exchange rate dependencies. Since then, many studies have presented various ways to model time-varying copula. For example, Da Costa Dias and Embrechts (2004) test for structural breaks in copula parameters, Giacomini et al. (2009) adopt a sequence of breakpoint tests to determine intervals of constant dependence, Hafner and Reznikova (2010) considered the copula parameter as a smooth function of time and estimate it by the local maximum likelihood, while Hafner and Manner (2012) and Almeida and Czado (2012) proposed a model in which the copula parameter is the transformation of the first order latent Gaussian autoregressive process. Creal et al. (2013) proposed an observer-driven autoregressive model in which scaled score drive dependence parameters. Manner and Reznikova (2012) provided an overview and comparison of (bivariate) time varying copula models. To best of our knowledge, only few papers allow for time varying parameters in high dimensions. Heinen and Valdesogo (2008) allowed the parameters of a vine copula to be driven by a variation dynamic conditional correlation (DCC) model of Engle (2002), So and Yeung (2014) introduced a vine copula model with dynamic dependence similar with a DCC model, and Creal and Tsay (2015) extend the factor copula model proposed by Oh and Patton (2017b) by allowing for stochastic factor loadings. On the other hand, Oh and Patton (2017a) introduced the time variation into the factor copula model by specifying it as a generalized autoregressive score (GAS) model.

4.3 Review of Vine Copula

Although the symmetrised Joe-Clayton copula proposed by Patton (2006b) and the skewed-t copula proposed by Ammann and Süss (2009) can capture both the symmetric and asymmetric tail dependence, they are less suitable for modelling the special cases where there is only upper tail dependence or only lower tail dependence, leading to biased results due to possible misspecification of the model. In order to circumvent these drawbacks and more precisely describe the dependence structure of stock indices and volatility indices, in our study, we employ the high dimensional vine copula to capture the asymmetric dependence between pairs of variables. Vine copula is a type of high dimensional copula which can choose their building blocks from a wide range of bivariate copula family so as to capture the asymmetric dependence characteristics easily. In this section, following Nikoloulopoulos et al. (2012), we briefly review the vine copula construction and inference.

4.3.1 Construction of Vine Copula

A d-variate copula $C(u_1, ..., u_d)$ is a cumulative distribution function (cdf) with uniform marginals on the unit interval, see examples in Joe (1997) and Nelsen (2007). Regarding the theorem of Sklar (1959) for multivariate case, if $F_j(y_j)$ is the cdf of a univariate continuous random variable Y_j, then $C(F_1(y_1), ..., F_d(y_d))$ is a d-variate distribution for $\mathbf{Y} = (Y_1, ..., Y_d)$ with marginal distributions $F_j, j = 1, ..., d$. Conversely, if H is a continuous d-variate cdf with univariate marginal cdfs $F_1, ..., F_d$, then there exists a unique d-variate copula C satisfy that

$$F(\mathbf{y}) = C(F_1(y_1), ..., F_d(y_d)), \quad \forall \mathbf{y} = (y_1, ..., y_d). \tag{4.1}$$

The corresponding density is

$$f(\mathbf{y}) = \frac{\partial^d F(\mathbf{y})}{\partial y_1 \dots \partial y_d} = c(F_1(y_1), ..., F_d(y_d)) \prod_{j=1}^{d} f_j(y_j), \tag{4.2}$$

where $c(u_1, ..., u_d)$ is the d-variate copula density and $f_j, j = 1, ..., d$, are the corresponding marginal densities. As we know, a copula C has reflection symmetry if $(U_1, ..., U_d) \sim$ C implies that $(1 - U_1, ..., 1 - U_d)$ has the same distribution C. When we require the copula models have the characteristics of reflection asymmetry and flexible lower or upper tail dependence, then vine copulas (see Bedford and Cooke (2001); Bedford and Cooke (2002); Kurowicka and Cooke (2006) and Joe (1997)) become the best choice.

A d-dimensional vine copulas are constructed through sequential mixing of $d(d-1)/2$ linked bivariate copulas by trees and their cdfs involve lower dimensional integrals. Since the densities of multivariate vine copulas can be factorized in terms of linked bivariate copulas and lower dimension marginals, they shows the advantage of computationally tractable.

According to the different types of tree structures, various vine copulas can be constructed. Two special cases are D-vines and C-vines while R-vines is their more general format.

With respect to the d-dimensional C-vine copula, the pairs at tree 1 are 1, i, for $i = 2, ..., d$, and for tree $l(2 \leq l < d)$, the (conditional) pairs are $l, i|1, ..., l-1$ for $i = l+1, ..., d$, the conditional copulas are specified for variables l and i given those indexed as 1 to $l-1$. For C-vines density is given by (Aas et al. (2009)),

$$f(y) = \prod_{k=1}^{d} f_k(y_k) \prod_{j=1}^{d-1} \prod_{i=1}^{d-j} c_{i,i+j|i+1,...,i+j-1}(F_{i|i+1...i+j-1}(y_i|y_{i+1:i+j-1}), F_{i+j|i+1,...,i+j-1}(y_{i+j}|y_{i+1:i+j-1})), \tag{4.3}$$

where $y_{k_1:k_2} = (y_{k_1}, ..., y_{k_2})$, index j denotes the tree, while i runs over the edges in each tree.

Regarding the d-dimensional D-vine copula, the pairs at tree 1 are $i, i+1$, for $i = 1, ..., d-1$, and for tree $l(2 \leq l < d)$, the (conditional) pairs are $i, i+l|i+1, ..., i+l-1$ for $i = 1, ..., d-l$, the conditional copulas are specified for variables i and $i+l$ given the variables indexed in between,

$$f(y) = \prod_{k=1}^{d} f_k(y_k) \prod_{j=1}^{d-1} \prod_{i=1}^{d-j} c_{j,j+i|1,...,j-1}(F_{j|1...j-1}(y_j|y_1, ..., y_{j-1}), F_{j+i|1...j-1}(y_{j+i}|y_1, ..., y_{j-1})), \tag{4.4}$$

where $y_{k_1:k_2} = (y_{k_1}, ..., y_{k_2})$, index j denotes the tree, while i runs over the edges in each tree.

For more general d-dimension regular vines, there are $d-1$ pairs at tree 1, $d-2$ pairs in tree 2 where each pair has one element in common, and for $l = 2, ..., d-1$, there are $d-l$ pairs in level l where each pair has $l-1$ elements in common. Other conditions for regular vines can be found in Bedford and Cooke (2001) and Bedford and Cooke (2002).

4.3.2 Inference of Vine Copula

In this part we discuss the parameter estimate of the C-vine (canonical vine copula) density given by (20). We omit the discussion of estimate of D-vine (drawable vine copula) density because we don't employ D-vine in modeling the dependence structure of risk factors in our analysis. Inference for the general regular vine is also feasible though not straightforward, details of R-vine inference can be found in Dissmann et al. (2013).

Here we follow the inference method of Aas et al. (2009). Assume that we observe n variables at time T time. Let $\mathbf{x}_i = (x_{i,1}, ..., x_{i,T}); i = 1, ..., n$, denote the data set. For simplicity, we assume that the T observations of each variable are independent over time. Independence assumption is not a limiting condition, in our empirical analysis, we will adopt univariate time series model fit to the margins and analyze the obtained residuals.

Since the margins are unknown, the parameter estimation must rely on the normalised ranks of the data. The approximate uniform and independence means what is being maximised is a pseudo-likelihood maximization. We extend the method of maximum pseudo-likelihood originally proposed for copula by Oakes (1994), and proved to be asymptotically normal and consistent both by Genest et al. (1995) and Shih and Louis (1995). Moreover, by adopting simulation method, Kim et al. (2007) indicate that the maximum pseudo-likelihood method outperform the maximum likelihood method when the marginal distributions are unknown.

For the canonical vine, the log-likelihood is given by

$$\sum_{j=1}^{n-1}\sum_{i=1}^{n-j}\sum_{t=1}^{T} log[c_{j,j+i|1,...,j-1}\{F(x_{j,t}|x_{1,t}, ..., x_{j-1,t}), F(x_{j+i,t}|x_{1,t}, ..., x_{j-1,t})\}]. \tag{4.5}$$

For each bivariate copula there is at least one parameter to be estimated which depends on which kind of bivariate copula is chosen. The log-likelihood must be numerically maximised over all parameters.

The marginal conditional distribution in vine copula construction is given by Joe (1997), for each j,

$$F(x|\mathbf{v}) = \frac{\partial C_{x,v_j|\mathbf{v}_{-j}}\{F(x|\mathbf{v}_{-j}), F(v_j|\mathbf{v}_{-j})\}}{\partial F(v_j|\mathbf{v}_{-j})} \tag{4.6}$$

where $C_{ij|k}$ is a bivariate copula distribution function. For the special case where v is univariate, we have

$$F(x|v) = \frac{\partial C_{x,v}\{F(x), F(v)\}}{\partial F(v)} \tag{4.7}$$

Then we introduce h function (Aas et al. (2009)), $h(x, v, \Theta)$ denotes this conditional distribution function when x and v are uniform, i.e., $f(x) = f(v) = 1$, $F(x) = x$ and $F(v) = v$.

That is,

$$h(x, v, \Theta) = F(x|v) = \frac{\partial C_{x,v}(x, v, \Theta)}{\partial v}, \tag{4.8}$$

where the parameter v denotes the conditioning variable and Θ represents the set of parameters for the copula of the joint distribution function of x and v. Let $h^{-1}(x, v, \Theta)$ be the inverse of the h-function with respect to the first variable u, or say the inverse of the conditional distribution function. $\Theta_{j,i}$ is the set of parameters of the corresponding copula density $c_{j,j+i|1,...,j-1}(\cdot,\cdot)$, $h(\cdot)$ is given by (23), and element t of $\mathbf{v}_{j,i}$ is $v_{j,i,t} = F(x_{i+j,t}|x_{1,t}, ..., x_{j,t})$. Further, $L(\mathbf{x}, \mathbf{v}, \Theta)$ is the log-likelihood of the chosen bivariate copula with parameters Θ given the data vectors $\mathbf{x}$ and $\mathbf{v}$. Which is,

$$L(\mathbf{x}, \mathbf{v}, \Theta) = \sum_{t=1}^{T} logc(x_t, v_t, \Theta). \tag{4.9}$$

where $c(u, v, \Theta)$ is the density of the bivariate copula with parameters Θ. According to the setting above, we can first estimate the parameters of the copula of tree 1 with the original data, then compute conditional distribution functions for tree 2 using the copula parameters from tree 1 and the h-function, repeat the process, estimate the parameters of the copula of tree 2 using the observations in last step, and then continue to repeat last step process until obtain all parameters. Finally, we can obtain the starting value of the parameters for numerical maximisation.

4.4 Generalized Autoregressive Score Regular Vine Dynamic Copula Model Setting

In our paper, we aim to construct a model that allow for high dimension vine copula parameters to be time varying. In this sense, following the generalized autoregressive score model (GAS) proposed by Creal et al. (2013), we construct a model combine the regular vine copula with GAS model. We assume that,

$$\mathbf{u}_t \sim c(\mathbf{u}_t; \omega, \mathcal{F}_{t-1}), \tag{4.10}$$

where c is the copula density, ω is the vector of time-independent parameters of our model, and $\mathcal{F}_{t-1}$ is the information set available at time $t-1$. Then we specify our dynamic Generalised Autoregressive Score regular vine copula model, we consider the bivariate time series process $(u_{i,t}, u_{j,t})$ for $t = 1, ..., T$. We assume that its distribution follows,

$$(u_{i,t}, u_{j,t}) \sim C(\cdot, \cdot; \theta_t^{ij}) \tag{4.11}$$

where the $\theta_t^{ij} \in \Theta$ represents the time-varying parameter of the copula C. In order to be able to compare our copula parameters that have different domains, the copula can equivalently be parameterized in terms of Kendall's $\tau \in (1, 1)$. This specification is based on a fact that, for all bivariate copulas, copula parameter and Kendall's τ exist a one-to-one relationship, which can be expressed as $\theta_t^{ij} = r(\tau_t^{ij})$. And assume τ_t^{ij} is driven by the process $\lambda_t^{ij} \in (-\infty, +\infty)$. Due to the fact that λ_t^{ij} takes values on the real line, we map it into (1, 1) by employing the inverse Fisher transform, the domain of τ_t^{ij} can be expressed as:

$$\tau_t^{ij} = \frac{exp(2\lambda_t^{ij}) - 1}{exp(2\lambda_t^{ij}) + 1} =: \psi(\lambda_t^{ij}). \tag{4.12}$$

The time-varying parameter is able to be specified in several different ways; see Almeida and Czado (2012) for a survey on different specifications. We employ the specification of the GAS model proposed by Creal et al. (2013) for the latent process. As the observation-driven model, it assumes an autoregressive structure for λ_t^{ij}, and also drive the latent process by employing the weighted score of the underlying model. The model of order one is given as

$$\lambda_t^{ij} = \omega_{ij} + \phi_{ij}\lambda_{t-1}^{ij} + \delta_{ij}s_{t-1}^{ij}, \tag{4.13}$$

where s_{t-1}^{ij} is the scaled score vector

$$s_{t-1}^{ij} = S_{ij,t}\nabla_{ij,t}, \tag{4.14}$$

with

$$\nabla_{ij,t} = \frac{\partial lnc(u_{i,t}, u_{j,t}; \omega_{ij}, \mathcal{F}_{t-1})}{\partial\theta_t^{ij}} \tag{4.15}$$

is the score and $\omega_{ij} = (\omega_{ij}, \phi_{ij}, \delta_{ij})$, and the scaling matrix $S_{ij,t}$ is the square root matrix of the inverse of the information matrix which is defined as

$$S_{ij,t} = \mathcal{J}_{t|t-1} \tag{4.16}$$

with

$$\mathcal{J}'_{t|t-1}\mathcal{J}_{t|t-1} = \mathcal{I}^{-1}_{t|t-1}, \tag{4.17}$$

where $\mathcal{I}_{t|t-1} = E_{t-1}[\nabla_{ij,t}\nabla'_{ij,t}]$ is the information matrix. For above specification we follow Creal et al. (2013), details and properties can be found there. Stationarity conditions are studied by Blasques et al. (2012). Blasques et al. (2015) show optimality properties of GAS models, whereas Koopman et al. (2016) compare the forecasting performance of a wide range of parameter-driven and observation-driven models and draw conclusion that both kinds of models perform equally well.

Next step, we combine bivariate dynamic copula models with the R-vine tree structure in order to construct the multivariate time-varying GAS R-vine copula model. In particular, above bivariate dynamic copula model is adopted as the building blocks of our R-vine copula. Till here, we set up our Generalized Autoregressive Score Regular Vine Dynamic Copula Model. Therefore, we obtain the dynamic R-vine copula density, here we present the density by using the format of C-vine copula density as follows, the complex general R-vine copula density representation can be found in Theorem 2.5. Dissmann et al. (2013),

$$c(u_1,...,u_d;\theta_t) := \prod_{j=1}^{d-1}\prod_{i=1}^{d-j} c_{l(i,j)}(F(u_i|\mathbf{u}_{i+1:i+j-1};\theta_t^{l(i,j)}), F(u_{i+j}|\mathbf{u}_{i+1:i+j-1};\theta_t^{l(i,j)})), \tag{4.18}$$

where $l(i,j) := i, i+j|i+1 : i+j-1$ and $\theta_t := \{\theta_t^{l(i,j)}; j = 1,...,d-1, i = 1,...,d-j\}$ is the time-varying copula parameter vector. Here, $c_{l(i,j)}(\cdot,\cdot;\theta_t^{l(i,j)}))$ is the bivariate copula density corresponding to the bivariate dynamic copula given in (4), where $\theta_t^{l(i,j)}$ satisfies

$$\theta_t^{l(i,j)} = r(\tau_t^{l(i,j)}) = r(\psi(\lambda_t^{l(i,j)})) \tag{4.19}$$

for the latent process $\lambda_t^{l(i,j)}$ defined by equations (7). The bivariate copula family corresponding to $l(i,j)$ can be chosen arbitrarily and independently of any other index $l(r,s)$.

4.5 Modelling Marginal Model

The first step of modelling dependence is to fit the marginal distribution to our returns, and estimate parameters of univariate models for the conditional mean and variance. It is well known that financial returns exhibit stylized facts, such as mean reversion, time-varying volatility and conditional heteroscedasticity. Taking into account these stylised facts, we suggest estimating an asymmetric AR-GJR-GARCH model to fit marginal distribution due to its simplicity and its successful application commonly reported in the literature. We employ ARMA models with the lag length chosen in order to minimize the BIC for the conditional mean. Regarding conditional variance, the standardized residuals are modeled by a GARCH(1,1) model with skewed Student t errors. As we are also focusing on volatility index returns, we have to pay special attention to fat tails and skewness due to possible leverage effects (see Ammann and Süss (2009)). There is an overview of volatility models by Poon and Granger (2003), alternative approaches could be the EGARCH or the TGARCH.

Let the random process r_t denote the index return which can be characterized by an autoregressive moving-average (ARMA) model as follows

$$r_t = a_0 + \sum_{i=1}^{p} a_i r_{t-i} + \sum_{j=1}^{q} b_j \epsilon_{t-j} + \epsilon_t \tag{4.20}$$

where a_0 is a constant; p and q are the order of autoregressive and moving average processes respectively for the conditional mean. The error term ϵ_t can be splitted into a stochastic part x_t and a time-dependent standard deviation σ_t so that $\epsilon_t = \sigma_t x_t$. The conditional variance σ_t^2 is characterized by an asymmetric GARCH model, namely GJR-GARCH(1,1)(see Glosten et al. (1993)).

$$\sigma_t^2 = \omega_i + \alpha_i \epsilon_{i,t-1}^2 + \beta_i \sigma_{i,t-1}^2 + \gamma_i \epsilon_{i,t-1}^2 I_{i,t-1} \tag{4.21}$$

where $I_{i,t-1} = 1$ if $\epsilon_{i,t-1} < 0$, and $I_{i,t-1} = 0$ if $\epsilon_{i,t-1} \geq 0$, γ indicates the presence of the leverage effect, e.g. bad news generates larger effect on the volatility compared to good news.

The filtered returns $x_t = \epsilon_t/\sigma_t$, $t = 1, ..., T$; follow a strong white noise process with a zero mean and unit variance. In our empirical work, we adopt Hansen (1994)'s skewed

Student's t distribution $x_t \sim skT(0,1;\nu,\zeta)$, with $\nu > 2$ and ζ denoting the degrees of freedom (dof) and asymmetry parameters, respectively. It has the PDF [1]

$$f(x;\nu,\zeta) = \begin{cases} bc\left(1+\frac{1}{\nu-2}\left(\frac{bz+a}{1-\zeta}\right)^2\right)^{-\frac{\nu+1}{2}}, & if \;\; z < -\frac{a}{b} \\ bc\left(1+\frac{1}{\nu-2}\left(\frac{bz+a}{1+\zeta}\right)^2\right)^{-\frac{\nu+1}{2}}, & if \;\; z \geq -\frac{a}{b} \end{cases}$$

where $a = 4\zeta c\frac{\nu-2}{\nu-1}$, $b^2 = 1 + 3\zeta^2 - a^2$, $c = \frac{\Gamma(\frac{\nu+1}{2})}{\sqrt{\pi(\nu-2)}\Gamma(\frac{\nu}{2})}$. The skewed Student's t distribution is quite general as it nests the Student's t distribution and the Gaussian density. Previous studies advocate this parametrization for the margins as able to capture the autocorrelation, volatility clustering, skewness and heavy tails exhibited typically by financial asset returns; see e.g. Jondeau and Rockinger (2006) and Kuester et al. (2006). In our empirical work, we adopt GJR-GARCH(1,1,1) and select the best ARMA p and q among 1, 2,..., 10 by minimizing the Akaike Information Criterion (AIC). The model parameters are estimated by quasi-maximum likelihood (QML). Uniform (0, 1) margins denoted $u_n = F_n(x_n)$, n = 1, 2, can be obtained from each filtered return series via the probability integral transform. Once the vector $\mathbf{u} = (u1, u2)'$ is formed, the copula parameter vector can be estimated by maximizing the corresponding copula log-likelihood function.

Many dependence measures can be expressed in terms of copula function, see Embrechts et al. (2002) for details. Here, we focus on the tail dependence indices, which describe the asymptotic dependence. Tail dependence indices measure the dependence in extreme values of the variables, capturing the dependence in the joint tails of the bivariate distributions. Specifically, the upper (lower) tail dependence is the probability of one variable having a higher (lower) value and being close to 1(0), given that the other variable has a higher (lower) value. The definition of tail dependence coefficients is given by,

Definition (Tail dependence coefficients). For a copula C of a random vector $(U, V)^T$ with marginal distribution function F_1 and F_2, we define its upper and lower tail dependence (TDC) via

$$\lambda_U(C) = \lim_{u\to 1^-} P(V > F_2^{-1}(u) | U > F_1^{-1}(u)) = \lim_{u\to 1^-} \frac{1-2u+C(u,u)}{1-u}, \qquad (4.22)$$

[1]There are other Student t distribution that the skewness is introduced in different ways, see Fernández and Steel (1998) and Aas and Haff (2006).

$$\lambda_L(C) = \lim_{u \to 0^+} P(V \leq F_2^{-1}(u) | U \leq F_1^{-1}(u)) = \lim_{u \to 0^+} \frac{C(u,u)}{u}, \tag{4.23}$$

The student t copula is symmetric, with the upper and lower tail dependence coefficients as $2t_{\nu+1}(-\sqrt{\nu+1}\sqrt{\frac{1-\theta}{1+\theta}})$, θ is the copula parameter for the student and ν denotes the degrees of freedom of the the student-t copula. BB1 copula SBB1 copula

The copula models above are static because their parameters are time invariant. However, empirical evidence suggests that financial returns exhibit time varying conditional distributions and, therefore, time varying dependence. In this sense, we combine dynamic Generalised Autoregressive Score model with Vine copula method to modeling time varying multivariate dependence.

4.6 Estimation of GAS Vine Copula Parameters

In previous section, we discussed the vine copula parameter estimate in general, here we present the detailed copula parameters estimate of vine copula GAS model. Since the joint density of copula model is the product of the marginal and the copula densities, we need to estimate the parameters the marginal model and the stochastic copula models separately,

$$g(\epsilon_{1,t}, ..., \epsilon_{d,t}) = c(F_i(\epsilon_{1,t}), ..., F_d(\epsilon_{d,t})) \cdot f_i(\epsilon_{1,t}) \cdot ... \cdot f_d(\epsilon_{d,t}), \tag{4.24}$$

where g represents the joint distribution densities, c denotes the copula density, and f represents the marginal densities. Taking logarithms of both sides, we obtain the joint loglikelihood as the sum of the marginal and the copula loglikelihood function. Then we adopt two-step IMF (inference functions for margins) method to estimate parameters, which is usually used in copula parameter estimate. In IFM method, the parameters of the marginal distributions are separated from each other and from those of the copula. In this sense, the first step, we estimate the marginal density. The returns is able to transformed to standardized residuals by either parametric (Joe (2005)) or nonparametric probability integral transformation (Genest et al. (1995)). If the marginal model is specified well, the parametric probability transformation can provide good approximation to the original copula data, but if the marginal models are misspecified (Kim et al. (2007)) there may some problems. Here, we rely on the concept of empirical marginal transformation which approximates an unknown parametric margin with the (uniform) empirical distribution function $\hat{u}_{1t} = \hat{F}_1(z_{1t}) = \frac{1}{T+1}\sum_{t=1}^{T}\mathbf{1}_{\{Z_{1t}\leq z_{1t}\}}$, and likewise for $\hat{u}_{2t} = \hat{F}_2(z_{2t})$,..., where $(z_{1t}, z_{2t}, ...)$, $t = 1, ..., T$, are the filtered standardized residuals.

In the second step, we estimate the copula parameters, as presented above, R vine copula density is the product of bivariate (conditional) copulas. Due to the infeasible computation of large number of parameters in one step, we employ the sequential estimation method to estimate the copula parameters in spite of adopting the sequential method for parameters estimate will result in a small loss in statistical efficiency and intractable forms for the standard errors of the parameter estimates.

For the bivariate model, the log-likelihood for observation t is given by

$$LL(\omega_{ij}; u_{i,t}, u_{j,t}) = lnc(u_{i,t}, u_{j,t}; \omega_{ij}, \mathcal{F}_{t-1}) = lnc(u_{i,t}, u_{j,t}; \theta_t^{ij}). \tag{4.25}$$

For the GAS model, θ_t^{ij} can be computed for a given value of ω_{ij} using the recursion (19), and therefore, the estimation is straightforward.

4.7 Data

We consider twelve indices from three major financial markets: US, Europe and Asia. In total, we have six equity indices as well as their corresponding implied volatility indices (cf. Table 1). We choose Standard and Poor's 500 Index and NASDAQ 100 Index representing US markets, FTSE 100 Index, DAX 30 Index and Euro Stoxx 50 Index representing European market, Nikkei 225 Stock Average Index representing Japanese market. The considered time period covers roughly 15 years, starting in particular on 1 January 2002 and ending on 30 Jun 2017. Excluding non-trading days, this results in 4044 observations of daily closing prices in US dollar. We notice that when the US stock market is closed, the considered Asian stock markets are open, while the UK and US stock markets have few trading hours in common. This lack of synchronicity and time zone differences constitutes a problem when studying the linkages between daily returns, it would significantly affect the estimated results, especially between the Japanese markets and the US/European markets. Therefore, we consider the Japanese market returns led one day. The daily return is calculated as $y_t = ln(P_t) - ln(P_{t-1})$, where P_t is the closing price of the index at day t. All data are downloaded from Datastream.

Figure 1 and Figure 2 show the time series returns of the all stock indices returns

Table 4.1: **Considered Indices separated by Regions**

Shortcut	Index Description	Currency	Sources
USA			
SPX	Standard and Poors 500 Index	USD	Datastream
VIX	Implied Volatility Index of the SPX	USD	Datastream
NDX	NASDAQ 100 Index	USD	Datastream
VNX	Implied Volatility Index of the NDX	USD	Datastream
Europe			
FTSE	FTSE Index	USD	Datastream
VFTSE	Implied Volatility Index of the FTSE Index	USD	Datastream
DAX	Deutscher Aktien Index (German Stock Index)	USD	Datastream
VDAX	Implied Volatility Index of the DAX	USD	Datastream
SX5E	Euro Stoxx 50 Index	USD	Datastream
VSX5E	Implied Volatility Index of the SX5E	USD	Datastream
Asia			
NKY	Nikkei-225 Stock Average Index	USD	Datastream
VNKY	Implied Volatility Index of the NKY	USD	Datastream

Note: This table lists considered six equity indices as well as corresponding volatility indices.

and their volatility indices returns. From the figure, it is possible to identify periods of high volatility, which correspond to the main crises, such as the Internet bubble bursting around 2002, the global financial crisis of September 2008 and the euro debt crisis in 2011. During these crisis periods, the stock index series reach local peak positions, and the volatility index returns continually show extreme values.

Table 1 provides the summary statistics for the daily stock index returns and their corresponding volatility index returns. From the skewness results, the stock index returns all have positive skewness from during the considered period, except the Nikkei 225 which has a negative skewness. In addition, all of the volatility index returns demonstrate a more positive skewness than their corresponding stock index returns, with the VXJ demonstrating the highest one. Moreover, both the kurtosis results of stock indices and volatility indices are much larger than 3 except the kurtosis of VDAX which is very close to 3, so that both stock index returns and volatility index returns exhibit fat tails characteristics; in particular, compared to other markets in the US and Europe, the volatility index of the Nikkei 225, the VXJ has the highest kurtosis. The Jarque-Bera test indicates that all these returns are rejected for normal distribution. The augmented Dickey-Fuller (ADF) unit

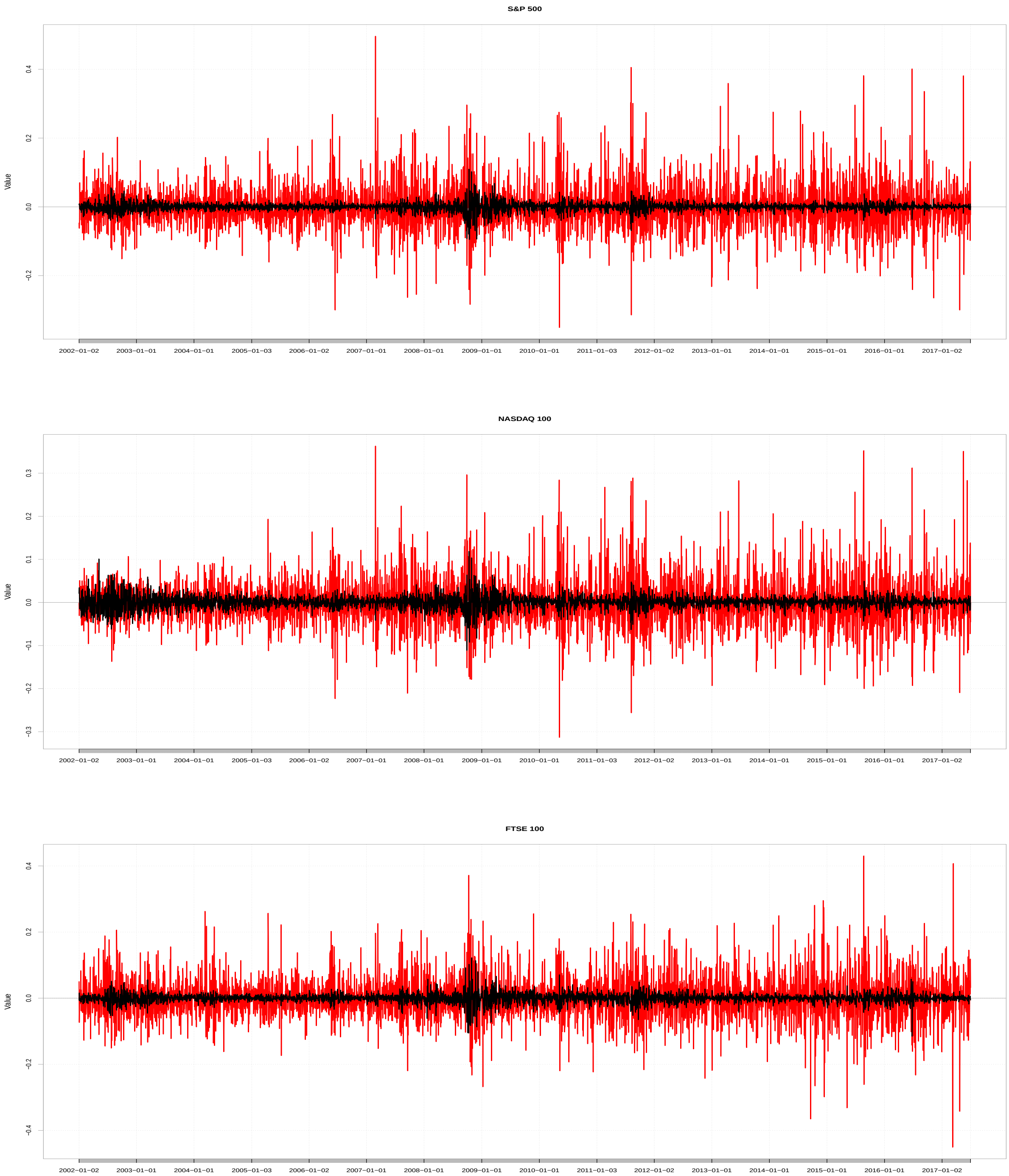


Figure 4.1: Time series of Stock index returns and their volatility index returns

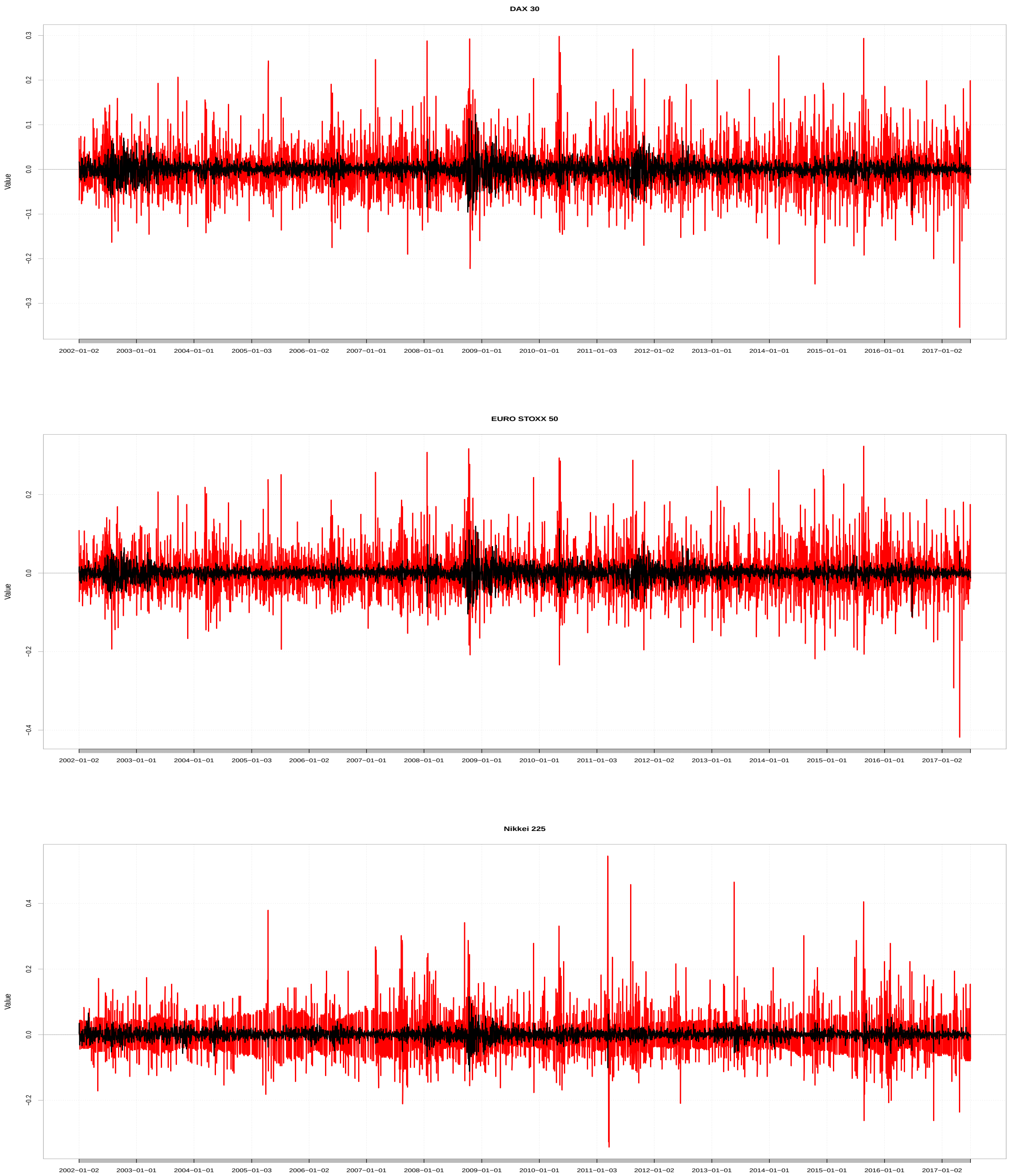


Figure 4.2: Time series of Stock index returns and their volatility index returns

root tests are all rejected as well.

4.7.1 Descriptive statistics

As mentioned above we first fit a univariate time series model for each return series in order to obtain the standardised returns for the subsequent estimation of the dynamic copulas. The estimated results for marginal distributions with skewed-t AR-GJR-GARCH models are presented in Table 4. The similar structures of the model coefficients confirm that the AR-GJR-GARCH model is generally suitable for all return series. All our GARCH coefficients β_1 are significant with values around 0.9, implying the persistence of volatility. Hence, not only the market volatility but also the volatility of volatility itself exhibits time-varying clustering effects. For the stock index returns, the variance is almost only positively related with negative return innovations, indicating the asymmetric return volatility phenomenon. For the volatility index returns, the variance of volatility is positively related with positive innovations and negatively related with negative innovations, and this relationship is almost symmetric, indicating that the volatility risk will be high when the market crashes and low when the market recovers. In Table 4 the Ljung-Box Q-test of lags equal to 1,2,5 shows that the residuals of the AR-GJR-GARCH model are unautocorrelated, which implies that the dependence in the following copula estimations (if existing) can only arise from the cross-market dependence instead of originating from the autocorrelations of each single return series. The skewed-t distribution estimation shows that the residuals of stock index returns are slightly negatively skewed, while the residuals of volatility index returns are a little more positively skewed, indicating the existence of the leverage effects in these markets. The Komogorov-Smirnov (KS) tests confirm that these residuals follow the skewed-t distributions.

4.8 Selection of Vine Copula Structure

We then select the best-fitting bivariate copula from ample candidate bivariate copula families as building blocks for vine copula model (See Table 27 candidate bivariate copulas). Given the size and complexity of our model, as well as the difficulty to estimate parameters precisely on higher trees, we decided to rely on the BIC to find more parsimonious model specifications and to minimize the estimation errors. For model selection we

Table 4.2: **Summary statistics of daily stock index returns and volatility index returns**

	S&P 500	Nasdaq 100	FTSE100	DAX30	Euro Stoxx 50	Nikkei 225
Panel A: Stock index returns						
min	-0.0947	-0.111	-0.115	-0.0960	-0.1110	-0.1119
max	0.1096	0.1185	0.1222	0.1237	0.1197	0.1164
mean	0.0001	0.0003	0.00005	0.0002	0.00003	0.0001
std.dev	0.0119	0.0143	0.0140	0.0163	0.0164	0.0145
skewness	-0.0234	0.0140	-0.0242	-0.0835	-0.0822	-0.0289
kurtosis	10.2609	6.1014	9.7809	5.2136	5.9302	5.1060
JB test Stat.	17796	6280.8	16176	4590.8	5937.5	4455.1
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
ADF test Stat.	-16.421	-16.053	-16.452	-15.105	-15.298	-16.281
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
	VIX	VXN	VFTSE	VDAX	VSTOXX	VXJ
Panel B: Volatility index returns						
min	-0.3506	-0.3130	-0.4506	-0.3540	-0.4184	-0.3429
max	0.4960	0.3629	0.4302	0.2982	0.3234	0.5447
mean	-0.0002	-0.0002	-0.0001	-0.00007	-0.00004	-0.0001
std.dev	0.0667	0.0564	0.0663	0.0539	0.0585	0.0626
skewness	0.6915	0.6889	0.3080	0.4901	0.5152	1.1189
kurtosis	4.5138	4.1408	3.3420	2.9093	3.1840	7.1734
JB test Stat.	3760.3	3213.4	1949	1590.7	1890	9524.8
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
ADF test Stat.	-18.728	-18.824	-17.909	-17.966	-18.217	-16.732
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)

Note: This table reports the Summary statistics of daily stock index returns and volatility index returns.

Table 4.3: **Estimated marginal model for Stock index returns**

	S&P 500	Nasdaq 100	FTSE100	DAX30	Euro Stoxx 50	Nikkei 225
AR-GJR-GARCH						
mu	0.000216	0.000463	0.000088	0.000408	0.000145	0.000183
	(0.000103)	(0.000141)	(0.000145)	(0.000176)	(0.000177)	(0.000238)
ar1	-0.074794	-0.049748	-0.038443	-0.029631	-0.044498	-0.130536
	(0.015288)	(0.015627)	(0.016224)	(0.015782)	(0.015745)	(0.013830)
omega	0.000001	0.000002	0.000003	0.000002	0.000003	0.000005
	(0.000002)	(0.000001)	(0.000001)	(0.000001)	(0.000002)	(0.000014)
alpha1	0.000000	0.000001	0.016700	0.003740	0.003073	0.031704
	(0.017231)	(0.005575)	(0.010362)	(0.007302)	(0.007868)	(0.059725)
beta1	0.900613	0.920164	0.887234	0.926612	0.916136	0.897178
	(0.037273)	(0.009677)	(0.011840)	(0.012739)	(0.010795)	(0.035555)
gamma1	0.176589	0.143462	0.158179	0.115329	0.135695	0.092547
	(0.037511)	(0.008975)	(0.024556)	(0.021401)	(0.023469)	(0.064628)
Skewed t distribution						
skew	0.887449	0.893038	0.904403	0.917988	0.921547	0.915099
	(0.016167)	(0.018521)	(0.019970)	(0.020039)	(0.020725)	(0.017904)
shape	6.635332	7.059804	9.218266	8.786032	8.512862	9.475939
	(0.866219)	(0.729865)	(1.232801)	(1.171116)	(1.023388)	(3.042029)
LB Q-test						
p-value						
Lag[1]	0.7726	0.7420	0.5093	0.3831	0.4125	0.7355
Lag[2]	0.8693	0.9817	0.8604	0.8705	0.8121	0.7725
Lag[5]	0.6916	0.8997	0.5541	0.6726	0.5345	0.4862
KS test						
p-value	0.084	0.235	0.259	0.161	0.170	0.024

Note: This table reports the Summary statistics of daily stock index returns and volatility index returns.

Table 4.4: **Estimated marginal model for Volatility index returns**

	VIX	VXN	VFTSE	VDAX	VSTOXX	VXJ
AR-GJR-GACH						
mu	0.001519	0.001104	0.001146	0.000707	0.001135	0.000105
	(0.000795)	(0.000741)	(0.000878)	(0.000785)	(0.000822)	(0.000774)
ar1	-0.068836	-0.022107	-0.058179	-0.000502	-0.021525	-0.144878
	(0.015110)	(0.015573)	(0.015374)	(0.015397)	(0.015371)	(0.015251)
omega	0.000310	0.000159	0.000125	0.000141	0.000168	0.000228
	(0.000037)	(0.000029)	(0.000037)	(0.000030)	(0.000034)	(0.000051)
alpha1	0.233985	0.189177	0.117124	0.104414	0.126909	0.100487
	(0.002696)	(0.024802)	(0.019552)	(0.015404)	(0.017290)	(0.018015)
beta1	0.824046	0.860422	0.909472	0.898826	0.888071	0.888696
	(0.011749)	(0.017901)	(0.017581)	(0.015622)	(0.015257)	(0.022750)
gamma1	-0.255179	-0.195048	-0.104954	-0.105448	-0.127506	-0.115600
	(0.002984)	(0.027228)	(0.018704)	(0.017187)	(0.018594)	(0.020456)
Skewed t distribution						
skew	1.268015	1.215515	1.193032	1.203417	1.267682	1.135340
	(0.028028)	(0.026180)	(0.026910)	(0.027404)	(0.030136)	(0.023960)
shape	5.363764	5.126753	5.199195	5.510295	5.382457	4.726062
	(0.439992)	(0.432397)	(0.417932)	(0.473566)	(0.444941)	(0.409607)
LB Q-test						
p-value						
Lag[1]	6.374e-01	0.7598033	7.104e-01	6.382e-01	0.5819960	5.913e-01
Lag[2]	4.431e-05	0.0107005	1.906e-04	1.537e-03	0.0141651	1.451e-08
Lag[5]	6.488e-06	0.0002798	3.267e-06	5.482e-05	0.0001605	4.776e-08
KS test						
p-value	0.372	0.556	0.538	0.827	0.801	0.284

Note: This table reports the Summary statistics of daily stock index returns and volatility index returns.

aim to demonstrate the superior fit of vine copulas with individually chosen pair-copula families and assess the gain over vine copula with only bivariate Student *t* or with only Gaussian pair-copula. We need to select a copula family for every pair of variables. We take a large number of copula into consideration, nevertheless, it is necessary to indicate we only choose the bivariate copula which take asymmetric tail dependence for the reason that we focus on investigating the existence and direction of international financial contagion. (See Appendix). Given these bivariate options we still have to decide which copula fits "best". In this case, we adopt the AIC (Akaike (1974)) criteria which corrects the log likelihood of a copula for the number of parameters. Bivariate copula selection using the AIC has previously investigated by Manner (2007) and Brechmann (2010) who find that it is quite reliable criterion, in particular in comparison to alternative criteria such as copula goodness-of-fit tests. Selection proceeds by computing the AIC's for each possible family and then choosing the copula with smallest AIC.

In order to investigate which copula structure is preferred to describe the dependence of the stock indices and volatility indices, we also employ a likelihood ratio based goodness-of-fit test-Vuong test, to compare multivariate Gaussian copula with other vine copula model. Therefore, we set

Null hypothesis: M1 = Multivariate Gaussian copula

Alternatives: M2 = R-vine *t* copula, R-vine mixed copula, C-vine *t* copula, C-vine Mixed copula, R-vine independence mixed Copula, C-vine independence mixed Copula

$multivariate\ \ Gaussian$ $(R - vine\ \ Gaussian)$: R-vine with each pair-copula term chosen as bivariate Gaussian copula, i.e., this corresponds to a multivariate Gaussian copula, where unconditional correlations can be obtained from conditional ones by inverting a generalized version.

$R - vine\ \ t$: R-vine with each pair-copula term chosen as bivariate Student-t copula. If the degrees of freedom parameter of a pair is estimated to be larger than 30, we set the copula to the Gaussian.

$R - vine\ \ mixed$: R-vine with pair-copula terms chosen individually from ample bivariate copula types (see Appendix).

$C - vine\ \ t$: C-vine with each pair-copula term chosen as bivariate Student-t copula. If the degrees of freedom parameter of a pair is estimated to be larger than 30, we set the copula to the Gaussian.

$C-vine\ \ mixed$: C-vine with pair-copula terms chosen individually from ample bivariate copula types (see Appendix).

$R-vine\ \ independence\ \ mixed$: R-vine with pair-copula terms chosen individually from bivariate copula families with independence copula.

$C-vine\ \ independence\ \ mixed$: C-vine with pair-copula terms chosen individually from bivariate copula families with independence copula (see above).

The likelihood-ratio based test proposed by Vuong (1989) can be used for comparing non-nested models. For this let c_1 and c_2 be two competing vine copulas in terms of their densities and with estimated parameter sets θ_1 and θ_2. We then compute the standardized sum, ν, of the log differences of their pointwise likelihoods $m_i := log[\frac{c_1(u_i|\hat{\theta}_1)}{c_2(u_i|\hat{\theta}_2)}]$ for observations $u_i \in [0, 1]$, $i = 1, ..., N$, i.e.,

$$statistic := \nu = \frac{\frac{1}{n}\sum_{i=1}^{N} m_i}{\sqrt{\sum_{i=1}^{N}(m_i - \overline{m})^2}} \tag{4.26}$$

Vuong (1989) shows that ν is asymptotically standard normal. According to the null-hypothesis

$$H_0 : E[m_i] = 0 \ \ \forall i = 1, ..., N, \tag{4.27}$$

we hence prefer vine model 1 to vine model 2 at level α if

$$\nu > \Phi^{-1}(1 - \frac{\alpha}{2}), \tag{4.28}$$

where Φ^{-1} denotes the inverse of the standard normal distribution function. If $\nu < -\Phi^{-1}(1 - \frac{\alpha}{2})$ we choose model 2. If, however, $|\nu| < \Phi^{-1}(1 - \frac{\alpha}{2})$, no decision among the models is possible.

Like AIC and BIC, the Vuong test statistic may be corrected for the number of parameters used in the models. There are two possible corrections; the Akaike and the Schwarz corrections, which correspond to the penalty terms in the AIC and the BIC, respectively.

Goodness-of-fit test results of vine copula are presented in Table 5. We list the log-likelihood value and AIC, BIC value of each candidate Vine copula model fitting to our stocking indices and volatility indices. From the results of log likelihood, in both of stock indices and volatility indices cases, the value of R-vine mixed copula log likelihood is larger than other candidate copula models, which means the R-vine mixed copula is superior to other model for modelling our data. The R-vine mixed copula also take the smallest AIC and BIC value in both stock indices and volatility indices cases.

Vuong test copula selection results for all models are summarized in Table 6 which lists the test statistics together with the p-values in parentheses of a Vuong test with and without Akaike and Schwarz corrections respectively, testing the multivariate Gaussian model against the alternative vine copula setting indicated by the respective column. From the Vuong tests results we see that the R-vine mixed copula have the largest values of Vuong test statistics except in statistic (Schwarz corrections). According to Vuong test criterion, the R-vine mixed copula can be preferred over other vine copula setting and multivariate Gaussian copula. Overall Vuong test demonstrates the usefulness of vine copula with individually chosen copula types for each pair copula term.

Joe et al. (2010) show that vine copulas can have a different upper and lower tail dependence for each bivariate margin when asymmetric bivariate copulas with upper/lower tail dependence are chosen in tree 1 of the vine. In other words, in order for a vine copula to have tail dependence for all bivariate margins, it is necessary for the bivariate copulas in tree 1 to have tail dependence but it is not necessary for the conditional bivariate copulas in trees $2, ..., d-1$ to have tail dependence, too. At trees 2 or higher, Gaussian copulas might be adequate to model the dependency structure. Therefore, in our subsequent analysis, we focus on the tree 1 of our vine copula. Taking these into account, based on above Vuong test results, we select R-vine mixed copula to model the dependence of stock indices and volatility indices. The results of R-vine mixed copula tree 1 demonstrate that BB1 and Survival BB1 copula are selected to model the pair of indices. The advantage of vine copulas does not come solely from the flexible tree structure, but the flexibility of mixing different bivariate families is used to replace the classical Gaussian and Student t copula.

4.9 GAS Regular Vine Copula Parameters Estimate of Stock Indices and Volatility Indices

In Table 7, we present the results for the static tree structure of the selected R-vine copula fitting to the stock indices and volatility indices returns separately. As observed in

Table 4.5: **Best Fitting Vine Copula for Stock Indices and Volatility Indices**

Panel A: Stock index returns

type	logLik	AIC	BIC
R-vine Gauss Copula	12128.59	-24227.18	-24132.61
R-vine t Copula	12363.36	-24666.72	-24477.58
C-vine t Copula	12349.90	-24639.80	-24450.65
R-vine ind. mixed Copula	12389.26	-24726.53	-24562.60
C-vine ind. mixed Copula	12331.82	-24615.63	-24464.32
R-vine mixed Copula	12390.10	-24724.20	-24547.67
C-vine mixed Copula	12335.10	-24616.20	-24445.97

Panel B: Volatility index returns

type	logLik	AIC	BIC
R-vine Gauss Copula	9447.36	-18864.71	-18770.14
R-vine t Copula	9954.12	-19864.24	-19679.10
C-vine t Copula	9952.70	-19845.40	-19656.25
R-vine ind. mixed Copula	9953.77	-19863.54	-19724.84
C-vine ind. mixed Copula	9916.25	-19786.49	-19641.48
R-vine mixed Copula	9955.63	-19865.26	-19720.26
C-vine mixed Copula	9916.93	-19785.86	-19634.55

Note: This table reports the loglikelihood, AIC and BIC value of various vine copula fitting to our data.

Table 4.6: **Goodness-of-fit Test of Vine Copula for Stock Indices and Volatility Indices**

Panel A: Stock index returns

Vuong Test	statistic(no corr.)	statistic(Akaike corr.)	statistic(Schwarz corr.)
R-vine t Copula	6.852353	6.455709	5.205341
	(0.0000)	(0.0000)	(0.0000)
C-vine t Copula	6.66681	6.262208	4.986753
	(0.0000)	(0.0000)	(0.0000)
R-vine ind. mixed Copula	7.190643	6.887211	5.930682
	(0.0000)	(0.0000)	(0.0000)
C-vine ind. mixed Copula	5.648425	5.398282	4.609738
	(0.0000)	(0.0000)	(0.0000)
R-vine mixed Copula	7.220232	6.861307	5.729841
	(0.0000)	(0.0000)	(0.0000)
C-vine mixed Copula	5.650120	5.321803	4.286826
	(0.0000)	(0.0000)	(0.0000)

Panel B: Volatility index returns

Vuong Test	statistic(no corr.)	statistic(Akaike corr.)	statistic(Schwarz corr.)
R-vine t Copula	11.67371	11.33486	10.26667
	(0.0000)	(0.0000)	(0.0000)
C-vine t Copula	11.57845	11.23476	10.15135
	(0.0000)	(0.0000)	(0.0000)
R-vine ind. mixed Copula	12.42240	12.25069	11.70939
	(0.0000)	(0.0000)	(0.0000)
C-vine ind. mixed Copula	11.40203	11.20750	10.59424
	(0.0000)	(0.0000)	(0.0000)
R-vine mixed Copula	12.43466	12.23894	11.62198
	(0.0000)	(0.0000)	(0.0000)
C-vine mixed Copula	11.40335	11.18479	10.49581
	(0.0000)	(0.0000)	(0.0000)

Note: This table reports the Vuong test results of various vine copulas for stock indices and volatility indices.

Table 4.7: **Tree Structure of Vine Copula for Stock Indices and Volatility Indices**

Panel A: Stock index returns

tree	edge	No.	family	par	par2	tau	UTD	LTD
1	1,6	17	SBB1	0.21	1.26	0.28	0.07	0.27
	1,2	17	SBB1	0.51	2.71	0.71	0.60	0.71
	4,1	7	BB1	0.32	1.38	0.37	0.35	0.21
	5,3	17	SBB1	0.49	2.19	0.63	0.53	0.63
	5,4	17	SBB1	0.57	4.12	0.81	0.74	0.82
2	4,6;1	7	BB1	0.10	1.04	0.09	0.05	0.00
	4,2;1	40	BB8270°	-1.06	-0.98	-0.03	-	-
	5,1;4	10	BB8	1.40	0.73	0.07	-	-
	4,3;5	9	BB7	1.02	0.04	0.03	0.03	0.00
3	5,6;4,1	20	SBB8	1.16	0.84	0.04	-	-
	5,2;4,1	37	BB1270°	-0.07	-1.03	-0.06	-	-
	3,1;5,4	17	SBB1	0.03	1.05	0.06	0.00	0.06
4	2,6;5,4,1	5	F	0.10	0.00	0.01	-	-
	3,2;5,4,1	27	BB190°	-0.06	-1.02	-0.05	-	-
5	3,6;2,5,4,1	13	SC	0.01	0.00	0.01	0.00	-

Panel B: Volatility index returns

tree	edge	No.	family	par	par2	tau	UTD	LTD
1	1,2	7	BB1	0.34	2.49	0.66	0.68	0.44
	1,6	7	BB1	0.14	1.19	0.21	0.21	0.01
	5,1	17	SBB1	0.38	1.29	0.35	0.24	0.29
	5,3	7	BB1	0.24	2.09	0.57	0.61	0.25
	5,4	7	BB1	0.40	3.41	0.76	0.77	0.60
2	5,2;1	5	F	0.88	0.00	0.10	-	-
	5,6;1	7	BB1	0.05	1.05	0.07	0.06	0.00
	3,1;5	5	F	0.69	0.00	0.08	-	-
	4,3;5	10	BB8	1.57	0.82	0.12	-	-
3	6,2;5,1	5	F	0.28	0.00	0.03	-	-
	3,6;5,1	5	F	0.31	0.00	0.03	-	-
	4,1;3,5	5	F	0.50	0.00	0.06	-	-
4	3,2;6,5,1	13	SC	0.03	0.00	0.01	0.00	-
	4,6;3,5,1	5	F	0.20	0.00	0.02	-	-
5	4,2;3,6,5,1	9	BB7	1.03	0.05	0.04	0.03	0.00

Note: This table reports the tree structure of vine copula for stock indices and volatility indices and corresponding parameters.

the table, all of the estimated tail indices are highly significant. Focusing on Tree 1, as expected, all 5 pairs of equity indices and volatility indices dependence captured by asymmetric BB1 copula and Survival BB1 copula. BB1 copula and Survival BB1 copula as well, known as Gumbel-Clayton copula, take the asymmetric tail dependence which applies to our financial contagion question very well. In addition, we can observe that in the all pairs of indices the strongest asymptotic dependence measured by the tail indices λ_u and λ_l occurs for the Euro Stoxx 50 index and DAX 30 index pair. That means extreme gains and losses of the entire European market are more likely to move together with the corresponding gains and losses of the Germany market comparing to any other stock markets. Their corresponding volatility indices also present highest tail dependence and asymmetric dependence among all pairs of volatility indices, whose upper tail dependence is 0.77, and lower tail dependence is 0.60. In this sense, Germany, as an important economy in Europe, probably have a stronger financial linkage with the entire European markets. As observed, S&P 500 and Nikkei 225 stock indices pair presents the lowest asymptotic dependence in the gains and losses, and in volatility index, also the US VIX and Japanese VXJ index pair exhibits weakest asymptotic dependence. However, both the stock indices pair and volatility indices pair of US and Japan exhibits high asymmetric dependence and tail dependence, which means probably there exist financial contagion transmit from US to Japan. Similarly, the stock indices pair of US and Germany take upper tail dependence of 0.35 and lower tail dependence of 0.21 also demonstrate there probably financial contagion transmit from US to Germany.

Though the static vine copula fitting results provide us some evidence of financial contagion, it is important to be aware that the asymptotic dependence behaviour is not necessarily related to a financial crisis. Which means markets can crash together as a result of bad news in which the impact only last for a short period (even a few days). In this sense, constant tail dependence parameters do not bring any definite confirmation about the behaviour of the dependence during turmoil periods, therefore, it is necessary to assessed by models with time-varying tail dependence parameters. Thus, once a period of crisis is identified, if the tail parameter increases after the crisis, we probably can say the dependence becomes stronger and possibly we can draw the conclusion that contagion exists.

Therefore, in this step, we fit our dynamic GAS R-vine copula to both equity indices and volatility indices, the corresponding parameter estimate results reports in Table 8. The evolution of the time-varying dependence measures of GAS R-vine copula, λ_u and λ_l, and Kendall's τ are presented in Figures 3 to 8. The evolution of dependence is clearer when observing the behaviour of both the upper and lower tail dependence coefficients. Thus, there is evidence of an increase in the dependence in most of the bivariate results for several periods. For the indices pairs, we found an increase in the dependence, at the beginning of 2002 (when the Internet bubble burst) and in the middle of 2011 (Euro crisis). An additional interesting finding is that indices pairs, the tail dependence increases after the first half of 2006, i.e. approximately one year before the beginning of the subprime crisis.

In particular, as discussed in static R vine copula model fitting, the joint dependence for volatility index returns of the S&P 500 and NASDAQ 100 varies more heavily than stock index returns. In terms of the evolution of tail dependence of stock indices pair, the tail dependence observed from the first four pairs of indices, which are S&P 500-Nasdaq 100, DAX30-S&P500, Europe Stoxx 50-DAX30, and FTSE-EURO Stoxx 50, significant increase after global market crises are found, such as, 9.11 in 2001, the global markets tumble at the end of February 2007, the most recent financial crisis at the end of 2008 (the Lehman Brother bankruptcy), and the 2011 Europe crisis. For example, during the Internet bubble burst in the late of 2001, the stock index pair DAX30-S&P 500 and volatility index pair of VSTOXX-VIX pair show more significant increase in dependence, which can be considered as an evidence of financial contagion transmit between US and Europe. Regarding the more serious Global Financial Crisis and Europe Debt Crisis, it is obviously to observe from the figures that all main markets dependence all increase significantly, which provide powerful evidence of financial contagion. The Nikkei225-FTSE 100 pair show strong tail dependence continuously comparing to the first four pairs, especially the period of after crisis. This results indicate that the strong linkage of Europe market and Asian market, and again provide the evidence of contagion transmit from UK or European market to Asian market. From volatility indices tail dependence evolution results, the increase of tail dependence not only has the similar trends with stock indices also more observable even if the dependence of market returns does not increase significantly comparing to stock indices, which indicate the volatility indices markets are more sensitive to crisis compared with stock markets, and reveals that contagion can also exist in cross-market volatilities. All these increasing tail dependence for these events are highly observable for the joint international markets, such as, the US and Germany, the US and the UK, and Germany and the Japan, which indicates that financial contagion exists. Most of the previous literature which only focuses on analysing contagion by investigating stock market returns is insufficient and may lead to wrong conclusions.

4.10 Assessing Contagion

The financial contagion normally refers to a significant increase of cross market linkage after a shock to one country or a group of countries. As discussed in above section, we employ dynamic GAS R-vine copula to investigate the existence of financial contagion. In this section, we devote to assess the presence of financial contagion based on the estimated GAS R vine copula models from the previous sections and relating our results with other proposals in the literature. As described in above section, there are several definitions of financial contagion in the academy. In this paper, we consider contagion to be defined as the significant increase of dependence between markets after a crisis, and two measures of dependence discussed above are employed: time-varying correlation Kendall's τ and time varying tail dependence indices-upper tail dependence λ_u and lower tail dependence λ_l. In our study, we investigate the presence of financial contagion by focusing on the analysis of the unconditional results of our GAS R vine Tree 1.

In this section, we adopt a hypothesis test framework in order to test the increasing tail dependence for crisis and post crisis periods. We set the bankruptcy of Lehman Brother as the event of shock, and define the pre-crisis period from 1st June 2008 to 14th September 2008, the crisis period is from 15th September 2008 to 15th October 2008, and the post-crisis period is from 16th October 2008 to 31st January 2009. λ_1 denotes the tail dependence coefficients for the pre-crisis period, λ_2 for the crisis period and λ_3 for the post-crisis period. Following Chen and Poon (2007), the null hypothesis used for the test of contagion is $H_0 : \lambda_2 = \lambda_1$ against $H_1 : \lambda_2 > \lambda_1$. If the null hypothesis is rejected at the 90% confidence level, which means the dependence of crisis period larger than pre-crisis periods, the existence of financial contagion be proved. The null hypothesis for the test of increasing tail dependence is set as $H_0 : \lambda_3 = \lambda_1$ against $H_1 : \lambda_3 > \lambda_1$. If the null hypothesis is rejected at the 90% confidence level, we can draw the conclusion that there

Table 4.8: **GAS R-vine Copula Parameters Estimates of Stock Indices and Volatility Indices**

Panel A: Stock index returns

tree	edge	No.	family	ω	ϕ	δ
1	1,6	17	SBB1	0.3883	0.0001	0.9778
				(0.0051)	(0.0000)	(0.0009)
	1,2	17	SBB1	0.5004	0.0023	0.9777
				(0.0046)	(0.0013)	(0.0000)
	4,1	7	BB1	0.4787	0.0001	0.9757
				(0.0080)	(0.0000)	(0.0005)
	5,3	17	SBB1	0.4304	0.0001	0.9831
				(0.0039)	(0.0000)	(0.0000)
	5,4	17	SBB1	0.5007	0.1377	0.9991
				(0.0004)	(0.0009)	(0.0000)

Panel B: Volatility index returns

tree	edge	No.	family	ω	ϕ	δ
1	1,2	7	BB1	0.5705	0.0182	0.9435
				(0.0000)	(0.0000)	(0.0000)
	1,6	7	BB1	0.5008	0.0042	0.9546
				(0.0051)	(0.0034)	(0.0000)
	5,1	17	SBB1	0.5327	0.0031	0.8372
				(0.0005)	(0.0000)	(0.0008)
	5,3	7	BB1	0.3767	0.0002	0.9829
				(0.0092)	(0.0000)	(0.0000)
	5,4	7	BB1	0.5002	0.0008	0.9666
				(0.0046)	(0.0019)	(0.0000)

Note: This table reports the GAS R-vine Copula Parameters Estimates of Stock Indices and Volatility Indices.

Figure 4.3: Evolution of upper tail dependence of stock indices

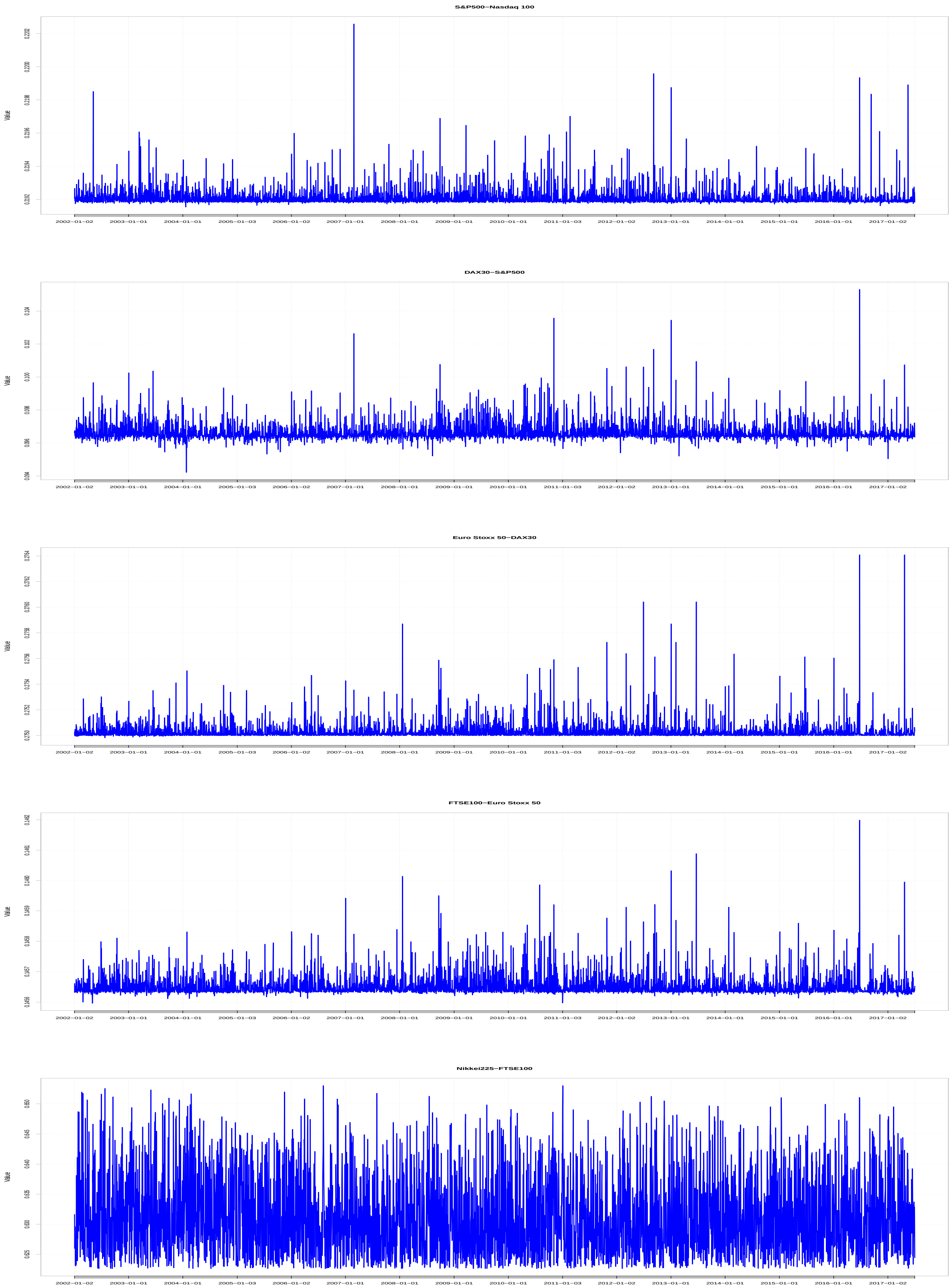


Figure 4.4: Evolution of upper tail dependence of volatility indices

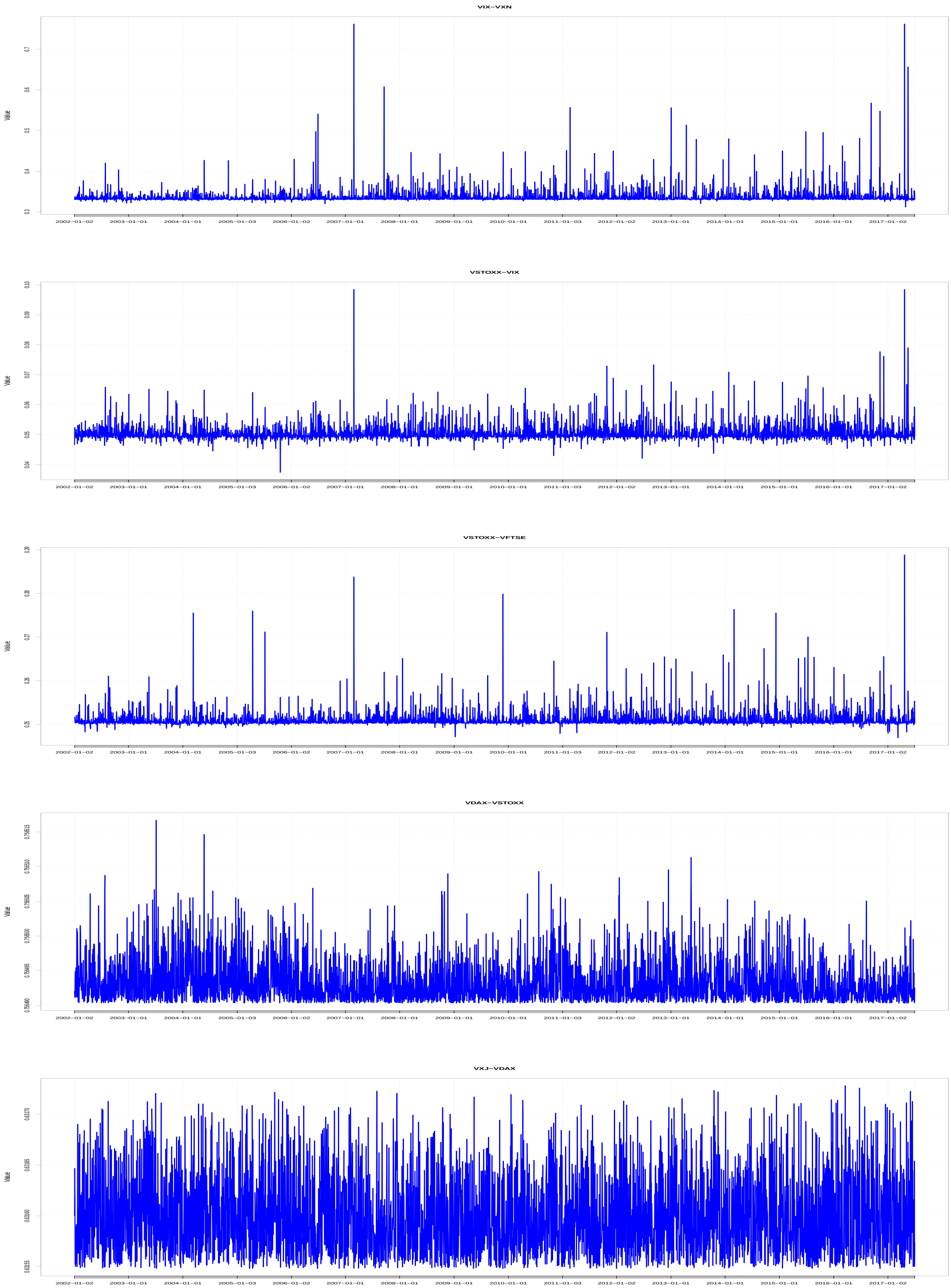


Figure 4.5: Evolution of lower tail dependence of stock indices

Figure 4.6: Evolution of lower tail dependence of volatility indices

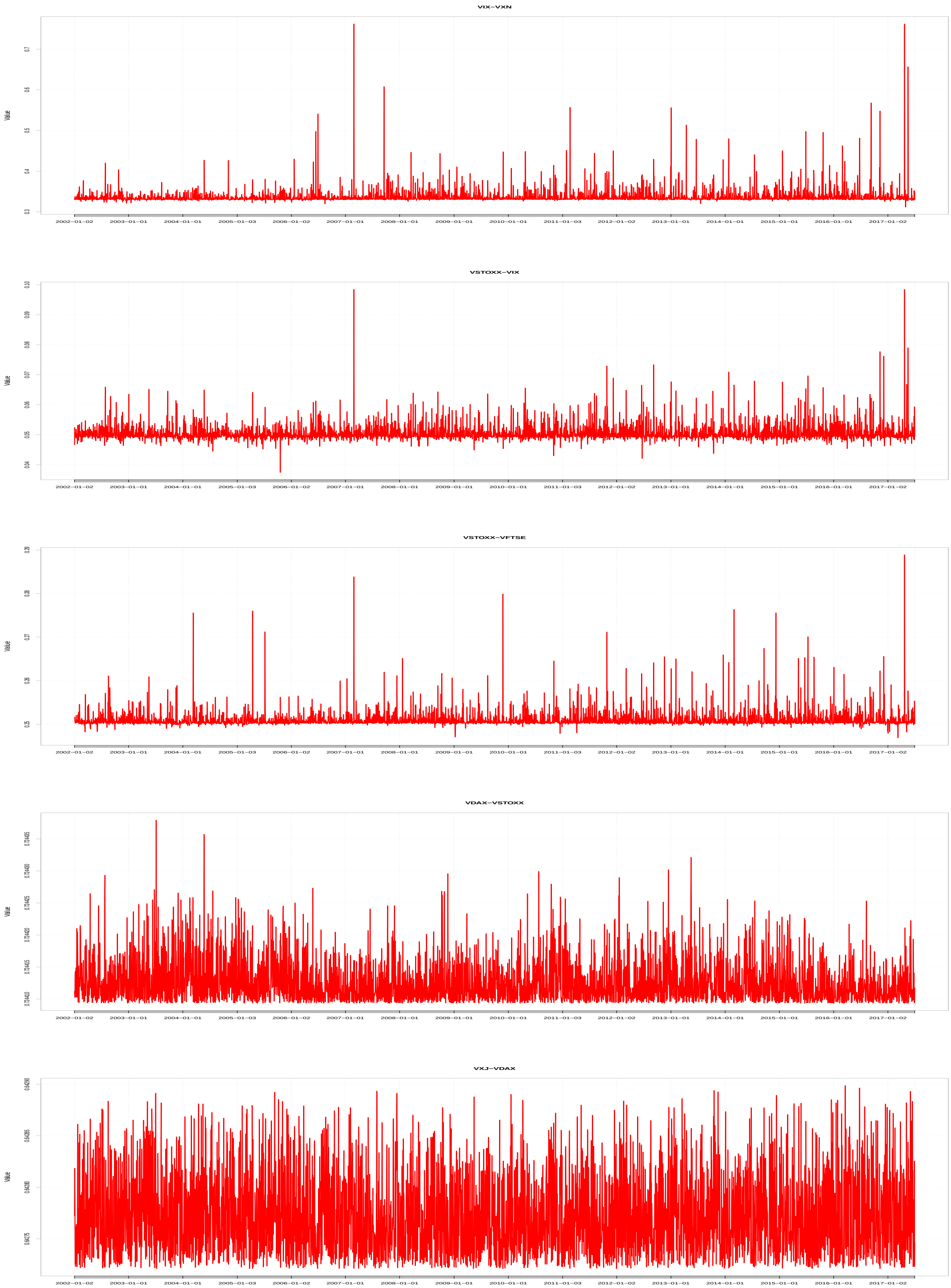


Figure 4.7: Evolution of Kendall's τ of stock indices

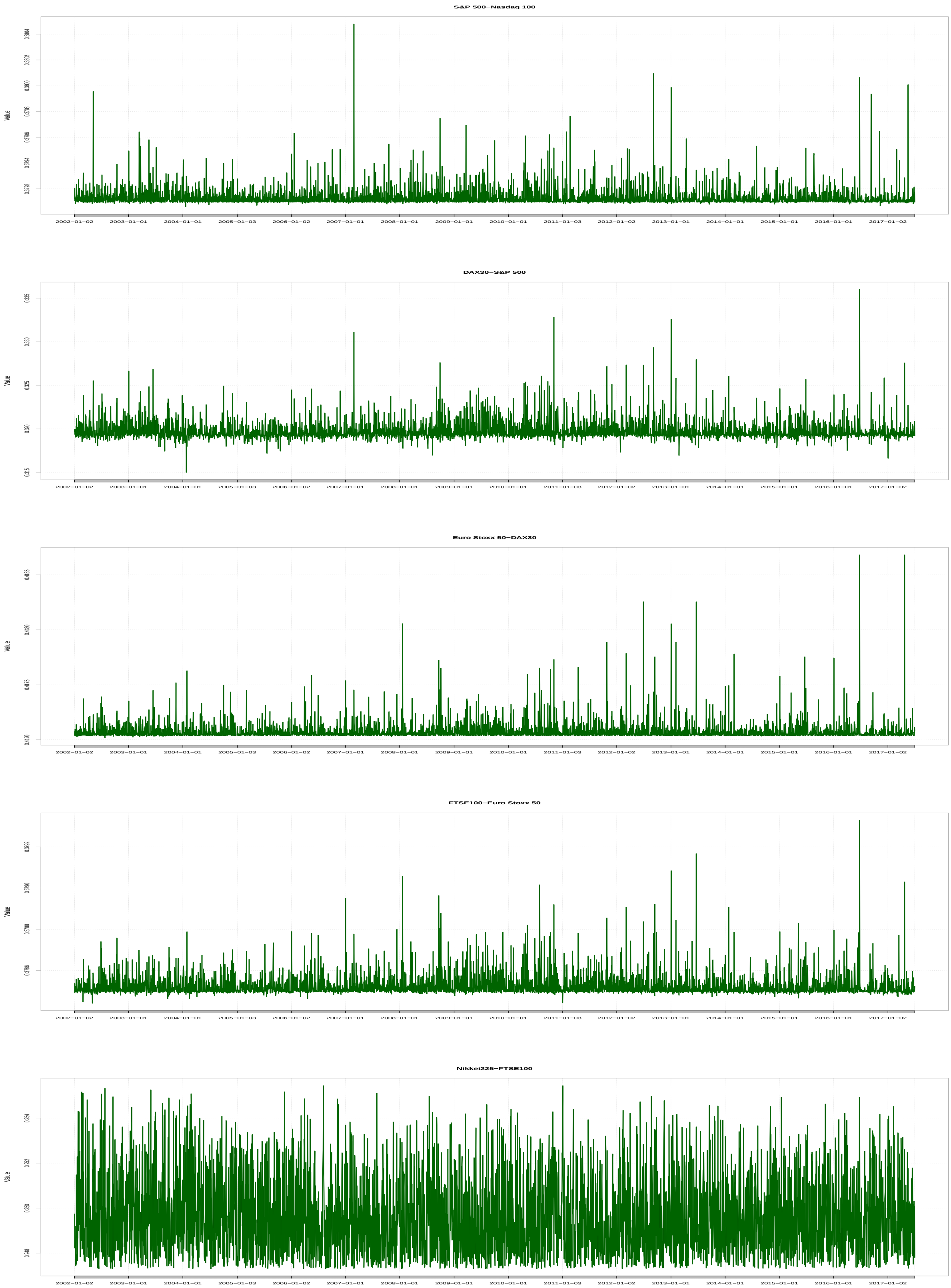
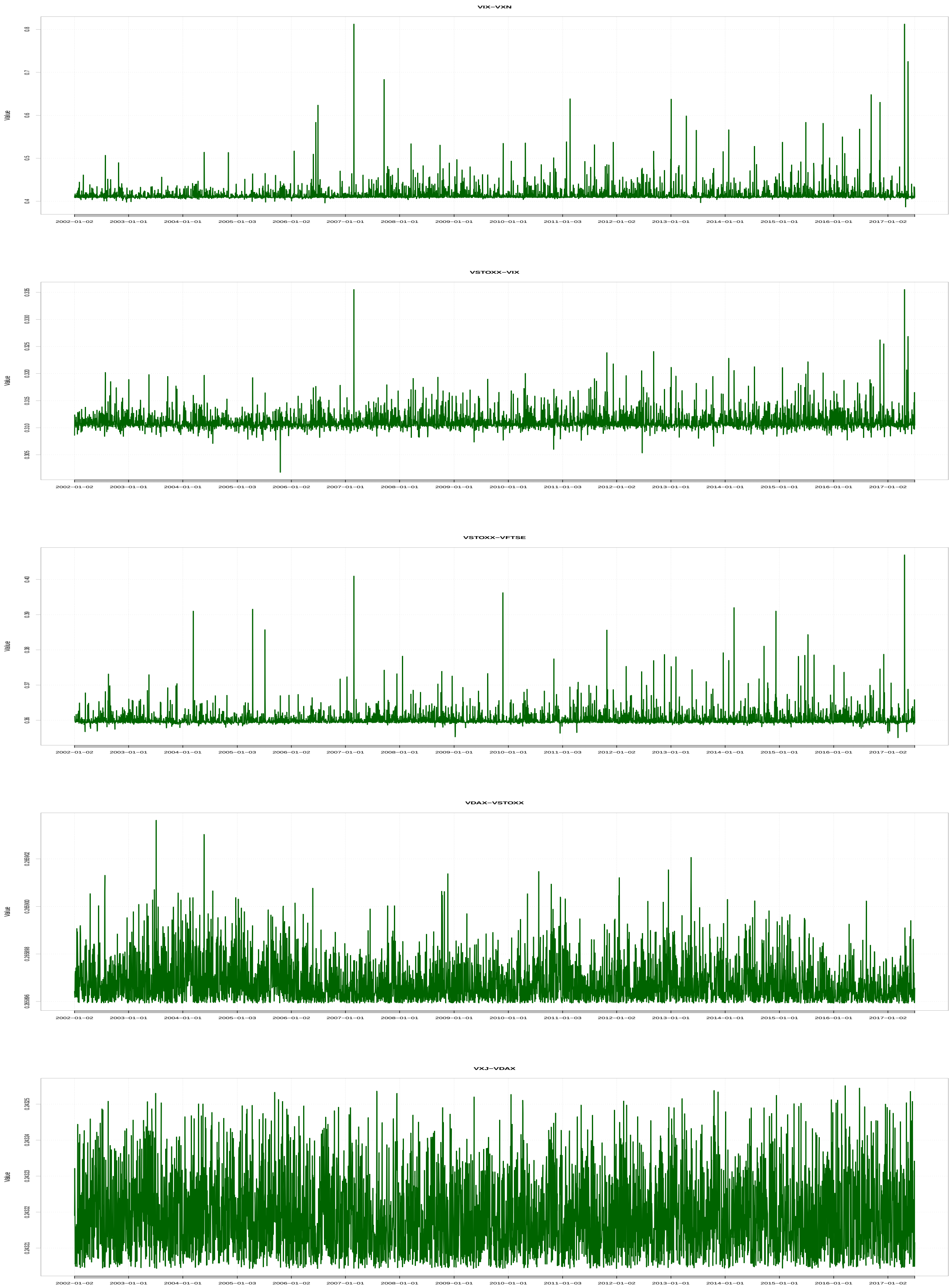


Figure 4.8: Evolution of Kendall's τ of volatility indices



is a significant increase in tail dependence during the post-crisis period.

Table 9 and Table 10 summarise the hypothesis tests results of our dynamic GAS R vine copula model. Contagion represents financial contagion is found during the crisis period, and Increase represents the tail dependence increases during the post-crisis period. We find there are several return pairs exhibiting contagion during the crisis periods and increasing tail dependence coefficients during the post-crisis periods. Regarding the stock index return pairs, more evidence of contagion and increasing tail dependence come from the lower tail dependence coefficients which also consistent with the static copula results. While for volatility index return pairs, more contagion evidence is found in the upper tail dependence coefficients, which also in line with the static results. These results confirm the fact that the dynamic tail dependence coefficients are asymmetric for both stock index and volatility index return pairs. The contagion evidence from lower tail dependence coefficients of stock index return pairs and upper tail dependence coefficients of volatility index return pairs require portfolio manager pay more attention to hedge the risk come from market dependence since the cross-market dependence significantly increases when market crash.

4.11 Backtesting

Since the dynamic DCC model, t-DCC model are widely used for time-varying modelling, it is necessary to investigate whether our GAS R-vine model is superior to other dynamic model. Therefore, we consider five competing models. For comparison, we first consider the traditional multivariate Gaussian copula with time-varying correlation matrix using the DCC dynamic of Engle (2002) previously adopted by Heinen and Valdesogo (2008). Moreover, due to the characteristics of tail dependence, multivariate Student *t* copula with DCC dynamic, denoted as *t*-DCC, is also included in our dynamic high-dimension model comparison. The third competing model is the constant dependence parameters static R-vine Copula model studied by Brechmann et al. (2012) and Dissmann et al. (2013). The purpose of choosing static R-vine model is to investigate whether it is necessary to employ a complex dynamic copula model. The structure of the R-vine is selected by the algorithms mentioned in Brechmann et al. (2012). The last competing model is our GAS R-vine model. For our GAS R-vine copula, either we can first rank our variables based on

Table 4.9: **Hypothesis tests for contagion and for increasing tail dependence**

Crisis Period													
Stock index returns							Volatility index returns						
Upper Tail													
	S&P 500	Nasdaq 100	FTSE100	DAX30	Euro Stoxx 50	Nikkei 225		VIX	VXN	VFTSE	VDAX	VSTOXX	VXJ
S&P 500	-	Increase	-	-	-	Increase	VIX	-	Crisis	Crisis/Increase	-	-	Crisis/Increase
Nasdaq 100	Increase	-	Crisis/Increase	-	-	-	VXN	Crisis	-	Crisis	-	-	Crisis
FTSE100	-	Crisis/Increase	-	-	-	Crisis	VFTSE	Crisis/Increase	Crisis	-	-	-	Crisis/Increase
DAX30	-	-	-	-	-	Crisis/Increase	VDAX	Increase	-	-	-	-	Crisis/Increase
Euro Stoxx 50	-	-	-	-	-	-	VSTOXX	-	-	-	-	-	-
Nikkei 225	Increase	-	Crisis	Crisis/Increase	Crisis/Increase	-	VXJ	Crisis/Increase	Crisis	Crisis/Increase	Crisis/Increase	Crisis/Increase	-
Lower Tail													
	S&P 500	Nasdaq 100	FTSE100	DAX30	Euro Stoxx 50	Nikkei 225		VIX	VXN	VFTSE	VDAX	VSTOXX	VXJ
S&P 500	-	-	Crisis	-	-	-	VIX	-	-	-	-	-	-
Nasdaq 100	-	-	-	-	-	Crisis/Increase	VXN	-	-	Increase	-	-	Increase
FTSE100	Crisis	-	-	Crisis/Increase	Crisis/Increase	Increase	VFTSE	-	Increase	-	-	-	-
DAX30	-	-	Crisis/Increase	-	-	-	VDAX	-	-	-	-	-	-
Euro Stoxx 50	-	-	-	-	-	-	VSTOXX	-	-	-	-	-	-
Nikkei 225	-	Crisis/Increase	Increase	-	-	-	VXJ	-	Increase	-	-	-	-

Note: This table reports the hypothesis tests for contagion and for increasing tail dependence of Crisis period.

Table 4.10: **Hypothesis tests for contagion and for increasing tail dependence**

Post Crisis Period													
Stock index returns							Volatility index returns						
Upper Tail													
	S&P 500	Nasdaq 100	FTSE100	DAX30	Euro Stoxx 50	Nikkei 225		VIX	VXN	VFTSE	VDAX	VSTOXX	VXJ
S&P 500	-	-	-	-	-	Increase	VIX	-	Crisis/Increase	Crisis	-	-	Crisis/Increase
Nasdaq 100	-	-	-	-	-	Crisis/Increase	VXN	Crisis/Increase	-	Increase	-	-	Crisis/Increase
FTSE100	-	-	-	-	-	Increase	VFTSE	Crisis	Increase	-	Crisis	Crisis	Crisis
DAX30	-	-	-	-	-	Increase	VDAX	Increase	Increase	Crisis	-	-	Crisis
Euro Stoxx 50	-	-	-	-	-	-	VSTOXX	-	-	-	-	-	-
Nikkei 225	Increase	Crisis/Increase	Increase	Increase	Increase	-	VXJ	Crisis/Increase	Crisis/Increase	Crisis	Crisis	Crisis	-
Lower Tail													
	S&P 500	Nasdaq 100	FTSE100	DAX30	Euro Stoxx 50	Nikkei 225		VIX	VXN	VFTSE	VDAX	VSTOXX	VXJ
S&P 500	-	Crisis/Increase	Increase	Increase	Increase	Increase	VIX	-	-	-	Crisis	Crisis	-
Nasdaq 100	Crisis/Increase	-	Increase	Increase	Increase	Crisis/Increase	VXN	-	-	Increase	Crisis	Crisis	-
FTSE100	Increase	Increase	-	Crisis/Increase	Crisis/Increase	Increase	VFTSE	Increase	-	-	-	-	-
DAX30	Increase	Increase	Crisis/Increase	-		-	VDAX	Crisis	Crisis	-	-	-	-
Euro Stoxx 50	-	-	-	-	-	-	VSTOXX	-	-	-	-	-	-
Nikkei 225	Increase	Crisis/Increase	-	-		-	VXJ	-	Increase	-	-	-	-

Note: This table reports the hypothesis tests for contagion and for increasing tail dependence of Post-Crisis period.

maximizing the overall pairwise dependence measured by Kendall's τ, which means the pair of variables with highest empirical Kendall's τ will be selected firstly. In a similar process, connect the next variable that has highest pairwise Kendall's τ with one of the previously chosen variables. In particular, we expect to capture the overall time variation of the dependence, as we discussed above, it turns out that time variation is most relevant on the first tree. The GAS R-vine model fit and forecasting performance are compared with Gaussian DCC copula model, Student t DCC copula model, and with a time-constant parameters Regular vine model. For the Gaussian and Student t copulas, we specify that the linear dependence parameter ρ_t evolves over time as in the DCC(1,1) model of Engle (2002):

$$Q_t = (1 - \bar{\alpha} - \bar{\beta}) \cdot \bar{Q} + \bar{\alpha} \cdot \epsilon_{t-1} \cdot \epsilon_{t-1}^T + \bar{\beta} \cdot Q_{t-1}, \tag{4.29}$$

$$\rho_t = Q_t^{*-1} Q_t Q_t^{*-1}, \tag{4.30}$$

where Q_t is the covariance matrix of the vector of first-step standardized residuals(ϵ_t)and $\bar{Q}$ is the unconditional covariance. Q_t^* is a square matrix with zeros as off-diagonal elements and the square root of those Q_t as diagonal elements.

We consider twelve indices from three major financial markets: US, Europe and Asia. In total, we have six equity indices as well as their corresponding implied volatility indices (cf. Table 1). We choose Standard and Poor's 500 Index and NASDAQ 100 Index representing US markets, FTSE 100 Index, DAX 30 Index and Euro Stoxx 50 Index representing European market, Nikkei 225 Stock Average Index representing Japanese market. The considered time period covers roughly 15 years, starting in particular on 1 January 2002 and ending on 30 Jun 2017. Excluding non-trading days, this results in 4044 observations of daily closing prices in US dollar. We split the sample into an in-sample period consisting of the first 3000 returns, covering the period until 1 July 2013, and an out-of-sample period covering the remaining 1044 observations.

We perform one-step ahead forecasts, and we do not re-estimate the models. For the out-of-sample fit of our model, we construct an equally weighted portfolio from the six stock market indices and volatility indices separately and estimate its value-at-risk at the 10%, 5%, and 1% level based on our four competing model specifications respectively. In Table 10, we report the exceedance rate of Kupiec (1995) unconditional coverage test, as well as the p-values of the dynamic quantile test by Engle and Manganelli (2004), which tests the correct coverage of the VaR and the independent identically distributed of the exceedances. Kupiec (1995) unconditional coverage test has been discussed in our Chapter 2, and the DQ test is essentially a Wald test for the overall significance of a linear probability model $\mathbf{H} - \alpha\mathbf{1} = \mathbf{X}\beta + \epsilon$ where $\mathrm{H} - \alpha\mathbf{1}$ with $\mathbf{H} = (H_{t+1})$ the demeaned hit variable, $\mathbf{1}$ is a vector of ones, $\mathbf{X} = (H_t, ..., H_{t-k}, VaR^{\alpha}_{t+1})'$ the regressor vector, and $\beta = (\beta_1, ..., \beta_{k+2})'$ the corresponding slope coefficients. The null hypothesis is $H_0 : \beta = 0$ and it can be tested using the Wald type test statistic,

$$DQ = \frac{\hat{\beta}'\mathbf{X}'\mathbf{X}\hat{\beta}}{\alpha(1-\alpha)} \sim \chi^2_{k+2} \tag{4.31}$$

We apply the test with 0 lags in order to test the unconditional coverage of the VaR and allow for four lags to additionally test the i.i.d.'ness. The results show that all models except the time-constant R-vine model perform well in terms of the unconditional coverage, in which our GAS R-vine copula model perform comparatively best. However, the i.i.d.ness of the VaR is rejected for all four models for the 1% VaR. Thus, it seems that choice of the dependence model not has a significant influence on the quality of the VaR forecasts as long as we allow for time variation in the dependence parameters, nevertheless, GAS R-vine copula model still demonstrate the superiority to some extent.

4.12 Conclusion

The common observation from the cross-market analysis reveals that all markets are interrelated, implying that events occur in one market have an impact on other markets. Cross-market dependence shows dynamic and asymmetric characteristics. Therefore, in this paper, we analysed the cross-market dependence for both stock index returns and volatility index returns by employing an innovative dynamic GAS R-vine copula approach and then investigate the existence of international financial contagion. To our best knowledge, there has been few tail dependence analysis both on dependence between different stock indices and volatility indices in the literature, and our analysis provides a new perspective to investigate the international financial contagion and asymmetric market dependence.

In this study, we first fit a skewed-t AR-GJR-GARCH model for the marginal distributions. We then fit a static constant parameters R-vine copula to the stock index and volatility index data, and then apply the dynamic GAS R-vine copula to measure the tail

Table 4.11: **Value at risk evaluation of Stock Indices and Volatility Indices**

Panel A: Stock index returns									
Model	Unconditional coverage test			DQ test 0 lags			DQ test 4 lags		
	10%	5%	1%	10%	5%	1%	10%	5%	1%
GAS R-vine	0.097	0.063	0.014	0.538	0.451	0.894	0.976	0.512	0.000
DCC	0.111	0.064	0.014	0.839	0.554	0.561	0.955	0.489	0.000
t-DCC	0.124	0.076	0.017	0.420	0.122	0.533	0.587	0.245	0.000
R-vine (constant)	0.118	0.088	0.021	0.541	0.043	0.068	0.910	0.049	0.000
Panel B: Volatility index returns									
Model	Unconditional coverage test			DQ test 0 lags			DQ test 4 lags		
	10%	5%	1%	10%	5%	1%	10%	5%	1%
GAS-RVine	0.091	0.059	0.012	0.534	0.446	0.887	0.969	0.508	0.000
DCC	0.107	0.061	0.013	0.832	0.550	0.551	0.951	0.483	0.000
t-DCC	0.118	0.074	0.015	0.417	0.119	0.531	0.584	0.240	0.000
R-vine (constant)	0.110	0.085	0.019	0.535	0.038	0.065	0.898	0.047	0.000

This table presents the evaluation of the Value at risk forecasts for both stock indices and volatility indices returns based on the out-of-sample period July 2013 until June 2017. DQ test refers to the p-value of the dynamic quantile test of Engle and Manganelli (2004).

dependence for both stock index returns and volatility index returns from financial markets indices in the US, Europe and Asia. From the static vine copula results, we primarily found some evidence of international financial contagion, the evolution of tail dependence coefficients and correlation Kendall's τ estimated from our GAS R-vine copula model provide us more dependable evidence of financial contagion, and reveal the direction of the contagion.

Comparing the volatility index returns and stock index returns, we find that the tail dependence change in the volatility indices are more easily observable, which means the dependence between the volatility indices is more sensitive and easily affected by market shocks, and reflects the instantaneous information (and the investors' predictions for future market movements) more rapidly than the stock indices. This is also consistent with the common observation in the literature that the volatility of market volatility is much greater than the market volatility itself. Different from the effect of news which can last a long time in stock indices, the shock to volatility indices can disappear completely within several hours, which is also important for hedging risk. The existence of contagion and tail dependence coefficients increasing in value during the period after crisis are found for both stock index and volatility index return pairs.

Our backtesting results demonstrate our GAS R-vine copula model outperform the Gaussian DCC, Student t DCC and static R-vine copula model from out-of-sample VaR forecasting.

In general, the results of our GAS R-vine copula model fitting to stock indices returns and volatility indices returns demonstrate the existence of international financial contagion between US, Europe and Asia financial markets, and our GAS R-vine copula model show the superiority to other competing dynamic models.

In sum, through our GAS R-vine copula model, we find the evidence supporting financial contagion, which possibly decrease the benefits of international portfolio diversification of both equity and volatility financial products. Both of the dependence structure of equity indices and volatility indices returns are asymmetric or has tail dependence characteristics. Volatility indices returns demonstrate that tail dependence lead to higher risk of turbulent markets. Comparing to the investors behaviour in calm period, the investors anticipation of trend of future markets movements tend to be similar in financial turmoil. The results of stock markets also support that portfolio managers are supposed to pay attention market downside risk. Moreover, the dependence of volatility indices returns is more sensitive than stock indices returns, and it can reflect instantaneous market turbulence quicker.

Table 12: **Bivariate copula family employed in Vine copula construction**

0 = independence copula
1 = Gaussian copula
2 = Student t copula (t-copula)
3 = Clayton copula
4 = Gumbel copula
5 = Frank copula
6 = Joe copula
7 = BB1 copula
8 = BB6 copula
9 = BB7 copula
10 = BB8 copula
13 = rotated Clayton copula (180 degrees; survival Clayton)
14 = rotated Gumbel copula (180 degrees; survival Gumbel)
16 = rotated Joe copula (180 degrees; survival Joe)
17 = rotated BB1 copula (180 degrees; survival BB1)
18 = rotated BB6 copula (180 degrees; survival BB6)
19 = rotated BB7 copula (180 degrees; survival BB7)
20 = rotated BB8 copula (180 degrees; survival BB8)
23 = rotated Clayton copula (90 degrees)
24 = rotated Gumbel copula (90 degrees)
26 = rotated Joe copula (90 degrees)
27 = rotated BB1 copula (90 degrees)
28 = rotated BB6 copula (90 degrees)
29 = rotated BB7 copula (90 degrees)
30 = rotated BB8 copula (90 degrees)
33 = rotated Clayton copula (270 degrees)
34 = rotated Gumbel copula (270 degrees)
36 = rotated Joe copula (270 degrees)
37 = rotated BB1 copula (270 degrees)
38 = rotated BB6 copula (270 degrees)
39 = rotated BB7 copula (270 degrees)
40 = rotated BB8 copula (270 degrees)
104 = Tawn type 1 copula
114 = rotated Tawn type 1 copula (180 degrees)
124 = rotated Tawn type 1 copula (90 degrees)
134 = rotated Tawn type 1 copula (270 degrees)
204 = Tawn type 2 copula
214 = rotated Tawn type 2 copula (180 degrees)
224 = rotated Tawn type 2 copula (90 degrees)
234 = rotated Tawn type 2 copula (270 degrees)

Note: This table lists all bivariate copula families we employ as Vine copula building blocks.

Chapter 5

Conclusion

This PhD thesis primarily deals with the dependence modelling of multivariate distributions of financial data and introduces novel vine copulas method to address significant financial modelling challenges in the credit portfolio risk management, asset allocation and financial contagion topic. In this respect, each chapter of the PhD thesis explores different research questions, focuses on multiple aspects of the finance hot topic and employs different modelling techniques that take into account the stylised features and complex dependence dynamics of financial data.

In particular, in our credit portfolio study, we compare various copula setting approaches both from a statistical and economic perspective. Vine copulas enable us to model a more flexible and less restricted dependence structure compared to classical Gaussian copula, as replacing the latter by the former leads to an increased AIC. The better statistical fit to the data suggests that the modeled dependence structure is a more realistic model of the actual dependence structure and, consequently, vine copula should be preferred to conventional Gaussian copula. When classic Gaussian copula is replaced by vine copula structures, the VaR and CVaR are all increased. C-vine mixed copula and R-vine mixed copula in turn lead to a higher risk measure than multivariate Gaussian copula. Flexible building blocks chosen from bivariate copula families in a vine structure results in more accurate and reliable estimate for VaR and CVaR. Therefore, we obtain statistically well-founded arguments that support the criticism of the role of the Gaussian copula in the financial crisis. We present the convenient and applicable alternative model-vine copula mixed model-which are supposed to be adopted by risk managers in order to improve the methodology of credit portfolio risk modelling.

Given the importance of alternative investments as an investment vehicle for investors to gain portfolio diversification benefits, and as traditional mean-variance portfolio strategy does not account for asymmetry in returns distributions, it is quite plausible that there is a need for more advanced portfolio management strategies that incorporate asymmetries especially when market regime changes over time. Therefore, our paper introduces a Markov regime switching regular vine copula asset allocation model in international assets markets and focuses on investigating, as the presence of regimes, whether the regime switching vine copula model is able to produce superior investment performance in the multi-asset case which including alternative investments compared to traditional models.

Through our GAS R-vine copula model, we find the evidence supporting financial contagion, which possibly decrease the benefits of international portfolio diversification of both equity and volatility financial products. Both of the dependence structure of equity indices and volatility indices returns are asymmetric or has tail dependence characteristics. Volatility indices returns demonstrate that tail dependence lead to higher risk of turbulent markets. Comparing to the investors behaviour in calm period, the investors anticipation of trend of future markets movements tend to be similar in financial turmoil. The results of stock markets also support that portfolio managers are supposed to pay attention to market downside risk. Moreover, the dependence of volatility indices returns is more sensitive than stock indices returns, and it can reflect instantaneous market turbulence quicker.

In general, our research results demonstrate that the extensively employed Gaussian dependence structures by practitioners and regulators is found definitely underestimate financial risk and lack the ability of capturing tail dependence and fat tail characteristics of the financial returns. The symmetric assumption of Gaussian copula or Student t copula and their lack of lower tail dependence coefficient are over simplistic leading to a systematic underestimation of financial risk and, in turn, endangering financial system. Therefore, it is crucial to incorporate tail dependence consideration. Against the above background of these criticism, it is very important to introduce vine copulas (also referred to as pair-copula constructions) to the financial returns dependence modelling.

Bibliography

Aas, K., Czado, C., Frigessi, A., and Bakken, H. (2009). Pair-copula constructions of multiple dependence. *Insurance: Mathematics and economics*, 44(2):182–198.

Aas, K. and Haff, I. H. (2006). The generalized hyperbolic skew students t-distribution. *Journal of financial econometrics*, 4(2):275–309.

Acar, E. F., Genest, C., and NešLehová, J. (2012). Beyond simplified pair-copula constructions. *Journal of Multivariate Analysis*, 110:74–90.

Adam, A., Houkari, M., and Laurent, J.-P. (2008). Spectral risk measures and portfolio selection. *Journal of Banking & Finance*, 32(9):1870–1882.

Äijö, J. (2008). Implied volatility term structure linkages between vdax, vsmi and vstoxx volatility indices. *Global Finance Journal*, 18(3):290–302.

Akaike, H. (1974). A new look at the statistical model identification. *Automatic Control, IEEE Transactions on*, 19(6):716–723.

Alaganar, V. T. and Bhar, R. (2002). Information and volatility linkage under external shocks: Evidence from dually listed australian stocks. *International review of financial analysis*, 11(1):59–71.

Almeida, C. and Czado, C. (2012). Efficient bayesian inference for stochastic time-varying copula models. *Computational Statistics & Data Analysis*, 56(6):1511–1527.

Amin, G. S. and Kat, H. M. (2002). Diversification and yield enhancement with hedge funds. *The Journal of Alternative Investments*, 5(3):50–58.

Amin, G. S., Kat, H. M., et al. (2003). Hedge fund performance 1990-2000: Do the" money machines" really add value? *Journal of financial and quantitative analysis*, 38(2):251–274.

Ammann, M. and Süss, S. (2009). Asymmetric dependence patterns in financial time series. *The European Journal of Finance*, 15(7-8):703–719.

Andersson, F., Mausser, H., Rosen, D., and Uryasev, S. (2001). Credit risk optimization with conditional value-at-risk criterion. *Mathematical Programming*, 89(2):273–291.

Ang, A. and Bekaert, G. (2002a). International asset allocation with regime shifts. *Review of Financial studies*, 15(4):1137–1187.

Ang, A. and Bekaert, G. (2002b). Regime switches in interest rates. *Journal of Business & Economic Statistics*, 20(2):163–182.

Ang, A. and Chen, J. (2002). Asymmetric correlations of equity portfolios. *Journal of financial Economics*, 63(3):443–494.

Arakelian, V. and Dellaportas, P. (2012). Contagion determination via copula and volatility threshold models. *Quantitative Finance*, 12(2):295–310.

Arora, N., Bohn, J. R., and Zhu, F. (2005). Reduced form vs. structural models of credit risk: A case study of three models. *Journal of Investment Management*, 3(4):43.

Artzner, P., Delbaen, F., Eber, J.-M., and Heath, D. (1999). Coherent measures of risk. *Mathematical finance*, 9(3):203–228.

Balduzzi, P. and Lynch, A. W. (1999). Transaction costs and predictability: Some utility cost calculations. *Journal of Financial Economics*, 52(1):47–78.

Barnett-Hart, A. K. (2009). The story of the cdo market meltdown: An empirical analysis. *Bachelor of Arts (Hons), Harvard University*.

Basel (2004). International convergence of capital measurement and capital standards. *A revised framework. BIS*.

Bauwens, L., Laurent, S., and Rombouts, J. V. (2006). Multivariate garch models: a survey. *Journal of applied econometrics*, 21(1):79–109.

Bedford, T. and Cooke, R. M. (2001). Probability density decomposition for conditionally dependent random variables modeled by vines. *Annals of Mathematics and Artificial intelligence*, 32(1-4):245–268.

Bedford, T. and Cooke, R. M. (2002). Vines: A new graphical model for dependent random variables. *Annals of Statistics*, pages 1031–1068.

Bekiros, S. D. and Georgoutsos, D. A. (2008). The extreme-value dependence of asia-pacific equity markets. *Journal of Multinational Financial Management*, 18(3):197–208.

Bellini, F. and Gianin, E. R. (2008). On haezendonck risk measures. *Journal of Banking & Finance*, 32(6):986–994.

Bernardi, M. and Catania, L. (2015). Switching-gas copula models for systemic risk assessment. *arXiv preprint arXiv:1504.03733*.

Blair, B. J., Poon, S.-H., and Taylor, S. J. (2010). Forecasting s&p 100 volatility: the incremental information content of implied volatilities and high-frequency index returns. In *Handbook of Quantitative Finance and Risk Management*, pages 1333–1344. Springer.

Blasques, F., Koopman, S. J., and Lucas, A. (2012). Stationarity and ergodicity of univariate generalized autoregressive score processes.

Blasques, F., Koopman, S. J., and Lucas, A. (2015). Information-theoretic optimality of observation-driven time series models for continuous responses. *Biometrika*, 102(2):325–343.

Bollerslev, T. (1990). Modelling the coherence in short-run nominal exchange rates: a multivariate generalized arch model. *The review of economics and statistics*, pages 498–505.

Bollerslev, T. (1995). Modelling the coherence in short run nominal exchange rates: a multivariate generalized arch model. *RF Engle*, pages 300–313.

Boyer, B. H., Gibson, M. S., Loretan, M., et al. (1997). *Pitfalls in tests for changes in correlations*, volume 597. Board of Governors of the Federal Reserve System Washington, DC.

Brechmann, E. (2010). Truncated and simplified regular vines and their applications diploma thesis. *University of Technology, Munich, Germany*.

Brechmann, E. C. and Czado, C. (2015). Coparmultivariate time series modeling using the copula autoregressive model. *Applied Stochastic Models in Business and Industry*, 31(4):495–514.

Brechmann, E. C., Czado, C., and Aas, K. (2012). Truncated regular vines in high dimensions with application to financial data. *Canadian Journal of Statistics*, 40(1):68–85.

Caporin, M. and McAleer, M. (2014). Robust ranking of multivariate garch models by problem dimension. *Computational Statistics & Data Analysis*, 76:172–185.

Cespedes, J. C. G., de Juan Herrero, J. A., Kreinin, A., and Rosen, D. (2006). A simple multi-factor factor adjustment for the treatment of credit capital diversification. *Journal of Credit Risk*, 2(3):57–85.

Chandrashekaran, V. (1999). Time-series properties and diversification benefits of reit returns. *Journal of Real Estate Research*, 17(1):91–112.

Changqing, L., Yanlin, L., and Mengzhen, L. (2015). Credit portfolio risk evaluation based on the pair copula var models. *J. Financ. Econ*, 3:15–30.

Chen, P., Baierl, G. T., and Kaplan, P. D. (2002). Venture capital and its role in strategic asset allocation. *The Journal of Portfolio Management*, 28(2):83–89.

Chen, Q. and Gerlach, R. H. (2013). The two-sided weibull distribution and forecasting financial tail risk. *International Journal of Forecasting*, 29(4):527–540.

Chen, S. and Poon, S.-H. (2007). Modelling international stock market contagion using copula and risk appetite.

Cherubini, U., Luciano, E., and Vecchiato, W. (2004). *Copula methods in finance*. John Wiley & Sons.

Chiang, K. and National, M.-L. (2007). Spanning tests on public and private real estate. *Journal of Real Estate Portfolio Management*, 13(1):7–15.

Chollete, L., Heinen, A., and Valdesogo, A. (2009). Modeling international financial returns with a multivariate regime-switching copula. *Journal of financial econometrics*, 7(4):437–480.

Chowdhury, I. and Sarno, L. (2004). Time-varying volatility in the foreign exchange market: New evidence on its persistence and on currency spillovers. *Journal of Business Finance & Accounting*, 31(5-6):759–793.

Christoffersen, P. and Pelletier, D. (2004). Backtesting value-at-risk: A duration-based approach. *Journal of Financial Econometrics*, 2(1):84–108.

Christoffersen, P. F. (1998). Evaluating interval forecasts. *International economic review*, pages 841–862.

Christoffersen, P. F. (2012). *Elements of financial risk management*. Academic Press.

Ciccarelli, M. and Rebucci, A. (2007). Measuring contagion and interdependence with a bayesian time-varying coefficient model: An application to the chilean fx market during the argentine crisis. *Journal of Financial Econometrics*, 5(2):285–320.

Claessens, S. and Forbes, K. (2001). International financial contagion: An overview of the issues and the book. In *International financial contagion*, pages 3–17. Springer.

Conover, C. M., Jensen, G. R., Johnson, R. R., and Mercer, J. M. (2010). Is now the time to add commodities to your portfolio? *The Journal of Investing*, 19(3):10–19.

Creal, D., Koopman, S. J., and Lucas, A. (2013). Generalized autoregressive score models with applications. *Journal of Applied Econometrics*, 28(5):777–795.

Creal, D. D. and Tsay, R. S. (2015). High dimensional dynamic stochastic copula models. *Journal of Econometrics*, 189(2):335–345.

Crosbie, P. and Bohn, J. (2003). Modeling default risk.

Crosbie, P. J. and Bohn, J. R. (2002). Modeling default risk, 2002. *KMV, San Francisco*.

Crouhy, M. G., Jarrow, R. A., and Turnbull, S. M. (2008). The subprime credit crisis of 2007. *The Journal of Derivatives*, 16(1):81–110.

Czado, C. (2010). Pair-copula constructions of multivariate copulas. In *Copula theory and its applications*, pages 93–109. Springer.

Da Costa Dias, A. and Embrechts, P. (2004). Change-point analysis for dependence structures in finance and insurance.

Daskalaki, C. and Skiadopoulos, G. (2011). Should investors include commodities in their portfolios after all? new evidence. *Journal of Banking & Finance*, 35(10):2606–2626.

Daul, S., De Giorgi, E. G., Lindskog, F., and McNeil, A. (2003). The grouped t-copula with an application to credit risk. *Available at SSRN 1358956*.

Demarta, S. and McNeil, A. J. (2005). The t copula and related copulas. *International Statistical Review/Revue Internationale de Statistique*, pages 111–129.

DeMiguel, V., Garlappi, L., and Uppal, R. (2009). Optimal versus naive diversification: How inefficient is the 1/n portfolio strategy? *Review of Financial Studies*, 22(5):1915–1953.

Dempster, A. P., Laird, N. M., and Rubin, D. B. (1977). Maximum likelihood from incomplete data via the em algorithm. *Journal of the royal statistical society. Series B (methodological)*, pages 1–38.

Denault, M. (2001). Coherent allocation of risk capital. *Journal of risk*, 4:1–34.

di Basilea per la vigilanza bancaria, C. (2004). *International convergence of capital measurement and capital standards: a revised framework*. Bank for International Settlements.

Diebold, F. X. and Mariano, R. S. (2002). Comparing predictive accuracy. *Journal of Business & economic statistics*, 20(1):134–144.

Dissmann, J., Brechmann, E. C., Czado, C., and Kurowicka, D. (2013). Selecting and estimating regular vine copulae and application to financial returns. *Computational Statistics & Data Analysis*, 59:52–69.

Dorfleitner, G., Fischer, M., and Geidosch, M. (2012). Specification risk and calibration effects of a multifactor credit portfolio model. *The Journal of Fixed Income*, 22(1):7–24.

Duffie, D. and Singleton, K. J. (1999). Modeling term structures of defaultable bonds. *Review of Financial studies*, 12(4):687–720.

Embrechts, P., McNeil, A., and Straumann, D. (2002). Correlation and dependence in risk management: properties and pitfalls. *Risk management: value at risk and beyond*, 176223.

Engle, R. (2002). Dynamic conditional correlation: A simple class of multivariate generalized autoregressive conditional heteroskedasticity models. *Journal of Business & Economic Statistics*, 20(3):339–350.

Engle, R. F. (1982). Autoregressive conditional heteroscedasticity with estimates of the variance of united kingdom inflation. *Econometrica: Journal of the Econometric Society*, pages 987–1007.

Engle, R. F. and Manganelli, S. (2004). Caviar: Conditional autoregressive value at risk by regression quantiles. *Journal of Business & Economic Statistics*, 22(4):367–381.

Engle, R. F. and Sheppard, K. (2001). Theoretical and empirical properties of dynamic conditional correlation multivariate garch. Technical report, National Bureau of Economic Research.

Ennis, R. M. and Sebastian, M. D. (2005). Asset allocation with private equity. *The Journal of Private Equity*, 8(3):81–87.

Erb, C. B. and Harvey, C. R. (2006). The strategic and tactical value of commodity futures. *Financial Analysts Journal*, 62(2):69–97.

Erdorf, S., Hartmann-Wendels, T., Heinrichs, N., et al. (2011). Diversification in firm valuation: A multivariate copula approach. Technical report, Cologne Graduate School in Management, Economics and Social Sciences.

Fabozzi, F. J., Chen, R.-R., Hu, S.-Y., and Pan, G.-G. (2010). Tests of the performance of structural models in bankruptcy prediction. *The journal of credit risk*, 6(2):37.

Fang, H.-B., Fang, K.-T., and Kotz, S. (2002). The meta-elliptical distributions with given marginals. *Journal of Multivariate Analysis*, 82(1):1–16.

Fermanian, J.-D. and Scaillet, O. (2005). Sensitivity analysis of var and expected shortfall for portfolios under netting agreements. *Journal of Banking & Finance*, 29(4):927–958.

Fernández, C. and Steel, M. F. (1998). On bayesian modeling of fat tails and skewness. *Journal of the American Statistical Association*, 93(441):359–371.

Fischer, M., Köck, C., Schlüter, S., and Weigert, F. (2009). An empirical analysis of multivariate copula models. *Quantitative Finance*, 9(7):839–854.

Fleming, J., Ostdiek, B., and Whaley, R. E. (1995). Predicting stock market volatility: A new measure. *Journal of Futures Markets*, 15(3):265–302.

Forbes, K. J. and Rigobon, R. (2002). No contagion, only interdependence: measuring stock market comovements. *The journal of Finance*, 57(5):2223–2261.

Frahm, G., Junker, M., and Szimayer, A. (2003). Elliptical copulas: applicability and limitations. *Statistics & Probability Letters*, 63(3):275–286.

Frey, R., McNeil, A. J., McNeil, A. J., and McNeil, A. J. (2001). *Modelling dependent defaults*. ETH, Eidgenössische Technische Hochschule Zürich, Department of Mathematics.

Frye, J. (2008). Correlation and asset correlation in the structural portfolio model. *The Journal of Credit Risk*, 4(2):75–96.

Fung, J. K., Lien, D., Tse, Y., and Tse, Y. K. (2005). Effects of electronic trading on the hang seng index futures market. *International Review of Economics & Finance*, 14(4):415–425.

Garcia, R. and Tsafack, G. (2011). Dependence structure and extreme comovements in international equity and bond markets. *Journal of Banking & Finance*, 35(8):1954–1970.

Genest, C., Ghoudi, K., and Rivest, L.-P. (1995). A semiparametric estimation procedure of dependence parameters in multivariate families of distributions. *Biometrika*, 82(3):543–552.

Giacomini, E., Härdle, W., and Spokoiny, V. (2009). Inhomogeneous dependence modeling with time-varying copulae. *Journal of Business & Economic Statistics*, 27(2):224–234.

Giacomini, R. and White, H. (2006). Tests of conditional predictive ability. *Econometrica*, 74(6):1545–1578.

Giot, P. (2005). Implied volatility indexes and daily value at risk models. *The Journal of derivatives*, 12(4):54–64.

Glasserman, P. (2004). Tail approximations for portfolio credit risk. *The Journal of Derivatives*, 12(2):24–42.

Glasserman, P. and Li, J. (2005). Importance sampling for portfolio credit risk. *Management science*, 51(11):1643–1656.

Glosten, L. R., Jagannathan, R., and Runkle, D. E. (1993). On the relation between the expected value and the volatility of the nominal excess return on stocks. *The journal of finance*, 48(5):1779–1801.

González-Rivera, G., Lee, T.-H., and Mishra, S. (2004). Forecasting volatility: A reality check based on option pricing, utility function, value-at-risk, and predictive likelihood. *International Journal of forecasting*, 20(4):629–645.

Gordy, M. B. (2000). A comparative anatomy of credit risk models. *Journal of Banking & Finance*, 24(1):119–149.

Gordy, M. B. (2003). A risk-factor model foundation for ratings-based bank capital rules. *Journal of financial intermediation*, 12(3):199–232.

Gorton, G. and Rouwenhorst, K. G. (2006). Facts and fantasies about commodity futures (digest summary). *Financial Analysts Journal*, 62(2):47–68.

Gourieroux, C., Laurent, J. P., and Scaillet, O. (2000). Sensitivity analysis of values at risk. *Journal of empirical finance*, 7(3):225–245.

Gray, S. F. (1996). Modeling the conditional distribution of interest rates as a regime-switching process. *Journal of Financial Economics*, 42(1):27–62.

Gueyie, J.-P. and Amvella, S. P. (2006). Optimal portfolio allocation using funds of hedge funds. *The Journal of Wealth Management*, 9(2):85–95.

Guidolin, M. and Timmermann, A. (2006a). An econometric model of nonlinear dynamics in the joint distribution of stock and bond returns. *Journal of Applied Econometrics*, 21(1):1–22.

Guidolin, M. and Timmermann, A. (2006b). Term structure of risk under alternative econometric specifications. *Journal of Econometrics*, 131(1):285–308.

Guidolin, M. and Timmermann, A. (2008). International asset allocation under regime switching, skew, and kurtosis preferences. *Review of financial studies*, 21(2):889–935.

Haff, I. H. (2012). Comparison of estimators for pair-copula constructions. *Journal of Multivariate Analysis*, 110:91–105.

Haff, I. H. et al. (2013). Parameter estimation for pair-copula constructions. *Bernoulli*, 19(2):462–491.

Hafner, C. M. and Manner, H. (2012). Dynamic stochastic copula models: Estimation, inference and applications. *Journal of Applied Econometrics*, 27(2):269–295.

Hafner, C. M. and Reznikova, O. (2010). Efficient estimation of a semiparametric dynamic copula model. *Computational Statistics & Data Analysis*, 54(11):2609–2627.

Hamilton, J. D. (1989). A new approach to the economic analysis of nonstationary time series and the business cycle. *Econometrica: Journal of the Econometric Society*, pages 357–384.

Hansen, B. E. (1994). Autoregressive conditional density estimation. *International Economic Review*, pages 705–730.

Hansen, P. R. (2005). A test for superior predictive ability. *Journal of Business & Economic Statistics*, 23(4):365–380.

Hansen, P. R. and Lunde, A. (2005). A forecast comparison of volatility models: does anything beat a garch (1, 1)? *Journal of applied econometrics*, 20(7):873–889.

Hansen, P. R., Lunde, A., and Nason, J. M. (2003). Choosing the best volatility models: the model confidence set approach. *Oxford Bulletin of Economics and Statistics*, 65(s1):839–861.

Hansen, P. R., Lunde, A., and Nason, J. M. (2011). The model confidence set. *Econometrica*, 79(2):453–497.

Harvey, A., Ruiz, E., and Shephard, N. (1994). Multivariate stochastic variance models. *The Review of Economic Studies*, 61(2):247–264.

Heinen, A. and Valdesogo, A. (2008). Asymmetric capm dependence for large dimensions: The canonical vine autoregressive copula model. *Available at SSRN 1297506*.

Hofert, M. (2011). Efficiently sampling nested archimedean copulas. *Computational Statistics & Data Analysis*, 55(1):57–70.

Hofmann, M. and Czado, C. (2010). Assessing the var of a portfolio using d-vine copula based multivariate garch models. *Submitted for publication*, pages 1–36.

Hu, J. (2010). Dependence structures in chinese and us financial markets: a time-varying conditional copula approach. *Applied Financial Economics*, 20(7):561–583.

Hu, L. (2006). Dependence patterns across financial markets: a mixed copula approach. *Applied financial economics*, 16(10):717–729.

Huang, J.-z. and Zhong, Z. K. (2013). Time variation in diversification benefits of commodity, reits, and tips. *The Journal of Real Estate Finance and Economics*, 46(1):152–192.

Hudson-Wilson, S., Fabozzi, F. J., and Gordon, J. N. (2003). Why real estate? *The Journal of Portfolio Management*, 29(5):12–25.

Hull, J. C. (2008). The credit crunch of 2007: What went wrong? why? what lessons can be learned?

Hull, J. C. and White, A. D. (2004). Valuation of a cdo and an n-th to default cds without monte carlo simulation. *The Journal of Derivatives*, 12(2):8–23.

Jarque, C. M. and Bera, A. K. (1980). Efficient tests for normality, homoscedasticity and serial independence of regression residuals. *Economics letters*, 6(3):255–259.

Jarrow, R. and Protter, P. (2004). Structural versus reduced form models: a new information based perspective. *Journal of Investment management*, 2(2):1–10.

Jarrow, R. A. and Turnbull, S. M. (1995). Pricing derivatives on financial securities subject to credit risk. *The journal of finance*, 50(1):53–85.

Joe, H. (1997). *Multivariate models and multivariate dependence concepts*. CRC Press.

Joe, H. (2005). Asymptotic efficiency of the two-stage estimation method for copula-based models. *Journal of Multivariate Analysis*, 94(2):401–419.

Joe, H. and Kurowicka, D. (2011). *Dependence modeling: vine copula handbook*. World Scientific.

Joe, H., Li, H., and Nikoloulopoulos, A. K. (2010). Tail dependence functions and vine copulas. *Journal of Multivariate Analysis*, 101(1):252–270.

Jondeau, E. and Rockinger, M. (2006). The copula-garch model of conditional dependencies: An international stock market application. *Journal of international money and finance*, 25(5):827–853.

Kalkbrener, M. (2005). An axiomatic approach to capital allocation. *Mathematical Finance*, 15(3):425–437.

Kealhofer, S. and Bohn, J. R. (2001). Portfolio management of default risk. *Net Exposure*, 1(2):12.

Kim, C.-J., Nelson, C. R., et al. (1999). State-space models with regime switching: classical and gibbs-sampling approaches with applications. *MIT Press Books*, 1.

Kim, G., Silvapulle, M. J., and Silvapulle, P. (2007). Comparison of semiparametric and parametric methods for estimating copulas. *Computational Statistics & Data Analysis*, 51(6):2836–2850.

Koch, P. D. and Koch, T. W. (1991). Evolution in dynamic linkages across daily national stock indexes. *Journal of International Money and Finance*, 10(2):231–251.

Kooli, M. (2007). The diversification benefits of hedge funds and funds of hedge funds. *Derivatives Use, Trading & Regulation*, 12(4):290–300.

Koopman, S. J., Lucas, A., and Scharth, M. (2016). Predicting time-varying parameters with parameter-driven and observation-driven models. *Review of Economics and Statistics*, 98(1):97–110.

Koyluoglu, U. and Stoker, J. (2002). Risk decomposition honour your contribution. *RISK-LONDON-RISK MAGAZINE LIMITED-*, 15(4):90–94.

Kroner, K. F. and Ng, V. K. (1998). Modeling asymmetric comovements of asset returns. *The review of financial studies*, 11(4):817–844.

Krylova, E., Nikkinen, J., and Vähämaa, S. (2009). Cross-dynamics of volatility term structures implied by foreign exchange options. *Journal of Economics and Business*, 61(5):355–375.

Kuester, K., Mittnik, S., and Paolella, M. S. (2006). Value-at-risk prediction: A comparison of alternative strategies. *Journal of Financial Econometrics*, 4(1):53–89.

Kupiec, P. H. (1995). Techniques for verifying the accuracy of risk measurement models. *The journal of Derivatives*, 3(2):73–84.

Kurowicka, D. and Cooke, R. M. (2006). *Uncertainty analysis with high dimensional dependence modelling*. John Wiley & Sons.

Kurowicka, D. and Joe, H. (2011). Dependence modeling-handbook on vine copulae.

Lagrado, R. and Osher, S. (1997). Reconciling differences. *RISK-LONDON-RISK MAGAZINE LIMITED-*, 10:79–83.

Lando, D. (1998). On cox processes and credit risky securities. *Review of Derivatives research*, 2(2-3):99–120.

Ledford, A. W. and Tawn, J. A. (1996). Statistics for near independence in multivariate extreme values. *Biometrika*, 83(1):169–187.

Lhabitant, F.-S. and Learned, M. (2002). Hedge fund diversification: How much is enough? *The Journal of Alternative Investments*, 5(3):23–49.

Li, D. X. (2000). On default correlation: A copula function approach. *The Journal of Fixed Income*, 9(4):43–54.

Longin, F. and Solnik, B. (1995). Is the correlation in international equity returns constant: 1960–1990? *Journal of international money and finance*, 14(1):3–26.

Longin, F. and Solnik, B. (2001). Extreme correlation of international equity markets. *The journal of finance*, 56(2):649–676.

Lopez, J. A. (1998). Regulatory evaluation of value-at-risk models.

Lopez, J. A. (1999). Methods for evaluating value-at-risk estimates. *Economic Review-Federal Reserve Bank of San Francisco*, (2):3.

Loretan, M. and English, W. B. (2000). Evaluating correlation breakdowns during periods of market volatility.

Low, C. (2004). The fear and exuberance from implied volatility of s&p 100 index options. *The Journal of Business*, 77(3):527–546.

Manner, H. (2007). *Estimation and model selection of copulas with an application to exchange rates*. Citeseer.

Manner, H. and Reznikova, O. (2012). A survey on time-varying copulas: specification, simulations, and application. *Econometric Reviews*, 31(6):654–687.

Markowitz, H. (1952). Portfolio selection. *The journal of finance*, 7(1):77–91.

Mausser, H. and Rosen, D. (2007). Economic credit capital allocation and risk contributions. *Handbooks in Operations Research and Management Science*, 15:681–726.

McNeil, A. J., Frey, R., and Embrechts, P. (2015). *Quantitative risk management: Concepts, techniques and tools*. Princeton university press.

Mendes, B. V. and De Melo, E. F. (2010). Local estimation of dynamic copula models. *International Journal of Theoretical and Applied Finance*, 13(02):241–258.

Mercier, G. and Frison, P.-L. (2009). Statistical characterization of the sinclair matrix: Application to polarimetric image segmentation. In *Geoscience and Remote Sensing Symposium, 2009 IEEE International, IGARSS 2009*, volume 3, pages III–717. IEEE.

Merino, S. and Nyfeler, M. A. (2004). Applying importance sampling for estimating coherent credit risk contributions. *Quantitative Finance*, 4(2):199–207.

Merton, R. C. (1974). On the pricing of corporate debt: The risk structure of interest rates. *The Journal of finance*, 29(2):449–470.

Min, A. and Czado, C. (2010). Bayesian inference for multivariate copulas using pair-copula constructions. *Journal of Financial Econometrics*, 8(4):511–546.

Morgan, J. (1997). Creditmetrics-technical document. *JP Morgan, New York*.

Nelsen, R. B. (2005). Dependence modeling with archimedean copulas.

Nelsen, R. B. (2007). *An introduction to copulas*. Springer Science & Business Media.

Nikkinen, J. and Sahlström, P. (2004). International transmission of uncertainty implicit in stock index option prices. *Global Finance Journal*, 15(1):1–15.

Nikoloulopoulos, A. K., Joe, H., and Li, H. (2012). Vine copulas with asymmetric tail dependence and applications to financial return data. *Computational Statistics & Data Analysis*, 56(11):3659–3673.

Oakes, D. (1994). Multivariate survival distributions. *Journaltitle of Nonparametric Statistics*, 3(3-4):343–354.

Oh, D. H. and Patton, A. J. (2017a). Modeling dependence in high dimensions with factor copulas. *Journal of Business & Economic Statistics*, 35(1):139–154.

Oh, D. H. and Patton, A. J. (2017b). Time-varying systemic risk: Evidence from a dynamic copula model of cds spreads. *Journal of Business & Economic Statistics*, pages 1–15.

Okhrin, O., Okhrin, Y., and Schmid, W. (2013). On the structure and estimation of hierarchical archimedean copulas. *Journal of Econometrics*, 173(2):189–204.

Okimoto, T. (2008). New evidence of asymmetric dependence structures in international equity markets. *Journal of financial and quantitative analysis*, 43(03):787–815.

Patton, A. J. (2004). On the out-of-sample importance of skewness and asymmetric dependence for asset allocation. *Journal of Financial Econometrics*, 2(1):130–168.

Patton, A. J. (2006a). Estimation of multivariate models for time series of possibly different lengths. *Journal of applied econometrics*, 21(2):147–173.

Patton, A. J. (2006b). Modelling asymmetric exchange rate dependence. *International economic review*, 47(2):527–556.

Pelletier, D. (2006). Regime switching for dynamic correlations. *Journal of econometrics*, 131(1):445–473.

Poon, S.-H. and Granger, C. W. (2003). Forecasting volatility in financial markets: A review. *Journal of economic literature*, 41(2):478–539.

Poon, S.-H., Rockinger, M., and Tawn, J. (2003). Extreme value dependence in financial markets: Diagnostics, models, and financial implications. *The Review of Financial Studies*, 17(2):581–610.

Puzanova, N. (2011). A hierarchical model of tail dependent asset returns for assessing portfolio credit risk. *Discussion Paper Series 2: Banking and Financial Studies*.

Pykhtin, M. (2004). Portfolio credit risk multi-factor adjustment. *RISK-LONDON-RISK MAGAZINE LIMITED-*, 17(3):85–90.

Ramchand, L. and Susmel, R. (1998). Volatility and cross correlation across major stock markets. *Journal of Empirical Finance*, 5(4):397–416.

Rigobon, R. (2003). On the measurement of the international propagation of shocks: is the transmission stable? *Journal of International Economics*, 61(2):261–283.

Rockafellar, R. T. and Uryasev, S. (2000). Optimization of conditional value-at-risk. *Journal of risk*, 2:21–42.

Rodriguez, J. C. (2007). Measuring financial contagion: A copula approach. *Journal of Empirical Finance*, 14(3):401–423.

Romano, J. P. and Wolf, M. (2005). Stepwise multiple testing as formalized data snooping. *Econometrica*, 73(4):1237–1282.

Salinas-Gutiérrez, R., Hernández-Aguirre, A., and Villa-Diharce, E. R. (2010). D-vine eda: a new estimation of distribution algorithm based on regular vines. In *Proceedings of the 12th annual conference on Genetic and evolutionary computation*, pages 359–366. ACM.

Salmon, F. (2009). Recipe for disaster: The formula that killed wall street, 2009.

Salmon, F. (2012). The formula that killed wall street. *Significance*, 9(1):16–20.

Savu, C. and Trede, M. (2010). Hierarchies of archimedean copulas. *Quantitative Finance*, 10(3):295–304.

Schmidt, D. (2003). Private equity-, stock-and mixed asset-portfolios: A bootstrap approach to determine performance characteristics, diversification benefits and optimal portfolio allocations. Technical report, CFS Working Paper.

Shih, J. H. and Louis, T. A. (1995). Inferences on the association parameter in copula models for bivariate survival data. *Biometrics*, pages 1384–1399.

Skiadopoulos, G. (2004). The greek implied volatility index: construction and properties. *Applied Financial Economics*, 14(16):1187–1196.

Sklar, M. (1959). *Fonctions de répartition à n dimensions et leurs marges*. Université Paris 8.

Smith, M., Min, A., Almeida, C., and Czado, C. (2012). Modeling longitudinal data using a pair-copula decomposition of serial dependence. *Journal of the American Statistical Association*.

So, M. K. and Yeung, C. Y. (2014). Vine-copula garch model with dynamic conditional dependence. *Computational Statistics & Data Analysis*, 76:655–671.

Sobol, I. M. (1993). Sensitivity estimates for nonlinear mathematical models. *Mathematical Modelling and Computational Experiments*, 1(4):407–414.

Soriano, P. and Climent, F. J. (2005). Volatility transmission models: a survey.

Stephen, L. and Simon, S. (2005). The case for reits in the mixed-asset portfolio in the short and long run. *Journal of Real Estate Portfolio Management*, 11(1):55–80.

Stöber, J. and Czado, C. (2012). Sampling pair copula constructions with applications to mathematical finance. *Simulating Copulas: Stochastic Models, Sampling Algorithms, and Applications*, 4.

Suisse, C. (1997). Creditrisk+: a credit risk management framework. *Credit Suisse Financial Products*, pages 18–53.

Tasche, D. (1999). Risk contributions and performance measurement. *Report of the Lehrstuhl für mathematische Statistik, TU München*.

Tasche, D. (2006). Measuring sectoral diversification in an asymptotic multi-factor framework.

Tasche, D. (2007). Capital allocation to business units and sub-portfolios: the euler principle. *arXiv preprint arXiv:0708.2542*.

Tse, Y. K. (2002). Residual-based diagnostics for conditional heteroscedasticity models. *The Econometrics Journal*, 5(2):358–374.

Van der Vaart, A. W. (2000). *Asymptotic statistics*, volume 3. Cambridge university press.

Vasicek, O. (2002). The distribution of loan portfolio value. *Risk*, 15(12):160–162.

Vasicek, O. A. (1987). *Probability of loss on loan portfolio*. Wiley Online Library.

Vasicek, O. A. (1991). Limiting loan loss probability distribution. *Finance, Economics and Mathematics*, pages 147–148.

Vuong, Q. H. (1989). Likelihood ratio tests for model selection and non-nested hypotheses. *Econometrica: Journal of the Econometric Society*, pages 307–333.

Wagner, N. and Szimayer, A. (2004). Local and spillover shocks in implied market volatility: evidence for the us and germany. *Research in international Business and Finance*, 18(3):237–251.

West, K. D. (1996). Asymptotic inference about predictive ability. *Econometrica: Journal of the Econometric Society*, pages 1067–1084.

White, H. (2000). A reality check for data snooping. *Econometrica*, 68(5):1097–1126.

Wu, C. J. (1983). On the convergence properties of the em algorithm. *The Annals of statistics*, pages 95–103.

Xu, Q. and Li, X.-M. (2009). Estimation of dynamic asymmetric tail dependences: an empirical study on asian developed futures markets. *Applied Financial Economics*, 19(4):273–290.

Yu, J. and Meyer, R. (2006). Multivariate stochastic volatility models: Bayesian estimation and model comparison. *Econometric Reviews*, 25(2-3):361–384.