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# Essays on Strategic Information Acquisition


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A Mamma e Babbo, a Gianfranco, grazie per la persona che sono

# Essays on Strategic Information Acquisition

by

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**Abstract**

This thesis studies information acquisition in settings where agents can strategically acquire, at cost or freely, some informative signals about the underlying state of the world prior to make a decision.

Chapter 2 studies how agents select information sources in a model with potentially delusional agents. Agents with anticipatory utility must decide whether to undertake a common project. Ex-ante, they can select which information sources to pay attention to. When choosing the information sources, agents take into account the fact that they may ex-post have to engage in costly denial. We show that multiple equilibria coexist: one in which agents are fully informed and one where agents pay attention only to the information source most likely to reveal favourable information.

Chapter 3 studies endogenous information acquisition in an investment trading-game à la Angeletos, Lorenzoni, and Pavan (2010). In such a game if agents have dispersed information, endogenous strategic complementarity in actions emerges owing to the information spillover between real sector and financial sector and generates inefficiency in the economy. By introducing endogenous information acquisition, this chapter aims at studying what information is acquired and how it affects the equilibrium outcome. It is shown that there exists complementarity in entrepreneurs' information acquisition. It also investigates the conditions under which information is not acquired at all.

Chapter 4 studies information acquisition in a network-formation game. It investigates how the desire to coordinate with some people and anti-coordinate with some others determines the information acquired and shapes the network formed in equilibrium. In an economy populated by N agents divided into two groups, in the first period agents can acquire informative signals about the state of the world by forming costly connections with other players. In the second period each agent chooses an action balancing the desire to be close to the fundamental, be close to the average action of players in his own group and be far from the average action of players in the opposite group.

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# Declaration

Chapter 2 is joint work with Fabrizio Adriani and Chapter 4 is joint work with Matteo Foschi. Chapter 3 is single authored.

A version of Chapter 3 has been presented at the following conferences:

- 22$^{\text{nd}}$ Spring Meeting of Young Economist (2017)
- Stony Brook 27$^{\text{th}}$ International Conference on Game Theory (2017).

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# Chapter 1

# Introduction

This thesis studies how agents strategically acquire information, at a cost or freely, before making a decision. It consists of three self-contained chapters, each considering a different setting. In Chapter 2 we study how agents select information sources in a model with potentially delusional agents; in Chapter 3 we study endogenous information acquisition in an investment-trading game; in Chapter 4 we study information acquisition within an endogenous network where agents have both coordination and anti-coordination motives.

Chapter 2 aims to provide a theoretical explanation of the interaction between the echo chamber and the selective exposure phenomena. The term “echo chamber" is commonly used to describe a phenomenon in which consumers of information and opinions get stuck in a chamber with like-minded people. The selective exposure phenomenon is a psychological mechanism according to which individuals tend to favour information that aligns with their pre-existing views while avoiding contradictory information. The theoretical model used in this chapter is based on and extends Benabou (2013). In that model, agents with anticipatory utility must decide whether to undertake a common project. Ex-ante, they receive information about the project's benefits but can strategically decide to engage in denial at a psychological cost. We augment Benabou's model by adding an initial stage where agents can select which information sources to pay attention to. For instance, an agent may select a subset of newspapers among all available newspapers, or may follow some pundits on twitter but not others. Crucially, when choosing the information sources to pay attention to, agents take into account the fact that they may ex-post have to engage in costly denial. This may create an incentive to avoid information sources that are more prone to convey "bad news". In other words, neglecting some information sources can spare the agent from bearing the psychological cost of suppressing bad news ex-post. Moreover, they also take into account that others may be similarly selective in their choices of information sources.

In Chapter 3 we study information acquisition in an investment trading game where (i) entrepreneurs base their investment decisions on their expectation about both an unknown underlying economic fundamental and the price at which they may sell their capital to the financial markets in the future; and (ii) traders operating in the financial market use the aggregate investment to learn about the fundamental. Our model is based on Angeletos, Lorenzoni, and Pavan (2010), but we change the information structure. Agents have some freely available public information about the profitability of the project and, before the investment decision, can acquire some private information at a cost. In particular the framework is the following. At the beginning of the game a new investment opportunity with unknown profitability (the fundamental) arises. Two sectors operate in this economy: the real sector populated by entrepreneurs and the financial sector populated by traders. In the first period each entrepreneur has to decide how much to invest in the new project. In the second period before the profitability of the project is revealed, a fraction $\lambda$ of entrepreneurs is hit by a liquidity shock and sells its capital to the financial sector. None of the agents in the economy is fully informed about the underlying state of the world. Entrepreneurs have free access to a public signal about the fundamental value of the project and they can also acquire at some cost a private signal by paying attention to listen to it. The effect of paying attention to the signal is the following, as attention increases the overall precision of the private signal increases. In the financial sector, traders also have free access to a public signal about the fundamental value and they also observe the aggregate capital invested in the economy. We characterise the information acquired and the individual investment decision. However our focus is on the entrepreneurs' information acquisition policy and in particular on the conditions under which entrepreneurs prefer to not acquire private information.

Chapter 4 aims at studying how information acquisition shapes networks. Our objective is to show how the desire to coordinate with some people and anti-coordinate with others determines the information acquired in equilibrium (its publicity) and shapes the network formed in equilibrium. Information acquisition in networks has been studied under several different assumptions. In the most related contributions to ours, Denti (2017) models "flexible" information acquisition in a coordination games by players arranged on a network; Myatt and Wallace (2017) study how asymmetries in games with quadratic payoffs affects how players arranged in a network use and acquire information; Herskovic and Ramos (2017) study the case of agents who acquire information from the same peers they want to coordinate. We look at the case where agents acquire information from their peers while wanting to coordinate with some of them and anti-coordinate with others. We develop a two period model in an economy populated by N players divided into two groups. Each player is endowed with a private signal about an underlying economic fundamental. Signals are identically distributed within group but differ in precision across groups. In the first period agents can form connections with other players. If a player connects to another one, he pays a cost and observes the signal of that player but not vice-versa. In the second period each agent chooses an action balancing the desire to be i) close to the fundamental, ii) close to the average action of players in his own group and iii) far from the average action of players in the opposite group.

# Chapter 2

# Information Avoidance, Echo Chambers and Uninformed Decisions

## 2.1 Introduction

Technologies such as the internet have eased the access to information and news to a continuously growing fraction of the society. This is so because the internet has dramatically reduced the cost of acquiring information from a wide range of sources. In fact, with the increasing variety of new media choices, the scale at which individuals are exposed to opinions, news and information is larger than what was possible with the traditional media. On the one hand, the growth of online news and social media allows individuals to be exposed to diverse viewpoints and opinions as well as to a variety of information. On the other hand, it might induce people to focus their attention only on a subset of information sources. As such, the way information is transmitted, processed and consumed has brought to the attention of academics and scholars the negative consequences of the increased variety of media platforms. In particular, it is claimed that social media and Internet "filter bubbles" can create echo chambers, (Pariser, 2011). Although there is not consensus in the literature on a formal definition of "echo chamber", the term is commonly used to describe a phenomenon in which consumers of information and opinions get stuck in a chamber with like-minded people. In the chamber, the opinions, information and beliefs get repeated and confirmed like an echo, rather than foster dialogue and critical reasoning (Jamieson and Cappella, 2008). Echo chambers are considered harmful for societies to the extent that the information and beliefs shared can produce more extreme opinions and increase polarisation (Sunstein, 2002). Echo chambers and the polarisation of views may shape decisions regarding many aspects of social life, ranging from those based on opinions (about politics or religion, for example) to those based on objective facts that have a well established consensus.

The phenomenon of echo chambers has been recently discussed in relation to the UK Brexit referendum and the US presidential elections. Considering objective facts, such as global warming evidence or the usefulness of vaccinations, there is evidence of a fervent debate on the validity of these scientific arguments, with a fraction of the population showing ideological division and disagreement about the importance of implementing greenhouse gas emission reduction's policies or vaccinating to reduce the risk of the spread of diseases. Many scholars are persuaded that these phenomena are due to the existence of echo chambers.

From a psychological perspective, the echo chamber phenomenon seems connected to the psychological mechanism called *selective exposure*. According to the selective exposure hypothesis, individuals tend to favour information that aligns with their pre-existing views while avoiding contradictory information.

In this paper, we aim to provide a theoretical explanation of the interaction between the echo chamber and the selective exposure phenomena. Our theory is based on and extends Benabou (2013). In that model, agents with anticipatory utility must decide whether to undertake a common project. Ex-ante, they receive information about the project's benefits but can strategically decide to engage in denial at a psychological cost. We augment Benabou's model by adding an initial stage where agents can select which information sources to pay attention to. For instance, an agent may select a subset of newspapers among all available newspapers, or may follow some pundits on twitter but not others. Crucially, when choosing the information sources to pay attention to, agents take into account the fact that they may ex-post have to engage in costly denial. This may create an incentive to avoid information sources that are more prone to convey "bad news". In other words, neglecting some information sources can spare the agent from bearing the psychological cost of suppressing bad news ex-post. Moreover, they also take into account that others may be similarly selective in their choices of information sources. As in Benabou's model, multiple equilibria coexist (for some parameter values) in our framework. There is typically an equilibrium where agents look at all available information and undertake the project only if it is worthwhile given the available information. However, there also exists an equilibrium where agents only pay attention to a subset of information sources (those that are less likely to convey bad news). This equilibrium is similar to Benabou's Mutually Assured Delusion (MAD) equilibrium, although we stress an important difference. The MAD equilibrium can be tested only indirectly, since it is not easy to observe whether agents engage in self delusion. In contrast, the sources of information that people look at are in principle observable. In terms of welfare, we show that the equilibrium where agents pay attention to all information sources dominates the equilibrium where agents are selective. While this result might appear obvious at first glance, we show that this is the case even when we include agents' anticipatory feelings in the welfare measure.

The multiple equilibria result is in line with the empirical studies about the presence of echo chamber and polarisation in social media in the debate about politics, climate change policy and the importance of vaccination. Multiple equilibria can explain contradictory empirical evidence about the existence of echo chambers. For example Williams et al. (2015) show that social media discussions on climate change often occur within polarising "echo chambers", but also within "open forums", namely mixed-attitude communities that reduce polarisation and stimulate debate. Examinations of selective exposure have shown that individuals do tend to confront with information and ideas they find supportive and consistent with their existing beliefs (Iyengar and Hahn, 2009). Garrett, Carnahan, and Lynch (2013) study Americans' use of online sources of political information. According to their empirical results, they argue that even though individuals seek ideologically consistent news sites, they are not systematically avoiding other news sites. Other scholars have found evidence of echo chambers on Twitter (Barbera et al., 2015; Himelboim, McCreery, and Smith, 2013), while others have shown that the trend does not persist on Facebook (Bakshy, Messing, and Adamic, 2015). Lawrence, Sides, and Farrell (2010) examine political polarisation among blog readers and find that they gravitate toward blogs that accord with their political beliefs. Few read blogs on both the left and right of the ideological spectrum. This empirical evidence provides strong support to our results. In fact, in our paper we show that selective exposure (in terms of information avoidance) is the result of an active role of the agents that strategically choose which information to pay attention to. At the investment stage, our equilibrium results can be interpreted as situations in which individuals, trapped inside the echo chamber, undertake uninformed decisions such as sustaining policies that do not reduce the greenhouse gas emissions, or deciding not to vaccinate. In the politics sphere, uninformed decisions may favour extreme candidates, populism and potentially harmful decisions such as leaving the EU in the Brexit referendum.

## 2.2 Related literature

This paper contributes to the broad literature on information avoidance. Information avoidance has been extensively studied in many research areas, such as medicine, communication, organisational behaviour and psychology. Sweeny et al. (2010) provide a survey of these literatures and define information avoidance as any behaviour intended to prevent or delay the acquisition of available but potentially unwanted information. According to them three reasons are at the base of why people may chose to avoid information. More information may induce a change in beliefs, it may demand undesired action or it may cause unpleasant emotions. There exists also empirical research in psychology that documents the tendency of people not to attend, i.e to ignore, information. A long-standing body of work links this phenomenon to the *selective exposure hypotesis*. According to this hypothesis people tend to selectively process, interpret and recall data in a way that leads to more favourable beliefs about their personal characteristics or future prospects. Several recent papers show in a rigorous way that people tend to respond in a asymmetrical way to good and bad news. For example Karlsson, Loewenstein, and Seppi (2009) examine the degree to which people choose to expose themselves differentially to additional information after conditioning on prior positive and negative news. They develop a model of selective attention in which individuals first receive incomplete information and then decide whether to acquire and attend to definitive information. Their results show that for reasonable parameter values, individuals exhibit an ostrich effect. That is, agents avoid exposing themselves to information that might cause psychological discomfort. A comprehensive review, both theoretical and empirical, about information avoidance has been recently documented by Golman, Hagmann, and Loewenstein (2017). Their focus is on situations in which people avoid information even when it is free and could improve decision making. In particular, they refer to a phenomenon which they call 'active' information avoidance. To be classified as such, information avoidance requires that the individual is aware of the existence of that information and that information if freely accessible.

Our work is in some sense related to the growing literature on models of opinion polarisation. Dixit and Weibull (2007) show how the beliefs of Bayesians with different priors can polarise when new information arrives. Benoit and Dubra (2016) argue that findings of group attitude polarisation in psychological studies can be rationalised using purely Bayesian models. Fryer, Harms, and Jackson (2018) show that opinion polarisation can persist when Bayesian agents have limited memory. Ortoleva and Snowberg (2015) explore how overconfidence drives polarisation and affects political behaviour. In a recent work Gentzkow, Wong, and Zhang (2018) argue that ideological divisions, like the ones displayed by recent debates over global warming, evolution, and vaccination, may arise when Bayesian agents have small biases in information processing and they are uncertain which sources they can trust. In this scenario, increasing the amount of information available may deepen ideological differences. All these papers show that polarisation is a result of Bayesian agents that either have limited memory, bias in information processing or different prior.

Our paper, however, does not intend to explain opinion polarisation but provides a rationale for the formation of echo chambers in which opinions may later on polarise. In fact, our paper can explain why "information bubbles" or "echo chambers", in which agents pay attention to only the same information or "voice", emerge, inducing agents to undertake uninformed decisions. The formation of echo chambers is not the result of biases or limited memory of Bayesian agents. It is instead the result of agents that strategically choose not to pay attention to potentially bad news in order to avoid the psychological cost of denying them.

## 2.3 The Model

Our model builds on the framework by Benabou (2013). In particular we use the same model set-up but with a different information structure. Specifically, while in Benabou (2013) agents receive only one exogenous signal about an underlying state of the world, in our set-up we allow agents to choose the information source they want to pay attention to.

**Technology.** A group of risk neutral agents, $i \in \{1, \ldots\ldots, n\}$, are engaged in a joint project or other activities generating spillovers. Time is discrete and covers three periods, $t = 0, 1, 2$. At $t = 1$, each agent chooses effort $e^i = \{0, 1\}$, which costs $ce^i$, $c > 0$. At $t = 2$, she will reap utility

$$U_2^i = \theta \left[ \alpha e^i + (1-\alpha) e^{-i} \right] \tag{2.1}$$

where $e^{-i} \equiv \frac{1}{n-1} \sum_{j \neq i} e^j$ is the average effort of others and $(1-\alpha) \in [0, 1-1/n]$ represents the degree of interdependence, reflecting the joint nature of the enterprise. The payoff structure of the final period is very simple and it is exactly the same of

Benabou (2013): there is no interdependence between effort decisions. This implies that there are not in built complementarities in the payoff of the agent, but only externalities, given by $1-\alpha$, without strategic interaction.

The state of nature is uncertain and it is $H$ (high) with (prior) probability $\mu$ and $L$ (low) with probability $(1-\mu)$. The project productivity $\theta$ is uncertain with expected value equal to $\theta_H$ conditional to the state being $H$ and $\theta_L$ conditional to the state being $L$. We denote $\Delta\theta \equiv \theta_H - \theta_L > 0$ and we assume $\theta_H > 0$ without loss of generality.

**Information structure.** There are two different information sources about the state of the world, information source $I_1$ and information source $I_2$. Each information source delivers either a signal about the state of the world or it is silent about it. We assume that signals are perfectly correlated within the same information source, that is agents observing the same information source receive the same signal. These information sources are costless, in the sense that the information is freely available to any agent that would like to observe it.

Specifically, $I_1$ sends the following signals

- $s_H$ with probability $p_1$ and $\varnothing$ (the empty signal) with probability $(1-p_1)$ if the state is $H$,
- $\varnothing$ with probability 1 if the state is $L$,

and $I_2$ sends

- $\varnothing$ with probability 1 if the state is $H$,
- $s_L$ with probability $p_2$ and $\varnothing$ with probability $(1-p_2)$ if the state is $L$.

Information source $I_1$ is the favourable information in the sense that either provides good news or leaves the agent uncertain about the state of the world. On the contrary, information source $I_2$ is potentially disappointing because it may convey bad news to the agent.

At the beginning of period 0 each agent chooses the information source to which she will pay attention. We assume that an agent pays always attention to the more favourable information[1]. However, because this information source is noisy, she can opt to pay attention to more information to reduce this noise. Formally she can *(i)* choose $I_1$ or *(ii)* choose both $I_1$ and $I_2$. Notice that when an agent $i$ chooses to pay attention to both information sources, she will receive the following combination of signals

- $(\varnothing, \varnothing)$ with probability $1-\mu p_1-(1-\mu)p_2$,
- $(s_H, \varnothing)$ with probability $\mu p_1$ and
- $(\varnothing, s_L)$ with probability $(1-\mu)p_2$,

where the first signal refers always to a signal delivered by $I_1$ and the second one refers always to a signal delivered by $I_2$.

**Preferences.** Period 1 payoff (the investment stage) includes the cost of effort, $-ce^i$, but also the *anticipatory utility* experienced from thinking about one's future prospects, $sE_1^i[U_2^i]$, where $s \geq 0$ parametrises the psychological and health effects of hopefulness, dread, and similar emotions.

At the start of period 1, an agent $i$ chooses effort to maximise the expected present value of payoffs, discounted at rate $\delta \in (0,1]$:

$$U_1^i = -ce^i + sE_1^i\left(U_2^i\right) + \delta E_1^i\left(U_2^i\right) \tag{2.2}$$

Actual beliefs in period 1 will depend on the information source chosen in period 0 and how objectively or subjectively the agent processed the signals received as described in the next paragraph. Therefore, the strategic interaction between agents is not at the effort decision stage, but at the information source choice stage. In period 0, an agent $i$ aims to maximise the discounted utility of all payoffs by choosing

[1] This is a simplifying assumption that we impose for tractability of the model. We could also allow agents to choose only $I_2$. However our results would not be affected.

which information source to pay attention to, that is

$$U_0^i = -M^i + \delta E_0^i\left[-c e^i + s E_1^i\left(U_2^i\right)\right] + \delta^2 E_0^i\left(U_2^i\right), \tag{2.3}$$

where $E_t^i$ denotes expectations at $t = 0, 1$ and $M^i$ the date-1 costs of her cognitive strategy.

**Cognitive Strategy and Beliefs.** In period 0, once the information source has been chosen, agents receive a signal. Upon observing the signal, each agent chooses how to interpret it, whether to keep it in mind or not to think about it, etc. Denoting with $\sigma \in \{s_H, s_L, \varnothing\}$ any signal received at period 0 and with $\hat{\sigma}^i \in \{\hat{s}_H, \hat{s}_L, \hat{\varnothing}\}$ the signal recalled at the beginning of period 1, formally an agent can:

i. accept the facts realistically, truthfully encoding $\hat{\sigma}^i_j = \sigma_j$, into memory or awareness[2].

ii. engage in denial, censoring or rationalisation, encoding

   - $\hat{s}_H$ when she receives $\varnothing$ from $I_1$,
   - $\hat{\varnothing}$ when she receives $s_L$ from $I_2$.

   We assume that denial is costly and that for each signal censored the agent bears an immediate cost $m_j \geq 0$, with $j \in \{1, 2\}$. In particular $m_1$ is the cost of censoring a signal that comes from information source $I_1$; while $m_2$ is the cost of denying a signal from information source $I_2$. We do not put any restriction on the cost of denial, allowing for both $m_1 = m_2$ and $m_1 \neq m_2$.

It is worth highlighting that, differently from Benabou (2013), in our set-up we consider the possibility that an agent engages in denial only when the signal received is the less favourable one among the two possible she can receive from each information source. That is, agents can change the signal from no signal to good signal

[2] So for example if an agent receive the signal $s_H$ and encodes it truthfully, at the beginning of period 1 she will observes $\hat{s}_H$. Thus $\hat{s}_H = s_H$.

if she observes information source $I_1$ and from bad to no signal if she observes information source $I_2$, but not vice-versa. However, based on Benabou (2013) results, we could generalise the model to a framework where the cognitive strategy considers also the case of censoring the more favourable signal. He shows that it is never optimal to deny a good signal for a positive cost of denial.

Specifically, in our model, the agent's cognitive strategy functions as follows

- $\lambda_1^i \equiv Pr(\hat{\varnothing}|\varnothing)$ is the probability that the agent will process correctly the signal $\varnothing$ received from information source $I_1$. Thus, $\lambda_1^i = 1$ means that from period 0 to period 1 the agent carries the same information.

- $\lambda_2^i \equiv Pr(\hat{s}_L|s_L)$ is the probability that the agent will process correctly the the signal $s_L$ received from information source $I_2$[3]. So, $\lambda_2^i = 1$ means that from period 0 to period 1 the agent carries the same information.

Although the model allows cognitive mixed strategy, we restrict our attention to equilibria in pure strategies. That is, we only look at the case where the agents either deny the signal with probability 1 or are completely realist.

Thus, given period-0 agents' cognitive strategy, the period-1 information set of an agent $i$ may be different from her period-0 information set. We assume that agents are rational, in the sense that they are aware of their tendency to deny bad signals and they will take this into account when they formulate their posterior beliefs. We also assume that, when an agent observes both information sources, the denial strategies on $I_1$ and on $I_2$ are set independently ex-ante. That is, the decision of whether to deny a signal received from $I_1$ is independent of the signal received from $I_2$; symmetrically, the decision of whether to deny a signal received from $I_2$ is independent of the signal received from $I_1$. Therefore at the beginning of period 1, henceforth the recalling stage, an agent $i$'s posterior belief when observing $I_1$ and recalling $\hat{s}_H$ is

$$Pr(s_H|\hat{s}_H, \lambda_1^i) = \frac{\mu p_1}{\mu p_1 + (1-\mu p_1)(1-\lambda_1^i)} \equiv r(\lambda_1^i).$$

[3]The complement of these two probabilities is respectively $(1-\lambda_1^i) \equiv Pr(\hat{s}_H|\varnothing)$ and $(1-\lambda_2^i) \equiv Pr(\hat{\varnothing}|s_L)$.

In the case where the agent observes both $I_1$ and $I_2$, given the assumption that denial strategies on $I_1$ and on $I_2$ are set independently ex-ante, she might recall $(\hat{\varnothing},\hat{\varnothing})$, $(\hat{s}_H,\hat{\varnothing})$ or $(\hat{s}_H,\hat{s}_L)$. Therefore if she recalls $(\hat{s}_H,\hat{\varnothing})$ her posterior beliefs are

$$Pr(s_H,\varnothing|\hat{s}_H,\hat{\varnothing})=$$

$$\frac{\mu p_1}{\mu p_1+(1-\mu p_1-(1-\mu)p_2)(1-\lambda_1^i)+(1-\mu)p_2(1-\lambda_1^i)(1-\lambda_2^i)} \equiv v(\lambda_1^i,\lambda_2^i) \tag{2.4}$$

$$Pr(\varnothing,\varnothing|\hat{s}_H,\hat{\varnothing})=$$

$$\frac{(1-\mu p_1-(1-\mu)p_2)(1-\lambda_1^i)}{\mu p_1+(1-\mu p_1-(1-\mu)p_2)(1-\lambda_1^i)+(1-\mu)p_2(1-\lambda_1^i)(1-\lambda_2^i)} \equiv q(\lambda_1^i,\lambda_2^i) \tag{2.5}$$

$$Pr(\varnothing,s_L|\hat{s}_H,\hat{\varnothing})=$$

$$\frac{(1-\mu)p_2(1-\lambda_1^i)(1-\lambda_2^i)}{\mu p_1+(1-\mu p_1-(1-\mu)p_2)(1-\lambda_1^i)+(1-\mu)p_2(1-\lambda_1^i)(1-\lambda_2^i)} \equiv p(\lambda_1^i,\lambda_2^i). \tag{2.6}$$

If she recalls $(\hat{\varnothing},\hat{\varnothing})$ her posterior beliefs are

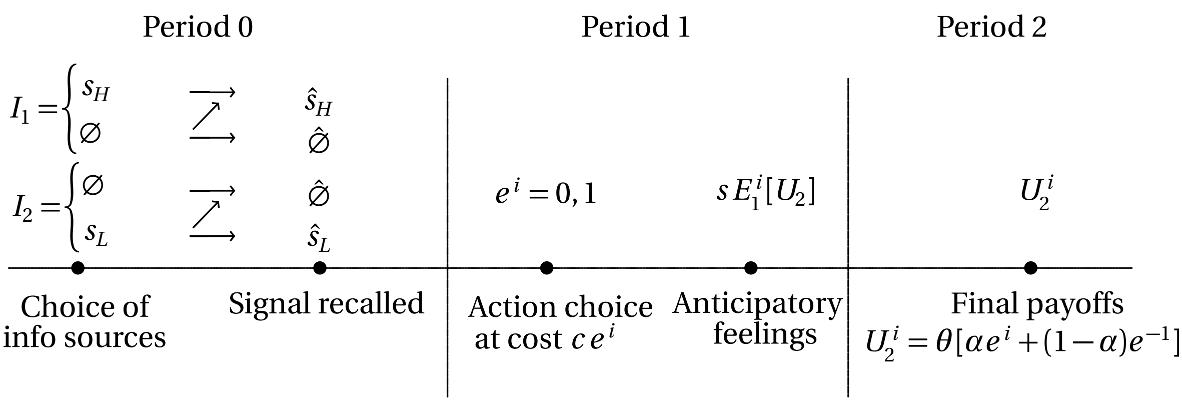
$$Pr(\varnothing,\varnothing|(\hat{\varnothing},\hat{\varnothing}),\lambda_2^i)=\frac{1-\mu p_1-(1-\mu)p_2}{1-\mu p_1-(1-\mu)p_2+(1-\mu)p_2(1-\lambda_2^i)} \equiv q(\lambda_2^i). \tag{2.7}$$

Finally, if she recalls $(\hat{s}_H,\hat{s}_L)$, the agent understands immediately that she has been delusional, inferring that the state is L.

**Timing.** At $t=0$ agents choose which information source to pay attention to and after they receive the corresponding signal they decide how to process it[4]. At $t=1$ agents choose an action $e=\{0,1\}$ at cost $ce$ and have anticipatory feelings about $U_2$. At $t=2$ agents get their final payoff $U_2$.

[4]Notice that at $t=0$ two events take place. Each agent (i) chooses the information source to pay attention to and (ii) decides how to process the signal received. We could consider these two events as belonging to two different periods, but it is without loss of generality that we consider these two events both happening at $t=0$, rather we split the first period into two sub-periods. The reason for using this approach is that each period identifies a particular stage of the agent's decision process, with the first period referring to the information acquisition/manipulation phase, the second period referring to the action phase and the last period referring to the realisation of the state.

Figure 2.1: Timeline




## 2.4 Model Solution

### 2.4.1 The effort choice

We solve the model by backward induction. In period $t = 1$, conditional on the recalled signal, an agent $i$ chooses effort $e$ to maximise (2.2). Notice that, given $U_2$, an agent $i$'s effort decision only depends on her beliefs about $\theta$. That is, she exerts effort only if $(s+\delta)\alpha E_1(\theta) > c$, independently of the effort decision of the other agents. The following parametric restriction

**Assumption 2.1.**

$$\theta_L < \frac{c}{(\delta+s)\alpha} < \frac{c}{\delta\alpha} < \mu\theta_H + (1-\mu)\theta_L \tag{2.8}$$

ensures that, without denial taking place, if the agent knew the true state, she would not exert effort in the bad-news state and she would exert effort in the good-news state. In contrast, if she were to choose an action based only on the prior, she would exert effort. In this set-up, as we allow agents to select the information sources they want to listen to, if denial does not take place, an agent $i$'s incentive to exert effort, conditional on having received a signal from the more favourable information source $I_1$, is

- $e = 1$ if she receives the signal $s_H$, because she knows that the state is $H$ w.p. 1 and therefore $\theta = \theta_H$ and

- $e = 1$ if she receives the signal $\varnothing$, as long as

$$\frac{c}{(s+\delta)\alpha} < \underbrace{Pr(H|\varnothing)\theta_H + Pr(L|\varnothing)\theta_L}_{E(\theta|\varnothing)}. \tag{2.9}$$

If she chooses to listen to both information sources, then

- $e = 0$ if she receives $(\varnothing, s_L)$, because she knows that the state is $L$ and therefore $\theta = \theta_L$ and
- $e = 1$ if she receives both $(s_H, \varnothing)$ and $(\varnothing, \varnothing)$ as long as condition (2.9) holds.

To make things simple we restrict our attention to situations where effort is not exerted only when the agent receives the bad signal, i.e. when she receives $s_L$.

**Assumption 2.2.** *Condition* (2.9) *is always satisfied.*

This assumption implies that an agent $i$ exerts effort whenever she receives an empty signal, that is both in the case she pays attention to only $I_1$ and in the case she pays attention to both $I_1$ and $I_2$, and encode it truthfully. Moreover notice that with the above assumption, an agent $i$ exerts effort also whenever she engages in denial of the signal received.

**Lemma 2.1** (Optimal Action)**.** *For all recalled signals profiles which do not include $\hat{\sigma}^i = \hat{s}_L$, an agent $i$ always exerts effort.*

**Proof.** A formal proof is provided in Appendix A.1. □

To understand the undergoing mechanism of the effort choice, notice that at the recalling stage the agent does not remember the signal received the period before but is aware of her tendency to deny signals. She just remembers the information sources observed. Given that the agent is a Bayesian updater, she also takes into account her tendency to deny signals when updating her beliefs about the state of the world. Therefore, when denial occurs with probability 1, the conditional expected productivity of $\theta$ is equivalent to what would be the expected value of $\theta$ having not observed the information source of the denied signal. Moreover, assumptions 2.1 and 2.2 imply that effort is not exerted only when agents know that the state is low with probability one. Thus, systematic denial of the signal $s_L$ is equivalent, for the agent, to not knowing when the state is low.

### 2.4.2 Ignorance as equilibrium

At the beginning of period 0 agent $i$ chooses which information source to observe in order to maximise the discounted utility of all payoffs given by (3). When selecting an information source, an agent will take into account: a) the fact that she might ex-post engage in denial, b) how other agents process the information they receive. In particular, agent $i$ needs to form beliefs about what information sources other agents will pay attention to and how they will process the signals received in order to predict whether others will exert effort or not. As argued above, the way an agent processes the information received affects her own effort choice, which in turn affects the final payoffs of all the other agents in the economy and, through the anticipatory utility term, their anticipatory feelings. This is because the final period payoff - expression (1) - depends both on $i$'s effort and on the effort of other agents. This spillover effect determines agent $i$'s anticipatory feelings. For instance, agent $i$ will feel upbeat if she expects both the state of nature to be high and the others to exert effort. On the other hand, she will be less enthusiastic if others exert effort when she expects the low state of nature. In particular, in case of negative expected value of the low state, the agent suffers from realising that others are making mistakes, that is, they exert effort when they should not because $\theta_L < 0$.

In Benabou (2013), if an agent expects others to suppress bad news (and thus refrains from exerting effort), she may have higher incentives to suppress bad news. This happens when $\theta_L < 0$. In this case, the fact that others are suppressing bad news induces them to exert effort, which in turn generates a negative spillover. Confronted with anticipatory feelings, the agent thus chooses to suppress bad news (which in turn leads her to exert effort as well). Benabou (2013) calls this mechanism the MAD principle.

In our model, agents receive a signal only after having chosen the information source. If the signal delivered by the information source chosen is a bad news and if an agent expects the other agents to exert effort, then she has an incentive to deny the bad news. However denying a signal is costly. If, on the one hand, the agent increases her anticipatory utility by being delusional and exerting effort, on the other hand she bears the psychological cost of suppressing the bad signal. However, the agent could avoid the risk of facing the bad news altogether by carefully selecting her information sources. This mechanism, as in Benabou (2013), spills over onto the other agents who become more willing to avoid the information source that might potentially carry the bad news and thus they become more likely to exert effort. This is so because, beliefs about what the others will be observing and doing enter into agent $i$'s utility function, inducing the agent to suppress information to avoid the psychological costs of realising that others make mistakes. Thus, the origin of complementarity in information acquisition lies in the fact that beliefs about other agents behaviuor affect an agent $i$'s utility function.

This is the case of agents observing only $I_1$, which represents the information source more favourable. Any agent that chooses to observe $I_1$ will always exert effort regardless of how the signal received is processed. Moreover, for a cost of denial sufficiently high, an agent will always encode truthfully the signal received by $I_1$. Clearly, an agent that were to observe $I_2$ together with $I_1$ will become more informed about the state of the world, and in particular this will help her to better predict the bad state and to choose the “right" action, on the ex-post perspective. However, due to the anticipatory preferences of the agent, when she receives a bad signal, she has the tendency to deny it and this is more likely to happen if the agent thinks that the other agents in the economy will always exert effort. Thus, in the end, an agent trying to become more informed will remain trapped by her way of processing the extra information she decides to receive; she benefits by being delusional because she perceives the extra information as good news but at the same time she immediately pays the mental cost of suppressing the bad news. The only way an agent has to avoid this trap is to decide to remain ignorant observing only $I_1$. We formalise this equilibrium in the following proposition.

**Proposition 2.1.** *Define* $\underline{\theta}_L \equiv \frac{1}{\alpha(s+\delta)}\left[c - s\Delta\theta(\frac{2(1-p_1)\mu}{1-\mu p_1} - 1)\right]$ *and* $\bar{\theta}_L \equiv \frac{c}{\alpha(s+\delta)}$ *and assume* $\mu > \frac{1}{2-p_1}$ *to ensure* $\overline{\theta}_L > \underline{\theta}_L$*.*
*Then, for any* $\theta_L \in (\underline{\theta}_L, \bar{\theta}_L)$ *there exists a non empty interval* $[\underline{m}, \overline{m}]$ *such that for any cost of denial* $m_1, m_2 \in [\underline{m}, \overline{m}]^2$ *there exists an equilibrium where all agents (i) look only at information source* $I_1$ *and (ii) exert effort.*

**Proof.** Proof in the Appendix A.1 □

Formal proof is provided in the Appendix. Here we just describe how we constructed this equilibrium. We first assume that there exists an equilibrium in which all agents observe only information source $I_1$ and study agents' optimal cognitive strategy. We then check the optimal cognitive strategy of an agent $i$ if she were to deviate and observe both $I_1$ and $I_2$. Finally, we compare the utility from the deviation to the utility in the candidate equilibrium, and we show that the deviation is not profitable.

This proposition thus shows that agents prefer to restrict attention only to some information, the more favourable one, if paying attention to more information would bring them to costly deny bad news leaving them to lower ex-ante utility. In other words, it is pointless to deviate and acquire more information if one then has to deny it. Neglecting information saves the psychological cost of denial. This happens *(i)* for values of $\theta_L$ that can be both negative and positive and *(ii)* when the prior probability of a high state is high enough.

A phenomenon that this result can explain is the echo chamber of climate change deniers and its consequences. In fact, despite the broad scientific consensus on the mechanisms and causes of climate change, there exists a considerable debate and diversity of opinion about this topic in public discourse. An empirical study confirm the presence of echo-chambers of climate change sceptics (Williams et al., 2015). Echo-chambers of this kind can be problematic whereby their presence weaken support for political action on strategies of mitigation or adaptation. For instance, actions against climate change are effective as long as a larger fraction of the population adopt them. From the perspective of a single individual, her own action is not effective if others do not act the same way. In such a contest, where climate change affects everybody lives, acquiring information on the anthropogenic causes of climate change would impact negatively individual well-being, more so when other actions do not conform to this view. Accordingly, individuals tend to avoid being informed so not to become aware of the negative externality caused by other individuals' actions.

An example of this could be represented by an individual's decision whether to buy a petrol car or an electrical one. Before taking a decision she would like to acquire information on the impact that these two engines have on the environment. Clearly, acquiring information on the negative impact of petrol cars on the environment would prevent her to buy that engine, but will leave her with the discomfort that her action is negligible if others individuals do the opposite. Therefore, selectively choosing only news that do not convey bad news, or specifically, reading only blogs or following twitter users that deny anthropogenic climate change turns out to be the optimal strategy.

### 2.4.3 Full information as equilibrium

In this section we provide sufficient conditions under which there exists an equilibrium where agents are fully informed, that is they observe both information sources and are always realist.

If an agent decides to listen to all the information available, that is both $I_1$ and $I_2$, she can receive the following pairs of signals: $(s_H, \varnothing)$, $(\varnothing, \varnothing)$ and $(\varnothing, s_L)$. In the first two cases the agent will always exert effort regardless the way she processes the signals. In the latter case instead, she does not exert effort if she does not deny the signal $s_L$ and she exerts effort if the signal $s_L$ is denied. Moreover, when the agent receives bad news, she would like the other agents to exert effort if $\theta_L > 0$ and abstain from it if $\theta_L < 0$. If the cost of denying the signals received is sufficiently high, the agent knows that she will process correctly any signal received, implying that if the state is bad she will not exert effort. If she expects the other agents to observe both information sources as well, then she knows that also the other agents will not exert effort when the signal delivered by $I_2$ is a bad news. Therefore if the agent cannot deny the bad news because it is too costly to do so, than it is better for her to become fully informed rather than observing only $I_1$.

**Proposition 2.2.** *For any $\theta_L < 0$, there exists a threshold cost of denial $\hat{m}$ such that, for any $m_1, m_2 \geq \hat{m}$ there exists an equilibrium where all agents (i) look at both information sources and (ii) are always realist.*

**Proof.** Proof in the Appendix A.1. □

This proposition shows that, when in the low state future prospects are bad, if the cost of denying both information sources is sufficiently large, agents choose to be fully informed and accept reality. The logic of this equilibrium mirrors the one for the selective exposure equilibrium presented in the previous section. Under this scenario, an agent still would like to avoid bad news. However, different from the previous section, other agents now pay attention to $I_2$. If others are realist, then they will refrain from exerting effort when faced with bad news. This implies that there will be no negative spillover. As a result, the gain in anticipatory utility from denial or avoidance of $I_2$ is smaller than in the previous case. In other word, selective exposure is contagious. An agents want to be selective only if others are selective. And this happens when $\theta_L < 0$. This creates strategic complementarity and thus ultimately multiple equilibria.

**Proposition 2.3.** *Assume $\mu > \frac{1}{2-p_1}$. For any $\theta_L \in (\underline{\theta}_L, \overline{\theta}_L)$ if the following condition holds*

$$-\theta_L(1-\alpha) > (\theta_H - \theta_L) r(I_1, I_2) \tag{2.10}$$

*then for any* $m_1, m_2 \in [\underline{m}, \overline{m}]^2$ *there exists an equilibrium where all agents observe only* $I_1$ *and are realist and an equilibrium where all agents observe both* $I_1$ *and* $I_2$ *and are realist.*

**Proof.** Proof in Appendix A.1. □

This result, which shows multiplicity of equilibria in terms of different awareness of the same reality, reflects the MAD principle of Benabou (2013). According to the MAD principle, $\theta_L < 0$ generates complementarity in the cognitive strategy of the agents leading to multiple equilibria. While in Benabou (2013) the multiplicity arises in terms of the cognitive strategy, in our paper the multiplicity arises in terms of the information sources observed. The explanation of this phenomenon has the same intuition as in Benabou (2013). Looking at equation (2.10), the RHS can be interpreted as the net benefit of an agent $i$, from observing both information sources, to believe that the project is highly productive ($\theta_H$ rather than $\theta_L$) when there are no spillovers (i.e. when $\alpha = 1$) or, equivalently, fixing everyone's action $e = 1$. In fact, when an agent $i$ observes both $I_1$ and $I_2$ and receives ($\varnothing$,$\varnothing$), she does not know the state of the nature, but she only knows that with probability $r(I_1, I_2)$ the productivity is $\theta_H$ and with probability $(1 - r(I_1, I_2))$ the productivity is $\theta_L$. Therefore the net expected gain of thinking the state is $H$ when it is not is $r(I_1, I_2)\theta_H + (1 - r(I_1, I_2))\theta_L - \theta_L \equiv r(I_1, I_2)(\theta_H - \theta_L)$. The LHS of (2.10) represents the expected loss that is inflicted to the agent $i$ by the other agents, through the spillover effect $(1 - \alpha)$, if the state is $L$ and they exert effort anyway because they observe only information source $I_1$. Therefore, when the expected loss inflicted by others is higher than the expected benefit an agent will get being more informed, ignorance becomes contagious and the agent $i$ prefers to remain ignorant too. This "complementarity" therefore leads to multiple equilibria.

Our result of complementarity in information acquisition that leads to the equilibrium of selective exposure to sources of information, has different implications from the complementarity in cognitive strategy of Benabou (2013). The main difference between the two is that our model allows us to derive empirical implication that can be tested directly, while it is not the case in Benabou (2013). In Benabou (2013), in principle, we can only observe the action taken by agents, but we are not able to observe the cognitive strategy of the agents, that is whether they are delusional. In our model, instead, not only we can observe agents' action, but we can also observe the information these agents look at. In fact, in recents years are becoming available data on what people look at, that is which news are shared. For instance, data on twitter following can be used to test our theory, but cannot be used to test Benabou (2013). Therefore our model is more rich in terms of empirical implications and data we can use to test it.

Another future of our model, relative to Benabou (2013) is that it might have insight in terms of policy implications. It is in fact vivid nowadays the debate on more effective regulation to combat the phenomenon of echo chambers and polarisation in the contest of online media and social media. Even though we do not do this kind of analysis, we might consider the possibility of manipulating an information sources. For instance imposing balance requiring that, if the state is bad, the more favourable information source, i.e. $I_1$, has to also deliver bad news with a positive probability. Intuitively, it seems that in such circumstances, whereby an info sources can convey bad news when the state is bad, there will be parameter space under which our agents would be delusional.

Notice that, in terms of the action taken, whether agents choose selectively a source of information or deny a bad signal, the outcome is the same. Therefore, a regulator that would like to intervene imposing balance on the signals conveyed by an information source, with the aim of preventing agents being selective in choosing only the information sources biased towards good news, would not be able to achieve his goal. By forcing a balance of the information sources, the regulation seems having no effect, because agents would end-up denying.

## 2.5 Welfare Analysis

In this section we rank the equilibria obtained in the previous section in terms of welfare. Welfare is computed at period $t = 0$.

Consider the case of the low state of the world. In the equilibrium where agents observe only $I_1$ they will receive the signal $\varnothing_1$ and equilibrium welfare is

$$U^*_{L,I_1} = \delta[-c + (s+\delta)(r(I_1)\Delta\theta + \theta_L)] \tag{2.11}$$

where $r(I_1) = \frac{\mu(1-p_1)}{1-\mu p_1}$. In the other equilibrium where all agents observe both $I_1$ and $I_2$, agents receive either $(\varnothing_1, \varnothing_2)$ or $(\varnothing_1, s_L)$ and equilibrium welfare is

$$U^*_{L,I_1\&I_2} = (1-p_2)\delta[-c + (s+\delta)(r(I_1,I_2)\Delta\theta + \theta_L)] \tag{2.12}$$

where $r(I_2) = \frac{\mu(1-p_1)}{1-\mu p_1-(1-\mu)p_2}$. A comparison of the equilibrium welfare in the two equilibria shows that welfare is higher in the equilibrium where both $I_1$ and $I_2$ are observed provided the following holds

$$-(1-\alpha)\theta_L > r(I_1,I_2)r(I_1)(\theta_H - \theta_L) - \left(\frac{c}{(s+\delta)} - \alpha\theta_L\right). \tag{2.13}$$

Consider now the case of the high state of the world. In the equilibrium where agents observe only $I_1$, they receive $s_H$ with probability $p_1$ and $\varnothing_1$ with probability $1-p_1$ ,with equilibrium welfare corresponding to

$$U^*_{I_1} = p_1\delta(-c + (s+\delta)\theta_H) + (1-p_1)\delta[-c + (s+\delta)(\Delta\theta\, r(I_1) + \theta_L)]. \tag{2.14}$$

In the equilibrium where agents observe both information sources, they receive $(s_H, \varnothing_2)$ and $(\varnothing_1, \varnothing_2)$ with probability $p_1$ and $1-p_1$ respectively with equilibrium welfare being equal to

$$U^*_{I_1,I_2} = p_1\delta(-c + (s+\delta)\theta_H) + (1-p_1)\delta[-c + (s+\delta)(\Delta\theta\, r(I_1,I_2) + \theta_L)]. \tag{2.15}$$

**Proposition 2.4.** *(i) If condition* (2.10) *holds, welfare in the low state is higher in the equilibrium where agents observe both information sources.*
*(ii) In the high state welfare is always higher in the equilibrium were agents observe both information sources.*

**Proof.** Proof in Appendix A.1. □

The result of part (i) seems counter intuitive. In fact, the anticipatory preferences of an agent should balance out the loss she will incur in the future when wrongly choosing $e = 1$ with the benefit she receives from the anticipatory utility, proportional to the value of the anticipatory preferences parameter $s$. However, notice that we are considering only values of $\theta_L$ negative and parameters space of the cost of denials that sustain multiplicity of equilibria. In this situation being more informed always dominate ignorance, because full information will impede agents to undertake the wrong action, i.e. $e = 1$, with some positive probability, which has negative externalities. Moreover, the cost of denial that makes realism or denial sustainable, is negatively related to the anticipatory preferences parameter $s$. That is, lower values of the cost of denial that make denial sustainable, correspond to higher values of $s$. However, our multiple equilibria exists for parameters space of the cost of denial small enough, or equivalently anticipatory preferences larger enough, such that agents choose information sources whose signals will not be denied. Therefore, in our equilibria agents are always realist because, for the same parameter space of the cost of denial, it they were to observe a signal that later on would be denied, that information source wouldn't be chosen in equilibrium. Thus, in the low state the equilibrium with full information Pareto dominates the equilibrium with ignorance, because in the former equilibrium there is a positive probability that agents do not exert effort when it is better not to do it. Intuitively, if anticipatory feelings are very strong, then the selective exposure could be optimal. However, we are considering a parameter space where the two equilibria coexist. In this parameter space, anticipatory utility cannot be too large, otherwise the equilibrium with full information cannot exist. We cannot prove this result directly though, as we are able to characterise the equilibrium only under the circumstance of $\theta_L < 0$.

## 2.6 Conclusions and Final Remarks

This paper develops a model that explains how strategic information avoidance leads to the formation of echo chambers, where ignorance spreads inside them. In settings where avoidance of bad news have negative externalities, these news become harder to accept, resulting in a contagious collective ignorance in which agents undertake “harmful” uninformed decisions. Examples of this phenomenon include the well known echo chamber of no-vaccination movement and the many ideological echo chambers in the political sphere, which leads to polarisation of opinion and extremism. This paper, to the best of our knowledge, is the first that explains the mechanism that links the formation of echo chambers to the selective exposure theory. In fact, in our model the echo chamber emerges in equilibrium as the result of the strategic selection of the information to pay attention to, in order to avoid the psychological cost of denying ex-post uncomfortable news.

This paper belongs to a broad research agenda that looks at the recent debate on how social media and the variety of news’ markets affect individual choice of news consumption. However, our results are not robust enough to be able to explain why in the real world, even though we do observe the presence of echo chambers, we also observe that a fraction of the society relies on diversified information. Possible explanations of this evidence is that people might have different beliefs about the state of the world, or that people and groups are heterogeneous in many other aspects that shape their information choice. Our model is not rich enough to capture these elements.

The model could be further extended to account for different groups with some heterogeneity between groups. Such a model, would be a step further able to explain ideological polarisation. Under this perspective, when agents have anticipatory preferences and can engage in denial of bad signals, we could address whether group polarisation occurs and whether it is symmetrical or not. For example we expect to find that only one group polarises while the other group does not. The opposite scenario would be one where one group polarises towards one information source, or “opinion", and the other group polarises towards the other information source.

In the context of social media and plurality of online and physical outlets in which information is consumed, the providers of news play an active role in selecting the signals to deliver through their platforms. Along this direction, another possible extension of the model is to introduce endogenous information sources in which a sender has to optimally select the signal to deliver, taking into account that consumers of information have anticipatory preferences and can engage in denial of the signal received. The model would allow to identify not only how news consumption shapes individual ignorance, but also how and whether ignorance or full awareness depends on the equilibrium production of news.

# Chapter 3

# Endogenous Information Acquisition in an Investment-Trading Game

## 3.1 Introduction

In this paper we study information acquisition in an investment trading game where (i) entrepreneurs base their investment decisions on their expectation about both an unknown underlying economic fundamental and the price at which they may sell their capital to the financial markets in the future; and (ii) traders operating in the financial market use the aggregate investment to learn about the fundamental. According to Angeletos, Lorenzoni, and Pavan (2010) in such a framework with dispersed information and an exogenous information structure, the information spillover generates inefficiency, calling for policy intervention aimed at improving welfare.

Within the framework of Angeletos, Lorenzoni, and Pavan (2010) a few questions arise: (i) What is the equilibrium outcome if agents were to buy the information they need? (ii) What is the optimal amount of private information acquired in equilibrium? We address these questions, by introducing endogenous information acquisition in the model of Angeletos, Lorenzoni, and Pavan (2010), hereafter ALP. Differently from them, in our model, agents not only take the investment decision according to the information available, but also choose how much costly attention to pay to an informative private signal before any investment decision is taken. This way of modelling information may be more appropriate: when new investment opportunities arise, it is reasonable to think that agents look for new information to learn about the profitability of the investment and this activity is costly.

In particular the framework is the following. At the beginning of the game a new investment opportunity with unknown profitability (the fundamental) arises. Two sectors operate in this economy: the real sector populated by entrepreneurs and the financial sector populated by traders. In the first period each entrepreneur has to decide how much to invest in the new project. In the second period before the profitability of the project is revealed, a fraction $\lambda$ of entrepreneurs is hit by a liquidity shock and sells its capital to the financial sector. Information is incomplete in the sense that each agent in the economy does not know the profitability of the project, but it knows the prior distribution of this fundamental. In addition, entrepreneurs have free access to a public signal about the fundamental value of the project. Before they make their investment decision they can also acquire at some cost a private signal by paying attention to listen to it. The effect of paying attention to the signal is the following, as attention increases the overall precision of the private signal increases. In the financial sector, traders also have free access to a public signal about the fundamental value and they also observe the aggregate capital invested in the economy.

We first characterise the benchmark economy. In the benchmark economy only the entrepreneurs have incomplete information about the fundamental, while traders are perfectly informed. Consistent with the result of ALP we show that in this case there does not exist any information spillover between the real and financial sectors. This is so because traders are informed and thus do not need to infer the profitability of the project from aggregate investment. As a result, the market clearing price in the financial sector is always equal to the fundamental value of the project. We then characterise the equilibrium outcome in terms of the information acquired and how this information affects agents' actions in the incomplete information case, that is when there are information spillovers between the real and financial sectors.

Our results show that, relative to the benchmark economy, entrepreneurs pay less attention to private information implying that the equilibrium precision of the private signal is lower than in the benchmark case. From the perspective of the forces that govern the amount of capital invested, in line with the findings of ALP, the entrepreneurs' investment decision depends more on the public signal and less on the private signal compared to what they would do in the benchmark case with perfectly informed traders. It is important to stress that, even though aggregate capital in our model futures the same characteristics of ALP model, our model differs from theirs. In ALP model agents over-react to signals with correlated noise, while here they acquire too little private information in the first phase. The two phenomena seem manifestation of the same mechanism.

Differently from ALP we show that under some circumstances, agents do not rely at all on private information when investing in the new project, both in the benchmark and in the incomplete information case. This is so when the exogenous precision of the private signal is sufficiently small compared to the sum of the precisions of the overall public information. This means that if the private signal is not informative enough by default, the benefit of acquiring the private signal is lower than the cost of paying attention to it.

We also show that there exists an equilibrium with no information acquisition even for intermediate values of the exogenous precision of the private signal. This happens if entrepreneurs expect to sell their capital to the financial market with high probability. However this result is strictly dependent on the precision of traders' public signal. Namely, if traders are less informed, entrepreneurs are more likely to not acquire information. By contrast, if the precision of entrepreneurs' private signal is sufficiently high, entrepreneurs always acquire private information.

The paper is organised as follows. In the next section we briefly discuss the related literature. Section 3.3 describes the model. Section 3.4 characterises both the benchmark economy with no information spill-over and the economy with information spill-over. In Section 3.5 we discuss the information acquisition policy and identify the condition under which the private signal is not acquired. Section 3.6 presents a summary of the main results of the paper and Section 3.7 concludes.

## 3.2 Literature Review

Following the seminal work of Morris and Shin (2002a) a growing literature has investigated the social welfare effects of different information structures in economies with strategic complementarity or substitutability in actions and incomplete information. In these economies, the value of an underlying economic fundamental is unknown and agents would like to take actions that are closer to the realisation of the fundamental and greater access to information helps agents to do so. A key question in these papers is whether more public or private information is desirable in such economies. In these models, public information is any signal about the unknown fundamental which is common across agents. In these cases the noise of the signal is correlated across agents; private information on the contrary, is any signal about the unknown fundamental that is privately observed by each agent. In this case the noise of the signal is idiosyncratic across agents.

Some papers focus on welfare analysis where the information available in the economy, either public and private, is exogenous and agents can only make decisions based on it, but cannot affect the information they get (Angeletos and Pavan, 2004a, 2007b). Other research, instead, considers economies with an endogenous information structure (Colombo and Femminis, 2008a; Colombo, Femminis, and Pavan, 2014a; Hellwig and Veldkamp, 2009a; Myatt and Wallace, 2012a). That is, when information is costly and agents have to acquire it, the information they obtain is endogenous in the sense that they choose which information to obtain either by choosing which signals to purchase or by paying a cost in order to increase the precision of the signals.

A class of economies with strategic complementarity widely studied in the literature can be captured by the beauty contest game. In such a class of games, if the information structure is exogenous and public information is the only source of information, an increase of its precision is always beneficial for social welfare. Conversely, when agents can also access private information, public information may be detrimental for welfare (Morris and Shin, 2002a). However, in a beauty contest framework where agents are allowed to choose the precision of their private signal, an increase in the precision of the public information is always welfare enhancing (Colombo and Femminis, 2008a).

In the case of economies with investment complementarities, where the coordination between agents is both privately and socially valuable, better precision of public information always increases welfare, while the opposite may occur with an increase in the precision of private information (Angeletos and Pavan, 2004a). However, when private information is not freely available to agents and it needs to be acquired, even if agents optimally coordinate according to the information they get, the information acquired in equilibrium may be inefficient, i.e. less precise than optimal (Colombo, Femminis, and Pavan, 2014a). Thus, according to whether strategic complementarities in actions are valuable only privately or also socially and whether the information structure is exogenous or endogenous, public and private information affect welfare differently.

Models with quadratic payoffs and strategic complementarity or substitutability have been extensively applied to: investment games (Angeletos and Pavan, 2004a), monopolistic competition (Hellwig, 2005), financial markets (Allen, Morris, and Shin, 2006a), political leadership (Dewan and Myatt, 2008a), Lucas-Phelps economy (Myatt and Wallace, 2014a), Cournot competition (Myatt and Wallace, 2015a).

Strategic complementarity may arise also endogenously as a result of an information spillover from one economic sector to another, such as in the case of real sector and financial market interacting with each other (Angeletos, Lorenzoni, and Pavan, 2010; Goldstein, Ozdenoren, and Yuan, 2013). Angeletos, Lorenzoni, and Pavan (2010) model a two-way feedback between investment decisions and asset prices in a financial market with incomplete information about investment opportunities. They show that a beauty-contest may arise from the interaction between the real sector, that has to decide how much to invest, and the financial market interested in the price of the asset related to that investment. From a social point of view the equilibrium outcome is inefficient: the existence of an information spillover induces investors to react too much to the correlated signal (public information) and too little to the idiosyncratic signal (private information).

Our paper is closely related to the paper of Angeletos, Lorenzoni, and Pavan (2010) as it extends their model adding endogenous information acquisition. Our main results are qualitatively the same of their paper. However, the focus of our paper is on the information acquisition in an economy with feedback effects between real sector and financial sector. In addition to complementarity in investment decisions, our paper shows that there exists also (endogenous) complementarity in information acquisition. This result is in line with Hellwig and Veldkamp (2009a) which shows that, when there is complementarity in actions there is also complementarity in information acquisition. Our paper is also related to Myatt and Wallace (2014a) because information acquisition is modelled as the attention paid to listen to the signal which in turns increases endogenously the precision of the signal acquired. Our way of modelling information acquisition however is different from their paper, because we allow the presence of a pure correlated signal and a pure idiosyncratic signal with endogenous precision, while in their paper all signals have a correlated component and an endogenous idiosyncratic component.

## 3.3 The Model

We introduce endogenous private information acquisition in the Angeletos, Lorenzoni, and Pavan (2010) model. Endogenous information acquisition is modeled similarly to Myatt and Wallace (2012a).

**Timing, information structure and key choices**. We consider an economy with a real sector and a financial sector. The economy is populated by two types of agents: entrepreneurs and traders. Each type is of measure $1/2$, where entrepreneurs are indexed by $i \in [0,1/2]$ and traders are indexed by $i \in (1/2,1]$. There are four periods, $t = \{0,1,2,3\}$.

In period $t = 0$ a new investment technology becomes available. The profitability of this new technology is uncertain and determined by the random variable $\theta \sim N(\mu, \sigma_\theta^2)$. Agents do not know $\theta$, they know only its distribution.

In period $t = 1$ only the real sector operates. Each entrepreneur $i$ invests $k_i$ unit of capital in the new technology. Investing in this technology costs $\frac{k_i^2}{2}$. Before deciding how much capital to invest, each entrepreneur has access to a public signal that has perfectly correlated noise

$$\bar{x} = \theta + \eta, \tag{3.1}$$

where $\eta \sim N(0, \kappa^2)$ is common across entrepreneurs and $\eta$ is independent of $\theta$. The signal thus has precision $\pi_{\bar{x}} = \frac{1}{\kappa^2}$. By paying a cost $C(z_i)$ entrepreneurs have also the possibility to acquire a private signal

$$x_i = \theta + \epsilon_i, \tag{3.2}$$

where $\epsilon_i \sim N(0, \frac{\xi^2}{z_i})$ with $\epsilon_i$ independent of $\eta$, $\theta$ and $\epsilon_j$ for any $j \neq i$. The overall precision of the private signal depends on two different components: the exogenous precision $\pi_{x_i} = \frac{1}{\xi^2}$ and the endogenous precision $z_i$. Following an argument similar to Myatt and Wallace (2012a) we refer to these elements of signal precision as the clarity and the attention paid to listen to the private signal, respectively. The way private information acquisition works is the following: each entrepreneur can pay attention $z_i \in R^+$ to listen to the signal and by doing so he can increase the total precision of the private signal. $z_i = 0$ is taken to mean that the entrepreneur does not acquire the private signal. In this case the signal $x_i$ is pure noise, that is $x_i \sim N(0, \infty)$.

In period $t = 2$ each entrepreneur is hit by a liquidity shock with probability $\lambda$, which forces him to sell the capital invested to the financial sector before the realisation of $\theta$. $\lambda$ is common knowledge to both entrepreneurs and traders. We assume that only entrepreneurs hit by the shock sell their capital, the rest of them do not sell it. In this period the financial market starts to operate because some entrepreneurs sell their capital. The financial market is perfectly competitive and its market clearing price is denoted $p$. Traders observe only the fraction of the capital entrepreneurs sell to the traders. Because $\lambda$ is known, by the law of large numbers, traders can infer the aggregate level of investment $K \equiv \int_0^{1/2} k_i di$. The information available to traders about the profitability $\theta$ is given by a public signal

$$y = \theta + \omega \tag{3.3}$$

with $\omega \sim N(0, \tau^2)$ where $\omega$ is independent of $\theta, \eta, \epsilon_i$. We denote its precision $\pi_y = \frac{1}{\tau^2}$. Moreover traders use the observation about aggregate capital to update their beliefs about $\theta$. Finally at $t = 3$ the fundamental value of $\theta$ is publicly revealed and production takes place assuming that each unit of capital delivers $\theta$ units of the consumption good.

Throughout the paper we restrict our analysis to a particular functional form of the cost of acquiring information.

**Assumption 3.1.** *The cost of acquiring information is a linear function of the attention $z_i$ and equal to $C(z_i) = \frac{z_i}{2}$*

With this assumption we are imposing linearity of the cost of acquiring information, the factor $\frac{1}{2}$ simplifies the algebra and is just for exposition purposes, but it does not affect any result.

**Preferences and endowments**. All agents receive an exogenous endowment $e$ of the (non-storable) consumption good in each period. Moreover, they are risk neutral and their discount rate is zero: preferences are given by $u_i = c_{i1} + c_{i2} + s_i c_{i3}$, where $c_{it}$ denotes agent i's consumption in period t, while $s_i$ is a random variable that takes value 0 if the agent is an entrepreneur hit by a liquidity shock and value 1 otherwise. Because there is no discounting and agents have linear preferences, agent's expected utility reduces to the expected present value of their net income flows. The net income flows of the entrepreneurs hit by a liquidity shock is equal to $3e + pk_i - k_i^2/2$, while entrepreneurs that are not hit by the liquidity shock receive net income flows equal to $3e + \theta k_i - k_i^2/2$. Therefore the expected utility of entrepreneur

$i$ from investing $k_i$ units of capital in the new technology, conditional on observing the signals $\bar{x}$ and $x_i$ with attention $z_i$ is given, up to a constant, by

$$E(u_i|\bar{x},x_i) = E\Big[(1-\lambda)\theta k_i + \lambda p k_i - \frac{k_i^2}{2}\Big|\bar{x},x_i\Big] - C(z_i). \quad (3.4)$$

A trader's net income flows is given by $3e+\theta q_i - p q_i$, where $p$ is the market clearing price in the financial market and $q_i$ is the the amount of capital bought by trader $i$. Given that a trader, at the time of trading, observes the exogenous signal $y$ and the aggregate capita $K$, his expected utility $t=2$ is, up to a constant,

$$E(u_i|K,y) = (E[\theta|K,y]-p)q_i, \quad (3.5)$$

## 3.4 Equilibrium

First we solve the last stage of the game, that is we consider a trader's expected utility at the time of trading. Notice that in this game we assume financial markets are perfectly competitive and markets always clear. The market clearing price in the financial market is therefore given by the traders' expectation of the fundamental: $p = E[\theta|K,y]$. We first solve for the market clearing price in the financial market, then we plug the optimal price function into the entrepreneur's expected utility and solve for optimal attention and investment. Before proceeding to solve the model we first define our equilibrium solution concept.

**Definition 3.1.** *A linear REE (rational expectation equilibrium) is an individual information acquisition policy $z$ and investment strategy $k(\bar{x},x_i)$, an aggregate investment function $K(\theta,\eta)$ and a price function $p(\theta,\eta,\omega)$ that jointly satisfy the following conditions:*

*i.* $z \in \underset{z\geq 0}{\operatorname{argmax}}\Big\{E\Big[(1-\lambda)\theta k + \lambda p k - \frac{k^2}{2}\Big] - C(z)\Big\}$

*ii. for all* $(\bar{x},x_i)$*,*

$$k(\bar{x}, x_i) \in \underset{k}{\operatorname{argmax}} \left\{ E\left[ (1-\lambda)\theta k + \lambda p k - \tfrac{k^2}{2} \middle| \bar{x}, x_i \right] - C(z) \right\}$$

*iii. for all* $(\theta, \eta)$,

$$K(\theta, \eta) = \int k(\bar{x}, x_i) d\Phi(\bar{x}, x_i | \theta, \eta);$$

*with* $\Phi(\bar{x}, x_i|\theta, \eta)$ *joint cdf of* $\bar{x}$ *and* $x_i$, *given* $\theta$ *and* $\eta$;

*iv. for all* $(\theta, \eta, \omega)$,

$$p(\theta, \eta, \omega) = E\left[\tilde{\theta} \middle| K(\theta, \eta), y\right];$$

*v. there exist scalars* $\delta_0$, $\delta_\theta$ *and* $\delta_\eta$ *such that, for all* $(\theta, \eta)$,

$$K(\theta, \eta) = \delta_0 \mu + \delta_\theta \theta + \delta_\eta \eta.$$

Condition (i) requires that the information acquisition policy should be optimal, that is each entrepreneur chooses $z$ to maximise his expected utility before any signal is observed. Condition (ii) requires that the entrepreneur's investment strategy is rational, taking as given the equilibrium price function. Condition (iii) defines the aggregate capital. Condition (iv) is just the market clearing condition in the financial markets. Condition (v) imposes linearity of the aggregate capital and therefore of the individual investment decision.

### 3.4.1 Benchmark

In this section we characterise the equilibrium of the benchmark economy. The benchmark is an economy in which no uncertainty about the profitability of the asset (fundamental) exists on traders' side, that is traders can perfectly observe the value of $\theta$. In such an environment, traders do not have to form any expectations about the profitability of the asset, therefore the market clearing price is equal to $\theta$. This in turn, implies that entrepreneurs do not form expectations on traders expectations about the fundamental $\theta$. Therefore asset prices do not affect the entrepreneur's expected utility.

**Lemma 3.1.** *In the benchmark economy, where uncertainty about the fundamental $\theta$ lies only on entrepreneurs' side, there is no information spill-over between the real and financial sectors.*

We now characterise the individual information acquisition policy and the optimal individual investment decision. Conditional on observing the public signal and the acquired private signal, an entrepreneur's expected utility reduces to

$$E(u_i|\bar{x}, x_i) = E\left(\theta k - \frac{k^2}{2}\Big|\bar{x}, x_i\right) - C(z). \tag{3.6}$$

The FOC for the maximisation of entrepreneur's expected utility (3.6) obtains that optimal investment decision is $k(\bar{x}, x_i) = E(\theta|\bar{x}, x_i)$. By Bayesian updating, given the normality assumptions the above expectation is linear and implies that the entrepreneur's expectation about the fundamental is a weighted average of the information available to them.

$$k = \beta_0\mu + \beta_{\bar{x}}\bar{x} + \beta_{x_i} x_i. \tag{3.7}$$

The parameters $\beta s$ represent the weight that each piece of information has on the entrepreneur's investment decision. Before any investment decision is taken, each entrepreneur has to choose the attention $z$ he wants to pay to the private signal he acquires. This implies that the parameters $\beta_0$, $\beta_{\bar{x}}$, and $\beta_{x_i}$ depend on the entrepreneur's information acquisition policy. We solve the entrepreneur's sequential problem described above as if each entrepreneur simultaneously chose the attention to listen to the signal and the weight to assign to each information source. Notice that we can do so because each entrepreneur would not change his decision about $z$ and the weight he attaches to each signal after having observed them. Moreover, the information acquisition policy and thus the weights attached to each source of information of an agent $i$ are not observed by anyone else, before any action is taken. Thus we can solve this two-stage entrepreneur's problem as if it were a one shot game.

**Proposition 3.1.** *In the economy without information spill-overs between the real*

*and financial sectors, the equilibrium information acquisition policy and individual investment strategy are unique.*
*Under Assumption 3.1 they are characterised as follows*

i. *the information acquisition policy is:*

$$z^B = \begin{cases} \frac{\sqrt{\pi_{x_i}} - (\pi_\theta + \pi_{\bar{x}})}{\pi_{x_i}} & \text{if } \pi_{x_i} > (\pi_\theta + \pi_{\bar{x}})^2 \\ 0 & \text{otherwise}; \end{cases} \tag{3.8}$$

ii. *whenever $z^B > 0$ individual capital investment is $k = \beta_0 \mu + \beta_{\bar{x}} \bar{x} + \beta_{x_i} x_i$, where*

$$\beta_0 = \frac{\pi_\theta}{\sqrt{\pi_{x_i}}}, \quad \beta_{\bar{x}} = \frac{\pi_{\bar{x}}}{\sqrt{\pi_{x_i}}} \quad \text{and} \quad \beta_{x_i} = \frac{\sqrt{\pi_{x_i}} - (\pi_{\bar{x}} + \pi_\theta)}{\sqrt{\pi_{x_i}}}; \tag{3.9}$$

iii. *whenever $z^B = 0$ individual capital investment is $k = \beta_0 \mu + \beta_{\bar{x}} \bar{x}$,*
*where $\beta_0 = \frac{\pi_\theta}{\pi_\theta + \pi_{\bar{x}}}$ and $\beta_{\bar{x}} = \frac{\pi_{\bar{x}}}{\pi_\theta + \pi_{\bar{x}}}$*

**Proof.** Proof in Appendix B.1 □

The above proposition clearly shows that in the benchmark economy individuals investment decision is not driven by asset prices, but only by their expectation about the fundamental. However, this proposition highlights an important feature of the benchmark economy: under what condition each entrepreneur acquires the private signal. First of all notice that the prior $\theta$ and the signal $\bar{x}$ represent the total amount of public information that each entrepreneur has access to. Then we can define $\pi_\theta + \pi_{\bar{x}}$ as the entrepreneurs' overall precision of the public sources of information. Thus entrepreneurs acquire private information only if the value of the exogenous precision of the private signal is sufficiently high relative to the overall precision of their public information. Moreover, notice that the existence of an equilibrium with no information acquisition comes from the linearity in the cost of acquiring information.

From Proposition 3.1 it follows that the aggregate capital is equal to $K(\theta,\eta) = \beta_0\mu + \beta_\theta\theta + \beta_\eta\eta$, with $\beta_\theta \equiv \beta_{\bar{x}} + \beta_{x_i}$ and $\beta_\eta \equiv \beta_{\bar{x}}$. The weight $\beta_\theta$ represents the response of aggregate capital to fundamental shocks, while $\beta_\eta$ represents the response of aggregate capital to the correlated shock, that is the response to the common shock $\eta$. In other words the aggregate capital function tells us how much of the aggregate capital is driven by fundamental motive, which reflects the profitability of the investment, and how much of the aggregate capital is driven by the common noise, which represents the volatile part of aggregate capital. Notice that when the private signal is not acquired, that is $z^B = 0$, the fundamental shock and common shock have equal weight in determining aggregate capital.

It is important to highlight that in this benchmark economy without information spillover everything is efficient, both the acquisition and the usage of information.

### 3.4.2 Incomplete Information

We now analyse the equilibrium in the case of incomplete information in which both entrepreneurs and traders do not know the value of $\theta$. We first analyse the last stage of the game, the traders' stage, and then we go back to study what happens at the entrepreneurs' stage.

**Traders' stage.**

The market clearing price in the financial market is $p = E[\theta|K,y]$. All the information traders have about the fundamental comes from the public signal $y$ and the aggregate capital $K$. Let us assume for the moment that the aggregate investment, given the information available in the economy, takes the following linear form $K(\theta,\eta) = \delta_0\mu + \delta_\theta\theta + \delta_\eta\eta$. We later verify that this corresponds to the true one. Observing the aggregate capital $K$ is equivalent to observe the following signal:

$$s = \theta + \varphi\eta \equiv \frac{K - \delta_0\mu}{\delta_\theta}. \tag{3.10}$$

Conditional on the prior $\theta$, the signal $s$ has precision $\frac{\pi_{\bar{x}}}{\varphi^2}$, where $\varphi = \frac{\delta_\eta}{\delta_\theta}$. Thus, traders' expectation of $\theta$ conditional on $K$ and $y$, using Bayesian updating is

$$E[\theta|K,y] = E[\theta|s,y] = \gamma_0\mu + \gamma_s s + \gamma_y y. \tag{3.11}$$

For the traders, the conditional expected value of $\theta$ is thus a linear combination of all the signals available to them, with the parameter $\gamma s$ representing how much each signal contributes to their expectation about the fundamental and thus to the assets' price. Accordingly, by substituting (3.3) and (3.10) into (3.11), the price function is a linear function of the prior, the fundamental and the noise terms $\eta$ and $\omega$

$$p = \gamma_0\mu + \gamma_\theta\theta + \gamma_\eta\eta + \gamma_\omega\omega \tag{3.12}$$

where parameters $\gamma_0, \gamma_\theta, \gamma_\eta$ and $\gamma_\omega$ are calculated explicitly in Appendix B.1. Therefore, the price function is a linear combination of all the signals available to the traders, as well. In particular, $\gamma_\theta$ represents the response of the price function to the fundamental shock $\theta$, $\gamma_\eta$ represents the response of the price function to the entrepreneurs' common shock $\eta$ and $\gamma_\omega$ represents the response of the price function to the traders' common shock $\omega$.

**Entrepreneur's stage**

At $t = 1$ each entrepreneur has to choose how much attention to pay to acquire the private signal $x_i$ and how much to invest. Given the normality of the prior and the signals and the quadratic payoffs, we can infer that individual investments can be expressed as a linear function of the information available to the entrepreneur:

$$k(\bar{x}, x_i) = \delta_0\mu + \delta_{\bar{x}}\bar{x} + \delta_{x_i}x_i \tag{3.13}$$

Using the fact that the signals are as given by equations (3.1) and (3.2) and substituting equations (3.12) and (3.13) into equation (3.4), the entrepreneur's problem reduces to choosing attention $z$ and weights $\delta_0$, $\delta_{\bar{x}}$ and $\delta_{x_i}$ to maximise his unconditional expected utility.

**Lemma 3.2.** *Assume that $\pi_{x_i} > (\pi_\theta + \pi_{\bar{x}})^2$ holds. Under Assumption 3.1, if $z > 0$ then*

*i. any investment strategy is characterized as follows*

$$\delta_0 = \frac{\pi_\theta}{\sqrt{\pi_{x_i}}} + \lambda \frac{\pi_\theta \varphi^2}{\pi_{\bar{x}} + \varphi^2(\pi_\theta + \pi_y)}, \tag{3.14}$$

$$\delta_{\bar{x}} = \frac{\pi_{\bar{x}}}{\sqrt{\pi_{x_i}}} + \lambda \frac{\varphi \pi_{\bar{x}}}{\pi_{\bar{x}} + \varphi^2(\pi_\theta + \pi_y)}, \tag{3.15}$$

$$\delta_{x_i} = \frac{\sqrt{\pi_{x_i}} - (\pi_{\bar{x}} + \pi_\theta)}{\sqrt{\pi_{x_i}}} - \lambda \frac{\varphi(\pi_{\bar{x}} + \varphi \pi_\theta)}{\pi_{\bar{x}} + \varphi^2(\pi_\theta + \pi_y)}; \tag{3.16}$$

*ii. and the information acquisition policy is given by*

$$z^* = \frac{1}{\sqrt{\pi_{x_i}}} \left\{ \frac{\sqrt{\pi_{x_i}} - (\pi_{\bar{x}} + \pi_\theta)}{\sqrt{\pi_{x_i}}} - \lambda \frac{\varphi(\pi_{\bar{x}} + \varphi \pi_\theta)}{\pi_{\bar{x}} + \varphi^2(\pi_\theta + \pi_y)} \right\}, \tag{3.17}$$

*where $\varphi$ is an endogenous parameter that in equilibrium must satisfy:*

$$\varphi = \frac{\delta_{\bar{x}}}{\delta_{\bar{x}} + \delta_{x_i}}; \tag{3.18}$$

**Proof.** Proof in Appendix B.1. □

The above proposition simply characterises the individual investment decision and the information acquisition policy when there exists information spill-over between the real and financial sectors.

**Proposition 3.2.** *For any of the equilibria identified in Lemma 3.2, relative to the benchmark economy with no information spill-over, entrepreneurs (i) pay less attention to private information and (ii) put more weight on the prior and on the public signal and less on the private signal.*

**Proof.** This can be seen by comparing $\beta_{\bar{x}}$ and $\beta_{x_i}$ in (3.9) with (3.15) and (3.16) respectively, and noticing that (3.15) can be written as $\delta_{\bar{x}} = \beta_{\bar{x}} + G$ and (3.16) can be written as $\delta_{x_i} = \beta_{x_i} - F$, where $G$ and $F$ are positive. Moreover, with the presence of information spill-over, in the case of acquisition of the private signal, entrepreneurs pay less attention to private information than in the benchmark. This can be seen by comparing (3.8) with (3.17) and noticing that $z^* = z^B - F$. □

Notice that, because the equilibrium attention paid to the private signal is lower than in the benchmark, under incomplete information the equilibrium precision of the private signal is lower than in the benchmark case. This comes from the fact that the total precision of the private signal, which is $z\pi_{x_i}$, is endogenously determined in equilibrium by $z^*$. Thus, as entrepreneurs pay less attention to the private signal than in the benchmark case, they acquire a signal that in equilibrium is less precise. Moreover, because $z^*$ is proportional to $\delta_{x_i}$, a decrease in the attention paid to the private signal relative to the benchmark, corresponds to less weight put on the private signal and consequently to an higher weight on the public signal. In other words, agents over-respond to sources with correlated noise. This mechanism can be explained by the fact that such sources of information permit the agents to better predict mis-pricing in financial markets.

It follows that the equilibrium value of aggregate capital is equal to

$$K = \delta_0 \mu + \delta_\theta \theta + \delta_\eta \eta, \tag{3.19}$$

where $\delta_\theta \equiv \delta_{\bar{x}} + \delta_{x_i}$ and $\delta_\eta \equiv \delta_{\bar{x}}$.

From equation (3.17) it can be immediately seen that $z^*$ may fail to be positive. Under some circumstances entrepreneurs may find it optimal not to acquire the private signal, i.e. $z^* = 0$. In the case entrepreneurs do not acquire any private signal, the information available to entrepreneurs is fully conveyed to traders through aggregate capital. In such situations, the information spill-over is at its maximum, that is $\varphi = 1$, and the economy is characterised are as follows:

$$k = \delta_0 \mu + \delta_{\bar{x}} \bar{x}, \tag{3.20}$$

where $\delta_0 = \frac{\pi_\theta}{\pi_\theta + \pi_{\bar{x}}}$ and $\delta_{\bar{x}} = \frac{\pi_{\bar{x}}}{\pi_\theta + \pi_{\bar{x}}}$;

$$K = \delta_0 \mu + \delta_\theta \theta + \delta_\eta \eta, \tag{3.21}$$

where $\delta_\theta = \delta_\eta \equiv \delta_{\bar{x}}$;

$$p = \gamma_0 \mu + \gamma_\theta \theta + \gamma_\eta \eta + \gamma_\omega \omega, \tag{3.22}$$

where: $\gamma_0 = \frac{\pi_\theta}{\pi_\theta + \pi_{\bar{x}} + \pi_y}$ $\quad \gamma_\theta = \frac{\pi_{\bar{x}} + \pi_y}{\pi_\theta + \pi_{\bar{x}} + \pi_y}$ $\quad \gamma_\eta = \frac{\pi_{\bar{x}}}{\pi_\theta + \pi_{\bar{x}} + \pi_y}$ and $\gamma_\omega = \frac{\pi_y}{\pi_\theta + \pi_{\bar{x}} + \pi_y}$.

Notice that, if entrepreneurs do not acquire private information, that is $z^* = 0$, then their investment decisions are first best efficient given the information at their disposal. However, it turns out that entrepreneurs' actions, i.e. their investment decisions, are less sensitive to fundamentals. This shows that there seems to be a trade-off between increasing the sensitivity of actions to fundamentals and reducing the exposure to correlated noise. In fact, if entrepreneurs acquire private information then they invest inefficiently but the aggregate capital conveys more info about the fundamental. If, instead, entrepreneurs do not acquire private information, they invest efficiently, but the amount of information about the fundamental contained in the aggregate capital is lower.

## 3.5 Information Acquisition Policy

In this section we study the conditions under which entrepreneurs do not pay attention to the private signal. First of all we state an important relationship between the optimal information acquisition policy $z^*$ and the parameter $\varphi$.

**Lemma 3.3.** *If $\pi_{\bar{x}} + \pi_\theta \geq \pi_y$, then $z^*$ is monotonically decreasing in $\varphi$.*

The proof is straightforward. We just study the sign of the first derivative of equation (3.17) with respect to $\varphi$. For complete proof see Appendix B.1.

$z^*$ represents the best response of an entrepreneur, in terms of the attention to pay to the private signal $x_i$, taking as given attention paid by the other entrepreneurs and the aggregate investment in the economy. The parameter $\varphi$ is a measure of information spillover from the real sector to the financial sector. High values of $\varphi$ correspond to aggregate capital that relies more on public information relative to the private one. This is given by the weights $\delta_{x_i}$ and $\delta_{\bar{x}}$. On the traders side, high values of $\varphi$ correspond to a signal conveyed to traders through aggregate capital that is less informative about the fundamental $\theta$. Therefore, when an entrepreneur expects traders to be less informed about the fundamental through aggregate capital, that is via $\varphi$, his incentive to acquire private information reduces. But entrepreneurs's expectation about traders' information depends on the aggregate behaviour of all entrepreneurs. Thus, this shows that there exists (endogenous) complementarity in attention. That is, if other entrepreneurs pay less attention to their private signal, traders become less informed ($\varphi$ is high) and this reduces the incentive of an entrepreneur to pay attention to his private signal.

**Lemma 3.4.** *$z^*$ is increasing in the precision of traders' public signal $y$.*

**Proof.** The first derivative of equation (3.17) with respect to $\pi_y$ is positive. □

This Lemma shows that there is a positive relationship between entrepreneurs' incentive to acquire the private information and the informativeness of traders' public signal. Therefore, if entrepreneurs expect traders to be more informed, then it becomes important for entrepreneurs to acquire more information. On the other hand, when entrepreneurs expect traders to have poor information about the fundamental value of the investment project, entrepreneurs' attention to the private signal reduces.

This sort of complementarity between traders' information and entrepreneurs' acquisition of private information has the following explanation. If traders are better informed, that is they have a signal with higher precision, their signal is more informative about the fundamental value of the assets. When traders are more informed about the fundamental, their price function will give less weight to the signal conveyed by the aggregate capital and more to traders' signal. In such scenario, the higher is the information at traders' hands, the higher is the incentive of entrepreneurs to invest in private information and to know more about the fundamental, as there is less room for them to count on traders' pricing error caused by looking at the aggregate capital.

As an example, suppose there are two different technologies in which entrepreneurs can invest, one well established and the other one not. For the established one, we might expect traders being well informed about the returns of investing in such technology, while in the case of the less established technology, traders might have more uncertainty. According to the above lemma, we should expect less information acquisition in the market with the less established technology relatively to the other market. Therefore, firms and investments in established markets tend to be fairly evaluated, not only because there is more (precise) information available but also because more information generates further information, i.e. firms acquire more private information. On the contrary in markets where there is less (precise) information to start with, i.e. there is more uncertainty about the returns of new technologies, information acquisition exacerbate the asset mis-pricing. Entrepreneurs do acquire less private information and asset prices tend to be more distant from the fundamental value.

For instance, the dot-com bubble can be an example of these phenomena. In fact, during the dot-com period – which lasted from 1997 to 2000 – has been observed an increasing number of firms investing in this new high-tech sector with a corresponding high evaluation in financial markets, of the capital of these firms. The subsequent burst of the bubble at the beginning of the new millennium, revealed that the the asset price of these firms was not reflecting its fundamental value.

From Lemmas 3.3 and 3.4 we know that $z^*$ is decreasing in $\varphi$ and increasing in $\pi_y$. Moreover from Proposition 3.2 we know that $\varphi = \frac{\delta_{\bar{x}}}{\delta_{\bar{x}}+\delta_{x_i}}$ and $z^* \propto \delta_{x_i}$. Our objective is then to identify under which conditions the information acquisition policy $z^*$ is equal to zero.

First of all, as it is shown in Proposition 3.1 whenever the exogenous precision of entrepreneurs' private signal is smaller than $(\pi_\theta + \pi_{\bar{x}})^2$ entrepreneurs do not acquire the private signal under complete information. From Proposition 3.2 and equation (3.17) we see that the same is true in the incomplete information case. Therefore an inspection of this equation allows us to identify the conditions under which its right hand side (RHS) is equal to zero. But then if the RHS is equal to zero then both $z^*$ and $\delta_{x_i}$ are equal to zero which imply $\varphi = 1$.

**Proposition 3.3.** *Assume $\pi_{\bar{x}} + \pi_\theta \geq \pi_y$. Let $\underline{\pi_{x_i}} \equiv (\pi_\theta + \pi_{\bar{x}})^2$. For any value of the precision of the traders' public signal $y$ there exists a range of values $\left[\underline{\pi_{x_i}}, \overline{\pi_{x_i}}\right]$ of the exogenous precision of the entrepreneurs' private signal $x_i$, such that:*

*i. If $\pi_{x_i} \leq \underline{\pi_{x_i}}$, then $z^* = 0$ is the unique equilibrium.*

*ii. If $\pi_{x_i} \in \left(\underline{\pi_{x_i}}, \overline{\pi_{x_i}}\right)$, then there exists a threshold value $\hat{\lambda} < 1$ such that $z^* = 0$ is an equilibrium if and only if $\lambda \in [\hat{\lambda}, 1)$.*

*iii. If $\pi_{x_i} > \overline{\pi_{x_i}}$, then $z^* = 0$ is not an equilibrium.*

**Proof.** Proof in Appendix B.1. □

Part i. of the above proposition highlights an important effect of incomplete information. Private information becomes less valuable. If it is not convenient to acquire the private signal when traders are fully informed, then it is not convenient to acquire it when traders have noisy signals about the fundamental.

Part ii. shows that for intermediate values of the exogenous precision of the private signal and high probability of a liquidity shock, entrepreneurs do not acquire the private signal about the profitability of the new project. Thus, when the liquidity shock is highly likely entrepreneurs are less concerned about learning the value of the productivity of the project and more concerned about the future asset prices. This is easy to see from (3.4), which is the conditional expected utility of an entrepreneur. Treating all the other parameters of that equation as fixed, the conditional expected value of $\theta$ is decreasing in $\lambda$, while the conditional expected value of asset prices is increasing in $\lambda$.

Part iii. shows instead that when the exogenous precision of the private signal is sufficiently high, private information is valuable, no matter how likely is the liquidity shock, and therefore they always acquire it.

Notice that the interval $(\underline{\pi_{x_i}}, \overline{\pi_{x_i}})$ shrinks as the precision of traders' public signal increases. In particular, while the lower bound of this interval is fixed, the upper bound is a decreasing function of $\pi_y$. This can be seen from the proof of Proposition 3.3, where $\overline{\pi_{x_i}} \equiv \left[\frac{(\pi_\theta+\pi_{\bar{x}})^2}{\pi_y} + (\pi_\theta + \pi_{\bar{x}})\right]^2$. Moreover, the precision $\pi_y$ not only affects the aforementioned interval but it also determines the threshold value $\hat{\lambda}$ of the probability of the liquidity shock. The following lemma describes the relationship between $\hat{\lambda}$ and the precision of traders' public signal.

**Lemma 3.5.** *For any precision $\pi_{x_i} \in \left(\underline{\pi_{x_i}}, \overline{\pi_{x_i}}\right)$, the threshold $\hat{\lambda}$ decreases as $\pi_y$ becomes smaller.*

**Proof.** Proof in Appendix B.1. □

The informativeness of traders' public signal seems to play a central role in determining the entrepreneurs' equilibrium information acquisition policy. Hence, the above results shed light on an important relation between the precision of traders' public signal $\pi_y$, the threshold value of the probability of a liquidity shock and the exogenous precision of entrepreneurs' private signal $\pi_{x_i}$. A lower value of $\pi_y$ has two effects. On the one hand, it increases the interval of the values of the exogenous precision of entrepreneurs private signal under which no information is acquired. On the other hand, a smaller probability of a liquidity shock is sufficient to make entrepreneurs not acquire the private signal.

## 3.6 Final Remarks

In the previous sections we have characterised the equilibrium investment decision both under positive and zero information acquisition. However uniqueness of the equilibrium is not always guaranteed. Proposition 3.3.(i) shows that $z^* = 0$ is the unique information acquisition policy for low value of $\pi_{x_i}$ and therefore unique are also the individual investment decision and the aggregate capital. By contrast, part (ii) identifies the conditions under which an equilibrium with zero information acquisition exists. However it does not say that this equilibrium is unique. Part (iii) instead identifies the conditions under which an equilibrium with zero information acquisition does not exist. Therefore, given proposition 3.3 we might have the following

- for any $\pi_{x_i} \in \left(\underline{\pi_{x_i}}, \overline{\pi_{x_i}}\right)$ and $\lambda < \hat{\lambda}$ multiple $z^* > 0$ might exist;
- for any $\pi_{x_i} > \overline{\pi_{x_i}}$ multiple $z^* > 0$ might exist ;
- for any $\pi_{x_i} \in \left(\underline{\pi_{x_i}}, \overline{\pi_{x_i}}\right)$ and $\lambda > \hat{\lambda}$ $z^* = 0$ is not the only equilibrium. We might also have equilibria with $z^* > 0$.

We can summarise the findings and the implications of incomplete information in the following Corollary.

**Corollary 3.1.** *With respect to the benchmark economy, under incomplete information:*

*i.* *entrepreneurs pay less attention to the private signal, that is $z^B > z^*$;*

*ii.* *for intermediate values of the exogenous precision of entrepreneurs' private signal and high probability of a liquidity shock there exists an equilibrium in which entrepreneurs do not acquire the private signal;*

*iii.* *a more precise private signal has two effects: (a) asset prices are more informative about the fundamental but (b) entrepreneurs' investments are less efficient* *iv. there exists complementarity between entrepreneurs' acquisition of private information and the precision of traders' public signal*

These results represent the contribution of this paper and add new insight to the framework analysed by Angeletos, Lorenzoni, and Pavan (2010) even though the two models have the same qualitative results. This means that whether the private signal is exogenously given or is acquired endogenously, the economy behaves in the same way, in the sense that the information spillovers drive the incentives of entrepreneurs to rely more on public information and less on private information. The main difference of this result lies in the fact that in our model agents acquire to little private information in the first phase, while in ALP agents simply over-react to signals with correlated noise.

However, environments with endogenous information acquisition might, under certain circumstances, exacerbate the result. In fact, in all the situations in which entrepreneurs do not pay attention to the private signal, the outcome in terms of individual investment and aggregate investment is worse than in the case of exogenous private information, both in the benchmark economy and in the incomplete information case.

In all the other cases in which entrepreneurs pay attention to the private signal, it is difficult two compare quantitatively the equilibrium with endogenous private information with the one with exogenous private information. A preliminary comparison of the two benchmark cases shows that under some parameter space of the precisions of the signals, under information acquisition the weight that entrepreneurs put on the public signal is lower than in the case of exogenous private signal[1]. In the incomplete information case a clear answer cannot be provided given the difficulty in comparing the endogenous parameter $\varphi$ in the two models. Further analysis needs to be done to qualitatively compare the two models.

[1]Notice that although some analysis has been carried out on this aspect, we do not provide a formal proof of it.

## 3.7 Conclusions

This paper studies endogenous information acquisition in an investment trading game between a real sector and a financial sector in which entrepreneurs first make their investment decisions about a new project and successively a fraction $\lambda$ of this capital is traded in the financial market. The profitability (the fundamental value) of the project is unknown to both entrepreneurs and traders and the information they have access to are only noisy signals about profitability. These signals may be both public and private. Public information is freely available to every agent while private information is agent specific and can be acquired only by entrepreneurs and only conditional on the fact that they pay attention to it. Information spillovers arise from the financial sector to the real sector from a two way feedback between entrepreneurs and traders. Entrepreneurs, by conveying a positive signal about the profitability of the new project, induce an increase in asset prices, which in turn raise their incentives to invest.

In line with the paper of Angeletos, Lorenzoni, and Pavan (2010) this effect creates endogenous complementarity in investment decisions, making entrepreneurs sensitive to high-order beliefs. As a consequence, the impact of fundamental shocks on aggregate capital is reduced, while common expectational shocks amplify their impact on aggregate capital.

The main difference from the paper of Angeletos, Lorenzoni, and Pavan (2010) is that with endogenous information acquisition of the private signal, the economy exhibits two different types of equilibrium. In one type of equilibrium entrepreneurs acquire the private signal and in the other type of equilibrium entrepreneurs rely only on public information. Moreover it is worth highlighting that the exogenous precision of the private signal $x_i$ and the precision of traders' signal $y$ play an important role on entrepreneurs' decision to pay attention to the private signal.

For very low value of the exogenous precision of the private signal entrepreneurs never acquire it, neither in the benchmark economy nor under incomplete information. For intermediate values of this precision, the probability of the liquidity shock matters to determine whether entrepreneurs optimally pay attention to the private signal. Whenever instead the exogenous precision of the private signal is very high, entrepreneurs always acquire the private signal regardless the probability of the liquidity shock. Moreover the parameters space under which both the exogenous precision of entrepreneurs' private signal takes intermediate value and the threshold value $\hat{\lambda}$ of the liquidity shock above which we have an equilibrium with no information acquisition, depend on the precision of traders' public signal. Specifically, as the precision of traders' signal decreases, the value of $\hat{\lambda}$ decreases and the interval of the intermediate values of the exogenous precision of the private signal $x_i$ under which no information acquisition is an equilibrium, increases. This implies that when traders are less informed the incentive for the entrepreneurs to acquire information is reduced.

Future research will focus on the existence of a unique equilibrium, the social value of information acquisition and the quantitative difference between this model of endogenous information acquisition and the model of Angeletos, Lorenzoni, and Pavan (2010) with exogenous private information.

# Chapter 4

# Information Acquisition and Endogenous Network Formation in an (Anti)-Coordination Game

## 4.1 Introduction

Social networks represent an important channel for the formation of opinions as well as for the decision making process. There are many situations in which individuals rely on information acquired through other individuals with whom they have ties and this information affects individual decisions later on. Oftentimes individuals identify themselves belonging to a particular group with which they share common sociodemographic, behavioural and intra-personal characteristics.

In the decision making process, group identity might play an important role. Individuals want to do the right thing and do it together with their peers but at the same time they may want to differentiate themselves from other groups' behaviour. In such a context, information gathered through other individuals is informative not only about the "right thing" but also about what others know and thus enables to predict what others do.

In this paper we capture this framework and study information acquisition and endogenous network formation in a game where agents have both coordination and anti-coordination motives. In particular, each agent seeks to take an action (i) close to some unknown state of the world, (ii) close to the action of his peers, and (iii) far away from the action of his rivals. Before taking an action an agent can acquire information from other agents by establishing a costly link. This project is motivated by our interest to show how the desire to coordinate with some people and anti-coordinate with some others (i) shapes the network formed in equilibrium and (ii) determines the equilibrium informational structure of an economy. Notice that the network structure and the informational structure of the economy will be determined in equilibrium at the same time. However they have a different interpretation. One refers to the shape of the network, i.e. complete, core-periphery, etc. The other refers to the information that is shared in the network, i.e whether some information is more public than other, and this is determined endogenously.

We develop a two stage model with $N$ players divided into two groups in which there is uncertainty about the state of the world. Each player is endowed with a noisy private signal about the underlying state. Signals are independent and identically distributed within group but differ in precision across groups. In the first period each agent has the option to form connections with other players. Connecting to another player is costly and a connection allows the player to observe the signal of that player. That is, each player acquires information through other players and at the same time contributes to the formation of the network. The way information spreads in the network is the following. Information can be observed only through a direct link: only the player that bears the cost of linking is able to observe the signal of the player with whom he is linked and not vice-versa. Moreover the agent that forms a link is able to observe only the signal of the player with whom he has formed a link but he is not able to observe the signals, if any, acquired by that player. In the second period each player uses the information acquired through the network to make a decision. Specifically, each player picks an action balancing three different motives *i)* being close to the underlying state of the world, *ii)* being close to the average action of players in his own group and *iii)* being far from the average action of players in the opposite group.

An example that can fit this model is the "competition" and interaction between two political parties seeking to design their electoral program. A group represents a political party and the members of a group represent the party members or in general, partisans of that party. This example does not intend to explain electoral competition between political party in order to win the elections. What we have in mind is how political activists contribute to the formation of the best electoral program of their party in view of a future electoral competition. In doing so each activist seeks to support the best policy, but at the same time she would like the party to display unit and also to differentiate from the electoral program of the other party. Following an argument similar to  Herskovic and Ramos (2017), we can think of partisans as having access to different sources of information about what would be the best electoral program. A partisan may prefer a particular source but she would rather focus on (i) the same sources as other party members in order to coordinate and display unit and (ii) the information sources of the members of the other party in order to learn their program and "possibly" differentiate from them. Therefore a partisan's action would represent her support to a particular policy and a partisan's signal would represent the information from her preferred source. Thus, an equilibrium information structure would specify which sources each partisan decides to follow. Partisans acquire signals from information sources other than their preferred ones, because other partisans are acquiring them as well.

We solve the game by backwards induction. We first solve the last period of the game, where players chose their optimal action given the network structure and hence the information acquired. Once the optimal action is characterised we can then solve the first period, that is we can identify the network formed and the information acquired in equilibrium. The analysis of the network formation for a generic network of $N$ players turned out to be more complicated than expected and at this stage of the research we are not able to fully characterise the endogenous network formation. Therefore, in order to understand players' incentives of linking to other players, which lead to the network formation, we start by studying the network formation in a 3-players network with two different configurations. In each configuration we have a majority group composed of two players and a minority group composed of one player. In one configuration the majority group has access to the more precise signals, in the other it is the minority that has the more precise signal. For each network configuration we first calculate the ex-ante expected utility of each player in the empty network and study the incentives to form a link with any other player in the network, of both of groups. We then calculate the ex-ante expected utility of each player in the complete network and study their incentive to keep the “last" link with any other player in the network.

We derive cost thresholds for any link in the empty and complete network such that a player prefers to form or keep the link if the cost of the link falls below the threshold. These thresholds therefore characterise the implicit value of a given link. We then compare the ordering of link values both within a given configuration (i.e. within configurations where the majority has access to the more and less precise signal, respectively) and across configurations (i.e. compare the values for example of specific links formed by the majority between a configuration where the majority has access to more precise information with a configuration where the majority has access to less precise information).

When comparing the relative values of specific links in a given configuration, the ordering of link valuations reverses between the empty and the complete network. For example, in the complete network, the majority always values links to individuals with high precision signals more. By contrast, in the empty network, the value of a given link to the majority always depends on the relative signal precisions in conjunction with the coordination and anti-coordination motives. The latter do not play a role regarding the ordering because in the complete network, coordination and anti-coordination can be facilitated via all other links.

When comparing the relative values across configurations, this reversal sometimes materialises, but not always. For the value of a within majority link, we show that it is higher when the group has less precise information in the empty network, but lower in the complete network. This is driven by the fact that if the information is less precise, forming the link is more valuable due to the coordination motive in the empty network - the outside option of not forming the link is less valuable. In the complete network, coordination and anti-coordination can be facilitated via other links and the precision of the signal itself matters more. Hence, a within-majority link is more valuable when the majority has access to more precise signals. For cross-group links, no reversal takes place and links to more precise signals are more valuable in both the empty and complete network.

Overall, these findings suggest that the interplay between the presence of other links in the information network and the coordination and anti-coordination motives is sufficiently complex to warrant further investigation. We view our results as an initial step towards better understanding the formation of these endogenous information structures.

## 4.2 Related Literature

This paper relates to two different literatures. It contributes to the literature on information acquisition in games with Gaussian-quadratic payoffs and also to the literature on information acquisition in networks and network formation. The literature on information acquisition in games with Gaussian-quadratic payoffs was initiated by the seminal paper of (Morris and Shin, 2002b). This literature investigates the use of information and its welfare consequences for the class of games with quadratic payoffs with either strategic complementarity (coordination games) or strategic substitutability (anti-coordination games) in economies where the information is exogenously given (Angeletos and Pavan, 2004b, 2007a). Games with quadratic payoffs have been applied to a variety of settings such as investment games with complementarities, business cycles, oligopoly games, political leadership, and financial markets (Allen, Morris, and Shin, 2006b; Angeletos and Pavan, 2004b, 2007a; Dewan and Myatt, 2008b; Myatt and Wallace, 2014b, 2015b). Other papers instead investigate the use of information and its welfare consequences under costly information acquisition (Colombo and Femminis, 2008b; Colombo, Femminis, and Pavan, 2014b; Hellwig and Veldkamp, 2009b; Myatt and Wallace, 2012b, 2018).

As already mentioned, the existing literature considers either coordination games or anti-coordination games. By contrast our paper considers a game with linear Gaussian-quadratic payoffs that accounts for both coordination and anti-coordination in actions at the same time. Our approach is new to the literature as no other paper considers such a set-up.

The literature on information acquisition in networks can be divided into two subcategories. Some papers investigate the impact of different network structures on information use (Denti, 2017; Leister, 2017; Myatt and Wallace, 2017), while others explores the impact that information use has on the network structure (Galeotti and Goyal, 2010; Herskovic and Ramos, 2017). Our paper contributes to the latter subcategory and is closely related to the work by Herskovic and Ramos (2017). They study information acquisition in a framework where agents acquire information from the same peers they want to coordinate with, thus allowing for the endogenous formation of the network. By contrast in our paper we study information acquisition in a framework where agents can acquire information from players with which they want to both coordinate and anti-coordinate.

Leister (2017) studies both the efficient acquisition and use of information under general heterogeneous network effects employing the familiar quadratic-payoffs setup of Ballester, Calvo-Armengol, and Zenou (2006). Players have access to a single “perfectly private" signal. Players can control the precision of this signal at some cost. He also considers a variant in which the precision choices of players are publicly observed prior to play. He develops measures of marginal strategic values to information and informational externalities, as functions of network position. The paper shows that disparities in equilibrium information investments are inefficiently low relative to the benevolent planner’s solution. All players face only positive strategic values, and thus gain from publicly increasing their informativeness in order to influence the information responses of others.

Denti (2017) models "flexible" information acquisition by players arranged on a network. Information is endogenous: players can reduce the uncertainty they face by acquiring costly information. Before taking action, each player observes the realisation of a signal, which has been previously chosen (at a cost) from some feasible set. Information acquisition is flexible in a sense that it allows each player to choose a signal that is arbitrarily correlated with the signals received by others and the state. He investigates how the network of relations shapes the endogenous information structure. He shows that network effects in action choice induce externalities in information acquisition. The analysis shows that these externalities can be measured by Bonacich centralities and provide new sources of multiple equilibria.

Myatt and Wallace (2017) study how asymmetries in games with quadratic payoffs affect how player arranged in a network use and acquire information. Asymmetries are represented as the weights that link players to neighbours and represents the desire of a player to coordinate (or anti-coordinate) with the corresponding neighbour. Each player can acquire and use information by paying costly attention to multiple sources of information. They show that relatively central players (in the sense of Bonacich) acquire fewer signals from relatively clear information sources; they acquire less information in total; and they place more emphasis on relatively public signals. An important message of their paper is that relatively clear information, which is equivalent to say relatively endogenous public information, has greater influence on players that are more central to a network. Here centrality is meant as a player that is more influenced rather than more influential.

Herskovic and Ramos (2017) develop a two stage game where agents make two decisions. First, they form their social connections, and, second, they choose an action. Each agent receives a signal about the state of the world. In addition, agents can, at a cost, form social connections to observe the signals received by other agents. In the second stage when choosing an action, agents balance their need to adapt to an unknown state of the world and their need to coordinate actions. An important result of their paper is that information is not perfectly substitutable. All agents have signals with the same precision, but in equilibrium some signals are more informative than others about the average action. This depends on the position of the agent in the network. They also show that there is strategic complementarity in the decision of agents to form a connection. As a player receives more links, his signal becomes more public thus, becoming more useful for others players that want to coordinate. Thus, that signal has a higher influence on the average action, and this influence emerges endogenously. Thinking of such players as“opinion makers", they relate their result to the origin of leadership.

## 4.3 The Model

### 4.3.1 Agents and Payoffs

We consider an economy populated by $N$ agents, divided into two groups: $A$ and $B$. Each agent $i \in N$ identifies himself to belong to a certain group, e.g. because he shares the same political views or ideals of that group. Thus, the population of agents is partitioned into two groups of size $N_A$ and $N_B$ respectively, where $N = N_A + N_B$. Given that agents can be re-ordered at will, let the "first" $N_A$ agents ($i = 1, 2, 3, ..., N_A$) belong to group $A$ and the following $N_B$ ones ($i = N_A + 1, N_A + 2, ...N$) belong to group $B$. Whether an agent $i \in N$ belongs to group $A$ or $B$ is assumed to be common knowledge. This assumption reflects the fact that individuals recognise themselves to belong to a particular group based on some sociodemographic characteristics or ideology and that they are also aware of other groups with different sociodemographic characteristics or ideology.

Each agent $i \in N$ seeks to maximise a quadratic loss function by choosing an action that is as close as possible to a target action. We call this action the *bliss* action. The bliss action balances the three different forces that agent $i$ cares about: (i) the need to match an underlying state of the world, $\theta$ (ii) the need to coordinate with agents in his own group and (iii) the need to anti-coordinate with agents in the other group. Let us denote the action of each agent $i \in A$ as $a_i$ and the one of each agent $i \in B$ as $b_i$. Each agent in $A$ and $B$ choses his action in order to maximise respectively:

$$
\begin{aligned}
& u_i^A = -(a_i - a_i^*)^2, && (4.1)\\
\text{where} \qquad & a_i^* = \theta - \alpha_A(\theta - \bar{a}_{-i}) + \beta_A\left(\theta - \bar{b}\right)
\end{aligned}
$$

and

$$
\begin{aligned}
& u_i^B = -(b_i - b_i^*)^2, && (4.2)\\
\text{where} \qquad & b_i^* = \theta - \alpha_B\left(\theta - \bar{b}_{-i}\right) + \beta_B(\theta - \bar{a}),
\end{aligned}
$$

and where:

$$
\bar{a} = \frac{1}{N_A}\sum_{j=1}^{N_A} a_j, \quad \bar{b} = \frac{1}{N_B}\sum_{j=N_A+1}^{N} b_j, \quad \bar{a}_{-i} = \frac{1}{N_A - 1}\sum_{j \neq i} a_j, \quad \bar{b}_{-i} = \frac{1}{N_B - 1}\sum_{j \neq i} b_j,
$$
$$
\alpha_A = \alpha\frac{N_A - 1}{N}, \quad \alpha_B = \alpha\frac{N_B - 1}{N}, \quad \beta_A = \beta\frac{N_B}{N}, \quad \beta_B = \beta\frac{N_A}{N} \quad \text{and} \quad \alpha, \beta \in [0,1].
$$

The parameters $\alpha$ and $\beta$ are respectively, the *coordination* and *anti-coordination* motives and express how much agents care about coordinating with agents in their same group and anti-coordinating with agents in the other group. Both parameters are weighted respectively by the relative size of the relevant group. Given this, agent $i$'s bliss action is the result of a weighted average between (i) the fundamental, (ii) the average action of the rest of the agents in his group and (iii) the average action of all the agents in the other group[1]. It is worth to highlight that, in the extreme case

[1] Notice that in the current version we are using a simplified version for the bliss action. General formula for bliss action should incorporate a parameter $\eta$ that measures the tradeoff between anti-coordination and coordination. Specifically, the bliss action for an agent in group A should be

$$
a_i^* = \theta - \alpha_A(\theta - \bar{a}_{-i}) + \beta_A\left(\theta - \bar{b}\right) + \eta(\bar{a}_{-i} - \bar{b})
$$

where $\alpha$ measures the tradeoff between coordination and fundamental, $\beta$ measures the tradeoff between anti-coordination and fundamental and $\eta$ measures the tradeoff between anti-coordination and coordination. Symmetrically we can write the bliss action for an agent in group B. Throughout where a group is composed only by one agent, e.g. $N_A = 1$, the coordination motive disappears for that agent ( and hence for that group).

### 4.3.2 Information structure

Information in the economy is incomplete, meaning that the value of the fundamental $\theta$ is unknown but all agents share a common prior about it,

$$\theta \sim N(0,1) \tag{4.3}$$

Each agent is endowed with a noisy private signal, $e_i$, about $\theta$,

$$e_i = \theta + \epsilon_i, \tag{4.4}$$

where $\epsilon_i \sim N(0, \sigma_J^2)$ with $J \in \{A, B\}$. That is, noises $\epsilon_i$ are i.i.d. within each group. Without loss of generality we assume that the signals observed by agents in group $B$ are more precise than the signals observed by agents in group $A$, that is $\sigma_A^2 > \sigma_B^2$. From now on we refer to $\pi_J = \frac{1}{\sigma_J^2}$ as the precision of the signals in group $J \in \{A, B\}$.

### 4.3.3 Network

Before taking any action, each agent $i \in N$ can acquire the private signal of any other agent $j \in N$ with $j \neq i$, by establishing a link with him. We refer to this mechanism as "tapping into j's signal". Linking to agents is costly and the cost is an increasing function of the number of links established by each agent. In this respect, the cost of establishing links is the cost of acquiring information. We denote by $K_i^J$ the number of links with agents belonging to group $J$ established by agent $i$, and define $C(K_i^A + K_i^B)$ the cost of acquiring information. Notice that it does not matter with whom a link is formed, only how many links an agent $i$ has formed. Moreover at the moment we do not put any restriction on the cost function. Hence the new payoff function

the paper we will consider the case of $\eta = 0$.

of an agent in $A$ is given by

$$u_i^A = -(a_i - a_i^*)^2 - C(K_i^A + K_i^B)$$

In the network each agent corresponds to a node. The interconnections between the agents, made by establishing links, constitute a directed network and the latter represents, therefore, the information structure of the economy.

The network $G = \{g_{ij}\}_{ij}$ is a list of ordered pair of agents, such that if agent $i$ observes agent $j's$ signal then $g_{ij} = 1$. Otherwise $g_{ij} = 0$. Forming a link is a unilateral decision and has, therefore, unilateral implications. That is, the links formed by an agent $i$ are not symmetric. If agent $i$ taps into $j$'s signal, agent $j$ does not observe $i$'s signal unless he pays the link-formation cost. Once agents have chosen which private signals to observe, the information set of an agent $i \in N$ is composed of the common prior, his own signal and the signals he has chosen to observe. To simplify notation we denote the common prior with $e_0$. Given that each agent $i \in N$ always observes at no cost the prior and his own signal, we write $g_{i0} = g_{ii} = 1$. Therefore we can describe the information set by $I_i = \{e_j \text{ with } j = 0, 1, ...., N, \text{ such that } g_{ij} = 1\}$.

### 4.3.4 Timing

The model consists of two periods. In the first period agents simultaneously acquire information, that is they form the network $G$. In the second period, once the network is formed and signals are observed, agents simultaneously choose their action to maximise their conditional expected payoff.

## 4.4 Model Solution

We solve the game by backwards induction. We first solve the last period of the game where players choose their optimal action given the network structure and hence the information acquired. Once the optimal action is characterised for a generic network of $N$ players, we can in principle solve the first period. That is, we can identify the network formed in equilibrium. At this stage we do not provide a full characterisation but analyse the incentive to form or drop a link in specific 3-player networks.

### 4.4.1 Second Stage: Optimal Action

Conditional on his information set $I_i$, in the second stage of the game agent $i$ solves $\max_a E(u_i^A|I_i)$ if he is in group $A$, and $\max_b E(u_i^B|I_i)$ otherwise. The first order conditions of these two problems yield:

$$a_i = E(a_i^*|I_i) = (1-\alpha_A+\beta_A)E(\theta|I_i)+\alpha_A E(\bar{a}_{-i}|I_i)-\beta_A E(\bar{b}|I_i) \tag{4.5}$$

$$b_i = E(b_i^*|I_i) = (1-\alpha_B+\beta_B)E(\theta|I_i)+\alpha_B E(\bar{b}_{-i}|I_i)-\beta_B E(\bar{a}|I_i). \tag{4.6}$$

The above equations show that agent $i$'s optimal action is a linear combination of the best predictor of the true state, the best predictor of the average action of all other agents in his group and the best predictor of the average action of all the agents in the other group.

The weight attached to the true state of the world is determined by (i) the coordination motive $\alpha$, times the relative size of the group he belongs to, and (ii) the anti-coordination motive $\beta$, times the relative size of the opposite group. The weight attached to the best predictor of the average action of all other agents in his group depends only on the coordination motive times agent $i$'s group relative size, while the weight attached to the best predictor of the average action of all the agents in the other group depends only on the anti-coordination motive times the relative size of the opposite group.

Following the standard approach in the literature, we focus on linear action strategy equilibria. In the appendix we prove that a linear action strategy equilibrium exists and is unique.

In a linear action strategy equilibrium each player chooses an action consisting of a linear combination of the signals he observes, that is, the signal he tapped into, his own signal and the common prior. Obviously, the signals not observed by an agent have, on his individual action, a coefficient equal to zero. Given that individual actions are linear combinations of each signal in the economy, the same is true for the average action of agents in group $A$ and the average action of agents in group $B$, as well as for the average action of all agents in a group excluding agent $i$. Specifically we have:

$$\bar{a} = \frac{1}{N_A}\sum_{i\in A} a_i = \sum_{k=0}^{N} \gamma_k^A e_k, \tag{4.7}$$

$$\bar{b} = \frac{1}{N_B}\sum_{i\in B} b_i = \sum_{k=0}^{N} \gamma_k^B e_k \tag{4.8}$$

$$\bar{a}_{-i} = \frac{1}{N_A - 1}\sum_{i\in A, j\neq i} a_j = \sum_{k=0}^{N} \gamma_{-ik}^A e_k, \tag{4.9}$$

$$\bar{b}_{-i} = \frac{1}{N_B - 1}\sum_{j\in B, j\neq i} b_j = \sum_{k=0}^{N} \gamma_{-ik}^B e_k \tag{4.10}$$

In the appendix we also prove that these linear coefficients sum up to 1.

The linear coefficients $\gamma_k^J$ and $\gamma_{-ik}^J$ represent respectively the influence that a signal $k$ has on the average action of agents of group $J$ and the average action of agents of group $J$ not including agent $i$. It is important to notice that for each signal there exist two measures of influence, one for the average action of group $A$ and another one for the average action of group $B$. These two measures might not be the same. Given their crucial role, in what follows, we formally define the following:

**Definition 4.1.** *The equilibrium group level influence of signal $k$, $\gamma_k^J$, represents how influential is the signal of agent $k$ on the average action of group $J = \{A, B\}$*

In the linear action strategy equilibrium, the equilibrium group level influence

$\gamma_k^A$ and $\gamma_k^B$ are equal to:

$$\gamma_k^A = \begin{cases} \dfrac{\pi_A}{N_A}\displaystyle\sum_{i=1}^{N_A}\dfrac{g_{ik}}{\pi_A(K_i^A+1)+\pi_B K_i^B+1}+\dfrac{\left(\alpha N_A\gamma_k^A-\beta N_B\gamma_k^B\right)}{N_A(N+\alpha)}(\bar{K}_A^k+1)+ \\ -\dfrac{\pi_A}{N_A(N+\alpha)}\displaystyle\sum_{s=0}^{N}\sum_{i=1}^{N_A}\dfrac{\left(\alpha N_A\gamma_s^A-\beta N_B\gamma_s^B\right)g_{is}g_{ik}}{\pi_A(K_i^A+1)+\pi_B K_i^B+1} \qquad \text{if} \quad \boldsymbol{k}\in\boldsymbol{A}, \\ \\ \dfrac{\pi_B}{N_A}\displaystyle\sum_{i=1}^{N_A}\dfrac{g_{ik}}{\pi_A(K_i^A+1)+\pi_B K_i^B+1}+\dfrac{\left(\alpha N_A\gamma_k^A-\beta N_B\gamma_k^B\right)}{N_A(N+\alpha)}\bar{K}_A^k+ \\ -\dfrac{\pi_B}{N_A(N+\alpha)}\displaystyle\sum_{s=0}^{N}\sum_{i=1}^{N_A}\dfrac{\left(\alpha N_A\gamma_s^A-\beta N_B\gamma_s^B\right)g_{is}g_{ik}}{\pi_A(K_i^A+1)+\pi_B K_i^B+1} \qquad \text{if} \quad \boldsymbol{k}\in\boldsymbol{B}, \end{cases} \tag{4.11}$$

$$\gamma_k^B = \begin{cases} \dfrac{\pi_A}{N_B}\displaystyle\sum_{i\in B}\dfrac{g_{ik}}{\pi_A K_i^A+\pi_B(K_i^B+1)+1}+\dfrac{(\alpha N_B\gamma_k^B-\beta N_A\gamma_k^A)}{N_B(N+\alpha)}\bar{K}_B^k+ \\ -\dfrac{\pi_A}{N_B(N+\alpha)}\displaystyle\sum_{s=0}^{N}\sum_{i\in B}\dfrac{(\alpha N_B\gamma_s^B-\beta N_A\gamma_s^A)g_{is}g_{ik}}{\pi_A K_i^A+\pi_B(K_i^B+1)+1} \qquad \text{if} \quad \boldsymbol{k}\in\boldsymbol{A} \\ \\ \dfrac{\pi_B}{N_B}\displaystyle\sum_{i\in B}\dfrac{g_{ik}}{\pi_A K_i^A+\pi_B(K_i^B+1)+1}+\dfrac{\left(\alpha N_B\gamma_k^B-\beta N_A\gamma_k^A\right)}{N_B(N+\alpha)}(\bar{K}_B^k+1)+ \\ -\dfrac{\pi_B}{N_B(N+\alpha)}\displaystyle\sum_{s=0}^{N}\sum_{i\in B}\dfrac{\left(\alpha N_B\gamma_s^B-\beta N_A\gamma_s^A\right)g_{is}g_{ik}}{\pi_A K_i^A+\pi_B(K_i^B+1)+1} \qquad \text{if} \quad \boldsymbol{k}\in\boldsymbol{B}, \end{cases} \tag{4.12}$$

where $\bar{K}_J^j = \sum_{i\in J, i\neq j} g_{ij}$, and $\pi_J = \sigma_J^{-2}$, with $J = \{A, B\}$. Details on the derivations of the above formulas are provided in Appendix C.1. Moreover notice that the parameter $\gamma_{-ik}^J$, for $J = \{A, B\}$, which is the equilibrium group level influence of signal $k$ without the link of agent $i$ , can be derived from the $\gamma_k^J$ using:

$$\gamma_{-ik}^J = \frac{N_J}{N_J-1}\gamma_k^J - \frac{1}{N_J-1}\lambda_{ik}^J, \tag{4.13}$$

where $\lambda_{ik}^J$ represent the individual weight an agent $i$ belonging to group $J$ attributes to signal $k$. The equation for $\lambda_{ik}^J$ is derived in (C.34) and (C.35) for $J = A$ and $J = B$

respectively, as shown in the appendix.

This characterisation of the equilibrium group level influence of a signal is a direct consequence of the presence of both coordination and anti-coordination motive. The interaction between coordination and anti-coordination motive has therefore a fundamental implication on how influential a signal is on the average action of both groups. In Herskovic and Ramos (2017), which is the closest paper to ours, but with only a coordination motive, as a signal receives more and more links, that signal becomes more influential on the average action of the group. In our model, when players in the two groups start looking to each other signals, the influence of a signal on the average action of a group is affected not only by how many players are looking to that particular signal, but also by whether that signal is observed by a player in its own group or by a player in the opposite one.

In particular, for a given network structure, if a signal receives an extra intra-group link, then that signal becomes more informative about the average action of its own group. If instead the extra link comes from the opposite group, then that signal becomes less influential on the average action of its own group, but at the same time it becomes more informative about the average action of the opposite group.

The above would not happen if the anti-coordination motive did not exist. In such a case, a signal influence would change only on the average action of the group that observes that signal, without affecting the influence that the same signal has on the average action of the other group.

Thus, when coordination and anti-coordination motives coexist, it seems that a more observed signal not necessarily is more informative about the average action of a group. Moreover, when the groups starts looking to the same signals, those signals might become less relevant to predict the average action of a group.

### 4.4.2 First Stage

In the first stage of the game, agents choose what signals to tap into in order to maximise their expected utility. Here below we derive the ex-ante expected payoff of an agent in group $A$ as function of other agents connections (for a given network $G$) and the resulting optimal actions. By substituting the optimal action of the second stage, that is $a_i = E(a_i^*|I_i)$, into the ex-ante expected utility of the first stage of the game we have:

$$E(u_i^A|G) = -E[(E(a_i^*|I_i) - a_i^*)^2|G] - c(K_i^A + K_i^B). \tag{4.14}$$

By solving expectations of the above equations (see Appendix C.2), we can write the value of the ex-ante expected utility as function of the other agents connections.

$$E(u_i^A|G) = -\frac{1}{\phi_i^A}\left(1 - \sum_{k=1}^{N} \delta_{ik}^A g_{ik}\right)^2 - \sum_{k=1}^{N}(1 - g_{ik})\left(\delta_{i,k}^A\right)^2 \sigma_k^2 - c(K_{i,A}^A + K_{i,A}^B), \tag{4.15}$$

where $\phi_i^A = 1 + \pi_A(K_i^A + 1) + \pi_B K_i^B$ and $\delta_{ik}^A = \left(\alpha \frac{N_A - 1}{N} \gamma_{-ik}^A - \beta \frac{N_B}{N} \gamma_k^B\right)$.

Symmetrical results apply for an agent in group $B$.

## 4.5 Network formation analysis

In this section we study the incentive of a player to form or to keep a link in specific 3-player networks. This analysis aims to understand the mechanism behind the network formation. Specifically, we consider the empty network and the complete network and we consider only the deviation of a player to form a link in the empty network and the deviation of the player to drop a link from the complete network. In essence, we derive conditions under which the empty and complete network, respectively, are stable. Because we allow for size heterogeneity between groups, we consider two different configurations: (i) two players in group A and one player in group B and (ii) one player in group A and two players in group B. We will refer to these two network configurations as $2A1B$ and $1A2B$ respectively. This is of interest because the two groups have signals with different precisions. In interpreting the results, it is important to keep in mind that the coordination motive never matters for the minority. As the minority in our configurations consists of a single agent, the coordination motive for the agent is given by $\alpha\frac{N_J-1}{N}=0$, where $J$ is the minority group.

In what follows we calculate the ex-ante expected utility of an agent $i$ in the empty and in the complete network. Starting from the empty network we also calculate the ex-ante expected utility of an agent $i$ of forming a link with any other agent in the network; and starting from the complete network we calculate the ex-ante expected utility of an agent $i$ when she cuts a link with any other agent in the network. Once we calculate these ex-ante expected utilities for each player in the network we analyse the incentive of each player to form or to keep a link. The analysis will be conducted in terms of the threshold costs below which forming and keeping a link renders the agent better off. The ex-ante expected utility conditional on a given graph is calculated using equation (4.15) for an agent in group $A$. In a symmetrical way calculate the the ex-ante expected utility for a player in group $B$. In order to simplify the notation, without loss of generality, we assume that the cost of forming links is linear and each link costs $c$. Therefore, $c$ is the cost of forming one link and $2c$ is the cost of forming 2 links. Moreover we denote the ex-ante expected utility, conditional on a given network $G$, simply only as $E(U_i^J)$ omitting the condition on G, where $i=\{1,2,3\}$ is a player and $J$ indicates the group to which the player belongs to.

### 4.5.1 Link-Formation Incentives. Network 2A1B: $i \in \{1,2\} \in A$ and $i = 3 \in B$

**Empty network.** In the empty network no player links to any other in the network. The graph associated with the empty network is represented by the following matrix

$$G^E = \begin{bmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \\ g_{31} & g_{32} & g_{33} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{4.16}$$

The ex-ante expected utility of a player $i \in A$ given the network $G^E$ is equal to

$$\begin{aligned}
E(U_i^A) &= -\frac{1}{\pi_A}\left(1-\sum_{k=1}^{3}\delta_{1k}^A g_{ik}\right)^2 - \sum_{k=1}^{3}(1-g_{ik})(\delta_{1k}^A)^2\pi_k^{-1} \\
&= \frac{1}{\pi_A}(1-\delta_{i1}^A)^2 - \left[(\delta_{i2}^A)^2\pi_A^{-1} + (\delta_{13}^A)^2\pi_B^{-1}\right] \\
&= -\frac{1}{\pi_A}\left[1-\frac{\alpha\gamma_{-11}^A-\beta\gamma_1^B}{3}\right]^2 - \left[\frac{1}{\pi_A}\left(\frac{\alpha\gamma_{-12}^A-\beta\gamma_2^B}{3}\right)^2 + \frac{1}{\pi_B}\left(\frac{\alpha\gamma_{-13}-\beta\gamma_3^B}{3}\right)^2\right] \\
&= -\frac{1}{\pi_A}\left(1+\frac{\alpha^2}{9}\right) - \frac{\beta^2}{9}\frac{1}{\pi_B}
\end{aligned} \tag{4.17}$$

where $\gamma_k^J$ is calculated using equations (4.11)-(4.12) and $\gamma_{-ik}^J$ are calculated using (4.13) together with equations (C.34)-(C.35).

In a similar way we calculate the ex-ante expected utility of agent $i = 3 \in B$. Utility calculations for this agent as well as the value of the parameters $\gamma_k^J$ and $\gamma_{-ik}^J$ are shown in Appendix C.3.1.

**From empty network to one link** In this paragraph we study the incentive of a player $i$ to link to another player. With the network configuration we are considering, that is two agents in group A an one agent in group B, for an agent in group A we study the incentive to link to an agent (i) of the same group (link to majority) and (ii) of the opposite group (link to minority). On the contrary for an agent in group B, we can only study the incentive to link to an agent of the opposite group (link to the majority).

If a player $i \in A$, say player 1, links to the majority when the other players are not linked to anyone, the graph related to this network is given by the following matrix

$$G^{+M} = \begin{bmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \\ g_{31} & g_{32} & g_{33} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{4.18}$$

and her ex-ante expected utility is equal to

$$\begin{aligned} E(U_1^A) &= -\frac{1}{2\pi_A}\left[1-\left(\frac{\alpha\left(\gamma_{-11}^A+\gamma_{-12}^A\right)}{3}-\frac{\beta\left(\gamma_1^B+\gamma_2^B\right)}{3}\right)\right]^2-\left(\frac{\alpha\gamma_{-13}^A}{3}-\frac{\beta\gamma_3^B}{3}\right)^2\frac{1}{\pi_B}-c \\ &= -\frac{(3-\alpha)^2}{18\pi_A}-\frac{\beta^2}{9\pi_B}-c \end{aligned} \tag{4.19}$$

The ex-ante expected utility of a player $i \in A$ from linking to the minority, and the ex-ante expected utility of player $3 \in B$ are shown in the appendix C.3.1.

By comparing the expected utility of a player $i$ in the empty network with the utility he will get by linking to another player, we can derive, in terms of the cost of linking, the condition under which an agent $i$ prefers linking to another player to the empty network.

**Definition 4.2.** *$c_{JK}$ is the threshold cost below which a player $i$ in group $J$ has an incentive to establish a link with K, where $J = \{A, B, b, a\}$ and $K = \{A, B, a, b\}$. $J = A$ means that the agent belongs to group A and that group is the majority group, while $J = b$ means that the player that establishes the link belongs to group $B$ and is the minority group. $K = A$ and $K = b$ have the same interpretation but they refer to the player to whom the player $i$ links to. So for example $c_{Ab}$ refers to the threshold cost below which a player $i$ in the majority group $A$ has an incentive to link to the player in the minority group $B$.*

Definition 4.2 is written in this fashion because we will use the same notation when we analyse the other configuration in the next section 4.5.2.

**Lemma 4.1.** *The threshold cost below which a player in the majority group A has an incentive to link to a player in the majority group A is*

$$c_{AA} = \frac{(3+\alpha)^2}{18\pi_A}. \tag{4.20}$$

*The threshold cost below which a player in the majority group A has an incentive to link to a player in the minority group B is*

$$c_{Ab} = \frac{(3\pi_B - \beta\pi_A)^2}{9(\pi_A + \pi_B)\pi_A\pi_B}. \tag{4.21}$$

*The threshold cost below which the player in the minority group B has an incentive to link to a player in the majority group A is*

$$c_{bA} = \frac{(3\pi_A - \beta\pi_B)^2}{9(\pi_A + \pi_B)\pi_A\pi_B} \tag{4.22}$$

**Proof.** Proof in Appendix C.3.3. □

Lemma 4.1 shows the threshold costs below which the empty network is unstable. We will use these later to compare and interpret a player's incentives to form a link.

**Complete network.** In the complete network each player links to all others and the graph associated to it is represented by the following matrix

$$G^C = \begin{bmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \\ g_{31} & g_{32} & g_{33} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \tag{4.23}$$

Notice that the diagonal entries do not represent a link but the fact that each players observes his own signal. The ex-ante expected utility of player $i \in A$ given the network $G^C$ is equal to

$$
\begin{aligned}
E(U_i^A) =& \frac{1}{2\pi_A+\pi_B}\left(1-\sum_{k=1}^{3}\delta_{ik}^A g_{ik}\right)^2 - \sum_{k=1}^{3}(1-g_{ik})(\delta_{ik}^A)^2\pi_k^{-1} - 2c \\
=& \frac{1}{2\pi_A+\pi_B}\left(1-\sum_{k=1}^{3}\alpha\frac{1}{3}\gamma_{-ik}^A - \beta\frac{1}{3}\gamma_k^B\right)^2 - 2c \\
=& -\frac{1}{2\pi_A+\pi_B}\left(1-\frac{\alpha-\beta}{3}\right)^2 - 2c
\end{aligned}
\tag{4.24}
$$

In a similar way we calculate the ex-ante expected utility of agent $i = 3 \in B$. Utility calculations for this agent as well as the value of the parameters $\gamma_k^J$ and $\gamma_{-ik}^J$ are shown in Appendix C.3.1.

**From complete to -1 link.** In this paragraph we study the incentive of a player in a complete network to keep the "last" link. For an agent in group A we study the incentive to keep a link with an agent (i) of the opposite group (keep a link with minority) and (ii) of the same group (keep link with majority). For an agent in group B, we can only study the incentive to keep a link with an agent of the opposite group (keep a link with the majority).

If a player $i \in A$, say player 1, drops a links with the minority, the graph associated to this network is represented by the following matrix

$$
G^{-m} = \begin{bmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \\ g_{31} & g_{32} & g_{33} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}
\tag{4.25}
$$

and her ex-ante expected utility is equal to

$$
\begin{aligned}
E(U_1^A) =& -\frac{1}{\pi_A+\pi_B}\left[1-\frac{\alpha(\gamma_{-11}^A+\gamma_{-12}^A)}{3}+\frac{\beta(\gamma_1^B+\gamma_2^B)}{3}\right]^2 - \left(\frac{\alpha}{3}\gamma_{-13}^A - \frac{\beta}{3}\gamma_3^B\right)^2\frac{1}{\pi_A} - c \\
=& -\frac{(3-\alpha+\beta)^2}{9(2\pi_A+\pi_B)} - \frac{(3+\alpha)^2(9-3\alpha-2\beta^2)^2\pi_B}{18(9-\beta^2)^2(2\pi_A+\pi_B)\pi_A} - c
\end{aligned}
\tag{4.26}
$$

The ex-ante expected utility of a player $i \in A$ from dropping a link with the majority and the ex-ante expected utility of player $3 \in B$ are shown in the appendix.

We now identify the conditions, in terms of the cost of linking, under which an agent $i$ prefers the complete network to dropping a link with any other player.

**Definition 4.3.** *$\kappa_{JK}$ is the threshold cost below which a player $i$ in group $J$ has an incentive to keep a link with K, where $J \in \{A, B, a, b\}$ and $K \in \{A, B, a, b\}$. $J = A$ means that the player belongs to group A and that group is the majority group, while $J = b$ means that the player $i$ belongs to group $B$ and is the minority group. $K = A$ and $K = b$ have the same interpretation but they refer to the player with whom the player $i$ prefers to keep the link. So for example $\kappa_{Ab}$ refers to the threshold cost below which a player $i$ in the majority group $A$ has an incentive to keep a link with the player in the minority group $B$.*

Similar to Definition 4.2, we write Definition 4.3 in this fashion because will we use the same notation when we analyse the other configuration in the next section 4.5.2.

**Lemma 4.2.** *The threshold cost below which a player in the majority group A prefers the complete network to dropping a link with a player in the majority group A is*

$$\kappa_{AA} = \frac{(3+\alpha)^2(9-3\alpha-2\beta^2)^2\pi_A}{9(9-\beta^2)^2(\pi_A+\pi_B)(2\pi_A+\pi_B)} \tag{4.27}$$

*The threshold cost below which a player in the majority group A prefers the complete network to dropping a link with the player in the minority group $B$ is*

$$\kappa_{Ab} = \frac{(3+\alpha)^2(9-3\alpha-2\beta^2)^2\pi_B}{18(9-\beta^2)^2(2\pi_A+\pi_B)\pi_A} \tag{4.28}$$

*The threshold cost below which the player in the minority group $B$ prefers the complete network to dropping a link with a player in the majority group A is*

$$\kappa_{bA} = \frac{(9-3\alpha-2\beta^2)^2\pi_A}{9(3-\alpha)^2(\pi_A+\pi_B)(2\pi_A+\pi_B)} \tag{4.29}$$

**Proof.** Proof in Appendix C.3.3. □

Lemma 4.2 shows the threshold costs below which the complete network is stable. We will use these later to compare and interpret a player's incentives to keep a link.

### 4.5.2 Link-Formation Incentives. Network 1A2B: $i = 1 \in A$ and $i \in \{2,3\} \in B$

In this section we proceed in the same way of the previous section and derive, for the configuration 1A2B, the threshold costs below which an agent has an incentive to form a link and to keep a link from the empty and the complete network respectively. Detailed calculations for this configuration are provided in Appendix C.3.2. The following Lemmas summarise the results.

**Lemma 4.3.** *The threshold cost below which the player in the minority group $A$ has an incentive to link to a player in the majority group $B$ is*

$$c_{aB} \equiv \frac{(3\pi_B - \beta\pi_A)^2}{9(\pi_A + \pi_B)\pi_A\pi_B} \tag{4.30}$$

*The threshold cost below which a player in the majority group $B$ has an incentive to link to the player in the majority group $B$ is*

$$c_{BB} \equiv \frac{(3+\alpha)^2}{18\pi_B}. \tag{4.31}$$

*The threshold cost below which a player in the majority group $B$ has an incentive to link to the player in the minority group $A$ is*

$$c_{Ba} \equiv \frac{(3\pi_A - \beta\pi_B)^2}{9(\pi_A + \pi_B)\pi_A\pi_B}. \tag{4.32}$$

**Proof.** Proof in Appendix C.3.3. □

Lemma 4.3 shows the threshold costs below which the empty network is unstable. We will use these later to compare and interpret a player's incentives to form a link.

**Lemma 4.4.** *The threshold cost below which the player in the minority group $A$ prefers the complete network to dropping a link with a player in the majority group $B$ is*

$$\kappa_{aB} = \frac{(9-3\alpha-2\beta^2)^2\pi_B}{9(3-\alpha)^2(\pi_A+2\pi_B)(\pi_A+\pi_B)} \tag{4.33}$$

*The threshold cost below which a player in the majority group $B$ prefers the complete network to dropping a link with a player in the majority group $B$ is*

$$\kappa_{BB} = \frac{(3+\alpha)^2(9-3\alpha-2\beta^2)^2\pi_B}{9(9-\beta^2)^2(\pi_A+2\pi_B)(\pi_A+\pi_B)} \tag{4.34}$$

*The threshold cost below which a player in the majority group $B$ prefers the complete network to dropping a link with the player in the minority group $A$ is*

$$\kappa_{Ba} = \frac{(3+\alpha)^2(9-3\alpha-2\beta^2)^2\pi_A}{18(9-\beta^2)^2(\pi_A+2\pi_B)\pi_B} \tag{4.35}$$

**Proof.** Proof in Appendix C.3.3. □

Lemma 4.4 shows the threshold costs below which the complete network is stable. We will use these later to compare and interpret a player's incentives to keep a link.

### 4.5.3 Analysis of the threshold costs

We now use the previously established threshold costs to better understand what drives agents' incentives to form or keep a link. To do so we compare these threshold costs both within group and between group within the same network configuration, as well as across network configurations. Given that these thresholds are calculated computing the incentive to either form a link or keep a link, we can interpret these threshold costs as the marginal value of a given link. Specifically, in the empty network we will talk about the marginal value of an extra link, while in the complete network we will talk about the marginal value of keeping a given link. Therefore, a greater value of the threshold cost is synonymous with a greater (marginal) value of a given link.

**Instability of the empty network**

We first analyse the threshold costs below which there is an incentive to form a link, that is we look at the following thresholds: $c_{Ab}$, $c_{AA}$, $c_{bA}$, $c_{aB}$, $c_{Ba}$ and $c_{BB}$.

**Lemma 4.5.** *A player $i \in \{A, B\}$ values equally a link with the opposite group, regardless whether the opposite group represents the majority or the minority, that is*

$$c_{Ab} = c_{aB} \qquad \textit{and} \qquad c_{bA} = c_{Ba}.$$

**Proof.** It simply follows by comparing (4.21) with (4.30) and (4.22) with (4.32). □

This lemma compares the marginal value of linking to the opposite group for the two different network configurations. The results follow from the fact that, because no player is linked to anyone else, the size of the group does not play any role and the first link helps to predict the state of the world and what the other player does. This is equal in both configurations. Lemma 4.5 thus states that the value of cross-group links does not depend on majority/minority.

**Lemma 4.6.** *For a player $i \in A$ a link to the opposite group is more valuable than for a player $i \in B$, that is, $c_{Ab} = c_{aB} > c_{bA} = c_{Ba}$.*

**Proof.** It simply follows by comparing (4.21) with (4.22) and (4.30) with (4.32) and also from Lemma 4.5. □

This lemma compares the value of cross-party links, i.e. linking to the opposite group, within the same network configuration, that is $c_{Ab} > c_{bA}$ and $c_{aB} > c_{Ba}$. It also compares the cross-party links to the minority and majority, respectively across the two configurations, i.e $c_{Ab} > c_{Ba}$ and $c_{aB} > c_{bA}$. In both cases, within and across configurations, we see that the marginal value of the first link is greater if this link is established with the player that has the more precise signal. A higher signal precision allows to learn more about the fundamental and what that agent does. Therefore, a player in group $A$ has a higher incentive to link to the opposite group, relative to a player in group $B$. From the above discussion we derive the following Corollary.

**Corollary 4.1.** *(a) Comparing the value of cross-party links within the same configuration (2A1B and 1A2B respectively), we obtain that the more precise signal is always more valuable.* $c_{Ab} > c_{bA}$ *and* $c_{aB} > c_{Ba}$.
*(b) Comparing the value of links to the minority and majority respectively across the two configurations, the link to the more precise signal is similarly always valuable.* $c_{Ab} > c_{Ba}$ *and* $c_{aB} > c_{bA}$.

We next compare the value of within-majority links across configurations.

**Lemma 4.7.** *Comparing the value of within majority links across configurations, a within-majority link is more valuable when the majority is group A then when it is B, that is,* $c_{AA} > c_{BB}$.

**Proof.** It simply follows by comparing (4.20) with (4.31). □

Lemma 4.7 compares within-group links (i.e. with the majority) across network configurations. The value of within-group link for the majority A is higher than for the majority B. This is so because, for a player in group $A$, who has a less precise signal than players in group $B$, linking to her own group helps to coordinate and to know $\theta$ more than players in group $B$, who already have a more precise information about $\theta$. In other words, a player in A has more to gain from coordinating better and learning more about the fundamental; a player in B has a higher "outside" option.

Given the result in lemma 4.7, considering only the difference in signal precisions, and abstracting from the coordination and anti-coordination motives, we would expect to find

$$c_{Ab} > c_{AA} > c_{BB} > c_{Ba}.$$

That is, we might expect that, within a given network, it is more valuable to link to the more precise signal, i.e. $c_{Ab} > c_{AA}$ and $c_{BB} > c_{Ba}$. As we will show next, this is not necessarily the case and depends on the relative precision of the signals together with the coordination and anti-coordination motives. In what follows, we consider these two inequalities separately. For the first inequality we can calculate

$$c_{Ab} > c_{AA} \iff \underbrace{2(3\pi_B - \beta\pi_A)^2 - (3+\alpha)^2(\pi_A + \pi_B)\pi_B}_{LHS} > 0. \tag{4.36}$$

It is immediate to see that the LHS of (4.36) has no clear sign, but we can show the following about it.

$$\frac{\partial LHS}{\partial \pi_A} = \beta^2\pi_A - \left[12\beta + (3+\alpha)^2\right]\pi_B < 0,$$
$$\frac{\partial LHS}{\partial \pi_B} = \pi_B\left[36 - 2(3+\alpha)^2\right] - \pi_A\left[12\beta + (3+\alpha)^2\right] > 0.$$

This implies that, in order for $c_{Ab} > c_{AA}$ to hold, the distance $\pi_B - \pi_A$ has to be big enough. It is not enough for $\pi_B > \pi_A$.[2] This result shows how players in an empty network do *not* necessarily value more, as a first connection, a link that grants them a *more* precise signal, because of their coordination and anti-coordination motives.

To better understand the result above, we can also study how the coordination and anti-coordination motives enter this comparison. It is immediate to see that

$$\frac{\partial LHS}{\partial \alpha} < 0, \quad \frac{\partial LHS}{\partial \beta} < 0.$$

The above confirms our intuitions. When (4.36) holds, a player $i \in A$ in an empty

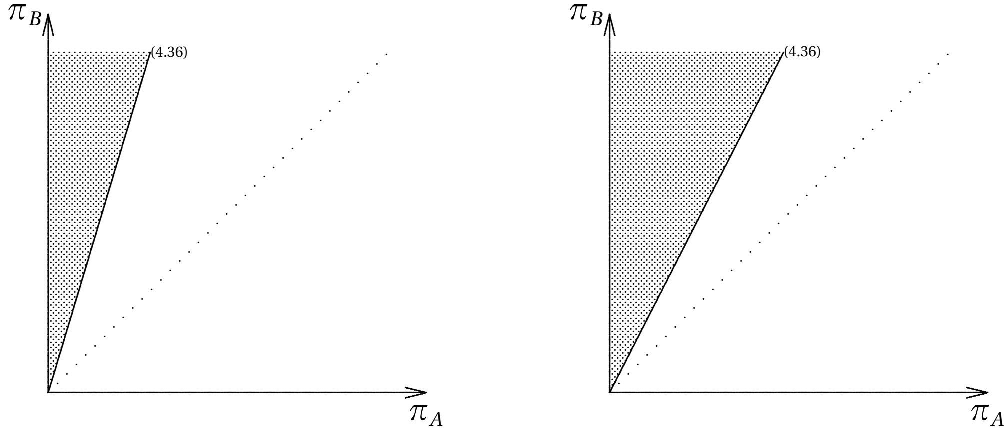
[2] To see why this statement holds, notice that if the inequality only depended on the difference in precision, at $\pi_B = \pi_A$ we would have $c_{Ab} = c_{AA}$. However, at $\pi_B = \pi_A$ the comparison between thresholds boils down to

$$(3-\beta)^2 - (3+\alpha)^2 > 0$$

which depends on the motives of the agent.

network $2A1B$ values more, as a first connection, a link that grants her a more precise signal only if i) the coordination motive is low enough — otherwise she is better off connecting to her own group-mate to ease coordination — ii) the anti-coordination motive is low enough — otherwise she runs the risk to be "stuck" following the player in the other group — iii) the precision of her own (and her group-mate) signal is low enough compared to the precision of the agent in $B$. In figure 4.1 we present two examples for the case of $\alpha < \beta$.

Figure 4.1: Network 2A1B: comparisons of the thresholds majority-minority versus majority-majority. Case $\alpha < \beta$.




(a) $c_{Ab} > c_{AA}, \alpha = 0.5, \beta = 0.8$.

(b) $c_{Ab} > c_{AA}, \alpha = 0.2, \beta = 0.7$.

A similar comparison for the thresholds of a player in group $B$ in a $1A2B$ network, is slightly more complicated. First of all, notice that the complication comes from the fact that

$$\frac{\partial c_{Ba}}{\partial \beta} \begin{cases} \leq 0 & \text{if } \beta \leq 3\frac{\pi_A}{\pi_B} \\ > 0 & \text{if } \beta > 3\frac{\pi_A}{\pi_B} \end{cases}$$

When the anti-coordination motive is large, a further increase in it features a larger $c_{Ba}$. This has two potential sources. On the one hand, in order to *really* anti-coordinate with the other group, the player must know what the other group plays. On the other hand, this comes from the special case we analyse: the one where the trade-off between anti-coordination and coordination is zero[3]. When the player wants to be further away from the other group's action, he wants to be even closer to the fundamental. When this effect is particularly strong, *any* information becomes good information. Hence, the player becomes interested in the (unprecise) signal of $A$.

Consider now the second inequality. Comparing $c_{Ba}$ to $c_{BB}$ we get:

$$
\begin{aligned}
& c_{BB} > c_{Ba} \\
\iff & (3+\alpha)^2(\pi_A+\pi_B)\pi_A - 2(3\pi_A - \beta\pi_B)^2 > 0 \\
\iff & \pi_A[\alpha(\alpha+6)(\pi_A+\pi_B)+9(\pi_B-\pi_A)] + 2\beta\pi_B(6\pi_A-\beta\pi_B) > 0 \qquad (4.37)
\end{aligned}
$$

The first term of (4.37) is obviously positive, the second one also is for $\beta \in \left[0, 6\frac{\pi_A}{\pi_B}\right]$. When the latter fails, there still exists a value of $\beta$ small enough for (4.37) to hold. Instead of focusing on that, however, notice that if $\pi_A > \frac{1}{6}\pi_B$, (4.37) always holds.

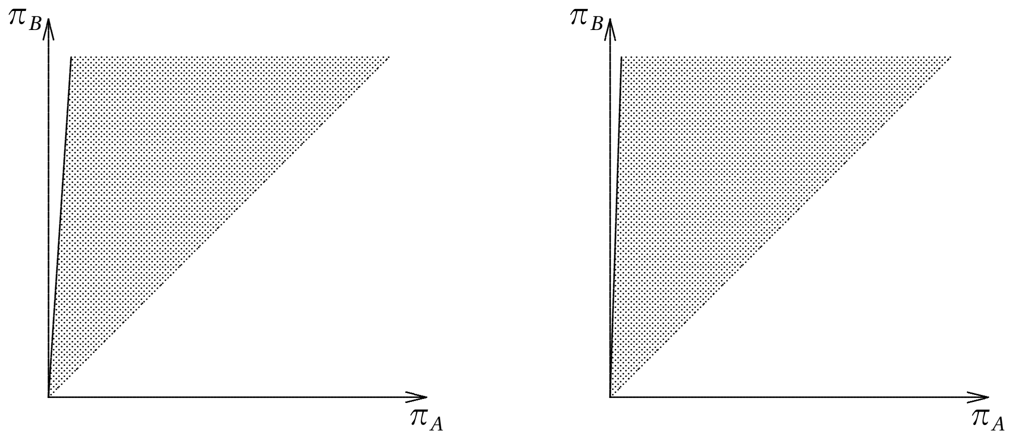
To sum up, in order for a player in $B$ in an empty network where the majority is group $B$, to be more interested in the signal of his group companion it has to be that either i) the signal of the other group is precise enough or ii) the anti-coordination motive is not too large. The intuition behind ii) is the same of the case of $\frac{\partial c_{Ba}}{\partial \beta}$. The reason behind i), instead, could be that when $\pi_A$ is sufficiently close to $\pi_B$, the player in $B$ has a good enough estimate of the other group's action and therefore does not need to know where group $A$'s action will be.

In any case (4.37) does not always hold. We can see it in figure 4.2 , where we present two numerical examples for the case of $\alpha < \beta$.

Combining the two conditions (4.37) and (4.36) together generates three areas in $(\pi_A, \pi_B)$ space. The size of the areas clearly depends on the value of the parameters $\alpha$ and $\beta$ but, abstracting from the size of the areas we can generalise what we discussed above with the graph in Figure 4.3 for the case of $\alpha < \beta$. In Figure 4.4 we plot instead the generalisation of conditions (4.37) and (4.36) for the case of $\alpha > \beta$, still abstracting from the size of the areas.

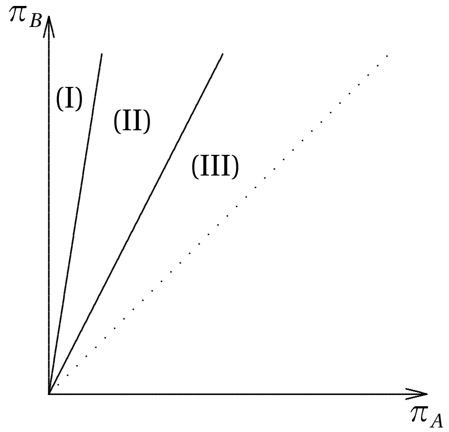
[3]As we pointed out earlier, throughout the paper we are considering a simplified version of the bliss action where $\eta = 0$.

Figure 4.2: Network 1A2B: comparisons of the thresholds majority-minority versus majority-majority. Case $\alpha < \beta$.




(a) $c_{Ba} < c_{BB}, \alpha = 0.5, \beta = 0.8.$ (b) $c_{Ba} < c_{BB}, \alpha = 0.2, \beta = 0.7.$

Figure 4.3: The three areas generated by conditions (4.36) and (4.37). In this case we assume $\beta > \alpha$.




**Remark 1.** *In the Figures 4.3 and 4.4,*

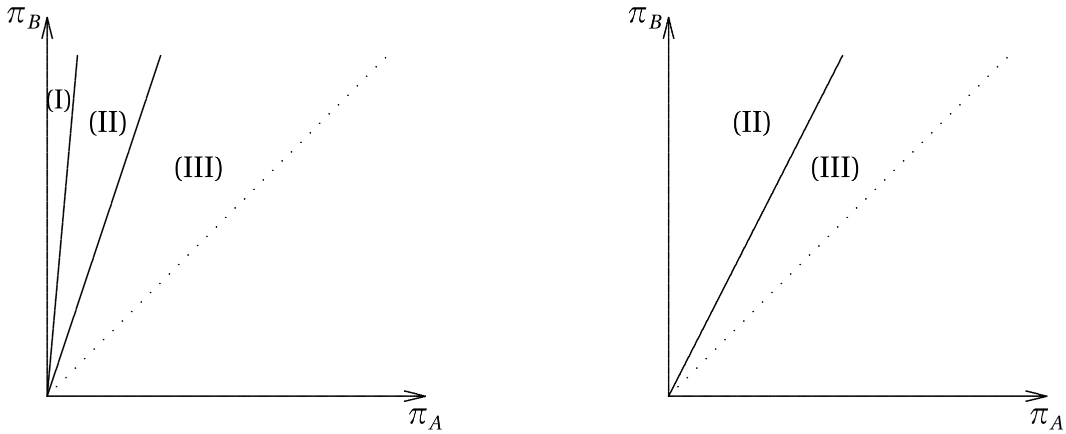
- *area (I) features* $c_{Ab} > c_{AA} > c_{BB} \quad \& \quad c_{Ba} > c_{BB}$
- *area (II) features* $c_{Ab} > c_{AA} > c_{BB} > c_{Ba}$ *and*
- *area (III) features* $c_{AA} > c_{Ab} \quad \& \quad c_{AA} > c_{BB} > c_{Ba}$.

*When* $\alpha > \beta$ *we still have three areas in* $(\pi_A, \pi_B)$ *space, though area (I) is very tiny and there are cases where the area shrinks to (II) and (III) area.*

Remark 1 is a statement about within configuration comparisons and compares the value of a majority-majority link versus a majority-minority link for the two network configurations. Notice that the distance $\pi_B - \pi_A$ increases from area (III) to area (I). In area (III), it is not sufficiently large for an agent in $A$ in a(n empty) network $2A1B$ to be interested more in the signal of $B$. In area (II), it is such that it drives fully the thresholds comparisons, as anticipated above. In area (I), finally, it is so large, than even an agent $B$ in an empty network $1A2B$ becomes interested in the other group's action, in order to anti-coordinate with it. When $\alpha > \beta$ region (I) vanishes for very small values of the parameter $\beta$ regardless the value of the coordination parameter. $\alpha$ has just the role of reducing the size of area (II).

Summarising the results of Remark 1, we can see that there exist three possible scenarios. When the signal precisions of the two groups are roughly the same, the majority always wants to look at their own signal ($c_{AA} > c_{Ab}$ and $c_{BB} > c_{Ba}$). Here the coordination motive always dominates, as the anti-coordination motive has an issue of a tension between the fundamental and anti-coordinating. By observing the other party's signal, it allows a better estimate of the fundamental. As the other party also wishes to be close to the fundamental, however, the anti-coordination motive provides an incentive to not fully move closer to the predicted value of the fundamental. By contrast, if $\pi_B > \pi_A$ the majority A prefers to look at the minority B, but the majority B prefers to look within ($c_{Ab} > c_{AA}$ and $c_{BB} > c_{Ba}$). Here everybody wants to look at the more precise signal. Finally, if $\pi_B >> \pi_A$ and the anti-coordination parameter $\beta$ is strong enough, the majority B prefers to look at A. We have $c_{Ab} > c_{AA}$ and $c_{Ba} > c_{BB}$. The reason why the majority B wants to look at the minority A is because of the anti-coordination motive. The tension with the fundamental disappears as B already has much more precise information. Observing the less precise signal of the member of group A allows the member of group B to anti-coordinate without moving away from the fundamental – his own signal is so much more precise that the impact of observing A's signal on predicting the fundamental is marginal.

Figure 4.4: The three areas generated by conditions (4.36) and (4.37). Case $\alpha > \beta$.




(a) $\alpha = 0.7, \beta = 0.5$. (b) $\alpha = 0.5, \beta \leq 0.1$.

**Stability of the complete network**

We now analyse the threshold costs below which an agent $i$ in the complete network has the incentive to keep a link with any another player in the network. Notice that, the larger is the threshold value, the more "valuable" is the link to the player.

**Lemma 4.8.** *Comparing the values of the links formed by the majority within a given configuration, we have that (i) $\kappa_{AA} < \kappa_{Ab}$ and (ii) $\kappa_{BB} > \kappa_{Ba}$*

**Proof.** (i) Subtracting (4.28) from (4.27), it reduces to $\pi_A(\pi_A - \pi_B) - (\pi_B^2 - \pi_A^2) < 0$ because $\pi_A < \pi_B$. (ii) Subtracting (4.35) from (4.34), it reduces to $(\pi_B^2 - \pi_A^2) - \pi_B(\pi_A - \pi_B) > 0$ because $\pi_A < \pi_B$ □

The above lemma shows that in the network configuration where the majority is group $A$, a player in the majority finds it more valuable to keep a link with the minority. On the contrary, in the network configuration where the majority group is $B$, a player in the majority finds it more valuable to keep a link with the majority. This always holds, regardless the values of the coordination motive $\alpha$ and the anti-coordination motive $\beta$. The precision of the signals seems to be the sole determinant of the relative values, i.e. links to group B are always more valuable than to A *for the majority.*

This result is in contrast to Remark 1, where we have shown that the relative value of a within-group link (i.e. link to the majority) compared to a link to the minority depends on the relative precisions as well as the relative strength of coordination and antic-coordination motives. The reason of the difference between the empty and the complete network is the following. Because in the complete network players are able to coordinate and anti-coordinate via all other observed signals, the signal precision is what remains. At the margin an agent in $A$ will lose more dropping a link with her opponent, who has a more precise signal, than dropping a link with her own group. On the contrary for a player in $B$, dropping a link with a player in the opposite group, who has a signal less precise, is not as bad as dropping a link with his own group.

**Lemma 4.9.** *Comparing the value of links to the minority and majority respectively across the two configurations, the link to the more precise signal is always more valuable.* $\kappa_{Ab} > \kappa_{Ba}$ *and* $\kappa_{aB} > \kappa_{bA}$.

**Proof.** This follows by comparing (4.35) with (4.28) and (4.29) with (4.33). The two comparisons holds because $\pi_A < \pi_B$. □

This lemma compares links across the two configurations. Namely it compares incentives to form cross-group links from majority to minority and minority to majority respectively. In the network configuration where the majority is B, keeping a link with the minority is less valuable than in the network configuration where the majority is group $A$. On the contrary, in the network configuration where the minority is group $B$ keeping a link with the majority is less valuable than in the network configuration where the minority is group $A$. This result suggests that it is not about who is the majority and the minority group, rather about who has the most precise signal. A more precise signal is more valuable. An analogue to these results is in Corollary 4.1 part (b) where we compare the same links but for the case of the empty network. There, it was also the precision driving the selective ordering.

**Lemma 4.10.** *Comparing the value of within majority links across configurations, a within-majority link is less valuable when the majority is group A than when it is group B. That is,* $\kappa_{BB} > \kappa_{AA}$.

**Proof.** This follows by comparing (4.34) with (4.27). This holds comparisons holds because $\pi_A < \pi_B$. □

This lemma refers to the value of within-majority links across the two configurations. It shows that, in the network configuration where the majority is group $B$ keeping a link with the majority is more valuable than in the network configuration where the majority is group $A$. The explanation for this result is the same we provided for Lemma 4.8.

It is worth to point out a difference of the marginal value of an extra link within group, when we compare the empty with the complete network. If we compare Lemma 4.7 and Lemma 4.10 the relationship between the two thresholds is reversed. In the complete network everybody else is linked to everybody, and the coordination and anti-coordination motives are balanced out via all the links. Thus, at the margin, what matters is only the precision of the signal. Because group B has the most precise signal, keeping a link with his own group is more valuable than for a player in $A$ that keeps a link with her own group. On the contrary, in the empty network (see Lemma 4.7), nobody is linked to anybody and there, the marginal value of a (first) link is a combination of both the precision of the signal and the coordination and anti-coordination motive. In this case, for players in group A a link with her own group is more valuable than for players in group B. This is because without the additional link, members of group A have access to a less precise signal and therefore a worse “outside option” if no link is formed than members of group B, both in terms of coordinating – without observing the other group member’s information, their own signals will in expectation be further apart.

**Lemma 4.11.** *In the network configuration where the majority is group A,*

- $\kappa_{Ab} > \kappa_{bA}$ *if* $\alpha \leq \beta$ *or if* $\alpha > \beta$ *and* $|\pi_B - \pi_A|$ *sufficiently big.*

- *$\kappa_{Ab} < \kappa_{bA}$ if $\alpha > \beta$ and $|\pi_B - \pi_A|$ sufficiently small.*

**Proof.** By subtracting (4.29) from (4.28) the sign of the difference is determined by

$$\pi_A[(9-\alpha^2)^2\pi_B - (9-\beta^2)^2\pi_A] + [(9-\alpha^2)^2\pi_B^2 - (9-\beta^2)^2\pi_A^2].$$

Given that $\pi_B > \pi_A$ by assumption, when $\alpha < \beta$ the terms inside the two squared brackets are positive. On the contrary, when $\alpha > \beta$, the sign of the terms inside the two squared brackets depends on the magnitude of the difference $|\pi_B - \pi_A|$. □

**Lemma 4.12.** *In the network configuration where the majority is group $B$,*

- *$\kappa_{Ba} > \kappa_{aB}$, if $\alpha < \beta$ and $|\pi_B - \pi_A|$ sufficiently small.*
- *$\kappa_{Ba} < \kappa_{aB}$, if $\alpha \geq \beta$ or if $\alpha < \beta$ and $|\pi_B - \pi_A|$ sufficiently high.*

**Proof.** By subtracting (4.33) from (4.35) the sign of the difference is determined by

$$\pi_B[(9-\alpha^2)^2\pi_A - (9-\beta^2)^2\pi_B] + [(9-\alpha^2)^2\pi_A^2 - (9-\beta^2)^2\pi_B^2].$$

Given that $\pi_B > \pi_A$ by assumption, when $\alpha > \beta$ the terms inside the two squared brackets are negative. On the contrary, when $\alpha < \beta$, the sign of the terms inside the two squared brackets depends on the magnitude of the difference $|\pi_B - \pi_A|$. □

Lemmas 4.11 and 4.12 both refer to the values of cross-group links within the same configurations and compare, for the two network configurations, the value of linking from the majority to the minority with the value of linking from the minority to the majority. Notice that, if we do not consider differences in the precision of the signals, that is if $\pi_A \approx \pi_B$, the two lemmas give the same predictions. That is, if $\alpha > \beta$ then majority-minority link is less valuable than the minority-majority link in both network configurations; with the opposite being true if $\alpha < \beta$. For further interpretation it is useful to look at extreme values of $\alpha$ and $\beta$, while keeping $\pi_A \approx \pi_B$.

If $\alpha = 1 > \beta = 0$ only coordination matters other than the fundamental, but it matters only for the majority group. The minority group finds more valuable to keep a link with the opposite (majority) group compared to the majority that keeps the link with the opposite (minority) group. This is because when the coordination motive is this strong, the majority group cares about the fundamental and coordinating, while the minority only cares about the fundamental. As such, the additional link is valuable to the minority as it allows a better prediction of the fundamental. In contrast, it holds little value to the majority as coordination can also be achieved via the other jointly observed information sources.

If $\alpha = 0 < \beta = 1$ anti-coordination and fundamental matter for both groups. Notice however that the anti-coordination parameter is rescaled by the relative size of the groups, implying that the minority group has a stronger anti-coordination motive. This could explain why the majority finds it more valuable than the minority to link with the opposite group.

By contrast, when we also consider differences in the precision of the signals, the results and intuitions discussed above do not apply anymore. This suggests that when considering differences in both coordination/anti-coordination parameters and precision of the signals, there is a trade-off between the two forces, which seems to be affected also by the relative size of the group. In such a case, the interpretation of the results is not clear and a deeper investigation is needed in order to understand the incentives of a player to keep a link.

The results of Lemmas 4.11 and 4.12 contrast with Corollary 4.1 part (a) which does the same comparison, but for the empty network case. The difference is that in the empty network, when forming the first link only the precision matters, while here it is the relative precision and the relative strength of coordination and anti-coordination motives that matter.

**Lemma 4.13.** *Comparing the values of links to the majority within a given configuration, we obtain the following. (i) If the coordination motive is weaker than the anti-coordination motive, i.e. if $\alpha < \beta$, then the within-majority link is more valu-*

*able than the link from the minority to the majority.* $\kappa_{AA} > \kappa_{bA}$ *and* $\kappa_{BB} > \kappa_{aB}$. *(ii) If the coordination motive is stronger than the anti-coordination motive, i.e. if* $\alpha > \beta$, *then the within-majority link is less valuable than the link from the minority to the majority.* $\kappa_{AA} < \kappa_{bA}$ *and* $\kappa_{BB} < \kappa_{aB}$. *(iii) If coordination and anti-coordination motives are balanced, i.e. if* $\alpha = \beta$, *then the within-majority link is as valuable as the link from the minority to the majority.* $\kappa_{AA} = \kappa_{bA}$ *and* $\kappa_{BB} = \kappa_{aB}$.

**Proof.** It follows from subtracting (4.29) from (4.27) and (4.33) from (4.34) and noticing that in both cases the sign of the difference is determined by

$$(\alpha^2 - \beta^2)(-18 + \alpha^2 + \beta^2).$$

This depends on whether $\alpha$ is greater or smaller than $\beta$ plus the fact that the term in the second brackets is negative, because $\alpha, \beta \in [0, 1]$. $\square$

Lemma 4.13 compares the marginal value of keeping a link with the majority group, between a player that is in the majority group and the player that represents the minority group. We can see that when coordination and anti-coordination are balanced, i.e. $\alpha = \beta$, different players value that link in the same way. When instead is the anti-coordination that matters more, the value of the link to a person in the majority group is higher than to a person in the minority group. The reverse holds when coordination matters more than anti-coordination.

The main message of Lemma 4.13 is twofold. First, whether the majority prefers to (keep the) link with the majority over the minority depends on the strength of coordination and anti-coordination motives, and is not affected by the relative signal precisions. Second, higher coordination than anti-coordination leads to relatively less value for the within-group link.

Notice that this lemma does not have a direct analogue to the empty network case, as we have not analysed the relative value of these links for the empty network scenario. Nonetheless it is of interest as it provides at first glance counter-intuitive results. We would have expected a higher coordination motive, relative to anti-coordination, to increase the value of the link more for those that link to the own group than for those that link to the opposite one. However, in interpreting this result, the first thing to bear in mind is that the coordination parameter $\alpha$ matters only for the majority group. The minority group (which is represented only by one player) cares only about anti-coordinating and the fundamental. If we look at the extreme cases, $\alpha = 1 > \beta = 0$ and $\alpha = 0 < \beta = 1$ an interpretation of these results becomes easier.

For the case of $\alpha = 1 > \beta = 0$, the majority group cares about the fundamental and coordinating, while for the minority group it is only the fundamental that matters. Thus, for the minority group any link is a "valuable" information as it helps to better predict the fundamental, while for the majority that link trades off between coordinating and being close to the fundamental. The force at work here, that makes that link for the majority less valuable relatively to the other group is the following. As the other player in the majority is linked to his own group, the majority is still able to coordinate, by putting weight on the jointly observed signals, and loosing one link doesn't cause much harm relative to the complete network.

For the case $\alpha = 0 < \beta = 1$, both the majority group and the minority group care about the fundamental and anti-coordinating. However, for the majority group, keeping a link with the majority adds value only to the fundamental. By contrast, for the minority, the value of keeping a link with the majority trades off between the fundamental and anti-coordinating. This trade-off could explain why the majority group finds more valuable a link with the majority compared to the value that the minority group attaches to the same link.

## 4.6 Final Remarks

In this paper we study information acquisition and endogenous network formation in a Gaussian-quadratic game where players care about both coordinating with some players and anti-coordinating with some others. There are N players divided into two groups, and each player is endowed with a private signal about the underlying state of the world. Signals are independently distributed and differs in their precision between groups but are identically distributed within group. Agents can acquire further information about the underlying state of the world by acquiring the signals of the other players and thus forming costly links in the network. Once the information is acquired and the network is formed, each player takes an action to maximise his expected payoff which balances three different forces: (i) being as close as possible to the state of the world, (ii) being close to the average action of his own group and (iii) being far away for the average action of the other group.

We first solve for the optimal action and following the standard in the literature we focus on actions in linear strategy. We then substitute the optimal action in the ex-ante expected utility and study the incentive of each player to form a link. We restrict the analysis to the case of a 3-players network allowing for two different network configurations by changing the size of the two groups. We focus on the incentive to form the first link in the case of an empty network and the incentive to keep a link in the case of a complete network.

We compare the incentives of a player to form a link both within the same network configuration and across configurations. When we look at the incentives to form a link within the same network configuration, we see that in the full network the majority's ordering of link valuations is driven solely by the signal precisions, while in the empty network the coordination and the anti-coordination motives also play a huge role. By contrast, when comparing the cross-link incentives, the signal precisions determine the order in the empty network, but interact with the coordination and anti-coordination motives in the complete network.

In the comparison of the incentives to form a link across network configurations, we see that the value of the within group links reverses between the empty and the full network. The order of the relative value of the cross-group links however, is preserved and determined solely by the signal precisions.

Finally, in the complete network the incentives to form a link to the majority display a counter-intuitive behaviour.

Overall, this analysis displays some of the incentives agents face and highlights that the understanding of the whole mechanism that leads to the formation of the network is still far from being achieved. However, there are interesting aspects which warrant further study.

# Chapter 5

# Conclusions

This thesis studied, under three different settings, how agents strategically acquire information, at a cost or freely, prior to making a decision.

Chapter 2 developed a model that explains how strategic information avoidance leads to the formation of echo chambers, where ignorance spreads inside them. It is shown that there is typically an equilibrium where agents look at all available information and undertake the project only if it is worthwhile given the available information. However, there also exists an equilibrium where agents only pay attention to a subset of information sources (those that are less likely to convey bad news). This equilibrium is similar to Benabou's Mutually Assured Delusion (MAD) equilibrium. In settings where avoidance of bad news have negative externalities, these news become harder to accept, resulting in a contagious collective ignorance in which agents undertake "harmful" uninformed decisions. The results of this chapter represent a first step towards a theoretical model that could explain the link between the echo chamber and opinion polarisation and the selective exposure hypothesis. Towards this direction a possible extension of this model could be to account for different groups with some heterogeneity between groups. Such a model, would be a step further able to explain ideological polarization.

Chapter 3 studied endogenous information acquisition in an investment trading game between a real sector and a financial sector, in which entrepreneurs first make their investment decision about a new research project and successively a fraction $\lambda$ of the total capital invested is traded in the financial market. $\lambda$ represents the probability that entrepreneurs are hit by a liquidity shock. It is shown that there exists complementarity in information acquisition and that for some parameter space entrepreneurs may not acquire private information. This happens when the probability of the liquidity shock is sufficiently high and the exogenous precision of the private signal is not too high. Moreover the probability of the liquidity shock above which information is not acquired is decreasing in the precision of traders public signal. Traders' public signal also positively affects the entrepreneurs' attention paid to the private signal. This suggests that the extent to which traders are informed affect entrepreneurs' incentive to acquire information. Thus, a possible extension to this model would be to consider information acquisition also on traders' side. This extension is of interest, because information acquisition on traders' side could mitigate the incentive of entrepreneurs to acquire less precise information.

Chapter 4 studied information acquisition and endogenous network formation in a Gaussian-quadratic game where players balance their action between the desire to be close to the underlying state of the world, to coordinate with some players and anti-coordinate with some others. The analysis is restricted to the case of a 3-players network under two different network configurations where the difference is in size of the two groups. The focus is on the incentives to form the first link in the case of an empty network and the incentive to keep a link in the case of a complete network. Incentives of a player to form a link both within the same network configuration and across configurations are compared and interpreted. Overall, the analysis displays some of the incentives agents face and highlights that the understanding of the whole mechanism that leads to the formation of the network is still far from being achieved. However, there are interesting aspects which warrant further study.

# Appendices

# Appendix A

# Appendix to Chapter 2

## A.1 Omitted Proofs

**Proof of Lemma 2.1.** Lemma 2.1 is an if and only if statement. That is, for any recalled signal profile that does not include $\hat{s}_L$ an agent $i$ exerts effort and for any recalled profile that does include $\hat{s}_L$ the agent does not exert effort. At the recalling stage, depending on the information source chosen, an agent recalls one of the following signal profiles: $\hat{\varnothing}$, $\hat{s}_H$, $(\hat{\varnothing},\hat{\varnothing})$, $(\hat{s}_H,\hat{\varnothing})$, $(\hat{\varnothing},\hat{s}_L)$ or $(\hat{s}_H,\hat{s}_L)$. At the recalling stage, when she decides whether to exert effort, she also takes into account her period $t=0$ denial strategy. Notice that, given that an agent can change the signal from bad to good and not vice-versa, an agent $i$ recalling both $\hat{\varnothing}$ and $(\hat{\varnothing},\hat{s}_L)$ is never in denial. In all the other recalled signal profiles an agent $i$ may or may not be in denial of the signal/s received.

We first prove that effort is always exerted if the recalled signal profile does not include $\hat{s}_L$, that is when the agent recalls one of the following $\hat{\varnothing}$, $\hat{s}_H$, $(\hat{\varnothing},\hat{\varnothing})$ or $(\hat{s}_H,\hat{\varnothing})$. If the agent recalls $\hat{\varnothing}$, by Assumption 2.2 she will exert effort because $E(\theta|\hat{\varnothing}) = E(\theta|\varnothing) > \frac{c}{\alpha(s+\delta)}$.

In the other cases in which the agent may have denied the signal, given (2.2) an agent $i$ exerts effort whenever

$$E(\theta|\hat{\sigma}_1,\lambda_1^i) > \frac{c}{\alpha(s+\delta)}$$

if she pays attention only to Information source $I_1$ and

$$E(\theta|\hat{\sigma}_1,\hat{\sigma}_2,\lambda_1^i,\lambda_2^i) > \frac{c}{\alpha(s+\delta)} \tag{A.1}$$

if she pays attention to both information sources.

Therefore, if an agent $i$ recalls $\hat{s}_H$, then

$$\begin{aligned} E(\theta|\hat{s}_H,\lambda_1^i) = \quad & Pr(s_H|\hat{s}_H)E(\theta|s_H) + Pr(\varnothing|\hat{s}_H)E(\theta|\varnothing) \\ = \quad & \tfrac{\mu p_1}{\mu p_1+(1-\mu p_1)(1-\lambda_1^i)}\theta_H + \tfrac{(1-\mu p_1)(1-\lambda_1^i)}{\mu p_1+(1-\mu p_1)(1-\lambda_1^i)}E(\theta|\varnothing). \end{aligned} \tag{A.2}$$

Equation (A.2) is increasing in $\lambda_1^i$, and at $\lambda_1^i = 0$ it reduces to $E(\theta)$, which by Assumption 1 implies $e = 1$. Consider now the case of an agent $i$ observing both information sources. If she recalls $(\hat{\varnothing}_1,\hat{\varnothing}_2)$, it might be that either she is recalling the true signal or she has denied $s_L$. Thus the expected productivity of $\theta$ will be

$$\begin{aligned} E[\theta|(\hat{\varnothing}_1,\hat{\varnothing}_2),\lambda_2^i] = & Pr[(\varnothing_1,\varnothing_2)|(\hat{\varnothing}_1,\hat{\varnothing}_2),\lambda_2^i]E(\theta|(\varnothing_1,\varnothing_2)) + \\ & + Pr[(\varnothing_1,s_L)|(\hat{\varnothing}_1,\hat{\varnothing}_2),\lambda_2^i]E(\theta|(\varnothing_1,s_L)) \\ = & \frac{1-\mu p_1-(1-\mu)p_2}{1-\mu p_1-(1-\mu)p_2+(1-\mu)p_2(1-\lambda_2^i)}E(\theta|(\varnothing_1,\varnothing_2)) \\ & + \frac{(1-\mu)p_2(1-\lambda_2^1)}{1-\mu p_1-(1-\mu)p_2+(1-\mu)p_2(1-\lambda_2^i)}\theta_L \end{aligned} \tag{A.3}$$

Equation (A.3) is increasing in $\lambda_2^i$ and at $\lambda_2^i = 0$ it reduces to $E(\theta|\varnothing_1)$ which, by Assumption 2, implies $e = 1$. With the same argument we can show that effort is exerted when an agent $i$ recalls the signal $(\hat{s}_H,\hat{\varnothing}_2)$. In this case it might be that either he has received the true signal, he has denied only $\varnothing_1$ or he has denied both $\varnothing_1$ and $s_L$. It is easy to see that $E(\theta|(\hat{s}_H,\hat{\varnothing}_2),\lambda_1^i,\lambda_2^i)$ is increasing both in $\lambda_1^i$ and $\lambda_2^i$ with $E[\theta|(\hat{s}_H,\hat{\varnothing}_2),0,0] = E(\theta)$. Therefore, Assumption 1 implies $e = 1$ when agent $i$ recalls $(\hat{s}_H,\hat{\varnothing}_2)$.

We now prove that effort is not exerted if the recalled signal profiles includes $\hat{s}_L$, that is when an agent $i$ recalls both $(\hat{\varnothing},\hat{s}_L)$ and $(\hat{s}_H,\hat{s}_L)$. If the agent recalls $(\hat{\varnothing},\hat{s}_L)$, according to her posterior beliefs she knows that the state of the world is L, therefore she does not exert effort because $E(\theta|\hat{\varnothing}, \hat{s}_L) = \theta_L$ and by Assumption 2.1 $\theta_L < \frac{c}{\alpha(s+\delta)}$. Consider now the remaining case $(\hat{s}_H, \hat{s}_L)$. This signal profile might be recalled given our assumption that, when an agent $i$ chooses both information sources, the denial strategy on the signal received from one information source is independent from the signal received from the other information source. In this case the agent is in denial only of the signal $\varnothing_1$ and the conditional expected value of $\theta$ is

$$\begin{aligned}E(\theta|\hat{s}_H, \hat{s}_L) =& Pr[(s_H, s_L)|(\hat{s}_H, \hat{s}_L), \lambda_1^i]E(\theta|(s_H, s_L))+\\ &+ Pr[(\varnothing_1, s_L)|(\hat{s}_H, \hat{s}_L), \lambda_1^i]E(\theta|(\varnothing_1, s_L))\\ =& \theta_L\end{aligned}$$

because, $Pr[(s_H, s_L)|(\hat{s}_H, \hat{s}_L), \lambda_1^i] = \frac{0}{0+(1-\mu)p_2(1-\lambda_1^i)} = 0$. Therefore an agent $i$ recalling $(\hat{s}_H, \hat{s}_L)$ will not exert effort. □

**Proof of Proposition 2.1.** Suppose that an agent $i$ chooses to observe information source $I_1$ only. Consider a symmetric equilibrium, where everybody else is observing $I_1$. An agent $i$ will receive the signal $s_H$ with probability $\mu p_1$ and $\varnothing$ with probability $1-\mu p_1$. Upon receiving $s_H$, the agent knows the state is H. Accordingly, she has no incentive to deny the signal. On the contrary, if she receives the empty signal $\varnothing$ she might deny it, recalling $\hat{s}_H$ at $t = 1$.

The optimal cognitive strategy of an agent $i$ when receiving $s_L$ is as follows; if she decides to be realist, she obtains the inter-temporal utility

$$\begin{aligned}U^i_{0R|(\varnothing)} &= \delta[-c + (s+\delta)E(\theta|\varnothing)]\\ &= \delta\{-c + (s+\delta)[Pr(H|\varnothing)\theta_H + Pr(L|\varnothing)\theta_L)]\}\\ &= \delta\left\{-c + (s+\delta)\left[\frac{(1-p_1)\mu}{1-\mu p_1}\theta_H + \frac{(1-\mu)}{1-\mu p_1}\theta_L\right]\right\}.\end{aligned} \tag{A.4}$$

Whereas if the agent reacts denying the signal, she obtains

$$U^i_{0D|(\varnothing)} = -m_1 + \delta\big[-c + s\big[r(\lambda_1^i)\theta_H + (1-r(\lambda_1^i))E(\theta|\varnothing)\big] + \delta E(\theta|\varnothing)\big]. \tag{A.5}$$

Notice that, for an agent, realism or denial of a signal from $I_1$ is independent of other agents' cognitive strategy. This is so because, when observing only information set $I_1$, agents always exert effort independently of the recalled signal. An agent's net incentive to deny reality is thus:

$$\begin{aligned} U^i_{1D|(\varnothing)} - U^i_{1R|(\varnothing)} &= -m_1 + \delta s\big[r(\lambda_1^i)(\theta_H - E(\theta|\varnothing))\big] \\ &= -m_1 + \delta s\, r(\lambda_1^i)\left[\theta_H - \left(\frac{\mu(1-p_1)}{1-\mu p_1}\theta_H + \frac{1-\mu}{1-\mu p_1}\theta_L\right)\right] \\ &= -m_1 + \delta s\, r(\lambda_1^i)\left(\frac{1-\mu}{1-\mu p_1}\theta_H - \frac{1-\mu}{1-\mu p_1}\theta_L\right) \\ &= -m_1 + \delta s\, r(\lambda_1^i)\left(\Delta\theta\frac{1-\mu}{1-\mu p_1}\right). \end{aligned} \tag{A.6}$$

Let us define $\Gamma(\lambda_1^i, m_1)$ the RHS of equation (A.6). Thus, the optimal strategy for agent $i$ is:

a) $\lambda_1^i = 1$ if $\Gamma(1, m_1) \leq 0$, which means

$$m_1 \geq \delta s \Delta\theta \frac{1-\mu}{1-\mu p_1} \equiv \underline{m}(I_1) \tag{A.7}$$

b) $\lambda_1^i = 0$ if $\Gamma(0, m_1) \geq 0$, which means

$$m_1 \leq \delta s \Delta\theta \frac{\mu p_1(1-\mu)}{1-\mu p_1} \equiv \overline{m}(I_1). \tag{A.8}$$

Notice that $\overline{m}(I_1) < \underline{m}(I_1)$. This means that denial is always optimal if the cost of denial $m_1$ is very small; vice-versa if the cost of denial is high, then the optimal strategy is to be realist. Moreover, notice that when the probability of receiving $s_H$ is very small, that is $p_1 \to 0$, denial is never optimal.

**Lemma A.1.** *In any equilibrium where all agents observe $I_1$ only, the equilibrium cognitive strategy is to be realist, that is $\lambda = 1$, if $m_1 > \underline{m}(I_1)$.*

Suppose that observing $I_1$ is not an equilibrium. Then observing both $I_1$ and $I_2$ must be a profitable deviation. Suppose then the agent deviates and observes both information sources; her cognitive strategy will depend on the signal received. In particular when receiving $(s_H, \varnothing)$ she will be realist, when receiving $(\varnothing,\varnothing)$ she may deny $\varnothing$ and when receiving $(\varnothing, s_L)$ she may deny both $\varnothing$ and $s_L$, only $s_L$ or only $\varnothing$. Focusing on the optimal cognitive strategy on the signal received from $I_1$, the utility of the agent being realist is

$$U_{0,R|(\varnothing,\varnothing)} = \delta[-c + (s+\delta)[\Delta\theta\, r(I_1, I_2) + \theta_L]] \tag{A.9}$$

where $r(I_1, I_2) \equiv \frac{(1-p_1)\mu}{1-\mu p_1-(1-\mu)p_2}$. The utility from denial is

$$\begin{aligned} U_{0,D|(\varnothing,\varnothing)} =& \delta\left\{s[v(\lambda_1^i,\lambda_2^i)\theta_H + q(\lambda_1^i,\lambda_2^i)[\Delta\theta\, r(I_1,I_2)+\theta_L] + p(\lambda_1^i,\lambda_2^i)\theta_L]\right\} + \\ &+ \delta[-c+\delta(\Delta\theta\, r(I_1,I_2)+\theta_L)] - m_1. \end{aligned} \tag{A.10}$$

The agent's net incentive to be realist on $I_1$ when she observes $(\varnothing,\varnothing)$ is then

$$\begin{aligned} &U_{0,R|(\varnothing,\varnothing)} - U_{0,D|(\varnothing,\varnothing)} = \\ &\delta\left\{s[(1-q(\lambda_1^i,\lambda_2^i))[\Delta\theta\, r(I_1,I_2)+\theta_L] - v(\lambda_1^i,\lambda_2^i)\theta_H - p(\lambda_1^i,\lambda_2^i)\theta_L]\right\} + m_1. \end{aligned} \tag{A.11}$$

Denoting $\Gamma(\lambda_1^i, \lambda_2^i, m_1)$ the RHS of equation (A.11), the optimal cognitive strategy is then

- $\lambda_1^i = 1$ if $\Gamma(1, \lambda_2^i, m_1) \geq 0$, that is if

$$m_1 \geq \delta s \Delta\theta(1 - r(I_1, I_2)) \equiv \underline{m}_1^d(\varnothing,\varnothing) \tag{A.12}$$

- $\lambda_1^i = 0$ if $\Gamma(0, \lambda_2^i, m_1) \leq 0$, that is if

$$\begin{aligned} m_1 \leq & \delta s \left[ \frac{\mu}{1-(1-\mu)p_2\lambda_2^i}\theta_H + \frac{(1-\mu)(1-p_2\lambda_2^i)}{1-(1-\mu)p_2\lambda_2^i}\theta_L - \Delta\theta\, r(I_1, I_2) - \theta_L \right] \\ \equiv & \overline{m}_1^d(\varnothing_1, \varnothing_2; \lambda_2^i) \end{aligned} \tag{A.13}$$

Notice that the threshold calculated in equation (A.12) does not depend on $\lambda_2^i$. On the contrary, the RHS of equation (A.13) is increasing in $\lambda_2^i$ and $\overline{m}_1^d(\varnothing, \varnothing; 1) < \underline{m}_1^d(\varnothing, \varnothing)$. Therefore full realism and full denial occur for disjoint sets of parameters.

**Lemma A.2.** *In any equilibrium where all agents observe only $I_1$, the optimal cognitive strategy of an agent deviating and observing both $I_1$ and $I_2$, conditional on receiving $(\varnothing, \varnothing)$, prescribes $\lambda_1^i = 1$ if $m_1 > \underline{m}_1^d(\varnothing, \varnothing)$*

This lemma just says that, if the cost of denying $I_1$ is sufficiently large, the agent will not deny it.

Consider the case in which the agent receives $(\varnothing, s_L)$. If the agent is realist on both signals, she knows that the state is low and she does not exert effort. However, she receives utility

$$U^i_{0,R|(\varnothing, s_L)} = \delta(\delta + s)(1-\alpha)\theta_L \tag{A.14}$$

because the other $n-1$ agents are observing $I_1$ and there they always exert effort. By contrasts if the agents denies only $s_L$, she obtains ex-ante intertemporal utility

$$\begin{aligned} U^i_{0,D}(\varnothing, s_L)|_{\hat{\varnothing}, \hat{\varnothing}} = \quad & \delta\{s[q(\lambda_2^i)(r(I_1, I_2)\theta_H + (1-r(I_1, I_2))\theta_L) \\ & +(1-q(\lambda_2^i))\theta_L]\} + \delta(\delta\theta_L - c) - m_2, \end{aligned} \tag{A.15}$$

and in case she denies both $\varnothing$ and $s_L$ she obtains utility

$$\begin{aligned} U^i_{0,D}(\varnothing, s_L)|_{\hat{s}_H, \hat{\varnothing}} = \quad & \delta\{-c + s[v(\lambda_1^i, \lambda_2^i)\theta_H + q(\lambda_1^i, \lambda_2^i)[r(I_1, I_2)\Delta\theta + \\ & +\theta_L] + p(\lambda_1^i, \lambda_2^i)\theta_L] + \delta\theta_L\} - (m_1 + m_2). \end{aligned} \tag{A.16}$$

Focusing only on the optimal cognitive strategy on the signal received from $I_1$, upon receiving $(\varnothing, s_L)$, using equations (A.15) and (A.16) we calculate the net incentive of being realist on $I_1$. Denoting $\Gamma(\lambda_1^i, \lambda_2^i, s|(\varnothing, s_L)) \equiv U_{0,D}^i(\varnothing, s_L)|_{\hat{\varnothing},\hat{\varnothing}} - U_{0,D}^i(\varnothing, s_L)|_{\hat{s}_H,\hat{\varnothing}}$, then the optimal cognitive strategy is

- $\lambda_1^i = 1$ if $\Gamma(1, \lambda_2^i, s|(\varnothing, s_L)) \geq 0$ or equivalently if

$$m_1 \geq \delta s \Delta\theta[1 - r(I_1, I_2)q(\lambda_2^i)] \equiv \underline{m}_1^d(\varnothing, s_L; \lambda_2^i) \tag{A.17}$$

Notice that $q(\lambda_2^i)$ is increasing in $\lambda_2^i$ therefore the threshold $\underline{m}_1^d(\varnothing, s_L; \lambda_2^i)$ is decreasing in $\lambda_2^i$. The following lemma summarises the conditions under which realism on $I_1$ is always optimal.

**Lemma A.3.** *In any equilibrium where all agents observe only $I_1$, the optimal cognitive strategy of an agent deviating and observing both $I_1$ and $I_2$ prescribes $\lambda_1^i = 1$ if $m_1 > \underline{m}(I_1)$.*

**Proof.** Using equation (2.7), if $\lambda_2^i = 0$ equation (A.17) becomes

$$\overline{m}_1^d(\varnothing, s_L; 0) = \delta s \Delta\theta \frac{1-\mu}{1-\mu(p_1)} = \underline{m}(I_1).$$

If instead $\lambda_2^i = 1$ equation (A.17) becomes

$$\underline{m}_1^d(\varnothing, s_L; 1) = \delta s \Delta\theta(1 - r(I_1, I_2) = \underline{m}_1^d(\varnothing,\varnothing).$$

Moreover $\underline{m}(I_1) > \underline{m}_1^d(\varnothing,\varnothing)$. Therefore for any $m_1 > \underline{m}(I_1)$ the deviating agent is always realist on $I_1$, that is $\lambda_1^i = 1$, regardless his cognitive strategy on the signal $s_L$. □

The logic is the usual one. This lemma shows that when the cost of denying $I_1$ is sufficiently large no agent will never deny it.

Now we analyse the optimal cognitive strategy on the signal received from $I_2$.

Using equations (A.14) and (A.15) the agent's net incentive of denial is equal to

$$U^i_{0,D|(\varnothing,s_L)} - U^i_{0,R|(\varnothing,s_L)} = -m_2 + \delta[-c + (s+\delta)\alpha\theta_L] + \delta s q(\lambda^i_2) r(I_1, I_2)\Delta\theta. \tag{A.18}$$

Denoting $\Gamma(\lambda^i_2, m_2|(\varnothing, s_L))$ the RHS of equation (A.18), then the optimal cognitive strategy is

- $\lambda^i_2 = 0$ if $\Gamma(0, m_2|(\varnothing, s_L)) \geq 0$, which is equivalent to

$$m_2 \leq \delta\,[s\,(\Delta\theta\, r(I_1) + \alpha\theta_L) + \delta\alpha\theta_L - c] \equiv \overline{m}^d_2(\varnothing, s_L) \tag{A.19}$$

  where $r(I_1) \equiv \frac{\mu(1-p_1)}{1-\mu p_1}$

- $\lambda^i_2 = 1$ if $\Gamma(1, m_2)|(\varnothing_1, s_L)) \leq 0$, which is equivalent to

$$m_2 \geq \delta\,[s\,(\Delta\theta\, r(I_1, I_2) + \alpha\theta_L) + \delta\alpha\theta_L - c] \equiv \underline{m}^d_2(\varnothing, s_L) \tag{A.20}$$

Notice that $\overline{m}^d_2(\varnothing, s_L) < \underline{m}^d_2(\varnothing, s_L)$, which means that the optimal $\lambda^i_2 = 0$ and $\lambda^i_2 = 1$ belong to two different ranges of the cost of denial. The following summarises the denial strategy on $s_L$ of an agent that deviates and observes both $I_1$ and $I_2$.

**Lemma A.4.** *In any equilibrium where all gents observe only $I_1$, the optimal cognitive strategy of an agent $i$ deviating and observing both $I_1$ and $I_2$ prescribes $\lambda^i_2 = 0$ if $m_2 \leq \overline{m}^d_2(\varnothing, s_L)$.*

In other words, if the cost of denying $I_2$ is sufficiently small, then the agent will deny it if she deviates.

We are now able to fully characterise the optimal cognitive strategy of an agent that deviates and observes both $I_1$ and $I_2$ when all other agents are observing only $I_1$. From Lemma A.1 to Lemma A.3 we know that if $m_1 \geq \underline{m}(I_1)$ then $\lambda^i_1 = 1$ and from Lemma A.4 that if $m_2 \leq \overline{m}^d_2(\varnothing, s_L)$ then $\lambda^i_2 = 0$.

**Lemma A.5.** *If the following condition holds*

$$\theta_L > \frac{1}{\alpha(\delta+s)}\left\{c - s\Delta\theta\left[\frac{2(1-p_1)\mu}{1-\mu p_1} - 1\right]\right\} \quad and \quad \mu > \frac{1}{2-p_1} \tag{A.21}$$

*then* $\underline{m}(I_1) < \overline{m}_2^d(\varnothing, s_L)$ *and, for any* $m_1$ *and* $m_2$ *in this range, (i) if all agents observe* $I_1$ *then they are realist and (ii) if an agent deviates and observes both* $I_1$ *and* $I_2$*, she never denies* $I_1$ *and would always denies* $I_2$*.*

Notice that the condition $\mu > \frac{1}{2-p_1}$ is required to guarantee that the parameter space of $\theta_L$ under which $\underline{m}(I_1) < \overline{m}_2^d(\varnothing, s_L)$ does exist. By Assumption 2.1, $\theta_L$ is assumed to be smaller than $\frac{c}{\alpha(s+\delta)}$. Therefore in order to have a non empty for $\theta_L$ such that the above optimal cognitive strategies are feasible, the prior probability of the high state must be sufficiently high.

The last thing we need to check is whether the deviation is profitable. By combining the results from Lemmas A.1 to A.4, for any $m_1, m_2 \in [\underline{m}(I_1), \overline{m}_2^d(\varnothing, s_L)]^2$ the ex-ante intertemporal utility of an agent $i$ from observing $I_1$ when everybody else is observing $I_1$ is

$$U^i_{0|I_1} = \delta[-c + (s+\delta)(\mu\Delta\theta + \theta_L)], \tag{A.22}$$

and the ex-ante inter-temporal utility of an agent $i$ that deviates to $I_1$ and $I_2$ is

$$U^{i,dev}_{0|I_1,I_2} = \delta[-c + (s+\delta)(\mu\Delta\theta + \theta_L)] - (1-\mu)p_2 m_2. \tag{A.23}$$

A comparison between (A.22) and (A.23) demonstrates that an agent $i$ does not find profitable to deviate from $I_1$. Therefore, observing only $I_1$ is an equilibrium. □

**Proof of Proposition 2.2.** We prove this proposition in multiple steps. We first assume that in equilibrium all agents observe both $I_1$ and $I_2$ and are always realist and we calculate the ex-ante inter-temporal utility of an agent i. In the second step we assume that, for the same parameters space for which an agent $i$ observes both information sources and is realist, she deviates to observe only $I_1$ and her optimal cognitive strategy is to be realist. We then calculate the agent $i$'s ex-ante inter-temporal utility of deviation and we compare it to the inter-temporal utility of the guessed equilibrium strategy. We then show that the deviation is not profitable. In the last step we characterise the parameters space under which an agent $i$ (i) is realist on both $I_1$ and $I_2$ when observing both information sources and (ii) is realist on $I_1$ when deviating and observing only $I_1$.

When an agent $i$ observes both information sources and is realist, she will receive the signal profile $(\varnothing, s_L)$ with probability $(1-\mu)p_2$ and from Lemma 2.1 she will not exert effort. Thus, the ex-ante inter-temporal utility of an agent $i$ that observes both $I_1$ and $I_2$ and is realist, if all other agents observe both $I_1$ and $I_2$ and are realist, is

$$
\begin{aligned}
U^i_{0|I_1,I_2} &= Pr(s_H,\varnothing)[\delta(-c+(\delta+s)E(\theta|s_H,\varnothing)] + Pr(\varnothing,\varnothing)[\delta(-c+(s+\delta)E(\theta|\varnothing,\varnothing)] \\
&= \delta\big\{\big[(s+\delta)(\mu\theta_H+(1-\mu)(1-p_2)\theta_L)\big]-[1-(1-\mu)p_2]c\big\}. \qquad \text{(A.24)}
\end{aligned}
$$

Suppose now that an agent $i$ deviates to observes only $I_1$ and in the continuation game she is realist. In this case, from Lemma 1, we know that the agent that deviates will always exert effort both when she receives $s_H$ and when she receives $\varnothing$. However, because the other $n-1$ agents are observing both $I_1$ and $I_2$ and are always realist, the deviating agent is aware of the fact that, when she receives $\varnothing$ the other $n-1$ agents might receive $(\varnothing,\varnothing)$ or $(\varnothing, s_L)$. In the first case both the deviating agents and the $n-1$ agents take the same action $e=1$. In the latter case, the deviating agent exert efforts while the $n-1$ agents do not exert effort because they know the state is $L$ with probability 1. Therefore, the deviating agent, when calculating her ex-ante intertemporal expected utility of deviation will take this fact into account, obtaining

$$
\begin{aligned}
U^i_0|_{I_1} =&(1-\mu p_1)\{\delta[-c+(s+\delta)(r(I_1)\theta_H+(1-r(I_1))[1-Pr(s_L|\varnothing)(1-\alpha)]\theta_L)]\}+ \\
&+\mu p_1[\delta(-c+(s+\delta)\theta_H)] \\
=&\delta\left\{(s+\delta)\left[\mu\theta_H+(1-\mu)\left(1-\frac{p_2}{2-p_1}(1-\alpha)\right)\theta_L\right]-c\right\}, \qquad \text{(A.25)}
\end{aligned}
$$

where $\frac{p_2}{2-p_1} = Pr(s_L|\varnothing)$ is the probability that the other $n-1$ agents receive $s_L$ from information sources $I_2$ when the deviating agent receives $\varnothing$ form $I_1$.

**Lemma A.6.** *If all agents are observing both information sources and are realist, then it is never profitable to deviate to observe only $I_1$ and be realist if $\theta_L < 0$.*

**Proof.** By subtracting equation (A.25) from (A.24), the difference reduces to

$$U_0^i|_{I_1,I_2} - U_0^{dev}|_{I_1} \equiv (s+\delta)\underbrace{\left(\frac{(1-\alpha)}{2-p_1} - 1\right)}_{<0}\theta_L + (1-\mu)c.$$

We can see that, whether this difference is positive or negative depends on the sign of $\theta_L$. In the case of $\theta_L > 0$ we cannot say that the deviation is profitable, unless under some further parameter restrictions. On the contrary, if $\theta_L < 0$ this difference is always positive, therefore the deviation is not profitable. □

We now identify the sufficient conditions for an agent $i$ to be realist on $I_1$ and $I_2$ when observing both information sources. Remember that when observing both $I_1$ and $I_2$, an agent $i$ can receive $(s_H,\varnothing)$, $(\varnothing,\varnothing)$ or $(\varnothing, s_L)$. When the agent receives $(s_H,\varnothing)$ she will be always realist on both signals. In the other two cases she can deny either both signals or only the signal from $I_2$. We first consider the case in which agents receive $(\varnothing,\varnothing)$. The utility of an agent $i$ being realist is then

$$U^i_{0,R|(\varnothing,\varnothing)} = \delta\{-c + (s+\delta)[r(I_1,I_2)\theta_H + (1-r(I_1,I_2))\theta_L]\}. \tag{A.26}$$

If the agent $i$ denies the signal $\varnothing$, she obtains utility

$$\begin{aligned} U^i_{0,D|(\varnothing,\varnothing)} &= \delta\{-c + s[v(\lambda_1,\lambda_2)\theta_H + q(\lambda_1,\lambda_2)[\Delta\theta\, r(I_1,I_2)\theta_H + \theta_L] + \\ &\quad + p(\lambda_1,\lambda_2)\alpha\theta_L] + \delta(r(I_1,I_2)\Delta\theta + \theta_L)]\} - m_1. \end{aligned} \tag{A.27}$$

Denoting $\Gamma(\lambda_1,\lambda_2,m_1) \equiv U^i_{0,D|(\varnothing,\varnothing)} - U^i_{0,R|(\varnothing,\varnothing)}$ the agent $i$'s net incentive of denying $\varnothing$ from $I_1$ when receiving $(\varnothing,\varnothing)$, then her optimal cognitive strategy is

- $\lambda_1 = 0$ if $\Gamma(0, \lambda_2, m_1) \geq 0$ which is equivalent to

$$m_1 \leq \begin{cases} \delta s[\Delta\theta(r(I_2) - r(I_1, I_2))] \equiv \overline{m}_1(\varnothing, \varnothing; 1) & \text{if } \lambda_2 = 1 \\ \\ \delta s(\Delta\theta[\mu - r(I_1, I_2)] - (1-\alpha)(1-\mu)p_2\theta_L) \equiv \overline{m}_1(\varnothing, \varnothing; 0) & \text{if } \lambda_2 = 0 \end{cases}$$

- $\lambda_1^i = 1$ if $\Gamma(1, \lambda_2, m_1) \leq 0$ which is equivalent to

$$m_1 \geq \delta s[\Delta\theta(1 - r(I_1, I_2))] \equiv \underline{m}_1(\varnothing, \varnothing). \tag{A.28}$$

Notice that $\underline{m}_1(\varnothing, \varnothing) > max\{\overline{m}_1(\varnothing, \varnothing; 1), \overline{m}_1(\varnothing_1, \varnothing_2; 0)\}$.

Consider now the case in which agents receive $(\varnothing, s_L)$. In this case the state of the world is low with probability one and if agent $i$ remains realist on both signals, then she obtains ex-ante intertemporal utility $U^i_{0,R}(\varnothing, s_L) = 0$. On the other hand, if the agent $i$ is realist on $\varnothing$ and delusional on $s_L$, she obtains utility

$$\begin{aligned} U^i_{0,D}(\varnothing, s_L)|_{\hat{\varnothing},\hat{\varnothing}} &= \delta\{s[q(\lambda^i_2)(r(I_1, I_2)\theta_H + (1 - r(I_1, I_2))\theta_L) + \\ &+ (1 - q(\lambda^i_2))\alpha\theta_L] - c + \delta\alpha\theta_L\} - m_2. \end{aligned} \tag{A.29}$$

Finally, if the agent denies both signals, she obtains utility

$$\begin{aligned} U^i_{0,D}(\varnothing, s_L)|_{\hat{s}_H,\hat{\varnothing}} &= \delta\{s[v(\lambda_1, \lambda_2)\theta_H + q(\lambda_1, \lambda_2)(\Delta\theta(r(I_1, I_2) + \theta_L) + \\ &+ p(\lambda_1, \lambda_2)\alpha\theta_L] - c + \delta\alpha\theta_L\} - (m_1 + m_2). \end{aligned} \tag{A.30}$$

Denoting $\Gamma(\lambda_1, \lambda_2, m_1 | \varnothing, s_L) \equiv U^i_{0,D}(\varnothing, s_L)|_{\hat{s}_H,\hat{\varnothing}} - U^i_{0,D}(\varnothing, s_L)|_{\hat{\varnothing},\hat{\varnothing}}$ the agent $i$'s net incentive of denying the signal from $I_1$ when receiving $(\varnothing, s_L)$, then her optimal cognitive strategy $\lambda^i_1$ is

- $\lambda_1^i = 1$ if $\Gamma(1, \lambda_2^i, m_1 | \varnothing, s_L) \leq 0$ which is equivalent to

$$m_1 \geq \begin{cases} \delta s[\Delta\theta(1 - r(I_1, I_2))] \equiv \underline{m}_1(\varnothing, s_L; 1) & \text{if } \lambda_2 = 1 \\ \\ \delta s(1 - r(I_1))[\Delta\theta + (1-\alpha)p_2\theta_L] \equiv \underline{m}_1(\varnothing, s_L; 0) & \text{if } \lambda_2 = 0 \end{cases} \tag{A.31}$$

- $\lambda_1 = 0$ if $\Gamma(0, \lambda_2^i, m_1 | \varnothing, s_L) \geq 0$ which is equivalent to

$$m_1 \leq \begin{cases} \delta s(1 - r(I_1))\mu p_1[\Delta\theta + (1-\alpha)p_2\theta_L] \equiv \overline{m}_1(\varnothing, s_L; 0) & \text{if } \lambda_2 = 0 \\ \\ \delta s[(r(I_2) - r(I_1, I_2))\Delta\theta] \equiv \overline{m}_1(\varnothing, s_L; 1) & \text{if } \lambda_2 = 1 \end{cases} \tag{A.32}$$

**Lemma A.7.** *Define* $\underline{m}_1(I_1, I_2) \equiv max\left\{\underline{m}_1(\varnothing, s_L; 0), \underline{m}_1(\varnothing, s_L; 1)\right\}$. *Whenever agents observe both* $I_1$ *and* $I_2$ *in the first stage, if* $m_1 \geq \underline{m}_1(I_1, I_2)$*, there exists an equilibrium of the continuation game where all agents are realist on* $I_1$*, that is* $\lambda_1 = 1$.

**Proof.** It follows from

$$\underline{m}_1(I_1, I_2) \geq \max\{\underline{m}_1(\varnothing, s_L; 0), \underline{m}_1(\varnothing, s_L; 1)\} \geq \underline{m}_1(\varnothing, s_L; 1) \quad =$$
$$\underline{m}_1(\varnothing, \varnothing) \geq \max\{\underline{m}_1(\varnothing, \varnothing; 0), \underline{m}_1(\varnothing, \varnothing; 1)\}.$$

□

Thus, for a cost of denial sufficiently high, all agents are realist on $I_1$ when observing both information sources.

We now analyse the conditions under which realism of the signal received from $I_2$ is the optimal cognitive strategy of an agent $i$ when everybody else is realist on both $I_1$ and $I_2$. Denoting $\Gamma(\lambda_2^i, m_2 | \varnothing, s_L) \equiv U_{0,D}^i(\varnothing, s_L)|_{\hat{\varnothing},\hat{\varnothing}} - U_{0,R}^i(\varnothing, s_L)$ the net incentive of an agent $i$ of denying the signal from $I_2$ when receiving $(\varnothing, s_L)$, then the optimal cognitive strategy $\lambda_2^i$ for an agent $i$ is

- $\lambda_2^i = 1$ if $\Gamma(1, m_2|\varnothing, s_L) \leq 0$ which is equivalent to

$$m_2 \geq \delta\{s[\Delta\theta\, r(I_1, I_2) + \theta_L] + \delta\alpha\theta_L - c\} \equiv \underline{m}_2(I_1, I_2) \tag{A.33}$$

- $\lambda_2^i = 0$ if $\Gamma(1, m_2|\varnothing, s_L) \geq 0$ which is equivalent to

$$m_2 \leq \delta\{s[r(I_1)\Delta\theta + (1-(1-r(I_1))p_2(1-\alpha))\theta_L] + \delta\alpha\theta_L - c\} \equiv \overline{m}_2(I_1, I_2) \tag{A.34}$$

**Lemma A.8.** *Whenever agents observe both $I_1$ and $I_2$ in the first stage, if if $m_2 \geq \underline{m}_2(I_1, I_2)$, there exists an equilibrium of the continuation game where all agents are realist on $I_2$, that is $\lambda_2 = 1$.*

**Proof.** This follows from equation (A.33). □

Combining lemmas A.7 and A.8 we are now able to characterise the social equilibrium cognitive strategies $\lambda_1$ and $\lambda_2$ when agents observe both information sources.

**Lemma A.9.** *Whenever agents observe both $I_1$ and $I_2$ in the first stage, for any $m_1 \geq \underline{m}_1(I_1, I_2)$ and $m_2 \geq \underline{m}_2(I_1, I_2)$ there exists an equilibrium of the continuation game where all agents are realist on both $I_1$ and $I_2$, that is $\lambda_1 = \lambda_2 = 1$.*

We now identify the parameter space under which an agent that deviates and observe only $I_1$ will be realist. When an agent $i$ deviates and observes only $I_1$, she will be always realist if she receives $s_H$. On the contrary if she receives $\varnothing$, being realist she will obtain utility:

$$\begin{aligned} U^d_{I_1,R} &= \delta\big\{-c + (s+\delta)\big[r(I_1)\theta_H + (1-r(I_1))[1 - Pr(s_L|\varnothing)(1-\alpha)]\theta_L\big]\big\} \\ &= \delta\left\{-c + (s+\delta)\left[r(I_1)\theta_H + (1-r(I_1))\left[1 - \frac{p_2}{2-p_1}(1-\alpha)\right]\theta_L\right]\right\}. \end{aligned} \tag{A.35}$$

Whereas if she denies the signal she obtains utility

$$U^d_{I_1,D} = \delta\left\{-c + s\left[r(\lambda^i_1)\theta_H + (1-r(\lambda^i_1))[r(I_1)\theta_H + (1-r(I_1))[1-Pr(s_L|\varnothing)(1-\alpha)]\theta_L]\right]\right\}+$$
$$+\delta^2\{r(I_1)\theta_H + (1-r(I_1))[1-Pr(s_L|\varnothing)(1-\alpha)]\theta_L\}$$
$$= \delta\left\{-c + s\left[r(\lambda^i_1)\theta_H + (1-r(\lambda^i_1))\left[r(I_1)\Delta\theta + \left[1-(1-r(I_1))\frac{p_2}{2-p_1}(1-\alpha)\right]\theta_L\right]\right]\right\}+$$
$$+\delta^2\left\{r(I_1)\Delta\theta + \left[1-(1-r(I_1))\frac{p_2}{2-p_1}(1-\alpha)\right]\theta_L\right\}. \tag{A.36}$$

Denoting $\Gamma(\lambda^i_1, m_1|I^d_1) \equiv U^d_{I_1,D} - U^d_{I_1,R}$ the agent $i$'s net incentive of denying the signal $\varnothing$ when deviating to $I_1$, then her optimal cognitive strategy of deviation is

- $\lambda^i_1 = 1$ if $\Gamma(1, m_1|I^d_1) \leq 0$, which is equivalent to

$$m_1 \geq \delta s(1-r(I_1))\left(\Delta\theta + (1-\alpha)\frac{p_2}{2-p_1}\theta_L\right) \equiv \underline{m}^d_1(I_1), \tag{A.37}$$

- $\lambda^i_1 = 0$ if $\Gamma(1, m_1|I^d_1) \geq 0$, which is equivalent to

$$m_1 \leq \delta s(1-r(I_1))\mu p_1\left(\Delta\theta + (1-\alpha)\frac{p_2}{2-p_1}\theta_L\right) \equiv \overline{m}^d_1(I_1). \tag{A.38}$$

Notice that $\overline{m}^d_1(I_1) < \underline{m}^d_1(I_1)$ for any value of $\theta_L$ and that $\underline{m}^d_1(I_1) < \underline{m}_1(\varnothing, s_L; 0)$ if $\theta_L > 0$ and the inequality is reversed if $\theta_L < 0$.

**Lemma A.10.** *Define $\hat{m} \equiv max\{\underline{m}^d_1(I_1), \underline{m}_1(I_1, I_2), \underline{m}_2(I_1, I_2)\}$. In any equilibrium where all agents are observing both information sources and are always realist, then for any $m_1, m_2 \geq \hat{m}$ (i) if an agent $i$ observes both $I_1$ and $I_2$ she will be always realist on both information sources and (ii) if she deviates and observes only $I_1$ she will be always realist.*

Summarising the results from Lemmas A.6 to A.10 we get Proposition 2.2. □

**Proof of Proposition 2.3.** From lemma A.5 and Proposition 2.1 we know that $\underline{m} \equiv \underline{m}(I_1) = \delta s\Delta\theta(1-r(I_1))$. Under condition (2.10), for any $\theta_L > \underline{\theta}_L$ the threshold value

$\hat{m}$ defined in lemma A.10 reduces to $\hat{m} \equiv max\{\underline{m}_1^d(I_1), \underline{m}_1(I_1, I_2)\}$, where $\underline{m}_1^d(I_1) = \delta s(1 - r(I_1))\left(\Delta\theta + (1-\alpha)\frac{p_2}{2-p_1}\theta_L\right)$ and $\underline{m}_1(I_1, I_2) = \delta s \Delta\theta(1 - r(I_1, I_2))$. It can be verified that $\underline{m} > max\{\underline{m}_1(I_1, I_2), \underline{m}_1^d(I_1)\}$, which implies that $\hat{m} < \underline{m}$. Therefore for any value of $m_1$ and $m_2$ in the range $[\underline{m}, \overline{m}]$ both equilibria exist, with the condition $\mu > \frac{1}{2-p_2}$ to guarantee that $[\underline{m}, \overline{m}]$ is a non empty interval. □

**Proof of Proposition 2.4.** In order to prove part (i), first of all notice that (2.10) implies $\theta_L < 0$. A sufficient condition for welfare being higher in the low state when both information sources are observed is that (2.13) holds. Thus, when (2.10) holds, the LHS of (2.13) is positive and the first part of the RHS of (2.13) is smaller than the RHS of condition (2.10). In addition the second term of the RHS of (2.13) is positive by Assumption 2.1. Therefore (2.10) implies (2.13).

The proof of part (ii) follows directly from the comparison of (2.14) with (2.15). It is straightforward to see that welfare is always higher in the equilibrium where agents observe both information sources given that $r(I_1, I_2) > r(I_1)$. □

# Appendix B

# Appendix to Chapter 3

## B.1 Omitted Proofs

**Proof of Proposition 3.1.** As explained in Section 3.4, rather then solving the entrepreneur's problem in two steps - that is first he decides whether to pay attention to listen to a private signal and second conditional on observing the public and the private signal eventually acquired, he chooses how much to invest in the new project - we solve the problem simultaneously. The entrepreneur's conditional expected utility, equation (3.6), is equivalent to the ex-ante entrepreneur's expected utility where each entrepreneur chooses $z_i$ and $k_i$ to maximise his payoff. By substituting equation (3.7) into equation (3.6) we obtain ex-ante expected utility

$$E(u_i) = E\Big[\theta(\beta_0\mu + \beta_{\bar{x}}\bar{x} + \beta_{x_i}x_i) - \frac{(\beta_0\mu + \beta_{\bar{x}}\bar{x} + \beta_{x_i}x_i)^2}{2}\Big] - C(z_i). \tag{B.1}$$

Taking into account the information structure as given by equations (3.1) and (3.2) and solving the expectation on the right-hand side of the above equation, ex-ante expected utility is equal to:

$$\begin{aligned} E(u_i) = \beta_0\mu^2 + (\beta_{\bar{x}} + \beta_{x_i})(\sigma_\theta^2 + \mu^2) - \frac{\beta_0^2\mu^2}{2} - \frac{\beta_{\bar{x}}^2}{2}(\sigma_\theta^2 + \mu^2 + \kappa^2) - \\ -\frac{\beta_{x_i}^2}{2}(\sigma_\theta^2 + \mu^2 + \frac{\xi^2}{z_i}) - \mu^2\beta_0(\beta_{\bar{x}} + \beta_{xi}) - \beta_{\bar{x}}\beta_{x_i}(\mu^2 + \sigma_\theta^2) - C(z_i). \end{aligned} \tag{B.2}$$

Under Assumption 3.1 the FOCs of the above equation with respect to $\beta_0$, $\beta_{\bar{x}}$, $\beta_{x_i}$ and $z_i$ are:

$$\beta_0 = 1 - \beta_{\bar{x}} - \beta_{x_i} \tag{B.3}$$

$$\beta_{\bar{x}} = \frac{\sigma_\theta^2 + \mu^2 - \beta_0 \mu^2 - \beta_{x_i}(\sigma_\theta^2 + \mu^2)}{(\sigma_\theta^2 + \mu^2 + \kappa^2)} \tag{B.4}$$

$$\beta_{x_i} = \frac{\sigma_\theta^2 + \mu^2 - \beta_0 \mu^2 - \beta_{\bar{x}}(\sigma_\theta^2 + \mu^2)}{(\sigma_\theta^2 + \mu^2 + \xi^2/z_i)} \tag{B.5}$$

$$z_i = \xi \beta_{x_i} \tag{B.6}$$

Solving equations (B.3)-(B.6) we obtain:

$$\beta_0 = \frac{\xi}{\sigma_\theta^2} \tag{B.7}$$

$$\beta_{\bar{x}} = \frac{\xi}{\kappa^2} \tag{B.8}$$

$$\beta_{x_i} = \frac{\sigma_\theta^2 \kappa^2 - \xi(\sigma_\theta^2 + \kappa^2)}{\sigma_\theta^2 \kappa^2} \tag{B.9}$$

$$z_i = \xi \beta_{x_i} \tag{B.10}$$

Rewriting equations (B.7)-(B.10) as functions of signal precisions rather than signal variances yeld the equations of Proposition 3.1. Given that we are focusing on symmetric equilibrium, we drop the subscript $i$ from the information acquisition policy $z_i$ and from the individual investment decision $k_i$. □

**Derivation of parameters of equation** (3.12)**.** The parameters $\gamma_0$, $\gamma_\theta$, $\gamma_\eta$, $\gamma_\omega$ are derived as follows:

$$E[\theta|s,y] = E(\theta) + \frac{1}{\frac{1}{\sigma_\theta^2}+\frac{1}{\varphi^2\kappa^2}+\frac{1}{\tau^2}}\left[\frac{1}{\varphi^2\kappa^2}(s-E(\theta)) + \frac{1}{\tau^2}(y-E(\theta))\right]$$

$$= \frac{\left[\frac{1}{\sigma_\theta^2}+\frac{1}{\varphi^2\kappa^2}+\frac{1}{\tau^2}\right]\mu + \frac{1}{\varphi^2\kappa^2}s + \frac{1}{\tau^2}y - \mu[\frac{1}{\varphi^2\kappa^2}+\frac{1}{\tau^2}]}{\frac{1}{\sigma_\theta^2}+\frac{1}{\varphi^2\kappa^2}+\frac{1}{\tau^2}}$$

$$= \frac{\frac{1}{\sigma_\theta^2}}{\frac{1}{\sigma_\theta^2}+\frac{1}{\varphi^2\kappa^2}+\frac{1}{\tau^2}}\mu + \frac{\frac{1}{\varphi^2\kappa^2}}{\frac{1}{\sigma_\theta^2}+\frac{1}{\varphi^2\kappa^2}+\frac{1}{\tau^2}}s + \frac{\frac{1}{\tau^2}}{\frac{1}{\sigma_\theta^2}+\frac{1}{\varphi^2\kappa^2}+\frac{1}{\tau^2}}y \tag{B.11}$$

Substituting (3.10) and (3.3) into (B.11) and using the definition of signals' precision we obtain the price function in the financial market

$$p = \underbrace{\frac{\pi_\theta}{\pi_\theta + \frac{\pi_{\bar{x}}}{\varphi^2} + \pi_y}}_{\gamma_0}\mu + \underbrace{\frac{\frac{\pi_{\bar{x}}}{\varphi^2} + \pi_y}{\pi_\theta + \frac{\pi_{\bar{x}}}{\varphi^2} + \pi_y}}_{\gamma_\theta}\theta + \underbrace{\frac{\frac{\pi_{\bar{x}}}{\varphi}}{\pi_\theta + \frac{\pi_{\bar{x}}}{\varphi^2} + \pi_y}}_{\gamma_\eta}\eta + \underbrace{\frac{\pi_y}{\pi_\theta + \frac{\pi_{\bar{x}}}{\varphi^2} + \pi_y}}_{\gamma_\omega}\omega \tag{B.12}$$

□

**Proof of Proposition 3.2.** Substituting (3.12) and (3.13) into equation (3.4) we transform the conditional expected utility into the following unconditional ex-ante expected utility

$$\begin{aligned} E(u_i) = E\Big[\theta(1-\lambda)(\delta_0\mu + \delta_{\bar{x}}\bar{x} + \delta_{x_i}x_i) + \\ +\lambda(\gamma_0\mu + \gamma_\theta\theta + \gamma_\eta\eta + \gamma_\omega\omega)(\delta_0\mu + \delta_{\bar{x}}\bar{x} + \delta_{x_i}x_i) + \\ -\frac{(\delta_0\mu + \delta_{\bar{x}}\bar{x} + \delta_{x_i}x_i)^2}{2}\Big] - C(z_i) \end{aligned} \tag{B.13}$$

Solving the expectation on the right hand side of equation (B.13) after substituting equation (3.1) and (3.2) into it, the entrepreneur's unconditional expected utility is

$$\begin{aligned}E(u_i) = {} & \delta_0\mu^2[(1-\lambda+\lambda(\gamma_0+\gamma_\theta)]+(1-\lambda)(\sigma_\theta^2+\mu^2)(\delta_{\bar{x}}+\delta_{x_i})+ \\ & +\delta_{\bar{x}}\lambda[\mu\gamma_0+\gamma_\theta(\sigma_\theta^2+\mu^2)+\gamma_\eta\kappa^2)]+\delta_{x_i}\lambda[\mu\gamma_0+\gamma_\theta(\sigma_\theta^2+\mu^2)]+ \\ & -\frac{\delta_0^2\mu^2}{2}-\frac{\delta_{\bar{x}}^2}{2}(\sigma_\theta^2+\mu^2+\kappa^2)-\frac{\delta_{x_i}^2}{2}(\sigma_\theta^2+\mu^2+\frac{\xi^2}{z_i})+ \\ & -\mu^2\delta_0(\delta_{\bar{x}}+\delta_{x_i})-\delta_{\bar{x}}\delta_{x_i}(\sigma_\theta^2+\mu^2)-C(z_i).\end{aligned} \tag{B.14}$$

Under Assumption 3.1, the FOCs with respect to $\delta_0$, $\delta_{\bar{x}}$, $\delta_{x_i}$ and $z_i$ of the equation above are:

$$\delta_0 = (1-\delta_{\bar{x}}-\delta_{x_i})-\lambda[1-\gamma_\theta-\gamma_0] \tag{B.15}$$

$$\delta_{\bar{x}} = \frac{(1-\lambda)(\sigma_\theta^2+\mu^2)+\lambda(\gamma_0\mu^2+\gamma_\theta(\sigma_\theta^2+\mu^2)+\gamma_\eta\kappa^2)-\delta_0\mu^2-\delta_{x_i}(\sigma_\theta^2+\mu^2)}{(\sigma_\theta^2+\mu^2+\kappa^2)} \tag{B.16}$$

$$\delta_{x_i} = \frac{(1-\lambda)(\sigma_\theta^2+\mu^2)+\lambda(\gamma_0\mu^2+\gamma_\theta(\sigma_\theta^2+\mu^2))-\delta_0\mu^2-\delta_{\bar{x}}(\sigma_\theta^2+\mu^2)}{(\sigma_\theta^2+\mu^2+\frac{\xi^2}{z_i})} \tag{B.17}$$

$$z_i = \xi\delta_{x_i} \tag{B.18}$$

Solving equations (B.15)-(B.18) and rewriting them as function of precisions rather than variances we obtain

$$\delta_0 = \frac{\pi_\theta}{\sqrt{\pi_{x_i}}}+\lambda\gamma_0 \tag{B.19}$$

$$\delta_{\bar{x}} = \frac{\pi_{\bar{x}}}{\sqrt{\pi_{x_i}}}+\lambda\gamma_\eta \tag{B.20}$$

$$\delta_{x_i} = \frac{\sqrt{\pi_{x_i}}-(\pi_{\bar{x}}+\pi_\theta)}{\sqrt{\pi_{x_i}}}-\lambda[1-(\gamma_\theta-\gamma_\eta)] \tag{B.21}$$

$$z_i = \frac{1}{\sqrt{\pi_{x_i}}}\left\{\frac{\left[\sqrt{\pi_{x_i}}-(\pi_{\bar{x}}+\pi_\theta)\right]}{\sqrt{\pi_{x_i}}}-\lambda[1-(\gamma_\theta-\gamma_\eta)]\right\} \tag{B.22}$$

Equations (3.14)-(3.17) are obtained by substituting the value of parameters $\gamma_0$, $\gamma_\theta$,

$\gamma_\eta$ into equations (B.19)-(B.22).

The parameter $\varphi = \frac{\delta_{\bar{x}}}{\delta_{\bar{x}}+\delta_{x_i}}$ solve the fixed point between the real sector and the financial sector. That is the signal $s$ observed by traders must coincide with the signal sent by traders through aggregate capital. □

**Proof of Lemma 3.3.** The first derivative of equation (3.17) with respect to $\varphi$ is

$$\frac{\partial z_i}{\partial \varphi} = \frac{\lambda \pi_{\bar{x}} \sqrt{\pi_{x_i}}[\varphi^2(\pi_\theta+\pi_y)-\varphi 2\pi_\theta-\pi_{\bar{x}}]}{[\pi_{\bar{x}}+\varphi^2(\pi_\theta+\pi_y)]^2}$$

The above equation is negative so long as $\varphi^2(\pi_\theta+\pi_y)-\varphi 2\pi_\theta-\pi_{\bar{x}}<0$. This quadratic equation, which discriminant is positive, has a minimum and two real roots. Let's define $\underline{\varphi} \equiv \frac{\pi_\theta-\sqrt{\pi_\theta^2+\pi_{\bar{x}}(\pi_\theta+\pi_y)}}{\pi_\theta+\pi_y}<0$ and $\overline{\varphi} \equiv \frac{\pi_\theta+\sqrt{\pi_\theta^2+\pi_{\bar{x}}(\pi_\theta+\pi_y)}}{\pi_\theta+\pi_y}>0$ its two roots. Therefore the quadratic equation is negative for $\varphi \in (\underline{\varphi},\overline{\varphi})$ and positive for $\varphi \in (-\infty,\underline{\varphi}) \cup (\overline{\varphi},+\infty)$. Given that the parameter $\varphi$ is restricted to take values between $[0,1]$, the quadratic equation is always negative $\forall\ \varphi \in [0,1]$ if $\overline{\varphi} \geq 1$, that is if $\pi_{\bar{x}}+\pi_\theta \geq \pi_y$. Therefore the fact that $\pi_{\bar{x}}+\pi_\theta \geq \pi_y$ implies that (3.17) is monotonically decreasing in $\varphi$. □

**Proof of Proposition 3.3.** The proof of part (i) is straightforward. If if $\pi_{x_i} \leq (\pi_{\bar{x}}+\pi_\theta)^2$ then the RHS of equation (3.17) is negative. Because $z_i$ is constrained to be greater or equal than zero, whenever the RHS of (3.17) $\leq 0$, we set $z_i=0$.

In order to prove part (ii.) first of all we identify the value $\overline{\pi_{x_i}}$. Let set the RHS of equation (3.17) equal to zero, which is equivalent to say that $z_i=0$. Notice that if $z_i=0$ then $\varphi=1$. Suppose now that $\lambda=1$. Then

$$\text{RHS of equation (3.17)} \equiv \sqrt{\pi_{x_i}}-(\pi_{\bar{x}}+\pi_\theta)-\frac{\sqrt{\pi_{x_i}}(\pi_{\bar{x}}+\pi_\theta)}{\pi_{\bar{x}}+\pi_\theta+\pi_y}=0. \tag{B.23}$$

By solving equation (B.23) for $\pi_{x_i}$ we find that it is satisfied for $\pi_{x_i}=\left[\frac{(\pi_\theta+\pi_{\bar{x}})^2}{\pi_y}+(\pi_\theta+\pi_{\bar{x}})\right]^2$. This value correspond to upper bound $\overline{\pi_{x_i}}$.

**Lemma B.1.** *Assume $\lambda=1$ and $\pi_{x_i}=\overline{\pi_{x_i}}$. Substituting these values into equation*

(3.17) *we get that* $z_i = 0$. *Then, if* $z_i = 0$ *at* $\lambda = 1$ *and* $\pi_{x_i} = \overline{\pi_{x_i}}$, *the RHS of* (3.17) *is negative for any* $\pi_{x_i} < \overline{\pi_{x_i}}$. *Then by continuity there always exists a value* $\hat{\lambda} \in (0,1)$ *such that for any* $\pi_{x_i} < \overline{\pi_{x_i}}$, *the RHS of equation* (3.17) *is equal to zero.*

Now we prove the sufficient condition of part (ii), that is for any $\pi_{x_i} \in \left(\underline{\pi_{x_i}}, \overline{\pi_{x_i}}\right)$, $z_i = 0$ is sufficient for $\lambda \in [\hat{\lambda}, 1)$.

Suppose $z_i = 0$ is an equilibrium. Then $\varphi = 1$ and RHS of equation (3.17) $\leq 0$ which implies $\lambda \geq \hat{\lambda}$.

Now we show that $\lambda \in [\hat{\lambda}, 1)$ is necessary for $z_i = 0$ to be an equilibrium. That is $\lambda \geq \hat{\lambda} \Rightarrow \varphi = 1$ and $z_i = 0$. Suppose this is not true. Then $\exists\ \lambda < \hat{\lambda}$ such that $z_i = 0$ and $\varphi = 1$. But then this contradicts lemma B.1.

The proof of part (iii.) is straightforward. From the proof of part (ii.) we know that if $\pi_{x_i} = \overline{\pi_{x_i}}$ and $\lambda = 1$ then $z_i = 0$. Equation (3.17) is decreasing in $\lambda$, therefore if $\pi_{x_i} > \overline{\pi_{x_i}}$ the RHS of equation (3.17) is always positive, and therefore does not exist an equilibrium with $z_i = 0$. □

**Proof of Lemma 3.5.** Consider the case when in equilibrium $z_i = 0$ and $\pi_{x_i} \in \left(\underline{\pi_{x_i}}, \overline{\pi_{x_i}}\right)$. Then by setting the RHS of equation (3.17) equal to zero we get the threshold value

$$\hat{\lambda} \equiv \frac{(\sqrt{\pi_{x_i}} - \pi_{\bar{x}} - \pi_\theta)(\pi_{\bar{x}} + \pi_\theta + \pi_y)}{\sqrt{\pi_{x_i}}(\pi_{\bar{x}} + \pi_\theta)}. \tag{B.24}$$

Its first derivative with respect to $\pi_y$ is positive. □

# Appendix C

# Appendix to Chapter 4

## C.1 Proof of Section 4.4.1

### C.1.1 Average action

As defined in section 4.3.4 the average action of group $A$ is $\bar{a} \equiv \frac{1}{N_A}\sum_{j=1}^{N_A} a_j$ and the average action of group $B$ is $\bar{b} \equiv \frac{1}{N_B}\sum_{j=N_A+1}^{N} b_j$. The average action of group $A$ excluding one agent is $\bar{a}_{-i} \equiv \frac{1}{N_A-1}\sum_{j\neq i} a_j = \frac{N_A}{N_A-1}\bar{a} - \frac{1}{N_A-1}a_i$ and the average action of group $B$ excluding one agent is $\bar{b}_{-i} \equiv \frac{1}{N_B-1}\sum_{j\neq i} b_j = \frac{N_B}{N_B-1}\bar{b} - \frac{1}{N_B-1}b_i$.

We now verify the following guess about the average actions:

$$\bar{a} = \sum_{j=0}^{N} \gamma_j^B e_j = \gamma^{A\prime} e \tag{C.1}$$

$$\bar{b} = \sum_{j=0}^{N} \gamma_j^B e_j = \gamma^{B\prime} e \tag{C.2}$$

We now focus only on the problem of an agent $i$ of group $A$. Given symmetry of the utility functions between the two groups, the problem of an agent $i$ of group $B$ is exactly the same.

From FOC, the optimal action of an agent $i \in A$ satisfies:

$$a_i = \left(1-\alpha_A+\beta_B\right)E\left(\theta|I_i\right)+\alpha_A E\left(\bar{a}_i|I_i\right)-\beta_B E\left(\bar{b}|I_i\right)$$

$$= \left(1-\alpha_A+\beta_B\right)E\left(\theta|I_i\right)+\alpha_A E\left(\frac{N_A}{N_A-1}\bar{a}-\frac{1}{N_A-1}a_i|I_i\right)-\beta_B E\left(\bar{b}|I_i\right)$$

$$= \left(\tilde{1}-\tilde{\alpha}_{A-1}+\tilde{\beta}_A\right)E\left(\theta|I_i\right)+\tilde{\alpha}_A E\left(\bar{a}|I_i\right)-\tilde{\beta}_A E\left(\bar{b}|I_i\right),$$

where $\tilde{1}=\frac{N}{N+\alpha}$, $\tilde{\alpha}_{A-1}=\alpha\frac{N_A-1}{N+\alpha}$, $\tilde{\alpha}_A=\alpha\frac{N_A}{N+\alpha}$ and $\tilde{\beta}_A=\beta\frac{N_B}{N+\alpha}$.

Using Bayesian updating, for an agent $i \in N$ the expected value of the state of the world given his information set is equal to $E(\theta|I_i)=\sum_{j=0}^{N}\tilde{g}_{ij}e_j=\bar{e}_i$, where $\tilde{g}_{i,j}=\frac{g_{i,j}\sigma_j^{-2}}{\sum_{s=0}^{N}g_{is}\sigma_s^{-2}}=\frac{g_{ij}\pi_j}{\sum_{s=0}^{N}g_{is}\pi_s}$, with $\pi_j$ for $j=0,1,....,N$ being the precision of signal $j$.

Using equation (C.1) and (C.2) about average actions, the expected value of average actions $\bar{a}$ and $\bar{b}$, conditional on $i$'s information set is

$$E\left(\bar{a}|I_i\right)=\sum_{j=0}^{N}\gamma_j^A E(e_j|I_i)=\sum_{j=0}^{N}\gamma_j^A g_{ij}e_j+\sum_{j=0}^{N}\gamma_j^A(1-g_{ij})\bar{e}_i, \tag{C.3}$$

$$E(\bar{b}|I_i)=\sum_{j=0}^{N}\gamma_j^B E(e_j|I_i)=\sum_{j=0}^{N}\gamma_j^B g_{ij}e_j+\sum_{j=0}^{N}\gamma_j^B(1-g_{ij})\bar{e}_i \tag{C.4}$$

Thus player $i$'s action for $i \in A$ is:

$$\begin{aligned} a_i = \left(\tilde{1}-\tilde{\alpha}_{A-1}+\tilde{\beta}_A\right)\bar{e}_i+\tilde{\alpha}_A &\left[\sum_{j=0}^{N}\gamma_j^A g_{ij}e_j+\sum_{j=0}^{N}\gamma_j^A(1-g_{ij})\bar{e}_i\right]+ \\ &-\tilde{\beta}_A\left[\sum_{j=0}^{N}\gamma_j^B g_{ij}e_j+\sum_{j=0}^{N}\gamma_j^B(1-g_{ij})\bar{e}_i\right] \end{aligned} \tag{C.5}$$

Equivalently, for a player $i \in B$ optimal action is:

$$
\begin{aligned}
b_i = (\tilde{1} - \tilde{\alpha}_{B-1} + \tilde{\beta}_B)\bar{e}_i + \tilde{\alpha}_B \left[ \sum_{j=0}^{N} \gamma_j^B g_{ij} e_j + \sum_{j=0}^{N} \gamma_j^A (1 - g_{ij}) \bar{e}_i \right] + \\
- \tilde{\beta}_A \left[ \sum_{j=0}^{N} \gamma_j^B g_{ij} e_j + \sum_{j=0}^{N} \gamma_j^B (1 - g_{ij}) \bar{e}_i \right].
\end{aligned}
\tag{C.6}
$$

We now verify the initial guess; using the fact that $\bar{a} = \frac{1}{N_A} \sum_{i=1}^{N_A} a_i$ and $\bar{b} = \frac{1}{N_B} \sum_{i=N_A+1}^{N} b_i$, then:

$$
\begin{aligned}
N_A \bar{a} = & \sum_{i=1}^{N_A} \Bigg\{ (\tilde{1} - \tilde{\alpha}_{A-1} + \tilde{\beta}_A)\bar{e}_i + \tilde{\alpha}_A \left[ \sum_{j=0}^{N} \gamma_j^A g_{ij} e_j + \sum_{j=0}^{N} (1 - g_{ij}) \bar{e}_i \right] + \\
& - \tilde{\beta}_A \left[ \sum_{j=0}^{N} \gamma_j^B g_{ij} e_j + \sum_{j=0}^{N} (1 - g_{ij}) \bar{e}_i \right] \Bigg\} \\
= & \tilde{1} \sum_{i=1}^{N_A} \bar{e}_i - \tilde{\alpha}_{A-1} \sum_{i=1}^{N_A} \bar{e}_i + \tilde{\beta}_A \sum_{i=1}^{N_A} \bar{e}_i + \tilde{\alpha}_A \sum_{j=0}^{N} \gamma_j^A \sum_{i=1}^{N_A} g_{ij} e_j + \tilde{\alpha}_A \sum_{j=0}^{N} \gamma_j^A \sum_{i=1}^{N_A} \bar{e}_i + \\
& - \tilde{\alpha}_A \sum_{j=0}^{N} \gamma_j^A \sum_{i=1}^{N_A} g_{ij} \bar{e}_i - \tilde{\beta}_A \sum_{j=0}^{N} \gamma_j^B \sum_{i=1}^{N_A} g_{ij} e_j - \tilde{\beta}_A \sum_{j=0}^{N} \gamma_j^B \sum_{i=1}^{N_A} \bar{e}_i - \tilde{\beta}_A \sum_{j=0}^{N} \gamma_j^B \sum_{i=1}^{N_A} g_{ij} \bar{e}_i.
\end{aligned}
\tag{C.7}
$$

Let us define the following vectors matrices:

$$
\boldsymbol{\tilde{\alpha}_{A-1}} = \begin{bmatrix} \frac{\alpha N_A - 1}{N+\alpha} \\ \frac{\alpha N_A - 1}{N+\alpha} \\ \vdots \\ \vdots \\ \frac{\alpha N_A - 1}{N+\alpha} \end{bmatrix}_{N_A \times 1}, \quad
\boldsymbol{\tilde{1}_A} = \begin{bmatrix} \frac{N}{N+\alpha} \\ \frac{N}{N+\alpha} \\ \vdots \\ \vdots \\ \frac{N}{N+\alpha} \end{bmatrix}_{N_A \times 1}, \quad
\boldsymbol{\tilde{\alpha}_A} = \begin{bmatrix} \frac{\alpha N_A}{N+\alpha} \\ \frac{\alpha N_A}{N+\alpha} \\ \vdots \\ \vdots \\ \frac{\alpha N_A}{N+\alpha} \end{bmatrix}_{N_A \times 1}
$$

$$\tilde{\boldsymbol{\beta}}_{\boldsymbol{A}} = \begin{bmatrix} \frac{\beta N_B}{N+\alpha} \\ \frac{\beta N_B}{N+\alpha} \\ \vdots \\ \vdots \\ \frac{\beta N_B}{N+\alpha} \end{bmatrix}_{N_A\times 1}, \quad \mathbf{1} = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ \vdots \\ 1 \end{bmatrix}_{(N+1)\times 1}, \quad \gamma^A = \begin{bmatrix} \gamma_0^A \\ \gamma_1^A \\ \vdots \\ \vdots \\ \gamma_N^A \end{bmatrix}_{(N+1)\times 1}$$

$$\gamma^B = \begin{bmatrix} \gamma_0^B \\ \gamma_1^B \\ \vdots \\ \vdots \\ \gamma_N^B \end{bmatrix}_{(N+1)\times 1}, \quad \bar{e} = \begin{bmatrix} \bar{e}_1 \\ \bar{e}_2 \\ \vdots \\ \vdots \\ \bar{e}_{N_A} \end{bmatrix}_{N_A\times 1}, \quad e = \begin{bmatrix} e_0 \\ e_1 \\ \vdots \\ \vdots \\ e_N \end{bmatrix}_{(N+1)\times 1},$$

$$G^A = \begin{bmatrix} g_{10} & g_{11} & \cdots & g_{1N} \\ g_{20} & g_{21} & \cdots & g_{2N} \\ \vdots & \vdots & & \vdots \\ \vdots & \vdots & & \vdots \\ g_{N_A0} & g_{N_A1} & \cdots & g_{N_AN} \end{bmatrix}_{N_A\times N}$$

Then equation (C.7) can be written in matrix form as following:

$$\begin{aligned} N_A\bar{a} = \tilde{\mathbf{1}}'_{\boldsymbol{A}}\bar{e}^A - \tilde{\boldsymbol{\alpha}}'_{\boldsymbol{A-1}}\bar{e}^A + \tilde{\boldsymbol{\beta}}'_{\boldsymbol{A}}\bar{e}^A + \gamma^{A'}\mathrm{diag}(G^{A'}\tilde{\boldsymbol{\alpha}}_{\boldsymbol{A}})e + \gamma^{A'}\mathbf{1}\tilde{\boldsymbol{\alpha}}'_{\boldsymbol{A}}\bar{e}^A + \\ -\gamma^{A'}G^{A'}\mathrm{diag}(\tilde{\boldsymbol{\alpha}}_{\boldsymbol{A}})\bar{e}^A - \gamma^{B'}\mathrm{diag}(G^{A'}\tilde{\boldsymbol{\beta}}_{\boldsymbol{A}})e - \gamma^{B'}\mathbf{1}\tilde{\boldsymbol{\beta}}'_{\boldsymbol{A}}\bar{e}^A + \\ +\gamma^{B'}G^{A'}\mathrm{diag}(\tilde{\boldsymbol{\beta}}_{\boldsymbol{A}})\bar{e}^A \end{aligned} \tag{C.8}$$

Notice that the vector $\bar{e}^A$ can be written in the following way:

$$\bar{e}^A = \begin{bmatrix} \sum_{k=0}^{N} \tilde{g}_{1k} e_k \\ \sum_{k=0}^{N} \tilde{g}_{2k} e_k \\ \vdots \\ \vdots \\ \sum_{k=0}^{N} \tilde{g}_{N_A k} e_k \end{bmatrix} = \begin{bmatrix} \tilde{g}_{10} & \tilde{g}_{11} & \cdots & \tilde{g}_{1N} \\ \tilde{g}_{20} & \tilde{g}_{21} & \cdots & \tilde{g}_{2N} \\ \vdots & \vdots & & \vdots \\ \vdots & \vdots & & \vdots \\ \tilde{g}_{N_A 0} & \tilde{g}_{N_A 1} & \cdots & \tilde{g}_{N_A N} \end{bmatrix} \begin{bmatrix} e_0 \\ e_1 \\ \vdots \\ \vdots \\ e_N \end{bmatrix} = \tilde{G}^A e \tag{C.9}$$

Then by using (C.9), equation (C.8) becomes:

$$N_A \bar{a} = \Big[\tilde{\mathbf{1}}_{\boldsymbol{A}}' \tilde{G}^A - \tilde{\boldsymbol{\alpha}}_{\boldsymbol{A-1}}' \tilde{G}^A + \tilde{\boldsymbol{\beta}}_{\boldsymbol{A}}' \tilde{G}^A + \gamma^{A'} diag(G^{A'} \tilde{\boldsymbol{\alpha}}_{\boldsymbol{A}}) e + \gamma^{A'} \mathbf{1} \tilde{\boldsymbol{\alpha}}_{\boldsymbol{A}}' \tilde{G}^A +$$
$$- \gamma^{A'} G^{A'} diag(\tilde{\boldsymbol{\alpha}}_{\boldsymbol{A}}) \tilde{G}^A - \gamma^{B'} diag(G^{A'} \tilde{\boldsymbol{\beta}}_{\boldsymbol{A}}) e - \gamma^{B'} \mathbf{1} \tilde{\boldsymbol{\beta}}_{\boldsymbol{A}}' \tilde{G}^A + \gamma^{B'} G^{A'} diag(\tilde{\boldsymbol{\beta}}_{\boldsymbol{A}}) \tilde{G}^A \Big] e$$

The final equation for the average action of group $A$ is then:

$$\bar{a} = \frac{1}{N_A} \Bigg\{ \gamma^{A'} \Big[ diag(G^{A'} \tilde{\boldsymbol{\alpha}}_{\boldsymbol{A}}) + \mathbf{1} \tilde{\boldsymbol{\alpha}}_{\boldsymbol{A}}' \tilde{G}^A - G^{A'} diag(\tilde{\boldsymbol{\alpha}}_{\boldsymbol{A}}) \tilde{G}^A \Big] + \tag{C.10}$$
$$- \gamma^{B'} \Big[ diag(G^{A'} \tilde{\boldsymbol{\beta}}_{\boldsymbol{A}}) + \mathbf{1} \tilde{\boldsymbol{\beta}}_{\boldsymbol{A}}' \tilde{G}^A - G^{A'} diag(\tilde{\boldsymbol{\beta}}_{\boldsymbol{A}}) \tilde{G}^A \Big] + (\tilde{\mathbf{1}}_{\boldsymbol{A}} - \tilde{\boldsymbol{\alpha}}_{\boldsymbol{A-1}} + \tilde{\boldsymbol{\beta}}_{\boldsymbol{A}})' \tilde{G}^A \Bigg\} e$$

Equivalently, average action $\bar{b}$ can be written in matrix form as follows:

$$\bar{b} = \frac{1}{N_B} \Bigg\{ \gamma^{B'} \Big[ diag(G^{B'} \tilde{\boldsymbol{\alpha}}_{\boldsymbol{B}}) + \mathbf{1} \tilde{\boldsymbol{\alpha}}_{\boldsymbol{B}}' \tilde{G}^B - G^{B'} diag(\tilde{\boldsymbol{\alpha}}_{\boldsymbol{B}}) \tilde{G}^B \Big] + \tag{C.11}$$
$$- \gamma^{A'} \Big[ diag(G^{B'} \tilde{\boldsymbol{\beta}}_{\boldsymbol{B}}) + \mathbf{1} \tilde{\boldsymbol{\beta}}_{\boldsymbol{B}}' \tilde{G}^B - G^{B'} diag(\tilde{\boldsymbol{\beta}}_{\boldsymbol{B}}) \tilde{G}^B \Big] + (\tilde{\mathbf{1}}_{\boldsymbol{B}} - \tilde{\boldsymbol{\alpha}}_{\boldsymbol{B-1}} + \tilde{\boldsymbol{\beta}}_{\boldsymbol{B}})' \tilde{G}^B \Bigg\} e$$

where $G^B$, and $\tilde{G}^B$ are $N_B \times (N+1)$ matrices and $\tilde{1}_B$ is $N_B \times 1$ vector for any agent $i \in B$ defined in a similar way of those for $i \in A$ as above. The vectors $\tilde{\alpha}_B$, $\tilde{\alpha}_{B-1}$ and

$\tilde{\beta}_B$ are of dimension $N_B \times 1$ defined as follows:

$$\tilde{\boldsymbol{\alpha}}_{\boldsymbol{B-1}} = \begin{bmatrix} \frac{\alpha N_B - 1}{N+\alpha} \\ \frac{\alpha N_B - 1}{N+\alpha} \\ \dots \\ \dots \\ \frac{\alpha N_B - 1}{N+\alpha} \end{bmatrix}, \qquad \tilde{\boldsymbol{\beta}}_{\boldsymbol{B}} = \begin{bmatrix} \frac{\beta N_A}{N+\alpha} \\ \frac{\beta N_A}{N+\alpha} \\ \dots \\ \dots \\ \frac{\beta N_A}{N+\alpha} \end{bmatrix}, \qquad \tilde{\boldsymbol{\alpha}}_{\boldsymbol{B}} = \begin{bmatrix} \frac{\alpha N_B}{N+\alpha} \\ \frac{\alpha N_B}{N+\alpha} \\ \dots \\ \dots \\ \frac{\alpha N_B}{N+\alpha} \end{bmatrix}$$

We now verify our guesses. Substituting equation (C.1) into equation (C.10) and equation (C.2) into equation (C.11), we obtain respectively:

$$\gamma^{A'} = \frac{1}{N_A}\Big\{\gamma^{A'}\big[diag(G^{A'}\tilde{\boldsymbol{\alpha}}_{\boldsymbol{A}}) + \mathbf{1}\tilde{\boldsymbol{\alpha}}'_{\boldsymbol{A}}\tilde{G}^A - G^{A'}diag(\tilde{\boldsymbol{\alpha}}_{\boldsymbol{A}})\tilde{G}^A\big] + \\ -\gamma^{B'}\big[diag(G^{A'}\tilde{\boldsymbol{\beta}}_{\boldsymbol{A}}) + \mathbf{1}\tilde{\boldsymbol{\beta}}'_{\boldsymbol{A}}\tilde{G}^A - G^{A'}diag(\tilde{\boldsymbol{\beta}}_{\boldsymbol{A}})\tilde{G}^A\big] + (\tilde{\mathbf{1}}_{\boldsymbol{A}} - \tilde{\boldsymbol{\alpha}}_{\boldsymbol{A-1}} + \tilde{\boldsymbol{\beta}}_{\boldsymbol{A}})'\tilde{G}^A\Big\} \tag{C.12}$$

$$\gamma^{B'} = \frac{1}{N_B}\Big\{\gamma^{B'}\big[diag(G^{B'}\tilde{\boldsymbol{\alpha}}_{\boldsymbol{B}}) + \mathbf{1}\tilde{\boldsymbol{\alpha}}'_{\boldsymbol{B}}\tilde{G}^B - G^{B'}diag(\tilde{\boldsymbol{\alpha}}_{\boldsymbol{B}})\tilde{G}^B\big] + \\ -\gamma^{A'}\big[diag(G^{B'}\tilde{\boldsymbol{\beta}}_{\boldsymbol{B}}) + \mathbf{1}\tilde{\boldsymbol{\beta}}'_{\boldsymbol{B}}\tilde{G}^B - G^{B'}diag(\tilde{\boldsymbol{\beta}}_{\boldsymbol{B}})\tilde{G}^B\big] + (\tilde{\mathbf{1}}_{\boldsymbol{B}} - \tilde{\boldsymbol{\alpha}}_{\boldsymbol{B-1}} + \tilde{\boldsymbol{\beta}}_{\boldsymbol{B}})'\tilde{G}^B\Big\} \tag{C.13}$$

By post-multiplying both LHS and RHS of equations (C.12) and (C.13) by a vector of ones, after some algebraic steps we obtain respectively:

$$\gamma^{A'}\mathbf{1}(N_A - \tilde{\boldsymbol{\alpha}}'_{\boldsymbol{A}}\mathbf{1}) = \frac{N_A N}{N+\alpha} - \tilde{\boldsymbol{\alpha}}'_{\boldsymbol{A-1}}\mathbf{1} + \tilde{\boldsymbol{\beta}}'_{\boldsymbol{A}}\mathbf{1} - \gamma^{B'}\mathbf{1}\tilde{\boldsymbol{\beta}}'_{\boldsymbol{A}}\mathbf{1} \tag{C.14}$$

$$\gamma^{B'}\mathbf{1}(N_B - \tilde{\boldsymbol{\alpha}}'_{\boldsymbol{B}}\mathbf{1}) = \frac{N_B N}{N+\alpha} - \tilde{\boldsymbol{\alpha}}'_{\boldsymbol{B-1}}\mathbf{1} + \tilde{\boldsymbol{\beta}}'_{\boldsymbol{B}}\mathbf{1} - \gamma^{A'}\mathbf{1}\tilde{\boldsymbol{\beta}}'_{\boldsymbol{B}}\mathbf{1} \tag{C.15}$$

Noticing that $\tilde{\boldsymbol{\alpha}}'_{\boldsymbol{J-1}}\mathbf{1} = \frac{\alpha N_J(N_J-1)}{N+\alpha}$, $\tilde{\boldsymbol{\alpha}}'_{\boldsymbol{J}}\mathbf{1} = \frac{\alpha N_J^2}{N+\alpha}$ and $\tilde{\boldsymbol{\beta}}'_{\boldsymbol{J}}\mathbf{1} = \frac{\beta N_A N_B}{N+\alpha}$ for $J = A, B$, we can further simplify the two equations above to obtain:

$$\gamma^{A'}\mathbf{1}[N_A(N+\alpha)-\alpha N_A^2] = N_A N - \alpha N_A^2 + \alpha N_A + \beta N_A N_B - \gamma^{B'}\mathbf{1}\beta N_A N_B \tag{C.16}$$

$$\gamma^{B'}\mathbf{1}[N_B(N+\alpha)-\alpha N_B^2] = N_B N - \alpha N_B^2 + \alpha N_B + \beta N_A N_B - \gamma^{A'}\mathbf{1}\beta N_A N_B \tag{C.17}$$

The system of this two equations represents the solution of the problem. By substituting equation (C.16) into equation (C.17) we get

$$\gamma^{B'}\mathbf{1} = 1 \tag{C.18}$$

Substituting (C.18) into equation (C.16) we get

$$\gamma^{A'}\mathbf{1} = 1 \tag{C.19}$$

Equations (C.18) and (C.19) show that the sum of weights for the average action of agent of group $A$ and for the average action of agent of group $B$ is equal to 1. Therefore we have proved our initial guess. Given that agent i's action and the average actions are linear combination of signals, it must be that the average action of all other agents excluding agent $i$, is also a linear combination of the signal:

$$\bar{a}_{-i} = \frac{1}{N_A - 1}\sum_{j\in A, j\neq i} a_j = \sum_{j=0}^{N} \gamma^A_{-ij} e_j \tag{C.20}$$

$$\bar{b}_{-i} = \frac{1}{N_B - 1}\sum_{j\in B, j\neq i} b_j = \sum_{j=0}^{N} \gamma^B_{-ij} e_j \tag{C.21}$$

Equations (4.11) of section 4.4.1 are derived from equation (C.12), while equations (4.12) are derived from equation (C.13). In particular we now write in sum notation a single element of equation (C.12) for a generic signal $h \in N$:

$$N_A\gamma_h^A = \Bigg[\frac{N-\alpha(N_A-1)+\beta N_B}{N+\alpha}\sum_{i=1}^{N_A}\tilde{g}_{ih} + \frac{\alpha N_A}{N+\alpha}\gamma_h^A\sum_{i=1}^{N_A} g_{ih} + \frac{\alpha N_A}{N+\alpha}\sum_{i=1}^{N_A}\tilde{g}_{ih} + \\ -\frac{\alpha N_A}{N+\alpha}\sum_{s=0}^{N}\sum_{i=1}^{N_A}\gamma_s^A g_{is}\tilde{g}_{ih} - \frac{\beta N_B}{N+\alpha}\gamma_h^B\sum_{i=1}^{N_A} g_{ih} - \frac{\beta N_B}{N+\alpha}\sum_{i=1}^{N_A}\tilde{g}_{ih} + \\ + \frac{\beta N_B}{N+\alpha}\sum_{s=0}^{N}\sum_{i=1}^{N_A}\gamma_s^B g_{is}\tilde{g}_{ih}\Bigg] \tag{C.22}$$

Recall that we previously defined $\tilde{g}_{ih} = \frac{g_{ih}\pi_h}{\sum_{s=0}^N g_{is}\pi_s}$, $K_i^A = \sum_{k\in N_A, k\neq i} g_{ik}$ and $K_i^B = \sum_{k\in N_B, k\neq i} g_{ik}$. In addition notice that the signal precision for any agents in group $A$ is $\pi_A$, the signal precision for any agent in group $B$ is $\pi_B$ and the prior has precision 1. Then

$$\sum_{i=i}^{N_A}\tilde{g}_{ij} = \begin{cases} \pi_A \sum_{i=1}^{N_A} \frac{g_{ij}}{\pi_A(K_i^A+1)+\pi_B K_i^B+1} & \text{if } j\in A \\ \pi_B \sum_{i=1}^{N_A} \frac{g_{ik}}{\pi_A(K_i^A+1)+K_i^B\pi_B+1} & \text{if } j\in B \end{cases} \tag{C.23}$$

Let us now define $\bar{K}_J^s$ the number of agents in group $J = A, B$ to tap into signal $s \in N$, that is

$$\sum_{i\in N_A, i\neq s} g_{is} = \bar{K}_A^s, \tag{C.24}$$

$$\sum_{i\in N_B, i\neq s} g_{is} = \bar{K}_B^s \tag{C.25}$$

Then the influence of a signal $j \in A$ and of a signal $k \in B$ on the average action of group $A$ is respectively

$$\gamma_j^A = \frac{\pi_A}{N_A}\sum_{i=1}^{N_A}\frac{g_{ij}}{\pi_A(K_i^A+1)+\pi_B K_i^B+1} + \frac{(\alpha N_A\gamma_j^A - \beta N_B\gamma_j^B)}{N_A(N+\alpha)}(\bar{K}_A^j+1) + \\ -\frac{\pi_A}{N_A(N+\alpha)}\sum_{s=0}^{N}\sum_{i=1}^{N_A}\frac{(\alpha N_A\gamma_s^A - \beta N_B\gamma_s^B)g_{is}g_{ij}}{\pi_A(K_i^A+1)+\pi_B K_i^B+1} \tag{C.26}$$

$$\gamma_k^A = \frac{\pi_B}{N_A}\sum_{i=1}^{N_A}\frac{g_{ik}}{\pi_A(K_i^A+1)+\pi_B K_i^B+1} + \frac{(\alpha N_A\gamma_k^A - \beta N_B\gamma_k^B)}{N_A(N+\alpha)}\bar{K}_A^k + \\ - \frac{\pi_B}{N_A(N+\alpha)}\sum_{s=0}^{N}\sum_{i=1}^{N_A}\frac{(\alpha N_A\gamma_s^A - \beta N_B\gamma_s^B)g_{is}g_{ik}}{\pi_A(K_i^A+1)+\pi_B K_i^B+1} \tag{C.27}$$

Equivalently from equation (C.13) we can derive the influence of a signal $j \in A$ and of a signal $k \in B$ on the average action of group $B$:

$$\gamma_j^B = \frac{\pi_A}{N_B}\sum_{i=N_A+1}^{N}\frac{g_{ij}}{\pi_A K_i^A+\pi_B(K_i^B+1)+1} + \frac{(\alpha N_B\gamma_j^B - \beta N_A\gamma_j^A)}{N_B(N+\alpha)}\bar{K}_B^j + \\ - \frac{\pi_A}{N_B(N+\alpha)}\sum_{s=0}^{N}\sum_{i=N_A+1}^{N}\frac{(\alpha N_B\gamma_s^B - \beta N_A\gamma_s^A)g_{is}g_{ij}}{\pi_A K_i^A+\pi_B(K_i^B+1)+1} \tag{C.28}$$

$$\gamma_k^B = \frac{\pi_B}{N_B}\sum_{i=N_A+1}^{N}\frac{g_{ik}}{\pi_A K_i^A+\pi_B(K_i^B+1)+1} + \frac{(\alpha N_B\gamma_k^B - \beta N_A\gamma_k^A)}{N_B(N+\alpha)}\bar{(}K_B^k+1) + \\ - \frac{\pi_B}{N_B(N+\alpha)}\sum_{s=0}^{N}\sum_{i=N_A+1}^{N}\frac{(\alpha N_B\gamma_s^B - \beta N_A\gamma_s^A)g_{is}g_{ik}}{\pi_A K_i^A+\pi_B(K_i^B+1)+1} \tag{C.29}$$

### C.1.2 Individual action

From equations (C.5) and (C.6) we can write the individual actions $a$ and $b$ in matrix form, where $a$ and $b$ represent the vector of the individual action of agents in groups $A$ and $B$ respectively.

$$a = \big[diag(\tilde{\mathbf{1}}_{\boldsymbol{A}})\bar{e}^A - diag(\tilde{\boldsymbol{\alpha}}_{\boldsymbol{A-1}})\bar{e}^A + diag(\tilde{\boldsymbol{\beta}}_{\boldsymbol{A}})\bar{e}^A + diag(\tilde{\boldsymbol{\alpha}}_{\boldsymbol{A}})G^A diag(\gamma^A)e + \\ + diag(\tilde{\boldsymbol{\alpha}}_{\boldsymbol{A}})\bar{e}^A - diag(\tilde{\boldsymbol{\alpha}}_{\boldsymbol{A}})diag(G^A\gamma^A)\bar{e}^A - diag(\tilde{\boldsymbol{\beta}}_{\boldsymbol{A}})G^A diag(\gamma^B)e + \\ - diag(\tilde{\boldsymbol{\beta}}_{\boldsymbol{A}})\bar{e}^A + diag(\tilde{\boldsymbol{\beta}}_{\boldsymbol{A}})diag(G^A\gamma^B)\bar{e}^A\big] \tag{C.30}$$

$$b = \Big[diag(\tilde{\mathbf{1}}_{\boldsymbol{B}})\bar{e}^B - diag(\tilde{\boldsymbol{\alpha}}_{\boldsymbol{B-1}})\bar{e}^B + diag(\tilde{\boldsymbol{\beta}}_{\boldsymbol{B}})\bar{e}^B + diag(\tilde{\boldsymbol{\alpha}}_{\boldsymbol{B}})G^B diag(\gamma^B)e + \tag{C.31}$$

$$+ diag(\tilde{\boldsymbol{\alpha}}_{\boldsymbol{B}})\bar{e}^B - diag(\tilde{\boldsymbol{\alpha}}_{\boldsymbol{B}})diag(G^B\gamma^B)\bar{e}^B - diag(\tilde{\boldsymbol{\beta}}_{\boldsymbol{B}})G^B diag(\gamma^A)e +$$

$$- diag(\tilde{\boldsymbol{\beta}}_{\boldsymbol{B}})\bar{e}^B + diag(\tilde{\boldsymbol{\beta}}_{\boldsymbol{B}})diag(G^B\gamma^A)\bar{e}^B\Big]$$

Recall that $\bar{e}^A = \tilde{G}^A e$ and $\bar{e}^B = \tilde{G}^B e$, then the vector of actions $a$ and $b$ can be written as follows:

$$a = \Big[diag(\tilde{\mathbf{1}}_{\boldsymbol{A}})\tilde{G}^A - diag(\tilde{\boldsymbol{\alpha}}_{\boldsymbol{A-1}})\tilde{G}^A + diag(\tilde{\boldsymbol{\beta}}_{\boldsymbol{A}})\tilde{G}^A + diag(\tilde{\boldsymbol{\alpha}}_{\boldsymbol{A}})G^A diag(\gamma^A)e + \tag{C.32}$$

$$+ diag(\tilde{\boldsymbol{\alpha}}_{\boldsymbol{A}})\tilde{G}^A - diag(\tilde{\boldsymbol{\alpha}}_{\boldsymbol{A}})diag(G^A\gamma^A)\tilde{G}^A - diag(\tilde{\boldsymbol{\beta}}_{\boldsymbol{A}})G^A diag(\gamma^B)e +$$

$$- diag(\tilde{\boldsymbol{\beta}}_{\boldsymbol{A}})\tilde{G}^A + diag(\tilde{\boldsymbol{\beta}}_{\boldsymbol{A}})diag(G^A\gamma^B)\tilde{G}^A\Big]e$$

The expression between the squared brackets, which we define $\Lambda_A$, is the matrix of weights of dimension $N_A \times N+1$ matrix.

$$b = \Big[diag(\tilde{\mathbf{1}}_{\boldsymbol{B}})\tilde{G}^B - diag(\tilde{\boldsymbol{\alpha}}_{\boldsymbol{B-1}})\tilde{G}^B + diag(\tilde{\boldsymbol{\beta}}_{\boldsymbol{B}})\tilde{G}^B + diag(\tilde{\boldsymbol{\alpha}}_{\boldsymbol{B}})G^B diag(\gamma^B)e + \tag{C.33}$$

$$+ diag(\tilde{\boldsymbol{\alpha}}_{\boldsymbol{B}})\tilde{G}^B - diag(\tilde{\boldsymbol{\alpha}}_{\boldsymbol{B}})diag(G^B\gamma^B)\tilde{G}^B - diag(\tilde{\boldsymbol{\beta}}_{\boldsymbol{B}})G^B diag(\gamma^A)e +$$

$$- diag(\tilde{\boldsymbol{\beta}}_{\boldsymbol{B}})\tilde{G}^B + diag(\tilde{\boldsymbol{\beta}}_{\boldsymbol{B}})diag(G^B\gamma^A)\tilde{G}^A\Big]e$$

The expression between the squared brackets, which we define $\Lambda_B$, is the matrix of weights of dimension $N_B \times N+1$ matrix.

Thus from $\Lambda_A$ and $\Lambda_B$ we can derive respectively the weight that an individual $i \in A$ and an individual $h \in B$ attaches to any signal $j \in N$:

$$\lambda_{ij}^{A} = g_{ij}\left[\frac{\alpha N_A\gamma_j^A - \beta N_B\gamma_j^B}{N+\alpha} + \frac{\pi_j}{\pi_A(K_i^A+1)+\pi_B K_i^B+1}\left(1-\sum_{s=0}^{N}\frac{(\alpha N_A\gamma_s^A-\beta N_B\gamma_s^B)}{N+\alpha}g_{is}\right)\right] \tag{C.34}$$

$$\lambda_{hj}^{B} = g_{hj}\left[\frac{(\alpha N_B\gamma_j^B - \beta N_A\gamma_j^A}{N+\alpha} + \frac{\pi_j}{\pi_A K_i^A+\pi_B (K_i^B+1)+1}\left(1-\sum_{s=0}^{N}\frac{(\alpha N_B\gamma_s^B-\beta N_A\gamma_s^A)}{N+\alpha}g_{hs}\right)\right] \tag{C.35}$$

where $\pi_j = \pi_A$ if signal $j \in A$ and $\pi_j = \pi_B$ if signal $j \in B$.

## C.2 Derivation of ex-ante expected utility

$$E(u_i^A|G) = -E[(a_i - a_i^*)^2|G] = -E[E(a_i - a_i^*|I_i)^2]|G] = -E[Var(a_i^*|I_i)|G], \tag{C.36}$$

where the second equation follows from the law of iterated expectations and the last one follows from the optimal action of agent $i$, as derived in (4.5).

Following Herskovic and Ramos (2017) we derive the value of $E(u_i^A|G)$ as a function of the group level influence of the signals and of agent $i$'s connections, $g_{ij}$.

We start writing the bliss action in sum notation using equations (4.4), (C.20) and (C.21):

$$
\begin{aligned}
a_i^* &= \left(1-\alpha\frac{N_A-1}{N}+\beta\frac{N_B}{N}\right)\theta+\alpha\frac{N_A-1}{N}\sum_{k=0}^{N}\gamma_{-ik}^{A}e_k-\beta\frac{N_B}{N}\sum_{k=0}^{N}\gamma_k^B e_k \\
&= \left(1-\alpha\frac{N_A-1}{N}+\beta\frac{N_B}{N}\right)\theta+\alpha\frac{N_A-1}{N}\theta\underbrace{\sum_{k=0}^{N}\gamma_{-ik}^{A}}_{=1}-\beta\frac{N_B}{N}\theta\underbrace{\sum_{k=0}^{N}\gamma_k^B}_{=1} \\
&\qquad +\alpha\frac{N_A-1}{N}\sum_{k=1}^{N}\gamma_{-ik}^{A}\sigma_k\epsilon_k-\beta\frac{N_B}{N}\sum_{k=1}^{N}\gamma_k^B\sigma_k\epsilon_k \\
&= \theta+\alpha\frac{N_A-1}{N}\sum_{k=1}^{N}\gamma_{-ik}^{A}\sigma_k\epsilon_k-\beta\frac{N_B}{N}\sum_{k=1}^{N}\gamma_k^B\sigma_k\epsilon_k, \\
&= \theta+\sum_{k=1}^{N}\left(\alpha\frac{N_A-1}{N}\gamma_{-ik}^{A}-\beta\frac{N_B}{N}\gamma_k^B\right)\sigma_k\epsilon_k \\
&= \theta+\sum_{k=1}^{N}\delta_k^A\sigma_k\epsilon_k
\end{aligned}
$$

where we have used the fact that $\epsilon_0 = 0$.

In matrix notation the bliss action becomes:

$$a_i^* = F_A'\omega \tag{C.37}$$

$$F_A = \begin{bmatrix} 1 \\ \delta_1^A\sigma_1 \\ \delta_2^A\sigma_2 \\ \dots \\ \delta_N^A\sigma_N \end{bmatrix}_{(N+1)\times 1}, \quad \omega = \begin{bmatrix} \theta \\ \epsilon_1 \\ \epsilon_2 \\ \dots \\ \epsilon_N \end{bmatrix}_{(N+1)\times 1}. \tag{C.38}$$

Notice that in order to simplify our calculations, we assume that the vector $\omega$ is a vector of independent standard normal variables.

Thus using equation (C.37) the ex-ante expected value of equation (C.36) reduce to the solution of the following equation:

$$E(u_i^A|G) = -F_A'\mathrm{Var}(\omega|I_i)F_A. \tag{C.39}$$

The solution of the above equation follows the same logic used in Proposition 3 of Herskovic and Ramos (2017).

First of all we need to solve the conditional variance. Given normality of all random variables, $\mathrm{Var}(\omega|I_i)$ is the conditional normal variance:

$$\mathrm{Var}(\omega|I_i) = \mathrm{Var}(\omega) - \mathrm{Cov}(\omega, X_{i,A}\Gamma\omega)\mathrm{Var}(X_{i,A}\Gamma\omega)^{-1}\mathrm{Cov}(\omega, X_{i,A}\omega)', \tag{C.40}$$

where:

$$\begin{aligned}
\mathrm{Var}(\omega) &= I \\
\mathrm{Cov}(\omega, X_{i,A}\Gamma\omega)' &= X_{i,A}\Gamma \\
\mathrm{Var}(X_{i,A}\Gamma\omega) &= X_{i,A}\Gamma\Gamma' X'_{i,A}
\end{aligned}$$

We need to invert the matrix $\mathrm{Var}(X_{i,A}\Gamma\omega)$. To do so we need to redefine the matrix $\Gamma$ in order to be able to apply the Shermann-Morrison theorem.

Let's define $\Gamma$ as:

$$\Gamma = \begin{bmatrix} \mathbf{1} & \Phi \end{bmatrix}$$

where **1** is a column vector of ones and

$$\Phi = \begin{bmatrix} \sigma_1 & 0 & \dots & 0 \\ 0 & \sigma_2 & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \dots & 0 & \sigma_N \end{bmatrix} \tag{C.41}$$

Then $\mathrm{Var}(X_{i,A}\Gamma\omega) = X_{i,A}\Phi\Phi' X'_{i,A} + \mathbf{11'}$, and we can apply the Sherman-Morrison theorem:

$$\mathrm{Var}(X_{i,A}\Gamma\omega)^{-1} = \left[X_{i,A}\Phi\Phi' X'_{i,A}\right]^{-1} - \frac{1}{\phi_i^A}\left[X_{i,A}\Phi\Phi' X'_{i,A}\right]^{-1}\mathbf{11'}\left[X_{i,A}\Phi\Phi' X'_{i,A}\right]^{-1} \tag{C.42}$$

where $\phi_i^A = 1 + \mathbf{1'}\left[X_{i,A}\Phi\Phi' X'_{i,A}\right]^{-1}\mathbf{1} = 1 + \sum_{j=1}^{N}\sigma_j^{-2}$.

Using the above, we can solve the following:

$$\begin{aligned}
\text{Cov}(\omega, X_{i,A}\Gamma\omega)\text{Var}(X_{i,A}\Gamma\omega)^{-1}\text{Cov}(\omega, X_{i,A}\omega)' &= \begin{bmatrix} \mathbf{1'} \\ \Phi' X_{i,A} \end{bmatrix} \text{Var}(X_{i,A}\Gamma\omega)^{-1}\begin{bmatrix} \mathbf{1} & X_{i,A}\Phi \end{bmatrix} \\
&= \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \equiv A
\end{aligned}$$

where

$$\begin{aligned}
A_{11} &= \mathbf{1'}\left[X_{i,A}\Phi\Phi' X'_{i,A}\right]^{-1}\mathbf{1} - \frac{1}{\phi_i^A}\mathbf{1'}\left[X_{i,A}\Phi\Phi' X'_{i,A}\right]^{-1}\mathbf{1}\mathbf{1'}\left[X_{i,A}\Phi\Phi' X'_{i,A}\right]^{-1}\mathbf{1} \\
A_{12} &= \mathbf{1'}\left[X_{i,A}\Phi\Phi' X'_{i,A}\right]^{-1}X_{i,A}\Phi - \frac{1}{\phi_i^A}\mathbf{1'}\left[X_{i,A}\Phi\Phi' X'_{i,A}\right]^{-1}\mathbf{1}\mathbf{1'}\left[X_{i,A}\Phi\Phi' X'_{i,A}\right]^{-1}X_{i,A}\Phi \\
A_{21} &= \Phi' X'_{i,A}\left[X_{i,A}\Phi\Phi' X'_{i,A}\right]^{-1}\mathbf{1} - \frac{1}{\phi_i^A}\Phi' X'_{i,A}\left[X_{i,A}\Phi\Phi' X'_{i,A}\right]^{-1}\mathbf{1}\mathbf{1'}\left[X_{i,A}\Phi\Phi' X'_{i,A}\right]^{-1}\mathbf{1} \\
A_{22} &= \Phi' X'_{i,A}\left[X_{i,A}\Phi\Phi' X'_{i,A}\right]^{-1}X_{i,A}\Phi - \frac{1}{\phi_i^A}\Phi' X'_{i,A}\left[X_{i,A}\Phi\Phi' X'_{i,A}\right]^{-1}\mathbf{1}\mathbf{1'}\left[X_{i,A}\Phi\Phi' X'_{i,A}\right]^{-1}X_{i,A}\Phi
\end{aligned}$$

Notice that $\mathbf{1}'\left[X_{i,A}\Phi\Phi' X'_{i,A}\right]^{-1}\mathbf{1} = \phi_i^A - 1$. Then we car write the matrix $A$ as follows:

$$A = \begin{bmatrix} \frac{\phi_i^A - 1}{\phi_i^A} & \frac{1}{\phi_i^A}\mathbf{1}\left[X_{i,A}\Phi\Phi' X'_{i,A}\right]^{-1}X_{i,A}\Phi \\ \frac{1}{\phi_i^A}\Phi' X'_{i,A}\left[X_{i,A}\Phi\Phi' X'_{i,A}\right]^{-1}\mathbf{1} & A_{22} \end{bmatrix} \tag{C.43}$$

Notice that the matrix $X_{i,A}$ is a selector matrix whose entries are only zero and ones. It has a number of rows equal to the number of signals observed by agent $i$ and a number of column equal to the total number of signal available in the economy, that is $N$. So each raw is composed by zeros and only one 1 at column corresponding to the $j-th$ signal observed by agent $i$.

Then, entry $A_{21}$ is equal to:

$$\frac{1}{\phi_i^A}\Phi' X'_{i,A}(X_{i,A}\Phi\Phi' X'_{i,A})^{-1}\mathbf{1} = \frac{1}{\phi_i^A}\begin{bmatrix} \frac{g_{i1}}{\sigma_1} \\ \frac{g_{i2}}{\sigma_2} \\ \vdots \\ \frac{g_{iN}}{\sigma_N}, \end{bmatrix} \tag{C.44}$$

with some entries equal to 0 if agent $i$ does not observe the signal $j = 1,...N$. Entry $A_{12}$ is equal to:

$$\frac{1}{\phi_i^A}\Phi' X'_{i,A}\left[X_{i,A}\Phi\Phi' X'_{i,A}\right]^{-1}\mathbf{1} = \frac{1}{\phi_i^A}\begin{bmatrix} \frac{g_{i1}}{\sigma_1} & \frac{g_{i2}}{\sigma_2} & \cdots & \frac{g_{iN}}{\sigma_N} \end{bmatrix} \tag{C.45}$$

Let's now analyse entry $A_{22}$. Let define $\alpha \equiv \Phi' X'_{i,A}\left[X_{i,A}\Phi\Phi' X'_{i,A}\right]^{-1} X_{i,A}\Phi$ and $\beta \equiv \Phi' X'_{i,A}\left[X_{i,A}\Phi\Phi' X'_{i,A}\right]^{-1}\mathbf{1}\mathbf{1'}\left[X_{i,A}\Phi\Phi' X'_{i,A}\right]^{-1} X_{i,A}\Phi$. Then

$$\alpha = \begin{bmatrix} g_{i1} & 0 & \cdots & \cdots & 0 \\ 0 & g_{i2} & 0 & \cdots & 0 \\ \vdots & & \ddots & & \\ \vdots & & & \ddots & \\ 0 & \cdots & \cdots & 0 & g_{iN} \end{bmatrix} \tag{C.46}$$

$$\beta = \begin{bmatrix} \frac{g_{i1}g_{i1}}{\sigma_1\sigma_1} & \frac{g_{i1}g_{i2}}{\sigma_1\sigma_2} & \cdots & \frac{g_{i1}g_{iN}}{\sigma_1\sigma_N} \\ \frac{g_{i2}g_{i1}}{\sigma_2\sigma_1} & \frac{g_{i2}g_{i2}}{\sigma_2\sigma_2} & \cdots & \frac{g_{i2}g_{iN}}{\sigma_2\sigma_N} \\ \vdots & \vdots & \ddots & \\ \frac{g_{iN}g_{i1}}{\sigma_N\sigma_1} & \cdots & & \frac{g_{iN}g_{iN}}{\sigma_N\sigma_N} \end{bmatrix} \tag{C.47}$$

Notice that some entries of the matrices $\alpha$ and $\beta$ might be zero if a link with agent

$j = 1, ..., N$ is not established. Given the above entry $A_{22}$ is equal to:

$$\frac{1}{\phi_i^A}\begin{bmatrix} \frac{(\phi_i^A\sigma_1^2-1)g_{i1}}{\sigma_1\sigma_1} & -\frac{g_{i1}g_{i2}}{\sigma_1\sigma_2} & \cdots & \cdots & -\frac{g_{i1}g_{iN}}{\sigma_1\sigma_N} \\ -\frac{g_{i1}g_{i2}}{\sigma_2\sigma_1} & \frac{(\phi_i^A\sigma_1^2-1)g_{i2}}{\sigma_2\sigma_2} & -\frac{g_{i2}g_{i3}}{\sigma_2\sigma_3} & \cdots & -\frac{g_{i2}g_{iN}}{\sigma_2\sigma_N} \\ \vdots & \vdots & \ddots & & \vdots \\ \vdots & \vdots & & \ddots & \\ -\frac{g_{iN}g_{i1}}{\sigma_N\sigma_1} & \cdots & \cdots & \cdots & \frac{(\phi_i^A\sigma_N^2-1)g_{iN}}{\sigma_N\sigma_N} \end{bmatrix} \tag{C.48}$$

Finally matrix $A$ is equal to:

$$A = \frac{1}{\phi_i^A}\begin{bmatrix} \phi_i^A-1 & \frac{g_{i1}}{\sigma_1} & \frac{g_{i2}}{\sigma_2} & \cdots & & \frac{g_{iN}}{\sigma_N} \\ \frac{g_{i1}}{\sigma_1} & \frac{(\phi_i^A\sigma_1^2-1)g_{i1}}{\sigma_1\sigma_1} & -\frac{g_{i1}g_{i2}}{\sigma_1\sigma_2} & \cdots & \cdots & -\frac{g_{i1}g_{iN}}{\sigma_1\sigma_N} \\ \frac{g_{i2}}{\sigma_2} & -\frac{g_{i1}g_{i2}}{\sigma_2\sigma_1} & \frac{(\phi_i^A\sigma_1^2-1)g_{i2}}{\sigma_2\sigma_2} & -\frac{g_{i2}g_{i3}}{\sigma_2\sigma_3} & \cdots & -\frac{g_{i2}g_{iN}}{\sigma_2\sigma_N} \\ \vdots & \vdots & \vdots & \ddots & & \vdots \\ \vdots & \vdots & \vdots & & \ddots & \vdots \\ \frac{g_{iN}}{\sigma_N} & -\frac{g_{iN}g_{i1}}{\sigma_N\sigma_1} & \cdots & \cdots & \cdots & \frac{(\phi_i^A\sigma_N^2-1)g_{iN}}{\sigma_N\sigma_N} \end{bmatrix}_{(N+1)\times(N+1)}$$

The conditional variance $\mathrm{Var}(\omega|I_i)$ is then equal to

$$\mathrm{Var}(\omega) - \mathrm{Cov}(\omega, X_{i,A}\Gamma\omega)\mathrm{Var}(X_{i,A}\Gamma\omega)^{-1}\mathrm{Cov}(\omega, X_{i,A}\omega)' = \mathrm{I}-\mathrm{A}$$

where

$$\mathrm{I{-}A} = \frac{1}{\phi_i^A}\begin{bmatrix} 1 & -\frac{g_{i1}}{\sigma_1} & \cdots & \cdots & \cdots & -\frac{g_{iN}}{\sigma_N} \\ -\frac{g_{i1}}{\sigma_1} & \phi_i(1-g_{i1})+\frac{g_{i1}}{\sigma_1^2} & \frac{g_{i1}g_{i2}}{\sigma_1\sigma_2} & \cdots & \cdots & \frac{g_{i1}g_{iN}}{\sigma_1^2} \\ -\frac{g_{i2}}{\sigma_2} & \frac{g_{i1}g_{i2}}{\sigma_2\sigma_1} & \phi_i(1-g_{i2})+\frac{g_{i2}}{\sigma_2^2} & \frac{g_{i2}g_{i3}}{\sigma_2\sigma_3} & \cdots & \frac{g_{i2}g_{iN}}{\sigma_2\sigma_N} \\ \vdots & \vdots & & \phi_i(1-g_{i3})+\frac{g_{i3}}{\sigma_3^2} & & \vdots \\ \vdots & \vdots & & & \ddots & \vdots \\ \vdots & \vdots & & & & \\ -\frac{g_{iN}}{\sigma_N} & \frac{g_{iN}g_{i1}}{\sigma_N\sigma_1} & \cdots & \cdots & \cdots & \phi_i(1-g_{iN})+\frac{g_{iN}}{\sigma_N^2} \end{bmatrix}$$

We now calculate the ex-ante expected payoff of an agent $i \in A$. Given the above we can write equation (C.39) as

$$
\begin{aligned}
E(u_i^A|G) &= -F_A'(I-A)F_A \\
&= \frac{1}{\phi_i^A}\begin{bmatrix} 1-\sum_{k=1}^N \delta_k^A g_{ik} \\ -\frac{g_{i1}}{\sigma_1}+\sum_{k=1}^N \delta_k^A \frac{g_{i1}g_{ik}}{\sigma_1}+\sigma_1\phi_i(1-g_{i1})\delta_1^A \\ \vdots \\ \vdots \\ \vdots \\ -\frac{g_{iN}}{\sigma_N}+\sum_{k=1}^N \delta_k^A \frac{g_{iN}g_{ik}}{\sigma_N}+\sigma_N\phi_i(1-g_{iN})\delta_N^A \end{bmatrix} \begin{bmatrix} 1 \\ \delta_1^A\sigma_1 \\ \vdots \\ \vdots \\ \vdots \\ \delta_N^A\sigma_N \end{bmatrix} \\
&= -\frac{1}{\phi_i^A}\left[1-2\sum_1^N \delta_{ik}^A g_{ik}+\sum_{s=1}^N\sum_{k=1}^N \delta_{ik}g_{is}g_{ik}\right]-\sum_{k=1}^N(1-g_{ik})\delta_{ik}^2\sigma_k^2 \\
&= \frac{1}{\phi_i^A}\left(1-\sum_{k=1}^N \delta_{ik}^A g_{ik}\right)^2-\sum_{k=1}^N(1-g_{ik})\delta_{ik}^2\sigma_k^2
\end{aligned}
$$

Remember that $\phi_i^A = 1+\sum_{k=1}^N g_{ij}\pi_j$ and it can be written as

$$\phi_i^A = 1+\pi_A+\sum_{k=1,k\neq i}^{N_A} g_{ik}\pi_A+\sum_{k=N_A+1}^{N} g_{ik}\pi_B = 1+\pi_A(K_i^A+1)+\pi_B K_i^B. \tag{C.49}$$

Then ex-ante expected utility is equal to:

$$E(u_i^A|G) = -\frac{1}{1+\pi_A(K_i^A+1)+\pi_B K_i^B}\left(1-\sum_{k=1}^N \delta_{ik}^A g_{ik}\right)^2-\sum_{k=1}^N(1-g_{ik})\delta_{ik}^2\pi_k^{-1}-c\left(K_i^A+K_i^B\right) \tag{C.50}$$

## C.3 Network Formation Analysis

### C.3.1 Configuaration 2A1B: $i = \{1,2\} \in A$ and $i = 3 \in B$

**Empty Network** In the empty network the group level influence $\gamma_k^J$ for $J = \{A, B\}$ and $k = 1,2,3$ are

$$\gamma_1^A = \gamma_2^A = 1/2 \qquad \gamma_3^A = 0$$
$$\gamma_1^B = \gamma_2^B = 0 \qquad \gamma_3^B = 1$$

and the group level influence without the link of agent $i$ are

$$\begin{aligned}
\gamma_{-11}^A &= 2\gamma_1^A - \lambda_{11}^A = 0 \\
\gamma_{-12}^A &= 2\gamma_2^A - \lambda_{12}^A = 1 \\
\gamma_{-13}^A &= 2\gamma_3^A - \lambda_{13}^A = 0.
\end{aligned}$$

The above parameters are derived using equation (4.13) and the fact that $\lambda_{11}^A = 1$ and $\lambda_{12}^A = \lambda_{13}^A = 0$.

The ex-ante expected utility of the player $3 \in B$ conditional on the network $G^E$ is equal to

$$\begin{aligned}
E(U_3^B) &= -\frac{1}{\pi_B}\left(1 - \sum_{k=1}^{3} \delta_{3k}^B g_{3k}\right)^2 - \sum_{k=1}^{3}(1-g_{3k})(\delta_{3k}^B)^2 \pi_k^{-1} \\
&= -\frac{1}{\pi_B}(1-\delta_{33}^B)^2 - \left[(\delta_{31}^B)^2\pi_A^{-1} + (\delta_{32}^B)^2\pi_A^{-1}\right] \\
&= -\frac{1}{\pi_B}\left[1 - \left(-\frac{2}{3}\beta\gamma_3^A\right)\right]^2 - \left[\left(-\frac{2}{3}\beta\gamma_1^A\right)^2 + \left(-\frac{2}{3}\beta\gamma_2^A\right)^2\right]\pi_A^{-1} \\
&= -\frac{1}{\pi_B} - \frac{2}{9}\frac{\beta^2}{\pi_A}
\end{aligned} \tag{C.51}$$

**One link network.** If a player $i \in A$, say player 1, links to the minority, the graph associated to this network is represented by the following matrix

$$G^{+m} = \begin{bmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \\ g_{31} & g_{32} & g_{33} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

and her ex-ante expected utility is equal to

$$\begin{aligned} E(U_1) &= -\frac{1}{\pi_A + \pi_B}\left[1 - \frac{\alpha(\gamma^A_{-11} + \gamma^A_{-13})}{3} + \frac{\beta(\gamma^B_1 + \gamma^B_3)}{3}\right]^2 - \left(\frac{\alpha}{3}\gamma^A_{-12} - \frac{\beta}{3}\gamma^B_2\right)^2 \frac{1}{\pi_A} - c \\ &= -\frac{(3+\beta)^2}{9(\pi_A + \pi_B)} - \frac{\alpha^2}{9\pi_A} - c \end{aligned} \tag{C.52}$$

For player $3 \in B$ when he links to a player in group A, say to player 1, the graph associated to this network is given by the matrix

$$G^{+M} = \begin{bmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \\ g_{31} & g_{32} & g_{33} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

and his ex-ante expected utility is equal to

$$\begin{aligned} E(U^B_3) &= -\frac{1}{\pi_A + \pi_B}\left[1 - \left(-\frac{2}{3}\beta\gamma^A_2 - \frac{2}{3}\beta\gamma^A_3\right)\right]^2 - \left(-\frac{2}{3}\beta\gamma^A_1\right)^2 \frac{1}{\pi_A} - c \\ &= -\frac{1}{9}\left(\frac{(3+\beta)^2}{\pi_A + \pi_A} + \frac{\beta^2}{\pi_A}\right) - c \end{aligned} \tag{C.53}$$

**Complete Network** In the complete network the group level influence $\gamma^J_k$ for $J = \{A, B\}$ and $k = 1,2,3$ as defined in equations (4.11)-(4.12) are

$$\gamma^A_1 = \gamma^A_2 = \gamma^B_1 = \gamma^B_1 = \gamma^B_2 = \frac{\pi_A}{2\pi_A - \pi_B} \tag{C.54}$$

$$\gamma^A_3 = \gamma^B_3 = \frac{\pi_B}{2\pi_A - \pi_B} \tag{C.55}$$

The ex-ante expected utility of the player $3 \in B$ conditional on the network $G^C$ is equal to

$$E(U_3^B) = -\frac{1}{2\pi_A + \pi_B}\left(1 + \frac{2}{3}\beta\right)^2 - 2c \tag{C.56}$$

**Complete minus one link network.** If an agent $i \in A$, say player 1, drops a link from the complete network with a player in the majority, the graph to associated this network is represented by the following matrix

$$G^{-M} = \begin{bmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \\ g_{31} & g_{32} & g_{33} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \tag{C.57}$$

and her ex-ante expected utility is equal to

$$\begin{aligned} E(U_1^A) &= -\frac{1}{\pi_A + \pi_B}\left[1 - \frac{\alpha(\gamma_{-11}^A + \gamma_{-13}^A)}{3} + \frac{\beta(\gamma_1^B + \gamma_3^B)}{3}\right]^2 - \left(\frac{\alpha}{3}\gamma_{-12}^A - \frac{\beta}{3}\gamma_2^B\right)^2 \frac{1}{\pi_A} - c \\ &= -\frac{[(9-\beta^2)((6+\beta)\pi_A + \pi_B(3-\alpha+\beta)) - \alpha(9+\beta^2)\pi_A - 3(\alpha^2+\beta^2)\pi_A]^2}{9(9-\beta^2)^2(2\pi_A+\pi_B)^2(\pi_A+\pi_B)} + \\ &\quad - \frac{[\beta(9-\beta^2) + 3(\alpha^2+\beta^2) - 3\alpha(3-\beta^2)]^2\pi_A}{9(9-\beta^2)^2(2\pi_A+\pi_B)^2} - c \end{aligned} \tag{C.58}$$

If player $3 \in B$ drops a links with the majority, the graph associated to this network is represented by the following matrix

$$G^{-M} = \begin{bmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \\ g_{31} & g_{32} & g_{33} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \tag{C.59}$$

and his ex-ante expected utility is equal to

$$
\begin{aligned}
E(U_3^B) &= -\frac{1}{\pi_A+\pi_B}\left(1+\frac{2}{3}\beta(\gamma_1^A+\gamma_3^A)\right)^2 - \frac{4}{9}\frac{\beta^2}{\pi_A}(\gamma_2^A)^2 - c \\
&= -\frac{[(3-\alpha)(2\pi_A(3+\beta)+\pi_B(3+2\beta))-2\beta^2\pi_A]^2}{9(3-\alpha)^2(2\pi_A+\pi_B)^2(\pi_A+\pi_B)} + \\
&\quad -\frac{4\beta^2(3-\alpha+\beta)^2\pi_A}{9(3-\alpha)^2(2\pi_A+\pi_B)^2} - c
\end{aligned}
\tag{C.60}
$$

### C.3.2 Configuaration 1A2B: $i = 1 \in A$ and $i = \{2, 3\} \in B$

**Empty network.** When no player links to any other in the network, the group level influence $\gamma_k^J$ for $J = \{A, B\}$ and $k = 1, 2, 3$ are

$$
\begin{aligned}
&\gamma_1^A = 1 \qquad \gamma_2^A = \gamma_3^A = 0 \\
&\gamma_1^B = 0 \qquad \gamma_2^B = \gamma_3^B = 1/2
\end{aligned}
$$

The graph associated to the empty network is equal to the one in (4.16) and the ex-ante expected utility of player $1 \in A$ given the network $G^E$ is equal to

$$
\begin{aligned}
E(U_1^A) &= -\frac{1}{\pi_A}(1-\delta_{11}^A)^2 - \frac{1}{\pi_B}(\delta_{12}^A)^2 - \frac{1}{\pi_B}(\delta_{13}^A)2 \\
&= -\frac{1}{\pi_A}\left(1+\frac{2}{3}\beta\gamma_1^B\right)^2 - \left[\left(-\frac{2}{3}\beta\gamma_2^B\right)^2 + \left(-\frac{2}{3}\beta\gamma_3^B\right)^2\right] \\
&= -\frac{1}{\pi_A} - \frac{2}{9}\frac{\beta^2}{\pi_B}
\end{aligned}
\tag{C.61}
$$

where $\gamma_k^J$ is calculated using equations (4.11)-(4.12) together with equations (C.34)-(C.35). The ex-ante expected utility of a player $i \in B$, say player2, conditional on the network $G^E$ is equal to

$$
\begin{aligned}
E(U_2^B) &= -\frac{1}{\pi_B}\left(1-\delta_{22}^B\right)^2 - \frac{1}{\pi_A}\left(\delta_{21}^B\right)^2 - \frac{1}{\pi_B}\left(\delta_{23}^B\right)^2 \\
&= -\frac{1}{\pi_B}\left(1-\left(\alpha\frac{1}{3}\gamma_{-22}^B - \beta\frac{1}{3}\gamma_2^A\right)\right)^2 - \frac{1}{\pi_A}\left(\alpha\frac{1}{3}\gamma_{-21}^B - \beta\frac{1}{3}\beta\gamma_1^A\right)^2 - \frac{1}{\pi_B}\left(\alpha\frac{1}{3}\gamma_{-23}^B - \beta\frac{1}{3}\gamma_3^A\right)^2 \\
&= -\frac{1}{\pi_B} - \frac{1}{\pi_A}\frac{\beta^2}{9} - \frac{1}{\pi_B}\frac{\alpha^2}{9} \\
&= -\frac{1}{\pi_B}\left(1+\frac{\alpha^2}{9}\right) - \frac{1}{\pi_A}\frac{\beta^2}{9}
\end{aligned}
\tag{C.62}
$$

where $\gamma_{-ik}^J$ are calculated using (4.13).

**From empty network to one link** In this paragraph we study the incentive of a player $i$ to link to another player. With the network configuration we are considering, that is one player in group A and two players in group B, for the player in group A we study the incentive to link to the opposite group (link to the majority). On the contrary for a player in group B we can study the incentive to link to a player I) of the same group (link to majority) and II) of the opposite group (link to minority).

If player $1 \in A$ links to the majority when the other players are not linked to anyone, the graph related to this network is equal to the one in (4.18) and her ex-ante expected utility is equal to

$$
\begin{aligned}
E(U_1^A) &= -\frac{1}{\pi_A+\pi_B}\left(1+\frac{2}{3}\beta(\gamma_1^B+\gamma_2^B)\right)^2 - \frac{1}{\pi_B}\left(-\frac{2}{3}\beta\gamma_3^B\right)^2 - c \\
&= -\frac{1}{9}\left(\frac{\beta^2}{\pi_B} + \frac{(3+\beta)^2}{\pi_A+\pi_B}\right) - c
\end{aligned}
\tag{C.63}
$$

If a player $i \in B$, say player 2, links to the majority, the graph associated to this network configuration is represented by the following matrix

$$G^{+M} = \begin{bmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \\ g_{31} & g_{32} & g_{33} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

and his ex-ante expected utility is equal to

$$\begin{aligned} E(U_2^B) &= -\frac{1}{2\pi_B}\left(1-\left(\frac{\alpha}{3}\gamma_{-22}^B-\frac{\beta}{3}\gamma_2^A\right)-\left(\frac{\alpha}{3}\gamma_{-23}^B-\frac{\beta}{3}\gamma_3^A\right)\right)^2-\left(\frac{\alpha}{3}\gamma_{-21}^B-\frac{\beta}{3}\gamma_1^A\right)^2\frac{1}{\pi_A}-c \\ &= -\frac{1}{9}\left(\frac{(3-\alpha)^2}{2\pi_B}+\frac{\beta^2}{\pi_A}\right)-c \end{aligned} \tag{C.64}$$

If a player $i \in B$, say player 2, links to the minority, the graph associated to this network configuration is represented by the following matrix

$$G^{+M} = \begin{bmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \\ g_{31} & g_{32} & g_{33} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

and his ex-ante expected utility is equal to

$$\begin{aligned} E(U_2^B) &= -\frac{1}{\pi_A+\pi_B}\left(1-\left(\frac{\alpha}{3}\gamma_{-21}^B-\frac{\beta}{3}\gamma_1^A\right)-\left(\frac{\alpha}{3}\gamma_{-22}^B-\frac{\beta}{3}\gamma_2^A\right)\right)^2-\left(\frac{\alpha}{3}\gamma_{-23}^B-\frac{\beta}{3}\gamma_3^A\right)^2\frac{1}{\pi_B}-c \\ &= -\frac{1}{9}\left(\frac{(3+\beta)^2}{\pi_A+\pi_B}+\frac{\alpha^2}{\pi_B}\right)-c \end{aligned} \tag{C.65}$$

**Complete network.** In the complete network each player links to all others and the graph associated to this network is equal to the matrix in (4.23) and the group level influence $\gamma_k^J$ for $J = \{A, B\}$ and $k = 1, 2, 3$ are:

$$\gamma_1^A = \gamma_1^B = \frac{\pi_A}{\pi_A - 2\pi_B}$$

$$\gamma_2^A = \gamma_3^A = \gamma_2^B = \gamma_3^B = \frac{\pi_B}{\pi_A - 2\pi_B}$$

The ex-ante expected utility of player $1 \in A$ given the complete network is equal to

$$
\begin{aligned}
E(U_1^A) &= -\frac{1}{\pi_A + 2\pi_B}\left(1 - \sum_{k=1}^{3} \delta_{1k}^A g_{1k}\right)^2 - \sum_{k=1}^{3}(1 - g_{1k})(\delta_{1k}^A)^2 \pi_k^{-1} - 2c \\
&= -\frac{1}{\pi_A + 2\pi_B}(1 - \delta_{11}^A - \delta_{12}^A - \delta_{13}^A)^2 - 2c \\
&= -\frac{1}{\pi_A + 2\pi_B}\left(1 + \frac{2}{3}\beta\left(\gamma_1^A + \gamma_2^B + \gamma_3^A\right)\right)^2 - 2c \\
&= -\frac{1}{\pi_A + 2\pi_B}\left(1 + \frac{2}{3}\beta\right)^2 - 2c
\end{aligned} \tag{C.66}
$$

The ex-ante expected utility of a player $i \in B$, say player 2, conditional on the network $G^C$ is equal to

$$
\begin{aligned}
E(U_2^B) &= -\frac{1}{\pi_A + 2\pi_B}\left(1 - \sum_{k=1}^{3} \delta_{2k}^B g_{2k}\right)^2 - \sum_{k=1}^{3}(1 - g_{2k})(\delta_{2k}^B)^2 \pi_k^{-1} - 2c \\
&= -\frac{1}{\pi_A + 2\pi_B}(1 - \delta_{21}^B - \delta_{22} - \delta_{23})^2 - 2c \\
&= -\frac{1}{\pi_A + 2\pi_B}\left(1 - \left(\alpha\frac{1}{3}(\gamma_{-21}^B + \gamma_{-22}^B + \gamma_{-23}^B) - \beta\frac{1}{3}(\gamma_1^A + \gamma_2^A + \gamma_3^A)\right)\right)^2 - 2c \\
&= -\frac{1}{\pi_A + 2\pi_B}\left(1 - \frac{\alpha - \beta}{3}\right)^2 - 2c
\end{aligned} \tag{C.67}
$$

**From complete to -1 link.** In this paragraph we study the incentive of a player in a complete network to keep the “last" link. For the player in group A we study the incentive to keep a link with agent of the opposite group (cut link with the majority). On the contrary for a player in group B we study the incentive to keep a link with a player I) of the opposite group (cut link with minority) and II) of the same group (cut link with majority). If player $1 \in A$, drops a links with the majority, the graph associated to this network is equal to the one represented in the matrix (4.25) and her ex-ante expected utility is equal to

$$
\begin{aligned}
E(U_1^A) &= -\frac{1}{\pi_A+\pi_B}\left(1+\frac{2\beta}{3}(\gamma_1^B+\gamma_2^B)\right)^2 - \frac{1}{\pi_B}\left(\frac{2\beta}{3}\gamma_3^B\right) - c \\
&= -\frac{4\beta^2(3-\alpha+\beta)^2\pi_B}{9(3-\alpha)^2(\pi_A+2\pi_B)^2} - \frac{[(3-\alpha)(2\pi_B(3+\beta)+\pi_A(3+2\beta))-2\beta^2\pi_B]^2}{9(3-\alpha)^2(\pi_A+2\pi_B)^2(\pi_A+\pi_B)} - c
\end{aligned}
\tag{C.68}
$$

If an agent $i \in B$, say player 2, drops a link with a player in the majority, the graph to associated this network is represented by the following matrix

$$
G^{-M} = \begin{bmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \\ g_{31} & g_{32} & g_{33} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \tag{C.69}
$$

and his ex-ante expected utility is equal to

$$
\begin{aligned}
E(U_2^B) &= \frac{1}{\pi_A+\pi_B}\left(1-\left(\frac{\alpha}{3}\gamma_{-21}^B-\frac{\beta}{3}\gamma_1^A\right)-\left(\frac{\alpha}{3}\gamma_{-22}^B-\frac{\beta}{3}\gamma_2^A\right)\right)^2 - \left(\frac{\alpha}{3}\gamma_{-23}^B-\frac{\beta}{3}\gamma_3^A\right)\frac{1}{\pi_B} - c \\
&= -\frac{[(9-\beta^2)((6+\beta)\pi_B+\pi_A(3-\alpha+\beta))-\alpha(9+\beta^2)\pi_B-3(\alpha^2+\beta^2)\pi_B]^2}{9(9-\beta^2)^2(\pi_A+2\pi_B)^2(\pi_A+\pi_B)} + \\
&\quad -\frac{[\beta(9-\beta^2)+3(\alpha^2+\beta^2)-3\alpha(3-\beta^2)]^2\pi_B}{9(9-\beta^2)^2(\pi_A+2\pi_B)^2} - c
\end{aligned}
\tag{C.70}
$$

If an agent $i \in B$ in the complete network,say player 2, drops a link with a player in the minority, the graph to associated this network is represented by the following matrix

$$
G^{-M} = \begin{bmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \\ g_{31} & g_{32} & g_{33} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \tag{C.71}
$$

and his ex-ante expected utility is equal to

$$E(U_2^B) = \frac{1}{2\pi_B}\left(1 - \left(\frac{\alpha}{3}\gamma_{-22}^B - \frac{\beta}{3}\gamma_2^A\right) - \left(\frac{\alpha}{3}\gamma_{-23}^B - \frac{\beta}{3}\gamma_3^A\right)\right)^2 - \left(\frac{\alpha}{3}\gamma_{-21}^B - \frac{\beta}{3}\gamma_1^A\right)\frac{1}{\pi_A} - c$$

$$= -\frac{(3-\alpha+\beta)^2}{9(\pi_A+2\pi_B)} - \frac{(3+\alpha)^2(9-3\alpha-2\beta^2)^2\pi_A}{18(9-\beta^2)^2(\pi_A+2\pi_B)\pi_B} - c \tag{C.72}$$

### C.3.3 Proofs of Section 4.5

**Proof of lemma 4.1.** An agent $i \in A$ in the empty network will link to the majority if the utility from linking to the majority is higher than the utility from the empty network, that is if

$$-\frac{(3-\alpha)^2}{18\pi_A} - \frac{\beta^2}{9\pi_B} - c(1) \geq -\frac{1}{\pi_A}\left(1+\frac{\alpha^2}{9}\right) - \frac{\beta^2}{9}\frac{1}{\pi_B}$$

which is equivalent to

$$c \leq \frac{(3+\alpha)^2}{18\pi_A} \tag{C.73}$$

An agent $i \in A$ in the empty network will link to the minority if the utility from linking to the majority is higher than the utility from the empty network, that is if

$$-\frac{(3+\beta)^2}{9(\pi_A+\pi_B)} - \frac{\alpha^2}{9\pi_A} - c \geq -\frac{1}{\pi_A}\left(1+\frac{\alpha^2}{9}\right) - \frac{\beta^2}{9}\frac{1}{\pi_B}$$

which is equivalent to

$$c \leq \frac{(3\pi_B - \beta\pi_A)^2}{9(\pi_A+\pi_B)\pi_A\pi_B} \tag{C.74}$$

The agent $3 \in B$ in the empty network will link to the majority if the utility from linking to the majority is higher than the utility from the empty network, that is if

$$-\frac{1}{9}\left(\frac{(3+\beta)^2}{\pi_A+\pi_A} + \frac{\beta^2}{\pi_A}\right) - c \geq -\frac{1}{\pi_B} - \frac{2\beta^2}{9\pi_A}$$

which is equivalent to

$$c \leq \frac{(3\pi_A - \beta\pi_B)^2}{9(\pi_A+\pi_B)\pi_A\pi_B} \tag{C.75}$$

□

**Proof of Lemma 4.2.** Using equations (4.24) and (C.58) a player $i \in A$ has the incentive not to drop a link with the majority if

$$c \leq \frac{(3+\alpha)^2(9-3\alpha-2\beta^2)^2\pi_A}{9(9-\beta^2)^2(\pi_A+\pi_B)(2\pi_A+\pi_B)} \tag{C.76}$$

Using (4.24) and (4.26), an agent $i \in A$ will not drop a link with the minority if

$$-\frac{1}{2\pi_A+\pi_B}\left(1-\frac{\alpha-\beta}{3}\right)^2-2c > -\frac{(3-\alpha+\beta)^2}{9(2\pi_A+\pi_B)}-\frac{(3+\alpha)^2(9-3\alpha-2\beta^2)^2\pi_B}{18(9-\beta^2)^2(2\pi_A+\pi_B)\pi_A}-c \tag{C.77}$$

which is equivalent to

$$c \leq \frac{(3+\alpha)^2(9-3\alpha-2\beta^2)^2\pi_B}{18(9-\beta^2)^2(2\pi_A+\pi_B)\pi_A} \tag{C.78}$$

Using equations (C.56) and (C.70) player $3 \in B$ has an incentive not to drop a link with the majority if

$$c \leq \frac{(9-3\alpha-2\beta^2)^2\pi_A}{9(3-\alpha)^2(\pi_A+\pi_B)(2\pi_A+\pi_B)}$$

□

**Proof of Lemma 4.3.** Using (C.63) and (C.61), the agent $1 \in A$ will link to the majority if

$$-\frac{1}{9}\left(\frac{(3+\beta)^2}{\pi_A+\pi_B}+\frac{\beta^2}{\pi_B}\right)-c \geq -\frac{1}{\pi_A}-\frac{2\beta^2}{9\pi_B}$$

which is equivalent to

$$c \leq \frac{(3\pi_B-\beta\pi_A)^2}{9(\pi_A+\pi_B)\pi_A\pi_B}. \tag{C.79}$$

For a player $i \in B$, say player 2, using (C.64) and (C.62), linking to the majority is better than the empty network if

$$-\frac{1}{9}\left(\frac{3-\alpha)^2}{2\pi_B}+\frac{\beta^2}{\pi_A}\right)-c \geq -\frac{1}{\pi_B}\left(1+\frac{\alpha^2}{9}\right)-\frac{\beta^2}{9\pi_A}$$

which is equivalent to

$$c \leq \frac{(3+\alpha)^2}{18\pi_B} \tag{C.80}$$

For a player $i \in B$, say player 2, using (C.65) and (C.62), linking to the minority is better than the empty network if

$$-\frac{1}{9}\left(\frac{(3+\beta)^2}{\pi_A+\pi_B}+\frac{\alpha^2}{\pi_B}\right)-c \geq -\frac{1}{9}\left(\frac{9+\alpha^2}{\pi_B}+\frac{\beta^2}{\pi_A}\right)$$

which is equivalent to

$$c \leq \frac{(3\pi_A-\beta\pi_B)^2}{9(\pi_A+\pi_B)\pi_A\pi_B} \tag{C.81}$$

□

**Proof of Lemma 4.4.** Using (C.66) and (C.68) agent $1 \in A$ will not drop a link with the majority if

$$\begin{gathered}-\tfrac{1}{\pi_A+2\pi_B}\left(1+\tfrac{2}{3}\beta\right)^2-2c > \\ -\tfrac{4\beta^2(3-\alpha+\beta)^2\pi_B}{9(3-\alpha)^2(\pi_A+2\pi_B)^2}-\tfrac{[(3-\alpha)(2\pi_B(3+\beta)+\pi_A(3+2\beta))-2\beta^2\pi_B]^2}{9(3-\alpha)^2(\pi_A+2\pi_B)^2(\pi_A+\pi_B)}-c\end{gathered}$$

which is equivalent to

$$c \leq \frac{(9-3\alpha-2\beta^2)^2\pi_B}{9(3-\alpha)^2(\pi_A+2\pi_B)(\pi_A+\pi_B)} \tag{C.82}$$

A player $i \in B$, say player 2, using (C.67) and (C.70), has the incentive not to drop a link with the majority if

$$-\frac{1}{\pi_A+2\pi_B}\left(1-\frac{\alpha-\beta}{3}\right)^2-2c \geq -\frac{[\beta(9-\beta^2)+3(\alpha^2+\beta^2)-3\alpha(3-\beta^2)]^2\pi_B}{9(9-\beta^2)^2(\pi_A+2\pi_B)^2}$$
$$-\frac{[(9-\beta^2)((6+\beta)\pi_B+\pi_A(3-\alpha+\beta))-\alpha(9+\beta^2)\pi_B-3(\alpha^2+\beta^2)\pi_B]^2}{9(9-\beta^2)^2(\pi_A+2\pi_B)^2(\pi_A+\pi_B)}-c$$

which is equivalent to

$$c \leq \frac{(3+\alpha)^2(9-3\alpha-2\beta^2)^2\pi_B}{9(9-\beta^2)^2(\pi_A+2\pi_B)(\pi_A+\pi_B)} \tag{C.83}$$

A player $i \in B$, say player 2, using (C.67) and (C.72), has the incentive not to drop a link with the minority if

$$-\frac{1}{\pi_A+2\pi_B}\left(1-\frac{\alpha-\beta}{3}\right)^2-2c \geq$$
$$-\frac{(3-\alpha+\beta)^2}{9(\pi_A+2\pi_B)}-\frac{(3+\alpha)^2(9-3\alpha-2\beta^2)^2\pi_A}{18(9-\beta^2)^2(\pi_A+2\pi_B)\pi_B}-c$$

which is equivalent to

$$c \leq \frac{(3+\alpha)^2(9-3\alpha-2\beta^2)^2\pi_A}{18(9-\beta^2)^2(\pi_A+2\pi_B)\pi_B} \tag{C.84}$$

□

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